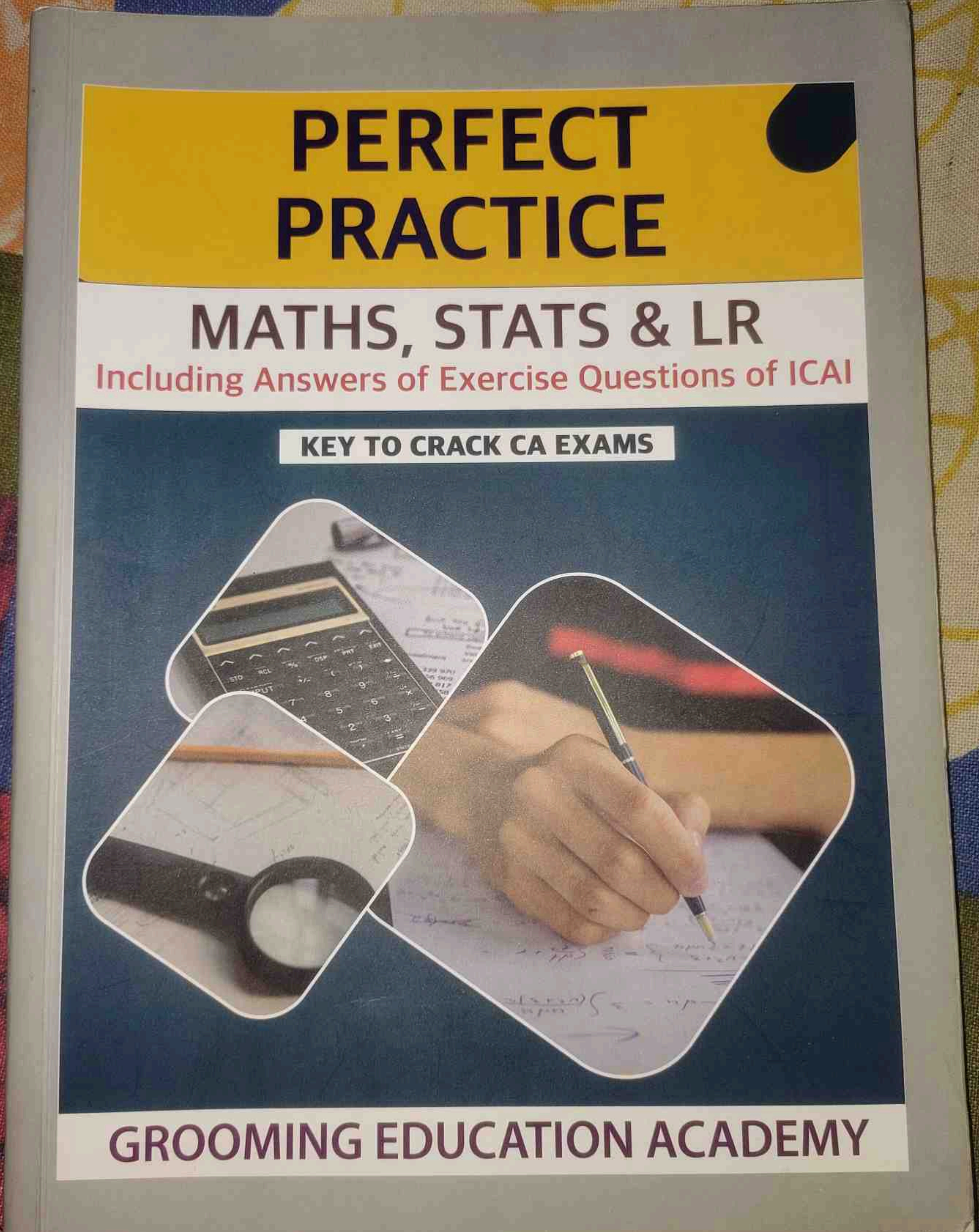

PERFECT PRACTICE

MATHS, STATS & LR

Including Answers of Exercise Questions of ICAI

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DEDICATED TO
KHATU SHYAM BABA

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* * *

"Never disrespect your mother or disappoint her. Do not hurt her feelings. Try to satisfy her in all respects. Only then the seed of devotion will sprout in you. Everyone should follow the dictum "matru devo bhava" in letter and in spirit and be recipient of his mothers love."

* * *

"The journey of 1000 miles begins with a single step"
- Ancient Chinese Proverb -

* * *

"Never tell some one they are wrong, that's a disastrous tactic" *- Dale Carnegie -*

* * *

"There is no stronger force known to mankind than for a human being to get down on his knees and ask God for Guidance."
- Stanley Arnold -

* * *

"I am grateful for all my problem. As each of then was overcome. I become strong and more able to meet these yet to come. I grew on my difficulties." *- J.C. Penny -*

* * *

"If a man can write a better book, preach a better sermon, or make a better mouse trap than his neighbors, though he builds his house in the woods,the world will make a beaten path to his door." *- R.W. Emerson -*

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Business Mathematics

Ratio, Proportion, Indices and Logarithms Exercise: 1A

Sol.1 (a) Inverse ratio of 11:15 = **15:11**

Sol.2 (d) $\frac{Antecedent}{Consequent} = \frac{15}{x}$

$\therefore \frac{15}{x} = \frac{3}{4} \Rightarrow x = \frac{4}{3} \times 15 = \mathbf{20}$

Sol.3 (c) Inverse of $\frac{5}{7} = \frac{7}{5} = \frac{Antecedent}{Consequent}$

⇒ Antecedent is **7**

Sol.4 (a) Compounded ratio $= \frac{2}{3} \times \frac{9}{4} \times \frac{5}{6} \times \frac{8}{10}$

= **1:1**

Sol.5 (c) Duplicate ratio of a:b $= a^2 : b^2$

∴ duplicate ratio of 3:4 = **9:16**

Sol.6 (d) Sub-duplicate ratio of a:b= $\sqrt{a} : \sqrt{b}$

Sub-duplicate ratio of 25:36 = $\sqrt{25} : \sqrt{36}$

= 5 : 6

Sol.7 (a) Triplicate ratio of a:b = $a^3 : b^3$

Triplicate ratio of 2:3 = $2^3 : 3^3$

= 8:27

Sol.8 (c) Sub triplicate ratio of a:b = $\sqrt[3]{a} : \sqrt[3]{b}$

Sub triplicate ratio of 8:27 = $\sqrt[3]{8} : \sqrt[3]{27}$

$= \sqrt[3]{2 \times 2 \times 2} : \sqrt[3]{3 \times 3 \times 3}$

= 2:3

Sol.9 (a) Duplicate ratio a : b = $a^2 : b^2$

Duplicate ratio 3 : 4 = $3^2 : 4^2$

= 9 : 16

Now, Compounded ratio:

$= \frac{4}{9} \times \frac{9}{16} = {}^1\!/_4$

∴ **1:4**

Sol.10 (c) Duplicate ratio a : b = $a^2 : b^2$

Duplicate ratio 3 : 4 = $3^2 : 4^2$

= 9 : 16

Triplicate ratio a : b = $a^3 : b^3$

Triplicate ratio 2 : 3 = $2^3 : 3^3$

= 8 : 27

Now, Compounded ratio = $\frac{4}{9} \times \frac{9}{16} \times \frac{8}{27} \times \frac{9}{7}$

$= \frac{2}{21} =$ **2:21**

Sol.11 (d) Duplicate ratio a : b = $a^2 : b^2$

Duplicate ratio 4 : 5 = $4^2 : 5^2$

= 16 : 25

Triplicate ratio a : b = $a^3 : b^3$

Triplicate ratio 1 : 3 = $1^3 : 3^3$

= 1 : 27

Sub-duplicate ratio of a:b= $\sqrt{a} : \sqrt{b}$

Sub-duplicate ratio of 81:256 = $\sqrt{81} : \sqrt{256}$

= 9 : 16

Sub triplicate ratio of a:b = $\sqrt[3]{a} : \sqrt[3]{b}$

Sub triplicate ratio of 125 : 512

$= \sqrt[3]{125} : \sqrt[3]{512}$

= 5 : 8

Now, Compounded ratio

$= \frac{16}{25} \times \frac{1}{27} \times \frac{9}{16} \times \frac{5}{8} = \frac{1}{120}$

= 1:120

Sol.12 (d) $\therefore a : b = 3 : 4 \Rightarrow \frac{a}{b} = {}^3\!/_4$

$a = 3k \ and \ b = 4k$

∴ (2a+3b): (3a+4b)

$= \frac{2a+3b}{3a+4b} = \frac{2\,(3k)+3(4k)}{3(3k)+4(4k)}$

$= \frac{6k+12k}{9k+16k} = \frac{18k}{25k} =$ **18:25**

Sol.13 (a) Let the nos. be $2x$ & 3x

<u>ATQ</u>

$\therefore \frac{2x-4}{3x-4} = \frac{3}{5} \Rightarrow 10x - 20 = 9x - 12$

$9 : 16$

$a^3 : b^3$

$2^3 : 3^3$

$: 27$

$\frac{9}{16} \times \frac{8}{27} \times \frac{9}{7}$

2:21

$a^2 : b^2$

$: 5^2$

$: 25$

$: b^3$

3^3

7

$\sqrt{a} : \sqrt{b}$

$56 = \sqrt{81} : \sqrt{256}$

$= 9 : 16$

$a : \sqrt[3]{b}$

12

$25 : \sqrt[3]{512}$

8

$\Rightarrow x = 8$

$\Rightarrow \therefore 2x = 16 \; \& \; 3x = 24$

$\therefore (2x, 3x) = $ **(16, 24)**

Sol.14 (c)

Let the angles of the triangle be 2k, 7k & 11k

We know, Sum of all angles in a triangle is 180°

$\Rightarrow 2k + 7k + 11k = 180°$

$\Rightarrow 20k = 180°$

$\Rightarrow k = 9°$

∴ Required angles are **18°, 63°, 99°**

Sol.15 (d) Given the total rupees is 324

Divided 324 into two parts x and y in ratio 11:7

Let x gets 11k rupees & y gets 7k rupees

ATQ.

$\Rightarrow 11k + 7k = 324$

$\Rightarrow 18k = 324$

$\Rightarrow x = \frac{324}{18} = 18$

∴ x gets = 11k

= 11(18) = **₹ 198**

∴ y gets = 7k

= 7(18) = **₹ 126**

Sol.16 (a)

Anand earns ₹ 80 in 7 hours, Pramod earns ₹ 90 in 12 hours

Per hour earning=80/7:90/12

compounded ratio = $\frac{80}{7} : \frac{90}{12}$

$= \frac{80}{7} \times \frac{12}{90} = \frac{32}{21} =$ **32:21**

Sol.17 (c) Let the nos. be $7x$ &$10x$

Difference between $7x$ & $10x$ = 105

The difference is positive it means

the greater number – smaller number = 105

$\Rightarrow 10x - 7x = 105$

$\Rightarrow 3x = 105$

$\Rightarrow x = \frac{105}{3}$

$\Rightarrow x = 35$

∴ Required nos. are

$= 7x = 7(35) = 245$

$= 10x = 10(35) = 350$

= **(245, 350)**

Sol .18 (b) $\frac{P}{Q} = \frac{11}{12}$ & $\frac{P}{R} = \frac{9}{8}$

∴ Make equal p in both ratio.

$\frac{P}{Q} = \frac{11\times9}{12\times9} = \frac{99}{108}$, $\quad \frac{P}{R} = \frac{9\times11}{8\times11} = \frac{99}{88}$

∴ Q:R = 108:88⟹**27:22**

Sol.19 (b) $\frac{x}{y} = 3:4$ ∴ let $x = 3k$ & $y = 4k$

Now, $\frac{x^2y+xy^2}{x^3+y^3} = \frac{(3k)^2 4k + 3k(4k)^2}{(3k)^3+(4k)^3}$

$= \frac{3k\times3k\times4k+3k\times4k\times4k}{3k\times3k\times3k+4k\times4k\times4k}$

$= \frac{36k^3+48k^3}{27k^3+64k^3}$

$= \frac{84k^3}{91k^3} = \mathbf{\frac{12}{13}}$

Sol.20 (c) $\frac{\sqrt{p-x^2}}{\sqrt{q-x^2}} = \frac{p}{q}$

Square both sides

$\Rightarrow \frac{p-x^2}{q-x^2} = \frac{p^2}{q^2} \Rightarrow pq^2 - q^2x^2 = p^2q - p^2x^2$

$\Rightarrow p^2x^2 - q^2x^2 = p^2q - pq^2$

$\Rightarrow (p^2 - q^2)x^2 = pq(p-q)$

We know $(a^2 - b^2) = (a+b)(a-b)$

$\Rightarrow x^2 = \frac{pq(p-q)}{(p+q)(p-q)} = \mathbf{\frac{pq}{p+q}}$

Sol.21 (a) $\frac{(2s-p)^2}{(3t-p)^2} = \frac{2s}{3t}$

$\Rightarrow \frac{4s^2-4sp+p^2}{9t^2-6pt+p^2} = \frac{2s}{3t}$

$\Rightarrow 12s^2t - 12pst + 3p^2t = 18st^2 - 12pst + 2p^2s$

$\Rightarrow$ 6st (2s -3t) = p^2 (2s-3t)

$\Rightarrow p^2$ = 6st

Sol.22 (c)

$\frac{p}{q} = \frac{2}{3}$ & $\frac{x}{y} = \frac{4}{5}$

Let $= p = 2k$ *and* $q = 3k$

$= x = 4t$ *and* $y = 5t$

$\therefore \frac{5px+3qy}{10px+4qy} = \frac{5(2k)(4t)+3(3k)(5t)}{10(2k)(4t)+4(3k)(5t)}$

$= \frac{40kt+45kt}{80kt+60kt} = \frac{85kt}{140kt}$

$= \frac{17}{28}$

Sol.23 (a) Give terms be $\frac{19}{31}$

ATQ.

$\Rightarrow \frac{19-x}{31-x} = \frac{1}{4}$

$\Rightarrow 76 - 4x = 31 - x$

$\Rightarrow 3x = 76 - 31$

$\Rightarrow x = \frac{45}{3}$

$= x =$ **15**

Sol.24 (c) Let the two persons be a and b

Let the earning be x

And the expenses be y

Let the daily earning of two person be $4x$ and $5x$ and expenses be 7y & 9y

We know, Earning = Expenses + Saving

$\therefore 4x = 7y + 50$ *and* $5x = 9y + 50$

$\therefore \frac{4x}{5x} = \frac{7y+50}{9y+50}$

$\therefore 36y + 200 = 35y + 250$

$= y = 50$

Now, $4x = 7y + 50$

$= 4x = 7(50) + 50 = 400$

$= x = \frac{400}{4} = 100$

$\Rightarrow x = 100$

Now the earning of the persons is

$5x = 500$

$4x = 4(100) = 400$

Their earning is **(₹ 400, ₹ 500)**

Sol .25 (c) Let the speed of first train be x km/hr

The speed of 2nd train = $\frac{400}{5}$ km/h = 80 km/h $(Speed = \frac{Distance}{Time})$

$\frac{x}{80} = \frac{7}{8} \Rightarrow x = \frac{7}{8} \times 80 = \mathbf{70}$

****

Ratio, Proportion, Indices and Logarithms Exercise: 1B

Sol.1 (a) Let the fourth proportion be x

$4 : 6 :: 8 : x$

We know that product of the mean proportion = product of external proportions

$\frac{4}{6} = \frac{8}{x}$

$4 \times x = 8 \times 6$

$\Rightarrow x = \frac{8\times6}{4}$

$= x =$ **12**

Sol.2 (b) Let the third proportion be x

In this ratio second and third number are the same

$12 : 18 :: 18 : x$

$\therefore \frac{12}{18} = \frac{18}{x}$

$\Rightarrow x = \frac{18\times18}{12} =$ **27**

Sol.3 (c) Let the mean proportion be x

In this ratio second and third number are the same

$25 : x :: x : 81$

$\therefore \frac{25}{x} = \frac{x}{81} \Rightarrow x^2 = 25 \times 81$

$\Rightarrow x = \sqrt{25 \times 81}$

$= x = 5 \times 9 = \mathbf{45}$

Alternative method

Mean proportion = $\sqrt{25 \times 81}$

$= 5 \times 9$

$= 45$

Sol..4 (d) In such problems, one has to set ratio equation between the given values.

Let us imagine that the number be x

The ratio of 6 to 13 is $\frac{6}{13}$

This ratio should be equal to ratio of number x and 26 means $\frac{x}{26}$

$\Rightarrow \frac{6}{13} = \frac{x}{26}$

$\Rightarrow x = \frac{6 \times 26}{13}$

$\Rightarrow x = \mathbf{12}$

Sol.5 (a) Let the fourth proportional be x

$2a : a^2 :: c : x$

$\therefore \frac{2a}{a^2} = \frac{c}{x}$

$\Rightarrow x = \frac{a^2 c}{2a}$

$\Rightarrow x = \mathbf{\frac{ac}{2}}$

Sol.6 (c) $\frac{1}{2} : \frac{1}{3} :: {}^{1}/_{5} : {}^{1}/_{x}$

$\Rightarrow 3:2 :: x:5 \;\; \Rightarrow \frac{3}{2} = \frac{x}{5}$

$\Rightarrow x = \frac{3 \times 5}{2} = \mathbf{{}^{15}/_{2}}$

Sol.7 (a) Mean proportion = $\sqrt{12x^2 \times 27y^2}$

$= \sqrt{2^2 \times 3 \times x^2 \times 3^3 \times y^2}$

$= 2 \times 3^2 \times x \times y = \mathbf{18xy}$

Sol.8 (c) $\therefore$ A= $\frac{B}{2} = \frac{C}{5}$ = k(Let)

$\Rightarrow$ A = k, B =2k & C = 5k

$\therefore$ A: B: C =k: 2k: 5k

$\therefore$ A: B: C = **1: 2: 5**

Sol.9 (c) $\therefore {}^{a}/_{3} = {}^{b}/_{4} = {}^{c}/_{7}$ = k(Let)

$\therefore a = 3k, b = 4k \;\&\; c = 7k$

$\therefore \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = \frac{14k}{7k} = \mathbf{2}$

Sol.10 (b) $\therefore \frac{p}{q} = \frac{r}{s} = \frac{2.5}{1.5}$

It means that $p = r = 2.5$

and $q = s = 1.5$

$\Rightarrow ps = qr$

$\Rightarrow \frac{ps}{qr} = \frac{2.5 \times 2.5}{1.5 \times 1.5} = \frac{1}{1}$

$\therefore ps : qr = \mathbf{1:1}$

Sol.11 (c) $\therefore \frac{x}{y} = \frac{z}{w} = \frac{2.5}{1.5}$

(It means $x = z = 2.5$ *and* $y = w = 1.5$)

$\Rightarrow \frac{x}{y} = \frac{z}{w} = \frac{5}{3}$

$\Rightarrow \frac{x+z}{y+w} = \frac{5+5}{3+3} = \frac{10}{6} = \frac{5}{3}$ **(By Addendo)**

Sol.12 (d) $\therefore \frac{5x-3y}{5y-3x} = \frac{3}{4}$

$\Rightarrow 20x - 12y = 15y - 9x$

$\Rightarrow 29x = 27y \Rightarrow \frac{x}{y} = \frac{27}{29} \Rightarrow x:y = \mathbf{27:29}$

Sol.13 (a) A:B =3:2 , B:C=3:5

$\Rightarrow \frac{A}{B} = \frac{3}{2}, \quad \frac{B}{C} = \frac{3}{5}$

(Make B equal in both ratios)

$\Rightarrow \frac{A}{B} = \frac{3 \times 3}{2 \times 3} = \frac{9}{6}, \quad \frac{B}{C} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$

(Now, B becomes the same.)

$\Rightarrow$ A:B:C=**9:6:10**

Sol.14 (d)

$\therefore \frac{x}{2} = \frac{y}{3} = \frac{z}{7}$ = k(Let) $\Rightarrow x = 2k$, y=3k & z =7k

$\therefore \frac{2x-5y+4z}{2y} = \frac{4k-15k+28k}{6k} = \frac{17k}{6k} = \mathbf{\frac{17}{6}}$

Sol.15 (d) x:y = 2:3 , y:z= 4:3

$= \frac{x}{y} = \frac{2}{3}, \quad \frac{y}{z} = \frac{4}{3}$

(make y equal in both ratios)

$=\frac{x}{y}=\frac{2\times4}{3\times4}=\frac{8}{12}, \quad \frac{y}{z}=\frac{4\times3}{3\times3}=\frac{12}{9},$ **8:12:9**

Sol.16 (a) The given ratio 4: 5: 6

1st Part $=\frac{4}{15}\times$ ₹750 = **₹200**

2nd Part $=\frac{5}{15}\times$ ₹750 = **₹250**

3rd Part $=\frac{6}{15}\times$ ₹750 = **₹300**

Sol.17 (a)

Let the present ages of three person in years be 7k +10, 8k + 10 & 9k+10

∴ (7k+10) + (8k +10) + (9k+10) = 150

⟹ 24k + 30 = 150 ⟹ 24k =120

$\Rightarrow k=\frac{120}{24}=5$

∴ 7k +10 = **45**

8k +10 = **50**

9k + 10 = **55**

Sol.18 (b) Let fourth term will be x

We know 14 : 16 : : 35 : x

Then $\frac{14}{16}=\frac{35}{x}\Rightarrow x=\frac{35\times16}{14}=\mathbf{40}$

Sol.19 (d) $\frac{x}{y}=\frac{z}{w}\Rightarrow\frac{y}{x}=\frac{w}{z}$ in called **invertendo**

Sol.20 (a) $\frac{p}{q}=\frac{r}{s}=\frac{p-r}{q-s}$, it is called **subtrahendo**

Sol.21 (c) $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$ is called **componendo** and **dividendo**

Sol.22 (d)

Sol.23 (c) Let the * be x

Now, 12 : 16 : : x : 20

$\therefore\frac{12}{16}=\frac{x}{20}\Rightarrow x=\frac{12\times20}{16}=\mathbf{15}$

Sol.24 (a) Now, 4 : x : : 9 : $13\frac{1}{2}$

$\frac{4}{x}=\frac{9}{13\frac{1}{2}}\Rightarrow x\,\frac{4\times\frac{27}{2}}{9}=\mathbf{6}$

Sol.25 (b) Mean proportion $=\sqrt{1.4\times5.6}$ gm

$=\sqrt{\frac{2\times7\times2\times2\times2\times7}{10\times10}}$ gm $=\frac{2\times2\times7}{10}$ gm = **2.8 gm**

Sol.26 (b) $\frac{a}{4}=\frac{b}{5}=\frac{c}{9}$ = k(Let) a=4k, b= 5k, c= 9k

$\therefore\frac{a+b+c}{c}=\frac{4k+5k+9x}{9k}=\frac{18k}{9k}=\mathbf{2}$

Sol.27 (c) Let the nos. are $3x$ and 4x

$\frac{3x+6}{4x+6}=\frac{4}{5}$

$\Rightarrow 15x+30=16x+24$

$\Rightarrow x=6$

$\therefore 3x = 18$ & $4x = \mathbf{24}$

Sol.28 (b) $\frac{a}{4}=\frac{b}{5}$ then by (C & D) $\frac{a+4}{a-4}=\frac{b+5}{b-5}$

Sol.29 (a) a:b = 4:1 ∴ $\sqrt{a/b}+\sqrt{b/a}=\sqrt{\frac{4}{1}}+\sqrt{\frac{1}{4}}=2+\frac{1}{2}$

$=5/2$

Sol.30 (b)

$\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$ = k (Let)

$\Rightarrow x=k(b+c-a),\ y=k(c+a-b),$

$z=k(a+b-c)$

ATQ.

$\therefore (b-c)\,x+(c-a)\,y+(a-b)\,z$

Put the value of x, y and z

$=k\,[(b-c)\,(b+c-a)$

$+(c-a)\,(c+a-b)+(a-b)\,(a+b-c)$

Taking k common

$=k[b^2-c^2-ab+ac+c^2-a^2-bc+ab+$
$a^2-b^2-ca+bc]$

$=k\times0=\mathbf{0}$

****

Ratio, Proportion, Indices and Logarithms Exercise: 1C

Sol.1 (c) $4\,x^{-1/4} = \frac{4}{x^{1/4}}$

Sol.2 (c) $8^{1/3} = 2^{3\times\frac{1}{3}} = 2$

Sol.3 (c) $2 \times (32)^{1/5} = 2 \times 2^{5\times\frac{1}{5}} = 2 \times 2 = \mathbf{4}$

Sol.4 (b) $4 / (32)^{1/5} = \frac{4}{2^{5\times\frac{1}{5}}} = \frac{4}{2} = \mathbf{2}$

Sol.5 (a) $\left(\frac{8}{27}\right)^{1/3} = \left(\frac{2}{3}\right)^{3\times\frac{1}{3}} = \frac{2}{3}$

Sol.6 (a) $2\,(256)^{-1/8} = 2 \times 2^{8\times\left(\frac{-1}{8}\right)} = 2 \times 2^{-1}$

$= \frac{2}{2} = \mathbf{1}$

Sol.7 (b)

$2^{1/2} \times 4^{3/4} = 2^{1/2} \times 2^{2\times\frac{3}{4}} = 2^{1/2} \times 2^{3/2} =$
$2^{\frac{1}{2}+\frac{3}{2}} = 2^{\frac{4}{2}} = 2^2 = \mathbf{4}$

Sol.8 (d) $\left(\frac{81x^4}{y^{-8}}\right)^{1/4} = (3^4\,x^4\,y^8)^{1/4}$

$= (3\,xy^2)^{4\times\frac{1}{4}} = \mathbf{3\,xy^2}$

Sol.9 (b)

$x^{a-b} \times x^{b-c} \times x^{c-a} = x^{a-b+b-c+c-a} = x^0 = \mathbf{1}$

Sol.10 (c) $\left(\frac{2\,p^2\,q^3}{3\,xy}\right)^0 = \mathbf{1}$

($\because$ any nos except 0 to the power 0 is 1)

Sol.11 (d)
$\{(3^3)^2 \times (4^2)^3 \times (5^3)^2\} / \{(3^3)^2 \times (4^3)^2 \times (5^2)^3\}$

$= \frac{3^6\times4^6\times5^6}{3^6\times4^6\times5^6} = \mathbf{1}$

Sol.12 (c) $\because\ 2^0 = 1\ \&\ \left(\frac{1}{2}\right)^0 = 1$

$\therefore\ 2^0 = \left(\frac{1}{2}\right)^{\mathbf{0}}$

Sol.13 (b) Let $x^{1/p} = y^{1/q} = z^{1/r} = k$

Now, $\Rightarrow x^{1/p} = k^1$

Multiply by p in the power both sides

$x^{\frac{1}{p}\times p} = k^{1\times p}$

$x = k^p$

$\Rightarrow y^{1/q} = k^1$

Multiply by q in the power both sides

$y^{\frac{1}{q}\times q} = k^{1\times q}$

$y = k^q$

$\Rightarrow z^{1/r} = k^1$

Multiply by r in the power both sides

$z^{\frac{1}{r}\times r} = k^{1\times r}$

$z = k^r$

$\because\ xyz = 1$

$\Rightarrow k^P \times k^q \times k^r = 1\ \Rightarrow k^{p+q+r} = k^0$

$\Rightarrow p + q + r = \mathbf{0}$

Sol.14 (d) $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$

$= y^{a-b+b-c+c-a-a-b} = y^{-a-b} = \frac{1}{y^{a+b}}$

Sol.15 (a) $x^{2\times\frac{1}{3}} = \sqrt[3]{\mathbf{x^2}}$

Sol.16 (c) $16\,x^{-3}y^2 \times 8^{-1}\,x^3\,y^{-2} = \frac{16y^2}{x^3} \times \frac{x^3}{8y^2}$

$= 2\,x^0\,y^0 = 2 \times 1 \times 1 = \mathbf{2}$

Sol.17 (a)

$(8/27)^{-1/3} \times \left(\frac{32}{243}\right)^{-1/5} = \left(\frac{2}{3}\right)^{3\times\left(\frac{-1}{3}\right)} \times \left(\frac{2}{3}\right)^{5\times(-1/5)}$

$= \left(\frac{2}{3}\right)^{-1} \times \left(\frac{2}{3}\right)^{-1} = \frac{3}{2} \times \frac{3}{2} = \mathbf{9/4}$

Sol.18 (c)
$\left\{(x+y)^{2/3}\,(x-y)^{3/2} / \sqrt{x+y} \times \sqrt{(x-y)^3}\right\}^6$

$= \left\{\frac{(x+y)^{2/3}\,(x-y)^{3/2}}{(x+y)^{1/2}\,(x-y)^{3/2}}\right\}^6 = \left\{(x+y)^{\frac{2}{3}-\frac{1}{2}}\right\}^6$

$= (x+y)^{\frac{1}{6}\times 6} = \mathbf{x+y}$

Sol.19 (d) $(125)^{2/3} \times \sqrt{25} \times \sqrt[3]{5^3} \times 5^{1/2}$

$$= 5^{3\times\frac{2}{3}} \times 5^{2\times\frac{1}{2}} \times 5^{3\times\frac{1}{3}} \times 5^{1/2}$$

$$= 5^2 \times 5^1 \times 5^1 \times 5^{\frac{1}{2}} = 5^{2+1+1+1/2}$$

$$= \mathbf{5^{9/2}}$$

Sol.20 (b)

$$\left[\left\{(2)^{1/2} \,.\, (4)^{3/4} \,.\, (8)^{5/6} \,.\, (16)^{7/8} \,.\, (32)^{9/10}\right\}^4\right]^{3/25}$$

$$= \left[\left\{2^{\frac{1}{2}} \times 2^{2\times 3/4} \times 2^{3\times\frac{5}{6}} \times 2^{4\times\frac{7}{8}} \times 2^{5\times\frac{9}{10}}\right\}^4\right]^{3/25}$$

$$= \left[\left(2^{\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+\frac{7}{2}+\frac{9}{2}}\right)^4\right]^{3/25}$$

$$= \left(2^{\frac{25}{2}\times 4}\right)^{3/25} = 2^{50\times\frac{3}{25}} = 2^6 = \mathbf{64}$$

Sol.21 (a) $[1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$

$$= \left[1 - \left\{1 - \frac{1}{1 - x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1 - \left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-1/2} = \left[1 - \frac{x^2-1}{x^2}\right]^{-1/2}$$

$$= \left[\frac{x^2-x^2+1}{x^2}\right]^{-1/2} = x^{\left(-2\times\frac{-1}{2}\right)} = x^1 = \mathbf{x}$$

Sol.22 (c) $\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$

$$= \left[x^{n\times\frac{n^2-1}{n}}\right]^{\frac{1}{n+1}} = x^{(n^2-1)\times\frac{1}{n+1}}$$

$$= x^{\frac{(n+1)\,(n-1)}{(n+1)}} = \mathbf{x^{n-1}}$$

Sol.23 (b)

$$\left[\frac{x^l}{x^m}\right]^{l^2+\,lm+m^2} \times \left(\frac{x^m}{x^n}\right)^{m^2+\,mn+\,n^2} \left(\frac{x^n}{x^l}\right)^{n^2+nl+l^2}$$

$$= x^{(l-m)\,(l^2+\,lm+m^2)} \times x^{(m-n)\,(m^2+mn+n^2)} \times r^{(n-l)\,(n^2+nl+l^2)}$$

$[\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2]$

$$= x^{l^3-m^3+m^3-n^3+n^3-l^3} = x^0 = \mathbf{1}$$

Sol.24 (b) $x = p^{\frac{1}{3}} - p^{-\frac{1}{3}}$

$$= x^3 = \left[p^{\frac{1}{3}} - p^{-\frac{1}{3}}\right]^3$$

$[\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)]$

$$= x^3 = p - \frac{1}{p} - 3p^{\frac{1}{3}}p^{-\frac{1}{3}}(p^{\frac{1}{3}} - p^{-\frac{1}{3}})$$

$$= x^3 = p - \frac{1}{p} - 3p^{\frac{1}{3}}p^{-\frac{1}{3}}(x)$$

$$= x^3 = p - \frac{1}{p} - 3x$$

$$= x^3 + 3x = \mathbf{p - \frac{1}{p}}$$

Sol.25 (c)

$$\frac{1}{1+a^{m-n}+a^{m-p}} + \frac{1}{1+a^{n-m}+a^{n-p}} + \frac{1}{1+a^{p-m}+a^{p-n}}$$

$$= \frac{1\times(a^{-m})}{(1+a^{m-n}+a^{m-p})\times(a^{-m})} + \frac{1\times(a^{-n})}{(1+a^{n-m}+a^{n-p})\times(a^{-n})} + \frac{1\times(a^{-p})}{(1+a^{p-m}+a^{p-n})\times(a^{-p})}$$ [multiply and divide by a^{-m}, a^{-n} and a^{-p}]

$$= \frac{a^{-m}}{a^{-m}+a^{-n}+a^{-p}} + \frac{a^{-n}}{a^{-n}+a^{-m}+a^{-p}} + \frac{a^{-p}}{a^{-p}+a^{-m}+a^{-n}}$$

$$= \frac{a^{-m}+a^{-n}+a^{-p}}{a^{-m}+a^{-n}+a^{-p}} = \mathbf{1}$$

Sol.26 (a) $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = \mathbf{x^0 = 1}$$

Sol.27 (b)

$\because x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$

$[\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)]$

$$\therefore x^3 = \left(3^{1/3} + 3^{-1/3}\right)^3$$

$$= \left(3^{1/3}\right)^3 + \left(3^{-1/3}\right)^3 + 33^{\frac{1}{3}}\,3^{\frac{-1}{3}}\left(3^{1/3} + 3^{-1/3}\right)$$

$\Rightarrow x^3 = 3 + 3^{-1} + 3\left(3^{1/3} + 3^{-1/3}\right)$

Put $x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$

$\Rightarrow x^3 = 3 + \frac{1}{3} + 3x$ [Multiply by 3 both sides]

$\Rightarrow 3x^3 = 9 + 1 + 9x$

$\Rightarrow 3x^3 - 9x = \mathbf{10}$

Sol.28 (a) Given $a^x = b$ and $b^y = c$

Put the value of b

$$\Rightarrow (a^x)^y = c \Rightarrow a^{xy} = c$$

$$c^z = a \quad \text{(Given)}$$

Put the value of c

$$\Rightarrow (a^{xy})^z = c$$

$$\Rightarrow c^{xyz} = c^1 \Rightarrow xyz = \mathbf{1}$$

Sol.29 (a)

$$\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

$$= x^{(a-b)\,(a^2+ab+b^2)} \times x^{(b-c)\,(b^2+bc+c^2)} \times x^{(c-a)\,(c^2+ca+a^2)}$$

$[\therefore a^3 - b^3 = (a-b)(a^2+ab+b^2)]$

$$= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = \mathbf{1}$$

Sol.30 (b) $2^x = 3^y = 6^{-z} = k\ (let)$

$$\therefore 2 = k^{1/x} \_\_\_\_(1)\ [a^n = k \therefore a = k^{\frac{1}{n}}]$$

$$3 = k^{1/y} \_\_\_\_(2)$$

$$6 = k^{-1/z}$$

$$\Rightarrow 2 \times 3 = k^{-1/z}$$

(Put the value of 2 and 3)

$$\Rightarrow k^{1/x} \times k^{1/y} = k^{-1/z}$$

$$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{-1/z}\ [a^m = a^n \ \therefore m = n]$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{-1}{z} \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \mathbf{0}$$

****

Ratio, Proportion, Indices and Logarithms Exercise: 1D

Sol.1 (b) $\log 6 + \log 5 = \log(6 \times 5) = \mathbf{\log 30}$

Sol.2 (c) $log_2 8 = log_2 2^3 = \mathbf{3}$

Sol.3 (b) $\log(^{32}/_4) = \log 32 - \log 4$

Sol.4 (a) $\log(1 \times 2 \times 3) = \mathbf{\log 1 + \log 2 + \log 3}$

Sol.5 (b) $log_{0.1} 0.0001 = log_{.1}(.1)^4$

$$= \frac{4}{1}\, log_{.1}(.1) = 4 \times 1 = \mathbf{4}$$

$$\left[\because log_{b^n} a^m = \frac{m}{n}\, log_b a\right]$$

Sol.6 (b) $2 \log x = 4 \log 3$

$$\Rightarrow \log x = \frac{4}{2} \log 3 \Rightarrow \log x = 2 \log 3 \Rightarrow$$

$$\log x = log\,(3)^2$$

$$\Rightarrow \log x = log 9$$

$$\Rightarrow x = \mathbf{9}$$

Sol.7 (a) $log_{\sqrt{2}} 64 = log_{2^{1/2}} 2^6$

$$= \frac{6}{1/2}\, log_2 2\ [\therefore \log_a a = 1]$$

$$= 12 \times 1 = \mathbf{12}$$

Sol.8 (c)

$$log_{\,2\sqrt{3}} 1728 = log_{2\sqrt{3}} 2^6 \times 3^3$$

$$= log_{2\sqrt{3}} (2\sqrt{3})^6 = 6 \Rightarrow \frac{6}{1} log_{2\sqrt{3}} (2\sqrt{3})$$

$$= 6 \times 1 = \mathbf{6}\ [\therefore \log_a a = 1]$$

Sol.9 (c) $log_9 (^1/_{81}) = log_9 9^{-2} = \mathbf{-2}$

Sol.10 (d)

$$log_2 0.0625 \Rightarrow log_2(.5)^4 \Rightarrow 4\, log_2 \left(\frac{1}{2}\right)$$

$$= 4\, log_2(2^{-1}) = \mathbf{-4}$$

Sol.11 (c) $\log 6 = \log(2 \times 3) = \log 2 + \log 3$

$$= 0.3010 + 0.4771 = \mathbf{0.7781}$$

Sol.12 (c)

$$log_2\, log_2\, log_2 16 = log_2\, log_2\, log_2\, 2^4$$

$$= log_2\, log_2 4 = log_2\, log_2\, 2^2 = log_2 2 = \mathbf{1}$$

Sol.13 **(a)** $log_9\ ^1/_3 = log_{3^2} 3^{-1} = \frac{-1}{2}\, log_3 3$

$[\therefore \log_a a = 1]$

$$= \frac{-1}{2} \times 1 = \mathbf{^{-1}/_2}$$

Sol.14 (c) $\log x + \log y = \log(x+y)$

$\log(xy) = \log(x+y) \Rightarrow xy = x + y$

$\Rightarrow y\,(x-1) = x \quad \Rightarrow y = \frac{x}{x-1}$

Sol.15 (c) $log_2\,[log_2\,\{log_3\,(log_3\,27^3)\}]$

$= log_2\,[log_2\,\{log_3\,(log_3\,3^9)\}]$

$= log_2\,[log_2\,\{log_3 9\,\}]$

$= log_2\,[log_2\,\{log_3\,3^2\}]$

$= log_2\,[log_2\,2] \qquad [\because \log_a a = 1]$

$= log_2\,1 = \mathbf{0}$

Sol.16 (a)

$log_2\,x + log_4 x + log_{16} x = \frac{21}{4} \Rightarrow log_2\,x + log_{2^2}\,x + log_{2^4} x = \frac{21}{4}$

$\Rightarrow log_2 x + \frac{1}{2}\,log_2\,x + \frac{1}{4}\,log_2\,x = \frac{21}{4}$

$\Rightarrow \frac{4log_2 x + 2log_2 x + log_2 x}{4} = \frac{21}{4}$

$\Rightarrow \frac{7}{4}\,log_2\,x = \frac{21}{4} \Rightarrow log_2 x = \frac{21}{4} \times \frac{4}{7}$

$\Rightarrow log_2 x = 3 \Rightarrow x = 2^3 = \mathbf{8}$

$\{log_{a^b} = x \;\Rightarrow\; a^x = b\}$

Sol.17 (b) $\because log_{10}\,2 = x \;\&\; log_{10} 3 = y$

$\therefore\; log_{10} 60 = log_{10}\,(2 \times 3 \times 10) = log_{10} 2 + log_{10} 3 + log_{10} 10$

$= \boldsymbol{x + y + 1}$

Sol.18 (c) $log_{10} 2 = x,\; log_{10} 3 = y$

$\therefore\; log_{10} 1.2 = log_{10}\left(\frac{2^2 \times 3}{10}\right) = 2\,log_{10} 2 + log_{10} 3 - log_{10} 10$

$= \boldsymbol{2x + y - 1}$

Sol.19 (a) $\log x = m + n \;\&\; \log y = m - n$

$\therefore \log \frac{10x}{y^2} = \log 10 + \log x - \log y^2$

$= 1 + \log x - 2 \log y$

$= 1 + m + n - 2(\mathrm{m} - \mathrm{n})$

$= 1 + m + n - 2m + 2n$

$= \boldsymbol{1 - m + 3n}$

Sol.20 (c)

$2\,log_{10} 5 + log\,_{10} 8 - \frac{1}{2}\,log_{10} 4$

$= 2log_{10} 5 + log_{10}\,2^3 - \frac{1}{2}\,log_{10}\,2^2 = log_{10}(5)^2 + log_{10}(8) - log_{10}(4)^{1/2}$

$= log_{10}(200) - log_{10}(2) \Rightarrow log_{10}\left(\frac{200}{2}\right) \Rightarrow log_{10}(100)$

$= log_{10}(10)^2 = 2\,log_{10} 10 = 2 \times 1 = \mathbf{2}$

Sol.21 (b)

$\log\;[1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$

$= \log\left[1 - \left\{1 - \frac{1}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \log\left[1 - \left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-1/2} = \log\left[1 - \left\{\frac{x^2}{x^2-1}\right\}^{-1}\right]^{-1/2}$

$= \log\left[1 - \frac{x^2-1}{x^2}\right]^{-1/2} = \log\left[\frac{x^2-x^2+1}{x^2}\right]^{-1/2}$

$= \log\left(\frac{1}{x^2}\right)^{-1/2} = \log\left[x^{-2 \times (-1/2)}\right]$

$= \boldsymbol{\log x}$

Sol.22 (a) $\log \sqrt[4]{729\,\sqrt[3]{9^{-1} \times 27^{-4/3}}}$

$\Rightarrow \log\,[729(3^{-2} \times 3^{-\frac{4}{3} \times 3})^{\frac{1}{3}}]^{\frac{1}{4}}$

$\Rightarrow \log[3^6(3^{-6})^{\frac{1}{3}}]^{\frac{1}{4}}$

$\Rightarrow \log\,(3^6 3^{-2})^{\frac{1}{4}} = \log\,(3^4)^{\frac{1}{4}}$

$\Rightarrow \mathbf{\log 3}$

Sol.23 (c) $(log_b a \times log_c b \times log_a c)^3$

$= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right)^3 = 1^3 = \mathbf{1}$

Sol.24 (d) $log_{2\sqrt{2}} 64 = log_{2^{3/2}} 2^6$

$\Rightarrow \frac{6}{\frac{3}{2}}\,log_2 2 = 6 \times \frac{2}{3} \times 1 = \mathbf{4}$

Sol.25 (c) $log_8 25 = log_{(2)^3}(\frac{100}{4})$

$$= \frac{1}{3}(log_2 100 - log_2(2)^2)$$

$$= \frac{1}{3}(2log_2 10 - 2log_2 2)$$

$$= \frac{1}{3}\left(\frac{2log10}{log2} - 2(1)\right)$$

$$= \frac{1}{3}(\frac{2\times1}{.3010} - 2(1)) \quad = \mathbf{1.5481}$$

****

Ratio, Proportion, Indices and Logarithms Exercise: Additional Question

Sol.1 (b) $\left(\frac{6^{-1}\times7^2}{6^2\times7^{-4}}\right)^{7/2} \times \left(\frac{6^{-2}\times 7^3}{6^3\times7^{-5}}\right)^{-5/2}$

$$= \left(\frac{7^{2+4}}{6^{2+1}}\right)^{7/2} \times \left(\frac{7^{3+5}}{6^{3+2}}\right)^{-5/2}$$

$$= \frac{7^{6\times\frac{7}{2}}}{6^{3\times\frac{7}{2}}} \times \frac{7^{8\times(-5/2)}}{6^{5\times(-5/2)}}$$

$$= \frac{7^{21}\times7^{-20}}{6^{21/2}\times6^{-25/2}} = \frac{7}{6^{-4/2}} = 7\times6^2 = \mathbf{252}$$

Sol.2 (a)

$$\frac{x^{2/7}}{z^{-1/2}} \times \frac{x^{2/5}}{z^{2/3}} \times \frac{x^{-9/7}}{z^{2/3}} \times \frac{z^{5/6}}{x^{-3/5}}$$

$$= \frac{x^{2/7+2/5-9/7+3/5}}{z^{-1/2+2/3+2/3-5/6}} = \frac{x^{\frac{10+14-45+21}{35}}}{z^{\frac{-3+4+4-5}{6}}} = \frac{x^0}{z^0} = \frac{1}{1} = \mathbf{1}$$

Sol.3 (c)

$$\frac{2^{x+3}\times3^{2x-y}\times5^{x+y+3}\times6^{y+1}}{6^{x+1}\times10^{y+3}\times15^x}$$

$$= \frac{2^{x+3}\times3^{2x-y}\times5^{x+y+3}\times2^{y+1}\times3^{y+1}}{2^{x+1}\times3^{x+1}\times2^{y+3}\times5^{y+3}\times3^x\times5^x}$$

$$= 2^{x+3+y+1-x-1-y-3} \times 3^{2x-y+y+1-x-1-x} \times 5^{x+y+3-y-3-x}$$

$$= 2^0\times3^0\times5^0 = 1\times1\times1 = \mathbf{1}$$

Sol.4 (b) $\frac{9^y.3^2.(3^{-y})^{-1} - 27^y}{3^{3x}.2^3} = \frac{1}{27}$

$$\Rightarrow \frac{3^{2y}.3^2.3^y - 3^{3y}}{3^{3x}.2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3y+2} - 3^{3y}}{3^{3x}.2^3} = \frac{1}{27} \Rightarrow \frac{3^{3y}(3^2-1)}{3^{3x}.8} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3y}\times8}{3^{3x}\times8} = \frac{1}{27} \Rightarrow \frac{1}{3^{3(x-y)}} = \frac{1}{3^3}$$

$$\Rightarrow 3(x-y) = 3 \Rightarrow x-y = \mathbf{1}$$

Sol.5 (a) $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} . \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} . \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$

$$= x^{\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}}$$

$$= x^{\frac{-1}{(a-b)(c-a)} - \frac{1}{(a-b)(b-c)} - \frac{1}{(b-c)(c-a)}}$$

$$= x^{\frac{-[b-c+c-a+a-b]}{(a-b)(b-c)(c-a)}} = x^0 = \mathbf{1}$$

Sol.6 (a) $\frac{16(32)^x - 2^{3x-2}.4^{x+1}}{15(2)^{x-1}(16)^x} - \frac{5(5)^{x-1}}{\sqrt{5^{2x}}}$

$$= \frac{2^4\times2^{5x} - 2^{3x-2}.2^{2x+2}}{15.2^{x-1}.2^{4x}} - \frac{5^{1+x-1}}{5^{2x\times\frac{1}{2}}}$$

$$= \frac{2^{4+5x} - 2^{3x-2+2x+2}}{15\times2^{x-1+4x}} - \frac{5^x}{5^x}$$

$$= \frac{2^{5x+4} - 2^{5x}}{15\times2^{5x-1}} - 1$$

$$= \frac{2^{5x}(2^4-1)}{15\times2^{5x}\times2^{-1}} - 1$$

$$= \frac{15}{15}\times2 - 1 = 2 - 1 = \mathbf{1}$$

Sol.7 (d) $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

$$= x^{(a-b)(a+b)}.x^{(b-c)(b+c)}.x^{(c-a)(c+a)}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = \mathbf{1}$$

Sol.8 (a) $\sqrt[(a+b)]{\frac{x^{a^2}}{x^{b^2}}} \times \sqrt[(b+c)]{\frac{x^{b^2}}{x^{c^2}}} \times \sqrt[c+a]{\frac{x^{c^2}}{x^{a^2}}}$

$$= \left(x^{a^2-b^2}\right)^{\frac{1}{a+b}}.\left(x^{b^2-c^2}\right)^{\frac{1}{b+c}}\left(x^{c^2-a^2}\right)^{\frac{1}{c+a}}$$

$$= x^{\frac{a^2-b^2}{a+b}}.x^{\frac{b^2-c^2}{b+c}}.x^{\frac{c^2-a^2}{c+a}}$$

$$= x^{a-b}.x^{b-c}.x^{c-a} = x^{a-b+b-c+c-a}$$

$$= x^0 = \mathbf{1}$$

Sol.9 (a) $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}} . \left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}} \left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$

$$= x^{\frac{b+c}{c-a}\times\frac{1}{a-b}} \quad x^{\frac{c+a}{a-b}\times\frac{1}{b-c}} \quad x^{\frac{a+b}{b-c}\times\frac{1}{c-a}}$$

$$= x^{\frac{b+c}{(c-a)(a-b)} + \frac{c+a}{(a-b)(b-c)} + \frac{a+b}{(b-c)(c-a)}}$$

$$= x^{\frac{b^2-c^2+c^2-a^2+a^2-b^2}{(a-b)(b-c)(c-a)}} = x^0 = 1$$

Sol.10 (a)

$$\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \left(\frac{x^a}{x^b}\right)^c = x^{(b-c)a} . x^{(c-a)b} . x^{(a-b)c}$$

$$= x^{ab-ac+bc-ab+ac-bc} = x^0 = \mathbf{1}$$

Sol.11 (c) $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \times \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \times \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$

$$= x^{\frac{b-c}{bc}} . x^{\frac{c-a}{ca}} . x^{\frac{a-b}{ab}}$$

$$= x^{\frac{1}{c} - \frac{1}{b} + \frac{1}{a} - \frac{1}{c} + \frac{1}{b} - \frac{1}{a}} = x^0 = 1$$

Sol.12(a)

$$\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} . \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} . \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$$

$=$

$$x^{(a-b)(a^2+ab+b^2)} . x^{(b-c)(b^2+bc+c^2)} . x^{(c-a)(c^2+ca+a^2)}$$

$[\therefore a^3 - b^3 = (a-b)(a^2+ab+b^2]$

$$= x^{(a^3-b^3)} . x^{(b^3-c^3)} . x^{(c^3-a^3)}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = \mathbf{1}$$

Sol.13 (b) $2^{x+y} = 4 \times 8 \times 16$

$$= 2^{x+y} = 2^2 \times 2^3 \times 2^4$$

$$= 2^{x+y} = 2^9$$

$$= x + y = 9$$

$$= (x+y)^2 = \mathbf{81}$$

Sol.14 (a)

$$\left(\frac{x^b}{x^c}\right)^{b+c-a} . \left(\frac{x^c}{x^a}\right)^{c+a-b} . \left(\frac{x^a}{x^b}\right)^{a+b-c}$$

$$= x^{(b-c)(b+c-a)} \quad x^{(c-a)(c+a-b)} \quad x^{(a-b)(a+b-c)}$$

$$= x^{(b-c)(b+c-a)+(c-a)(c+a-b)+(a-b)(a+b-c)}$$

$\{a^2 - b^2 = (a+b)(a-b)\}$

$$= x^{b^2-c^2-ab+ac+c^2-a^2-bc+ab+a^2-b^2-ac+bc}$$

$$= x^0 = \mathbf{1}$$

Sol.15 (c)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

$$= x^{(a+b)(a^2-ab+b^2)} \, x^{(b+c)(b^2-bc+c^2)} \, x^{(c+a)(c^2-ca+c^2)}$$

$$= x^{(a+b)(a^2-ab+b^2)+(b+c)(b^2-bc+c^2)+(c+a)(c^2-ca+c^2)}$$

$\{a^3 + b^3 = (a+b)(a^2+b^2-ab)\}$

$$= x^{a^3+b^3} \quad x^{b^3+c^3} \quad x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3} = x^{2(a^3+b^3+c^3)}$$

Sol.16 (c)

$$x^{a^2 b^{-1} c^{-1}} . x^{b^2 c^{-1} a^{-1}} . x^{c^2 . a^{-1} b^{-1}} - x^3 = 0$$

$$\Rightarrow x^{\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}} - x^3 = 0$$

$$\Rightarrow x^{\frac{a^3+b^3+c^3}{abc}} = x^3$$

$$\Rightarrow \frac{a^2+b^3+c^3}{abc} = 3 \Rightarrow a^3 + b^3 + c^2 = 3\,abc$$

If is only possible when a + b + c =**0**

Sol.17 (d) $z^{z\sqrt{z}} = \left(z\sqrt{z}\right)^z$

$$\Rightarrow z^{z\sqrt{z}} = \left(z^{3/2}\right)^z$$

$$\Rightarrow z^{z\sqrt{z}} = z^{\frac{3z}{2}} \Rightarrow z\sqrt{z} = \frac{3z}{2}$$

$$\Rightarrow z\sqrt{z} = \frac{3}{2}z$$

$$\Rightarrow \sqrt{z} = \frac{3}{2} \Rightarrow z = \left(\frac{3}{2}\right)^2 = \mathbf{9/4}$$

Sol.18 (b)

$$\frac{1}{x^b + x^{-c}+1} + \frac{1}{x^c + x^{-a}+1} + \frac{1}{x^a + x^{-b}+1} = 1$$

$$\Rightarrow \frac{x^c}{x^{b+c}+1+x^c} + \frac{x^a}{x^{c+a}+1+x^a} + \frac{x^b}{x^{a+b}+1+x^b} = 1$$

$\therefore$ If $x^c = 1 \Rightarrow c = 0$

$x^{b+c} = 1 \Rightarrow b + c = 0 \Rightarrow b = 0 \; x^a = 1 \Rightarrow a = 0$

$x^{c+a} = 1 \Rightarrow c + a = 0 \Rightarrow c = 0$

$x^b = 1 \Rightarrow b = 0$

$x^{a+b} = 1 \Rightarrow a + b = 0 \Rightarrow a = 0$

Hence a + b + c = **0**

Sol.19 (c) $\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$

$$= \frac{z^{-a}}{(1+z^{a-b}+z^{a-c})z^{-a}} + \frac{z^{-b}}{(1+z^{b-c}+z^{b-a})z^{-b}} + \frac{z^{-c}}{(1+z^{c-a}+z^{c-b})z^{-c}}$$

$$= \frac{z^{-a}}{z^{-a}+z^{-b}+z^{-c}} + \frac{z^{-b}}{z^{-b}+z^{-c}+z^{-a}} + \frac{z^{-c}}{z^{-c}+z^{-a}+z^{-b}}$$

$$= \frac{z^{-a}+z^{-b}+z^{-c}}{z^{-a}+z^{-b}+z^{-c}} = \mathbf{1}$$

Sol.20 (b)

$(5.678)^x = (0.5678)^y = 10^z = k\ (let)$

$\therefore 5.678 = k^{1/x}, 0.5678 = k^{1/y}\ \&\ 10 = k^{1/z}$

Now $5.678 = 0.5678 \times 10$

$\Rightarrow k^{1/x} = k^{1/y} . k^{1/z} \Rightarrow k^{\frac{1}{x}} = k^{\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow \frac{1}{x} = \frac{1}{y} + \frac{1}{z} \Rightarrow \frac{1}{x} - \frac{1}{y} - \frac{1}{z} = \mathbf{0}$

Sol.21 (d) $x = 4^{1/3} + 4^{-1/3}$

$\therefore x^3 = \left(4^{1/3} + 4^{-1/3}\right)^3$

$= \left(4^{1/3}\right)^3 + \left(4^{-1/3}\right)^3 +$
$3.4^{1/3}.4^{-1/3}\left(4^{1/3} + 4^{-1/3}\right)$

$x^3 = 4 + 4^{-1} + 3x \quad \Rightarrow \quad x^3 = 4 +$
$\frac{1}{4} + 3x$

$\Rightarrow x^3 = \frac{16+1+12x}{4}$

$\Rightarrow 4x^3 - 12x = \mathbf{17}$

Sol.22 (b) $x = 5^{1/3} + 5^{-1/3}$

$\therefore x^3 = 5 + 5^{-1} + 3\left(5^{1/3} + 5^{-1/3}\right)$

$\Rightarrow \quad x^3 = 5 + \frac{1}{5} + 3x$

$\Rightarrow 5x^3 = 25 + 1 + 15x$

$\Rightarrow 5x^3 - 15x = \mathbf{26}$

Sol.23 (a)

$ax^{2/3} + b\,x^{1/3} + c = 0$

$\Rightarrow a\,x^{2/3} + b\,x^{1/3} = -c$

$\therefore \left(a\,x^{2/3} + b\,x^{1/3}\right)^3 = (-c)^3$

$\Rightarrow a^3x^2 + b^3x + 3abx\left(a\,x^{2/3} + b\,x^{1/3}\right) = -c^3$

$\Rightarrow a^3x^2 + b^3x + 3abx\,(-c) = -c^3$

$\Rightarrow a^3x^2 + b^3x - 3abcx + c^3 = 0$

$\Rightarrow a^3x^2 + b^3x + c^3 = \mathbf{3abc\,x}$

Sol.24 (b) $a^P = b,\ b^q = c, c^r = a$

$= a^P = b$

$= (c^r)^P = b = c^{rp} = b$

$= (b^q)^{rp} = b = b^{pqr} = b^1$

$\Rightarrow \mathbf{pqr = 1}$

Sol.25 (c) $a^p = b^q = c^r = k\ (let) \Rightarrow a = k^{1/p}$

$b = k^{1/q}$

$\&\ c = k^{1/r}$

Now $b^2 = ac \Rightarrow \left(k^{1/q}\right)^2 = k^{\frac{1}{p}} \times k^{\frac{1}{r}}$

$\Rightarrow k^{\frac{2}{q}} = k^{\frac{1}{p}+\frac{1}{r}}$

$\Rightarrow \frac{2}{q} = \frac{1}{p} + \frac{1}{r}$

$\Rightarrow \frac{2}{q} = \frac{r+p}{pr}$

$\Rightarrow \frac{q\,(p+r)}{pr} = \mathbf{2}$

Sol.26 (d) $\left[\frac{x^{\frac{a}{a-b}}}{x^{\frac{a}{a+b}}} \div \frac{x^{\frac{b}{b-a}}}{x^{\frac{b}{b+a}}}\right]^{a+b}$

$= \left(x^{\frac{a}{a-b}-\frac{a}{a+b}} \div x^{\frac{b}{b-a}-\frac{b}{a+b}}\right)^{a+b}$

$= \left(x^{\frac{2ab}{a^2-b^2}} \div x^{\frac{2\,ab}{b^2-a^2}}\right)^{a+b}$

$= \left(x^{\frac{2ab}{a^2-b^2}-\frac{2ab}{b^2-a^2}}\right)^{a+b} = \left(x^{\frac{4\,ab}{a^2-b^2}}\right)^{a+b}$

$= x^{\frac{4ab}{a^2-b^2}\times(a+b)} = \boldsymbol{x^{\frac{4ab}{a-b}}}$

Sol.27 (a)

$\left(\frac{x^{ab}}{x^{a^2+b^2}}\right)^{a+b} \times \left[\frac{x^{b^2+c^2}}{x^{bc}}\right]^{b+c} \left(\frac{x^{ca}}{x^{c^2+a^2}}\right)^{c+a}$

$= \left(\frac{1}{x^{a^2+b^2-ab}}\right)^{(a+b)} \times \left(x^{b^2+c^2-bc}\right)^{b+c} \times$
$\left(\frac{1}{x^{c^2+a^2-ca}}\right)^{c+a}$

$= \frac{1}{x^{(a^3+b^3)}} \times x^{(b^3+c^3)} \times \frac{1}{x^{(c^3+a^3)}}$

$= x^{-a^3-b^3+b^3+c^3-c^3-a^3} = \boldsymbol{x^{-2a^3}}$

Sol.28 (c)

$$\left[\frac{x^{ab}}{x^{a^2+b^2}}\right]^{a+b} \times \left[\frac{x^{bc}}{x^{b^2+c^2}}\right]^{b+c} \times \left[\frac{x^{ca}}{x^{c^2+a^2}}\right]^{c+a}$$

$$=\left(\frac{1}{x^{a^2+b^2-ab}}\right)^{(a+b)} \times \left(\frac{1}{x^{b^2+c^2-bc}}\right)^{b+c} \times \left(\frac{1}{x^{c^2+a^2-ca}}\right)^{c+a}$$

$$= x^{-(a^2+b^2-ab)(a+b)} \times x^{-(b^2+c^2-bc)(b+c)} \times x^{-(c^2+a^2-ca)(c+a)}$$

$$= x^{-(a^3+b^3)-(b^3+c^3)-(c^3+a^3)}$$

$$= \boldsymbol{x^{-2(a^3+b^3+c^3)}}$$

Sol.29 (d) $\left(\frac{m^x}{m^y}\right)^{x+y} \times \left(\frac{m^y}{m^z}\right)^{y+z} \div 3\,(m^x.m^z)^{x-z}$

$$= m^{x^2-y^2} \times m^{y^2-z^2} \div 3\,m^{x^2-z^2}$$

$$= \frac{m^{x^2-y^2+y^2-z^2-x^2+z^2}}{3} = \frac{m^0}{3} = \boldsymbol{\frac{1}{3}}$$

Sol.30 (c) $\frac{1}{1+a^{y-x}} + \frac{1}{1+a^{x-y}}$

$$= \frac{a^x}{a^x+a^y} + \frac{a^y}{a^y+a^x} = \frac{a^x+a^y}{a^x+a^y} = \mathbf{1}$$

Sol.31 (a) $\because xyz = 1$

$$\therefore \frac{1}{1+x+y^{-1}} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+x^{-1}}$$

$$= \frac{y}{y+xy+1} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+x^{-1}}$$

$$= \frac{y}{y+z^{-1}+1} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+yz}$$

[put xy=1/z, and 1/x=yz]

$$= \frac{y+1}{y+z^{-1}+1} + \frac{z^{-1}}{z^{-1}+1+y} = \frac{y+1+z^{-1}}{y+z-1+1} = \mathbf{1}$$

Sol.32 (b)

$2^a = 3^b = 12^c = k\ (let)$

$\therefore 2 = k^{1/a}, 3 = k^{1/b}$

$\& \; 12 = k^{1/c}$

$\Rightarrow 2^2 \times 3 = k^{1/c} \Rightarrow \left(k^{1/a}\right)^2 \times k^{1/b} = k^{1/c}$

$\Rightarrow k^{\frac{2}{a}+\frac{1}{b}} = k^{1/c} \Rightarrow \frac{2}{a}+\frac{1}{b} = \frac{1}{c}$

$\Rightarrow \frac{2}{a}+\frac{1}{b}-\frac{1}{c} = 0 \Rightarrow \frac{1}{c}-\frac{1}{b}-\frac{2}{a} = \mathbf{0}$

Sol.33 (a) $2^a = 3^b = 6^{-c} = k\ (let)$

$\Rightarrow 2 = k^{1/a} \;\&\; 3 = k^{1/b}$

$\& \; 6 = k^{-1/c}$

$\Rightarrow 2 \times 3 = k^{-1/c} \Rightarrow k^{1/a} \times k^{1/b} = k^{-1/c}$

$\Rightarrow k^{\frac{1}{a}+\frac{1}{b}} = k^{-1/c} \Rightarrow \frac{1}{a}+\frac{1}{b} = -\frac{1}{c}$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \mathbf{0}$

Sol.34 (b) $3^a = 5^b = 75^c = k\ (let)$

$\Rightarrow 3 = k^{1/a}, 5 = k^{1/b}$

$\& \; 75 = k^{1/c}$

$\Rightarrow 3 \times 5^2 = k^{1/c} \Rightarrow k^{\frac{1}{a}} \times k^{2/b} = k^{1/c}$

$\Rightarrow k^{\frac{1}{a}+\frac{2}{b}} = k^{\frac{1}{c}} \Rightarrow \frac{1}{a}+\frac{2}{b} = \frac{1}{c}$

$\Rightarrow \frac{b+2a}{ab} = \frac{1}{c} \Rightarrow c\,(b+2a) = ab$

$\Rightarrow ab - c\,(2a+b) = \mathbf{0}$

Sol.35 (a)

$2^a = 3^b = (12)^c = k\ (let)$

$\therefore 2 = k^{1/a}\; 3 = k^{1/b}$

$\& \; 12 = k^{1/c} \Rightarrow 2^2 \times 3 = k^{1/c}$

$\Rightarrow k^{2/a}.k^{\frac{1}{b}} = k^{1/c} \Rightarrow k^{1/c} \Rightarrow k^{\frac{2}{a}+\frac{1}{b}} = k^{\frac{1}{c}}$

$\Rightarrow \frac{2}{a}+\frac{1}{b} = \frac{1}{c} \Rightarrow ab - c(a+2b) = \mathbf{0}$

Sol.36 (c)

$2^a = 4^b = 8^c$

$\Rightarrow 2^a = 2^{2b} = 2^{3c} \Rightarrow a = 2b = 3c = k\ (let)$

$\therefore a = k, b = \frac{k}{2} \;\&\; c = \frac{k}{3}$

$\because abc = 288 \Rightarrow \frac{k^3}{6} = 288$

$\Rightarrow k^3 = 288 \times 6$

$\Rightarrow k^3 = (12)^3 \Rightarrow k = 12$

$\therefore a = 12, b = 6 \;\&\; c = 4$

$\therefore \frac{1}{2a}+\frac{1}{4b}+\frac{1}{8c} = \frac{1}{24}+\frac{1}{24}+\frac{1}{32}$

$= \frac{4+4+3}{96} = \frac{11}{96}$

Sol.37 (c)

$a^p = b^q = c^r = d^s = k\ (let)$

$\therefore a = k^{1/p}, b = k^{1/q}, c = k^{1/r}, d = k^{1/s}$

Now, $ab = cd \Rightarrow k^{1/p} \times k^{1/q} = k^{\frac{1}{r}} \times k^{1/s}$

$\Rightarrow k^{\frac{1}{p}+\frac{1}{q}} = k^{\frac{1}{r}+\frac{1}{s}} \Rightarrow \frac{1}{p}+\frac{1}{q} = \frac{1}{r}+\frac{1}{s}$

$\Rightarrow \frac{1}{p}+\frac{1}{q}-\frac{1}{r}-\frac{1}{s} = \mathbf{0}$

Sol.38 (c) $\because a^b = b^a$

$\Rightarrow a = b^{\frac{a}{b}}$

$\therefore \left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\left(\frac{a}{b}-1\right)} = \left(\frac{b^{\frac{a}{b}}}{b}\right)^{\frac{a}{b}} - \left(b^{\frac{a}{b}}\right)^{\left(\frac{a}{b}-1\right)}$

$= \left(b^{\frac{a}{b}-1}\right)^{\frac{a}{b}} - \left(b^{\frac{a}{b}}\right)^{\left(\frac{a}{b}-1\right)} = \mathbf{0}$

Sol.39 (c)

$m = b^x,\ n = b^y$

$\Rightarrow (m^y . n^x) = b^2 \Rightarrow (b^x)^y . (b^y)^x = b^2$

$\Rightarrow b^{xy}.b^{yx} = b^2 \Rightarrow b^{2xy} = b^2$

$\Rightarrow 2xy = 2 \Rightarrow xy = 1$

Sol.40 (a)

$a = xy^{m-1},\ b = xy^{n-1}, c = xy^{p-1}$

$\therefore a^{n-p} \times b^{p-m} \times c^{m-n} = (xy^{m-1})^{n-p} \times (xy^{n-1})^{p-m} \times (xy^{p-1})^{m-n}$

$= x^{n-p}\, y^{mn-mp-n+p}\, x^{p-m} y^{np-mn-p+m}\, x^{m-n} y^{pm-np-m+n}$

$= x^{n-p+p-m+m-n} . y^{(m-1)(n-p)+(n-1)(p-m)+(p-1)(m-n)}$

$= x^0 \times y^{mn-mp-n+p+np-nm-p+m+pm-np-m+n}$

$= x^0 . y^0 = 1 \times 1 = \mathbf{1}$

Sol.41 (b)

$a = x^{n+p}\, y^m, b = x^{p+m}\, y^n, c = x^{m+n}\, y^p$

$\therefore a^{n-p} \times b^{p-m} \times c^{m-n} = (x^{n+p}\, y^m)^{n-p} . (x^{p+m}\, y^n)^{p-m}\, (x^{m+n}\, y^p)^{m-n}$

$= x^{n^2-p^2+p^2-m^2+m^2-n^2}\, y^{mn-mp+np-mn+pm-np}$
$[a^2 - b^2 = (a+b)(a-b)]$

$= x^0 . y^0 = 1 \times 1 = \mathbf{1}$

Sol.42 (b)

$a = (\sqrt{2}+1)^{1/3} - (\sqrt{2}-1)^{1/3}$

Taking cubic on both sides

$\therefore a^3 = (\sqrt{2}+1) - (\sqrt{2}-1) - 3(2-1)^{1/3}\left\{(\sqrt{2}+1)^{1/3} - (\sqrt{2}-1)^{1/3}\right\}$

$= a^3 = 2 - 3a$

$\Rightarrow a^3 + 3a - 2 = \mathbf{0}$

Sol.43 (a)

$a = x^{1/3} + x^{-1/3}$

$\therefore a^3 = x + x^{-1} + 3\ \left(x^{1/3} + x^{-1/3}\right) x^{1/3} \times x^{-1/3}$

$= a^3 = x + x^{-1} + 3\,a$

$\Rightarrow a^3 - 3a = \boldsymbol{x + x^{-1}}$

Sol.44 (c) $a = 3^{1/4} + 3^{-1/4}$

$\Rightarrow a = 3^{\frac{1}{4}} + \frac{1}{3^{\frac{1}{4}}} = \frac{3^{\frac{1}{2}}+1}{3^{\frac{1}{4}}}$

$\therefore a^2 = \frac{(\sqrt{3}+1)^2}{\sqrt{3}}$

$=$b$=$ $3^{1/4} - 3^{-1/4}$

$\Rightarrow b = 3^{\frac{1}{4}} - \frac{1}{3^{\frac{1}{4}}} = \frac{3^{\frac{1}{2}}-1}{3^{\frac{1}{4}}}$

$\therefore b^2 = \frac{(\sqrt{3}-1)^2}{\sqrt{3}}$

$\Rightarrow 3(a^2+b^2)^2 = [\frac{(\sqrt{3}+1)^2}{\sqrt{3}} + \frac{(\sqrt{3}-1)^2}{\sqrt{3}}]^2$

$\Rightarrow 3\,(a^2+b^2)^2 = 3\left[\frac{64}{3}\right] = \mathbf{64}$

Sol.45 (d) $x = \sqrt{3} + \frac{1}{\sqrt{3}} \quad \therefore x^2 = 3 + \frac{1}{3} + 2$

$y = \sqrt{3} - \frac{1}{\sqrt{3}} \quad \therefore y^2 = 3 + \frac{1}{3} - 2$

$\therefore x^2 - y^2 = \mathbf{4}$

Sol.46 (c) $a = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$$\Rightarrow a(\sqrt{2}+\sqrt{3}) = 4\sqrt{6}\_\_\_\_\_(I)$$

$$\therefore \frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}}$$

$$= \frac{a^2-2a(\sqrt{3}-\sqrt{2})-4\sqrt{6}+a^2+2a(\sqrt{3}-\sqrt{2})-4\sqrt{6}}{a^2-2a(\sqrt{2}+\sqrt{3})+4\sqrt{6}}$$

$$= \frac{2a^2-8\sqrt{6}}{a^2-2\times4\sqrt{6}+4\sqrt{6}} = \frac{2(a^2-4\sqrt{6})}{a^2-4\sqrt{6}} = \mathbf{2}$$

Sol.47 (a)

$$P+\sqrt{3}\,Q+\sqrt{5}\,R+\sqrt{15}\,S = \frac{1}{1+\sqrt{3}+\sqrt{5}}$$

$$= \frac{\sqrt{5}-(\sqrt{3}+1)}{\{\sqrt{5}+(\sqrt{3}+1)\}\{\sqrt{5}-(\sqrt{3}+1)\}}$$

$$= \frac{\sqrt{5}-\sqrt{3}-1}{5-4-2\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}-1}{1-2\sqrt{3}} \times \frac{1+2\sqrt{3}}{1+2\sqrt{3}}$$

$$= \frac{2\sqrt{15}-6-2\sqrt{3}+\sqrt{5}-\sqrt{3}-1}{1-12}$$

$$= \frac{7}{11} + \frac{3\sqrt{3}}{11} - \frac{\sqrt{5}}{11} - \frac{2\sqrt{15}}{11}$$

$$\Rightarrow P+\sqrt{3}\,Q+\sqrt{5}\,R+\sqrt{15}\,S = \frac{7}{11} + \frac{3}{11}\sqrt{3} - \frac{1}{11}\sqrt{5} - \frac{2}{11}\sqrt{15}$$

$$\therefore P = \frac{7}{11}.$$

Sol.48 (c) $a = 3+2\sqrt{2}$

$$\therefore a^{1/2} + a^{-1/2} = \sqrt{3+2\sqrt{2}} + \frac{1}{\sqrt{3+2\sqrt{2}}}$$

$$= \sqrt{(\sqrt{2}+1)^2} + \frac{1}{\sqrt{(\sqrt{2}+1)^2}}$$

$$= \sqrt{2}+1 + \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \sqrt{2}+1 + \frac{\sqrt{2}-1}{2-1} = \sqrt{2}+1+\sqrt{2}-1$$

$$= \mathbf{2\sqrt{2}}$$

Sol.49 (b)

$$a = 3+2\sqrt{2} \quad \therefore a^{1/2} = \sqrt{3+2\sqrt{2}}$$

$$\therefore a^{1/2} = \sqrt{(\sqrt{2}+1)^2} = \sqrt{2}+1$$

$$\therefore a^{1/2} - a^{-1/2} = \sqrt{2}+1 - \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \sqrt{2}+1-(\sqrt{2}-1) = \sqrt{2}+1-\sqrt{2}+1 = \mathbf{2}$$

Sol.50 (a)

$$a = \frac{1}{2}(5-\sqrt{21})$$

$$\Rightarrow 2a-5 = -\sqrt{21}$$

$$\therefore (2a-5)^2 = (-\sqrt{21})^2 \Rightarrow 4a^2-20a+25 = 21$$

$$\Rightarrow 4a^2-20a+4 = 0 \Rightarrow a^2-5a+1 = 0\_\_(I)$$

Now

$$a^3+a^{-3}-5a^2-5a^{-2}+a+a^{-1}$$
$$= (a^3-5a^2+a)+(a^{-3}-5a^{-2}+a^{-1})$$
$$= a(a^2-5a+1)+a^{-3}(1-5a+a^2)$$
$$= a\times0+a^{-3}\times0 \quad from\ (I)$$
$$= 0+0 = \mathbf{0}$$

Sol.51 (d) $a = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})^2}{(2-\sqrt{3})^2}} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{(2+\sqrt{3})^2}{4-3} = 7+4\sqrt{3}$$

$$= \text{a} = 7+4\sqrt{3} = a-7 = 4\sqrt{3}$$

$$\Rightarrow (a-7)^2 = (4\sqrt{3})^2$$

$$a^2-14a+49 = 48$$

$$a^2-14a = -1 \Rightarrow a(a-14) = -1$$

$$\therefore [a(a-14)]^2 = (-1)^2 = \mathbf{1}$$

Sol.52 (c)

$$a = 3-\sqrt{5} \Rightarrow (a-3) = -\sqrt{5}$$

$$\therefore (a-3)^2 = (-\sqrt{5})^2 \Rightarrow a^2-6a+9 = 5$$

$$\Rightarrow a^2-6a+4 = 0\_\_\_\_\_(I)$$

Now,

$$a^4-a^3-20a^2-16a+24$$

$$= a^2(a^2-6a+4)+5a(a^2-6a+4)+6(a^2-6a+4)$$

$$= (a^2+5a+6)(a^2-6a+4)$$

$$= (a^2+5a+6)\times0\ [form\ (I)]$$

$$= \mathbf{0}$$

Sol.53 (b)

$a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3+2+2\sqrt{6}}{3-2}$

$\Rightarrow a = 5 + 2\sqrt{6} \Rightarrow 5 - a = -2\sqrt{6}$

$\therefore (5-a)^2 = \left(-2\sqrt{6}\right)^2 \Rightarrow 25 - 10a + a^2 = 24$

$\Rightarrow a^2 - 10a + 1 = 0$_____$(I)$

Now $2a^4 - 21a^3 + 12a^2 - a + 1$
$= 2a^4 - 20a^3 + 2a^2 2 - a^3 + 10a^2 - a + 1$
$= 2a^2(a^2 - 10a + 1) - a(a^2 - 10a + 1) + 1$
$= (2a^2 - a)(a^2 - 10a + 1) + 1$
$= (2a^2 - a) \times 0 + 1\ [From\ (I)]$
$= 0 + 1 = \mathbf{1}$

Sol.54 (a) $\sqrt{3+\sqrt{5}} = \sqrt{\frac{6+2\sqrt{5}}{2}}$

[multiply and divide by 2 to make them square]

$= \sqrt{\frac{\left(\sqrt{5}+1\right)^2}{2}} = \sqrt{\left(\frac{\sqrt{5}+1}{\sqrt{2}}\right)^2}$

$= \frac{\sqrt{5}+1}{\sqrt{2}} = \sqrt{5/2} + \sqrt{1/2}$

Sol.55 (b) $\because x = \sqrt{2-\sqrt{2-\sqrt{2 - - - -\infty}}}$

$\therefore x^2 = 2 - \sqrt{2-\sqrt{2-\sqrt{2 - - - \infty}}}$

$= 2 - x$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x + 2 = 0\ or\ x - 1 = 0$

$\Rightarrow x = -2\ \ or\ x = 1$

rejected

As it is not possible

$\therefore x = \mathbf{1}$

Sol.56 (a) $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\therefore a + b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{3+2+2\sqrt{6}+3+2-2\sqrt{6}}{3-2} = \frac{10}{1} = \mathbf{10}$

Sol.57 (c) $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$a^2 + b^2 = (a+b)^2 - 2ab$

$= \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)^2 - 2\ \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \left(\frac{3+2+2\sqrt{6}+3+2-2\sqrt{6}}{3-2}\right)^2 - 2$

$= \left(\frac{10}{1}\right)^2 - 2 = 100 - 2 = \mathbf{98}$

Sol.58 (c)

$a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\therefore \frac{1}{a^2} + \frac{1}{b^2} = \left(\frac{1}{a} + \frac{1}{b}\right)^2 - 2\ \frac{1}{a} \times \frac{1}{b}$

$= \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2 - 2 \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \left(\frac{3+2-2\sqrt{6}+3+2+2\sqrt{6}}{3-2}\right)^2 - 2$

$= (10)^2 - 2 = 100 - 2 = \mathbf{98}$

Sol.59 (a)

$\sqrt{x+\sqrt{x^2-y^2}} = \sqrt{\frac{1}{2}\ \left[2x + 2\sqrt{x^2-y^2}\right]}$

$= \sqrt{\frac{1}{2}\left[\left(\sqrt{x+y}\right)^2 + \left(\sqrt{x-y}\right)^2 + 2\sqrt{(x+y)(x-y)}\right]}$

$= \sqrt{\frac{1}{2}\ \left(\sqrt{x+y} + \sqrt{x-y}\right)^2}$

$= \frac{1}{\sqrt{2}}\ \left[\boldsymbol{\sqrt{x+y} + \sqrt{x-y}}\right]$

Sol.60(b)

$\sqrt{11-\sqrt{120}} = \sqrt{\left(\sqrt{6}\right)^2 + \left(\sqrt{5}\right)^2 - 2\sqrt{6\times5}}$

$= \sqrt{\left(\sqrt{6}-\sqrt{5}\right)^2} = \boldsymbol{\sqrt{6}-\sqrt{5}}$

Sol.61 (c)

$\log\ (1+2+3) = \log\ 6$

$= \log(1\times2\times3) = \mathbf{\log 1 + \log 2 + \log 3}$

Sol.62 (a)

$log_{2\sqrt{7}} 21952$

$= log_{2\sqrt{7}} 2^6 \times 7^3 = log_{2\sqrt{7}} (2\sqrt{7})^6 = 6$

Now $log_{3\sqrt{3}} 19683 = log_{3\sqrt{3}} 3^9$

$= 9\, log_{3^{\frac{3}{2}}} 3 = 9 \times \frac{1}{3/_2}\, log_3 3$

$= 9 \times \frac{2}{3} = 6$

Sol.63 (a)

$4\log\left(\frac{8}{25}\right) - 3\log\frac{16}{125} - \log 5$

$= 4\,[\log 8 - \log 25] - 3\,(\log 16 - \log 125) - \log 5$

$= 4\,(3\log 2 - 2\log 5) - 3\,(4\log 2 - 3\log 5) - \log 5$

$= 12\log 2 - 8\log 5 - 12\log 2 + 9\log 5 - \log 5 \;= \mathbf{0}$

Sol.64 (a) $a^{\log b - \log c} \times b^{\log c - \log a} \times c^{\log a - \log b}$

$= a^{\log\frac{b}{c}} \times b^{\log\frac{c}{a}} \times c^{\log\frac{a}{b}} \qquad [a^{\log n} = n]$

$= \frac{b}{c} \times \frac{c}{a} \times \frac{a}{b} = \mathbf{1}$

Sol.65 (c) $\frac{1}{log_{ab}\,(abc)} + \frac{1}{log_{bc}\,(abc)} + \frac{1}{log_{ca}\,(abc)}$

$= \frac{1}{\frac{logabc}{logab}} + \frac{1}{\frac{logabc}{logbc}} + \frac{1}{\frac{logabc}{logca}}$

$= \frac{logab}{logabc} + \frac{logbc}{logabc} + \frac{logca}{logabc}$

$= \frac{logab + logbc + logca}{logabc} = \frac{log(ab \times bc \times ca)}{logabc}$

$= \frac{log(abc)^2}{logabc} = \frac{2\,logabc}{logabc} = \mathbf{2.}$

Sol.66 (b) $\frac{1}{1 + log_a\, bc} + \frac{1}{1 + log_b\, ca} + \frac{1}{1 + log_c\, ab}$

$= \frac{1}{log_{a\,abc}} + \frac{1}{log_b\, abc} + \frac{1}{log_c\, abc}$

$[\because \log_a a = 1\ \log_b b = 1 = \log_c c]$

$= \frac{1}{1 + \frac{\log bc}{\log a}} + \frac{1}{1 + \frac{\log ca}{\log b}} + \frac{1}{1 + \frac{\log ab}{\log c}}$

$= \frac{\log a}{\log a + \log bc} + \frac{\log b}{\log b + \log ca} + \frac{\log c}{\log c + \log ab}$

$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$

$= \frac{\log a + logb + \log c}{\log abc} = \mathbf{1.}$

Sol.67 (a) $\frac{1}{log_{\frac{a}{b}}(x)} + \frac{1}{log_{\frac{b}{c}}(x)} + \frac{1}{log_{\frac{c}{a}}(x)}$

$= \frac{\log\frac{a}{b}}{\log x} + \frac{\log\frac{b}{c}}{\log x} + \frac{\log\frac{c}{a}}{\log x} = \frac{\log\left(\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}\right)}{\log x}$

$= log_x\left(\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}\right) = log_x 1 = \mathbf{0}$

Sol.68 (b) $log_b\,(a)\,.\,log_c(b)\,.\,log_a(c)$

$= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} = \mathbf{1}$

[from base change formula]

Sol.69 (b)

$log_b\, a^{1/_2}\,.\,log_c\, b^3\,.\,log_a\, c^{2/_3}$

$= \frac{\log a^{\frac{1}{2}}}{\log b} \times \frac{\log b^3}{\log c} \times \frac{\log c^{\frac{2}{3}}}{\log a} = \frac{1\log a}{2\log b} \times \frac{3\log b}{\log c} \times \frac{2\log c}{3\log a} = \mathbf{1}$

Sol.70 (b) $a^{\log b/c}\,.\,b^{\log c/a}\ c^{\log a/b}$

$= \frac{b}{c} \times \frac{b}{a} \times \frac{a}{b} = \mathbf{1.} \qquad [a^{\log n} = n]$

Sol.71 (b) $(bc)^{\log^{b}/_{c}}\,.\,(ca)^{\log^{c}/_{a}}\,.\,(ab)^{\log^{a}/_{b}}$

$= \frac{b}{c} \times \frac{c}{a} \times \frac{a}{b} = \mathbf{1} \quad [a^{\log n} = n]$

Sol.72 (a) $\log\frac{a^n}{b^n} + \log\frac{b^n}{c^n} + \log\frac{c^n}{a^n}$

$= \log\left(\frac{a^n}{b^n} \times \frac{b^n}{c^n} \times \frac{c^n}{a^n}\right) = \log 1 = \mathbf{0}$

Sol.73 (a) $\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab}$

$= \log\left(\frac{a^2}{bc} \times \frac{b^2}{ca} \times \frac{c^2}{ab}\right) = \log 1 = \mathbf{0}$

Sol.74 (b)

$\log\,(a^9) + \log\, a = 10 \Rightarrow 9\log a + \log a = 10$

$\Rightarrow 10\log a = 10 \Rightarrow \log a = 1 \Rightarrow a = 10^1 = \mathbf{10}$

[∵ Under stood base 10]

Sol.75 (b) $\frac{\log a}{y - z} = \frac{\log b}{z - x} = \frac{\log c}{x - y} = k\ (let)$

$\therefore \log a = k\,(y - z) \Rightarrow e^{k(y-z)} = a$

$\log b = k\,(z - x) \Rightarrow e^{k(z-x)} = b$

$\log c = k\,(x - y) \Rightarrow e^{k(x-y)} = c$

$= e^{k(y-z)} \times e^{k(z-x)} \times e^{k(x-y)} \Rightarrow \mathbf{1}.$

Sol.76 (b) $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y} = k\ (let)$

$\therefore \log a = k\ (y-z), \log b = k(z-x)\ \&\ \log c = k\ (x-y)$

$= e^{k(y-z)} = a, e^{k(z-x)} = b, e^{k(x-y)} = c$

$= e^{k(y-z)(y+z)} \times e^{k(z-x)(z+x)} \times e^{k(x-y)(x+y)}$

$= e^{k(y^2-z^2+z^2-x^2+x^2-y^2)} \Rightarrow e^0 = \mathbf{1}.$

Sol.77 (b) $\log a = \frac{1}{2}\log b = \frac{1}{5}\log c = k\ (let)$

$\Rightarrow \log(a) = k, \log b = 2k\ \&\ \log c = 5k$

$= e^k = a, e^{2k} = b, e^{5k} = c$

$= (e^k)^4 \times (e^{2k})^3 \times (e^{5k})^{-2} \Rightarrow e^0 = \mathbf{1}.$

Sol.78 (a)

$\frac{1}{2}\log a = \frac{1}{3}\log b = \frac{1}{5}\log c = k\ (let)$

$\therefore \log a = 2k \Rightarrow e^{2k} = \text{a}$

$\log b = 3k \Rightarrow e^{3k} = b$

$\log c = 5k \Rightarrow e^{5k} = c$

Now $a^4 - bc = (e^{2k})^4 - e^{3k} \times e^{5k} \Rightarrow e^{8k} - e^{8k} = \mathbf{0}$

Sol.79 (b)

$\frac{1}{4}\log_2 a = \frac{1}{6}\log_2 b = -\frac{1}{24}\log_2 c = k\ (let)$

$\therefore \log_2 a = 4k \Rightarrow a = 2^{4k}$

$\log_2 b = 6k \Rightarrow b = 2^{6k}$

$\log_2 c = -24k \Rightarrow c = 2^{-24k}$

Now, $a^3\, b^2\, c = (2^{4k})^3 . (2^{6k})^2 . 2^{-24k}$

$= 2^{12k+12k-24k} = 2^0 = \mathbf{1}$

Sol.80 (b)

$\frac{1}{\log_a(ab)} + \frac{1}{\log_b(ab)}$

$= \frac{\log a}{\log ab} + \frac{\log b}{\log ab} \Rightarrow \frac{\log a + \log b}{\log ab} \Rightarrow \frac{\log ab}{\log ab} \Rightarrow \mathbf{1}.$

Sol.81 (a)

$\frac{1}{\log_a t} + \frac{1}{\log_b t} + \frac{1}{\log_c t} = \frac{1}{\log_z t}$

$= \frac{\log a}{\log t} + \frac{\log b}{\log t} + \frac{\log c}{\log t} = \frac{\log z}{\log t}$

$\Rightarrow \frac{\log a + \log b + \log c}{\log t} = \frac{\log z}{\log t}$

$= \log(\text{abc}) = \log(\text{z}) \Rightarrow \text{z} = \boldsymbol{abc}.$

Sol.82 (a)

$l = 1 + \log_a bc$

$m = 1 + \log_b ca$

$n = 1 + \log_c ab$

Now $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} - 1$

$= \frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ca} + \frac{1}{1+\log_c ab} - 1$

$= \frac{\log_a}{\log a + \log bc} + \frac{\log b}{\log b + \log ca} + \frac{\log c}{\log c + \log ab} - 1$

$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc} - 1 \Rightarrow \frac{\log(abc)}{\log(abc)} - 1 = \mathbf{0}.$

Sol.83 (a) $a = b^2 = c^3 = d^4$

$= \text{a}=b^2 \Rightarrow b = a^{\frac{1}{2}}, c = a^{\frac{1}{3}},\ d = a^{\frac{1}{4}}$

$\Rightarrow \log_a\left(a . a^{\frac{1}{2}} . a^{\frac{1}{3}} . a^{\frac{1}{4}}\right) \Rightarrow \log_a\left(a^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}\right)$

$\Rightarrow (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}) \log_a a$

$\Rightarrow \mathbf{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$

Sol.84 (a)

$\log_a b + \log_{a^2} b^2 + \log_{a^3} b^3 + \cdots + \log_{a^n} b^n$

$= \log_a b + \frac{2}{2}\log_a b + \frac{3}{3}\log_a b + \cdots + \frac{n}{n}\log_a b$

$\left[\because \log_{b^n} a^m = \frac{m}{n}\log_b a\right]$

$= \log_a b + \log_a b + \log_a b + \cdots to\ n\ times$

$= \log_a (b \times b \times \ldots..) = \boldsymbol{\log_a b^n}$

Sol.85 (b) $a^{\left(\frac{1}{\log_b a}\right)}$

$=$ Let $y = a^{\left(\frac{1}{\log_b a}\right)}$

$= \log y = \frac{1}{\log_b a}\log a \Rightarrow \log y = \frac{\log b}{\log a} \times \log a$

$= \text{Y} = \mathbf{b}.$

Sol.86 (d) $a^{log_a b \times log_b c \times log_c d \times log_d t}$

$= a^{\log\left(\frac{b}{a}\times\frac{c}{b}\times\frac{d}{c}\times\frac{t}{d}\right)}$

$\Rightarrow a^{\log\frac{t}{a}} = \frac{t}{a}$

Sol.87 (a)

$\because x, y$ & z are three consecutive integers

$\therefore y = x + 1$ & $z = x + 2$

$\therefore 1 + xz = 1 + x\,(x + 2) = 1 + x^2 + 2x$

$= (x + 1)^2$

$\Rightarrow 1 + xz = y^2$

Taking log on both sides

$\therefore \log\ (1 + xz) = \log y^2$

$\Rightarrow \log\ (1 + xz) = 2 \log y$

$\Rightarrow \log(1 + xz) - 2\log y = \mathbf{0}$

which is true

Sol.88 (c) $\log \frac{\mathrm{a+b}}{3} = \frac{1}{2}\ (\log \mathrm{a} + \log \mathrm{b})$

$\Rightarrow \log\left(\frac{a+b}{3}\right) = \frac{1}{2}\log ab$

$\Rightarrow \log\ (\frac{a+b}{3}) = \log(ab)^{\frac{1}{2}}$

$\Rightarrow\ (\frac{a+b}{3}) = \sqrt{ab} \ \ \Rightarrow a + b = 3\sqrt{ab}$

$\Rightarrow\ (a + b)^2 - 9\ ab = 0$

$\Rightarrow\ a^2 + b^2 - 7\ ab = 0$

$\Rightarrow\ a^2 + b^2 = 7\ ab$

$\Rightarrow\ \frac{a}{b} + \frac{b}{a} = \mathbf{7}$

Sol.89 (a) $a^2 + b^2 = 7ab$

$\Rightarrow (a + b)^2 = 9\ ab \Rightarrow \left(\frac{a+b}{3}\right)^2 = ab \Rightarrow \frac{a+b}{3} = \sqrt{ab}$

ATQ.

$\Rightarrow \log\ \left(\frac{a+b}{3}\right) - \frac{1}{2}\log a + \frac{1}{2}\log b$

$\Rightarrow\ \log\ \left(\frac{a+b}{3}\right) - [\log a^{\frac{1}{2}} + \log b^{\frac{1}{2}}]$

$\Rightarrow \log\ \left(\frac{a+b}{3}\right) - \log(\sqrt{ab} \Rightarrow \log(\frac{a+b}{3\sqrt{ab}})$

$\Rightarrow \log\ (\frac{3\sqrt{ab}}{3\sqrt{ab}}) \Rightarrow \log(1) = \mathbf{0}.$

Sol.90 (a)

$a^3 + b^3 = 0\ \Rightarrow (a + b)^3 - 3ab\ (a + b) = 0$

$\Rightarrow (a + b)\ [(a + b)^2 - 3ab] = 0$

$\Rightarrow (a + b)^2 - 3ab = 0\quad (\because \mathrm{a + b} \neq 0)$

$\Rightarrow (a + b)^2 = 3ab$

$\therefore \log(a + b)^2 = \log(3ab)$

$\Rightarrow 2\log(a + b) = \log 3 + \log a + \log b$

$\Rightarrow \log(a + b) - \frac{1}{2}\ (\log a + \log b + \log 3) = \mathbf{0}$

Sol.91 (d)

$x = log_a bc\ ; y = log_b ca; z = log_c ab$

$\therefore 1 + x = 1 + log_a bc = log_a abc$

$\therefore\ \frac{1}{1+x} = log_{abc} a$

Similarly, $\frac{1}{1+y} = log_{abc} b$

& $\frac{1}{1+z} = log_{abc} c$

$\therefore \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = log_{abc} a + log_{abc} b + log_{abc} c$

$= \log_{\mathrm{abc}}\ abc = 1$

$\Rightarrow \frac{y+1+1+x}{(1+x)\,(1+y)} \qquad = 1 - \frac{1}{1+z}$

$\Rightarrow\ \frac{x+y+2}{1+x+y+xy} = \frac{z}{z+1}$

$\Rightarrow xz + yz + 2z + x + y + 2 = z + xz + yz + xyz$

$\Rightarrow xyz - x - y - z = \mathbf{2}$

Sol.92 (a)

$\log t + \log(t - 3) = 1 \Rightarrow \log t(t - 3) = 1$

$\Rightarrow t\ (t - 3) = 10^1\ \Rightarrow t^2 - 3t - 10 = 0$

$\Rightarrow (t - 5)\ (t + 2) = 0\ \ \Rightarrow t - 5 = 0$

[$\because$ t + 2 ≠ 0 as it is not possible]

$\Rightarrow t = \mathbf{5}$

Sol.93 (d)

$log_3\ [log_2\ (log_3 t)] = 1$

$\Rightarrow log_2\,(log_3 t) = 3^1 \Rightarrow log_2\,(log_3 t) = 3$

$\Rightarrow log_3 t = 2^3 \Rightarrow log_3 t = 8$

$\Rightarrow t = 3^8 = \mathbf{6561}$

Sol.94 (c) $log_{1/2}\,[log_t\,(log_4 32)] = 2$

$\Rightarrow log_t\,(log_{2^2} 2^5) = \left(\frac{1}{2}\right)^2$

$\Rightarrow log_t\,\left(\frac{5}{2}\right) = \frac{1}{4}$

$\Rightarrow \frac{5}{2} = t^{1/4} \Rightarrow t = \left(\frac{5}{2}\right)^4 = \mathbf{\frac{625}{16}}$

Sol.95 (c)

$(4.8)^x = (0.48)^y = 1000$

$\Rightarrow 4.8 = (1000)^{1/x} \Rightarrow 4.8 = 10^{3/x}$ ______(I)

And $(0.48)^y = 1000 \Rightarrow .48 = 10^{3/y}$ _____(II)

$\therefore from\ [(I) \div (II)]$

$10 = 10^{\frac{3}{x} - \frac{3}{y}}$

$\therefore \log 10 = \log\left(10^{\left(\frac{3}{x} - \frac{3}{y}\right)}\right)$

$\Rightarrow 1 = \frac{3}{x} - \frac{3}{y} \Rightarrow \frac{1}{x} - \frac{1}{y} = 1/3$

Sol.96 (a) $x^{2a-3}\,y^{2a} = x^{6-a}\,y^{5a}$

$\Rightarrow x^{2a-3-6+a} = y^{5a-2a}$

$\Rightarrow x^{3a-9} = y^{3a}$

$\Rightarrow \left(\frac{x^{3a}}{y^{3a}}\right) = x^9$

Taking log on both sides

$\Rightarrow \log\left(\frac{x}{y}\right)^{3a} = log x^9 \Rightarrow 3a\,log\left(\frac{x}{y}\right) =$ $9 log x$

$\Rightarrow a \log\left(\frac{x}{y}\right) = 9\,\frac{log x}{3}$

$\Rightarrow a \log\frac{x}{y} = \mathbf{3 \log x}$

Sol.97 (a) $x = \frac{e^n - e^{-n}}{e^n + e^{-n}}$

$\Rightarrow xe^{-n} + xe^n = e^n - e^{-n}$

$\Rightarrow xe^{-n} + e^{-n} = e^n - xe^n$

$\Rightarrow \frac{e^{-n}}{e^n} = \frac{1-x}{1+x}$

$\Rightarrow \frac{e^n}{e^{-n}} = \frac{1+x}{1-x}$ (reciprocal)

$= e^{n-n} = \frac{1+x}{1-x} \Rightarrow e^{2n} = \frac{1+x}{1-x}$

$= log_e\left(\frac{1+x}{1-x}\right) = 2\text{n} \Rightarrow \frac{1}{2} log_e\left(\frac{1+x}{1-x}\right) = \mathbf{n.}$

……

Equations

Exercise: 2A

Sol.1 (b)

$\therefore$ The Equation $-7x + 1 = 5 - 3x$

Put the option in the equation

If $x = -1$ then $7+1 = 5+3 \Rightarrow 8 = 8$

<u>**Option: B**</u> Which is true (✓)

If $x=1$ then $-7+1 = 5 - 3 \Rightarrow -6 = \mathbf{2}$

Sol. 2(a)

The equation is $\frac{x+4}{4} + \frac{x-5}{3} = 11$

Here $x = 20$ Satisfy the equation

$\frac{20+4}{4} + \frac{20-5}{3} = 11$

$\Rightarrow 6 + 5 = 11 \Rightarrow 11 = \mathbf{11}$

Sol.3 (c)

The equation is $\frac{x}{30} = \frac{2}{45}$

If we put

$x = 1\frac{1}{3} = \frac{4}{3}$ in the equation, then we have

$\frac{4/3}{30} = \frac{2}{45} \Rightarrow \frac{4}{3 \times 30} = \frac{2}{45} \Rightarrow \frac{2}{45} = \mathbf{\frac{2}{45}}$

Which is true whereas other do not satisfy equation.

Sol.4 (c)

The equation $\frac{x+24}{5} = 4 + \frac{x}{4}$

If we put $x = 16$ in the equation then $\frac{16+24}{5} = 4 + \frac{16}{4}$

$\Rightarrow \frac{40}{5} = 4 + 4 \Rightarrow 8 = \mathbf{8}$ **which is true**

Sol.5 (b)

If put $x = 8$ in the option then $\frac{x+4}{2} + \frac{x+10}{9} = 8 \Rightarrow \frac{8+4}{2} + \frac{8+10}{9} = 8$

$\Rightarrow 6+2 = 8 \Rightarrow 8 = \mathbf{8}$

Which is true

Sol.6 (d)

$$\frac{y+11}{6} - \frac{y+1}{9} = \frac{y+7}{4}$$

If we put y = $^{-1}/_{7}$ in the equation

We have

$$\frac{-\frac{1}{7}+11}{6} - \frac{-\frac{1}{7}+1}{9} = \frac{-\frac{1}{7}+7}{4} \Rightarrow \frac{-1+77}{7\times6} - \left(\frac{-1+7}{7\times9}\right) = \frac{-1+49}{7\times4}$$

$$\Rightarrow \frac{76}{7\times6} - \frac{6}{7\times9} = \frac{48}{7\times4} \Rightarrow \frac{76}{6} - \frac{6}{9} = \frac{48}{4}$$

$$\Rightarrow \frac{38-2}{3} = 12 \Rightarrow 12 = \mathbf{12}$$

Which is true, whereas other option don't satisfy the equation.

Sol.7 (a)

(P+2) (P-3) + (P+3) (P-4) = P(2P-5)

For option

a) P = 6 then

$8 \times 3 + 9 \times 2 = 6\ (12\text{-}5)$

$\Rightarrow 24 + 18 = 6 \times 7$

$\Rightarrow 42 = \mathbf{42}$ (✓)

b) P = 7 then

$9 \times 4 + 10 \times 3 = 7\ (14\text{-}5)$

$\Rightarrow 36+30 = 7\times9$

$\Rightarrow 66 = 63$ (×)

c) P = 5 then

$7\times 2 + 8\times 1 = 5\ (10\text{-}5)$

$\Rightarrow 14+8 = 5\times5$

$\Rightarrow 22 = 25$ (×)

Sol.8 (d)

The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$

Put the value of x in the equation from the option.

Option

a) $x = 1$

$$\frac{12+1}{4} = \frac{15-1}{5} + \frac{2-5}{3-1}$$

$$\Rightarrow \frac{13}{4} = \frac{14}{5} + \frac{-3}{2}$$

$$\Rightarrow \frac{13}{4} = \frac{13}{10}$$ (×)

b) $x = 2$

$$\frac{24+1}{4} = \frac{30-1}{5} + \frac{4-5}{6-1}$$

$$\Rightarrow \frac{25}{4} = \frac{29}{5} + \frac{-1}{5}$$

$$\Rightarrow \frac{25}{4} = \frac{28}{5}$$ (×)

c) $x = 5$

$$\frac{60+1}{4} = \frac{75-1}{5} + \frac{10-5}{15-1}$$

$$= \frac{61}{4} = \frac{74}{5} + \frac{5}{14}$$

$$\Rightarrow \frac{61}{4} = \frac{1036+25}{5\times14} \Rightarrow \frac{61}{4} = \frac{1061}{70}$$ (×)

d) $x = 7$

$$\frac{84+1}{4} = \frac{105-1}{5} + \frac{14-5}{21-1}$$

$$\Rightarrow \frac{85}{4} = \frac{104}{5} + \frac{9}{20}$$

$$\Rightarrow \frac{85}{4} = \frac{416+9}{20}$$

$$\Rightarrow \frac{85}{4} = \frac{425}{20}$$

$$\Rightarrow \frac{85}{4} = \frac{85}{4}$$ (✓)

Sol.9 (c)

$$\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$$

Put the value of x given in option to the equation; we have four options

a) $x = 0$

$$0 - \frac{1}{0.5} + 0 - \frac{1}{0.0005} = 0$$ (×)

b) $x = 1$

$$\frac{1}{0.5} - \frac{1}{0.05} + \frac{1}{0.005} - \frac{1}{0.0005} = 0$$

$\Rightarrow 2 - 20 + 200 - 2{,}000 = 0$ (×)

c) $x = 10$

$\frac{10}{0.5} - \frac{1}{0.05} + \frac{10}{0.005} - \frac{1}{0.0005} = 0$

$\Rightarrow 20 - 20 + 2{,}000 - 2{,}000 = 0$

$\Rightarrow 0 = \mathbf{0}$ (✓)

****

Equations

Exercise: 2B

Sol.1 (c) Observation the option carefully we find
For the option

$(a)\ 17 + 15 = 32 \neq 52$ (×)

$(b)\ 12 + 10 = 22 \neq 52$ (×)

$(c)\ 27 + 25 = 52$

Also 27-25 =**2** (✓)

Sol.2 (b) For Pythagoras theorem
$(b) = \sqrt{5^2 - 4^2}\ cm = \sqrt{25 - 16}\ cm = \sqrt{9}\ cm = 3cm$

$\therefore Area = lb = 4 \times 3\ cm^2 = \mathbf{12cm^2}$

Sol.3 (a)
For the option

$(a)\ 20 + 36 = 56$

$3 \times 20\ \frac{-36}{3} = 60 - 12 = \mathbf{48}$ (✓)

$(b)\ 25 + 31 = 56$

$3 \times 25 - \frac{1}{3} \times 31 = 75 - \frac{31}{3} \neq 48$ (×)

$(c)\ 24 + 32 = 56$

$\&\ 3 \times 24 - \frac{1}{3} \times 32 = 72 - \frac{32}{3} \neq 48$ (×)

Sol.4 (b) For the option

(a) 37,

$3 + 7 = 10$

$\&\ 37 - 18 = 19\ so\ 1 \neq 9$ (×)

(b) 73,

$7 + 3 = 10$

$73 - 18 = 55\ \therefore 5 = \mathbf{5}$ (✓)

(c) 75,

$7 + 5 = 12\ \neq 10$ (×)

Sol.5 (c) For the option

$(a)\ \frac{1}{4} \times 84 - \frac{1}{6} \times 84 = 21 - 14 = 7\ \neq 4$ (×)

$(b)\ \frac{1}{4} \times 44 - \frac{1}{6} \times 44 = 11 - \frac{22}{3} = \frac{11}{3} \neq 4$ (×)

$(c)\ \frac{1}{4} \times 48 - \frac{1}{6} \times 48 = 12 - 8 = \mathbf{4}$ (✓)

Sol.6 (a) For the option

$(a)\ (50, 20)$

10 years before

$50 - 10 = 4 \times (20 - 10) \Rightarrow 40\ \ = 40$

10 years after

Also, $50 + 10 = 2\ (20 + 10)$ (✓)

$60 = \mathbf{60}$

$(b)\ (60{,}20)$

10 years ago

$60 - 10 = 4\ (20 - 10) \Rightarrow 50 = 40$ (×)

$(c)\ (55{,}25)$

10 years ago

$50 - 10 = 4\ (25 - 10) \Rightarrow 40 = 60$ (×)

Sol.7 (d) For the option

$(a)\ (16{,}200)$

$16 \times 200 = 3200$

Also $\frac{25}{16} = \frac{25}{2} = 12\ \frac{1}{2}$ (×)

Here Quotient is $12 \neq 2$

$(b)\ (160, 20)$

$160 \times 20 = 3{,}200$

Also $\frac{160}{20} = 8\ \ \neq 2$ (×)

$(c)\ (60, 30)$

$60 \times 30 = 1800 \neq 3,200$ (×)

(d) (80,40)

Here $80 \times 40 = 320$

Also $\frac{80}{40} = \mathbf{2}$ (✓)

Sol.8 (d) For the option

(a) $\frac{5}{7}$

Here 7 - 5 =2

Also, $\frac{5+5}{7} - \frac{5}{7} = \frac{10}{7} - \frac{5}{7} = \frac{5}{7} \neq 1$ (×)

(b) $\frac{1}{3}$

Here $3 - 1 = 2$

Also $\frac{1+5}{3} - \frac{1}{3} = \frac{5}{3} \neq 1$ (×)

(c) $\frac{7}{9}$

Here 9 - 7 =2

Also $\frac{7+5}{9} - \frac{7}{9} = \frac{5}{9} \neq 1$ (×)

(d) $\frac{3}{5}$,

$5 - 3 = 2$

Also $\frac{3+5}{5} - \frac{3}{5} = \frac{5}{5} = \mathbf{1}$ (✓)

Sol.9

(a) Read the Question carefully & check the option
For the option

(a) (₹20, ₹16, ₹15)Here $20 + 16 + 15 = 51$

Also $20 - 16 = \mathbf{4}$

20 - 15 =**5** (✓)

(b) (₹15, 20, ₹16)

Here $15 + 20 + 16 = 51$

Also, $15 - 20 = -5 \neq 4$ (×)

(c) (₹25, ₹11, ₹15)

Here $25 + 11 + 15 = 51$

$11 - 25 = -14 \neq 4$ (×)

Sol.10 (c) For the option

(a) 39

Here $3 = 3 \times 9$ (×)

(b) 92, $9 = 3 \times 2$ (×)

(c) 93, $9 = 3 \times 3$

Also $93 - 54 = \mathbf{39}$ (✓)

(d) 94, $9 = 3 \times 4$ (×)

Sol.11 (c) For the option

(a) Number = 320

Now $\frac{1}{6}\left(\frac{1}{2} \times 320\right) + \frac{1}{4}\left(\frac{1}{2} \times 320\right) = \frac{1}{5}\,(320) + 4$

$\Rightarrow \frac{80}{3} + 40 = 64 + 4$ (×)

(b) 400,

$\frac{1}{6}\left(\frac{1}{2} \times 400\right) + \frac{1}{4}\left(\frac{1}{2} \times 400\right) = \frac{1}{5}\,(400) + 4$

$\Rightarrow \frac{100}{3} + 50 = 80 + 4$ (×)

(c) 480

$\frac{1}{6}\left(\frac{1}{2} \times 480\right) + \frac{1}{4}\left(\frac{1}{2} \times 480\right) = \frac{1}{5}\,(480) + 4$

$\Rightarrow 40 + 60 = 96 + 4$

$\Rightarrow 100 = \mathbf{100}$ (✓)

Sol.12 (a) For the option

(a) 50,

$\frac{1}{2}\,(50) = \frac{1}{5}\,(50) + 15$

$\Rightarrow 25 = 10 + 15$

$\Rightarrow 25 = \mathbf{25}$ (✓)

(b) 40,

$\frac{1}{2}\,(40) = \frac{1}{5}\,(40) + 15$

$\Rightarrow 20 \quad = 23$ (×)

(c) 80,

$\frac{1}{2}\,(80) = \frac{1}{5}\,(80) + 15$

$\Rightarrow 40 \quad = 31$ (×)

****

Equations

Exercise: 2C

Sol.1 (b) $3x + 4y = 7,\ 4x - y = 3$

For the option

(a) $(1, -1)$

$3 \times 1 + 4 \times (-1) = 7$ & $4 \times 1 - (-1) = 3$

$\Rightarrow -1 = 7$ & $5 = 3$ (×)

(b) (1, 1)

$3 \times 1 + 4 \times 1 = 7$ & $4 \times 1 - 1 = 3$

$\Rightarrow 7 = 7$ & $3 = \mathbf{3}$ (✓)

(c) (2, 1)

$3 \times 2 + 4 \times 1 = 7$ & $4 \times 2 - 1 = 3$

$\Rightarrow 10 = 7$ & $7 = 3$ (×)

(d) (1, -2)

$3 \times 1 + 4\,(-2) = 7$ & $4 \times 1 - (-2) = 3$

$\Rightarrow -5 = 7$ & $6 = 3$ (×)

Sol.2 (c) $\frac{x}{2} + \frac{y}{3} = 2,\ x + 2y = 8$

Put the option, i.e., the value of (x, y) in the option into the equation, those values which satisfy both the equation is the answer

Here option (c) (2, 3) satisfy the equation

i.e $\frac{2}{2} + \frac{3}{3} = 2$ & $2 + 2 \times 3 = 8$

$\Rightarrow 1 + 1 = 2$ & $2 + 6 = 8$

$\Rightarrow 2 = 2$ & $8 = 8$ (✓)

Sol.3 (a) $\frac{x}{p} + \frac{y}{q} = 2,\ x + y = p + q$

For the option

(a) $(x = p, y = q)$

$\frac{P}{P} + \frac{q}{q} = 2$ & $P + q = P + q$

$\Rightarrow 1 + 1 = 2$ & $P + q = \boldsymbol{P + q}$ (✓)

Other options don't satisfy

Sol.4 (a) $\frac{1}{16x} + \frac{1}{15y} = \frac{9}{20}, \frac{1}{20x} - \frac{1}{27y} = \frac{4}{45}$

For the option

(a) $\left(\frac{1}{4}, \frac{1}{3}\right)$,

$\frac{1}{4} + \frac{1}{5} = \frac{9}{20}, \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$

$\Rightarrow \frac{5+4}{20} = \frac{9}{20}, \frac{9-5}{45} = \frac{4}{45}$ (✓)

(b) $\left(\frac{1}{3}, \frac{1}{4}\right)$

$\frac{3}{16} + \frac{4}{15} = \frac{9}{20}, \frac{3}{20} - \frac{4}{27} = \frac{4}{45}$ (×)

(c) $(3, 4)$

$\frac{1}{48} + \frac{1}{60} = \frac{9}{20}, \frac{1}{60} - \frac{1}{108} = \frac{4}{45}$ (×)

(d) $(4, 3)$

$\frac{1}{64} + \frac{1}{45} = \frac{9}{20}, \frac{1}{80} - \frac{1}{81} = \frac{4}{45}$ (×)

Sol.5 (d) $\frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10}$ *and* $3xy = 10\,(y - x)$

For the option

(a) $(5, 2)$

$\frac{4}{5} - \frac{5}{2} = \frac{5+2}{5 \times 2} + \frac{3}{10}$ & $3 \times 5 \times 2 = 10(2 - 5)$

$(+ve) \neq (-ve)$ (×)

(b) $(-2, -5)$

$\frac{4}{-2} - \frac{5}{-5} = \frac{-2-5}{-2 \times (-5)} + \frac{3}{10}$ & $3 \times (-2)\,(-5) =$ $10(-5 + 2)$ (×)

(c) $(2, -5)$

$\frac{4}{2}\ \frac{-5}{-5} = \frac{2-5}{2 \times (-5)} + \frac{3}{10}$ & $3 \times 2 \times (-5) = 10(-5 - 2)$

$\Rightarrow 2 + 1 = \frac{-3}{-10} + \frac{3}{10}$ & $-30 = -70$ (×)

(d) $(2, 5)$

$\frac{4}{2} - \frac{5}{5} = \frac{2+5}{2 \times 5} + \frac{3}{10}$ & $3 \times 2 \times 5 = 10\,(5 - 2)$

$\Rightarrow 2 - 1 = \frac{7}{10} + \frac{3}{10}$ & $30 = 30$

$\Rightarrow 1 = 1$ & $30 = \mathbf{30}$ (✓)

Sol.6 (a) $x + 5y = 36,\ \frac{x+y}{x-y} = \frac{5}{3}$

For the option

(a) $(16, 4)$,

$16 + 5 \times 4 = 36,\ \frac{16+4}{16-4} = {}^{5}/_{3}$

$\Rightarrow 36 = 36,\ \frac{20}{12} = {}^{5}/_{3}$

$\Rightarrow \frac{5}{3} = {}^{5}/_{3}$ (✓)

In a similar way, other options don't satisfy the equation

Sol.7 (b) $x - 3y = 0,\ x + 2y = 20$

For the option

$(a)\ x = 4,\ y = 12$

$4 - 3 \times 12 = 0,\ 4 + 2 \times 12 = 20$ (×)

$(b)\ x = 12,\ y = 4$

$12 - 3 \times 4 = 0\ \&\ 12 + 2 \times 4 = 20$

$\Rightarrow 0 = 0\ \&\ 20 = \mathbf{20}$ (✓)

In a similar way to check, other options don't satisfy the equation on

Sol.8 (c) $7x - 3y = 31,\ \ 9x - 5y = 41$

For the option

$(a)\ (-4, -1)$

$-7 \times (-4) - 3 \times (-1) = 31,\ 9 \times (-4) - 5(-1) = 41$ (×)

$(b)\ (-1, 4)$

$7 \times (-1) - 3 \times 4 = 31, 9 \times (-1) - 5 \times 4 = 41$(×)

$(c)\ (4, -1)$

$7 \times 4 - 3 \times (-1) = 31,\ 9 \times 4 - 5 \times (-1) = 41$

$\Rightarrow 28 + 3 = 31,\ 36 + 5 = \mathbf{41}$ (✓)

$(d)\ (3, 7)$

$7 \times 3 - 3 \times 7 = 31,\ 9 \times 3 - 5 \times 7 = 41$ (×)

Sol.9 (b) $1.5x + 2.4y = 1.8,\ 2.5\ (x + 1) = 7y$

For the option

$(a)\ (0.5, 0.4)$

$1.5 \times 0.5 + 2.4 \times 0.4 = 1.8,\ 2.5(0.5 + 1) = 7 \times 0.4$

$\Rightarrow 0.75 + 0.96 = 1.8,\ 3.75 = 2.8$ (×)

$(b)\ (0.4, 0.5)$

$1.5 \times 0.4 + 2.4 \times 0.5 = 1.8,\ 2.5(.4 + 1) = 7 \times 0.5$

$\Rightarrow .6 + 1.2 = 1.8,\ 3.50 = \mathbf{3.5}$ (✓)

In a similar way, other options don't satisfy the equation

Sol.10 (d) $\frac{3}{x+y} + \frac{2}{x-y} = 3,\ \ \frac{2}{x+y} + \frac{3}{x-y} = 3\frac{2}{3}$

For the option

$(a)\ (1, 2)$

$\frac{3}{1+2} + \frac{2}{1-2} = 3,\ \frac{2}{1+2} + \frac{3}{1-2} = 3\frac{2}{3}$

$\Rightarrow 1 - 2 = 3,\ \frac{2}{3} - 3 = 3\frac{2}{3}$ (×)

$(b)\ (-1, -2)$

$\frac{3}{-1-2} + \frac{2}{-1+2} = 3,\ \frac{2}{-1-2} + \frac{2}{-1+2} = 3\frac{2}{3}$

$\Rightarrow -1 + 2 = 3,\ \frac{-2}{3} + 3 = 3\frac{2}{3}$ (×)

$(c)\ (1, {}^{1}/_{2})$

$\frac{3}{1+\frac{1}{2}} + \frac{2}{1-{}^{1}/_{2}} = 3,\ \frac{2}{1+{}^{1}/_{2}} + \frac{3}{1-{}^{1}/_{2}} = 3\frac{2}{3}$

$\Rightarrow 2 + 4 = 3,\ \frac{4}{3} + 6 = 3\frac{2}{3}$ (×)

$(d)\ (2,1)$

$\frac{3}{2+1} + \frac{2}{2-1} = 3,\ \frac{2}{2+1} + \frac{3}{2-1} = 3\frac{2}{3}$

$\Rightarrow 1 + 2 = 3,\ \ \ \ \frac{2}{3} + 3 = 3\frac{2}{3}$

$\Rightarrow 3 = 3,\ 3\frac{2}{3} = 3\frac{2}{3}$ (✓)

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Equations

Exercise: 2D

Sol.1 (a) $1.5x + 3.6y = 2.1,\ 2.5(x + 1) = 6y$

For the option

$(a)\ (0.2, 0.5)$

$1.5 \times 0.2 + 3.6 \times 0.5 = 2.1,\ 2.5\ (.2 + 1) = 6 \times .5$

$\Rightarrow .3 + 1.8 = 2.1\ \ ,\ \ 3.0 = 3$

$\Rightarrow 2.1 = 2.1\ ,\ 3 = \mathbf{3}$ (✓)

Check the other option in a similar way; we find other options don't satisfy the equation

Sol.2 (c) $\frac{x}{5}+\frac{y}{6}+1=\frac{x}{6}+\frac{y}{5}=28$

For the option

(*a*) (6, 9)

$\frac{6}{5}+\frac{9}{6}+1=\frac{6}{6}+\frac{9}{5}=28$

$\Rightarrow \frac{81}{30}+1=1+\frac{9}{5}=28$ (×)

(*b*) (9, 6)

$\frac{9}{5}+\frac{6}{6}+1=\frac{9}{6}+\frac{6}{5}=28$ (×)

(*c*) (60, 90)

$\frac{60}{5}+\frac{90}{6}+1=\frac{60}{6}+\frac{90}{5}=28$

$\Rightarrow 12+15+1=10+18=28$

$\Rightarrow 28 = 28 = 28$ (✓)

(*d*) (90, 60)

$\frac{90}{5}+\frac{60}{6}+1=\frac{90}{6}+\frac{60}{5}=28$

$\Rightarrow 18+10+1=15+12=28$

$\Rightarrow 29 = 27 = 28$ (×)

Sol.3 (a) $\frac{x}{4}=\frac{y}{3}=\frac{z}{2}, 7x+8y+5z=62$

For the option

(*a*) (4, 3, 2)

$\frac{4}{4}=\frac{3}{3}=\frac{2}{2}, 7\times4+8\times3+5\times2=62$

$\Rightarrow 1=1=1,\ 28+24+10=62$

$\Rightarrow 62=\mathbf{62}$ (✓)

Check the other option in a similar way; we find, other options don't satisfy the equation

Sol.4 (d) $\frac{xy}{x+y}=20,\ \frac{yz}{y+z}=40,\ \frac{zx}{z+x}=24$

For the option

(*a*) (120, 60, 30)

$\frac{120\times60}{120+60}=20,\ \frac{60\times30}{60+30}=40,\ \frac{30\times120}{30+120}=24$

$\Rightarrow \frac{120\times60}{180}=20,\ \frac{60\times30}{90}=40,\ \frac{30\times120}{150}=24$

$\Rightarrow 40=20,\ 20=40,\ 24=24$ (×)

(*b*) (60, 30, 120)

$\frac{60\times30}{60+30}=20,\ \frac{30\times120}{30+120}=40,\ \frac{120\times60}{120+60}=24$

$\Rightarrow 20=20,\ 24=40,\ 40=24$ (×)

(*c*) (30, 120, 60)

$\frac{30\times120}{30+120}=20,\ \frac{120\times60}{120+60}=40,\ \frac{60\times30}{60+30}=24$

$\Rightarrow 24=20,\ 40=40,\ 20=24$ (×)

(*d*) (30, 60, 120)

$\frac{30\times60}{30+60}=20,\ \frac{60\times120}{60+120}=40,\ \frac{120\times30}{120+30}=24$

$\Rightarrow 20=20,\ 40=40,\ 24=\mathbf{24}$ (✓)

Sol.5 (a) $2x+3y+4z=0,\ x+2y-5z=0,\ 10x+16y-6z=0$

For the option

(*a*) (0, 0, 0)

$0+0+0=0,\ 0+0-0=0,\ 0+0-0=0$

$\Rightarrow 0=0,\quad 0=0,\quad 0=\mathbf{0}$ (✓)

Check the other option in a similar manner, which don't satisfy the equation

Sol.6 (c) $\frac{1}{3}(x+y)+2z=21, 3x-\frac{1}{2}(y+z)=65, x+\frac{1}{2}(x+y-z)=38$

For the option

(*a*) (4, 9, 5),

$\frac{1}{3}(4+9)+2\times5=21,\ 3\times4-\frac{1}{2}(9+5)=65,\ 4+\frac{1}{2}(4+9-5)=38$ (×)

(*b*) (2, 9, 5),

$\frac{1}{3}(2+9)+2\times5=21,\ 3\times2-\frac{1}{2}(9+5)=65,\ 2+\frac{1}{2}(2+9-5)=38$ (×)

(*c*) (24, 9, 5),

$\frac{1}{3}(24+9)+2\times5=21,\ 3\times24-\frac{1}{2}(9+5)=65,\ 24+\frac{1}{2}(24+9-5)=38$

$\Rightarrow 11+10=21,\ 72-7=65,\ 24+14=38$

$\Rightarrow 21=21,\ 65=65,\ 38=\mathbf{38}$ (✓)

$(d)\ (5,24,9),$

$\frac{1}{3}(5+24)+2\times 9=21,\ 3\times 5-\frac{1}{2}(24+9)=65, 5+\frac{1}{2}(5+24-9)=38$ (×)

Sol.7 (a) $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10},\ \ 3xy=10\,(y-x)$

For the option
$(a)\ (2,5)$

$\frac{4}{2}-\frac{5}{5}=\frac{2+5}{2\times5}+\frac{3}{10}, 3\times 2\times 5=10\,(5-2)$

$\Rightarrow 2-1=\frac{7}{10}+\frac{3}{10},\ 30=10\times 3$

$\Rightarrow 1=1\ ,\ 30=\mathbf{30}$ (✓)

Check the other option in the same way, which don't satisfy the equation

Sol.8 (c) $\frac{x}{0.01}+\frac{y+0.03}{0.05}=\frac{y}{0.02}+\frac{x+0.03}{0.04}=2$

For the option

$(a)\ (1,2)$

$\frac{1}{0.01}+\frac{2+0.03}{0.05}=\frac{2}{0.02}+\frac{1+0.03}{0.04}=2$ (×)

$(b)\ (0.1,0.2)$

$\frac{0.1}{0.01}+\frac{0.2+0.03}{0.05}=\frac{0.2}{0.02}+\frac{0.1+0.03}{0.04}=2$ (×)

$(c)\ (0.01,0.02)$

$\frac{0.01}{0.01}+\frac{0.02+0.03}{0.05}=\frac{0.02}{0.02}+\frac{0.01+0.03}{0.04}=2$

$\Rightarrow 1+1\ =1+1=2$

$\Rightarrow 2\ =2\ =\mathbf{2}$ (✓)

$(d)\ (0.02,\ 0.01)$

$\frac{0.02}{0.01}+\frac{0.01+0.03}{0.05}=\frac{0.01}{0.02}+\frac{0.02+0.03}{0.04}=2$ (×)

Sol.9 (b) $\frac{xy}{y-x}=110,\ \frac{yz}{z-y}=132,\ \frac{zx}{z+x}=\frac{60}{11}$

For the option

$(a)\ (12,\ 11,\ 10)$

$\frac{12\times11}{11-12}=110\ \ \frac{11\times10}{10-11}=132,\ \frac{10\times12}{10+12}=\frac{60}{11}$ (×)

$(b)\ (10,11,\ 12)$

$\frac{10\times11}{11-10}=110,\ \frac{11\times12}{12-11}=132,\ \frac{10\times12}{10+12}=\frac{60}{11}$

$\Rightarrow 110=110,\ 132=132,\ \frac{120}{22}=\frac{60}{11}$

$\Rightarrow \frac{60}{11}=\frac{\mathbf{60}}{\mathbf{11}}$ (✓)

Check the other option in the same way, which don't satisfy the equation

Sol.10 (d) $3x-4y+70z=0,\ 2x+3y-10z=0,\ x+2y+3z=13$

For the option

$(a)\ (1,3,7)$

$3\times1-4\times3+70\times7=0,\ 2\times1+3\times3-10\times7=0,\ 1+2\times3+3\times7=13$ (×)

$(b)\ (1,7,3)$

$3\times1-4\times7+70\times3=0,\ 2\times1+3\times7-10\times3=0,\ 1+2\times7+3\times3=13$ (×)

$(c)\ (2,4,3)$

$3\times2-4\times4+70\times3=0,\ 2\times2+3\times4-10\times3=0,\ 2+2\times4+3\times3=13$ (×)

$(d)\ (-10,10,1)$

$3\times(-10)-4\times10+70\times1=0,\ 2\times(-10)+3\times10-10\times1=0,-10+2\times10+3\times1=13$

$\Rightarrow 0=0,\ 0=0,\ 13=\mathbf{13}$ (✓)

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Equations
Exercise: 2E

Sol.1 (b) Find the ratio of income given in option then the ratio for option

$(a)\ \frac{500}{400}=\frac{5}{4}=5:4$
$(b)\ \frac{400}{500}=\frac{4}{5}=\mathbf{4:5}$
$(c)\ \frac{300}{600}=\frac{1}{2}=1:2$
$(d)\ \frac{350}{550}=\frac{7}{11}=7:11$

Here only option (b) satisfies the condition

Sol.2 (a) Check the option

For the option

$(a)\ \frac{3+2}{8+2}=\frac{5}{10}=\frac{\mathbf{1}}{\mathbf{2}}$ (✓)

$\frac{3+12}{8+12} = \frac{15}{20} = \frac{3}{4}$

Check the other option in the same manner which don't satisfy the condition

Sol.3 (d) Let the present age of the person be y years, and the two sons are $x_1\ yrs\ \&\ x_2$ yrs. respectively

$\therefore y = 2\,(x_1 + x_2) \Rightarrow x_1 + x_2 = \frac{y}{2}$_____$(I)$

Five years ago

$y - 5 = 3\ [(x_1 - 5) + (x_2 - 5)]$

$\Rightarrow y - 5 = 3\,(x_1 + x_2 - 10)$

$\Rightarrow y - 5 = 3\ \left[\frac{y}{2} - 10\right]\quad [from\ (I)]$

$\Rightarrow 2y - 10 = 3\ (y - 20)$

$\Rightarrow 2y - 10 = 3y - 60\ \ \Rightarrow -10 + 60 = 3y - 2y$

$\Rightarrow \mathbf{50 = y}$

Sol.4 (c)

The option lies between 10 & 100.

(a) 54,

$54 + 9 = 63$ (×)

(b) $53 + 9 = 62$ (×)

(c) $45 + 9 = \mathbf{54}$ (✓)

(d) $55 + 9 = 64$ (×)

Sol.5 (b) Let the wages of each man and each boy be Rs. x & Rs. y respectively

Then the equation be

$8x + 6y = 33$_____(I)

& $4x = 5y + 4.5$

$\Rightarrow 4x - 5y = 4.5$_____$(II)$

Now,

Check the option

(a) $8 \times 1.5 + 6 \times 3 = 33\ \ \Rightarrow 12 + 18 = 33$ (×)

(b) $8 \times 3 + 6 \times 1.5 = 33\ \ \Rightarrow 24 + 9 = 33\ \Rightarrow$ $33 = 33$

Also $4 \times 3 - 5 \times 1.5 = 4.5\ \ \Rightarrow 12 - 7.5 = 4.5$

$\Rightarrow 4.5 = \mathbf{4.5}$ (✓)

In a similar way, check other options which don't satisfy the equation

Sol.6 (c) Let the ten's place digit be x & unit place digit be y

$\therefore Number = 10x + y$

$\because 10x + y = 4\,(x + y) \Rightarrow 6x - 3y = 0$

$\Rightarrow 2x - y = 0$_____(I)

Also $10x + y + 27 = 10y + x$

$\Rightarrow 9x - 9y = -27$

$\Rightarrow x - y = -3$_____(II)

Now, Check the option,

For the option

(a) 63, $2 \times 6 - 3 = 0$ (×)

(b) 35, $2 \times 3 - 5 = 0$ (×)

(c) 36, $2 \times 3 - 6 = 0 \Rightarrow 0 = 0$

Also, $3 - 6 = -3\ \Rightarrow -3 = \mathbf{-3}$ (✓)

(d) 60, $2 \times 6 - 0 = 0$ (×)

Sol.7 (a) Let the greater number *be* x & smaller be $y\ i.e\ (x > y)$

$\therefore\ \frac{1}{5}\,x = \frac{1}{3}\,y$_____$(I)$

Also $x + y = 16$_____(II)

Check the option carefully. The option only satisfy the condition

Checking of the option of Q-7

(a) (6,10), $\frac{1}{5} \times 10 = \frac{1}{3} \times 6\ \ \Rightarrow 2 = 2$

Also $6 + 10 = \mathbf{16}$ (✓)

(b) (9,7), $\frac{1}{5} \times 9 = \frac{1}{3} \times 7$ (×)

(c) (12,4), $\frac{1}{5} \times 12 = \frac{1}{3} \times 4$ (×)

(d) (11,5), $\frac{1}{5} \times 11 = \frac{1}{3} \times 5$ (×)

Sol.8 (a) $y - x = 7$____(I)

Also $x - 15 = \frac{3}{4}(y - 15)$

$\Rightarrow 4x - 3y = 15$____(II)

Check out the option,

For the option

$(a)\ (x = 36, y = 43),\ 43 - 36 = 7\ \Rightarrow 7 = 7$

Also $4 \times 36 - 3 \times 43 = 15\ \Rightarrow 144 - 129 = 15\ \Rightarrow 15 = \mathbf{15}$ (✓)

In a similar way. Check the other options which don't satisfy the equation

Sol.9 (c) Let the digit at 100th, 10th and unit places be x, y & z respectively

$\therefore Number = 100x + 10y + z$

∴ the equation is

$x + y + z = 12$____(I)

$100z + 10y + x = 100x + 10y + z + 495$

$99z - 99x = 495$

$\Rightarrow z - x = 5$____(II)

$100x + 10z + y = 100x + 10y + z + 36$

$\Rightarrow 9(z - y) = 36$

$\Rightarrow z - y = 4$____(III)

Now, Check out the option,

For the option

$(a)\ 327, 3 + 2 + 7 = 12 \Rightarrow 12 = 12$

$7 - 3 = 5\ \Rightarrow 4 = 5$ (×)

$(b)\ 372,\ 3 + 7 + 2 = 12\ \Rightarrow 12 = 12$

$2 - 3 = 5\ \Rightarrow -1 = 5$ (×)

$(c)\ 237,\ 2 + 3 + 7 = 12\ \Rightarrow 12 = 12$

$7 - 2 = 5\ \Rightarrow 5 = 5$

And $7 - 3 = 4\ \Rightarrow 4 = \mathbf{4}$ (✓)

$(d)\ 273,\ 2 + 7 + 3 = 12\ \Rightarrow 12 = 12$

$3 - 2 = 5\ \Rightarrow 1 = 5$ (×)

Sol.10 (a) Let the greater number be x & smaller be y

∴ Equation is

$2x - 2y = 18\ \Rightarrow x - y = 9$____$(I)$

$Also\ \frac{1}{3}y + \frac{1}{5}x = 21$____$(II)$

Now, check the options

$(a)\ (36, 45),\ 45 - 36 = 9\ \Rightarrow 9 = 9$

$\&\ \frac{1}{3} \times 36 + \frac{1}{5} \times 45 = \mathbf{21}$ (✓)

$\Rightarrow 12 + 9 = 21\ \Rightarrow 21 = \mathbf{21}$

In a similar way, check out the other option which doesn't satisfy the equation

Sol.11 (a) $4q + 7p = 17$____(I)

$and\ P = \frac{q}{3} + \frac{7}{4}$____$(II)$

Check the option,

For the option

$(a)\ (p, q) = \left(2, \frac{3}{4}\right), 4 \times \frac{3}{4} + 7 \times 2 = 17\ \Rightarrow 17 = 17$

$\&\ 2 = \frac{3/4}{3} + \frac{7}{4}\ \Rightarrow 2 = \mathbf{2}$ (✓)

$(b)\ \left(3, \frac{1}{2}\right),\ 4 \times \frac{1}{2} + 7 \times 3 = 17\ \Rightarrow 23 = 17$ (×)

$(c)\ \left(5, \frac{3}{5}\right), 4 \times \frac{3}{5} + 7 \times 5 = 17$ (×)

****

Equations
Exercise: 2F

Sol.1 (d) $2x^2 + 8x - m^3 = 0$

Here $\alpha = \beta$

$\alpha + \beta = \frac{-b}{a} \Rightarrow 2\alpha = \frac{-8}{2} \Rightarrow \alpha = -2$

$\alpha\beta = \frac{c}{a}\ \Rightarrow\ \alpha^2 = \frac{-m^3}{2}\ \Rightarrow (-2)^2 = \frac{-m^3}{2}$

$\Rightarrow 8 = -m^3\ \Rightarrow m^3 = -8$

$\Rightarrow m^3 = (-2)^3\ \Rightarrow m = \mathbf{-2}$

Sol.2 (d) $2^{2x+3} - 3^2 . 2^x + 1 = 0$

$\Rightarrow 2^{2x} . 2^3 - 3^2 . 2^x + 1 = 0 \Rightarrow 8\,(2^x)^2 - 9 \times 2^x + 1 = 0$

$Let\ 2^x = y$

∴ Equation we have

$8\,y^2 - 9y + 1 = 0$

$\Rightarrow 8\,y^2 - 8y - y + 1 = 0$

$\Rightarrow 8y\,(y-1) - 1\,(y-1) = 0 \ \Rightarrow (8y-1)\,(y-1) = 0$

$\Rightarrow 8y - 1 = 0 \ or \ y - 1 = 0$

$\Rightarrow y = \frac{1}{8} \ or \ y = 1$

$\Rightarrow 2^x = 2^{-3} \ or \ 2^x = 2^0$

$\Rightarrow x = -3 \ or \ x = \mathbf{0}$

Sol.3 (b) $Let\ x = 4 + \frac{1}{4+\frac{1}{4+\frac{1}{4+---\infty}}}$

$\Rightarrow x = 4 + \frac{1}{x}$

$\Rightarrow x^2 - 4x - 1 = 0$

$\Rightarrow x = \frac{-b \pm \sqrt{D}}{2a} = \frac{4 \pm \sqrt{16+4}}{2\times1} \qquad [\because D = b^2 - 4ac]$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

But x can't be-ve

$\therefore \boldsymbol{x = 2 + \sqrt{5}}$

Sol.4 (b) $2x^2 - 4x - 3 = 0$

$\therefore \alpha + \beta = \frac{-(-4)}{2} = 2$

$\alpha\beta = \frac{-3}{2}$

$\therefore \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 2^2 - 2 \times \left(\frac{-3}{2}\right)$

$= 4 + 3 = \mathbf{7}$

Sol.5 (a) $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

Here $\alpha + \beta = \left(\frac{1}{\alpha}\right)^2 + \left(\frac{1}{\beta}\right)^2 = \frac{\beta^2+\alpha^2}{\alpha^2\,\beta^2}$

$\Rightarrow (\alpha+\beta) = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\Rightarrow \frac{-b}{a} = \frac{\left(\frac{-b}{a}\right)^2 - 2\times\frac{c}{a}}{\left(\frac{c}{a}\right)^2}$

$\Rightarrow \frac{-b}{a} = \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2}$

$\Rightarrow -b\,c^2 = ab^2 - 2\,a^2c$

$\Rightarrow 2a^2c = ab^2 + b\,c^2$

$\Rightarrow 2 = \frac{ab^2 + b\,c^2}{a^2c}$

$\Rightarrow 2 = \frac{b^2}{ac} + \frac{bc}{a^2}$

$\Rightarrow \frac{b^2}{ac} + \frac{bc}{a^2} = \mathbf{2}$

Sol.6 (c) $x^2 - (P+4)\,x + 2P + 5 = 0$

$Here\ \alpha = \beta$

$\alpha + \beta = \frac{-b}{a} \Rightarrow 2\alpha = P + 4$

$\Rightarrow \alpha = \frac{P+4}{2}$ __(I)

$Also\ \alpha\beta = \frac{c}{a} \Rightarrow \alpha^2 = 2P + 5$

$\Rightarrow \left(\frac{P+4}{2}\right)^2 = 2P + 5$

$\Rightarrow P^2 + 8P + 16 = 8P + 20$

$\Rightarrow P^2 = 4 \ \Rightarrow P = \pm\mathbf{2}$

Sol.7 (d) $x^2 + (2P-1)\,x + P^2 = 0$

$\therefore Roots\ are\ real\ \therefore D \geq 0 \quad [\because D = b^2 - 4ac]$

$\Rightarrow (2P-1)^2 - 4 \times 1 \times P^2 \geq 0$

$\Rightarrow 4\,P^2 - 4P + 1 - 4P^2 \geq 0$

$\Rightarrow -4P + 1 \geq 0 \ \Rightarrow -4P \geq -1$

$\Rightarrow P \leq \frac{-1}{-4} \ \Rightarrow \boldsymbol{P \leq \frac{1}{4}}$

Sol.8 (b) $2x^2 + 5\,x - m = 0$

$\because x = m\ is\ the\ solution$

$\therefore 2\,m^2 + 5m - m = 0$

$\Rightarrow 2\,m^2 + 4m = 0 \ \Rightarrow 2m\,(m+2) = 0$

$\Rightarrow m = 0 \ or \ m + 2 = 0$

$\Rightarrow m = 0 \ or \ m = -\mathbf{2}$

Sol.9 (b) $x^2 + 2x + 1 = 0$

$\because P \,\&\, q$ *are roots*

$\therefore P + q = -2$

$\& \; Pq = 1$

$\therefore\; P^3 + q^3 = (P + q)^3 - 3Pq\,(P + q)$

$= (-2)^3 - 3 \times 1\,(-2)$

$= -8 + 6 = -2$

Sol.10 (b) $\because$ L + M + N = 0____(I)

$(M + N - L)\,x^2 + (N + L - M)\,x + (L + M - N) = 0$

$\Rightarrow\; -2\,L\,x^2 - 2Mx - 2N = 0\;\; [From\ (I)]$

$\Rightarrow L\,x^2 + M\,x + N = 0$

$\therefore D = \;b^2 - 4ac = M^2 - 4\,NL$

$= (-L - N)^2 - 4NL \quad [from\ (I)]$

$\Rightarrow\; (L + N)^2 - 4\,NL$

$= (L - N)^2\; \boldsymbol{\geq 0}$

$\therefore$ **Roots are real and rational**

Sol.11 (a) & (d) $x^2 = x + 1 \;\Rightarrow x^2 - x - 1 = 0$

$\therefore\; \alpha + \beta = 1,\; \alpha\beta = -1$

$\therefore\; \frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha} = \frac{\alpha^3 - \beta^3}{\alpha\beta}$

$= \frac{(\alpha-\beta)^3 + 3\alpha\beta\,(\alpha-\beta)}{\alpha\beta}$

$[\because\; \alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 + 4\alpha\beta}\;\; = \pm\sqrt{1+4} = \pm\sqrt{5}]$

$= \frac{(\pm\sqrt{5})^3 + 3\,(-1)\,(\pm\sqrt{5})}{-1}$

$= \frac{\pm\sqrt{5}\,(5-3)}{-1} = \frac{\pm\sqrt{5}\times 2}{-1}$

$= \boldsymbol{\pm 2\sqrt{5}}$

Sol.12 (c) $P \neq q,\; P^2 = 5\,P - 3, q^2 = 5q - 3$

$\therefore\; P^2 - q^2 = 5\,(P - q) \Rightarrow (P + q)(p - q) = 5(p - q) \Rightarrow p + q = 5$____$(I)$

Also

$\therefore\; (P + q)^2 = 5^2 \;\Rightarrow P^2 + q^2 + 2pq = 25 \;\Rightarrow 5p - 3 + 5q - 3 + 2pq = 25$

$\Rightarrow 5\,(p + q) - 6 + 2\,pq = 25$

$\Rightarrow 5 \times 5 - 6 + 2pq = 25$

$\Rightarrow pq = \frac{6}{2} = \boldsymbol{3}$

For required equation
Sum of roots $= \frac{P}{q} + \frac{q}{P}$

$= \frac{P^2 + q^2}{Pq}$

$= \frac{5P - 3 + 5q - 3}{Pq}$

$= \frac{5\,(P+q) - 6}{Pq}$

$= \frac{5\times 5 - 6}{3}$

$= \frac{19}{3}$

Product of roots $= \frac{P}{q} \times \frac{q}{P} = 1$

$\therefore$ The equation we have.

$x^2 -\; (Sum\ of\ the\ roots)\; x + Product\ of\ the\ roots = 0$

$\Rightarrow\; x^2 - \frac{19}{3}x + 1 = 0$

$\Rightarrow\; 3\,x^2 - 19x + 3 = 0$

Sol.13 (b) $5\,x^2 + 13x + P = 0$

Here $\alpha = \frac{1}{\beta} \Rightarrow \alpha\beta = 1$

$\Rightarrow\; \frac{P}{5} = 1 \;\Rightarrow \boldsymbol{P = 5}$

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Equations
Exercise: 2G

Sol.1 (b) $(a + b - 2c)\,x^2 + \;(2a - b - c)\,x + c + a - 2b = 0$

Check the option,

For the option

$(a)\; x = 1$

$a + b - 2c + 2a - b - c + c + a - 2b = 0$

$\Rightarrow 2\,(2a - b - c) = 0$ (×)

$(b)\ x = -1$

$a + b - 2c - 2a + b + c + c + a - 2b = 0$

$\Rightarrow 0 = \mathbf{0}$ (✓)

In similar way, other option don't satisfy the equation

Sol.2 (d)

$x^2 - 8x + m = 0 \ \therefore \alpha + \beta = 8 \ \&\ \alpha\beta = m$

$\alpha = \beta + 4 \ \Rightarrow \alpha - \beta = 4$

$\Rightarrow (\alpha - \beta)^2 = 4^2 \ \Rightarrow (\alpha + \beta)^2 - 4\,\alpha\beta = 16$

$\Rightarrow 8^2 - 4m = 16 \ \Rightarrow 64 - 16 = 4m$

$\Rightarrow m = \frac{48}{4} = \mathbf{12}$

Sol.3 (a) $7\,(x + 2P)^2 + 5P^2 = 35\,xP + 117\,P^2$

$\Rightarrow 7\,(x^2 + 4Px + 4\,P^2) + 5\,P^2 - 35\,Px - 117P^2 = 0$

$\Rightarrow 7\,x^2 - 7\,Px - 84\,P^2 = 0$

$\Rightarrow x^2 - Px - 12\,P^2 = 0 \ \Rightarrow x^2 - 4Px + 3Px - 12P^2 = 0$

$\Rightarrow x\,(x - 4P) + 3P\,(x - 4P) = 0$

$\Rightarrow (x + 3P)\,(x - 4P) = 0 \Rightarrow x + 3P = 0 \ or\ x - 4\,P = 0$

$\Rightarrow x = -3P \ \ \mathbf{or}\ \ \boldsymbol{x = 4P}$

Sol.4 (d) $\frac{6\,x}{x+1} + \frac{6\,(x+1)}{x} = 13$

$\Rightarrow 6\,x^2 + 6\,(x + 1)^2 = 13\,x\,(x + 1)$

$\Rightarrow 6\,x^2 + 6\,x^2 + 12x + 6 - 13x^2 - 13x = 0$

$\Rightarrow -x^2 - x + 6 = 0 \ \Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x + 3)\,(x - 2) = 0 \Rightarrow x + 3 = 0 \ or\ x - 2 = 0$

$\Rightarrow x = -3 \ or\ x = \mathbf{2}$

Sol.5 (b) $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$

$\Rightarrow \frac{1}{x+p+q} - \frac{1}{x} = \frac{1}{p} + \frac{1}{q}$

$\Rightarrow \frac{x-x-p-q}{(x+p+q)\,x} = \frac{q+p}{pq}$

$\Rightarrow -(p + q)\,pq = (q + p)\,x\,(x + p + q)$

$\Rightarrow x^2 + (p + q)\,x + pq = 0$

$\Rightarrow x^2 + px + qx + pq = 0 \Rightarrow x\,(x + p) + q(x+p) = 0$

$\Rightarrow (x + p)\,(x + q) = 0$

$\Rightarrow x + p = 0 \ or\ x + q = 0$

$\Rightarrow x = -p \ or\ x = -\boldsymbol{q}$

Sol.6 (b) $x^2 + 9x + 18 = 6 - 4x$

$\Rightarrow x^2 + 13x + 12 = 0 \ \Rightarrow x^2 + 12x + x + 12 = 0 \ \Rightarrow \ (x + 12)\ (x + 1) = 0$

$\Rightarrow x + 12 = 0 \ or\ x + 1 = 0$

$\Rightarrow x = -12 \ or\ x = -1$

Sol.7 (a) $\sqrt{2\,x^2 + 5x - 2} - \sqrt{2\,x^2 + 5x - 9} = 1$

$\Rightarrow \sqrt{2\,x^2 + 5x - 2} = 1 + \sqrt{2\,x^2 + 5x - 9}$

Squaring both sides

$2\,x^2 + 5x - 2 = 1 + 2\,x^2 + 5x - 9 + 2\sqrt{2\,x^2 + 5x - 9}$

$\Rightarrow 6 = 2\sqrt{2\,x^2 + 5x - 9}$

$\Rightarrow 3 = \sqrt{2\,x^2 + 5x - 9}$

Again squaring both sides

$9 = 2\,x^2 + 5x - 9 \ \Rightarrow 2\,x^2 + 5x - 18 = 0$

$\Rightarrow 2\,x^2 + 9x - 4x - 18 = 0$
$\Rightarrow x\,(2x + 9) - 2\,(2x + 9) = 0$
$\Rightarrow (x - 2)\,(2x + 9) = 0 \Rightarrow x - 2 = 0 \ or\ 2x + 9 = 0$
$\Rightarrow x = 2 \ or\ x = {}^{-\mathbf{9}}/_{\mathbf{2}}$

Sol.8 (c) $3\,x^2 - 17x + 24 = 0 \ \Rightarrow 3\,x^2 - 9x - 8x + 24 = 0$

$\Rightarrow 3x\,(x - 3) - 8\,(x - 3) = 0$

$\Rightarrow (3x - 8)\,(x - 3) = 0 \Rightarrow 3x - 8 = 0 \ or\ x - 3 = 0$

$\Rightarrow x = {}^{8}/_{3} \ or\ x = 3 \ \Rightarrow x = 2\tfrac{2}{3} \ or\ \boldsymbol{x = 3}$

Sol.9 (c) $\frac{3\,(3\,x^2+15)}{6} + 2\,x^2 + 9 = \frac{2\,x^2+96}{7} + 6$

$\Rightarrow \frac{3\,x^2+15+4\,x^2+18}{2} = \frac{2\,x^2+96+42}{7}$

$\Rightarrow (7\,x^2 + 33) \times 7 = (2\,x^2 + 138) \times 2$

$\Rightarrow 49\,x^2 + 231 = 4\,x^2 + 276$

$\Rightarrow 45\,x^2 = 45 \Rightarrow x^2 = 1$

$\Rightarrow x = \pm\mathbf{1}$

Sol.10 (a) $\left(\frac{l-m}{2}\right) x^2 - \left(\frac{l+m}{2}\right) x + m = 0$

$\Rightarrow (l-m)\,x^2 - (l+m)\,x + 2m = 0$

$D = b^2 - 4ac = (l+m)^2 - 4\,(l-m) \times 2m$

$= l^2 + m^2 + 2\,lm - 8\,lm + 8\,m^2$

$= l^2 + 9\,m^2 - 6\,lm$

$= (l - 3\,m)^2$

$\therefore x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{l+m\pm(l-3\,m)}{2\,(l-m)}$

$= \frac{l+m+l-3m}{2\,(l-m)}$ or $\frac{l+m-l+3\,m}{2\,(l-m)}$

$= \frac{2\,(l-m)}{2\,(l-m)}$ or $\frac{4m}{2\,(l-m)}$

$= \mathbf{1}$ ***or*** $\frac{\mathbf{2\,m}}{\mathbf{l-m}}$

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Equations
Exercise: 2H

Sol.1 (c) Let one number be x & other be $= 8 - x$

$\therefore x^2 + (8-x)^2 = 34$

$\Rightarrow x^2 + 64 - 16x + x^2 = 34$

$\Rightarrow 2\,x^2 - 16\,x + 30 = 0$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow (x-5)\,(x-3) = 0 \Rightarrow x = 5$ *or* $x = 3$

Or

Check the option

For the option

$(a)\ (7, 10),\ 7 + 10 = 8 \ \Rightarrow 17 = 8$ (×)

$(b)\ (4, 4),\ 4 + 4 = 8 \ \Rightarrow 8 = 8$

Also $4^2 + 4^2 = 34 \ \Rightarrow 16 + 16 = 34 \ \Rightarrow 32 = 34$ (×)

$(c)\ (3, 5),\ 3 + 5 = 8 \ \Rightarrow 8 = 8$

Also $3^2 + 5^2 = 34 \ \Rightarrow 9 + 25 = 34 \ \Rightarrow \mathbf{34 = 34}$ (✓)

$(d)\ (2, 6),\ 2 + 6 = 8 \ \Rightarrow 8 = 8$

$2^2 + 6^2 = 34 \ \Rightarrow 40 = 34$ (×)

Sol.2 (b) $x^2 + (x+3)^2 = 89$

Check, the option

For the option

$(a)\ (7, 4),\ x = 4$ *(as smaller is x)*

$4^2 + (4+3)^2 = 89 \ \Rightarrow 16 + 49 = 89$ (×)

$(b)\ (5, 8),\ x = 5$

$\therefore 5^2 + (5+3)^2 = 89 \ \Rightarrow 25 + 64 = 89 \ \Rightarrow 89 = \mathbf{89}$ (✓)

In a similar way, check out the other options which don't satisfy the equation

Sol.3 (a) Let the number be x

$\therefore 5x = 2\,x^2 - 3$

$\Rightarrow 2\,x^2 - 5x - 3 = 0$

For the option

$(a)\ x = 3,\ 2 \times 3^2 - 5 \times 3 - 3 = 0 \ \Rightarrow 18 - 15 - 3 = 0 \ \Rightarrow 0 = \mathbf{0}$ (✓)

In a similar way, check out the other options which don't satisfy the equation

Sol.4 (b) $l = x,\ 2(l + b) = 180 \ \Rightarrow b = 90 - x$

$A = lb \ \Rightarrow x\,(90 - x) = 2000$

$\Rightarrow x^2 - 90x + 2000 = 0$

∴ For the option

$(a)\ (205m, 80m)\ \therefore x = 205$

$\therefore (205)^2 - 90 \times 205 + 2000 = 0$ (×)

$(b)\ (50m, 40m), x = 50$

∴ $50^2 - 90 \times 50 + 2000 = 0$

$\Rightarrow 2500 - 4500 + 2000 = 0$

$\Rightarrow 0 = \mathbf{0}$ (✓)

In a similar way, check out the other options which don't satisfy the equation

Sol.5(c) $P^2 + (P+5)^2 = 625,$

$Difference\ in\ sides = \{(P+5) - P\}cm = 5cm$

Check out the option

In option (C) has diff. is sides =5cm

Also $15^2 + 20^2 = 625 \Rightarrow 225 + 400 = 625$

$\Rightarrow 625 = \mathbf{625}$ (✓)

Sol.6 (d) Let the two parts be x & y

$\therefore x + y = 50$ ____(I)

Also $\frac{1}{x} + \frac{1}{y} = {}^{1}/_{12}$ ___(II)

∴ For the option

(a) $(24, 26),\ 24 + 26 = 50 \ \Rightarrow 50 = 50$

Also $\frac{1}{24} + \frac{1}{26} = \frac{1}{12}$ (×)

(b) $(28, 22),\ 28 + 22 = 50 \ \Rightarrow 50 = 50$

Also $\frac{1}{28} + \frac{1}{22} = \frac{1}{12}$

$\Rightarrow \frac{11+14}{2\times14\times11} = \frac{1}{12} \Rightarrow \frac{25}{2\times14\times11} = \frac{1}{12}$ (×)

(c) $(27, 23),\ 27 + 23 = 50 \ \Rightarrow 50 = 50$

Also $\frac{1}{27} + \frac{1}{23} = \frac{1}{12}$ (×)

(d) $(20, 30),\ 20 + 30 = 50$

Also $\frac{1}{20} + \frac{1}{30} = \frac{1}{12} \Rightarrow \frac{3+2}{60} = \frac{1}{12}$

$\Rightarrow \frac{5}{60} = \frac{1}{12} \Rightarrow \frac{1}{12} = \mathbf{\frac{1}{12}}$ (✓)

Sol.7 (a)

Let the two consecutive nos. be x & $x + 1$

$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{240}$

Check out the option

For the option

(a) $(15, 16),\ \therefore x = 15$

$\frac{1}{15} - \frac{1}{16} = \frac{1}{240} \Rightarrow \frac{16-15}{240} = \frac{1}{240} \Rightarrow \frac{1}{240} = \mathbf{\frac{1}{240}}$ (✓)

In a similar way, the other options which don't satisfy the equation

Sol.8 (b) Let the sides are $x\ cm$ & $(x + 4)cm$

$\therefore\ x^2 + (x+4)^2 = 20^2$

∴ For the option

(a) $(11cm, 15cm)\ \therefore x = 11$

$\therefore\ 11^2 + 15^2 = 20^2 \ \Rightarrow 121 + 225 = 400$ (×)

(b) $(12cm, 16cm),\ x = 12$

$12^2 + 16^2 = 20^2 \ \Rightarrow 144 + 256 = 400 \Rightarrow 400 = \mathbf{400}$ (✓)

In a similar way, check out the other options which don't satisfy the equation

Sol.9 (c) Let the numbers be x & y

$x + y = 45$___(I)

Also $\sqrt{xy} = 18$

$\Rightarrow xy = 324$ ____(II)

∴ For the option

(a) $(15, 30),\ 15 + 30 = 45 \ \Rightarrow 45 = 45$

$15 \times 30 = 324$ (×)

(b) $(32, 13),\ 32 + 13 = 45 \Rightarrow 45 = 45$

$32 \times 13 = 324$ (×)

(c) $(36, 9),\ 36 + 9 = 45 \ \Rightarrow 45 = 45$

$36 \times 9 = 324 \ \Rightarrow 324 = \mathbf{324}$ (✓)

(d) $(25, 20),\ 25 + 20 = 45$

$25 \times 20 = 324$ (×)

Sol.10 (a) Let the sides of an equilateral triangle be x units

∴ The sides of a right triangle are

$x - 12,\ x - 13,\ x - 14$

$\therefore (x-12)^2 = (x-13)^2 + (x-14)^4$

Now, for the option

$(a)\ x = 17$

$(17-12)^2 = (17-13)^2 + (17-14)^2$

$\Rightarrow 5^2 = 4^2 + 3^2$

$\Rightarrow 25 = \mathbf{25}$ (✓)

In a similar way, check out the other options which don't satisfy the equation

Sol.11 (a) $-2000\,P^2 + 2000P + 17000 = 5000$

∴ For the option

$(a)\ p = 3, -2000 \times 3^2 + 2000 \times 3 + 17000 = 5000 \Rightarrow 5000 = \mathbf{5000}$ (✓)

$(b)\ p = 5, -2000 \times 5^2 + 2000 \times 5 + 17000 = 5000$

$\Rightarrow -23000 = 5000$ (×)

$(c)\ p = 2$

$\therefore -2000 \times 2^2 + 2000 \times 2 + 17000 = 5000$

$\Rightarrow 13000 = 5000$ (×)

Sol.12 (c) Check out the option

For the option

$(a)\ (3\sqrt{2}, 2\sqrt{3}),\ 3\sqrt{2} \times (3\sqrt{2} + 2\sqrt{3}) = 70$ (×)

$(b)\ (5\sqrt{2},\ 3\sqrt{5}),\ 5\sqrt{2}\,(5\sqrt{2} + 3\sqrt{5}) = 70$ (×)

$(c)\ (2\sqrt{2}, 5\sqrt{2}),\ 5\sqrt{2}\,(2\sqrt{2} + 5\sqrt{2}) = 70$

$\Rightarrow 5\sqrt{2} \times 7\sqrt{2} = 70 \Rightarrow 70 = 70$

Also $2\sqrt{2}\,(5\sqrt{2} - 2\sqrt{2}) = 12$

$\Rightarrow 2\sqrt{2} \times 3\sqrt{2} = 12 \Rightarrow 12 = 12$ (✓)

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Equations
Exercise: 2I

Sol.1 (b) $x^3 - 6x^2 + 11x - 6 = 0$

Here $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{6}{1} = 6$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{11}{1} = 11$

$\alpha\beta\gamma = \frac{-d}{a} = \frac{6}{1} = 6$

For the option

a) (-1, 1, -2), $-1 + 1 + (-2) = 6 \Rightarrow -2 = 6$ (×)

b) (1, 2, 3), $1 + 2 + 3 = 6 \Rightarrow 6 = 6$

And $1\times2+2\times3+3\times1 = 11 \Rightarrow \mathbf{11 = 11}$ (✓)

Also $1\times2\times3 = \mathbf{6}$

c) (-2, 2, 3), $-2 + 2 + 3 = 6 \Rightarrow 3 = 6$ (×)

d) (0, 4, -5), $0 + 4 + (-5) = 6 \Rightarrow -1 = 6$ (×)

Sol.2 (b) $x^3 + 2x^2 - x - 2 = 0$

$\therefore \alpha + \beta + \gamma = -2$

$\alpha\beta + \beta\gamma + \gamma\alpha = -1$

$\alpha\beta\,\gamma = 2$

∴ For the option

a) (1, -1, 2), $1+(-1)+2 = -2 \Rightarrow 2 = -2$ (×)

b) (-1, 1, -2), $-1+1+(-2) = -2 \Rightarrow -2 = -2$

And $(-1) \times 1 + 1 \times (-2) + (-2) \times (-1) = -1$

$\Rightarrow -1 - 2 + 2 = -1 \Rightarrow -1 = \mathbf{-1}$ (✓)

Also $(-1) \times 1 \times (-2) = 2 \Rightarrow 2 = \mathbf{2}$

In similar way, check out the option which don't satisfy the equation.

Sol.3 (b) Zeros of the equation are 0,4, -5 as x, $x - 4$, & $x+5$ are the factors LHS of the equation

$\therefore \alpha + \beta + \gamma = 0+4-5 = -1$

$\alpha\beta + \beta\gamma + \gamma\alpha = 0-20+0 = -20$

$\alpha\beta\,\gamma = 0$

∴ Required equation is

$x^3 - (\alpha + \beta + \gamma)\,x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\,x - \alpha\beta\,\gamma = 0$

$\Rightarrow x^3 + x^2 - 20x - 0 = 0$

$\Rightarrow \mathbf{x^3 + x^2 - 20x = 0}$

Sol.4 (b) $3x^3 + 5x^2 - 3x - 5 = 0$

$\alpha + \beta + \gamma = {}^{-5}/_{3}, \alpha\beta + \beta\gamma + \gamma\alpha = \frac{-3}{3} = -1$

$\alpha\beta\gamma = \frac{5}{3}$

For the option

a) $x - 1, x - 2, x - {}^{5}/_{3} \therefore \alpha = 1, \beta = 2, \gamma = {}^{5}/_{3}$

$\therefore \alpha + \beta + \gamma = {}^{-5}/_{3} \Rightarrow 1 + 2 + {}^{5}/_{3} = {}^{-5}/_{3}$ (×)

b) $x - 1, x + 1, 3x + 5 \therefore \alpha = 1, \beta = -1, \gamma = {}^{-5}/_{3}$

$\therefore \alpha + \beta + \gamma = {}^{-5}/_{3} \Rightarrow 1 + (-1) + \left({}^{-5}/_{3}\right) = {}^{-5}/_{3}$

$\Rightarrow {}^{-5}/_{3} = {}^{-5}/_{3}$

And $\alpha\beta + \beta\gamma + \gamma\alpha = -1 \Rightarrow -1 + {}^{5}/_{3} - {}^{5}/_{3} = -1 \Rightarrow -1 = -1$

Also $\alpha\beta\gamma = \frac{5}{3} \Rightarrow 1 \times (-1)\left({}^{-5}/_{3}\right) = {}^{5}/_{3}$

$\Rightarrow \frac{5}{3} = {}^{5}/_{3}$ (✓)

In a similar way, check out the options which don't satisfy the equation.

Sol.5 (b) $x^3 + 7x^2 - 21x - 27 = 0$

$\therefore \alpha + \beta + \gamma = -7, \ \alpha\beta + \beta\gamma + \gamma\alpha = -21$

$A\beta\gamma = 27$

$\therefore$ For the option

a) (-3, -9, -1), $-3 + (-9) + (-1) = -7 \Rightarrow -13 = -7$(×)

b) (3, -9, -1), $3 + (-9) + (-1) = -7 \Rightarrow -7 = -7$

And $3 \times (-9) + (-9) \times (-1) + (-1) \times 3 = -21$

$\Rightarrow -27 + 9 - 3 = -21 \Rightarrow -21$

Also $3 \times (-9) \times (-1) = 27 \Rightarrow 27 = \mathbf{27}$ (✓)

In a similar way, check out the other options which don't satisfy the equation.

Sol.6 (a) $x^3 + x^2 - x - 1 = 0$

$\Rightarrow x^2(x+1) - 1(x+1) = 0 \Rightarrow (x^2 - 1)(x+1) = 0$

$\Rightarrow (x+1)(x-1)(x+1) = 0$

$\Rightarrow x+1 = 0, x-1 = 0, x+1 = 0$

$\Rightarrow x = -1, x = 1, x = \mathbf{-1}$

Sol. 7(d) $x^3 + x^2 - 20x = 0$

$\Rightarrow x(x^2 + x - 20) = 0 \Rightarrow x(x+5)(x-4) = 0$

$\Rightarrow x = 0, x+5 = 0, x-4 = 0$

$\Rightarrow x = 0, x = -5, x = \mathbf{4}$

Sol.8 (b) $x^3 + 6x^2 + 9x + 4 = 0$

Here $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-6}{1} = -6$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{9}{1} = 9$

$\alpha\beta\gamma = \frac{-d}{a} = -\frac{4}{1} = -4$

For the option

a) (4, 1, -1), $4 + 1 + (-1) = -6 \Rightarrow 4 = 6$ (×)

b) (-4, -1, -1), $(-4) + (-1) + (-1) = -6 \Rightarrow -6 = -6$

And $(-4) \times (-1) + (-1) \times (-1) + (-4) \times (-1) = 9 \Rightarrow 9 = \mathbf{9}$ (✓)

Also $(-4) \times (-1) \times (-1) = \mathbf{-4}$

In a similar way, other options which don't satisfy the equation

Sol.9 (a) $4x^3 + 8x^2 - x - 2 = 0$

$\Rightarrow 4x^2(x+2) - 1(x+2) = 0$

$\Rightarrow (4x^2 - 1)(x+2) = 0 \Rightarrow (2x+1)(2x-1)(x+2) = 0$

$\therefore 2x + 1 = 0, 2x - 1 = 0$, or $x + 2 = 0$

$\Rightarrow x = {}^{-1}/_{2}\ x = {}^{1}/_{2}$ or $x = -2$

If $x = {}^{-1}/_{2}$ $\therefore 2x + 3 = -1 + 3 = 2$

If $x = {}^{1}/_{2}$ $\therefore 2x + 3 = 1 + 3 = 4$

If $x = -2$ $\therefore 2x + 3 = \mathbf{-4 + 3 = -1}$

Sol.10 (a) $2x^3 - x^2 - 4x + 2 = 0$

$\therefore$ For the option

a) $x = {}^{1}/_{2}, 2 \times \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 - 4 \times \left(\frac{1}{2}\right) + 2 = 0$

$\Rightarrow \frac{1}{4} - \frac{1}{4} - 2 + 2 = 0$

$\Rightarrow 0 \Rightarrow \mathbf{0}$ (✓)

b) $x = {}^{-1}/_{2}$, $2\left(\frac{-1}{2}\right)^3 - \left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{4}\right) + 2 = 0$

$\Rightarrow -\frac{1}{4} - \frac{1}{4} + 2 + 2 = 0$ (×)

In similar way, other option don't satisfy the equation

****

Time Value of Money
Exercise: 4A

Sol.1 (b) $S.I = Pit \ or \ \frac{PRT}{100}$ (∵ i = R%)

$= ₹3500 \times \frac{12}{100} \times 3$

$= ₹\,\mathbf{1260}$

Sol.2 (a) $S.I = \frac{PRT}{100} = \frac{5000\times15\times\frac{9}{2}}{100}$

$= \frac{5000\times9\times15}{200}$

$= ₹\,\mathbf{3375}$

Sol.3 (c) $S.I = \frac{PRT}{100} \Rightarrow R = \frac{S.I\times100}{PT}$

$= \frac{300\times100}{5000\times1}$

$\Rightarrow$ **R = 6% p.a.**

Sol.4 (d) *Amount = Principal + Simple Interest*

$\Rightarrow$ *Simple Interest = Amount – Principal*

$= ₹(7200 - 4500) = ₹2700$

Sol.5 (a) *Simple Interest = Amount – Principal*

$= ₹(16500 - 12000) = ₹4500$

$T = 2\frac{1}{2} = {}^{5}/_{2}$ years

$\therefore R = \frac{S.I\times100}{PT} = \frac{4500\times100\times2}{12000\times5} = \mathbf{15\%\ p.a.}$

Sol.6 (b) $S.I = \frac{PRT}{100}$

$\Rightarrow T = \frac{S.I\times100}{PR} = \frac{2500\times100\times2}{10000\times25}$

= **2 years**

Sol.7 (a) $T = \frac{SI\times100}{PR} = \frac{1700\times100\times2}{8500\times25}$

(∵ *Simple Interest = Amount – Principal* = ₹10200 – ₹8500 = ₹1700)

$T = \frac{8}{5}$ *years*

$= 1\frac{3}{5}$ *yrs* = ***1yr 7month*** **(approx)**

Sol.8 (c) $SI = ₹1200$

Rate = 18% *P.a*

$= \frac{18}{12}\%\ P.m$

$= \frac{3}{2}\%\ P.m$

$\Rightarrow R = \frac{3}{2}$

T = 1month

$\therefore P = \frac{S.I\times100}{RT} = \frac{1200\times100}{{}^{3}/_{2}\times1}$

$= \frac{1200\times100\times2}{3}$

$= ₹\,\mathbf{80000}$

Sol.9 (a)

Amount = Principal + Interest $= (P + \frac{PRT}{100})$

$\Rightarrow A = P\left(1 + \frac{RT}{100}\right)$

= Amount at end of 2 years= ₹6,200

= Amount at end of 3 years= ₹7,400

∴ Difference of amount of 2 year and 3 year is simple interest= ₹1200

∴ Simple interest for two years =1200× 2 = ₹2400

$\therefore$Principal = Amount at end of 2year - Simple interest of 2 years.

Principal=6,200-2,400=₹3,800

Rate of interest = $\frac{1,200}{3,800} \times 100 = \mathbf{31.57\%}$.

Sol.10 (c) $A = 2P,\ T = 10$ years

$\therefore S.I = A - P$

$\therefore S.I = 2P - P = P$

$\therefore R = \frac{S.I \times 100}{P \times T} = \frac{P \times 100}{P \times 10} = 10\ \%$

Now

$T = \frac{S.I \times 100}{PR}$

$= \frac{2P \times 100}{P \times 10}\ (\because I = 3P - P = 2P)$

= **20 years**

****

Time Value of Money
Exercise: 4B

Sol.1 (a) $A = P\,[(1+i)^n]$

$= 1{,}000\,[(1+0.05)^4]$

$= 1{,}000 \times ((1.05)^4)$

$= \textbf{₹ 1215.50}$

Sol.2 (d) $A = P\,(1+i)^n$

$= 100\,(1+0.05)^{20}$

$= 100\,(1.05)^{20}$

= **₹ 265.33**

Sol.3 (c)
$i = 3\%\ P.a = \frac{3}{2}\%\ Per\ six\ month =$
$0.015\ per\ six\ month$
$E = (1+i)^n - 1 = (1+0.015)^2 - 1 = 0.030225$

= 3.0225 % P.a.

Sol.4 (b) $Scrap\ value = cost\ of\ assets\ (1-i)^n$

$= 30{,}000 = 1{,}00{,}000\,(1-0.2)^n$

$\Rightarrow 30{,}000 = 1{,}00{,}000(0.8)^n$

$\Rightarrow \frac{30{,}000}{1{,}00{,}000} = (0.8)^n \Rightarrow 0.3 = (0.8)^n$

$\Rightarrow$ **n= 5.4(approx.)**

Sol.5 (a) $A = P\,[1+i]^n$

$\Rightarrow ₹\,1{,}000 = P\,(1+0.03)^4$

$(\because i = 6\%\ P.a = 3\%\ P.half\ annum\ n = 2 \times 2 = 4)$

$\Rightarrow\ 1{,}000 = P\,(1.03)^4$

$\Rightarrow P = \frac{1000}{(1.03)^4} = \textbf{₹ 888.48 (approx)}$

Sol.6 (c) Let the initial population= 100
Final population =140

A = P $(1+i)^n \Rightarrow$ 140= 100$(1+.02)^n$
= 1.4 = $(1.02)^n \Rightarrow n = \mathbf{17}\ \boldsymbol{years}.$

Sol.7 (d) $C.I - S.I = ₹\,110.16$

$\Rightarrow\ \mathrm{P} = \frac{d \times (100)^3}{r^2 \times (r+300)} \Rightarrow \frac{110.16 \times (100)^3}{36 \times (306)} =$
₹10,000 ***(approx.)***

Sol.8 (a) $Scrap\ value = Cost\ of\ assets\ (1-i)^n$

= Scrapped value = $10{,}000\,(1-0.1)^{10}$

$= 10000 \times (0.9)^{10}$

= **₹ 3,486.78 (approx.)**

Sol.9 (d) $e = \left(1+\frac{i}{m}\right)^{n \times m} - 1$

$= e = \left(1+\frac{i}{4}\right)^{1\times4} - 1 \Rightarrow \left(1+\frac{0.07}{4}\right)^4 - 1$

$= (1.0175)^4 - 1 = 0.07185903$

$= \mathbf{7.18\%}$ **(approx)**

Sol.10 (b)
Principal = 16,000 and Rate of interest = 10%.

$\because C.I = P\,[(1+i)^n - 1]$

$= 16{,}000\,[(1+0.05)^3 - 1]\left[\because i = \frac{10}{2}\%\ \&\ n = \frac{3}{2} \times 2\right]$

$= 16{,}000\,[(1.05)^3 - 1]$

$=$ ₹ 2,522

Sol.11 (c) $C.I = P\,[(1+i)^n - 1]$

$\left[\because i = \frac{10}{4}\% = 0.025\; n = 1 \times 4 = 4\right]$

$= 40{,}000[(1.025)^4 - 1]$

$=$ ₹ 4152.51 (approx.)

Sol.12 (d) $P = \frac{d\times(100)^2}{r^2}$

$= 2{,}400 = \frac{d\times(100)^2}{25} \Rightarrow 6.$

$=$ ₹ 6

Sol.13 (a)

Annual increment in the population

$= (39.4 - 19.4) = 20$ Per thousand $= 2\%$

Let the initial population = 100

Final population = 200

$A = P\,(1+i)^n$

$\Rightarrow 2 \times 100 = 100\left(1 + \frac{2}{100}\right)^n$

$\Rightarrow 2 = (1.02)^n \;\; \Rightarrow \boldsymbol{n = 35\ years\ (approx.)}$

Sol.14 (a) P = ₹ 4000

$n = \frac{6}{3} = 2$

($\because$ each quarter has three months)

$i = 12\%\,P.a = \frac{12}{4}\%P.Q. = 0.03$ Per quarter

$C.I = P\,[(1+i)^n - 1]$

$= 4{,}000\,[(1+0.03)^2 - 1]$

$= 4{,}000\,[(1.03)^2 - 1]$

$=$ ₹ 243.60

****

Time Value of Money
Exercise: 4C

Sol.1 (d) $P.V = R\,(\frac{1-(1+i)^{-n}}{i})$

$= P.V = 3{,}000(\frac{1-(1+.045)^{-15}}{0.045})$

$=$ P.V= ₹32,218. 64

Sol.2 (a) $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$= 150\left[\frac{(1+0.035)^{12}-1}{0.035}\right]$

$=$ ₹ 2190. 28 (approx)

Sol.3 (c) $P.V = R\,(\frac{1-(1+i)^{-n}}{i})$

$= 10{,}000 = R\,(\frac{1-(1+0.04)^{-30}}{0.04})$

$= 10{,}000 = R\,(17.292)$

$=$ R = ₹578.30

Sol.4 (d) $V = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{1{,}200}{0.08}\left[1 - \frac{1}{(1+0.08)^{12}}\right]$

$= \frac{1{,}200}{0.08}\left[\frac{(1.08)^{12}-1}{(1.08)^{12}}\right]$

$= \frac{1{,}20{,}000}{8}\left(\frac{2.518-1}{2.518}\right)$

$=$ ₹ 9043.29

Sol.5 (a) $FV = \frac{A}{i\,\{(1+i)^n - 1\}^{-1}}\;\;\left[\because FV = \frac{A\,[1+i)^n - 1]}{i}\right]$

$= \frac{100}{0.05\,\{(1.05)^{10} - 1\}^{-1}}$

$= \frac{100\times 20}{(0.628)^{-1}}$

$= 2{,}000 \times 0.628$

$=$ ₹ 1,258 (approx.)

Sol.6 (b) $\because A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$\Rightarrow 50{,}000 = R\left[\frac{(1.05)^{25}-1}{0.05}\right]$

$\Rightarrow 50{,}000 = R\,(47.727)$

$\Rightarrow \;\; \boldsymbol{R} = $ **₹ 1047. 62**

Sol.7 (b) $P.V = R\left[\frac{(1+i)^n - 1}{i}\right]$

$\Rightarrow 3{,}137.12 = 100\left[\frac{(1+0.045)^n - 1}{0.045}\right]$

$\Rightarrow (1.045)^n - 1 = \frac{3{,}137.12\times 0.045}{100}$

$\Rightarrow \;\; (1.045)^n - 1 = 1.4117$

$\Rightarrow (1.045)^n = 2.4117 \Rightarrow (1.045)^n = (1.045)^{20}$ (approx)

$\Rightarrow$ n =20 years

Sol.8 (a) $P.V = R\left(\frac{1-(1+i)^{-n}}{i}\right)$

$\Rightarrow$ 10,000= 1,000$\left(\frac{1-(1+0.05)^{-n}}{0.05}\right)$

$\Rightarrow$ $10\times 0.05 = 1-(1+0.05)^{-n}$

$\Rightarrow$ $0.5 = (1.05)^{-n}$

$\Rightarrow$ $0.5 = \frac{1}{(1.05)^n}$

$\Rightarrow$ $(1.05)^n = \frac{1}{0.5}$

$\Rightarrow$ $(1.05)^n = 2$

= n = 14.2 years

Sol.9 (b) $C.I. = P\,[(1+i)^n - 1]$

$= 5{,}120\,[(1+0.125)^3 - 1]$
= ₹ 2,170

Sol.10 (d) $P.V = R\left(\frac{1-(1+i)^{-n}}{i}\right)$

$\Rightarrow$ 20,000= 2,000$\left(\frac{1-(1+0.05)^{-n}}{0.05}\right)$

$\Rightarrow$ $10\times 0.05 = 1-(1.05)^{-n}$

$\Rightarrow$ $0.5 = (1.05)^{-n}$

$\Rightarrow$ $0.5 = \frac{1}{(1.05)^n} \Rightarrow (1.05)^n=2$

= n = 14.2 years

Sol.11 (a) R= 500 and r=10% (or i= 0.1)

F. V= R $\left(\frac{(1+i)^n-1}{i}\right)$

F.V= 500 $\left(\frac{(1.1)^{12}-1}{.1}\right)$

F.V = ₹10,692.14

SI of the next year

$SI = PRT$ = 10,692.14(0.1)(1) = ₹1,069.214

Amount after 1year after 12th instalment= 10,692.14 + 1069.214 = **₹11,761.36**

Sol.12 (d) $P.V = R\left(\frac{1-(1+i)^{-n}}{i}\right)$

P.V =5,000$\left(\frac{1-(1.04)^{-12}}{0.04}\right)$

P. V= 5,000(9.385074) $\Rightarrow$

P.V= **₹46,925.37**

Sol.13 (c) $V = \frac{a}{i} = \frac{300}{0.1}$

= ₹ 3,000

****

Time Value of Money
Exercise: 4D

Sol.1 (b) $A = P\left(1+\frac{R}{100}\right)^n = P(1+i)^n$

$\Rightarrow$ $5{,}200 = P\left(1+\frac{5}{100}\right)^6$

$\Rightarrow$ $5{,}200 = P[1.05]^6$

$\Rightarrow$ $5{,}200 = P \times 1.3401$ *(approx)*

$\Rightarrow$ $P = \frac{5200}{1.3401}$ = ₹ **3,880** ***(approx)***

Sol.2 (a) $C.I = P[(1+i)^n - 1]$

$C.I = 1{,}000[(1.05)^4 - 1]$

$C.I = 1{,}000\,[1.2155 - 1]$

C.I = ₹ **215.50**

Sol.3 (c) $C.I = P[(1+i)^n - 1]$

$\Rightarrow \cancel{P} = \cancel{P}\,[(1.05)^n - 1]$
$[\because C.I = A - P = 2P - P]$

$\Rightarrow 1 = (1.05)^n - 1 \Rightarrow (1.05)^n = 2$

= **14.2** ***(approx)***

Sol.4 (d) $A = P\,(1+i)^n$

= 10,000= P(1.04)^{18}

= 10,000 = P (2.0258)

P= ₹ 4, 936. 32

Sol.5 (a) $A = P\,(1+i)^n$

= A = 3P

$\Rightarrow 3P = P\,(1+0.08)^n \Rightarrow 3 = (1.08)^n$

= P = 14.28 years (approx.)

Sol.6 (a) **P. V=** $R\left(\frac{1-(1+i)^{-n}}{i}\right)$

$P.V = 80\left(\frac{1-(1.05)^{-20}}{0.05}\right)$

P. V= ₹ 997 (approx.)

Sol.7 (c) **P. V=** $R\left(\frac{1-(1+i)^{-n}}{i}\right)$

$P.V = 4{,}000\left(\frac{1-(1.05)^{-25}}{0.05}\right)$

P.V= ₹56,375.77

Total cash down payment =56,375.77+20,000=**₹76,375.77**

Sol.8 (a)

Balance Amount (V) = (3,00,000 − 2,00,000)

= ₹ 1,00,000

$= i = \frac{12}{2}\% = 0.06$

$= n = 20$

P.V= $R\left(\frac{1-(1+i)^{-n}}{i}\right)$

$= 1{,}00{,}000 = R\left(\frac{1-(1.06)^{-20}}{0.06}\right)$

= 1,00,000 = R (11.47)

= R = **₹ 8,718.40 (approx.)**

****

Time Value of Money
Exercise: Additional Question

Sol.1 (d) $S.I = \frac{20{,}000 \times 5 \times 4}{100} = ₹\,4000$

$C.I = ₹\,20{,}000\,[(1+0.05)^4 - 1]$

$= 20{,}000\,[(1.05)^4 - 1]$

$= 20{,}000 \times (1.21550625 - 1)$

$= ₹\,4{,}310$ (approx)

∴ Required difference $= ₹\,(4{,}310 - 4{,}000) =$ **₹ 310**

Sol.2 (d)

$C.I = ₹\,10{,}000\,[(1+0.03)^4\,(1+0.045)^2 - 1]$

$= 10{,}000\,[(1.03)^4\,(1.045)^2 - 1]$

$=$ **₹ 2, 290. 83**

Sol.3 (a) $P = \frac{A}{(1+i)^n}$

$= p = A(1+i)^{-n}$

$= 10{,}000\,(1.05)^{-2}$

$=$ **₹ 9, 070 (approx)**

Sol.4 (d) $P = \frac{A}{(1+i)^n} = A\,(1+i)^{-n}$

$= 10{,}000\,(1+0.025)^{-4}$

$[\because i = 5\%\,P.a =$ $2.5\%\ Per\ half\ yearly\ n = 2 \times 2 =$ $4\ half\ yearly]$

$= 10{,}000 \times (1.025)^{-4}$

$= 10{,}000 \times 0.9059$

$=$ **₹ 9, 059**

Sol.5 (d) Let the amount received by each son after 20 years is ₹ x

$$\frac{x}{(1.035)^{16}} + \frac{x}{(1.035)^{+13}} + \frac{x}{(1.035)^{+10}} = 1{,}00{,}000$$

$\Rightarrow x\,[(1.035)^{-16} + (1.035)^{-13} + (1.035)^{-10}] = 1{,}00{,}000$

$\Rightarrow x\,[0.5767 + 0.6394 + 0.7089] = 1{,}00{,}000$

$\Rightarrow x \times 1.925 = 1{,}00{,}000$

$\Rightarrow x = \frac{1{,}00{,}000}{1.925} =$ **51, 948 (approx)**

Sol.6 (b) $A = P\,(1+i)^n$

$\Rightarrow 2P = P\,(1+0.05)^n$

$\Rightarrow 2 = (1.05)^n$

$\Rightarrow n = 14.2\ yrs$ (approx)

$\Rightarrow$ ***n = 14 years and 2 months*** **(approx)**

Sol.7 (d) $A = P\,(1+i)^n$

$= 3P = P\,(1+0.025)^n\ \left(\because n = nos.of\ half\ i = \frac{5}{2}\%\ Per\ half\ yrs\right)$

$\Rightarrow 3 = (1.025)^n$

$= 44.5$ years(approx.)

$\therefore$ Required time $= \frac{44.5}{2}\ yrs = 22.25\ years(approx).$

$= \mathbf{22\ yrs\ \&\ 3\ month}$ **(approx)**

Sol.8(c)

$= Scrap\ value = Cost\ of\ asset\ (1-i)^n$

$(\because i = Rate\ of\ depreciation)$

$\Rightarrow 9{,}000 = 23240\ (1-0.1)^n$

$\Rightarrow 9{,}000 = 23240 \times (0.9)^n$

$\Rightarrow \frac{9{,}000}{23{,}240} = (0.9)^n \Rightarrow (0.3874) = (0.9)^n$

⇒ n = 9 years(approx.)

Sol.9 (d)

$Scrap\ value = Cost\ of\ asset\ (1-i)^n$

$= 2{,}00{,}000 = 4{,}90{,}740\ (1-0.15)^n$

$\Rightarrow \frac{2{,}00{,}000}{4{,}90{,}740} = (1-.15)^n \Rightarrow (0.40754) = (0.85)^n$

⇒ n= 5.543 years(approx.)

Required time = 5 years 7 months (approx.)

Sol.10 (d)

$4{,}90{,}740 \times (100-90)\% = 4{,}90{,}740\ (1-0.15)^n$

$\Rightarrow 0.1 = (0.85)^n$

$\Rightarrow n = 14.17$ years

∴ Required time =**14 years 2 month**

Sol.11 (c) P.V= $R(\frac{1-(1+i)^{-n}}{i})$

$= 6{,}00{,}000 = R(\frac{1-(1.06)^{-20}}{0.06})$

$= 6{,}00{,}000 = R(11.47)$

R= ₹ 52,310

Sol.12 (a) $F.V = R\left[\frac{(1+i)^n-1}{i}\right]$

$\Rightarrow 5{,}00{,}000 = R\left[\frac{(1.04)^{25}-1}{0.04}\right]$

$\Rightarrow 20{,}000 = A\ [2.6658-1]$ (approx)

$\Rightarrow A = \frac{20{,}000}{1.6658} = ₹\ \mathbf{12{,}006}$ **(approx)**

Sol.13 (c) Amount for sinking fund

$= \left[5{,}20{,}000 \times \left(\frac{100+25}{100}\right) - 25{,}000\right]$

$= \left(5{,}20{,}000 \times \frac{5}{4} - 25{,}000\right)$

$= (6{,}50{,}000 - 25{,}000)$

$= ₹\ 6{,}25{,}000$

Now $6{,}25{,}000 = R\left[\frac{(1.035)^{25}-1}{0.035}\right]$

$= \quad 6{,}25{,}000 = R\left(\frac{2.3632-1}{0.035}\right)$

$\Rightarrow R = \frac{6{,}25{,}000}{38.95} = \mathbf{16{,}046}$ **(approx)**

Sol.14 (d) $F.V = R\left[\frac{(1+i)^n-1}{i}\right]$

$\Rightarrow 40{,}00{,}000 = R\left[\frac{(1.03)^{30}-1}{0.03}\right]$

$\Rightarrow 40{,}00{,}000 = R(47.575)$

$\Rightarrow ₹\ \mathbf{84{,}078}$ **(approx)**

Sol.15 (b) P.V= $R(\frac{1-(1+i)^{-n}}{i})$

$\left[\because n = 13 \times 2 = 26\ and\ i = \frac{4}{2}\% = 0.02\right]$

$= A = \frac{14{,}400}{2} = ₹7{,}200$

$=$ P.V= $7{,}200\ (\frac{1-(1.02)^{-26}}{0.02})$ = **₹1,44,871**

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Permutation and Combination

Exercise: 5A

Sol.1 (c) $4_{P_3} = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{24}{1} = \mathbf{24}$

Sol.2 (b) $4_{P_4} = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{24}{1} = \mathbf{24}$

Sol.3 (a) $\lfloor 7 = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \mathbf{5{,}040}$

Sol.4 (b) $\lfloor 0 = 0! = \mathbf{1}$

Sol.5 (c) In n_{P_r}, n is always a positive integer

Sol.6 (b) $n \geq r$

Sol.7 (d) r

Sol.8 (a)

${}^{n}P_{r} = \frac{\lfloor n}{\lfloor n-r} = n(n-1)(n-2)\ldots\ldots\ldots n(n-r+1)$

Sol.9 (b) $n_{P_4} = 12 \times n_{P_2}$

$\Rightarrow \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!}$

$\Rightarrow \frac{(n-2)!}{(n-4)!} = 12$

$\Rightarrow \frac{(n-2)(n-3)(n-4)!}{(n-4)!} = 12$

$\Rightarrow n^2 - 5n + 6 - 12 = 0$

$\Rightarrow n^2 - 5n - 6 = 0$

$\Rightarrow n^2 - 6n + n - 6 = 0$

$\Rightarrow (n-6)(n+1) = 0$

$\Rightarrow n - 6 = 0 \; or \; n + 1 = 0$

$\Rightarrow \boldsymbol{n = 6 \; or} \; \boxed{\boldsymbol{n = -1}}$ **rejected as it is not possible**

Sol.10 (c) $n_{P_3} : n_{P_2} = 3:1$

$\Rightarrow \frac{n!}{(n-3)!} \times \frac{(n-2)!}{n!} = \frac{3}{1}$

$\Rightarrow \frac{(n-2)(n-3)!}{(n-3)!} = 3 \Rightarrow n - 2 = 3 \Rightarrow$ $\boldsymbol{n = 5}$

Sol.11 (b) ${}^{m+n}P_2 = 56 \Rightarrow \frac{(m+n)!}{(m+n-2)!} = 56$

$$\Rightarrow \frac{(m+n)(m+n-1)(m+n-2)!}{(m+n-2)!} = 56$$

$\Rightarrow (m+n)(m+n-1) = 56$____$(I)$

$\Rightarrow m - n_{P_2} = 30 \Rightarrow \frac{(m-n)!}{(m-n-2)!} = 30$

$$\Rightarrow \frac{(m-n)(m-n-1)(m-n-2)!}{(m-n-2)!} = 30$$

$\Rightarrow (m-n)(m-n-1) = 30$____$(II)$

From equation (I)

$(m+n)(m+n-1) = 8 \times 7$

$\Rightarrow m + n = 8$____(III)

From equation (II)

$(m-n)(m-n-1) = 6 \times 5$

$\Rightarrow m - n = 6$____(IV)

From equations $[(III) + (IV)]$

$$\begin{array}{c} m+n=8 \\ \underline{m-n=6} \\ 2m=14 \Rightarrow m=7 \end{array}$$

$\therefore \boldsymbol{n = 1}$

Sol.12 (a) $5_{P_r} = 60$

$\Rightarrow \frac{5!}{(5-r)!} = 60$

$\Rightarrow (5-r)! = \frac{5!}{60}$

$\Rightarrow (5-r)! = \frac{120}{60}$

$\Rightarrow (5-r)! = 2$

Go through the options

Option (a) r=3

$\Rightarrow (5-3)! = 2! = 2$ *(Correct Answer)*

In a similar way, you can check the other options don't satisfy the equation.

Sol.13 (c) ${}^{n_1+n_2}P_2 = 132$

$\Rightarrow \frac{(n_1+n_2)!}{(n_1+n_2-2)!} = 132$

$\Rightarrow (n_1+n_2)(n_1+n_2-1) = 12 \times 11$

$\Rightarrow n_1 + n_2 = 12$____$(I)$

Now, ${}^{n_1-n_2}P_2 = 30$

$\Rightarrow \frac{(n_1-n_2)!}{(n_1-n_2-2)!} = 30$

$\Rightarrow (n_1-n_2)(n_1-n_2-1) = 6 \times 5$

$\Rightarrow n_1 - n_2 = 6$____$(II)$

From $[(I) + (II)]$

$$\begin{array}{c} n_1+n_2=12 \\ \underline{n_1-n_2=6} \\ 2n_1=18 \end{array}$$

$\Rightarrow \boldsymbol{n_1 = 9}$

$\therefore \boldsymbol{n_2 = 12 - 9 = 3}$

Sol.14 (b)

Total nos. of letters in the word computer is 8, and all are distinct

$\therefore$ Required arrangements = ${}^8c_8 \times 8!$

= 40,320-1= 40,319.

s

Correct Answer)

check the other

equation.

1) = 12 × 11

= 6 × 5

er is 8, and

Sol.15 (a) FAILURE

Total nos. of letters in the word FAILURE is 7, and all are different

Also, Nos. of vowels =4

∴ Required arrangement

∴ 4! × 4! (∵Taking all vowels as a single unit so there are 4 units can be arranged in 4! Ways (external arrangement) & also vowels are arranged in 4! ways(internal arrangement))

= 24 × 24 = **576**

Sol.16 (c) Arrangement of 10 papers = 10!

$\Rightarrow Best\ and\ worst\ come\ together = 2!$

$\Rightarrow Arrangement\ of\ 10\ papers\ when\ best\ and$

$worst\ come\ together = 2!\,9!$

$\Rightarrow Never\ come\ together = 10! - 2!\,9! = 9!\,(10 - 2) = \mathbf{8.9!}$

Sol.17 (a) Required nos. of arrangement =

Total arrangement of n articles – nos. of arrangement taking two particular articles are together

$= n! - (n-1)! \times 2!$

$= \mathbf{(n-2)\ (n-1)!}$

Sol.18 (b) Required nos. = $^{12}c_3 \times 3!$

$= 12 \times 11 \times 10 = \mathbf{1,320}$

Sol.19 (d) Step 1- $4 \times 3 \times 2 \times 1 = 24$

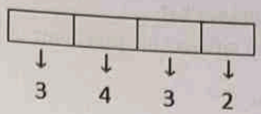
Step-2- $\frac{Step\ 1}{Total\ Digits} = \frac{24}{4} = 6$

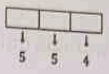
Step -3 Sum of digits= 2+4+6+8= 20

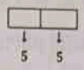
Step-4 – $Step\ 2 \times Step\ 3 = 6 \times 20 = 120.$

Step 5- $120 \times 1,000 + 120 \times 100 + 120 \times 10 + 120 \times 1 = 1,33,320.$

Sol.20 (a)







∴ Required no. = 3 × 4 × 3 × 2 = 72

Or

A thousand places can be filled with anyone out of 5, 6 or 7 in $^3P_1$ ways, and the remaining 3 places can be filled without of remaining 4 digits can be done in $^4P_3$ ways

∴ Required nos. of ways

$= {}^3P_1 \times {}^4P_3 = \frac{3!}{2!} \times \frac{4!}{1!}$

= 3 × 24 = **72**

Sol.21 (c) Required nos. of 4 digit numbers 4× 4 × 3 × 2 =**96.**

Sol.22 (c) Taking angle as a single unit and remaining three letters as 3 unit

∴ Total nos. of ways = Total nos. of arrangement of 4 units

= 4! = **24**

Sol.23 (a)

Total nos. of letters of word DAUGHTER = 8

∴ Total nos. of odd place = 4

Total nos. of vowels in word daughter = 3

∴ Required nos. of different word

$= {}^4c_3 \times 3!\ {}^5c_5 \times 5!$

$= \frac{4!}{3!} \times 3! \times \frac{5!}{0!} = 24 \times 120$ **= 2,880**

****

Permutation and Combination

Exercise: 5B

Sol.1 (c) Total nos. of a circular arrangement of n things = $(n - 1)!$

∴ Required nos. of ways

$= (7 - 1)! = 6! = \mathbf{720}$

Sol.2 (a) Taking particular two boys as a single unit and the other 5 boys as 5 units

∴ 6 units arranged around the table in

$= (6 - 1)! = 5!$ Ways.

And two particular boys arrange in 2! Ways

∴ Required nos. of ways $= 5! \times 2! = \mathbf{240}$

Sol.3 (b) For a necklace

The number of ways $= \frac{1}{2}\,(n-1)!$

$= \frac{1}{2}\,(50-1)! = \frac{1}{2} \times 49!$

$= \frac{1}{2}\,\lfloor\mathbf{49}$

Sol.4 (c)

Required nos. of ways = Total seat arrangement of 6 person (i.e., 3 ladies & 3 gents) in round table

[(nos. of ways no two ladies sit together × (nos. of ways 2 gents seat together)] × arrangements

$= \ {}^3C_2 \times 2! \times {}^3C_2 \times 2! \times 2!$

$= \ 3 \times 2 \times 3 \times 2 \times 2 = \mathbf{72}.$

Sol.5 (b)

Total nos. of letter in word 'DOGMATIC' = 8

∴ Required nos. of arrangements

$= {}^8c_8 \times 8! = 8! = \mathbf{40{,}320}$ $\qquad [\because {}^nc_{n=1}]$

Sol.6 (b) Required nos. of arrangement

$= {}^1C_1 \times {}^9C_3 \times 4!$

$= 1 \times \frac{9\times8\times7}{3\times2\times1} \times 4 \times 3 \times 2 \times 1$

$= \mathbf{2{,}016}$

Sol.7 (c) Required arrangement

$= {}^{10}P_4 - {}^1C_1 \times {}^9C_3 \times 4! = 10 \times 9 \times 8 \times 7 - 1 \times 9 \times 8 \times 7 \times 4$

$= 5{,}040 - 2016 = \mathbf{3{,}024}$

Sol.8 (d) Required nos. of way in

Which they occupy the seats

$= {}^6c_2 \times 2! = 6 \times 5 = \mathbf{30}$

Sol.9 (a) Required number of numbers

$= \ 7 \times 6 \times 5 = \mathbf{210}.$

Sol.10 (c) Required nos. of number

$= 5 \times 5 \times 4 + 5 \times 5$

$= 100 + 25 = \mathbf{125}$

Sol.11 (c) Let the nos. of boys in the group be n

${}^nP_4 = 12 \times {}^nP_2$

$\Rightarrow \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!}$

$\Rightarrow \frac{(n-2)!}{(n-4)!} = 12$

$\Rightarrow (n-2)\,(n-3) = 12$

$\Rightarrow n^2 - 5n - 6 = 0$

$\Rightarrow (n-6)\,(n+1) = 0$

$\Rightarrow n - 6 = 0 \ or \ n + 1 = 0$

$\Rightarrow n = 6 \quad or \quad n =$
-1 *Rejected as it is not possible*
$\therefore \boldsymbol{n = 6}$

Sol.12 (b) $1^1p_1 + 2^2p_2 + 3^3p_3 + \ldots\ldots\ldots\ldots 10^{10}p_{10}$

$= 1! + 2(2!) + 3(3!) \ldots\ldots\ldots\ldots\ldots\ldots 10(10!)$
$(n(n!) = (n+1)! - n!$

$= 2! - 1! + 3! - 2! + 4! - 3! + \ldots\ldots\ldots\ldots + 11! - 10!$

$= \ 11! - 1$

$= \ \mathbf{11_{P_{11}} - 1}$

Sol.13 (c) There are 10 digits 0, 1, 2, 3, ---, 9

Extreme left position of the number can be filled with anyone out of 9 digits, i.e., 1, 2, 9, ---, 9 in ${}^9P_1$ ways and remaining 8 positions of 9 digit number can be filled with any digit of remaining 9 digits because 0 (zero) can be placed after the extreme left position of a number. So it can be done in ${}^9P_8$ ways

∴ Required number of 9 digit number

$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

$= 9 \times 9! = 9(9!)$

$= 9 \lfloor 9$

Sol.14 (b) Let three particular men as a single unit and remain 3 men as 3 unit

∴ 4 Units can be arranged externally 4! and three particular men can be arranged internally 3!.

∴ Required nos. of ways $= 4! \times 3!$

$= {}^4\mathbf{P}_4 \times {}^3\mathbf{P}_3$

Sol.15 (a) Taking AB as a single unit and the other 3 as 3 units, so external arrangement = 4! and internal arrangement is not possible.

∴ Required nos. of arrangement = 4!

$= 4! = \mathbf{24}$

Sol.16 (b) Total nos. of ways of going = ${}^{10}c_1$ ways and returning = ${}^{9}c_1$ ways.

∴ Total nos. of ways to go and return

$= {}^{10}c_1 \times {}^{9}c_1 = \frac{10!}{9!} \times \frac{9!}{8!}$

$= 10 \times 9 \;\; = \mathbf{90}$

Sol.17 (b) we have to arrange the rest 7 sweets because larger sweet is going to the younger students.

Required nos. of ways $= 7! \times {}^7c_7$

$= 1 \times 7! = \mathbf{5,040}$

Sol.18 (c) In the word Monday M is fixed, and for the last alphabet, we have 4 options. And for the 2^{nd} alphabet also 4 options and so on.

Required no. of ways = $4\times 3 \times 2 \times 1 \times 4 = \mathbf{96.}$

Sol.19 (c) . + . + . + . + . + . + .

There are 7 dot positions in which any four positions are filled with four '-, signs in

${}^7P_4 / 4!$ (∵ four '-, signs are identical)

${}^7c_4 \frac{6!}{6!} \times \frac{4!}{4!} = \mathbf{35.}$

Sol.20 (a) Here, the word MOBILE has 6 letter

∴ It has 3 odd places and also three consonant is placed in ${}^3C_3 \times 3!$

And remaining three even places can be filled with the remaining three vowels in ${}^3c_3 \times 3!$ways

∴ Required nos. of ways

$= {}^3C_3 \times 3! \times {}^3C_3 \times 3! = 3! \times 3! = 6 \times 6 = \mathbf{36}$

Sol.21 (a) Taking shortest & tallest person as a single unit and another individual as a unit

∴ There are 4 units is arrange in the round table be done in

(4 – 1)! Ways = 3! Ways = **6**

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Permutation and Combination

Exercise: 5C

Sol.1 (a) ${}^{12}C_4 + {}^{12}C_3 = {}^{12+1}C_4 = {}^{13}C_4$

$(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_r)$

$= \frac{13!}{4! \times 9!}$

$= \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1}$

= 715

Sol.2 (b) ${}^nP_r = 336 \Rightarrow \frac{n!}{(n-r)!} = 336$____(*I*)

${}^nC_r = 56 \Rightarrow \frac{n!}{r!\,(n-r)!} = 56$____(*II*)

from $[(I) \div (II)]\; r! = \frac{336}{56} = 6 = 3!$

$\Rightarrow r = 3$

$\therefore {}^nP_3 = 336 \Rightarrow \frac{n!}{(n-3)!} = 336$

$\Rightarrow n\,(n-1)\,(n-2) = 8 \times 7 \times 6$

$\therefore \mathbf{n = 8}$

Sol.3 (c) $\because {}^{18}C_r = {}^{18}C_{r+2}$

$\Rightarrow {}^{18}C_r = {}^{18}C_{18-(r+2)}\; [\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow r = 18 - (r+2)$

$\Rightarrow r = 16 - r$

$\Rightarrow 2r = 16 \Rightarrow r = 8$

$\therefore {}^rC_5 = {}^8C_5 = \frac{8!}{5! \times 3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = \mathbf{56}$

Sol.4 (b) ${}^nC_{r-1} = 56 \Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!} = 56$____(*I*)

${}^nC_r = 28 \Rightarrow \frac{n!}{r!\,(n-r)!} = 28$____(*II*)

${}^nC_{r+1} = 8 \Rightarrow \frac{n!}{(r+1)!\,(n-r-1)!} = 8$____(*III*)

$from\ [(I) \div (II)]$

$$\frac{n!}{(r-1)!(n-r+1)} \times \frac{r!(n-r)!}{n!} = \frac{56}{28}$$

$$\Rightarrow \frac{r}{n-r+1} = 2 \Rightarrow r = 2n - 2r + 2$$

$\Rightarrow 2n = 3r - 2$____(*IV*)

$from\ [(II) \div (III)]$

$$\frac{n!}{(r)!\,(n-r)!} \times \frac{(r+1)!\,(n-r-1)!}{n!} = \frac{28}{8}$$

$$\Rightarrow \frac{r+1}{n-r} = \frac{7}{2} \Rightarrow 2r + 2 = 7n - 7r$$

$$\Rightarrow 7n = 9r + 2$$

$\Rightarrow 7\left(\frac{3r-2}{2}\right) = 9r + 2 \Rightarrow 21r - 14 = 18r + 4$

$\Rightarrow 3r = 18 \ \Rightarrow r = \mathbf{6}$

Sol.5 (b) Required nos. of ways to invite one or more

$= {}^8C_1 + {}^8C_2 + {}^8C_3 + \text{---} + {}^8C_8$

$= 2^8 - 8_{C_0} = 2^8 - 1$

$= 256 - 1 = \mathbf{255}$

Sol.6 (a) Required nos. of ways

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - {}^4C_0 = 2^4 - 1 = 15$

= 15

Sol.7 (b) $\because {}^nC_{10} = {}^nC_{14}$

$\Rightarrow {}^nC_{n-10} = {}^nC_{14}\ (\because {}^nC_r = {}^nC_{n-r})$

$\Rightarrow n - 10 = 14 \ \Rightarrow n = 24$

$\therefore {}^{25}C_n = {}^{25}C_{24} = \frac{25!}{24!\times 1!} = \mathbf{25}$

Sol.8 (c) Required nos. of committees

$= {}^4C_1 \times {}^7C_4 + {}^4C_2 \times {}^7C_3 + {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

$= 4 \times 35 + 6 \times 35 + 4 \times 21 + 1 \times 7$

$= 140 + 210 + 84 + 7 = 441$

Sol.9 (a) $\because {}^{28}C_{2r} : {}^{24}C_{2r-4} = 225:11$

$$\Rightarrow \frac{28!}{(2r)!\,(28-2r)!} \times \frac{(2r-4)!(28-2r)!}{24!} = \frac{225}{11}$$

$$\Rightarrow \frac{28\times27\times26\times25\times24!\times(2r-4)!}{24!\times(2r)\,(2r-1)\,(2r-2)\,(2r-3)\,(2r-4)!} = \frac{225}{11}$$

$\Rightarrow 7 \times 3 \times 26 \times 11 = 2r\,(2r-1)\,(2r-2)\,(2r-3)$

$\Rightarrow 14 \times 13 \times 12 \times 11 = (2r)\,(2r-1)\,(2r-2)\,(2r-3)$

$\Rightarrow 2r = 14 \ \Rightarrow r = \mathbf{7}$

Sol.10 (b) Nos. of diagonal in a polygon with n sides

$\therefore$ Required nos. of diagonals $\frac{n(n-3)}{2} = \frac{10\times7}{2} = \mathbf{35.}$

Sol.11 (c) Required nos. of triangles

$= {}^{12}C_3 - {}^5C_3 = \frac{12!}{3!\times9!} - \frac{5!}{3!\times2!}$

$= \frac{12\times11\times10}{3\times2\times1} - \frac{5\times4}{2\times1} = 220 - 10$

= 210

Sol.12 (a) Required nos. of lines

$= {}^{16}C_2 = \frac{16\times15}{2\times1} = \mathbf{120}$

Sol.13 (c) Required nos. of ways

$= {}^5C_1 + {}^5C_2 + {}^5C_3 = 5 + 10 + 10$

= 25

Sol.14 (b) Let the nos. of guest be n

$\therefore {}^nC_2 = 66$

$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \Rightarrow \frac{n\,(n-1)}{2\times1} = 66$

$\Rightarrow n^2 - n - 132 = 0$

$\Rightarrow\ n^2 - 12n + 11n - 132 = 0$

$\Rightarrow (n-12)\,(n+11) = 0$

$\Rightarrow n - 12 = 0\ or\ n + 11 = 0$

$\Rightarrow n = 12\ or\ [n = -11]$ rejected as it is not possible

$\therefore$ n = **12**

Sol.15 (b) Required nos. of parallelogram

$= \frac{mn(m-1)(n-1)}{4} = \frac{4\times3\times3\times2}{4}$

$= 6 \times 3 = \mathbf{18}$

Sol.16 (a) Required nos. of ways

$= \frac{(12!)}{3!(4!)^3} = \frac{12\times11\times10\times9\times8\times7\times6\times5\times4!}{3\times2\times1\times4!\times4!\times4!}$

$= \frac{12\times11\times10\times8\times7\times6\times5}{6\times24\times24} = 11 \times 5 \times 3 \times 7 \times 15 =$ **5,775**

Sol.17 (b) ∵ n similar divided into r group with equal nos. p things then the

Required number of ways

$= \frac{15!}{(5!)^3}$

Sol.18 (c) Required nos. of chords = $^8C_2$

$= \frac{8!}{2!\times6!} = \frac{8\times7}{2\times1} = \mathbf{28}$

Sol.19 (d) Required nos. of committees

Situation 1- Mr y is a member- $^6c_3 \times {}^7c_3$

Situation 2- when Mr. y is not a member- $^6c_4 \times {}^8c_3$

$^6c_3 \times {}^7c_3 + {}^6c_4 \times {}^8c_3 = \mathbf{1,540}$

Sol.20 (d) $({}^nC_r + {}^nC_{r+1} = {}^{n+1}C_r)$

Therefore n= **499.**

Sol.21 (a) Total nos. of ways in which majority decision reversing the lower court

$= {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$

$= 126 + 84 + 36 + 9 + 1$

= 256

Sol.22 (d) Number of trials shall be lighted

Situation1) both are non-defective= $^2c_2$

Situation 2) one defective and one non-defective $^3c_1 \times {}^2c_1$

= 1+6 = **7**

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Permutation and Combination

Exercise: 5D

Sol.1 (b) Total nos. of ways of arranging the letters of the ward 'CALCUTTA'

$= \frac{8!}{2!\times2!\times2!}$

Total nos. of ways of arranging the letters of the word 'AMERICA'

$= \frac{7!}{2!}$

∴ Required ratio = $\frac{\frac{8!}{2!\times2!\times2!}}{7!/2!}$

$= \frac{8!}{2!\times2!\times2!} \times \frac{2!}{7!}$

$= \frac{8\times7!}{2\times2\ \times7!} = \frac{2}{1} = \mathbf{2:1}$

Sol.2 (a) EXAMINATION

A	E	I	M	N	O	X	T
↓	↓	↓	↓	↓	↓	↓	↓
2	1	2	1	2	1	1	1

∴ Number of ways in which all four letter are different = $^8C_4$

Numbers of ways in which two letters are same & two are distinct = $^3C_1 \times {}^7C_2$

Number of ways in 2 pair of two similar letter = $^3C_2$

∴ Required nos. of ways of selecting 4 letters from the word 'EXAMINATION'

$= {}^8C_4 + {}^3C_1 \times {}^7C_2 + {}^3C_2 = 70 + 63 + 3 = \mathbf{136}$

Sol.3 (c) Required nos. of different words

$= {}^{12}C_4 \times {}^5C_3 \times (4 + 3)!$

$= \frac{12!}{4!\times8!} \times \frac{5!}{3!\times2!} \times 7!$

$= \frac{12\times11\times10\times9}{4\times3\times2\times1} \times \frac{5\times4}{2\times1} \times 7!$

$= 495 \times 10 \times 7! = 4{,}950 \times 7!$

$= 4{,}950 \times \underline{|7}$

Sol.4 (b) [∵ Let two sides of table be A & B then 2 particular sit one side A then 3 on side B ∴ from remaining 3, 2 sit on side A and 1 sit on side B be done in ${}^3C_2 \times {}^1C_1$ ways and each 4 guests on each side can be arrange in 4! ways]

Total nos. of sitting arrangement

$= {}^3C_2 \times {}^1C_1 \times (4!) \times (4!)$

$= 3 \times 1 \times 24 \times 24$

= 1,728

Sol.5 (b) Total numbers of ways an examine can answer 1 or more questions

$= {}^6C_1 \times 2 + {}^6C_2 \times 2^2 + {}^6C_3 \times 2^3 + {}^6C_4 \times 2^4 + {}^6C_5\, 2^5 + {}^6C_6 \times 2^6$

= 12+60+160+240+192+64=**728.**

Sol.6 (a) ${}^{51}C_{31} = {}^{51}C_{51-31} = {}^{51}C_{20}$

$(\because {}^nC_r = {}^nC_{n-r})$

Sol.7 (c) There are 6 letters in the of these APURNA

∴ 3 odd and 3 even position and also 3 vowels and 3 consonants are here

∴ Required arrangement that vowels and consonants appear alternate

$= {}^3C_3 \times {}^3C_3 \times 3! \times 3! = 3! \times 3! = 6 \times 6 =$ **36**

Sol.8 (b) & (c) Here letters appear in the word COMMERCE

C	E	M	O	R
↓	↓	↓	↓	↓
2	2	2	1	1

∴ Required number of arrangements

$= \frac{8!}{2! \times 2! \times 2!} = \frac{8 \times 7!}{2 \times 2 \times 2} = 7!$

Sol.9 (b) Required nos. of choice

$= {}^6C_4 \times {}^6C_2 + {}^6C_3 \times {}^6C_3 + {}^6C_2 \times {}^6C_4$

$= 15 \times 15 + 20 \times 20 + 15 \times 15 = 225 + 400 + 225 =$ **850**

Sol.10 (c) 6 correct prediction out 8 matches be ${}^8C_6$ and for other two matches two wrong prediction can be done in 2 × 2 ways. because two option out of 3 (win, loss or Drawn) are wrong

∴ Required number of different forecasts $= {}^8C_6 \times 2 \times 2$

$= 28 \times 2 \times 2 =$ **112**

Sol.11 (b) Number of ways

$= \frac{1}{2} \times (n-1)! = \frac{1}{2} \times (8-1)! = \frac{1}{2} \times 7!$

$= \frac{1}{2} \times 5040 =$ **2,520**

Sol.12 (c) For find number of factors just break the numbers into prime numbers. Then power of prime number of every number add 1 and multiply powers.

Here

$75600 = 2^4 \times 3^3 \times 5^2 \times 7^1$

∴ Total nos. of factors

$= (4+1) \times (3+1) \times (2+1) \times (1+1)$

$= 5 \times 4 \times 3 \times 2 = 120$

Hence the different factors

The number 75600 has is

$= 120 - 1 =$ **119**

Sol.13 (d) Numbers of ways in which all 4 digit different $= {}^4C_4 \times 4! = 24$

Number of ways in which 2 digits are like and other two are different

$= {}^2C_1 \times {}^3C_2 \times \frac{4!}{2!} = 2 \times 3 \times 12 = 72$

Number of ways in which 2 pairs of like digits $= {}^2C_2 \times \frac{4!}{2! \times 2!} = 6$

∴ Required number of 4 digit numbers = 24 + 72 + 6 = **102**

Sol.14 (a) ∵ Here there are four different types of note

∴ Required nos. of ways

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 2^4 - {}^4C_0 = 16 - 1 =$ **15**

Sol.15 (b) Required nos. of ways.

$$= \frac{(p+q+r)!}{p!\times q!\times r!} = \frac{9!}{2!\times 3!\times 4!}$$

$$= \frac{9\times 8\times 7\times 6\times 5\times 4!}{2\times 6\times 4!} = \mathbf{1,260}$$

Sol.16 (c) $^{n-1}P_r + r.\, ^{n-1}P_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!} + r \times \frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-1-r)!} \times \left(1 + \frac{r}{n-r}\right)$$

$$= \frac{(n-1)!}{(n-1-r)!} \times \left(\frac{n-r+r}{n-r}\right)$$

$$= \frac{n\,(n-1)!}{(n-r)\,(n-r-1)!} = \frac{n!}{(n-r)!}$$

$$= {}^{n}P_r$$

Sol.17 (a) $L2n. = (2n)\,(2n-1)\,(2n-2) \ldots 3.2.1$

$$= \{1.3.5 \ldots (2n-1)\}\,\{2.4.6 \ldots (2n)\}$$

$$= \{1.3.5 \ldots (2n-1)\}\,2^n\,(1.2.3 \ldots n)$$

$$= \mathbf{2^n\,\{1.3.5 \ldots (2n-1)\}\,n!}$$

Sol.18 (c)

3 or 5		2 or 4
↓	↓	↓
2 ×	3 ×	2

4		2
↓	↓	↓
1	3	1

nos. of 3 digit even no. greater than 300

$= 2\times 3\times 2 + 1\times 3\times 1 = 12 + 3 = 15$

			2 or 4
↓	↓	↓	↓
4	3	2	2

nos. of 4 digit even $no = 4\times 3\times 2\times 2 = 48$

				2 or 4
↓	↓	↓	↓	↓
4	3	2	1	2

nos. of 5 digit even $no = 4\times 3\times 2\times 1\times 2 = 48$

∴ Required number of number $= 15 + 48 + 48 =$ **111**

Sol.19 (b) Required nos. of ways the letters can be dropped = $^5C_5 \times 5!$

= 5! =**120**

Sol.20 (a) $\because {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \text{---} + {}^nC_n = 2^n$

$\therefore {}^nC_1 + {}^nC_2 + {}^nC_3 + \text{---} + {}^nC_n = 2^n - {}^nC_0$

$= \mathbf{2^n - 1}$

****

Permutation and Combination
Exercise: Additional Question

Sol.1 (c) From the principle of fundamental counting the required ways

$^6C_1\times {}^6C_1 = \mathbf{36.}$ $[\therefore {}^nC_1 = n]$

Sol.2 (a) Nos. of route to go $= {}^6C_1 = 6$

Nos. of route to return = 1

∴ Required nos. of ways

$= 6\times 1 = \mathbf{6}$

Sol.3 (d) Number of ways to go $= {}^6C_1 = 6$

Number of ways to return $= {}^5C_1 = 5$.

∴ Required number of ways

$= 6\times 5 = \mathbf{30}$

Sol.4 (a) Required nos. of telephone connections

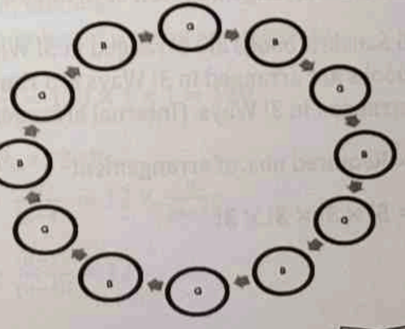
= Numbers of ways of filling 8 block any digit out of 10 digits and no boundary in repetition

$= 10\times 10\times \text{---}8\ times = \mathbf{10^8}$

Sol.5 (a) Total nos. of possible events

$= {}^{10}C_1\times {}^{10}C_1\times {}^{10}C_1$

$= 10\times 10\times 10 = 1{,}000$



No of successful event =1

∴ Required nos. of different ways 3 ring of a lock cannot combine

$= 1000 - 1 = \mathbf{999}$

Sol.6 (c) By the principle of fundamental counting

Required nos. choice ${}^2C_1 \times {}^2C_1 \times {}^5C_1$

$= 2 \times 2 \times 5 = \mathbf{20}$

Sol.7 (a) Required nos. of ways to occupy

The seats = ${}^8C_3 \times 3! = {}^8\mathbf{P}_3$

Sol.8 (a) The word 'LOGARITHMS' has 10 letter and all are different

∴ Required nos. of word

= Arranging 5 letter out of 10 letters

$= {}^{10}C_5 \times 5! = {}^{10}\mathbf{p}_5$

Sol.9 (c)

↓	↓	↓	↓
3	4	3	2

1st block can be filled with anyone from 7, 8, 9

And remaining 3 blocks can be filled with anyone out of 4 digits

∴ Required nos. of 4 digits numbers = $3 \times 4 \times 3 \times 2 =$ **72.**

Sol.10 (a) Taking the same language together

∴ There is 3 unit as every language as a unit so it is external arrangement can be arranged in 3! Ways.

5 Sanskrit books are arranged in 5! Ways 3 English books are arranged in 3! Ways & 3 Hindi books are arranged in 3! Ways. (Internal arrangement)

∴ Required nos. of arrangement

$= 5! \times 3! \times 3! \times 3!$

Sol.11 (b) Firstly, 6 girls are sitting in 6 alternate seats around a table in (6-1)! Ways and each boy out of 6 boys sit between the two girls as six seats available be in ${}^6P_6$, i.e. 6!

∴ Required nos. of ways can 6 boys & 6 girls are seated

$= (6-1)! \times 6! = \mathbf{5! \times 6!}$

Sol.12 (a) Required nos. of ways 4 Americans & 4 English men be seated at round table = $(4-1)! \times 4! = \mathbf{3! \times 4!}$

Sol.13 (a) Taking Kerela & Bengal chief ministers as a single unit and another individual as a unit

∴ There is 16 unit to sit at a round table in (16-1) ways.

= 15!

And Kerela & Bengal chief minister sit together in 2! Ways.

∴ Required sitting arrangement

$= \mathbf{15! \times 2!}$

Sol.14 (a) The word 'ACCOUNTANT' has

A	C	N	O	T	U
↓	↓	↓	↓	↓	↓
2	2	2	1	2	1

∴ Required nos. of permutation

$= \frac{10!}{2! \times 2! \times 2! \times 2!} = \frac{10!}{(2!)^4}$

Sol.15 (a) The word 'ENGINEERING' has

E	G	I	N	R
↓	↓	↓	↓	↓
3	2	2	3	1

∴ Required number of permutation of the word 'ENGINEERING.'

$= \frac{11!}{3! \times 2! \times 2! \times 3!} = \mathbf{11!} \div [(3!)^2 \ (2!)^2]$

Sol.16 (a) The word 'ASSASSINATION' has

A	I	N	O	S	T
↓	↓	↓	↓	↓	↓
3	2	2	1	4	1

∴ Required arrangement = $\frac{13!}{3! \times 2! \times 2! \times 4!}$

$= 13! \div [3! \times 4! \times (2!)^2]$

Sol.17 (b) Required numbers higher than a million $= \frac{7!}{3! \times 2!} - \frac{6!}{3! \times 2!}$

[∵ Total arrangement of the digits with 0 the no. of number start]

$= \frac{6!}{3! \times 2!} \times (7-1) = \frac{720}{6 \times 2} \times 6$

= 360

Sol.18 (d) The word 'ALLAHABAD' has

$$\begin{array}{ccccc} A & B & D & H & L \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 1 & 1 & 1 & 2 \end{array}$$

∴Required permutation of the word 'ALLAHABAD' $= \frac{9!}{4! \times 2!} = \mathbf{7,560}$

Sol.19 (b) The word 'ALLAHABAD' has

$$\begin{array}{ccccc} A & B & D & H & L \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 1 & 1 & 1 & 2 \end{array}$$

Here, there are 9 letters are arranged in 9 positions in which 4 positions are even and 4 vowels as 4A

4A can be placed at 4 even positions in $\frac{4!}{4!} = 1$and remaining 5 letters in 2 L, 1B, 1D & 1H be arranged in $\frac{5!}{2!}$ *ways* = **60.**

∴ Required arrangement in which are occupy the even position.

Sol.20 (a) The word 'MATHEMATICS' has

$$\begin{array}{cccccccc} A & C & E & I & H & M & S & T \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \end{array}$$

∴ Required arrangement $= \frac{11!}{2! \times 2! \times 2!}$

$= \mathbf{11!} \div (2!)^3$

Sol.21 (b) The word 'MATHEMATICS' has

$$\begin{array}{cccccccc} A & C & E & H & I & M & S & T \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \end{array}$$

Taking all 4 vowels together as a single unit and others as individual units

External arrangement $= \frac{8!}{2! \times 2!}$, internal arrangement $= \frac{4!}{2!}$

∴ Required arrangement $= \frac{8!}{2! \times 2!} \times \frac{4!}{2!}$

$= \mathbf{(8! \times 4!)} \div \mathbf{(2!)^3}$

Sol.22 (c) The word 'ARRANGE' has

$$\begin{array}{ccccc} A & E & G & N & R \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 1 & 1 & 2 \end{array}$$

∴ Required arrangement $= \frac{7!}{2! \times 2!}$

$= \frac{5,040}{2 \times 2} = \mathbf{1,260}$

Sol.23 (c) The word 'ARRANGE' has

$$\begin{array}{ccccc} A & E & G & N & R \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 1 & 1 & 2 \end{array}$$

Taking 2 'R's as a single unit and other letters individually as a single unit

∴ Total unit =6

∴ Required arrangement

$= \frac{6!}{2!} \times \frac{2!}{2!} = \frac{720}{2} = 360$

Sol.24 (b)

Required nos. of ways = Total arrangement - (The nos. of arrangement in which 2 'R's come together)

$= \frac{7!}{2! \times 2!} - \frac{6!}{2!} \times \frac{2!}{2!}$

$= \frac{5040}{2 \times 2} - \frac{720}{2}$

$= 1260 - 360 = \mathbf{900}$

Sol.25 (a)

Taking 2 'R's as a single unit and also 2 'A's as a single unit and another individual as a single unit

∴ Total nos. of unit = 5

∴ Required number of ways of arrangement

$= 5! \times \frac{2!}{2!} \times \frac{2!}{2!} = 120 \times 1 \times 1 = \mathbf{120}$

Sol.26 (b) ${}^{n}P_4 = 12\ {}^{n}P_2$

$$\Rightarrow \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{(n-2)!}{(n-4)!} = 12$$

$\Rightarrow (n-2)(n-3) = 12$

$\Rightarrow n^2 - 5n - 6 = 0$

$\Rightarrow n^2 - 5n - 6 = 0$

$\Rightarrow (n-6)(n+1) = 0$

$\Rightarrow n - 6 = 0 \; or \; n + 1 = 0$

$\Rightarrow \boldsymbol{n = 6 \; or \; n = -1}$

But n =1 is not possible as n can't be negative.

Sol.27 (d)

$4\,{}^{n}P_3 = 5\,{}^{(n-1)}P_3 \Rightarrow 4 \times \frac{n!}{(n-3)!} = 5 \times \frac{(n-1)!}{(n-4)!}$

$\Rightarrow 4\,\frac{n\,(n-1)!}{(n-3)\,(n-4)!} = 5 \times \frac{(n-1)!}{(n-4)!}$

$\Rightarrow \frac{4n}{n-3} = 5 \;\Rightarrow 4n = 5n - 15$

$\Rightarrow 15 = 5n - 4n \Rightarrow \boldsymbol{n = 15}$

Sol.28 (a) ${}^{n}P_r \div {}^{n-1}P_{r-1}$

$= \frac{n!}{(n-r)!} \times \frac{[(n-1)-(r-1)]!}{(n-1)!}$

$= \frac{n \times (n-1)!}{(n-r)!} \times \frac{(n-r)!}{(n-1)!}$

$= \boldsymbol{n}$

Sol.29 (c)

Required number of numbers less than

$1000 = 1 + 9 \times 1 + 8 \times 1 + 9 \times 8 \times 1 + 8 \times 8 \times 1$

$= 1 + 9 + 8 + 72 + 64$

$= \mathbf{154}$

Sol.30 (a) Taking the best and the worst papers as a single unit, and they are arranged in 2! ∴ Required numbers of arrangement = Total arrangement - (Numbers of arrangements taking the best and worst together)

$= 8! - 7! \times 2! = \mathbf{8! - 2 \times 7!}$

Sol.31 (a) × B × B × B × B ×

4 Boys can be arranged in 4! Ways and the 3 girls can be seated at × position in ${}^{5}C_3$ ways

∴ Required arrangement

$= {}^{4}C_4 \times 4! \times {}^{5}C_3\, 3!$

$= \frac{5 \times 4}{2} \times 3! \times 4!$

$= \mathbf{5! \times 4! \div 2!}$

Sol.32 (b) Taking all three boys as a single unit and each individual girl as a single units

∴ Total nos. of units(external arrangement) = 5!

= 3 boys are sit(internal arrangement)= 3!

∴ Required number of arrangement

$= \mathbf{5! \times 3!}$

Sol.33 (b) Required number of

Six digit numbers = ${}^{6}C_6 \times 6!$

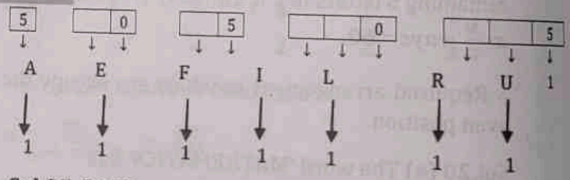
$= \frac{6!}{0!} = \frac{6!}{1} = \mathbf{6!}$

Sol.34 (a) Required number of numbers

= Total 6 digits number = 6!

The number of numbers divisible by 5 for the first digit we have 5 digits for the second digit we have 4 digits and so on and for the last digit must be 5 to divisible by 5. So total numbers are divisible by 5=5!

$= 6! - 5!$



Sol.35 (b) The word 'FAILURE' has

There are 7 letters in the word FAILURE in which 3 letters are consonants, and 4 positions are odd in 7 positions.

∴ Required number of arrangement

$= {}^{4}P_3 \times {}^{4}P_4$

$= \frac{4!}{1!} \times \frac{4!}{0!} = 4! \times 4! = (4!)^2$

Sol.36 (a)

The word 'STRANGE' has

2 vowels and 5 consonant

Taking 2 vowels a single unit, and they can be arranged in 2! Ways.

and 5 consonants as 5 units in each one is one unit

∴ Total unit =6.

∴ Required numbers of arrangement in which vowels are never separated

$= 6! \times 2!$

Sol.37 (a) Required number of arrangement =

Total arrangement - The number of ways in which vowels come together

$= 7! - 6! \times 2!$

Sol.38 (c) The word STRANGE has 7 letters in which 4 are odd positions

∴ 2 vowels are arranged in 4 positions in ${}^4P_2$ ways, or we can write as ${}^4C_2 \times 2!$

And remaining 5 letters are arranged in the remaining 5 positions in ${}^5P_5$ ways or 5!

∴ Required numbers of ways

$= {}^5P_5 \times {}^4P_2$

Sol.39 (a) Required nos. of four digits number = ${}^7P_4$ or ${}^7C_4 \times 4!$

Sol.40 (c) Four digits number greater than 3400 and less than 4000 =

3	4 or more		
↓	↓	↓	↓
1 ×	4 ×	5 ×	4

$= 1 \times 4 \times 5 \times 4 = 80$

For digits number greater than 4,000

$$\left(\begin{array}{cccc} \text{4 or More} & & & \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 6 & 5 & 4 \end{array}\right) = 4 \times 6 \times 5 \times 4 = 480$$

∴ Required number of four digits number greater than $3{,}400 = 80 + 480 = \mathbf{560}$

Sol.41 (a) Required number of ways $= {}^6P_6$ ($\because$ 6 letters in word ZENITH be arranged).

Sol.42 (c) In the order of letter in ZENITH EHINTZ

Numbers of word start with the letter $\frac{ZENITH}{EHINTZ}$

$5! \times ((5) \times 4!\,(0) \times 3!\,(2) \times 2!\,(1) \times 1!\,(1) = 600 + 0 + 12 + 2 + 1 = 615$

Therefore, position of zenith is **616.**

Sol.43 (a) ${}^{n-1}P_3 \div {}^{n+1}P_3 = \frac{5}{12}$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-2)!}{(n+1)!} = \frac{5}{12}$$

$$\Rightarrow \frac{(n-1)! \times (n-2)\,(n-3)\,(n-4)!}{(n-4)! \times (n+1)\,(n)\,(n-1)!} = \frac{5}{12}$$

$$\Rightarrow 12\,(n^2 - 5n + 6) = 5\,(n^2 + n)$$

$$\Rightarrow 7n^2 - 65n + 72 = 0$$

$$\Rightarrow 7\,n^2 - 56n - 9n + 72 = 0$$

$$\Rightarrow 7n\,(n-8) - 9\,(n-8) = 0$$

$$\Rightarrow (7n-9)\,(n-8) = 0$$

$$\Rightarrow 7\,n - 9 = 0 \text{ or } n - 8 = 0$$

$$\Rightarrow n = {}^{9}/_{7} \text{ or } n = 8$$

$n = {}^{9}/_{7}$ Rejected as it is not possible

$\therefore \boldsymbol{n = 8}$

Sol.44 (b) ${}^{n+3}P_6 \div {}^{n+2}P_4 = 14$

$$\Rightarrow \frac{(n+3)!}{(n-3)!} \times \frac{(n-2)!}{(n+2)!} = 14$$

$$\Rightarrow \frac{(n+3)\,(n+2)! \times (n-2)\,(n-3)!}{(n-3)!\,(n+2)!} = 14$$

$$\Rightarrow n^2 + n - 6 - 14 - 0$$

$$\Rightarrow n^2 + n - 20 = 0$$

$$\Rightarrow (n+5)\,(n-4) = 0$$

$$\Rightarrow n + 5 = 0 \text{ or } n - 4 = 0$$

$$\Rightarrow n = -5 \text{ or } n = 4$$

$\Rightarrow n = -5$ Rejected as it is not possible

$\therefore \boldsymbol{n = 4}$

Sol.45 (c) ${}^{7}P_{n} \div {}^{7}P_{n-3} = 60$

$$\Rightarrow \frac{7!}{(7-n)!} \times \frac{(7-n+3)!}{7!} = 60$$

$$\Rightarrow \frac{(10-n)(9-n)(8-n)(7-n)!}{(7-n)!} = 60$$

$$\Rightarrow (10-n)(9-n)(8-n) = 5 \times 4 \times 3$$

$$\therefore 10 - n = 5 \Rightarrow 10 - 5 = n$$

$$\Rightarrow n = 5$$

Sol.46 (c) From the fundamental principle of counting, the required nos. ways

$= 4 \times 5 = \mathbf{20}$

Sol.47 (b) Required numbers of ways can 5 people occupy 8 vacant chairs

$= {}^{8}C_{5} \times 5! = 8 \times 7 \times 6 \times 5 \times 4$
$= \mathbf{6{,}720}$

Sol.48 (b) Required nos. of ticket

$= {}^{50}P_{2} = \frac{50!}{48!} = 50 \times 49$

$= \mathbf{2{,}450}$

Sol.49 (a) Required numbers of six digits $5 \times 5 \times 4 \times 3 \times 2 \times 1 = \mathbf{600}$.

Sol.50 (c) ∵ 0 is fixed at ten's place, and the remaining 5 places can be filled with the remaining 5 digits can be done in $5 \times 4 \times 3 \times 2 \times 1 = 120$

∴ Required number of numbers will have 0's in ten's place

$= 1 \times 120 = \mathbf{120}$

Sol.51 (a) Required nos. of words formed from the letter of word 'SUNDAY' which have 6 letters and all are different $= {}^{6}C_{6} \times 6! = \mathbf{6!}$

Sol.52 (b) When 'N' is fixed at the first position and the remaining 5 positions are filled with the remaining 5 letters in ${}^{5}C_{5}$ 5! Ways.

∴ Required nos. of a word beginning with 'N' $= 1 \times {}^{5}\mathbf{P}_{5} = \mathbf{5!}$

Sol.53 (c) In the word 'SUNDAY' has 6 letters in which 'N' is fixed at 1st position and 'A' at last position

∴ The remaining 4 letters are arranged at 4 positions in ${}^{4}C_{4} \times 4! = \mathbf{4!}$

Sol.54 (a) The word 'MONDAY' has 6 letters, and all are distinct

∴ Required number of arrangements

$= {}^{6}C_{6} \times 6! = \mathbf{6!}$

Sol.55 (b) The word 'ORIENTAL' has 8 letters, and all are distinct

∴ Required numbers of arrangements

$= {}^{8}C_{8} \times 8! = \mathbf{8!}$

Sol.56 (c) The word 'MONDAY' has 6 letters in which 1st position is fixed with A and last position by N

∴ Required number of arrangements

$= {}^{4}C_{4} \times 4! = \mathbf{4!}$

Sol.57 (a) The word 'ORIENTAL' has 8 letters, and all are distinct in which 'A' is fixed at 1st position and 'N' at the last position

∴ Remaining 6 letters are arranged in 6 positions $= {}^{6}C_{6} \times 6 = \mathbf{6!}$

Sol.58 (a) The word LOGARITHM has 6 consonants and 3 vowels letters

∴ Required nos. of ways of choosing

$= {}^{6}C_{1} \times {}^{3}C_{1} = 6 \times 3 = \mathbf{18}$

Sol.59 (b) The word EQUATION has 3 consonant and 5 vowels letters

∴ Required numbers of chosen consonant and one vowel are done in ${}^{3}C_{1} \times {}^{5}C_{1}$

$= 3 \times 5 = \mathbf{15}$

Sol.60 (a) The word 'TRIANGLE' has 8 letters, and all are distinct
∴ Required numbers of words
$= {}^{8}c_{8} \times 8! = \mathbf{8!}$

Sol.61 (b) The word TRIANGLE has 8 letters in T is fixed at 1st position and remaining 7 can be arranged in 7 positions in ${}^{7}c_{7} \times 7! = \mathbf{7!}$

Sol.62 (b) The word 'TRIANGLE' has 8 letters in which E is fixed at 1st position, and the remaining 7 letters can be arranged at 7 positions in ${}^{7}c_{7} \times 7! = \mathbf{7!}$

Sol.63 (c) The word 'TRIANGLE' has 8 letters in which 2 letters are fixed and the remaining 6 letters are arranged in 6 positions be done in $^6c_6 \times 6! = \mathbf{6!}$

Sol.64 (d) Two letters 'T' and 'E' can be placed in 2! Ways and the remaining 6 letters can be arranged in 6! Ways

∴ Required number of ways

$= 2! \times 6!$

Sol.65 (a) Required number of arrangements in which consonants never together =

Total arrangement of letter of the word TRIANGLE – Number of arrangements in which consonant letters are together

$= 8! - {}^4C_4 \times 4! \times {}^5c_5\ 5!$ (Taking all 5 consonants as a single unit and another individual as a unit)

$= \mathbf{8! - 4! \times 5!}$

Sol.66 (b) In the word TRIANGLE has 8 letters in which 5 consonants and 3 vowels

$\times C \times C \times C \times C \times C \times$

∴ 5 Consonants placed at 5 'C' position in 5! Ways and place for 3 vowels are 6'X' position

If can be done in $^6C_3 \times 3!$ways

∴ Required number of ways in which no two vowels are together

$= {}^6C_3 \times 5! \times 3! = \mathbf{{}^6P_3 \times 5!}$

Sol.67 (c) Taking all 5 consonants as a single unit and the other 3 vowels as another unit

∴ Required number of arrangements

$= 2! \times 5! \times 3!$ [2 units arranged in 2! Ways & 5 consonants in 5! Ways and 3 vowels in 3! Ways]

Sol.68 (d) The word 'TRIANGLE' has 8 letters

∴ 4 positions are odd, and 4 positions are even.

∴ 3 vowels are placed at odd places in $^4P_3$ ways. And remaining 5 consonants are placed at the remaining 5 places in

$= {}^5P_5$, i.e., 5!

∴ Required number of arrangements

$= \mathbf{{}^4C_3 \times 3! \times 5!}$

Sol.69 (d) Consonants and vowels positions remain the same can be done in 5 consonant places in 5 positions, and 3 vowels can be placed at 3 positions **5!× 3!** ***ways***.

Sol.70 (a)

Taking four vowels as a single unit and the other 3 consonants in the word failure as individually as a unit.

∴ 4 units can be arranged in 4! Ways(external arrangement) and 4 vowels also can be arranged (internal arrangement) in 4! Ways.

Required number of ways= $4! \times 4! = (4!)^2$

Sol.71 (a) Required number of arrangements

= Total number of arrangements – (number of arrangements in which two particulars books are together)

$= n! - 2!(n-1) \Rightarrow \mathbf{(n-2)(n-1)!}$.

Sol.72 (a) Taking all maths books as a single unit and all English books as a single unit

∴ Two units are arranged in 2! Ways (external arrangement) and three maths books arranged in 3! Ways and 5 English books are arranged in 5! (Internal arrangement)

Required number of ways $= \mathbf{2! \times 3! \times 5!}$

Sol.73 (b) Taking two maths papers as a single unit. And remaining four books as an individual unit are arranged in 5! Ways (external arrangement). And two maths books also arranged in 2! Ways (internal arrangement)

Required number of arrangements $= 5! \times 2! = \mathbf{240}$.

Sol.74 (c) $\times\ 0 \times 0 \times 0 \times 0 \times$

Required number of ways- 4 other papers are placed at 4 '0' position in 4! Ways. And 2 mathematics papers can be placed at any two positions out of available 5 positions in $^5C_2 \times 2!$ Ways.

$= 4! \times {}^5C_2 \times 2! = \mathbf{480}$

Sol.75 (d)

In the word 'SIGNAL' has 6 letters 3 positions are odd at which 2 vowels can be placed in $^3C_2 \times 2!$ Ways and remaining 4 placed at remaining 4 positions in 4! Ways.

∴ Required number of ways $= {}^3C_2 \times 2! \times 4! = \mathbf{144}$.

Sol.76 (d) The word 'VIOLENT' has 7 letters in which 3 are vowels, and 4 are consonants. ∴ Total even placed are 3 at which 3 vowels are placed in $^3C_3$ ×3! Ways. The remaining 4 consonants at the remaining 4 positions in 4! Ways.

∴ Required number of ways= **3!× 4!**

Sol.77 (d) We have to form 4 digits number with 9 digits. 1,2,3,4,____9

∴ Required number of ways $9 \times 8 \times 7 \times 6 = 3024$.

Sol.78 (b) For finding 4 digit numbers between 3,000 and 4,000 at thousand places, we can put only 3, at 100th place we have 5 options at tens place, we have 4 options at the unit place we have 3options. Required numbers between 3,000 and 4,000 = $1 \times 5 \times 4 \times 3 = \mathbf{60}$.

Sol.79 (d) We have 5 digits we have to make number greater than 23,000, so we have two situations

Situation 1- In which number starting with 2 and for next digit, we have 3 digits available because we have to make number greater than 23,000 so available digits for 2nd digit are 3,4,5. Which is selected in $^3C_1$ ways. And the rest 3 digits we have 3 digits they can be arranged in $^3C_3$× 3!

Required number from situation 1 = $^3C_1$× $^3C_3$ × 3!=18

Situation 2- in which number is starting with 3,4 or 5. Required number of ways in situation 2 = $^3C_1$ ×$^4C_4$ ×4!=72.

The total required number of numbers greater than 23,000 is **90.**

Sol.80 (a) Required number of arrangements.

$$= \frac{(5+4\times2+6\times3+1\times8)!}{5!\times(4!)^2\times(6!)^3} = \frac{39!}{5!\times(4!)^2\times(6!)^3}$$

Sol.81 (a) In the word permutation, we have 11 letters in which 2 letters are similar.

Required number of arrangements = $\frac{11!}{2!}$ which we can also write as **$^{11}p_{11}/2$.**

Sol.82 (a) In 1 million 7 digits, and also we have 7 digits.

Required number of ways = arrangement of 7 digits – numbers start with 0

$$= \frac{7!}{2!\times3!} - \frac{6!}{2!\times3!} \Rightarrow \frac{6!}{2!\times3!} \times (7-1)$$

$$= \frac{720\times6}{2\times6} = \mathbf{360}.$$

Sol.83 question is wrong.

Sol.84 (c) In the word HARYANA 7 letters are there.

∴ Required number of arrangement= $\frac{7!}{3!} = \mathbf{840}$.

Sol.85 (b) In the word, HARYANA has 7 letters. Taking H and N as a single unit and rest as an individual unit. They can be arranged in 6! Ways(external arrangement) and h and n are arranged in 2! Ways(internal arrangement). ∴Required numbers of arrangement= $\frac{6!}{3!} \times 2! = \mathbf{240}$.

Sol.86 (d) In the word, HARYANA has 7 letters in which 2 letters in which H and N are fixed at 1st and last remaining 5 letters are arranged in which 3 'A'.

∴ Required number of arrangements. =$\frac{5!}{3!} = \mathbf{20}$.

Sol.87(c) Required number of signals

$= (4)^5 - 1 = \mathbf{1,023}$.

Sol.88 (a) There are 4 letterboxes are available for each letter.

∴ Required number of ways can 9 letters be posted = $(4)^9$.

Sol.89 (a)

Required number of ways = $\frac{1}{2} \times (n-1)! = \frac{1}{2} \times (8-1)! = \mathbf{7!/2}$.

Sol.90 (b) Total number of circular arrangement of 8 boys = (8-1)!=**7!**

Sol.91 (a)

6 men can sit at a round table in (6-1)! =5!

But the clockwise and anticlockwise arrangement has the same neighbour.

∴ Required number of arrangements = $\frac{1}{2} \times 5!$.

Sol.92 (c) Firstly, 6 women can sit at the alternate position in a round table in (6-1)! = 5! and also remaining 6 alternate vacant seats can be seated by 6 men in a round table in $^6C_6$ 6! = 6!

∴ Required number of ways = **5!× 6!**

Sol.93 (d) Total arrangement in a round table =(7-1)!=6!

When women sit together, then the total arrangement in round table =(5-1)!× 3! (taking all women as a single unit, so total unit become 4+1=5 also women can arrange in 3! Ways) = 4!× 3!

Required arrangement = 6!- 4! × 3!⟹ (6× 5 – 3!)4! = 24× 4! = **576**.

Sol.94 (d) women are sitting together, so taking all women as a single unit. The total number of units is 5, and it can arrange into (5-1)! Ways and 3 women are arranged mutually in 3! Ways.

∴ Required number of arrangements =**4!**× **3!**

Sol.95 For children= $^4C_2$× 2!× ^{6}C2× 2!
For others =7!

⟹Required arrangements = $^4C_2$× 2!× $^6C_2$× 2!× 7!
Solutions are not possible with available options.

Sol.96 (a) Taking 3 particular people as a single unit, they sit in a particular order in one way. Then 5 units can be arranged in **5! Ways**.

Sol.97 (b) Taking 3 persons as a single unit and they can also arrange in 3! Ways(internal arrangement).
∴ The total number of units =5.
∴ Required number of arrangements =3!× 5! = 6 × 5! = 6!

Sol.98 (c) Two people who take end seats in 2! And the remaining 5persons can be seated in 5! Ways.
∴ Required number of sitting arrangements = **2!× 5!** ***ways***.

Sol.99 (b) One person takes the middle seat, and the remaining 6 people sit in 6 seats in 6! Ways

∴ Required number of arrangements = 6!

Sol.100 (c) The word CHALK can be arrange $\frac{CHALK}{ACHKL}$
= 4!(1)+ 3!(1)+2!(0)+1!(1)= 24+6+0+1= 31

The rank of **CHALK is 32.**

Sol.101 (b) Taking 2 boys, 2 girls and 2 men as 1 unit each. (External arrangement) ∴ the total number of the unit is arranged in 3! Ways. Also, 2 boys in 2! Ways, 2 girls in 2! Ways and 2 men in 2! Ways (internal arrangement)

∴ Required number of arrangements = 3!× 2! × 2! × 2! = 6 × 2 × 2 × 2 = **48.**

Sol.102 (a) selecting 7 questions out of 10 questions
Required number of ways = $^{10}\mathbf{C_7}$

Sol.103 (b) Taking exactly 2 girls from 4 girls and 3 boys from 6 boys.

Required number of selection= $^6C_3$ ×$^4C_2$

= $\frac{6!}{3!\times3!}\times\frac{4!}{2!\times2!}$= **120.**

Sol.104 (a) Required number of ways of selection = $^1C_1$ ×$^{30}C_3$ = $^{30}C_3$

Sol.105 (b) If one person is always excluded ∴ We have to select 4 out of 30 candidates.

= $^{30}\mathbf{C_4}$.

Sol.106 (a) We have to select 1 particular ball and rest two balls from the remaining 7 balls.

∴ Required number of ways of selection =

$^1C_1$ × $^7C_2$ = $^7\mathbf{C_2}$

Sol.107 (b) we have to select 3 balls out of 8 balls.

∴ Required number of selections = $^8\mathbf{C_3}$

Sol.108 (a) Selecting 3 candidates out of 5 candidates, and we can select any number of the candidate but not exceeding the number to be elected.
∴ Required number of ways of selection = $^5C_1$ + $^5C_2$ + $^5C_3$ = 5+10+10 =**25.**

Sol.109 (c) Selecting 6 questions from 2 groups, each group have 5 questions, and we have to select at least two questions from each group.

∴ Required number of ways to select a question=

$^5C_2$ ×$^5C_4$ + $^5C_3$ ×$^5C_3$ + $^5C_4$ ×$^5C_2$

= $\frac{5\times4}{2}\times5+\frac{5\times4}{2\times1}\times\frac{5\times4}{2\times1}+5\times\frac{5\times4}{2\times1}$

$= 50+100+50 =$ **200.**

Sol.110 (b) Arrangement of 6 consonants out of 10 consonants and 3 vowels out of 4 vowels. Firstly we have to the selection of letters and then arrangement.

Required number of ways $= {}^{10}C_6 \times {}^{4}C_3 \times 9!$.

Sol.111 (a) There are 8 men, so on each side, numbers of men can row =4.
3 men are already selected on 1 side and 2 men on the other side.

∴ 1 man can be seated out of the remaining 3 men for one side be done in ${}^{3}C_1$, and 2 men for the other side out of the remaining 2 in ${}^{2}C_2$ ways (internal arrangement), and each side men can be arranged in 4! Ways. (External arrangement).

∴ Required number of arrangements $= {}^{3}C_1 \times {}^{2}C_2 \times 4! \times 4! = {}^{3}C_1 \times (4!)^2$

Sol.112 (d) There are 7 women in which two women refuse to join the party.
∴ 3 women are selected out of the remaining 5 women in ${}^{5}C_3$ ways. And 3 men is selected out of 10 men in ${}^{10}C_3$ ways.

∴ Required number of ways a party of 6 is to be formed $= {}^{5}C_3 \times {}^{10}C_3$

$= \frac{5!}{3! \times 2!} \times \frac{10!}{3! \times 7!} = \frac{5 \times 4}{2 \times 1} \times \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$
$= 10 \times 120 =$ **1, 200.**

Sol.113 (a) Selecting the first 11 players out of 16 players. 3 bowlers to be selected out of 4 and 1 wicket wicket-keeper out of 2.

Total player =16, number of bowler =4, number of wicket-keeper= 2, number of batsman=10.

∴ Required number of ways $= {}^{4}C_3 \times {}^{2}C_1 \times {}^{10}C_7 = 4 \times 2 \times \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 4 \times 2 \times 120 =$ **960.**

Sol.114 (a) Total player =16, number of bowler =4, number of wicket-keeper= 2, number of batsman=10.

We have to select 11 players.

∴ Required number of selection=

${}^{4}C_4 \times {}^{2}C_2 \times {}^{10}C_5 + {}^{4}C_4 \times {}^{2}C_1 \times {}^{10}C_6 + {}^{4}C_3 \times {}^{2}C_2 \times {}^{10}C_6 + {}^{4}C_3 \times {}^{2}C_1 \times {}^{10}C_7$

$= 1 \times 1 \times \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} + 1 \times 2 \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + 4 \times 1 \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + 4 \times 2 \times \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$

$= 252 + 420 + 840 + 960 =$ **2,472.**

Sol.115 (c) selection of 12 candidates out of 'n' person but two-person are fixed so we have to select 10 persons out of n-2.

∴ Required number of ways of selection $= {}^{n-2}C_{10}$

Sol.116 (d) selection of 12 candidates out of 'n' persons, but their persons are already selected, so we have to select 9 candidates out of n-3 candidates.

∴ Required number of ways of selection ${}^{n-3}C_9$

Sol.117 (a) according to question if a and b are three times as often together c, d and e are together.

Required number of ways ${}^{n-2}C_{10} = {}^{n-3}C_9 \times 3$

$\Rightarrow \frac{(n-2)!}{10! \times (n-12)!} = 3 \times \frac{(n-3)!}{9! \times (n-12)!}$

$\Rightarrow \frac{(n-2)(n-3)!}{10 \times 9!} = \frac{3 \times (n-3)!}{9!} \Rightarrow n - 2 = 30$

$\Rightarrow$ **n =32.**

Sol.118 (d) The word 'COMBINATION' has -

$C \Rightarrow 1$, $O \Rightarrow 2$, $M \Rightarrow 1$, $B \Rightarrow 1$, $I \Rightarrow 2$, $N \Rightarrow 2$, $A \Rightarrow 1$, $T \Rightarrow 1$

Number of ways in which selecting all 4 different letters$= {}^{8}C_4$ ways $= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70$.

Selecting 2 letters same and 2 are different

$= {}^{3}C_1 \times {}^{7}C_2$

Selecting 2 pairs of same letters $= {}^{3}C_2 = 3$

∴ Required number of 4 letters = 70+63+3=**136**

Sol.119 (c) ${}^{18}C_n = {}^{18}C_{n+2} \Rightarrow {}^{18}C_n = {}^{18}C_{18-(n+2)}$

$\Rightarrow n = 18 - n - 2 \Rightarrow 2n = 16 \Rightarrow$ **n=8.**

Sol.120 (d) ${}^{n}C_6 / {}^{n-2}C_3 = 91/4$

$\Rightarrow \frac{n!}{6! \times (n-6)!} \times \frac{3! \times (n-5)!}{(n-2)!} = \frac{91}{4}$

$\Rightarrow \frac{n(n-1)(n-2)! \times 6 \times (n-5)(n-6)!}{720 \times (n-6)!(n-2)!} = \frac{91}{4}$

$\Rightarrow n(n-1)(n-5) = 30 \times 91$

$\Rightarrow$ n(n-1)(n-5) =5× 2 × 3 ×7× 13.

$\Rightarrow n(n-1)(n-5) = 15 \times 14 \times 13.$

Which is not possible of any value of **n.**

Sol.121 (c) In order to pass the examination, a student have score minimum marks in each of the subjects, so a student fails if he fails in 1, 2, 3, up to 7 subjects.

∴ Required number of

ways = $^7C_1 + ^7C_2 + ^7C_3 + ^7C_4 + ^7C_5 + ^7C_6 + ^7C_7$

$= (2)^7 - 1 = \mathbf{127}.$

Sol.122 (a) Selecting one or more questions out of 6 questions, each having an alternative.

∴ Required number of ways of answering = $^6C_1 \times 2 + ^6C_2 \times (2)^2 + \dots\dots\dots + ^6C_6 \times (6)^6$

= 12+60+ 160+240+192+64 = **728.**

Sol.123 (c) There are 12 points in a plane in which 6 points are collinear. The number of different straight lines are-

Required number of straight lines = $^{12}C_2 - ^6C_2 + 1$

$= \frac{12\times11}{2\times1} - \frac{6\times5}{2\times1} + 1$

= 66-15+1= **52.**

Sol.124 (c) There are 12 points in a plane in which 6 points are collinear. The number of different triangles formed by joining the straight lines-

∴ Required numbers of triangles = $^{12}C_3 - ^6C_3$

$= \frac{12\times11\times10}{3\times2\times1} - \frac{6\times5\times4}{3\times2\times1} = 220 - 20 = \mathbf{200}$

Sol.125 (a) Selecting 2 teachers out of 10 teachers in $^{10}c_2$ ways and 3 students out of 20 students in $^{20}c_3$ ways.

Required number of ways = $\mathbf{^{10}C_2 \times ^{20}C_3}$.

Sol.126 (b) If selecting 1 teacher out of the remaining 9 teachers because the particular teacher is included in $^9C_1$ ways. And 3 students are selected out of 20 students in $^{20}C_3$ ways.

∴Required number of ways = $\mathbf{^9C_1 \times ^{20}C_3}$

Sol.127 (c) If a particular student is excluded, then we have to select 3 students out of 19 students in $^{19}C_3$ ways. And 2 teachers are selected out of 10 teachers in $^{10}C_2$ ways.

Required number of ways = $\mathbf{^{10}C_2 \times ^{19}C_3}$

Sol.128 (a) Selecting 21 red balls and 19 blue balls so that no two blue balls are together. So firstly 21 balls are arranged and then in space between then blue balls are arranged as shown in $\frac{21!}{21!}$ ways and 22 space available x positions filled by 19 balls in $^{22}C_{19}$ 19!/19! Ways (all balls are identical)

$\times R \times R \times R \times \dots\dots\dots\dots R \times$

∴ Required number of ways of arrangement

$= \frac{21!}{21!} \times ^{22}C_{19} \times \frac{19!}{19!}$

$= 1 \times \frac{22!}{3! \times 19!} = \frac{22\times21\times20}{3\times2\times1} = \mathbf{1,540}.$

Sol.129 (a) Selecting 3 males out of 5 males and 2 females out of 6 females to make a committee of 5 people.

∴ Required number of ways

$= ^5C_3 \times ^6C_2 = \frac{5!}{3!\times2!} \times \frac{6!}{2!\times4!}$

$= \frac{5\times4}{2\times1} \times \frac{6\times5}{2\times1} = \mathbf{150}.$

Sol.130 (b) Selecting 2 males out of 5 males and 3 females out of 6 females to make a committee of 5 people.

Required number of ways to make a committee of 5 people = $^5C_2 \times ^6C_3$

$= \frac{5!}{2!\times3!} \times \frac{6!}{3!\times3!}$

$= \frac{5\times4}{2\times1} \times \frac{6\times5\times4}{3\times2\times1} = \mathbf{200}$

Sol.131 (c) If there is no female, then we have to select 5 out of 5 males in $^5C_5$ ways and arranged in 5! Ways.

∴Required number of ways of selection = $^5C_5$ = **1**

Sol.132 (d) If there must be a single female then, it may be 1 female ,2 female, 3 female, 4 female or 5 female.

∴Required number of choices

$= {}^6C_1 \times {}^5C_4 + {}^6C_2 \times {}^5C_3 + {}^6C_3 \times {}^5C_2 + {}^6C_4 \times {}^5C_1 + {}^6C_5 \times {}^5C_0$

$= 6 \times 5 + \frac{6\times5}{2\times1} \times \frac{5\times4}{2\times1} + 20 \times 10 + 15 \times 5 + 6 \times 1$

$= 30 + 150 + 200 + 75 + 6 = \mathbf{461}$.

Sol.133 (d) If we have to select not more than 3 males then it may be 0 male, 1 male, 2male and 3 male.

∴Required number of choices $= {}^5C_0 \times {}^6C_5 + {}^5C_1 \times {}^6C_4 + {}^5C_2 \times {}^6C_3 + {}^5C_3\,{}^6C_2$

$= 1 \times 6 + 5 \times 15 + 10 \times 20 + 10 \times 15$

$= 6 + 75 + 200 + 150 = \mathbf{431}$.

Sol.134 (a) If we have to make a committee of 5 out of 7 men and 4 women there should be at least 1 woman so it may be 1 ,2,3 and 4 women.

∴Required number of ways

$= {}^7C_4 \times {}^4C_1 + {}^7C_3 \times {}^4C_2 + {}^7C_2 \times {}^4C_3 + {}^7C_1 \times {}^4C_4$

$= 35 \times 4 + 35 \times 6 + 21 \times 4 + 7 \times 1$

$= 140 + 210 + 84 + 7 = \mathbf{441}$.

Sol.135 (a) If a red ball is always included, it means we have to select 3 balls out of 11 balls in which 1 blue and 10 white balls.

∴ Required number of ways = **$^{11}C_3$ways.**

Sol.136 (b) If a red ball is always included, but the blue ball is excluded, so we have to select 3 balls out of the remaining 10 white balls. In **$^{10}C_3$ ways**

Sol.137 (c) If we have to select 4 balls out of 10 balls because blue and red balls are excluded. So required number of ways = **$^{10}C_4$ ways.**

Sol.138 (b) If party A has a majority, it means at least 3 persons out 5 persons are from party A.

∴ Required number of ways

$= {}^6C_5 \times {}^4C_0 + {}^6C_4 \times {}^4C_1 + {}^6C_3 \times {}^4C_2$

$= 6 \times 1 + 15 \times 4 + 20 \times 6 = \mathbf{186}$.

Sol.139 (c) Required number of ways of selection= $^3C_1 \times {}^4C_1$

Sol.140 (a) Firstly, we have to select I vowel out of 3 and 2 consonants out of 7 and arrangement in – C V C

∴ Required number of arrangements

$= {}^3C_1 \times {}^7C_2 \times 2! = \mathbf{3 \times 7 \times 6}$.

Sol.141 (a) Selecting 4 at a time, there being at least one odd and even-numbered counter in each combination.

∴ Required numbers of combination

$= {}^4C_1 \times {}^4C_3 + {}^4C_2 \times {}^4C_2 + {}^4C_3 \times {}^4C_1$

$= 4 \times 4 + 6 \times 6 + 4 \times 4 = 16 + 36 + 16 = \mathbf{68}$.

Sol.142 (d) The word MATHEMATICS has M⟹2, A⟹2, T⟹2, H⟹1, E⟹1, I⟹1, C⟹1, S⟹1.

Selecting 4 letters all are different= $^8C_4$ ways

Selecting two different letters and two are same letters = $^3C_1 \times {}^7C_2$

Selecting two different pairs of same letters = $^3C_2$ ways

∴ Required number of ways of selection

$= {}^8C_4 + {}^3C_1 \times {}^7C_2 + {}^3C_2$

$= \frac{8\times7\times6\times5}{4\times3\times2\times1} + 3 \times 21 + 3$

$= 70 + 63 + 3 = \mathbf{136}$.

Sol.143 (d) The word MATHEMATICS has M⟹2, A⟹2, T⟹2, H⟹1, E⟹1, I⟹1, C⟹1, S⟹1.

Arrangement of 4 letters and all are different= $^8C_4 \times 4!$

Arrangement two different letters and two are same letters = $^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$

Arrangement of two different pairs of same letters $= {}^3C_2 \times \frac{4!}{2! \times 2!}$

∴ Required number of ways of selection $= {}^8C_4 \times 4! + {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} + {}^3C_2 \times \frac{4!}{2! \times 2!}$

$= \frac{8!}{4! \times 4!} \times 4! + 3 \times 21 \times 12 + 3 \times \frac{24}{2\times2}$

$= 8 \times 7 \times 6 \times 5 + 756 + 18 = \mathbf{2,454}$

Sol.144 (a) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \Rightarrow$ for 12 words

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \Rightarrow \mathit{for\ 8\ words} \Rightarrow (2)^8$

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Sequence and Series

Exercise: 6A

Sol.1 (b) $a_n = a + (n-1)d$

$= 1 + (n-1) \times 2 = \mathbf{2n-1}$

Or

put n=2 if we put n=2 in option b, it gives 3, so L.H.S and R.H.S satisfy.

Sol.2 (a) $a_n = ar^{n-1} = (-1)(-2)^{n-1}$

$= (-1)(-1)^{n-1}2^{n-1}$

$= (-1)^n 2^{n-1}$

Or

Put n =2 in the options, and when we get 2, then it is the nth term, but a and b both options satisfy, so we put n=3, then only option a satisfy.

Sol.3 (a) $\sum_{i=4}^{7} \sqrt{2i-1}$

$$= \sqrt{2 \times 4 - 1} + \sqrt{2 \times 5 - 1} + \sqrt{2 \times 6 - 1} + \sqrt{2 \times 7 - 1}$$

$$= \mathbf{\sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}}$$

Sol.4 (a) $a_k = ar^{k-1} = -5(-5)^{k-1} = (-5)^k$

$\therefore$ Required sum$= \sum_{k=1}^{\infty} a_k = \mathbf{\sum_{k=1}^{\infty}(-5)^k}$

Sol.5 (a) $t_n = n^2 - 2n$

$\therefore t_1 = 1^2 - 2 \times 1 = -1$

$t_2 = 2^2 - 2 \times 2 = 0$

$t_3 = 3^2 - 2 \times 3 = \mathbf{3}$

Sol. 6 (b) $a = -1, d = -2$

$a_n = -39$

$\Rightarrow -1 + (n-1)(-2) = -39$

$\Rightarrow -1 - 2n + 2 = -39$

$\Rightarrow -2n = -40$

$$\Rightarrow n = \frac{-40}{-2} = \mathbf{20}$$

Sol. 7 (c) $8x+4, 6x-2, 2x+7$ are in A.P.

$\therefore 2 \times (6x-2) = (8x+4) + (2x+7)$

$\Rightarrow 12x - 4 = 10x + 11$

$$\Rightarrow 2x = 15 \Rightarrow x = \mathbf{\frac{15}{2}}$$

Sol.8 (d) $a_m = n, a_n = m$

$a_r = m + n - r$

(Shortcut 5 when $a_p = q, a_q = p$ *then* $a_r = p + q - r$

or

Let 1st term & common diff. of an A.P. be a & d respectively

$a_m = n \Rightarrow a + (m-1)d = n$ ______ (I)

$a_n = m \Rightarrow a + (n-1)d = m$ ______ (II)

from [(I) - (II)]

$(m-n)d = n - m$

$$\Rightarrow d = \frac{-(m-n)}{m-n} = -1$$

$\therefore a = n + m - 1$

$\therefore a_r = a + (r-1)d$

$= m + n - 1 + (r-1)(-1)$

$= m + n - 1 - r + 1$

$= \mathbf{m + n - r}$

Sol. 9 (a,b)

$a = 10,\ d = 9\frac{2}{3} - 10 = -\frac{1}{3}$

$s_n = 155$

$$\Rightarrow \frac{n}{2}\left\{2 \times 10 + (n-1)\left(-\frac{1}{3}\right)\right\} = 155$$

$\Rightarrow n(60 - n + 1) = 155 \times 6$

$\Rightarrow n^2 - 61n + 930 = 0$

$\Rightarrow n^2 - 61n + 30 \times 31 = 0$

$\Rightarrow n^2 - 30n - 31n + 30 \times 31 = 0$

$\Rightarrow n(n-30) - 31(n-30) = 0$

$\Rightarrow (n-31)(n-30) = 0$

$\Rightarrow n - 31 = 0$ or $n - 30 = 0$

$\Rightarrow \mathbf{n = 31}$ **or** $\mathbf{n = 30}$

Sol. 10 I $s_n = 5n^2 + 2n$

Put n =1

$= a_1 = s_1 = 5(1)^2 + 2(1) = 7$

$= a_1 + a_2 = s_2 = 5(2)^2 + 2(2) =$ 24

$a_2 = 17$

$\therefore$ *put* $n = 2$ *in options and option c satisfy.*

=$10n - 3$

Sol. 11 (a) $a = 1, d = 3$

$a_{20} = a + 19d = 1 + 19 \times 3 = 58$

Sol. 12 I $a = 5, d = 2$

$a_{21} = a + 20d = 5 + 20 \times 2 = \mathbf{45}$

Sol. 13 (b) $a = 0.6, d = 1.2 - 0.6 = 0.6$

$a_{13} = a + 12d = 0.6 + 12 \times 0.6 = \mathbf{7.8}$

Sol. 14 (a) $a = 9, d = -4$

$\therefore S_{100} = \frac{100}{2}\{2 \times 9 + 99 \times (-4)\}$

$= 50 \times (18 - 396)$

$= 50 \times (-378) = \mathbf{-18,900}$

Sol. 15 (b) -6,_ _14

= a_1 = -6, a_4 = a+3d=14, -6+3d=14.

= d =20/3

$\Rightarrow \ a_2 = -6 + \frac{20}{3} = 2/3$

$\Rightarrow \ a_3 = \frac{2}{3} + \frac{20}{3} = \frac{22}{3} = \mathbf{7\frac{1}{3}}$

Sol. 16 (c, d) Let the integers are $a - d, a \,\&\, a + d$

$a - d + a + a + d = 15 \Rightarrow 3a = 15$

$\Rightarrow a = 5$

Also $(a - d)\ a(a + d) = 80$

$\Rightarrow (a^2 - d^2)a = 80 \Rightarrow (25 - d^2)5 = 80$

$\Rightarrow 25 - d^2 = 16 \Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

$\therefore$ **Nos. are 2, 5, 8 or 8, 5, 2**

Sol. 17 (b) $S_n = 3n^2 + 5n$

When n=1$\Rightarrow$ $S_1 = 3(1)^2 + 5 = 8$, *when* $n = 2\ s_2 = 3(2)^2 + 5(2) = \mathbf{22}$.

When n=3 $\Rightarrow S_3 = 3(3)^2 + 5(3) =$ **42**. *option b satisfy.*

Sol. 18 (b) $a = 75,\ d$=5

a_n =25555

$\Rightarrow 75 + (n - 1) \times 5 = 25555$

$\Rightarrow 5n = 25555 - 70$

$\Rightarrow n = \frac{25485}{5} = \mathbf{5,097}$

Sol. 19 (b) $a_p = \frac{3p-1}{6}$

when p=1 $\Rightarrow \frac{3(1)-1}{6} = \frac{2}{6} = \frac{1}{3}$

When p=2 $\Rightarrow a_2 = \frac{3(2)-1}{6} = \frac{5}{6}$

when $n = 3 \Rightarrow a_3 = \frac{3(3)-1}{6} = \frac{4}{3}$

Sum of first 3 terms = $\frac{1}{3} + \frac{5}{6} + \frac{4}{3} = \frac{2+5+8}{6} = \frac{15}{6} = \mathbf{\frac{5}{2}}$

Put n=3 in options, then option b satisfy.

Sol. 20 I A.M= $\frac{33+77}{2} = \frac{110}{2} = \mathbf{55}$

Sol. 21 I A= -2 , A+ 5d=23$\Rightarrow$ -2+5d=23 $\Rightarrow$ d= 5

$\therefore A_1 = a + d = -2 + 5 = 3$

$A_2 = a + 2d = -2 + 10 = 8$

$A_3 = a + 3d = -2 + 15 = 13$

$A_4 = a + 4d = -2 + 20 = \mathbf{18}$

Sol. 22 (a) $a = 14$

Also, $S_5 + S_{10} = 0$

$\Rightarrow \frac{5}{2}[2a + 4d] = -\frac{10}{2}[2a + 9d]$

$\Rightarrow 10a + 20d + 20a + 90d = 0$

$\Rightarrow 30a + 110d = 0$

$\Rightarrow d = \frac{-15a}{55}$

= d = $\frac{\overset{3}{\cancel{-15}} \times 14}{\underset{11}{\cancel{55}}} = \frac{-42}{11}$

$\therefore a_3 = a + 2d = 14 - \frac{84}{11} = \frac{154-84}{11}$

$= \frac{70}{11} = \mathbf{6\frac{4}{11}}$

Sol. 23 (b) $S_n = 52$

$\Rightarrow \frac{n}{2}\{2 \times (-8) + (n - 1)2\} = 52$

$\Rightarrow \frac{n}{2} \times 2(-8+n-1) = 52$

$\Rightarrow n^2 - 9n - 52 = 0$

$\Rightarrow (n-13)(n+4) = 0$

$\Rightarrow n - 13 = 0 \Rightarrow \boldsymbol{n = 13}$

($\because n$ can't be ve)

Sol. 24 (a) $a = -4, a_n = 146$

$S_n = 7171$

$\Rightarrow \frac{n}{2}\{a + an\} = 7171$

$\Rightarrow \frac{n}{2}(-4 + 146) = 7171$

$\Rightarrow \frac{n}{2}(142) = 7171$

$\Rightarrow 71n = 7171 \Rightarrow n = \frac{7171}{71} = \mathbf{101}$

Sol. 25I $S_{17} = \frac{17}{2}\left\{2 \times 3\frac{1}{2} + 16 \times 3\frac{1}{2}\right\}$

$= \frac{17}{2}\{7 + 56\}$

$= \frac{17}{2} \times 63 = \frac{1071}{2} = \mathbf{535\frac{1}{2}}$

……

Sequence and Series

Exercise: 6B

Sol.1 (a) $t_7 = ar^6 = 6 \times 2^6 = \mathbf{384}$

Sol.2 (b) $t_8 = ar^7 = 6 \times 2^7 = \mathbf{768}$

Sol.3 I $t_{12} = ar^{11} = -128 \times \left(\frac{-1}{2}\right)^{11}$

$=(-2)^7 \times (-2)^{-11} = (-2)^{-4} = \frac{1}{(-2)^4} = \mathbf{\frac{1}{16}}$

Sol.4 I $t_4 = ar^3 = 0.04 \times 5^3 = \mathbf{5}$

Sol.5 (a) $a_{10} = ar^9 = 1 \times 2^9 = \mathbf{512}$

Sol.6 (b) $a_7 = ar^6 = 1 \times (-3)^6 = \mathbf{729}$

Sol.7 I $a_{31} = ar^{30} = x^2 \times \left(\frac{1}{x}\right)^{30} = \mathbf{\frac{1}{x^{28}}}$

Sol.8 (a) $s_7 = \frac{a(r^7-1)}{r-1} = \frac{(-2)[(-3)^7-1]}{-3-1}$

$= \frac{\cancel{-2} \times (-2{,}187 - 1)}{\underset{2}{\cancel{-4}}}$

$= \frac{-2{,}188}{2} = \mathbf{-1{,}094}$

Sol.9 (d) $s_8 = \frac{a(1-r^8)}{1-r}$

$= \frac{243\left[1 - \left(\frac{1}{3}\right)^8\right]}{1 - \frac{1}{3}}$

$=243 \times \frac{3}{2}\left(1 - \frac{1}{3^8}\right)$

$= \frac{729}{2}\ (\frac{6560}{6561})$

$= \frac{\overset{3280}{\cancel{6560}}}{\underset{9}{\cancel{18}}} = \frac{3280}{9} = \mathbf{364\frac{4}{9}}$

Sol.10 (a) $s_{18} = \frac{a(r^{18}-1)}{r-1}$ $\qquad r_1 = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$= \frac{\frac{1}{\sqrt{3}}\left[(\sqrt{3})^{18} - 1\right]}{\sqrt{3}-1}$

$= \frac{1}{\sqrt{3}(\sqrt{3}-1)}(3^9 - 1) = \frac{1}{\sqrt{3}(\sqrt{3}-1)}(19683 - 1)$

$= \frac{19682\sqrt{3}+1}{\sqrt{3}(\sqrt{3}-1)(\sqrt{3}+1)} = \mathbf{\frac{9841(\sqrt{3}+1)}{\sqrt{3}}}$

Sol.11 I $ar = 24$ ______(I)

$ar^4 = 81$______(II)

Equation II divide by equation I

$\therefore \frac{ar^4}{ar} = \frac{\overset{27}{\cancel{81}}}{\underset{8}{\cancel{24}}} \Rightarrow r^3 = \left(\frac{3}{2}\right)^3 \Rightarrow r = \frac{3}{2}$

$\therefore a = \cancel{24}^{8} \times \frac{2}{\cancel{3}} = 16$

∴Required series **16, 24, 36, 54**, ...

Sol.12 I Do it by option since 1st and second options are not in G.P and then go through the third option then it is in G.P

= 3+9+27= 39.

=3× 9 × 27

$= 729.$ *and c options satisfy both conditions.*

Or

$\frac{a}{r} + a + ar = 39$

$\Rightarrow \frac{a}{r}(1 + r + r^2) = 39$______(I)

Also $\frac{a}{r} . a . ar = 729$

$\Rightarrow a^3 = 9^3 \Rightarrow a = 9$______(II)

From (I) & (II)

$$\frac{9^3}{r}(1 + r + r^2) = \cancel{39}^{13}$$

$$\Rightarrow 3r^2 + 3r + 3 = 13r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow (3r - 1)(r - 3) = 0 \Rightarrow 3r - 1 = 0 \text{ or } r - 3 = 0$$

$$\Rightarrow r = \frac{1}{3} \; or \; r = 3$$

Then the numbers are **27, 9, 3, or 3, 9, 27**

Sol. 13 (a) Let the three terms in G.P. be $\frac{a}{r}$, a & ar

$$\frac{a}{r} . a . ar = \frac{27}{8} \Rightarrow a^3 = \left(\frac{3}{2}\right)^2$$

$$\Rightarrow a = \frac{3}{2}$$

Sol. 14 I $= (1 + 2 + 4 + \cdots to\ 14$ terms)

$$= \frac{1(2^{14} - 1)}{2 - 1}$$

= ₹ 163.83

Sol. 15 (a) $s_n = 4 + 44 + 444 + \cdots$ to n terms

$$= \frac{4}{9}[9 + 99 + 999 + \cdots to\ n\ terms]$$

$$= \frac{4}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \cdots to\ n\ terms]$$

$$= \frac{4}{9}[(10 + 10^2 + 10^3 + \cdots to\ n\ terms) - n]$$

$$= \frac{4}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] = \boldsymbol{\frac{4}{9}\left\{\frac{10}{9}(10^n - 1) - n\right\}}$$

Sol.16(b) $s_n = 0.1 + 0.11 + 0.111 + \cdots to\ n\ terms$

$$= \frac{1}{9}[.9 + .99 + .999 + \cdots to\ n\ terms]$$

$$= \frac{1}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \cdots to\ n\ terms]$$

$$= \frac{1}{9}[n - (0.1 + 0.01 + 0.001 + \cdots to\ n\ terms)]$$

$$= \frac{1}{9}\left[n - \frac{(0.1)(1-(0.1)^n)}{(1-0.1)}\right]$$

$$= \frac{1}{9}[\text{n-} \frac{0.1(1-(0.1)^n}{0.9}]$$

$$= \boldsymbol{\frac{1}{9}\left[n - \frac{1}{9}\{1 - (0.1)^n\}\right]}$$

Sol. 17 (a) $s_{20} = 244 s_{10}$

$$\Rightarrow \frac{\cancel{a}(r^{20} - 1)}{\cancel{r - 1}} = 244 \times \frac{\cancel{a}(r^{10} - 1)}{\cancel{r - 1}}$$

$$\Rightarrow \frac{r^{20}-1}{r^{10}-1} = 244 \Rightarrow \frac{(r^{10}+1)(r^{10}-1)}{r^{10}-1} = 244$$

$$\Rightarrow r^{10} + 1 = 244$$

$$\Rightarrow r^{10} = 243 \Rightarrow (r^2)^5 = 3^5$$

$$\Rightarrow r^2 = 3 \Rightarrow r = \pm\boldsymbol{\sqrt{3}}$$

Sol. 18 (b) $s_n = 364$, r=3

$$\Rightarrow \frac{1(3^n - 1)}{3 - 1} = 364 \Rightarrow 3^n - 1 = 728$$

$$\Rightarrow 3^n = 729 \Rightarrow 3^n = 3^6 \Rightarrow \boldsymbol{n = 6}$$

Sol. 19 I Do this question by option, and option a and b are not in G.P. Only option c is in G.P.

$\Rightarrow 3^2 + 9^2 + 27^2 =$
$819\ so\ option\ c\ satify\ conditions, therefore,$

$it\ is\ correct.$

Or

Let the three nos. in G.P. be $\frac{a}{r}$, a & r

<u>ATP</u> $\frac{a}{r} . a . ar = 729 \Rightarrow a^3 = 9^3$

$\Rightarrow a = 9$______(I)

Also $\frac{a^2}{r^2} + a^2 + a^2r^2 = 819$

$$\Rightarrow \frac{a^2}{r^2}(1 + r^2 + r^4) = 819$$

$$\Rightarrow 1 + r^2 + r^4 = \cancel{819}^{91} \times \frac{r^2}{\cancel{81}_9}$$

$$\Rightarrow 9r^4 - 82r^2 + 9 = 0$$

$$\Rightarrow 9r^4 - 81r^2 - r^2 + 9 = 0$$

$\Rightarrow 9r^2(r^2-9)-1(r^2-9)=0$

$\Rightarrow (9r^2-1)(r^2-9)=0 \Rightarrow 9r^2-1=0 \; or \; r^2-9=$

$\Rightarrow r^2 = \frac{1}{9} \; or \; r^2 = 9$

$\Rightarrow r = \pm\frac{1}{3} \; or \; r = \pm 3$

$\therefore$ Numbers are **27, 9, 3 or 3, 9, 27**

Sol. 20 (a) $s_n = \frac{a(r^n-1)}{r-1} = \frac{1(2^n-1)}{2-1} = \mathbf{2^n - 1}$

Sol. 21 (b) $s_\infty = \frac{a}{1-r}$, r= $\frac{-1}{7}$

$= \frac{14}{1-\frac{1}{7}} = \frac{\overset{7}{\cancel{14}} \times 7}{\underset{4}{\cancel{8}}} = \frac{49}{4} = \mathbf{12\frac{1}{4}}$

Sol. 22 I $s_\infty = \frac{a}{1-r}$ r= $\frac{-1}{3}$

$= \frac{1}{1-\frac{1}{3}} = \frac{3}{4} = \mathbf{0.75}$

Sol. 23 (b) $s_n = 8191 \Rightarrow \frac{1(2^n-1)}{2-1} = 8191$

$\Rightarrow 2^n - 1 = 8191 \Rightarrow 2^n = 8192$

$\Rightarrow 2^n = 2^{13} \Rightarrow \boldsymbol{n = 13}$

Sol. 24 (a) 4,_ __ _972

$\Rightarrow a_6 = ar^5 = 972 \Rightarrow 4r^5 = 972$

$\Rightarrow r^5 = 243, \; r = 3.$

$\Rightarrow a_2 = a_1 \times r = 4 \times 3 = 12$

$\Rightarrow a_3 = a_2 \times r = 12 \times 3 = 36$

$\Rightarrow a_4 = a_3 \times r = 36 \times 3 = 108$

$\Rightarrow a_5 = a_4 \times r = 108 \times 3 = \mathbf{324}$

……

Sequence and Series
Exercise - 6C

Sol.1 (a) Do it by options in option a 5,7,9 is in A.P so first condition satisfy and sum= 1+5+15= 21

Add 1 in 1st term 5 in the second term, and 15 in the third term. The numbers are 6,12 and 24 $r_1 = 2\,r_2 = 2$

$\therefore$ 5,7 and 9 options satisfy all conditions.
Or

Let the three nos. in A.P. be $a-d, a \;\&\; a+d$

<u>ATQ</u>

$a-d+a+a+d=21 \Rightarrow 3a{=}21 \Rightarrow a=7$ ______(I)

Also $a-d+1, a+5 \;\&\; a+d+15$ are in G.P.

$\therefore (a+5)^2 = (a-d+1)(a+d+15)$
$(b^2 = ac)$

$\Rightarrow (12)^2 = (8-d)(22+d)$[from (I)]

$\Rightarrow 144 = 176 - 14d - d^2$

$\Rightarrow d^2 + 14d - 32 = 0 \Rightarrow (d+16)(d-2) = 0$
$\Rightarrow d + 16 = 0 \; or \; d - 2 = 0$
$\Rightarrow d = -16 \; or \; d = 2$
$\therefore$ Required nos. are **5, 7 & 9**

Sol.2 (d) r =$\frac{1}{3}$

$1 + \frac{1}{3} + \frac{1}{3^2} + \; ... + \frac{1}{3^{n-1}}$

$= \frac{1\left[1-\left(\frac{1}{3}\right)^n\right]}{1-\frac{1}{3}} = \frac{\mathbf{3}}{\mathbf{2}}\left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{3^n}}\right)$

Sol.3 (b) $s_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = \mathbf{3}$

Sol.4 (b, c)

$a + ar = \frac{5}{3} \Rightarrow a(1+r) = \frac{5}{3}$______ (I)

$s_\infty = 3 \Rightarrow \frac{a}{1-r} = 3$______(II)

From [(I) ÷ (II)]

$\frac{a(1+r)}{a} \times (1-\mathrm{r}) = \frac{5}{3} \times \frac{1}{3}$

$\Rightarrow 1 - r^2 = \frac{5}{9} \Rightarrow r^2 = 1 - \frac{5}{9}$

$\Rightarrow r^2 = \frac{4}{9} \Rightarrow r = \pm\frac{2}{3}$

Sol.5 I $\because p, q, r \; in \; A.P. \Rightarrow p + r = 2q$

x, y, z are in $G.P.$

$\therefore \frac{y}{x} = \frac{z}{y} = k(let)$

$\therefore y = kx, z = y\,k \qquad \mathrm{z} = kx(\mathrm{k}) \Rightarrow \mathrm{z} = k^2x$

$\therefore x^{q-r}.y^{r-p}.z^{p-q} = x^{q-r}.(xk)^{r-p}.(x\,k^2)^{p-q}$

$= x^{q-r+r-p+p-q}\,k^{r-p+2p-2q}$

$= x^{\circ} \times k^{\circ} = 1 \times 1 = \mathbf{1}$

Sol.6 (b, c) Do it by options b and c both are in G.P then in option b multiply 4 with 10 and 40 the new series is 40, 100 and 160 so this series is in A.P, and when we do the same procedure in option c the new series is 160, 100, 40, so this is also in A.P.

Or

Let the three nos. in G.P. be $\frac{a}{r}, a$ & ar

$\frac{a}{r} + a + ar = 70 \Rightarrow \frac{a}{r}(1 + r + r^2) = 70$ ______(I)

ATQ

$4\frac{a}{r}, 5a$ & $4ar$ are in A.P.

$\therefore\ 2 \times 5a = \frac{4a}{r} + 4ar$

$\Rightarrow 10 = \frac{4}{r} + 4r$

$\Rightarrow 4r^2 - 10r + 4 = 0 \Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (2r - 1)(r - 2) = 0 \Rightarrow 2r - 1 = 0\ or\ r - 2 = 0$

$\Rightarrow r = \frac{1}{2}\ or\ r = 2$

If $r = \frac{1}{2}$ than $a = \frac{70\times\frac{1}{2}}{1+\frac{1}{2}+\frac{1}{4}} = \frac{35\times4}{4+2+1}$

$= \frac{\overset{5}{\cancel{35}} \times 4}{\cancel{7}} = 20$

$\Rightarrow a = 20$

$\therefore$ Numbers are 40, 20 & 10

If $r = 2$ than $a = \frac{\overset{10}{\cancel{70}}\times2}{\cancel{7}} = 20$

Then the numbers are **10, 20, 40**

Sol.7 (a,b) Options a and b both are in A.P add 1, 4 and 19 respectively in the first, second, and the third term of option a so we get new series is 27, 9, and 3, which is in G.P and when we add 1,4 and 19 respectively in option b, so we get new series is 3, 9 and 27 which is also in G.P so both option a and b are correct.

Or

Let the nos. are $a - d, a$ & $a + d$

$a - d + a + a + d = 15 \Rightarrow 3a = 15 \Rightarrow a = 5$

$\Rightarrow (a - d + 1), (a + 4) \& (a + d + 19)$ are in G.P.

$\Rightarrow (a + 4)^2 = (a - d + 1) \times (a + d + 19)$

$\Rightarrow 9^2 = (6 - d)(24 + d)$

$\Rightarrow 81 = 144 - 18d - d^2$

$\Rightarrow d^2 + 18d - 63 = 0 \ \Rightarrow d^2 + 21d - 3d - 63 = 0$

$\Rightarrow$ d(d+21) -3(d+21)=0

$\Rightarrow (d + 21)(d - 3) = 0$

$\Rightarrow d + 21 = 0\ or\ d - 3 = 0$

$\Rightarrow d = -21\ or\ d = 3$

If $a = 5$ & $d = 3$, then

$\therefore$ Numbers are 2, 5 & 8

If $a = 5$ & $d = -21$

Then the numbers are = **26, 5, −16**

Sol.8 (a) x, y, z are in G.P.

$\Rightarrow \frac{y}{x} = \frac{z}{y} \Rightarrow y^2 = xz$ ______(I)

Now $x^p = y^q = z^\sigma = k\ (let)$

$\Rightarrow x = k^{1/p}, y = k^{1/q}$ & $z = k^{\frac{1}{\sigma}}$ ____(II)

From (I) & (*II*)

$k^{2/q} = \mathrm{k}^{1/\mathrm{p}}.\ \mathrm{k}^{1/\sigma}$

$\Rightarrow k^{2/q} = \mathrm{k}^{\frac{1}{\mathrm{p}}\cdot\frac{1}{\sigma}} \Rightarrow \frac{2}{\mathrm{q}} = \frac{1}{\mathrm{p}} + \frac{1}{\sigma}$

$\therefore \frac{1}{p}, \frac{1}{q}$ & $\frac{1}{\sigma}$ **are in A.P.**

Sol.9 I $\because\ 2x, (x + 10)$and $(3x + 2)$are in A. P.

$\Rightarrow 2(x + 10) = 2x + 3x + 2$

$\Rightarrow 2x + 20 = 5x + 2$

$\Rightarrow 3x = 18 \Rightarrow \boldsymbol{x = 6}$

Sol.10 I Positive and equal number

⇒ A.M =G.M=H.M

⇒ unequal = A.M is greater than G.M

$A = \frac{x+y}{2}$ & $G = \sqrt{xy}$

$A - G = \frac{x+y}{2} - \sqrt{xy} \Rightarrow$ A-G = $\frac{x+y-2\sqrt{xy}}{2}$

$$= \frac{\left(\sqrt{x}-\sqrt{y}\right)^2}{2} \geq 0$$

$\Rightarrow A - G \geq 0$

$\Rightarrow \boldsymbol{A \geq G}$

Sol.11 (a) Do this question by option A.M =40 and G.M = 24. A.M is 40 in a, b and c options, and G.M is 24 only in option a = $\sqrt{72 \times 8} = 24$ *so option is correct.*

Or

Let the nos. be a & b

$\therefore \frac{a+b}{2} = 40 \Rightarrow a + b = 80$

$\Rightarrow b = 80 - a$ ______(I)

Also $\sqrt{ab} = 24$

$\Rightarrow ab = 576 \Rightarrow a(80 - a) = 576$ [From I]

$\Rightarrow a^2 - 80a + 576 = 0 \Rightarrow a^2 - 72a - 8a + 576 = 0$

⇒ a(a-72) -8(a-72) =0

$\Rightarrow a = 8$ or 72 if $a = 8$ then $b = 72$

If $a = 72$ *then* $b = 8$

Sol.12 I Do through options. Only option c numbers are in A.P., and when we add 8, 6, and 4, the series is in G.P, so option c is correct.

Or

Let the three nos. in A.P. be $a - d, a$ & $a + d$

$a - d + a + a + d = 15 \Rightarrow 3a = 15$

$\Rightarrow a = 5$______(I)

Also $a - d + 8, a + 6, a + d + 4$ are in G.P.

$\therefore (a+6)^2 = (a-d+8)(a+d+4)$

$\Rightarrow (11)^2 = (13-d)(9+d)$[from (I)]

$\Rightarrow 121 = 117 + 4d - d^2$

$\Rightarrow d^2 - 4d + 4 = 0$

$\Rightarrow (d-2)^2 = 0 \Rightarrow d - 2 = 0 \Rightarrow d = 2$

∴ Numbers are **3, 5 & 7**

Sol.13 (a) Do through options. In series 4, 8, 16, 32 $r_1 = 2, r_2 = 2$ so only in option a) series are in G.P., and all conditions are satisfied.

Or

Let the four nos. in G.P. be a, ar, ar^2 & ar^3

$a + ar + ar^2 + ar^3 = 60$

$\Rightarrow a(1 + r + r^2 + r^3) = 60$_____(I)

Also $\frac{a+ar^3}{2} = 18 \Rightarrow a(1 + r^3) = 36$_____(II)

From [(I) ÷ (II)]

$$\frac{a(1+r+r^2+r^3)}{a(1+r^3)} = \frac{\overset{5}{\cancel{60}}}{\underset{3}{\cancel{36}}}$$

$\Rightarrow 3 + 3r + 3r^2 + 3r^3 = 5 + 5r^3$

$\Rightarrow 2r^3 - 3r^2 - 3r + 2 = 0$

$\Rightarrow (r+1)(2r^2 - 5r + 2) = 0$

$\Rightarrow (r+1)(2r^2 - 4r - r + 2) = 0$

$\Rightarrow (r+1)[2r(r-2) - 1(r-2)] = 0$

$\Rightarrow (r+1)(2r-1)(r-2) = 0$

$\Rightarrow r + 1 = 0, 2r - 1 = 0$ *or* $r - 2 = 0$

$\Rightarrow r = -1, r = \frac{1}{2}$ *or* $r = 2$

but $r = -1$ is not possible, so it is rejected.

If $r = \frac{1}{2}$ then $a = 32$

∴ Numbers are 32, 16, 8, 4

If $r = 2$ *than* $a = 4$

Numbers are **4, 8, 16 & 32**

Sol.14 (d)

$S_n = 6240$

$\Rightarrow \frac{\overset{15}{\cancel{30}}}{\cancel{2}}\{2a + (30-1) \times 10\} = 6240$

$\Rightarrow 2a + 290 = \frac{6240}{15}$

$\Rightarrow 2a = 416 - 290$

$\Rightarrow 2a = 126$

$\Rightarrow a = \frac{126}{2} = \mathbf{63}$

Sol.15 (b) r = 1.03

$S_n = \frac{a(r^n-1)}{r-1} = \frac{1.03[(1.03)^n-1]}{1.03-1}$

$= \frac{103}{3}[(1.03)^n - 1]$

Sol.16 (a)

x, y, z are in A.P.

$\therefore 2y = x + z$_______(I)

Also $x, y, (z+1)$ are in G.P.

$\therefore y^2 = x(z+1)$

$\Rightarrow \left(\frac{x+z}{2}\right)^2 = xz + x$ [From I]

$\Rightarrow (x+z)^2 = 4xz + 4x$

$\Rightarrow (x+z)^2 - 4xz = 4x$

$\Rightarrow \boldsymbol{(x-z)^2 = 4x}$

Sol. 17 (a,b) Do it by options when we put option a in A.P then series is -8,-8,-8 and when in G.P the new series is -8,8,-8 so r =-1. And when we put option b in the A.P. series, then the new series is 16, 4,-12, which is also an A.P series and when we put in G.P, so r =1/2. So both a and b options are correct.

Or

$\because x, 8 \,\&\, y$ are in G.P.

$\therefore xy = 8^2 \Rightarrow xy = 64$_______(I)

and $x, y, -8$ are in A. P.

$\Rightarrow 2y = x - 8$

$\therefore y = \frac{x-8}{2}$_______(II)

From (I) & (II) $x\left(\frac{x-8}{2}\right) = 64$

$\Rightarrow x^2 - 8x - 128 = 0 \Rightarrow x^2 - 16x + 8x - 128 = 0$

$\Rightarrow x(x-16) + 8(x-16) = 0 \quad \Rightarrow (x-16)(x+8) = 0$

$\Rightarrow x - 16 = 0 \text{ or } x + 8 = 0 \Rightarrow x = 16 \text{ or } x = -8$

If $x = 16$ then $y = \frac{16-8}{2} = 4$

If $x = -8$ then $y = \frac{-8-8}{2} = \mathbf{-8}$

Sol.18 (c) $a_n = ar^{(n-1)}$

$t_n = 1/2^{17}$

$\Rightarrow 16 \times \left(\frac{1}{2}\right)^{n-1} = 1/2^{17}$

$\Rightarrow 2^4 \times \frac{1}{2^{n-1}} = \frac{1}{2^{17}} \Rightarrow \frac{1}{2^{n-5}} = \frac{1}{2^{17}}$

$\Rightarrow n - 5 = 17 \Rightarrow \boldsymbol{n = 22}$

Sol.19(b)

$\because a = 1, r = \frac{1}{2}$

$S_n = 1\frac{127}{128} \Rightarrow \frac{a(1-r^n)}{1-r} = \frac{255}{128}$

$\Rightarrow \frac{1\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = \frac{255}{128} \qquad \Rightarrow \frac{2\left(1-\frac{1}{2^n}\right)}{1}$

$= \frac{255}{128} \qquad \Rightarrow \quad 1 - \frac{1}{2^n} = \frac{255}{256}$

$\Rightarrow \frac{1}{2^n} = 1 - \frac{255}{256} \Rightarrow \frac{1}{2^n} = \frac{1}{256}$

$\Rightarrow \frac{1}{2^n} = \frac{1}{2^8} \Rightarrow \boldsymbol{n = 8}$

Sol.20 (c)

$ar^3 = x, ar^9 = y, ar^{15} = z$

$zx = ar^{15}.ar^3 = a^2r^{18} = (ar^9)^2 = y^2$

$\Rightarrow \boldsymbol{y^2 = zx}$

Sol.21(a)

x, y, z are in $G.P.$

$\therefore \frac{y}{x} = \frac{z}{y} \Rightarrow y^2 = \mathbf{xz}$

Sol.22 (c)

$a = 201, d = 2$

$an = 299$

$\Rightarrow 201 + (n-1) \times 2 = 299$

$\Rightarrow (n-1) \times 2 = 98 \Rightarrow n - 1 = 49 \Rightarrow x = 50$

$\therefore s_n = \frac{n}{2}\{a + an\} = \frac{50}{2}\{201 + 299\}$

$=25\times500 = \mathbf{12,500}$

Sol.23 (a)

$a = 507, d = 13, a_n = 988$

$\Rightarrow 507 + (n-1) \times 13 = 988$

$\Rightarrow (n-1) \times 13 = 481 \Rightarrow n - 1 = 37 \Rightarrow n = 38$

$\therefore S_n = \frac{n}{2}(a + an) = \frac{38}{2}\{507 + 988\}$

$=19\times1495 = \mathbf{28,405}$

Sol.24 (b)

$s_n = 3 + 5 + 7 + 9 + \cdots$ to n terms

$= \frac{n}{2}\{2 \times 3 + (n-1) \times 2\} \Rightarrow \frac{n}{2}(2 \times 3 + 2n - 2)$

$= \frac{n}{2}(2n + 4) = n(n+2) = n^2 + 2n$

$\therefore sn + 1 = n^2 + 2n + 1 = \boldsymbol{(n+1)^2}$

Sol.25 (c) 'n' for numbers divisible by 4= 75-25+1=51 , N for numbers divisible by 5 = 60-20+1=41, N for numbers divisible by 20 = 15-5+1=11.

Required sum

$$= (100 + 104 + 108 + \cdots + 300) + (100 + 105 + \cdots + 300) - (100 + 120 + \cdots + 300)$$

$$= \frac{51}{2}(100 + 300) + \frac{41}{2}(100 + 300) - \frac{11}{2}(100 + 300)$$

$= \frac{1}{2}(400)[51 + 41 - 11]$

$= 200 \times 81 = \mathbf{16,200}$

Sol.26 (a) Numbers which are divisible by 4 and 5 means numbers divisible by 20.

$\Rightarrow$ 300/20 - 100/20+1= 11

Required sum = $(100 + 120 + 140 + \cdots + 300)$

$= \frac{11}{2}(100 + 300) = \frac{11}{\cancel{2}} \times \cancel{400}^{200} = \mathbf{2,200}$

Sol.27 (b)

$a = 100, d = -5$

$s_n = 975$

$\Rightarrow \frac{n}{2}\{2 \times 100 + (n-1)(-5)\} = 975$

$\Rightarrow \frac{n}{2}\{200 - 5n + 5\}$

$\Rightarrow \frac{n}{2}\{-5n + 205\} = 975$

$\Rightarrow -5n^2 + 205n = 1950$

$\Rightarrow 5n^2 - 205n + 1950 = 0$

$\Rightarrow n^2 - 41n + 390 = 0$

$\Rightarrow (n-15)(n-26) = 0 \Rightarrow n - 15 = 0$ *or* $n - 26 = 0$

$\Rightarrow \boldsymbol{n = 15}$ ***or*** $\boldsymbol{n = 26}$

Sol.28 (c)

$n = 10, d = 100$

$s_n = \frac{10}{2}\{2a + (10-1)100\} \Rightarrow 16500 = 5(2a + 900)$

$\Rightarrow \frac{16500}{5} = 2a + 900 \Rightarrow 3300 - 900 = 2a$

$\Rightarrow a = \frac{2,400}{2} = \mathbf{1,200}$

Sol.29 (a)

$A = P(1+i)^n$

$\Rightarrow ₹9,625 = P(1 + 0.1)^5$

$\Rightarrow P = \frac{₹9,625}{(1.1)^5} = \mathbf{₹5,976.37}\boldsymbol{(approx)}$

Sol.30 (d) A=P$(1+i)^n$

=$55(1+0.02)^{10}$

= **67.0446931 Crores (approx.)**

****

Sequence and Series
Exercise: Additional Questions

Sol.1 (c)

a, b, c are in A.P.

$\therefore 2b = a + c$

$\therefore$ b=$\frac{a+c}{2}$ __________(1)

Also $b^2 = ac\ (\because a, b\ \&\ c\ are\ in\ G.P.)$

$\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac$ [From I]

$\Rightarrow (a+c)^2 = 4ac \Rightarrow (a+c)^2 - 4ac = 0$
$\Rightarrow (a-c)^2 = 0 \Rightarrow a - c = 0$

$\Rightarrow a = c \therefore a = b = c$

$\therefore \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore \frac{1}{a}, \frac{1}{b}$ & $\frac{1}{c}$ are **in A.P.**

$\therefore a, b, \& c$ are also **in $H.P.$**

Sol.2 (a) Let A & D be the 1st term & common difference of an A.P.

$a_p = a \Rightarrow A + (p-1)D = a$ _____(I)

$a_q = b \Rightarrow A + (q-1)D = b$ _____(II)

$a_r = c \Rightarrow A + (r-1)D = c$ _____(III)

$\therefore\ a(q-r) + b(r-p) + c(p-q)$

$$= [A + (p-1)D](q-r) + [A + (q-1)D](r-p) + [A + (r-1)D](p-q)$$

$$= A(q - r + r - p + p - q) + D[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$= A \times 0 + D \times 0 = 0 + 0 = \mathbf{0}$

Sol.3 (b) $a_p = q,\ a_q = p$

a_r= p+q-r $(a_m = n, a_n = m\ then\ a_r = m + n - r)$

Sol.4 (a) $a_p = q\ , a_q = p$

$\therefore\ a_{p+q} = \mathbf{0}$

$(a_m = n, a_n = m, a_{m+n} = n + m - m - n)$

Sol.5 (a)

$1 + 2 + 3 + \cdots + n = \frac{n}{2}\{2 \times 1 + (n-1) \times 1\}$

$= \frac{\boldsymbol{n}}{\mathbf{2}}(\boldsymbol{n} + \mathbf{1})$

Sol.6 (b)

$1^2 + 2^2 + 3^2 + \cdots + n = \frac{\boldsymbol{n(n+1)(2n+1)}}{\mathbf{6}}$

Sol.7 (c)

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left\{\frac{\boldsymbol{n(n+1)}}{\mathbf{2}}\right\}^2$

Sol.8 (c)

$s_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 72 = \frac{n}{2}\{2 \times 17 + (n-1)(-2)\}$

$\Rightarrow \frac{n}{2}(34 - 2n + 2) = 72 \Rightarrow \frac{n}{2}(36 - 2n)$=72

$\Rightarrow 72 = n(18 - n)$

$\Rightarrow n^2 - 18n + 72 = 0 \Rightarrow n^2 - 12n - 6n + 72 = 0$

$\Rightarrow (n-12)(n-6) = 0$

$\Rightarrow n - 12 = 0\ or\ n - 6 = 0 \Rightarrow n = 12\ or\ \boldsymbol{n} = \mathbf{6}$

Sol.9 (a) put n=2

$= (1 - \frac{1}{n}) + (1 - \frac{2}{n}) \Rightarrow (1 - \frac{1}{2}) = \frac{1}{2}$

$at\ n = 2\ sum\ is\ \frac{1}{2}\ so\ put\ n = 2\ in\ options\ and\ which\ option\ gives\ sum = \frac{1}{2}\ at\ n = 2\ is\ the\ answer.$

Option a) $(1 \frac{1}{2}) + \left(1 - \frac{2}{2}\right) = \frac{1}{2}$.

Sol.10 (a)

$s_n = 2n^2 + 3n\ \Rightarrow s_1 = 2(1) + 3 = 5, s_2 = 2(2)^2 + 6 = 14.$

$t_1 = s_1 = 2 + 3 = 5$

$t_2 = s_2 - s_1 = 14 - 5 = 9$

$t_3 = s_3 - s_2 = 27 - 14 = 13$

Here $t_2 - t_1 = 9 - 5 = 4$

$\&t_3 - t_2 = 13 - 9 = 4$

$\therefore \boldsymbol{t_2 - t_1 = t_3 - t_2}$

Hence the series is in A.P.

Sol.11 (b)

$a = 203, d = 7 \text{ and } a_n = 399$

$\Rightarrow 203 + (n-1) \times 7 = 399$

$\Rightarrow (n-1) \times 7 = 196 \Rightarrow n - 1 = 28 \Rightarrow n = 29$

$\therefore s_n = \frac{n}{2}\{a + a_n\} = \frac{29}{2}\{203 + 399\} = 29 \times 301$

=8,729

Sol.12 (c) Numbers divisible by 5 = $\frac{200}{5} = 40$

Required sum

$= (1 + 2 + 3 + 4 + \cdots + 200) - (5 + 10 + \cdots + 200)$

$= \frac{n(n+1)}{2} - \frac{40}{2}(10 + 39 \times 5)$

$= \frac{200 \times 201}{2} - \frac{40 \times 210}{2} =$ 20,100 −4,100 = **16,000**

Sol.13 (a) Let 1st term & common difference of an A.P. be A & D, respectively

$s_p = a \Rightarrow \frac{p}{2}\{2A + (p-1)D\} = a$

$\Rightarrow A + (p-1)\frac{D}{2} = \frac{a}{p}$_____(I)

Similarly $s_q = b \Rightarrow A + (q-1)\frac{D}{2} = \frac{b}{q}$_____(II)

$s_r = c \Rightarrow A + (r-1)\frac{D}{2} = \frac{c}{r}$_____(III)

$\therefore \left(\frac{a}{p}\right)(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$

$=A(q - r + r - p + p - q) + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$

$= A \times 0 + \frac{D}{2} \times 0 = 0 + 0 = \mathbf{0}$

Sol.14 (c)

$s_1 = \frac{n}{2}(2a + (n-1)d)$

$s_2 = \frac{2n}{2}(2a + (2n-1)d)$

$s_2 - s_1 = \frac{n}{2}\{4a - 2a + (4n - 2 - n + 1)d\}$

$= \frac{n}{2}\{2a + (3n-1)d\}$

$s_3 = \frac{3n}{2}\{2a + (3n-1)d\}$

$\therefore s_3 \div (s_2 - s_1) = \frac{3\frac{n}{2}\cancel{\{2a+(3n-1)d\}}}{\frac{n}{2}\cancel{\{2a+(3n-1)d\}}}$ **=3**

Sol.15 (b) If the ratio of s_n of two series is given in the question, the ratio of a_n Of the two-term will replace n from 2n-1.

$\therefore s_n = \frac{7n-5}{5n+17} \Rightarrow s_n = \frac{7(2n-1)-5}{5(2n-1)+17}$

$= \frac{14n-12}{10n+12} = 1$

(Two terms are equal means ratio is 1)
= 14n-12=10n+12 ⇒ 4n=24 ⇒ **n = 6.**

Sol.16 (a) Do it by options only option a) sum =6 and all numbers are in A.P

Or

Let the numbers be $a - d, a \,\&\, a + d$

Now, $a - d + a + a + d = 6 \Rightarrow 3a = 6 \Rightarrow a = 2$

Also $(a - d)\, a\, (a + d) = -24$

$\Rightarrow (2 - d)2(2 + d) = -24$

$\Rightarrow 4 - d^2 = -12 \Rightarrow d^2 = 16$

$\Rightarrow d = \pm 4$

$\therefore$ *numbers are* $\mathbf{-2, 2, 6}$ **Or** $\mathbf{6, 2, -2}$

Sol. 17 (a) Do it by option only option a sum is 6, and the sum of the square of terms =44, so option a) is correct.

Or

Let the numbers be $a - d, a \,\&\, a + d$

$a - d + a + a + d = 6 \Rightarrow 3a = 6 \Rightarrow a = 2$

$(a - d)^2 + a^2 + (a + d)^2 = 44$

$\Rightarrow 3a^2 + 2d^2 = 44$

$\Rightarrow 2d^2 = 44 - 12 \Rightarrow d^2 = \frac{32}{2} = 16$

$\Rightarrow d = \pm 4$

$\therefore$ *Numbers are* $-\mathbf{2, 2, 6}$ **or** $\mathbf{6, 2, -2}$

Sol.18 (a) Do it by option only option a sum is 6, but this question is the wrong sum of the cubes of the terms =216, not 232, which satisfy option a.

Or

Let the three numbers in A.P. be $a - d, a \,\&\, a + d$

ATQ

$a - d + a + a + d = 6 \Rightarrow 3a = 6 \Rightarrow a = 2$

$(a - d)^3 + a^3 + (a + d)^3 = 216$

$\Rightarrow 3a^3 + 6ad^2 = 216$

$\Rightarrow 24 + 12d^2 = 216 \Rightarrow 12d^2 = 192$

$\Rightarrow d^2 = 16 \Rightarrow d = \pm 4$

$\therefore$ ***Numbers are*** $\mathbf{-2, 2, 6}$ ***or*** $\mathbf{6, 2, -2}$

Sol.19 (a) Do it by option. Options b and d are negative, which is not possible only option a) sum is 12.5, so option a) is the answer.

Or

Let the five parts be

ATQ

$a - 2d, a - d, a, a + d$ and $a + 2d$

Now $a - 2d + a - d + a + a + d + a + 2d =$ 12.50

$\Rightarrow 5a = 12.50 \Rightarrow a = \frac{12.50}{5} = 2.50$

Also $\frac{a-2d}{a+2d} = \frac{2}{3}$

$\Rightarrow 3a - 6d = 2a + 4d$

$\Rightarrow 10d = a \Rightarrow d = \frac{2.50}{10} = 0.25$

$\therefore$ *Required Parts are*

$\mathbf{2, 2.25, 2.50, 2.75 \,\&\, 3}$

Sol. 20 (c)

a, b, c in $A.P.$

Put $a = 1, b = 2 \,\&\, c = 3$

$\therefore \frac{a^3+4b^3+c^3}{b(a^2+c^2)} = \frac{1+32+27}{2(1+9)} = \frac{60}{20} = \mathbf{3}$

Sol.21 (b)

Put $a = 1, b = 2 \,\&\, c = 3$ ($\because a, b, c$ are in $A.P.$)

$\therefore \frac{a^2+4ac+c^2}{ab+bc+ca} = \frac{1+12+9}{2+6+3} = \frac{22}{11} = \mathbf{2}$

Sol.22 (a) put $a = 1, b = 2 \,\&\, c = 3$

$\therefore \frac{a}{bc}(b + c) = \frac{1 \times 5}{6} = \frac{5}{6}$

$\frac{b}{ca}(c + a) = \frac{8}{3}$

$\frac{c}{ab}(a + b) = \frac{9}{2}$

Here $\frac{8}{3} - \frac{5}{6} = \frac{16-5}{6} = \frac{11}{6}$

$\frac{9}{2} - \frac{8}{3} = \frac{27 - 16}{6} = \frac{\mathbf{11}}{\mathbf{6}}$

$\because$ Diff. between consecutive terms same hence it is in A.P.

Sol.23 (a)

Put $a = 1, b = 2 \,\&\, c = 3$
$\therefore a^2(b + c) = 5, b^2(c + a) = 16, c^2(a + b) = 27$

Here $16 - 5 = 27 - 16$

$\therefore$ it is **in A.P.**

Sol.24 (a)

$\because (b + c)^{-1}, (c + a)^{-1}, (a + b)^{-1}$ are in A.P.

$\therefore\ b + c = 1, c + a = 1/2 \,\&\, a + b = 1/3$

$\therefore 2(a + b + c) = \frac{6+3+2}{6} = \frac{11}{6}$

$\Rightarrow a + b + c = \frac{11}{12}$

$\therefore a = \frac{-1}{12}, b = \frac{5}{12} \,\&\, c = \frac{7}{12}$

$\therefore\ a^2 = \frac{1}{144}, b^2 = \frac{25}{144}, c^2 = \frac{49}{144}$

$b^2 - a^2 = \frac{24}{144} = c^2 - b^2$

$\therefore$ **a^2, b^2, c^2 are in A.P.**

Sol.25 (c) put $a = 1, b = 5$ & $c = 7$

$a^2 = 1$

$b^2 = 25 \qquad \therefore b^2 - a^2 = c^2 - b^2 = 24$

$c^2 = 49$

Now $b + c = 12, c + a = 8, a + b = 6$

$\frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$

$\frac{1}{6} - \frac{1}{8} = \frac{4-3}{24} = \frac{1}{24}$

$\therefore$ **$(b + c), (c + a)$ & $(a + b)$ are in H.P**

Sol.26 (a)

Put $a = 1, b = 5$ & $c = 7$

$\therefore \frac{a}{b+c} = \frac{1}{12}, \frac{b}{c+a} = \frac{5}{8}, \frac{c}{a+b} = \frac{7}{6}$

$\frac{5}{8} - \frac{1}{12} = \frac{15-2}{24} = \frac{13}{24}, \qquad \frac{7}{6} - \frac{5}{8} = \frac{28-15}{24} = \frac{13}{24}$

$\therefore \frac{a}{b+c}, \frac{b}{c+a}$ & $\frac{c}{a+b}$ are in A. P.

Sol.27 (c)

$\because \frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.

Adding 2 in each term, we have

$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.

Dividing each term by $a + b + c$

$\therefore$ **$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.**

$\therefore$ **a, b, c are in H.P.**

Sol.28 (c)

$(b - c)^2, (c - a)^2, (a - b)^2$ are in A.P.

$\therefore (c - a)^2 - (b - c)^2 = (a - b)^2 - (c - a)^2$

$$\Rightarrow [(\cancel{c} - a) + (b - \cancel{c})][(c - a) - (b - c)] = [(\cancel{a} - b) + (c - \cancel{a})][(a - b) - (c - a)]$$

$\Rightarrow$[(b-a)] [(c-a) -(b-c)] = [(c-b)] [(a-b)-(c-a)]

$\Rightarrow \frac{(c-a)-(b-c)}{c-b} = \frac{(a-b)-(c-a)}{b-a}$

$\Rightarrow \frac{(c-a)-(b-c)}{-(b-c)(c-a)} = \frac{(a-b)-(c-a)}{-(a-b)(c-a)}$

(Dividing c-a both sides)

$\Rightarrow \frac{1}{b-c} - \frac{1}{c-a} = \frac{1}{c-a} - \frac{1}{a-b}$

$\Rightarrow \frac{1}{c-a} - \frac{1}{b-c} = \frac{1}{a-b} - \frac{1}{c-a}$

$\therefore \frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in A.P.

$\therefore$ **$(b - c), (c - a), (a - b)$ are in H.P.**

Sol.29 (a) put a, b and c = 1,2 and 3 respectively

$\because a, b, c$ are in A.P.

$\Rightarrow$ (b+c) = (2+3)=5

$\Rightarrow$(c+a) =(3+1)= 4

$\Rightarrow$ (a+b)= (2+1) = 3

$\therefore$ **they are in A.P**

Sol.30 (c)

$s_n = 3 + 5 + 7 + \cdots$ to n terms

$= \frac{n}{2}\{2 \times 3 + (n-1)2\}$

$= \frac{n}{2}\{2n + 4\} = \frac{n}{\cancel{2}} \times \cancel{2}(n + 2) = n^2 + 2n$

$= (n + 1)^2 - 1 \Rightarrow s_n + 1 = \boldsymbol{(n+1)^2}$

Sol.31 (a) Put n=1 at n=1 $a_1 = s_1$

$s_n = 2n^2 + 3n$

$s_1 = a_1 = 2(1) + 3 = \mathbf{5}$

Now put n=1 in the options, and when we put n=1 in option a, it is 5

Sol.32 (a) $a_p = \frac{1}{q}, a_q = \frac{1}{p}$

$\therefore s_{pq} = \frac{1}{2}(pq + 1)$

Sol.33 (a) $s_p = q, s_q = p$

$\therefore s_{p+q} = -(p + q) \qquad$ [by shortcut]

Or

$= s_p = q \Rightarrow 2ap + p(p-1)d = 2q$ [*Equation I*]

$= s_q = p \Rightarrow 2aq + q(q-1)d = 2p$ [*Equation II*]

Equation I- Equation II

$\Rightarrow 2a(p-q) + [p(p-1) + q(q-1)]d = 2q - 2p$

$\Rightarrow 2a + (p+q-1)d = -2$

$$s_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$\Rightarrow s_{p+q} = \frac{p+q}{2} \times (-2) = \boldsymbol{-(p+q)}$

Sol.34 (b)

$$s_1 = \frac{n}{2}\{2 \times 1 + (n-1) \times 1\} = \frac{n}{2}(n+1)$$

$$s_2 = \frac{n}{2}\{2 \times 1 + (n-1) \times 2\} = \frac{n}{2} \times 2n = n^2$$

$$s_3 = \frac{n}{2}\{2 \times 1 + (n-1) \times 3\} = \frac{n}{2}(3n-1)$$

$$\therefore \frac{s_1 + s_3}{s_2} = \frac{\frac{n}{2}(n+1) + \frac{n}{2}(3n-1)}{n^2}$$

$$= \frac{n}{2n^2}(n + 1 + 3n - 1)$$

$$= \frac{1}{2n} \times 4n = \mathbf{2}$$

Sol.35 (b)

$a = 507, \quad d = 13$

$a_n = 988 \Rightarrow 507 + (n-1) \times 13 = 988$

$\Rightarrow (n-1)13 = 481 \Rightarrow n - 1 = 37 \Rightarrow n = 38$

$$\therefore s_n = \frac{n}{2}\{a + a_n\} = \frac{38}{2}(507 + 988)$$

$= 19 \times 1495 = \mathbf{28,405}$

Sol.36

$a = 104, \quad d = 4$

$a_n = 296 \Rightarrow 104 + (n-1) \times 4 = 296$

$\Rightarrow 4n + 100 = 296 \Rightarrow 4n = 196$

$$\Rightarrow n = \frac{196}{4} = 49$$

$$\therefore s_n = \frac{n}{2}\{a + a_n\} = \frac{49}{2}\{104 + 296\}$$

$$= \frac{49}{2} \times 400 = 49 \times 200$$

$= \mathbf{9,800}$

Sol.37 (b)

$= (100 + 101 + \cdots + 300)$, a= 100, d=1, $a_n = 300$

$\Rightarrow$ 300= 100+(n-1)1 $\Rightarrow$ 200=n-1$\Rightarrow$ n=201

$\Rightarrow \quad s_n = \frac{201}{2}(200 + (201-1)1) \Rightarrow \frac{201}{2}(400) =$ 40,200

= 100+104+108...... +300

a= 100, d=4 $a_n = 300$

$\Rightarrow$ 300 = 100 +(n-1)4$\Rightarrow$ n=51

$\Rightarrow s_n = \frac{51}{2}(200 + 50 \times 4) = 10{,}200$

∴Required sum = 40,200-10,200=**30,000**

Sol.38 (c)

$a = 100, \quad d = 5$

$a_n = 300 \Rightarrow 100 + (n-1) \times 5 = 300$

$\Rightarrow (n-1) \times 5 = 200 \Rightarrow n - 1 = 40 \Rightarrow n = 41$

$$\therefore s_n = \frac{n}{2}\{a + a_n\} = \frac{41}{2}(100 + 300)$$

$$= \frac{41}{\cancel{2}} \times \cancel{400}^{200} = \mathbf{8,200}$$

Sol.39 (d) Numbers which are divisible by 4 and 5 means divisible by 20.

$a = 100, \quad d = 20$

$a_n = 300 \Rightarrow 100 + (n-1) \times 20 = 300$

$\Rightarrow (n-1) \times 20 = 200 \Rightarrow n - 1 = 10 \Rightarrow n = 11$

$$\therefore s_n = \frac{n}{2}\{a + a_n\} = \frac{11}{2}(100 + 300)$$

$$\frac{11}{2} \times 400 = 11 \times 200 = \mathbf{2,200}$$

Sol.40 (d)

Required sum

$$= (100 + 104 + \cdots + 300) + (100 + 105 + \cdots + 300) - (100 + 120 + 140 + \cdots + 300)$$

$$= \frac{51}{2}(100 + 300) + \frac{41}{2}(100 + 300) - \frac{11}{2}(100 + 300)$$

$=\left(\frac{51+41-11}{\cancel{2}}\right)(\cancel{400})200$

$= 81 \times 200 = \mathbf{16,200}$

Sol.41 (a)

$\frac{t_n}{t_n^1} = \frac{3n+4}{n+4}$

Required ratio=

$\frac{t_4}{t_4^1} = \frac{3 \times 4 + 4}{4+4}$

$= \frac{16}{8} = \frac{2}{1} = 2:1$

Sol. 42 (a)

a, b, c & d are in A.P. put a, b, c and d =1,2, 3 and 4 in options

$\therefore a^2 - 3b^2 + 3c^2 - d^2 = 0 \Rightarrow 1^2 - 3(2)^2 + 3(3)^2 - 4^2 = \mathbf{0}$

Sol.43 (d)

a, b, c, d, e are in A.P. then put a ,b ,c , d, e = 1,2,3,4,5 in options

$\therefore$ *option a)* $1 - 2 - 4 + 5 = 0$

$\therefore$ *option b)* $1 - 6 + 5 = 0$

$\therefore$ *option c)* $2 - 6 + 4$

$= 0$ *all options satisfy therefore option d is correct.*

Sol.44 (d) Do it by option, b option is negative, so it is not possible. Options a and c both are in A.P and sum is 18 and product is 192, so both are correct, so option d is the correct answer.

Or

Let the nos. be $a - d, a$ & $a + d$

Now, $a - d + a + a + d = 18 \Rightarrow 3a = 18 \Rightarrow a = 6$

Also $(a - d)a(a + d) = 192$

$\Rightarrow (6 - d)6(6 + \mathrm{d}) = 192$

$\Rightarrow 36 - d^2 = \frac{192}{6}$

$\Rightarrow d^2 = 36 - 32 \Rightarrow \mathrm{d}^2 = 4$

$\Rightarrow \mathrm{d} = \pm 2$

$\therefore$ Numbers are 4, 6, 8 or **8, 6, 4**

Sol.45 (c) Do it by options, a and c both options are in A.P and sum is 27 and square is 341. So option c is the correct answer.

Or

Let the nos. are $a - d, a$ & $a + d$

ATQ

$a - d + a + a + d = 27 \Rightarrow 3a = 27 \Rightarrow a = 9$

Also $(a - d)^2 + a^2 + (a + d)^2 = 341$

$\Rightarrow 3a^2 + 2d^2 = 341 \Rightarrow 2d^2 = 341 - 243$

$\Rightarrow d^2 = \frac{98}{2} \Rightarrow d^2 = 49 \Rightarrow d = \pm 7$

$\therefore$ **Numbers are 2,9,16 or 16, 9, 2**

Sol.46 (a) Do it by option. The only option a) sum is 24, and their product is 945. And all numbers are in A.P.

Or

Let the nos. be $a - 3d, a - d, a + d$ & $a + 3d$

$a - 3d + a - d + a + d + a + 3d = 24$

$\Rightarrow 4a = 24 \Rightarrow a = 6$

Also $(a - 3d)(a - d)(a + d)(a + 3d) = 945$

$\Rightarrow (a^2 - 9d^2)(a^2 - d^2) = 945$

$\Rightarrow (36 - 9d^2)(36 - d^2) = 945$

$\Rightarrow (4 - d^2)(36 - d^2) = 105$

$\Rightarrow d^4 - 40d^2 + 144 - 105 = 0$

$\Rightarrow d^4 - 40d^2 + 39 = 0 \Rightarrow (d^2 - 39)(d^2 - 1) = 0$

$\Rightarrow d^2 - 39 = 0 \text{ or } d^2 - 1 = 0$

$\Rightarrow d = \pm\sqrt{39} \text{ or } d = \pm 1$

If $d = \pm 1$ & $a = 6$ then the nos. are **3, 5, 7, 9 or 9,7,5,3**

Sol.47 (b) Do it by options. Only option b, all numbers sum is 24 and sum of squares is 120. So option b is the correct answer.

Or

Let the four nos. in A.P. be $a - 3d, a - d, a + d$ & $a + 3d$

<u>ATQ</u>

$a - 3d + a - d + a + d + a + 3d = 20$

$\Rightarrow 4a = 20 \Rightarrow a = 5$

Also

$$(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$$

$\Rightarrow 4a^2 + 20d^2 = 120 \Rightarrow a^2 + 5d^2 = 30$

$\Rightarrow 25 + 5d^2 = 30 \Rightarrow 5d^2 = 5 \Rightarrow d = \pm 1$

∴ Numbers are **2, 4, 6, 8 or 8, 6, 4, 2**

Sol.48 (c) Do it by option sum of 2nd and 3rd term is 22 ∴ $9 + 13 = 22$ and product of 1st and 5th term is 85. So $5\times 17 = 85$. *Only* Option c satisfy all conditions. It is the correct answer.

Or

Let the four nos. in A.P. be $a - 3d, a - d, a + d$ & $a + 3d$

$a - d + a + d = 22 \Rightarrow 2a = 22 \Rightarrow a = 11$

and $(a - 3d) \times (a + 3d) = 85$

$(11 - 3d)(11 + 3d) = 85$

$\Rightarrow 121 - 9d^2 = 85 \Rightarrow 9d^2 = 36$

$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

∴ Numbers are **5, 9, 13, 17 or 17, 13, 9, 5**

Sol.49 (a) Do it by options. C and d options are negative, so that's not possible. Option a) sum = 3+4+5+6+7=25. And the sum of square $=3^2 + 4^2 + 5^2 + 6^2 + 7^2 =$ *135 so option a is the correct answer.*

Or

Let the five nos. in A.P. be $a - 2d, a - d, a, a + d$ & $a + 2d$

<u>ATQ</u>

$a - 2d + a - d + a + a + d + a + 2d = 25$

$\Rightarrow 5a = 25 \Rightarrow a = 5$

Also $(a - 2d)^2 + (a - d)^2 + a^2 + (a + d)^2 + (a + 2d)^2 = 135$

$\Rightarrow 5a^2 + 10d^2 = 135 \Rightarrow 10d^2 = 135 - 125$

$\Rightarrow d^2 = \frac{10}{10} = 1 \Rightarrow d = \pm 1$

Numbers are **3, 4, 5, 6, 7 or 7, 6, 5, 4, 3**

Sol.50 (b) Do it by options, b option sum =3+3.5+4+4.5+5=20 and product of first and last term is 15, so option b is correct. Options c and d are negative, so that's not possible. Only you have to choose from a and b.

Or

Let the nos. be $a - 2d, a - d, a, a + d$ & $a + 2d$

$a - 2d + a - d + a + a + d + a + 2d = 20$

$\Rightarrow 5a = 20 \Rightarrow a = 4$

Also $(a - 2d)(a + 2d) = 15$

$\Rightarrow a^2 - 4d^2 = 15 \Rightarrow 4d^2 = 16 - 15$

$\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2} = \pm 0.5$

Numbers are **3, 3.5, 4, 4.5, 5 or 5, 4.5, 4, 3.5, 3**

Sol.51 (a)

$s_2 = 2 + 4 = 6$

∴ put $n = 2$

$n(n + 1) = \mathbf{6}$

Sol.52 (d)

$s_2 = a + b + 2a = 3a + b$

∴ put $n = 2$ in the option

$n(a - b) + 2b = 2a$

$n(a + b) = \mathbf{2a + 2b}$

Sol.53 (b)

$s_2 = (x + y)^2 + x^2 + y^2 = 2x^2 + 2y^2 + 2xy$

∴ put $n = 2$ option (a)

$$(x + y)^2 - 2(n - 1)xy = (x + y)^2 - 2xy = (x - y)^2$$

∴ put $n = 2$ option (b)

$n(x+y)^2 - n(n-1)xy$

$= 2(x+y)^2 - 2xy = \mathbf{2x^2 + 2y^2 + 2xy}$

Sol.54 (b) put n=2, option (b)

$s_2 = \frac{n-1}{n} + \frac{n-2}{n} = \frac{2n-3}{n} = \frac{2\times 2-3}{2} = \frac{1}{2}$

$\frac{1}{2} \times (n-1) = \frac{2-1}{2} = \mathbf{\frac{1}{2}}$

Sol.55 (a)

$s_n = 1\times 4 + 3\times 7 = 4 + 21 = 25$

$\therefore$ put $n = 2$ option (a)

$\therefore \frac{n}{2}(4n^2 + 5n - 1) = \frac{2}{2}(16 + 10 - 1) = \mathbf{25}$

Sol.56 (a)

$s_2 = 1^2 + 3^2 = 1 + 9 = 10$

Put $n = 2$ in the option

$\frac{n}{3}(4n^2 - 1) = \frac{2}{3}(16 - 1) = \frac{2}{3}\times 15 = \mathbf{10}$

Sol.57 (d)

$s_2 = 1 + (1+2) = 4$

Put $n = 2$ option (a)

$\frac{n}{3}(n+1)(n-2) = \frac{2}{3}\times 3\times 0 = 0$

Put $n = 2$ option (b)

$\frac{n}{3}(n+1)(n+2) = \frac{2}{3}\times 3\times 4 = 8$

Put $n = 2$ option (c)

$n(n+1)(n+2) = 2\times 3\times 4 = \mathbf{24}$

Sol.58 (a)

$s_2 = \frac{1^2}{1} + \frac{1^2+2^2}{2} = 1 + \frac{5}{2} = \frac{7}{2}$

Put $n = 2$

$\therefore \frac{n}{36}(4n^2 + 15n + 17) = \frac{1}{18}(16 + 30 + 17) = \frac{63}{18} = \frac{7}{2}$

Sol.59 (a)

$s_1 = a_1 = 2\times 4\times 6 + 4\times 6\times 8 = 240$

Put $n = 2$

$\therefore 2n(n^3 + 6n^2 + 11n + 6)$

$\underline{= 2\times 2}(8 + 24 + 22 + 6) = 4\times 60 = \mathbf{240}$

Sol.60 (a)

$s_2 = 1\times 3^2 + 4\times 4^2 = 73$

Put $n = 2$

$\frac{n}{12}(n+1)(9n^2 + 49n + 44) - 8n$

$= \frac{2}{12}\times 3\times (36 + 98 + 44) - 16 = \mathbf{73}$

Sol.61 (a)

$s_2 = 4 + 6 = 10$

Put $n = 2$

$\frac{n}{6}(n^2 + 3n + 20) = \frac{2}{6}(4 + 6 + 20) = \frac{2}{\cancel{6}}\times \cancel{30} = \mathbf{10}$

Sol.62 (a)

$s_2 = 11 + 23 = 34$

Put $n = 2$

$\therefore 3^{n+1} + 5n - 3 = 27 + 10 - 3 = \mathbf{34}$

Sol.63 (a)

$s_2 = \frac{1}{4\times 9} + \frac{1}{9\times 14} = \frac{1}{28}$

Put $n = 2$

$\therefore \frac{n}{4}(5n+4)^{-1} = \frac{2}{4}\times (10+4)^{-1} = \frac{1}{2}\times \frac{1}{14} = \mathbf{\frac{1}{28}}$

Sol.64 (a)

$s_2 = 1 + 3 = 4$

Put $n = 2$

$n^2 = 2^2 = \mathbf{4}$

Sol.65 (a)

$s_2 = 2 + 6 = 8$

Put $n = 2$

$\therefore 2n^2 = 2\times 2^2 = \mathbf{8}$

Sol.66 (a)

$s_2 = 1\times 2 + 2\times 3 = 8$

$\therefore \frac{n}{3}(n+1)(n+2) = \frac{2}{3} \times 3 \times 4 = \mathbf{8}$

Sol.67 (a)

$s_1 = 1 \times 2 \times 3 + 2 \times 3 \times 4 = 30$

Put $n = 2$

$\therefore \frac{n}{4}(n+1)(n+2)(n+3) = \frac{2}{4} \times 3 \times 4 \times 5 = 30$

Sol.68 (a)

$s_2 = 1 \times 2 + 3.2^2 = 14$

Put $n = 2$ in the option

$\therefore (n-1)2^{n+2} - 2^{n+1} + 6 = 1 \times 16 - 8 + 6 = \mathbf{14}$

Sol.69 (a)

$s_2 = \frac{1}{3 \times 8} + \frac{1}{13 \times 8} = \frac{2}{39}$

Put $n = 2$ in the option

$\frac{n}{3}(5n+3)^{-1} = \frac{2}{3} \times (10+3)^{-1} = \frac{2}{3} \times \frac{1}{13} = \frac{\mathbf{2}}{\mathbf{39}}$

Sol.70 (a)

$s_2 = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$

Put $n = 2$ in the option

$2n(n+1)^{-1} = 2 \times 2(3)^{-1} = 4 \times \frac{1}{3} = \frac{\mathbf{4}}{\mathbf{3}}$

Sol.71 (a)

$s_2 = 2^2 + 5^2 = 29$

Put $n = 2$ in the option

$\frac{n}{2}(6n^2 + 3n - 1) = \frac{2}{2}(24 + 6 - 1) = \mathbf{29}$

Sol.72 (a)

$s_2 = 1^2 + 3^2 = 10$

Put $n = 2$ in the option

$\frac{n}{3}(4n^2 - 1) = \frac{2}{3}(16 - 1) = \frac{2}{\cancel{3}} \times \cancel{15} = \mathbf{10}$

Sol.73 (a)

$s_1 = 1 \times 4 + 3 \times 7 = 25$

Put $n = 2$ in the option

$= \frac{n}{2}(4n^2 + 5n - 1)$

$= \frac{2}{2}(16 + 10 - 1) = \frac{25}{\cancel{2}} \times \cancel{2} = \mathbf{25}$

Sol.74 (a)

$s_1 = 2 \times 3^2 + 5 \times 4^2 = 98$

Put $n = 2$ in the option

$\therefore \frac{n}{12}(9n^3 + 62n^2 + 123n + 22)$

$= \frac{2}{12}(72 + 248 + 264 + 22) = \mathbf{98}$

Sol.75 (a)

$s_2 = 1 + 4 = 5$

Put $n = 2$ in the option

$\therefore \frac{n}{6}(n+1)(2n+1) = \frac{2}{6} \times 3 \times 5 = \mathbf{5}$

Sol.76 (a)

$s_2 = 1^2 + 1^2 + 2^2 = 6$

Put $n = 2$ in the option

$\therefore \frac{n}{12}(n+1)^2(n+2) = \frac{2}{12} \times 3^2 \times 4 = \mathbf{6}$

Sol.77 (b)

$s_2 = 1 + \frac{4}{3} = \frac{7}{3}$

Put $n = 2$ in the option

$\therefore \frac{3}{2}\left[n - \frac{1}{2}(1 - 3^{-n})\right] = \frac{3}{2}\left[2 - \frac{1}{2}\left(1 - \frac{1}{9}\right)\right]$

$= \frac{3}{2}\left[2 - \frac{1}{2} \times \frac{8}{9}\right] = \frac{3}{2}\left[2 - \frac{4}{9}\right] = \frac{3}{2} \times \frac{14}{9} = \frac{\mathbf{7}}{\mathbf{3}}$

Sol.78 (a)

Put $n = 2$

$s_2 = n \times 1 + (n-1) \times 2 = 3n - 2 = 4$

Put $n = 2$ in the option

$\frac{n}{6}(n+1)(n+2) = \frac{2}{6} \times 3 \times 4 = \mathbf{4}$

Sol.79 (a)

$s_2 = 1 + 5 = 6$

Put $n = 2$ in the option

$\frac{n^2}{2}(n+1) = \frac{4}{2} \times 3 = \mathbf{6}$

Sol.80 (a)

$s_2 = 4 + 14 = 18$

Put $n = 2$ in the option

$n(n+1)^2 = 2 \times 3^2 = \mathbf{18}$

Sol.81 (a)

$s_2 = 3 + 6 = 9$

Put $n = 2$ in the option

$\therefore\ 2^{n+1} + \frac{n}{2}(n+1) - 2 = 2^3 + \frac{2}{2}(3) - 2$

$= 8 + 3 - 2 = \mathbf{9}$

Sol.82 (a)

$a_1 = \frac{1}{4 \times 7} = \frac{1}{28}$

Put $n = 1$ in the option

$\frac{1}{3}[(3n+1)^{-1} - (3n+4)^{-1}]$

$= \frac{1}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \frac{1}{3} \times \frac{7-4}{28} = \frac{1}{3} \times \frac{3}{28} = \mathbf{\frac{1}{28}}$

Sol.83 (a)

$s_2 = \frac{1}{4 \times 7} + \frac{1}{10 \times 7} = \frac{1}{20}$

$\therefore \frac{n}{4}(3n+4)^{-1} = \frac{2}{4}\left(\frac{1}{10}\right) = \mathbf{\frac{1}{20}}$

Sol.84 (a)

$s_1 = \frac{1^2}{1} + \frac{1^2 + 2^2}{1+2} = 1 + \frac{5}{3} = \frac{8}{3}$

Put $n = 2$ in the option

$\frac{n}{3}(n+2) = \frac{2}{3} \times 4 = \mathbf{\frac{8}{3}}$

Sol.85 (a)

$s_1 = \frac{1^3}{1} + \frac{1^3 + 2^3}{2} = \frac{11}{2}$

Put $n = 2$ In the option

$\frac{n}{48}(n+1)(n+2)(3n+5)$

$= \frac{2}{48} \times 3 \times 4 \times 11 = \mathbf{\frac{11}{2}}$

Sol.86 (a)

$n^2 + 2n[1 + 2 + 3 + \cdots + (n-1)]$

$= n^2 + 2n\frac{(n-1)\{n-1+1\}}{2}$

$= n^2 + n(n-1)n = \cancel{n^2} + n^3 - \cancel{n^2}$

$= \mathbf{n^3}$

Sol.87 (a)

Let $p(n) = 2^{4n} - 1$

Put $n = 1$

$\therefore p(1) = 2^{4n} - 1 = 16 - 1 = \mathbf{15}$

Sol.88 (b)

Let $p(n) = 3^n - 2n - 1$

Put $n = 1$

$\therefore p(1) = 3^1 - 2 \times 1 - 1 = 3 - 2 - 1 = 0$

$p(2) = 3^2 - 2 \times 2 - 1 = 9 - 4 - 1 = \mathbf{4}$

Sol.89 (c)

$= n(n-1)(2n-1)$

Put $n = 1$

$p(1) = 1 \times 0 \times 1 = 0$

$p(2) = 2 \times 1 \times 3 = \mathbf{6}$

Sol.90 (d) $7^{2n} + 16n - 1$

Put $n = 1$

$p(1) = 7^2 + 16 \times 1 - 1 = 49 + 16 - 1 = \mathbf{64}$

Sol.91 (a)

$a_n = 3n^2 + 2n$

$\therefore a_2 = 3(2)^2 + 2 \times 2 = 16$

$\Rightarrow S_2 = 5 + 16 = 21$

Put $n = 2$ in the option

$\frac{n}{2}(n+1)(2n+3) = \frac{2}{2} \times 3 \times 7 = \mathbf{21}$

Sol.92 (a)

$a_n = n\,.\,2^n$

$\therefore a_2 = 2 \times 2^2 = 8$

$\therefore s_2 = 2 + 8 = 10$

Put $n = 2$ in the option

$(n-1)2^{n+1} + 2 = 1 \times 8 + 2 = \mathbf{10}$

Sol.93 (a)

$a_n = 5 \times 3^{n+1} + 2n$

$\therefore a_2 = 5 \times 3^3 + 2 \times 2 = 139$

$\Longrightarrow s_2 = 47 + 139 = 186$

Put $n = 2$ in the option

$\therefore \frac{5}{2}(3^{n+2} - 9) + n(n+1) = \frac{5}{2}(81 - 9) + 2 \times 3$

$= \frac{5}{\cancel{2}} \times \cancel{72}^{36} + 6 = \mathbf{186}$

Sol.94 (c) Do it by option. In both options, a and b third term is the square of the first term. And 5th term is 64.

$\therefore$ Option c is correct.

Or

Let the series be $a + ar + ar^2 + \cdots$
$ar^2 = a^2 \Rightarrow r^2 = a$______(I)

Also $ar^4 = 64 \Rightarrow a\,.\,a^2 = 64$

$\Rightarrow a^3 = 4^3 \Rightarrow a = 4$
$\therefore r^2 = 4 \Rightarrow r = \pm 2$
$\therefore$ Required series is
$4 + 8 + 16 + 32 + \cdots$
Or
$\mathbf{4 - 8 + 16 - 32 + \cdots}$

Sol.95 (c) Do it by the option.

$\therefore$ 2+5+8= 15 and 2+1=3, 5+4 =9, 8+19=27. 3,9,27 are in G.P series.

$\therefore$ 26+5-16=15 and 26+1=27, 5+4=9, -16+19=3. 27,9, 3 are in G.P series

Or

Let the numbers be $a - d, a \,\&\, a + d$

$a - d + a + a + d = 15 \Rightarrow 3a = 15$

$\Rightarrow a = 5$

Also $a - d + 1, a + 4 \,\&\, a + d + 19$ are in G.P.

$\Rightarrow 6 - d, 9 \,\&\, 24 + d$ are in G.P.

$\Rightarrow 9^2 = (6-d)(24+d) = 144 - 18d - d^2$

$\Rightarrow d^2 + 18d - 63 = 0 \Rightarrow (d+21)(d-3) = 0$

$\Rightarrow d + 21 = 0 \; or \; d - 3 = 0 \Rightarrow d = -21 \; or \; d = 3$

$\therefore$ Numbers are $\mathbf{26, 5, -16} \; \boldsymbol{or} \; \mathbf{2, 5, 8}$

Sol.96 (b) Let 1st term & common ratio of a G.P. be A & R, respectively.

$a = AR^{p-1}$

$b = AR^{q-1}$

$c = AR^{r-1}$

$\therefore a^{q-r}.\,b^{r-p}.\,c^{p-q}$

$= A^{q-r+r-p+p-q}.\,R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$=A^0 \times R^0 = 1 \times 1 = \mathbf{1}$

Sol. 97 (b)

Let $a = -1, b = 0, c = 1$ as a, b, c are in A.P. & $x = 2, y = 4 \,\&\, z = 8, as\; x, y, z$ are in G.P.

$\therefore x^{b-c}.\,y^{c-a}\,.\; z^{a-b} = 2^{-1}.\,4^2.\,8^{-1} = \mathbf{1}$

Sol.98 (b)

Let $a = -1, b = 0, c = 1$ as a, b, c are in A.P.

$x = 2, y = 4 \,\&\, z = 8$ as x, y, z are in G.P.

Now $(x^b.\,y^c.\,z^a) \div (x^c.\,y^a.\,z^b)$

$= x^{b-c}.\,y^{c-a}.\,z^{a-b} = 2^{-1} \times 4^2 \times 8^{-1} = \mathbf{1}$

Sol.99 (a)

$s_2 = 7 + 77 = 84$

Put $n = 2$ in the option

$\left(\frac{7}{9}\right)\left[\frac{1}{9}(10^{n+1} - 10) - n\right]$

$= \frac{7}{9}\left[\frac{1}{9}(10^3 - 10) - 2\right]$

$= \frac{7}{9}\left(\frac{1}{9} \times 990 - 2\right) = \frac{7}{9}(110 - 2) = \frac{7}{9}(108) = \mathbf{84}$

Sol.100 (a)

$1+3+3^2+\cdots$ to n term > 7000

$\Rightarrow \frac{1(3^n-1)}{3-1} > 7000 \Rightarrow 3^n - 1 > 14000$

$\Rightarrow 3^n > 14001$

$\therefore 3^9 > 14001$

$\therefore$ The least value of $\boldsymbol{n} = \mathbf{9}$

Sol.101 (b)

$$S = \frac{a(r^n-1)}{r-1}$$

$$R = \frac{\frac{1}{a}\left[1-\left(\frac{1}{r}\right)^n\right]}{1-\frac{1}{r}} = \frac{r^n-1}{a\,r^{n-1}(r-1)}$$

P= $a.ar.ar^2$ ... to n terms

$=a^n r^{1+2+3+\cdots+n-1}$

$=a^n r^{\frac{(n-1)n}{2}}$

$\Rightarrow P^2 = a^{2n} r^{(n-1)n}$

Now $S^n R^{-n} = \frac{a^n (r^{n-1})^n}{(r-1)^n} \times \frac{(r^{n-1})^{-n}}{a^{-n} r^{(n-1)(n)} (r-1)^{-n}}$

$= \frac{a^n \cancel{(r^n-1)^n}}{\cancel{(r^n-1)^n}} \times \frac{a^n (r^{n-1})^n \cancel{(r-1)^n}}{\cancel{(r^{n-1})^n}}$

$= a^{2n}\, r^{(n-1)n}$

$=P^2$

Hence P is the G.M. between S^n & R^{-n}

Sol.102 (a) $r = \frac{1}{\sqrt{2}}$

$$S_\infty = \frac{a}{1-r} = \frac{8}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{8\sqrt{2}}{\sqrt{2}-1} = \frac{8\sqrt{2}(\sqrt{2}+1)}{2-1}$$

$= \mathbf{8(2+\sqrt{2})}$

Sol.103 (a)

$$S_\infty = \frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\cdots\infty$$

$$= \left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\cdots\infty\right)+\left(\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}+\cdots\infty\right)$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2^2}} + \frac{\frac{1}{3^2}}{1-\frac{1}{3^2}}$$

$$= \frac{1}{2}\times\frac{4}{3}+\frac{1}{9}\times\frac{9}{8} = \frac{2}{3}+\frac{1}{8} = \mathbf{\frac{19}{24}}$$

Sol.104 (a) $S_\infty = \frac{a}{1-r}$

$x = \frac{1}{1-a} \Rightarrow a = 1-\frac{1}{x}$

$y = \frac{1}{1-b} \Rightarrow b = 1-\frac{1}{y}$

Now, $1+ab+a^2b^2+\cdots\infty = \frac{1}{1-ab} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$

$$= \frac{xy}{xy-(x-1)(y-1)} = \frac{xy}{x+y-1}$$

$$\Rightarrow 1+ab+a^2b^2+\cdots\infty = \boldsymbol{\frac{xy}{x+y-1}}$$

Sol.105 (c) Do it by options a and b both options are in G.P.

$\therefore$ 20+10+5=35

5+10+20=35. Product of both options = $20\times 10 \times 5 = 1000$. So option c is correct.

Or

Let the number be $\frac{a}{r}$, a & ar

$\frac{a}{r}+a+ar = 35$

$\Rightarrow a(1+r+r^2) = 35r$ _______(I)

Also $\frac{a}{r}\times a\times ar = 1000$

$\Rightarrow a^3 = 1000 \Rightarrow a = 10$_____(II)

From (I) & (II)

$\therefore 10\,(1+r+r^2) = 35r$

$\Rightarrow 10r^2 - 25r + 10 = 0$

$\Rightarrow 10r^2 - 20r - 5r + 100 = 0$

$\Rightarrow (r-2)(10r-5) = 0$

$\Rightarrow r = 2 \text{ or } r = \frac{1}{2}$

$\therefore$ Numbers are **5, 10, 20 or 20, 10, 5**

Sol.106 (c) Do it by option
Here 3+6+12=21

& $3^2 + 6^2 + 12^2 = 9 + 36 + 144 = 189$

Also 12+6+3=21 & $12^2 + 6^2 + 3^2 = \mathbf{189}$

Sol.107 (a)
Let $a = 1, b = 2, c = 4$

Now $a(b^2 + c^2) - c(a^2 + b^2)$

$= 1(4 + 16) - 4(1 + 4)$

$= 20 - 20 = \mathbf{0}$

Sol.108 (a)
Let $a = \frac{1}{2}, b = 1, c = 2, d = 4$

$\therefore b(ab - cd) - (c + a)(b^2 - c^2)$

$= 1\left(\frac{1}{2} - 8\right) - \left(2 + \frac{1}{2}\right)(1 - 2^2)$

$= \frac{-15}{2} - \frac{5}{2} \times (-3) = \frac{-15}{2} + \frac{15}{2} = \mathbf{0}$

Sol.109 (a)
Let $a = 1, b = 2, c = 4, d = 8$
Now $(ab + bc + cd)^2 - (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$
$= (2 + 8 + 32)^2 - (1 + 4 + 16)(4 + 16 + 64)$
$= (42)^2 - 21 \times 84 = 1764 - 1764 = \mathbf{0}$

Sol.110 (b)
Let $a = 1,\ b = 2,\ c = 4,\ d = 8$

$\therefore a + b = 3$

$b + c = 6$

$c + d = 12$

$\because$ **3, 6, 12 are in G.P.**

Sol.111 (b)
$a = 1, b = 2, c = 4$

$\therefore a^2 + b^2 = 1 + 4 = 5$

$ab + bc = 2 + 8 = 10$

$b^2 + c^2 = 4 + 16 = 20$

$\because$ **5, 10, 20 are in G. P.**

Sol.112 (a)
$x = \frac{a+b}{2},\ y = \sqrt{ab} \Rightarrow y^2 = ab$

$z = \frac{2ab}{a + b} = \frac{y^2}{x} \Rightarrow y^2 = zx$

$\therefore$ ***x, y, z are in G.P.***

Sol.113 (a)
Let $a = 1, b = 2$ & $c = 4$

Then $(a - b + c)(a + b + c)^2 - (a + b + c)(a^2 + b^2 + c^2)$

$=3(7)^2 - 7(1 + 4 + 16) = 147 - 147 = \mathbf{0}$

Sol.114 (a)

Let $a = 1, b = 2$ & $c = 4$

$\therefore a(b^2 + c^2) - c(a^2 + b^2) = 1(4 + 16) - 4(1 + 4)$

$= 20 - 20 = \mathbf{0}$

Sol. 115 (a)
Let $a = 1, b = 2$ & $c = 4$

Now $a^2b^2c^2(a^{-3} + b^{-3} + c^{-3}) - (a^3 + b^3 + c^3)$

$= 1 \times 4 \times 16\left(\frac{1}{1} + \frac{1}{8} + \frac{1}{64}\right) - (1 + 8 + 64)$

$= 64\left(\frac{64+8+1}{64}\right) - (1 + 8 + 64)$

$=(64 + 8 + 1) - (1 + 8 + 64) = \mathbf{0}$

Sol.116 (b)

Let $a = 1, b = 2$ & $c = 4, d = 8$

$(a - b)^2 = (1 - 2)^2 = (-1)^2 = 1$

$(b - c)^2 = (2 - 4)^2 = (-2)^2 = 4$

$(c - d)^2 = (4 - 8)^2 = (-4)^2 = 16$

$\because$ **1, 4, 16 are in *G.P.***

Sol.117 (a)

Let $a = 1, b = 2,\ c = 4$ & $d = 8$

Now $(b - c)^2 + (c - a)^2 + (d - b)^2 - (a - d)^2$

$=(-2)^2 + 3^2 + 6^2 - (-7)^2$

$= 4 + 9 + 36 - 49 = 49 - 49 = \mathbf{0}$

Sol.118 (a)

$a-b,\ b-c\ \&\ c-a$ are in G.P.

$\therefore \frac{b-c}{a-b}=\frac{c-a}{b-c}=r(let)$

$\therefore\ b-c=r(a-b)$

$c-a=r(b-c)=r^2(a-b)$

$b-c+c-a=r(a-b)+r^2(a-b)$

$\Rightarrow -(a-b)=(r+r^2)(a-b)$

$\Rightarrow (r^2+r+1)(a-b)=0$

$\Rightarrow a-b=0$______(I)

$\therefore (a+b+c)^2-3(ab+bc+ca)$

$=a^2+b^2+c^2-ab-bc-ca$

$=\frac{1}{2}[2a^2+2b^2+2c^2-2ab-2bc-ca]$

$=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

$=\frac{1}{2}[(a-b)^2+r^2(a-b)^2+r^4(a-b)^2]$

$=\frac{1}{2}[(a-b)^2(1+r^2+r^4)]$

$=\frac{1}{2}[0]$ [From (I)]

$=0$

Sol.119 (a)

$a^{1/x}=b^{1/y}=c^{1/z}=k(let)$

$\therefore a=k^x, b=k^y\ \&\ c=k^z$

$\because a,b,c$ are in $G.P.$

$\therefore b^2=ac\Rightarrow (k^y)^2=k^x.k^z$

$\Rightarrow k^{2y}=k^{x+z}$

$\Rightarrow 2y=x+z$

$\therefore$ ***x, y, z* are in A.P.**

Sol.120 (a)

$x=\frac{a}{1-\frac{1}{r}}\Rightarrow 1-\frac{1}{r}=\frac{a}{x}\ \Rightarrow\ \frac{1}{r}=1-\frac{a}{x}$

$$y=\frac{b}{1+\frac{1}{r}}\Rightarrow 1+\frac{1}{r}=\frac{b}{y}\ \Rightarrow\ \frac{1}{r}=\frac{b}{y}-1$$

and $z=\frac{c}{1-\frac{1}{r^2}}$

$\therefore \frac{xy}{z}=\frac{\frac{a}{1-\frac{1}{r}}\times\frac{b}{1+\frac{1}{r}}}{\frac{c}{1-\frac{1}{r^2}}}$

$$=\frac{ab}{\cancel{1-\frac{1}{r^2}}}\times\frac{\cancel{1-\frac{1}{r^2}}}{c}$$

$$=\frac{ab}{c}$$

$$\Rightarrow \frac{ab}{c}-\frac{ab}{c}=\mathbf{0}$$

Sol.121 (a)

a,b,c are in A.P. $\Rightarrow b=\frac{a+c}{2}$

a,x,b are in G.P. $\Rightarrow x=\sqrt{ab}$

b,y,c are in G.P. $\Rightarrow y=\sqrt{bc}$

$x^2+y^2=ab+bc=b(a+c)$

$=b\times 2b$

$\Rightarrow x^2+y^2=2b^2$

$\therefore$ **x^2, b^2 & y^2 are in A.P.**

Sol.122 (a)

$a, b-a, c-a$ are in G.P.

$a={}^{b}/_{3}={}^{c}/_{5}=k\ (let)$

$\Rightarrow a=k, b=3k, c=5k$

$\Rightarrow$ a= k , b-a = 2k, c-a =4k. k, 2k, 4k are in G.P

$\therefore$ ***a*, *b*, & *c* are in A.P.**

Sol.123 (a) $a, b, (c+1)$ are in G. P.

Let a , b , c+1 =1,2, 4 respectively because they are in A.P.

$\therefore$ **a, b and c =1,2 and 3 so it is in A.P.**

Sol.124 (a)

$$\because s_\infty=\frac{a}{1-r}$$

$\therefore s_1=\frac{1}{1-\frac{1}{2}}=2,\ s_2=\frac{2}{1-\frac{1}{3}}=3, s_3=\frac{3}{1-\frac{1}{4}}=4$

$$s_n=\frac{n}{1-\frac{1}{n+1}}=n+1$$

$\therefore s_1 + s_2 + s_3 + \cdots + s_n = 2 + 3 + 4 + \cdots + (n+1)$

$= \frac{n}{2}[2 \times 2 + (n-1) \times 1] = \frac{n}{2}(n+3)$

Sol.125 (a) Do it by option.

4,-2,1. r= -1/2. 4th term will be $1 \times \frac{-1}{2} = \frac{-1}{2}$. 5th term will be $1 \times \frac{-1}{2} \times \frac{-1}{2} = \frac{1}{4}$, 6th term will be $\frac{1}{4} \times \frac{-1}{2} \times 1 = \frac{-1}{8}$. Option satisfied.

Or

Let the G.P. be $a, ar, ar^2, \ldots$

$$a_3 = 1 \Rightarrow ar^2 = 1$$

$$a_6 = -\frac{1}{8} \Rightarrow ar^5 = -\frac{1}{8}$$

$\therefore \frac{ar^5}{ar^2} = \frac{-1/8}{1} \Rightarrow r^3 = \left(-\frac{1}{2}\right)^3 \Rightarrow r = -\frac{1}{2}$

$\therefore a = 4$

$\therefore$ **Required $G.P.$ is $4, -2, 1, \ldots$**

Sol.126 (a)

$$a_{p+q} = m \Rightarrow ar^{p+q-1} = m$$

$$a_{p-q} = n \Rightarrow ar^{p-q-1} = n$$

$$\therefore ar^{p+q-1} \times ar^{p-q-1} = mn$$

$$\Rightarrow a^2 r^{2p-2} = mn$$

$$\Rightarrow (ar^{p-1})^2 = mn \Rightarrow (a_p)^2 = mn$$

$$\Rightarrow a_p = \sqrt{mn}$$

Sol.127 (a) At n=2 $s_2 = \frac{1}{\sqrt{3}} + 1$

Put n = 2 in option

a) $\frac{1}{6}(3+\sqrt{3})(3^{\frac{2}{2}} - 1) \Rightarrow \frac{1}{6}\sqrt{3}(\sqrt{3}+1)(3-1)$

$= \frac{1}{3}\sqrt{3}(\sqrt{3}+1)$

Or

$$s_n = \frac{a(r^n - 1)}{r - 1} = \frac{\frac{1}{\sqrt{3}}\left[\left(\sqrt{3}\right)^n - 1\right]}{(\sqrt{3} - 1)}$$

$$= \frac{3^{\frac{n}{2}} - 1}{3 - \sqrt{3}} = \frac{\left(3^{\frac{n}{2}} - 1\right)}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{(3+\sqrt{3})(3^{n/2} - 1)}{9 - 3}$$

$$= \frac{(3+\sqrt{3})(3^{n/2} - 1)}{6}$$

Sol.128 (c)

$$s_n = \frac{a(1 - r^n)}{1 - r} = \frac{\frac{5}{2}\left[1 - \left(\frac{-2}{5}\right)^n\right]}{1 - \left(\frac{-2}{5}\right)}$$

$$= \frac{5}{2}\left(\frac{5^n - (-2)^n}{5^n}\right) \times \frac{5}{7} = \frac{1}{14 \times 5^{n-2}}[5^n - (-2)^n]$$

If n is even, then $s_n = \frac{1}{14 \times 5^{n-2}}(5^n - 2^n)$

If n is odd, then $s_n = \frac{1}{14(5^{n-2})}(5^n + 2^n)$

Sol.129 (a) At n=2 $s_2 = 0.3 + 0.03 = 0.33$

Put n=2 $\frac{1}{3}\left(1 - \frac{1}{10^2}\right) \Rightarrow \frac{1}{3}\left(\frac{99}{100}\right) = \frac{33}{100} = 0.33$

Or

$$s_n = \frac{0.3[1 - (0.1)^n]}{1 - 0.1}$$

$$= \frac{0.3\left(1 - \frac{1}{10^n}\right)}{0.9} = \frac{1}{3}\left(1 - \frac{1}{10^n}\right)$$

Sol.130 (c)

$$s_8 = 5 \times s_4$$

$$\Rightarrow \frac{\cancel{a}(r^8 - 1)}{\cancel{r-1}} = 5\frac{\cancel{a}(r^4 - 1)}{\cancel{r-1}}$$

$\Rightarrow \frac{r^8 - 1}{r^4 - 1} = 5 \Rightarrow \frac{(r^4 + 1)(r^4 - 1)}{(r^4 - 1)} = 5 \Rightarrow r^4 + 1$

$= 5$

$$\Rightarrow r^4 = 4 \Rightarrow r^2 = 2 \Rightarrow r = \pm\sqrt{2}$$

Sol.131 (a)

$$a = 1, r = 1/2$$

$$s_n = 1 + \frac{127}{128}$$

$$\Rightarrow \frac{a(1 - r^n)}{1 - r} = \frac{255}{128}$$

$$\Rightarrow \frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}} = \frac{255}{128} \Rightarrow 2\left(1 - \frac{1}{2n}\right) = \frac{255}{128}$$

$$\Rightarrow 1 - \frac{1}{2^n} = \frac{255}{256}$$

$\Rightarrow \frac{1}{2^n} = 1 - \frac{255}{256}$

$\Rightarrow \frac{1}{2^n} = \frac{1}{256} \Rightarrow \frac{1}{2^n} = \frac{1}{2^8} \Rightarrow \boldsymbol{n = 8}$

Sol.132 (a)

$r = 2, \quad a_n = 128$

$\Rightarrow a\, 2^{n-1} = 128$

$\Rightarrow a = 2^{7-n+1} \Rightarrow a = 2^{8-n}$

Also $s_n = 255 \Rightarrow \frac{a(r^n-1)}{r-1} = 255$

$\Rightarrow \frac{2^{8-n}(2^n-1)}{2-1} = 255$

$\Rightarrow 2^8 - 2^{8-n} = 255 \Rightarrow 2^{8-n} = 1$

$\Rightarrow 8 - n = 0 \Rightarrow \boldsymbol{n = 8}$

Sol.133 (b)

$a = 1, \quad r = 4$

$s_n = 341 \Rightarrow \frac{1(4^n-1)}{4-1} = 341$

$\Rightarrow 4^n - 1 = 1023 \Rightarrow 4^n = 1024$

$\Rightarrow 2^{2n} = 2^{10} \Rightarrow 2n = 10 \Rightarrow \boldsymbol{n = 5}$

Sol.134 (a) Do it by option

At n=2 $s_2 = 5 + 55 = 60$

Put n =2

$= \frac{50}{81}(100-1) - \frac{10}{9} \Rightarrow \frac{550}{9} - \frac{10}{9} = \frac{540}{9} = 60$

Or

$s_n = 5 + 55 + 555 + \cdots$ to n terms

$\Rightarrow s_n = \frac{5}{9}(9 + 99 + 999 + \cdots \text{to n terms})$

$\Rightarrow s_n = \frac{5}{9}[(10-1) + (10^2-1) + (10^3-1)$
$+ \cdots \text{to n terms}]$

$= \frac{5}{9}[(10 + 10^2 + 10^3 + \cdots \text{to n terms}) - n]$

$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$

$= \frac{5}{9}\left[\frac{10}{9}(10^n-1) - n\right] = \boldsymbol{\frac{50}{81}(10^n-1) - \frac{5}{9}n}$

Sol.135 (a) Do it by option, at n =2

$s_2 = 0.5 + 0.55 = 1.05$

$= \frac{5}{9}(2) - \frac{5}{81}\left(1 - \frac{1}{10^2}\right) \Rightarrow \frac{10}{9} - \frac{5}{81}\left(\frac{99}{100}\right)$

$\Rightarrow \frac{5}{9}(2 - \frac{11}{100}) = 1.05$

Or

$s_n = 0.5 + 0.55 + 0.555 + \cdots$ to n terms

$\Rightarrow s_n = \frac{5}{9}[.9 + .99 + .999 + \cdots \text{to n terms}]$

$= \frac{5}{9}[(1-.1) + (1-0.01) + (1-0.001)$
$+ \cdots \text{to n terms}]$

$= \frac{5}{9}[n - (0.1 + 0.01 + 0.001 + \cdots \text{to n terms})]$

$= \frac{5}{9}\left\{n - \frac{0.1[1-(0.1)^n]}{1-0.1}\right\}$

$= \frac{5}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right] = \boldsymbol{\frac{5}{9}n - \frac{5}{81}(1 - 10^{-n})}$

Sol.136 (a) Do it by options, at n =2 $s_2 = 2.0909$

$\Rightarrow$ put n =2, $= \frac{103}{3}(1.03^2 - 1)$

$\Rightarrow \frac{103}{3}(0.0609) = 2.0909$

Or

$s_n = \frac{1.03[(1.03)^n - 1]}{1.03 - 1}$

$\Rightarrow \boldsymbol{s_n = \frac{103}{3}[(1.03)^n - 1]}$

Sol.137 (a)

$s_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{3}}$

$s_\infty = \frac{1}{2} \times \frac{3}{2} = \boldsymbol{\frac{3}{4}}$

Sol.138 (a)

$s_\infty = \frac{a}{1-r} = \frac{4}{1-0.2}$

$= \frac{4}{0.8} = \frac{40}{8} = \boldsymbol{5}$

Sol.139 (a)

$$s_\infty=\frac{a}{1-r}=\frac{\sqrt{2}}{1-\frac{1}{2}}=2\sqrt{2}$$

Sol.140 (a)

$$s_\infty=\frac{2}{3}+\frac{5}{9}+\frac{2}{27}+\frac{5}{81}+\cdots\infty=\frac{2/3}{1-\frac{1}{9}}+\frac{5/9}{1-\frac{1}{9}}$$

$$\Rightarrow\frac{2}{3}\times\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{8}}}+\frac{5}{9}\times\frac{9}{8}=\frac{3}{4}+\frac{5}{8}=\frac{11}{8}$$

Sol.141 (a)

$$s_\infty=\frac{a}{1-r}=\frac{\sqrt{2}+1}{1-\frac{1}{\sqrt{2}+1}}=\frac{\left(\sqrt{2}+1\right)^2}{\sqrt{2}}=\frac{3+2\sqrt{2}}{\sqrt{2}}$$

$$=\frac{1}{2}(3\sqrt{2}+4)$$

Sol.142 (a)

$$s_\infty=(1+2^{-2})(2^{-1}+2^{-4})+(2^{-2}+2^{-6})+\cdots\infty$$

$$=(1+2^{-1}+2^{-2}+\cdots\infty)$$

$$+(2^{-2}+2^{-4}+2^{-6}+\cdots\infty)$$

$$=\frac{1}{1-\frac{1}{2}}+\frac{\frac{1}{4}}{1-\frac{1}{4}}=2+\frac{1}{4}\times\frac{4}{3}$$

$$=\frac{6+1}{3}=\frac{7}{3}$$

Sol.143 (a)

$$s_\infty=\frac{4}{7}-\frac{5}{7^2}+\frac{4}{7^3}-\frac{5}{7^4}+\cdots\infty$$

$$=\left(\frac{4}{7}+\frac{4}{7^3}+\cdots\infty\right)-\left(\frac{5}{7^2}+\frac{5}{7^4}+\cdots\infty\right)$$

$$=\frac{4/7}{1-\frac{1}{49}}-\frac{5/49}{1-\frac{1}{49}}=\frac{4}{\cancel{7}}\times\frac{\overset{7}{\cancel{49}}}{48}-\frac{5}{\cancel{49}}\times\frac{49}{48}$$

$$=\frac{28-5}{48}=\frac{23}{48}$$

Sol.144 (a) Do it by options.

= 1 , ½ , ¼ then r=1/2 $s_n=\frac{1}{1-\frac{1}{2}}=2$

Sum of squares of series $=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$

Or

$\frac{a}{1-r}=2$______(I)

Also $\frac{a^2}{1-r^2}=\frac{4}{3}$______(II)

From (I) & (II)

$$\frac{[2(1-r)]^2}{1-r^2}=\frac{4}{3}\Rightarrow\frac{4(1-r)^2}{(1+r)\cancel{(1-r)}}=\frac{4}{3}$$

$$\Rightarrow 3-3r=1+r\Rightarrow 4r=2$$

$$\Rightarrow r=\frac{1}{2}$$

$$\therefore\ a=2\left(1-\frac{1}{2}\right)=1$$

∴ required series is $\mathbf{1+\frac{1}{2}+\frac{1}{2^2}+\cdots}$

Sol.145 (a)

$$a=\frac{1}{4}$$

$$s_\infty=\frac{1}{3}\Rightarrow\frac{a}{1-r}=\frac{1}{3}$$

$$\Rightarrow\frac{1}{4(1-r)}=\frac{1}{3}$$

$$\Rightarrow 1-r=\frac{3}{4}\Rightarrow r=1-\frac{3}{4}=\frac{1}{4}$$

∴ required series is $\mathbf{\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}}+\cdots\infty$

Sol.146 (a) Do it by options.

r= 2 $s_n=\frac{10}{1-\frac{4}{5}}=50.\ option\ satisfied.$

Or

$a-ar=2\Rightarrow a(1-r)=2$______(I)

Also $\frac{a}{1-r}=50$______(II)

From [(I) ÷ (II)]

$$\Rightarrow(1-r)^2=\frac{2}{50}=\frac{1}{25}$$

$$\Rightarrow r=1-\frac{1}{5}=\frac{4}{5}$$

$$\therefore a=\frac{2}{1-r}=\frac{2}{\frac{1}{5}}=10$$

$\therefore$ Required series is $\mathbf{10+8+\frac{32}{5}+\cdots}$

Sol.147 (c) Do it by options.

a) 10, 30, 90. r=3. Sum of numbers=10+90+30=130 product =27,000

b) 90, 30, 10. r=1/3 sum = 130 product =27,000

Or

Let three numbers in G.P. be

$\frac{a}{r}, a \, \& \, ar$

$$\frac{a}{r}+a+ar=130 \Rightarrow a(1+r+r^2)$$
$$=130r \_\_\_\_\_ \text{(I)}$$

$$\frac{a}{r}.a.ar=27000 \Rightarrow a^3=(30)^3$$

$$\Rightarrow a=30 \_\_\_\_\_ \text{(II)}$$

From (I) & (II) $3\cancel{0}(1+r+r^2)=13\cancel{0}r$

$$\Rightarrow 3+3r+3r^2-13r=0 \Rightarrow 3r^2-10r+3=0$$

$$\Rightarrow (3r-1)(r-3)=0 \Rightarrow r=\frac{1}{3} \text{ or } 3$$

$\therefore$ Numbers are **90, 30, 10 or 10, 30, 90**

Sol.148 (c) In this case, verify with option

Here $\frac{1}{3}+1+3=\frac{1+3+9}{3}=\frac{13}{3}$

Also $\left(\frac{1}{3}\right)^2+1^2+3^2=\frac{1+9+81}{9}=\frac{91}{9}$

Sol.149 (a)

In this case, verify with option

$\frac{2}{\cancel{9}} \times \frac{2}{\cancel{3}} \times 2 \times \cancel{6}^2 \times \cancel{18}^2 = \mathbf{32}$

Also 6×18=108

Sol.150 (c)

In this case, verify with option

Here 1×3×9=27

1×3+3×9+9×1=3+27+9=39

Also, $9\times3\times1=27 \;\&\; 9\times3+3\times1+9\times1=\mathbf{39}$

Sol.151 (a) Do it by option.

x, 8, y are in G.P let x and y= 16 and 4. $r_1=\frac{16}{8}=2, r_2=\frac{8}{4}$=2 so it is in G.P

16, 4. -8 are in A.P, so both conditions are satisfied.

Or

$x, 8, y,$ are in G. P.: $xy=8^2 \Rightarrow xy=64 \_\_\_\_\_$ (I)
Also $x, y, -8$ are in A. P.
$\therefore y=\frac{x-8}{2} \_\_\_\_\_$ (II)

From (I) & (II)

$$x\left(\frac{x-8}{2}\right)=64$$

$$\Rightarrow x^2-8x-128=0$$

$$\Rightarrow (x-16)(x+8)=0 \Rightarrow x=16 \text{ or } x=-8$$

If $x=16$ *then* $y=\frac{16-8}{2}=\mathbf{4}$

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Sets Relation and Functions

Exercise: 7A

Sol.1 (b) Let A={2, 3, 5}
$n=3$

$\therefore$ Total nos. of subset $=2^n=2^3=\mathbf{8}$

Sol.2 (a) Total nos. the subset of a set has n elements = $\mathbf{2^n}$

Sol.3 (c) Set=∅ or { }

Sol. 4 (a) $A=\{2,3,5,7\}$

$B=\{4,6,8,10\}$ ∴ A∩B = { } or ∅

Sol.5 (b) $\because x \in z \,\&\, \{x|0<x<5\}=\{1,2,3,4\}$

Sol.6 (c) $\{0,2,4,6,8,10\}=\{2x: 0 \le x \le 5 \,\&\, x \in z\}$

Sol.7 (b) $P \cap Q=\{1,3\} \therefore n(P \cap Q)=\mathbf{2}$

Sol.8 (c) $P \cup Q=\{1,2,3,5,6,7,10,15\}$

$n(P \cup Q)=\mathbf{8}$

Sol.9 (a) $n(P)=5$

$n(S)=15$

$\therefore n(P')=n(S)-n(P)=15-5=\mathbf{10}$

Sol.10 (b) $n(Q) = 5, n(S) = 15$

$\therefore n(Q') = 15 - 5 = \mathbf{10}$

Sol.11 (b) $\{1, 8, 27, 64, \ldots\}$

Sol.12 (a)

Sol.13 (c) If x is odd, then
$1 - (-1)^x = 1 - (-1) = 1 + 1 = 2$
If x is even then $1 - (-1)^x = 1 - 1 = 0$

Required set$= \{\mathbf{0}, \mathbf{2}\}$

Sol.14 (b) $E = \{2, 4, 6, \ldots\}, O = \{1, 3, 5, 7, \ldots\}$

$\therefore E \cup O = \{1, 2, 3, 4, 5, \ldots\} = N$

Sol.15 (b) R⊂E

Sol.16 (a) N = $\{1, 2, 3, \ldots\}$

$I = \{1, 2, 3, \ldots\} \therefore$ N $= I$

Sol.17 (b)

Sol.18 (c)

Sol.19 (b) $\left\{\frac{n(n+1)}{2} : n \text{ is a positive integer}\right\}$

$= \{1, 2, 3, \ldots.\} = \boldsymbol{N}$

Sol.20 (c) $n(A) = 5$ *and* $n(B) = \mathbf{5}$

$\therefore n(A) = n(B)$

Sol.21 (a) $A \cup A = A$

Sol.22 (b) $A \cap A = A$

Sol.23 (c) $(A \cup B)' = A' \cap B'$

Sol.24 (b) $(A \cap B)' = A' \cup B'$

Sol. 25 (b) $E \supset A = A \cup E = E$

Sol.26 (a) $E \supset A = A \cap E = A$

Sol.27 (a) $E \cup E = E$

Sol. 28 (b) $E \supset A$

$\therefore$ All elements of A belong to E

$\therefore$ None of the elements belongs to E'

$\therefore A \cap E' = \emptyset$

Sol. 29 (c) $A \cap \emptyset = \emptyset$

Sol.30 (a) $A \cup A' = E$ where E is a universal set

Sol.31 (b) $5 + x > 10 \Rightarrow x > 10 - 5 \Rightarrow x > 5$

$\therefore$ Required set $= \{6, 7, 8, 9\}$

Sol.32 (a) $A \Delta B = (A - B) \cup (B - A)$

$= \{1, 2, 4\} \cup \{5, 7\} = \{\mathbf{1, 2, 4, 5, 7}\}$

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Sets Relation and Functions

Exercise: 7B

Sol. 1 (b, d) $\because$ For a function, every domain has a range.

Sol. 2 (c) $x + y = 5 \Rightarrow y = 5 - x \Rightarrow f(x) = y = 5 - x$

{(2,3), (3,2),(1,4),(4,1),(0,5),(5,0)}

$\therefore$ **It is one − one function**

Sol. 3(a) Here $x = 4$ & $x \in R$

= {(4,1),(4,2),(4,3),.......}

$\therefore$ *common function* (x)

$\therefore$ **It is not a function**

Sol. 4 (b) Let $f(x) = y = x^2$

= {(x, y) y=x^2}

= {(1,1),(2,4), (3,9)......}

$\therefore$ It is not a one-one function

$\therefore$It is many-one function

Sol. 5(a) It is not a function

$\because (2, 3), (2, 4), (2, 5)$ all belongs in the relation

Sol. 6(c) Domain = $\{1, 2\}$

Sol. 7(b) Range = $\{0\}$

Sol. 8(b) Domain = R and range = R^+& $\{0\}$

Sol. 9(a) Option a satisfy the condition that $f(x) = g(x) i.e\, f(1) = g(1)$

Sol. 10 (b) $\because f(x) = \frac{1}{1-x}$

$$\therefore f(-1) = \frac{1}{1-(-1)} = \frac{\mathbf{1}}{\mathbf{2}}$$

Sol. 11 (d) $g(x) = \frac{x-1}{x} \therefore g\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}-1}{-1/2} = -\frac{3}{2} \times \frac{2}{1} = 3$

Sol. 12 (a) $f(x) = \frac{1}{1-x}, g(x) = \frac{x-1}{x}$

$\therefore fog(x) = f\{g(x)\} = f\left(\frac{x-1}{x}\right)$

$$= \frac{1}{1-\frac{x-1}{x}} = \frac{1}{\frac{x-x+1}{x}} = \frac{x}{1} = x$$

Sol. 13(b) $f(x) = \frac{1}{1-x}$ and $g(x) = \frac{x-1}{x}$

$\therefore gof\,(x) = g\{f(x)\} = g\left(\frac{1}{1-x}\right) = \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}$

$$= \frac{1-1+x}{1-x} \times \frac{1-x}{1} = x$$

Sol. 14 (a) $f(x) = 2^x$

$\Rightarrow \{(2,2),(2,4),(2,8),.......)$

$\Rightarrow 2^{x_1} = 2^{x_2} \Rightarrow x_1 = x_2$

$\therefore$ **It is one − one function or one − one mapping**

Sol.15 (c) $\because 0 \le x \le 9 \Rightarrow 1 \le 1 + x \le 10$

$\therefore \log_{10} 1 \le \log_{10}(1+x) \le \log_{10} 10$

$\Rightarrow 0 \le \log_{10}(1+x) \le 1$

$\therefore$ **Range of the function = [0, 1]**

Sol. 16 (b) $\because f(x) = 2x \therefore let\ y = f(x) = 2x$

$\Rightarrow \quad x = \frac{y}{2} \Rightarrow f^{-1}(x) = \frac{x}{2}$

Sol. 17 (a) $f(x) = x + 3, g(x) = x^2$

$\therefore fog\,(x) = f\{g(x)\} = f(x^2) = \boldsymbol{x^2 + 3}$

Sol. 18 (c) $f(x) = x + 3, g(x) = x^2$

$\therefore f(x).g(x) = (x+3)x^2 = \boldsymbol{x^3 + 3x^2}$

Sol. 19 (b) Let $y = h(x) = log_{10}x$

$\Rightarrow \quad y = log_{10}x$

$\Rightarrow 2^3 = 8$

$\Rightarrow \log 2^8 = 3$

$\Rightarrow 10^y = x \Rightarrow 10^x = \boldsymbol{f^{-1}(x)}$

Sol. 20(a) $0 \le x \le 9 \Rightarrow 1 \le 1 + x \le 10$

$\therefore 10^1 \le 10^{1+x} \le 10^{10}$

$\Rightarrow \boldsymbol{10 \le h(x) \le 10^{10}}\ (\because h(x) = 10^{1+x})$

****

Sets Relation and Functions
Exercise: 7C

Sol.1 (a) $a_{R_a} \Rightarrow a < a$ which not true $\therefore$ It is not reflexive

$a_{R_b} \Rightarrow a < b \quad \therefore b$ is not less than a

$\therefore$ It is not symmetric

a_{R_b} & $b_{R_c} \Rightarrow a < b$ & $b < c$

$\Rightarrow a < c \Rightarrow a_{R_c}$

$\therefore$ **It is Transitive**

Sol. 2 (d) $a_{R_a} \Rightarrow a = a$ which is true

$\therefore$ It is reflexive

$a_{R_b} \Rightarrow a = b \Rightarrow b = a \Rightarrow b_{R_a}$

$\therefore$ It is symmetric

a_{R_b} & $b_{R_c} \Rightarrow a = b$ & $b = c \Rightarrow a = c \Rightarrow a_{R_c}$

$\therefore$ It is transitive

Hence it is equivalence relation

Sol.3 (d) $a_{R_a} \Rightarrow a$ & a has same father is true

$\therefore$ It is reflexive

$a_{R_b} \Rightarrow a$ & b has same father $\Rightarrow b$ & a has same father $\Rightarrow b_{R_a}$

$\therefore$ It symmetric

a_{R_b} & $a_{R_c} \Rightarrow a$ & b has same father & $(b$ & $c)$ has same father $\Rightarrow a$ & c has same father $\Rightarrow a_{R_c}$

$\therefore$ **It is transitive**

Sol.4 (b) No line is perpendicular to itself

$\therefore$ no reflexive

$a_{R_b} \Rightarrow a$ line $\perp$ b line $\Rightarrow b$ line $\perp$ a line

$\Rightarrow b_{R_a} \therefore$ It is symmetric

$a_{R_b} \Rightarrow a \perp b$ and $b_{R_c} \Rightarrow b \perp c\, b$

$\therefore a \parallel c \Rightarrow (a,c) \notin R$

$\therefore$ **Is it not transitive**

Sol.5 (a) $(a,a) \in R \Rightarrow a = \frac{1}{a}$ which is not true
$\therefore$ It is not reflexive
$(a, b)\,) \in R \Rightarrow a = \frac{1}{b} \Rightarrow b = \frac{1}{a} \Rightarrow (b,a) \in R$

$\therefore$ It is symmetric

$(a,b) \in R \;\&\; (b,c) \in R \Rightarrow a = \frac{1}{b} \;\&\; b = \frac{1}{c}$

$\Rightarrow a \neq \frac{1}{c} \Rightarrow (a,c) \notin R$

$\therefore$ **It is not transitive**

Sol.6 (d) $(x,y) \in R \Rightarrow x = y$ which is true

$\therefore$It is reflexive

$(x,y) \in R \Rightarrow x = y \Rightarrow y = x \Rightarrow (y,x) \in R$

$\therefore$ It is symmetric relation

$(x,y) \in R \;\&\; (y,z) \in R \Rightarrow x = y \;\&\; y = z$

$\Rightarrow x = z \Rightarrow (x,z) \in R$

$\therefore$ It is transitive $\therefore$ **It is an equivalence relation.**

Sol.7 (d) $x + y = 2x \Rightarrow y = x$

$\therefore (x,x) \in R \Rightarrow x = x$ $\therefore$ It is reflexive
$(x,y) \in R \Rightarrow y = x \Rightarrow x = y \Rightarrow (y,x) \in R$
$\therefore$ It is symmetric
$(x,y) \in R \;\&\; (y,z) \in R \Rightarrow y = x \;\&\; z = y$
$\Rightarrow z = x \Rightarrow (x,z) \in R$
$\therefore$ **It is transitive**
$\therefore$ **It is an equivalence relation**

Sol.8 (d) $(x,x) \in R \Rightarrow x = x^2$ which is not true

$\therefore$ It is not reflexive

$(x,y) \in R \Rightarrow x = y^2 \Rightarrow y \neq x^2$

$\Rightarrow (y,x) \in R$ $\therefore$ It is not symmetric

$(x,y) \in R \;\&\; (y,z) \in R \Rightarrow x = y^2 \;\&\; y = z^2$

$\Rightarrow x \neq z^2 \Rightarrow (x,z) \notin R$

$\therefore$ **It is not transitive**

Sol.9 (a) $n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow 62 = 32 + 42 - n(A \cup B)$

$\Rightarrow n(A \cup B) = 74 - 62 = \mathbf{12}$

Sol.10 (b) T⟶ Tea, C ⟶ Coffee

$n(T \cup C) = 20, n(T - C) = 8, n(T) = 13$

$\Rightarrow 20 = 13 + n(C - T)$
$\Rightarrow n(C - T) = 20 - 13 = \mathbf{7}$

Sol.11 (c) $n = 3$ ($\because$ number of element in a set=n)

$\therefore$Total nos. of subset = $2^n = 2^3 = \mathbf{8}$

Sol.12 (a) $V = \{-2\}, R = \{-2, 0\}$

$s = \{\mathbf{-2, 1}\}$

Sol.13 (a,b) $(A \cap B)' = A' \cup B'$

$(A \cup B)' = A' \cap B'$

Sol.14 (c) $n(P) = 3,\; n(Q) = 4,\; n(R) = 2$

$\therefore n(P \times Q \times R) = n(P) \times n(Q) \times n(R)$

$=3\times4\times2=24$

Sol.15 (a) $B \cap C = \{5\}$

$A \times (B \cap C) = \{(2,5), (3,5)\}$

Sol.16 (b) $n(U) = 52{,}000$

$n(x)=28{,}000, n(y)=23{,}000, n(x \cap y) = 4000$

$\therefore\; n(x \cup y) = 28{,}000 + 23{,}000 - 4{,}000 = 47{,}000$

$n(x \cup y)' = 50{,}000 - 47{,}000 = \mathbf{3,000}$

Sol.17 (a) $n(A) = 5,\; n(B) = 5$

$n\,(A \cap B) = 2$

$n(A - B) = n(A) - n(A \cap B) = 5 - 2 = 3$

Sol.18 (b)

Sol.19 (c) M ⟶ Men

W ⟶ Women

D ⟶ Doctor

$n(M) = 23, \quad n(W) = 29, \quad n(D) = 4$

$n(M \cup D) = 24$

$\Rightarrow n(M) + n(D) - n(M \cap D) = 24$

$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$2 + 42 - n(A \cup B)$

$B) = 74 - 62 = \mathbf{12}$

$T \rightarrow$ Tea, $C \rightarrow$ Coffee

$20, n(T - C) = 8, n(T) = 13$

$+ n(C - T)$

$= 20 - 13 = \mathbf{7}$

$= 3$ ($\because$ number of element in a set=n)

f subset $= 2^n = 2^3 = \mathbf{8}$

$= \{-2\}, R = \{-2, 0\}$

$A \cap B)' = A' \cup B'$

B'

$= 3,\ n(Q) = 4,\ n(R) = 2$

$= n(P) \times n(Q) \times n(R)$

$= \{5\}$

$), (3,5)\}$

52,000

$=23,000, n(x \cap y) = 4000$

$00 + 23,000 - 4,000 = 47,000$

$- 47,000 = \mathbf{3,000}$

$, n(B) = 5$

$(A \cap B) = 5 - 2 = 3$

$n(D) = 4$

$= 29,$

$\cap D) = 24$

$$\Rightarrow 23 + 4 - n(M \cap D) = 24 \Rightarrow n(M \cap D) = 27 - 24 = 3$$

$$n(M \cap D) + n(W \cap D) = 4 \Rightarrow 3 + n(W \cap D) = 4$$

$$\Rightarrow n(W \cap D) = 1$$

Sol.20 (b) $n(A) = 2$

$n(P(A)) = 2^2 = \mathbf{4}$

Sol.21 (a) $a + b + c + d = 48\%$

$b + c + f + g = 54\%$

$c + d + e + f = 64\%$

$b + c = 28\%$

$c + d = 30\%$

$h = 6\%$

$a + b + c + d + e + f + g + h = 100\%$

$a + b + c + d + e + f + g = 94\%$

$$\Rightarrow 48\% + 54\% + 64\% - 28\% - 32\% - 30\% + c = 94\%$$

$\Rightarrow 166\% - 90\% + c = 94\%$

$\Rightarrow 76\% + c = 94\% \Rightarrow c = 18\%$

$\therefore$ **Required value = 18% 2,000=360**

Sol.22 (b) $f = (32 - 18)\% = 14\%$

$\therefore$ Required no. = 14% of 2000=**280**

Sol.23 (c) $n(C - T - S)$

$= n(C) - n(C \cap T) - (C \cap S) + n(C \cap T \cap S)$

=(48 − 28 − 30 + 18)%

=(66 − 58)% = 8%

$\therefore$ Required no. =8% of 2000

=**160**

Sol.24 (a) $f(x) = x + 3, g(x) = x^2$

$go\,f(x) = g\{f(x)\} = g(x + 3) = (x + 3)^2$

Sol.25 (b) Let $y = f(x) = \frac{1}{1-x}$

$$\Rightarrow 1 - x = \frac{1}{y} \Rightarrow x = 1 - \frac{1}{y}$$

$$\Rightarrow f^{-1}(x) = \frac{x-1}{x}$$

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Sets Relation and Functions
Exercise: Additional Questions

Sol.1 (a) A is a proper subset of B; x is not an element of A; A contains B; singleton with an only element zero; A is not contained in B.

Sol.2 (b) A={x: x is an alphabet in English}, I= {x: x is an odd integer <25}, {1, 3, 5, 7 .....} I = {x: $x^2 + 5x + 7 = 0$}

Sol.3 (d) A ={x: x is a vowel}, B={x: x is a natural number}, C={x:-15<x<15∧x is a whole number }

Sol.4 (a){3, 5, 7}, {0, 2, 4, 6, 8}, {0, 1, 2.....9}

Sol.5 (b) $[\because (XUY) \cap Z = \{0,2,3,4,5,6,7,8\} \cap \{3,7\} = \&(\emptyset UV) \cap \emptyset = V \cap \emptyset = \emptyset]$

Sol.6 (c) $V = \{x: x + 2 = 0\} \Rightarrow V = \{-2\}$

$R = \{x: x^2 + x - 2 = 0, x < 0\} = \{-2\}$

$S = \{-2\}$

Sol.7 (a) $A = \{f, l, o, w, e, r\}$

$B = \{f, l, o, w\}, C = \{w, o, l, f\}$

$D = \{f, o, l, w\}$

Sol.8 (a) $\because$ i) Correct ii) Incorrect
iii) Correct iv) Incorrect

Sol.9 (a) Correct → i), ii), iii), ix), x), xiii)

Incorrect → iv), v), vi), vii), viii), xi), xii)

Sol.10 (a) $A = \{0\}, B = \{0,1\}, C = \emptyset, D = \{\emptyset\}, E = \emptyset, F = \{0\},$

True → i), iii), iv), v)

False → ii), vi), vii)

Sol.11 (a) True → i), iv), vii)

False → ii), iii), v), vi)

Sol.12 (a) Finite --- (i)

Infinite --- (ii), (iii)

Empty --- (iv)

Sol.13 (a) $\begin{bmatrix} \because A \cap B = \{2,3\},\ B \cap C = \{7,9\} \\ A \cap C = \{1,4\},\ A \cap B \cap C = \emptyset \end{bmatrix}$

Sol.14 (b) $x = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A \cup \{B \cap C\} = \{1{,}9{,}10\{\cup\}6\}$

$= \{\mathbf{1, 6, 9, 10}\}$

Sol.15 (b) $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$

$= \{\mathbf{1, 6, 9, 10}\}$

Sol.16 (d)

Income		**C → 6,000**	**D → 6,000 - 10,999**	**E → 11,000 -15,999**	**>16,000**
	0	70	50	20	50
B →	1	152	308	114	46
A →	>2	10	174	84	94

i) $C \cap B = 152$ ii) $A \cup E = (10 + 174 + 84 + 94 + (20 + 114 + 84) - 84 = \mathbf{496}$

Sol.17) (d) i) $(A \cap B)' \cap E = 20$

ii) $(C \cup D \cup E) \cap (A \cup B)' = 70 + 50 + 20 = \mathbf{140}$

Sol.18 (c) i) 152+308 = 460 = $(C \cup D) \cap B$

ii) $(A \cup B)' \cap (C' \cup D' \cup E')$

Sol.19 (c) i) $A \cup E$

ii) $(A \cup B)'$

Sol.20 (d) $A = \{a, b, c, d\}$

The element of Power set P (A)

The power set of a is set of all possible sub set of set A. As set A has 4 element, P(A) will have $2^4 = 16$ element and the elements are:

$\therefore P\,|A| = \{\emptyset, \{a\}, \{b\}, \{C\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}$

$\{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}$

$\{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}\,\}$

Sol.21 (a) In a meeting resolution is passed on majority. It means it should have more than 50%.

Given that a, b, c, d own 50%, 20%, 15% and 15% share each.

For {a+b} = 50%+20% = 70% > 50%

Similarly {a+c} = 50%+15% = 65% > 50%

So, we will calculate for every elements of option a, each one >50%.

While {b,c,d} =20%+15%+15% = 50%

Which is not greater than 50%, so it is not possible.

While for option c, each value <50%

So that is also not correct option.

So Option a is correct answer.

Sol.22 (b) The resolution is blocked when it is exactly 50%.

In option a, each one >50%.

In option b, it is exactly 50%. So, we will block the decision. So, option b is the correct answer.

Sol.23 (d) The resolution will lose when it has less than 50%.

In option a, each one has value more than 50%.

In option b, it is equal to 50%.

In option c, for {a,b,c} >50% other are ≤ 50%.

So, it is also not proper losing.

So, option d, none of these is correct answer. In module it is given option c which is not correct. It will be correct if {a,c,d} not given in the option.

Sol.24 (a) If A ={a, b, c, d, e, f,}, B ={a, e, i, o, u} and C ={m, n, o, p, r, s, t, u}

A∪B ={a, b, c, d, e, f, i, o, u}, So the option a is the correct answer.

Sol.25 (a) If A ={a, b, c, d, e, f,}, B ={a, e, i, o, u} and C ={m, n, o, p, r, s, t, u}

A∪C ={a, b, c, d, e, f, m, n, o, p, q, r, s, t, u}

So the option a is the correct answer.

Sol.26 (a) If A ={a, b, c, d, e, f,}, B ={a, e, i, o, u} and C ={m, n, o, p, r, s, t, u}

B∪C = {a, e, i, o, u, m, n, p, q, r, s, t}

So the option a is the correct answer.

Sol.27 (a) A-B = {b,c,d,f}

Sol.28 (a) If A =$\{a, b, c, d, e, f,\}$, B =$\{a, e, i, o, u\}$ and C =$\{m, n, o, p, r, s, t, u\}$

$A \cap B = \{a, b, c, d, e, f,\} \cap \{a, e, i, o, u\}$

$A \cap B = \{a, e\}$ So the option a is the correct answer.

Sol.29 (c) If A =$\{a, b, c, d, e, f,\}$, B =$\{a, e, i, o, u\}$ and C =$\{m, n, o, p, r, s, t, u\}$

$B \cap C = \{a, e, i, o, u\} \cap \{m, n, o, p, r, s, t, u\}$

$B \cap C = \{o, u\}$ So the option c is the correct answer.

Sol.30 (a) $A = \{a, b, c, d, e, f\}$

$B - C = \{a, e, i\}$

$\therefore A \cup (B - C) = \{a, b, c, d, e, f, i\}$

Sol.31 (a) If A =$\{a, b, c, d, e, f,\}$, B =$\{a, e, i, o, u\}$ and C =$\{m, n, o, p, r, s, t, u\}$

$A \cup B \cup C = \{a, b, c, d, e, f,\} \cup \{a, e, i, o, u\} \cup \{m, n, o, p, r, s, t, u\}$

$A \cup B \cup C = \{a, b, c, d, e, f, i, o, u, m, m, p, q, r, s, t\}$

So the option a is the correct answer.

Sol.32 (a) If A =$\{a, b, c, d, e, f,\}$, B =$\{a, e, i, o, u\}$ and C =$\{m, n, o, p, r, s, t, u\}$

$A \cap B \cap C = \{a, b, c, d, e, f,\} \cap \{a, e, i, o, u\} \cap \{m, n, o, p, r, s, t, u\}$

$A \cap B \cap C = \{\,\} = \emptyset$

Sol.33 (a) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$A' = \{x : x \in U, x \notin A\}$

$A' = \{7,8,9,10,11,12,13\}$

Sol.34 (b) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$B' = \{x : x \in U, x \notin B\}$

$B' = \{4,6,8,10,11,12,13\}$

Sol.35 (c) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$C' = \{x : x \in U, x \notin C\}$

$C' = \{4,5,9,11,13\}$

Sol.36 (a) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$A' = \{x : x \in U, x \notin A\}$

$A' = \{7,8,9,10,11,12,13\}$

$(A')' = \{x : x \in U, x \notin A'\}$

$(A')' = \{3,4,5,6\}$

Sol.37 (b) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$B' = \{x : x \in U, a \notin B\}$

$B' = \{4,6,8,10,11,12,13\}$

$(B')' = \{x : x \in U, x \notin B'\}$

$(B')' = \{3,7,9,5\}$

Sol.38 (c) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$A \cup B = \{3,4,5,6,7,9\}$

$(A \cup B)' = \{x : x \in U, x \notin (A \cup B)\}$

$(A \cup B)' = \{8,10,11,12,13\}$

Sol.39 (b) If A $\{3,4,5,6\}$, B = $\{3,7,9,5\}$ and

C =$\{6,8,10,12,7\}$, U = $\{1,2,3, \ldots\ldots, 11,12,13\}$

$A \cap B = \{3,5\}$

$(A \cap B)' = \{x : x \in U, x \notin (A \cap B)\}$

$(A \cap B)' \{4,6,7,8,9,10,11,12,13\}$

Sol.40 (c) $A' \cup C' = (A \cap C)' = \{3,4,5,7, \ldots .13\}$

Sol.41 (a) If A = $\{1,2,3,4, \ldots\ldots, 9\}$ B = $\{2,4,6,8\}$, C = $\{1,3,5,7,9\}$, D = $\{3,4,5\}$ and E = $\{3,5\}$

$S \subset D$ and $S \not\subset B$

So, S should have elements 3,4 or 5

$S \not\subset B$, so S = $\{3,5\}$

Sol.42(b) If A = $\{1,2,3,4, \ldots\ldots, 9\}$ B = $\{2,4,6,8\}$, C = $\{1,3,5,7,9\}$, D = $\{3,4,5\}$ and E = $\{3,5\}$

$S \subset B$ and $S \not\subset C$

So, S = $\{2,4\}$

Sol.43 (a) If U = {1,2,3,.......,9} be the universal set A = {1,2,3,4} and B = {2,4,6,8}

$A \cup B = \{1,2,3,4\} \cup \{2,4,6,8\}$

$A \cup B = \{1,2,3,4,6,8\}$

Sol.44 (b) If U = {1,2,3,.......,9} be the universal set A = {1,2,3,4} and B = {2,4,6,8}

$A \cap B = \{1,2,3,4\} \cap \{2,4,6,8\}$

$A \cap B = \{2,4\}$

Sol.45 (c) If U = {1,2,3,.......,9} be the universal set A = {1,2,3,4} and B = {2,4,6,8}

$A' = \{x : x \in U, x \notin A\}$

$A' = \{5,6,7,8,9\}$

Sol.46 (d) If U = {1,2,3,.......,9} be the universal set A = {1,2,3,4} and B = {2,4,6,8}

$A \cup B = \{1,2,3,4\} \cup \{2,4,6,8\}$

$A \cup B = \{1,2,3,4,6,8\}$

$(A \cup B)' = \{5,7,9\}$

Sol.47 (d) If U = {1,2,3,.......,9} be the universal set A = {1,2,3,4} and B = {2,4,6,8}

$A \cap B = \{1,2,3,4\} \cap \{2,4,6,8\}$

$A \cap B = \{2,4\}$

$(A \cap B)' = \{1,3,5,6,7,8,9\}$

Sol.48 (a) $P \times Q = \{1,2,x\} \times \{a,x,y\}$

$= \{1,a\}, \{1,x\}, \{1,y\}, \{2,a\}, \{2x\}, \{2y\}$

$\{x,a\}, \{x,x\}, \{x,y\}$

Sol.49 (b) $P \times R = \{1,2,x\} \times \{x,y,z\} = \{(1,x),(1,y),(1,z)(2,x),(2,y),(2,z),(x,x)(x,y),$

$(x,z)\}$

Sol.50 (c) $Q \times R = \{a,x,y\} \times \{x,y,z\}$

$= \{(a,x),(a,y),(a,z)(x,x),$

$(x,y),(x,z),(y,x)(y,y),(y,z)\}$

Sol.51 (d) $(P \times Q) \cap (P \times R)$

$= P \times (Q \cap R) = \{1,2,x\} \times \{x,y\}$

$= \{(1,x),(1.y),(2,x),(2,y),(x,x),(x,y)\}$

Sol.52 (c) $(R \times Q) \cap (R \times P) = R \times (Q \cap P)$

$= \{x,y,z\} \cap \{x\} = \{(x,x),(y,x),(z,x)\}$

Sol.53 (d) $P \times Q =$
$\{(1,a),(1,x),(1,y),(2,a)(2,x),(2,y),(x,a),(x,x),(x,y)\}$

$R \times P =$
$\{(x,1),(x,2),(x,x),(y,1),(y,2),(y,x),(2,1),(2,2),(2,x)\}$

$\therefore (P \times Q) \cup (R \times P)$

$= \{(1,a),(1,x),(1,y),(2,a)(2,x),(2,y),(x,a),$
$(x,x),(x,y),(x,1),(x,2),(y,1),(y,2),(y,x),(z,1),(z,2),$

Sol.54 (a) $n(P \times Q \times R) = n(P) \times n(Q) \times n(R)$

$= 4 \times 3 \times 2 = \mathbf{24}$

Sol.55 (b) P = {4, 5, 6}

Sol.56 (a) $A \times (B \; U \; C) = \{2,3\} \times \{4,5,6\}$

$= \{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

Sol.57 (b) $A = \{2,3\},\ B \cap C = \{5\}$

$A \times (B \cap C) = \{(2,5),(3,5)\}$

Sol.58 (c) $(A \times B) \cup (B \times C) =$

$\{(2,4),(2,5),(3,4),(3,5),(4,5),(4,6),(5,5),(5,6)\}$

Sol.59 (c)

$n(A) = 32,\ n(B) = 42,\ n(A \cup B) = 62$

$\Rightarrow n(A) + n(B) - n(A \cup B) = 62$

$\Rightarrow 32 + 42 - n(A \cup B) = 62$

$\Rightarrow$ n(A ∪ B) = 74 − 62 = **12**

Sol.60 (a) $A \rightarrow Telegraph$

$B \rightarrow Times\ of\ India$

$n(t)50{,}000, n(A) = 28{,}000$

$n(B) = 23{,}000, n(A \cap B) = 4{,}000$

$n(A \cup B) = 28{,}000 + 23{,}000 - 4{,}000 = 47{,}000$

$\therefore n(A \cup B)' = 50{,}000 - 47{,}000 = \mathbf{3{,}000}$

Sol.61 (a) $A \rightarrow Coffee,\ B \rightarrow Tea,\ C \rightarrow Cocoa$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$\Rightarrow (100\% - 6\%) = 48\% + 54\% + 64\% - 28\% - 30\% - 32\% + n(A \cap B \cap C)$

$\Rightarrow 94\% = 76\% + \mathrm{n}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$

$\Rightarrow \mathrm{n}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = (94 - 76)\% = 18\%$

$\therefore$ *Required no.* $= 18\%$ *of* $2{,}000 =$ **360**

Sol.62 (b) $n(B \cap C \cap A') = n(B \cap C) - n(B \cap C \cap A)$

$= (32 - 18)\% = 14\%$

$\therefore$ *Required no.* $= 14\%$ *of* $2000 =$ **280**

Sol.63 (c) $n(A - B - C) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= (48 - 28 - 30 + 18)\% = 8\%$

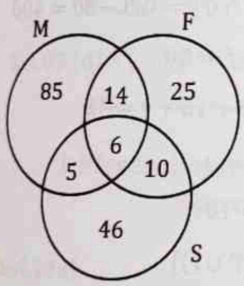
$\therefore$ *Required no.* $= 8\%$ *of* $2{,}000 =$ **160**

Sol.64 (a) $n(M) = 110,\ n(F) = 55,\ n(S) = 67$

$n(M \cap F \cap S') = n(M \cap F) - P(M \cap F \cap S) = 20$

$n(M \cap S \cap F') = n(M \cap S) - n(M \cap F \cap S) = 11$

$n(F \cap S \cap M') = n(F \cap S) - P(M \cap F \cap S) = 16$



$n(M \cup S \cup F) = 173$

Let $n(M \cap S \cap F) = x$

Now,

$n(M \cup S \cup F) = n(M) + n(F + n(s) - n(M \cap F) - n(F \cap S) - n(S \cap M) + n(M \cap F \cap S)$

$\Rightarrow 173 = 110 + 55 + 67 - (20 + x) - (16 + x) - (11 + x) + x$

$\Rightarrow 232 - 47 - 2x = 173$

$\Rightarrow 2x = 185 - 173 = 12 \Rightarrow \boldsymbol{x = 6}$

Sol.65 (c) Required No. $= 14 + 6 + 5 + 10 =$ **35**

Sol.66 (b) $A \rightarrow$ *Passed in Account*

$B \rightarrow$ *Passed in Maths*

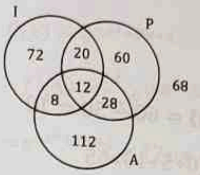
$C \rightarrow$ *Passed in Costing*

$P(A \cup B \cup C) = 45 + 50 + 30 - 30 - 32 - 35 + 25 =$ **53**

Sol.67 (a) $I \rightarrow$ *Industry*

$P \rightarrow$ *Practice*

$A \rightarrow$ *Assistants*



$= \mathrm{n}(\mathrm{P} \cap A)$
$= n(I \cup P \cup A) = 112 + 120 + 160 - 32 - 40 - 20 + 12 = 312$

$\therefore n(I \cup P \cup U)' = n(t) - n(I \cup P \cup A) = 400 - 312 =$ **88**

Sol.68 (b) *Required no.* $= 72 + 60 + 112 = 244$

Sol.69 (a) $n = (A \cup B \cup C) = 42 + 17 + 27 - 7 - 13 - 18 + 3 = 51 > 50$

Sol.70 (b) *Required no.* $= n(W \cap R \cap B)$

$n(W \cup R \cup B) - n(W) - n(R) - n(B) + n(W \cap R) + n(R \cap B) + n(W \cap B)$

$= 100 - 50 - 40 - 30 + 20 + 15 + 10 = 25$

Sol.71 (a) $100 - 10 = 50 + 40 + 30 - 20 - 15 - 10 + 20$

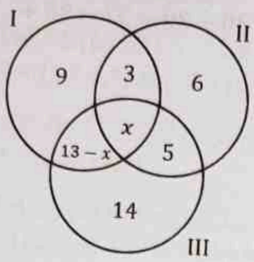
$\Rightarrow 90 \neq 95$

Which is not true.

Sol.72 (a)

Let n (all three) = x

$= n(II) = 24 \Rightarrow 3 + x + 5 + 6 = 24$



$\Rightarrow x = 24 - 14 = \mathbf{10}$

Sol.73 (a) (Passed in all three) $= 60 - 50 = 10$

Sol.74 (d) $n(A)' = 20+15+5+10+5+10=65$

Sol.75 (d) $A \cap C = \Phi$

Sol.76 (c) $n(Y \cup N)' = 5 + 5 + 10 = 20$

Sol.77 (d) $n[A \cap (Y \cap N)]' = \Phi$

Sol.78 (a) $A \rightarrow April$

$J \rightarrow June$

$M \rightarrow May$

$n(A \cup M \cup J) = n(A) + n(M) + n(J) - n(A \cap M) - n(M \cap J) - n(A \cap J) + n(A \cap M \cap J)$

$= 59 + 62 + 62 - 35 - 33 - 31 + 22$

$= 205 - 99 = 106 \neq \mathbf{100}$

Sol.79 (a) $n(S) = 35,\ n(F) = 40,\ n(R) = 18$

$n(S \cap F) = 7,\ n(S \cap R) = 11,\ n(F \cap R) = 12$

$n(S \cap F \cap R) = 3$

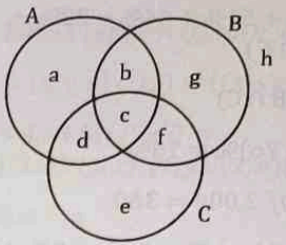
$n(S \cup F \cup R) = 35 + 40 + 18 - 7 - 11 - 12 + 3 = 66$

Sol.80 $a = 180$

$d + c = 80$

$d + c + e + f = 480$

$a + d = 230$



$a + b + c + d = 360,\ c + f = 80,\ h = 140$

$\therefore d = 50,\ c = 30,\ b = 100,\ f = 50,\ e = 350$

$a + b + c + d + e + f + g + h = 1000 \Rightarrow g = 1000 - 800 = 200$

$\therefore n(B) = b + c + f + g = 100 + 30 + 50 + 200 = \mathbf{380}$

Sol.81 (b) From the information

$n(A \cap B' \cap C') = 180, n(A \cap B') = 230,$

$n(U) = 1000, n(A \cap C) = 80,$

$n(A \cup B \cup C)' = 140, n(A) = 360, n(C) = 480$

$n(B \cap C) = 80, n(A \cap B') = 230,$

$n(C \cap B') = n(C) - n(B \cap C) = 480 - 80 = 400$

Sol.82 (c) $(B \cap C \cap A') = f = 50$

Sol.83 (a) $n(M) = 7 + 10 + 16 + 9 = 42$

Sol.84 (b) $n(L \cap I) = 8$

Sol.85 (c) $n(S \cap T \cap I) = 10$

Sol.86 (d) $n\{(M \cup L) \cap (T \cup I)\}$

$= (7 + 3) + (16 + 8) + (9 + 0) = \mathbf{43}$

Sol.87 (d) $n\{S' \cup (S' \cap I)'\} = n\{S' \cup (S \cup I')\}$

$n\{(S' \cup S) \cup (S' \cup I')\} = n\{(U) \cup (S' \cup I')\}$

$= 44 + 42 + 13 = \mathbf{99}$

Sol.88 (c) $n(S \cup M)' = n(L)$

Sol.89 (a) $n(I \cap T)' = 65$

$n\{S - (I \cap S') = n(S \cap (I \cap S'))$

$= n\left(S \cap (I' \cup S)\right) = n(S) = 44$

$\therefore n\,(I \cap T)' > n\,(S - (I \cap S'))$

Sol.90 (a) $A \rightarrow Aggregate$

$B \rightarrow Ist$

$C \rightarrow II$

$n\,(A \cap B \cap C) = 1000 - (658 + 372 + 590 - 166 - 434 - 126) = \mathbf{106}$

Sol.91 (b) $n\,(A \cap C') = n(A) - n(A \cap C) = 658 - 434 = \mathbf{224}$

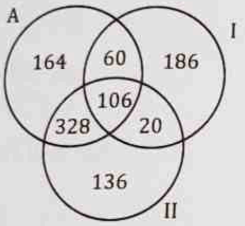
Sol.92 (c) $n\,(B \cap A') = n(B) - n(B \cap A)$

$= 372 - 166 = \mathbf{206}$

Sol.93 (d) $n\,(B' \cap C) = n(C) - n(B \cap C)$

$= 590 - 126 = \mathbf{464}$

Sol.94 (c) $164 + 328 + 136 = 628$



Sol.95 (d) $n\,(A \cap B' \cap C') = \mathbf{164}$

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Differentiation and Integration
Exercise: 8A

Sol.1 (a)

$y = 2x^3 - 3x^2 - 12x + 8$

$\therefore \dfrac{dy}{dx} = 6x^2 - 6x - 12$

$\therefore \left(\dfrac{dy}{dx}\right)_{x=0} = 6 \times 0 - 6 \times 0 - 12 = \mathbf{-12}$

Sol.2 (b)

$y = 2x^3 - 5x^2 - 3x$

$\dfrac{dy}{dx} = 6x^2 - 10x - 3$

$\therefore \left(\frac{dy}{dx}\right)_{x=0} = 0 - 0 - 3 = \mathbf{-3}$

Sol.3 (c)

$y = \sqrt{x+1}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\sqrt{x+1}$

$= \dfrac{1}{2\sqrt{x+1}} \times (1+0)$

$= \mathbf{\dfrac{1}{2\sqrt{x+1}}}$

Sol.4 (b)

$f(x) = e^{ax^2+bx+c}$

$\therefore f'(x) = \frac{d}{dx} e^{(ax^2+bx+c)}$

$= \boldsymbol{e^{ax^2+bx+c} \times (2ax+b)}$

Sol.5 (a)

$f(x) = \dfrac{x^2+1}{x^2-1}$

Quotient Rule

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$

$$= \frac{(x^2-1)\dfrac{d}{dx}(x^2+1) - (x^2+1)\dfrac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$\therefore f'(x) = \dfrac{(x^2-1) \times 2x - (x^2+1) \times 2x}{(x^2-1)^2}$

$= \dfrac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2} = \mathbf{\dfrac{-4x}{(x^2-1)^2}}$

Sol.6 (a)

$y = x(x-1)(x-2)$

$$\frac{dy}{dx} = (x-1)(x-2)\frac{d}{dx}x + x(x-2)\frac{d}{dx}(x-1) + x(x-1)\frac{d}{dx}(x-2)$$

$\therefore\ \frac{dy}{dx} = (x-1)(x-2) \times 1 + x(x-2) \times 1 + x(x-1) \times 1$

$= x^2 - 3x + 2 + x^2 - 2x + x^2 - x$

$= \mathbf{3x^2 - 6x + 2}$

Sol.7 (d)

$y - xy + 2px + 3qy = 0$

Diff. both sides

$$\frac{dy}{dx} - \left(y \times 1 + x\frac{dy}{dx}\right) + 2p + 3q\frac{dy}{dx} = 0$$

$$\Rightarrow (1 - x + 3q)\frac{dy}{dx} = y - 2p$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-2p}{1-x+3q}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=3 \text{ and } y=2} = \frac{-2}{3}$$

$$\Rightarrow \frac{2-2P}{1-3+3q} = \frac{-2}{3}$$

$$\Rightarrow \frac{\cancel{2}(1-p)}{-2+3q} = \frac{-\cancel{2}}{3}$$

$$\Rightarrow 3 - 3p = 2 - 3q$$

$\Rightarrow 3p - 3q = 1$_____(I)

Also (3, 2) lies on the curve

$$\therefore 2 - 6 + 6p + 6q = 0$$

$\Rightarrow 6p + 6q = 4$_____(II)

From [(I) ×2 + (II)]

$$6p - \cancel{6q} = 2$$

$$6p + \cancel{6q} = 4$$

$$12p = 6 \Rightarrow p = 1/2$$

$$\therefore q = \frac{\left(\frac{3}{2}-1\right)}{3} = \mathbf{1/6}$$

Sol.8 (b)

$y^2 = ux^3 + v$

Diff. both sides w.r. to x

$$2y\frac{dy}{dx} = 3x^2u$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2u}{2y}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=2 \text{ and } y=3} = 4$$

$\Rightarrow \frac{\cancel{12}^{2}u}{\cancel{6}} = 4 \Rightarrow u = \frac{4}{2} = 2$_____(I)

Also (2, 3) lies on the curve

$$\therefore 9 = 8u + v$$

$$\Rightarrow 9 = 16 + v \Rightarrow v = -7$$

Hence $u = 2$ & $v = -7$

Sol. 9 (d)

$$y + px + qy = 0$$

$$\therefore \frac{d}{dx}(y + px + qy) = 0$$

$$\Rightarrow \frac{dy}{dx} + p + q\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-p}{1+q}$$

$\therefore \left(\frac{dy}{dx}\right)_{x=1 \text{ and } y=1} = \frac{1}{2} \Rightarrow \frac{-p}{1+q} = \frac{1}{2} \Rightarrow 2p + q = -1$_____(I)

Also (1, 1) lies on the curve

$$\therefore 1 + p + q = 0$$

$\Rightarrow p + q = -1$_____(II)

From [(I)−(II)]

$$2p + q = -1$$

$$-p \pm q = \mp 1$$

$$p = 0$$

$$\therefore \mathbf{q = -1}$$

Sol.10 (b)

$xy = 1$

Diff. both sides w.r. to x

$$y + x\frac{dy}{dx} = 0$$

$\therefore y^2 + xy\frac{dy}{dx} = 0$ (multiply by 'y' both side)

$$\Rightarrow y^2 + 1 \times \frac{dy}{dx} = 0 \Rightarrow y^2 + \frac{dy}{dx} = 0$$

Sol. 11 (c)

$$\frac{d}{dx}\left(\sqrt{x + \sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x + \sqrt{x}}} \times \left(1 + \frac{1}{2\sqrt{x}}\right)$$

Sol. 12 (a)

$$e^{-xy} - 4xy = 0$$

$$\therefore \frac{d}{dx}(e^{-xy} - 4xy) = 0$$

$$e^{-xy} \times \left\{-\left(y + x\frac{dy}{dx}\right)\right\} - 4\left(y + x\frac{dy}{dx}\right) = 0$$

$$\left(y + x\frac{dy}{dx}\right)(-e^{-xy} - 4) = 0$$

$$\Rightarrow y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

Sol. 13 (a)

$$\frac{x^2}{a^2} - \frac{-y^2}{a^2} = 1$$

$$\therefore \frac{2x}{a^2} - \frac{2y}{a^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

Sol.14 (a)

$$log(^x/_y) = x + y$$

Diff. both sides w.r. to x

$$\frac{1}{^x/_y} \times \left(\frac{y \times 1 - x\frac{dy}{dx}}{y^2}\right) = 1 + \frac{dy}{dx}$$

$$\Rightarrow y - x\frac{dy}{dx} = xy + xy\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - xy}{x + xy} = \frac{y(1-x)}{x(1+y)}$$

Sol. 15 (b)

$$\because x^3 + y^3 - 3axy = 0$$

Diff. both sides w.r. to x

$$3x^2 + 3y^2\frac{dy}{dx} - 3a\left(y + x\frac{dy}{dx}\right) = 0$$

$$\Rightarrow (3y^2 - 3ax)\frac{dy}{dx} = 3ay - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(ay - x^2)}{3(y^2 - ax)} = \frac{ay - x^2}{y^2 - ax}$$

Sol. 16 (c)

$x = at^2, y = 2at$

Diff. both side w. r. to t

$$\frac{dx}{dt} = 2at, \qquad \frac{dy}{dt} = 2a$$

$$\therefore \frac{dy/dt}{dx/dt} = \frac{dy}{dx} = \frac{2a}{2at} = \frac{\mathbf{1}}{\mathbf{t}}$$

Sol.17 (a)

$x = 2t + 5, \ y = t^2 - 2$

Diff. both side w. r. to t

$$\frac{dx}{dt} = 2, \qquad \frac{dy}{dt} = 2t$$

$$\therefore \frac{dy/dt}{dx/dt} = \frac{dy}{dx} = \frac{2t}{2} = \mathbf{t}$$

Sol. 18 (c)

$$y = \frac{1}{\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}x^{-\frac{1}{2}} = \frac{-1}{2}x^{-\frac{3}{2}} = \frac{\mathbf{-1}}{\mathbf{2x\sqrt{x}}}$$

Sol. 19 (a)

$x = 3t^2 - 1, \qquad y = t^3 - t$

Diff. both side w. r. to t

$$\frac{dx}{dt} = 6t, \qquad \frac{dy}{dt} = 3t^2 - 1$$

$$\therefore \frac{dy/dt}{dx/dt} = \frac{dy}{dx} = \frac{\mathbf{3t^2 - 1}}{\mathbf{6t}}$$

Sol. 20 (a)

$$y = \sqrt{4 - x^2}$$

When $y = x$

$$\therefore x = \sqrt{4 - x^2} \Rightarrow x^2 = 4 - x^2$$

$$\Rightarrow 2x^2 = 4 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

$$\therefore y = \sqrt{4 - 2} = \sqrt{2}$$

$\therefore$ point is $(\sqrt{2}, \sqrt{2})$

$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{4 - x^2}} = \frac{-x}{\sqrt{4 - x^2}} = \frac{-x}{y} = \frac{-\sqrt{2}}{\sqrt{2}} = \mathbf{-1}$$

Sol. 21 (b)

$$y = x^2 - x$$

$$\frac{dy}{dx} = 2x - 1$$

When $y = 2$ then $x^2 - x = 2$

$$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x = 2 \text{ or } x = -1$$

∴ Required point is (2, 2)

$$\therefore \left(\frac{dy}{dx}\right)_{x=2 \text{ and } y=2} = 2 \times 2 - 1 = \mathbf{3}$$

Sol. 22 (a)

$$x^2 + y^2 + 2gx + 2hy = 0$$

Diff. both sides w. r. to x

$$2x + 2y\frac{dy}{dx} + 2g + 2h\frac{dy}{dx} = 0$$

$$\Rightarrow 2(y+h)\frac{dy}{dx} = -2(x+g)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x+g)}{(y+h)}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0 \text{ and } y=0} = \frac{-(0+g)}{0+h} = \frac{-g}{h}$$

Sol. 23 (d)

$$y = \frac{e^{3x}-e^{2x}}{e^{3x}+e^{2x}} = \frac{e^{2x}(e^x-1)}{e^{2x}(e^x+1)} = \frac{e^x-1}{e^x+1}$$

$$\frac{dy}{dx} = \frac{(e^x+1)e^x - (e^x-1) \times e^x}{(e^x+1)^2}$$

$$= \frac{e^{2x}+e^x-e^{2x}+e^x}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$$

Sol. 24 (b)

$$x^y.\ y^x = M$$

∴ Taking logarithm both sides

$$y \log x + x \log y = \log M$$

Diff. both sides w. r. to x

$$\frac{y}{x} + \log x\ \frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$$

$$\Rightarrow \left(\log x + \frac{x}{y}\right)\frac{dy}{dx} = -\left(\frac{y}{x} + \log y\right)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\mathbf{y}}{\mathbf{x}}\left(\frac{\mathbf{y + x \log y}}{\mathbf{y \log x + x}}\right)$$

Sol. 25 (c)

$x = t + t^{-1}$ and $y = t - t^{-1}$

Diff. both side w. r. to t

$$\therefore \frac{dx}{dt} = 1 - \frac{1}{t^2} \ \&\ \frac{dy}{dt} = 1 + \frac{1}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{t^2+1}{\cancel{t^2}} \times \frac{\cancel{t^2}}{t^2-1}$$

$$\therefore \frac{dy/dt}{dx/dt} = \left(\frac{dy}{dx}\right)_{t=2} = \frac{4+1}{4-1} = \mathbf{5/3}$$

Sol. 26 (a)

$$x^3 - 2x^2y^2 + 5x + y - 5 = 0$$

Diff. both sides w. r. to x

$$3x^2 - 4xy^2 - 4x^2y\frac{dy}{dx} + 5 + \frac{dy}{dx} = 0$$

$$\Rightarrow (1 - 4x^2y)\frac{dy}{dx} = 4xy^2 - 3x^2 - 5$$

$$\Rightarrow \frac{dy}{dx} = \frac{4xy^2 - 3x^2 - 5}{1 - 4x^2y}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1 \text{ and } y=1} = \frac{4-3-5}{1-4} = \frac{-4}{-3} = \mathbf{{}^4/_3}$$

Sol. 27 (b)

$$\frac{d}{dx}(x^2 \log x) = x^2 \times \frac{1}{x} + 2x \log x$$

$$= x + 2x \log x = \mathbf{x(1 + 2\log x)}$$

Sol. 28 (c)

$$\frac{d}{dx}\left(\frac{3-5x}{3+5x}\right)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

$$= \frac{(3+5x)\frac{d}{dx}(3-5x) - (3-5x) \times \frac{d}{dx}(3+5x)}{(3+5x)^2}$$

$$= \frac{(3+5x)(-5) - (3-5x) \times 5}{(3+5x)^2}$$

$$= \frac{-15 - \cancel{25x} - 15 + \cancel{25x}}{(3+5x)^2} = \frac{\mathbf{-30}}{\mathbf{(3+5x)^2}}$$

Sol. 29 (a)

$$y = \sqrt{2x} + 3^{2x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{2x}} \times \cancel{2} + 3^{2x} \log{}^{3}_{e} \times 2$$

$$= \frac{\mathbf{1}}{\mathbf{\sqrt{2x}}} + \mathbf{2 \times 3^{2x} \log{}^{3}_{e}}$$

Sol. 30 (b)

Let $y = log\left[e^{x}\left\{\frac{x-2}{x+2}\right\}^{3/4}\right]$

$= x + \frac{3}{4}[\log(x-2) - \log(x+2)]$

$\therefore \frac{dy}{dx} = 1 + \frac{3}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$

$= 1 + \frac{3}{4}\left(\frac{x+2-x+2}{x^2-4}\right)$

$= 1 + \frac{3}{4}\left(\frac{4}{x^2-4}\right)$

$= 1 + \frac{3}{x^2-4}$

$= \frac{x^2-4+3}{x^2-4} = \frac{\mathbf{x^2-1}}{\mathbf{x^2-4}}$

Sol. 31 (c)

$\frac{d}{dx}\left(e^{3x^2-6x+2}\right) = e^{3x^2-6x+2} \times (6x-6)$

$= \mathbf{6(x-1)e^{3x^2-6x+2}}$

Sol. 32 (a)

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x+1}{e^x-1}\right)$

$= \frac{(e^x-1)\frac{d}{dx}(e^x+1) - (e^x+1)\frac{d}{dx}(e^x-1)}{(e^x-1)^2}$

$= \frac{(e^x-1)e^x - (e^x+1).e^x}{(e^x-1)^2}$

$= \frac{\cancel{e^{2x}} - e^x - \cancel{e^{2x}} - e^x}{(e^x-1)^2} = \frac{\mathbf{-2e^x}}{\mathbf{(e^x-1)^2}}$

Sol. 33 (b)

$f(x) = \left\{\frac{(a+x)}{(1+x)}\right\}^{a+1+2x}$

$= \log f(x) = (a+1+2x).\log\frac{(a+x)}{(1+x)} = (a+1+2x)\,[\log(a+x) - \log(1+x)]$

$= \frac{1}{f(x)}f'(x) = (a+1+2x)\{\frac{1}{a+x}.\frac{d}{dx}(a+x) - \frac{1}{1+x}.\frac{d}{dx}(1+x)\} + 2[\log(a+x) - \log(1+x)]$

$= (a+1+2x)\left[\frac{1}{a+x} - \frac{1}{1+x}\right] + 2[\log(a+x) - \log(1+x)]$

$f'(x) = f(x)[(a+1+2x)\left[\frac{1}{a+x} - \frac{1}{1+x}\right] + 2[\log(a+x) - \log(1+x)]]$

$f'(x) = \left\{\frac{(a+x)}{(1+x)}\right\}^{a+1+2x} \{(a+1+2x)\left[\frac{1}{a+x} - \frac{1}{1+x}\right] + 2[\log(a+x) - \log(1+x)]\}$

$f'(0) = a^{a+1}\{(a+1)[\frac{1}{a} - 1] + 2[loga - log1]\}$

$f'(0) = a^{a+1}\left\{(a+1)\left(\frac{1-a}{a}\right) + 2loga\right\}$

$f'(0) = \boldsymbol{a^{a+1}}\{\boldsymbol{\frac{1-a^2}{a} + 2loga}\}$

Sol. 34 (a)

$x = at^2 \;\therefore\; \frac{dx}{dt} = 2at$ (Diff. w. r. to t)

$y = 2at \;\therefore\; \frac{dy}{dt} = 2a$ (Diff. w. r. to t)

$\therefore \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

$\therefore \left(\frac{dy}{dx}\right)_{t=2} = \frac{1}{2}$

Sol. 35 (a)

$f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$\therefore f'(x) = 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}\right)$

$\therefore f'(2) = 2\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}}\right)$

$= \cancel{2} \times \frac{3}{\sqrt{2}} \times \frac{1}{4\sqrt{2}} = \mathbf{3/4}$

Sol. 36 (b)

$f(x) = x^2 - 6x + 8$

$\therefore f'(x) = 2x - 6$

$\therefore f'(5) = 2(5) - 6 = 4$

$\therefore f'(x) = 2(8) - 6 = 10$

$\therefore f'(5) - f'(8) = 4 - 10 = -6$

$\therefore f'(x) = 2(2) - 6 = -2 = \mathbf{3\,f'\,(2)}$

Sol. 37 (b)

$y = \left(x + \sqrt{x^2+m^2}\right)^n$

$\therefore \frac{dy}{dx} = n\left(x + \sqrt{x^2+m^2}\right)^{n-1} \times \left(1 + \frac{1}{\cancel{2}\sqrt{x^2+m^2}} \times \cancel{2}\,x\right)$

$$= n\left(x+\sqrt{x^2+m^2}\right)^{n-1}\left(\frac{\sqrt{x^2+m^2}+x}{\sqrt{x^2+m^2}}\right)$$

$$= \frac{n\left(x+\sqrt{x^2+m^2}\right)^n}{\sqrt{x^2+m^2}} = \frac{ny}{\sqrt{x^2+m^2}}$$

Sol. 38 (a)

$$y = \sqrt{x/m} + \sqrt{\frac{m}{x}}$$

$$\Rightarrow y = \frac{x+m}{\sqrt{mx}} \Rightarrow y^2 = \frac{(x+m)^2}{mx}$$

$$\therefore m\,x\,y^2 = (x+m)^2$$

Diff. both sides w.r. to x

$$2\,mx\,y\frac{dy}{dx} + my^2 = 2(x+m)\times 1$$

$$\therefore 2xy\,\frac{dy}{dx} + y^2 = 2\left(\frac{x+m}{m}\right)$$

$$\Rightarrow 2xy\,\frac{dy}{dx} + \frac{x^2+m^2+2mx}{mx} = 2\left(\frac{x}{m}\right) + 2$$

$$\Rightarrow 2xy\,\frac{dy}{dx} + \frac{x}{m} + \frac{m}{x} + \cancel{2} = 2\left(\frac{x}{m}\right) + \cancel{2}$$

$$\Rightarrow 2xy\,\frac{dy}{dx} + \frac{m}{x} - \frac{x}{m} = 0$$

$$\Rightarrow 2xy\,\frac{dy}{dx} - \frac{x}{m} + \frac{m}{x} = \mathbf{0}$$

Sol. 39 (c)

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + ..$$

$$\therefore \frac{dy}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \cdots \frac{n\,x^{n-1}}{n!} + ..$$

$$=1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{(n-1)!} + ..$$

$$\Rightarrow \frac{dy}{dx} = y$$

$$\Rightarrow \frac{dy}{dx} - y = \mathbf{0}$$

Sol. 40 (a)

$$f(x) = x^k \;\therefore f'(x) = k\,x^{k-1}$$

$$\therefore f'(1) = k \times 1^{k-1}$$

$$\Rightarrow 10 = k \Rightarrow k = \mathbf{10}$$

Sol. 41 (d)

$$y = \sqrt{x^2+m^2}$$

$$\therefore \frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+m^2}} = \frac{x}{y}$$

Sol. 42 (c)

$$y = e^x + e^{-x}$$

$$\therefore \frac{dy}{dx} = e^x - e^{-x} = \sqrt{(e^x - e^{-x})^2}$$

$$=\sqrt{(e^x + e^{-x})^2 - 4} = \sqrt{y^2 - 4}$$

$$=\frac{dy}{dx} - \sqrt{y^2-4} = \mathbf{0}$$

Sol. 43 (a)

$$\frac{d}{dx}\left(\frac{x^2-1}{x}\right) = \frac{d}{dx}\left(x - \frac{1}{x}\right) = \mathbf{1 + \frac{1}{x^2}}$$

Sol. 44 (b)

$$\frac{d}{dx}\left(\frac{x^2+1}{x}\right) = \frac{d}{dx}\left(x + \frac{1}{x}\right) = 1 - \frac{1}{x^2}$$

Sol. 45 (a and c)

$$y = e^{\sqrt{2x}} \;\; \therefore \; \frac{dy}{dx} = e^{\sqrt{2x}} \times \frac{1}{\cancel{2}\sqrt{2x}} \times \cancel{2}$$

$$= \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Sol. 46 (b)

$$y = \sqrt{x}^{\sqrt{x}\ldots\infty}$$

$$\Rightarrow y = \left(\sqrt{x}\right)^y$$

$$\Rightarrow \log y = \frac{y}{2}\log x$$

Diff. both sides w.r.t to x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{y}{x} + \log x\,\frac{dy}{dx}\right]$$

$$\Rightarrow \left(\frac{1}{y} - \frac{1}{2}\log x\right)\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{x}\right)$$

$$\Rightarrow \left(\frac{2 - y\log x}{\cancel{2}y}\right)\frac{dy}{dx} = \frac{y}{\cancel{2}x}$$

$$\Rightarrow \frac{dy}{dx} = \mathbf{\frac{y^2}{x(2 - y\log x)}}$$

Sol. 47 (c)

$$x = \frac{1-t^2}{1+t^2}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

Diff. w. r. to t

$$\frac{dx}{dt} = \frac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)\times -2t - (1-t^2)\times 2t}{(1+t^2)^2}$$

$$= \frac{-4t}{(1+t^2)^2}$$

$$y = \frac{2t}{1+t^2}$$

Diff. w. r. to t

$$\frac{dx}{dt} = \frac{(1+t^2)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{(1+t^2)\times 2 - 2t\times 2t}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2}$$

$$\therefore \left(\frac{dy}{dx}\right)_{t=1} = \frac{2\times 0}{(1+1)^2} = \mathbf{0}$$

Sol. 48 (b)

$$f(x) = \frac{x^2}{e^x}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

$$\therefore f'(x) = \frac{e^x\frac{d}{dx}(x^2) - x^2\frac{d}{dx}e^x}{(e^x)^2}$$

$$\therefore f'(x) = \frac{2x\,e^x - x^2e^x}{(e^x)^2} = \frac{2x-x^2}{e^x}$$

$$\therefore f'(1) = \frac{2-1}{e} = \frac{\mathbf{1}}{\mathbf{e}}$$

Sol. 49 (c)

$$y = \left(x+\sqrt{x^2-1}\right)^m$$

$$\frac{dy}{dx} = m\left(x+\sqrt{x^2-1}\right)^{m-1}\left(1+\frac{2x}{2\sqrt{x^2-1}}\right)$$

$$= m\left(x+\sqrt{x^2-1}\right)^{m-1}\left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right)$$

$$= \frac{m\left(x+\sqrt{x^2-1}\right)^m}{\sqrt{x^2-1}} = \frac{my}{\sqrt{x^2-1}}$$

$$\therefore \left(\frac{dy}{dx}\right)^2 = \frac{m^2y^2}{x^2-1}$$

$$\Rightarrow (x^2-1)\left(\frac{dy}{dx}\right)^2 - m^2y^2 = \mathbf{0}$$

Sol. 50 (d)

$$f(x) = \frac{4-2x}{2+3x+3x^2}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

$$\therefore f'(x) = \frac{(2+3x+3x^2)\frac{d}{dx}(4-2x) - (4-2x)\frac{d}{dx}(2+3x+3x^2)}{(2+3x+3x^2)^2}$$

$$= \frac{(2+3x+3x^2)\times(-2) - (4-2x)\times(3+6x)}{(2+3x+3x^2)^2}$$

$$\because f'(x) = 0$$

$$\Rightarrow \frac{-4-6x-6x^2-12-18x+12x^2}{(2+3x+3x^2)^2} = 0$$

$$\Rightarrow 6x^2-24x-16 = 0$$

$$\Rightarrow 3x^2-12x-8 = 0$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2\times a} = \frac{-(-12)\pm\sqrt{144+96}}{2\times 3}$$

$$\Rightarrow x = \frac{12\pm\sqrt{240}}{2\times 3}$$

$$= \frac{12\pm 4\sqrt{15}}{6}$$

$$= \frac{\mathbf{2(3\pm\sqrt{15})}}{\mathbf{3}}$$

……

Differentiation and Integration
Exercise: 8B

Sol.1 (b) $\int 5x^2\,dx = 5\int x^2\,dx = 5 \times \frac{x^3}{3} + k$

$= \frac{5x^3}{3} + k$

Sol.2 (b)

$$\int (3 - 2x - x^4)\,dx = 3\int dx - 2\int x\,dx - \int x^4 dx$$

$$= 3x - 2\frac{x^2}{2} - \frac{x^5}{5} + k$$

$$= 3x - x^2 - \frac{x^5}{5} + k$$

Sol.3 (b)

$$\int f(x)dx = \int (4x^3 + 3x^2 - 2x + 5)\,dx$$

$$= 4\int x^3\,dx + 3\int x^2dx - 2\int xdx + 5\int dx$$

$$= 4 \times \frac{x^4}{4} + 3 \times \frac{x^3}{3} - 2 \times \frac{x^2}{2} + 5x + k$$

$$= x^4 + x^3 - x^2 + 5x + k$$

Sol.4 (b)

$\int (x^2 - 1)dx = \int x^2dx - \int dx = \frac{x^3}{3} - x + k$

Sol.5 ©

$$\int (1 - 3x)(1 + x)\,dx = \int (1 - 2x - 3x^2)\,dx$$

$$= x - 2 \times \frac{x^2}{2} - 3 \times \frac{x^3}{3} + k = x - x^2 - x^3 + k$$

Sol.6 (a)

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + k$$

$$= \frac{2}{3}x^{3/2} - 2x^{1/2} + k$$

Sol.7 (d) $\int \left(px^3 + qx^2 + rk + \frac{w}{x}\right) dx$

$$= p\int x^3dx + q\int x^2dx + rk\int dx + w\int \frac{1}{x}dx$$

$$= p\frac{x^4}{4} + q\frac{x^3}{3} + rkx + w\log|x| + k$$

Sol.8 (a) $let\ I = \int (4x + 5)^6 dx$

$Put\ 4x + 5 = t$

$Diff.w.r.t.x.$

$\therefore 4 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{4}$

$\therefore \mathrm{I} = \int t^6 \times \frac{dt}{4} = \frac{1}{4}\int t^6\,dt$

$= \frac{1}{4} \times \frac{t^7}{7} + k$

$= \frac{1}{28}(4x + 5)^7 + k$

Sol.9 (b)

$let\ \mathrm{I} = \int x(x^2 + 4)^5 dx$

put $x^2 + 4 = t$

$Diff.w.r.t.x.$

$\therefore 2x = \frac{dt}{dx}$

$\Rightarrow x\,dx = \frac{dt}{2}$

$= \mathrm{I} = \int xt^5.\ \frac{dt}{2x}$

$\therefore \mathrm{I} = \int t^5.\ \frac{dt}{2} = \frac{1}{2}\int t^5 dt$

$= \frac{1}{2} \times \frac{t^6}{6} + k$

$= \frac{1}{12}(x^2 + 4)^6 + k$

Sol.10 (a) Let I= $\int (x + a)^n dx$

Put $x + a = t$

(Diff. w. r. t. x.)

$\therefore dx = dt$

$\therefore \mathrm{I} = \int t^n dt$

$= \frac{t^{n+1}}{n+1} + k$

$= \frac{(x+a)^{n+1}}{n+1} + k$

Sol.11 (b) Let I=$\int 8x^2/(x^3+2)^3 dx = 8\int \frac{x^2 dx}{(x^3+2)^3}$

Put $x^3 + 2 = t$

Diff. w. r. t. x.

$\therefore 3x^2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{3x^2}$

$\therefore \text{I} = 8\int \frac{x^2}{t^3} \cdot \frac{dt}{3x^2}$

$= 8\int \frac{1}{t^3} \cdot \frac{dt}{3}$

$= \frac{8}{3}\int t^{-3}\, dt = \frac{8}{3}\left(\frac{t^{-2}}{-2}\right) + k$

$= \frac{-4}{3t^2} + k$

$= \boldsymbol{\frac{-4}{3(x^3+2)^2} + k}$

Sol.12 (c)

$let\ \text{I} = \int \frac{1}{x^2 - a^2} dx = \int \frac{1}{(x+a)(x-a)} dx$

Put $\frac{1}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a}$

$\Rightarrow 1 = A(x-a) + B(x+a)$

$\Rightarrow 1 = (A+B)x + (-aA + aB)$

Comparing co-efficient of x & constant both sides

$A + B = 0 \Rightarrow B = -A$ _____(I)

$-aA + aB = 1$

$\Rightarrow -aA - aA = 1$[from (I)]

$\Rightarrow -2\,aA = 1$

$\Rightarrow \text{A} = \frac{-1}{2a}$

$\therefore B = \frac{1}{2a}$

$\therefore \text{I} = \frac{-1}{2a}\int \frac{dx}{x+a} + \frac{1}{2a}\int \frac{dx}{x-a}$

$= \frac{-1}{2a}\log|x+a| + \frac{1}{2a}\log|x-a| + k$

$= \frac{1}{2a}(\log|x-a| - \log|x+a|) + k$

$= \boldsymbol{\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + k}$

Sol.13 (a) $\int x^2 e^{3x} dx$

$\therefore \int f(x) \times g(x) dx = g(x)\int f(x)dx - \int[\frac{d}{dx}g(x)\int f(x)dx]dx$ [Using ILATE]

$= x^2\int e^{3x}dx - \int\left\{\frac{dx^2}{dx}.\int e^{3x}dx\right\}dx$

$= x^2 \times \frac{e^{3x}}{3} - \int 2x \times \frac{e^{3x}}{3} dx$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\int x\, e^{3x} dx$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x\int e^{3x}dx - \int\left\{\frac{d(x)}{dx}.\int e^{3x}dx\right\}dx\right]$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x.\frac{e^{3x}}{3} - \int\left\{1 \times \frac{e^{3x}}{3}\right\}dx\right]$

$= \frac{x^2e^{3x}}{3} - \frac{2}{9}x\, e^{3x} + \frac{2}{9}\int e^{3x}dx$

$= \boldsymbol{\frac{x^2e^{3x}}{3} - \frac{2}{9}x\, e^{3x} + \frac{2}{27}e^{3x} + k}$

Sol.14 (d) $\int \log x\, dx$

$= \int (\log x \times 1)\, dx$

$= \log x\int 1\, dx - \int\left\{\frac{d(\log x)}{dx}\int 1\, dx\right\}dx$

$= \log x \times x - \int \frac{1}{x} \times x\, dx$

$= x\log x - \int dx$

$= \boldsymbol{x\log x - x + c}$

Sol.15 (a) $\int x\, e^x dx$

$\therefore \int f(x) \times g(x) dx = g(x)\int f(x)dx - \int[\frac{d}{dx}g(x)\int f(x)dx]dx$ [Using ILATE]

$x.\int e^x dx - \int\left\{\frac{dx}{dx}.\int e^x dx\right\}dx$

$= x\, e^x - \int 1 \times e^x$

$= x\, e^x - e^x + k$

$= \mathbf{(x-1)e^x + k}$

Sol.16 (a) $\int (\log x)^2 dx = \int (\log x)^2 . 1 dx$

$\qquad\qquad\qquad\qquad \text{I} \qquad \text{II}$

$\therefore \int f(x) \times g(x) dx = g(x) \int f(x) dx - \int [\frac{d}{dx} g(x) \int f(x) dx] dx$ [Using ILATE]

$$= (\log x)^2 \int dx - \int \left\{ \frac{d}{dx} (\log x)^2 . \int dx \right\} dx$$

$$= (\log x)^2 . x - \int 2 \log x \times \frac{1}{x} \times x \, dx$$

$$= x(\log x)^2 - 2 \int \log x \times 1 \, dx$$

$$= x(\log x)^2 - 2 \left[\log x \int dx - \int \left\{ \frac{d}{dx} (\log x) . \int dx \right\} dx \right]$$

$$= x(\log x)^2 - 2 \left[(\log x) \times x - \int \frac{1}{\cancel{x}} \times \cancel{x} \, dx \right]$$

$$= \mathbf{x(\log x)^2 - 2 x \log x + 2x + k}$$

Sol.17 (a) Let I=$\int \frac{(x+5)dx}{(x+1)(x+2)^2}$

Put $\frac{x+5}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$\Rightarrow x + 5 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$

$= A(x^2 + 4x + 4) + B(x^2 + 3x + 2) + C(x + 1)$

$\Rightarrow x + 5 = (A + B)x^2 + (4A + 3B + C)x + (4A + 2B + C)$

Comparing co-efficient of x^2, x & constant both sides

$A + B = 0 \Rightarrow B = -A$ ____ (I)

$4A + 3B + C = 1 \Rightarrow A + C = 1$ ____(II)[Using of equation (I)]

$4A + 2B + C = 5 \Rightarrow 2A + C = 5$ ____(III) (ii)

From [(III)-(II)]

$2A + C = 5$

$-A \pm C = -1$

$A = 4$

$\therefore B = -4$

and $C = 1 - 4 = -3$

$\therefore \text{I} = 4 \int \frac{dx}{x+1} - 4 \int \frac{dx}{x+2} - 3 \int \frac{dx}{(x+2)^2}$

$$= 4 \, log|x+1| - 4 \log|x+2| - 3 \times \left(\frac{-1}{x+2} \right) + k$$

$$= \mathbf{4 \, log|x+1| - 4 \log|x+2| + \frac{3}{(x+2)} + k}$$

Sol.18 (b) $\int_0^1 (2x^2 - x^3) dx$

$$= \left[2 \times \frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= \left(\frac{2}{3} \times 1^3 - \frac{1^4}{4} \right) - (0 - 0)$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \mathbf{\frac{5}{12}}$$

Sol.19 (a) $\int_2^4 (3x-2)^2 dx = \int_2^4 (9x^2 - 12x + 4) dx$

$$= \left[9 . \frac{x^3}{3} - 12 . \frac{x^2}{2} + 4x \right]_2^4$$

$$= [3x^3 - 6x^2 + 4x]_2^4$$

$$= (3 \times 4^3 - 6 \times 4^2 + 4 \times 4) - (3 \times 2^3 - 6 \times 2^2 + 4 \times 2)$$

$$= (192 - 96 + 16) - (24 - 24 + 8)$$

$$= 112 - 8 = \mathbf{104}$$

Sol.20 (d) $\int_0^1 x \, e^x dx$

$\therefore \int f(x) \times g(x) dx = g(x) \int f(x) dx - \int [\frac{d}{dx} g(x) \int f(x) dx] dx$ [Using ILATE]

$$= \left[x \int e^x dx - \int \left\{ \frac{dx}{dx} . \int e^x dx \right\} dx \right]_0^1$$

$$=[x \, e^x - \int 1 \times e^x dx]_0^1$$

$$= [x \, e^x - e^x]_0^1 = (1 \times e - e) - (0 \times e^0 - e^0)$$

$$= (e - e) - (0 - 1) = 0 + 1 = \mathbf{1}$$

Sol.21 (d) Let I= $\int x^x (1 + \log x) \, dx$

$$\int e^{x \log x} . (1 + \log x) \, dx$$

$\left[\because x^x = e^{\log_e^{(x^x)}} = e^{x\log_e^x}\right]$

Put $x \log x = t$

$Diff.w.r.to\ x$

$\therefore \left(\log x \times 1 + x \times \frac{1}{x}\right) dx = dt$

$\Rightarrow (\log x + 1)dx = dt$

$\therefore I = \int e^t dt = e^t = e^{x\log x} + k$

$= \boldsymbol{x^x + k}$

Sol.22 (b) $\int f(x)dx = \sqrt{1+x^2}$

$=\int \sqrt{1^2+x^2}\, dx$

$=\frac{x}{2}\sqrt{1^2+x^2} + \frac{1^2}{2} log\left|x + \sqrt{1^2+x^2}\right| + k$

$$\left[\because \int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2} log\left|x+\sqrt{a^2+x^2}\right| + k\right]$$

$= \boldsymbol{\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2} log\left|x+\sqrt{1+x^2}\right| + k}$

Sol.23 (a) $\int \frac{\sqrt{2}(x^2+1)}{\sqrt{x^2+2}} dx$

$=\sqrt{2}\int \frac{x^2+2-1}{\sqrt{x^2+2}}\, dx$

$=\sqrt{2}\left[\int \sqrt{x^2+2}dx - \int \frac{1}{\sqrt{x^2+2}} dx\right]$

$=\sqrt{2}\left[\int \sqrt{x^2+\left(\sqrt{2}\right)^2}dx - \int \frac{1}{\sqrt{x^2+\left(\sqrt{2}\right)^2}} dx\right]$

$=\sqrt{2} \left[\frac{x}{2}\sqrt{x^2+\left(\sqrt{2}\right)^2} + \frac{\left(\sqrt{2}\right)^2}{2} log\left|x + \sqrt{x^2+\left(\sqrt{2}\right)^2}\right| - \log\left|x+\sqrt{x^2+\left(\sqrt{2}\right)^2}\right|\right]$

$=\sqrt{2}\left[\frac{x\sqrt{x^2+2}}{2} + log\left|x+\sqrt{x^2+2}\right| - log\left|x + \sqrt{x^2+2}\right|\right]$

$= \boldsymbol{\frac{x}{\sqrt{2}}\sqrt{x^2+2} + k}$

Sol.24 (a) Let I$= \int (e^x + e^{-x})^2(e^x - e^{-x})\, dx$

put $e^x + e^{-x} = t$

$Diff.w.r.to\ x$

$\therefore (e^x - e^{-x})dx = dt$

$\therefore$ I$= \int t^2 dt$

$= \frac{t^3}{3} + k$

$=\boldsymbol{\frac{1}{3}(e^x+e^{-x})^3 + k}$

Sol.25 (b) Let I$= \int_0^a [f(x) + f(-x)]dx$

$= \int_0^a f(x)dx + \int_0^a f(-x)dx$

$= \int_0^a f(x)dx + I_1$

Put $-x = t$

$\therefore -dx = dt$

$\Rightarrow dx = -dt$

$\therefore I_1 = \int_0^a f(-x)dx = \int_0^{-a} f(t)(-dt)$

$= -\int_0^{-a} f(t)dt$

$= \int_{-a}^0 f(t)dt$

$= \int_{-a}^0 f(x)dx \left[\therefore \int_a^b f(x)dx = \int_a^b f(t)dt\right]$

$\therefore$ I$= \int_0^a f(x)dx + \int_{-a}^0 f(x)dx$

$= \int_{-a}^0 f(x)dx + \int_0^a f(x)dx$

$= \boldsymbol{\int_{-a}^a f(x)dx}$

Sol.26 (a) $\int \frac{xe^x}{(x+1)^2} dx$

$=\int \left[\frac{x+1-1}{(x+1)^2}\right] e^x\, dx$

$=\int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] e^x\, dx$

$= \frac{1}{x+1} e^x + k$

$\left[\because \int (f(x) + f'(x))e^x\, dx = e^x f(x) + k\right]$

$= \boldsymbol{\frac{e^x}{(x+1)} + k}$

Sol.27 (b) $\int (x^4 + 3/x)\, dx = \int x^4 dx + 3\int \frac{1}{x} dx$

$= \frac{x^5}{5} + 3\log|x| + k$

Sol.28 (d) $\int \frac{(1-x)^3}{x} dx$

$= \int \frac{1-3x+3x^2-x^3}{x} dx$

$= \int \left(\frac{1}{x} - 3 + 3x - x^2\right) dx$

$= \boldsymbol{log\, x - 3x + \frac{3}{2}x^2 - \frac{x^3}{3} + k}$

Sol.29 (a) $\because y = f(x)$

$\therefore \frac{dy}{dx} = f'(x)$

$\therefore dy = \int f'(x)\, dx$

$\therefore \int dy = \int f'(x)\, dx$

$\Rightarrow y = \int (2x-1)\, dx \; (\because f'(x) = 2x - 1)$

$= \cancel{2} . \frac{x^2}{\cancel{2}} - x + k$

$\Rightarrow y = x^2 - x + k$

It passes through (1, 0)

$\therefore 0 = 1 - 1 + k$

$\Rightarrow k = 0$

∴ Required equation of the curve is

$\boldsymbol{y = x^2 - x}$

Sol.30 (c) $\int_1^4 (2x+5)\, dx$

$= \left[2 . \frac{x^2}{2} + 5x\right]_1^4$

$= [x^2 + 5x]_1^4$

$= (4^2 + 5 \times 4) - (1^2 + 5 \times 1)$

$= (16 + 20) - (1 + 5)$

$= 36 - 6 = \mathbf{30}$

Sol.31 (a) Let $\mathrm{I} = \int_1^2 \frac{2x}{1+x^2} dx$

Put $1 + x^2 = t$

$\therefore 2x\, dx = dt$

$\therefore \mathrm{I} = \int_2^5 \frac{dt}{t}$

$= [log_e|t|]_2^5$

$= log_e^5 - log_e^2$

$= \boldsymbol{log5/2}$

Sol.32 (b) Let $\mathrm{I} = \int_0^4 \sqrt{3x+4}$

Put $3x + 4 = t$

Diff. w. r. t. x

Upper limit = 3x+4 = 3(4) + 4 = 16

Lower limit = 3(0) + 4 = 4

$\therefore 3dx = dt$

$= dx = dt/3$

$\therefore \mathrm{I} = \int_4^{16} t^{1/2} . \frac{dt}{3}$

$= \frac{1}{3}\int_4^{16} t^{1/2}\, dt$

$= \frac{1}{3}\left[\frac{t^{3/2}}{3/2}\right]_4^{16}$

$= \frac{2}{9}\left[t^{3/2}\right]_4^{16} = \frac{2}{9}[64 - 8]$

$= \frac{2}{9} \times 56 = \mathbf{\frac{112}{9}}$

Sol.33 (b) $\int_0^2 \frac{x+2}{x+1}\, dx = \int_0^2 \left(1 + \frac{1}{x+1}\right) dx$

$= [x + log_e|x+1|]_0^2$

$= (2 + \log_e 3) - (0 + 0)$

$= \mathbf{2 + \log_e 3}$

Sol.34 (d) Let $\mathrm{I} = \int_1^{e^2} \frac{dx}{x\,(1+\log x)^2}$

Put $1 + \log x = t$

Diff. w. r. t. x

$\therefore \frac{1}{x} dx = dt$

Upper limit $= t = 1 + \log x = 1 + loge^2 = 3$

Lower limit $= 1 + log0^2 = 1$

$\therefore$ I$= \int_1^3 \frac{dt}{t^2}$

$= \left[-\frac{1}{t}\right]_1^3$

$= \frac{-1}{3} + 1 = \mathbf{2/3}$

Sol.35 (d) $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}\, dx$

$= \int_0^4 \frac{x^2 + 5x + 4}{\sqrt{x}}\, dx$

$= \int_0^4 (x^{3/2} + 5x^{1/2} + 4x^{-1/2})\, dx$

$= \left[\frac{x^{5/2}}{5/2} + 5\,\frac{x^{3/2}}{3/2} + 4\frac{x^{1/2}}{1/2}\right]_0^4$

$= \left[\frac{2}{5}x^{5/2} + \frac{10}{3}x^{3/2} + 8x^{1/2}\right]_0^4$

$= \left(\frac{64}{5} + \frac{80}{3} + 16\right) - (0 + 0 + 0)$

$= \frac{192+400+240}{15} = \frac{832}{15}$

$= 55\frac{7}{15}$

Sol.36 (b) Slope of the curve at $(x, y) = \frac{dy}{dx}$

$\Rightarrow 4x - 3 = \frac{dy}{dx}$

$\Rightarrow \int dy = \int (4x - 3)\, dx$

$\Rightarrow y = 4\,\frac{x^2}{2} - 3x + k$

$\Rightarrow y = 2x^2 - 3x + k$

It passes through $(1, 3)$

$\therefore 3 = 2\times 1^2 - 3 \times 1 + k$

$\Rightarrow 3 = 2 - 3 + k \Rightarrow k = 4$

$\therefore$ Equation we have

$\boldsymbol{y = 2x^2 - 3x + 4}$

Sol.37 (b) Let I$= \int_2^3 f(5 - x) - \int_2^3 f(x)\, dx$

$=$ I$_1 -$ I$_2$

I$_1 = \int_2^3 f(5 - x)\, dx$

Put $5 - x = t$

$\therefore -dx = dt$

$\Rightarrow dx = -dt$

$\therefore$ I$_1 = \int_3^2 f(t)(-dt)$

$= -\int_3^2 f(t)dt$

$= \int_2^3 f(t)dt \;\left[\because \int_a^b f(x)dx = -\int_b^a f(x)dx\right]$

$= \int_2^3 f(x)dx \;\left[\because \int_a^b f(x)dx = -\int_a^b f(t)dx\right]$

$\Rightarrow$I$_1 =$ I$_2$

$\Rightarrow$I$_1 =$ I$_2 = 0$

$\Rightarrow$**I = 0**

Sol.38 (a) $\int (x - 1)\, e^x/x^2\, dx$

$= \int \left(\frac{x-1}{x^2}\right) e^x\, dx = \int \left(\frac{1}{x} - \frac{1}{x^2}\right) e^x\, dx$

$= \frac{1}{x}e^x + k\ [\because \int [f(x) + f'(x)]e^x dx = e^x f(x) + k]$

Sol.39 (a) $\int \frac{e^x(x\log x+1)}{x} dx$

$= \int e^x \left(\log x + \frac{1}{x}\right) dx$

$= \quad e^x \log x + k \qquad [\because \int \boldsymbol{e^x[f(x) + f'(x)]dx =}$ $\boldsymbol{e^x f(x) + k}]$

Sol.40 (b) $\int \log x^2\, dx$

$= 2 \log x\, dx$

$= 2\int 1 . \log x\, dx$

$= 2\left[\log x \times \int dx - \int \left\{\frac{d(\log x)}{dx} . \int dx\right\} dx\right]$

$= 2\left[x \log x - \int \frac{1}{x} \times x\, dx\right]$

$= 2[x \log x - x] + k$

$= \boldsymbol{2x\,(\log x - 1) + k}$

Sol.41(c) $\int_1^2 x\ log\ x\ dx = \Big[\log x \int x\ dx - \int \left\{\frac{d}{dx}\log x \int x\ dx\right\} dx\Big]_1^2$

$$= \left[\frac{x^2}{2}\log x - \int \frac{1}{x}.\frac{x^2}{2}\ dx\right]_1^2$$

$$= \left[\frac{x^2}{2}\log x - \frac{1}{2}\int x\ dx\right]_1^2$$

$$= \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_1^2$$

$$= \left(\frac{2^2}{2}\log 2 - \frac{2^2}{4}\right) - \left(\frac{1^2}{2}\log 1 - \frac{1^2}{4}\right)$$

$$= (2\log 2 - 1) - \left(0 - \frac{1}{4}\right)$$

$$= 2\log 2 - 1 + \frac{1}{4}$$

$$= \mathbf{2\log 2 - \frac{3}{4}}$$

Sol.42 (a) $\int_1^2 \left(\frac{x^2-1}{x^2}\right) e^{x+\frac{1}{x}} dx$

$=\int_1^2 \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}dx}$

Put $x + \frac{1}{x} = \mathrm{t}$

Diff. w. r. to x

$\therefore \left(1 - \frac{1}{x^2}\right) dx = dt$

Upper limit $= t = x + \frac{1}{x} = 2 + \frac{1}{2} = \frac{5}{2}$

Lower limit = 1+1= 2

$\therefore \mathrm{I}=\int_2^{5/2} e^t\ \mathrm{dt} = [\mathrm{e^t}]_2^{5/2}$

$=e^{5/2} - \mathrm{e}^2 = \mathbf{e^2(\sqrt{e} - 1)}$

Sol.43 (c) $\int_0^2 3x^2 dx = [3\int x^2 dx]_0^2$

$=\left[3 \times \frac{x^3}{3}\right]_0^2 = 2^3 - 0^3 = 8 - 0 = \mathbf{8}$

Sol.44 (a) Let $\mathrm{I}= \int \frac{(2-x)^{e^x}}{(1-x)^2} dx = \int \left[\frac{1+1-x}{(1-x)^2}\right] e^x dx$

$= \int \left[\frac{1}{(1-x)^2} + \frac{1}{1-x}\right] e^x dx$

$= \boldsymbol{\frac{1}{1-x} e^x + k}$

$= \left[\because \int f(x) + f'(x) e^x dx = e^x f(x) + k\right]$

Sol.45 (b) $\int x^3 \log x\ dx$

II I

$=\log x\ .\frac{x^4}{4} - \int \frac{1}{x}\ .\frac{x^4}{4} dx$

$$= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3 dx$$

$$= \frac{x^4}{4}\log x - \frac{x^4}{16} + k$$

$$= \boldsymbol{\frac{x^4}{16}(4\log x - 1) + k}$$

Sol.46 (c) Let $\mathrm{I}= \frac{\int \log(\log x)}{x} dx$

Put $\log x = t$ (Diff. w. r. to x)

$\therefore \frac{1}{x} dx = dt$

$\therefore I = \int \log t\ dt = \log t \int dt - \int \left\{\frac{d}{dt}(\log t)\ .\int dt\right\} dt$

$= t\ log\ t - \int \frac{1}{\cancel{t}} \times \cancel{t}\ dt$

$=t \log t - \int dt$

$=t \log t - t + k$

$= \log x . \log(\log x) - \log x + k$

$= \boldsymbol{log\ x(\log(\log x) - 1) + k}$

Sol.47 (a) $\int (\log x)^2\ x\ dx$

I II

$$= (\log x)^2 . \frac{x^2}{2} - \int \left\{\frac{d}{dx}(\log x)^2 . \int x\ dx\right\} dx$$

$= \frac{x^2}{2}(\log x)^2 - \int \cancel{2}\log x \times \frac{1}{\cancel{x}} \times \frac{x^{\cancel{2}}}{\cancel{2}} dx$

$$= \frac{x^2}{2}(\log x)^2 - \int x \log x\ dx$$

$=\frac{x^2}{2}(\log x)^2-\left[\log x\,.\frac{x^2}{2}-\int\frac{1}{x}\times\frac{x^2}{2}dx\right]$

$=\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}(\log x)+\frac{1}{2}\int x\,dx$

$=\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}(\log x)+\frac{x^2}{4}+k$

$=\frac{x^2}{2}\left[(\log x)^2-\log x+\frac{1}{2}\right]+k$

Sol.48 (b) Let $I=\int\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$

Put $e^x+e^{-x}=t$

$\therefore(e^x-e^{-x})dx=dt$

$\therefore I=\int\frac{dt}{t}$

$=log|t|+k$

$=log|e^x+e^{-x}|+k$

Sol.49 (a) Let $I=\int\frac{3x}{x^2-x-2}dx$

$=\int\frac{3x}{(x-2)(x+1)}dx$

Put $\frac{3x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$

$\Rightarrow 3x=A(x+1)+B(x-2)$

$\Rightarrow 3x=(A+B)x+(A-2B)$

Comparing co-efficient of x & constant,

$A+B=3$ _____ (I)

$A-2B=0$ ______ (II)

From [(I) - (II)]

$A+B=3$

$A-2B=0$

$-\ \ +\ \ \ -$

3B=3⟹B=1

∴ A=2

$\therefore I=2\int\frac{1}{x-2}dx+1\int\frac{1}{x+1}dx$

$=2\,log|x-2|+\log|x+1|+k$

Sol.50 (c)

$\therefore f'(x)=x-1$

$\therefore y=f(x)=\int f'(x)dx$

$=\int(x-1)\,dx$

$\Rightarrow y=\frac{x^2}{2}-x+k$

It passes through (1, 0)

$\therefore 0=\frac{1}{2}-1+k$

$\Rightarrow 0=-\frac{1}{2}+k$

$\Rightarrow k=\frac{1}{2}$

$\therefore y=\frac{x^2}{2}-x+\frac{1}{2}$

****

Business Statistics
Measure of Central Tendency
Exercise: Set-B

Sol. 1 (a)

$\bar{X} = \frac{15+20+25}{3} = \frac{60}{3} = 20$

∴ Sum of deviation of the obs. from A.M. i.e. $\bar{X}$

= (15-20) + (20 – 20) + (25 – 20)

= -5 + 0 + 5 = **0**

Sum of deviation of the obs. from their mean is always zero.

Sol. 2(b) 4,5,6,8,9,11

$\left(\frac{n+1}{2}\right)^{th} \Rightarrow 3.5^{th}$

Median = $3^{rd} + 0.5[4^{th} - 3^{rd}]$

Median= 6+0.5 [2] = **7**

Sol. 3 (b)

Numbers 5, 8, 6, 4, 10, 15, 18, 10

Here 10 occurs the most time

∴ Modal value = **10**

Sol. 4 (c)

G.M for the nos. 8, 24, and 40

$= (8 \times 24 \times 40)^{1/3}$

$= (2^3 \times 2^3 \times 3 \times 2^3 \times 5)^{1/3}$

$= 2 \times 2 \times 2 \times (3 \times 5)^{1/3}$

$= \mathbf{8\sqrt[3]{15}}$

Sol. 5 (c)

$\text{H.M.} = \frac{n}{\sum_{i=1}^{n}\frac{1}{xi}} = \frac{3}{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}} = \frac{3}{\frac{15+10+6}{30}}$

$= \frac{3\times30}{31} = \frac{90}{31} = \mathbf{2.90\ (approx.)}$

Sol. 6 (b)

Let the nos. be a & b

A.M. = 6.50

$\Rightarrow \frac{a+b}{2} = 6.50 \Rightarrow a + b = 13$

⇒ b = 13 – a____ (I)

Also, G.M = 6

$\Rightarrow \sqrt{ab} = 6 \Rightarrow ab = 36$

⇒ a (13 – a) = 36

$\Rightarrow a^2 - 13a + 36 = 0$

⇒ (a – 9) (a - 4) = 0

⇒ a – 9 = 0 or a – 4=0

⇒ a = 9 or a = 4

If a = 9 then b = 4

If a = 4 then b = 9

Sol. 7 (d)

Let the numbers be a & b

$\text{A.M} = 5 \Rightarrow \frac{a+b}{2} = 5 \Rightarrow a + b = 10$______ (I)

$\text{H.M} = 3.2 \Rightarrow \frac{2ab}{a+b} = 3.2$

$\Rightarrow \frac{2ab}{10} = 3.2 \Rightarrow ab = 16$

$\therefore \sqrt{ab} = 4 \Rightarrow \mathbf{G.M = 4}$

Sol. 8 (c)

Arrange in ascending order

10, 12, 15, 16, 18, 20, 23, 28

n =8

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ *observation* = $\left(\frac{8+1}{4}\right)^{th}$ *observation*

= $(2.25)^{th}$*observation*

= 2^{nd} observation + 0.25 × ($3^{rd} - 2^{nd}$) *obs.*

= 12 + 0.25 × (15 – 12)

= 12 + 0.25 × 3

= **12.75**

Sol. 9 (b)

Arrange in ascending order

9, 10, 11, 12, 15, 18, 20, 25

n = 8

$D_3 = 3\left(\frac{n+1}{10}\right)^{th} = \left(3 \times \frac{9}{10}\right)^{th}$ *observation*

= $(2.7)^{th} observation$ = 2nd observation. + 0.7 $(3^{rd} - 2^{nd})$

= 10 + 0.7 (11 − 10) = 10 + 0.70 = **10.70**

Sol. 10 (c)

$\bar{X}_{12} = \frac{n_1\overline{X_1}+n_2\overline{X_2}}{n_1+n_2} = \frac{30\times50+20\times60}{30+20}$

$= \frac{1500+1200}{50} = \frac{2700}{50} = \mathbf{54}$

Sol. 11 (a)

Let the no. of skilled worker be n_2

$\bar{X}_{12} = \frac{n_1\overline{X_1}+n_2\overline{X_2}}{n_1+n_2}$

$\Rightarrow 12000 = \frac{n_1\times10000+n_2\times15000}{n_1+n_2}$

$\Rightarrow 12000n_1 + 12000n_2 = 10000n_1 + 15000n_2$

$\Rightarrow 2000n_1 = 3000n_2$

$\Rightarrow n_1/n_2 = \frac{3000}{2000}$

$\Rightarrow \frac{n_1}{n_2} = \frac{3}{2}$

∴ Required % of skilled worker

$= \frac{n_2}{n_1+n_2} \times 100\% = \frac{2}{3+2} \times 100\%$

$= \frac{2}{5} \times 100\% = \mathbf{40\%}$

Sol. 12 (c)

Combined H.M $= \frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_1}}$

$= \frac{15+13}{\frac{15}{75}+\frac{13}{65}} = \frac{28}{\frac{1}{5}+\frac{1}{5}} = \frac{28}{2/5}$

$= 28 \times \frac{5}{2} = \mathbf{70}$

Sol. 13 (c)

H.M. $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+--\mp\frac{1}{x_n}}$

$\Rightarrow \frac{n}{\frac{1}{1}+\frac{1}{1/2}+\frac{1}{1/3}+\frac{1}{1/4}+---+n}$

$\Rightarrow \frac{n}{1+2+3+4+---+n}$

$\Rightarrow \frac{n}{\frac{n(n+1)}{2}}$ [∴ 1+2+3+4+ ---- +n = $\frac{n(n+1)}{2}$]

$\Rightarrow \frac{1}{\frac{(n+1)}{2}} \Rightarrow \frac{2}{n+1}$

Sol. 14(b) Speed is measured in km/hour

	Km	hour
Constant	(✓)	(×)
Given	(×)	(×)

Since first variable i.e. km is constant ∴ we will apply Harmonic Mean

x

500 H.M. $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}}$

700 $= \frac{2}{\frac{1}{500}+\frac{1}{700}} = \frac{2\times500\times700}{1200}$

= 583.33 km/hour

How to solve

1) $\frac{1}{500}$ $in\ M^+$
2) $\frac{1}{700}$ $in\ M^+$
3) 2 ÷ MRC & then

Press =

Sol.15 (a)

x	f	fx
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25
	15	55 = $\sum fx$

$\bar{X} = \frac{\sum fx}{\sum f}$

$= \frac{55}{15} = \frac{11}{3}$

Sol. 16 (c)

Y = 2X − 3

$\therefore Y_M = 2X_M - 3$

= 2 × 20 - 3 = **37**

Sol.17(c)

∴ 2u + v + 7 = 0 ⇒ v = -2u -7

When A.M. of u = 10

∴ A.M. of v= -2× 10 − 7 = **−27**

Sol.18 (b)

∴ $x - y - 10 = 0 \Rightarrow y = x - 10$

When $x = 23$ then y = 23-10 = **13**

Sol.19 (a)

G.M. of xy = (G.M. of x) × $(G.M.\ of\ y)$

= 10 × 15 = **150**

Sol. 20 (c)

(A.M) $\frac{x_1+x_2+--+x_{10}}{10} = 15$

(G.M.) $(x_1\ x_2 --- x_{10})^{1/10} = 15$

H. M. = $\frac{10}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+--+\frac{1}{x_{10}}}$

∴ A.M., G.M & H.M. are G.P.

∴ (G.M.)² = A.M × H.M

⇒ $(15)^2 = 15 \times H.M.$

⇒ H.M = **15**

****

Measure of Central Tendency
Exercise: Set-C

Sol. 1 (c)

C.I.	Frequency	C.F.	X	$d' = \frac{x-39.5}{10}$	fd'
4.5 – 14.5	10	10	9.5	−3	−30
14.5 – 24.5	18	28	19.5	−2	−36
24.5 – 34.5	**32**	**60**	29.5	−1	−32
34.5 – 44.5	26	86	**39.5** **A**	0	0
44.5 – 54.5	14	100	49.5	1	14
54.5 – 64.5	10	110	59.5	2	20

$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times i \Rightarrow 39.5 + \frac{(-64)}{110} \times 10$

$\bar{X} = 33.68$

$Median = \frac{N^{th}}{2} = 55^{th} \Rightarrow l + \frac{N/2 - C.F_{p}}{l} \times i$

$= 24.5 + \frac{55-28}{32} \times 10$

= 24.5 + 8.44

Median = 32.94

Sol. 2 (c)

C.I.	Frequency	X	$d' = \frac{X-A}{i}$	fd'
349.5 – 369.5	15	359.5	−3	−45
369.5 – 389.5	27 f_0	379.5	−2	−54
389.5 – 409.5	**31 f_1**	399.5	−1	−31
409.5 – 429.5	19 f_2	**419.5**	0	0
429.5 – 449.5	13	439.5	1	13
449.5 – 469.5	6	459.5	2	12
				−105

$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

$= 389.5 + \frac{31-27}{62-27-19} \times 20$

$389.5 + \frac{4}{16} \times 20 = 394.5$

Mode = 394.5

Mean $= A + \frac{\Sigma fd'}{\Sigma f} \times i$

$419.5 + \frac{(-105)}{111} \times 20$

Mean = 400.58

Sol. 3 (c)

Profit	C.F.	C.I.	F
Below 5,000	10	0-5,000	10
Below 10,000	25	5,000-10,000	15 f_0
Below 15,000	45	**10,000-15,000**	**20 f_1**
Below 20,000	55	15,000-20,000	10 f_2
Below 25,000	62	20,000-25,000	7
Below 30,000	65	25,000-30,000	3
			65

Mode = $l + \frac{f_1 + f_0}{2f_1 - f_0 - f_2} \times i$

$= 10{,}000 + \frac{20-15}{40-15-10} \times 5000$

$= 10{,}000 + \frac{5}{15} \times 5{,}000$

Mode = 11,667

Median $= l + \frac{N/_2 - CF_P}{l} \times i$

$= 10{,}000 + \frac{65/_2 - 25}{20} \times 5000$

$\boldsymbol{Median = 10{,}000 + 1{,}875}$

Median = 11,875

Sol. 4 (d)

C.I.	f	x	fx
0-20	5	10	50
20-40	18	30	540
40-60	X	**50**	**1,000**
60-80	12	70	840
80-100	5	90	450
Mode= 44	**60**		**2,880**

Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

$44 = 40 + \frac{x-18}{2x-18-12} \times 20$

$= 4 = \frac{x-18}{2x-30} \times 20$

$\frac{4}{20} = \frac{x-18}{2(x-15)}$

$\frac{2}{5} = \frac{x-18}{x-15}$

$2x - 30 = 5x - 90$

$60 = 3x$

$\boldsymbol{x = 20}$

Mean = $\frac{\Sigma fx}{\Sigma f}$

$\bar{X} = \frac{2,880}{60}$

$\boldsymbol{\bar{X} = 48}$

Sol. 5 (c)

Wage Groups	**Totally Marks**	**No. of column (f)**	***Profits*** $\frac{x}{m}$	fx
50-55	|||	3	100	300
55-60	卌	5	200	1,000
60-65	卌|	6	300	1,800
65-70	||||	4	400	1,600
70-75	||	2	500	1,000
				5,700

Average Bonus = $\frac{\Sigma fx}{\Sigma f} = \frac{5700}{20} = ₹\,285$

Sol. 6 (a)

C.I.	f	$C.F.$
1-9500	5	5
9500-19,500	18	23
19,500-29,500	38	61
29,500-39,500	**20**	**81**
39,500-49,500	9	90
49,500-59,500	2	92

$\boldsymbol{Q_3 = K\left(\frac{N}{4}\right) = 3\left(\frac{N}{4}\right)} \Rightarrow 3 \times \frac{92}{4}$

$Q_3 = l + \frac{K\left(\frac{N}{4}\right) - C.F._p}{f} \times i$

$Q_3 = 29,500 + \frac{69-61}{20} \times 10,000$

$Q_3 = 29,500 + 4,000$

$\boldsymbol{Q_3 = 33,500}$

$P_{65} = l + \frac{K\left(\frac{N}{10}\right) - C.F._p}{f} \times i$

$P_{65} = 65 \times \left(\frac{92}{1000}\right) = \boldsymbol{59.80th}$

$P_{65} = 19,500 + \frac{59.80-23}{38} \times 10,000$

$19,500 + 9684.21$

$\boldsymbol{P_{65} = 29,184}$

Sol. 7 (c)

C.I.	No. of student	C.F.	x	f	fx
0-10	10	10	5	10	50
10-20	x	10+x	15	9	135
20-30	25	35+x	25	25	625
30-40	**30**	**65+x**	35	30	1,050
40-50	y	65+$x + y$	45	16	720
50-60	10	75+$x + y$	55	10	550
	100				**3,130**

Median = $\boldsymbol{l + \frac{N/2 - C.F._p}{f} \times i}$

$32 = 30 + \frac{50-(35+x)}{30} \times 10$

$2 = \frac{15-x}{30} \times 10$

$6 = 15 - x$

$x = 15 - 6$

$\boldsymbol{x = 9}$

$75 + x + y = 100$

$75 + 9 + y = 100$

$\boldsymbol{y = 16}$

Mean = $\frac{\Sigma fx}{\Sigma f}$

$\bar{X} = \frac{3,130}{100}$

$\boldsymbol{\bar{X} = 31.3}$

Sol. 8 (c)

C.I.	No. of student	C.F.
30-40	8	8
40-50	16	24
50-60	22	46
60-70	28	74
70-80	$\boldsymbol{x}$	98
80-90	12	110

Mode = $\boldsymbol{l} + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

$66 = 60 + \frac{28-22}{56-22-x} \times 10$

$6 = \frac{6}{34-x} \times 10$

$6 \times (34 - x) = 60$

$34 - 10 = x$

$x = 24$

Median $= l + \frac{N/2 - C.F._p}{f} \times i$

$Median = 60 + \frac{55-46}{28} \times 10$

$Median = 60 + \frac{9}{28} \times 10$

$Median = \mathbf{63.21}$

****

Measure of Dispersion

Exercise: Set-B

Sol.1 (d) Coefficient Of range $\Rightarrow \frac{H-L}{H+L} \times 100$

$$= \frac{90-60}{90+60} \times 100 \Rightarrow \frac{30}{150} \times 100 = ₹\,20$$

Sol.2 (c)$3x + 2y + 10 = 0$

$3x = -10 - 2y$

$x = \frac{-10}{3} - \frac{2}{3}y$

$R_X = |b|R_Y \;\Rightarrow\; R_X = \frac{2}{3}R_Y \;\Rightarrow\; 3R_X = 2R_Y$

Sol.3 (c) Coefficient Of range $\Rightarrow \frac{H-L}{H+L} \times 100$

$= \frac{59.5-9.5}{59.5+9.5} \times 100 \Rightarrow \frac{50}{69} \times 100 = \mathbf{72.46}$

Sol.4 (b) $R_X = 2$

$Y = -3x + 50$

[Range of Y = |-3| Range of X]

$R_Y = |b|R_X \Rightarrow R_Y = 3R_X = 2 \times 3 = \mathbf{6}$

Sol.5(c)

X	$\lvert X - \bar{X}\rvert$
5	0.2
8	2.8
6	0.8
3	2.2
4	1.2
	7.2

$M.D = \frac{7.2}{5} = 1.44$

$\bar{X} = \frac{26}{5} = \mathbf{5.2}$

Sol.6 (a)

X	F	FX	$\lvert X - \bar{X}\rvert$	F $\lvert X - \bar{X}\rvert$
50	7	350	5	35
60	7	420	5	35
	14	770		70

$$M.D. = \frac{\sum F|X - \bar{X}|}{\sum F} = \frac{\cancel{70}}{\cancel{14}} = 5$$

$\bar{X} = \frac{\sum FX}{\sum F} = \frac{\cancel{770}}{\cancel{14}} = \mathbf{55}$

Sol.7 (c)

X	$\lvert X - \bar{X}\rvert$
1	4
2	3
3	2
4	1
5	0
6	1
7	2
8	3
9	4
	20

$M.D. \Rightarrow \frac{\Sigma|X-\bar{X}|}{n} = \frac{20}{9}$

$Coefficient\ of\ mean\ deviation = \frac{\frac{20}{9}}{5} \times 100 \Rightarrow$
$\frac{20}{9} \times \frac{1}{5} \times 100 = \frac{400}{9}$

$\bar{X} = \frac{45}{9} = 5$

Sol.8 (a) $5y - 3x = 10$

$MD_X = 12$

$5y = 10 + 3x$

$y = 2 + \frac{3x}{5}$

$M.D._y = \frac{3}{5} \times 12 \Rightarrow \frac{36}{5} = 7.2$

Sol.9 (b) $2x + 3y - 7 = 0$

$3y = 7 - 2x$

$y = \frac{7}{3} - \frac{2}{3}x$

$\bar{Y} = \frac{7}{3} - \frac{2(1)}{3} \Rightarrow \frac{5}{3}$

$M.D._Y = \frac{2}{3} \times \frac{3}{10} \Rightarrow \frac{1}{5}$

$Coefficient. of\ M.D._Y \Rightarrow \frac{\frac{1}{5}}{\frac{5}{3}} \times 100$

$\Rightarrow \frac{1}{5} \times \frac{3}{5} \times 100 = 12$

Sol.10 (a)

	X	|X-Mode|
Mode	$\frac{4}{11}$ $\frac{6}{11}$ $\frac{8}{11}$ $\frac{9}{11}$ $\frac{12}{11}$ $\frac{8}{11}$	$\frac{4}{11}$ $\frac{2}{11}$ 0 $\frac{1}{11}$ $\frac{4}{11}$ 0
		$\frac{11}{11} = 1$

M.D. = $\frac{\sum\lvert X-Mode\rvert}{n} = \frac{1}{6}$

Mode =$\frac{8}{11}$

Sol.11 (b)

X	f	X^2	fX^2	fX
5	3	25	75	15
9	3	81	243	27
10	3	100	300	30
	9		**618**	**72**

$\bar{X} = \frac{72}{9} = 8$

$\sigma = \sqrt{\frac{\sum fX^2}{\sum f} - (\bar{X})^2} \Rightarrow \sqrt{\frac{618}{9} - (8)^2} \Rightarrow \sqrt{\frac{618-576}{9}} \Rightarrow$
$\sqrt{\frac{42}{9}} = \frac{\sqrt{42}}{3}$

Sol.12 $(b)\ \bar{X} = a$

$\sigma_x = b$

$Find\ \sigma\ of\ \frac{x-a}{b}$

$y = \frac{x-a}{b}$

$y = \frac{x}{b} - \frac{a}{b} \Rightarrow y = \frac{1}{b} \times x - \frac{a}{b}$

$\sigma_y = \frac{1}{b} \times \sigma_x$

$\sigma_y = \frac{1}{\cancel{b}} \times \cancel{b} = \mathbf{1}$

Sol.13 (a)

X	$(X-\bar{X})$	$(X-\bar{X})^2$
53	-5	25
52	-6	36
61	3	9
60	2	4
64	6	36
290		110

$\sigma = \sqrt{\sum\frac{(X-\bar{X})^2}{n}} = \sqrt{\frac{110}{5}} = \sqrt{22} = 4.69$

$\bar{X} = \frac{290}{5} = 58$

$C.V. = \frac{4.69}{58} \times 100 = \mathbf{8.09}$

Sol.14 (a) $\sigma_x = 3$
Variance of 5-2x

$y = 5 - 2x$

$\sigma_y = 3 \times |2| \Rightarrow 6$

Variance = $(\sigma_y)^2$

$= (6)^2$

$= 36$

Sol.15 (b) $2x + 3y + 4 = 0$

$\sigma_x = 6$

$\sigma_y = ?$

$3y = -4 - 2x$

$y = \frac{-4}{3} - \frac{2}{3}x$

$\sigma_y = \frac{2}{3} \times 6 = 4$

Sol.16 (a)

$Q_1 = 45, Q_2 = 52\ and\ Q_3 = 65$

$= QD = \frac{65-45}{2} = \frac{20}{2} = \mathbf{10}$

Sol.17 (d) $3x + 4y = 20$

$QD_X = 12$

$QD_Y = ?$

$4y = 20 - 3x$

$y = \frac{20}{4} - \frac{3}{4}x$

$QD_Y = \left|\frac{-3}{4}\right| \times QD_X$

$= \frac{3}{4} \times 12 = \mathbf{9}$

Sol.18 (b) $\sqrt{\frac{n^2-1}{12}} = 2$

$\frac{n^2-1}{12} = 4$

$n^2 - 1 = 48$

$n^2 = 49$

$n = \mathbf{7}$

Sol.19 (c) $y = 2x + 5$

$\sigma_x = 5$

$\bar{Y} = 2(10) + 5 \Rightarrow 25$

$\sigma_y = 2(5) = 10$

$C.V. of\ y = \left(\frac{\sigma_y}{\bar{Y}} \times 100\right) = \frac{10}{25} \times 100 \Rightarrow \mathbf{40}$

Sol.20 (c)

X	X^2
a	a^2
b	b^2
2	4
a+b+2	$a^2 + b^2 + 4$

$\bar{X} = 3$

$\sigma = \frac{2}{\sqrt{3}}$

$\bar{X} = \frac{a+b+2}{3}$

$3 = \frac{a+b+2}{3}$

$9 = a + b + 2$

7 = a + b

$\frac{2}{\sqrt{3}} = \sqrt{\frac{a^2+b^2+4}{3} - (3)^2}$

$\frac{4}{3} = \frac{a^2 + b^2 + 4}{3} - 9$

$\frac{4}{3} = \frac{a^2 + b^2 + 4 - 27}{3}$

$4 = a^2 + b^2 - 23$

$27 = a^2 + b^2$

$(a + b)^2 - 2ab = 27$

$(7)^2 - 2ab = 27$

49 -2ab =27

2ab = 49-27

ab = $\frac{22}{2}$ = **11**

****

Measure of Dispersion
Exercise: Set–C

Sol.1 (c)

X	Frequency	Fx	$\lvert X - \bar{X}\rvert$	$f\lvert X - \bar{X}\rvert$
5	3	15	11.52	34.56
10	4	40	6.52	26.08
15	6	90	1.52	9.12
20	5	100	3.48	17.4
25	3	75	8.48	25.44
30	2	60	13.48	26.96
	23	**380**		**139.56**

$\bar{X} = \frac{\Sigma fx}{\Sigma f}$

$\bar{X} = \frac{380}{23}$

$\bar{X} = 16.52$

Mean Deviation from Mean = $\frac{\Sigma F|X-\bar{X}|}{\Sigma F}$

$M.D. from\ Mean = \frac{139.56}{23}$

$M.D. from\ Mean = 6.07$

Sol. 2 (d)

X	Frequency	C.F.	$\lvert X - Median\rvert$	$f\lvert X - Median\rvert$
3	2	2	6	12
5	8	10	4	32
7	9	19	2	18
9	16	35	0	0
11	14	49	2	28
13	7	56	4	28
15	4	60	6	24
				142

Median = $\left(\frac{N+1}{2}\right)^{th} = \left(\frac{60+1}{2}\right)^{th} = 30.5$

$Median = 9$

$Mean\ deviation\ from\ Median = \frac{\Sigma F|X-Median|}{\Sigma F}$

$M.D. from\ Median = \frac{142}{60}$

$M.D. from\ Median = 2.37\ approx$

Sol. 3 (a)

C.I.	F	X	$d' = \frac{x-d}{i}$	fd'	$\lvert X-\bar{X}\rvert$	$F\lvert X-\bar{X}\rvert$
59.5-62.5	5	61	−2	−10	5.64	28.2
62.5-65.5	22	64	−1	−22	2.64	58.08
65.5-68.5	28	67	0	0	0.36	10.08
68.5-71.5	17	70	1	17	3.36	57.12
71.5-74.5	3	73	2	6	6.36	19.08
				−9		172.56

$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times i \Rightarrow 67 + \frac{(-9)}{75} \times 3$

$\bar{X} = 66.64$

$Mean\ devliation\ from\ Mean = \frac{\Sigma F|X-\bar{X}|}{\Sigma F}$

$M.D.\ from\ Mean = \frac{172.56}{75}$

$M.D.\ from\ Mean = 2.30$

$Coefficient\ of\ Mean\ deviation$

$= \frac{Mean\ deviation}{\bar{X}} \times 100 = \frac{2.30}{66.64} \times 100 = 3.45$

Sol. 4 (a)

C.I.	f	C.F.	X	$\lvert X - \text{Median}\rvert$	$F\lvert X - \text{Median}\rvert$
130.5-140.5	3	3	135.5	25.67	77.01
140.5-150.5	8	11	145.5	15.67	125.36
150.5-160.5	13	24	155.5	5.67	73.71
160.5-170.5	**15**	**39**	165.5	4.33	64.95
170.5-180.5	6	45	175.5	14.33	85.98
180.5-190.5	5	50	185.5	24.33	121.65
					548.66

Median $= l + \frac{N/2 - C.F._p}{f} \times i$

$Median = 160.5 + \frac{25-24}{15} \times 10$

$Median = 160.5 + \frac{10}{15}$

$\mathbf{Median = 161.17}$

$M.D.\ from\ Median = \frac{\Sigma F|X - Median|}{\Sigma F}$

$M.D.\ from\ Median = \frac{548.66}{50}$

$\mathbf{M.D.\ from\ Median = 10.97}$

Sol.5 (b)

Age	f	$d' = \frac{X-A}{i}$	fd'	$\lvert X-\bar{X}\rvert$
20	13	−3	−39	−30.95
30	28	−2	−56	−20.95
40	31	−1	−31	−10.95
50	46	0	0	-0.95
60	39	1	39	9.05
70	23	2	46	19.05
80	20	3	60	29.05
	200		19	

$\lvert X-\bar{X}\rvert^2$	$f\lvert X-\bar{X}\rvert^2$
957.90	12452.732
438.90	12289.27
119.90	3716.9775
0.9025	41.515
81.90	3194.1975
362.90	8346.7575
843.90	16878.05
	56919.5

$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times i$

$50 + \frac{19}{200} \times 10$

Mean= 50.95

$\sigma = \sqrt{\frac{\Sigma f(X-\bar{X})^2}{\Sigma f}}$

$\sigma = \sqrt{\frac{56919.5}{200}}$

$\sigma = 16.87$

Sol. 6 (c)

C.I.	f	x	$d' = \frac{X-A}{i}$	fd'	$(X-\bar{X})$	$(X-\bar{X})^2$	$f(X-\bar{X})^2$
30-40	17	35	−2	−34	−19.7	388.09	6597.53
40-50	28	45	−1	−28	−9.7	94.09	2634.52
50-60	21	55	0	0	0.3	0.09	1.89
60-70	15	65	1	15	10.3	106.09	1591.35
70-80	13	75	2	26	20.3	412.09	5357.17
80-90	6	85	3	18	30.3	918.09	5508.54
	100			−3			21,691

$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times i$

$\bar{X} = 55 + \frac{(-3)}{100} \times 10$

$\bar{X} = 54.7$

$\sigma = \sqrt{\frac{\Sigma f(X-\bar{X})^2}{\Sigma f}}$

$\sigma = \sqrt{\frac{21,691}{100}} = 14.73$

Coefficient of Variation $= \frac{\sigma}{\bar{X}} \times 100 \Rightarrow \frac{14.73}{54.7} \times 100$

C.V. = 26.93

Sol. 7 (a)

X Dividend Paid by A	Y Dividend Paid by B	x^2	y^2
5	4	25	16
9	8	81	64
6	7	36	49
12	15	144	225
15	18	225	324
10	9	100	81
8	6	64	36
10	6	100	36
75	**73**	**775**	**831**

$\bar{X} = \frac{75}{8} = \mathbf{9.375}$

$\bar{Y} = \frac{73}{8} = \mathbf{9.125}$

$\sigma_X = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{775}{8} - (9.375)^2}$

$= \sqrt{96.875 - 87.89}$

$= \sqrt{8.985}$

$\sigma_x = 2.997$

$\sigma_y = \sqrt{\frac{\Sigma y^2}{N} - (\bar{y})^2}$

$\sigma_y = \sqrt{\frac{831}{8} - (9.125)^2}$

$= \sqrt{103.875 - 83.26} \Rightarrow \sigma_y = \sqrt{20.615}$

$\sigma_y = 4.54$

$C.V. of\ A = \frac{\sigma}{x} \times 100 \Rightarrow \frac{2.997}{9.375} \times 100 = 31.968$

$C.V. of\ B = \frac{\sigma}{x} \times 100 \Rightarrow \frac{4.54}{9.125} \times 100 = \mathbf{49.75}$

Company A is more consistent for Payment of Dividend concerned.

Sol. 8 (c)

$\bar{X}_{12} = 65$ $\quad \bar{X}_1 = 70$ $\quad \bar{X}_2 = \ x$

$\sigma_{12} = 7.03$ $\quad \sigma_1 = 3$ $\quad n_2 = 40$

$Combined\ Mean = \frac{n_1\bar{X}_1 + n_2 + \bar{X}_2}{n_1 + n_2}$

$65 = \frac{60 \times 70 + 40x}{60 + 40}$

$6500 = 4200 + 40x \Rightarrow 2300 = 40x$

$x = \mathbf{57.5}$

$\bar{X}_2 = \mathbf{57.5}$

$d_1 = \bar{X}_{12} - \bar{X}_1 \Rightarrow 65 - 70 = -5$

$d_2 = \bar{X}_{12} - \bar{X}_2 \Rightarrow 65 - 57.5 = 7.5$

$\boldsymbol{Combined\ S.D} = \sqrt{\frac{n_1{\sigma_1}^2 + n_2{\sigma_2}^2 + n_1{d_1}^2 + n_2{d_2}^2}{n_1 + n_2}}$

$\sigma_{12} = \sqrt{\frac{60 \times (3)^2 + 40(y)^2 + 60 \times (-5)^2 + 40 \times (7.5)^2}{60 + 40}}$

$7.03 = \sqrt{\frac{60 \times 9 + 40 \times y^2 + 60 \times 25 + 40 \times 56.25}{100}}$

$7.03 = \sqrt{\frac{540 + 40y^2 + 1500 + 2250}{100}}$

$7.03 = \sqrt{\frac{40y^2 + 4290}{100}}$

$49.4209 \times 100 = 40y^2 + 4290$

$4942.09 = 40y^2 + 4290$

$652.09 = 40y^2$

$y^2 = 16.30225$

$y = 4.03$

$\boldsymbol{\sigma_y = 4}$

Sol. 9 (b)

$n_1 = 30$

$n_2 = 20$

$\bar{X}_1 = 55$

$\bar{X}_2 = 60$

$Variance\ of\ Series\ 1 =$
16 $\quad S.D. of\ Series\ 1 = 4$

$Variance\ of\ Series\ 2 =$
25 $\quad S.D. of\ Series\ 2 = 5$

$\bar{X}_{12} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2} \Rightarrow \bar{X}_{12} = \frac{30 \times 55 + 20 \times 60}{50}$

$\bar{X}_{12} = \frac{1650 + 1200}{50} \Rightarrow 57$

$\bar{X}_{12} = 57$

$d_1 = \bar{X}_{12} - \bar{X}_1 \Rightarrow d_1 = 57 - 55 = 2$

$d_2 = \bar{X}_{12} - \bar{X}_2 \Rightarrow d_2 = 57 - 60 = -3$

$\sigma_{12} = \sqrt{\frac{n_1{\sigma_1}^2 + n_2{\sigma_2}^2 + n_1{d_1}^2 + n_2{d_2}^2}{n_1 + n_2}}$

$\sigma_{12} = \sqrt{\frac{30 \times (4)^2 + 20(5)^2 + 30 \times (2)^2 + 20 \times (-3)^2}{30 + 20}}$

$\sigma_{12} = \sqrt{\frac{30 \times 16 + 20 \times 25 + 30 \times 4 + 20 \times 9}{50}}$

$$\sigma_{12} = \sqrt{\frac{480+500+120+180}{50}}$$

$$\sigma_{12} = \sqrt{\frac{1280}{50}} \Rightarrow \sigma_{12} = \sqrt{25.6}$$

$$\boldsymbol{\sigma_{12} = 5.06}$$

Sol.10 (b)

$n = 100$

$\bar{X} = 40 \qquad \bar{X} = \frac{\sum x}{n}$

$\sigma = 5.1 \qquad 40 = \frac{\sum x}{100} \Rightarrow \sum x =$

$4000\ (Incorrect)$

$\sum x\,(Corect) = \sum x\,(Incorect) -$

$Wrong\ observation + Correct\ observation$

$\sum x\,(Corect) = 4{,}000 - 50 + 40$

$\sum x\,(Corect) = 3990$

$Corrected\ A.M. = \frac{3990}{100} = 39.90$

$\sigma = \sqrt{\frac{\sum x^2}{N} - (\bar{X})^2} \Rightarrow 5.1 = \sqrt{\frac{\sum x^2}{100} - (40)^2} \Rightarrow$

$26.01 = \frac{\sum x^2}{100} - 1600$

$1626.01 = \frac{\sum x^2}{100}$

$162601 = \sum X^2$ (Incorrect)

$\sum X^2\ (Correct) = \sum X^2\ (Incorrect) -$

$(Incorrect)^2 + (Correct)^2$

$\sum X^2\ (Correct) = 162601 - (50)^2 + (40)^2$

$\sum X^2\ (Correct) = 162601 - 2500 + 1600$

$\sum X^2\ (Correct) = 161701$

$$\sigma = \sqrt{\frac{161701}{100} - (39.9)^2}$$

$\sigma = \sqrt{1617.01 - 1592.01}$

$\sigma = \sqrt{25} \Rightarrow \boldsymbol{\sigma = 5}$

Sol. 11(a)

C.I.	f	C.f.
L-29.5	5	5
29.5-39.5	7	12
39.5-49.5	18	30
49.5-59.5	32	62
59.5-79.5	28	90
79.5-U	10	100

$Q_1 = K\left(\frac{N}{4}\right)^{th} \Rightarrow \left(1 \times \frac{100}{4}\right)^{th} \Rightarrow 25th$

$Q_3 = 3\left(\frac{10}{4}\right) = 75th$

$Q_1 = l + \frac{K\left(\frac{N}{4}\right) - C.f._p}{f} \times i$

$Q_1 = 39.5 + \frac{25-12}{18} \times 10$

$Q_1 = 39.5 + \frac{13}{18} \times 10$

$Q_1 = 46.72$

$Q_3 = 59.5 + \frac{75-62}{28} \times 10$

$Q_3 = 59.5 + \frac{13}{28} \times 20$

$Q_3 = 68.79$

$Q.D. = \frac{Q_3 - Q_1}{2} \Rightarrow \frac{68.79-46.72}{2}$

$\boldsymbol{Q.D. = 11.035}$

****

Correlation and Regression
Exercise: Set- B

Sol.1 (b)

$$r = \frac{cov\,(x,y)}{S_x . S_y} = \frac{40}{\sqrt{16}\sqrt{256}} = \frac{40}{4 \times 16} \qquad [\because S = \sigma]$$

$= \frac{5}{8} = \mathbf{0.625}$

Sol.2 (b) $\because -1 \le r \le 1$

$\therefore \frac{cov\,(x,y)}{S_x\, S_y} \le 1$

$\Rightarrow S_x\, S_y \ge cov\,(x,y)$

$\boldsymbol{\Rightarrow S_x\, S_y \ge 15}$

Sol.3 (b) $Cov(x,y) \le S_x\, S_y$

$\Rightarrow 20 \le \sqrt{16}\, S_y \Rightarrow S_y \ge \frac{20}{\sqrt{16}}$

$\Rightarrow S_y \ge 5$

$\boldsymbol{\Rightarrow S_y^2 \ge 25}$

Sol.4 (c) Y = a+bx is the equation having relation between X and Y. It is perfect liner relation. So, r= 1 or r=-1 according as b>0 or b<0.

Sol.5 (d) Co-efficient of non-determination = 1 − r^2

$= 1 - (0.6)^2 = 1 - 0.36 = \mathbf{0.64}$

Sol.6 (b) $u + 5x = 6 \Rightarrow u = 6 + (-5)x$

$\therefore b = -5$

$3y - 7v = 20 \Rightarrow v = \frac{-20}{7} + \left(\frac{3}{7}\right)y$

$\therefore d = \frac{3}{7}$

$r_{xy} = \frac{b\,d}{|b||d|}\, r_{uv}$

Here *b & d have opposite sign*

$\therefore r_{xy} = -r_{uv} \Rightarrow r_{uv} = -r_{xy} = \mathbf{-0.58}$

Sol.7 (c)

$3x + 4u + 7 = 0$ $\qquad r_{xy} = -0.6$

$= 4u = -3x - 7$

$= u = -\frac{3}{4}x - \frac{7}{4}$

$r_{uy} = -ve$

$r_{uy} = \mathbf{0.6}$

Sol.8 (b) $r_{uv} = r_{xy}$ ($\because$ Correlation co-efficient is invariant with change of origin)

= 0.93

Sol.9 (c) $r_{uv} = r_{xy}$ (It change by it sign when scale is change by opposite sign)

= - 0.93

Sol.10 (c) $r_R = 1 - \frac{6\sum d_i^2}{n\,(n^2-1)}$

$= 1 - \frac{6\times21}{8\,(8^2-1)} = 1 - \frac{6\times21}{8\times63}$

$= 1 - 0.25 = \mathbf{0.75}$

Sol.11 (a) $r_R = 1 - 6\frac{\sum d_i^2}{n\,(n^2-1)}$

$\Rightarrow 0.6 = 1 - \frac{6\times66}{n\,(n^2-1)}$

$\Rightarrow \frac{6\times66}{n\,(n^2-1)} = 0.4$

$\Rightarrow n(n^2-1) = \frac{6\times66\times10}{4}$

$\Rightarrow (n+1)n(n-1) = 3\times3\times11\times10$

$\Rightarrow (n+1)n(n-1) = 11\times10\times9$

$\Rightarrow n = \mathbf{10}$

Sol.12 (b) $r_R = 1 - \frac{6\sum d_i^2}{n\,(n^2-1)}$

$\Rightarrow 0.4 = 1 - \frac{6\sum d_i^2}{6\,(6^2-1)}$

$\Rightarrow 0.6 = \frac{6\sum d_i^2}{6\,(6^2-1)} \Rightarrow \sum d_i^2 = 0.6\times35 = 21$

$\therefore$ Rectified $\sum d_i^2 = 21 + 4^2 - 3^2$

$= 21 + 16 - 9 = 28$

$\therefore$ Recfified $r_R = 1 - \frac{6\times28}{6\,(6^2-1)} = 1 - \frac{28}{35} = 1 - 0.8 =$ 0.2

Sol.13 (d) Here n =10, c = 4

$\therefore m = n - 1 = 9$

$\therefore r = \pm\sqrt{\frac{\pm(2c-m)}{m}} = -\sqrt{\frac{\pm(8-9)}{9}} = -\sqrt{1/9}$

$= -\frac{1}{3}$

Sol.14 (a) $r = \pm\sqrt{\pm\frac{(2\,c-m)}{m}}$

$\therefore \frac{1}{\sqrt{3}} = \sqrt{\frac{2\times6-(p-1)}{(p-1)}}$ $\qquad [\because m = p - 1]$

$\therefore \frac{1}{3} = \frac{12-p+1}{p-1}$

$\Rightarrow p - 1 = 39 - 3p$

$\Rightarrow 4p = 40 \Rightarrow p = \frac{40}{4} = \mathbf{10}$

Sol.15 (c)

X	Y	XY	x^2	y^2
-5	27	-135	25	729
-4	18	-72	16	324
-3	11	-33	9	121
-2	6	-12	4	36
-1	3	-3	1	9
0	2	0	0	4
1	3	3	1	9
2	6	12	4	36
3	11	33	9	121
4	18	72	16	324
5	27	135	25	729
0	**132**	**0**	**110**	**2,442**

$r = \frac{n\sum xy - \sum X \sum Y}{\sqrt{n\sum x^2-(\sum X)^2}\,\sqrt{n\sum y^2-(\sum Y)^2}}$

$= \frac{11\times0-0\times132}{\sqrt{11\times110-0^2}\,\sqrt{11\times2442-(132)^2}}$

$= \mathbf{0}$

Sol.16 (*c*) $5a + 10b = 40$______(I)

$10a + 25b = 95$______(II)

Multiply 2 in the first equation we get 10a+20b=80
Then solve 10a+20b=80 and 10a+25b=95
We get a=2 and b=3
Regression line of y on x is y = a + bx
= y = 2 + 3x

Sol.17 (a) $2x + 3y = -1$______(I)

$5x + 6y = -1$______(II)

From $[(I) \times 2 - (II)]$

$$\begin{array}{r} 4x+6y=-2 \\ 5x+6y=-1 \\ - \quad - \quad + \\ \hline -x=-1 \Rightarrow x=1 \end{array}$$

$$\therefore y = \frac{-1-2}{3} = \frac{-3}{3} = -1$$

$$(\bar{X}, \bar{Y}) = (\mathbf{1}, \mathbf{-1})$$

Sol.18 (b)

$3x + y = 13$______(I)

$2x + 5y = 20$______(II)

Let the line of regression x *on* y be 1st equation

$$\therefore x = \frac{-1}{3}y + \frac{13}{3}$$

$$\therefore b_{xy} = {}^{-1}/_{3}$$

and line of regression Y on X be

$$2x + 5y = 20 \Rightarrow y = \frac{-2}{5}x + 4$$

$$\therefore b_{yx} = {}^{-2}/_{5}$$

Now $r = \pm\sqrt{b_{xy} \times b_{yx}}$

$$\therefore r = -\sqrt{\left(\frac{-1}{3}\right)\left({}^{-2}/_{5}\right)} = -\sqrt{\frac{2}{15}} > -1$$

Which is correct

Hence the line of regression

***y* on *x* be $\mathbf{2x + 5y = 20}$**

Sol.19 (d) $2x - 3y = 10$______(I)

$3x + 4y = 15$______(II)

Let the equation x on y be equation (II)

i.e $3x + 4y = 15 \Rightarrow x = \frac{-4}{3}y + 5$

$$\therefore b_{xy} = {}^{-4}/_{3}$$

$\therefore$ Equation line of regression y *on* x be equation (I)
i.e; $2x - 3y = 10$

$$\Rightarrow y = \frac{2}{3}x - \frac{10}{3}$$

$$\therefore b_{yx} = {}^{2}/_{3}$$

Here $\boldsymbol{b_{xy}}$ & $\boldsymbol{b_{yx}}$ have different sign

Sol.20 (b) $b_{yx} = 2.4$

$u = 2x + 5 \ \therefore Scale\ u = 2$

$v = -3y - 6 \ \therefore Scale\ v = -3$

$$b_{vu} = \frac{Scale\ v}{Scale\ u}\, b_{yx}$$

$$= \frac{-3}{2} \times 2.4 = \mathbf{-3.6}$$

Sol.21 (a) $\because$ Line of regression y on x be

$$4y - 5x = 15 \Rightarrow y = \frac{5}{4}x + \frac{15}{4}$$

$$\therefore b_{yx} = {}^{5}/_{4} = 1.25$$

$$r^2 = b_{xy} \times b_{yx}$$

$$\Rightarrow (0.75)^2 = b_{xy} \times 1.25$$

$$\Rightarrow b_{xy} = \frac{0.5625}{1.25} = \mathbf{0.45}$$

Sol.22 (c) $y = -2x + 3 \ \therefore b_{yx} = -2$

$$8x = -y + 3 \Rightarrow x = {}^{-1}/_{8}\,y + {}^{3}/_{8}$$

$$\therefore b_{xy} = {}^{-1}/_{8}$$

$$r = \pm\sqrt{b_{xy}\, b_{yx}}$$

$$\therefore r = -\sqrt{-2 \times \left(\frac{-1}{8}\right)} = -\sqrt{{}^{1}/_{4}}$$

$$= {}^{-1}/_{2} = \mathbf{-0.5}$$

Sol.23 (b) $b_{yx} = r\frac{\sigma_y}{\sigma_x} \Rightarrow \frac{-3}{4} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{4}}{\sigma_x}$

$$\Rightarrow \sigma_x = \frac{\sqrt{3}}{2} \times \frac{2}{-3} \times 4$$

$$\Rightarrow \sigma_x{}^2 = \left(\frac{-4}{\sqrt{3}}\right)^2 = \frac{16}{3}$$

Sol.24 (a) The line of regression $y\ on\ x\ be$

$$y = 3x + 4$$

$$\therefore \bar{Y} = 3\bar{X} + 4$$

$$= 3 \times (-1) + 4$$

$$= -3 + 4$$

$$= 1$$

$$\therefore \bar{Y} = 1$$

****

Correlation and Regression
Exercise: Set-C

Sol. 1 (c)

x	y	xy	x^2	y^2
1	8	8	1	64
2	6	12	4	36
3	7	21	9	49
4	5	20	16	25
5	5	25	25	25
15	**31**	**86**	**55**	**199**

$$r = \frac{N\sum xy - \sum x \sum y}{\sqrt{N\sum x^2 - (\sum x)^2}\ \sqrt{N\sum y^2 - (\sum y)^2}}$$

$$r = \frac{5\times86 - 15\times31}{\sqrt{5\times55-(15)^2}\ \sqrt{5\times199-(31)^2}}$$

$$r = \frac{430-465}{\sqrt{275-225}\ \sqrt{995-961}}$$

$$r = \frac{-35}{\sqrt{50}\ \sqrt{34}} \Rightarrow r = \frac{-35}{41.23}$$

$$\boldsymbol{r = -0.85}$$

Sol. 2 (a)

x	y	$dx = (x - 64)$	$dy = (y - 65)$	$dx\,dy$	dx^2	dy^2
64	57	0	−8	0	0	64
60	60	−4	−5	20	16	25
67	73	3	8	24	9	64
59	62	−5	−3	15	25	9
69	68	5	3	15	25	9
		−1	**−5**	**74**	**75**	**171**

$$r = \frac{N\sum dx\,dy - \sum dx . \sum dy}{\sqrt{N\sum dx^2 - (\sum dx)^2}\ \sqrt{N\sum dy^2 - (\sum dy)^2}}$$

$$r = \frac{5\times74-(-5)(-1)}{\sqrt{5\times75-(-1)^2}\ \sqrt{5\times171-(-5)^2}}$$

$$r = \frac{370-5}{\sqrt{375-1}\ \sqrt{855-25}}$$

$$r = \frac{365}{\sqrt{374}\ \sqrt{830}}$$

$$r = \frac{365}{557.15}$$

$$\boldsymbol{r = 0.655}$$

Sol.3 (b)

x	y	$dx = (x - 33)$	$dy = (x - 27)$	dx^2	dy^2	$dx\,dy$
46	37	13	10	169	100	130
45	35	12	8	144	64	96
42	31	9	4	81	16	36
40	28	7	1	49	1	7
38	30	5	3	25	9	15
35	25	2	−2	4	4	−4
32	23	−1	−4	1	16	4
30	19	−3	−8	9	64	24
27	19	−6	−8	36	64	48
25	18	−8	−9	64	81	72
360	**265**	**30**	**−5**	**582**	**419**	**428**

$$r = \frac{N\sum dx\,dy - \sum dx \sum dy}{\sqrt{N\sum dx^2 - (\sum dx)^2}\ \sqrt{N\sum dy^2 - (\sum dy)^2}}$$

$$r = \frac{10\times428 - 30\times(-5)}{\sqrt{10\times582-(30)^2}\ \sqrt{10\times419-(-5)^2}}$$

$$r = \frac{4280+150}{\sqrt{5820-900}\ \sqrt{4190-25}}$$

$$r = \frac{4430}{\sqrt{4920}\ \sqrt{4165}}$$

$$r = \frac{4430}{4526.78}$$

$$\boldsymbol{r = 0.98\ (Approx)}$$

Sol.4 (c)

$\sum xy = 414,\ \sum x = 120,\ \sum y = 90,\ \sum x^2 = 600,\ \sum y^2 = 300\ n = 30$

$\sum xy\,(correct) = 414 - 12 \times 11 - 6 \times 8 + 10 \times 9 + 8 \times 10$

$414 - 132 - 48 + 90 + 80$

$\boldsymbol{\sum xy\,(correct) = 404}$

$\sum x^2\,(correct) = \sum x^2\,(Incorrect) - (Incorrect)^2 + (correct)^2$

$= 600 - (12)^2 - (6)^2 + (10)^2 + 8^2$

$= 600 - 144 - 36 + 100 + 64$

$\sum x^2\,(correct) = 584$

$\sum y^2\,(correct) = \sum y^2\,(Incorrect) - (Incorrect)^2 + (correct)^2$

$\sum y^2\,(correct) = 300 - 11^2 - 8^2 + 9^2 + 10^2$

$\sum y^2\,(correct) = 300 - 121 - 64 + 81 + 100$

$\sum y^2\,(correct) = 296$

$\sum x(correct) = \sum x(Incorrect) - Incorrect + Correct$

$= 120 - 12 - 6 + 10 + 8$

$\sum x(correct) = 120$

$\sum y(correct) = \sum y(Incorrect) - Incorrect + Correct$

$\sum y(correct) = 90 - 11 - 8 + 9 + 10$

$\sum y(correct) = 90$

$\bar{X} = \frac{\sum x}{N} = \frac{120}{30} \Rightarrow \bar{X} = 4$

$\bar{Y} = \frac{\sum y}{N} \Rightarrow \bar{Y} = \frac{90}{30} \Rightarrow \bar{Y} = 3$

$\sigma_x = \sqrt{\frac{\sum x^2}{n} - (\bar{X})^2} \;\Rightarrow\; \sigma_x \sqrt{\frac{584}{30} - 16}$

$\sigma_x \sqrt{19.47 - 16}$

$\sigma_x = 1.8627$

$\sigma_y = \sqrt{\frac{\sum y^2}{n} - (\bar{Y})^2} \;\Rightarrow\; \sigma_y \sqrt{\frac{296}{30} - 9}$

$\sigma_y \sqrt{9.87 - 9} \;\Rightarrow 0.93$

$\sigma_y = 0.93$

$Cov.(xy) = \frac{\sum xy}{n} - \bar{x}\bar{y}$

$Cov.(xy) = \frac{404}{30} - 4 \times 3 \Rightarrow Cov.(xy) = 1.4667$

$r = \frac{Cov.(xy)}{\sigma_x \sigma_y} \Rightarrow r = \frac{1.4667}{0.93 \times 1.8627}$

$\boldsymbol{r = 0.846}$

Sol.5 (d)

C.I.	x (Mid value)	No. of items	No. of Defec -tives	No. of Defectives Per 100(y)	xy	x^2	y^2
9-11	10	250	25	10	100	100	100
11-13	12	350	70	20	240	144	400
13-15	14	400	60	15	210	196	225
15-17	16	300	45	15	240	256	225
17-19	18	150	20	13.33	240	324	177.69
	70			73.33	1030	1020	1127.69

$r = \frac{N\sum xy - \sum x \sum y}{\sqrt{N\sum x^2 - (\sum x)^2}\,\sqrt{N\sum y^2 - (\sum y)^2}}$

$r = \frac{5\times1030 - 70\times73.33}{\sqrt{5\times1020-(70)^2}\,\sqrt{5\times1127.69-(73.33)^2}}$

$r \frac{5150-5133.10}{\sqrt{5100-4900}\,\sqrt{5638.45-5377.2889}}$

$r = \frac{16.9}{\sqrt{200}\,\sqrt{261.17}}$

$r = \frac{16.9}{228.55}$

$\boldsymbol{r = 0.07}$

Sol.6 (d)

$r = 0.4$

$Cov.(xy) = 8$

$Variance\ of\ x = 16$

$\sum(y - \bar{y})^2 = 250$

$(\sigma_x)^2 = Variance$

$(\sigma_x)^2 = 16$

$\sigma_x = 4$

$r = \frac{Cov.(x,y)}{\sigma_x \sigma_y}$

$0.4 = \frac{8}{4\times\sigma_y}$

$\sigma_y = 5$

$\sigma_y = \sqrt{\frac{\sum(y-\bar{y})^2}{N}}$

$5 = \sqrt{\frac{250}{N}}$

$25 = \frac{250}{N}$

$\boldsymbol{N = 10}$

Sol.7 (d)

S. No.	R_x	R_y	$d = R_x - R_y$	d^2
1	7	5	2	4
2	6	4	2	4
3	2	6	−4	16
4	4	3	1	1
5	5	8	−3	9
6	3	2	1	1
7	1	1	0	0
8	8	7	1	1
			0	**36**

$r = 1 - \frac{6\sum d^2}{N(N^2-1)}$

$r = 1 - \frac{6\times36}{8(8^2-1)} \Rightarrow r = 1 - \frac{6\times36}{8\times63}$

$\boldsymbol{r = 0.57}$

Sol. 8 (c)

S. No.	x	y	R_x	R_y	d	d^2
1	58	62	3	6	−3	9
2	43	63	5	5	0	0
3	50	79	4	1	3	9
4	19	56	10	8	2	4
5	28	65	8	4	4	16
6	24	54	9	10	−1	1
7	77	70	1	2	−1	1
8	34	59	6	7	−1	1
9	29	55	7	9	−2	4
10	75	69	2	3	−1	1
						46

$r = 1 - \frac{6\sum d^2}{N(N^2-1)}$

$r = 1 - \frac{6\times46}{10\times99}$

$\boldsymbol{r = 0.72}$

Sol. 9(d)

S. No.	x	y	R_x	R_y	$d = R_x - R_y$	d^2
1	25	30	6	5	1	1
2	30	25	4.5	6	−1.5	2.25
3	46	50	3	2.5	0.5	0.25
4	30	40	4.5	4	0.5	0.25
5	55	50	2	2.5	0.5	0.25
6	80	78	1	1	0	0

$$r = 1 - \frac{6\left[\frac{\sum d^2 + \sum((m_1)^3 - m_1) + \sum((m_2)^3 - m_2)}{12}\right]}{n(n^2-1)}$$

$= \frac{\sum((m_1)^3 - m_1) + \sum((m_2)^3 - m_2)}{12}$

$= \frac{(2^3-2)+(2^3-2)}{12}$

$= \frac{6+6}{12} = 1$

$r = 1 - \frac{6(5)}{6\times35}$

$\boldsymbol{r = 0.857}$

Sol.10(c)

Supply (x)	Demand (y)	Sign. of Deviation of (x)	Sign. of Deviation of (y)	Product of Deviation
68	65			
43	60	−	−	+
38	55	−	−	+
78	61	+	+	+
66	35	−	−	+
83	75	+	+	+
38	55	−	−	+
23	40	−	−	+
83	85	+	+	+
63	80	−	−	+
53	85	−	+	−

$m = n - 1$

$m = 11 - 1 = 10$

$m = 10$

C = No. of Positive Sign.

C = 9

$r = \pm\sqrt{\pm\frac{(2c-m)}{m}} \Rightarrow r = \pm\sqrt{\pm\left(\frac{18-10}{10}\right)}$

$r = +\sqrt{\frac{8}{10}}$

$\boldsymbol{r = 0.89}$

Sol. 11(a)

Year	Price	Sign. of Deviation of Price	Demand	Sign. of Deviation of Demand	Product of Deviation
1996	35		36		
1997	38	+	35	−	-
1998	40	+	31	−	-
1999	33	−	36	+	-
2000	45	+	30	−	-
2001	48	+	29	−	-
2002	49	+	27	−	-
2003	52	+	24	−	-

$m = n - 1$

$m = 8 - 1 \Rightarrow m = 7$

$C = 0$

$r = \pm\sqrt{\pm\frac{(2c-m)}{m}} \Rightarrow r = \pm\sqrt{\pm\frac{(0-7)}{7}}$

$r = -\sqrt{\frac{-7}{7}}$

$r = -1$

Sol.12 (c)

x	y	$dx = x - 80$	$dy = y - 60$	dx^2	$dx\ dy$
41	28	−39	−32	1,521	1,248
82	56	2	−4	4	−8
62	35	−18	−25	324	450
37	17	−43	−43	1,849	1,849
58	42	−22	−18	484	396
96	85	16	25	256	400
127	105	47	45	2,209	2,115
74	61	−6	1	36	−6
123	98	43	38	1,849	1,634
100	73	20	13	400	260
800	**600**	**0**	**0**	**8,932**	**8,338**

$\bar{X} = \frac{800}{10} = 80$

$\bar{Y} = \frac{600}{10} = 60$

$b_{yx} = \frac{N\sum dxdy - \sum dx \sum dy}{N\sum dx^2 - (\sum dx)^2}$

$b_{yx} = \frac{10 \times 8338 - 0}{10 \times 8932 - (0)^2} = b_{yx} = \frac{83380}{89320}$

$b_{yx} = 0.933$

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$y - 60 = 0.93(x - 80)$

$y - 60 = 0.93x - 74.64$

$\mathbf{y = 0.93x - 14.64}$

Sol. 13(b)

x	y	$dx(x - 170)$	$dy(y - 172)$	$dx\,dy$	dx^2
175	173	5	1	5	25
172	172	2	0	0	4
167	171	−3	−1	3	9
168	171	−2	−1	2	4
172	173	2	1	2	4
171	170	1	−2	−2	1
174	173	4	1	4	16
176	175	6	3	18	36
169	170	−1	−2	2	1
170	173	0	1	0	0
1,714	**1,721**	**14**	**1**	**34**	**100**

$\bar{X} = \frac{1,714}{10} = 171.4$

$\bar{Y} = \frac{1,721}{10} = 172.1$

$b_{yx} = \frac{N\sum dxdy - \sum dx \sum dy}{N\sum dx^2 - (\sum dx)^2}$

$b_{yx} = \frac{10 \times 34 - 14 \times 1}{10 \times 100 - 196}$

$b_{yx} = \frac{340 - 14}{1000 - 196} = \frac{326}{804}$

$b_{yx} = 0.405$

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$y - 172.1 = 0.405\ (x - 171.4)$

$y - 172.1 = 0.405x - 69.417$

$\mathbf{y = 0.405x + 102.683}$

Sol. 14 (a)

x	y	xy	x^2	y^2
38	28	1,064	1,444	784
23	23	529	529	529
43	43	1,849	1,849	1,849
33	38	1,254	1,089	1,444
28	8	224	784	64
165	**140**	**4920**	**5,695**	**4,670**

Regression Coefficient of x and y

$b_{xy} = \frac{N\sum xy - \sum x \sum y}{N\sum y^2 - (\sum y)^2}$

$b_{xy} = \frac{5 \times 4,920 - 165 \times 140}{5 \times 4,670 - (140)^2}$

$b_{xy} = \frac{24,600 - 23,100}{23,350 - 19,600}$

$b_{xy} = \frac{1,500}{3,750} = 0.4$

$b_{yx} = \frac{N\sum xy - \sum x \sum y}{N\sum x^2 - (\sum x)^2}$

$b_{yx} = \frac{5 \times 4,920 - 165 \times 140}{5 \times 5,695 - (165)^2}$

$b_{yx} = \frac{24,600 - 23,100}{2,8475 - 27,225}$

$b_{yx} = \frac{1,500}{1,250} = \mathbf{1.2}$

Sol.15 (c)

x	y	xy	y^2
11	21	231	441
12	15	180	225
15	13	195	169
16	12	192	144
18	11	198	121
19	10	190	100
21	9	189	81
112	**91**	**1,375**	**1,281**

Regression Coefficient of x and y

$b_{xy} = \frac{N\sum xy - \sum x \sum y}{N\sum y^2 - (\sum y)^2}$

$b_{xy} = \frac{7\times1,375 - 112\times91}{7\times1,281 - (91)^2}$

$b_{xy} = \frac{9,625 - 10,192}{8,967 - 8,281}$

$b_{xy} = \frac{-567}{686}$

$b_{xy} = -0.8265$

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$(x - 16) = -0.8265(y - 13)$

$x - 16 = -0.8265y + 10.7445$

$x = -0.8265(25) + 10.7445 + 16$

$x = -20.6625 + 26.7445$

$x = 6.08$

Sol. 16(d)

$\bar{X} = 80, \quad \bar{Y} = 98$

$Variance\ of\ x = 4, \quad Variance\ of\ y = 9$

$r = 0.6$

$\sigma_x = 2, \quad \sigma_y = 3$

$b_{yx} = r\frac{\sigma_y}{\sigma_x}$

$b_{yx} = 0.6 \times \frac{3}{2}$

$\mathbf{b_{yx} = 0.9}$

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$y - 98 = 0.9\,(x - 80)$

$y - 98 = 0.9x - 72$

$y = 0.9x - 72 + 98$

$y = 0.9(90) + 26$

$\mathbf{y = 107}$

Sol. 17 (b)

$8x + 10y = 25, \quad 16x + 5y = 12$

$10y = 25 - 8x$ [*Assume it is y on x*]

$y = \frac{25}{10} - \frac{8}{10}x$

$y = \frac{5}{2} - \frac{4}{5}x$

$16x = 12 - 5y$ [*Assume it on x on y*]

$x = \frac{12}{16} - \frac{5}{16}y$

$x = 0.75 - \frac{5}{16}y$

$r = \pm\sqrt{b_{xy} \times b_{yx}}$

$r = -\sqrt{\frac{4}{5} \times \frac{5}{16}}$

$r = -0.5$

$b_{yx} = r\frac{\sigma_y}{\sigma_x} \Rightarrow \frac{-4}{5} = -0.5 \times \frac{\sigma_y}{5} \Rightarrow \sigma_y = 8$

Sol. 18 (a)

$Mean\ of\ x = 62$ $Mean\ of\ y = 25$

$\sigma_x = 5$ $\sigma_y = 6$

$r = 0.92$

$b_{yx} = r \times \frac{\sigma_y}{\sigma_x} \Rightarrow b_{yx} = 0.92 \times \frac{6}{5}$

$b_{yx} = 1.104$

$b_{xy} = r \times \frac{\sigma_x}{\sigma_y} \Rightarrow b_{xy} = 0.92 \times \frac{5}{6}$

$b_{xy} = 0.767$

Sum of regression coefficient = 1.104 + 0.767

= 1.871

Sol. 19(c)

x	y	xy	x^2	y^2
75	35	2,625	5,625	1,225
81	45	3,645	6,561	2,025
85	59	5,015	7,225	3,481
105	75	7,875	11,025	5,625
93	43	3,999	8,649	1,849
113	79	8,927	12,769	6,241
121	87	10,527	14,641	7,569
125	95	11,875	15,625	9,025
798	**518**	**54,488**	**82,120**	**37,040**

$$r=\frac{n\sum xy-\sum x\sum y}{\sqrt{n\sum x^2-(\sum x)^2}\sqrt{n\sum y^2-(\sum y)^2}}$$

$$r=\frac{8\times54{,}488-798\times518}{\sqrt{8\times82{,}120-(798)^2}\sqrt{8\times37{,}040-(518)^2}}$$

$$r=\frac{43{,}5904-41{,}3364}{\sqrt{65{,}6960-63{,}6804}\sqrt{29{,}6320-26{,}8324}}\Rightarrow\frac{22{,}540}{\sqrt{20{,}156}\sqrt{27{,}996}}$$

$$r=\frac{22{,}540}{23{,}754.42}=0.9488=0.95$$

****

Probability
Exercise: Set-B

Sol.1 (a) No. of white ball = 5

No. of blackball =7

P (drawing two different balls)

$=\frac{5_{C_1}\times7_{C_1}}{12_{C_2}}=\frac{5\times7}{\frac{12\times11}{2\times1}}=\frac{5\times7}{6\times11}$

$=\frac{35}{66}$

Sol.2 (b) Single cast with two dice means single throw of two dice.

$n\,(S)=6\times6=36$

At least seven numbers means sum = 7, 8, 9 10, 11, 12

$n(E)=6+5+4+3+2+1=21$

E = { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (2,6), (3,5), (4,4), (5,3), (6,2), (3,6), (4,5), (5,4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6) }

$P\,(E)=\frac{n(E)}{n\,(S)}=\frac{21}{36}=\frac{7}{12}$

Sol.3 (c) P (getting at least one defective item)

= 1 − P (getting non defective items)

$=1-\frac{4_{C_3}}{6_{C_3}}=1-\frac{4}{\frac{6\times5\times4}{3\times2}}$

$=1-\frac{1}{5}={}^4/_5=\mathbf{0.80}$

Sol.4 (d) $n\,(S)=6\times6=36$

$n\,(E)=6$ [(1,1), (2,2), (3,3), (4,4), (5,5), & (6,6)]

$\therefore P(E)=\frac{6}{36}=\mathbf{{}^1/_6}$

Sol.5 (c) $P\,(A\cap B)=0$

$P(B\cap C)=0$

$P\,(A\cap C)=0$

$P\,(A\cup B\cup C)=1$

$\&\ P(A)+P(B)+P(C)=1$

$\therefore P(A\cap B\cap C)=\mathbf{0}$

Sol.6 (d) P (odd number greater than 4) = 5, 7, 9

$=\frac{3_{C_1}}{10_{C_1}}=\frac{3}{10}=\mathbf{0.30}$

Sol.7 (b) Wages are 50, 62, 40, 70, 45, 56, 32, 45

$\bar{X}=\frac{50+62+40+70+45+56+32+45}{8}$

$=\frac{400}{8}=50$

P (wages lower than average)

$=\frac{4}{8}=\frac{1}{2}=\mathbf{0.5}$

Sol.8 (b) $P(A)=2P(B)=3P(C)=k(Let)$

$\therefore P(A)=k$

$P(B)=\frac{k}{2}$

$P(C)=\frac{k}{3}$

∵ A, B & C are mutually exclusive & exhaustive events

$\therefore P\,(A\cup B\cup C)=1$

$\Rightarrow P(A)+P(B)+P(C)=1$

$\Rightarrow k+\frac{k}{2}+\frac{k}{3}=1$

$\Rightarrow \frac{6k+3k+2k}{6} = 1$

$\Rightarrow k = \frac{6}{11}$

$\therefore P(B) = \frac{k}{2} = \frac{3}{11}$

Sol.9 (d) $P(B) = 0.3, P(A\ but\ not\ B) = 0.4$
$\Rightarrow P(A) - P(A \cap B) = 0.4$
$P(\bar{A}) = 0.6\ \therefore P(A) = 1 - 0.6 = 0.4$
$\therefore P(A \cap B) = 0$ [Mutually exclusive]
$P(A \cup B) = P(A) + P(B) = 0.4 + 0.3 = \mathbf{0.7}$

Sol.10 (d) $n\,(S) = 12$

$n\,(E) = 4\,(5, 6, 10, 12)$

$P(E) = \frac{n(E)}{n(S)} = \frac{4}{12} = \mathbf{{}^{1}/_{3}}$

Sol.11 (d)

$P(A) = {}^{3}/_{5}, P(B) = {}^{2}/_{3}, P(A \cup B) = {}^{3}/_{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{4} = \frac{3}{5} + \frac{2}{3} - P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{19}{15} - \frac{3}{4} = \frac{76-45}{60} = \frac{31}{60}$

$P(A/B) = \frac{P\,(A \cap B)}{P(B)} = \frac{{}^{31}/_{60}}{{}^{2}/_{3}} = \frac{31}{40} = \mathbf{0.775}$

Sol.12 (b) $\therefore P(A + B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \times P(B)$

$= \frac{3}{5} + \frac{2}{3} - \frac{3}{5} \times \frac{2}{3} = \frac{9+10-6}{15} = \mathbf{\frac{13}{15}}$

Sol.13 (a) $P(A) = p\ and\ P(B) = q$

$\therefore P({}^{A}/_{B}) = \frac{P\,(A \cap B)}{P(B)} \leq \frac{P(A)}{P(B)}$

$\Rightarrow P\,({}^{A}/_{B}) \leq \frac{p}{q}$

Sol.14 (c) $P\,(\bar{A} \cup \bar{B}) = {}^{5}/_{6} \Rightarrow P(\overline{A \cup B}) = {}^{5}/_{6}$

$\therefore P(A \cap B) = 1 - {}^{5}/_{6} = {}^{1}/_{6}$

$P(A) = {}^{1}/_{2}$

$P\,(\bar{B}) = {}^{2}/_{3}\ \therefore P(B) = 1 - {}^{2}/_{3} = {}^{1}/_{3}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6}$

$= \mathbf{{}^{2}/_{3}}$

Sol.15 (b) $Let\ P(B) = x$

$P(A) = {}^{2}/_{5}$

$P(A \cap B) = P(A) \times P(B) = \frac{2}{5}x$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow {}^{2}/_{3} = \frac{2}{5} + x - \frac{2}{5}x$

$\Rightarrow \frac{2}{3} - \frac{2}{5} = \frac{3}{5}x \Rightarrow x = \frac{4}{15} \times \frac{5}{3} = \mathbf{{}^{4}/_{9}}$

Sol.16 (c) $P(A) = {}^{2}/_{3}, P(B) = {}^{3}/_{4}$

$P\,({}^{A}/_{B}) = {}^{2}/_{3}$

$\Rightarrow \frac{P\,(A \cap B)}{P\,(B)} = {}^{2}/_{3} \Rightarrow P\,(A \cap B) = \frac{2}{3} \times P(B) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

$\therefore P({}^{B}/_{A}) = \frac{P\,(A \cap B)}{P(A)} = \frac{{}^{1}/_{2}}{{}^{2}/_{3}} = \frac{1}{2} \times \frac{3}{2} = \mathbf{{}^{3}/_{4}}$

Sol.17 (d) $P(A) = a, P(B) = b, P(A \cap B) = c$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = a + b - c$

Now, $P\,(A' \cap B') = P\,(A \cup B)' = 1 - P(A \cup B)$

$= 1 - (a + b - c) = \mathbf{1 - a - b + c}$

Sol.18 (d) P (only A occur) $= P\,(A \cap B' \cap C')$

Sol.19 (c) $S = \{BB, GG, GB, BG\}$

$A \rightarrow$ Family has a girl child ={BG, GB, GG}

P=(A) = ¾

$B \rightarrow$ Family has 2nd child girl {GG}

P(B) = ¼

A∩B = {GG}, P(A∩B) = ¼

$$P\left(\frac{B}{A}\right) = \frac{p(A \cap B)}{p(B)} = \frac{\mathbf{\frac{1}{4}}}{\mathbf{\frac{3}{4}}} = \mathbf{\frac{1}{3}}$$

Sol.20 (a) $S = \{HH, HT, TH, TT\}$

$A \rightarrow$ 1st coin shows the head

$B \rightarrow$ 2nd coin is tail

$P(A) = {}^{2}/_{4} = {}^{1}/_{2}$

$P(B) = {}^{2}/_{4} = {}^{1}/_{2}$

$P(A \cap B) = {}^{1}/_{4}$

$\therefore P\left({}^{B}/_{A}\right) = \frac{{}^{1}/_{4}}{{}^{1}/_{2}} = \frac{1}{2} = 0.50$

Sol.21 (c) $E(X) = p_1x_1 + p_2x_2 + p_3x_3$

$= 0.30 \times 0 + 0.50 \times 1 + 0.20 \times 2$

$= 0 + 0.50 + 0.40 = \mathbf{0.90}$

Sol.22 (b) $y = -3x + 4$

$\sigma_X = 2$

$\therefore \sigma_Y = |-3|\sigma_X = 3 \times 2 = \mathbf{6}$

Sol.23 (a) $2x + 3y + 4 = 0$

$\Rightarrow y = -\frac{2}{3}x - {}^{4}/_{3}$

$\& V_{(X)} = \sigma_x^2 = 6$

$\therefore V_{(Y)} = \left(\frac{-2}{3}\right)^2 \times 6 = \frac{4}{9} \times 6 = \mathbf{{}^{8}/_{3}}$

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Probability
Exercise: Set-C

Sol.1 (b)

$S = \{(S, M), (M, T), (T, W), (W, TH),$
$(TH, F), (F, Sat), (Sat, S)\}$

$n\,(S) = 7$

$n(E) = 2$

$\therefore P\,(E) = \mathbf{{}^{2}/_{7}}$

Sol.2 (c) $n\,(S) = 2 \times 2 \times 2 = \mathbf{8}$

Favorable outcomes = {(HTH), (THH), (HHT), (HHH)}

$n\,(E) = 4$

$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \mathbf{{}^{1}/_{2}}$

Sol.3 (c) $n\,(S) = 6 \times 6 = 36$

$E =$
$\{(1,5), (5,1), (2,4), (4,2), (3,3), (6,3), (3,6), (5,4), (4,5)\}$

$n\,(E) = 9$

$n\,(E) = \frac{9}{36} = {}^{1}/_{4} = 0.25$

$\therefore P(E') = 1 - P(E) = 1 - 0.25 = \mathbf{0.75}$

Sol.4 (a) $n\,(S) = (365)^4$

$n\,(E) = 365 \times 364 \times 363 \times 362$

$\therefore P(E) = \frac{365\times364\times363\times362}{(365)^4}$

$= \frac{\mathbf{364\times363\times362}}{\mathbf{(365)^3}}$

Sol.5 (d) (i) $\frac{5_{C_3}}{12_{C_3}} \times \frac{7_{C_3}}{12_{C_3}} = \frac{\frac{\frac{5\times4}{2}}{12\times11\times10}}{3\times2} \times \frac{\frac{\frac{7\times6\times5}{3\times2}}{12\times11\times10}}{3\times2} = \frac{10}{220} \times \frac{35}{220}$

$= \frac{7}{968}$

(ii) $\frac{5_{C_3}}{12_{C_3}} \times \frac{7_{C_3}}{9_{C_3}}$ ($\because$ Without replacement remaining balls =12-3=9)

$\frac{\frac{\frac{5\times4}{2}}{12\times11\times10}}{3\times2} \times \frac{\frac{\frac{7\times6\times5}{3\times2}}{9\times8\times7}}{3\times2} = \frac{10}{220} \times \frac{35}{84} = \frac{1}{22} \times \frac{5}{12} = \frac{\mathbf{5}}{\mathbf{264}}$

Sol.6 (a) $R \rightarrow Red, W \rightarrow white, B \rightarrow Blue$

Required Probability = P(RRR) + P(WWW) + P(BBB)

$= \frac{5}{18} \times \frac{4}{18} \times \frac{3}{9} + \frac{7}{18} \times \frac{8}{18} \times \frac{4}{9} + \frac{6}{18} \times \frac{6}{18} \times \frac{2}{9}$

$= \frac{60+224+72}{18\times18\times9}$

$= \frac{356}{18\times18\times9} = \frac{\mathbf{89}}{\mathbf{729}}$

Sol.7 (c) Let A → Multiple of 7

B → Multiple of 11

$n\,(S) = 1000$

$P\,(A) = \frac{142}{1000}$

$P\,(B) = \frac{90}{1000}$

$P\ (A \cap B) = \frac{12}{1000}$

$\therefore P\ (A \cup B) = P(A) + P(B) - P\ (A \cap B)$

$= \frac{142}{1000} + \frac{90}{1000} - \frac{12}{1000} = \frac{220}{1000} = \frac{11}{50}$

= 0.22

Sol.8 (c) Required Probability

$\therefore P\ (A \cap B) = P(A) \times P(B) = \frac{5_{C_3}}{13_{C_3}} \times \frac{8_{C_3}}{10_{C_3}}$

$= \frac{\frac{5\times4}{2}}{\frac{13\times12\times11}{3\times2}} \times \frac{\frac{8\times7\times6}{3\times2}}{\frac{10\times9\times8}{3\times2}} = \frac{10}{13\times22} \times \frac{56}{120} = \frac{10\times7}{13\times22\times15}$

$= \frac{7}{429}$

Sol.9 (c) E_1 → Selecting bag 1st

E_2 → Selecting bag 2nd

A → drawing a blue ball from selected bag Required Probability = $P(E_1) \times \left({}^{A}/_{E_1}\right) + P(E_2) \times P\left({}^{A}/_{E_2}\right)$

$= \frac{1}{2} \times \frac{6}{11} + \frac{1}{2} \times \frac{7}{10} = \frac{3}{11} + \frac{7}{20}$

$= \frac{60+77}{220} = \frac{137}{220}$

Sol.10 (d) $P(A) = {}^{1}/_{3}, P(B) = {}^{1}/_{5}, P(C) = {}^{1}/_{2}$

$\therefore P(A \cap B) = \frac{1}{3} \times \frac{1}{5} = {}^{1}/_{15}, P(B \cap C) = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

$P\ (A \cap C) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

$P(A \cap B \cap C) = \frac{1}{3} \times \frac{1}{5} \times \frac{1}{2} = \frac{1}{30}$

$\therefore P\ (A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{1}{3} + \frac{1}{5} + \frac{1}{2} - \frac{1}{15} - \frac{1}{10} - \frac{1}{6} + \frac{1}{30}$

$= \frac{10+6+15-2-3-5+1}{30} = \frac{22}{30} = \frac{11}{15}$

Sol.11 (d) A → Person aged 60

B → person aged 65

C → person aged 70

$P(A) = 0.7, P(B) = 0.4, P(C) = 0.2$

$P(A') = 0.3, P(B') = 0.6, P(C') = 0.8$

∴ Required Probability

$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A \cap B \cap C)$

$= 0.7 \times 0.4 \times 0.8 + 0.7 \times 0.6 \times 0.2 + 0.3 \times 0.4 \times 0.2 + 0.7 \times 0.4 \times 0.2$

$= 0.224 + 0.084 + 0.024 + 0.056$

= 0.388

Sol.12 (b) Required Probability

$= \frac{30}{100} \times \frac{75}{100} + \frac{70}{100} \times \frac{25}{100} = \frac{9+7}{40} = \frac{16}{40} = \frac{2}{5}$

= 0.4

Sol.13 (b) Let A → 1st transferred ball is red

B → 1ST transferred ball is white

E → 2nd ball is red

$P(E) = P(A) \times P({}^{E}/_{A}) + P(B) \times P({}^{E}/_{B})$

$= \frac{3}{8} \times \frac{5}{11} + \frac{5}{8} \times \frac{4}{11} = \frac{15+20}{88} = \frac{35}{88}$

Sol.14 (a) A → Failed in Physics

B → Failed in chemistry

$P(A) = \frac{30}{100}$

$P(B) = \frac{40}{100}$

$P(A \cup B) = \frac{50}{100}$

$P\ \left({}^{A'}/_{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{P(A \cup B) - P(A)}{P(B)} = \frac{\frac{50}{100} - \frac{30}{100}}{{}^{40}/_{100}}$

$= \frac{20}{100} \times \frac{100}{40} = {}^{1}/_{2}$

Sol.15 (c) Required probability
= 1 − P (Both defective)

$= 1 - \frac{8_{C_2} \times 2_{C_2}}{10_{C_4}}$

$= 1 - \frac{8\times7}{2\times1} \times 1 \times \frac{4\times3\times2\times1}{10\times9\times8\times7}$

$= 1 - \frac{2}{15} = {}^{13}/_{15}$

Sol.16 Every ball can be placed in

(B_1) or (B_2) or (B_3)

Means Total Possible outcome $= 3^8 = 6561$

Now, Possible outcomes are that the first bag contains three balls & the rest balls are in other bags.

$\binom{B,}{1,2,3}$ (B_2) (B_2)

$$\underset{\downarrow}{8_{C_3}} \times 1 \times 1 \times 1 \times 2 \times 2 \times 2 \times 2 \times 2$$

Selection of 3 balls for B_1 Bag

For 3 balls placement as these balls will go in B_1 only

Rest of balls have two options other B_2 or B_3

$8_{C_3} \times 1 \times 2^5 = 1{,}792$

$P = \frac{1792}{6561} = \mathbf{0.2731}$

Sol.17 (c) Required Probability

$= \frac{2!\times 6_{P_3} \times 4!}{8!} = \frac{2!\times 6!\times 4!}{3!\times 8!}$

$= \frac{2\times 6!\times 4\times 3!}{3!\times 8\times 7\times 6!} = \mathbf{{}^1/_7}$

Sol.18 (c) $P(A) = {}^1/_2, P(B) = {}^1/_3, P(A \cap B) = {}^1/_4$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6+4-3}{12} = \frac{7}{12}$

$Now\ P\left({}^{A'}/_{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{1-P(B)}$

$= \frac{1-P(A\cup B)}{1-P(B)} = \frac{1-\frac{7}{12}}{1-{}^1/_3} = \frac{5}{12} \times \frac{3}{2} = \frac{5}{8}$

Sol.19 (d) $n(S) = 4! = 24$

$n(E) = 2\times1\times1\times1 + 2\times1\times1\times1 + 2\times1\times1\times1 + 2\times1\times1\times1$

Last digits possible are 12, 24, 16, 64

$= 2+2+2+2 = 8$

$\therefore P(E) = \frac{8}{24} = \mathbf{{}^1/_3}$

Sol.20 (c)

X:	1	2	4	5	6	Total
P:	0.15	0.25	0.20	0.30	0.10	
XP	0.15	0.50	0.80	1.5	.60	3.55
X^2 P	0.15	1.0	3.2	7.5	3.6	15.45

Required S.D $= \sqrt{\sum X^2P - (\sum X\,P)^2}$

$= \sqrt{15.45 - (3.55)^2}$

$= \sqrt{15.45 - 12.6025}$

$= \sqrt{2.8475}$

= 1.69 (approx.)

Sol.21 (a) $P(X=0) = \frac{7_{C_4}}{10_{C_4}} = \frac{\frac{7\times6\times5}{3\times2}}{\frac{10\times9\times8\times7}{4\times3\times2}} = \frac{1}{6}$

$P(X=1) = \frac{7_{C_3}\times 3_{C_1}}{10_{C_4}} = \frac{7\times6\times5\times3\times4\times3\times2\times1}{3\times2\times1\times10\times9\times8\times7} = \frac{1}{2}$

$P(X=2) = \frac{7_{C_2}\times 3_{C_2}}{10_{C_4}} = \frac{21\times3\ \ \times4\times3\times2\times1}{10\times9\times6\times7} = \frac{3}{10}$

$P(X=3) = \frac{7_{C_1}\times 3_{C_3}}{10_{C_4}} = \frac{7\times1}{10\times9\times8\times7} \times 4 \times 3 \times 2 \times 1 = \frac{1}{30}$

∴ Required distribution is

X	0	1	2	3	Total
P (X)	$^1/_6$	$^1/_2$	$^3/_{10}$	$^1/_{30}$	
X P (X)	0	$^1/_2$	$^6/_{10}$	$^3/_{30}$	1.2

$E(X) = 0 + \frac{1}{2} + \frac{6}{10} + \frac{3}{30} = \frac{15+18+3}{30} = \frac{36}{30} = \mathbf{1.20}$

Sol.22 (c) Correct $E(X) = (1 - 0.2) \times 60 + (1 - 0.3) \times 70 + (1 - 0.1) \times 90$

$= 0.8 \times 60 + 0.7 \times 70 + 0.9 \times 90$

$= 48 + 49 + 81 = \mathbf{178}$

Sol.23 (d)

X	(R R) 40	R W 30	W W 20	Total
P (X)	$^2/_{15}$	$^8/_{15}$	$^5/_{15}$	
X P (X)	$\frac{80}{15}$	$\frac{240}{15}$	$\frac{100}{15}$	$\frac{420}{15}$

$E(X) = \frac{420}{15} = 28$

Sol.24 (b)

X	1	2	4	6	8	Total
P	k	2 k	3 k	3 k	k	
X P	k	4 k	12 k	18 k	8 k	43 k
$X^2 P$	k	8 k	48 k	108 k	64 k	229 k

$\sum P = 1$

$\Rightarrow 10\,k = 1 \Rightarrow k = {}^1/_{10}$

$\sum X\,P = 43 \times \frac{1}{10} = 4.3$

$\sum X^2\,P = 229 \times \frac{1}{10} = 22.9$

Variance of $X = \sum X^2 P - (\sum X\,P)^2$

$= 22.9 - (4.3)^2$

$= 22.90 - 18.49$

$= \mathbf{4.41}$

****

Probability
Exercise: Additional Questions

Sol.1 (b) All possible outcomes of a random experiment forms the sample space

Sol.2 (d) Equally likely events

Sol.3 (a) Mutually exclusive events

Sol.4 (b) $n\ (A) = 0$

$\therefore P(A) = 0$

Sol.5 (b) $n\ (S) = 52$

$n\ (E) = 13$

$\therefore P\ (E) = \frac{n\ (E)}{n\ (S)} = \frac{13}{52} = {}^{1}/_{4}$

Sol.6 (a) $n\ (S) = 52$

$n(E) = 4$

$P\ (E) = \frac{n\ (E)}{n\ (S)} = \frac{4}{52} = {}^{1}/_{13}$

Sol.7 (c) $n\ (S) = 52$

$n\ (E) = 1$
$\therefore P\ (E) = \frac{n\ (E)}{n\ (S)} = {}^{1}/_{52}$

Sol.8 (a) $n\ (S) = 6 \times 6 = 36$

$E = \{(1,4), (4,1), (2,3), (3,2)\}$

$n(\mathrm{E}) = 4$

$\therefore P\ (E) = \frac{n(E)}{n(S)} = \frac{4}{36} = {}^{1}/_{9}$

Sol.9 (b) $n\ (S) = 6 \times 6 = 36, n(E) = 5$

$\mathrm{E} = \{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)\}$

$$P(E) = \frac{n\ (E)}{n\ (S)} = \frac{\mathbf{5}}{\mathbf{36}}$$

Sol.10 (b) $S = \{HH, HT, TH, TT\}$
$n\ (S) = 4$

$n\ (E) = 2$

$\therefore P\ (E) = \frac{2}{4} = \mathbf{{}^{1}/_{2}}$

Sol.11 (b) $n\ (S) = \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\} = 8$

E = at least one head appears on upper face. $\{HHT, HTT, THT, HHH, HTH, THH, TTH\}$ =7

$n\ (E) = 7$ [∵ All cases except all there tail]
$\therefore P\ (E) = \frac{7}{8}$

Sol.12 (a) It is true

Sol.13 (b) $n\ (S) = 100$

$n\ (E) = 9$ [from the table we get, favourable to E]

$P(E) = {}^{9}/_{100}$

Sol.14 (c) $n(S) = 100, \quad n(E) = 20 + 9 = 29$

$\therefore P(E) = \frac{29}{100}$

Sol.15 (d) $n(S) = 100$

$n(E) = 8 + 35 + 18 = 61$

$\therefore P(E) = \frac{61}{100}$

Sol.16 (d)
$n(S) = 100, \quad n(E) = 0 + 10 + 8 = 18$

$P(E) = \frac{18}{100}$

Sol.17 (a) $n(S) = 1{,}000$

$n(E) = 60$

$P(E) = \frac{60}{1000}$

Sol.18 (a) It is true

Sol.19 (d) Let $A \rightarrow$ Spade

$P(A) = {}^{1}/_{4}$

$\therefore P(A') = 1 - P(A) = 1 - {}^{1}/_{4} = {}^{3}/_{4}$

Sol.20 (c) $n(E) = n(S)$

$\therefore P(E) = \frac{n(E)}{n(S)} = 1$

Sol.21 (d) Sum of all probabilities of mutually exclusive and exhaustive events is equal to 1

Sol.22 (b) $P(X_1) + P(X_2) + P(X_3) = 1$

Sol.23 (a) $P(X_1) = {}^{1}/_{4}, P(X_3) = {}^{1}/_{3}, P(X_2) = ?$

$\because P(X_1) + P(X_2) + P(X_3) = 1$

$\Rightarrow \frac{1}{4} + P(X_2) + \frac{1}{3} = 1$

$\Rightarrow P(X_2) = 1 - \left(\frac{1}{4} + \frac{1}{3}\right)$

$= 1 - \frac{7}{12} = {}^{5}/_{12}$

Sol.24 (b) $n(S) = 6 \times 6 = 36$

$n(E) = 3 \qquad [\because E = \{(4,6), (6,4), (5,5)\}]$

$P(E) = \frac{3}{36} = {}^{1}/_{12}$

Sol.25 (d) $n(S) = 6 \times 6 = 36$

$n(E) = 5$ E= {(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)}
$P(E) = {}^{5}/_{36}$

Sol.26 (a) It is true

Sol.27 (b) Exhaustive

Sol.28 (c) P (Sure event) =1

Sol.29 (d) Mutually exclusive, exhaustive and equal likely cases.

Sol.30 (a) $n(S) = 2 \times 2 = 4$ [∵ S = {H H, H T, T H, T T}]
$n(E) = 1$
$P(E) = {}^{1}/_{4}$

Sol.31 (b) $n(S) = 2 \times 2 = 4$

$n(E) = 2$ [∵ E = {H T, T H}]

$\therefore P(E) = {}^{2}/_{4}$

Sol.32 (c) $n(S) = 2 \times 2 = 4$

$n(E) = 1$

$\therefore P(E) = {}^{1}/_{4}$

Sol.33 (c) $n(S) = 2 \times 2 = 4$

Favourable outcomes = {(HT), (TH), (HH)} $n(E) = 3$ $\therefore P(E) = {}^{3}/_{4}$

Sol.34 (d) $n(S) = 2 \times 2 = 4$

$n(E) = 0$

$\therefore P(E) = \frac{0}{4} = \mathbf{0}$

Sol.35 (c) $n(S) = 2 \times 2 = 4$

$n(E) = 4$

$\therefore P(E) = \frac{4}{4} = \mathbf{1}$

Sol.36 (a) $n(S) = 2 \times 2 = 4,$ $n(E) = 2$

$\therefore P(E) = \frac{2}{4} = {}^{1}/_{2}$

Sol.37 (a) $n(S) = 6 \times 6 = 36$

$n(E) = 4$ [∵ E = {(2,6), (6,2), (4,3), (3,4)}]

$\therefore P(E) = \frac{4}{36}$

Sol.38 (c) $n(S) = {}^{11}C_1 = 11$

$n(E) = {}^{6}C_1$ $\therefore P(E) = \frac{6}{11}$

Sol.39 (b) $P(A \cup B) = P(A + B)$

Sol.40 (a) $P(A \cap B) = P(AB)$

Sol.41 (b) $P(A^C) = 1 - P(A)$

$\therefore P(A) + P(A^C) = \mathbf{1}$

Sol.42 (d) $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$
$\Rightarrow P(A + B) = \mathbf{P(A) + P(B)}$

Sol.43 (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A + B) = \mathbf{P(A) + P(B) - P(AB)}$

Sol.44 (c) $P(A \cap B) = P(A) \times P(B)$

$\Rightarrow P(AB) = \mathbf{P(A) \times P(B)}$

Sol.45 (b) $\because P(AB) = P(A) \times P(B) \Rightarrow P(A \cap B) = P(A) \times P(B)$

A & B are independent events

Sol.46 (a and b) $P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(B/A) = \frac{P(AB)}{P(A)}$

Sol.47 (b) $P(A) = 1/2, P(B) = \frac{1}{3}, P(AB) = 1/4$

$\therefore P(A + B) = P(A) + P(B) - P(AB)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6+4-3}{12} = \mathbf{\frac{7}{12}}$

Sol.48 (d) $P(A) = 1/2, P(B) = \frac{1}{3}, P(AB) = 1/4$

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(AB)}{P(B)} = \frac{1/4}{1/3} = \mathbf{3/4}$

Sol.49 (a) $P(A) = 1/3, P(B) = \mathbf{1/4}$

$\therefore P(A) \neq P(B)$

Sol.50 (d) A & B are independent

$\therefore P(A \cap B) = P(A) \times P(B)$

$\therefore P(A^c \cap B) = P(A^C) \times P(B)$

$P(A \cap B^C) = P(A) \times P(B^C)$

$P(A^C \cap B^C) = \mathbf{P(A^C) \times P(B^C)}$

Sol.51 (b) $A \rightarrow Ace$

$\therefore P(A) = 4/52 = 1/13$

$\therefore P(A^C) = 1 - 1/13 = \frac{12}{13}$

P (At least one Ace) = 1- P (none of two is ace)

$= 1 - P\,(A^C) \times P\,(A^C)$

$= 1 - \frac{12}{13} \times \frac{12}{13} = \frac{169-144}{169} = \frac{25}{169}$

Sol.52 (c) $S = \{1, 2, 3, 4, 5, 6\}$

Sol.53 (d) $\because$ P (A + B) = P (A) + P (B) − P (AB)

$\Rightarrow \frac{1}{2} = \frac{1}{4} + \frac{2}{5} - P\,(AB)$

$\Rightarrow P\,(AB) = \frac{13}{20} - \frac{1}{2} = \frac{13-10}{20} = \frac{3}{20}$

Sol.54 (a) $P\,(AB) = P\,(A) \times P(B) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

$\therefore P\,(A + B) = P\,(A) + P\,(B) - P\,(AB) = \frac{2}{3} + \frac{3}{5} - \frac{2}{5}$

$= \frac{10+9-6}{15} = \frac{13}{15}$

Sol.55 (b) $n = 100$

P = P (Getting a head) $= {}^{1}/_{2}$

$\therefore E\,(X) = np = 100 \times \frac{1}{2} = 50$

Sol.56 (a) $P\,(A + B) = P\,(A) + P\,(B) - P\,(AB)$

$\Rightarrow \frac{1}{2} = \frac{1}{3} + \frac{1}{4} - P\,(AB)$

$\Rightarrow P(AB) = \frac{7}{12} - \frac{1}{2} = \frac{1}{12}$

$\therefore P\,({}^{B}/_{A}) = \frac{P\,(AB)}{P\,(A)} = \frac{{}^{1}/_{12}}{{}^{1}/_{3}} = \frac{1}{12} \times \frac{3}{1} = \frac{1}{4}$

Sol.57 (c) Greater than equal to 0

Sol.58 (c) 1

Sol.59 (b) Probability density function

Sol.60 (b) $P\,(a_1) + P\,(a_2) + P\,(a_3) = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{2+1+3}{6} = \frac{6}{6} = 1$

Sol.61 (a) $P\,(a_1) + P\,(a_2) + P\,(a_3) = 0 + \frac{1}{3} + \frac{2}{3} = 1$

$\therefore S = \{a_1, a_2, a_3\}$

Sol.62 (c) $P\ (A \cap B) = P\ (A) \times P(B)\quad (\because A\&B\ are\ independent)$

$\therefore P\ (^{B}/_{A}) = \frac{P\ (A \cap B)}{P\ (A)} = \frac{P\ (A) \times P(B)}{P\ (A)} = P\ (B)$

Sol.63 (b) $[\because\ E(X) < 0]$

Sol.64 (d)

X	2	3	4	5	6	7	8	9	10	11	12	Total
P (X)	$^1/_{36}$	$^2/_{36}$	$^3/_{36}$	$^4/_{36}$	$^5/_{36}$	$^6/_{36}$	$^5/_{36}$	$^4/_{36}$	$^3/_{36}$	$^2/_{36}$	$^1/_{36}$	
X P (X)	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{12}{36}$	$\frac{20}{36}$	$\frac{30}{36}$	$\frac{42}{36}$	$\frac{40}{36}$	$\frac{36}{36}$	$\frac{30}{36}$	$\frac{22}{36}$	$\frac{12}{36}$	$\frac{252}{36} = 7$

$\therefore E(X) = 7$

Sol.65 (a) $P\ (A/B) = \frac{P\ (AB)}{P\ (B)} = \frac{^1/_{12}}{^1/_4} = \mathbf{^1/_3}$

Sol.66 (c) $P\ (B/A) = \frac{P\ (AB)}{P\ (A)} = \frac{^1/_{12}}{^2/_3} = \frac{1}{12} \times \frac{3}{2} = \mathbf{\frac{1}{8}}$

Sol.67 (d) Let $A \rightarrow$ One student passing a test

$B \rightarrow$ Another student passing a test

$\frac{P\ (A)}{P\ (\bar{A})} = \frac{3}{7} \qquad \therefore P\ (A) = \frac{3}{10}$

$\frac{P\ (\bar{B})}{P\ (B)} = \frac{3}{5} \quad \Rightarrow P\ (B) = \frac{5}{8}$

$\therefore P\ (A \cap B) = P\ (A) \times P(B) = \frac{3}{10} \times \frac{5}{8} = \mathbf{\frac{3}{16}}$

Sol.68 (b) $A \rightarrow$ One Student passing a test

$B \rightarrow$ Other student passing a test

$\frac{P\ (A)}{P\ (\bar{A})} = \frac{3}{7} \Rightarrow P\ (A) = \frac{3}{10}$

$\frac{P\ (\bar{B})}{P\ (B)} = \frac{3}{5} \Rightarrow P\ (B) = \frac{5}{8}$

$\therefore P\ (\bar{A} \cap \bar{B}) = P\ (\bar{A}) \times P\ (\bar{B}) = \frac{7}{10} \times \frac{3}{8} = \mathbf{\frac{21}{80}}$

Sol.69 (a) $\because \mathrm{P}\ (^{\mathrm{B}}/_{\mathrm{A}}) = \frac{\mathrm{P}\ (\mathrm{A \cap B})}{\mathrm{P}\ (\mathrm{A})},\ \mathrm{P}\ (\mathrm{A}) \neq 0$

$\therefore \boldsymbol{P\ (A) > 0}$

Sol.70 (b) Disjoint

Sol.71 (c) $n\ (S) = 20$

$n\ (E) = 10$

$P\ (E) = \frac{10}{20} = \mathbf{^1/_2}$

Sol.72 (c) $n\ (S) = 6 \times 6 = 36$

$n\ (E) = 6 \qquad [\because \mathrm{E} = \{(1{,}1),\ (2{,}2),\ (3{,}3),$
$(4{,}4),\ (5{,}5),\ (6{,}6)]$

$P\ (E) = \frac{6}{36} = \mathbf{^1/_6}$

Sol.73 (b) $n\ (S) = 6 \times 6 = 36, \quad E = \{(1{,}2),$
$(2{,}1),$
$(2{,}3), (3{,}2), (3{,}4), (4{,}3), (4{,}5), (5{,}4), (5{,}6), (6{,}5)\}$

$n\ (E) = 10$

$P\ (E) = \frac{10}{36} = \mathbf{\frac{5}{18}}$

Sol.74 (a) $n\ (S) = 6 \times 6 = 36,\ n(E) = 8$

$P\ (E) = \frac{8}{36} = \mathbf{\frac{2}{9}}$

Sol.75 (c) {(H,H), (T,T), (T,H), (H,T)}

Sol.76 (b) $n\ (S) = 3 + 8 = \mathbf{11}$

$n\ (E) = 3$

$P\ (E) = \mathbf{^3/_{11}}$

Sol.77 (a) $n\ (S) = 6 \times 6 = 36$

$n\ (E) = 36 - (1 + 2 + 3 + 4) = 36 - 10 = 26$

$P\ (E) = \frac{26}{36} = \frac{13}{18}$

Sol.78 (b) $n\ (S) = 1000$

$n\ (E) = 600$

$P\ (E) = \frac{600}{1000} = {}^{3}/_{5}$

Sol.79 (c) $n\ (S) = \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\} = 8$

E = the event that all three tails occur {TTT} = 1

$n\ (E) = 1$

$P\ (E) = {}^{1}/_{8}$

Sol.80 (b) $n\ (S) = \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\} = 8$

E= the event that exactly two heads occur = {(HHT), (THH), ((HTH)} = 3

$n\ (E) = 3 \quad \therefore P\ (E) = {}^{3}/_{8}$

Sol.81 (a) $n\ (S) = \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\} = 8$

E = the event that at least two head occur = {(HHT), (THH), (HTH), (HHH)} = 4

$n\ (E) = 4$

$P\ (E) = {}^{4}/_{8} = {}^{1}/_{2}$

Sol.82 (b) $n\ (S) = 2 \times 2 \times 2 \times 2 = 16$

Favourable outcomes = {(HHTT), (THHT), (TTHH), (HTTH), (HTHT), (THTH)}

$P\ (E) = \frac{6}{16} = {}^{3}/_{8}$

Sol.83 (b) $n\ (S) = 2^4 = 16$

Favourable outcomes = {(HHTT), (THHT), (TTHH), (HTTH), (HTHT), (THTH)}

$\therefore P\ (E) = \frac{6}{16} = {}^{3}/_{8}$

Sol.84 (b) $P\ (A') = 1 - P\ (A)$

Sol.85 (a) $P\ (A') = 1 - P\ (A) = 1 - {}^{3}/_{8} = {}^{5}/_{8}$

Sol.86 (c) $P\ (\bar{B}) = 1 - P\ (B) = 1 - {}^{1}/_{3} = {}^{2}/_{3}$

Sol.87 (b) $P\ (A + B) = P\ (A) + P(B) - P(AB)$

$= \frac{3}{8} + \frac{1}{3} - \frac{1}{4} = \frac{9+8-6}{24} = \frac{11}{24}$

Sol.88 (d) $\because$ A&B are mutually exclusive

$\therefore P\ (A \cap B) = 0 \Rightarrow P\ (AB) = 0$

Sol.89 (b) $n\ (S) = 6$

$n\ (E) = 2 \qquad \therefore P\ (E) = \frac{2}{6} = {}^{1}/_{3}$

Sol.90 (a) $n\ (S) = 52, \quad n\ (E) = 4 + 4 = 8$

$P\ (E) = {}^{8}/_{52} = {}^{2}/_{13}$

Sol.91 (c) $n\ (S) = 6 \times 6 = 36$

Favourable outcomes = {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (5, 6), (6, 5)}

$n\ (E) = 6 + 2 = 8$

$P\ (E) = {}^{8}/_{36} = {}^{2}/_{9}$

Sol.92 (a) $P\ (A) = {}^{1}/_{6},\ P\ (B) = {}^{1}/_{4}$

P (one of the horse win) $= \frac{1}{6} + \frac{1}{4} = \frac{2+3}{12} = {}^{5}/_{12}$

Sol.93 (b) $P\ (\overline{A + B}) = 1 - P\ (A + B)$

$= 1 - \left(\frac{1}{6} + \frac{1}{4}\right) = 1 - \frac{5}{12} = \frac{7}{12}$

Sol.94 (d) $P\ (A') = 1 - P\ (A) = 1 - \frac{7}{8} = {}^{1}/_{8}$

Sol.95 (c) $P\ (S) = \frac{n\ (S)}{n\ (S)} = 1$

Sol.96 (b) P (Bird not killed) =1- P (Bird killed)

$= 1 - {}^{1}/_{3} = {}^{2}/_{3}$

Sol.97 (c) $n\,(S) = 10,\ n\,(E) = 9$

$P\,(E) = \mathbf{{}^{9}/_{10}}$

Sol.98 (a) Required Probability = 1- (Ship does not return safely)

$= 1 - {}^{9}/_{10} = \mathbf{{}^{1}/_{10}}$

Sol.99 (b) Required expectation = $\frac{6}{11} \times 77$

=₹ 42

Sol.100 (d) $S = \{BB,\ BG,\ GB,\ GG\}$

Total outcomes = {BB, BG, GB}, Favourable outcomes = {BB}

$P = \mathbf{\frac{1}{3}}$

Sol.101 (c) A → only even nos. occur= {2, 4, 6}

B → Number greater than 2 = {4, 6}

$P\,({}^{B}/_{A}) = \mathbf{{}^{2}/_{3}}$

Sol.102 (a) A → red card

B → king

$P\,(A) = {}^{5}/_{7}$

$P\,(A \cap B) = \frac{2}{7}$

$P\,({}^{B}/_{A}) = \frac{P\,(A \cap B)}{P\,(A)} = \frac{{}^{2}/_{7}}{{}^{5}/_{7}} = \mathbf{{}^{2}/_{5}}$

Sol.103 (b) Let M → Mathematics & B → Biology

$P\,(M) = \frac{40}{100}, P(B) = {}^{25}/_{100}, P\,(M \cap B) = {}^{15}/_{100}$

$\therefore P\,({}^{M}/_{B}) = \frac{P\,(M \cap B)}{P\,(B)} = \frac{{}^{15}/_{100}}{{}^{25}/_{100}} = \frac{15}{25} = \mathbf{{}^{3}/_{5}}$

Sol.104 (c) Let M → Mathematics & B → Biology

$P\,(M) = {}^{40}/_{100}, P(B) = {}^{25}/_{100}, P\,(M \cap B) = {}^{15}/_{100}$

$\therefore P\,({}^{M}/_{B}) = \frac{P\,(M \cap B)}{P\,(M)} = \frac{{}^{15}/_{100}}{{}^{40}/_{100}} = \frac{15}{40} = \mathbf{{}^{3}/_{8}}$

Sol.105 (a) $n\,(S) = 6, n(E) = 3,\ P\,(E) = \frac{3}{6} = \mathbf{{}^{1}/_{2}}$

Sol.106 (a) P (A) = P (sure event) =**1**

Sol.107 (c) Impossible

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Theoretical Distribution Exercise: Set-B

Sol.1 (d) $n = 48$

$p = 0.75$

$q = 1 - p = 1 - 0.75 = 0.25$

$\therefore\ \sigma = \sqrt{n\,p\,q} = \sqrt{48 \times 0.75 \times 0.25}$

$= \sqrt{9} = \mathbf{3}$

Sol.2 (b) $\frac{\mu}{n} = {}^{1}/_{2}$

$\Rightarrow \mu = \frac{1}{2}\,n = \frac{1}{2} \times 20 = \mathbf{10}$

Sol.3 (b) When the variance is the greatest

$\therefore p = q = {}^{1}/_{2}$

$\sigma^2 = npq = 16 \times \frac{1}{2} \times \frac{1}{2} = \mathbf{4}$

Sol.4 (b) $(n + 1)\,p = (15 + 1) \times \frac{1}{3} = \frac{16}{3} = 5.33$

Which is fraction

$\therefore$ Mode = [5.33] = **5**

{∵ Greatest integer of 5.33 = 5}

Sol.5 (d) $\mu = 3 \Rightarrow np = 3$

$\sigma = 1.5 \Rightarrow \sigma^2 = (1.5)^2$

$\Rightarrow npq = 2.25 = {}^{9}/_{4}$

$\Rightarrow 3 \times q = {}^{9}/_{4} \Rightarrow q = {}^{3}/_{4}$

$\therefore p = 1 - \frac{1}{q} = 1 - {}^{3}/_{4} = {}^{1}/_{4}$

$\therefore n \times \frac{1}{4} = 3$

$\Rightarrow \mathbf{n = 12}$

Sol.6 (c) $n = 6$

$p = {}^{1}/_{2}, q = {}^{1}/_{2}$

$P(X = 3) = 6_{C_3} p^3 q^3$

$= \frac{6\times5\times4}{3\times2\times1} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3$

$= 20 \times \frac{1}{8} \times \frac{1}{8} = \frac{5}{16} = \mathbf{0.3125}$

Sol.7 (b) n = 4

$p = \frac{60}{100} = 0.6$

$q = 1 - p = 0.4$

$p(X \geq 1) = 1 - P(X = 0)$

$= 1 - 4_{C_0} p^0 q^4 = 1 - (0.4)^4$

$= 1 - 0.0256 = \mathbf{0.9744}$

Sol.8 (a) n = 5

$p = {}^{1}/_{2}$

$q = {}^{1}/_{2}$

$P(X = 3) = 5_{C_3} p^3 q^2 = \frac{5\times4}{2\times1} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

$= 10 \times \frac{1}{8} \times \frac{1}{4} = \frac{5}{16} = \mathbf{0.3125}$

Sol.9 (d) $\sigma = 2 \Rightarrow \sigma^2 = 2^2$

$\Rightarrow m = 4$ (∵ In Poisson distribution Mean = Variance = m)

Now $P(1.5 < X < 2.9)$

$= P(X = 2) = \frac{e^{-m} m^2}{2!}$

$= \frac{e^{-4}\times4^2}{2} = 8\, e^{-4} = 8 \times 0.018$ (approx)

$= \mathbf{0.144}$

Sol.10 (c) Here, the mean of the Poisson distribution is 1, So m = 1

$P(X = x) = \frac{e^{-m} \times m^x}{x!}$

$P(X = 0) = \frac{e^{-1} \times 1^0}{0!} = e^{-1} = \frac{1}{2.71282} = 0.3686$

$\therefore P(X \geq 1) = 1 - P(X = 0)$

$= 1 - 0.36788 = \mathbf{0.63212}$

Sol.11 (b) $C.V. = 50 \Rightarrow \frac{\sigma}{\mu} \times 100 = 50$

$\Rightarrow \frac{\sigma}{\mu} = \frac{1}{2}$

$\Rightarrow \mu = 2\,\sigma$

$\Rightarrow m = 2\sqrt{m} \Rightarrow \sqrt{m} = 2$

(∵ Mean = m & Variance = m)

$\Rightarrow m = 4$

P (non-zero)

$= 1 - P(X = 0)$

$= 1 - e^{-m} = 1 - e^{-4}$

$= 1 - 0.018 = \mathbf{0.982}$

Sol.12 $m = \mu = np = 200 \times \frac{1.5}{100} = 3$

$\therefore P(X = 0) = e^{-m} = e^{-3} = 0.04979$

$= \mathbf{0.05}$

Sol.13 (c) $P(X = 1) = P(X = 2)$

$\Rightarrow \frac{e^{-m}\times m^1}{1!} = \frac{e^{-m}\times m^2}{2!}$

$\Rightarrow m = 2$

$\therefore mean\ of\ X = \mathbf{m = 2}$

Sol.14 (b) n = 100, $p = \frac{1}{100}$

$m = \mu = np = 100 \times \frac{1}{100} = 1$

$P(X = 2) = \frac{e^{-m}\times m^2}{2!} = \frac{e^{-1}\times 1^2}{2} = \frac{e^{-1}}{2}$

$= \frac{1}{2} \times 0.36788 = 0.18394 = \mathbf{0.184}$

Sol.15 (a) $f(2) = 3f(4)$

$\Rightarrow \frac{e^{-m} \times m^2}{2!} = 3 \times \frac{e^{-m} \times m^4}{4!}$

$\Rightarrow m^2 = \frac{4!}{2! \times 3} = \frac{4 \times 3!}{3!} = 4$

$\Rightarrow m = 2$

$\therefore$ Variance of $X = \mathbf{m = 2}$

Sol.16 (c) $f(x) = \frac{1}{4\sqrt{2\pi}} e^{-(x-10)^2/32}$

But $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2/(\sigma)^2}$

Comparing these two, we have u =10

$\& \; 2\sigma^2 = 32 \Rightarrow \sigma^2 = 16$

$\therefore co-efficient\ of\ variance = \frac{\sigma}{\mu} \times 100$

$= \frac{4}{10} \times 100 = \mathbf{40}$

Sol.17 (c) $f(x) = \frac{1}{\sqrt{72\pi}} e^{-(x-10)^2/72}$

$= \frac{1}{6\sqrt{2\pi}} e^{-(x-10)^2/2\times 6^2}$

$\therefore \mu = 10$

$\& \; \sigma = 6$

$\therefore$ 1st quartile $(Q_1) = \mu - 0.675\sigma$

$= 10 - 0.675 \times 6$

$= 10 - 4.05 = \mathbf{5.95}$

Sol.18 (d) $Q_1 = 14.6 \Rightarrow \mu - 0.675\sigma = 14.6$___$(I)$

$Q_3 = 25.4 \Rightarrow \mu + 0.675\sigma = 25.4$____$(II)$

From $[(II - (I)]$ $2 \times 0.675\sigma = 10.8$

$\Rightarrow \sigma = \frac{10.8}{2 \times 0.675} = \mathbf{8}$

Sol.19 (b) $\because$ M. D. = 16 $\Rightarrow 0.8\sigma = 16$

$\Rightarrow \sigma = \frac{16}{0.8} = 20$

$\therefore Q.D. = 0.675\,\sigma = 0.675 \times 20$

= 13.50

Sol.20 $\mu - \sigma = 40$______(I)

$\mu + \sigma = 60$_____(II)

From $[(II) - (I)]$ $2\sigma = 20$

$\Rightarrow \sigma = 10$

$\therefore M.D = 0.8 \times \sigma = 0.8 \times 10 = \mathbf{8}$

Sol.21 (d) $Q.D = 4.05 \quad \Rightarrow 0.675 \times \sigma = 4.05$

$\Rightarrow \sigma = \frac{4.05}{0.675} = 6$

$\therefore M.D. = 0.8 \times \sigma = 0.8 \times 6$

= 4.8

Sol.22 (a) $Q_1 = 13.25 \quad \Rightarrow \mu - 0.675\sigma = 13.25$_____$(I)$

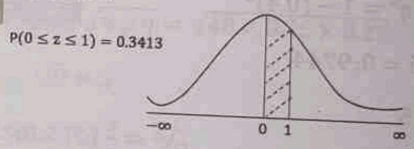
$M.D. = 8 \quad \Rightarrow 0.8\,\sigma = 8 \Rightarrow \sigma = 10$

$\therefore \mu = 13.25 + 0.675 \times 10$

= 20

$\therefore$ Mode = Mean = μ = **20**

Sol.23 (b)



But $P(-\infty < z < 0) = 0.5$

$\therefore \varphi = P(Z \le 1) = 0.5 + 0.3413$

= 0.8413

Sol.24 (c) Required mean $= \mu_1 + \mu_2 = 10 + 12 = 22$

$\& \; SD = \sqrt{\sigma_1^2 + \sigma_2^2}$

$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

= 5

****

Theoretical Distribution
Exercise: Set-C

Sol.1 (d) n = 10

$p = {}^1/_8$

$q = 1 - \frac{1}{8} = \frac{7}{8}$

$P\ (X \geq 2) = 1 - \ [P\ (X = 0) + P(X = 1)]$

$= 1 - \left(10_{C_0}\, p^0\, q^{10} + 10_{c_1} \times P^1 \times q^9\right)$

$= 1 - \left[\left(\frac{7}{8}\right)^{10} + 10 \times \frac{1}{8} \times \left(\frac{7}{8}\right)^9\right]$

$= 1 - \left(\frac{7}{8}\right)^9 \left(\frac{7}{8} + \frac{10}{8}\right)$

$= 1 - \frac{17}{8} \times \left(\frac{7}{8}\right)^9 = 1 - 2.125 \times 0.30066$ (aprrox)

$= 1 - 0.6389 = \mathbf{0.3611}$

Sol.2 (b) $\mu = \frac{10}{3} \Rightarrow np = \frac{10}{3}$____$(I)$

Also $2P\ (X = 2) = P\ (X = 3)$

$\Rightarrow 2 \times n_{C_2} p^2\, q^{n-2} = n_{C_3}\, p^3\, q^{n-3}$

$\Rightarrow p = \frac{2\ n_{C_2}}{n_{C_3}} q$

$\Rightarrow p = 2 \times \frac{n\,!}{2\,! \times (n-2)!} \times \frac{3\,! \times (n-3)!}{n\,!}\ q$

$\Rightarrow p = \frac{2 \times 3 \times 2\,! \times (n-3) 1\ q}{2\,! \times (n-2)\ (n-3)!}$

$\Rightarrow (n-2)p = 6q$

$\Rightarrow \frac{10}{3} - 2p = 6(1-p)$

$\Rightarrow 4p = 6 - \frac{10}{3}$

$\Rightarrow 4p = \frac{8}{3}$

$\Rightarrow p = {}^2/_3 \quad \therefore q = 1 - {}^2/_3 = {}^1/_3$

$\therefore n \times \frac{2}{3} = \frac{10}{3} \Rightarrow n = 5$

$\therefore P\ (X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 5_{C_0}\, p^0\, q^5 + 5_{C_1}\, p\, q^4 + 5_{C_2} p^2\, q^3$

$= \left(\frac{1}{3}\right)^5 + 5 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^4 + 10 \times \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3$

$= \frac{1+10+40}{243} = \frac{51}{243} = \frac{17}{81}$

Sol.3 (c) n = 8

$P = {}^1/_3$

$q = 1 - {}^1/_3 = {}^2/_3$

$P\ (X \geq 5) = P\ (X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$

$= 8_{C_5}\, p^5\, q^3 + 8_{C_6}\, p^6\, q^2 + 8_{C_7}\, p^7 q + 8_{C_8}\, p^8\, q^0$

$= 56 \times \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^3 + 28 \times \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^2 +$
$8 \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right) + 1 \times \left(\frac{1}{3}\right)^8 \times 1$

$= \frac{56 \times 8 + 28 \times 4 + 8 \times 2 + 1}{3^8}$

$= \frac{577}{6561}$

$\therefore$ Required number of enumerators

$= \frac{577}{6561} \times 1000$

= 88

Sol.4 (c) $\mu = 5 \Rightarrow np = 5$_____$(I)$

$10\ P(X = 0) = \ P(X = 1)$

$\Rightarrow 10 \times n_{C_0}\, p^0\, q^n = n_{C_1} \times p\, q^{n-1}$

$\Rightarrow 10\ q^n = np\ q^{n-1}$

$\Rightarrow 10q^n = 5\ q^{n-1}\ [From\ (I)]$

$\Rightarrow q = \frac{5}{10} = {}^1/_2$

$\therefore p = 1 - q = 1 - {}^1/_2 = {}^1/_2$

$\therefore n = 10$

Now $P\left(X \geq {}^1/_X > 0\right) = \frac{P\ (X \geq 1 \cap X > 0)}{P\ (X > 0)}$

$= \frac{P\ (X \geq 1)}{P\ (X \geq 1)} = \mathbf{1}$

Sol.5 (d) P (at least one boy & one girl) = P (one boy & 3 Girls) + P (2 boys & 2 Girls) +p(3 boys &1 girl)

$$=\frac{4\,!}{3\,!}\times\frac{1}{2}\times\left(\frac{1}{2}\right)^3+\frac{4\,!}{2\,!\times 2\,!}\times\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^2+\frac{4\,!}{3\,!}\times\left(\frac{1}{2}\right)^3\times\frac{1}{2}$$

$$=\frac{4}{16}+\frac{6}{16}+\frac{4}{16}=\frac{14}{16}=\frac{7}{8}$$

$\therefore$ Required numbers of families

$$=\frac{7}{8}\times 128=\mathbf{112}$$

Sol.6 (a) When n = 10

$$P\,(X=5)=2\;\;P(X=4)$$

$$\Rightarrow\;10_{C_5}\,p^5\,q^5=2\times 10_{C_4}\,p^4\,q^6$$

$$\Rightarrow p=\frac{2\times 10_{C_4}}{10_{C_5}}\,q=2\times\frac{10\,!}{4\,!\times 6\,!}\times\frac{5\,!\times 5\,!}{10!}\times q$$

$$\Rightarrow p=\frac{2\times 5}{6}\,q={}^{5}\!/_{3}\,q$$

$$\Rightarrow\frac{p}{q}={}^{5}\!/_{3}$$

$$\therefore p=\frac{5}{8}\;\&\;q={}^{3}\!/_{8}$$

Now,

If $n=8,\;p={}^{5}\!/_{8}\;\&\;q={}^{3}\!/_{8}$

The $P\,(X=2)=\;8_{C_2}\,p^2\,q^6$

$$=28\times\left(\frac{5}{8}\right)^2\left(\frac{3}{8}\right)^6=\mathbf{0.0304\ (approx)}$$

Sol.7 (d) $\therefore m=\bar{X}=\frac{0\times16+1\times25+2\times32+3\times17+4\times10}{16+25+32+17+10}$

$$=\frac{0+25+64+51+40}{100}=\frac{180}{100}=1.8$$

$$f\,(X)=\frac{e^{-m}\,m^X}{X\,!}$$

$$\therefore f\,(2)=\frac{e^{-1.8}\times(1.8)^2}{2\,!}=\frac{0.1653\times 3.24}{2}$$

$=0.267786$ $\quad(e^{-1.8}=0.36788\times 0.4493=0.1653\,(approx))$

$\therefore$ expected frequency for $X=2$

$$=N\,f(X)=100\times 0.267786$$

$$=27$$

$$f\,(3)=\frac{e^{-1.8}\times(1.8)^3}{3\,!}=0.1606716$$

$\therefore$ Expected frequency for $X=3$

$$=0.1606716\times 100=16$$

$$f\,(4)=\frac{e^{-1.8}\times(1.8)^4}{4\,!}=0.07230222$$

$\therefore$ Expected frequency for $X=4$

$$=100\times 0.07230222=8$$

$\therefore$ Sum of the expected frequencies for $X=2,3 and\ 4$

$$=27+16+8=\mathbf{61}$$

Sol.8 (b) When $X=50\ then\ z=\frac{X-\mu}{\sigma}=\frac{50-50}{10}=0$

When $X=60\ then\ z=\frac{60-50}{10}=1$

$$\therefore P\,(X\le 60/x>50)\;=P\,(Z\le 1/z>0)$$

$$=\frac{P\,(0<z\le1)}{P\,(z>0)}=\frac{P\,(0<z\le1)}{P\,(z>0)}$$

$$=\frac{0.3413}{0.5}=\mathbf{0.6826}$$

Sol.9 (c) $\because$ 9 P(X = 4) + 90 P(X = 6) = P(X = 2)

$$\Rightarrow 9\,\frac{e^{-m}\,m^4}{4\,!}+90\,\frac{e^{-m}\,m^6}{6\,!}=\frac{e^{-m}\,m^2}{2\,!}$$

$$\Rightarrow\;\frac{9\,m^2}{4\,!}+\frac{90\,m^4}{6\,!}=\frac{1}{2\,!}$$

$$\Rightarrow\frac{9\,m^2}{24}+\frac{90\,m^4}{720}=\frac{1}{2}$$

$$\Rightarrow\;\frac{3\,m^2+m^4}{8}=\frac{1}{2}$$

$$\Rightarrow\;m^4+3\,m^2-4=0$$

$$\Rightarrow\;(m^2+4)\;\;(m^2-1)=0$$

$$\Rightarrow\;m^2-1=0\qquad(\because\;m^2+4\neq 0)$$

$$\Rightarrow m=1$$

$$\therefore P\,(X\le 1)=\;\;P(X=0)+P(X=1)$$

$$=e^{-m}+\frac{e^{-m}.m}{1\,!}$$

$$=e^{-1}+e^{-1}=2\;e^{-1}=2\times 0.36788$$

$$=\mathbf{0.73576}$$

Sol.10 (c) $C.V. = 50 \Rightarrow \frac{\sigma}{\mu} \times 100 = 50$

$\Rightarrow \frac{\sigma}{\mu} = {}^{1}/_{2}$

$\Rightarrow \frac{\sigma^2}{\mu^2} = {}^{1}/_{4} \Rightarrow \frac{m}{m^2} = {}^{1}/_{4}$

$\Rightarrow \frac{1}{m} = {}^{1}/_{4} \Rightarrow m = 4$

$\therefore P\,(X > 1/x > 0)$

$= \frac{P\,(X>1 \cap X>0)}{P\,(X>0)} = \frac{1-[P\,(X=0)+P(X=1)]}{1-P(X=0)}$

$= \frac{1-(e^{-m}+m\,e^{-m})}{1-e^{-m}}$

$= \frac{1-(m+1)e^{-m}}{1-e^{-m}} = \frac{1-5\,e^{-4}}{1-e^{-4}}$

$= \frac{1-5\times0.01832}{1-0.01832} = \frac{1-0.0916}{0.98168}$

$= \frac{0.9084}{0.98168} = 0.925352$

$= \mathbf{0.9254}$ **(approx)**

Sol.11 (a) $m = np = 200 \times \frac{1}{100} = 2$

$P\,(X > 3) = 1 - \;[P\,(X = 0) + P\,(X = 1) + P\,(X = 2) + P\,(X = 3)]$

$= 1 - \;\left[e^{-m} + \frac{e^{-m}.m}{1\,!} + \frac{e^{-m}.m^2}{2\,!} + \frac{e^{-m}.m^3}{3\,!}\right]$

$= 1 - e^{-2}\left[1 + 2 + \frac{2^2}{2} + \frac{2^3}{6}\right]$

$= 1 - e^{-2}\,(1 + 2 + 2 + 1.3333)$

$= 1 - 0.13534 \times 6.3333$

$= 0.1428$ (approx)

Sol.12 (c) $m = 1.20$

$P\,(X > 2) = 1 - \;[P\,(X = 0) + P\,(X = 1) + P\,(X = 2)]$

$= 1 - \left(e^{-m} + e^{-m}.\,m + \frac{e^{-m}.m^2}{2\,!}\right)$

$= 1 - e^{-1.20}\left[1 + 1.2 + \frac{(1.2)^2}{2}\right]$

$= 1 - 0.3012 \times 2.92 = 1 - 0.879504$

= 0.12 (approx)

Sol.13 (b) $m = \bar{X} = \frac{0\times76+1\times74+2\times29+3\times17+4\times3+5\times1}{76+74+29+17+3+1}$

$= \frac{0+74+58+51+12+5}{200}$

$= \frac{200}{200} = 1$

$f\,(0) = e^{-m} = e^{-1} = 0.36788$

The expected frequency for $X = 0$ is

$= N\,f(x) = 200 \times 0.36788$

$= 74$

$f\,(1) = e^{-m}\,\frac{m}{1\,!} = e^{-1} \times 1 = 0.36788$

$\therefore$ Expected frequency for $X = 1$ is

$= 200 \times f(1) = 200 \times 0.36788$

$= 73.576 = 73$

$f\,(2) = e^{-m}\,\frac{m^2}{2\,!} = \frac{e^{-1}}{2} = \frac{0.36788}{2} = 0.18394$

$\therefore$ expected frequency for $X = 2$ is

$= 200 \times f\,(2) = 200 \times 0.18394$

$= 36.788 = 37$

$\therefore$ Required sum of expected frequency for $X =$ 0,1,2

$= 74 + 73 + 37 = 184$

Or

Sum of expected frequencies

for $X = 0, 1$ *and* 2

$= N\,f(0) + N\,f(1) + N\,f(2)$

$= N\,[f(0) + f(1) + f(2)]$

$= N\,\left[e^{-m} + m\,e^{-m} + \frac{m^2}{2}\,e^{-m}\right]$

$= 200\,\left(e^{-1} + 1 \times e^{-1} + \frac{1}{2}\,e^{-1}\right)$

$= 200 \times \frac{5}{2}\,e^{-1}$

$= 500 \times 0.36788$

$= \mathbf{183.94} \approx \mathbf{184}$

Sol.14 (a) $m = 2$

$P\ (X \geq 3) = 1 - [P\ (X = 0) + P\ (X = 1) + P\ (X = 2)]$

$= 1 - \left(e^{-m} + \frac{e^{-m} \times m}{1!} + \frac{e^{-m}.m^2}{2!}\right)$

$= 1 - e^{-m}\left(1 + m + \frac{m^2}{2}\right)$

$= 1 - e^{-2}\ (1 + 2 + 2)$

$= 1 - 0.13534 \times 5 = 1 - 0.6767$

$= 0.3233$

$\therefore$ Required nos. of drivers

$= 500 \times 0.3233$

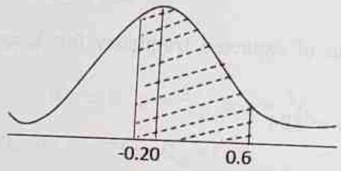
$= \mathbf{161.65 \approx 162}$

Sol.15 (b) $\mu = 50, \sigma = 20$

$z_1 = \frac{46-50}{20} = \frac{-4}{20} = -0.20$

$z_2 = \frac{62-50}{20} = \frac{12}{20} = 0.60$

$P\ (z_1 \leq z \leq z_2) = P\ (-0.20 \leq z \leq 0.60)$



$= P\ (0 \leq z \leq 0.20) + P\ (0 \leq z \leq 0.6)$

$[\because\ P\ (-0.20 \leq z \leq 0) = P(0 \leq z \leq 0.20)]$

$= 0.0793 + 0.2257$

$= 0.305$

$\therefore$ Required Number student weighing between 46 kg and 62 kg

$= 0.305 \times 800 = \mathbf{244}$

Sol.16 (a) $\mu = 10{,}000,\ \sigma = 2{,}000$

If $X = 14{,}000$

$\therefore z = \frac{14000-10000}{2000}$

$= \frac{4{,}000}{2{,}000} = 2$

$P\ (Z > 2) = 0.5 - P(0 \leq Z \leq 2)$

$0.5 - 0.4772 = 0.0228$

Now

$N \times P\ (Z > 2) = 50$

$\Rightarrow N \times 0.0228 = 50$

$\Rightarrow N = \frac{50}{0.0228} = 2{,}192.982456$

$\approx \mathbf{2,193}$

Sol.17 (c) $\mu = 500,\ \sigma = 120$

$P\ (500 \leq x \leq k) = 40.32\%$

$\Rightarrow P\ \left(0 \leq Z \leq \frac{K-500}{120}\right) = 0.4032$

$\Rightarrow P\ \left(z \leq \frac{k-500}{120} = 0.5 + .4032\right)$

$= 0.9032$

$= z =\ (1.30)$

$\Rightarrow P\ \left(z \leq \frac{k-500}{120}\right) =\ P\ (z \leq 1.30)$

$\Rightarrow \frac{k-500}{120} = 1.30$

$\Rightarrow k = 500 + 1.30 \times 120$

$= 500 + 156 = 656$

Sol.18 (b) X follows normal distribution with $\mu = 1800\ and\ \sigma = 300$

$\phi(1) = 0.84$

$P(-\infty < z < 1) = 0.84$

So, $P(0 < z < 1) = 0.84 - 0.50 = 0.34$

Family has weekly food expenditure in excess of ₹ 1800 is given but the answer is calculated with ₹ 2100.

So, we will change the question to ₹2100 and solve, then only we have to use $\phi(1)$

Let P is the probability of success, that is possibility of expenses more then ₹2100.

$P(x > 2100)$

$$P\left(\frac{x-1800}{300} > \frac{2100-1800}{300}\right)$$

$P(z > 1) = 1 - P(z < 1) = 1 - 0.84 = 0.16$

So, p= 0.16 and q = 1 - p = 1-0.16 =0.84

Let y be the variable that selecting family have expense then 2100. It will follow Binomial distribution with n=5, p=0.16 and q=0.84

Required probability is at least one family have expenses more then 2100.

$P(y \geq 1) = 1 - P(y = 0) = 1 - {}^5C_0\,(0.16)^0(0.84)^5$

$1 - 0.418 = 0.582$

Sol.19 (c) $\mu = 700, \sigma = 50$

$\because z = \frac{X-\mu}{\sigma}$

$\therefore z_1 = \frac{660-700}{50} = \frac{-40}{50} = -0.8$

$z_2 = \frac{720-700}{50} = \frac{20}{50} = 0.4$

$P\,(-0.8 < z < 0.4) = P\,(-0.8 < z \leq 0) + P(0 \leq z < 0.4)$

$= P\,(0 \leq z < 0.8) + P\,(0 \leq z < 0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$

$\therefore$ Required expected number of workers = $5{,}000 \times 0.4435 = 2217.5$

$\approx$ **2218**

Sol.20 (a) $P\,(X \geq 60) = \frac{50}{100}$

$\Rightarrow P\left(z \geq \frac{60-\mu}{\sigma}\right) = 0.5$____$(I)$

and $P\,(X \leq 55) = \frac{10}{100}$

$\Rightarrow P\left(z \leq \frac{55-\mu}{\sigma}\right) = 0.1$____$(II)$

From equation (I)

$0.5 - P\left(0 \leq z \leq \frac{60-\mu}{\sigma}\right) = 0.5$

$\Rightarrow P\left(0 \leq z \leq \frac{60-\mu}{\sigma}\right) = 0$

$\therefore \frac{60-\mu}{\sigma} = 0 \Rightarrow \mu = 60$

Now from equation (II)

$0.5 - P\left(\left(\frac{55-\mu}{\sigma}\right) \leq z \leq 0\right) = 0.1$

$\therefore P\left(\left(\frac{55-\mu}{\sigma}\right) \leq z \leq 0\right) = 0.4$

$\Rightarrow P\left(0 \leq z \leq -\left(\frac{55-\mu}{\sigma}\right)\right) = 0.4$

$\therefore -\left(\frac{55-\mu}{\sigma}\right) = 1.28$

$\Rightarrow -(55 - 60) = 1.28\,\sigma$

$\Rightarrow \sigma = \frac{5}{1.28} \Rightarrow \sigma = 3.90$ (approx)

$\therefore variance = \sigma^2 = (3.9)^2 = \mathbf{15.21}$

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Theoretical Distribution Exercise: Additional Question

Sol.1 (c) Binomial Distribution

Sol.2 (a) Number of trials of the experiment

Sol.3 (b) Discrete

Sol.4 (b) Binomial Distribution

Sol.5 (a) Success

Sol.6 (b) When $P = 0.5 = q$

then the binomial distribution is symmetrical

Sol.7 (a) $P > 0.5 \;\; \therefore P > q$, then the binomial distribution asymmetrical.

Sol.8 (b) Mean = np

Sol.9 (a) Variance = npq

Sol.10 (b) Right

Sol.11 (a) $\because$ np = 9
$npq = 2.25$
$\Rightarrow 9q = 2.25 \Rightarrow q = \frac{2.25}{9} = \mathbf{0.25}$

Sol.12 (a) $mean > variance$

Sol.13 (b) $S.D. = \sqrt{variance} = \sqrt{npq}$

Sol.14 (b) Poisson

Sol.15 (b) Poisson

Sol.16 (c) 0

Sol.17 (a) $np \text{ } is \text{ } finite$

Sol.18 (c) In Poisson distribution mean = variance

Sol.19 (a) $Mean = \lambda$

Sol.20 (c) $S.D. = \sqrt{npq}$

Sol.21 (a) Normal distribution is used for continuous events

Sol.22 (b) Continuous random variables

Sol.23 (b) $\varphi(k) \geq 0$

Sol.24 (c) 1

Sol.25 (b) 1

Sol.26 (c) $p = q$

Sol.27 (a) High

Sol.28 (c) Normal

Sol.29 (b) Normal

Sol.30 (c) Mean = Median = Mode (same)

Sol.31 (b) Total area under the curve = 1

Sol.32 (b) Probability is maximum at mean

Sol.33 (a) True

Sol.34 (a) Normal

Sol.35 (c) Mean = 0 & S.D. = 1

Sol.36 (b) 2

Sol.37 (d) t distribution

Sol.38 (a) t distribution

Sol.39 (d) t distribution

Sol.40 (a) t distribution

Sol.41 (a) $p \approx 0 \text{ \& } q \approx 1$

Sol.42 (a) Increase infinitely

Sol.43 (b) Normal

Sol.44 (a) True

Sol.45 (c) $E\ (X) = mean$

Sol.46 (b) Variance

Sol.47 (a) Mean $= E(X) = 1 \times p + 0 \times (1-p) = p + 0 = p$

Sol.48 (a) It is a true statement

Sol.49 (b) n and p

Sol.50 (b) $n = 4,\ p = {}^{1}/_{3} \quad \therefore q = 1 - p = 1 - {}^{1}/_{3} = {}^{2}/_{3}$

$\therefore\ \sigma^2 = npq = 4 \times \frac{1}{3} \times \frac{2}{3} = {}^{8}/_{9}$

Sol.51 (d) Mean $= 20 \Rightarrow np = 20$____(I)

$\sigma\ (S.D) = 4 \Rightarrow variance\ (\sigma^2) = 16$

$\Rightarrow npq = 16 \quad \Rightarrow 20 \times q = 16 \quad [from\ (I)]$

$\Rightarrow q = \frac{16}{20} = {}^{4}/_{5}$

Sol.52 (c) Mean $= 20 \quad \Rightarrow np = 20$

S.D $= 4 \quad \Rightarrow \sigma = 4 \ \Rightarrow \sqrt{npq} = 4$

$\Rightarrow npq = 16 \quad \Rightarrow 20q = 16$

$\Rightarrow q = \frac{16}{20} = {}^{4}/_{5}$

$\therefore p = 1 - q = 1 - {}^{4}/_{5} = {}^{1}/_{5}$

Sol.53 (b) $np = 20 \ \ \& \sqrt{npq} = 4 \Rightarrow npq = 16$

$\therefore q = \frac{16}{20} = {}^{4}/_{5}$

$\therefore p = 1 - {}^{4}/_{5} = {}^{1}/_{5}$

$\therefore n = 20 \times 5 = 100$

Sol.54 (a) Discrete

Sol.55 (c) Normal distribution

Sol.56 (a) Poisson

Sol.57 (b) 0

Sol.58 (a) Mean = Median = Mode

Sol.59 (c) Mean

Sol.60 (b) Mean

Sol.61 (a) $P\,(X = 8) = \frac{n\,(E)}{n\,(S)} = {}^{1}/_{5}$

Sol.62 (d) $P\,(X = 12) = \frac{n\,(E)}{n\,(S)} = \frac{0}{5} = 0$

Sol.63 (b) $P\,(X \leq 12) = \frac{n\,(E)}{n\,(S)} = {}^{4}/_{5}$

Sol.64 (c) $P\,(X < 12) = \frac{3}{5}$

Sol.65 (a) $P\,(X > 10) \quad = \frac{n\,(E)}{n\,(S)} = {}^{3}/_{5}$

Sol.66 (c) The probability density function of a continuous random variable is define as follow

$F(x) = c \; where \; -1 \leq x \leq 1$

$= 0$ otherwise

$$\int_{-1}^{1} f(x)\,dx = 1$$

$$\int_{-1}^{1} C\,dx = 1$$

$= C[x]_{-1}^{1} = 1$

$= C[1 - (-1)] = 1$

$C = \frac{1}{2}$

Sol.67 (b) A continuous random variable x has the probability density .

$f(x) = \frac{1}{2} - ax \qquad [0 < x < 4]$

When 'a' is constant.

$$\int_{0}^{4} f(x)\,dx = 1$$

$$\int_{0}^{4} \left(\frac{1}{2} - ax\right) dx = 1$$

$$\left[\frac{1}{2}x - \frac{ax^2}{2}\right]_0^4 = \frac{1}{2}[4 - 0] - \frac{a}{2}[16 - 0] = 1$$

$2 - 8a = 1$

$8a = 1$

$a = \frac{1}{8}$

Sol.68 (b) The probability density function of a continuous random variable is define as follow

$f(x) = \frac{1}{2} \qquad [4 < x < 6]$

Then $[4 < x < 5]$

$$\int_{4}^{5} f(x)\,dx$$

$$\int_{4}^{5} \left(\frac{1}{2}\right) dx = \left[\frac{1}{2}x\right]_4^5 = \frac{1}{2}[5 - 4] = 0.5$$

Sol.69 (b) $n = 500 \;,\; p = {}^{1}/_{6}$

$\therefore Mean = np = 500 \times \frac{1}{6} = \frac{500}{6}$

Sol.70 (a) n = 500

$p = {}^{1}/_{6} \quad \therefore q = 1 - {}^{1}/_{6} = {}^{5}/_{6}$

$\therefore S.D = \sqrt{npq} \;=\; \sqrt{500 \times \frac{1}{6} \times \frac{5}{6}}$

$= \frac{50}{6}$

Sol.71 (c) np = 2

$npq = 1.2 \Rightarrow q = \frac{1.2}{2} = 0.6$

$\therefore p = 1 - q = 1 - 0.6 = 0.4$

$\therefore n = \frac{2}{0.4} = \mathbf{5}$

Sol.72 (a) $np = 2$

$npq = 1.6 \;\; \Rightarrow q = \frac{1.6}{2} = 0.8$

$\therefore p = 1 - q = 1 - 0.8 = 0.2 = {}^{\mathbf{1}}/_{\mathbf{5}}$

Sol.73 (d) $np = 5$

$\& \sqrt{npq} = 3 \quad \Rightarrow npq = 9$

$\Rightarrow \boldsymbol{q} = \frac{9}{5} > \mathbf{1}$

Which is not true

Sol.74 (a) $E\,(X) = K$

Sol.75 (c) Uniform distribution

Sol.76 (c) Theoretical distribution is a probability distribution

Sol.77 (a) Probability function is known as frequency function

Sol.78 (c) Uniform distribution

Sol.79 (c) $f\,(x) = {}^1\!/_6$

Sol.80 (a) $f\,(x) = {}^1\!/_n$

Sol.81 (d) $P\,(X = 9) = \frac{n\,(E)}{n\,(S)} = \frac{1}{6}$

Sol.82 (b) $P\,(X = 12) = \frac{n\,(E)}{n\,(S)} = \frac{0}{6} = 0$

Sol.83 (a) $P\,(X < 15) = \frac{n\,(E)}{n\,(S)} = \frac{3}{6} = {}^1\!/_2$

Sol.84 (a) $P\,(X \le 15) = \frac{4}{6} = {}^2\!/_3$

Sol.85 (b) $P\,(X > 15) = \frac{n\,(E)}{n\,(S)} = \frac{2}{6} = \frac{1}{3}$

Sol.86 (c) $P\,(|X - 14| < 5) = \frac{n\,(E)}{n\,(S)}$

$= \frac{3}{6} = {}^1\!/_2 \quad [x - 14 < 5 \Rightarrow x < 19\ or - x + 14 < 5 \Rightarrow x > 9 \quad \therefore x = 11, 15, 18]$

Sol.87 (b) Mean $= \frac{\Sigma x f(x)}{n}$

$= \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \cdots + \frac{n}{n}$

$= \frac{1+2+3+\cdots+n}{n} = \frac{\frac{n\,(n+1)}{2}}{n} = \frac{n+1}{2}$

Sol.88 (a) Area under the probability curve to the left of the vertical line at t

Sol.89 (b) Cumulative distribution function

Sol.90 (c) The probability density function of a continuous random variable is

$y = k(x - 1) \quad [1 < x < 2]$

As given function is probability function

$$\int_1^2 f(x)\,dx = 1$$

$$\int_1^2 k(x - 1)\,dx = 1$$

$= \frac{k}{2}[x^2]_1^2 - k[x]_1^2 = 1$

$= \frac{k}{2}(4 - 1) - k(2 - 1) = 1$

$= \frac{3k}{2} - k = 1$

$= k = 2$

****

Index Number

Exercise: Additional Questions

Sol.3 (c) The best average for constructing an index number is the geometric mean.

Sol.4 (a) The time-reversal test is satisfied by the fisher's index number.

Sol.5 (d) The factor reversal test is satisfied by the fisher's index number.

Sol.6 (d) Simple Geometric mean of price relative.

Sol.7 (d) Fisher's ideal index number is based on the geometric mean of Laspeyres and Paasche's index number.

Sol.8 (b) Paasche index is based on current year quantities.

Sol.9 (c) Fisher's ideal index number is the geometric mean of Laspeyres and Paasche's index numbers.

Sol.10 (c) Price relative is expressed in terms of p= $\frac{p_n}{p_0} \times 100$.

Sol.11 (c) Paasche index number is expressed in terms of $\frac{\Sigma P_n\,Q_n}{\Sigma P_0\,Q_n} \times 100$.

Sol.12 (c) Time reversal test is satisfied by marshall-edge worth formula.

Sol.13 (a) Cost of the Living index number is expressed in terms of $\frac{\Sigma P_n\,Q_0}{\Sigma P_0\,Q_0} \times 100$.

Sol.14 (b)

	P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_0	P_0Q_1	P_1Q_1
X	L	10	2	5	10L	20	5L	10
Y	L	5	P	2	5L	5P	2L	2P
Total					**15L**	**20 + 5P**	**7L**	**10 + 2P**

Now,

$$\frac{\text{Laspeyre's Index number}}{\text{Pasche's Index number}} = \frac{28}{27}$$

$$\Rightarrow \frac{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100}{\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times 100} = \frac{28}{27}$$

$$\Rightarrow \frac{20+5P}{15L} \times \frac{7L}{10+2P} = \frac{28}{27}$$

$$\Rightarrow 9(20 + 5P) = 20(10 + 2P)$$

$$\Rightarrow 180 + 45P = 200 + 40P$$

$$\Rightarrow 5P = 20 \quad \Rightarrow P = \frac{20}{5} = \mathbf{4}$$

Sol.15 (c) Let current prices ₹ x

$$\textit{Increase in price} = 1.25x$$

$$\textit{Increased price} = 2.25x$$

$\therefore$ Index Number $= \frac{P_n}{P_0} \times 100$

$$= \frac{2.25x}{x} \times 100$$

= 225

Sol.16 (b) Index number $= \frac{P_n}{P_0} \times 100$

$$\Rightarrow 250 = \frac{P_n}{P_0} \times 100$$

$$\Rightarrow P_n = 2.5\, P_0$$

$$\therefore \textit{Increased } \% = \frac{P_n - P_0}{P_0} \times 100$$

$$= \frac{(2.5-1)\, P_0}{P_0} \times 100$$

$$= 1.5 \times 100 = \mathbf{150\%}$$

Sol.17 (c) Index number $= \frac{P_n}{P_0} \times 100$

$$= \frac{\left(\frac{100-35}{100}\right) P_0}{P_0} \times 100$$

$$= \frac{65}{100} \times 100 = \mathbf{65}$$

Sol.18 (c) Link relative Index $= \frac{P_n}{P_{n-1}} \times 100$

Sol.19 (a) Fisher's Ideal Index number

$$= \sqrt{\textit{Laspeyre's Index} \times \textit{Paache's Index}}$$

Sol.20 (b) $P_{01} \times Q_{01} = V_{01} = \frac{\Sigma p_n\, q_n}{\Sigma p_0\, q_0}$

Sol.21 (a) Marshall-edge worth Index

$$= \frac{\Sigma P_n\,(q_0+q_n)}{\Sigma P_0\,(q_0+q_n)} \times 100$$

$\therefore$ After interchanging P & q then Marshall-edge worth Index

$$= \frac{\Sigma q_n\,(P_0+P_n)}{\Sigma q_0\,(P_0+P_n)} \times 100$$

Sol.22 (d) Dorbish and Bowley's Index number

$= \frac{1}{2}(L+P)$ [∵ L = Laspeyre's Index no. P = Paasche's Index no.]

$\Rightarrow 145 = \frac{1}{2}(L+150) \Rightarrow L = 290 - 150 = 140$

∴ Fisher's Ideal Index number

$= \sqrt{L \times P} = \sqrt{140 \times 150}$

$=$ **144.91 (approx)**

Sol.23 (b) $P_{01} = \frac{P_1}{P_0} \times 100$

$\Rightarrow P_0 = \frac{P_1}{P_{01}} \times 100 = \frac{160}{313} \times 100$

= **₹ 51.12 (approx)**

Sol.24 (a) $C.L.I = \frac{\Sigma p_n q_0}{\Sigma p_0 q_0} \times 100 = \frac{3850}{3500} \times 100$

= **110**

Sol.25 (b) A.M. of Price Index number

$= \frac{1}{4}\left[\frac{7}{5} \times 100 + \frac{10}{8} \times 100 + \frac{32}{25} \times 100 + \frac{12}{6} \times 100\right]$

$= \frac{1}{4}[140 + 125 + 128 + 200]$

$= \frac{1}{4} \times 593 =$ **148.25**

Sol.30 (c) Monthly Income in the year 1984

$= \frac{200}{160} \times 800 = 1{,}000$

∴ D. A. to be paid to the employee = 1,000 − 800

= ₹ 200 p.m.

Sol.31 (d) The simple geometric mean of price relative and weighted aggregative formula satisfy the circular test.

Sol.32 (d) Fisher's ideal index number is the only formula which satisfies both the time-reversal test and factor reversal test.

Sol.33 (a) "Neither Laspeyres nor Paasche's formula obeys" time-reversal and factor reversal tests of index number.

Sol.34 (a) Bowley's Index Number $= \frac{1}{2}(L+P)$

$\Rightarrow 150 = \frac{1}{2}(L+P)$ [∵ L → Laspeyre's I. N. P → Paasche's I. N]

$\Rightarrow L + P = 300 \Rightarrow L = 300 - P$ ____(I)

Also, Fisher's Index number $= \sqrt{LP}$

$\Rightarrow 149.95 = \sqrt{L\,P}$

$\Rightarrow L\,P = (149.95)^2$

$\Rightarrow (300 - P)\,P = 22485.0025$

$\Rightarrow P^2 - 300P + 22485.0025 = 0$

$\Rightarrow P = \frac{300 \pm \sqrt{90000 - 89940.01}}{2 \times 1}$

$= \frac{300 \pm \sqrt{59.99}}{2}$

$= \frac{300 \pm 7.75}{2}$ (approx)

$= \frac{307.75}{2}$ or $\frac{292.25}{2}$

= **153.88 (approx)** or **146.13 (approx)**

Sol.35 (b) Monthly salary in 1972 must be (to maintain std. living in 1960)

$P_1 = \frac{P_0 \times P_{01}}{100} = \frac{500 \times 250}{100}$

= ₹ 1250

∴ Extra allowances $= P_1 - 750$

= (1250 − 750) = ₹ **500**

Sol.36 (a)

	P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_0	P_0Q_1	P_1Q_1
A	4	3	6	2	12	18	8	12
B	5	4	6	4	20	24	20	24
C	7	2	9	2	14	18	14	18
D	2	3	1	5	6	3	10	5
Total					**52**	**63**	**52**	**59**

Fisher's Ideal Index = $\sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$

$\Rightarrow \sqrt{\frac{63}{52} \times \frac{59}{52}} \times 100$

= **117.3 (approx)**

Sol.38 (a) Factor reversal test is expressed in terms of $\frac{\sum P_1 q_1}{\sum P_0 q_0}$

Sol.39 (c) Circular test is satisfied by the simple geometric mean of price relative and weighted aggregative with fixed weight.

Sol.40 (b) General Index number

$= \frac{35\times425+15\times235+20\times215+8\times115+22\times150}{35+15+20+8+22}$

$= \frac{14875+3525+4300+920+3300}{100}$

= 269.2

Sol.41 (a) The Price per unit of commodity A

$= \frac{Values}{Quantity\ units} = \frac{500}{100} =$ **₹ 5**

Sol.42 (c) Increase in prices = $\frac{P_1 - P_0}{P_0} \times 100$

$= \frac{152\,P_0 - 100\,P_0}{100\,P_0} \times 100$

$\left[\because 152 = \frac{P_1}{P_0} \times 100 \quad \Rightarrow P_1 = \frac{152\,P_0}{100}\right]$

$= \frac{52\,P_0}{P_0} =$ **52%**

Sol.43 (a) Value Index = $\frac{\sum P_1 Q_1}{\sum P_0 Q_0} \times 100$

Sol.44 (a) Purchasing power of money is Reciprocal of price index number

Sol.45 (b) Index number = $\frac{P_1}{P_0} \times 100$

$= \frac{P_0\,(100+25)}{100\,P_0} \times 100$

= 125

Sol.46 (a) Increase in Price

$= \left(\frac{P_1-P_0}{P_0}\right) \times 100 = (Index\ number - 100)\%$

$= (355 - 100)\% = \mathbf{255\%}$

Sol.47 (c) Index number $= \frac{P_1}{P_0} \times 100$

$= \frac{P_0 \frac{(100+125)}{100}}{P_0} \times 100$

$= \frac{225}{100} \times 100 \ = \mathbf{225}$

Sol.48 (c) Percentage increases in Price

$= (Index\ no. -100)\%$

$= (280 - 100)\%$

$= \mathbf{180\%}$

Sol.49 (c) $P_1 = \left(\frac{100-35}{100}\right) P_0 = \frac{65\ P_0}{100}$

$\therefore$ Index number $= \frac{P_1}{P_0} \times 100$

$= \frac{65\ P_0}{100\ P_0} \times 100$

= 65

Sol.50 (a) Suitable Index Number $= \frac{125\times5+67\times2+250\times3}{5+2+3}$

$= \frac{625+134+750}{10} = \frac{1509}{10} = \mathbf{150.9}$

Sol.51 (a) Bowley's Index Number

$= \frac{\text{Laspeyre's Index + Paasche's Index}}{2}$

Sol.52 (b)

Commodity	P_0	P_1
Rice	35	42
Wheat	30	35
Pulse	40	38
Fish	107	120
Total	212	235

$\therefore$ Simple Aggregative Index $= \frac{\Sigma P_1}{\Sigma P_0} \times 100$

$= \frac{235}{212} \times 100$

$= \mathbf{110.849\ (approx)}$

Sol.53 (b) Laspeyres price index = Paasche's price index.

Sol.54 (b & d) The quantity index number using fisher's formula satisfies the time-reversal test and factor reversel test.

Sol.55 (d) For constructing consumer price index is used in Laspeyres method.

Sol.56 (a) The cost of living index is always a weighted index.

Sol.57 (c) The time-reversal test is not satisfied to Laspeyres and Paasche index.

Sol.58

Item	P_0	P_1	W_0	P_0W_0	P_1W_1
Wheat	0.50	0.75	2	1.00	1.50
Milk	0.60	0.75	5	3.00	3.75
Egg	2.00	2.40	4	8.00	9.60
Sugar	1.80	2.10	8	14.40	16.80
Shoes	8.00	10.00	1	8.00	10.00
Total				**34.40**	**41.65**

$\therefore$ A weighted average of price Relative Index $= \frac{\Sigma P_1 W_0}{\Sigma P_0 W_0} \times 100 = \frac{41.65}{34.40} \times 100$

$= \mathbf{121.08\ (approx)}$

Sol.59 (a) The factor Reversal test

$P_{01} \times Q_{01} = \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$

Sol.60 (b) $P_{02} = \frac{P_{01} \times P_{12}}{100} = \frac{150\times200}{100} = \mathbf{300}$

Sol.61 (c) Circular test is not by Laspeyres or Paasche's or fisher's ideal index no.

Sol.62 (a)

Commodity	P_0	q_0	P_1	q_1	$P_0\,q_0$	$P_1\,q_1$
A	4	3	6	2	12	12
B	5	4	6	4	20	24
C	7	2	9	2	14	18
D	2	3	1	5	6	5
Total					52	59

$\therefore$ Required value ratio $= \frac{\Sigma P_1\,q_1}{\Sigma P_0\,q_0} = \frac{59}{52}$

Sol.63 (b) The total sum of the values of a given year divided the sum of the values of the base year.

Sol.64 (b) Time Reversal Test

$P_{01} \times P_{10} = \mathbf{1}$

Sol.65 (a) Price in 1995 = 100

$\therefore$ Price in 1996 $= 100 \times \left(\frac{100+20}{100}\right) = 120$

Price in 1994 $= 120 \times \frac{100}{(100-20)}$

$= 120 \times \frac{100}{80} = 150$

Price in 1997 $= 120 \times \left(\frac{100}{100+50}\right)$

$= 120 \times \frac{100}{150} = 80$

$\therefore$ Required data from 1994 to 1997

$= (\mathbf{150, 100, 120, 80})$

Sol.66 (d)

Commodity	P_0	P_1
A	6	10
B	2	2
C	4	6
D	11	12
E	8	12
Total	**31**	**42**

$\therefore$ Price Index Number $= \frac{\Sigma P_1}{\Sigma P_0} \times 100$

$= \frac{42}{31} \times 100$

= 135.48 (approx)

Sol.67 (a)

Commodity	P_0	P_1
Rice	36	54
Pulse	30	50
Fish	130	155
Potato	40	35
Oil	110	110
	346	404

$\therefore$ Index number $= \frac{\Sigma P_1}{\Sigma P_0} \times 100$

$= \frac{404}{346} \times 100$

= 116.8 (approx.)

Sol.68 (a) The Bowley price index number is represented in terms of A.M of Laspeyres method and Paasche method.

Sol.69 (b) Fisher's price index number is G. M. of Laspeyres and Paasche's price index method.

Sol.70 (b)

The price index number using simple G.M. of the n relatives is given by

$\boldsymbol{log\,I_{on} = 2 + \frac{1}{n}\Sigma\,log\frac{P_n}{P_0}}$ **Sol.71 (b)**

Commodities	P_0	P_1	$\frac{P_1}{P_0} \times 100$	$\log\left(\frac{P_1}{P_0} \times 100\right)$
A	45	55	122.22	2.0871
B	60	70	116.67	2.0671
C	20	30	150.00	2.1761
D	50	75	150.00	2.1761
E	85	90	105.88	2.0249
F	120	130	108.33	2.0346
Total				12.5659

$\therefore$ Price Index (BY the method of price relative using G. M.) = Antilog $\left(\frac{1}{n}\,\Sigma(log\frac{P_1}{P_0} \times 100\right)$

$= \text{Antilog}\left(\frac{1}{6} \times 12.5659\right)$

$= \text{Antilog } 2.0943 \text{ (approx)}$

$= 1.243 \times 10^2$

$= \mathbf{124.3}$

Sol.72 (a)

Group	P_0	q_0	$P_0\,q_0$
A	120	6	720
B	132	3	396
C	98	4	392
D	115	2	230
E	108	1	108
F	95	4	380
		20	2226

$\therefore I = \frac{\Sigma P_0\,q_0}{\Sigma q_0} = \frac{2226}{20} = \mathbf{111.3}$

Sol.73 (b) Price ratio $= \frac{7.5}{5} = 1.5$

Quantity ratio $= \frac{90}{120} = 0.75$

$\therefore$ Required Product $= 1.5 \times 0.75$

$= \mathbf{1.125}$

Sol.74 (a) Time reversal test.

Sol.75 (d)

Group	Weight q_0	Index number P_0	$P_0\,q_0$
Food	50	241	12050
Cloth	2	21	42
Fuel & light	3	204	612
Rent	16	256	4096
Misc	29	179	5191
Total	100		21991

$\therefore$ Cost of living Index $= \frac{\Sigma P_0\,q_0}{\Sigma q_0}$

$= \frac{21991}{100} = \mathbf{219.91}$

Sol.76 (b) Worker salary should increase to $= \frac{200}{110} \times 325 = 590.91$ (approx)

$\therefore$ Required additional amount

$= 590.91 - 500$

$= \text{₹}\,\mathbf{90.91}$

Sol.77 (d) Price relative $= \frac{25}{30} \times 100$

$= 83.33$ (approx)

Sol.78 (b) Decrease in Price on the basis of 1982

$= \frac{120-60}{120} \times 100$

$= \frac{60}{120} \times 100 = \mathbf{50\%}$

Sol.79 (a) Cost of living index numbers are also used to find real wages by the process of deflating of index number.

Sol.80 (a)

Commodity	P_0	q_0	P_1	q_1	$P_0\,q_1$	$P_1\,q_1$
A	3	18	4	15	45	60
B	5	6	5	9	45	45
C	4	20	6	26	104	156
D	1	14	3	15	15	45
Total					209	306

$\therefore$ Paasche's Price Index $= \frac{\Sigma P_1\,q_1}{\Sigma P_0\,q_1} \times 100$

$= \frac{306}{209} \times 100 = \mathbf{146.41\ (approx.)}$

Sol.81 (b)

	P_0	q_0	P_1	q_1	$P_0\,(q_0+q_1)$	$P_1\,(q_0+q_1)$
A	7	17	13	25	294	546
B	6	23	7	25	288	336
C	11	14	13	15	319	377
D	4	10	8	8	72	144
	Total				973	1403

∴ Marshall Edge worth index

$= \frac{\Sigma P_1\,(q_0+q_1)}{\Sigma P_0\,(q_0+q_1)} \times 100$

$= \frac{1403}{973} \times 100 = \mathbf{144.19\ (approx)}$

Sol.82 (a) The circular test is an extension of the time-reversal test.

Sol.83 (c) $I_{01} \times I_{12} \times I_{20} = \frac{\Sigma P_1}{\Sigma P_0} \times \frac{\Sigma P_2}{\Sigma P_1} \times \frac{\Sigma P_0}{\Sigma P_2} = 1$

Sol.84 (b) Price relative of 1976 $= 100 \times \frac{(100+20)}{100} = 120$

Price relative of 1974 $= 120 \times \frac{100}{100-20}$

$= 120 \times \frac{100}{80} = 150$

Price relative of 1977 $= 120 \times \frac{100}{100+50}$

$= 120 \times \frac{100}{150} = \mathbf{80}$

Sol.85 (b) $I_{01} \times I_{10} = 1$

Sol.86 (a) Required Price relative $= \frac{P_1}{P_0} \times 100$

$= \frac{30}{25} \times 100 = 120$

Sol.87 (b) Chain Index for the year

$1993 = \frac{103}{100} \times 100 = \mathbf{103}$

$1995 = \frac{105}{100} \times 103 = \mathbf{108.15}$

$1996 = \frac{112}{100} \times 108.15 = \mathbf{121.13}$

$1997 = \frac{108}{100} \times 121.13 = \mathbf{130.82}$

Sol.88 (c) Real wages $= \frac{200}{110} \times 330$

= ₹ 600

∴ Real wages decreased by

= ₹ (600 – 500)

= **₹ 100**

Sol.89 (c) Salary in 1985 $= \frac{250}{100} \times 3{,}000$

= ₹ 7,500

∴ Required dearness allowances

= ₹ (7,500 – 3,000)

= **₹ 4,500**

Sol.90 (d) Salary in 1985 $= \frac{200}{160} \times 800$

= ₹ 1,000

∴ Required dearness to be paid to the employee = ₹ (1,000 – 800)

= **₹ 200**

Sol.91 (c) Let the cost of Tobacco initially be Rs. 100 then Increased cost of Tobacco

$\therefore 100 \times \frac{100+50}{100} = 150$

∴ Increase in Price Tobacco

= ₹ (150-100)

= ₹ 50

∵ ₹ 50 is the 5% of Index number

∴ ₹ 100 *is the* $\frac{5}{50} \times 100$ *Index number*

= **10%**

Sol.92 (a) Purchasing Power of money of 1950

$in\ 1960 = \frac{110.3}{98.4} = \mathbf{1.12\ (approx.)}$

Sol.93 (b) Laspeyre's Index number $= \frac{\sum P_n Q_0}{\sum P_0 Q_0}$

$= \frac{1900}{1360} = \mathbf{1.397\ (approx)}$

Sol.94 (a) Let the weight of food Index be x & other be y

$125\ (x + y) = 120x + 135y$

$\Rightarrow 5\,x = 10\,y \ \Rightarrow x = 2y$

$\therefore$ Required % $= \frac{x}{x+y} \times 100$

$= \frac{2y}{3y} \times 100 = \mathbf{66.67\ (approx.)}$

Sol.95 (b) Price Index for retained Input for 1967 taking 1960 as base $= \frac{100 \times 87.6}{62 \times 71.5} \times 100$

$= \mathbf{197.61\ (approx.)}$

Sol.96 (d) Raised salary should be $= \frac{200}{110} \times 330$

= ₹ 600

$\therefore$ Loss in salary = ₹ (600 − 500)

= ₹ 100

Sol.97 (d)

Commodity	Q_0	P_0	Q_1	P_1	$P_0\,Q_0$	$P_1\,Q_0$	$P_0\,Q_1$	$P_1\,Q_1$
A	2	2	6	18	4	36	12	108
B	5	5	2	2	25	10	10	4
C	7	7	4	24	49	168	28	96
					78	214	50	208

$\therefore$ Fisher's quantity Index number

$$= \sqrt{\frac{\sum Q_1\,P_0}{\sum Q_0\,P_0} \times \frac{\sum Q_1\,P_1}{\sum Q_0\,P_1}} \times 100$$

$$= \sqrt{\frac{50 \times 208}{78 \times 214}} \times 100$$

= 78.93 (approx.)

Sol.98 (a)

Commodity	P_0	P_1	P_1/P_0	$\log\left(\frac{P_1}{P_0}\right)$
A	25	55	2.2	0.3424
B	30	45	1.5	0.1761
				0.5185

$$P_{01} = Anti\ log\left[2+\frac{1}{n}\sum\log\left(\frac{P_1}{P_0}\right)\right]$$

$$= Anti\log\left[2+\frac{1}{2}\ (0.5185)\right]$$

$$= Anti\log\ (2.25925)$$

$$= 1.817 \quad \times 10^2 = \mathbf{181.7}$$

Sol.99 (d)

Commodity	P_0	q_0	P_1	q_1	$P_0\ q_1$	$P_1\ q_1$
X	4	10	6	15	60	90
Y	6	15	4	20	120	80
Z	8	5	10	4	32	40
		Total			212	210

Index number (from Paasche's formula)

$$= \frac{\sum P_1\ Q_1}{\sum P_0\ Q_1} \times 100 = \frac{210}{212} \times 100$$

= 99.06 (approx.)

Sol.100 (b) Group Index number = $\frac{\sum(\text{Price relative} \times \text{W})}{\sum \text{W}}$

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Logical Reasoning

Number Series

Sol.1 (c) 6, 11, 21, 36, 56, ?

Let the next no. is x

Here $11-6=5,\ 21-11=10,\ 36-21=15,\ 56-36=20$

So difference is in A.P.

$\therefore x-56=25 \quad \Rightarrow x=56+25=\mathbf{81}$

Sol.2 (d) 10, 100, 200, 310, ?

Let the next no. is x

Here $100-10=90,\ 200-100=100,\ 310-200=110$

$\therefore x-310=120 \quad \Rightarrow x=310+120=\mathbf{430}$

Sol.3 (c) 11, 13, 17, 19, 23, 25, 29 ?

Let the next no. is x

Here $13-11=2,\ 17-13=4,\ 19-17=2,\ 23-19=4$

$25-23=2,\ 29-25=4, x-29=2 \ \Rightarrow x=31$

Sol.4 (d) 6, 12, 21, 33, ?

Let the no. be x

$12-6=6,\ 21-12=9,\ 33-21=12$

$\therefore x-33=15 \quad \Rightarrow x=\mathbf{48}$

Sol.5 (a) 2, 5, 9, 14, ?, 27, Let the no. be x

$5-2=3,\ 9-5=4,\ 14-9=5,\ \therefore x-14=6 \ \Rightarrow x=20$

$\& \ 27-x=7 \quad \Rightarrow x=\mathbf{20}$

Sol.6 (b) 6, 11, 21, ?, 56, 81

Let the required no. be x

Here $11-6=5,\ 21-11=10,\ x-21=?,\ 56-x=?, 81-56=25$

$\therefore x-21=15 \quad \Rightarrow x=36$

$56-x=20 \ \Rightarrow x=56-20=36$

Sol.7 (a) 10, 18, 28, 40, 54, ?, 88

Let the no. be x

$\therefore 18-10=8,\ 28-18=10,\ 40-28=12,\ 54-40=14$

$\therefore x-54=16 \quad \Rightarrow x=70$

$88-x=18 \quad \Rightarrow x=\mathbf{70}$

Sol.8 (a) 120, 99, ?, 63, 48, 35

Let the no. be x

$\therefore 120-99=21, 99-x=?, x-63=?, 63-48=15, 48-35=13$

$\therefore 99-x=19 \quad \Rightarrow x=80$

$or\ x-63=17 \quad \Rightarrow x=\mathbf{80}$

Sol.9 (b) 22, 24, 28, 36, ?, 84

Let the no. be x

Here $24-22=2,\ 28-24=4,\ 36-28=8,\ x-36=?,\ 84-x=?$

$\therefore x-36=16 \quad \Rightarrow x=\mathbf{52}$

Also $84-x=32 \quad \Rightarrow x=84-32=52$

Sol.10 (a) 4,832, 5,840, 6,848, 7,856, ?

Let the no. be x

$5{,}840-4{,}832=1{,}008;\ 6{,}848-5{,}840=1{,}008;$

$7{,}856-6{,}848=1{,}008$

$\therefore x-7856=1008 \quad \Rightarrow x=\mathbf{8,864}$

Sol.11 (a) 10, 100, 200, 310, 430, ?

Let the no. be x

$100-10=90,\ 200-100=100,\ 310-200=110,\ 430-310=120,\ \therefore x-430=130$

$\Rightarrow x=\mathbf{560}$

Sol.12 (b) 28, 33, 31, 36, 34, ?

Let the no. be x

Here $33-28=5,\ 31-33=-2,\ 36-31=5,\ 34-36=-2$

$\therefore x-34=5 \ \Rightarrow x=\mathbf{39}$

Sol.13 (d) 120, 80, 40, 45, ?, 15

$\therefore 120 - 80 = 40, 80 - 40 = 40$

$45 - x = x - 15 \Rightarrow x = \mathbf{30}$

Sol.14 (d) 2, 15, 41, 80, 132, ?

Let the no. be x

$15 - 2 = 13,\ 41 - 15 = 26,\ 80 - 41 = 39,$
$132 - 80 = 52$

$\therefore x - 132 = 65\ \Rightarrow x = \mathbf{197}$

Sol.15 (a) 6, 17, 39, ?, 116

Let the no. be x

Here $17 - 6 = 11,\ 39 - 17 = 22,\ x - 39 =?,$
$116 - x =?$

$\therefore x - 39 = 33\ \Rightarrow x = 72$

Also $116 - x = 44\ \Rightarrow x = 116 - 44 = \mathbf{72}$

Sol.16 (a) 1, 4, 10, 22, ?, 94

Let the no. be x

$\therefore 4 - 1 = 3,\ 10 - 4 = 6,\ 22 - 10 = 12,\ x - 22 =?\ \&\ 94 - x =?$

$\therefore x - 22 = 24\ \Rightarrow x = 46$

Also $94 - x = 48\ \Rightarrow x = 94 - 48 = \mathbf{46}$

Sol.17 (b) 4, 9, 25, 49, ?, 169, 289, 361

Here all are perfect square nos. of prime numbers.

∴ Observing the option, 121 which is perfect square no.

Sol.18 (c) 4, 12, 36, ?, 324

Let the no. be x

Here $\frac{12}{4} = 3,\ \frac{36}{12} = 3,\ \frac{x}{36} =?,\ \frac{324}{x} =?$
$\therefore \frac{x}{36} = 3 \Rightarrow x = \frac{324}{3} = \mathbf{108}$

Sol.19 (a) 1, 1, 4, 8, 9, ?, 16, 64
Let the no. be x
Here $1 = 1^2, 1 = 1^3, 4 = 2^2, 8 = 2^3, 9 = 3^2$
$\therefore x = 3^3, 16 = 4^2\ \&\ 64 = 4^3$
$\therefore x = \mathbf{27}$

Sol.20 (b) 5760, 960, 192, ?, 16, 8

Let the no. be x

Here $\frac{960}{5760} = {}^1/_6, \frac{192}{960} = {}^1/_5, \frac{x}{192} =?, \frac{16}{x}, \frac{8}{16} = {}^1/_2$

$\therefore \frac{x}{192} = {}^1/_4 \Rightarrow x = \frac{192}{4} = \mathbf{48}$

Also $\frac{16}{x} = \frac{1}{3} \Rightarrow x = 48$

Sol.21 (c) 1, 2, 6, 7, 21, 22, 66, ?, 201

Let the no. be x

Here $2 = 1 + 1,\ 6 = 2 \times 3, 7 = 6 + 1, 21 = 7 \times 3, 22 = 21 + 1$

$66 = 22 \times 3, x = 66 + 1, 201 = x \times 3$

$\therefore x = 67\quad Also\ x = \frac{201}{3} = \mathbf{67}$

Sol.22 (a) 48, 24, 96, ?, 192

Let the no. be x

$\frac{24}{48} = {}^1/_2,\ \frac{96}{24} = 4,\ \frac{x}{96} =?,\ \frac{192}{x} =?$

$\therefore \frac{x}{96} = {}^1/_2\ \Rightarrow x = \frac{96}{2} = 48$

Also $\frac{192}{x} = 4 \Rightarrow x = \frac{192}{4} = \mathbf{48}$

Sol.23 (a) 165, 195, 255, 285, ?, 345

Let the number be x

Here $195 - 165 = 30,\ 255 - 195 = 60,\ 285 - 255 = 30,\ \therefore x - 285 = 60$

$\Rightarrow x = \mathbf{345}$

Sol.25 (a) 7, 26, 63, 124, 215, ?, 511

Let the number be x

$7 = 2^3 - 1,\ 26 = 3^3 - 1,\ 63 = 4^3 - 1,\ 124 = 5^3 - 1,\ 215 = 6^3 - 1$

$\therefore x = 7^3 - 1 = 343 - 1 = 342\ \&\ 511 = 8^3 - 1$

∴ Required number = **342**

Sol.26 (b) 3, 7, 15, 31, ?, 127

Let the number be x

$7 = 2 \times 3 + 1,\ 15 = 2 \times 7 + 1,\ 31 = 2 \times 15 + 1$

$\therefore x = 2 \times 31 + 1 = 63$

Also $127 = 2x + 1 \ \Rightarrow 2x = 126 \Rightarrow x = \mathbf{63}$

Sol.27 (d) 8, 28, 116, 584, ?

Let the no. be x

Here $28 = 8 \times 3 + 4,\ 116 = 28 \times 4 + 4,\ 584 = 116 \times 5 + 4$

$\therefore x = 584 \times 6 + 4$

$= 3{,}504 + 4 = \mathbf{3{,}508}$

Sol.28 (a) 6, 13, 28, 59, ?

Let the no. be x

Here $13 = 6 \times 2 + 1,\ 28 = 13 \times 2 + 2,\ 59 = 28 \times 2 + 3$

$\therefore x = 59 \times 2 + 4 = \mathbf{122}$

Sol.29 (a) 2, 7, 27, 107, 427, ?

Let the no. be x

Here $7 = 2 \times 4 - 1,\ 27 = 7 \times 4 - 1,\ 107 = 27 \times 4 - 1,\ 427 = 107 \times 4 - 1$

$\therefore x = 427 \times 4 - 1 = 1708 - 1 = \mathbf{1{,}707}$

Sol.30 (b) 5, 2, 7, 9, 16, 25, 41, ?

Let the no. be x

Here $7 = 5 + 2,\ 9 = 2 + 7,\ 16 = 7 + 9,\ 25 = 9 + 16,\ 41 = 16 + 25$

$\therefore x = 25 + 41 = \mathbf{66}$

Sol.31 (b)

M	A	D	R	A	S
↓	↓	↓	↓	↓	↓
N	B	E	S	B	T

∴ + 1 in the Letters.

D	E	L	H	I
↓	↓	↓	↓	↓
E	F	M	I	J

Sol.32 (a)

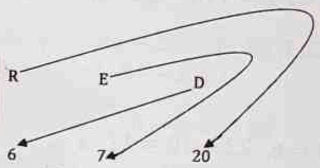
R	A	M	A	N
↓	↓	↓	↓	↓
1	2	3	2	5

D	I	N	E	S	H
↓	↓	↓	↓	↓	↓
6	7	5	4	8	9

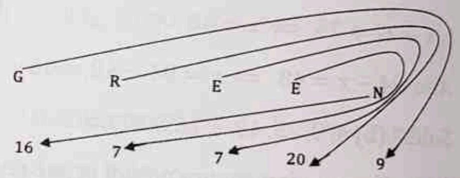
∴

H	A	M	A	M
↓	↓	↓	↓	↓
9	2	3	2	3

Sol.33 (c)



∴



The position Letter no. + 2

Sol.34 (a) A → 1

FAT → 6 + 1 + 20 = 27

∴ FAITH → 6 + 1 + 9 + 20 + 8

= 44

Sol.35 (a)

B	R	O	T	H	E	R
↓	↓	↓	↓	↓	↓	↓
2	4	5	6	7	8	4

S	I	S	T	E	R
↓	↓	↓	↓	↓	↓
9	1	9	6	8	4

∴

B	O	R	B	E	R	S
↓	↓	↓	↓	↓	↓	↓
2	5	4	2	8	4	9

Sol.36 (c)

D	E	L	H	I
↓	↓	↓	↓	↓
7	3	5	4	1

C	A	L	C	U	T	T	A
↓	↓	↓	↓	↓	↓	↓	↓
8	2	5	8	9	6	6	2

∴

C	A	L	I	C	U	T
↓	↓	↓	↓	↓	↓	↓
8	2	5	1	8	9	6

Sol.37 (a)

C	L	O	C	K
↓	↓	↓	↓	↓
3	4	2	3	5

T	I	M	E
↓	↓	↓	↓
8	6	7	9

∴

M	O	T	E	L
↓	↓	↓	↓	↓
7	2	8	9	4

Sol.38 (b)

P	A	L	E
↓	↓	↓	↓
2	1	3	4

E	A	R	T	H
↓	↓	↓	↓	↓
4	1	5	9	0

∴

P	E	A	R	L
↓	↓	↓	↓	↓
2	4	1	5	3

Sol.39 (a)

L	O	S	E
↓	↓	↓	↓
1	3	5	7

G	A	I	N
↓	↓	↓	↓
2	4	6	8

∴

8	2	1	4	6
↓	↓	↓	↓	↓
N	G	L	A	I

Sol.40 (c)

M	E	K	L	F
↓	↓	↓	↓	↓
9	1	7	8	2

(The position of the letters are -4 in each letter)

L	L	L	J	K
↓	↓	↓	↓	↓
8	8	8	6	7

(The position of the letters is -4 in each letter)

∴

I	H	J	E	D
↓	↓	↓	↓	↓
5	4	6	1	0

Sol.41 (d) ∵

N	A	M	E
↓	↓	↓	↓
4	2	5	8

∴

M	E	A	N
↓	↓	↓	↓
5	8	2	4

Sol.42 (a)

G	O	L	D
↓	↓	↓	↓
I	Q	N	F

(The position of the letters is +2 in each letter)

∴

W	I	N	D
↓	↓	↓	↓
Y	K	P	F

Sol.43 (a) ∵

R	O	S	E
↓	↓	↓	↓
T	Q	U	G

∴ (The position of the letters is +2 in each letter)

B	I	S	C	U	I	T
↓	↓	↓	↓	↓	↓	↓
D	K	U	E	W	K	V

Sol.44 (a)

Z	D	R	C	V	F
↓	↓	↓	↓	↓	↓
6	1	2	8	7	5

Sol.45 (d)

W	N	C	S	Z	V
↓	↓	↓	↓	↓	↓
3	4	8	9	6	7

Sol.46 (c)

R	D	N	F	V	S
↓	↓	↓	↓	↓	↓
2	1	4	5	7	9

Sol.47 (b) ∵

D	E	L	H	I
−1 ↓	−2 ↓	−3 ↓	−4 ↓	−5 ↓
C	C	I	D	D

∴

B	O	M	B	A	Y
−1 ↓	−2 ↓	−3 ↓	−4 ↓	−5 ↓	−6 ↓
A	M	J	X	V	S

Sol.48 (a)

R	I	P	P	L	E
↓	↓	↓	↓	↓	↓
6	1	3	3	8	2

L	I	F	E
↓	↓	↓	↓
8	1	9	2

∴

P	I	L	L	E	R
↓	↓	↓	↓	↓	↓
3	1	8	8	2	6

Sol.49 (a) PALAM → 16 + 1 + 12 + 1 + 13 = 43

∴ SANTACRUZ → 19 + 1 + 14 + 20 + 1 + 3 + 18 + 21 + 26 = **123**

Sol.50 (d)

Digit	7	2	1	5	3	9	8
Letter	W	L	M	S	I	N	D

∵

Sol.51 (c) 256 - You are good.

637 – We are bad.
358 – Good and bad

In the first and second lines, 6 is common in digits and 'are' is common in the alphabet, so the code of are is 6.
In the second and third lines, 3 is common in digits, and 'bad' is common in the alphabet, so code is code of bad is 3.
In the third and first lines, '5' is common in digits and good in the alphabet. So the code of 'good' is 5.

∴ **Code for "and" is 8 in the third line.**

Sol.52 (a) 3, 5, 7, 15, 17, 19

Here all nos. are prime except 15 which is composite

Sol.53 (b) 10, 14, 16, 18, 23, 24, 26

All are even except 23 which is odd

Sol.54 (b) 1, 4, 9, 16, 24, 25, 36

All are perfect square nos. except 24

Sol.55 (a) 41, 43, 47, 53, 61, 71, 73, 75

All are prime nos. except 75

Sol.56 (b) 16, 25, 36, 73, 144, 196, 225

All nos. are perfect square no. except 73

Sol.57 (a) 1, 4, 9, 16, 19, 36, 49

Here all are perfect square nos. except 19

Sol.58 (a) 1, 5, 14, 30, 49, 55, 91

Here all nos. are not perfect square nos. Except 49, which is perfect square no.

Sol.59(a) except 751, all numbers sum is even and the sum of 751 is 13, which is an odd number.

1	8	4	6	3	2
↓	↓	↓	↓	↓	↓
M	D	B	J	I	L

Sol.60 (c) 5-4=1, 7-5=2, 10-7=3 14-10 = 4 which is in the A.P so the next number should be 19 but here written is 18 so it is odd. 25-19=6, 32-25=7.

Sol.61 (c) All nos. are composite numbers except 43, which is a prime number.

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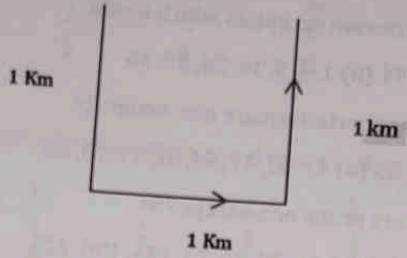
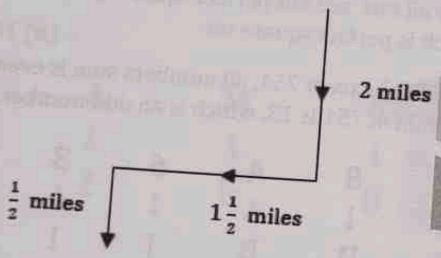
Sol.1 (c)



∴ He is facing north.

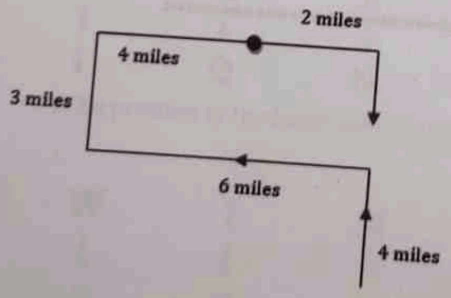
Sol.2 (d)

∴Final back direction is opposite of south which north

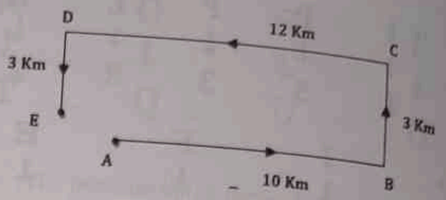


Sol.3 (b)

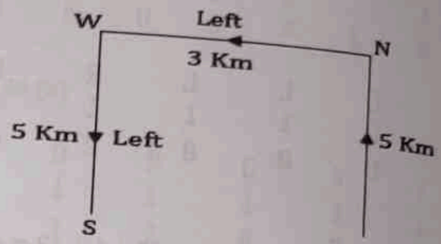
∴ The required direction he is facing in the South



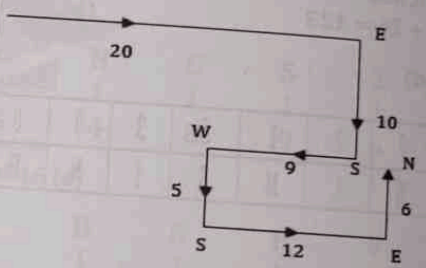
Sol.4 (c)



Sol.5 (b)

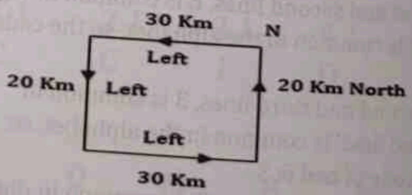


Sol.6 (b)



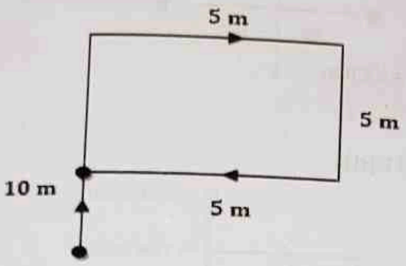
∴ Required direction he is facing in the North

Sol.7 (b)



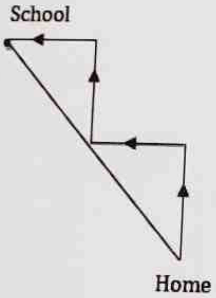
Sol.8 (c)

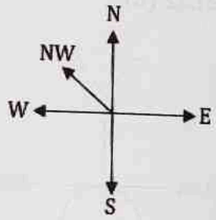
Required direction of final position with original position in the North.



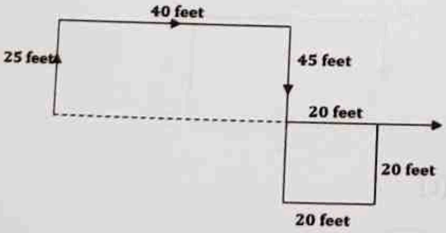
Sol.9 (b)

∴ Required direction is the North-west

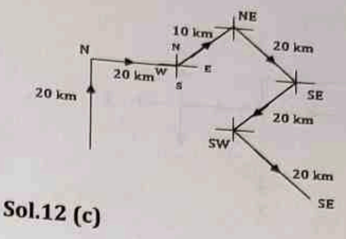




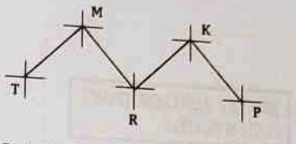
Sol.10 (d)



Sol.11 (a)

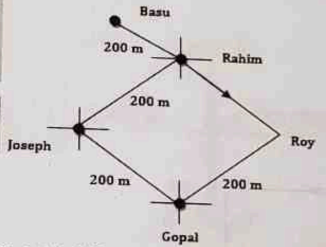


Sol.12 (c)



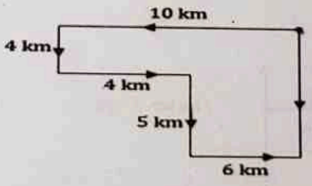
Sol.13 (a)

∴ Required direction is south-east

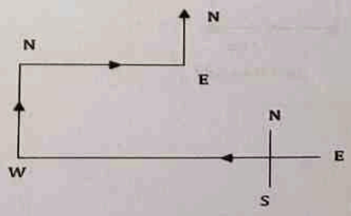


Sol.14 (d)

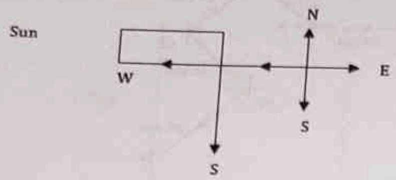
∴ Required direction is the South



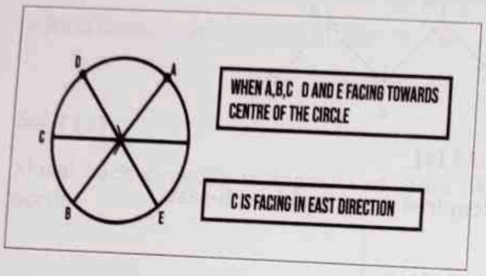
Sol.15 (a)



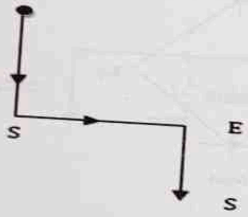
Sol.16 (a)



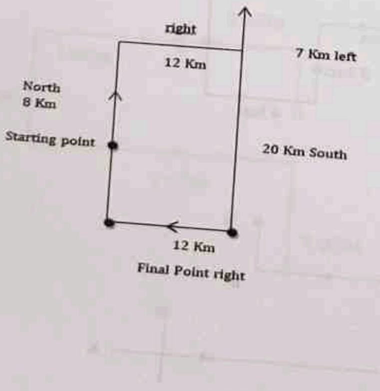
Sol.17 (d)



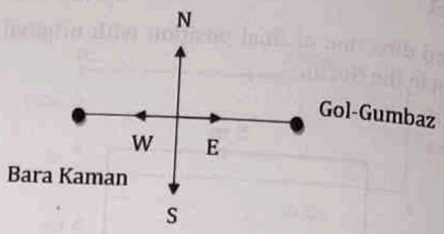
Sol.18 (d)



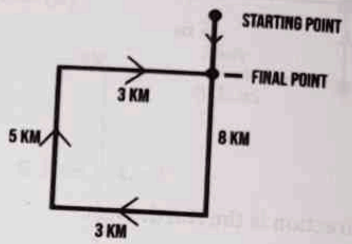
Sol.19 (b)



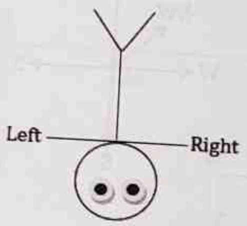
Sol.20(a)



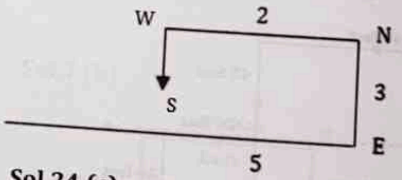
Sol.21 (d)



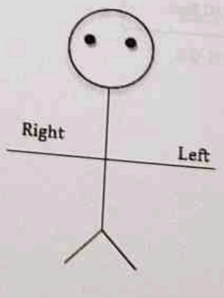
Sol.22 (b)



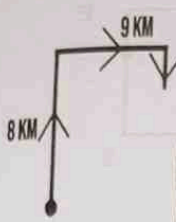
Sol.23 (a)



Sol.24 (c)

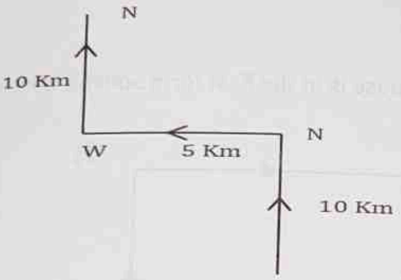


Sol.25 (a)

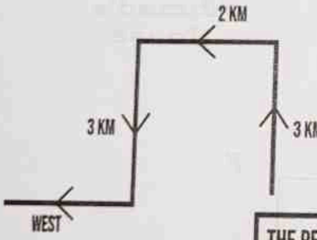


HE IS TRAVELLING FROM SOUTH IT MEANS GOING IN NORTH DIRECTION

Sol.26 (b)

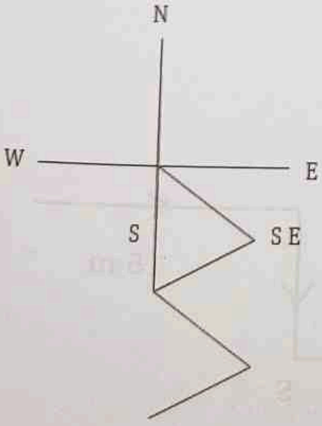


Sol.27 (b)



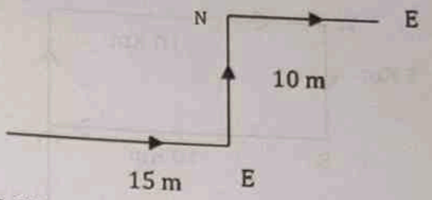
THE PERSON IS NOW WALKING IS WEST DIRECTION

Sol.28 (a)

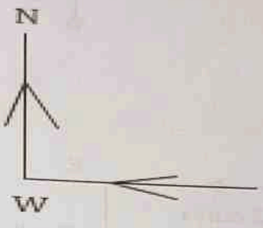


∴ He is in the South from the starting point.

Sol.29 (b)



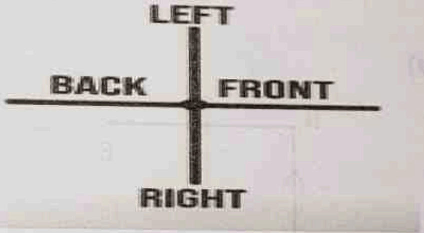
Sol.30 (b)



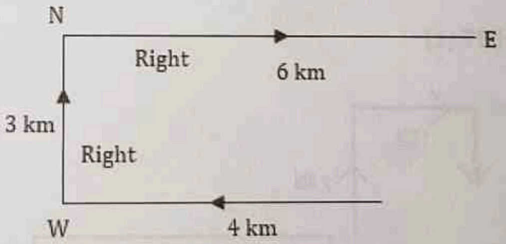
∴ He is walking in the North Direction.

Sol.31 (b)

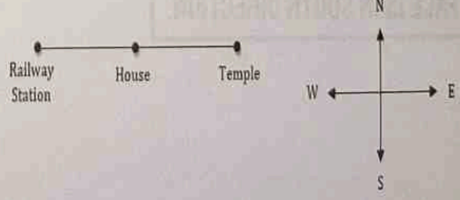
∴ Back in west



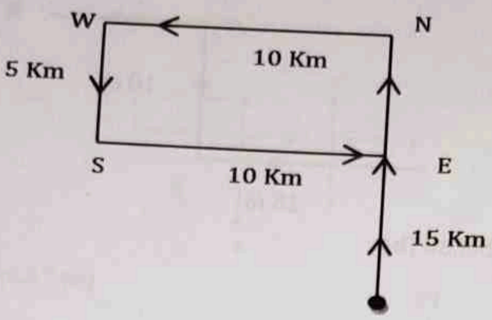
Sol.32 (a)



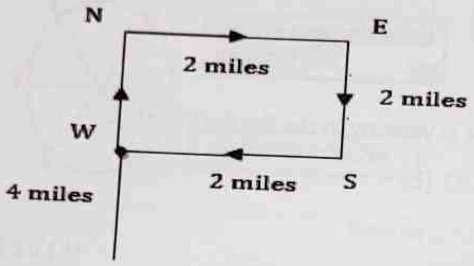
Sol.33 (c)



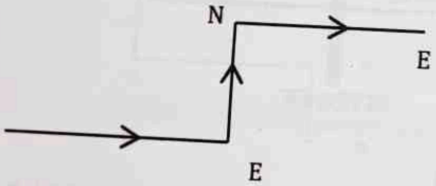
Sol.34 (c)



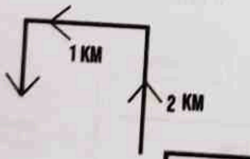
Sol.35 (a)



Sol.36 (a)



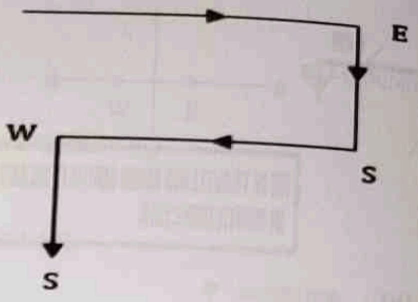
Sol.37 (c)



SHE IS WALKING IN SOUTH

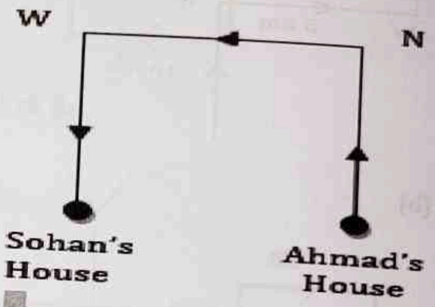
∴ HER FACE IS IN SOUTH DIRECTION.

Sol.38 (c)

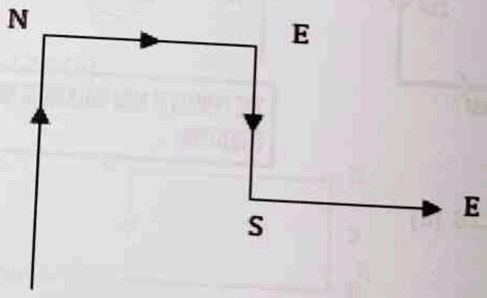


Sol.39 (a)

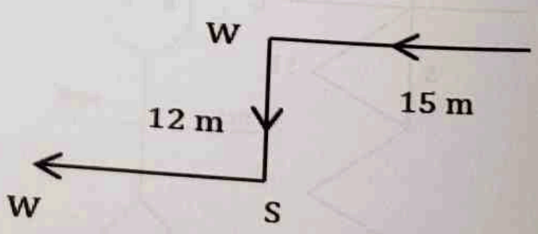
∴Ahmad's house is in the East form Sohan's house



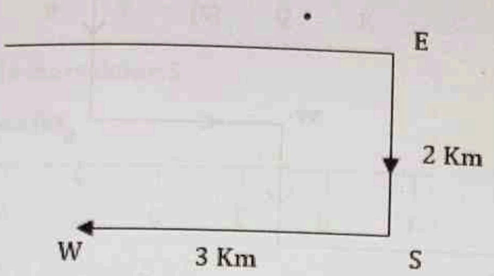
Sol.40 (b)



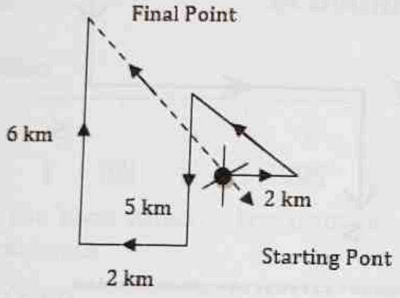
Sol.41 (b)



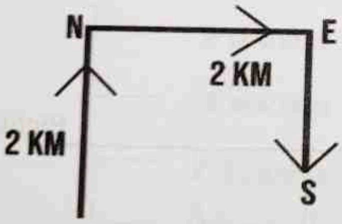
Sol. 42 (c)



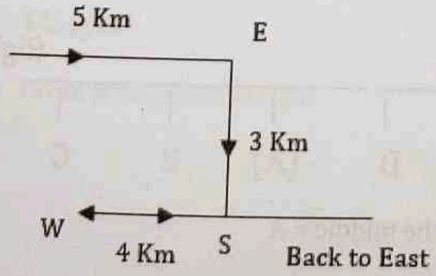
Sol.43 (c)



Sol.44 (c)

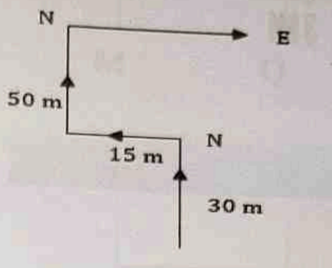


Sol.45 (a)



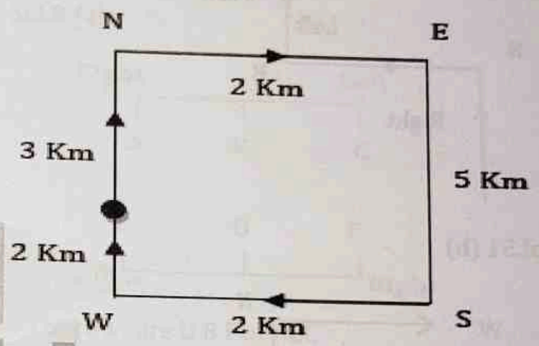
Sol.46 (b)

∴ Heading in the East direction



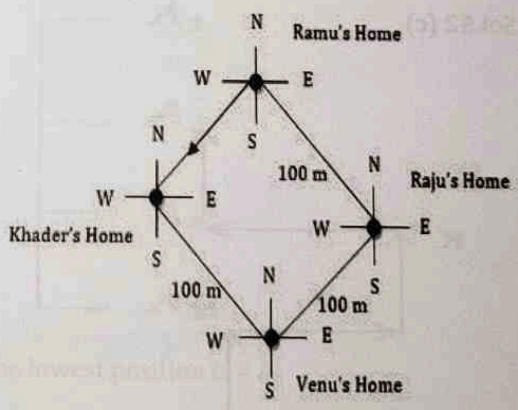
Sol.47 (a)

∴ Required direction is in the North

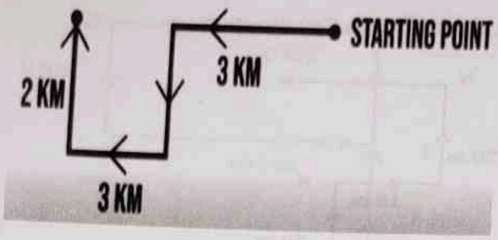


Sol.48 (b)

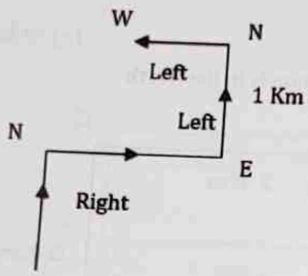
∴ Position of Khadar's home is on relation of Ramu's home is south west



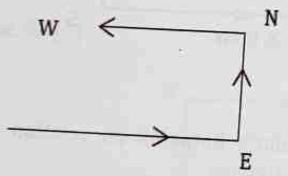
Sol.49 (b)



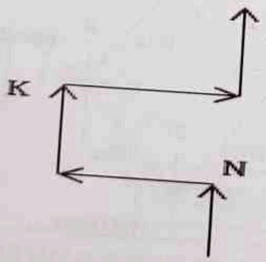
Sol.50 (b)



Sol.51 (b)

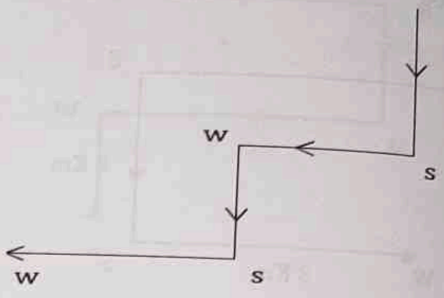


Sol.52 (c)

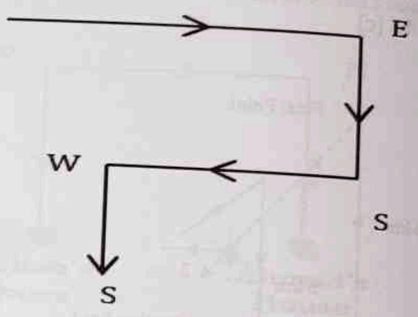


∴ Komal is walking in the North direction.

Sol.53 (c)



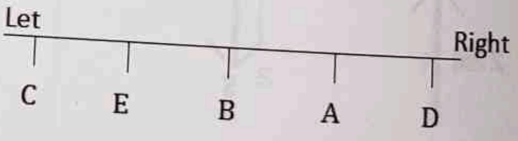
Sol.54 (b)



****

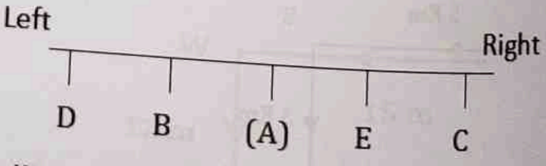
Seating Arrangement

Sol.1 (c)



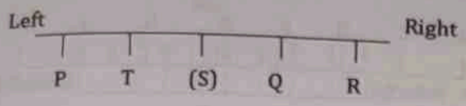
∴ Second, from the left end is E

Sol.2 (a)



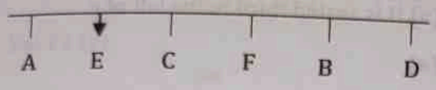
∴ House in the middle = A

Sol.3 (a)



∴ In the middle = S

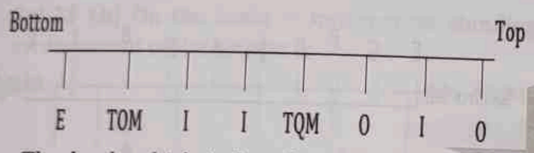
Sol.4 (b)



∴ F is between B & C

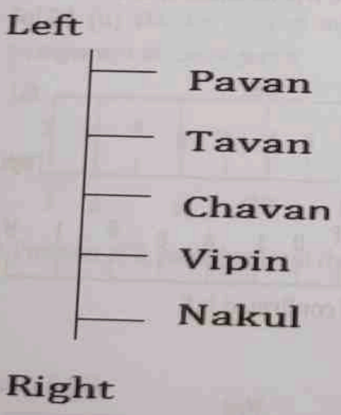
Sol.5 (a)

I → Industrial relation
O → Organization Behaviour
E → Economics



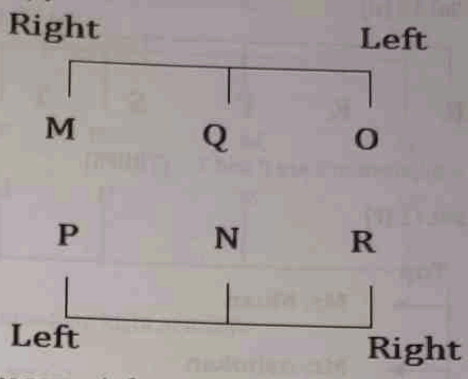
∴ The book which is last from the top = E, i.e. Economics

Sol.6 (d)



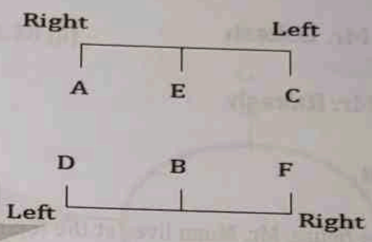
∴ Tavan is fourth from right

Sol.7 (b)



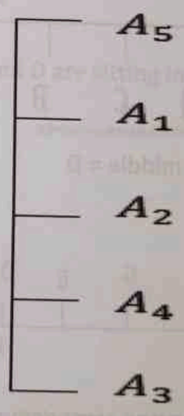
∴ The person in front of N is Q

Sol.8 (a)



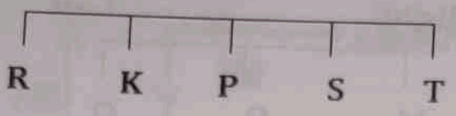
∴ The rows are D B F or AEC

Sol.9 (b)



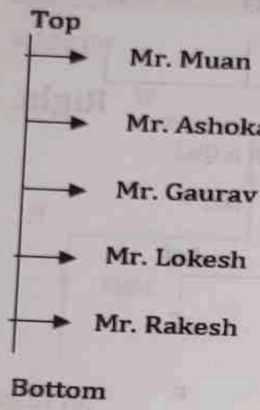
∴ The lowest position is = A_3

Sol.10 (d)



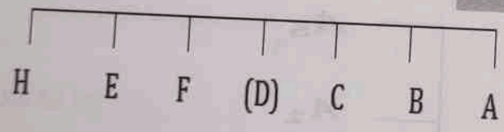
∴ Adjacent to S are P and T (TRKPS)

Sol.11 (c)



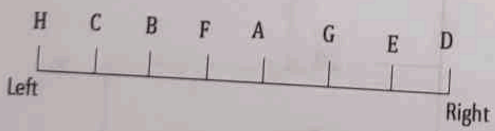
∴ From the figure, Mr. Muan lives at the top most flat.

Sol.12 (b)



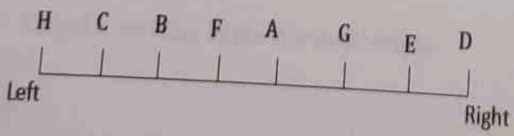
∴ The person sitting in the middle = D

Sol.13 (d)



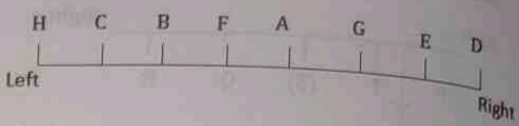
∴ Clearly, from the given seating arrangement

Sol.14 (c)



∴ Third to the right of C is A

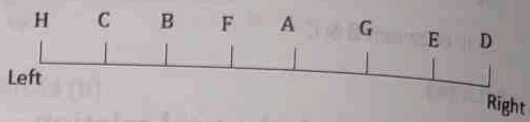
Sol.15 (d)



∴ Persons seated at the two extreme ends of the line are D&H

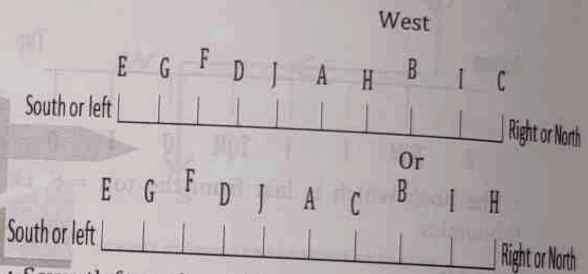
Sol.16 (a) H is seated third to the left of F

Sol.17 (a)



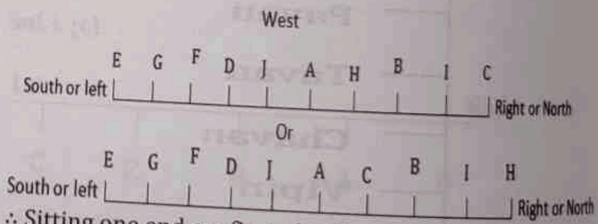
∴ Number of Person seated b/w A & F is one

Sol.18 (d)



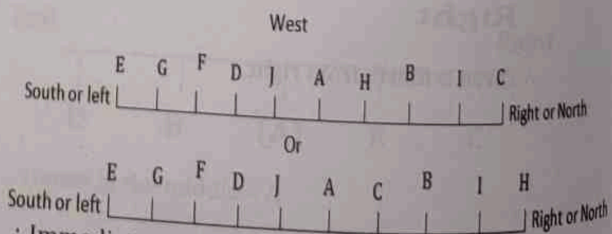
∴ Seventh from the will be either H or C

Sol.19 (c)



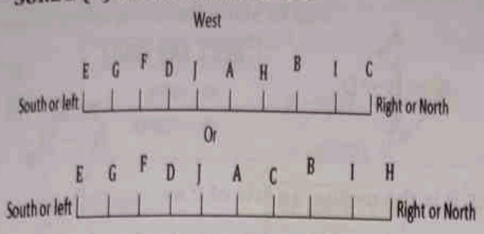
∴ Sitting one end confirmed is E

Sol.20 (d)

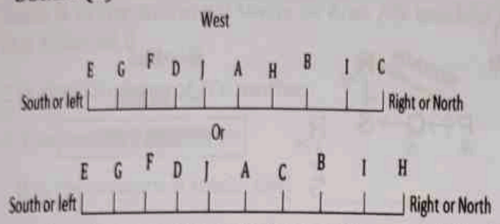


∴ Immediate neighbour of I will be BC or BH, which is not confirmed

Sol.21 (a) Second left of D is G

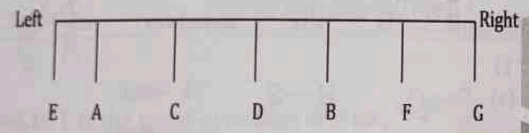


Sol.22 (c)



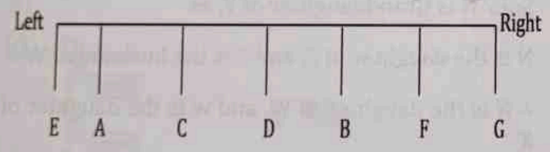
∴ After changing immediate neighbours of E is A

Sol.23 (b) On the basis of information standing arrangement of the singers is



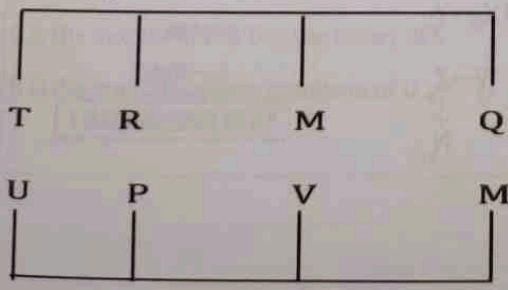
∴ The second extreme right is F

Sol.24 (d) On the basis of information standing arrangement of the singers is



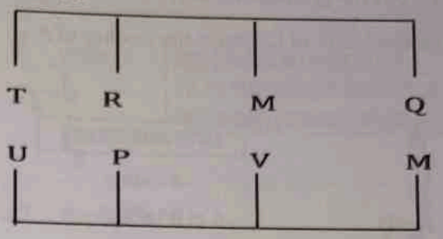
∴ Position of B, counting from the left, will be 5th

Sol. 25 (d)



∴ P is sitting in the front of R.

Sol. 26 (c)

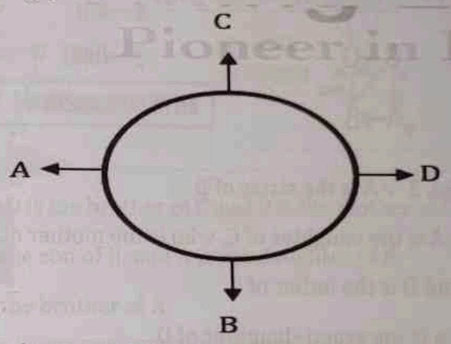


∴ Immediate Right of R is T.

Sol. 27 (a)

∴ M and V are sitting in front of each other.

Sol. 28 (a)



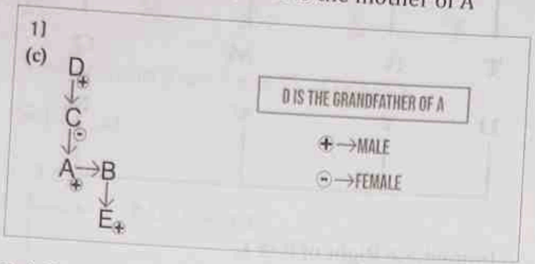
∴ A and D are sitting infront of each other.

****

Blood Relation

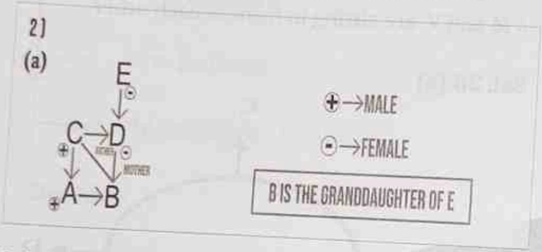
Sol.1. D is the grandfather of A

as D is the father of C, who is the mother of A



Sol. 2 B is the grand-daughter of E

as E is the mother of D, who is the mother of B

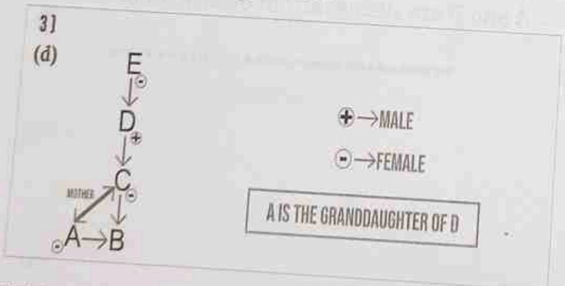


Sol. 3 ∵ A is the sister of B

∴ A is the daughter of C, who is the mother of B

And D is the father of C

∴ A is the grand-daughter of D



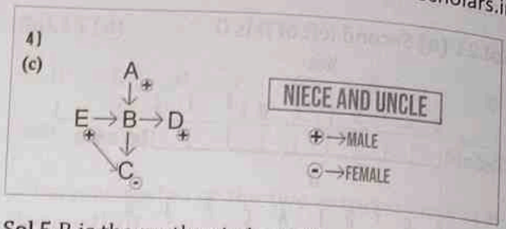
Sol.4 ∵E is the son of A

And A is the father of B

∴ E is the brother of B

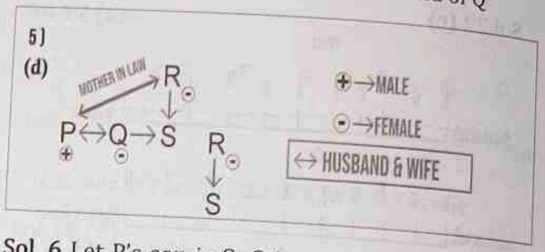
C is the daughter of B

∴ C is the niece of E & E is the uncle of C



Sol.5 R is the mother in law of P as

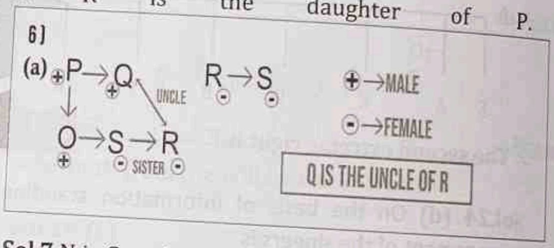
R is the mother of Q, and P is the husband of Q



Sol. 6 Let P's son is O. Q is uncle of R as Q is the brother of P and P's son is the brother of S

∴ P's son is the brother of R

∴ R is the daughter of P.



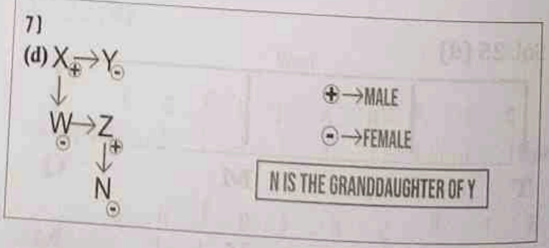
Sol.7 N is Granddaughter of Y, as

N is the daughter of Z, and Z is the husband of W

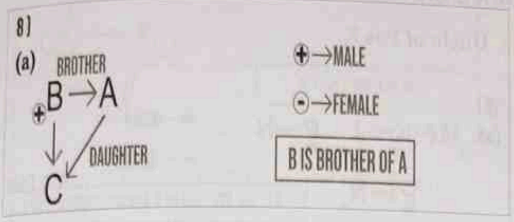
∴ N is the daughter of W. and w is the daughter of X

∴ N is the granddaughter of X

But X is the husband of Y



Sol. 8 B is the brother of A as C is the daughter of A, and B is the parental uncle of C.

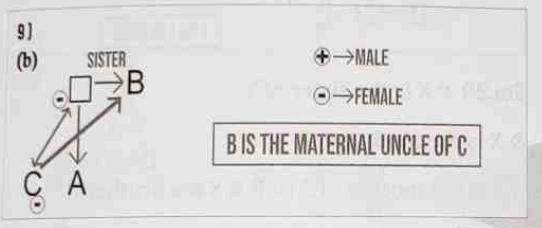


Sol.9 B is the maternal uncle of A as A's mother is the sister of B

C is the daughter of A's mother

∴ C is the sister of A

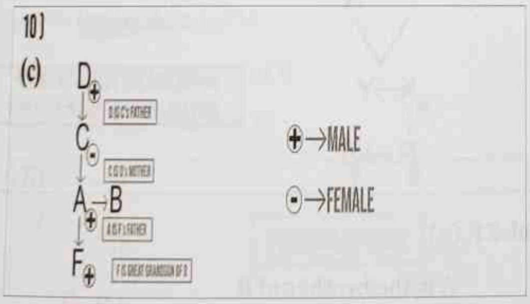
∴ B is the maternal uncle of C



Sol.10 F is the great-grandson of D

as F is the son of A & C is the mother of A

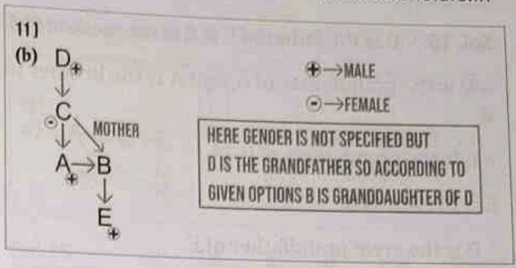
∴ F is the grandson of C and D is the father of C



Sol. 11 ∵ B's brother is A & C is the mother of A

∴ C is the mother of B & D is the father of C

∴ B is the granddaughter/grandson of D

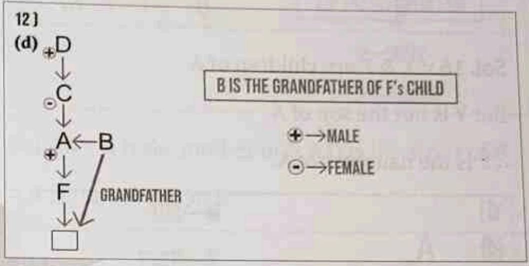


Sol.12 ∵ Brother of B is A

F is the son of A

∴ A is the grandfather of F's child

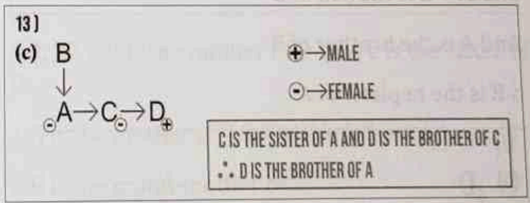
∴ B is the grandfather of F's child



Sol.13 D is the brother of C and B is the mother of C

∴ D is the son of B, and A is the daughter of B

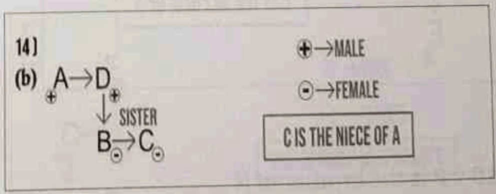
∴ D is the brother of A



Sol. 14 ∵ D is the father of B & C is the sister of B

∴ D is the father of C & A is the brother of D

∴ C is the Niece of A.



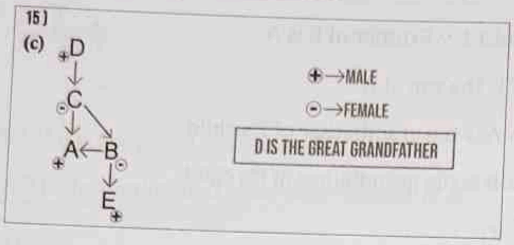
Sol. 15 ∵ D is the father of C & C is the mother of A

∴ D is the grandfather of A, and A is the brother of B

∴ D is the grandfather of B

E is the son of B

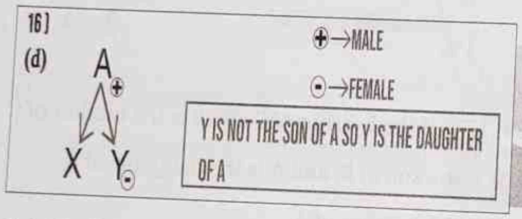
∴ D is the great grandfather of E



Sol. 16 ∵ *X* & *Y* are children of A

But Y is not the son of A

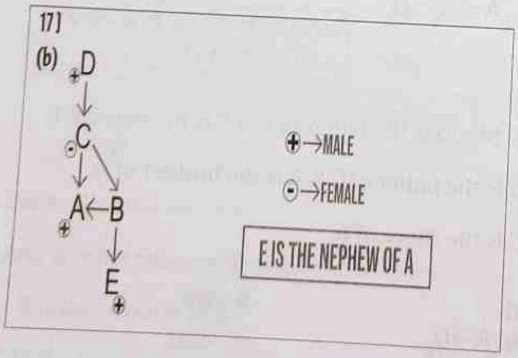
∴ Y is the daughter of A.



Sol. 17∵ E is the son of B

And A is the brother of B

∴ E is the nephew of A.



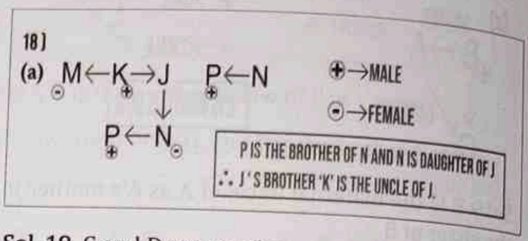
Sol. 18 ∵ P is the brother of N

And N is the daughter of J

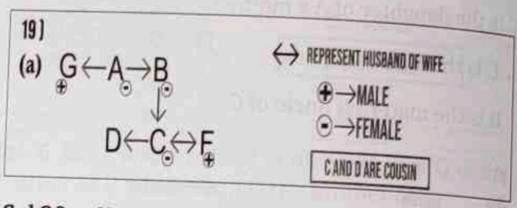
∴ P is the son of J

& K is the brother of J

∴ Uncle of P is K.



Sol. 19. C and D are cousins.

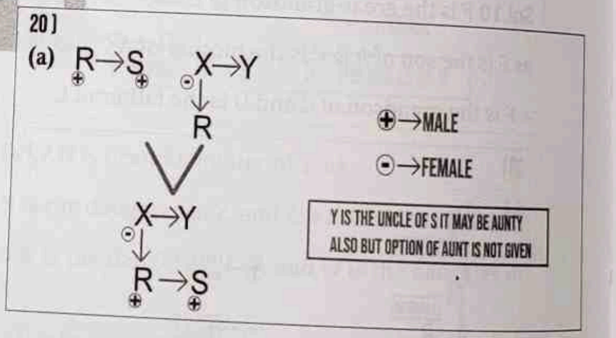


Sol.20 ∵ X is the sister of Y

& X is the mother of R

∴ X is the mother of S (∵ R & S are brothers)

∴ Y is the uncle of S



Sol. 21. (a)

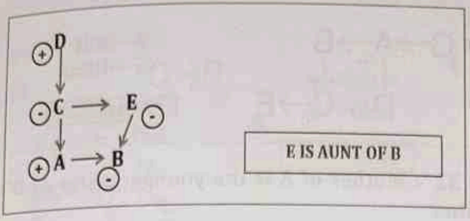
A is the brother of B

C is the mother of A & B

D is the father of C

E is Sister of C

∴ E is Aunt of B



Sol. 22 ∵ B & C are sisters to one another

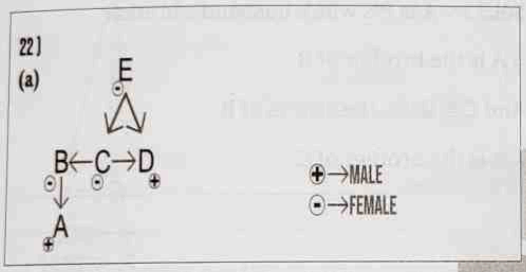
E is the mother of C

D is the son of E

∴ D is the brother of B and C

A is the son of B

∴ D is the maternal uncle of A



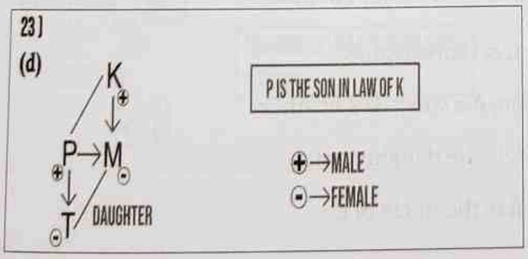
Sol. 23 P is the father of T

And T is the daughter of M

∴ P is the husband of M

And M is the daughter of K

∴ P is the son in law of K



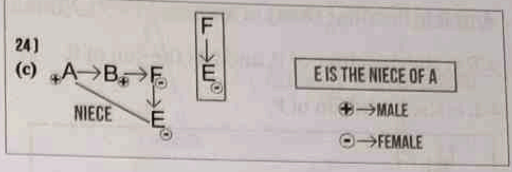
Sol. 24 ∵ E is the daughter of F

F is the wife of B

∴ E is the daughter of B

A is the brother of B

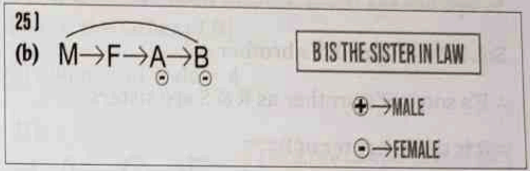
∴ E is the niece of A



Sol.25 ∵ A & B are sister A & A is the sister of F

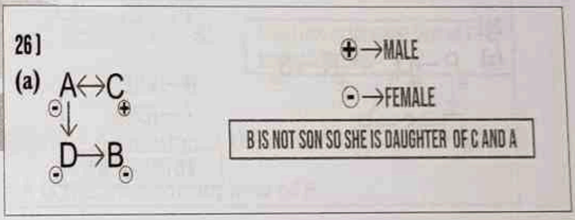
M & F are a married couple

∴ B is the sister-in-law of M



Sol. 26 ∵ A is the mother of D & D is the sister of B

∴ A is the mother of B



Sol. 27 ∴ E is the mother of D, and c is the sister of D

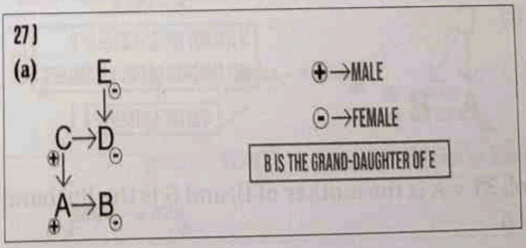
∴ E is the mother of C & C is the father of A

∴ E is the grand-mother of A

& B is the sister of A

∴ E is the grandmother of B

∴ B is the granddaughter of E.

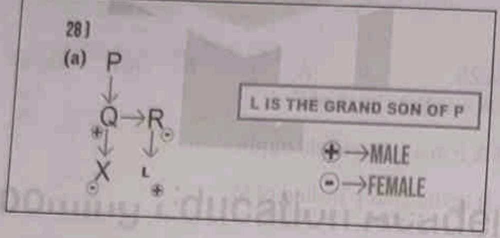


Sol. 28 ∵ Q is the son of P & X is the daughter of Q

And R is the Aunt (Bua) of X

∴ R is the daughter of P, and L is the son of R

∴ L is the grandson of P.



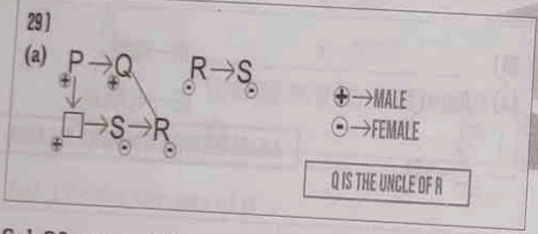
Sol. 29 ∵ P's son is S's brother

∴ P's son is R's brother as R & S are sisters

∴ R is the daughter of P.

And Q is the brother of P.

∴ Q is the uncle of R



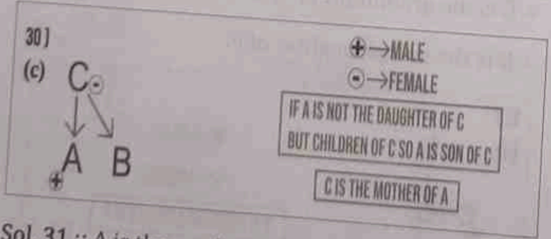
Sol. 30 ∵ A and B are the young ones of C

and C is the mother of B

∴ C is the mother of A, and A is not the daughter of C

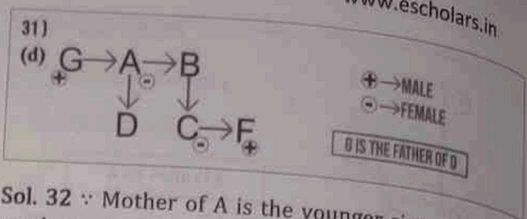
∴ A is the son of C

Hence C & A are mother & son



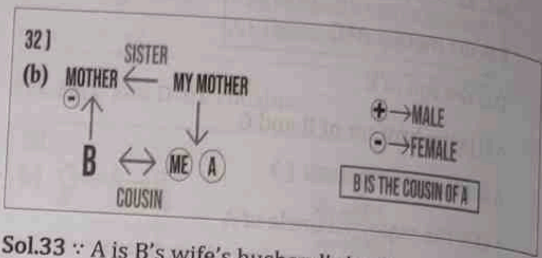
Sol. 31 ∵ A is the mother of D, and G is the husband of A

∴ G is the father of D



Sol. 32 ∵ Mother of A is the younger sister of B's mother

∴ A is the cousin of B

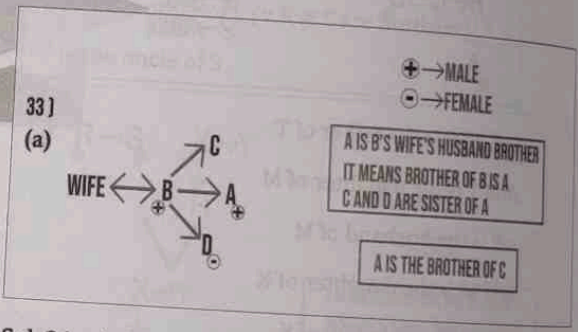


Sol.33 ∵ A is B's wife's husband's brother

∴ A is the brother of B

And C & D are the sisters of B

∴ A is the brother of C



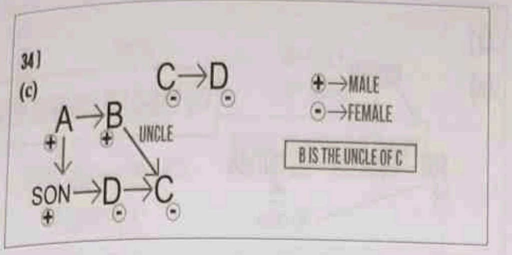
Sol. 34 ∵ A & B are brothers

& C & D are sisters

Also, A's son is D's brother

∴ C is the daughter of A

∴ B is the uncle of C

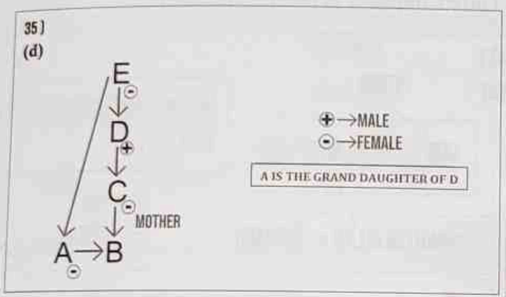


Sol. 35 ∵ A is B's sister & C is B's Mother

∴ A is the daughter of C

And D is C's father

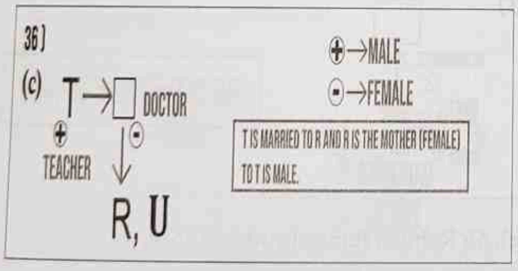
∴ A is the grand-daughter of D

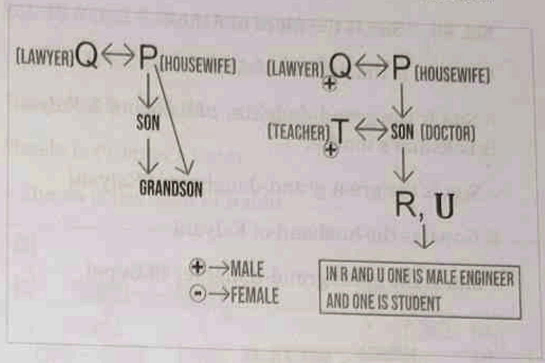


Sol. 36∵ The family has two married couple

∴ Grand-daughter is not a lawyer, not an engineer & not a doctor

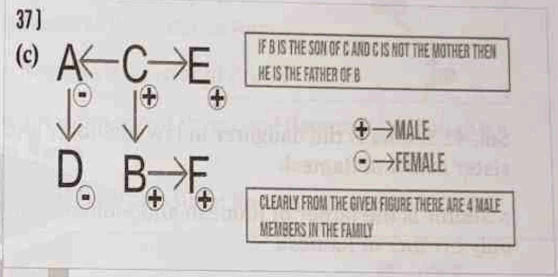
∴ Grand-daughter will be a student





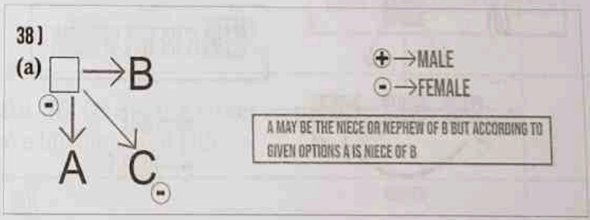
Sol. 37[∵ B is the son of C & C is not the mother of B ∴ C is the father of B]

∴ Total nos. of male = 4



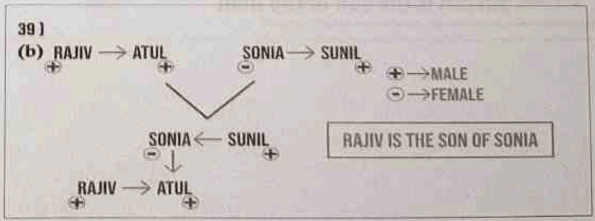
Sol. 38 ∵ A's mother is the sister of B

∴ A is the niece or nephew of B.



Sol. 39 ∵Rajiv is the bother of Atul and Atul is the son of Sonia

∴ Rajiv is the son of Sonia



Sol. 40 ∵ Sita is the niece of Ashok

Lakshmi is the mother of Ashok

∴ Sita is the grand-daughter of Lakshmi & Kalyani is Lakshmi's mother

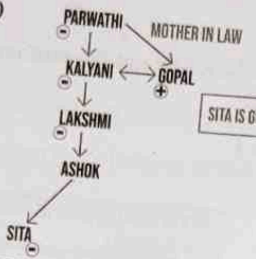
∴ Sita is the great grand-daughter of Kalyani

& Gopal is the husband of Kalyani

∴ Sita is the great grand-daughter of Gopal

40)

(c)



SITA IS GOPAL'S GREAT GRAND DAUGHTER

⊕→MALE
⊖→FEMALE

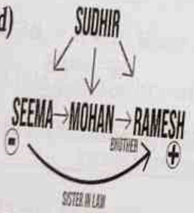
Sol. 41 Seema is the daughter in law of Sudhir and sister in law of Ramesh

∴ Sudhir is the father of Ramesh and Mohan is the only brother of Ramesh

∴ Seema is the wife of Mohan

41)

(d)



SEEMA IS THE WIFE OF MOHAN

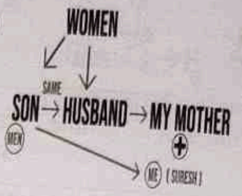
⊕→MALE
⊖→FEMALE

Sol. 42∵ The man is the son of a woman, and the woman is the mother of husband of Suresh's mother

∴ Suresh is the son of the man

42)

(a)



THE MAN IS THE FATHER OF SURESH

⊕→MALE
⊖→FEMALE

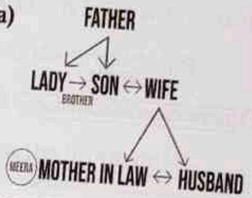
Sol. 43 ∵The lady's father's only son's wife is the mother in law of Meera

∴ Meera's husband is the lady's father's only son's son

∴ Lady's nephew is Meera's husband.

43)

(a)



MEERA'S HUSBAND IS NEPHEW OF THAT LADY

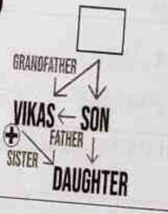
⊕→MALE
⊖→FEMALE

Sol.44 ∵ The daughter of Vikas's grand-father only son

∴ She is the sister of Vikas

44)

(c)



⊕→MALE
⊖→FEMALE

SHE IS THE SISTER OF VIKAS

Sol. 45 Rohit is Rani's brother's son

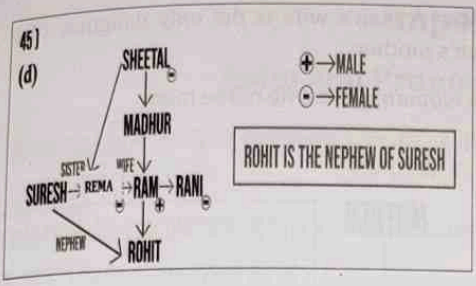
& Ram is the brother of Rani

∴ Rohit is the son of Ram

Suresh's sister is the wife of Ram

∴ Suresh's sister of the mother of Rohit

∴ Rohit is the nephew of Suresh

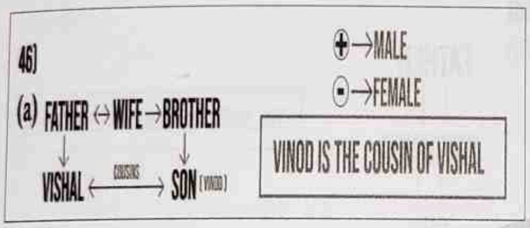


Sol.46∵ Vinod's father's wife will be Vinod's mother

Vishal is the son of the brother of Vinod's mother

Vishal will be the cousin of Vinod

∴ Vinod is the cousin of Vishal



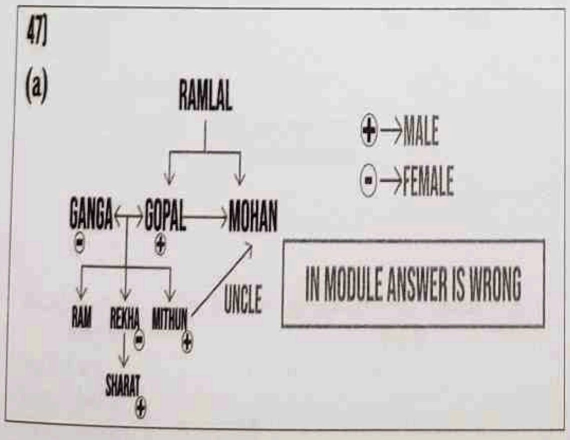
Sol.47 ∵ Mithun is the uncle of Sharat

Rekha is the mother of Sharat

Mohan is the brother of Gopal & Gopal is the husband of Ganga, and Ganga's husband's brother is Mohan

∴ Mohan is the uncle of Rekha

∴ Mohan is the uncle of Mithun.



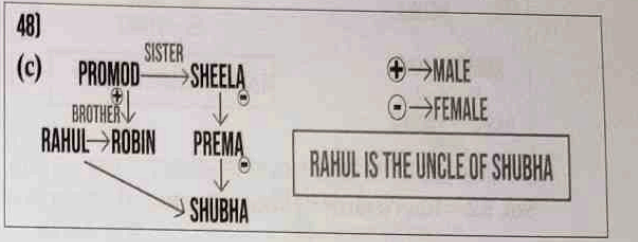
Sol. 48 Rahul & Robin are brother

Pramod is the father of Robin

∴ Promod is the father of Rahul

Sheela is Pramod's sister

∴ Sheela is the Aunt of Rahul



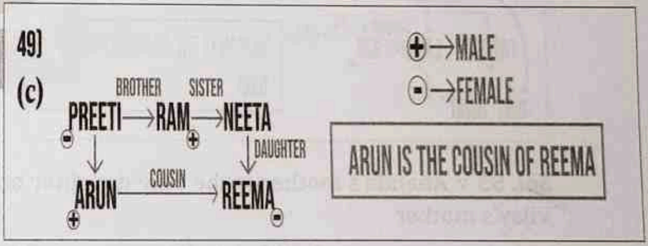
Sol. 49 Preeti's brother is Ram

& Neeta is the sister of Ram

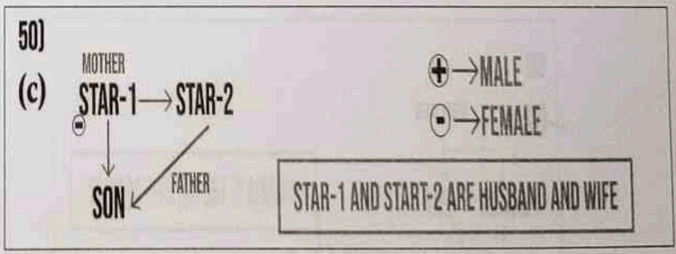
∴ Neeta is the sister of Preeti

Arun is the son of Preeti and Reema is the daughter of Neeta

∴ Arun is the cousin of Reema

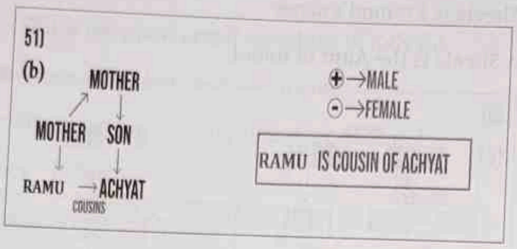


Sol. 50. One is the father of the other son, so they are husband and wife.



Sol.51 Achyat is the brother's son of Ramu's mother

∴ Ramu is the cousin of Achyat or Achyat is the cousin of Ramu



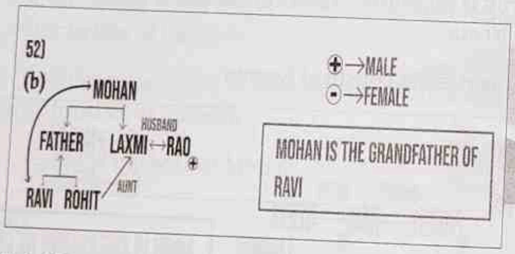
Sol. 52 ∵ Ravi's father's son is Rohit

& Rohit's Aunt is Laxmi

∴ Ravi's Aunt is Laxmi

Laxmi's husband's father in law is Mohan

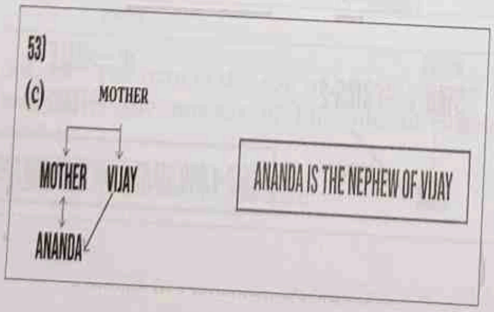
∴ Mohan is the grand-father of Ravi



Sol. 53 ∵ Ananda's mother is the only daughter of Vijay's mother

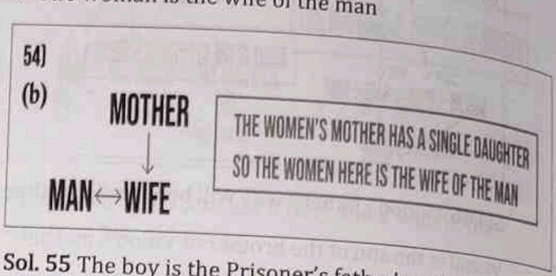
∴ Ananda's mother is the sister of Vijay

∴ Ananda is the nephew of Vijay



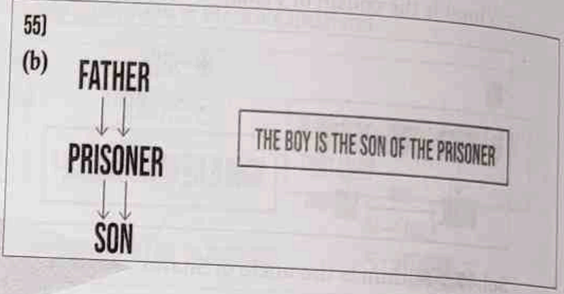
Sol. 54 ∵ A man's wife is the only daughter of a woman's mother

∴ The woman is the wife of the man



Sol. 55 The boy is the Prisoner's father's son's son

∴ The boy is the son of Prisoner



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Important Questions

Ratio and Proportion Indices and Logarithms

Q.1) Division of ₹324 between X and Y is in the ratio 11 : 7. X & Y would get Rupees;

a) ₹204, ₹120 **b)** ₹200, ₹124 **c)** ₹200, ₹124 [✓ **d)** ₹198, ₹126]

Sol. (d)

Let X gets 11k rupees & Y gets 7k rupees

$11k + 7k = 324 \Longrightarrow 18k = 324$

$\Longrightarrow k = \frac{324}{18} = 18$

$\therefore$ X gets $= 11k = 11(18) =$ ₹ 198

Y gets $= 7k = 7(11) =$ ₹ 126

Q.2) P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12, and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is;

a) 22 : 27 [✓ **b)** 27 : 22] **c)** 32 : 33 **d)** None of these

Sol. (b)

$\frac{P}{Q} = \frac{11}{12} \& \frac{P}{R} = \frac{9}{8}$

$\therefore$ Make P equal in both ratios

$= \frac{P}{Q} = \frac{11\times9}{12\times9} = \frac{99}{108}$

$= \frac{P}{R} = \frac{9\times11}{8\times11} = \frac{99}{88}$

$\therefore$ Q: R = 108: 88 $\Longrightarrow$ 27: 22

Q.3) The ratio compounded of the duplicate ratio of 4 : 5, the triplicate ratio of 1 : 3, sub duplicate ratio of 81 : 256 and the sub-triplicate ratio of 125 : 512 is;

a) 4 : 512 **b)** 3 : 32 **c)** 1 : 12 [✓ **d)** 1 : 120]

Sol. (d)

$$\text{Compounded ratio} = \frac{4^2}{5^2} \times \frac{1^3}{3^3} \times \frac{\sqrt{81}}{\sqrt{256}} \times \frac{\sqrt[3]{125}}{\sqrt[3]{512}}$$

$$= \frac{16}{25} \times \frac{1}{27} \times \frac{9}{16} \times \frac{5}{8} = \frac{1}{120}$$

$$= 1: 120$$

Q.4) If p : q = 2 : 3 and x : y = 4 : 5, then the value of 5px + 3qy : 10px + 4qy is;

a) 71 : 82 **b)** 27 : 28 [✓ **c)** 17 : 28] **d)** None of these

Sol. (c)

$\frac{p}{q} = \frac{2}{3} \ \& \ \frac{x}{y} = \frac{4}{5}$

$\therefore \frac{5px+3qy}{10px+4qy} \ \frac{5(2)(4)+3(3)(5)}{10(2)(4)+4(3)(5)}$

$= \frac{40+45}{80+60} = \frac{85}{140}$

$= \frac{17}{28}$

Q.5) 4, *, 9, 13½ are in proportion. Then * is;

[✓ **a)** 6] **b)** 8 **c)** 9 **d)** None of these

Sol. (a)

Let * be x

Proportion a : b :: c : d

$\frac{4}{x} = \frac{9}{13\frac{1}{2}} \Rightarrow x = \frac{4\times\frac{27}{2}}{9}$

$= x = 6$

Q.6) If $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{x}$ are in proportion then x *is*

a) $\frac{15}{2}$ **b)** $\frac{3}{15}$ **c)** $\frac{2}{15}$ **d)** $\frac{1}{15}$

Sol. (a)

Product of middle two terms = Product of extremes

So, $\frac{1}{2x} = \frac{1}{15}$; $x = \frac{15}{2}$

Q.7) The number which when subtracted from each of the terms of the ratio 19 : 31 reducing into 1 : 4 is ;

a) 15 **b)** 5 **c)** 1 **d)** None of these

Sol. (a)

$$\frac{19-x}{31-x} = \frac{1}{4}$$

$\Rightarrow 76 - 4x = 31 - x$

$\Rightarrow 3x = 76 - 31 \Rightarrow x = \frac{45}{3} = 15$

Q.8) A jar contains black and white marbles. If there are ten marbles in the jar, then which of the following could not be the ratio of black to white marbles?

a) 9 : 1 **b)** 7 : 3 **c)** 1 : 10 **d)** 1 : 4

Sol. (c)

1 : 10 ⇒ (There are 10 marbles in the jar, So the ration can't be more than 10)

Q.9) If 2A = 3B and 4B = 5C, then A : C is:

a) 4 : 3 **b)** 15 : 8 **c)** 8 : 15 **d)** 3 : 4

Sol. (b)

2A = 3B and 4B = 5C $\Rightarrow \frac{A}{B} = \frac{3}{2}$ *and* $\frac{B}{C} = \frac{5}{4} \Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{3}{2} \times \frac{5}{4}\right)$

$= \frac{15}{8} \Rightarrow A : C = 15 : 8$

Q.10) An alloy is to contain copper and zinc in the ratio 9 : 4. The zinc required to mix with 24kg of copper is:

a) $10\frac{2}{3}kg$ **b)** $10\frac{1}{3}kg$ **c)** $9\frac{2}{3}kg$ **d)** 9 kg

Sol. (a)

Let zinc in alloy is x kg

$\therefore \frac{9}{4} = \frac{24}{x} \therefore x = \frac{4\times24}{9} = \frac{32}{3}$

$\Rightarrow x = 10\frac{2}{3}kg$

Q.11) A three-digit number is such that this number itself is divisible by the sum of its digits. The sum of hundreds and unit digits is 6, while the sum of the tens and unit digit is 5. What is the ratio of the unit and tens digit:

a) 1:2 **b)** 3:4 **c)** 2:3 **d)** 2:7

Sol. (c)

Let the number be $100x + 10y + z$, then

$x + z = 6$ *and* $y + z = 5$

∴ from the given options only option (c) is suitable

i.e., $y + z = 5$

or 3+2 = 5

Q.12) The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 5 hours, the speed of the first train is;

a) 10 Km/hr **b)** 50 Km/hr [✓ **c)** 70 Km/hr] **d)** None of these

Sol. (c)

Let the speed of 1st train be x km/h

Speed of 2nd train $= \frac{400}{5}\frac{km}{h} = 80\ km/h$ $[\because speed = \frac{Distance}{Time}]$

$$= \frac{x}{80} = \frac{7}{8} \Rightarrow \frac{7}{8} \times 80 = 70\ km/h$$

Q.13) In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3:1. The quantity of water added in the mixture in order to make this ratio 2:1 is:

a) 15 litres **b)** 10 litres **c)** 8 litres [✓ **d)** 5 litres]

Sol. (d)

Glycerine $= \frac{40}{3+1} \times 3 = 30\ litres.$

Water $= \frac{40}{4} \times 1 = 10\ litres$

Let x litres of water is added to the mixture

Then, $\frac{30}{10+x} = \frac{2}{1}$

Or, $2x + 20 = 30$ or $x = 5$

Q.14) The income of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5:3. If each saves ₹ 1,500, then B's income is:

a) ₹ 6,000 **b)** ₹ 4,500 **c)** ₹ 3,000 **d)** ₹ 7,500

Sol. (a)

Let x is common in the ratio

$\therefore A's\ income = 3x$

B's income $= 2x$

$\therefore \frac{3x-1500}{2x-1500} = \frac{5}{3}$

$\Rightarrow 10x - 7500 = 9x - 4500$

$\Rightarrow 10x - 9x = 7500 - 4500$

$\Rightarrow x = 3000$

B's income $= 2x =$ ₹ 6000

Q.15) If x varies inversely as square of y and given that $y = 2$ for $x = 1$, then the value of x for $y = 6$ will be;

a) 3 **b)** 9 **c)** 1/3 **d)** 1/9

Sol. (d)

$x \propto \frac{1}{y^2} \Rightarrow x = k.\frac{1}{y^2} \Rightarrow x = \frac{k}{y^2}; where\ k = proportional\ constant$

When $x = 1$ and y = 2

$\therefore 1 = \frac{k}{2^2} \Rightarrow k = 4 \quad \therefore x = \frac{4}{y^2}$

When y = 6, Then $x = \frac{4}{6^2} = \frac{1}{9}$

$\therefore x = \frac{1}{9}$

Q.16) The ratio of the rate of flow of water in pipes varies inversely as the square of the radius of the pipes. What is the ratio of the rates of flow in two pipes of diameters 2 cm and 4 cm?

a) 1 : 2 **b)** 2 :1 **c)** 1 : 8 **d)** 4 : 1

Sol. (d)

The radii of the two pipes are 1 cm and 2 cm. The square of the radii of the two pipes is 1 cm and 4 cm.

∴ Rates of the flow of the pipes are in the ratio $1 : \frac{1}{4}, i.e., 4 : 1$

Q. 17) The ratio of the number of boys to the number of girls in a school of 720 students is 3:5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2:3.

a) 102 **b)** 21 **c)** 42 **d)** 40

Sol. (c)

The number of the boys $= 720 \times \frac{3}{8} = 270$

The number of the girls $= 720 \times \frac{5}{8} = 450$

Let the number of new boys admitted be x, and the total number of boys become $(270+ x)$
Total number of girls, after admitted 18 new girls be (450+18)= 468.

$= \frac{270 + x}{468} = \frac{2}{3}$

$= 810 + 3x = 936$

$= 3x = 126$

$= x = 42$

Q.18) The sum of the ages of 3 persons is 150 years. 10 years ago, their ages were in the ratio 7 : 8 : 9. Their present ages are;

a) (45, 50, 55) **b)** (40, 60, 50) **c)** (35, 45, 70) **d)** None of these

Sol. (a)

Let the present ages of three person in years be 7k +10, 8k + 10 & 9k+10

$\therefore (7k+10) + (8k +10) + (9k+10) = 150$

$\Rightarrow 24k + 30 = 150 \Rightarrow 24k = 120$

$\Rightarrow k = \frac{120}{24} = 5$

$\therefore 7k +10 = 45$

$8k +10 = 50$

$9k + 10 = 55$

Q.19) Total price of 7 bananas and 4 mangoes is equal to the total price of 5 mangoes and 3 apples. The price of two apples is equal to that of three bananas. Find the ratio of the price of one mango and one banana.

a) 3:2 **b)** 5:2 **c)** 4:3 **d)** 7:5

Sol. (b)

Let price of one banana be ₹ x

Price of one mango be ₹ y

Price of one apple be ₹ z

Given, $7x + 4y = 5y + 3z$

$\Rightarrow 7x + 4y = 5y + 3 \times \frac{3}{2}x$ $(2z = 3x)$

$\Rightarrow 7x - \frac{9}{2}x = y$ $(z = \frac{3}{2}x)$

$\Rightarrow 5x = 2y$

$\Rightarrow \frac{x}{y} = \frac{2}{5}$ $[x = 2,\ y = 5\ and\ z = \frac{3}{2} \times 2 = 3]$

$\Rightarrow y : x = 5 : 2$

∴ Required ratio is 5 : 2

Q.20) Half of the girls and one-third of the boys of a college reside in the hostel. What fractional part of the student body is hostel dwellers if the total number of girls in the college is 100 and is 1/4th of the total strength?

a) 2/ 5 **b)** 5 /12 **c)** 1/ 5 **d)** 3/ 8

Sol. (d)

Total number of girls = 100

Total girls = ¼ (Total no. of students)

Total number of students = $100 \times 4 = 400$

Number of boys = Total students – Total girls

$= 400 - 100 = 300$

∴ Number of boys = 300

∴ Number of hostel dwellers = 50 + 100 = 150

∴ Required ratio = $150 : 400 = 3 : 8 = \frac{3}{8}$

Q.21) The price of entry tickets at a fun park was increased in the ratio 7 : 9, due to which footfalls fell in the ratio 13 : 11. What is the new daily collection (in Rs.) if the daily collection before the price hike was ₹ 2,27,500?

a) ₹ 2,37,500 **b)** ₹ 2,47,500 **c)** ₹ 2,32,500 **d)** ₹ 2,42,500

Sol. (b)

Daily Collection = Price of one ticket × Footfall per day

∴ Ratio of daily collection before and after change = $(7 \times 13) : (9 \times 11) = 91 : 99$

The daily collection before the price hike was ₹ 2,27,500

∴ New daily collection = 2,27,500 × 99/91 = ₹ 2,47,500

Q.22) The ratio of the present ages of son and his father is 1: 5 and that of his mother and father is 4: 5, after two years the ratio of the age of the son to that of his mother becomes 3: 10, what is the present age of the father?

a) 35 years **b)** 28 years **c)** 37 years **d)** None of these

Sol. (a)

Ratio age of father : age of son = 5 : 1

And ratio of age of father and age of mother = 5 : 4

Hence, ratio of age of son : age of mother = 1 : 4

Let the present age of son, mother and father is x, $4\,x$ and $5\,x$

$\Rightarrow (x + 2) : (4x + 2) = 3 : 10$

$\Rightarrow 10x + 20 = 12x + 6$

$\Rightarrow x = 7$

∴ The present age of father is 5×7 = 35 years

Q.23) Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

a) 2 : 3 : 4 **b)** 6 : 7 : 8 **c)** 6 : 8 : 9 **d)** None of these

Sol. (a)

Let the original number of seats in Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.

The number of increased seats is: (140% of $5x$), (150% of $7x$) and (175% of $8x$)

$$i.e., \left[\frac{140}{100} \times 5x\right], \left[\frac{150}{100} \times 7x\right] \text{ and } \left[\frac{175}{100} \times 8x\right] \text{ or } 7x, \frac{21x}{2} \text{ and } 14x.$$

∴ **Required ratio** = $7x : \frac{21x}{2} : 14x = 14x : 21x : 28x = 2 : 3 : 4$

Q.24) If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ *then*

(b – c) x + (c – a) y+ (a – b) z is;

a) 1

b) 0

c) 5

d) None of these

Sol. (b)

$\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ = k(let)

$\therefore$ (b - c) x + (c - a) y + (a -b) z

$\Rightarrow x = k(b+c-a),\ y = k(c+a-b), z = k(a+b-c)$

$= k\,[(b-c)(b+c-a) + (c-a)(c+a-b) + (a-b)(a+b-c)$

$= k\,[b^2 - c^2 - ab + ac + c^2 - a^2 - bc + ab + a^2 - b^2 - ca + bc]$

$= k \times 0 = 0$

Q.25) What quantity must be added to the terms of the ratio $p+q : p-q$ to make it equal to $\frac{(p+q)^2}{(p-q)^2}$

a) $(q+p)/2p$

b) $(q-p)/2p$

c) $(q^2-p^2)/2p$

d) None of these

Sol. (c)

Let x be the required quantity: $\therefore \frac{(p+q)+x}{(p-q)+x} = \frac{(p+q)^2}{(p-q)^2}$

$\Rightarrow (p+q)(p-q)^2 + x(p-q)^2 = (p+q)^2(p-q) + x(p+q)^2$

$\Rightarrow (p+q)(p-q)^2 - (p+q)^2(p-q) = x\{(p+q)^2 - (p-q)^2\}$

$\Rightarrow (p+q)(p-q)[p-q-p-q] = x(4pq) \Rightarrow (p^2-q^2)(-2q) = x\,(4pq)$

$\Rightarrow (q^2-p^2) = x(2p) \Rightarrow x = \frac{q^2-p^2}{2p}.$

Q.26) If $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$ *then* $\frac{a+b+c}{c}$ *is;*

a) 4

b) 2

c) 7

d) None of these

Sol. (b)

let $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$ = k

$a = 4k, b = 5k, c = 9k$

$\therefore \frac{a+b+c}{c} = \frac{4k+5k+9k}{9k} = \frac{18k}{9k} = 2$

Or

$\frac{a}{4} = \frac{b}{5} = \frac{c}{9} \Rightarrow$ a:b:c = 4:5:9

$\therefore \frac{a+b+c}{c} = \frac{4+5+9}{9} = \frac{18}{9} = 2$

Q.27) Four numbers are in proportion. The sum of the squares of the four numbers is 50, and the sum of the means is 5. The ratio of the first two terms is 1:3. What is the ratio of the third and fourth terms?

a) 3:1

b) 1:3

c) 2:4

d) 4:2

Sol. (b)

Let the four numbers be $a : b :: c : d$

Given, $a^2 + b^2 + c^2 + d^2 = 50$

$b + c = 5$

and $a : b = 1 : 3$

It is consider $a : b = 1 : 3$ as it is, then

c = 2 (5 – 3 = 2)

$\Rightarrow$1 : 3 :: 2 : d

$\Rightarrow \frac{1}{3} = \frac{2}{d}$

$\Rightarrow d = 6$ (∵ a : b :: c : d)

∴ The ratio of c and d is 2:6 = 1:3

OR

The ratio of a and b is the same as ratio of c and d. Hence, the answer will be 1:3.

Q.28) Four friends A, B, C and D have some marbles with them. The ratio of the number of marbles with C and D is 7:6. B has one marble less than that with D. The ratio of the number of marbles with A and C is 12:7. Which of the following cannot be the total number of marbles with them?

a) 30 **b)** 185 **c)** 309 **d)** 155

Sol. (d)

Let the number of marbles with C and D be $7x$ and $6x$, respectively.

The number of marbles with B is $6x - 1$.

The number of marbles with A is $12x$

Total number of marbles with them = $12x + 6x - 1 + 7x + 6x = 31x - 1$

The number of marbles can be 30, 185, 309 when the value of x is 1, 6 and 10 respectively.

∴ Total number of marbles cannot be 155

Q.29) If a, b, c and d are proportional then mean proportion between $a^2 + c^2$ and $b^2 + d^2$ is:

a) ac/bd **b)** ab + cd **c)** a/b + d/c **d)** None of these

Sol. (b)

a : b : : c : d

⇒ ad = bc ---(i)

Now mean proportion between $(a^2 + c^2)$ and $(b^2 + d^2)$

$= \sqrt{(a^2 + c^2) \times (b^2 + d^2)}$

$= \sqrt{(ab)^2 + a^2d^2 + b^2c^2 + c^2d^2}$

$= \sqrt{(ab)^2 + (cd)^2 + 2a^2d^2}$

$= \sqrt{(ab)^2 + (cd)^2 + 2abcd}$ (∵ad = bc)

= (ab + cd)

Q.30) There are 19068 employees in 4 companies altogether. If two-third of the first, five-seventh of the second, one and half of the third and one-fourth of the fourth are the same number of employees, then find the ratio of the number of employees of A and D if A, B, C and D be the first, second, third and fourth companies respectively.

a) 8 : 3 **b)** 35 : 83 **c)** 83 : 35 [✓ **d)** 3 : 8]

Sol. (d)

Let $\frac{2}{3}A = \frac{5}{7}B = \frac{1}{2}C = \frac{1}{4}D = k$

$\Rightarrow A:B:C:D = \frac{3}{2}k:\frac{7}{5}k:\frac{2}{1}k:\frac{4}{1}k$

$\Rightarrow A:B:C:D = 15:14:20:40$

∴ A:D = 3 : 8

Q.31) In a province, the cities are divided into 3 categories, A, B and C. The ratio of the number of cities of type A and B is 6: 11, while that of cities of type B and C is 3: 12. If the number of cities of type A is 486, then what is the total number of cities in the province?

a) 4900 [✓ **b)** 4941] **c)** 4000 **d)** 4321

Sol. (b)

A: B = 6: 11 and B: C = 3: 12

Combined ratio of A:B:C = 18: 33: 132

Let the no. of cities of type A, B and C is $18x$, $33x$ and $132x$ respectively

So, $18x = 486$

$\Rightarrow x = 27$

∴ Total no. of cities = $18x + 33x + 132x = 183 \times 27 = 4941$

Q.32) Using the properties of proportion, the value of x in $\frac{x^3+3x}{3x^2+1} = \frac{341}{91}$ is:

a) 7 **b)** 11 **c)** 13 **d)** 15

Sol. (b)

If $\frac{a}{b} = \frac{c}{d}$

From componendo and dividendo,

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$

By applying the property

$\Rightarrow \frac{(x^3+3x)+(3x^2+1)}{(x^3+3x)-(3x^2+1)} = \frac{341+91}{341-91}$

$\Rightarrow \frac{x^3+1^3+3x+3x^2}{x^3-1^3+3x-3x^2} = \frac{432}{250}$

$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{216}{125}$

Taking cube both sides

$\Rightarrow \frac{(x+1)}{(x-1)} = \frac{6}{5}$

$\Rightarrow 5x + 5 = 6x - 6$

$\therefore x = 11$

Q.33) In a famous temple, every devotee offers fruits to the orphans. Every orphan receives bananas, oranges and grapes in the ratio of 3:2:7 in terms of dozen. But the weight of a grape is 24gm, and the weight of the banana and orange are in the ratio of 4:5, while the weight of orange is 150gm. Find the ratio of all the fruits in terms of weight that an orphan gets:

a) 30:25:14 **b)** 38:27:19 **c)** 75:42:90 **d)** 71:63:67

Sol. (a)

Let the weight of the banana be $4x$

The weight of orange be $5x = 150$

$\therefore x = 30$

Hence, the weight of banana = $4x = 4 \times 30 = 120gm$

The ratio of no. of fruits = 3:2:7

∴ The ratio of all three fruits in terms of weight, that orphan gets

$\Rightarrow (3 \times 120) : (2 \times 150) : (7 \times 24) = 30 : 25 : 14$

Q.34) A number is divided into three parts in the ratio of 8:12:5. 12 is added to the first part, 18 is subtracted from the second part, and the third part is increased by 20%, thus making the ratio 10:9:6. What is the new number?

Ans
Ratio Ko
Sum Kardo
to 150 aayega

a) 156 **b)** 168 **c)** 150 **d)** 180

Sol. (c)

Let the no. be $8x$, $12x$ and $5x$

According to given information

$(8x + 12) : (12x - 18) : 6x = 10 : 9 : 6$

Taking any two ratios we can find the value of x

$(12x - 18) : 6x = 9 : 6$

$\Rightarrow 6(12x - 18) = 9 \times 6x$

$\Rightarrow 72x - 108 = 54x$

$\Rightarrow 18x = 108$

$\Rightarrow x = 6$

∴ New number = $(8x + 12) + (12x - 18) + 6x$

$= (8(6) + 12) + (12(6) - 18) + 6(6) = 150$

Q.35) The ratio of the area of land and water of the earth respectively is 1 : 2. Ratio of the area of land and water in the northern hemisphere is 2 :3. Then find the ratio of area of land and water in southern hemisphere.

a) 3:5 **b)** 3:2 **c)** 4:11 **d)** 7:11

Sol. (c)

For the whole earth,

Land = $\frac{1}{1+2} = \frac{1}{3}$ and Water = $\frac{2}{1+2} = \frac{2}{3}$

∴ For the northern hemisphere,

Land = $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ and Water = $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$ as northern hemisphere is half of earth

∴ Southern hemisphere,

Land = $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$ and Water = $\frac{2}{3} - \frac{3}{10} = \frac{11}{30}$

∴ Required ratio = $\frac{2}{15} : \frac{11}{30} = 4 : 11$

Q.36) 1 kg Solder alloy A contains lead, tin, silver and copper in the ratio of 3:2:1:4 and another 800 gm Solder alloy B contains lead, tin, silver and copper in the ratio of 5:3:2:6. If both solder alloys are mixed together, then find the ratio between the total weight of silver in the resulting alloy and the total weight of tin in the resulting alloy?

a) 4:9 **b)** 3:7 **c)** 4:3 **d)** 4:7

Sol. (d)

Alloy A

Weight of silver in 1kg alloy = 1/10 of 1 kg=100gm

Weight of tin in 1kg alloy = 2/10 of 1 kg = 200 gm

Alloy B

Weight of silver in 800gm alloy = 2/16 of 800gm = 100gm

Weight of tin in 800gm alloy = 3/16 of 800gm = 150gm

When both alloy are mixed total weight of both alloy = 1000gm + 800gm = 1800gm

∴ The ratio between the weight of silver and weight of tin= 200: 350 = 4:7

Q.37) In a school, the student to teacher ratio is 40 : 1. However, as per rules, the ratio should be at most 25 : 1. If the minimum number of more teachers required to achieve the desired ratio is 30, then how many students are there in the school?

a) 1200 **b)** 1500 **c)** 1800 **d)** 2000

Sol. (d)

Suppose there are $40x$ students and x teachers

Given, if 30 more teachers are added, the ratio of students to teacher becomes 25 : 1

$\Rightarrow \frac{40x}{x+30} = \frac{25}{1}$

$\Rightarrow 40x = 25x + 750$

$\Rightarrow 19x = 750$

$\Rightarrow x = 50$

∴ Number of students in school = 40 × 50 = 2000

Q.38) The ratio of the prices of two houses was 16 : 23. Two years later when the price of the first has increased by 10% and that of second by ₹ 477, the ratio of the prices becomes 11:20. Find the original prices of the second houses.

a) ₹ 1200 **b)** ₹ 1219 **c)** ₹ 1319 **d)** None of these

Sol. (b)

Let the original price of two houses are 16x and 23x

ATQ

$$= \frac{16x + 10\% of\ 16x}{23x + 477} = \frac{11}{20}$$

$$= \frac{17.6x}{23x + 477} = \frac{11}{20}$$
$$= 352x = 253x + 5247$$
$$= 99x = 5247$$
$$= x = 53$$

The original price of the second house = 23(53) = ₹ 1219

Q.39) If $x : y = z : w = 2.5 : 1.5$, the value of $(x + z)/(y + w)$ is;

a) 1 **b)** 3/5 **c)** 5/3 **d)** None of these

Sol. (c)

$$\therefore \frac{x}{y} = \frac{z}{w} = \frac{2.5}{1.5}$$
$$\Rightarrow \frac{x}{y} = \frac{z}{w} = \frac{5}{3}$$
$$\Rightarrow \frac{x}{z} = \frac{y}{w} \therefore \frac{x+z}{z} = \frac{y+w}{w} \qquad \text{(componendo)}$$
$$\Rightarrow \frac{x+z}{y+w} = \frac{z}{w} = \frac{5}{3}$$

Q.40) The first, second and third terms of the proportion are 42, 36, 35. Find the fourth term.

a) 24 **b)** 26 [✓ **c)** 30] **d)** None of these

Sol. (c)

Let the fourth term be x.
Thus 42, 36, 35, x are in proportion.
Product of extreme terms = $42 \times x$
Product of mean terms = 36×35
Since the numbers make up a proportion
Therefore, $42 \times x = 36 \times 35$
or, $x = (36 \times 35)/42$
or, $x = 30$
Therefore, the fourth term of the proportion is 30.

Q.41) Find in what ratio will the total wages of the workers of a factory be increased or decreased if there be a reduction in the number of workers in the ratio 15:11 and an increment in their wages in the ratio 22:25.

a) 2:5 **b)** 6:5 **c)** 3:5 **d)** None of these

Sol. (b)

Let the number of workers = $15x$
And the average wage of the workers = $22y$
The total wage = $(15x)(22y) = 330xy$
ATQ.
The number of workers decreased to = $11x$
And the average wage of the worker increased to = 25y
Now, the total wage = $275xy$
The required ratio = old total wages: new total wages

$$= \frac{330xy}{275xy} = \frac{330}{275} = \frac{6}{5}$$

The ratio is 6: 5

Q.43) A dealer mixes tea costing ₹6.92 per kg. with tea costing ₹ 7.77 per kg and sell the mixture at ₹ 8.80 per kg and earns a profit of 17.5% on his sale price. In what proportion does he mix them?

a) 2:3 **b)** 2:5 [✓ **c)** 3:2] **d)** None of these

Sol. (c)

Cost price of the first type tea = 6.92/kg
Cost price of the second tea =7.77/kg

Sale price of mixture tea= 8.80/kg
Profit on sale is 17.5% (8.80 × 17.5%)
Now, *The proft per kg is* 1.54
Cost price of mixture tea = 8.80-1.54 = 7.26
Let x quantity mixed of the first type of tea and y quantity of the second type tea.
$= 7.26(x + y) = 6.92x + 7.77y$
$= 7.26x - 6.92x = 7.77y - 7.26y$
$= 0.34x = 0.51y$
$= \frac{x}{y} = \frac{0.51}{0.34}$
$= x : y = 3 : 2$

Q.44) The ratio of the speed of P, Q and R is 10:12:15 respectively. What is the ratio of the time taken by P, Q and R respectively to cover the same distance?

a) 10:12:15
b) 15:12:10
[✓ **c)** 6:5:4]
d) 4:5:6

Sol. (c)
Let the speed of P, Q and R be $10x$, $12x$ and $15x$ respectively
Distance travel by them is y km.
∴ Ratio of time taken = $\frac{y}{10x} : \frac{y}{12x} : \frac{y}{15x}$
$= \frac{1}{10} : \frac{1}{12} : \frac{1}{15}$
$= 6:5:4$

Q.44) A 35kg mixture of hydrocarbon contains methane and ethane in the ratio 4:3. On adding 10 kg of methane and x kg of ethane, the ratio of methane and ethane becomes 6:7. Then, what is the mean proportional of $(x + 5)$ and $(x - 4)$?

[✓ **a)** 20]
b) 25
c) 12
d) 30

Sol. (a)
Quantity of methane = 4/7 of 35 = 20kg
Quantity of ethane = 3/7 of 35 = 15kg
According to question,
$\Rightarrow \frac{20+10}{15+x} = \frac{6}{7}$
$\Rightarrow 6x + 90 = 210$
$\Rightarrow x = 20$
∴ Mean proportional of $(x + 5)$ and $(x - 4)$ = $\sqrt{(x+5)(x-4)}$
$\Rightarrow \sqrt{(25)(16)} = 20$

Q.45) If the present age of Q is 33.33% more than the present age of P and the present age of R is 35% more than the present age of Q. If the average of the present age of P, Q and R is $\frac{62}{3}$ years. After x years, the ratio of the age of P and R is 9:13. Find the value of x?

a) 10
b) 15
c) 12
d) 16

Sol. (c)
Let the present age of P is 3k, then the present age of Q will be 4k
The present age of R will be $4k + \frac{7}{20} of\ 4k = \frac{27k}{5}$
According to Question
$\Rightarrow \frac{3k+4k+\frac{27k}{5}}{3} = \frac{62}{3}$
$\Rightarrow 62k = 62 \times 5$
$\Rightarrow k = 5$
The present age of P is 15 years and R is 27 years
According to Question
$\Rightarrow \frac{15+x}{27+x} = \frac{9}{13}$

$\Rightarrow 195 + 13x = 243 + 9x$
$\Rightarrow 4x = 48$
$\therefore x = 12$

Q.46) A mocktail 'x' has alcohol and water in the ratio 3: 2. Another mocktail 'Y' has alcohol and water in the ratio 7: 4. If x and Y are mixed in the ratio 18: 11, then what will be the ratio of water to alcohol in the final ratio?

a) 56:89 **b)** 898:56 **c)** 83:37 **d)** 37:83

Sol. (a)
Ratio of mocktail x is 3:2 and base is 5
Ratio of mocktail Y is 7: 4 and base is 11
Making their base equal
$x \times 11 = 55, Y \times 5 = 55$
$x = 33:22, Y = 35:20$
Now, x and Y are mixed in the ratio 18: 11, ratio of alcohol to water will be
$\Rightarrow (33 \times 18 + 35 \times 11):(22 \times 18 + 20 \times 11) = 979:616$
$\therefore$ Ratio of water to alcohol = 56: 89

Q.47) When a number is added to another number the total becomes $133\frac{1}{3}$ percent of the second number. What is the ratio between the first and the second number?

a) 2: 3 **b)** 3: 2 **c)** 1: 3 **d)** Data inadequate

Sol. (c)
Let the first no. be x and second no. be y
$\therefore$ According to given condition
$\Rightarrow x + y = \frac{4y}{3}$
$\Rightarrow x = \frac{y}{3}$
$\Rightarrow \frac{x}{y} = \frac{1}{3}$
$\therefore$ The ratio between the first and second no. is 1 : 3

Q.48) A bag contains 50P, 25P and 10P coins in the ratio 4:10: 5, amounting to ₹ 150. Find the number of coins of each type respectively.

a) 80, 90, 100 **b)** 120, 300, 150 **c)** 140, 250, 400 **d)** None of these

Sol. (b)
Let the no. of coins of 50p, 25p and 10p be $4x$, $10x$ and $5x$ respectively.
According to ques
$\Rightarrow 4x \times 0.50 + 10x \times 0.25 + 5x \times 0.10 = 150$
$\Rightarrow 2x + 2.5x + 0.5x = 150$
$\Rightarrow 5x = 150$
$\Rightarrow x = 30$
$\therefore$ No. of 50p coins is 120, No. of 25p coins is 300 and 10p coins is 150

Q.49) In an examination, Payal got marks in Mathematics, Science and History in the ratio of 12:16:19. When she applies for rechecking of her answer sheets, 30 marks are increased in each three subjects and the final marks secured in Mathematics is 72% of the final marks secured in History. Then the ratio of final marks in Mathematics, Science and History is:

a) 19:21:24 b) 18:22:26 c) 18:22:25 d) 19:22:20

Sol. (c)
Let Payal got marks in Mathematics, Science and History are 12x, 16x & 19x respectively.
After increasing 30 marks in each subject
Mathematics = $12x + 30$
Science = $16x + 30$
History = $19x + 30$
According to ques.,

$\Rightarrow \frac{12x+30}{19x+30} \times 100 = 72$

$\Rightarrow \frac{12x+30}{19x+30} = \frac{18}{25}$

$\Rightarrow 300x + 750 = 342x + 540$

$\Rightarrow 342x - 300x = 750 - 540$

$\Rightarrow 42x = 210$

$\Rightarrow x = 5$

$\therefore$ The ratio of final marks in all three subjects

$\Rightarrow (60 + 30) : (80 + 30) : (95 + 30)$

$\Rightarrow 90 : 110 : 125$

$\Rightarrow 18 : 22 : 25$

Q.50) Three containers have their volumes in the ratio 3: 4: 5. They are full of mixtures of milk and water. The mixtures contain milk and water in the ratio of (4: 1), (3: 1) and (5: 2) respectively. The contents of all these three containers are poured into a fourth container. The ratio of milk and water in the fourth container is:

a) 4 : 1
b) 151 : 48
[✓ **c)** 157 : 53]
d) 5: 2

Sol. (c)

Let the volumes be $3x$, $4x$ and $5x$

Container with vol. $3x$

Milk = $\frac{4}{4+1} \times 3x$ and Water = $\frac{1}{4+1} \times 3x$

$\Rightarrow \frac{12x}{5} : \frac{3x}{5}$

Container with vol. $4x$

Milk = $\frac{3}{3+1} \times 4x$ and Water = $\frac{1}{3+1} \times 4x$

$\Rightarrow \frac{12x}{4} : \frac{4x}{4}$

Container with vol. $5x$

Milk = $\frac{5}{5+2} \times 5x$ and Water = $\frac{2}{5+2} \times 5x$

$\Rightarrow \frac{25x}{7} : \frac{10x}{7}$

Total milk = $\frac{12}{5}x + \frac{12}{4}x + \frac{25}{7}x = \frac{336x+420x+500x}{140} = \frac{1256}{140}x$

Total water = $\frac{3}{5}x + \frac{4}{4}x + \frac{10}{7}x = \frac{84x+140x+200x}{140} = \frac{424}{140}x$

Ratio of milk and water in fourth container = $\frac{1256}{424}$ = 157 : 53

Q.51) The value of $\left(\frac{2p^2 q^3}{-3xy}\right)^0$ Where, $q, x, y \neq 0$ is equal to;

a) 0
b) 2/3
[✓ **c)** 1]
d) None of these

Sol. (c)

$\left(\frac{2p^2 q^3}{3xy}\right)^0 = \mathbf{1}$ ($\because$ Any no. s except 0 to the power 0 is 1)

Q.52) If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of p + q + r is;

a) 1
[✓ **b)** 0]
c) 1/2
d) None of these

Sol. (b)

$x^{1/p} = y^{1/q} = z^{1/r} = k$ (let)

$\Rightarrow x = k^p,\ y = k^q\ \&\ z = k^r \qquad [a^x = k \therefore a = k^{1/x}]$

$\because xyz = 1$

$\Rightarrow k^p \times k^q \times k^r = 1 \Rightarrow k^{p+q+r} = k^0$

$\Rightarrow \boldsymbol{p + q + r = 0}$

Q.53) The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is;

a) y^{a+b} **b)** y **c)** 1 **d)** $1/y^{a+b}$

Sol. (d)

$$y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$$
$$= y^{a-b+b-c+c-a-a-b} = y^{-a-b} = \frac{1}{y^{a+b}}$$

Q.54) If $a^x = b$, $b^y = c$, $c^z = a$, then xyz is

a) 1 **b)** 2 **c)** 3 **d)** None of these

Sol. (a)

$$a^x = b$$
$$b^y = c \quad \Rightarrow (a^x)^y = c \Rightarrow a^{xy} = c$$
$$c^z = a \quad \Rightarrow (a^{xy})^z = a \text{ (put the value of c)}$$
$$\Rightarrow a^{xyz} = a^1 \Rightarrow \mathbf{xyz = 1}$$

Q.55) The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is;

a) 9/4 **b)** 4/9 **c)** 2/3 **d)** None of these

Sol. (a)

$$(8/27)^{-1/3} \times \left(\frac{32}{243}\right)^{-1/5} = \left(\frac{2}{3}\right)^{3\times\left(\frac{-1}{3}\right)} \times \left(\frac{2}{3}\right)^{5\times(-1/5)}$$
$$= \left(\frac{2}{3}\right)^{-1} \times \left(\frac{2}{3}\right)^{-1} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$

Q.56) Simplified value of $(125)^{2/3} \times \sqrt{25} \times \sqrt[3]{5^3} \times 5^{\frac{1}{2}}$ is;

a) 5 **b)** 1/5 **c)** 1 **d)** $5^{\frac{9}{2}}$

Sol. (d)

$$(125)^{2/3} \times \sqrt{25} \times \sqrt[3]{5^3} \times 5^{1/2}$$
$$= 5^{3\times\frac{2}{3}} \times 5^{2\times\frac{1}{2}} \times 5^{3\times\frac{1}{3}} \times 5^{1/2}$$
$$= 5^2 \times 5^1 \times 5^1 \times 5^{\frac{1}{2}} = 5^{2+1+1+1/2}$$
$$= 5^{\frac{9}{2}}$$

Q.57) $\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$

a) x^n **b)** x^{n+1} **c)** x^{n-1} **d)** None of these

Sol. (c)

$$\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$$
$$= \left[x^{n\times\frac{n^2-1}{n}}\right]^{\frac{1}{n+1}} = x^{(n^2-1)\times\frac{1}{n+1}}$$
$$= x^{\frac{(n+1)(n-1)}{(n+1)}} = \boldsymbol{x^{n-1}}$$

Q.58) Using $(a-b)^3 = a^3-b^3-3ab(a-b)$ tick the correct of these when $x = p^{1/3} - p^{-1/3}$

a) $x^3 + 3x = p + \frac{1}{p}$ **b)** $x^3 + 3x = p - \frac{1}{p}$ **c)** $x^3 + 3x = p + 1$ **d)** None of these

Sol. (b)

$$x = p^{\frac{1}{3}} - p^{-\frac{1}{3}}$$
$$= x^3 = [p^{\frac{1}{3}} - p^{-\frac{1}{3}}]^3$$
$$= x^3 = p - \frac{1}{p} - 3p^{\frac{1}{3}}p^{-\frac{1}{3}}(x)$$
$$= x^3 = p - \frac{1}{p} - 3x \Rightarrow x^3 + 3x = p - \frac{1}{p}$$

Q.59) If $2^x = 3^y = 6^{-z}$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is

a) 1 **b)** 0 **c)** 2 **d)** None of these

Sol. (b)

$$2^x = 3^y = 6^{-z} = k\ (let)$$
$$\therefore 2 = k^{1/x}$$
$$3 = k^{1/y}$$
$$6 = k^{-1/z}$$
$$\Rightarrow 2 \times 3 = k^{-1/z}$$
$$\Rightarrow k^{1/x} \times k^{1/y} = k^{-1/z}$$
$$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{-1/z}$$
$$\Rightarrow \frac{1}{x}+\frac{1}{y} = \frac{-1}{z} \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$$

Q.60) The value of $\frac{1}{343^{-2/3}}+\frac{1}{625^{-3/4}}+\frac{1}{64^{-1/6}}$ is:

a) 200 **b)** 176 **c)** 656 **d)** 182

Sol. (b)

$$\Rightarrow \frac{1}{343^{-2/3}}+\frac{1}{625^{-3/4}}+\frac{1}{64^{-1/6}}$$
$$\Rightarrow \frac{1}{7^{3(-\frac{2}{3})}}+\frac{1}{5^{4(-\frac{3}{4})}}+\frac{1}{2^{6(-\frac{1}{6})}}$$
$$\Rightarrow \frac{1}{7^{-2}}+\frac{1}{5^{-3}}+\frac{1}{2^{-1}}$$
$$\Rightarrow 7^2+5^3+2^1 = 176$$

Q.61) If $y^{x-2}(y^{2x+2}+y^{1-x}) = y^{-3}(y^9+y^2)$, then the value of x is:

a) a positive integer **b)** 0 **c)** a fraction **d)** a negative integer

Sol. (a)

$$\Rightarrow y^{x-2}(y^{2x+2}+y^{1-x}) = y^{-3}(y^9+y^2)$$
$$\Rightarrow y^{x-2+2x+2}+y^{x-2+1-x} = y^6+y^{-1}$$
$$\Rightarrow y^{3x}+y^{-1} = y^6+y^{-1}$$
$$\Rightarrow y^{3x} = y^6$$
$$\Rightarrow 3x = 6$$
$\Rightarrow x = 2$ positive integer

Q.62) If $(5.678)^x = (0.5678)^y = (10)^z$ then the value of $\frac{1}{x}+\frac{1}{z}-\frac{1}{y}$ is:

a) $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=1$ **b)** $\frac{1}{x}-\frac{1}{y}-\frac{1}{z}=0$ **c)** $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=-1$ **d)** None of these

Sol. (b)

Let $(5.678)^x = (0.5678)^y = (10)^z = k$

Then, $(5.678)^x = k \Rightarrow 5.678 = k^{1/x}$ ---(1)

$(0.5678)^y = k \Rightarrow 0.5678 = k^{1/y}$ ---(2)

$(10)^z = k \Rightarrow 10 = k^{1/z}$ ---(3)

Multiply (1) & (3) and divided by (2)

$$\Rightarrow \frac{5.678\times 10}{0.5678} = \frac{k^{1/x}\times k^{1/z}}{k^{1/y}} \Rightarrow 100 = k^{\frac{1}{x}+\frac{1}{z}-\frac{1}{y}}$$
$$\Rightarrow (k^{1/z})^2 = k^{\frac{1}{x}+\frac{1}{z}-\frac{1}{y}} = k^{\frac{2}{z}} \qquad [\therefore k^{1/z} = 10]$$
$$\Rightarrow \frac{2}{z} = \frac{1}{x}+\frac{1}{z}-\frac{1}{y}$$
$$\Rightarrow \frac{1}{x}-\frac{1}{z}-\frac{1}{y} = 0$$

Q.63) $x^a = y^b = z^c$ and $x^3y^4 = z$, find the value of c.

a) $\frac{2ab}{a^2+b^2}$ **b)** $\frac{ab}{4a+3b}$ **c)** $\frac{xy}{2x+3y}$ **d)** None of these

Sol. (b)

Let $x^a = y^b = z^c = k$

$x = k^{1/a}, y = k^{1/b}$ & $z = k^{1/c}$ $\qquad x^3y^4 = z$

$\Rightarrow k^{\frac{3}{a}} \times k^{\frac{4}{b}} = k^{\frac{1}{c}}$

$\Rightarrow \frac{3}{a} + \frac{4}{b} = \frac{1}{c}$

$\Rightarrow \frac{3b+4a}{ab} = \frac{1}{c}$

$\therefore c = \frac{ab}{4a+3b}$

Q.64) Find the value of

$$(1000)^{\frac{1}{3}} \div (0.0001)^{\frac{1}{2}}\ of\ (0.0025)^{\frac{1}{2}} + (0.00032)^{\frac{1}{5}}\ of\ (216)^0 + \left(\frac{25}{169}\right)^{-\frac{1}{2}} \div \left(\frac{169}{100}\right)^{\frac{1}{2}}$$

a) 22002.2 **b)** 20202.2 **c)** 20022.2 **d)** 20002.2

Sol. (d)

Given $(1000)^{\frac{1}{3}} \div (0.0001)^{\frac{1}{2}}\ of\ (0.0025)^{\frac{1}{2}} + (0.00032)^{\frac{1}{5}}\ of\ (216)^0 + \left(\frac{25}{169}\right)^{-\frac{1}{2}} \div \left(\frac{169}{100}\right)^{\frac{1}{2}}$

$\Rightarrow [10 \div (0.01 \times 0.05) + (0.2 \times 1) + \left(\frac{169}{25}\right)^{\frac{1}{2}} \div \frac{13}{10}]$

$\Rightarrow 10 \times \frac{1}{0.0005} + 0.2 + \frac{13}{5} \times \frac{10}{13}$

$\Rightarrow 20000 + 0.2 + 2$

$\Rightarrow 20002.2$

Q.65) Simplify $\left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2$

a) 0 **b)** 1 **c)** 4 **d)** 16

Sol. (b)

Given $\left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= \left[\frac{e^x+e^{-x}+e^x-e^{-x}}{2}\right]\left[\frac{e^x+e^{-x}-e^x+e^{-x}}{2}\right]$

$= e^x \times e^{-x} = e^0 = 1$

Q.66) The expression $\left(\frac{x^a}{x^{-b}}\right)^{(a^2-ab+b^2)} \times \left(\frac{x^b}{x^{-c}}\right)^{(b^2-bc+c^2)} \times \left(\frac{x^c}{x^{-a}}\right)^{(c^2-ca+a^2)}$ is equal to:

a) x **b)** $x^{2(a^3+b^3+c^3)}$ **c)** 1 **d)** x^{a^2}

Sol. (b)

$= \left(\frac{x^a}{x^{-b}}\right)^{(a^2-ab+b^2)} \times \left(\frac{x^b}{x^{-c}}\right)^{(b^2-bc+c^2)} \times \left(\frac{x^c}{x^{-a}}\right)^{(c^2-ca+a^2)}$

$= x^{(a+b)(a^2-ab+b^2)} \times x^{(b+c)(b^2-bc+c^2)} \times x^{(c+a)(c^2-ca+a^2)}$

$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$

$= x^{2(a^3+b^3+c^3)}$

Q.67) Which of the following statement is False?

i) $(0.9)^2 - (0.08)^2 + (0.07)^2 > 0.805$

ii) $(0.4 \times 0.05 \times 0.006 \div 0.0003) > 0.5$

iii) $(1/0.4)^2 + (1/0.1)^2 > (1/0.5)^2$

a) Only I [✓ **b)** Only II] **c)** Only III **d)** None of these

Sol. (b)

Considering statement, I

$\Rightarrow (0.9)^2 - (0.08)^2 + (0.07)^2 > 0.805$

$\Rightarrow 0.81 - 0.0064 + 0.0049 > 0.805$

$\Rightarrow 0.8085 > 0.805$, which is true

Q.68) Simplification of $\left(\frac{2^l}{2^m}\right)^{l^2+lm+m^2} \times \left(\frac{2^m}{2^n}\right)^{m^2+mn+n^2} \times \left(\frac{2^n}{2^l}\right)^{n^2+nl+l^2}$ gives:

a) 1/x **b)** 0 [✓ **c)** 1] **d)** None of these

Sol. (c)

Given $\left(\frac{2^l}{2^m}\right)^{l^2+lm+m^2} \times \left(\frac{2^m}{2^n}\right)^{m^2+mn+n^2} \times \left(\frac{2^n}{2^l}\right)^{n^2+nl+l^2}$

$\Rightarrow 2^{(l-m)(l^2+lm+m^2)} \times 2^{(m-n)(m^2+mn+n^2)} \times 2^{(n-l)(n^2+nl+l^2)}$

We know that $a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$\Rightarrow 2^{(l^2-m^3)} \times 2^{(m^3-n^3)} \times 2^{(n^3-l^3)}$

$\Rightarrow 2^0 = 1$

Q.69) $\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$

a) $\frac{1}{2}$ **b)** $\frac{3}{2}$ **c)** $\frac{2}{3}$ **d)** $\frac{1}{3}$

Sol. (b)

$$= \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$$

$$= \frac{2^n + \frac{2^n}{2}}{2 \times 2^n - 2^n}$$

$$= \frac{2^n(2+1)}{2^n \times 2(2-1)} = \frac{3}{2}$$

Q.70) If $u^{5x} = v^{5y} = w^{5z}$ and $u^2 = vw$ then $xy + zx - 2yz =$ __________.

[✓ **a)** 0] **b)** 1 **c)** 2 **d)** None of these

Sol. (a)

Let $u^{5x} = v^{5y} = w^{5z} = k$ (say)

$u^{5x} = k$

$u = k^{1/5x}$

$v^{5y} = k$

$v = k^{1/5y}$

$w^{5z} = k$

$w = k^{1/5z}$

Given $u^2 = vw$

$\left(k^{1/5x}\right)^2 = k^{1/5y} . k^{1/5z}$

$k^{\frac{2}{5x}} = k^{\frac{1}{5z}+\frac{1}{5y}}$

$\frac{2}{5x} = \frac{1}{5y} + \frac{1}{5z}$
$\frac{2}{5x} = \frac{y+z}{5yz}$
$x(y+z) = 2yz$
$= xy + xz = 2yz$
$\therefore xy + zx - 2yz = 0$

Q.71) If a and b are whole numbers such that $a^b = 1331, then\ the\ value\ of\ (a-1)^{b+1}$ is:
a) 25000
b) 40000
c) 65000
✓**d)** 10000

Sol. (d)
$\Rightarrow a^b = 11^3$
$\Rightarrow a = 11$
$\Rightarrow b = 3$
$\Rightarrow (a-1)^{b+1}$
$\Rightarrow (11-1)^{3+1}$
$\Rightarrow 10^4 = 10000$

Q.72) If $\frac{1}{1+m^{(q-p)}+m^{(r-p)}} + \frac{1}{1+m^{(p-q)}+m^{(r-q)}} + \frac{1}{1+m^{(q-r)}+m^{(p-r)}} = n$, then the value of n is:
a) m^p
b) m^{p-q-r}
c) m^{pqr}
✓**d)** 1

Sol. (d)
$\Rightarrow \frac{1}{1+m^{(q-p)}+m^{(r-p)}} + \frac{1}{1+m^{(p-q)}+m^{(r-q)}} + \frac{1}{1+m^{(q-r)}+m^{(p-r)}}$
$\Rightarrow \frac{1}{1+\frac{m^q}{m^p}+\frac{m^r}{m^p}} + \frac{1}{1+\frac{m^p}{m^q}+\frac{m^r}{m^q}} + \frac{1}{1+\frac{m^q}{m^r}+\frac{m^p}{m^r}}$
$\Rightarrow m^p/(m^p+m^q+m^r) + m^q/(m^p+m^q+m^r) + m^r/(m^p+m^q+m^r)$

$\Rightarrow (m^p+m^q+m^r)/(m^p+m^q+m^r)$
$\Rightarrow 1$

Q.73) If $5^{3n} - 5^{3n-1} = 500$, then the value of n is:
a) 17/30
b) 4
✓**c)** 4/3
d) 19/37

Sol. (c)
Given, $5^{3n} - 5^{3n-1} = 500$
$\Rightarrow 5^{3n-1}(5-1) = 500$
$\Rightarrow 5^{3n-1} = 125$
$\Rightarrow 5^{3n-1} = 5^3$
$\Rightarrow$ 3n - 1 = 3
$\Rightarrow$ n = 4/3

Q.74) If $(\frac{x^b}{x^c})^{b+c-a} \times (\frac{x^c}{x^a})^{c+a-b} \times (\frac{x^a}{x^b})^{a+b-c}$
✓**a)** 1
b) 3
c) 2
d) None of these

Sol. (a)
$\Rightarrow x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)} \times x^{(a-b)(a+b-c)}$
$\Rightarrow x^{(b-c)(b+c)-a(b-c)} \times x^{(c-a)(c+a)-b(c-a)} \times x^{(a-b)(a+b)-c(a-b)}$
$\Rightarrow x^{(b^2-c^2+c^2-a^2+a^2-b^2)[-a(b-c)-b(c-a)-c(a-b)]}$
$\Rightarrow x^0$
$\Rightarrow 1$

Q.75) On simplification $[\frac{x^{ab}}{x^{a^2+b^2}}]^{a+b} \times [\frac{x^{b^2+c^2}}{x^{bc}}]^{b+c} \times [\frac{x^{ca}}{x^{c^2+a^2}}]^{c+a}$ reduces to:
✓**a)** x^{-2a^3}
b) x^{2a^3}
c) $x^{-2(a^3+b^3+c^3)}$
d) $x^{2(a^3+b^3+c^3)}$

Sol. (a)

Given: $[\frac{x^{ab}}{x^{a^2+b^2}}]^{a+b} \times [\frac{x^{b^2+c^2}}{x^{bc}}]^{b+c} \times [\frac{x^{ca}}{x^{c^2+a^2}}]^{c+a}$

$\Rightarrow x^{-(a^2+b^2-ab)(a+b)} \times x^{(b^2+c^2-bc)(b+c)} \times x^{-(c^2+a^2-ca)(c+a)}$

$\Rightarrow x^{-(a^3+b^3)+(b^3+c^3)-(c^3+a^3)} = x^{-2a^3}$

Q.76) If 2 log x = 4 log 3, the x is equal to;

a) 3 **b)** 9 **c)** 2 **d)** None of these

Sol. (b)

$2\log x = 4\log 3$

$\Rightarrow \log x = \frac{4}{2}\log 3 \Rightarrow \log x = 2\log 3 \Rightarrow \log x = log\,(3)^2$

$\Rightarrow \log x = log 9$

$\Rightarrow \boldsymbol{x = 9}$

Q.77) If log x + log y = log (x +y), y can be expressed as;

a) x -1 **b)** x **c)** x / x -1 **d)** None of these

Sol. (c)

$\log x + \log y = \log(x+y)$

$\log(xy) = \log(x+y) \Rightarrow xy = x + y$

$\Rightarrow y\,(x-1) = x \quad \Rightarrow \boldsymbol{y = \frac{x}{x-1}}$

Q.78) If $\frac{logx}{logy} = \frac{log49}{log7}$, then the relation between x and y is:

a) $x = \sqrt{y}$ **b)** $x = y^3$ **c)** $x = y^2$ **d)** $y = x^2$

Sol. (c)

Given $\frac{logx}{logy} = \frac{log49}{log7}$

$\Rightarrow log_y x = log_7 49$

$\Rightarrow log_y x = 2log_7 7$

$\Rightarrow log_y x = 2$

$\therefore \boldsymbol{x = y^2}$

Q.79) If $x = log_{0.1}0.001, y = log_9 81$, then $\sqrt{x - 2\sqrt{y}} =$

a) $3 - 2\sqrt{2}$ **b)** $\sqrt{3} - 2$ **c)** $\sqrt{2} - 1$ **d)** $\sqrt{2} - 2$

Sol. (c)

$\Rightarrow x = log_{0.1}0.1^3$

$x = 3log_{0.1}0.1$

$x = 3$

$y = log_9 9^2$

$y = 2log_9 9$

$y = 2$

$\therefore \sqrt{x - 2\sqrt{y}} = \sqrt{3 - 2\sqrt{2}} = \sqrt{(2+1) - 2\sqrt{2 \times 1}}$

$= \sqrt{(\sqrt{2} - 1)^2} = \boldsymbol{\sqrt{2} - 1}$

Q.80) log (1 + 2 + 3) is exactly equal to

a) log 1 + log 2 + log 3 **b)** log (1×2×3) **c)** Both the above **d)** None

Sol. (c)

$\log\,(1+2+3) = \log\,6$

$= \log(1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$

Q.81) If $a^{logb} = 3$, then the value of $a^{logb} + b^{loga}$ is:

a) 3 **b)** 2 **c)** 6 **d)** 12

Sol. (c)

Given $a^{logb} = 3$

$\because x^{logy} = y^{logx}$

$\therefore a^{logb} + b^{loga} = 3 + 3 = 6$

Q.82) If $log_4(x^2 + x) - log_4(x + 1) = 2$, then the value of x is:

a) 2 **b)** 4 **c)** 8 **d)** 16

Sol. (d)

Given $log_4(x^2 + x) - log_4(x + 1) = 2$

$\Rightarrow log_4\left(\frac{x^2+x}{x+1}\right) = 2$

$\Rightarrow \left(\frac{x^2+x}{x+1}\right) = 4^2 \Rightarrow \frac{x(x+1)}{x+1} = 16$

$\therefore x = 16$

Q.83) The logarithm of 21952 to the base of $2\sqrt{7}$ and 19683 to the base of $3\sqrt{3}$ are

a) Equal **b)** Not equal **c)** Have a difference of 2269 **d)** None

Sol. (a)

$log_{2\sqrt{7}} 21952$

$= log_{2\sqrt{7}} 2^6 \times 7^3 = log_{2\sqrt{7}} (2\sqrt{7})^6$

$= 6\, log_{2\sqrt{7}} 2\sqrt{7} = 6$

Now $log_{3\sqrt{3}} 19683 = log_{3\sqrt{3}} 3^9$

$= 9\, log_{3^{\frac{3}{2}}} 3 = 9 \times \frac{1}{3/2}\, log_3 3$

$= 9 \times \frac{2}{3} = 6$

Q.84) The value of is $4\log\frac{8}{25} - 3\log\frac{16}{125} - \log 5$ is

a) 0 **b)** 1 **c)** 2 **d)** −1

Sol. (a)

$4\log\left(\frac{8}{25}\right) - 3\log\frac{16}{125} - \log 5$

$= 4\,[log_8 - log_{25}] - 3\,(\log 16 - \log 125) - \log 5$

$= 4\,(3\log 2 - 2\log 5) - 3\,(4\log 2 - 3\log 5) - \log 5$

$= 12\,\log 2 - 8\,\log 5 - 12\log 2 + 9\log 5 - \log 5$

$= 0$

Q.85) If $log_2 x + log_4 x + log_{16} x = \frac{21}{4}$, these x is equal to;

a) 8 **b)** 4 **c)** 16 **d)** None of these

Sol. (a)

$log_2 x + log_4 x + log_{16} x = \frac{21}{4} \Rightarrow log_2 x + log_{2^2} x + log_{2^4} x = \frac{21}{4}$

$\Rightarrow log_2 x + \frac{1}{2}\, log_2 x + \frac{1}{4}\, log_2 x = \frac{21}{4}$

$\Rightarrow \frac{4log_2 x + 2log_2 x + log_2 x}{4} = \frac{21}{4}$

$\Rightarrow \frac{7}{4}\, log_2 x = \frac{21}{4} \Rightarrow log_2 x = \frac{21}{4} \times \frac{4}{7}$

$\Rightarrow log_2 x = 3 \Rightarrow x = 2^3 = 8 \qquad \{log_{a^b} = x \;\Rightarrow\; a^x = b\}$

Q.86) Given that log x = m + n and log y = m − n, the value of log 10 x /y^2 is expressed in terms of mand n as;

a) 1 − m + 3n **b)** m − 1 + 3n **c)** m + 3n + 1 **d)** None of these

Sol. (a)

$\log x = m + n \;\&\; \log y = m - n$

$\therefore \log\frac{10x}{y^2} = \log 10 + \log x - \log y^2$

$= 1 + \log x - 2\log y = 1 + m + n - 2(\mathrm{m} - \mathrm{n})$
$= 1 + m + n - 2m + 2n$
$= 1 - m + 3n$

Q.87) The simplified value of log $\sqrt[4]{729.\sqrt[3]{9^{-1}.27^{-4/3}}}$ is;

a) log 3 **b)** log 2 **c)** log $^1/_2$ **d)** None of these

Sol. (a)

log $\sqrt[4]{729\sqrt[3]{9^{-1} \times 27^{-4/2}}}$
$= \log (729\sqrt[3]{9^{-1}27^{-4/3}})^{1/4}$
$\Rightarrow \log(729)^{\frac{1}{4}}(3^{-2} \times (3^3)^{-\frac{4}{3}})^{\frac{1}{3}\times\frac{1}{4}}$
$\Rightarrow \log(3^6)^{\frac{1}{4}}(3^{-2})^{\frac{1}{4}} \Rightarrow \log(3^4)^{\frac{1}{4}}$
= log 3

Q.88) What is log (a + $\sqrt{a^2}$+ 1) + log ($\frac{1}{a + \sqrt{a^2}+1}$) it is equal to?

a) 1 **b)** 0 **c)** 2 **d)** 1/2

Sol. (b)

Let log (a + $\sqrt{a^2}$+ 1) + log ($\frac{1}{a + \sqrt{a^2}+1}$)
= log (a + $\sqrt{a^2}$+ 1) + log1 – log (a + $\sqrt{a^2}$+ 1)
= log (a + $\sqrt{a^2}$+ 1) – log (a + $\sqrt{a^2}$+ 1) = 0

Q.89) If $\frac{loga}{p+q-2r} = \frac{logb}{q+r-2p} = \frac{logc}{p+r-2q}$, *then the value of* $a^2b^2c^2$ is:

a) 0 **b)** 1 **c)** 2 **d)** 3

Sol. (b)

Let $\frac{loga}{p+q-2r} = \frac{logb}{q+r-2p} = \frac{logc}{p+r-2q} = k$
$\Rightarrow \log a = k(p + q - 2r), \log b = k(q + r - 2p), \log c = k(p + r - 2q)$
Let $a^2b^2c^2 = m$
Taking log both sides
$\Rightarrow log(a^2b^2c^2) = logm$
$\Rightarrow loga^2 + logb^2 + logc^2 = logm$
$\Rightarrow$ 2loga + 2logb + 2logc = log m
$\Rightarrow 2(\mathrm{p + q - 2r})\mathrm{k} + 2(\mathrm{q + r - 2p})\mathrm{k} + 2(\mathrm{p + r - 2q})\mathrm{k} = \log m$
$\Rightarrow 2\mathrm{k}[\mathrm{p + q - 2r + q + r - 2p + p + r - 2q}] = \log m$
$\Rightarrow 2k[0] = \log m$
∴ log m = 0 = log 1
$\Rightarrow$ m = 1
$= a^2b^2c^2 = m = 1$

Q.90) If $log_4 5$ = a and $log_5 6$ = b then what is the value of $log_3 2$?

a) $\frac{1}{2a+1}$ **b)** $\frac{1}{2b+1}$ **c)** 2ab+1 **d)** $\frac{1}{2ab-1}$

Sol. (d)

$log_4 5$ = a and $log_5 6$=b
= $log_4 5 \times log_5 6$ = ab
= $log_4 6$ = ab ⇒ $\frac{1}{2}log_2 6$ = ab
= $\frac{1}{2}(log_2 2 + log_2 3)$=ab
= (1 + $log_2 3$) = 2ab
= $log_2 3$ = 2ab – 1
= $log_3 2 = \frac{1}{2ab-1}$

Q.91) If $a^3 - b^3 = 0$, then the value of $\log(a+b) - \frac{1}{2}(loga + logb + log3)$ is equal to

a) 1 **b)** -1 **c)** 3 **d)** 0

Sol. (d)

Given $\log\ (a+b) - \frac{1}{2}(loga + logb + log3)$

$\Rightarrow \log\ (a+b) - \log\ (3ab)^{\frac{1}{2}}$

$\Rightarrow \log\left(\frac{a+b}{\sqrt{3ab}}\right) \Rightarrow \log\left(\frac{\sqrt{3ab}}{\sqrt{3ab}}\right)=0$

Now, it is given that

$\Rightarrow a^3 - b^3 = 0$

$\Rightarrow (a+b)(a^2+b^2-ab) = 0$

$\Rightarrow (a+b)^2 - 2ab - ab = 0$

$\Rightarrow a + b = \sqrt{3ab}$

Q.92) If $a^2 + b^2 = 0$ *and* $a + b \neq 0$, then the value of log(a+b) is:

a) loga + logb + log2 **b)** $\frac{1}{2}$(loga + logb + log2) **c)** loga + logb **d)** None of these

Sol. (b)

$\Rightarrow \log(a+b) = \frac{1}{2}\log(a+b)^2$

$\Rightarrow \frac{1}{2}\log\ (a^2 + b^2 + 2ab) \quad \because (a^2+b^2) = 0$

$\therefore \log(a+b) = \frac{1}{2}log2ab = \frac{1}{2}(log2 + loga + logb)$

Q.93) If $\frac{1}{log_a t} + \frac{1}{log_b t} + \frac{1}{log_c t} = \frac{1}{log_z t}$, then the value of z is:

a) abc **b)** a+b+c **c)** a(b+c) **d)** (a+b)c

Sol. (a)

Given $\frac{1}{log_a t} + \frac{1}{log_b t} + \frac{1}{log_c t} = \frac{1}{log_z t}$

$\Rightarrow log_t a + log_t b + log_t c = log_t z$

$\Rightarrow log_t abc = log_t z$

$\therefore$ z = abc

Q.94) $\frac{1}{log_{ab}\,(abc)} + \frac{1}{log_{bc}\,(abc)} + \frac{1}{log_{ca}\,(abc)}$ is equal to;

a) 0 **b)** 1 **c)** 2 **d)** -1

Sol. (c)

$\frac{1}{log_{ab}\,(abc)} + \frac{1}{log_{bc}\,(abc)} + \frac{1}{log_{ca}\,(abc)}$

$= \frac{1}{\frac{logabc}{logab}} + \frac{1}{\frac{logabc}{logbc}} + \frac{1}{\frac{logabc}{logca}}$

$= \frac{logab}{logabc} + \frac{logbc}{logabc} + \frac{logca}{logabc}$

$= \frac{logab+logbc+logca}{logabc} = \frac{\log(ab\times bc\times ca)}{logabc}$

$= \frac{log(abc)^2}{logabc} = \frac{2\ logabc}{logabc} = 2.$

Q.95) If $\frac{logx}{q-r} = \frac{logy}{r-p} = \frac{logz}{p-q}$ then find the value of $x^{q+r} \times y^{r+p} \times z^{p+q}$ is:

a) 0 **b)** 1 **c)** -1 **d)** 3

Sol. (b)

Let $\frac{logx}{q-r} = \frac{logy}{r-p} = \frac{logz}{p-q} = k$

$logx = k(q-r), logy = k(r-p)$ *and* $logz = k(p-q)$

$\Rightarrow x^{q+r} \times y^{r+p} \times z^{p+q} = A$

Taking log both sides,

$\Rightarrow logx^{q+r} + logy^{r+p} + logz^{p+q} = logA$

$\Rightarrow (q+r)k(q-r) + (r+p)k(r-p) + (p+q)k(p-q) = logA$

$\Rightarrow k(q^2 - r^2 + r^2 - p^2 + p^2 - q^2) = logA$

$\therefore$ logA = 0

= A = 1

Equations

Q.1) The solution of the equation (p+2) (p-3) + (p+3) (p-4) = p(2p-5) is;

[✓ **a)** 6] **b)** 7 **c)** 5 **d)** None of these

Sol. (a)

(p+2) (p-3) + (p+3) (p-4) = p(2p-5)

For (option)

P = 6 then

$8 \times 3 + 9 \times 2 = 6\,(12\text{-}5)$

$\Rightarrow 24 + 18 = 6 \times 7$

$\Rightarrow 42 = 42$ (✓)

b) P = 7 then

$9 \times 4 + 10 \times 3 = 7\,(14\text{-}5)$

$\Rightarrow 36+30 = 7\times9$

$\Rightarrow 66 = 63$ (×)

c) P = 5 then

$7\times 2 + 8\times 1 = 5\,(10\text{-}5)$

$\Rightarrow 14+8 = 5\times5$

$\Rightarrow 22 = 25$ (×)

Q.2) The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for;

a) $x = 1$ **b)** $x = 2$ **c)** $x = 5$ [✓ **d)** $x = 7$]

Sol. (d)

The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$

Put the value of x in the equation from the option.

a) $x = 1$

$\frac{12+1}{4} = \frac{15-1}{5} + \frac{2-5}{3-1}$

$\Rightarrow \frac{13}{4} = \frac{14}{5} + \frac{-3}{2}$

$\Rightarrow \frac{13}{4} = \frac{28-15}{10}$

$\Rightarrow \frac{13}{4} = \frac{13}{10}$ (×)

b) $x = 2$

$\frac{24+1}{4} = \frac{30-1}{5} + \frac{4-5}{6-1}$

$\Rightarrow \frac{25}{4} = \frac{29}{5} + \frac{-1}{5}$

$\Rightarrow \frac{25}{4} = \frac{28}{5}$ (×)

c) $x = 5$

$\Rightarrow \frac{60+1}{4} = \frac{75-1}{5} + \frac{10-5}{15-1}$

$\Rightarrow \frac{61}{4} = \frac{74}{5} + \frac{5}{14}$

$\Rightarrow \frac{61}{4} = \frac{1036+25}{5\times14} \Rightarrow \frac{61}{4} = \frac{1061}{70}$ (×)

d) $x = 7$

$\frac{84+1}{4} = \frac{105-1}{5} + \frac{14-5}{21-1}$

$\Rightarrow \frac{85}{4} = \frac{104}{5} + \frac{9}{20}$

$\Rightarrow \frac{85}{4} = \frac{416+9}{20}$
$\Rightarrow \frac{85}{4} = \frac{425}{20}$
$\Rightarrow \frac{85}{4} = \frac{85}{4}$

(✓)

Q.3) The satisfying value of $x^3 + x^2 - 20x = 0$ are
a) (1, 4, - 5)
b) (2, 4, - 5)
c) (0, - 4, 5)
d) (0, 4, - 5)

Sol. (d)
$x^3 + x^2 - 20x = 0$
$\Rightarrow x\,(x^2 + x - 20) = 0 \Rightarrow x\,(x+5)\,(x-4) = 0$
$\Rightarrow x = 0, x+5 = 0, x-4 = 0$
$\Rightarrow x = 0, x = -5, x = 4$

Q.4) $2x + 3y + 4z = 0,\ x + 2y - 5z = 0,\ 10x + 16y - 6z = 0$. Find $x, y,$ and z
a) (0,0,0)
b) (1, −1, 1)
c) (3, 2, −1)
d) (1, 0, 2)

Sol. (a)
$2x + 3y + 4z = 0,\ x + 2y - 5z = 0,\ 10x + 16y - 6z = 0$
For the (option)
$(a)\ (0,0,0)$
$0 + 0 + 0 = 0,\ 0 + 0 - 0 = 0,\ 0 + 0 - 0 = 0$
$\Rightarrow 0 = 0,\ 0 = 0,\ 0 = 0$

(✓)

Check the other options in a similar manner, which don't satisfy the equation.

Q.5) The equation $x^2 - (P+4)x + 2P + 5 = 0$ has equal roots the values of P will be.
a) ± 1
b) 2
c) ± 2
d) −2

Sol. (c)
$x^2 - (P+4)\,x + 2P + 5 = 0$
$Here\ \alpha = \beta$
$\alpha + \beta = \frac{-b}{a} \Rightarrow 2\alpha = P + 4$
$\Rightarrow \alpha = \frac{P+4}{2}$ __(I)
$Also\ \alpha\beta = \frac{c}{a} \Rightarrow \alpha^2 = 2P + 5$
$\Rightarrow \left(\frac{P+4}{2}\right)^2 = 2P + 5$
$\Rightarrow P^2 + 8P + 16 = 8P + 20$
$\Rightarrow P^2 = 4 \ \Rightarrow P = \pm 2$

Q.6) If one root of $5x^2 + 13x + P = 0$ be reciprocal of the other then the value of P is;
a) -5
b) 5
c) 1/5
d) -1/5

Sol. (b)
$5x^2 + 13x + P = 0$
Here $\alpha = \frac{1}{\beta} \Rightarrow \alpha\beta = 1$
$\Rightarrow \frac{P}{5} = 1 \ \Rightarrow P = 5$

Q.7) $\frac{1}{3}(x + y) + 2z = 21, 3x - \frac{1}{2}(y + z) = 65, x + \frac{1}{2}\,(x + y - z) = 38$
a) (4, 9, 5)
b) (2, 9, 5)
c) (24, 9, 5)
d) (5, 24, 9)

Sol. (c)
$\frac{1}{3}\,(x + y) + 2z = 21, 3x - \frac{1}{2}\,(y + z) = 65, x + \frac{1}{2}\,(x + y - z) = 38$
For the (option)
$(a)\ (4,9,5),$

$\frac{1}{3}(4+9)+2\times 5=21,\quad 3\times 4-\frac{1}{2}(9+5)=65,\quad 4+\frac{1}{2}(4+9-5)=38$ (×)

$(b)\ (2,9,5),$

$\frac{1}{3}(2+9)+2\times 5=21,\quad 3\times 2-\frac{1}{2}(9+5)=65,\quad 2+\frac{1}{2}(2+9-5)=38$ (×)

$(c)\ (24,9,5),$

$\frac{1}{3}(24+9)+2\times 5=21,\ 3\times 24-\frac{1}{2}(9+5)=65,\ 24+\frac{1}{2}(24+9-5)=38$

$\Rightarrow 11+10=21,\ 72-7=65,\ 24+14=38$

$\Rightarrow 21=21,\ 65=65,\ 38=38$ (✓)

$(d)\ (5,24,9),$

$\frac{1}{3}(5+24)+2\times 9=21, 3\times 5-\frac{1}{2}(24+9)=65, 5+\frac{1}{2}(5+24-9)=38$ (×)

Q.8) $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10},\quad 3xy=10(y-x)$

[✓ **a)** (2, 5)] **b)** (5, 2) **c)** (2, 7) **d)** (3, 4)

Sol. (a)

$\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10},\quad 3xy=10\,(y-x)$

For the (option)

$(a)\ (2,5)$

$\frac{4}{2}-\frac{5}{5}=\frac{2+5}{2\times 5}+\frac{3}{10}, 3\times 2\times 5=10\,(5-2)$

$\Rightarrow 2-1=\frac{7}{10}+\frac{3}{10},\ 30=10\times 3$

$\Rightarrow 1=1\quad,\quad 30=30$ (✓)

Check the other options in the same way, which don't satisfy the equation.

Q.9) The satisfying values of x for the equation $\frac{1}{x+p+q}=\frac{1}{x}+\frac{1}{p}+\frac{1}{q}$ are

a) (p, q) [✓ **b)** (−p, −q)] **c)** (p, −p) **d)** (−p, q)

Sol. (b)

$$\frac{1}{x+p+q}=\frac{1}{x}+\frac{1}{p}+\frac{1}{q}$$

$$\Rightarrow \frac{1}{x+p+q}-\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$$

$$\Rightarrow \frac{x-x-p-q}{(x+p+q)\,x}=\frac{q+p}{pq}$$

$$\Rightarrow -(p+q)\,pq=(q+p)\,x\,(x+p+q)$$

$$\Rightarrow x^2+(p+q)\,x+pq=0$$

$$\Rightarrow x^2+px+qx+pq=0\Rightarrow x\,(x+p)+q(x+p)=0$$

$$\Rightarrow (x+p)\,(x+q)=0$$

$$\Rightarrow x+p=0\ or\ x+q=0$$

$$\Rightarrow x=-p\ or\ x=-q$$

Q10) The equation $\frac{3(3x^2+15)}{6}+2x^2+9=\frac{2x^2+96}{7}+6$

[✓ **a)** (1, 1)] **b)** (1/2, -1) [✓ **c)** (1, -1)] **d)** (2, -1)

Sol. (c)

$$\frac{3(3x^2+15)}{6} + 2x^2 + 9 = \frac{2x^2+96}{7} + 6$$
$$\Rightarrow \frac{3x^2+15+4x^2+18}{2} = \frac{2x^2+96+42}{7}$$
$$\Rightarrow (7x^2+33) \times 7 = (2x^2+138) \times 2$$
$$\Rightarrow 49x^2 + 231 = 4x^2 + 276$$
$$\Rightarrow 45x^2 = 45 \Rightarrow x^2 = 1$$
$$\Rightarrow x = \pm 1$$

Q.11) The solution for the pair of equations $\frac{1}{16x} + \frac{1}{15y} = \frac{9}{20}$, $\frac{1}{20x} - \frac{1}{27y} = \frac{4}{45}$ is given by

a) $\left(\frac{1}{4}, \frac{1}{3}\right)$
b) $\left(\frac{1}{3}, \frac{1}{4}\right)$
c) (3, 4)
d) (4, 3)

Sol. (a)

$\frac{1}{16x} + \frac{1}{15y} = \frac{9}{20}, \frac{1}{20x} - \frac{1}{27y} = \frac{4}{45}$

For the (option)

$(a)\ \left(\frac{1}{4}, \frac{1}{3}\right),$

$\frac{1}{4} + \frac{1}{5} = \frac{9}{20}, \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$

$\Rightarrow \frac{5+4}{20} = \frac{9}{20}, \frac{9-5}{45} = \frac{4}{45}$ (✓)

$(b)\ \left(\frac{1}{3}, \frac{1}{4}\right)$

$\frac{3}{16} + \frac{4}{15} = \frac{9}{20}, \frac{3}{20} - \frac{4}{27} = \frac{4}{45}$ (×)

$(c)\ (3, 4)$

$\frac{1}{48} + \frac{1}{60} = \frac{9}{20}, \frac{1}{60} - \frac{1}{108} = \frac{4}{45}$ (×)

$(d)\ (4, 3)$

$\frac{1}{64} + \frac{1}{45} = \frac{9}{20}, \frac{1}{80} - \frac{1}{81} = \frac{4}{45}$ (×)

Q.12) The simultaneous equations $7x - 3y = 31$, $9x - 5y = 41$ have solutions given by

a) (–4, –1)
b) (–1, 4)
c) (4, –1)
d) (3, 7)

Sol. (c)

$7x - 3y = 31, \ 9x - 5y = 41$

For the (option)

$(a)\ (-4, -1)$

$7 \times (-4) - 3 \times (-1) = 31, \ 9 \times (-4) - 5(-1) = 41$ (×)

$(b)\ (-1, 4)$

$7 \times (-1) - 3 \times 4 = 31, \ 9 \times (-1) - 5 \times 4 = 41$ (×)

$(c)\ (4, -1)$

$7 \times 4 - 3 \times (-1) = 31, \ 9 \times 4 - 5 \times (-1) = 41$

$\Rightarrow 28 + 3 = 31, \ 36 + 5 = 41$ (✓)

$(d)\ (3, 7)$

$7 \times 3 - 3 \times 7 = 31, \ 9 \times 3 - 5 \times 7 = 41$ (×)

Q.13) If $4x^3 + 8x^2 - x - 2 = 0$ then the value of $(2x + 3)$ is given by;

a) 4, –1, 2
b) –4, 2, 1
c) 2, –4, –1
d) None of these

Sol. (a)

$4x^3 + 8x^2 - x - 2 = 0$

$\Rightarrow 4x^2 (x+2) - 1(x+2) = 0$

$\Rightarrow (4x^2 - 1)(x+2) = 0 \Rightarrow (2x+1)(2x-1)(x+2) = 0$

$\therefore 2x + 1 = 0, 2x - 1 = 0, \text{ or } x + 2 = 0$

$\Rightarrow x = {}^{-1}/_{2}\ x = {}^{1}/_{2}$ or $x = -2$

If $x = {}^{-1}/_{2}$ $\therefore 2x + 3 = -1 + 3 = 2$

If $x = {}^{1}/_{2}$ $\therefore 2x + 3 = 1 + 3 = 4$

If $x = -2$ $\therefore 2x + 3 = -4 + 3 = -1$

Q.14) The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3 + 5x^2 - 3x - 5 = 0$ are

a) $x - 1, x - 2, x - 5/3$

b) $x - 1, x + 1, 3x + 5$

c) $x + 1, x - 1, 3x - 5$

d) $x - 1, x + 1, x - 2$

Sol. (b)

$3x^3 + 5x^2 - 3x - 5 = 0$

$\alpha + \beta + \gamma = -\frac{5}{3}, \alpha\beta + \beta\gamma + \gamma\alpha = \frac{-3}{3} = -1$

$\alpha\beta\gamma = \frac{5}{3}$

For the (option)

a) $x - 1, x - 2, x - {}^{5}/_{3} \therefore \alpha = 1, \beta = 2, \gamma = {}^{5}/_{3}$

$\therefore \alpha + \beta + \gamma = {}^{-5}/_{3} \Rightarrow 1 + 2 + {}^{5}/_{3} = {}^{-5}/_{3}$ (×)

b) $x - 1, x + 1, 3x + 5 \therefore \alpha = 1, \beta = -1, \gamma = {}^{-5}/_{3}$

$\therefore \alpha + \beta + \gamma = {}^{-5}/_{3} \Rightarrow 1 + (-1) + \left({}^{-5}/_{3}\right) = {}^{-5}/_{3} \Rightarrow {}^{-5}/_{3} = {}^{-5}/_{3}$

And $\alpha\beta + \beta\gamma + \gamma\alpha = -1 \Rightarrow -1 + {}^{5}/_{3} - {}^{5}/_{3} = -1 \Rightarrow -1 = -1$

Also $\alpha\beta\gamma = \frac{5}{3} \Rightarrow 1 \times (-1)\left({}^{-5}/_{3}\right) = {}^{5}/_{3}$

$\Rightarrow \frac{5}{3} = {}^{5}/_{3}$ (✓)

In a similar way, check out the other options which don't satisfy the equation.

Q.15) The values of x satisfying the equation $\sqrt{(2x^2 + 5x - 2)} - \sqrt{(2x^2 + 5x - 9)} = 1$ are

a) (2, −9/2) **b)** (4, −9) **c)** (2, 9/2) **d)** (−2, 9/2)

Sol. (a)

$\sqrt{2x^2 + 5x - 2} - \sqrt{2x^2 + 5x - 9} = 1$

$\Rightarrow \sqrt{2x^2 + 5x - 2} = 1 + \sqrt{2x^2 + 5x - 9}$

Squaring both sides

$2x^2 + 5x - 2 = 1 + 2x^2 + 5x - 9 + 2\sqrt{2x^2 + 5x - 9}$

$\Rightarrow 6 = 2\sqrt{2x^2 + 5x - 9}$

$\Rightarrow 3 = \sqrt{2x^2 + 5x - 9}$

Again, squaring both sides

$\Rightarrow 9 = 2x^2 + 5x - 9 \Rightarrow 2x^2 + 5x - 18 = 0$

$\Rightarrow 2x^2 + 9x - 4x - 18 = 0$

$\Rightarrow x(2x + 9) - 2(2x + 9) = 0$

$\Rightarrow (x - 2)(2x + 9) = 0 \Rightarrow x - 2 = 0 \; or \; 2x + 9 = 0$

$\Rightarrow x = 2 \; or \; x = {}^{-9}/_{2}$

Q.16) *The value of* $4 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + --- \infty}}}$

a) $1 \pm \sqrt{2}$ **b)** $2 + \sqrt{5}$ **c)** $2 \pm \sqrt{5}$ **d)** None of these

Sol. (b)

$Let\ x = 4 + \frac{1}{4+\frac{1}{4+\frac{1}{4+---\infty}}}$

$\Rightarrow x = 4 + \frac{1}{x}$

$\Rightarrow x^2 - 4x - 1 = 0$

$\Rightarrow x = \frac{-b \pm \sqrt{D}}{2a} = \frac{4 \pm \sqrt{16+4}}{2\times1}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

But x can't be -ve

$\therefore x = 2 + \sqrt{5}$

Q.17) If α and β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is;

a) $2\sqrt{5}$ **b)** $\sqrt{5}$ **c)** $3\sqrt{5}$ **d)** $-2\sqrt{5}$

Sol. (a) & d)

$x^2 = x + 1 \Rightarrow x^2 - x - 1 = 0$

$\therefore \alpha + \beta = 1,\ \alpha\beta = -1$

$\therefore \frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha} = \frac{\alpha^3 - \beta^3}{\alpha\beta}$

$= \frac{(\alpha-\beta)^3 + 3\alpha\beta\,(\alpha-\beta)}{\alpha\beta}$

$[\because \alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\sqrt{1+4} = \pm\sqrt{5}]$

$= \frac{(\pm\sqrt{5})^3 + 3\,(-1)\,(\pm\sqrt{5})}{-1}$

$= \frac{\pm\sqrt{5}\,(5-3)}{-1} = \frac{\pm\sqrt{5}\times2}{-1}$

$= \pm\, 2\sqrt{5}$

Q.18) If $2^{2x+3} - 3^2 . 2^x + 1 = 0$ then values of x are

a) 0, 1 **b)** 1, 2 **c)** 0, 3 **d)** 0, -3

Sol. (d)

$2^{2x+3} - 3^2 . 2^x + 1 = 0$

$\Rightarrow 2^{2x} . 2^3 - 3^2 . 2^x + 1 = 0 \Rightarrow 8\,(2^x)^2 - 9 \times 2^x + 1 = 0$

$Let\ 2^x = y$

$\therefore$ Equation we have

$= 8\,y^2 - 9y + 1 = 0$

$\Rightarrow 8\,y^2 - 8y - y + 1 = 0$

$\Rightarrow 8y\,(y-1) - 1\,(y-1) = 0 \Rightarrow (8y-1)\,(y-1) = 0$

$\Rightarrow 8y - 1 = 0\ or\ y - 1 = 0$

$\Rightarrow y = \frac{1}{8}\ or\ y = 1$

$\Rightarrow 2^x = 2^{-3}\ or\ 2^x = 2^0$

$\Rightarrow x = -3\ or\ \ x = 0$

Q.19) The sum of the digits of a two-digit number is 10. If 18 be subtracted from it, the digits in the resulting number will be equal. The number is;

a) 37 **b)** 73 **c)** 75 **d)** None of these

Sol. (b)

For the (option)

(a) 37,

$(a)\ 3 + 7 = 10$

$\&\ 37 - 18 = 19\ so\ 1 \neq 9$

$(b)\ 73,$

(×)

$7 + 3 = 10$

$73 - 18 = 55 \therefore 5 = 5$ (✓)

(c) 75,

$7 + 5 = 12 \neq 10$ (×)

Q.20) Ten years ago, the age of a father was four times of his son. Ten years hence the age of thefather will be twice that of his son. The present ages of the father and the son are.

[✓ **a)** (50, 20)] **b)** (60, 20) **c)** (55, 25) **d)** None of these

Sol. (a)

For the (option)

(a) (50,20)

10 years before

$50 - 10 = 4 \times (20 - 10)$

$\Rightarrow 40 = 40$

10 years after

Also, $50 + 10 = 2\,(20 + 10)$

$60 = 60$ (✓)

(b) (60,20)

10 years ago

$60 - 10 = 4\,(20 - 10) \Rightarrow 50 = 40$ (×)

(c) (55,25)

10 years ago

$50 - 10 = 4\,(25 - 10) \Rightarrow 40 = 60$ (×)

Q.21) The product of two numbers is 3200, and the quotient when the larger number is dividedby, the smaller is 2. The numbers are;

a) (16, 200) **b)** (160, 20) **c)** (60, 30) [✓ **d)** (80, 40)]

Sol. (d)

For the (option)

(a) (16,200)

$16 \times 200 = 3200$

Also $\frac{200}{16} = \frac{25}{2} = 12\,\frac{1}{2}$ (×)

Here Quotient is $12 \neq 2$

(b) (160,20)

$160 \times 20 = 3200$

Also $\frac{160}{20} = 8 \quad \neq 2$ (×)

(c) (60,30)

$60 \times 30 = 1800 \neq 3200$ (×)

(d) (80,40)

Here $80 \times 40 = 3200$

Also, $\frac{80}{40} = 2$ (✓)

Q.22) Three persons, Mr. Roy, Mr. Paul and Mr. Singh together, have ₹ 51. Mr. Paul has ₹ 4 less than Mr. Roy and Mr. Singh has got ₹ 5 less than Mr. Roy. They have the money as.

[✓ **a)** (₹ 20, ₹ 16, ₹ 15)] **b)** (₹ 15, ₹ 20, ₹ 16) **c)** (₹ 25, ₹ 11, ₹ 15) **d)** None of these

Sol. (a)

Read the question carefully & check the options

For the (option)

(a) (₹20, ₹16, ₹15)

Here $20 + 16 + 15 = 51$

Also, $20 - 16 = 4$ (✓)

$20 - 15 = 5$

(b) (₹15, ₹20, ₹16)
Here $15 + 20 + 16 = 51$
Also, $15 - 20 = -5 \neq 4$ (×)
(c) (₹25, ₹11, ₹15)
Here $25 + 11 + 15 = 51$
$11 - 25 = -14 \neq 4$ (×)

Q.23) Monthly incomes of two persons are in the ratio $4:5$, and their monthly expenses are in the ratio $7:9$. If each saves ₹50 per month, find their monthly incomes.
a) (500, 400) **b)** (400, 500) **c)** (300, 600) **d)** (350, 550)

Sol. (b)

Find the ratio of income given in (then the ratio for (option)

$(a)\ \frac{500}{400} = \frac{5}{4} = 5:4$ (×)

$(b)\ \frac{400}{500} = \frac{4}{5} = 4:5$ (✓)

$(c)\ \frac{300}{600} = \frac{1}{2} = 1:2$ (×)

$(d)\ \frac{350}{550} = \frac{7}{11} = 7:11$ (×)

Here, only ((b) satisfies the condition.

Q.24) y is older than x by 7 years 15 years back x's age was 3/4 of y's age. Their present ages are;
a) (x =36, y=43) **b)** (x =50, y=43) **c)** (x =43, y=50) **d)** (x =40, y=47)

Sol. (a)

$y - x = 7$_____(I)
Also $x - 15 = \frac{3}{4}(y - 15)$
$\Rightarrow 4x - 3y = 15$_____(II)
Check out the option,
For the (option)
$(a)\ (x = 36, y = 43),\ 43 - 36 = 7 \ \Rightarrow 7 = 7$
Also $4 \times 36 - 3 \times 43 = 15 \ \Rightarrow 144 - 129 = 15 \ \Rightarrow 15 = 15$ (✓)
In a similar way. Check the other options which don't satisfy the equation

Q.25) Find the fraction which is equal to 1/2 when both its numerator and denominator areincreased by 2. It is equal to 3/4 when both are increased by 12.
a) 3/8 **b)** 5/8 **c)** 2/8 **d)** 2/3

Sol. (a)

Check the option
For the option

$(a)\ \frac{3+2}{8+2} = \frac{5}{10} = \frac{1}{2}$

$\frac{3+12}{8+12} = \frac{15}{20} = \frac{3}{4}$ (✓)

Check the other options in the same manner which don't satisfy the condition.

Q.26) The sum of two numbers is 8, and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are;
a) (7, 10) **b)** (4, 4) **c)** (3, 5) **d)** (2, 6)

Sol. (c)

One number x & other $= 8 - x$
$\therefore x^2 + (8 - x)^2 = 34$
$\Rightarrow x^2 + 64 - 16x + x^2 = 34$
$\Rightarrow 2x^2 - 16x + 30 = 0$
$\Rightarrow x^2 - 8x + 15 = 0$
$\Rightarrow (x - 5)(x - 3) = 0 \Rightarrow x = 5 \ or \ x = 3$

Or
Check the option
For the option
$(a)\ (7,10),\ 7+10=8\ \Rightarrow 17=8$
$(b)\ (4,4),\ 4+4=8\ \Rightarrow 8=8$
Also $4^2+4^2=34\ \Rightarrow 16+16=34\ \Rightarrow 32=34$ (×)
$(c)\ (3,5),\ 3+5=8\ \Rightarrow 8=8$
Also $3^2+5^2=34\ \Rightarrow 9+25=34\ \Rightarrow 34=34$ (×)
$(d)\ (2,6),\ 2+6=8\ \Rightarrow 8=8$ (✓)
$2^2+6^2=34\ \Rightarrow 40=34$ (×)

Q.27) The area of a rectangular field is 2000 sq. m, and its perimeter is 180m. Form a quadratic equation by taking the length of the field as x and solve it to find the length and breadth ofthe field. The length and breadth are;

a) (205m, 80m) [✓ **b)** (50m, 40m)] **c)** (60m, 50m) **d)** None

Sol. (b)
$l=x,\ 2(l+b)=180\ \Rightarrow b=90-x$
$A=lb\ \Rightarrow x\,(90-x)=2000$
$\Rightarrow x^2-90x+2000=0$
$\therefore$ For the (option)
$(a)\ (205m, 80m)\ \therefore x=205$
$\therefore (205)^2-90\ \times 205+2000=0$ (×)
$(b)\ (50m, 40m), x=50$
$\therefore\ 50^2-90\times 50+2000=0$
$\Rightarrow 2500-4500+2000=0$
$\Rightarrow 0=0$ (✓)

In a similar way, check out the other options which don't satisfy the equation.

Q.28) The hypotenuse of a right–angled triangle is 20cm. The difference between its other two sides be 4cm. The sides are;

a) (11cm, 15cm) [✓ **b)** (12cm, 16cm)] **c)** (20cm, 24cm) **d)** None of these

Sol. (b)
Let the sides are $x\ cm$ & $(x+4)cm$
$\therefore\ x^2+(x+4)^2=20^2$
$\therefore$ For the (option)
$(a)\ (11cm, 15cm)\ \therefore x=11$
$\therefore\ 11^2+15^2=20^2\ \Rightarrow 121+225=400$ (×)
$(b)\ (12cm, 16cm),\ x=12$
$12^2+16^2=20^2\ \Rightarrow 144+256=400\ \Rightarrow 400=400$ (✓)

In a similar way, check out the other options which don't satisfy the equation

Q.29) The sides of an equilateral triangle are shortened by 12 units, 13 units, and 14 units, respectively and a right-angled triangle is formed. The side of the equilateral triangle is;

[✓ **a)** 17 units] **b)** 16 units **c)** 15 units **d)** 18 units

Sol. (a)
Let the sides of an equilateral triangle be x units
$\therefore$ The sides of the right triangle are
$x-12,\ x-13,\ x-14$
$\therefore (x-12)^2=(x-13)^2+(x-14)^4$
Now, for the (option)
$(a)\ x=17$

$(17-12)^2 = (17-13)^2 + (17-14)^2$
$\Rightarrow 5^2 = 4^2 + 3^2$
$\Rightarrow 25 = 25$

In a similar way, check out the other options which don't satisfy the equation (✓)

Q.30) A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience, it is known that demand D (in number of bottles) is given by D = $-2000p^2 + 2000p + 17000$. The price per bottle that will result in zero inventory is

a) ₹ 3 **b)** ₹ 5 **c)** ₹ 2 **d)** None of these

Sol. (a)

$-2000\,p^2 + 2000p + 17000 = 5000$
∴ For the option
$(a)\ p = 3, -2000 \times 3^2 + 2000 \times 3 + 17000 = 5000 \ \Rightarrow 5000 = 5000$ (✓)
$(b)\ p = 5, -2000 \times 5^2 + 2000 \times 5 + 17000 = 5000$
$\Rightarrow -23000 = 5000$
$(c)\ p = 2$ (×)
$\therefore -2000 \times 2^2 + 2000 \times 2 + 17000 = 5000$
$\Rightarrow 13000 = 5000$
(×)

Q.31) One student is asked to divide half of a number by 6 and the other half by 4 and add the two quantities. Instead of doing so, the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was;

a) 320 **b)** 400 **c)** 480 **d)** None of these

Sol. (c)

For the option
(a) Number = 320
Now $\frac{1}{6}\left(\frac{1}{2} \times 320\right) + \frac{1}{4}\left(\frac{1}{2} \times 320\right) = \frac{1}{5}\,(320) + 4$
$\Rightarrow \frac{80}{3} + 40 = 64 + 4$ (×)
(b) 400,
$\frac{1}{6}\left(\frac{1}{2} \times 400\right) + \frac{1}{4}\left(\frac{1}{2} \times 400\right) = \frac{1}{5}\,(400) + 4$
$\Rightarrow \frac{100}{3} + 50 = 80 + 4$ (×)
(c) 480
$\frac{1}{6}\left(\frac{1}{2} \times 480\right) + \frac{1}{4}\left(\frac{1}{2} \times 480\right) = \frac{1}{5}\,(480) + 4$
$\Rightarrow 40 + 60 = 96 + 4$
$\Rightarrow 100 = 100$
(✓)

Q.32) The sum of the digits in a three-digit number is 12. If the digits are reversed, the number is increased by 495 but reversing only of the ten's and unit digits increases the number by 36. The number is;

a) 327 **b)** 372 **c)** 237 **d)** 273

Sol. (c)

Let the digit at 100th, 10th and unit places be x, y & z respectively
$\therefore Number = 100x + 10y + z$
∴ the equation are
$x + y + z = 12$____(I)
$100z + 10y + x = 100x + 10y + z + 495$
$99z - 99x = 495$
$\Rightarrow z - x = 5$____(II)
$100x + 10z + y = 100x + 10y + z + 36$
$\Rightarrow 9\,(z - y) = 36$

$\Rightarrow z - y = 4$ ____ (III)

Now, Check out the option,

For the (option)

$(a)\ 327, 3 + 2 + 7 = 12 \Rightarrow 12 = 12$

$7 - 3 = 5 \Rightarrow 4 = 5$ (×)

$(b)\ 372,\ 3 + 7 + 2 = 12 \Rightarrow 12 = 12$

$2 - 3 = 5 \Rightarrow -1 = 5$ (×)

$(c)\ 237,\ 2 + 3 + 7 = 12 \Rightarrow 12 = 12$

$7 - 2 = 5 \Rightarrow 5 = 5$

And $7 - 3 = 4 \Rightarrow 4 = 4$ (✓)

$(d)\ 273,\ 2 + 7 + 3 = 12 \Rightarrow 12 = 12$

$3 - 2 = 5 \Rightarrow 1 = 5$ (×)

Q.33) The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p = \frac{q}{3} + \frac{7}{4}$, respectively where p is the market price, and q is the quantity, then the equilibrium price and quantity are:

[✓ **a)** $2, \frac{3}{4}$] **b)** $3, \frac{1}{2}$ **c)** $5, \frac{3}{5}$ **d)** None of these

Sol. (a)

$4q + 7p = 17$ ____ (I)

$and\ p = \frac{q}{3} + \frac{7}{4}$ ____ (II)

Check the option,

For the (opyion)

$(a)\ (p, q) = \left(2, \frac{3}{4}\right), 4 \times \frac{3}{4} + 7 \times 2 = 17 \Rightarrow 17 = 17$

$\&\ 2 = \frac{3/4}{3} + \frac{7}{4} \Rightarrow 2 = 2$ (✓)

$(b)\ \left(3, \frac{1}{2}\right),\ 4 \times \frac{1}{2} + 7 \times 3 = 17 \Rightarrow 23 = 17$ (×)

$(c)\ \left(5, \frac{3}{5}\right), 4 \times \frac{3}{5} + 7 \times 5 = 17$ (×)

Q.34) If the roots of the equations $x^3 - 15x^2 + kx - 45 = 0$ are in A.P., find value of k:

a) 56 **b)** 59 **c)** -56 **d)** -59

Sol. (b)

$\because$ Roots are in A.P.

Let roots are $a - d\ ; a\ ; a + d$

$So, (a - d) + a + (a + d) = 15$

$or;\ 3a = 15$

$or;\ a = 5$

And Product of roots

$(a - d).a.(a + d) = 45$

$or\ (5 - d).5.(5 + d) = 45$

$or\ 25 - d^2 = 9$

$or;\ d^2 = 25 - 9 = 16$

$or\ ; d = \sqrt{16} = 4$

Hence; roots are $a - d\ ; a\ ; a + d = 5 - 4\ ; 5; 5 + 4$

$= 1\ ; 5\ ; 9.$

The value of k

= Sum of the product of two roots in an order

$= (1 \times 5) + (5 \times 9) + (9 \times 1)$

$= 5 + 45 + 9 = 59$

Q.35) If $\alpha + \beta = -2$ and $\alpha\beta = -3$ where α and β are the roots of the equation, which is

a) $x^2 - 2x - 3 = 0$ [✓ **b)** $x^2 + 2x - 3 = 0$] **c)** $x^2 + 2x + 3 = 0$ **d)** $x^2 - 2x + 3 = 0$

Sol. (b)

Quadratic Eqn. having roots α and β is

$x^2 - (\alpha + \beta)x + \alpha\,\beta = 0$

$or;\ x^2 - (-2)x + (-3) = 0$

$or;\ x^2 + 2x - 3 = 0$

Q.36) Find the condition that one root is double the of $ax^2 + bx + c = 0$

a) $2b^2 = 3ac$ **b)** $b^2 = -3ac$ [✓ **c)** $2b^2 = 9.ac$] **d)** None

Sol. (c)

Let 1st root = 1

Then 2nd root = 2

Then Eqn. is

$x^2 - (1 + 2)x + 1 \times 2 = 0$

$or\ x^2 - 3x + 2 = 0$

Comparing it with $ax^2 + bx + c = 0$

We get;

$a = 1\,;\, b = -3;\, c = 2$

Go by choices (GBC)

$(a)\ 2b^2 = 3ac$

$\therefore 2.(-3)^2 = 3.1.2 = 6$ $(False)$

$(b)\ 2b^2 = -3ac$

$\therefore 2.(-3)^2 = -3.1.2 = -6$ $(False)$

$(c)\ 2b^2 = 9.ac$

$\therefore 2.(-3)^2 = 9.1.2$

$\Rightarrow 18 = 18$ $(True)$

Q.1) P = ₹8,500, A = ₹10,200, R = 12 ½ % SI, T will be.
a) 1 yr. 7 months. **b)** 2 yrs. **c)** 1 ½ yr. **d)** None of these

Sol. (a)

$T = \frac{S.I.\times100}{PR} = \frac{1700\times100\times2}{8500\times25}$ $(\because Simple\ interest = Amount - Principal = ₹1700)$

$T = \frac{8}{5}$ years

$T = 1\frac{3}{5}\ yrs = 1yr\ 7month$ (approx)

Q. 2) Amount and S.I. on ₹ 3,500 for 3 years at 12 % p.a. is
a) ₹ 3800 **b)** ₹ 4,760 **c)** ₹ 3500 **d)** None of these

Sol. (b) $S.I = Pit\ or\ \frac{PRT}{100}$ $(\because$ i = R%)

= ₹ 3500 × $\frac{12}{100}$ × 3

= ₹ 1260

$Amount = Principal + Interest$

= 3500 + 1260 = ₹ 4760

Q. 3) A sum of amount to ₹ 6200 in 2 years and ₹ 7400 in 3 years. The principal and rate of interest are
a) ₹ 3800, 31.57% **b)** ₹ 3,000, 31.57% **c)** ₹ 3500, 31.57% **d)** None of these

Sol. (a)

$Amount = Principal + Interest = (P + \frac{PRT}{100})$

$\Rightarrow A = P\left(1 + \frac{RT}{100}\right)$

= Amount at end of 2 years= ₹6,200

= Amount at end of 3 years= ₹7,400

∴ Difference of amount of 2 year and 3 year is simple interest= ₹1200

∴ Simple interest for two years =1200× 2 = ₹2400

∴Principal = Amount at end of 2year – Simple interest of 2 years.

Principal= 6,200 - 2,400= ₹ 3,800

Rate of interest = $\frac{1,200}{3,800} \times 100 = 31.57\%$.

Q.4) In what time will ₹ 8000 at 3% per annum produce the same interest as ₹ 6000 does in 5 years at 4% simple interest is?
a) 4 years **b)** 5 years **c)** 3 years **d)** None of these

Sol. (b)

Let the time be t

According to the question,

$\Rightarrow \frac{8000\times3\times t}{100} = \frac{6000\times5\times4}{100}$

$\Rightarrow 240t = 1200$

∴ t = 5 years

Q.5) A sum of money doubles itself in 10 years. The number of years it would triple itself is;
a) 25 years. **b)** 15 years. **c)** 20 years **d)** None of these

Sol. (c)

$A = 2P,\ T = 10$

$\therefore S.I = P$

$\therefore R = \frac{S.I\times100}{P\times T} = \frac{P\times100}{P\times10} = 10\ \%$

Now, $T = \frac{S.I.\times100}{PR}$

$= \frac{2P\times100}{P\times10}$ $(\because I = 3P - P = 2P)$

= 20 years

Q.6) If the simple interest for 6 years is equal to 30% of the principal, then interest will be equal to the principal after how many years?

a) 20 years **b)** 30 years **c)** 10 years **d)** 22 years

Sol. (a)

Let the principal be ₹ x, and the rate of interest be R%

Then, S.I. = 30% of $x = \frac{30}{100} \times x$

$\Rightarrow \frac{30}{100} \times x = \frac{x \times R \times 6}{100}$

$\Rightarrow \mathrm{R} = 5\%$

Let the time in which the principal is equal to simple interest be 't' years

$\Rightarrow x = \frac{x \times 5 \times t}{100}$

$\Rightarrow \mathrm{t} = \frac{100}{5}$

$\therefore \mathrm{t} = 20$ years

Q.7) A sum of ₹ 725 is lent at the beginning of the year at a certain rate of interest. After 8 months, a sum of ₹ 362.50 more is lent but at a rate twice the former. At the end of the year, ₹ 33.50 is earned as interest from both loans. What was the original rate of interest?

a) 2.77% **b)** 2.50% **c)** 6% **d)** 5%

Sol. (a)

The original rate is for 8 months, and the new rate is for only 4 months, i.e., 1/3years

$\Rightarrow 33.50 = \left[\frac{725 \times R \times 8}{100 \times 12}\right] + \left[\frac{(725 + 362.50) \times 2R \times 4}{100 \times 12}\right]$

$\Rightarrow 33.50 \times 300 = (1450 + 2175)R$

$\therefore R = 2.77\%$

Q.8) A father wants to divide 18750 between his two sons. One is 12 years old, and the other is 14 years old. Father wants that at the rate of 5% per annum, his both son will get the same amount at the age of 18. Find the sum that should be allotted to the elder son:

a) ₹ 9,000 **b)** ₹ 9,750 **c)** ₹ 9,500 **d)** ₹ 10,000

Sol. (b)

Let the younger son allotted amount to be ₹ x and the elder son ₹ $(18750 - x)$

$\Rightarrow x + \frac{x \times 5 \times 6}{100} = (18750 - x) + \frac{(18750 - x) \times 5 \times 4}{100}$

$\Rightarrow x + \frac{30x}{100} = (18750 - x) + 3750 - \frac{20x}{100}$

$\Rightarrow 2x + \frac{x}{2} = 22500$

$\Rightarrow x =$ ₹ 9,000

∴ The elder son allotted amount

= (18,750 – 9,000)

= ₹ 9,750

Q.9) A computer is available for ₹ 39,000 cash or ₹ 17,000 as cash down payment followed by five monthly instalments of ₹ 4800 each. What is the rate of interest under the instalment plan?

a) 35.71 %p.a. **b)** 36.71%p.a. **c)** 37.71%p.a. **d)** 38.71%p.a.

Sol. (d)

The total cost of computer =₹ 39000

Down payment =₹ 17000

Balance = 39000 – 17000 =₹ 22000

Let the rate of interest be R% p.a.

Amount of ₹ 22000 for 5 months

$\Rightarrow 22000 + 22000 \times \frac{5}{12} \times \frac{R}{100}$

$\Rightarrow 22000 + \frac{275}{3}R$ ...(i)

Customer pay ₹ 4800 per month up to 5 months

Sum of the amount of these instalments

$\Rightarrow 4800 \times 5 + \text{S.I. on } 4800 \text{ for } (4+3+2+1)$

$\Rightarrow 24000 + 4800 \times \frac{10}{12} \times \frac{R}{100}$

Amount = 24000 + 40R ...(ii)

by using (i) and (ii)

$\Rightarrow 22000 + \frac{275}{3}R = 24000 + 40R$

$\Rightarrow 2000 = \frac{275-120}{3}R$

$\therefore$ R = 38.71% p.a.

Q.10) A person invest an amount of ₹ 2200 in two parts. If the ratio of rates of two investments is 4:5 and ratio of their respective time is $2\frac{1}{2}:3\frac{1}{2}$ then the interest produced in both parts are equal find the 1st part?

a) ₹ 1000 **b)** ₹ 1200 **c)** ₹ 1400 **d)** None of these

Sol. (c)

Let the 1st part be x, and 2nd part be y

Let the rate of interest of both investment = 4r and 5r

Let the time of both investment = 5t/2 and 7t/2

Hence, $x + y = 2200$

$\Rightarrow y = 2200 - x$

According to question

$\Rightarrow \frac{x \times 4r \times 5t}{100 \times 2} = \frac{y \times 5r \times 7t}{100 \times 2}$

$\Rightarrow 20x = 35y$

$\Rightarrow 20x = 35(2200 - x)$

$\Rightarrow 55x = 77000$

$\therefore x$ = ₹ 1400

Q.11) A sum of ₹ 1440 is lent out in three parts in such a way that the interests on first parts at 2% for 3 years, the second part at 3% for 4 years and the third part at 4% for 5 years are equal. Then the difference between the largest and the smallest is

a) ₹ 200 **b)** ₹ 400 **c)** ₹ 560 **d)** ₹ 500

Sol. (c)

Let the first, second and third parts be x, y and z, respectively

According to question

$\Rightarrow \frac{x \times 2 \times 3}{100} = \frac{y \times 3 \times 4}{100} = \frac{z \times 4 \times 5}{100}$

$\Rightarrow 6x = 12y = 20z = 60K$

$\Rightarrow x = 10K, y = 5K, z = 3K$

$\Rightarrow 18K = 1440 \therefore K = 80$

$\Rightarrow x$ = ₹ 800, y = ₹ 400, z = ₹ 240

$\therefore$ Difference =₹ 560

Q.12) A boy aged 12 years is left with ₹ 1,00,000, which is under a trust. The trustees invest the money at 6% per annum and pay the minor by a sum of ₹ 2500 for his pocket money at the end of each year. The expenses of trust come out to be ₹ 500 per annum. Find the amount that will be handed over to the minor boy after he attains the age of 18 years.

a) ₹ 1,25,000 **b)** ₹ 1,18,000 **c)** ₹ 1,50,000 **d)** ₹ 1,20,000

Sol. (b)

P=₹1,00,000

r = 6%

Total Expenses = 2500+500= ₹ 3000

S.I. = $\frac{100000 \times 6 \times 1}{100}$ = ₹ 6000

Saving = 6000 – 3000 = ₹ 3000

Interest till 18 years = 3000×6= ₹ 18000

Amount handed over to the minor boy after he attains the age of 18 years

= 100000+18000= ₹ 1,18,000

Q.13) In order to buy a car, a man borrowed ₹ 1,80,000 on the condition that he had to pay 7.5% interest every year. He also agreed to repay the principal in equal annual instalments over 21 years. After a certain number of years, however, the rate of interest has been reduced to 7%. It is also known that at the end of the agreed period, he will have paid in all ₹ 2,70,900 in interest. For how many years does he pay at the reduced interest rate?

a) 7 years **b)** 12 years **c)** 14 years **d)** 16 years

Sol. (c)

if he pays 7.5% for n years and then 7% for the remaining 21-n years on ₹ 1,80,000, then he pays this much interest

$\Rightarrow \left(\frac{7.5}{100}\right)(180{,}000)(n) + \left(\frac{7}{100}\right)(180{,}000)(21 - n)$

That's equal to ₹ 270,900, so we have:

$\Rightarrow \left(\frac{7.5}{100}\right)(180{,}000)(n) + \left(\frac{7}{100}\right)(180{,}000)(21 - n) = 2{,}70{,}900$

$\Rightarrow (7.5)(1800)(n) + (7)(1800)(21 - n) = 2{,}70{,}900$

$\Rightarrow 7.5n + 147 - 7n = \frac{270{,}900}{1800} = \frac{301}{2}$

$\Rightarrow 15n + 294 - 14n = 301$

$\therefore n = 7$

Q.14) Three amounts P, Q and R such that Q is the simple interest on P and R is the simple interest on Q. If in all the cases, rate of interest per annum and the time for which interest is calculated in the same, then the relation between P, Q and R is;

a) PQR = 1 **b)** $P^2 = QR$ **c)** $Q^2 = PR$ **d)** $R = P^2Q$

Sol. (c)

Let the rate be r and the time be t in both cases

$\Rightarrow Q = \frac{P \times r \times t}{100}$ ---(1)

$\Rightarrow R = \frac{Q \times r \times t}{100}$ ---(2)

Dividing equation (2) from (1)

$\Rightarrow \frac{Q}{R} = \frac{P}{Q}$

$\therefore Q^2 = PR$

Q.15) A person invested ₹ 5,000 at some rate of simple interest and ₹ 4,000 at 1 per cent higher rate of interest. If the interest in both the case after 4 years is same, the rate of interest in the former case is

a) 4% **b)** 6% **c)** 8% **d)** 2%

Sol. (a)

Let the rate be x

According to question

$\Rightarrow \frac{5000 \times x \times 4}{100} = \frac{4000 \times (x+1) \times 4}{100}$

$\Rightarrow 4000x = 16{,}000$

$\therefore x = 4\%$

Q.16) An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes:

a) 10% **b)** 10.25% **c)** 10.5% **d)** None of these

Sol. (b)

Let the sum of ₹ 100

S.I. for first 6 months = $\frac{100 \times 10 \times 1}{100 \times 2}$ = ₹ 5

S.I. for last 6 months = $\frac{105 \times 10 \times 1}{100 \times 2}$ = ₹ 5.25

So, amount at the end of 1 year = (100+5+5.25) = ₹ 110.25

∴ Effective rate =110.25 – 100 = 10.25%

Q.17) Mr X borrowed ₹ 5,120 at 12 ½ % p.a. C.I. At the end of 3 years, the money was repaid alongwith interest accrued. The total amount paid by him is;

a) ₹ 7,100 **b)** ₹ 7,290 **c)** ₹ 7,000 **d)** None of these

Sol. (b)

$A = P\,[(1+i)^n]$

$A = 5120\,[(1+0.125)^3]$

A = ₹ 7,290

Q.18) ₹4,000 is invested at an annual rate of interest of 10%. What is the amount after two years if compounding is done (a) Annually, (b) Semi-annually, (c) Quarterly, (d) Monthly?

a) ₹4840, ₹ 4862, ₹ 4874, ₹ 4882 **b)** ₹ 4840, ₹ 4682, ₹ 4874, ₹ 4922

c) ₹ 4840, ₹ 4862, ₹ 4784, ₹ 4922 **d)** None

Sol. (a)

(a) $A = P\,[(1+i)^n]$

$A = 4{,}000\,[(1+0.1)^2]$

A = ₹ 4,840

(b) $n = 2 \times 2 = 4$ *and* $i = \frac{10}{2} = 5\%$

$A = P\,[(1+i)^n]$

$= 4{,}000\,[(1+0.05)^4]$

= ₹ 4,862

(c) $n = 4 \times 2 = 8$ *and* $i = \frac{10}{4} = 2.5\%$

$A = P\,[(1+i)^n]$

$= 4{,}000\,[(1+0.025)^8]$

= ₹ 4,873.6

(d) $n = 12 \times 2 = 24$ *and* $i = \frac{10}{12} = 0.833\%$

$A = P\,[(1+i)^n]$

$= 4{,}000\,[(1+0.00833)^{24}]$

= ₹ 4,881.56

Q.19) A bank pays interest at the rate of 8% p.a. compounded half-yearly. Find how much should be deposited in the bank at the beginning of the year in order to accumulate ₹ 12,000 for 3 years?

a) ₹ 11,200 **b)** ₹ 10,124 **c)** ₹ 9,486.16 **d)** ₹ 10,890

Sol. (c)

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 12000 = P\left(1+\frac{8}{2\times100}\right)^{2\times3}$$

$$\Rightarrow 12000 = P\left(\frac{26}{25}\right)^6$$

$$\Rightarrow 12000 = P(1.265)$$

$\Rightarrow P =$ ₹ 9,486.16

Q.20) A sum is being lent out at 20% p.a. compound interest. What is the ratio of increase in the amount of 4th year to 5th year?

a) 4:5 **b)** 5:4 **c)** 5:6 **d)** Can't determined

Sol. (c)

$$\Rightarrow \frac{P\left(1+\frac{r}{100}\right)^4}{P\left(1+\frac{r}{100}\right)^5}$$

$$\Rightarrow \frac{1}{\left(1+\frac{r}{100}\right)}$$

$$\Rightarrow \frac{100}{100+r}$$

$\Rightarrow \frac{100}{120}$

$\Rightarrow 5:6$

Q.21) In the compound interest, if the amount is 9 times its principal in two years, then the rate of interest is?

a) 300% [✓ **b)** 200%] **c)** 150% **d)** 100%

Sol. (b)

Given,

$A = P\left(1+\frac{r}{100}\right)^t$

Or; $9P = P\left(1+\frac{r}{100}\right)^2$

Or; $9 = \left(1+\frac{r}{100}\right)^2$

Or; $3^2 = \left(1+\frac{r}{100}\right)^2 \Rightarrow 3 = 1+\frac{r}{100}$

$\Rightarrow 2 = \frac{r}{100} \Rightarrow r = 200\%$

Q.22) Udit purchased a Maruti Van for ₹ 1,96,000, and the rate of depreciation was $14\frac{2}{7}\%$ per annum. Find the value of the van after two years.

[✓ **a)** ₹ 1,44,000] **b)** ₹ 1,40,000 **c)** ₹ 1,50,000 **d)** ₹ 1,60,000

Sol. (a)

Cost of Van = ₹ 1,96,000

Time = 2 years

Rate of depreciation = 100/7%

Amount = $1{,}96{,}000\left(1-\frac{100/7}{100}\right)^2$

$\Rightarrow 196000 \times \left(\frac{6}{7}\right)^2$

$\Rightarrow$ ₹ 1,44,000

After 2 years it's cost will be ₹ 1,44,000

Q.23) A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000, and its ultimate scrap value was ₹ 30,000. The effective life of the machine is;

a) 4.5 years (approx.) [✓ **b)** 5.4 years (appx.)] **c)** 5 years (appx.) **d)** None of these

Sol. (b)

$30000 = 100000\,(1-0.2)^n$

$\Rightarrow 30{,}000 = 1{,}00{,}000(.8)^n$

$\Rightarrow \frac{30{,}000}{1{,}00{,}000} = (0.8)^n \Rightarrow 0.3 = (0.8)^n$

$\Rightarrow$ n= 5.4 years (approx.)

Q.24) The least number of complete years in which a sum of money put at 20% CI will be more than doubled is:

a) 4 **b)** 5 **c)** 6 **d)** 8

Sol. (a)

According to question

$\Rightarrow P\left(1+\frac{20}{100}\right)^n > 2P$

$\Rightarrow \left(\frac{6}{5}\right)^n > 2$

$\Rightarrow \frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5} > 2$

$\therefore$ n = 4 years

Q.25) If a principal P becomes Q in 2 years when interest R% is compounded half-yearly. And if the same principal P becomes Q in 2 years when interest S% has compounded annually, then which of the following is true?

a) $R > S$ **b)** $R = S$ [✓ **c)** $R < S$] **d)** $R \leq S$

Sol. (c)

Since interest is compounded half-yearly at R% p.a., the value of R will be lesser than the value of S.

Q.26) The difference between C.I. and S.I. on a certain sum of money invested for 3 years at 6% p.a.is ₹ 110.16. The principle is;

a) ₹ 3,000 **b)** ₹ 3,700 **c)** ₹ 12,000 [✓ **d)** ₹ 10,000]

Sol. (d)

$C.I - S.I = ₹\ 110.16$

$\Rightarrow P = \frac{D\times(100)^3}{r^2\times(r+300)} \Rightarrow \frac{110.16\times(100)^3}{36\times(306)} = ₹\ 10{,}000\ (approx.)$

Q.27) A man gets a simple interest of ₹ 1000 on a certain principal at the rate of 5% in 4 years. What compound interest will the man get on twice the principal in 2 years at the same rate?

a) ₹1,000 **b)** ₹1,005 **c)** ₹1,012 [✓ **d)** ₹1,025]

Sol. (d)

R=5%, t=4 years

$\Rightarrow 1000 = \frac{P\times5\times4}{100}$

$\therefore P = 5000$

Twice of Principal = 2×5000 = ₹ 10,000

Amount = $10000\left(1+\frac{5}{100}\right)^2$

Amount = ₹ 11,025

$\therefore$ C.I. = 11025 – 10000 = ₹ 1,025

Q.28) A person invested some amount at the rate of 6% simple interest per annum and received ₹ 900 as an interest after 3 years. If interest is compounded every year, then how many rupees more did he receive on the same amount at the same rate of interest for the same time?

a) ₹ 38.13 **b)** ₹ 25.33 [✓ **c)** ₹ 55.08] **d)** ₹ 35.30

Sol. (c)

Let Principal amount be P

$\Rightarrow 900 = \frac{P\times6\times3}{100}$

$\Rightarrow P = ₹\ 5{,}000$

Amount = $5000(1+0.06)^3 = ₹\ 5{,}955.08$

C.I. = 5955.08 – 5000 = ₹ 955.08

$\therefore$ CI – SI =955.80 - 900 = ₹ 55.08

Q.29) Manav borrowed a certain amount from the bank for 4 years. The bank charges 20% S.I. for the first two years and 15% p.a. C.I. for the last 2 years. If the interest amount given by Manav to the bank is ₹ 2970 less than the borrowed amount, then what is the total amount given by Manav to the bank?

a) ₹ 28,090 **b)** ₹ 37,030 **c)** ₹ 41,070 **d)** ₹ 32,050

Sol. (b)

Let borrowed amount be $100x$

Total amount at the end of 2 years = $100x + (100x \times 20 \times 2)/100 = 140x$

Total amount at the end of 4 years = $140x\ (1 + 15/100)^2 = 185.15x$

According to question

$\Rightarrow 185.15x - 100x = 100x - 2970$

$\Rightarrow 100x - 85.15x = 2970$

$\Rightarrow 14.85x = 2970$

$\Rightarrow x = 200$

Total amount borrowed = $100x$ = ₹ 20,000
Total interest amount = 20000 – 2970 = ₹17,030
∴ Total amount given by Manav to bank = 20,000 + 17,030 = ₹ 37,030

Q.30) A sum of money is accumulating at compound interest at a certain rate of Interest. If simple interest instead of the compound were reckoned, the interest for the first two years would be diminished by ₹ 20 and that for the first three years by ₹ 61. Find the sum.

a) ₹ 6,000
b) ₹ 8,000
c) ₹ 7,500
d) ₹ 6,500

Sol. (b)

Difference between C.I. and S.I. for 2 years = 20

$\Rightarrow \frac{PR^2}{100^2} = 20$ ---(i)

Difference between C.I. and S.I. for 3 years = 61

$\Rightarrow \frac{PR^2}{100^2}\left[\frac{300+R}{100}\right] = 61$

$\Rightarrow 300 + R = \left(\frac{61}{20}\right) \times 100$

$\Rightarrow$ R = 5%

From (i)

P = $20 \times \frac{100^2}{5^2}$

∴ P = ₹ 8,000

Q.31) Vikas invested the sum of money in two schemes, A and B, offering compound interest at 8% and 9% per annum, respectively. If the total amount of interest accrued through two schemes together in two years was ₹ 4818.30 and the total amount invested was ₹ 27,000, what was the amount invested in Scheme A?

a) ₹ 12,000
b) ₹ 13,500
c) ₹ 15,000
d) None of these

Sol. (a)

Let the amount invested in scheme A be x, and in the scheme, B be $(27000 - x)$

$\Rightarrow 4818.30 = \left[x\{\left(1 + \frac{8}{100}\right)^2 - 1\}\right] + \left[(27000 - x)\{\left(1 + \frac{9}{100}\right)^2 - 1\}\right]$

$\Rightarrow \frac{481830}{100} = \left(x \times \frac{104}{625}\right) + \frac{1881(27000-x)}{10000}$

$\Rightarrow 217x = 2604000$

∴ x = ₹ 12,000

Q.32) Kavita borrowed ₹ 10,815, which is to be paid back in 3 equal half-yearly instalments. If the interest is compounded half-yearly at $\frac{40}{3}$% per annum, how much is each instalment?

a) ₹ 2,048
b) ₹ 3,150
c) ₹ 4,096
d) ₹ 5,052

Sol. (c)

Rate of interest = $\frac{40/3}{2}\% = \frac{20}{3}\%$ half-yearly

$P.V. = R(\frac{1-(1+i)^{-n}}{i})$

$\Rightarrow 10815 = R(\frac{1-(1+0.0667)^{-3}}{0.0667})$

$\Rightarrow 10815 = R(2.6402)$

$\Rightarrow R = \frac{10815}{2.6402}$ = ₹ 4096

Q.33) A sum of money is put at compound interest for 2 years at 20% p.a. It would earn ₹ 482 more if the interest were payable half-yearly than it was payable yearly; then the sum is

a) ₹ 20,000
b) ₹ 25,000
c) ₹ 26,000
d) None of these

Sol. (a)

CI if calculated annually

$C_1 = P\left[\left(1+\frac{20}{100}\right)^2 - 1\right] \Rightarrow P\left[\frac{36}{25} - 1\right] = \frac{11P}{25}$

CI if calculated semi-annually

$C_1 = P\left[\left(1+\frac{10}{100}\right)^4 - 1\right] = \frac{4641P}{10000}$

$\therefore \frac{4641P}{10000} - \frac{11P}{25} = 482 \Rightarrow \frac{241P}{10000} = 482$

$\therefore$ P = ₹ 20,000

Q.34) Neeraj bought a car and paid ₹ 12,000 as a down payment. He told the seller that he would pay ₹ 13,050 after 1 year and ₹ 22,680 after two years at $12\frac{1}{2}$% compound interest per annum. At what amount did he purchase the car?

a) ₹ 42,000 **b)** ₹ 40,000 **c)** ₹ 41,520 **d)** ₹ 42,510

Sol. (c)

Down payment = ₹ 12000

Principal for the first instalment to be P_1

$\Rightarrow 13050 = P_1\left(1+\frac{25}{2\times100}\right)$

$\Rightarrow P_1 =$ ₹ 11600

Principal for the second instalment to be P_2

$\Rightarrow 22680 = P_2\left(1+\frac{25}{2\times100}\right)^2$

$\Rightarrow P_2 =$ ₹ 17920

$\therefore$ Purchase value of car = 12000 + 11600 + 17920 = ₹ 41,520

Q.35) The annual birth and death rates per 1,000 are 39.4 and 19.4, respectively. The number of years in which the population will be doubled assuming there is no immigration or emigrationis;

a) 35 years. **b)** 30 years. **c)** 25 years. **d)** None of these

Sol. (a)

Annual increment in the population

= (39.4 − 19.4) = 20 Per thousand = 2%

Let the initial population = 100 and the number of years be n

Final population = 200

$A = P(1+i)^n$

$\Rightarrow 2\times100 = 100\left(1+\frac{2}{100}\right)^n$

$\Rightarrow 2 = (1.02)^n \Rightarrow n = 35$ *years.*

Q.36) Rishabh borrowed a certain sum from Anita at a certain rate of 10% simple interest for 2 years. He lent this sum to Sunil at the rate 50% more than the rate on which simple interest was accrued. Find the C.I. for 2 years Compounded annually at the new rate of interest if Rishabh paid ₹ 8000 as an interest to Anita?

a) ₹ 12,900 **b)** ₹ 11,000 **c)** ₹ 13,900 **d)** ₹ 14,000

Sol. (a)

$\because$ S.I. = ₹ 8000

$\Rightarrow \frac{P\times10\times2}{100} = 8000$

$\Rightarrow P =$ ₹ 40,000

New rate 50% more than 10 = 150/100× 10 = 15%

$\Rightarrow C.I. = 40000\times\left[\left(1+\frac{15}{100}\right)^2 - 1\right]$

$\Rightarrow$ C.I. $= 40000\times\left[\left(\frac{23}{20}\right)^2 - 1\right]$

$\therefore$ C.I. = ₹ 12,900

Q.37) Kavita has ₹ 48000 with her, a part of which she invested in a scheme for 3 years at 20% p.a. C.I., and from remaining, he purchased a laptop whose value decreases by 10% every year. At the end of 3 years, the price of the laptop will be ₹ 5508 more than the total amount received from the scheme, then at what price the laptop is purchased?

a) ₹ 40,000 **b)** ₹ 48,000 **c)** ₹ 32,000 **d)** ₹ 36,000

Sol. (d)

Let the amount invested in the scheme and price of the laptop be x and $48000 - x$, respectively.

Amount received after three years from scheme = $x \times 1.2^3 = 1.728x$

Price of the laptop after 3 years = $(48000 - x) \times 0.9^3 = (34992 - 0.729x)$

According to question

$34992 - 0.729x - 1.728x = 5508$

$\Rightarrow x =$ ₹ 12000

∴ Price at which laptop is purchased = 48000 – 12000 = ₹ 36,000

Q.38) Ankit and Anjali have equal amounts. Ankit invested all his amounts at 10% p.a. compounded annually for 2 years, and Anjali invested 1/4th amount at 10% p.a. compound interest annually and rested at r% per annum at simple interest for the same 2 years period. The amount received by both at the end of 2 years is the same. What is the value of r?

a) 14% **b)** 12.50% **c)** 10.50% **d)** 11%

Sol. (c)

Let the amount of Ankit and Anjali each has ₹ 100

Compound amount of Ankit's investment

$\Rightarrow 100\left(1 + \frac{10}{100}\right)^2 = 121$

Compound amount of Anjali's investment of 1/4th of the amount, i.e. 100/4 = 25

$\Rightarrow 25\left(1 + \frac{10}{100}\right)^2 = 30.25$

Simple interest of Anjali's rest of the amount, i.e. 75

$\Rightarrow \text{S.I.} = \frac{75 \times r \times 2}{100} = 1.5r$

The amount received by both at the end of 2 years is the same

$\Rightarrow 121 = 30.25 + 75 + 1.5r$

$\therefore r = 10.5\%$

Q.39) ₹ 2,60,200 is divided between Ram and Shyam so that the amount that Ram receives in 4 years is the same as that Shyam receives in 6 years. If the interest is compounded annually at the rate of 4% per annum, then Ram's share is;

a) ₹ 1,25,000 **b)** ₹ 1,35,200 **c)** ₹ 1,52,000 **d)** ₹ 1,08,200

Sol. (b)

Let Ram's share be ₹ x, and Shyam share is ₹ $(2,60,200 - x)$

According to the question,

$\Rightarrow x\left[1 + \left(\frac{4}{100}\right)\right]^4 = (260200 - x)\left[1 + \left(\frac{4}{100}\right)\right]^6$

$\Rightarrow x = (260200 - x)\left[1 + \left(\frac{4}{100}\right)\right]^2$

$\Rightarrow x = (260200 - x)\frac{676}{625}$

$\Rightarrow \frac{625x}{676} = 260200 - x$

$\Rightarrow \frac{625x}{676} + x = 260200$

$\Rightarrow 1301x = 260200 \times 676$

$\Rightarrow x =$ ₹ $1,35,200$

∴ Ram's share is ₹ 1,35,200

Q.40) The effective rate of interest corresponding a nominal rate of 7% p.a. convertible quarterly is;

a) 7% **b)** 7.5% **c)** 5% **d)** 7.18%

Sol. (c)

$E = (1 + \frac{i}{4})^{1 \times 4} - 1 \Rightarrow \left(1 + \frac{0.07}{4}\right)^4 - 1 = (1.0178)^4 - 1$

$= 0.07185903$

$= 7.18\%$ (approx.)

Q.41) Yash lent ₹ 12000 on a condition such that the rate of compound interest per annum is 5% for the first year, 6% for the next two years and 10% for the next three years. How much compound interest he will get at the end of the time period?

a) ₹ 6,843.35 **b)** ₹ 6,843.20 **c)** ₹ 6,483.75 **d)** ₹ 6,843.44

Sol. (d)

P = ₹ 12000

Amount $= 12000\left(1 + \frac{5}{100}\right)\left(1 + \frac{6}{100}\right)^2\left(1 + \frac{10}{100}\right)^3$

Amount $= 12000\left(\frac{21}{20}\right)\left(\frac{53}{50}\right)^2\left(\frac{11}{10}\right)^3$

A = ₹ 18,843.44

C.I.= 18,843.44 − 12000 = ₹ 6,843.44

Q.42) A company establishes a sinking fund to provide for the payment of ₹ 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if interest is 5% per annum.

a) ₹ 6142 **b)** ₹ 6048 **c)** ₹ 6052 **d)** ₹ 6152

Sol. (b)

$\Rightarrow A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$\Rightarrow 200000 = \frac{R}{0.05}(1.05)^{20} - 1) =$ ₹ 6048

Q.43) A = ₹100 n = 10, i = 5% find the FV of annuity.

Using the formula F.V. $A/i\,\{(1+i)^n - 1\}^{-1}$FV is equal to

a) ₹ 1,258 **b)** ₹ 2,581 **c)** ₹ 1,528 **d)** None of these

Sol. (a)

$$V = \frac{A}{i\,\{(1+i)^n - 1\}^{-1}} \quad \left[\because FV = \frac{A\,[(1+i)^n - 1]}{i}\right]$$

$$= \frac{100}{0.05\,\{(1.05)^{10} - 1\}^{-1}}$$

$$= \frac{100 \times 20}{(0.628)^{-1}}$$

$$= 2000 \times 0.628$$

= ₹ 1,258 (approx.)

Q. 44) If the amount of an annuity after 25 years at 5% p.a. CI is ₹ 50,000 the annuity will be

a) ₹ 1,406.90 **b)** ₹ 1,047.62 **c)** ₹1,146.90 **d)** None of these

Sol. (b) $\because A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$\Rightarrow 50000 = R\left[\frac{(1.05)^{25} - 1}{0.05}\right]$

$\Rightarrow$ 50,000 =R (47.727)

$\Rightarrow R =$ ₹ 1047.62

Q.45) The cost of a T.V. is ₹ 12,000. A customer bought it after paying ₹ 4,000 as a down payment, and he promises to pay the rest amount in three equal instalments at 5% compound interest per annum. Find the amount of each instalment paid by him?

a) ₹ 2,973.66 **b)** ₹ 2,937.66 **c)** ₹ 2,983.33 **d)** ₹ 2,837.33

Sol. (b)

Amount on which instalment has to be paid = 12,000 - 4,000 = ₹ 8,000

$\Rightarrow 8000 = R\left[\frac{1-(1+0.05)^{-3}}{0.05}\right]$

$\Rightarrow 8000 = R[2.7232]$

$\therefore$ R = ₹ 2937.66

Q.46) A loan of ₹10,000 is to be paid back in 30 equal instalments. The amount of each instalmentto cover the principal and at 4% p.a. CI is;

a) ₹ 587.87 **b)** ₹ 587 **c)** ₹ 578.30 **d)** None of these

Sol. (c)

$P.V. = R(\frac{1-(1+i)^{-n}}{i})$

$\Rightarrow 10,000 = R(\frac{1-(1+0.04)^{-30}}{0.04})$

$\Rightarrow 10,000 = R(17.292)$

= R = ₹ 578.30

Q. 47) Suppose your mom decides to gift you ₹ 10, 000 every year starting from today for the next five years. You deposit this amount in a bank as and when you receive and get 10% p.a. interest rate compound annually. What is the present value of this annuity?

a) ₹ 41,698.70 **b)** ₹ 41,958.70 **c)** ₹ 54,000 **d)** None of these

Sol. (a)

R = ₹ 10,000, n = 5 and i = 0.10 (Annuity due)

Present Value $= R\left[\frac{1-(1+i)^{-(n-1)}}{i}\right] + R = 10000\left[\frac{1-(1+0.1)^{-4}}{0.1}\right] + 10000$

$\therefore$ Present value = 31,698.70 + 10,000

= ₹ 41,698.70

Q.48) LIC India offers a 7 years annuity with a guaranteed rate of 6.35% compounded annually. How much should a person need to pay for one of these annuities if he wants to receive payments of ₹ 10000 annually over the 7 years period?

a) ₹ 55,135.98 **b)** ₹ 90,226.70 **c)** ₹ 59,000 **d)** None of these

Sol. (b)

R = ₹ 10000

i = 0.0635

t = 7 years

$$\Rightarrow F.V. = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

$$= \frac{10000[(1+0.0635)^7-1](1+0.0635)}{0.0635}$$

$\therefore$ V = ₹ 90,226.70

Q. 49) Z invests ₹ 10,000 every year starting from today for the next 10 years. Suppose the interest rate is 8% p.a. compound annually. Calculate the future value of the annuity.

a) ₹ 1,55,135.98 **b)** ₹ 1,56,454.8 **c)** ₹ 1,59,000 **d)** None of these

Sol. (b)

R = ₹ 10,000

i = 0.08

t = 10 years

$$\Rightarrow \text{F. V.} = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

$$= \frac{10000[(1+0.08)^{10}-1](1+0.08)}{0.08}$$

$\therefore$ V = ₹ 1,56,454.875

Q.50) A person invests ₹500 at the end of each year with a bank which pays interest at 10% p. a C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is.

a) ₹ 11,761.36 **b)** ₹ 10,000 **c)** ₹ 12,000 **d)** None of these

Sol. (a)

R= ₹ 500 r = 10% i=0.1

F.V. = R $\left(\frac{(1+i)^n-1}{i}\right)$

F.V. = 500 $\left(\frac{(1.1)^{12}-1}{0.1}\right)$

F.V. = ₹ 10,692.14

Amount after 1year after 12th instalment = 10,692.14 + 1069.22 = ₹ 11,761.36

Q.51) Ramesh wants to retire and receive ₹ 3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

a) ₹4,44,775 **b)** ₹4,49,775 **c)** ₹5,49,775 **d)** ₹2, 44,977

Sol. (b)

$R =$ ₹ 3000

$= i = 0.08/12 \text{ or } 0.00667$

$$P.V. = \frac{R}{i} = \frac{3000}{0.00667} = ₹\ 4{,}49{,}775$$

Q.52) Megha deposits ₹ 2000 annually into Retirement Pension Plan that earns 6.85% compounded annually. Due to a change in employment, these deposits stop after 10 years, but the account continues to earn interest until Megha retires 25 years after the last deposit is made. How much is in the account when Megha Retires?

a) ₹ 1,40,000.02 **b)** ₹ 1,43,785.10 **c)** ₹ 1,32,000.05 **d)** None of these

Sol. (b)

R = ₹ 2,000

i = 0.0685

t = 10

$$\Rightarrow \textit{Future value} = \frac{2000[(1+0.0685)^{10}-1]}{0.0685} = ₹\ 27{,}437.89$$

Now, the amount ₹ 27,437.89 earns interest for 25 years compounded annually:

$\Rightarrow A = 27437.89(1 + 0.0685)^{25} =$ ₹ 1,43,785.10

Q.53) A man purchased a house valued at ₹ 3,00,000. He paid ₹ 2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half-yearly in 20 equal half-yearly instalments. If the first instalment is paid after six months from the date of purchase, then the amount of each instalment is;

a) ₹ 8,718.40 **b)** ₹ 8,769.21 **c)** ₹ 7,893.13 **d)** None of these

Sol. (a)

Balance amount (V) = (3,00,000 – 2,00,000) = ₹ 1,00,000

$$i = \frac{12}{2}\% = 0.06$$

$$n = 20$$

P. V. = R $\left(\frac{1-(1+i)^{-n}}{i}\right)$

$\Rightarrow$1,00,000= R $\left(\frac{1-(1+0.06)^{-20}}{0.06}\right)$

$\Rightarrow$1,00,000 = R (11.47)
$\Rightarrow$ R = ₹ 8,718.40 (approx.)

Q.54) Due to some medical problems, Vinod takes premature retirement and gets 9,900 rupees quarterly. He wants to pass this quarterly payment to future generations after his death. He can earn an interest of 7.2% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

a) ₹ 3,50,000 **b)** ₹ 4,50,000 **c)** ₹ 5,50,000 **d)** None of these

Sol. (c)

i = $\frac{7.2}{100}$ = 0.072 p. a. and $\frac{0.072}{4}$ quarterly

$\Rightarrow P.V. = \frac{R}{i}$

$\Rightarrow P.V. = \frac{9900}{\frac{0.072}{4}} = 9900 \times \frac{4}{0.072}$

$\therefore$ P.V. = ₹ 5,50,000

Q. 55) Raja, aged 40 wishes his wife Rani to have ₹ 40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a., How much should he invest annually?

a) ₹ 84,448 **b)** ₹ 84,450 **c)** ₹ 84,449 **d)** ₹ 81,632.65

Sol. (d)

$$F.V = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

$\Rightarrow 4000000 = R\left[\frac{(1.03)^{30}-1}{0.03}\right](1.03)$

$\Rightarrow 40,00,000 = R(49)$

$\Rightarrow$ ₹ 81,632.65 (approx)

Q.56) A sinking fund is created for redeeming debentures worth ₹ 5 lakhs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

a) ₹ 12,001 **b)** ₹ 12,000 **c)** ₹ 12,006 **d)** None of these

Sol. (c)

F.V. = $R\left[\frac{(1+i)^n-1}{i}\right]$

$\Rightarrow 500000 = R\left[\frac{(1+0.04)^{25}-1}{0.04}\right]$

$\Rightarrow 500000 = R[41.646]$

$\Rightarrow$ R = ₹ 12,006

Q.57) A man decides to retire at the age of 50 years, and his employer gives him a pension of ₹ 20,000 per year for the rest of his life. Reckoning his expectation of life to be 12 years and that interest is at 4% per annum, what single sum is equivalent to his pension?

a) ₹1,87,701 **b)** ₹ 1,25,000 **c)** ₹ 1,26,000 **d)** none of these

Sol. (a)

R = ₹ 20,000, n = 12 and i = 0.04

Present Value = $R\left[\frac{1-(1+i)^{-n}}{i}\right] = 20000\left[\frac{1-(1+0.04)^{-12}}{0.04}\right]$

$Present\ value = 20000[9.3851]$

$\therefore$ Present value = ₹ 1,87,701

Q. 58) How much amount is required to be invested every year so as to accumulated ₹ 3,00,000 at the end of 10 years if interest is compounded annually at 10%?

a) ₹ 20,214 **b)** ₹ 20,140.17 **c)** ₹ 18,823.60 **d)** ₹ 20,214.61

Sol. (c)
A = ₹ 3,00,000   n = 10   i = 0.1

Future Value = $R\left[\frac{(1+i)^{n}+1}{i}\right] = 300{,}000 = R\left[\frac{(1+0.10)^{10}-1}{0.1}\right]$

$300{,}000 = R[15.9374]$

∴ R= ₹18,823.60

Q.59) The annual sales of a company in the year 2015 was Rs. 1000, and in the year 2020 was ₹ 2490. Find the compounded annual growth of sales in the given period of the same company:

a) 14.289% **b)** 10% **c)** 15% **d)** 20%

Sol. (d)
Given tn = 2020 and t_0 = 2015
V(tn) = 2490 and V(t_0) = 1000

$\Rightarrow \left[\left(\frac{2490}{1000}\right)^{\frac{1}{2020-2015}} - 1\right] \times 100$

$\Rightarrow \left[(2.49)^{\frac{1}{5}} - 1\right] \times 100$

$\Rightarrow (1.200 - 1) \times 100$

$\Rightarrow 20\%$

Q. 60) Appu retired at 60 years, receiving a pension of ₹ 14,400 a year paid in half-yearly instalments for the rest of his life after reckoning his life expected to be 13 years and that interest at 4% p.a. is payable half-yearly. What single sum is equivalent to his pension?

a) ₹ 1,45,000 **b)** ₹ 1,44,871 **c)** ₹ 1,44,800 **d)** ₹ 1,44,700

Sol. (b) P.V = R($\frac{1-(1+i)^{-n}}{i}$)

$\left[\because n = 13 \times 2 = 26 \ and \ i = \frac{4}{2}\% = 0.02\right]$

= A = $\frac{14400}{2}$ = ₹ 7200

= P.V= 7,200 ($\frac{1-(1.02)^{-26}}{0.02}$) = ₹1,44,871

Q.61) A merchant buys a house and a car for Rs.125000 and Rs.180000, respectively. If the value of the house increased at the rate of 20% per annum and the value of the car depreciated at the rate of 10% per annum that what is his profit or loss after two years?

a) ₹ 20,600 **b)** ₹ 20,800 **c)** ₹ 20,500 **d)** ₹ 20,700

Sol. (b)

Total Amount = $125000\left(1+\frac{20}{100}\right)^2 + 180000\left(1-\frac{10}{100}\right)^2$

Total Amount = ₹ 3,25,800

Initial price of house and Car = 1,25,000 + 1,80,000 = ₹ 3,05,000

Profit = 325800 – 305000 = ₹ 20,800

Q.62) A company may obtain a machine either by leasing it for 5 years (useful life) at an annual rent of ₹ 2,000 or by purchasing it for ₹ 8,100. If the company can borrow money at 10% p.a., which alternative is preferable?

a) Leasing is preferable **b)** Leasing is not preferable

c) Cannot say **d)** none of these

Sol. (a)

Present value of P (rest) of the annuity.

$\Rightarrow P = A\left[\frac{1-(1+i)^{-n}}{i}\right] \Rightarrow 2000\left[\frac{1-(1+0.1)^{-5}}{0.10}\right]$

$\Rightarrow P = 20000[1-(1.1)^{-5}]$

⇒ P = ₹ 7,581 which is less than the Purchase Price.

∴ Leasing is preferable

Q. 63) Mr Paul borrows ₹20,000 on condition to repay it with CI at 5% p.a. in annual instalments of ₹2,000 each. The number of years for the debt to be paid off is

a) 10years **b)** 12 years **c)** 11years **d)** 14.2years

Sol. (d) $P.V = R\,(\frac{1-(1+i)^{-n}}{i})$

$\Rightarrow$ $20{,}000 = 2000(\frac{1-(1+0.05)^{-n}}{0.05})$

$\Rightarrow$ $10\times 0.05 = 1-(1.05)^{-n}$

$\Rightarrow$ $0.5 \quad = (1.05)^{-n}$

$\Rightarrow$ $0.5 \quad = \frac{1}{(1.05)^{n}}$

$\Rightarrow (1.05)^{n} = 2$

$= n = 14.2$ years

Q. 64) An investor intends to purchase a three-year ₹ 1,000 par value bond having a nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

a) ₹ 674 **b)** ₹ 906 **c)** ₹ 1764 **d)** ₹ 1645

Sol. (b)

Step: -1

Find the interest receivable

Face value × Nominal rate of interest

1000 × 10% = ₹ 100

Step: - 2

Find present value of interest receivable

$= R\left[\frac{1-(1+i)^{-n}}{i}\right] = 100\left[\frac{1-(1+0.14)^{-3}}{0.14}\right] = ₹\ 232$

Step: -3

Find present value of redemption

(Since, repayment of Loan one time it is not on Annuity)

$A = P(1+r)^{n}$

$= 1000 = P(1+0.14)^{3} =$

$$P = \frac{1000}{(1+0.14)^{3}} = \frac{1000}{1.481522} = ₹\ 674$$

Step: -4 = Step: -2 + Step: -3

Purchased price of bond = 232 + 674 = ₹ 906

Q. 65) Compute the net present value for a project with a net investment of ₹ 1,00,000 and net cash flows year one is ₹ 55,000; for year two is ₹ 80,000 and for year three is ₹ 15,000. Further, the company's cost of capital is 10%

a) ₹14, 674 **b)** ₹15,764 **c)** ₹27,340 **d)** ₹20,1645

Sol. (c)

Year	Net cash Flows	PVIF@10%	Discounted cash flows
0	(1,00,000)	1.000	(1,00,000)
1	55,000	0.909	49,995
2	80,000	0.826	66,080
3	15,000	0.751	11,265
Net present value			27,340

Since the net present value of the project is positive, the company should accept the project.

Q.65) If the amount in 2.25 times of the sum after 2 years at compound interest (compounded annually), the rate of interest per annum is:

a) 25% **b)** 30% **c)** 45% **d)** 50%

Sol. (d)
Amount = 2.25P
$\Rightarrow 2.25P = P\left(1 + \frac{R}{100}\right)^2$
$\Rightarrow (1.5)^2 = \left(1 + \frac{R}{100}\right)^2$
$\Rightarrow 1.5 = 1 + \frac{R}{100}$
$\therefore$ Rate of interest = 50%

Q.66) At compound interest, if a certain sum of money doubles in n years, then the amount will be four times in:

a) n^2 years **b)** $2n^2$ years **c)** $2n$ years **d)** $4n$ years

Sol. (c)
Since, A = 2P, then
$\Rightarrow 2P = P(1 + i)^n$
$\Rightarrow 2 = (1 + i)^n$
Square both sides
$\Rightarrow 4 = (2)^2 = [(1 + i)^n]^2 = (1 + i)^{2n}$
$\therefore$ Time period = 2n

Permutations and Combinations

Q.1) If $n_1 + n_{2_{P_2}} = 132, n_1 - n_{2_{P_2}} = 30$ then,

a) $n_1 = 6, n_2 = 6$ **b)** $n_1 = 10, n_2 = 2$ **c)** $n_1 = 9, n_2 = 3$ **d)** None of these

Sol. (c)

$n_1 + n_{2_{P_2}} = 132$

$\Rightarrow \frac{(n_1+n_2)!}{(n_1+n_2-2)!} = 132$

$\Rightarrow (n_1 + n_2)\ (n_1 + n_2 - 1) = 12 \times 11$

$\Rightarrow n_1 + n_2 = 12$_____$(I)$

Now, $n_1 - n_{2_{P_2}} = 30$

$\Rightarrow \frac{(n_1-n_2)!}{(n_1-n_2-2)!} = 30$

$\Rightarrow (n_1 - n_2)\ (n_1 - n_2 - 1) = 6 \times 5$

$\Rightarrow n_1 - n_2 = 6$_____$(II)$

From $[(I) + (II)]$

$\Rightarrow n_1 + n_2 + n_1 - n_2 = 12 + 6 \Rightarrow 2n_1 = 18$

$\Rightarrow n_1 = 9$

$\therefore n_2 = 12 - 9 = 3$

Q.2) For what value of x, ${}^{1000}C_x$ is maximum?

a) 0 **b)** 1 **c)** 500 **d)** 999

Sol. (c)

${}^nC_r$ is maximum when $r = \frac{n}{2} = \frac{1000}{2}$

$x = \mathbf{500}$

Q.3) If ${}^{500}C_{91} = {}^{499}C_{92} + {}^{n}C_{91}$ then n is

a) 501 **b)** 500 **c)** 502 **d)** 499

Sol. (d)

$({}^nC_r + {}^nC_{r+1} = {}^{n+1}C_r)$

Therefore, n+1= 500

$\Rightarrow$ n = 499

Q.4) If ${}^{15}C_{3r} = {}^{15}C_{2r}$ then r =?

a) 1 **b)** 2 **c)** 3 **d)** 5

Sol. (c)

$\Rightarrow {}^nC_{r1} = {}^nC_{r2}$ then $n = r1 + r2$ (Use this property)

$\Rightarrow$ 15= 3r +2r

$\Rightarrow r = 3$

Q.5) What is the least possible value of n if:

${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$

a) 6 **b)** 8 **c)** 7 **d)** 12

Sol. (b)

Go through the option

= n= 6 then, ${}^5C_3 + {}^5C_4 > {}^6C_3$

$\Rightarrow$10+5>20 wrong answer

= n= 8 then, ${}^7C_3 + {}^7C_4 > {}^8C_3$

$\Rightarrow$35+35>56, which is correct answer (least value)

= n= 7 then, ${}^6C_3 + {}^6C_4 > {}^7C_3$

$\Rightarrow$20+15>35 wrong answer

= n= 12 then, $^{11}C_3 + {}^{11}C_4 > {}^{12}C_3$
⇒165+330>220

wrong answer

Q.6) $^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + ...+ {}^nC_n$ equals

a) $2^n - 1$ **b)** 2^n **c)** $2^n + 1$ **d)** None of these

Sol. (a)

$\because {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + --- + {}^nC_n = 2^n$
$\therefore {}^nC_1 + {}^nC_2 + {}^nC_3 + --- + {}^nC_n = 2^n - {}^nC_0$
$= 2^n - 1$

Q.7) If $^nC_{r-1} = 56$, $^nC_r = 28$ and $^nC_{r+1} = 8$, then r is equal to

a) 8 **b)** 6 **c)** 5 **d)** None of these

Sol. (b)

$^nC_{r-1} = 56 \Rightarrow \frac{n!}{(r-1)!\ (n-r+1)!} = 56$ ____(I)

$^nC_r = 28 \Rightarrow \frac{n!}{r!\ (n-r)!} = 28$ ____(II)

$^nC_{r+1} = 8 \Rightarrow \frac{n!}{(r+1)!\ (n-r-1)!} = 8$ ____(III)

$$from\ [(I) \div (II)]$$
$$\frac{n!}{(r-1)!(n-r+1)} \times \frac{r!(n-r)!}{n!} = \frac{56}{28}$$
$$\Rightarrow \frac{r}{n-r+1} = 2 \Rightarrow r = 2n - 2r + 2$$
$$\Rightarrow 2n = 3r - 2\_\_\_\_(IV)$$
$$from\ [(II) \div (III)]$$
$$\frac{n!}{(r)!\ (n-r)!} \times \frac{(r+1)!\ (n-r-1)!}{n!} = \frac{28}{8}$$
$$\Rightarrow \frac{r+1}{n-r} = \frac{7}{2} \Rightarrow 2r + 2 = 7\,n - 7r$$
$$\Rightarrow 7n = 9r + 2$$
$$\Rightarrow 7\left(\frac{3\,r-2}{2}\right) = 9r + 2 \Rightarrow 21r - 14 = 18r + 4$$
$$\Rightarrow 3r = 18 \ \Rightarrow r = 6$$

Q.8) How many 3-digit numbers can be formed from 4, 5,6,7,8 and 9? When repetition of the digit is allowed.

a) $^6c_3$ **b)** $^9c_3$ **c)** 216 **d)** None of these

Sol. (c)

Total digit is 6 and selection of all digit for 3-digit numbers (When repetition is allowed)
⇒ $^6C_1 \times {}^6C_1 \times {}^6C_1$
= 6^3 = 216

Q.9) How many can 8-digit telephone numbers be allotted using the digit 0 to 9 if each number starts with 28 and no digit appears more than once?

a) $^{10}C_8$ **b)** $^{10}C_6$ **c)** $^8C_6$ **d)** $^{10}C_{10}$

Sol. (c)

The total digit is 8 (2 and 8 is except from 0 to 9), and the selection of the digit is 6
= $^8C_6$

Q.10) The number of 4-digit numbers having at least one of their digits repeated is

a) 4,464 **b)** 4,664 **c)** 15,000 **d)** None

Sol. (a)

Total required arrangement
= Total arrangement of 4-digit when repetition of all digit – Total arrangement when repetition not allowed

Total arrangement of 4-digit when repetition of all = $9 \times 10 \times 10 \times 10$ =9000
Total arrangement when repetition not allowed = $9 \times 9 \times 8 \times 7 = 4536$
Required arrangements = 9000 − 4536= 4,464

Q.11) In how many ways of choosing 4 kings from a pack of playing cards when 2-Queen, 1-Jack and 1-King lost?

a) 270725 **b)** 270700 **c)** 27000 **d)** None

Sol. (d)

Total remaining cards = 48 (52 – 2 queen- 1jack, - 1 king)
In this case, only 3 kings
Therefore, we can't select 4 kings

Q.12) n articles are arranged in such a way that 2 particular articles never come together. Thenumber of such arrangements is

a) (n−2) (n-1)! **b)** (n+1)! (n-2) **c)** n **d)** None of these

Sol. (a)

Required nos. of arrangement
= Total arrangement of n articles – Nos. of arrangement taking two particular articles are together
$= n! - (n-1)! \times 2!$
$= (n-2)\,(n-1)!$

Q.13) There are 10 trains plying between Calcutta and Delhi. The number of ways in which a person can go from Calcutta to Delhi and return by a different train is

a) 99 **b)** 90 **c)** 80 **d)** None of these

Sol. (b)

Total nos. ways of going = ${}^{10}c_1$ ways and returning = ${}^{9}c_1$ ways.
∴ Total nos. of ways to go and return
$= {}^{10}C_1 \times {}^{9}C_1 = \frac{10!}{9!} \times \frac{9!}{8!}$
$= 10 \times 9 = 90$

Q.14) The total number of 9 digit numbers of different digits is

a) $10\lfloor 9$ **b)** $8\lfloor 9$ **c)** $9\lfloor 9$ **d)** None of these

Sol. (c)

There are 10 digits 0, 1, 2, 3, ---, 9
Extreme left position of the number can be filled with anyone out of 9 digits i.e., 1, 2, 9, ---, 9 in ${}^{9}P_1$ ways and the remaining 8 positions of 9 digit number can be filled with any digit of the remaining 9 digits because 0 (zero) can be placed after the extreme left position of a number. So it can be done in ${}^{9}P_8$ ways
∴ Required number of 9 digit number
$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 9 \times 9! = 9(9!)$
$= 9\lfloor 9$

Q.15) How many different straight lines can be formed by joining 16 different points on a plane, of which 4 are collinear, and the rest are non-collinear?

a) 115 **b)** 112 **c)** ${}^{16}C_4$ **d)** 120

Sol. (a)

Numbers of straight lines Apply formula = ${}^{n}C_2 - {}^{m}C_2 + 1$
= n = Total number of different points
= m = Total number of collinear points
Total points(n) = 16 and m = 4 points are collinear
$\Rightarrow {}^{16}C_2 - {}^{4}C_2 + 1$
= 115

Q.16) 8 points are marked on the circumference of a circle. The number of chords obtained by joining these in pairs is

a) 25 **b)** 27 **c)** 28 **d)** None of these

Sol. (c)

Required nos. of chords = $^8C_2$

$= \frac{8!}{2! \times 6!} = \frac{8 \times 7}{2 \times 1} = 28$

Q.17) If R parallel line in the plane which is intersected by a family of S parallel lines. Find the number of parallelograms formed.

a) $\frac{RS(R-1)(S-1)}{4}$ **b)** R^S **c)** $\frac{RS(R-1)(S-1)}{2}$ **d)** RS

Sol. (a)

If the family of 'm' parallel lines are intersected by the family of 'n' parallel lines, then the number of parallelograms will be $\frac{mn(m-1)(n-1)}{4}$

$\Rightarrow \frac{RS(R-1)(S-1)}{4}$

Q.18) In which one of the following polygons has twice diagonals as the number of sides?

a) 5 **b)** 6 **c)** 7 **d)** 8

Sol. (c)

Let n is the side of the polygon

Number of diagonals of a polygon = $\frac{n(n-3)}{2}$

ATQ

$\Rightarrow \frac{n(n-3)}{2} = 2n$

$\Rightarrow n(n-3) = 2n \times 2$

$\Rightarrow n - 3 = 4$

$\Rightarrow n = 7$

Q.19) Find the number of different poker hands in a pack of 52 playing cards.

a) $^{52}C_5$ **b)** $^{52}P_5$ **c)** 52! **d)** None

Sol. (a)

Poker hands means five playing cards

⇒Total playing cards = 52

⇒Total required arrangement = $^{52}C_5$

Q.20) Find the total number of 6-digit numbers not divisible by 2 which can be formed with the six digits 1, 2, 4, 5, 6, 7, when the repetition is allowed?

a) 23,328 **b)** 22338 **c)** 2000 **d)** 201

Sol. (a)

When a number is not divisible by 2 than unit place (1,5,7)

The digits repetition is allowed

All 6 nos.	All 6 nos.	All 6 nos.	All 6 nos.	All 6 nos.	Only 1, 5, 7

$\Rightarrow 6 \times 6 \times 6 \times 6 \times 6 \times 3$

$= 23{,}328$

Q.21) In how many ways 10 oranges can be divided into 4 persons, so they get 1, 2, 3 and 4 orange, respectively.

a) 120 **b)** 1230 **c)** 12500 **d)** 12600

Sol. (d)

The number of ways in which (m + n + o + p) things can be arranged into four groups containing m, n, o and p things, respectively, is

Where m denotes oranges received by the first person

n denotes oranges received by the second person
o denotes oranges received by the third person
p denotes oranges received by the fourth person

$= \frac{(m+n+o+p)!}{m!n!o!p!}$

$= \frac{10!}{1!2!3!4!}$

= 12,600

Q.22) The number of ways in which 8 different beads can be arranged to form a necklace is:

a) 2,520 **b)** 3,660 **c)** 36,502 **d)** 18,002

Sol. (c)

A necklace is circular
Let fix the position of one bead
Then 7 beads can be arranged into 7! Ways = 5040
Now as it is a necklace and it can be wear from both sides
Flipping its sides will reduce ways to half
∵ No. of ways = 5040/2 = 2520

Q.23) If 12 school teams are participating in a quiz contest, then the number of ways the first, second and third positions may be won is

a) 1,230 **b)** 1,320 **c)** 3,210 **d)** None of these

Sol. (b)

Required nos. = $^{12}C_3 \times 3!$
$= 12 \times 11 \times 10 = 1,320$

Q.24) 5 persons are sitting at a round table in such a way that the Tallest Person is always on the right–side of the shortest person. The number as such arrangement are

a) 6 **b)** 8 **c)** 24 **d)** None of these

Sol. (a)

Taking shortest & Tallest person as a single unit and another individual as a unit
∴ There are 4 units is arranged in a round table be done in
(4 – 1)! Ways
= 3! Ways = 6

Q.25) The number of diagonals in a decagon is

a) 30 **b)** 35 **c)** 45 **d)** None of these

Sol. (b)

Nos. of diagonal in a polygon with n sides = $\frac{n(n-3)}{2}$
∴ Required nos. of diagonals = $\frac{10 \times 7}{2} = 35.$

Q.26) The number of straight lines obtained by joining 16 points on a plane, no three of thembeing on the same line, is

a) 121 **b)** 110 **c)** 210 **d)** None of these

Sol. (a)

Numbers of straight lines Apply formula = $^nC_2 - {}^mC_2 + 1$
= n = Total number of different points
= m = Total number of collinear points
Required nos. of lines
$= {}^{16}C_2 - {}^0C_2 + 1 = \frac{16 \times 15}{2 \times 1} + 1 = 120 + 1 = 121$

Q.27) The number of 4-digit numbers formed with the digits 1, 1, 2, 2, 3, 4 is

a) 100 **b)** 101 **c)** 201 **d)** 102

Sol. (d)

Numbers of ways in which all 4-digit are different = ${}^4C_4 \times 4! = 24$

Number of ways in which 2-digits are like, and the other two are different

$= {}^2C_1 \times {}^3C_2 \times \frac{4!}{2!} = 2 \times 3 \times 12 = 72$

Number of ways in which 2 pairs of like digits = ${}^2C_2 \times \frac{4!}{2! \times 2!} = 6$

$\therefore$ Required number of 4-digit numbers = $24 + 72 + 6 = 102$

Q.28) The number of ways a person can contribute to a fund out of 1 ten-rupee note, 1 five-rupee note, 1 two-rupee and 1 one-rupee note is

a) 15 **b)** 25 **c)** 10 **d)** None of these

Sol. (a)

$\because$ Here there are four different types of notes

$\therefore$ Required nos. of ways

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 2^4 - {}^4C_0 = 16 - 1 = 15$

Q.29) The letter of the word COMBINATION and PERMUTATION are arranged in all possible ways. The ratio of the number of arrangements is:

a) 4:1 **b)** 1:4 **c)** 1:1 **d)** None

Sol. (b)

COMBINATION (2-I, 2-O, 2-N, 1-C, 1-M, 1-B, 1-A, 1-T) = ${}^{11}C_{11} \times \frac{11!}{2! \times 2! \times 2!}$

PERMUTATION (1-P, E-1, R-1, M-1, U-1, T-2, A-1, I-1, O-1, N-1) = ${}^{11}C_{11} \times \frac{11!}{2!}$

Ratio of combination and permutation = $\frac{11C11 \times \frac{11!}{2! \times 2! \times 2!}}{11C11 \times \frac{11!}{2!}}$

$$= \frac{\frac{1}{8}}{\frac{1}{2}}$$

$= \frac{1}{4} = \mathbf{1:4}$

Q.30) A person has 20 friends, of which 8 of them are relatives. He wishes to invite 7 persons so that 3 of them are relatives. In how many ways he can invites?

a) 28,450 **b)** 20,600 **c)** 28,120 **d)** 27,720

Sol. (d)

A person invites his 20 friends out of 8 are relatives

Selection of 7 friends out of exactly 3 are relatives

$\Rightarrow {}^{12}C_4 \times {}^8C_3$

$= \frac{12!}{4!8!} \times \frac{8!}{5!3!} = 495 \times 56$

$= \mathbf{27{,}720}$

Q.31) A box contains 2 green balls, 3 pink balls and 4 red balls. How many ways can 2 balls be drawn from the box if at least one pink ball is included in the draw?

a) 12 **b)** 21 **c)** 25 **d)** 6

Sol. (b)

Total number of balls = 2 + 3 + 4 = 9

Total no. of ways 2 balls can be drawn from 9 = ${}^9C_2$

No pink ball is drawn = 9 – 3 = 6 = ${}^6C_2$

Required no. of ways at least one pink ball is to be drawn = Total no. of ways – No pink ball is drawn

$\Rightarrow {}^9C_2 - {}^6C_2$

$\Rightarrow \frac{9!}{7!2!} - \frac{6!}{4!2!}$

$\therefore 36 - 15 = \mathbf{21}$

Q.32) In a 12th board examination, a student is required to pass the four different subjects. In how many ways can he fail?

a) 15 **b)** 120 **c)** 30 **d)** 45

Sol. (a)

Student failed in one subject $^4C_1 = 4$

Student failed in 2 subjects $^4C_2 = 6$

Student failed in 3 subjects $^4C_3 = 4$

Student failed in 4 subjects $^4C_4 = 1$

The number of ways in which he can fail = 4 + 6 + 4 + 1 = **15**

Q.33) A committee of 3 ladies and 4 gents is to be formed out of 8 ladies and 7 gents. Mrs. X refuses to serve on a committee in which Mr. Y is a member. The number of such committees is

a) 1,530 **b)** 1,500 **c)** 1,520 **d)** 1,540

Sol. (d)

Required nos. of committees

Situation 1- Mr Y is a member- $^6C_3 \times {}^7C_3$

Situation 2- when Mr Y is not a member- $^6C_4 \times {}^8C_3$

$= {}^6C_3 \times {}^7C_3 + {}^6C_4 \times {}^8C_3 = \mathbf{1,540}$

Q.34) If 12 school teams are participating in a quiz contest, then the number of ways the first, second and third positions may be won is

a) 1,230 **b)** 1,320 **c)** 3,210 **d)** None of these

Sol. (b)

Required nos. = $^{12}C_3 \times 3!$

$= 12 \times 11 \times 10 = \mathbf{1,320}$

Q.35) 3 ladies and 3 gents can be seated at a round table so that any two and only two ladies sit together. The number of ways is

a) 70 **b)** 27 **c)** 72 **d)** None of these

Sol. (c)

Required nos. of ways

= Total seat arrangement of 6 person (i.e., 3 ladies & 3 gents) in round table

[(Nos. of ways, no two ladies sit together × (Nos. of ways 3 ladies sit together)]

$= \; ^3C_2 \times 2! \times {}^3C_2 \times 2! \times 2!$

$= \; 3 \times 2 \times 3 \times 2 \times 2 = \mathbf{72}$

Q.36) The number of ways in which 6 men can be arranged in a row so that the particular 3 men sit together is

a) $^4P_4$ **b)** $^4P_4 \times {}^3P_3$ **c)** $(3)^2$ **d)** None of these

Sol. (b)

Let three particular men as a single unit and remain 3 men as 3 unit

∴ 4 Units can be arranged externally 4! and three particular men can be arranged internally 3!.

∴ Required nos. of ways =4!× 3!

$= \mathbf{{}^4P_4 \times {}^3P_3}$

Q.37) The total number of ways in which six '+' and four '–' signs can be arranged in a line suchthat no two '–' signs occur together is

a) 7/3 **b)** 6 × 7 / 3 **c)** 35 **d)** None of these

Sol. (c)

.+.+.+.+.+.+.

There are 7 dot positions in which any four positions are filled with four '-, signs in

${}^{7}P_4 / 4!$ (∵ four '-, signs are similar)

${}^{6}C_6 \times \frac{6!}{6!} \times {}^{7}C_4 \times \frac{4!}{4!} = \mathbf{35}$

Q.38) A question paper contains 6 questions, each having an alternative. The number of ways an examine can answer one or more questions is

a) 720 **b)** 728 **c)** 729 **d)** None of these

Sol. (b)

Total numbers of ways an examine can answer 1 or more questions

$= {}^{6}C_1 \times 2 + {}^{6}C_2 \times 2^2 + {}^{6}C_3 \times 2^3 + {}^{6}C_4 \times 2^4 + {}^{6}C_5\, 2^5 + {}^{6}C_6 \times 2^6$

$= 12 + 60 + 160 + 240 + 192 + 64 = \mathbf{728}$

Q.39) A candidate is required to answer 6 out of 12 questions which are divided into two groups containing 6 questions in each group. He is not permitted to attempt not more than four from any group. The number of choices are.

a) 750 **b)** 850 **c)** 800 **d)** None of these

Sol. (b)

Required nos. of choice

$= {}^{6}C_4 \times {}^{6}C_2 + {}^{6}C_3 \times {}^{6}C_3 + {}^{6}C_2 \times {}^{6}C_4$

$= 15 \times 15 + 20 \times 20 + 15 \times 15 = 225 + 400 + 225 = \mathbf{850}$

Q.40) The number of arrangements of the letters in the word `FAILURE', so that vowels are always coming together is

a) 576 **b)** 575 **c)** 570 **d)** None of these

Sol. (a)

FAILURE

Total nos. of letters in the word FAILURE are 7 and all are different

Also, Nos. of vowels = 4

∴ Required arrangement

∴ $4! \times 4!$ (∵Taking all vowels as a single unit so there are 4 units can be arranged in 4! Ways (external arrangement) & also vowels are arranged in 4! Ways (internal arrangement))

$= 24 \times 24 = \mathbf{576}$

Q.41) The number of ways the letters of the word 'TRIANGLE' to be arranged so that the word 'angle' will always be present is

a) 20 **b)** 60 **c)** 24 **d)** 32

Sol. (c)

Taking angle as a single unit and remaining three letters as 3 unit

∴ Total nos. of ways

= Total nos. of arrangement of 4 units

$= 4! = \mathbf{24}$

Q.42) If the letters word 'DAUGHTER' is to be arranged so that vowels occupy the odd places, then the number of different words are

a) 2,880 **b)** 676 **c)** 625 **d)** 576

Sol. (a)

Total nos. of letters of word DAUGHTER = 8

∴ Total nos. of odd place = 4

Total nos. of vowels in word daughter = 3

∴ Required nos. of different word

$= {}^{4}C_3 \times 3!\, {}^{5}C_5 \times 5!$

$= \frac{4!}{3!} \times 3! \times \frac{5!}{0! \times 5!} \times 5! = 24 \times 120$

$= \mathbf{2{,}880}$

Q.43) If all S's come together then in how many ways the letters of the word SUCCESSFUL be arranged?

a) 10,080 **b)** 10,098 **c)** 40,080 **d)** None

Sol. (a)

Total word = 10 (S-3, U-2, C-2, E-1, F-1 L-1)
(SSS) Count as a single unit, U-2, C-2, E-1, F-1 L-1
Total word = 8
External arrangement of 8-word × internal arrangement of words × $arragement\ of\ all\ S$

$\Rightarrow {}^3C_3 \times \frac{3!}{3!} \times {}^8C_8 \frac{8!}{2!2!}$

$= \mathbf{10,080}$

Q.44) The number of squares on a chessboard is

a) 30 **b)** 600 **c)** 150 **d)** 204

Sol. (d)

A chessboard has 8 squares on each side
If only one square box makes only 1 square
If 2 square boxes make 2^2
Similarly, 3 square boxes make 3^2
Similarly, 4 square boxes make 4^2
Similarly, 5 square boxes make 5^2
Similarly, 6 square boxes make 6^2
Similarly, 7 square boxes make 7^2
Similarly, 8 square boxes make 8^2

$\Rightarrow 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$

$= \mathbf{204}$

Q.45) Out of 7 gents and 4 ladies, a committee of 5 is to be formed. The number of committees such that each committee includes at least one lady is

a) 400 **b)** 440 **c)** 441 **d)** None of these

Sol. (c)

Required nos. of committees

$= {}^4C_1 \times {}^7C_4 + {}^4C_2 \times {}^7C_3 + {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

$= 4 \times 35 + 6 \times 35 + 4 \times 21 + 1 \times 7$

$= 140 + 210 + 84 + 7 = \mathbf{441}$

Q.46) There are 12 points in a plane, of which 5 are collinear. The number of triangles is

a) 200 **b)** 211 **c)** 210 **d)** None of these

Sol. (c)

Required nos. of triangles

$= {}^{12}C_3 - {}^5C_3 = \frac{12!}{3! \times 9!} - \frac{5!}{3! \times 2!}$

$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} - \frac{5 \times 4}{2 \times 1} = 220 - 10$

$= \mathbf{210}$

Q.47) Four men and 3 women are to be seated for dinner such that no 2 women sit together and no 2 men sit together. Find the numbers of ways in which this can be arranged?

a) 144 **b)** 72 **c)** 36 **d)** None

Sol. (a)

Only one arrangement can be formed when men sit at odd place and women sit at even place
Sitting arrangement of men and sitting arrangement of women

$\Rightarrow {}^4C_4\, 4! \times {}^3C_3\, 3!$

$= \mathbf{144}$

Q.48) There are 3 red, 4 green and 5 black balls in a bag. They are drawn one by one and arranged in a row, assuming that all the 12 balls are drawn, determine the number of different arrangements.

a) 22,770 **b)** 27,720 **c)** 22,077 **d)** 27,270

Sol. (b)

Total balls = 12 (3-green, 4-red and 5-black)

Selection of 12 balls and arrangements of 12 balls

$\Rightarrow {}^{12}C_{12}\,\frac{12!}{3!\times4!\times5!}$

$\Rightarrow \left(\frac{12!}{3!\times4!\times5!}\right)$

$= \mathbf{27,720}$

Q.49) There are 15 people, including 2 friends L and M. in how many ways can L and M be arranged around the circular table if they cannot be seated together?

a) 13! **b)** 12! **c)** 13! × 12 **d)** 14! × 13

Sol. (c)

Total required arrangements = Total possible arrangements – Total arrangements (when L and M sit together)

Total required arrangements = (15-1)! – (14-1)! × 2!

= 14! – 13! × 2!

= **13! × 12**

Q.50) At an election, there are 5 candidates, and 3 members are to be elected. A voter is entitled to vote for any number of candidates not greater than the number to be elected. The number of ways a voter chooses to vote is

a) 20 **b)** 22 **c)** 25 **d)** None of these

Sol. (c)

Required nos. of ways

$= {}^5C_1 + {}^5C_2 + {}^5C_3 = 5 + 10 + 10$

= **25**

Q.51) Every two persons shake hands with each other at a party, and the total number of handshakes is 66. The number of guests in the party is

a) 11 **b)** 12 **c)** 13 **d)** 14

Sol. (b)

Let the nos. of guests be n

$\therefore {}^nC_2 = 66$

$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \Rightarrow \frac{n\,(n-1)}{2\times1} = 66$

$\Rightarrow n^2 - n - 132 = 0$

$\Rightarrow\ n^2 - 12n + 11n - 132 = 0$

$\Rightarrow (n-12)\,(n+11) = 0$

$\Rightarrow n - 12 = 0\ or\ n + 11 = 0$

$\Rightarrow n = 12\ or\ [n = -11]$ rejected as it is not possible

$\therefore \mathbf{n = 12}$

Q.52) In how many ways of choosing two face cards and two other cards from a pack of 52 playing cards?

a) 51,480 **b)** 52,018 **c)** 2,138 **d)** 4,148

Sol. (a)

Face cards =12 and other cards = 40

Selection is 2 face card and 2 other cards

$\Rightarrow {}^{12}C_2 \times {}^{40}C_2$

$= \frac{12!}{2!10!} \times \frac{40!}{2!38!}$

$= \frac{12\times11\times10!}{2\times1\times10!} \times \frac{40\times39\times38!}{2\times1\times38!}$

= **51,480**

Q.53) In how many ways that 3 commerce books, 3 computer books and 5 economic books be arranged along a row so that books of the same subjects have come together is

a) 29,950 **b)** 25,940 **c)** 25,920 **d)** None of these

Sol. (c)

Commerce book = 3

Mathematics books = 3

Economics books = 5

Total required arrangements = External arrangement x Internal arrangement

$\Rightarrow 3! \times 3! \times 5! \times 3!$

$= \mathbf{25{,}920}$

Q.54) One die is rolled four times. The number of possible outcomes in which at least one die shows 3

a) 625 **b)** 1296 **c)** 671 **d)** 650

Sol. (c)

Total possible arrangement of a die rolled 4 time = 6^4

Total possible arrangement a die rolled 4 time when 3 not come = 5^4

Total required arrangement = $6^4 - 5^4$

$\Rightarrow 1296-625$

$= \mathbf{671}$

Q.55) A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways can be seated if all sisters can sit together?

a) 360 **b)** 720 **c)** 120 **d)** None of these

Sol. (b)

All sister sits together then they will become a single unit

Then, total arrangement = (4+1) =5 and selection also 5

$\Rightarrow {}^5C_5 \times 5! \times 3!$

$= 5! \times 3!$

$= \mathbf{720}$

Q.56) The way of selecting 4 letters from the word `EXAMINATION' is

a) 136 **b)** 130 **c)** 125 **d)** None of these

Sol. (a)

EXAMINATION

$$\begin{array}{cccccccc} A & E & I & M & N & O & X & T \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 1 \end{array}$$

∴ Number of ways in which all four letters are different = ${}^8C_4$

Numbers of ways in which two letters are same & two are distinct = ${}^3C_1 \times {}^7C_2$

Number of ways in 2 pair of two similar letter = ${}^3C_2$

∴ Required nos. of ways of selecting 4 letters from the word 'EXAMINATION'

$= {}^8C_4 + {}^3C_1 \times {}^7C_2 + {}^3C_2 = 70 + 63 + 3$

$= \mathbf{136}$

Q.57) The number of even numbers greater than 300 can be formed with the digits 1, 2, 3, 4, 5 without repetition is

a) 110 **b)** 112 **c)** 111 **d)** None of these

Sol. (c)

3 or 5		2 or 4
↓	↓	↓
2 ×	3 ×	2

4		2
↓	↓	↓
1	3	1

Nos. of 3 digits even no. greater than 300

$= 2 \times 3 \times 2 + 1 \times 3 \times 1 = 12 + 3 = 15$

			2 or 4
↓	↓	↓	↓
4	3	2	2

Nos. of 4 digit even $no = 4 \times 3 \times 2 \times 2 = 48$

				2 or 4
↓	↓	↓	↓	↓
4	3	2	1	2

Nos. of 5 digit even $no = 4 \times 3 \times 2 \times 1 \times 2 = 48$

$\therefore$ Required number of even numbers = 15 + 48 + 48 = **111**

Q.58) Find the total number of numbers not divisible by 2, which can be formed with the six digits 1, 2, 4, 5, 6, 7?

a) 23,328 **b)** 22,338 **c)** 2,000 **d)** 201

Sol. (a)

When a number is not divisible by 2 then unit place (1,5,7)

This question has repetition allowed

All 6 nos.	All 6 nos.	All 6 nos.	All 6 nos.	All 6 nos.	Only 1, 5, 7
⇒6 x	6 x	6 x	6 x	6 x	3

=23,328

Q.59) What is the rank or order of the word 'ZENITH' in the dictionary?

a) 706 **b)** 708 **c)** 616 **d)** 686

Sol. (c)

In the order of letter in ZENITH ⇒ EHINTZ

Numbers of word start with the letter $\frac{ZENITH}{EHINTZ}$

$5! \times ((5) \times 4!\,(0) \times 3!\,(2) \times 2!\,(1) \times 1!\,(1) = 600 + 0 + 12 + 2 + 1 = 615$

Therefore, position of zenith is **616**.

Q.60) How many four-digit numbers can be formed with the digits 2, 3, 5, 7, 9, which are lie between 3000 and 4000?

a) 125 **b)** 60 **c)** 24 **d)** 250

Sol. (a)

Thousandth place Only 3	Hundredth place All 5	Tenth place All 5	Unit place All 5
1 ×	5 ×	5 ×	5 = **125**

Q.61) A letter lock consists of 3 rings, and each ring contains 9 non-zero digits. This lock can be opened by setting a 3-digit code with the proper combination of each of the 3 rings. Maximum how many codes can be formed to open the lock?

a) 100 **b)** 180 **c)** 243 **d)** 729

Sol. (d)

Total non-zero digit =9

There is a 3-digit code

All 9-digit	All 9-digit	All 9-digit
9 ×	9 ×	9 = **729**

Q.62) How many five-digit numbers can be formed with the remaining 1,2,3,4,5, which are greater than 25000?

a) 1,800 **b)** 2,000 **c)** 1,825 **d)** 250

Sol. (b)

Case-i

2 5 All 5 All 5 All 5

5 × 5 × 5 = 125

Case-ii

3,4,5 All 5 All 5 All 5 All 5

3 × 5 × 5 × 5 × 5 = 1875

Total arrangement = 1875 + 125 = **2,000**

Q.63) The sum of the numbers of the nth terms of the series
(1) +(1+2) +(1+2+3) +(1+2+3+4) +.............................+(1+2+3+4+.......... +n)

a) $^{n+1}C_3$ **b)** $^{n+1}C_2$ **c)** $^{n}C_2$ **d)** $^{n+2}C_3$

Sol. (d)

Sum of n terms of natural numbers = $\frac{n(n+1)}{2} = \frac{n^2+n}{2}$

Sum of the series = $\frac{1}{2}\left(\sum n^2 + \sum n\right)$

$$= \frac{1}{2}\left[\frac{n\,(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$\Rightarrow \frac{1}{2}\left(\frac{n(n+1)}{2}\right)\left(\frac{2n+4}{3}\right)$$

$$\Rightarrow \frac{1}{6}[n(n+1)(n+2)]$$

Go through the (option)

$$^{n+2}C_3 = \frac{(n+2)!}{3!(n+2-3)!} = \frac{(n+2)!}{3!(n-1)!}$$

$$\frac{(n+2)(n+1)n\,(n-1)!}{3!\,(n-1)!}$$

$$\boldsymbol{\frac{1}{6}[n(n+1)(n+2)]}$$

Q.64) The sum of all 4- digit number containing the digits 2, 4, 6, 8, without repetitions is

a) 1,33,330 **b)** 1,22,220 **c)** 2,13,330 **d)** 1,33,320

Sol. (d)

Step 1- $4\times 3\times 2\times 1 = 24$

Step 2- $\frac{step\ 1}{total\ digits} = \frac{24}{4} = 6$

Step 3- sum of digits= 2+4+6+8= 20

Step 4 – $step\ 2 \times step\ 3 = 6\times 20 = 120$

Step 5- $120\times 1000 + 120\times 100 + 120\times 10 + 120\times 1 = \mathbf{1,33,320}$

Q.65) If all the words formed by the letter of the word RAINBOW are arranged in a dictionary form, then what is the rank that the first word starts with "R"?

a) 3600TH **b)** 3606TH **c)** 3000TH **d)** 3601TH

Sol. (d)

(A, B, I, N, O, R, W)

⇒Word starts with A = 6! = 720

⇒Word starts with B = 6! = 720

⇒Word starts with I = 6! = 720

⇒Word starts with N = 6! = 720

⇒Word starts with O = 6! = 720

⇒Word starts with R = 720+720+720+720+720+1

= **3601TH**

is

form, then

Q.66) A panel of 4 teachers, 4 advocates and one teacher who is also an advocate, how many committees of 3 can be made if it has to sure at least one teacher and one advocate?

a) 76 **b)** 78 **c)** 80 **d)** 68

Sol. (a)

Case 1- 2 teacher and 1 teacher-advocate $^4C_2 \times 1 = 6$
Case 2- 2 advocate and 1teacher-advocate $^4C_2 \times 1 = 6$
Case 3- 2 teacher and 1 advocate $^4C_2 \times {}^4C_1 = 24$
Case 4- 2 advocate and 1 teacher $^4C_2 \times {}^4C_1 = 24$
Case 5- 1 teacher, 1 advocate and 1 teacher-advocate $^4C_1 \times {}^4C_1 \times 1 = 16$
The sum of all cases = **76**

Q.67) What is the 50th word? If AGAIN is written in dictionary.

a) Again **b)** Naaig **c)** Giaan **d)** Naagi

Sol. (b)

Firstly, arrange the words in sequence (A, A, G, I, N)

Case-1

If A is fix as a first letter

$A - \times - \times - \times -$

$4 \times 3 \times 2 \times 1 = 24$

Case-2

If G is fix as a first letter

$G - \times - \times - \times -$

$= \frac{4!}{2!} \times \frac{4 \times 3 \times 2 \times 1}{2 \times 1}$

$= 12$

Case-3

If I is fix as a first letter

$I - \times - \times - \times -$

$= \frac{4!}{2!} \times \frac{4 \times 3 \times 2 \times 1}{2 \times 1}$

$= 12$

49th word will be start with N

That is NAAGI

50th word will be **NAAIG**

Sequence and Series

Q.1) The sum of n terms of an AP is $3n^2 + 5n$. The series is

a) 8, 14, 20, 26 **b)** 8, 22, 42, 68 **c)** 22, 68, 114, ... **d)** None of these

Sol. (a)

$\Rightarrow s_n = 3n^2 + 5n$

When put n=1 $s_1 = 3(1)^2 + 5 = 8$

$when\ put\ n = 2\ s_2 = 3(2)^2 + 5(2) = 22.$

$\Rightarrow a_2 = s_2 - s_1 = 22 - 8 = 14$

When put n=3 $s_3 = 3(3)^2 + 5(3) = 42$

$a_3 = s_3 - s_2 = 42 - 22 = \mathbf{20}.\ option\ a\ satisfy.$

Q.2) The number of numbers between 74 and 25,556 divisible by 5 is

a) 5,090 **b)** 5,097 **c)** 5,095 **d)** None of these

Sol. (b)

$a = 75,\ d$=5 and a_n =25555

$\Rightarrow a_n = a + (n-1)d$

$\Rightarrow 75 + (n-1) \times 5 = 25555$

$\Rightarrow 5n = 25555 - 70$

$\Rightarrow n = \frac{25485}{5} = \mathbf{5097}$

Q.3) The sum of all natural numbers from 100 to 300, which are divisible by 4 or 5, is___.

a) 10,200 **b)** 8,200 **c)** 2,200 **d)** 16,200

Sol. (d)

Required sum

$= (100 + 104 + \cdots + 300) + (100 + 105 + \cdots + 300) - (100 + 120 + 140 + \cdots + 300)$

$= \frac{51}{2}(100 + 300) + \frac{41}{2}(100 + 300) - \frac{11}{2}(100 + 300)$

$= \left(\frac{51 + 41 - 11}{\cancel{2}}\right)(\cancel{400})200$

$= 81 \times 200 = \mathbf{16,200}$

Q.4) The first and the last term of an A.P. are −4 and 146. The sum of the terms is 7171. The numberof terms is

a) 101 **b)** 100 **c)** 99 **d)** None of these

Sol. (a)

$a = -4, a_n = 146$ and $s_n = 7171$

$\Rightarrow s_n = \frac{n}{2}\{a + an\} = 7171$

$\Rightarrow \frac{n}{2}(-4 + 146) = 7171 \Rightarrow \frac{n}{2}(142)$

$\Rightarrow 71n = 7171 \Rightarrow n = \frac{7171}{71} = \mathbf{101}$

Q.5) A sum of ₹ 6240 is paid off in 30 instalments such that each instalment is ₹ 10 more than the proceeding instalment. The value of the 1^{st} instalment is

a) ₹ 36 **b)** ₹ 30 **c)** ₹ 60 **d)** None of these

Sol. (d)

$S_{30} = 6240\ and\ d = 10$

$\Rightarrow \frac{\overset{15}{\cancel{30}}}{\cancel{2}}\{2a + (30 - 1) \times 10\} = 6240$

$$\Rightarrow 2a + 290 = \frac{6240}{15}$$
$$\Rightarrow 2a = 416 - 290$$
$$\Rightarrow 2a = 126$$
$$\Rightarrow a = \frac{126}{2} = \mathbf{63}$$

Q.6) Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio of 2:3

a) 2, 2.25, 2.5, 2.75, 3
b) -2, -2.25, -2.5, -2.75, -3
c) 4, 4.5, 5, 5.5, 6
d) -4, -4.5, -5, -5.5, -6

Sol. (a)

Do it by option. (b and d are negative which is not possible only (a) sum is 12.5. So, (a) is the answer.

Or

Let the five parts be

<u>ATQ</u>

$a-2d, a-d, a, a+d$ and $a+2d$

Now, $a-2d+a-d+a+a+d+a+2d = 12.50$

$$\Rightarrow 5a = 12.50 \Rightarrow a = \frac{12.50}{5} = 2.50$$

Also, $\frac{a-2d}{a+2d} = \frac{2}{3}$

$$\Rightarrow 3a - 6d = 2a + 4d$$
$$\Rightarrow 10d = a \Rightarrow d = \frac{2.50}{10} = 0.25$$

$\therefore$ *Required Parts are*

$\mathbf{2, 2.25, 2.50, 2.75 \ \& \ 3}$

Q.7) The mth term of an A. P. is n and nth term is m. The rth term of it is

a) m + n +r
b) n + m - 2r
c) m + n + r/2
d) m + n - r

Sol. (d)

$a_m = n, a_n = m$

$a_r = m + n - r$ (Shortcut: -5 when $a_p = q, a_q = p$ *then* $a_r = p + q - r$)

or

Let 1st term & common diff. of an A.P. be a & d respectively

$a_m = n \Rightarrow a + (m-1)d = n$______(I)

$a_n = m \Rightarrow a + (n-1)d = m$______(II)

From [(I) - (II)]

$(m-n)d = n - m$

$$\Rightarrow d = \frac{-(m-n)}{m-n} = -1$$

$\therefore a = n + m - 1$

$\therefore a_r = a + (r-1)d$

$= m + n - 1 + (r-1)(-1)$

$= m + n - 1 - r + 1$

$= \boldsymbol{m + n - r}$

Q.8) The nth term of the series whose sum to n terms is $5n^2 + 2n$ is

a) 3n - 10
b) 10n - 2
c) 10n - 3
d) None of these

Sol. (c)

$s_n = 5n^2 + 2n$

Put n=1

$s_1 = 5(1)^2 + 2(1) = 7$

$s_2 = 5(2)^2 + 2(2) = \ 24$

$a_2 = s_2 - s_1 = 17$
$\therefore$ *put* $n = 2$ *in options and option c satisfy.*
$= \mathbf{10n - 3}$

Q.9) The two arithmetic means between –6 and 14 is

a) $\frac{2}{3}, \frac{1}{3}$ **b)** $\frac{2}{3}, 7\frac{1}{3}$ **c)** $-\frac{2}{3}, -7\frac{1}{3}$ **d)** None of these

Sol. (b)

-6, _ _ 14
$= a = -6, \quad a + 3d = 14, \quad -6 + 3d = 14.$
$= d = \frac{20}{3}$
$\Rightarrow \ a_2 = -6 + \frac{20}{3} = \frac{2}{3}$
$\Rightarrow \ a_3 = \frac{2}{3} + \frac{20}{3} = \frac{22}{3} = 7\frac{1}{3}$

Q.10) The sum of three integers in A.P. is 15 and their product is 80. The integers are

a) 2, 8, 5 **b)** 8, 2, 5 **c)** 2, 5, 8 **d)** 8, 5, 2

Sol. (c & d)

Let the integers are $a - d, a$ & $a + d$
$a - d + a + a + d = 15 \Rightarrow 3a = 15$
$\Rightarrow a = 5$
Also $(a - d)\, a(a + d) = 80$
$\Rightarrow (a^2 - d^2)a = 80 \Rightarrow (25 - d^2)5 = 80$
$\Rightarrow 25 - d^2 = 16 \Rightarrow d^2 = 9$
$\Rightarrow d = \pm 3$
$\therefore$ Nos. are **2, 5, 8 or 8, 5, 2**

Q.11) 7^{2n} +16*n*-1 is divisible by

a) 15 **b)** 4 **c)** 6 **d)** 64

Sol. (d)

$7^{2n} + 16n - 1$
Put $n = 1$
$p(n = 1) = 7^2 + 16 \times 1 - 1 = 49 + 16 - 1 = \mathbf{64}$

Q.12) The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3^{rd} term of the A.P. is

a) $6\frac{4}{11}$ **b)** 106 **c)** 4/11 **d)** None of these

Sol. (a)

$a = 14$
Also, $s_5 + s_{10} = 0$
$\Rightarrow \frac{5}{2}[2a + 4d] = -\frac{10}{2}[2a + 9d]$
$\Rightarrow 10a + 20d + 20a + 90d = 0$
$\Rightarrow 30a + 110d = 0$
$\Rightarrow d = \frac{-15a}{55}$
$= \cfrac{\overset{3}{\cancel{-15}} \times 14}{\underset{11}{\cancel{55}}} = \frac{-42}{11}$
$\therefore a_3 = a + 2d = 14 - \frac{84}{11} = \frac{154 - 84}{11}$
$= \frac{70}{11} = 6\frac{4}{11}$

Q.13) The sum of cubes of first n natural number is________.

a) $(n/2)(n+1)$ **b)** $(n/6)(n+1)(2n+1)$ **c)** $[(n/2)(n+1)]^2$ **d)** None

Sol. (c)

$$1^3+2^3+3^3+\cdots+n^3=\left\{\frac{n(n+1)}{2}\right\}^2$$

Q.14) If $a,\ b,\ c$ be the sum of $p,\ q,\ r$ terms, respectively of an A.P. the value of $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)$ is___.

a) 0 **b)** 1 **c)** -1 **d)** None

Sol. (a)

Let 1st term & common difference of an A.P. be A & D respectively

$$s_p=a\Rightarrow\frac{p}{2}\{2A+(p-1)D\}=a$$

$$\Rightarrow A+(p-1)\frac{D}{2}=\frac{a}{p} \qquad \text{\_\_\_\_(I)}$$

Similarly, $s_q=b\Rightarrow A+(q-1)\frac{D}{2}=\frac{b}{q}$ ____(II)

$$s_r=c\Rightarrow A+(r-1)\frac{D}{2}=\frac{c}{r} \qquad \text{\_\_\_\_\_\_(III)}$$

$$\therefore\left(\frac{a}{p}\right)(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)$$

$$=A(q-r+r-p+p-q)+\frac{D}{2}[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]$$

$$=A\times0+\frac{D}{2}\times0=0+0=\mathbf{0}$$

Q.15) The sum of n terms of $(x+y)^2, (x^2+y^2), (x-y)^2$, is

a) $(x+y)^2-2(n-1)xy$ **b)** $n(x+y)^2-n(n-1)xy$ **c)** Both (a) and (b) **d)** None

Sol. (b)

$s_2=(x+y)^2+x^2+y^2=2x^2+2y^2+2xy$

$\therefore$ put $n=2$ in the options

$(x+y)^2-2(n-1)xy=(x+y)^2-2xy=(x-y)^2$

$n(x+y)^2-n(n-1)xy$

$=2(x+y)^2-2xy=\mathbf{2x^2+2y^2+2xy}$

Q.16) The sum to ∞ of the series $-5,\ 25,\ -125,\ 625,\ \ldots..$ can be written as

a) $\sum_{k=1}^{\infty}(-5)^k$ **b)** $\sum_{k=1}^{\infty}5^k$ **c)** $\sum_{k=1}^{\infty}-5^k$ **d)** None of these

Sol. (a)

$a_k=ar^{k-1}=-5(-5)^{k-1}=(-5)^k$

$\therefore$ Required sum$=\sum_{k=1}^{\infty}a_k=\sum_{k=1}^{\infty}\mathbf{(-5)^k}$

Q.17) The nth element of the sequence $-1,\ 2,\ -4,\ 8\ldots..$ is

a) $(-1)^n\,2^{\,n-1}$ **b)** 2^{n-1} **c)** 2^n **d)** None of these

Sol. (a)

$=a_n=ar^{n-1}=(-1)(-2)^{n-1}=(-1)(-1)^{n-1}2^{n-1}$

$=\mathbf{(-1)^n2^{n-1}}$

Or

Put n =2 in the options and when we get 2 then it is the nth term but a and b both options satisfy so we put n=3 then only (a satisfy.

Q.18) The sum of the series $\frac{1}{\sqrt{3}} + 1 + \frac{3}{\sqrt{3}} + \ldots \ldots$ to 18 terms is

a) $9841\frac{(1+\sqrt{3})}{\sqrt{3}}$ **b)** 9841 **c)** $\frac{9841}{\sqrt{3}}$ **d)** None of these

Sol. (a)

$s_{18} = \frac{a(r^{18}-1)}{r-1}$ $r_1 = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$= \frac{\frac{1}{\sqrt{3}}\left[(\sqrt{3})^{18}-1\right]}{\sqrt{3}-1}$

$= \frac{1}{\sqrt{3}(\sqrt{3}-1)}(3^9 - 1) = \frac{1}{\sqrt{3}(\sqrt{3}-1)}(19683 - 1)$

$= \frac{(9841)\sqrt{3}+1}{\sqrt{3}(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\mathbf{9841(\sqrt{3}+1)}}{\sqrt{3}}$

Q.19) The sum of n terms of the series 7 + 77 + 777 + …… is

a) (7/9)[(1/9)(10^{n+1}-10)-n] **b)** (9/10)[(1/9)(10^{n+1}-10)-n]

c) (10/9)[(1/9)(10^{n+1}-10)-n] **d)** None

Sol. (a)

$s_2 = 7+77 = 84$

Put $n = 2$ in the ((a)

$= \left(\frac{7}{9}\right)\left[\frac{1}{9}(10^{n+1} - 10) - n\right]$

$= \frac{7}{9}\left[\frac{1}{9}(10^3 - 10) - 2\right]$

$= \frac{7}{9}\left(\frac{1}{9} \times 990 - 2\right) = \frac{7}{9}(110 - 2) = \frac{7}{9}(108) = \mathbf{84}$

Q.20) Three numbers are in A.P. and their sum is 21. If 1, 5, 15 are added to them respectively,they form a G. P. The numbers are

a) 5, 7, 9 **b)** 9, 5, 7 **c)** 7, 5, 9 **d)** None of these

Sol. (a)

Do it by options in (a

5,7,9 are in A.P so first condition satisfy and sum= 5+7+9= 21

Add 1 in 1st term 5 in second term and 15 in third term the numbers are 6,12 and 24

$r_1 = 2 \; and \; r_2 = 2$

∴ 5, 7 and 9 (satisfy all conditions.

Or

Let the three nos. in A.P. be $a - d, a \;\&\; a + d$

ATQ

$a - d + a + a + d = 21 \Longrightarrow 3a = 21 \Longrightarrow a = 7$ ______(I)

Also $a - d + 1, a + 5 \;\&\; a + d + 15$ are in G.P.

$\therefore (a + 5)^2 = (a - d + 1)(a + d + 15)$

$\Rightarrow (12)^2 = (8 - d)(22 + d)$[from (I)] $(b^2 = ac)$

$\Rightarrow 144 = 176 - 14d - d^2$

$\Rightarrow d^2 + 14d - 32 = 0 \Rightarrow (d + 16)(d - 2) = 0$

$\Rightarrow d + 16 = 0 \; or \; d - 2 = 0$

$\Rightarrow d = -16 \; or \; d = 2$

∴ Required nos. are **5, 7 & 9**

Q.21) If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, thenyour total savings in two weeks will be

a) ₹ 163 **b)** ₹ 183 **c)** ₹ 1633.83 **d)** None of these

Sol. (c)

$= (1 + 2 + 4 + \cdots \; to \; 14 \; terms)$ Paise

$= \frac{1(2^{14}-1)}{2-1}$ Paise
$=$ **₹ 163.83**

Q.22) Given x, y, z are in G.P. and $x^p = y^q = z^\sigma$, then 1/p, 1/q, 1/ σ are in
a) A.P. **b)** G.P. **c)** Both A.P. and G.P. **d)** None of these

Sol. (a)
x, y, z are in G.P.
$\Rightarrow \frac{y}{x} = \frac{z}{y} \Rightarrow y^2 = xz$______(I)
Now $x^p = y^q = z^\sigma = k\ (let)$
$\Rightarrow x = k^{1/p}, y = k^{1/q}\ \&\ z = k^{\frac{1}{\sigma}}$_____(II)
From (I) & (II)
$k^{2/q} = k^{1/p}.\ k^{1/\sigma}$
$\Rightarrow k^{\frac{2}{q}} = k^{\frac{1}{p}+\frac{1}{\sigma}} \Rightarrow \frac{2}{q} = \frac{1}{p} + \frac{1}{\sigma}$
$\therefore \frac{1}{p}, \frac{1}{q}\ \&\ \frac{1}{\sigma}$ **are in A.P.**

Q.23) The sum of 3 numbers of a G.P. is 39 and their product is 729. The numbers are
a) 3, 27, 9 **b)** 9, 3, 27 **c)** 3, 9, 27 **d)** None of these

Sol. (c)
Do it by (since 1st and second options are not in G.P and when go through third (then it is in G.P
= 3+9+27= 39.
= $3 \times 9 \times 27 = 729.$ *and c options satisfy both conditions.* (a and b are not in GP.
Or
$\frac{a}{r} + a + ar = 39$
$\Rightarrow \frac{a}{r}(1 + r + r^2) = 39$______(I)
Also $\frac{a}{r} \times a \times ar = 729$
$\Rightarrow a^3 = 9^3 \Rightarrow a = 9$______(II)
From (I) & (II)
$\frac{9^3}{r}(1 + r + r^2) = \cancel{39}^{13}$
$\Rightarrow 3r^2 + 3r + 3 = 13r$
$\Rightarrow 3r^2 - 10r + 3 = 0$
$\Rightarrow 3r^2 - 9r - r + 3 = 0$
$\Rightarrow 3r(r-3) - 1(r-3) = 0$
$\Rightarrow (3r-1)(r-3) = 0 \Rightarrow 3r - 1 = 0 \text{ or } r - 3 = 0$
$\Rightarrow r = \frac{1}{3}\ or\ r = 3$
Then the numbers are **27, 9, 3, or 3, 9, 27**

Q.24) Four geometric means between 4 and 972 are
a) 12, 36, 108, 324 **b)** 12, 24, 108, 320 **c)** 10, 36, 108, 320 **d)** None of these

Sol. (a)
4, _ _ _ _, 972
$\Rightarrow a_6 = ar^5 = 972 \Rightarrow 4r^5 = 972$
$\Rightarrow r^5 = 243,\ r = 3.$
$\Rightarrow a_2 = a_1 \times r = 4 \times 3 = 12$
$\Rightarrow a_3 = a_2 \times r = 12 \times 3 = 36$
$\Rightarrow a_4 = a_3 \times r = 36 \times 3 = 108$
$\Rightarrow a_5 = a_4 \times r = 108 \times 3 = \mathbf{324}$

Q.25) t_4 of a G.P. is x, $t_{10} = y$ and $t_{16} = z$. Then

a) $x^2 = yz$ **b)** $z^2 = xy$ **c)** $y^2 = zx$ **d)** None of these

Sol. (c)

$ar^3 = x, ar^9 = y, ar^{15} = z$

$zx = ar^{15}. ar^3 = a^2r^{18} = (ar^9)^2 = y^2$

$\Rightarrow \boldsymbol{y^2 = zx}$

Q.26) If the third term of a G.P. is the square of the first and the fifth term is 64 the series would be_____.

a) 4 + 8 + 16 + 32 + …. **b)** 4 – 8 + 16 – 32 + …….. **c)** both a and b **d)** None

Sol. (c)

Do it by option. In both options, a and b third term is the square of the first term. And 5th term is 64.

∴ (c is correct.

Or

Let the series be $a + ar + ar^2 + \cdots$

$ar^2 = a^2 \Rightarrow r^2 = a$______(I)

Also, $ar^4 = 64 \Rightarrow a \,.\, a^2 = 64$

$\Rightarrow a^3 = 4^3 \Rightarrow a = 4$

$\therefore r^2 = 4 \Rightarrow r = \pm 2$

∴ Required series is

$\boldsymbol{4 + 8 + 16 + 32 + \cdots}$ **Or** $\boldsymbol{4 - 8 + 16 - 32 + \cdots}$

Q.27) The sum of n terms of the series $1^2/1+(1^2+2^2)/(1+2)+(1^2+2^2+3^2)/(1+2+3)+$ ………….is

a) $(n/3)(n+2)$ **b)** $(n/3)(n+1)$ **c)** $(n/3)(n+3)$ **d)** None

Sol. (a)

$$= s_2 = \frac{1^2}{1} + \frac{1^2 + 2^2}{1 + 2} = 1 + \frac{5}{3} = \frac{8}{3}$$

Put $n = 2$ in the ((a)

$$= \frac{n}{3}(n + 2) = \frac{2}{3} \times 4 = \frac{8}{3}$$

Q.28) find the sum of the infinity of the following series: 1-1+1-1+1-1+1-1+1 …………….∞

a) 1 b) 0 c. ½ d) Does not exist

Sol. (c)

Common ratio = -1

Sum of infinity terms $= \frac{a}{1-r}$

$$= \frac{1}{1-(-1)} = \frac{1}{2}$$

Q.29) The sum of the infinite GP 14, – 2, + 2/7, – 2/49, + … is

a) $4\frac{1}{12}$ **b)** $12\frac{1}{4}$ **c)** 12 **d)** None of these

Sol. (b)

$$S_\infty = \frac{a}{1-r}\ , \quad r = \frac{a_2}{a_1} = -\frac{2}{14} = \frac{-1}{7}$$

$$= \frac{14}{1-\left(-\frac{1}{7}\right)} = \frac{\overset{7}{\cancel{14}}\times 7}{\underset{4}{\cancel{8}}} = \frac{49}{4} = \boldsymbol{12\frac{1}{4}}$$

Q.30) If p, q and r are in A.P. and x, y, z are in G.P., then $x^{q-r}. y^{r-p}. z^{p-q}$ is equal to

a) 0 **b)** –1 **c)** 1 **d)** None of these

Sol. (c)

$\because p, q, r$ *in* $A.P. \Rightarrow p + r = 2q$

x, y, z are in $G.P.$

$\therefore \frac{y}{x} = \frac{z}{y} = k(let)$

$\therefore y = kx, z = y\,k \qquad z = kx(k) \Rightarrow z = x\,k^2$

$\therefore x^{q-r}.y^{r-p}.z^{p-q} = x^{q-r}.(xk)^{r-p}.(x\,k^2)^{p-q}$
$= x^{q-r+r-p+p-q} \times k^{r-p+2p-2q}$
$= x^0 \times k^0 = 1 \times 1 = \mathbf{1}$

Q.31) If A be the A.M. of two positive unequal quantities x and y and G be their G. M, then

a) A < G **b)** A>G **c)** A ≥ G **d)** A ≤ G

Sol. (c)

Positive and equal number
$\Rightarrow$ A.M. = G. M. = H.M.
$\Rightarrow$ when numbers are unequal, then A.M. is greater than G.M.
$\Rightarrow A = \frac{x+y}{2}$ & $G = \sqrt{xy}$
$\Rightarrow A - G = \frac{x+y}{2} - \sqrt{xy}$
$\Rightarrow$ A-G = $\frac{x+y-2\sqrt{xy}}{2}$
$= \frac{\left(\sqrt{x}-\sqrt{y}\right)^2}{2} \geq 0$
$\Rightarrow A - G \geq 0$
$\Rightarrow \boldsymbol{A \geq G}$

Q.32) If x, y, z are in A.P. and $x, y, (z+1)$ are in G.P. then

a) $(x-z)^2 = 4x$ **b)** $z^2 = (x-y)$ **c)** $z = x - y$ **d)** None of these

Sol. (a)

x, y, z are in A.P.
$\therefore 2y = x + z$ ________(I)
Also $x, y, (z+1)$ are in G.P.
$\therefore y^2 = x(z+1)$
$\Rightarrow \left(\frac{x+z}{2}\right)^2 = xz + x$
$\Rightarrow (x+z)^2 = 4xz + 4x$
$\Rightarrow (x+z)^2 - 4xz = 4x$
$\Rightarrow (x-z)^2 = \mathbf{4x}$

Q.33) If $1 + a + a^2 + \cdots\infty = x$ and $1 + b + b^2 + \cdots\infty = y$ then $1 + ab + a^2b^2 + \cdots\infty = \frac{1}{1-ab}$ is given by _____.

a) $\frac{xy}{x+y-1}$ **b)** $\frac{xy}{x-y-1}$ **c)** $\frac{x-y}{x+y+1}$ **d)** None of these

Sol. (a)

$S_\infty = \frac{1}{1-r}$
$x = \frac{1}{1-a}, \quad y = \frac{1}{1-b}$
Now, $1 + ab + a^2b^2 + \cdots\infty = \frac{1}{1-ab} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$
$= \frac{xy}{xy-(x-1)(y-1)} = \frac{xy}{xy-xy+x+y-1} = \frac{xy}{x+y-1}$
$\Rightarrow 1 + ab + a^2b^2 + \cdots\infty = \frac{xy}{x+y-1}$

Q.34) If a, b, x, y, z are positive numbers such that a, x, b are in A.P., and a, y, b are in G.P. and z=(2ab)/(a+b) then

a) x, y, z are in G.P. **b)** x < y < z **c)** both **d)** None

Sol. (a)

$x = \frac{a+b}{2}, \; y = \sqrt{ab} \Rightarrow y^2 = ab$
$z = \frac{2ab}{a+b} = \frac{y^2}{x} \Rightarrow y^2 = zx$

$\therefore$ *x, y, z are in G.P.*

Q.35) If *a, b, c, d* are in G.P. then the value of (b-c)^2+(c-a)^2+(d-b)^2-(a-d)^2 is given by

a) 0 **b)** 1 **c)** -1 **d)** None

Sol. (a)

Let $a = 1, b = 2,\ c = 4\ \&\ d = 8$

Now $(b-c)^2 + (c-a)^2 + (d-b)^2 - (a-d)^2$

$=(-2)^2 + 3^2 + 6^2 - (-7)^2$

$= 4 + 9 + 36 - 49 = 49 - 49 = \mathbf{0}$

Q.36) If (a-b), (b-c), (c-a) are in G.P. then the value of (a+b+c)^2-3(ab+bc+ca) is given by

a) 0 **b)** 1 **c)** -1 **d)** None

Sol. (a)

$a-b,\ b-c\ \&\ c-a$ are in G.P.

$\therefore \frac{b-c}{a-b} = \frac{c-a}{b-c} = r\ (let)$

$\therefore\ b - c = r\,(a-b)$

$c - a = r(b-c) = r^2(a-b) \qquad [\therefore b - c = r\,(a-b)]$

$b - c + c - a = r(a-b) + r^2(a-b)$

$\Rightarrow -(a-b) = (r + r^2)(a-b)$

$\Rightarrow (r^2 + r + 1)(a-b) = 0$

$\Rightarrow a - b = 0$______(I)

$\therefore (a+b+c)^2 - 3(ab+bc+ca)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - ca] \qquad [(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$= \frac{1}{2}[(a-b)^2 + r^2(a-b)^2 + r^4(a-b)^2]$

$= \frac{1}{2}[(a-b)^2(1 + r^2 + r^4)]$

$= \frac{1}{2}[0]$ [From (I)]

$=0$

Q.37) If S$_1$, S$_2$, S$_3$,……….. S$_n$ are the sums of infinite G.P.s whose first terms are 1, 2, 3 …..n and whose common ratios are 1/2, 1/3, ……1/(n+1) then the value of S$_1$+S$_2$+S$_3$+ ……..S$_n$ is

a) (n/2) (n+3) **b)** (n/2) (n+2) **c)** (n/2) (n+1) **d)** n^2/2

Sol. (a)

$\because s_\infty = \dfrac{a}{1-r}$

$\therefore s_1 = \frac{1}{1-\frac{1}{2}} = 2,\ s_2 = \frac{2}{1-\frac{1}{3}} = 3, s_3 = \frac{3}{1-\frac{1}{4}} = 4$

$s_n = \dfrac{n}{1 - \dfrac{1}{n+1}} = n + 1$

$\therefore s_1 + s_2 + s_3 + \cdots + s_n = 2 + 3 + 4 + \cdots + (n+1)$

$= \frac{n}{2}[2 \times 2 + (n-1) \times 1] = \frac{n}{2}(4 + n - 1) = \frac{n}{2}\boldsymbol{(n+3)}$

Q.38) If the sum of infinite terms in a G.P. is 2 and the sum of their squares is 4/3 the series is

a) 1, 1/2, 1/4 …… **b)** 1, −1/2, 1/4 ……. **c)** −1, −1/2, − 1/4 …. **d)** None

Sol. (a)

Do it by options.

= 1, ½, ¼ then r = 1/2 $s_n = \frac{1}{1-\frac{1}{2}} = 2$

Sum of squares of series = $\frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

Or

$\frac{a}{1-r} = 2$______(I)

Also $\frac{a^2}{1-r^2} = \frac{4}{3}$______(II)

From (I) & (II)

$\frac{[2(1-r)]^2}{1-r^2} = \frac{4}{3} \Rightarrow \frac{4(1-r)^2}{(1+r)\cancel{(1-r)}} = \frac{4}{3}$

$\Rightarrow 3 - 3r = 1 + r \Rightarrow 4r = 2$

$\Rightarrow r = \frac{1}{2}$

$\therefore a = 2\left(1 - \frac{1}{2}\right) = 1$

∴ required series is

$1 + \frac{1}{2} + \frac{1}{2^2} + \cdots$

Q.39) The numbers $x, 8, y$ are in G.P. and the numbers $x, y, -8$ are in A.P. The values of x, y are ______.

a) 16, 4 **b)** 4, 16 **c)** both **d)** None

Sol. (a)

Do it by option.

$x, 8, y$ are in G.P

Let x and y = 16 and 4, $r_1 = \frac{16}{8} = 2, r_2 = \frac{8}{4} = 2$, so it is in G.P

16, 4. -8 are in A.P so both conditions satisfied.

Or

$x, 8, y$, are in G. P. $: \; xy = 8^2 \Rightarrow xy = 64$_____(I)

Also $x, y, -8$ are in A. P.

$\therefore y = \frac{x-8}{2}$______(II)

From (I) & (II)

$x\left(\frac{x-8}{2}\right) = 64$

$\Rightarrow x^2 - 8x - 128 = 0$

$\Rightarrow x^2 - 16x + 8x - 128 = 0$

$\Rightarrow x(x-16) + 8(x-16) = 0$

$\Rightarrow (x-16)(x+8) = 0 \Rightarrow x = 16 \; or \; x = -8$

If $x = 16 \; then \; y = \frac{16-8}{2} = \mathbf{4}$

Q.40) The sum of n terms of the series if $\log(x) + \log\frac{x^2}{y} + \log\frac{x^3}{y^2} \ldots\ldots\ldots\ldots is$

a) $\frac{n}{2}\left[2n\log\left(\frac{x}{y}\right) + \log xy\right]$ **b)** $\frac{n}{2}\left[n\log(xy) + \log\frac{x}{y}\right]$

c) $\frac{n}{2}\left[n\log\left(\frac{x}{y}\right) - \log xy\right]$ **d)** $\frac{n}{2}\left[n\log\left(\frac{x}{y}\right) + \log xy\right]$

Sol. (d)

Go through option

Put the value n =1

In (A $= \frac{1}{2}\left[2\ \log\frac{x}{y} + \log xy\right] = \frac{1}{2}\left[\log\frac{x^2}{y^2} \times xy\right] = \frac{1}{2}\left[\log\frac{x^3}{y}\right]$ wrong answer

Put the value n =1 In (A option)

$= \frac{1}{2}\left[\log xy + \log\frac{x}{y}\right] = \frac{1}{2}\left[\log xy \times \frac{x}{y}\right] = \frac{1}{2}[\log x^2] = \frac{1}{2} \times 2[\log x] = [\log x]$

Put the value n =2 In (B option)

$= \frac{2}{2}\left[2\ \log xy + \log\frac{x}{y}\right] = \left[\log\frac{x}{y} \times (xy)^2\right] = \left[\log\frac{x^3}{y}\right]$

Put the value n = 3 In (B

$= \frac{3}{2}\left[3\ \log xy + \log\frac{x}{y}\right] = \frac{3}{2}\left[\log(xy)^3 \times \frac{x}{y}\right] = \frac{3}{2}[\log x^4y^2]$ wrong answer

Put the value n =1 In (C option)

$= \frac{1}{2}\left[\log\frac{x}{y} - \log xy\right] = \frac{1}{2}\left[\log\frac{x}{y} \div xy\right] = \frac{1}{2}\left[\log\frac{1}{y^2}\right] = \log\frac{1}{y}$ wrong answer

Put the value n =1

In (D $= \frac{1}{2}\left[\log\frac{x}{y} + \log xy\right] = \frac{1}{2}\left[\log\frac{x}{y} \times xy\right] = \frac{1}{2}[\log x^2] = \frac{1}{2} \times 2[\log x] = [\log x]$

put the value n =2

$= \frac{2}{2}\left[2\ \log\frac{x}{y} + \log xy\right] = \left[\log\frac{x^2}{y^2} \times xy\right] = \left[\log\frac{x^3}{y}\right]$ answer

Put the value n = 3 In (D option)

$= \frac{3}{2}\left[3\ \log\frac{x}{y} + \log xy\right] = \frac{3}{2}\left[\log\frac{x^3}{y^3} \times xy\right] = \frac{3}{2}\left[\mathbf{\log\frac{x^4}{y^2}}\right]$ Correct answer

Q.41) If p, q, r, s and t are the first five terms of A.P. such that p+r+t = -9 and s.q.r = 21, find the first term.

a) 3 **b)** 2 **c)** 4 **d)** -11

Sol. (d)

Let five terms are $(p = a - 2d, q = a - d, r = a, s = a + d, t = a + 2d)$

$P + r + t = a - 2d + a + a + 2d = -9$

$= 3a = -9$

$\Rightarrow a = -3$

$s.q.r = 21$

$\Rightarrow (a-d)a(a+d) = 21$

$\Rightarrow (-3-d)(-3)(-3+d) = 21$

$\Rightarrow 9 - d^2 = -7$

$\Rightarrow d^2 = 16$

$\Rightarrow d = 4$

$first\ term = a - 2d$

$= -3 - 2(4) = \mathbf{-11}$

Q.42) If 9 times the 9th term of an A.P. is equal to 37 times its 37th term, find 46th term

a) 0 **b)** 1 **c)** mn **d)** -mn

Sol. (a)

If $n(a_n) = m(a_m)$

then $a_{m+n} = 0$

$\Rightarrow 9\ a_9 = 37\ a_{37}$

then $a_{37+9} = a_{46} = \mathbf{0}$

Q.43) The sums of n terms of two A.P. are in the ratio of $\frac{7n+1}{4n+27}$. Find the ratio of 11th terms :

a) $\frac{4}{5}$ **b)** $\frac{2}{3}$ **c)** $\frac{4}{3}$ **d)** $\frac{1}{1}$

Sol. (c)

If the ratio of S_n of 2 A.P. series given in the question,
Then ratio of a_n term will be replaced "n" from 2n-1

e first term.

$\frac{S1_n}{S2_n} = \frac{7n+1}{4n+27}$

$\frac{a1_n}{a2_n} = \frac{7(2n-1)+1}{4(2n-1)+27} = \frac{4}{3}$

$= \frac{7(2(11)-1)+1}{4(2(11)-1)+27} = \frac{148}{111} = \frac{\mathbf{4}}{\mathbf{3}}$

Q.44) Find the sum of all odd numbers of four-digit which are divisible by 9:

a) 27,540 **b)** 2,54,700 **c)** 5,87,420 **d)** 27,54,000

Sol. (d)

First term = 1017 (we can't take 1008 because it is even number)

Common difference = 18

Last term = 9999

$a_n = a + (n-1)d = 9999$

$n = 500$

$S_n = \frac{n}{2}(a + l)$

$= \frac{500}{2}(1017 + 9999)$

= **27,54,000**

Q.45) The sum of three numbers of A.P. is 24, and the sum of their cubes is 1968. Find the product of the numbers:

a) 70 **b)** 140 **c)** 440 **d)** 210

Sol. (c)

The first three terms are (a - d), a, (a + d)

Sum of first three numbers = a – d + a + a + d = 24

⇒ 3a =24

⇒ a=8

sum of their cube = $(a-d)^3 + a^3 + (a+d)^3 = 1968$

$\Rightarrow a^3 - d^3 - 3a^2d + 3ad^2 + a^3 + a^3 + d^3 + 3a^2d + 3ad^2 = 1968$

$\Rightarrow 512 - d^3 - 192d + 24d^2 + 512 + 512 + d^3 + 192d + 24d^2 = 1968$

$\Rightarrow 1536{+}48d^2 = 1968$

$\Rightarrow d^2 = 9$

$\Rightarrow d = 3$

First term = a-d = 8-3 = 5

Second term = 8

Third term = 8+3 =11

Product of first three terms = 5x8x11 = **440**

Q.46) If m times the m^th^ terms of an A.P. is equal to n times its nth term, find (m+n)^th^ term of A.P.:

a) 1 **b)** 0 **c)** -mn **d)** m+n

Sol. (b)

If $n(a_n) = m(a_m)$

then $a_{m+n} = \mathbf{0}$

Q.47) If 5th term is $\frac{1}{7}$ and 7th term is $\frac{1}{5}$ of an A.P. then the sum of 35 terms is: -

a) 18 **b)** 35 **c)** 105 **d)** None

Sol. (a)

If $a_p = \frac{1}{q}$ and $a_q = \frac{1}{p}$

Then $S_{pq} = \frac{1}{2}(pq + 1)$

$\Rightarrow S_{35} = \frac{1}{2}(35 + 1)$
$\Rightarrow S_{35} = \mathbf{18}$

Q.48) if the difference between two consecutive interior angles of a polygon is 5°. The smallest angle is 107.5°. Find the numbers of the side of a polygon.

a) 6 **b)** 15 **c)** 20 **d)**23

Sol. (a)
Go through the options
Sum of the interior angle of a polygon = 180(n-2) = $\frac{n}{2}[2a + (n-1)d]$
Put n= 6
Sum of the interior angle of a polygon = 180(6-2) = $\frac{6}{2}[2(107.5) + (6-1)5]$
$= 720 = 720$
(a satisfy the equation.
OR
Sum of the interior angle of a polygon = 180(n-2) = $\frac{n}{2}[2a + (n-1)d]$
$\Rightarrow a = 107.5\ and\ d = 5$
$\Rightarrow$ 180(n-2) = $\frac{n}{2}[2(107.5) + (n-1)5]$
$= 360n - 720 = 215n - 5n + 5n^2$
$= 5n^2 - 150n + 720 = 0$
$= n^2 - 24n - 6n + 144 = 0$
$= n(n-24) - 6(n-24) = 0$
$= (n-6)(n-24) = 0$
$= \boldsymbol{n = 24}$ **or 6**

Q.49) How many numbers between 200 and 900 are divisible by 2 and 3.

a) 111 **b)** 116 **c)** 119 **d)** None3

Sol. (b)
Required the number which is divisible by 2 and 3 means divisible by 6
First number 204 and last number 894
⇒ 894 = 204 + (n-1)6
⇒ n = 116

Q. 50) Find the sum of all 3-digit numbers when divisible by 7 yields 1 as remainder.

a) 70,024 **b)** 70,240 **c)** 70,464 **d)** 70,460

Sol. (c)
First term = 106 and last term = 995 common difference = 7
$\Rightarrow an = a + (n-1)d$
⇒ 995 = 106 + $(n-1)7$
$\Rightarrow n = 128$
Sn = $\frac{n}{2}(a + l)$
$= \frac{128}{2}(106 + 995)$
= 70,464

Q. 51) If the sum of the first four terms is 68 and the sum of the last 4 terms is 180. The first term is 11. Find the number of terms of A.P.

a) 12 **b)** 10 **c)** 9 **d)**11

Sol. (d)

Sum of first four terms = 11 + 11 + d + 11 + 2d + 11 + 3d = 68

⇒ 44+6d = 68

= d = 6

Sum of last four terms = 11+ (n-4) d + 11 + (n-3) d + 11 + (n-2) d + 11 + (n-1) d = 180

= 11 + (n-4)4 + 11 + (n-3)4 + 11 + (n-2)4 + 11 + (n-1)4 = 180

⇒ 44 – 16 – 12 – 8 – 4 + 16n = 180

⇒ 16n = 176

⇒ $\boldsymbol{n = 11}$

Q.52) A ball is dropped from a height of 180 feet, and it rebounds $\frac{2}{3}$ of the height, it falls. If it continues to fall and rebound. Find the total distance that the ball can travel before coming to rest.

a) 440 ft **b)** 360ft **c)** 400ft **d)** 900ft.

Sol. (d)

$$\Rightarrow a = 180 + 180\left(\frac{2}{3}\right) = 300$$

Sum of infinity terms = $\frac{a}{1-r}$

$$= \frac{300}{1-\frac{2}{3}} = \mathbf{900}$$

Q.53) The sum of the series $8^3 + 9^3 + 10^3 + 11^3 \dots\dots\dots 30^3$ is:

a) 766060875 **b)**766038973 **c)** 10,05,22,673 **d)** None

Sol. (c)

$$= 8^3 + 9^3 + 10^3 + 11^3 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 30^3$$

$$= (1^3 + 2^3 + 3^3 + 4^3 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 30^3) - (1^3 + 2^3 + 3^3 + 4^3 \dots\dots 7^3)$$

$$= \sum n^3 = \left[\frac{n(n+1)}{2}\right]^3 \textit{ Apply formula}$$

$$= \left[\frac{30(30+1)}{2}\right]^3 - \left[\frac{7(7+1)}{2}\right]^3$$

$$= \left[\frac{30(31)}{2}\right]^3 - \left[\frac{7(8)}{2}\right]^3$$

$$= \left[\frac{930}{2}\right]^3 - \left[\frac{56}{2}\right]^3$$

$$= [465]^3 - [28]^3$$

= **10,05,22,673**

Q.54) The number of divisors of 165, 308 and 451 is in

a) A.P. **b)** G.P. **c)** Both **d)** None

Sol. (a)

165 = 11x15

308 = 11x28

451 = 11x41

15, 28 and 41 are in A.P.

Q.55) The value of $0.1\overline{32}$ is:

a) 132 **b)** $\frac{132}{1000}$ **c)** $\frac{131}{999}$ **d)** $\frac{131}{990}$

Sol. (d)

$\Rightarrow x = 0.1\,\overline{32}$

$10\,x = 1.\,\overline{32}$...1

$1000\,x = 132.\overline{32}$..2

Equation 1 subtract from equation 2

$\Rightarrow 990\,x = 131$

$\Rightarrow x = \frac{131}{990}$

Q.56) If $\sum n = 78$, then $\sum n^2$ is equal to:

a) 550 **b)** 1250 **c)** 650 **d)** 1100

Sol. (c)

$$\sum n = 78$$

$$\therefore \frac{n(n+1)}{2} = 78$$

$$\therefore n(n+1) = 156$$

$$\therefore n^2 + n = 156$$

$$\therefore n^2 + n - 156 = 0$$

$$\therefore n = 12$$

$$\Rightarrow \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{12(12+1)(12 \times 2 + 1)}{6} = \frac{12 \times 13 \times 25}{6} = \mathbf{650}$$

Sets, Relation and Function

Q.1) The set $\{x \mid 0 < x < 5\}$ represents the set when x may take integral values only

a) $\{0, 1, 2, 3, 4, 5\}$ **b)** $\{1, 2, 3, 4\}$ **c)** $\{1, 2, 3, 4, 5\}$ **d)** None of these

Sol. (b)

$\because x \in z \,\&\, \{x|0 < x < 5\} = \{1,2,3,4\}$

Q.2) The set $\{2^x \mid x$ is any positive rational number$\}$ is

a) an infinite set **b)** a null set **c)** a finite set **d)** None of these

Sol. (a)

Q.3) $\{1-(-1)^x\}$ for all integral x is the set

a) $\{0\}$ **b)** $\{2\}$ **c)** $\{0, 2\}$ **d)** None of these

Sol. (c)

If x is odd then

$1-(-1)^x = 1-(-1) = 1+1 = 2$

If x is even then $1-(-1)^x = 1-1 = 0$

Required set= $\{0, 2\}$

Q.4) $\{n(n+1)/2 : n$ is a positive integer$\}$ is

a) a finite set **b)** a infinite set **c)** is an empty set **d)** None of these

Sol. (b)

$\left\{\frac{n(n+1)}{2} : n \text{ is a positive integer}\right\}$

$= \{1,2,3,\ldots.\} = N$

Q.5) If E = {1, 2, 3, 4, 5, 6, 7, 8, 9}, the subset of E satisfying $5 + x > 10$ is

a) $\{5, 6, 7, 8, 9\}$ **b)** $\{6, 7, 8, 9\}$ **c)** $\{7, 8, 9\}$ **d)** None of these

Sol. (b)

$5 + x > 10 \Rightarrow x > 10 - 5 \Rightarrow x > 5$

$\therefore$ Required set = $\{6,7,8,9\}$

Q.6) $\{(x, y), y = x^2\}$ where x, y c R is

a) not a function **b)** a function **c)** inverse mapping **d)** None of these

Sol. (b)

let $f(x) = y = x^2$

$= \{(x, y)\, y = x^2\}$

$= \{(1,1),(2,4),(3,9),(-1,1), \ldots\ldots\}$

$\therefore$ It is not a one-one function

$\therefore$ It is many-one function

Q.7) The domain and range of $\{(x, y) : y = x^2\}$ where x, y c R is

a) (reals, natural numbers) **b)** (reals, non-negative reals)

c) (reals, reals) **d)** None of these

Sol. (b)

Domain = R and range = $R^+ \,\&\, \{0\}$

Q.8) If $f(x) = 1/1-x$ *and* $g(x) = (x-1)/x$, then $fog(x)$ is
a) x **b)** $1/x$ **c)** $-x$ **d)** None of these

Sol. (a)

$f(x) = \frac{1}{1-x}$, $g(x) = \frac{x-1}{x}$

$\therefore fog(x) = f\{g(x)\} = f\left(\frac{x-1}{x}\right)$

$= \frac{1}{1-\frac{x-1}{x}} = \frac{1}{\frac{x-x+1}{x}} = \frac{x}{1} = x$

Q.9) The Inverse function f^{-1} if $f(x) = 2x$ is
a) $1/2\,x$ **b)** $\frac{x}{2}$ **c)** $1/x$ **d)** None of these

Sol. (b)

$\because f(x) = 2x \;\therefore\; let\; y = f(x) = 2x$

$\Rightarrow \quad x = \frac{y}{2} \Rightarrow f^{-1}(x) = \frac{x}{2}$

Q.10) The Inverse h^{-1} *when* $h(x) = log_{10}x$ is
a) $log_{10}x$ **b)** 10^x **c)** $\log_{10} 1/x$ **d)** None of these

Sol. (b)

let $y = h(x) = log_{10}x$

$\Rightarrow$ y= $log_{10}x$

$\Rightarrow 2^3 = 8$

$\Rightarrow \log 2^8 = 3$

$\Rightarrow 10^y = x \Rightarrow 10^x = f^{-1}(x)$

Q.11) If A has 32 elements, B has 42 elements and A∪B has 62 elements, the number of elements in A∪B is
a) 12 **b)** 74 **c)** 10 **d)** None of these

Sol. (a)

$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow 62 = 32 + 42 - n(A \cup B)$

$\Rightarrow n(A \cup B) = 74 - 62 = 12$

Q.12) If F : A → R is real valued function defined by $f(x) = \frac{1}{x}$ *then*
a) R **b)** R – {1} **c)** R – {0} **d)** R - N

Sol. (c)

$f(x) = \frac{1}{x}$ is defined to all $x \in$ R except $x = 0$

$\therefore A = R - \{0\}$

Q.13) The number of proper sub-set of the set {3, 4, 5, 6, 7} is
a) 32 **b)** 31 **c)** 30 **d)** 25

Sol. (b)

Number of proper sub – sets = $2^n - 1$

$= 2^5 - 1 = 31$

Q.14) If R is the set of real numbers such that the function f : R → R is defined by $f\ (x) = \ (x+1)^2$, then find (fof)

a) $(x+1)^2+1$ **b)** x^2+1 **c)** $\{(x+1)^2+1\}^2$ **d)** None of these

Sol. (c)

$\because f(x)(x+1)^2$

$fof = f\{f(x)\} = f\{(x+1)^2\}$

$= \{(x+1)^2+1\}^2$

Q.15) In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is

a) 6 **b)** 7 **c)** 1 **d)** None of these

Sol. (b)

T⟶ Tea, C ⟶ Coffee

$n(T \cup C) = 20, n(T - C) = 8, n(T) = 13$

$\Rightarrow 20 = 13 + n(C - T)$

$\Rightarrow n(C - T) = 20 - 13 = 7$

Q.16) The sets V = $\{x : \ x+2=0\}$, R=$\{x : \ x^2+2x=0\}$ and S = $\{x : \ x^2 \ + \ x - \ 2 \ = \ 0\}$ are equal to one another if x is equal to

a) –2 **b)** 2 **c)** ½ **d)** None of these

Sol. (a)

$V = \{-2\}, R = \{-2, 0\}$

$S = \{-2, 1\}$

Q.17) If the set P has 3 elements, Q four and R two then the set P × Q × R contains

a) 9 elements **b)** 20 elements **c)** 24 elements **d)** None of these

Sol. (c)

$n(P) = 3,\ n(Q) = 4,\ n(R) = 2$

$\therefore n(P \times Q \times R) = n(P) \times n(Q) \times n(R)$

=3×4×2=24

Q.18) A town has a total population of 50,000. Out of it 28,000 read the newspaper x and 23,000 read y while 4,000 read both the papers. The number of persons not reading x and y both is

a) 2,000 **b)** 3,000 **c)** 2,500 **d)** None of these

Sol. (b)

$n(U) = 52000$

$n(x)$=28000, $n(y)$=23000, $n(x \cap y) = 4000$

$\therefore\ n(x \cup y) = 28000 + 23000 - 4000 = 47000$

$n(x \cup y)' = 50000 - 47000 = 3000$

Q.19) At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreigndoctors. The number of women doctors attending the conference is

a) 2 **b)** 4 **c)** 1 **d)** None of these

Sol. (c)

M ⟶ Men

W ⟶ Women

D ⟶ Doctor

$n(M) = 23, \quad n(W) = 29, \quad n(D) = 4$

$n(M \cup D) = 24$

$\Rightarrow n(M) + n(D) - n(M \cap D) = 24$

$\Rightarrow 23 + 4 - n(M \cap D) = 24 \Rightarrow n(M \cap D) = 27 - 24 = 3$

$n(M \cap D) + n(W \cap D) = 4 \Rightarrow 3 + n(W \cap D) = 4$

$\Rightarrow n(W \cap D) = 1$

Q.20) Out of 2000 employees in an office 48% preferred Coffee (C), 54% liked Tea (T), 64% used to Smoke (S). Out of the total 28% used C and T, 32% used T and S and 30% preferred C and S, only 6% did none of these. The number having all the three is

a) 360 **b)** 300 **c)** 380 **d)** None of these

Sol. (a)

$a + b + c + d = 48\%$

$b + c + f + g = 54\%$

$c + d + e + f = 64\%$

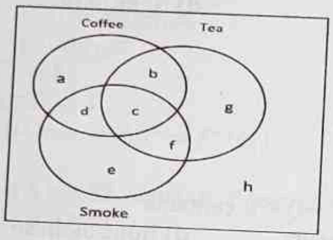
$b + c = 28\%$

$c + d = 30\%$

$h = 6\%$

$a + b + c + d + e + f + g + h = 100\%$

$a + b + c + d + e + f + g = 94\%$



$\Rightarrow 48\% + 54\% + 64\% - 28\% - 32\% - 30\% + c = 94\%$

$\Rightarrow 166\% - 90\% + c = 94\%$

$\Rightarrow 76\% + c = 94\% \Rightarrow c = 18\%$

∴ Required value = 18% 2000 = 360

Q.21) If $f(x) = 1/1-x$, then $f^{-1}(x)$ is

a) $1-x$ **b)** $(x-1)/x$ **c)** $x/(x-1)$ **d)** None of these

Sol. (b)

let $y = f(x) = \frac{1}{1-x}$

$\Rightarrow 1 - x = \frac{1}{y} \Rightarrow x = 1 - \frac{1}{y}$

$\Rightarrow f^{-1}(x) = \frac{x-1}{x}$

Q.22) If V={0, 1, 2, ...9}, X={0, 2, 4, 6, 8}, Y={3, 5, 7} and Z={3 7} then Y ∪ Z, (V ∪ Y) ∩ X, (X ∪ Z) ∪ V are respectively: –

a) {3, 5, 7}, {0, 2, 4, 6, 8}, {0, 1, 2, ...9}

b) {2, 4, 6}, {0, 2, 4, 6, 8}, {0, 1, 2, ...9}

c) {2, 4, 6}, {0, 1, 2, ...9}, {0, 2, 4, 6, 8}

d) None

Sol. (a)

Y ∪ Z = {3,5,7}, (V ∪ Y) ∩ X = {0,2,4,6,8}, (X ∪ Z) ∪ V= {0, 1, 2, ...9}

Q.23) What is the relationship between the following sets? A={x: x is a letter in the word *flower*} B={ x: x is a letter in the word *flow*} C={ x: x is a letter in the word *wolf*} D={ x: x is a letter in theword *follow*}

a) B=C=D and all these are subsets of the set A
b) B=C≠D
c) B≠C≠D
d) None

Sol. (a)

$= \{f, l, o, w, e, r\}$
$B = \{f, l, o, w\}, c = \{w, o, l, f\}$
$D = \{f, o, l, w\}$

Q.24) If A={a, b, c}, B={a, b}, C = {a, b, d}, D={c, d} and E={d} state which of the following statements are correct: – (i) B ⊂ A (ii) D ≠ C (iii) C ⊃ E (iv) D ⊂ E (v) D⊂B (vi) D = A (vii) B ⊄ C (viii) E ⊂ A (ix) E ≠B (x) a ∈ A (xi) a ⊂ A (xii) {a} ∈ A (xiii) {a} ⊂ A

a) (i) (ii) (iii) (ix) (x) (xiii) only are correct
b) (ii) (iii) (iv) (x) (xii) (xiii) only are correct
b) (i) (ii) (iv) (ix) (xi) (xiii) only are correct
d) None

Sol. (a)

Correct → i), ii), iii), ix), x), xiii)
Incorrect → iv), v), vi), vii), viii), xi), xii)

Q.25) If A = {0, 1} state which of the following statements are true: – (i) {1} ⊂ A (ii) {1} ∈ A (iii) ϕ ∈ A (iv) 0 ∈ A (v) 1⊂ A (vi) {0} ∈ A (vii) ϕ ⊂ A

a) (i) (iv) and (vii) only are true
b) (i) (iv) and (vi) only are true
b) (ii) (iii) and (vi) only are true
d) None

Sol. (a)

True → i), iv), vii)
False → ii), iii), v), vi)

Q.26) If A = {1, 2, 3, 4} B = {2, 3, 7, 9} and C = {1, 4, 7, 9} then

a) A ∩ B ≠ ϕ B ∩ C ≠ ϕ A ∩ C ≠ ϕ but A ∩ B ∩ C = ϕ
b) A ∩ B = ϕ B ∩ C = ϕ A ∩ C = ϕ A ∩ B ∩ C = ϕ
b) A ∩ B ≠ ϕ B ∩ C ≠ ϕ A ∩ C ≠ ϕ A ∩ B ∩ C ≠ ϕ
d) None

Sol. (a)

$$\begin{bmatrix} \because A \cap B = \{2,3\}, \; B \cap C = \{7,9\} \\ A \cap C = \{1,4\}, \; A \cap B \cap C = \emptyset \end{bmatrix}$$

Q.27) A sample of income group of 1172 families was surveyed and noticed that for income groups < ₹6000/-, ₹6000/- to ₹10999/-, ₹11000/-, to ₹15999/-, ₹16000 and above No. TV set is available to 70, 50, 20, 50 families one set is available to 152, 308, 114, 46 families and two or more sets are available to 10, 174, 84, 94 families.

If A = {$x|x$ is a family owning two or more sets}, B = { $x|x$ is a family with one set, }C = { $x|x$ is a family with income less than ₹6000/-}, D = { $x|x$ is a family with income ₹6000/- to ₹ 10999/-}, E = { $x|x$ is a family with income ₹ 11000/- to ₹15999/-}, find the number of families in each of the following sets (i) C ∩ B (ii) A ∪ E

a) 152, 580
b) 152, 20
c) 152, 50
d) None of these

Sol. (d)

	Income	**C → 6,000**	**D → 6000-10999**	**E → 11000-15999**	**>16000**
	0	70	50	20	50
B →	1	152	308	114	46
A →	>2	10	174	84	94

i) $C \cap B = 152$ ii) $A\,U\,E = (10 + 174 + 84 + 94 + (20 + 114 + 84) - 84 = 496$

Q.28) If A = {a, b, c, d} list the element of power set P (A)

a) {ϕ {a} {b}{c} {d} {a, b} {a, c} {a, d} {b, c} {b, d} {c, d}

b) {a, b, c} {a, b, d} {a, c, d} {b, c, d}

c) {a, b, c, d}

d) All the above elements are in P (a)

Sol. (d)

$A = \{a, b, c, d\}$

$\therefore P\ |A|\{\emptyset, \{a\}, \{b\}, \{C\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}\ \}$

$\{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}$

$\{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}$

Q.29) Identify the elements of P if set Q = {1, 2, 3} and P × Q = {(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)}

a) {3, 4, 5} **b)** {4, 5, 6} **c)** {5, 6, 7} **d)** None

Sol. (b)

P = {4, 5, 6}

Q.30) If A = {2, 3}, B = {4, 5}, C = {5, 6} then A × (B ∪ C) is

a) {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

b) {(2, 5), (3, 5)}

c) {(2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (4, 6), (5, 5), (5, 6)}

d) None

Sol. (a)

$A \times (B\ U\ C) = \{(2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Q.31) After qualifying out of 400 professionals, 112 joined industry, 120 started practice and 160 joined as paid assistants. There were 32, who were in both practice and service 40 in both practice and assistantship and 20 in both industry and assistantship. There were 12 who did all the three. Find how many could not get any of these.

a) 88 **b)** 244 **c)** 122 **d)** None of these

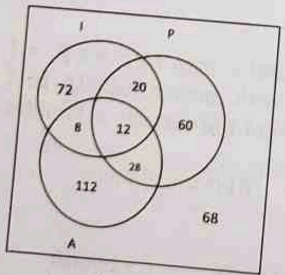
Sol. (a)

$I \rightarrow Industry$

$P \rightarrow Practice$

$A \rightarrow Assistants$

$n(P \cap A)$



$n = (I \cup P \cup A) = 112 + 120 + 160 - 32 - 40 - 20 + 12 = 312$

$\therefore n(I \cup P \cup U)' = n(t) - n(I \cup P \cup A) = 400 - 312 = 88$

Q.32) A marketing research team interviews 50 people about their drinking habits of tea coffee or milk or A B C respectively. Following data is obtained but the Manager is not sure whether these are consistent.

Category	No.	Category	No.
ABC	3	A	42
AB	7	B	17
BC	13	C	27
AC	18		

a) Inconsistent since $42 + 17 + 27 - 7 - 13 - 18 + 3 \neq 50$

b) Consistent

c) Cannot determine due to data insufficiency

d) None

Sol. (a)

$$n = (A \cup B \cup C) = 42 + 17 + 27 - 7 - 13 - 18 + 3 = 51 > 50$$

Q.33) Out of 60 students 25 failed in paper (1), 24 in paper (2), 32 in paper (3), 9 in paper (1) alone, 6 in paper (2) alone, 5 in papers (2) and (3) only and 3 in papers (1) and (2) only Find how many failed in all the three papers.

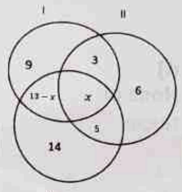
a) 10 **b)** 60 **c)** 50 **d)** None

Sol. (a)

Let n (all three) $= x$

$n(II) = 24 \Rightarrow 3 + x + 5 + 6 = 24$

$\Rightarrow x = 24 - 14 = 10$



Differentiation and Integration

Q.1) The differential coefficients of $(x^2+1)/x$ is

a) $1 + 1/x^2$ **b)** $1 - 1/x^2$ **c)** $1/x^2$ **d)** None of these

Sol. (b)

$$\frac{d}{dx}\left(\frac{x^2+1}{x}\right) = \frac{d}{dx}\left(x+\frac{1}{x}\right) = 1 - \frac{1}{x^2}$$

Q.2) $y = \sqrt{2x} + 3^{2x}$ then $\frac{dy}{dx}$ is equal to

a) $(1/\sqrt{2x}) + 2.3^{2x}\ \log_e 3$ **b)** $1/\sqrt{2x}$

c) $2.3^{2x}\ \log_e 3$ **d)** None of these

Sol. (a)

$$y = \sqrt{2x} + 3^{2x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{2x}} \times \cancel{2} + 3^{2x}\,(\log_e 3) \times 2$$

$$= \frac{1}{\sqrt{2x}} + 2 \times 3^{2x} \log_e 3$$

Q.3) If $f(x) = e^{ax^2+bx+c}$ the $f'(x)$ is

a) e^{ax^2+bx+c} **b)** $e^{ax^2+bx+c} \times (2ax+b)$ **c)** $2ax + b$ **d)** None of these

Sol. (b)

$$f(x) = e^{ax^2+bx+c}$$

$$\therefore f'(x) = \frac{d}{dx} e^{(ax^2+bx+c)}$$

$$= e^{ax^2+bx+c} \times (2ax+b)$$

Q.4) If $y = \sqrt{x^2+m^2}$ then $y\,y_1$ (where $y_1 = dy/dx$) is equal to

a) $-x$ **b)** x **c)** $1/x$ **d)** None of these

Sol. (b)

$$y = \sqrt{x^2+m^2}$$

$$\therefore \frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+m^2}} = \frac{x}{y}$$

$$\Rightarrow y_1 = \frac{x}{y} \Rightarrow y_1 y = x$$

Q.5) If $y = \frac{(2x+1)(3x+1)}{4x+1}$ then $\frac{dy}{dx}$ is

a) $\frac{24x^2+12x+1}{(4x+1)^2}$ **b)** $\frac{24x^2+12x+5}{(4x+1)^2}$ **c)** $\frac{24x^2+12x}{(4x+1)^2}$ **d)** $\frac{24x^2+12x+9}{(4x+1)^2}$

Sol. (a)

$$\Rightarrow y = \frac{(2x+1)(3x+1)}{4x+1} = \frac{6x^2+5x+1}{4x+1}$$

Apply quotient Rule

$$\Rightarrow \frac{dy}{dx} = \frac{(4x+1)\frac{d}{dx}(6x^2+5x+1) - (6x^2+5x+1)\frac{d}{dx}(4x+1)}{(4x+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(12x+5)(4x+1) - (6x^2+5x+1)4}{(4x+1)^2}$$

$$=\frac{48x^2+32x+5-24x^2-20x-4}{(4x+1)^2}$$
$$=\frac{24x^2+12x+1}{(4x+1)^2}$$

Q.6) If $f(x)=\frac{4-2x}{2+3x+3x^2}$ then the values of x for which $f'(x)=0$ is

a) $2\left(1\pm\sqrt{\frac{5}{3}}\right)$ **b)** $(1\pm\sqrt{3})$ **c)** $\frac{2}{3}(3\pm\sqrt{15})$ **d)** None of these

Sol. (c)

$$f(x)=\frac{4-2x}{2+3x+3x^2}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\frac{d}{dx}(f(x))-f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$
$$\therefore f'(x)=\frac{(2+3x+3x^2)\times(-2)-(4-2x)\times(3+6x)}{(2+3x+3x^2)^2}$$
$$f'(x)=0$$
$$\Rightarrow\frac{-4-6x-6x^2-12-18x+12x^2}{(2+3x+3x^2)^2}=0$$
$$\Rightarrow 6x^2-24x-16=0$$
$$\Rightarrow 3x^2-12x-8=0$$
$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\Rightarrow x=\frac{12\pm\sqrt{144+96}}{2\times3}$$
$$=\frac{12\pm\sqrt{240}}{6}$$
$$=\frac{12\pm4\sqrt{15}}{6}$$
$$=\frac{6\pm2\sqrt{15}}{3}=\frac{2}{3}(3\pm\sqrt{15})$$

Q.7) If $=\frac{e^{3x}-e^{2x}}{e^{3x}+e^{2x}},$ *then* $\frac{dy}{dx}$ *is equal to*

a) $2e^{5x}$ **b)** $\frac{2e^x}{(e^x+1)^2}$ **c)** $e^{5x}(e^{5x}+e^{2x})$ **d)** None of these

Sol. (b)

$$y=\frac{e^{3x}-e^{2x}}{e^{3x}+e^{2x}}\Rightarrow\frac{e^{2x}(e^x-1)}{e^{2x}(e^x+1)}\Rightarrow\frac{e^x-1}{e^x+1}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\frac{d}{dx}(f(x))-f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$
$$\Rightarrow\frac{dy}{dx}=\frac{(e^x+1)e^x-(e^x-1)\times e^x}{(e^x+1)^2}$$
$$\Rightarrow\frac{e^{2x}+e^x-e^{2x}+e^x}{(e^x+1)^2}=\frac{2e^x}{(e^x+1)^2}$$

Q.8) If $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1, \frac{dy}{dx}$ can be expressed as

a) $\frac{x}{y}$ **b)** $\frac{x}{x^2-a^2}$ **c)** $\frac{1}{\sqrt{\frac{x^2}{a^2}-1}}$ **d)** None of these

Sol. (a)

$\frac{d}{dx}\left(\frac{x^2}{a^2}\right) - \frac{d}{dx}\left(\frac{y^2}{a^2}\right) = \frac{d}{dx} 1$ (Diff. both sides w. r. t. x)

$\therefore \frac{2x}{a^2} - \frac{2y}{a^2}\frac{dy}{dx} = 0$

$\Rightarrow \frac{2y}{a^2}\frac{dy}{dx} = \frac{2x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$

Q.9) If $xy = 1$ then $y^2 + \frac{dy}{dx}$ is equal to

a) 1 **b)** 0 **c)** -1 **d)** None of these

Sol. (b)

$xy = 1$ (Diff. both sides w. r. to x)

Using product rule $\frac{d}{dx}(u \times v) = \frac{d}{dx}(u) \times v + \frac{d}{dx}(v) \times u$

$y + x\frac{dy}{dx} = 0$ (Multiplied by y both sides)

$\therefore y^2 + xy\frac{dy}{dx} = 0$

$\Rightarrow y^2 + 1 \times \frac{dy}{dx} = 0 \Rightarrow y^2 + \frac{dy}{dx} = 0$

Q.10) Find $\frac{dy}{dx}$ when $x^3 + y^3 = xy$

a) $\frac{y-3x^2}{3x}$ **b)** $\frac{y-3x^2}{3y^2-x}$ **c)** $\frac{y-3x^2}{3y^2}$ **d)** None

Sol. (b)

$\Rightarrow \frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}xy$

$= 3x^2 + 3y^2y' = y + xy'$

$= 3y^2y' - xy' = y - 3x^2$

$= (3y^2 - x)y' = y - 3x^2$

$= \frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$

Q.11) Given $x = 2t + 5, y = t^2 - 2; \frac{dy}{dx}$ is calculated as

a) t **b)** -1/t **c)** 1/t **d)** None of these

Sol. (a)

$x = 2t + 5, \; y = t^2 - 2$

$\frac{dx}{dt} = 2, \frac{dy}{dt} = 2t$ (Diff. both sides w. r. to t)

$\therefore \frac{dy}{dx} = \frac{2t}{2} = t$

Q.12) Given $x = t + t^{-1}$ and $y = t - t^{-1}$ the value of $\frac{dy}{dx}$ $at\ t = 2$ is

a) 3/5
b) -3/5
c) 5/3
d) None of these

Sol. (c)

$x = t + t^{-1}$ and $y = t - t^{-1}$

Diff. both sides w. r. to t

$\therefore \frac{dx}{dt} = 1 - \frac{1}{t^2} \& \frac{dy}{dt} = 1 + \frac{1}{t^2}$

$\therefore \frac{dy}{dx} = \frac{t^2+1}{t^2} \times \frac{t^2}{t^2-1} = \frac{t^2+1}{t^2-1}$

$\therefore \left[\frac{dy}{dx}\right]_{t=2} = \frac{(2)^2+1}{(2)^2-1}$

$= \frac{4+1}{4-1} = \frac{5}{3}$

Q.13) $x = \frac{3at}{1+t^3}, y = \frac{3at^2}{1+t^3}$ then $\frac{dy}{dx}$ is

a) $\frac{t(2-t^3)}{-1+t^3}$
b) $\frac{t(2-t^3)}{1-2t^3}$
c) $\frac{t(2+t^3)}{1+2t^3}$
d) None

Sol. (b)

$\Rightarrow \frac{d}{dt}x = \frac{d}{dt}\frac{3at}{1+t^3}$ (Diff. both sides w. r. to t)

Apply quotient Rule

$$\Rightarrow \frac{dx}{dt} = \frac{3a(1+t^3) - 9at^3}{(1+t^3)^2}$$

$$\Rightarrow \frac{d}{dt}(y) = \frac{d}{dt}\frac{3at^2}{1+t^3}$$

$$\Rightarrow \frac{dy}{dt} = \frac{6at(1+t^3) - 9at^4}{(1+t^3)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at(1+t^3) - 9at^4}{3a(1+t^3) - 9at^3}$$

$$\Rightarrow \frac{3at(2+2t^3-3t^3)}{3a(1+t^3-3t^3)} = \frac{t(2-t^3)}{1-2t^3}$$

Q.14) If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is

a) $\frac{x-y}{x(logx+1)}$
b) $\frac{y}{xlogx}$
c) $\frac{x}{xlogx}$
d) $\frac{x-y}{xlogx}$

Sol. (a)

$x^y = e^{x-y}$ (Taking log both sides)

$\Rightarrow log\ x^y = log\ e^{x-y}$

$\Rightarrow ylogx = (x-y)\log e$

$= y\ logx = x - y$

Differentiation both sides w. r. t. x

$$= \frac{d}{dx}(y\ logx) = \frac{d}{dx}x - \frac{d}{dx}y$$

$$= logx\frac{dy}{dx} + \frac{y}{x} = 1 - \frac{dy}{dx}$$

$$= \frac{dy}{dx}(\log x + 1) = 1 - \frac{y}{x}$$

$= \frac{dy}{dx} = \frac{x-y}{x(logx+1)}$

Q15) If $y = x^{1+\frac{1}{x}}$, *then* find $\frac{dy}{dx}$

a) $1 - \frac{1}{x}(\frac{x+1-logx}{x^2})$ **b)** $1 + \frac{1}{x}(\frac{x-1-logx}{x^2})$ **c)** $x^{1+\frac{1}{x}}(\frac{x+1-logx}{x^2})$ **d)** None

Sol. (c)

$\Rightarrow y = x^{1+\frac{1}{x}}$

(Taking log both sides)

$\Rightarrow \log y = \log x^{1+\frac{1}{x}}$

$\Rightarrow \log y = \left(1 + \frac{1}{x}\right) \log x$

Differentiation both sides w. r. t. x

$\Rightarrow \frac{d}{dx}(\log y) = \frac{d}{dx}\left(1 + \frac{1}{x}\right) \log x$

Using product rule $\frac{d}{dx}(u \times v) = \frac{d}{dx}(u) \times v + \frac{d}{dx}(v) \times u$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\left(1 + \frac{1}{x}\right) \times \log x + \frac{d}{dx} \log x \times \left(1 + \frac{1}{x}\right)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (\frac{-1}{x^2}) \times \log x + \frac{1}{x} \times \left(1 + \frac{1}{x}\right)$

$\Rightarrow \frac{dy}{dx} = y(\frac{x + 1 - logx}{x^2})$

$= x^{1+\frac{1}{x}}(\frac{x+1-logx}{x^2})$

Q.16) If $y = x^{\log(logx)}$ then $\frac{dy}{dx}$ is

a) $\frac{y}{x}[1 + \log(\log x)]$ **b)** $\frac{x}{y}(1 + \log x)$ **c)** $\frac{y}{x}(1 - \log x)$ **d)** None

Sol. (a)

$\Rightarrow y = x^{\log(logx)}$

Taking log both sides

$\Rightarrow \log y = logx^{\log(logx)}$

$= \log y = \log(logx) . \log x$

Differentiation both sides w. r. t. x

$= \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log(logx) \times \log x + \frac{d}{dx} logx \times \log(logx)$

$= \frac{1}{y}\frac{dy}{dx} = \frac{1}{xlogx} \times \log x + \frac{1}{x} \times \log(logx)$

$= \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{x} \times \log(logx)$

$= \frac{dy}{dx} = \frac{y}{x}[1 + \log(\log x)]$

Q.17) If $x^y = y^x$, *then* find $\frac{dy}{dx}$

a) $\frac{y(x\log y-y)}{x(y\log x-x)}$ **b)** $\frac{y(x\log y+y)}{x(y\log x-x)}$ **c)** $\frac{y(x\log y-y)}{x(y\log x+x)}$ **d)** $\frac{y(x\log y+y)}{x(y\log x+x)}$

Sol. (a)

$\Rightarrow x^y = y^x$

$\Rightarrow \log x^y = \log y^x$ (*Taking* log *both sides*)

$\Rightarrow y \log x = x \log y$

$\Rightarrow \frac{d}{dx}(y \log x) = \frac{d}{dx}(x \log y)$ \(Differentiation both sides w. r. t. x)

Using product rule $\frac{d}{dx}(u \times v) = \frac{d}{dx}(u) \times v + \frac{d}{dx}(v) \times u$

$\Rightarrow \frac{dy}{dx} \times (\log x) + \frac{d}{dx}(logx) \times y = \frac{d}{dx} x \times (\log y) + \frac{d}{dx}(logy) \times x$

$\Rightarrow \frac{dy}{dx} \times (\log x) + \frac{y}{x} = (logy) + \frac{x}{y}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} \times (\log x) - \frac{x}{y}\frac{dy}{dx} = \log y - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx}(\log x - \frac{x}{y}) = \log y - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$

$= \frac{y(x\log y - y)}{x(y\log x - x)}$

Q.18) If $y = x^3 logx, find\ the\ value\ f''(x)$

a) $xlogx + 5x$ **b)** $6xlogx + x$ **c)** $6xlogx + 5x$ **d)** $6xlogx$

Sol. (c)

$\Rightarrow y = x^3\ logx$

Differentiation both sides w. r. t. x

$\Rightarrow y' = 3x^2(logx) + x^2$

$\Rightarrow y' = x^2[3(logx) + 1]$

Second-order differentiation both sides w. r. t. x

$\Rightarrow y'' = 2x(3(logx) + 1) + 3 \times \frac{1}{x} \times x^2$

$= 6xlogx + 2x + 3x$

$= 6xlogx + 5x$

Q.19) If $y = x^m e^{nx}$ then $\frac{d^2y}{dx^2}$ *is*

a) $2nx^{m-1}e^{nx} + m(m-1)x^{m-2}e^{nx} - nmx^{m-1}e^{nx} + n^2\,x^{m-1}e^{nx}$

b) $2nx^{m-1}e^{nx} - m(m-1)x^{m-2}e^{nx} + nmx^{m-1}e^{nx} + n^2\,x^{m-1}e^{nx}$

c) $2nx^{m-1}e^{nx} + m(m-1)x^{m-2}e^{nx} + nmx^{m-1}e^{nx} + n^2\,x^{m-1}e^{nx}$

d) $m(m-1)x^{m-2}e^{nx} + 2nmx^{m-1}e^{nx} + n^2\,x^m e^{nx}$

Sol. (d)

$\Rightarrow y = x^m e^{nx}$

$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}x^m e^{nx}$

$\Rightarrow \frac{dy}{dx} = mx^{m-1}e^{nx} + nx^m e^{nx}$

$= (m + nx)\,x^{m-1}e^{nx}$

$\Rightarrow \frac{d^2y}{dx^2} = nx^{m-1}e^{nx} + (m-1)\,(m+nx)\,x^{m-2}e^{nx} + n(m+nx)\,x^{m-1}e^{nx}$

$\Rightarrow nx^{m-1}e^{nx} + m(m-1)x^{m-2}e^{nx} + nmx^{m-1}e^{nx} - nx^{m-1}e^{nx} + nmx^{m-1}e^{nx} + n^2x^m e^{nx}$

$= m(m-1)x^{m-2}e^{nx} + 2nmx^{m-1}e^{nx} + n^2\,x^m e^{nx}$

Q.20) If $e^y(x+1) = 1, then\ find\ y''$

a) y' **b)** 1 **c)** 0 **d)** $(y')^2$

Sol. (d)

$\Rightarrow e^y(x+1) = 1$

$\Rightarrow x + 1 = \frac{1}{e^y}$

$\Rightarrow e^{-y} = x + 1$ (Differentiation both sides w. r. t. x)

$\Rightarrow -e^{-y}y' = 1$

$\Rightarrow y' = -\frac{1}{e^{-y}} = -e^y$

Second-order differentiation both sides w. r. t. x

$\Rightarrow y'' = -e^y.y'$

$\Rightarrow y'' = (y')^2$

Q.21) If $x = \frac{1-t}{1+t}$ and $y = \frac{2t}{1+t}$ $then\ \frac{d^2y}{dx^2}\ is$

a) 1 **b)** 0 **c)** -1 **d)** 2

Sol. (b)

$\Rightarrow \frac{d}{dt}x = \frac{d}{dt}\frac{1-t}{1+t}$

Apply quotient Rule

$\Rightarrow \frac{dx}{dt} = \frac{-1(1+t)-1(1-t)}{(1+t)^2}$ (Differentiation both sides w. r. to t)

$= \frac{-1-t-1+t}{(1+t)^2} \Rightarrow \frac{-2}{(1+t)^2}$

$\Rightarrow \frac{d}{dt}y = \frac{d}{dt}\frac{2t}{1+t}$

$= \frac{2(1+t)-2t}{(1+t)^2} \Rightarrow \frac{2}{(1+t)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{2}{(1+t)^2}}{\frac{-2}{(1+t)^2}} = -1$

Second-order differentiation both sides w. r. t. x

$\Rightarrow \frac{d^2y}{dx^2} = 0$

Q.22) If $y = \left(\frac{1}{x}\right)^x$ then $\left(\frac{d^2y}{dx^2}\right)_{x=1}$ *is*

a) 1 **b)** -1 **c)** 0 **d)** -2

Sol. (c)

$y = \left(\frac{1}{x}\right)^x$

Taking log both sides

$= log y = log\left(\frac{1}{x}\right)^x$

$= log y = x \log\frac{1}{x} = -x \log x$

$= \frac{d}{dx} log y = -\frac{d}{dx}(x \log x)$

$= \frac{1}{y}\frac{dy}{dx} = -(\log x + 1)$

$= \frac{dy}{dx} = -y(\log x + 1)$

$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{dy}{dx}(\log x + 1) + \frac{y}{x}\right]$

put the value of $\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\left[-y(\log x + 1)(\log x + 1) + \frac{y}{x}\right]$

$\Rightarrow \frac{d^2y}{dx^2} = -y\left[\frac{1}{x} - (\log x + 1)^2\right]$

$\left(\frac{d^2y}{dx^2}\right)_{x=1} = -y\left[\frac{1}{1} - (\log(1) + 1)^2\right] \Rightarrow -y(1-1) = 0$

Q.23) If $y = ae^{2x} + bxe^{2x}$ where a and b are constants, the value of the expression $\frac{d^2y}{dx^2} - 4\frac{dy}{dx}$ is

a) 0 **b)** $e^{2x}(a + bx)$ **c)** -1 **d)** $-4e^{2x}(a + bx)$

Sol. (d)

$\Rightarrow y = ae^{2x} + bxe^{2x}$

$\Rightarrow \frac{d}{dx}y = a\frac{d}{dx}e^{2x} + b\frac{d}{dx}xe^{2x}$

$$\Rightarrow \frac{dy}{dx} = 2ae^{2x} + b(e^{2x} + 2xe^{2x})$$

$$\Rightarrow \frac{d^2y}{dx^2} = 4ae^{2x} + b(2e^{2x} + 2e^{2x} + 4xe^{2x})$$

$$\Rightarrow \frac{d^2y}{dx^2} - 4\frac{dy}{dx} \qquad \textit{put the value of } \frac{dy}{dx} \textit{ and } \frac{d^2y}{dx^2}$$

$$= 4ae^{2x} + b(4e^{2x} + 4xe^{2x}) - 4(2ae^{2x} + b(e^{2x} + 2xe^{2x}))$$

$$= 4ae^{2x} + 4be^{2x} + 4bxe^{2x} - 8ae^{2x} - 4be^{2x} - 8bxe^{2x}$$

$$= -4ae^{2x} - 4bxe^{2x} \Rightarrow -4e^{2x}(a + bx)$$

Q.24) If $y = \log[x + \sqrt{1 + x^2}]$the value of the expression $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

a) 0 **b)** 1 **c)** 2 **d)** -x

Sol. (a)

$$\Rightarrow y = \log\left[x + \sqrt{1 + x^2}\right]$$

$$\Rightarrow \frac{d}{dx}y = \frac{d}{dx}\log\left[x + \sqrt{1 + x^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{1 + x^2}} \times \left(1 + \frac{2x}{2\sqrt{1 + x^2}}\right)$$

$$= \frac{1}{x + \sqrt{1 + x^2}} \times \left(\frac{2(\sqrt{1 + x^2} + x)}{2\sqrt{1 + x^2}}\right) = \frac{1}{x + \sqrt{1 + x^2}} \times \left(\frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(1 + x^2)^{\frac{-1}{2}} = \frac{-2x}{2(1 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -x(1 + x^2)^{\frac{-3}{2}}$$

$$\Rightarrow (x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$$

Put the value of $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$

$$\Rightarrow -(x^2 + 1)x(1 + x^2)^{\frac{-3}{2}} + x\frac{1}{\sqrt{1 + x^2}}$$

$$\Rightarrow \frac{-1}{\sqrt{1 + x^2}} + \frac{1}{\sqrt{1 + x^2}} = 0$$

Q.25) If $y = \sqrt{logx + \sqrt{\log x + \sqrt{\log x + \cdots \ldots \ldots \ldots \ldots + \infty}}}$ find $(2y - 1)\frac{dy}{dx}$

a) y **b)** x **c)** $\frac{1}{x}$ **d)** $\frac{1}{y}$

Sol. (c)

$$\Rightarrow y = \sqrt{logx + \sqrt{\log x + \sqrt{\log x + \cdots \ldots \ldots \ldots \ldots + \infty}}}$$

$$\Rightarrow y = \sqrt{\log x + y}$$

Square both sides

$$\Rightarrow y^2 = \log x + y$$

$$\Rightarrow \frac{d}{dx}y^2 = \frac{d}{dx}\log x + \frac{d}{dx}y$$

$$= 2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\Rightarrow (2y - 1)\frac{dy}{dx} = \frac{1}{x}$$

Q.26) Find the fourth derivative of $\log(3x+4)^{1/2}$

a) $-243(3x+4)^{-4}$ **b)** $243(3x+4)^{-4}$ **c)** $27(3x+4)^{-4}$ **d)** None

Sol. (a)

$$\Rightarrow f(x) = \log(3x+4)^{\frac{1}{2}}$$

$$\Rightarrow f'(x) = \frac{1}{(3x+4)^{\frac{1}{2}}} \times \frac{3}{2\sqrt{3x+4}} = \frac{3}{2(3x+4)} = \frac{3}{2}(3x+4)^{-1}$$

$$\Rightarrow f''(x) = \frac{3}{2}(-3)(3x+4)^{-2} = \frac{-9}{2}(3x+4)^{-2}$$

$$\Rightarrow f'''(x) = 27(3x+4)^{-3}$$

$$\Rightarrow f''''(x) = -243(3x+4)^{-4}$$

Q.27) The slope of the tangent to the curve $y = x^2 - x$ at the point, where the line y=2 cuts thecurve in the first quadrant is

a) 2 **b)** 3 **c)** -3 **d)** None of these

Sol. (b)

$y = x^2 - x$

$\frac{dy}{dx} = 2x - 1$

When $y = 2$ then $x^2 - x = 2$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2$ *or* $x = -1$

$\therefore$ Required point is $(2, 2)$

$\therefore \left[\frac{dy}{dx}\right]_{x=2} = 2 \times 2 - 1 = 3$

Q.28) The gradient of the curve $y - xy + 2px + 3qy = 0$ at the point $(3, 2)$ is $\frac{-2}{3}$. The values of p and q are

a) $(1/2, 1/2)$ **b)** $(2, 2)$ **c)** $(-1/2, -1/2)$ **d)** $(1/2, 1/6)$

Sol. (d)

$y - xy + 2px + 3qy = 0$

Diff. both sides w. r. t. x

$$\frac{dy}{dx} - \left(y \times 1 + x\frac{dy}{dx}\right) + 2p + 3q\frac{dy}{dx} = 0$$

$\Rightarrow (1 - x + 3q)\frac{dy}{dx} = y - 2p$

$\Rightarrow \frac{dy}{dx} = \frac{y-2p}{1-x+3q}$

$\because \left[\frac{dy}{dx}\right]_{x=3,y=2} = \frac{-2}{3}$

$\Rightarrow \frac{2-2P}{1-3+3q} = \frac{-2}{3}$

$\Rightarrow \frac{2(1-p)}{-2+3q} = \frac{-2}{3}$

$\Rightarrow 3 - 3p = 2 - 3q$

$\Rightarrow 3p - 3q = 1$_____(I)

Also $(3, 2)$ lies on the curve

$\therefore\ 2 - 6 + 6p + 6q = 0$

$\Rightarrow 6p + 6q = 4$_____(II)

From [(I) ×2 + (II)]

$6p - \cancel{6q} = 2$

$6p + \cancel{6q} = 4$

$= 12p\ = 6 \Rightarrow p = 1/2$

$\therefore q = \frac{\left(\frac{3}{2}-1\right)}{3} = 1/6$

Q.29) For the curve $x^2 + y^2 + 2gx + 2hy = 0$, the value of $\frac{dy}{dx}$ at (0,0) is

a) g/h **b)** -g/h **c)** 1 **d)** -h/g

Sol. (b)

$x^2 + y^2 + 2gx + 2hy = 0$

Differentiation w. r. t. x

$\Rightarrow \frac{d}{dx}(x^2 + y^2 + 2gx + 2hy = 0)$

$= 2x + 2y\frac{dy}{dx} + 2g + 2h\frac{dy}{dx} = 0$

Put the value of $x = 0$ and $y = 0$

$= 2g + 2h\frac{dy}{dx} = 0$

$= 2h\frac{dy}{dx} = -2g$

$= \frac{dy}{dx} = -g/h$

Q.30) The curve $y^2 = ux^3 + v$ passes through point P (2,3) and $\frac{dy}{dx} = 4$ at P. The value of u and v are

a) u=2, v= 7 **b)** u=2, v= -7 **c)** u=-2, v= -7 **d)** u=0, v= -1

Sol. (b)

$y^2 = ux^3 + v$..............................1

$\Rightarrow \frac{d}{dx}y^2 = u\frac{d}{dx}x^3 + \frac{d}{dx}v$

$\Rightarrow 2y\frac{dy}{dx} = 3ux^2 + 0$

Put the value of $\frac{dy}{dx} = 4, x = 2$ and $y = 3$

$\Rightarrow 2(3)4 = 3u(2)^2$

$\Rightarrow u = 2$

$u = 2, x = 2$ and $y = 3$ put in equation 1

$\Rightarrow 3^2 = 2 \times 2^3 + v$

$\Rightarrow v = -7$

Q.31) Find the point at which the tangent to the curve $y = \sqrt{4x-3}-1$ has its slope $\frac{2}{3}$.

a) (2,3) **b)** (3,2) **c)** (3, -2) **d)** (-3, -2)

Sol. (b)

$\Rightarrow y = \sqrt{4x-3}-1$

$\Rightarrow \frac{dy}{dx} = \frac{4}{2\sqrt{4x-3}}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{4x-3}} = \frac{2}{3}$

$= \sqrt{4x-3} = 3$

$= 4x - 3 = 9$

$= x = 3$

$\Rightarrow y = \sqrt{4x-3}-1$ put $x = 3$

$= y = 2$ and -4

Q.32) Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x -axis.

a) (3, -20) **b)** (-1, 12) **c)** Both **d)** None

Sol. (c)

$\Rightarrow y = x^3 - 3x^2 - 9x + 7$

$\Rightarrow \frac{d}{dx}y = \frac{d}{dx}x^3 - 3\frac{d}{dx}x^2 - 9\frac{d}{dx}x + \frac{d}{dx}7$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 9$

Given the slope of the curve parallel to the x -axis

$\Rightarrow \frac{dy}{dx} = 0$

$= 3x^2 - 6x - 9 = 0$

$= 3x^2 - 9x + 3x - 9 = 0$

$= 3x(x - 3) + 3(x - 3) = 0$

$= 3(x - 3)(x + 1) = 0$

$\Rightarrow x = 3, -1$

$\Rightarrow y = x^3 - 3x^2 - 9x + 7$ put $x = 3$

$\Rightarrow y = 3^3 - 3(3)^2 - 9 \times 3 + 7$

$\Rightarrow y = -20$

$\Rightarrow y = x^3 - 3x^2 - 9x + 7$ put $x = -1$

$\Rightarrow y = 1^3 - 3(1)^2 - 9(1) + 7$

$= y = 12$

Q.33) The slope of the tangent to the curve $y = \sqrt{2 - x^2}$ at the point where the ordinate and abscissa are equal, is

a) 2 b) -1 c) 0 d) ½

Sol. (b)

$\Rightarrow y = \sqrt{2 - x^2}$

ATQ

$\Rightarrow x = y$

$\Rightarrow y = \sqrt{2 - y^2}$

$= y^2 = 2 - y^2$

$= 2y^2 = 2$

$\therefore y = 1 = x$

$\Rightarrow y = \sqrt{2 - x^2}$

Differentiation both sides w. r. t. x

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2\sqrt{2 - x^2}}$$

$= \frac{-x}{\sqrt{2-x^2}} = -1$

Q.34) Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x - coordinate is 2.

a) 6 b) 8 c) 10 d) 11

Sol. (d)

$\Rightarrow y = x^3 - x + 1$

$$\Rightarrow \frac{d}{dx}y = \frac{d}{dx}x^3 - \frac{d}{dx}x + \frac{d}{dx}1$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 1$$

Given $x = 2$

$\Rightarrow \frac{dy}{dx} = 3(2)^2 - 1 = 11$

Q.35) The total cost of 20 units of a commodity is ₹ 205, while the total cost of 10 units is ₹ 135. Assuming that the cost function is a linear function, find the marginal cost function.

a) 65 **b)** 14 **c)** 21 **d)** 7

Sol. (d)

Let the linear function

$C(x) = ax + b$

Here, x denote as unit, a denote the variable cost per unit and b denote the fixed cost.

205=20a+b 1

135 = 10a+b 2

Equation 2 subtract from equation 1

205-135 =20a+b-10a-b

70 = 10a

= a = 7

Put a = 7 in equation 1

205 = 20(7) +b

= b = 65

$\therefore C(x) = 7x + 65$

$\Rightarrow MC = \frac{d}{dx}C(x) = \frac{d}{dx}(7x + 65)$

$\Rightarrow MC(x) = 7$

Q.36) The cost function for x units of a commodity, $C(x) = \frac{1}{3}x^3 + 3x^2 - 7x + 16$. Find the marginal average cost.

a) $\frac{2}{3}x + 3 - \frac{16}{x^2}$
b) $\frac{2}{3}x - 3 - \frac{16}{x^2}$
c) $\frac{2}{3} + 3x - \frac{16}{x^2}$
d) $\frac{2}{3}x + 3 + \frac{16}{x^2}$

Sol. (a)

$\Rightarrow C(x) = \frac{1}{3}x^3 + 3x^2 - 7x + 16$

$\Rightarrow AC = \frac{TC}{x} = \frac{1}{3}\frac{x^3}{x} + 3\frac{x^2}{x} - 7\frac{x}{x} + \frac{16}{x}$

$\Rightarrow AC = \frac{1}{3}x^2 + 3x - 7 + \frac{16}{x}$

$\Rightarrow Marginal\ Average\ cost = \frac{d(AC)}{dx} = \frac{d}{dx}(\frac{1}{3}x^2 + 3x - 7 + \frac{16}{x})$

$= \frac{2}{3}x + 3 - \frac{16}{x^2}$

Q.37) The total cost $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.

a) 21
b) 20.967
c) 21.014
d) 21.171

Sol. (b)

$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$

$\Rightarrow MC = \frac{d}{dx}(TC) = \frac{d}{dx}(0.007x^3 - 0.003x^2 + 15x + 4000)$

$\Rightarrow MC = 0.021x^2 - 0.006x + 15$

$\Rightarrow MC_{x=17} = 0.021(17)^2 - 0.006(17) + 15$

$\Rightarrow MC_{x=17} = 20.967$

Q.38) The cost function of a company is given by:

$C(x) = 100x - 8x^2 + \frac{x^3}{3}$ Where x denotes the output. Find the level of output at which marginal cost is minimum

a) 8
b) 10
c) 6
d) 12

Sol. (a)

$C(x) = 100x - 8x^2 + \frac{x^3}{3}$

$\Rightarrow MC = \frac{d}{dx}C(x) = \frac{d}{dx}(100x - 8x^2 + \frac{x^3}{3})$

$\Rightarrow MC = 100 - 16x + x^2$

$\Rightarrow \frac{d}{dx}[MC(x)] = 0 - 16 + 2x$

$\Rightarrow \frac{d}{dx}[MC(x)] = 0$

$\Rightarrow -16 + 2x = 0$

$= x = 8$

$\Rightarrow \frac{d}{dx}MC(x)=2x-16$

$\Rightarrow \frac{d^2}{dx^2}MC(x)=2$

$\Rightarrow \frac{d^2}{dx^2}MC(x)>0$

$\therefore$ Marginal cost is minimum at $x = 8$ units.

Q.39) The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when $x = 5$.

a) 66 **b)** 12 **c)** 30 **d)** 60

Sol. (a)

$R(x) = 3x^2 + 36x + 5$

$\Rightarrow MR = \frac{dR}{dx} = \frac{d}{dx}(3x^2 + 36x + 5)$

$\Rightarrow MR = 6x + 36$

$\Rightarrow MR_{x=5} = 6(5) + 36$

$= 66$

Q.40) A manufacturer determines that t employees will produce a total of x units of a product per day, where x = 2t. if the demand equation for the product is p =-0.5 x +20, determine the marginal revenue when t = 5.

a) 20 **b)** 22 **c)** 25 **d)** 30

Sol. (a)

$\Rightarrow$Revenue = price × units

$\Rightarrow R(x) = (-0.5x + 20)x$

$\Rightarrow R(x) = -0.5x^2 + 20x$

$\Rightarrow R(t) = -0.5(2t)^2 + 20(2t)$

$\Rightarrow R(t) = -2t^2 + 40t$

$\Rightarrow MR = \frac{d}{dt}R(t) = \frac{d}{dt}(-2t^2 + 40t)$

$\Rightarrow MR = -4t + 40$

$\Rightarrow MR_{t=5} = -4(5) + 40 = 20$

Q.41) A company decided to set up a small production plant for manufacturing clocks. The total cost for the initial setup is ₹ 9 lakhs. The additional cost for producing each clock is ₹ 300. Each clock is sold at ₹ 750. During the first month, 1500 clocks are produced and sold. Determine the break-even point.

a) 1500 **b)** 1200 **c)** 2000 **d)** 1000

Sol. (c)

Fixed cost = 9 lakh

Variable cost = 300 per unit

Price per unit = 750

Let x is the number of units

Total cost = fixed cost + variable cost

$\Rightarrow TC(x) = 900000 + 300x$

Revenue = price × unit

$\Rightarrow R(x) = 750x$

ATQ.

At the break-even point,

$= R(x) = TC(x)$

$= 750x = 9{,}00{,}000 + 300x$

$= 450x = 9{,}00{,}000$

$= x = 2000$ *units*

$\therefore$ 2000 clock have to be sold to achieve the break-even point.

Q.42) $\int(1-3x)(1+x)dx$ is equal to

a) $x - x^2 - x^3$ **b)** $x^3 - x^2 + x$ **c)** $x - x^2 - x^3 + k$ **d)** None of these

Sol. (c)

$\int(1-3x)(1+x)\,dx = \int(1-2x-3x^2)\,dx$

$= x - 2\times\frac{x^2}{2} - 3\times\frac{x^3}{3} + k \Rightarrow x - x^2 - x^3 + k$

Q.43) Integrate w. r. t. x, $\sqrt{x}+\frac{1}{\sqrt{x}}$

a) $\frac{1}{3}x^{3/2}+2x^{-1/2}$+c b) $\frac{2}{3}x^{3/2}+2x^{1/2}$+c c) $\frac{2}{3}x^{3/2}-x^{1/2}+c$ d) None

Sol. (b)

$\int \sqrt{x}+\frac{1}{\sqrt{x}} = x^{\frac{1}{2}}+x^{-\frac{1}{2}}$

$=\int x^{\frac{1}{2}}+\int x^{-\frac{1}{2}}$

$=\frac{2}{3}x^{3/2}+2x^{1/2}$+c

Q.44) Integrate w. r. t. x $\frac{1}{xlogx.log(logx)}$

a) $\log(\log(logx))+c$ **b)** $\log(logx)+c$ **c)** log x **d)** None

Sol. (a)

$\int\frac{1}{xlogx.\log(logx)}dx$

Let $\log(logx)=t$

$\frac{d}{dx}\log(logx)=\frac{d}{dx}t$

$\frac{1}{x\,logx}=\frac{dt}{dx}$

$dx = x\,logx\,dt$

$\int\frac{1}{xlogx.t}x\,logx\,dt$

$\int\frac{1}{t}dt = logt + c$

$=\log(\log(logx))+c$

Q.45) $\int\frac{e^x(x\log x+1)}{x}dx$ is equal to

a) $e^x\log x+k$ **b)** e^x+k **c)** $\log x+k$ **d)** None of these

Sol. (a)

$\int\frac{e^x(x\log x+1)}{x}dx$

$=\int e^x\left(\log x+\frac{1}{x}\right)dx$

$=e^x\log x+k$ $[\because \int e^x[f(x)+f'(x)]dx = e^x f(x)+k]$

Q.46) Evaluate $\int\frac{x^4-x^2+1}{x^2-1}\,dx$

a) $\frac{x^3}{3}+\frac{1}{2}\log\frac{x+1}{x-1}+c$ **b)** $\frac{x^3}{3}+\log\frac{x+1}{x-1}+c$ **c)** $x+\frac{1}{2}\log\frac{x+1}{x-1}+c$ **d)** None

Sol. (a)

$\Rightarrow\int\frac{x^4-x^2+1}{x^2-1}\,dx = \int\frac{x^2(x^2-1)}{x^2-1}+\frac{1}{x^2-1}\,dx$

$=\int x^2+\frac{1}{x^2-1}\,dx$

Apply formula $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}log\frac{x+a}{x-a} + c$

$= \frac{x^3}{3} + \frac{1}{2}\log\frac{x+1}{x-1} + c$

Q.47) Use integration by parts to evaluate $\int x^2e^{3x}dx$

a) $x^2e^{3x}/3 - 2x\,e^{3x}/9 + 2/27\,e^{3x} + k$ **b)** $x^2e^{3x} - 2x\,e^{3x} + 2e^{3x} + k$

C) $\frac{e^{3x}}{3} - x\,e^{3x/9} + 2e^{3x}/k$ **d)** None of these

Sol. (a)

$\int x^2e^{3x}dx$

$= x^2\int e^{3x}dx - \int\left\{\frac{d}{dx}x^2 \times \int e^{3x}dx\right\}dx$

$= x^2 \times \frac{e^{3x}}{3} - \int 2x \times \frac{e^{3x}}{3}dx$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\int x\,e^{3x}dx$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x\int e^{3x}dx - \int\left\{\frac{d(x)}{dx}.\int e^{3x}dx\right\}dx\right]$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x.\frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}dx\right]$

$= \frac{x^2e^{3x}}{3} - \frac{2}{9}x\,e^{3x} + \frac{2}{9}\int e^{3x}dx$

$= \frac{x^2e^{3x}}{3} - \frac{2}{9}x\,e^{3x} + \frac{2}{27}e^{3x} + k$

Q.48) Using integration by parts $\int x^3\log x\,dx$

a) $\frac{x^4}{16} + k$ **b)** $\frac{x^4}{16}(4\log x - 1) + k$ **c)** $4\log x - 1 + k$ **d)** None of these

Sol. (b)

$\int x^3\log x\,dx$

II I

$=\log x.\frac{x^4}{4} - \int\frac{1}{x}.\frac{x^4}{4}dx$

$= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3dx$

$= \frac{x^4}{4}\log x - \frac{x^4}{16} + k$

$= \frac{x^4}{16}(4\log x - 1) + k$

Q.49) $\int xlogx^2dx$ *is equal to:*

a) $\frac{x^2}{2}(\log x^2 - 1)$ +c **b)** $\frac{x^2}{2}(\log x^2 - x)$ +c **c)** $\frac{x^2}{2}(\log x^2 + 1)$ +c **d)** None

Sol. (a)

$\Rightarrow \int xlogx^2\,dx$

Let $x^2 = t$

$\Rightarrow \frac{d}{dx}x^2 = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} = 2x$

$\Rightarrow dx = \frac{dt}{2x}$

$\Rightarrow \int xlog\,t\,\frac{dt}{2x}$

$\Rightarrow \frac{1}{2}\int log\,t\,dt$

Integration by part

$\Rightarrow \frac{1}{2}(\log t \int 1\, dt - \int \left[\int 1 dt \times \frac{d}{dx}\log t\right] dt\,)$

$\Rightarrow \frac{1}{2}(\text{t} \log t - \int 1\, dt)$

$\Rightarrow \frac{1}{2}(\text{t} \log t - t)$ +c

$\Rightarrow \frac{1}{2}(x^2 \log x^2 - x^2)$ +c

$\Rightarrow \frac{x^2}{2}(\ \log x^2 - 1)$ +c

Q.50) By the method of partial fraction $\int \frac{3x}{x^2-x-2} dx$ is

a) $2\ log_e|x-2| +\ log_e|x+1| + k$

b) $2\ log_e|x-2| -\ log_e|x+1| + k$

c) $log_e|x-2| +\ log_e|x+1| + k$

d) None of these

Sol. (a)

Let I=$\int \frac{3x}{x^2-x-2} dx$

=$\int \frac{3x}{(x-2)(x+1)} dx$

Put $\frac{3x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$\Longrightarrow 3x = A(x+1) + B(x-2)$

$\Longrightarrow 3x = (A+B)x + (A-2B)$

Comparing co-efficient of x & constant,

$A + B = 3$ ______ (I)

$A - 2B = 0$______ (II)

From [(I) - (II)]

$A + B = 3$

$A - 2B = 0$

$-\quad +\quad -$

3B=3$\Longrightarrow$B=1

$\therefore$ A=2

$\therefore$ I=2$\int \frac{1}{x-2} dx + 1 \int \frac{1}{x+1} dx$

= $2\ log|x-2| + \log|x+1| + k$

Q.51) Evaluate $\int \frac{1}{x-x^3} dx$ is

a) $\frac{1}{2}\log\frac{x^2}{1-x^2}$+c

b) $\frac{1}{2}\log\frac{x}{1-x^2}$+c

c) $\log\frac{x^2}{1-x^2}$+c

d) None

Sol. (a)

$\int \frac{1}{x-x^3} dx$

$= \int \frac{1}{x(1-x^2)} dx$

$= \int \frac{1}{x(1-x)(1+x)} dx = \int \frac{A}{x} dx + \int \frac{B}{1-x} dx + \int \frac{C}{1+x} dx$

$= 1 = A\,(1+x)(1-x) + Bx(1+x) + Cx(1-x)$

Put x = 1then B =1/2

Put x =-1 then c = -1/2

Put x = 0 then A = 1

$\int \frac{1}{x} dx + \frac{1}{2}\int \frac{1}{1-x} dx - \frac{1}{2}\int \frac{1}{1+x} dx$

$= \log x - \frac{1}{2}\log(1-x) - \frac{1}{2}\log(1+x)$+c

$=\frac{2}{2}logx-\frac{1}{2}\log(1-x)(1+x)$+c

$=\frac{1}{2}\log\frac{x^2}{1-x^2}$+c

Q. 52) $\int\frac{3x^2-2x+5}{(x+1)(x^2+5)}dx$ is equal to

a) $\log(x+1)(x^2+5)+c$ b) $\log(x+1)+c$ c) $\log(x^2+5)+c$ d) None

Sol. (a)

Let $\int\frac{3x^2-2x+5}{(x+1)(x^2+5)}=\frac{A}{x+1}+\frac{Bx+C}{(x^2+5)}$

$=3x^2-2x+5=A(x^2+5)+(Bx+C)(x+1)$

Equating the coefficient of x^2, x and constant terms.

$\Rightarrow A+B=3$ i

$\Rightarrow C-B=-2$ ii

$\Rightarrow 5A-C=5$ iii

Adding the equation (i) and (ii)

$\Rightarrow$A+B+C-B = 3-2

$\Rightarrow$A+C = 1 iv

Adding the equation (iii) and (iv)

$\Rightarrow$A+C+5A-C = 5+1

$\Rightarrow$6A = 6$\Rightarrow$A = 1

Put the value of A in equation (i) and (iv), we will get

$\Rightarrow$B = 2 and C = 0

$$\int\frac{3x^2-2x+5}{(x+1)(x^2+5)}=\int\frac{1}{x+1}dx+\int\frac{2x}{(x^2+5)}dx$$

$\Rightarrow\int\frac{1}{x+1}dx+\int\frac{2x}{(x^2+5)}dx=\log(x+1)+\log(x^2+5)+c$

$\Rightarrow\log(x+1)(x^2+5)+c$

Q.53) $\int_0^4\sqrt{3x+4}\,dx$ is equal to

a) $\frac{9}{112}$ **b)** $\frac{112}{9}$ **c)** $\frac{11}{9}$ **d)** None of these

Sol. (b)

let I=$\int_0^4\sqrt{3x+4}$

Put $3x+4=t$

$\therefore 3dx=dt$

$=dx=dt/3$

$\therefore \mathrm{I}=\int_4^{16}t^{1/2}\cdot\frac{dt}{3}$

$=\frac{1}{3}\int_4^{16}t^{1/2}\,dt$

$=\frac{1}{3}\left[\frac{t^{3/2}}{3/2}\right]_4^{16}$

$=\frac{2}{9}\left[t^{3/2}\right]_4^{16}=\frac{2}{9}[64-8]$

$=\frac{2}{9}\times 56=\frac{112}{9}$

Q.54) Evaluate $\int_1^2\left(\frac{-1}{x^2}\right)e^{1+\frac{1}{x}}dx$

a) $-e^2+e^{1.5}$ **b)** $-e^2-e^{1.5}$ **c)** $-e^2$ **d)** $e^{1.5}$

Sol. (a)

$$\int_1^2 \left(\frac{-1}{x^2}\right) e^{1+\frac{1}{x}}\, dx$$

Let $1+\frac{1}{x}=t$

$\Rightarrow \frac{d}{dx}(1+\frac{1}{x}) = \frac{d}{dx}\text{t}$

$= \frac{-1}{x^2} = \frac{dt}{dx}$

$= dx = -x^2 dt$

$$\int_1^2 \left(\frac{-1}{x^2}\right) e^{t}\, (-x^2) dt$$

$\Rightarrow \int_1^2 e^t\ dt = [e^t]_1^2$

Put the value of t

$= \left[e^{1+\frac{1}{x}}\right]_1^2$

$= -e^2 + e^{1.5}$

Q.55) Find the equation of the curve where slope at (x, y) is $9x^2$ and which passes through the origin.

a) $y = 3x^3$ **b)** $y = -3x^3$ **c)** $y = x^3$ **d)** $y = 9x^3$

Sol. (a)

$$\frac{dy}{dx} = 9x^2$$

$$\int 1\, dy = \int 9x^2\, dx$$

$\Rightarrow y = \frac{9x^3}{3} + c \quad \Rightarrow y = 3x^3 + c$

Put the value $x = 0$ and y = 0

$\Rightarrow 0 = 3(0) + c$

$= c = 0$

$\Rightarrow y = 3x^3$

Q.56) $MR = \frac{6}{(x+2)^2} + 5$, what is the $R(x)$ function?

a) $-\frac{6}{x+2} + 5x + 3$ b) $\frac{6}{x+2} + 5x + 3$ c) $-\frac{6}{x+2} - 5x + 3$ d) None of these

Sol. (a)

$$MR = \frac{6}{(x+2)^2} + 5$$

$$\Rightarrow R(x) = \int \frac{6}{(x+2)^2} + 5\, dx = -\frac{6}{x+2} + 5x + c$$

$$\Rightarrow R(x) = -\frac{6}{x+2} + 5x + c$$

$R(0) = 0$

$$= -\frac{6}{0+2} + 5(0) + c = 0$$

$= c = 3$

$\Rightarrow R(x) = -\frac{6}{x+2} + 5x + 3$

Business Statistics

Measure of Central Tendency

Q.1) Which of the following statements is wrong?

a) Mean is rigidly defined

b) Mean is not affected due to sampling fluctuations

c) Mean has some mathematical properties

d) All these

Sol. Option b)

Mean is affected due to sampling fluctuations.

Q.2) In the case of an even number of observations which of the following is median?

a) Any of the two middle-most value

b) The simple average of these two middle values

c) The weighted average of these two middle values

d) Any of these

Sol. Option b)

The simple average of two middle values in the case of an even number of observations is the median.

Q.3) The most commonly used measure of central tendency is

a) AM **b)** Median **c)** Mode **d)** Both GM and HM

Sol. Option a)

The most commonly used measure of central tendency is AM.

Q.4) For a moderately skewed distribution, which of the following relationship holds?

a) Mean – Mode = 3 (Mean – Median)

b) Median – Mode = 3 (Mean – Median)

c) Mean – Median = 3 (Mean – Mode)

d) Mean – Median = 3 (Median – Mode)

Sol. Option a)

Mean – Mode = 3(Mean – Median)

Q.5) Which of the following results hold for a set of distinct positive observations?

a) $AM \leq GM \leq HM$ **b)** $HM > GM > AM$ **c)** $AM > GM > HM$ **d)** $GM > AM > HM$

Sol. Option c)

$AM > GM > HM$

Q.6) Quartiles are the values dividing a given set of observations into
a) Two equal parts **b)** Four equal parts **c)** Five equal parts **d)** None of these

Sol. Option b)
Quartiles are the values dividing a set of observations into four equal parts.

Q.7) Which of the following measure(s) satisfies (satisfy) a linear relationship between two variables?
a) Mean **b)** Median **c)** Mode **d)** All of these

Sol. Option d)
Mean, median, and mode satisfy the linear relationship between two variables.

Q.8) For 889, 999, 391, 384, 390, 480, 485, 760, 111, 240 Rank of median is
a) 2.75 **b)** 5.5 **c)** 8.25 **d)** None

Sol. Option b)
Here No. of observation (N) = 10

Rank of Median $(m_e) = \left(\frac{N+1}{2}\right)^{th}$ observation

$= \left(\frac{10+1}{2}\right)^{th}\ term = 5.5^{th}\ term$

Rank of Median $(m_e) = \boldsymbol{5.5^{th}\ term}$

Q.9) The 3rd decile for values 15, 10, 20, 25, 18, 11, 9, 12 is
a) 13 **b)** 10.7 **c)** 11 **d)** 11.5

Sol. Option b)
Arranging in Ascending order
9, 10, 11, 12, 15, 18, 20, 25
N = 8

$\therefore D_3 = \left(\frac{N+1}{10}\right)^{th} obs. = 3\left(\frac{8+1}{10}\right) = 2.7th\ obs.$

= 2nd obs. + 0.7(3rd – 2nd obs.)

= 10 + 0.7 (11-10) = **10.7**

Q.10) What is the H.M. of $1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \frac{1}{n}$?
a) n **b)** 2n **c)** $\frac{2}{(n+1)}$ **d)** $\frac{n(n+1)}{2}$

Sol. Option c)

$$\text{H.M.} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + ---+\frac{1}{x_n}}$$

$$\Rightarrow \frac{n}{\frac{1}{1} + \frac{1}{1/2} + \frac{1}{1/3} + \frac{1}{1/4} + --- + n}$$

$$\Rightarrow \frac{n}{1+2+3+4+-----+n}$$

$$\Rightarrow \frac{n}{\frac{n(n+1)}{2}}$$

$$\Rightarrow \frac{1}{\frac{(n+1)}{2}} \Rightarrow \boldsymbol{\frac{2}{n+1}}$$

Q.11) The A.M. of the square of the first '2n' natural number is

a) $\frac{1}{6}(2n+1)(4n-1)$

b) $\frac{1}{6}(2n-1)(4n-1)$

c) $\frac{1}{6}(2n-1)(4n+1)$

d)) $\frac{1}{6}(2n+1)(4n+1)$

Sol. Option d)

The sum of the square of the natural number $(S_n) = \frac{n(n+1)(2n+1)}{6}$

$$S_{2n} = \frac{2n(2n+1)(2\times 2n+1)}{6}$$

$$= \frac{2n(2n+1)(4n+1)}{6}$$

$$A.M. = \frac{S_{2n}}{2n} = \frac{2n(2n+1)(4n+1)}{6\times 2n}$$

$$= \frac{1}{6}\boldsymbol{(2n+1)(4n+1)}$$

Q.12) If a variable assumes the value of 1, 2, 3....5 with frequencies as 1, 2, 3.......5, then what is the AM?

a) $\frac{11}{3}$ **b)** 5 **c)** 4 **d)** 4.50

Sol. Option a)

x	f	fx
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25
	15	$\sum fx = 55$

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{55}{15} = \frac{11}{3}$$

Q.13) Mean of n observation is $\bar{x}$, if the first observation is increased by 1, 2^{nd} observation is by 2, and so on, then the new mean is _________.

a) $\bar{x} + n.n$ **b)** $\bar{x} + \frac{n+1}{2}$ **c)** $\bar{x} + \frac{n}{2}$ **d)** $\bar{x} + n$

Sol. Option b)

$$\text{New Mean} = \frac{n\bar{x} + (1+2+3+\cdots....+n)}{n}$$

$$= \bar{x} + \frac{n(n+1)}{2n} = \bar{x} + \frac{n+1}{2}$$

Q.14) The mean and mode for the following frequency distribution

Class interval:	350–369	370–389	390–409	410–429	430–449	450–469
Frequency:	15	27	31	19	13	6

a) 400 and 390 **b)** 400.58 and 390 **c)** 400.58 and 394.50 **d)** 400 and 394

Sol. Option c)

C.I.	Frequency	X	$d' = \frac{X-A}{i}$	fd'
349.5 - 369.5	15	359.5	−3	−45
369.5 - 389.5	27 f_0	379.5	−2	−54
389.5 - 409.5	**31** f_1	399.5	−1	−31
409.5 - 429.5	19 f_2	**419.5**	0	0
429.5 - 449.5	13	439.5	1	13
449.5 - 469.5	6	459.5	2	12
	111			−105

$$Mode = l + \frac{f_1-f_0}{2f_1-f_0-f_2} \times i$$

$$= 389.5 + \frac{31-27}{62-27-19} \times 20$$

$$389.5 + \frac{4}{16} \times 20 = 394.5$$

Mode = 394.5

Mean $= A + \frac{\Sigma fd'}{\Sigma f} \times i$

$$= 419.5 + \frac{(-105)}{111} \times 20$$

Mean = 400.58

Q.15) Following is an incomplete distribution having a modal mark as 44

Marks:	0–20	20–40	40–60	60–80	80–100
No. of Students:	5	18	y	12	5

What would be the mean marks?

a) 45 **b)** 46 **c)** 47 **d)** 48

Sol. Option d)

C.I.	f	x	fx
0-20	5	10	50
20-40	18	30	540
40-60	**y = 20**	**50**	**1,000**
60-80	12	70	840
80-100	5	90	450
Mode = 44	**60**		**2,880**

$$\text{Mode} = l + \frac{f_1-f_0}{2f_1-f_0-f_2} \times i$$

$$44 = 40 + \frac{y-18}{2y-18-12} \times 20$$

$$= 4 = \frac{y-18}{2y-30} \times 20$$
$$\frac{4}{20} = \frac{y-18}{2(y-15)}$$
$$\frac{2}{5} = \frac{y-18}{y-15}$$
$$2y - 30 = 5y - 90$$
$$60 = 3y$$
$$\boldsymbol{y = 20}$$
$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$
$$\bar{X} = \frac{2{,}880}{60}$$
$$\boldsymbol{\bar{X} = 48}$$

Q.16) If there are two groups containing 30 and 20 observations and having 50 and 60 asarithmetic means, then the combined arithmetic mean is

a) 55 **b)** 56 **c)** 54 **d)** 52

Sol. Option c)

$$\bar{X} = \frac{n_1\,\bar{X_1} + n_2\,\bar{X_2}}{n_1+n_2} = \frac{30\times50+20\times60}{30+20}$$

$$= \frac{1500+1200}{50} = \frac{2700}{50} = \mathbf{54}$$

Q.17) The combined mean of the three groups is 12, and the combined mean of the first two groups is 3. If the first, second and third groups have 2, 3 and 5 items, respectively, then the mean of the third group is

a) 10 **b)** 21 **c)** 12 **d)** 13

Sol. Option b)

$$\bar{x}_{123} = \frac{n_1\bar{x}_1+n_2\bar{x}_2+n_3\bar{x}_3}{n_1+n_2+n_3}$$

$$\Rightarrow 12 = \frac{2\times3+3\times3+5\bar{x}_3}{2+3+5}$$

$$\Rightarrow 120 = 15 + 5\bar{x}_3$$

$$\therefore \bar{x}_3 = \frac{105}{5} = \mathbf{21}$$

Q.18) If there are 3 observations 15, 20, 25, and then the sum of deviation of the observations from their AM is

a) 0 **b)** 5 **c)** -5 **d)** None of these

Sol. Option a)

$$\bar{X} = \frac{15+20+25}{3} = \frac{60}{3} = 20$$

∴ Sum of deviation of the observation From A.M. i.e., $\bar{X}$

= (15-20) + (20 – 20) + (25 – 20)

= -5 + 0 + 5 = 0

Note: Sum of deviation of the observation From their mean is always zero.

Q.19) If the relationship between two variables u and v are given by 2u + v + 7 = 0 and if the AM of u is 10, then the AM of v is

a) 17 **b)** -17 **c)** -27 **d)** 27

Sol. Option c)

$\therefore 2u + v + 7 = 0 \Longrightarrow v = -2u - 7$

When A.M. of u = 10

$\therefore$ A.M. of $v = -2\times 10 - 7 = \mathbf{-27}$

Q.20) The average age of 15 students is 15 years. Out of these the average age of 5 students is 14 years, and that of other 9 students is 16 years, then the age of 15th student is ________.

a) 11 years **b)** 14 years **c)** 15 years **d)** None of these

Sol. Option a)

Age of 15th student $= 15 \times 15 - 5 \times 14 - 9 \times 16 = \mathbf{11\ years}$

Q.21) The mean salary of a group of 50 persons is ₹ 5,850. Later on, it is discovered that the salary of one employee has been wrongly taken as ₹ 8,000 instead of ₹ 7,800. The corrected mean salary is

a) ₹ 5,854 **b)** ₹ 5,846 **c)** ₹ 5,650 **d)** None of the above

Sol. Option b)

Correct mean $= \frac{Correct\ \Sigma x}{N}$

$\bar{x} = \frac{\Sigma x - wrong\ obs. + Correct\ obs.}{N}$

$(\because \Sigma x = N \times \bar{X})$

$\bar{x} = \frac{50\times5{,}850 - 8000 + 7800}{50} = \mathbf{5,846}$

Q.22) The average age of a group of 10 students was 20 years. The average age is increased by two years when two new students joined the group. What is the average age of two new students who joined the group?

a) 22 years **b)** 30 years **c)** 44 years **d)** 32 years

Sol. Option d)

Sum of the age of two boys

$= (10 + 2) \times (20 + 2) - 10 \times 20$

$= 264 - 200 = 64$

Average age of two boys $= \frac{64}{2} = \mathbf{32}$

Q.23) If in a moderately skewed distribution, the values of mode and mean are 32.1 and 35.4, respectively, then the value of the median is

a) 33.3 **b)** 34 **c)** 34.3 **d)** 33

Sol. Option c)

$M_o = 3M - 2\bar{X}$

Or $32.1 = 3M - 2\times 35.4$

Or $M = \frac{32.1 + 70.8}{3} = \mathbf{34.3}$

Q.24) The third quartile and 65th percentile for the following data are

Profits in '000.'	less than 10	10–19	20–29	30–39	40–49	50–59
No. of firms:	5	18	38	20	9	2

a) ₹33,500 and ₹29,184
b) ₹33,000 and ₹28,680
c) ₹33,600 and ₹29,000
d) ₹33,250 and ₹29,250

Sol. Option a)

C.I.	f	$C.F.$
0-9500	5	5
9500-19,500	18	23
19,500-29,500	38	61
29,500-39,500	**20**	**81**
39,500-49,500	9	90
49,500-59,500	2	92

$$\boldsymbol{Q_3 = 3\left(\frac{N}{4}\right)} = 3 \times \frac{92}{4} = 69$$

$$Q_3 = l + \frac{K\left(\frac{N}{4}\right) - C.F._p}{f} \times i$$

$$Q_3 = 29{,}500 + \frac{69-61}{20} \times 10{,}000$$

$$Q_3 = 29{,}500 + 4{,}000$$

$$\boldsymbol{Q_3 = 33,500}$$

$$P_{65} = l + \frac{K\left(\frac{N}{100}\right) - C.F._p}{f} \times i$$

$$P_{65} = 65 \times \left(\frac{92}{100}\right) = \boldsymbol{59.80th}$$

$$P_{65} = 19{,}500 + \frac{59.80-23}{38} \times 10{,}000$$

$$= 19{,}500 + 9684.21$$

$$\boldsymbol{P_{65} = 29,184}$$

Q.25) What is the G.M. for the numbers 8, 24 and 40?

a) 24
b) 12
c) $8.\sqrt[3]{15}$
d) 10

Sol. Option c)

G.M for the nos. 8, 24, and 40

$$= (8 \times 24 \times 40)^{1/3}$$

$$= (2^3 \times 2^3 \times 3 \times 2^3 \times 5)^{1/3}$$

$$= 2 \times 2 \times 2 \times (3 \times 5)^{1/3} = \boldsymbol{8\sqrt[3]{15}}$$

Q.26) Geometric Mean of three observations 40, 50 and x is 10. The value of x is

a) 2
b) 4
c) 1/2
d) None of these

Sol. Option c)

$$G.M. = (x \times y \times z)^{\frac{1}{3}}$$

$$10 = \sqrt[3]{40 \times 50 \times x}$$

Apply cube both sides

$$(10)^3 = 40 \times 50 \times x$$

$$1{,}000 = 40 \times 50 \times x$$

$$x = \frac{1000}{2000} = \frac{1}{2}$$

Q.27) If the AM and G.M. for 10 observations are both 15, then the value of H.M. is

a) Less than 15 **b)** More than 15 **c)** 15 **d)** Cannot be determined

Sol. Option c)

$(\text{A.M}) = \frac{x_1+x_2+--+x_{10}}{10} = 15$

$(\text{G.M.}) = (x_1\, x_2 --- x_{10})^{1/10} = 15$

$\text{H. M.} = \frac{10}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+--+\frac{1}{x_{10}}}$

∴ A.M., G.M & H.M. are G.P.

$\therefore (\text{G.M.})^2 = \text{A.M} \times \text{H.M}$

$\Rightarrow (15)^2 = 15 \times H.M.$

$\Rightarrow$ H.M. = **15**

Q.28) If the Harmonic mean of two numbers is 4 and Arithmetic mean (A) and Geometric mean (G) satisfy the equation $2A + G^2 = 27$ then the two numbers are

a) (1,3) **b)** (9,5) **c)** (6,3) **d)** (12,7)

Sol. Option c)

$H = \frac{2ab}{a+b} = \frac{2\times6\times3}{6+3} = 4\ (true)$

$A = \frac{6+3}{2} = 4.5$

$G = \sqrt{ab} = \sqrt{6\times3} = \sqrt{18}$

$It\ satisfies\ 2A + G^2 = \mathbf{27}$

Q.29) If the mean of first n natural numbers is equal to $\frac{n+7}{3}$, then n is equal to:

a) 10 **b)** 11 **c)** 12 **d)** None of these

Sol. Option b) Here, $\frac{1+2+3+\cdots+n}{n} = \frac{n+7}{3}$

$\Rightarrow \frac{\frac{n(n+1)}{2}}{n} = \frac{n+7}{3}$

$\Rightarrow \frac{n+1}{2} = \frac{n+7}{3} \Rightarrow 3n+3 = 2n+14$

$\Rightarrow n = \mathbf{11}$

Q.30) A candidate obtains the following percentages in an examination. English 46%; Mathematics 67%; Sanskrit 72%; Economics 58%; Political Science 53%. It is agreed to give double weights to marks in English and Mathematics as compared to other subjects. The weighted mean is:

a) 58.40 **b)** 58.43 **c)** 58.24 **d)** 58.45

Sol. Option b) ∵ Weighted mean = $\frac{\sum_{i=1}^{n} x_i\, w_i}{\sum_{i=1}^{n} w_i}$

$= \frac{2\times46+2\times67+1\times72+1\times58+1\times53}{2+2+1+1+1}$

$= \frac{92+134+72+58+53}{7} = \mathbf{58.43}$ (approx.)

Q.31) A person runs the first $\frac{1}{5}th$ of the distance at 2 km/hr, the next one half at 3 km/hr and the remaining distance at 1 km/hr. Find his average speed.

a) $\frac{15}{17}$ km/hr **b)** $\frac{30}{17}$ km/hr **c)** $\frac{17}{30}$ km/hr **d)** None of these

Sol. Option b)

Let the total distance be d

Average speed = $\frac{\frac{d}{5}+\frac{d}{2}+\frac{3d}{10}}{\frac{d}{10}+\frac{d}{6}+\frac{3d}{10}} = \frac{d}{17d} \times 30$

∴ Average speed = $\frac{30}{17}$ **km/hr**

Q.32) If two boxes of oranges sell at ₹ 10 and ₹ 20, respectively. The average price per orange in paise is:

a) 7.5 **b)** 7 **c)** 8 **d)** 6.4

Sol. Option a)

In this question variable is orange, and the constant is the rupee. So, the harmonic mean is applicable in calculating the average rate as it is based on the rate

$\therefore \text{H.M.} = \frac{n}{\frac{1}{n_1}+\frac{1}{n_2}+\_\_\_\_\_\_+\frac{1}{n_n}}$

$= \frac{2}{\frac{1}{10}+\frac{1}{20}} = 2 \times \frac{20}{3}$

$= \frac{40}{3}$

∴ Average rate of orange sold per ₹ is $\frac{40}{3}$

∴ Average Price per orange = $\frac{1}{40/3}$ ₹

$= \frac{3}{40}$ Rs $= \frac{3}{40} \times 100$ Paise

= **7.5 Paise**

Q.33) The percentage of items in a frequency distribution lying between upper and lower quartiles is

a) 70% **b)** 40% **c)** 50% **d)** 35%

Sol. Option c) Percentage of items in a frequency distribution lying between upper and lower quartiles is **50%**

Q.34) A person covers 12 km at 3 km/hr, 18 km at 9 km/hr and 24 km at 4 km/hr. Find the average speed in covering the whole distance.

a) 4.5 km/hr **b)** 5 km/hr **c)** 10 km/hr **d)** None of these

Sol. Option a)

Time to cover 12 km = $\frac{12}{3}$ = 4 hrs

Time to cover 18 km = $\frac{18}{9}$ = 2 hrs

Time to cover 24 km = $\frac{24}{4}$ = 6 hrs

∴ Average speed = $\frac{12+18+24}{4+2+6} = \frac{54}{12} = \mathbf{4.5\ km/hr}$

Q.35) If there are two groups with 125 and 120 as harmonic mean and containing 45 and 30 observations then the combined H.M. is given by

a) 102 **b)** 110.52 **c)** 109 **d)** 123

Sol. Option b) Given is $HM_1 = 125$, $HM_2 = 120$, $n_1 = 45$ and $n_2 = 30$

Combined H.M. = $\frac{n_1+n_2}{\frac{n_1}{HM_1}+\frac{n_2}{HM_2}} = \frac{45+30}{\frac{45}{125}+\frac{30}{120}} = \frac{75}{\frac{9}{25}+\frac{1}{4}} = \frac{75}{\frac{36+25}{100}} = \frac{75}{61} \times 100 = \mathbf{123}$

Q.36) The geometric mean of four values was calculated as 16. It was later discovered that one of the values was recorded wrongly as 32 when, in fact, it was 162. Calculate the correct geometric mean.

a) 18 **b)** 24 **c)** 28 **d)** none of these

Sol. Option b)

Let 4 values be X_1, X_2, X_3, X_4

G.M. = $(X_1 \times X_2 \times X_3 \times X_4)^{\frac{1}{4}} = 16$

$\Rightarrow X_1 \times X_2 \times X_3 \times X_4 = (16)^4 = 65536$

But this is not the correct product

Correct $(X_1 \times X_2 \times X_3 \times X_4) = \frac{65536 \times 162}{32} = 331776$

Corrected G.M. = $(331776)^{\frac{1}{4}} = \mathbf{24}$

Q.37) If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ harmonic mean of a and b, then n is

a) 3 **b)** 2 **c)** -1 **d)** None of these

Sol. Option c)

H.M. = $\frac{2}{\frac{1}{a}+\frac{1}{b}}$

$\Rightarrow \frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow \frac{2ab}{a+b} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow 2ba^{n+1} + 2ab^{n+1} = a^{n+2} + ba^{n+1} + ab^{n+1} + b^{n+2}$

$\Rightarrow ba^{n+1} + ab^{n+1} = a^{n+2} + b^{n+2}$

$\Rightarrow a^{n+1}(b-a) + b^{n+1}(a-b) = 0$

$\Rightarrow (a-b)(a^{n+1} - b^{n+1}) = 0$

$\Rightarrow a^{n+1} = b^{n+1}$

$\Rightarrow (\frac{a}{b})^n = (\frac{b}{a})^1 = (\frac{a}{b})^{-1}$

$\therefore$ **n = -1**

Q.38) If the relationship between x and y is given by $4x - 6y = 13$ and the median of x is 16. Find the median of y.

a) 6.2 **b)** 7.5 **c)** 8.1 **d)** 8.5

Sol. Option d)

Given, $4x - 6y = 13$

Put value of $x = 16$

$\Rightarrow 4(16) - 6y = 13$

$\Rightarrow 64 - 6y = 13$

$\Rightarrow -y = \frac{13-64}{6}$

$\therefore y = 8.5$

The median of y = **8.5**

Q.39) If the median of 5, 9, 11, 3, 4, x, 8 is 6, the value of x is equal to

a) 6 **b)** 5 **c)** 4 **d)** 3

Sol. Option a)

Arranging the given numbers in ascending order, we get:

3,4,5,8,9,11 and x, which can be at any place since we do not know its value

Here n=7, which is odd

$\therefore$ The median will be $\frac{(n+1)^{th}}{2}$ term

$\therefore$ Median = $\frac{(7+1)^{th}}{2}$ term = 4th term

But the median given is 6

The fourth term of the given data arranged in ascending order will be 6 only when x is placed there, and its value is 6

Therefore, $\boldsymbol{x = 6}$

Q.40) There are 50 numbers. If each number is subtracted from 53, then the mean of the numbers so obtained is -3.5. Find the mean of the given number.

a) 49.5 kg **b)** 45.5 kg **c)** 42.1 kg **d)** none of these

Sol. Option a)

Mean of 50 numbers

$\Rightarrow \overline{X} = \frac{n_1+n_2+n_3+\cdots+n_{50}}{50}$ ...(i)

Let y be the mean when 53 is deducted from every no.

$\Rightarrow \overline{y} = \frac{n_1-53+n_2-53+n_3-53+\cdots+n_{50}-53}{50}$ $(\because \overline{y} = -3.5)$

$\Rightarrow -3.5 = \frac{n_1+n_2+n_3+\cdots+n_{50}}{50} - \frac{50\times53}{50}$

$\Rightarrow -3.5 = \overline{X} - 53$

$\therefore \overline{X} = \mathbf{49.5}$

Q.41) The average of runs of a cricket player of 10 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4?

a) 4 **b)** 70 **c)** 76 **d)** 72

Sol. Option c)

Let the 10 innings runs be $a_1, a_2, \dots, a_{10}$

$\Rightarrow 32 = \frac{a_1 + a_2 \dots + a_{10}}{10}$

$\Rightarrow 320 = a_1 + a_2 \dots + a_{10}$ ...(i)

Let the next innings runs be a_{11}

$\Rightarrow 32 + 4 = \frac{a_1 + a_2 \dots + a_{10} + a_{11}}{11}$

$\Rightarrow 396 = a_1 + a_2 \dots + a_{10} + a_{11}$ ... (ii)

Substituting (i) and (ii)

$\Rightarrow 320 + a_{11} = 396$

$\therefore a_{11} = \mathbf{76}$

Q.42) If five times the geometric mean of two numbers 'a' and 'b' is equal to the arithmetic mean of those two numbers such that $a > b > 0$, then compute the value of $\frac{a+b}{a-b}$.

a) 17.2 **b)** $\frac{2\sqrt{6}}{5}$ **c)** $\frac{5}{2\sqrt{6}}$ **d)** none of these

Sol. Option c)

A.M. of two no. = $\frac{a+b}{2}$

G.M. of two no. = $\sqrt{ab}$

$\Rightarrow \frac{a+b}{2} = 5\sqrt{ab}$

$\Rightarrow a + b = 10\sqrt{ab}$

Squaring both sides

$\Rightarrow (a + b)^2 = 100ab$

Subtract 4ab from both the sides

$\Rightarrow (a - b)^2 = (a + b)^2 - 4ab = 96ab$

$\therefore \frac{a+b}{a-b} = \sqrt{\frac{100ab}{96ab}} = \sqrt{\frac{25}{24}} = \frac{5}{2\sqrt{6}}$

Q.43) In a coconut grove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year per tree be 100, then the value of x is-

a) 4 **b)** 6 **c)** 8 **d)** 2

Sol. Option a)

$\Rightarrow 100 = \frac{(x+2)60 + x \times 120 + (x-2)180}{(x+2) + x + x - 2} = \frac{60x + 120 + 120x + 180x - 360}{3x}$

$\Rightarrow 300x = 360x - 240$

$\Rightarrow 60x = 240$

$\therefore \boldsymbol{x = 4}$

Q. 44) A cyclist covers first three kms at an average speed of 10 km/h. Another two km at 4 km/h. and the last two km at 2 km/h. The average speed for the entire journey in kph is:

a) 2.43 **b)** 3.43 **c)** 3.89 **d)** None of these

Sol. Option c)

Average speed = Weighted H.M.

$\bar{X} = \frac{\Sigma f}{\frac{f_1}{x_1}+\frac{f_2}{x_2}+\frac{f_3}{x_3}}$

$\Rightarrow \frac{3+2+2}{\left(\frac{1}{10}\times 3\right)+\left(\frac{1}{4}\times 2\right)+\left(\frac{1}{2}\times 2\right)}$

∴ Average speed = **3.89 km/h.**

Q. 45) The A.M. of n observations is P. If the sum of n – 8 observations is a, then the mean of the remaining 8 observations is:

a) $\frac{nP+a}{8}$ **b)** $\frac{nP-a}{8}$ **c)** $\frac{nP+a}{n}$ **d)** $\frac{nP-a}{n}$

Sol. Option b)

Let n observation be $x_1,\ x_2, x_3, \dots\dots\dots\dots x_n$

Given, $\frac{x_1+x_2+x_3,\dots\dots\dots+x_n}{n} = P$ ...(i)

Also given, $x_1 + x_2 + x_3, \dots\dots\dots\dots + x_{n-8} = a$ ...(ii)

$\therefore x_{n-7} + x_{n-6} + x_{n-5} + x_{n-4} + x_{n-3} + x_{n-2} + x_{n-1} + x_n = nP - a$

Using (i) and (ii)

Mean of last eight observation = $\frac{x_{n-7}+x_{n-6}+x_{n-5}+x_{n-4}+x_{n-3}+x_{n-2}+x_{n-1}+x_n}{8} = \frac{nP-a}{8}$

Q. 46) The mean age of a combined group of men and women is 25 years. If the mean age of the group of men is 26 and that of the group of women is 21, then the percentage of men and women in the group is:

a) 60, 40 **b)** 80, 20 **c)** 20, 80 **d)** 40, 60

Sol. Option b)

Here, $\bar{x} = 25, \bar{x}_1 = 26, \bar{x}_2 = 21$

Let $n_1 + n_2 = 100$, where n_1 is no. of men and n_2 is no. of women.

Then, $n_2 = 100 - n_1$

$\therefore 25 = \frac{26n_1+21(100-n_1)}{100}$

$= 2500 = 26n_1 + 2100 - 21n_1$

$= 5n_1 = 400$

$\Rightarrow \boldsymbol{n_1 = 80, n_2 = 20}$

Q. 47) A truck travels along four sides of a square with 60 kmph, 70 kmph, 80 kmph and 90 kmph speed. The average speed is

a) 50 km/h **b)** 100 km/h **c)** 73.31 km/h **d)** 90 km/h

Sol. Option c)

$\bar{X} = \frac{\Sigma f}{\frac{f_1}{x_1}+\frac{f_2}{x_2}+\frac{f_3}{x_3}}$

H.M. = $\frac{4}{\frac{1}{60}+\frac{1}{70}+\frac{1}{80}+\frac{1}{90}}$

$\Rightarrow$ H.M. = $\frac{4}{0.016+0.014+0.0125+0.0111} = \mathbf{73.31\ km/h}$

Q. 48) 50th percentile is known as.

a) 50th decile **b)** 50th quartile

c) Mode **d)** Median

Sol. Option d) 50th percentile is known as median.

Q. 49) If $\frac{a^{m+1}+b^{m+1}}{a^m+b^m}$ is the G.M. between the numbers a and b, then the value of m is

a) $-\frac{1}{2}$ **b)** $\frac{1}{2}$ **c)** 1 **d)** 0

Sol. Option a)

$\sqrt{ab} = \frac{a^{m+1}+b^{m+1}}{a^m+b^m} \Rightarrow a^{m+\frac{1}{2}}.b^{\frac{1}{2}} + a^{\frac{1}{2}}.b^{m+\frac{1}{2}} = a^{m+1} + b^{m+1}$

$\Rightarrow a^{m+1} - a^{m+\frac{1}{2}}.b^{\frac{1}{2}} = a^{\frac{1}{2}}.b^{m+\frac{1}{2}} - b^{m+1}$

$\Rightarrow a^{m+\frac{1}{2}}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right] = b^{m+\frac{1}{2}}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right]$

$\Rightarrow \frac{a^{m+\frac{1}{2}}}{b^{m+\frac{1}{2}}} = 1$

$\therefore$ m+1/2=0 $\Rightarrow$ **m = -1/2**

Q. 50) If the arithmetic mean of two numbers is 10 and their geometric mean is 8, the numbers are:

a) 20, 5 **b)** 12, 8 **c)** 15, 5 **d)** 16, 4

Sol. Option d)

AM = $\frac{a+b}{2}$

$\Rightarrow 10 = \frac{a+b}{2} \Rightarrow a + b = 20$ ---(i)

G.M. = $\sqrt{ab}$

$8 = \sqrt{ab} \Rightarrow ab = 64$

$\therefore a(20 - a) = 64$

$\Rightarrow a^2 - 20a + 64 = 0$

$(a - 16)(a - 4) = 0$

$\therefore$ **a = 16, b = 4**

Q. 51) The width of each of ten classes in a frequency distribution is 2.5 and the lower-class boundary of the lowest class is 10.6. Which one of the following is the upper-class boundary of the highest class?

a) 35.6 **b)** 33.1 **c)** 30.6 **d)** none of these

Sol. Option a)

Lower class (s) = 10.6

Width = 2.5 (class interval)

Upper class (L) of lightest class

$\Rightarrow$ S = 10 × C.I.

$\Rightarrow$ 10.6 + 10 × 2.5 = **35.6**

Q. 52) If there are two groups with 125 and 115 as harmonic means and containing 25 and 23 observations then the combined HM is given by

a) 100 **b)** 120 **c)** 85.2 **d)** 71.3

Sol. Option a)

Combined H.M. = $\frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}$

$\Rightarrow \frac{25+23}{\frac{25}{125}+\frac{23}{115}} = \frac{48}{\frac{1}{5}+\frac{1}{5}} = 48 \times \frac{5}{2} = \mathbf{120}$

Measure of Dispersion

Q.1) If the range of a set of values is 65 and maximum value in the set is 83, then the minimum value in the set is

a) 74 **b)** 9 **c)** 18 **d)** None of the above

Sol. Option c)

Range = L – S

65 = 83 – S

$\therefore S = 83 - 65 = \mathbf{18}$

Q.2) If the range of x is 2, what would be the range of $-3x + 50$?

a) 2 **b)** 6 **c)** -6 **d)** 44

Sol. Option b)

$R_X = 2$

$Y = -3x + 50$

$R_Y = |-3|R_X = 2 \times 3 = \mathbf{6}$

Q.3) If R_a and R_b denote ranges of a and b respectively where a and b are related by 6a+ 4b+20=0, what would be the relation between a and b?

a) R_a=R_b **b)** $3R_a = 2R_b$ **c)** $3R_a = 4R_b$ **d)** None of these

Sol. Option b)

Given eq. 6a+ 4b+20=0

$\Rightarrow$ 6a = -20 – 4b

$\Rightarrow a = -\frac{20}{6} - \frac{4}{6}b = -\frac{10}{3} - \frac{2}{3}b$

$R_a = |b|\, R_b$

$\Rightarrow R_a = \frac{2}{3} R_b$

$\therefore$ The relation between x and y is $3R_a = 2R_b$

Q.4) What is the value of mean deviation about mean for the following observations?

50, 60, 50, 50, 60, 60, 60, 50, 50, 50, 60, 60, 60, 50.

a) 5 **b)** 7 **c)** 35 **d)** 10

Sol. Option a)

x	f	fx	$\lvert x - \bar{x}\rvert$	f $\lvert x - \bar{x}\rvert$
50	7	350	5	35
60	7	420	5	35
	14	770		70

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{\cancel{770}}{\cancel{14}} = 55$$

$$M.D. = \frac{\Sigma f|X - \bar{X}|}{\Sigma f} = \frac{\cancel{70}}{\cancel{14}} = \mathbf{5}$$

Q.5) What is the mean deviation about median for the following data?

x :	3	5	7	9	11	13	15
F :	2	8	9	16	14	7	4

a) 2.50 **b)** 2.46 **c)** 2.43 **d)** 2.37

Sol. Option d)

x	Frequency	C.F.	$\|x - Median\|$	$f\|x - Median\|$
3	2	2	6	12
5	8	10	4	32
7	9	19	2	18
9	16	35	0	0
11	14	49	2	28
13	7	56	4	28
15	4	60	6	24
				142

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} = \left(\frac{60+1}{2}\right)^{th} = 30.5th$$

$$Median = 9$$

$$Mean\ deviation\ from\ Median = \frac{\sum F|x-Median|}{\sum F}$$

$$M.D.from\ Median = \frac{142}{60}$$

$$M.D.from\ Median = \mathbf{2.37\ approx.}$$

Q.6) The equation of a line is $5x + 2y = 17$. Mean deviation of Y about mean is 5. Calculate mean deviation of x about mean.

a) -2 **b)** 2 **c)** -4 **d)** None

Sol. Option b)

$$x = 17 - \frac{2}{5}Y$$

$$MD_x = |-\frac{2}{5}|\ MD_y$$

$$MD_x = \frac{5\times2}{5}$$

$$MD_x = \mathbf{2}$$

Q.7) If the points of inflexion of a Normal Curve are 40 and 60 respectively, then its mean deviation is

a) 8 **b)** 45 **c)** 50 **d)** 60

Sol. Option a)

If the point of inflexion of a Normal Distribution are 40 and 60.

Formula, $\mu - \sigma = 40 \ldots\ldots\ldots(1)$

$\mu + \sigma = 60 \ldots\ldots\ldots(2)$

Solving eqns. (1) and (2) ; we get

$\mu = 50, \sigma = 10$

Then M.D. = $0.8.\sigma = 0.8 \times 10 = \mathbf{8}$

Q.8) The coefficient of mean deviation about mean for the first 9 natural numbers is

a) 200/9 **b)** 80 **c)** 400/9 **d)** 50

Sol. Option c)

x	$\lvert x-\bar{x}\rvert$
1	4
2	3
3	2
4	1
5	0
6	1
7	2
8	3
9	4
	20

$M.D. \Rightarrow \frac{\sum|x-\bar{x}|}{n} = \frac{20}{9}$

$$Coefficient\ of\ mean\ deviation = \frac{\frac{20}{9}}{5} \times 100$$

$\Rightarrow \frac{20}{9} \times \frac{1}{5} \times 100 = \frac{400}{9}$

$\bar{x} = \frac{45}{9} = \mathbf{5}$

Q.9) If two variables x and y are related by $2x + 3y - 7 = 0$ and the y mean and mean deviation about mean of x are 1 and 0.3 respectively, then the co-efficient of mean deviation of y about mean is:

a) -5 **b)** 4 **c)** 12 **d)** 50

Sol. Option c)

If Y = a + bx, a and b being constant, then $M.D_y = |b|(M.D_X)$

(Since M.D. changes due to change in scale)

$\therefore Y = -\frac{2}{3}x + \frac{7}{3}$

$M.D_y = \left|-\frac{2}{3}\right|(M.D_X) = \left(\frac{2}{3}\right) \times 0.3 = 0.2$

Also, $2x + 3y - 7 = 0$

$2\bar{X} + 3\bar{Y} - 7 = 0$ (Since, the A.M. is affected by change of origin as well as change of scale)

$\bar{Y} = -\frac{2}{3}\bar{X} + \frac{7}{3}$

$\bar{Y} = -\frac{2}{3} \times 1 + \frac{7}{3} = \frac{5}{3}$ $(Given\ \bar{X} = 1)$

Coefficient of mean deviation of Y about mean

$= \frac{M.D_{\bar{y}}}{\bar{y}} \times 100 = \frac{0.2}{5/3} \times 100 = \mathbf{12}$

Q.10) The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is

a) 10 **b)** 20 **c)** 25 **d)** 8.30.

Sol. Option a)

$Q_1 = 45, Q_2 = 52$ and $Q_3 = 65$

$$QD = \frac{65-45}{2} \Rightarrow \frac{20}{2} = \mathbf{10}$$

Q.11) The value of appropriate measure of dispersion for the following distribution of daily wages are given by

Wages (₹):	Below 30	30-39	40-49	50-59	60-79	Above 80
No. of workers	5	7	18	32	28	10

a) ₹ 11.033 **b)** ₹ 10.50 **c)** 11.8 **d)** ₹ 11.68

Sol. Option a)

C.I.	f	C.f.
L-29.5	5	5
29.5-39.5	7	12
39.5-49.5	18	30
49.5-59.5	32	62
59.5-79.5	28	90
79.5-U	10	100

$$Q_1 = K\left(\frac{N}{4}\right)^{th} = 1 \times \left(\frac{100}{4}\right)^{th} = 25th$$

$$Q_3 = 3\left(\frac{100}{4}\right)^{th} = 75th$$

$$Q_1 = l + \frac{K\left(\frac{N}{4}\right) - C.f._p}{f} \times i$$

$$= 39.5 + \frac{25-12}{18} \times 10$$

$$= 39.5 + \frac{13}{18} \times 10$$

$$Q_1 = 46.72$$

$$Q_3 = 59.5 + \frac{75-62}{28} \times 20$$

$$= 59.5 + \frac{13}{28} \times 20$$

$$Q_3 = 68.786$$

$$Q.D. = \frac{Q_3 - Q_1}{2} \Rightarrow \frac{68.786-46.72}{2}$$

$$\boldsymbol{Q.D. = 11.033}$$

Q.12) Coefficient of quartile deviation is ¼ then Q_3/Q_1 is

a) 5/3 **b)** 4/3 **c)** 3/4 **d)** 3/5

Sol. Option a)

Coeff. Of Q.D $= \frac{1}{4}$

$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1}{4}$ *[cross product]*

or; $4\,Q_3 - 4\,Q_1 = Q_3 + Q_1$

or; $4\,Q_3 - Q_3 = Q_1 + 4Q_1$

or; $3\,Q_3 = 5Q_1$

or; $\frac{Q_3}{Q_1} = \frac{5}{3}$

Q.13) Co-efficient of QD is equal to ________.

a) $\frac{QD}{M} \times 100$ **b)** $\frac{QD}{\bar{x}} \times 100$ **c)** $\frac{QD}{Z} \times 100$ **d)** None

Sol. Option a)

Co-efficient of QD = $\frac{Q_3-Q_1}{Q_3+Q_1} \times 100$

$= \frac{\frac{Q_3-Q_1}{2}}{\frac{Q_3+Q_1}{2}} \times 100 = \frac{QD}{M} \times 100$

When $\boldsymbol{M = \frac{Q_3+Q_1}{2}}$ **(Symmetrical)**

Q.14) If the quartile deviation of x is 10 and $3x + 6y = 30$, what is the quartile deviation of y.

a) 2 **b)** 3 **c)** 4 **d)** 5

Sol. Option d)

Given $3x + 6y = 30$

$= 6y = 30 + 3x$

$= y = 5 - \frac{x}{2}$

Then, $y = 5 - \frac{x}{2}$

$Q.D._{y} = |b| \times Q.D._{x}$

$\therefore Q.D._{y} = \left|-\frac{1}{2}\right| \times 10$

$\therefore \boldsymbol{Q.D._{y} = 5}$

Q.15) If the mean deviation of a normal variable is 16, what is its quartile deviation?

a) 10 **b)** 13.33 **c)** 15 **d)** 12.05

Sol. Option b)

4S.D. = 5 M.D.

= S.D. = $\frac{5}{4} \times M.D.$

= S.D. = $\frac{5}{4} \times 16 = 20$

Q.D. = $\frac{2}{3} \times S.D.$

= Q.D. = $\frac{2}{3} \times 20 = 13.33$

Q.16) The mean and standard deviation of a normal distribution are ₹ 70 and ₹ 8 respectively. Find the inter-quartile range of the distribution.

a) 7.75 **b)** 8.75 **c)** 9.75 **d)** None of these

Sol. Option b)

Given, $\bar{X} = 70$, $\sigma = 8$

Q.D. = 0.6745 (σ)

$= 0.6745 \times 8$

$= 5.396$

Q.D. = $\frac{Inter-Quartile\ Range}{2} = \frac{Q_3-Q_1}{2}$

$Q_3 - Q_1 = Q.D. \times 2$

$Q_3 - Q_1 = 5.396 \times 2 = \mathbf{10.792}$

Q.17) For a series the value of mean deviation is 15. Find the most likely value of its quartile deviation.

a) 18.6 **b)** 4.8 **c)** 2.6 **d)** 12.5

Sol. Option (d)
Applying the following relationship.

$Q.D. = \frac{5}{6} M.D.$

Between quartile deviation (Q.D) and mean deviation (M.D.) we obtain

Q.D. $= \frac{5}{6} \times 15 = \mathbf{12.5}$

Q.18) The variance of data: 3, 4, 5, 8 is

a) 4.5 **b)** 3.5 **c)** 5.5 **d)** 6.5

Sol. Option b)
$x: 3, 4, 5, 8$
$\sum x = 20;\ \sum x^2 = 9 + 16 + 25 + 64 = 114$
Variance $= \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$
$= \frac{114}{4} - 25 = \mathbf{3.5}$

Q.19) The standard deviation of the weights (in kg.) of the student of a class of 50 students was calculated to be 4.5 kg. Later on it was found that due to some fault in weighting machine, the weight of each student was under measured by 0.5 kg. The Correct standard deviation of the weight will be:

a) Less than 4.5 **b)** Greater than 4.5 **c)** Equal to 4.5 **d)** Cannot be determined

Sol. Option c)
RULE: S.D. remains unaffected due to a change of origin but changes with respect to scale.
So, correct S.D. of 50 students **= 4.5**

Q.20) Suppose a population A has 100 observations 101, 102, 103, 200 and another population B has 100 observations 151, 152, 153,............ 250. If V_A *and* V_B represent the variance of the two populations respectively then $V_A / V_B =$:

a) 9/4 **b)** 1 **c)** 4/9 **d)** 2/3

Sol. Option b)
Rule : SD doesn't change with respect to the change of origin(+/-).
Population A : S.D. of 101, 102, 103, ... 200.
Let Its SD = σ
50 is added to all observations; we get data : 151, 152, ..., 250.
Its SD = σ (Also)
Hence SD of data B = σ

$\frac{V_A}{V_B} = \frac{\sigma^2}{\sigma^2} = \mathbf{1}$

Q.21) If two random variables x and y are related by $Y = 2 - 3x$, then the SD of Y is given by

a) $-3 \times SD\ of\ x$ **b)** $3 \times SD\ of\ x$ **c)** $9 \times SD\ of\ x$ **d)** $2 \times SD\ of\ x$

Sol. Option b)

S.D. of y $(\sigma_y) = |b| S.D. of\ x\ (\sigma_x)$

$= |-3|\sigma_x = \mathbf{3\sigma_x}$

Q.22) What is the coefficient of variation of the following numbers?

53, 52, 61, 60, 64.

a) 8.09 **b)** 18.08 **c)** 20.23 **d)** 20.45

Sol. Option a)

X	$(X-\bar{X})$	$(X-\bar{X})^2$
53	-5	25
52	-6	36
61	3	9
60	2	4
64	6	36
290		110

$$\sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}} = \sqrt{\frac{110}{5}} = \sqrt{22} = 4.69$$

$\bar{X} = \frac{290}{5} = 58$

$C.V. = \frac{4.69}{58} \times 100 = \mathbf{8.09}$

Q.23) What is the standard deviation from the following data relating to the age distribution of200 persons?

Age (year) :	20	30	40	50	60	70	80
No. of people:	13	28	31	46	39	23	20

a) 15.29 **b)** 16.87 **c)** 18.00 **d)** 17.52

Sol. Option b)

Age	f	$d' = \frac{X-A}{i}$	fd'	$X-\bar{X}$	$(X-\bar{X})^2$	$f(X-\bar{X})^2$
20	13	−3	−39	−30.95	957.9025	12452.7325
30	28	−2	−56	−20.95	438.9025	12289.27
40	31	−1	−31	−10.95	119.9025	3716.9775
50	46	0	0	-0.95	0.9025	41.515
60	39	1	39	9.05	81.9025	3194.1975
70	23	2	46	19.05	362.9025	8346.7575
80	20	3	60	29.05	843.9025	16878.05
	200		19			**56919.5**

$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times i$

$= 50 + \frac{19}{200} \times 10$

Mean= 50.95

$\sigma = \sqrt{\frac{\Sigma f(X-\bar{X})^2}{\Sigma f}}$

$\sigma = \sqrt{\frac{56919.5}{200}}$

σ =16.87

Q.24) The sum of squares of deviation from mean of 10 observations is 250. Mean of the data is 10. Find the co-efficient of variation.

a) 10% **b)** 25% **c)** 50% **d)** 0%

Sol. Option c)

Given, N = 10; $\sum(X-\bar{X})^2 = 250$

Mean = 10

$S.D. = \sqrt{\frac{\sum(X-\bar{X})^2}{N}} = \sqrt{\frac{250}{10}} = 5$

So, $CV = \frac{S.D.}{Mean} \times 100 = \frac{5}{10} \times 100 = \mathbf{50\%}$

Q.25) If mean and coefficient of variation of the marks of n students is 20 and 80 respectively. What will be Variance of them

a) 256 **b)** 16 **c)** 25 **d)** None of these

Sol. Option a)

Given $\bar{X} = 20; and\ C.V = 80\%$

$\therefore C.V. = \frac{\sigma}{\bar{X}} \times 100$

$\therefore 80 = \frac{\sigma}{20} \times 100$

$\therefore \sigma = \frac{80\times20}{100} = 16$

Variance = $\sigma^2 = 16^2 = \mathbf{256}$

Q.26) If AM and CV of a random variable x are 10 & 40 respectively, them the variance of $\left(-15+\frac{3x}{2}\right)$:

a) 64 **b)** 81 **c)** 49 **d)** 36

Sol. Option d)

Given ; $\frac{\sigma}{10} \times 100 = 40 \quad \therefore \sigma = 4$

$\therefore SD\ of\ \left(-15+\frac{3}{2}x\right) = \frac{3}{2} \times SD(x) = \frac{3}{2} \times 4 = 6$

$\therefore Variance\ of\ \left(-15+\frac{3}{2}x\right) = 6^2 = \mathbf{36}$

Q.27) If the mean and SD of x are a and b respectively, then the SD of $\frac{x-a}{b}$ is

a) -1 **b)** 1 **c)** ab **d)** a/b

Sol. Option b)

$\bar{X} = a$

$\sigma = b$

$Find\ \sigma\ of\ \frac{x-a}{b}$

$y = \frac{x-a}{b}$

$y = \frac{x}{b} - \frac{a}{b} \Rightarrow y = \frac{1}{b} \times x - \frac{a}{b}$

$\sigma_y = \frac{1}{b} \times \sigma_x$

$\boldsymbol{\sigma_y = \frac{1}{b} \times b = 1}$

Q.28) If 5 is subtracted from each observation of some certain item then is co-efficient of variation is 10% and if 5 is added to each item them its coefficient of variation is 6%. Find original coefficient of variation.

a) 8% **b)** 7.5% **c)** 4% **d)** None of these

Sol. Option b)

Coefficient of variation C.V. = $\frac{\sigma}{Mean} \times 100$

RULE: S.D. does not change but Mean changes due to the change of origin.
Let original S.D. = σ and Mean = $\bar{x}$

$$\therefore Case\ I \quad \frac{\sigma}{\bar{x}-5} \times 100 = 10 \ldots\ldots (1)$$

$$\therefore Case\ II \quad \frac{\sigma}{\bar{x}+5} \times 100 = 6 \ldots\ldots (2)$$

Dividing (2) by (1), we get

$$\frac{\bar{x}-5}{\bar{x}+5} = \frac{6}{10}; Solving\ it, we\ get$$

$$\bar{x} = 20\ and\ \sigma = 1.5$$

Original C.V. = $\frac{1.5}{20} \times 100 = \mathbf{7.5\%}$

Q.29) If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the SD of the combined sample of size 50?

a) 5.00 **b)** 5.06 **c)** 5.23 **d)** 5.35

Sol. Option b)

$n_1 = 30$ $\quad \bar{X}_1 = 55$
$n_2 = 20$ $\quad \bar{X}_2 = 60$
$Variance\ of\ Series\ 1 = 16$ $\quad S.D.of\ Series\ 1 = 4$
$Variance\ of\ Series\ 2 = 25$ $\quad S.D.of\ Series\ 2 = 5$

$$\bar{X}_{12} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} \Rightarrow \bar{X}_{12} = \frac{30\times55+20\times60}{50}$$
$$\bar{X}_{12} = \frac{1650+1200}{50} \Rightarrow 57$$
$$\bar{X}_{12} = 57$$

$$d_1 = \bar{X}_{12} - \bar{X}_1 \Rightarrow d_1 = 57 - 55 = 2$$

$$d_2 = \bar{X}_{12} - \bar{X}_2 \Rightarrow d_2 = 57 - 60 = -3$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1{d_1}^2 + n_2{d_2}^2}{n_1+n_2}}$$
$$\sigma_{12} = \sqrt{\frac{30\times(4)^2+20(5)^2+30\times(2)^2+20\times(-3)^2}{30+20}}$$
$$\sigma_{12} = \sqrt{\frac{30\times16+20\times25+30\times4+20\times9}{50}}$$
$$\sigma_{12} = \sqrt{\frac{480+500+120+180}{50}}$$
$$\sigma_{12} = \sqrt{\frac{1280}{50}} \Rightarrow \sigma_{12} = \sqrt{25.6}$$
$$\boldsymbol{\sigma_{12} = 5.06}$$

Q.30) The mean and SD for a group of 100 observations are 65 and 7.03 respectively. If 60 of these observations have mean and SD as 70 and 3 respectively, what is the SD for the group comprising 40 observations?

a) 16 **b)** 25 **c)** 4 **d)** 2

Sol. Option c)

$\bar{X}_{12} = 65$ $\bar{X}_1 = 70$ $\bar{X}_2 = x$

$\sigma_{12} = 7.03$ $\sigma_1 = 3$ $n_2 = 40$

$n_1 = 60$ $\sigma_2 = y$

$$Combined\ Mean = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$

$$65 = \frac{60\times70+40x}{60+40}$$

$$6500 = 4200 + 40x \Rightarrow 2300 = 40x$$

$$\boldsymbol{x = 57.5}$$

$$\boldsymbol{\bar{X}_2 = 57.5}$$

$$d_1 = \bar{X}_{12} - \bar{X}_1 \Rightarrow 65 - 70 = -5$$

$$d_2 = \bar{X}_{12} - \bar{X}_2 \Rightarrow 65 - 57.5 = 7.5$$

$$\boldsymbol{Combined\ S.D} = \sqrt{\frac{n_1{\sigma_1}^2+n_2{\sigma_2}^2+n_1{d_1}^2+n_2{d_2}^2}{n_1+n_2}}$$

$$\sigma_{12} = \sqrt{\frac{60\times(3)^2+40(y)^2+60\times(-5)^2+40\times(7.5)^2}{60+40}}$$

$$7.03 = \sqrt{\frac{60\times9+40\times y^2+60\times25+40\times56.25}{100}}$$

$$7.03 = \sqrt{\frac{540+40y^2+1500+2250}{100}}$$

$$7.03 = \sqrt{\frac{40y^2+4290}{100}}$$

$$49.4209 \times 100 = 40y^2 + 4290$$

$$4942.09 = 40y^2 + 4290$$

$$652.09 = 40y^2$$

$$y^2 = 16.30225$$

$$y = 4.03$$

$$\boldsymbol{\sigma_y = 4}$$

Q.31) Mean and S.D. of a given set of observations is 1,500 and 400 respectively. If there is an increment of 100 in the first year and each observation is hiked by 20% in 2nd year, then find new mean and S.D.

a) 1920, 480 **b)** 1920, 580 **c)** 1600, 480 **d)** 1600, 400

Sol. Option a)

$\bar{X} = 1500, \sigma = 400$

After 1st year,

Mean	S.D.
1500+100 = 1600	400

After 2nd year,

Mean	S.D.
$1600 \times 1.20 = \mathbf{1,920}$	$400 \times 1.20 = \mathbf{480}$

Q.32) If standard deviation of first 'n' natural numbers is 2 then the value of 'n' is

a) 10 **b)** 7 **c)** 6 **d)** 5

Sol. Option b)

S.D. of 1st .n. natural Numbers $= \sqrt{\frac{n^2-1}{12}}$

$= \frac{2}{1} = \sqrt{\frac{n^2-1}{12}}, OR, 4 = \frac{n^2-1}{12}$

OR, $n^2 - 1 = 48$

OR, $n^2 = 49 \Rightarrow \boldsymbol{n = 7}$

Q.33) Standard deviation is ______ times of $\sqrt{MD \times QD}$

a) 2/3 **b)** 4/5 **c)** $\sqrt{\frac{15}{8}}$ **d)** $\sqrt{\frac{8}{15}}$

Sol. Option c)

4 S.D = 5 M.D = 6 Q.D

Let, 4 S.D = 5 M.D = 6 Q.D = LCM of 4,5,6 =60

S.D = 60/4 =15

MD = 60/5 =12

QD = 60/6 = 10

Let SD is x times of $\sqrt{MD \times QD}$

$SD = x\sqrt{MD \times QD}$

$15 = x\sqrt{12 \times 10}$

Squaring on both sides; we get

$225 = x^2.12 \times 10$

So, $x = \sqrt{\frac{225}{12\times10}} = \sqrt{\frac{15}{8}}$

Q.34) The arithmetic means and the standard deviation of a set of 9 items are 43 and 5 respectively. If an item of value 63 is added to the set, find the standard deviation of all the 10 items.

a) 4.21 **b)** 8.01 **c)** 7.65 **d)** None of these

Sol. Option c)

Given that,

n = 9, $\bar{x}$ = 43, σ = 5

$\bar{x} = \frac{\sum x}{n}$

$\Rightarrow \sum x = n\bar{x} = 9 \times 43 = 387$

$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{\sum x^2}{n} - (\bar{x})^2$

$=\sum x^2 = n(\sigma^2 + \bar{x}^2) = 9(25 + 1849) = 16866$

If a new item 63 is added then the new number of terms becomes 10.

$\because$ New $\sum x$ = (old $\sum x$) + 63 = 387 + 63 = 450

New mean = $\bar{x} = \frac{450}{10} = 45$

New $\sum x^2 = 16866 + (63)^2 = 16866 + 3969 = 20835$

$\therefore$ New S.D. $= \sqrt{\frac{\sum x^2}{10} - (\bar{x})^2} = \sqrt{\frac{20835}{10} - (45)^2} = \sqrt{58.5} = 7.65$

Q.35) Comment on the statement "after settlement the average weekly wage in a factory has increased from ₹ 16 to ₹ 24 and standard deviation has increased from 4 to 4.1. After the settlement, the wage has become higher and more uniform".

a) C.V. is more & Variation is more
b) C.V. is less, Variation is less
c) C.V. is more & Variation is less
d) C.V. is less & Variation is more

Sol. Option b)

C.V. before settlement = $\frac{\sigma_{\text{before}}}{\bar{x}_{\text{before}}} \times 100 = \frac{4}{16} \times 100 = 25\%$

C.V. after settlement = $\frac{\sigma_{\text{after}}}{\bar{x}_{\text{after}}} \times 100 = \frac{4.10}{24} \times 100 = 17.08\%$

∴ **After the settlement C.V. is less, so the variation is also less**

Q.36) Compute the SD of 7, 3, 5, 3, 2.

Without any more computation, obtain the SD of Sample −1, −5, −3, −5 −6,

a) 1.79
b) 2.45
c) 3.12
d) None of these

Sol. Option a)

Computation of SD

X	x^2
7	49
3	9
5	25
3	9
2	4
20	96

The SD of the original set of observations is given by

$$s = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{96}{5} - \left(\frac{20}{5}\right)^2}$$

$$= \sqrt{19.2 - 16}$$

$$= \sqrt{3.2} = \mathbf{1.79}$$

If we denote the original observations by x and the observations of sample by y, then we have

$y = -8 + x$

$y = (-8) + (1)\,x$

$\therefore s_y = |1| \times s_x$

$\therefore s_y = 1 \times 1.79 = \mathbf{1.79}$

Q.37) The mean and variance of 5 observations are 4.80 and 6.16 respectively. It three of the observations are 2,3 and 6, what are the remaining observations?

a) 4 & 9
b) 3 & 4
c) 7 & 9
d) None of these

Sol. Option a)

Let the remaining two observations be a and b, then as given

$\frac{2+3+6+a+b}{5} = 4.80$

$= 11 + a + b = 24$

$= a + b = 13$.......................(1)

And $\frac{2^2+a^2+b^2+3^2+6^2}{5} - (4.80)^2$

$= \frac{49+a^2+b^2}{5} - 23.04 = 6.16$

$= 49 + a^2 + b^2 = 146$

$= a^2 + b^2 = 97$……………………………(2)

From (1), we get $a = 13 - b$…………………(3)

Eliminating a from (2) and (3), we get

$= (13 - b)^2 + b^2 = 97$

$= 169 – 26b + 2b^2 = 97$

$= b^2 – 13\, b + 36 = 0$

$= (b–4)(b–9) = 0$

$= b = 4\ or\ 9$

From (3), $a = 9\ or\ 4$

Thus, the remaining observations are 4 and 9

Q.38) If the standard deviation of a data is 9.4 and if each value of the data is decreased by 6, then find the new standard deviation.

a) 3.4 **b)** 9.4 **c)** 1.2 **d)** None of these

Sol. Option b)

Given, σ = 9.4

Each value decreased by 6

The standard deviation will not change when we subtract some fixed constant to all the values.

∴ **New standard deviation is 9.4**

Q.39) Variance of ⋋, β and γ is 9, then variance of 5γ, 5β and 5γ is:

a) 45 **b)** $\frac{9}{5}$ **c)** $\frac{5}{9}$ **d)** 225

Sol Option d) ∵Variance of $\propto,\ \beta,\ \gamma$ is 9

∴S. D of $\propto,\ \beta,\ \gamma$ is $\sqrt{a}$ i.e, 3

∴S. D of $5\propto, 5\beta\ and\ 5\gamma$ is $= 5 \times S.D.of\ \propto,\ \beta\ and\ \gamma$

$= 5 \times 3 = 15$

∴Variance of $5\propto, 5\beta\ and\ 5\gamma = (15)^2 = \mathbf{225}$

Q.40) If n = 5, x̄ = 6, $\sum x^2 = 765$, then calculate the coefficient of variation.

a) 200.09% **b)** 185.28% **c)** 180.28% **d)** 190.21%

Sol. Option c)

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} = \sqrt{\frac{765}{5} - (6)^2}$$

$\Rightarrow \sqrt{153 - 36} = \sqrt{117}$

$\Rightarrow \sigma = 10.817$

C.V. $= \frac{\sigma}{\bar{x}} \times 100$

$\Rightarrow \frac{10.817}{6} \times 100$

∴ C.V. = 1.8028×100 = **180.28%**

Q.41) For a set of 150 observations, taking assumed mean as 4, the sum of the deviations is -16 cm. and the sum of the squares of these deviations is 357 cm^2. Find the coefficient of variation.

a) 15 **b)** 12.01 **c)** 28.22% **d)** 38.56%

Sol. Option d)

Required, mean & S.D.

Mean = $A + \frac{\sum fd}{n} = 4 - \frac{16}{150} = 3.89$

S.D. = $\sqrt{\frac{\sum fd^2}{n} - \left(\frac{\sum fd}{n}\right)^2} = \sqrt{\frac{357}{150} - \left(\frac{-16}{150}\right)^2} = \sqrt{2.38 - 0.011} = \sqrt{2.369} = 1.5$

$\therefore$ Coefficient of Variation = $\frac{\sigma}{\bar{x}} \times 100 = \frac{1.5}{3.89} \times 100 = \mathbf{38.56\%}$

Q.42) The mean and SD for a, b and 4 are 6 and $\frac{4}{\sqrt{6}}$ respectively. The value of ab would be

a) 72 **b)** 24 **c)** 48 **d)** 36

Sol. Option c)

X	X^2
a	a^2
b	b^2
4	16
a+b+4	$a^2 + b^2 + 16$

$\bar{X} = \frac{a+b+4}{3}$

$6 = \frac{a+b+4}{3}$

$\Rightarrow 18 = a + b + 4$

$\Rightarrow$ a + b = 14

$\sigma = \frac{4}{\sqrt{6}}$

$\Rightarrow \frac{4}{\sqrt{6}} = \sqrt{\frac{a^2+b^2+16}{3} - (6)^2}$

$\Rightarrow \frac{16}{6} = \frac{a^2+b^2+16}{3} - 36$

$\Rightarrow \frac{16}{6} = \frac{a^2+b^2-92}{3}$

$\Rightarrow 100 = a^2 + b^2$

$\Rightarrow (a+b)^2 - 2ab = 100$

$\Rightarrow$ 196 – 2ab = 100

$\therefore$ **ab = 48**

Q.43) The total marks scored by two students A and B in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

a) A>B **b)** A<B **c)** A = B **d)** A ≠ B

Sol. Option b)

n = 5

A	B
Total marks $\sum x = 460$	Total marks $\sum x = 480$
S.D. = 4.6	S.D. = 2.4
$\bar{x} = \frac{\sum x}{n} = \frac{460}{5} = 92$	$\bar{x} = \frac{\sum x}{n} = \frac{480}{5} = 96$
C.V. = $\frac{\sigma}{\bar{x}} \times 100$	C.V. = $\frac{\sigma}{\bar{x}} \times 100$
$= \frac{4.6}{92} \times 100 = 5\%$	$= \frac{2.4}{96} \times 100 = 2.5\%$

$\therefore$ **B is more consistent than A.**

Q.44) If the mean and standard deviation of 75 observations is 40 and 8 respectively, find the new standard deviation if each observation is multiplied by 5.

a) 20 **b)** 30 **c)** 40 **d)** 50

Sol. Option c)

Given: $\bar{x} = 40, \sigma = 8, N = 75$
$New\,\bar{x} = Old\,\bar{x} \times 5 = Old\,\bar{x} \times 5$
$= 40 \times 5 = 40 \times 5$
$= 200$
and,
$New\,\sigma = Old\,\sigma \times 5 = Old\,\sigma \times 5$
$= 8 \times 5 = \mathbf{40}$

Q.45) If the profits of a company remains the same for the last ten months, then the standard deviation of profits for these ten months would be?

a) Positive **b)** Negative **c)** Zero **d)** (a) or (c)

Sol. Option (c)
If the profits of a company remains the same for the last ten months, then the standard deviation of profits for these ten months would be Zero.

Q.46) If Coefficients of variation of two series are 60% and 80%. Their standard deviations are 24 and 20 respectively. What are their arithmetic means?

a) 40 & 25 **b)** 60 & 25 **c)** 40 & 35 **d)** None of these

Sol. Option (a)

We Know C.V. $= \frac{S.D.}{Mean} \times 100$

∴ For first series, we have $60 = \frac{24}{Mean} \times 100 \Rightarrow \text{Mean} = \frac{2400}{60} = \mathbf{40}$

For second series, $80 = \frac{20}{Mean} \times 100 \Rightarrow \text{Mean} = \frac{2000}{80} = \mathbf{25}$

Q.47) A wall clock strikes the bell once at 1 o'clock, 2 times at 2 o'clock, 3 times at 3 o'clock and so on. How many times will it strike on a particular day? Find the standard deviation of the number of strikes the bell make a day.

a) 6.90 **b)** 6.00 **c)** 6.88 **d)** 6.85

Sol. Option (a)
The number of strikes the bell make a day
$= 2(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12)$
Number of times strike in a day
$\Rightarrow 2\left[\frac{n(n+1)}{2}\right]$
$\Rightarrow 2\left(\frac{12\times13}{2}\right) = 156$
S.D. of first n natural numbers
$\sigma = \sqrt{\frac{n^2-1}{12}}$
S.D. of number of strikes in a day
$\Rightarrow 2\sqrt{\frac{n^2-1}{12}} = 2\sqrt{\frac{12^2-1}{12}} = 2\sqrt{\frac{143}{12}}$
$\Rightarrow 2 \times 3.45 = \mathbf{6.90}$

Q.48) The number of workers employed, the mean wage (in ₹) per month and standard deviation (in ₹) in each section of a factory are given below. Calculate the standard deviation of all workers taken together.

Section	No. of workers employed	Mean wage	Standard deviation
A	50	113	6
B	60	120	7
C	90	115	8

a) 7.75 **b)** 8.75 **c)** 9.75 **d)** None of these

Sol. Option (a)

$$\bar{X}_{123} = \frac{N_1\bar{X}_1+N_2\bar{X}_2+N_3\bar{X}_3}{N_1+N_2+N_3}$$

$$= \frac{(50\times113)+(60\times120)+(90\times115)}{50+60+90}$$

$$= \frac{5650+7200+10350}{200} = \frac{23200}{200} = 116$$

Combined standard deviation of three series:

$$\sigma_{123} = \sqrt{\frac{N_1\sigma_1{}^2+N_2\sigma_2{}^2+N_3\sigma_3{}^2+N_1d_1{}^2+N_2d_2{}^2+N_3d_3{}^2}{N_1+N_2+N_3}}$$

$$d_1 = |\bar{x}_1 - \bar{x}_{123}| = |113-116| = 3$$

$$d_2 = |\bar{x}_2 - \bar{x}_{123}| = |120-116| = 4$$

$$d_3 = |\bar{x}_3 - \bar{x}_{123}| = |115-116| = 1$$

$$\sigma_{123} = \sqrt{\frac{50(6)^2+60(7)^2+90(8)^2+50(3)^2+60(4)^2+90(1)^2}{50+60+90}}$$

$$\sigma_{123} = \sqrt{\frac{1800+2940+5760+450+960+90}{200}} = \sqrt{\frac{12000}{200}} = \mathbf{7.75}$$

Q.49) The coefficients of variation of wages of male workers and female workers 55% and 70% respectively. While standard deviations are 22 and 15.4 respectively Calculate the overall average wages of all workers given that 80% of the workers are male.

a) 37.02 **b)** 36.4 **c)** 40.1 **d)** None of these

Sol. Option b)

We are given the following information:

	Male Workers	Female Workers
Coefficient of variation	55	70
Standard deviation	22	15.4

Average wages of male workers:

$$C.V. = \frac{\sigma}{\bar{x}_1}\times 100 \quad \Rightarrow \quad \bar{x}_1 = \frac{\sigma}{C.V.}\times 100 = \frac{22}{55}\times 100 = 40$$

Average wages of female workers:

C.V. = $\frac{\sigma}{\bar{x}_2} \times 100$ ⇒ $\bar{x}_2 = \frac{\sigma}{C.V.} \times 100 = \frac{15.4}{70} \times 100 = 22$

Average wages of all workers: -

$\bar{x}_1$ =40 $\bar{x}_2 = 22$ n_1=80 n_2=20

$\therefore \bar{X}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = \frac{80\times40+20\times22}{100} = \frac{3200+440}{100} = \frac{3640}{100} = 36.4$

Q. 50) The mean and variance of 100 items were worked out as 40 and 25 respectively by a student. By mistake an item 50 was wrongly taken as 5 in calculating the above. You are required to find the correct standard deviation.

a) 2.5 **b)** 3.5 **c)** 4.5 **d)** 5.5

Sol. Option a)

$\bar{x} = \frac{\sum x}{n}$

$40 = \frac{\sum x}{100}$

Incorrect $\sum x = 40 \times 100 = 4000$

Correct $\sum x = 4000 + 50 - 5 = 4{,}045$

$\therefore$ Correct mean = $\frac{Correct \sum x}{n} = \frac{4045}{100} = 40.45$

Variance = 25

$\therefore \sigma = 5$

$\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2 \Rightarrow 5^2 = \frac{\sum x^2}{100} - (40)^2$

$\Rightarrow 25 + 1600 = \frac{\sum x^2}{100}$

Incorrect $\sum x^2 = 162500$

Correct $\sum x^2 = 162500 - 5^2 + 50^2 = 164975$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

$\Rightarrow \sqrt{\frac{164975}{100} - (40.45)^2} = \sqrt{1649.75 - 1636.2025} = \sqrt{13.5475} = 3.68$

Correlation and Regression

Q.1) If r is the Karl person's coefficient of correlation in a bivariate distribution, the two regression lines are at right angles when ______

a) $r = \pm 1$ **b)** $r = 0$ **c)** $r = \pm\infty$ **d)** None

Sol. Option b)

If $r = 0$; Two Regression Lines are perpendicular to each other.

Q.2) If the sum of the product of deviation of x and y series from their mean is zero, then the coefficient of correlation will be;

a) 1 **b)** -1 **c)** 0 **d)** None of these

Sol. Option c)

Given $\sum(X - \overline{X})(Y - \overline{Y}) = 0$

Formula, $r = \frac{\sum(X-\overline{X})(Y-\overline{Y})}{N\times\sigma_x\times\sigma_y}$

$= \frac{0}{N\times\sigma_x\times\sigma_y} = \mathbf{0}$

Q.3) The coefficient of correlation between x and y series from the following data:

	x Series	**y Series**
Number of pairs of observations	15	15
Arithmetic Mean	25	18
Standard Deviation	3.01	3.03
Sum of squares of deviation from mean	136	138

Sum of the product of the deviations of x and y series from their respective means = 122, is:

a) 0.89 **b)** 0.99 **c)** 0.69 **d)** 0.91

Sol. Option a)

Given; $\sigma_x = 3.01, \sigma_y = 3.03, \sum x^2 = 136,\ \sum y^2 = 138, \sum xy = 122$

Where $x = X - \overline{X}; y = Y - \overline{Y}$

Formula; $r = \frac{\sum xy}{\sum x^2\times\sum y^2} = \frac{122}{\sqrt{136\times138}} = \mathbf{0.89}$

Q.4) What is the value of correlation coefficient due to Pearson on the basis of the following data:

x:	−9	−8	−7	−6	−5	0	5	4	3	2	1
y:	25	16	12	5	4	2	4	5	12	16	25

a) -0.287 **b)** 0.237 **c)** −0.17 **d)** 0

Sol. Option a)

x	y	xy	x^2	y^2
-9	25	-225	81	625
-8	16	-128	64	256
-7	12	-84	49	144
-6	5	-30	36	25
-5	4	-20	25	16
0	2	0	0	4
5	4	20	25	16
4	5	20	16	25
3	12	36	9	144
2	16	32	4	256
1	25	25	1	625
$\sum x = -20$	$\sum y = 126$	$\sum xy = -354$	$\sum x^2 = 310$	$\sum y^2 = 2136$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$\Rightarrow \frac{11\times(-354)-(-20)(126)}{\sqrt{11\times310-(-20)^2}\sqrt{11\times2136-(126)^2}}$$

$$\Rightarrow \frac{-3894+2520}{54.86\times87.29} = \frac{-1374}{4788.72} = \mathbf{-0.287}$$

Q.5) Find the coefficient of correlation between the heights of brothers and sisters from the following data: -

Heights of brothers (in cm): (x)	65	66	67	68	69	70	71
Heights of sisters (in cm): (y)	67	68	66	69	72	72	69

a) 0.88 **b)** 0.14 **c)** 0.67 **d)** None of these

Sol. Option c)

Let the heights of brothers be denoted by x and that of sisters by y, then

$$\bar{x} = \frac{65+66+67+68+69+70+71}{7} = \mathbf{68}$$

$$\bar{y} = \frac{67+68+66+69+72+72+69}{7} = \mathbf{69}$$

Computation of Correlation Coefficient.

x	$dx = (x-68)$	$dx^2 = (x-68)^2$	y	$dy = (y-69)$	$dy^2 = (y-69)^2$	$dx\,dy$
65	−3	9	67	−2	4	6
66	−2	4	68	−1	1	2
67	−1	1	66	−3	9	3
68	0	0	69	0	0	0
69	1	1	72	3	9	3
70	2	4	72	3	9	6
71	3	9	69	0	0	0
	$\sum dx = 0$	$\sum dx^2 = 28$		$\sum dy = 0$	$\sum dy^2 = 32$	$\sum dxdy = 20$

$$\text{Now, } r = \frac{n\sum dx\,dy - \sum dy \sum dx}{\sqrt{n\sum dx^2 - (dx)^2}\sqrt{n\sum dy^2 - (dy)^2}} = \frac{7\sum dx\,dy - 0}{\sqrt{7\sum dx^2 - 0}\sqrt{7\sum dy^2 - 0}}$$

$$= \frac{7\sum dx\,dy}{7\sqrt{\sum dx^2 \times (\sum dy^2)}} = \frac{20}{\sqrt{28\times32}}$$

$$= \frac{5}{7.48} = \mathbf{0.67}$$

Q.6) The following results relate to bivariate data on (x, y):

$\sum xy = 414,\ \sum x = 120,\ \sum y = 90,\ \sum x^2 = 600,\ \sum y^2 = 300, n = 30$. Later on, it was known that two pairs of observations (12, 11) and (6, 8) were wrongly taken, the correct pairs of observations being (10, 9) and (8, 10). The corrected value of the correlation coefficient is

a) 0.752 **b)** 0.768 **c)** 0.846 **d)** 0.953

Sol. Option c)

$\sum xy = 414,\ \sum x = 120,\ \sum y = 90,\ \sum x^2 = 600,\ \sum y^2 = 300$

$\sum xy\,(correct) = 414 - 12\times11 - 6\times8 + 10\times9 + 8\times10$

$= 414 - 132 - 48 + 90 + 80$

$\mathbf{\sum xy\,(correct) = 404}$

$\sum x^2\,(correct) = \sum x^2\,(Incorrect) - (Incorrect)^2 + (correct)^2$

$= 600 - (12)^2 - (6)^2 + (10)^2 + 8^2$

$= 600 - 144 - 36 + 100 + 64$

$\sum x^2\,(correct) = 584$

$$\sum y^2\,(correct) = \sum y^2\,(Incorrect) - (Incorrect)^2 + (correct)^2$$
$$\sum y^2\,(correct) = 300 - 11^2 - 8^2 + 9^2 + 10^2$$
$$\sum y^2\,(correct) = 300 - 121 - 64 + 81 + 100$$
$$\sum y^2\,(correct) = 296$$
$$\sum x(correct) = \sum x(Incorrect) - Incorrect + Correct$$
$$= 120 - 12 - 6 + 10 + 8$$
$$\sum x(correct) = 120$$

$$\sum y(correct) = \sum y(Incorrect) - Incorrect + Correct$$
$$\sum y(correct) = 90 - 11 - 8 + 9 + 10$$
$$\sum y(correct) = 90$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$
$$\Rightarrow \frac{30 \times (404) - (90)(120)}{\sqrt{30 \times 584 - (120)^2}\sqrt{30 \times 296 - (90)^2}}$$
$$= \frac{12120 - 10800}{\sqrt{3120}\sqrt{780}}$$
$$= \frac{1320}{1560} = \mathbf{0.846}$$

Q.7) The coefficient of correlation r between x and y when: x and y when: $\text{Cov}(x, y) = -16.5$, $\text{Var}(x) = 2.89$, $\text{Var}(y) = 100$ is:

a) -0.97 **b)** 0.97 **c)** 0.89 **d)** -0.89

Sol. Option a)

Coefficient of correlation

$$r = \frac{Cov(x,y)}{\sigma_x.\sigma_y}$$

$$= \frac{-16.5}{\sqrt{2.89} \times \sqrt{100}}$$

$$= \frac{-16.5}{1.7 \times 10} = \mathbf{-0.97}$$

Q.8) If the covariance between x and y is 30, the variance of x is 25, and the correlation coefficient is 0.5, then what is the variance of y?

a) 169 b) 81 c) 144 d) None of these

Sol. Option c)

Given, $\text{cov}(x, y) = 30$

$\text{Var}(x) = 25$ and $\text{r}(x, y) = 0.5$

$$\Rightarrow \text{r}(x, y) = \frac{\text{cov}(x,y)}{\sqrt{\text{Var}(x) \times \text{Var}(y)}}$$

$$\Rightarrow 0.5 = \frac{30}{\sqrt{25 \times \text{Var}(y)}}$$

Check through the option

a) $0.5 \neq 0.46$
b) $0.5 \neq 0.66$
c) $0.5 = 0.5$ which is correct

$\therefore$ **Var(y) = 144**

Q.9) For two variables x and y, it is known that $\text{cov}(x,y) = 8, r = 0.4$, variance of x is 16 and sum of squares of deviation of y from its mean is 250. The number of observations for this bivariate data is;

a) 7 **b)** 8 **c)** 9 **d)** 10

Sol. Option d)

$r = 0.4$

$\text{Cov.}(x,y) = 8$

$Variance\ of\ x = 16$

$\sum(y-\bar{y})^2 = 250$

$(\sigma_x)^2 = Variance$

$(\sigma_x)^2 = 16$

$\sigma_x = 4$

$r = \frac{Cov.(x,y)}{\sigma_x \sigma_y}$

$0.4 = \frac{8}{4 \times \sigma_y}$

$\sigma_y = 5$

$\sigma_y = \sqrt{\frac{\sum(y-\bar{y})^2}{N}}$

$5 = \sqrt{\frac{250}{N}}$

$25 = \frac{250}{N}$

$\boldsymbol{N = 10}$

Q.10) From the following data

x:	2	3	5	4	7
y:	4	6	7	8	10

The coefficient of correlation was found to be 0.93. What is the correlation between u and vas given below?

u:	-3	-2	0	-1	2
v:	-4	-2	-1	0	2

a) −0.93 **b)** 0.93 **c)** 0.57 **d)** −0.57

Sol. Option b)

$r_{uv} = r_{xy}$ (∵ Correlation co-efficient is invariant with change of origin)

= 0.93

Q.11) If the relationship between two variables x and y in given by $2x + 3y + 4 = 0$, then the valueof the correlation coefficient between x and y is

a) 0 **b)** 1 **c)** -1 **d)** Negative

Sol. Option c)

$2x + 3y + 4 = 0$

$Put\ x = 1,$

$2(1) + 3y + 4 = 0 \Rightarrow 3y = -6 \Rightarrow y = -2.$

$Put\ x = 2,$

$2(2) + 3y + 4 = 0 \Rightarrow 3y = -8 \Rightarrow y = \frac{-8}{3} = -2.66$

∴ When the value of x is increased, then the value of y decreases, so the inverse relationship = −1.

Q.12) If the sum of squares of difference of ranks, given by two judges A and B, of 9 students is 27, what is the value of rank correlation coefficient?

a) 0.7 **b)** 0.65 **c)** 0.775 **d)** 0.75

Sol. Option c)

Given, N = 9, $\sum D^2 = 27$

$r_R = 1 - \frac{6\sum D^2}{N(N^2-1)}$

$r_R = 1 - \frac{6\times27}{9(9^2-1)}$

$r_R = 1 - \frac{6\times27}{9(9^2-1)}$

$\therefore r_R = 1 - \frac{162}{720} = \mathbf{0.775}$

Q.13) While computing rank correlation coefficient between profit and investment for the last 6 years of a company the difference in rank for a year was taken 3 instead of 4. What is the rectified rank correlation coefficient if it is known that the original value of rank correlation coefficient was 0.4?

a) 0.3 **b)** 0.2 **c)** 0.25 **d)** 0.28

Sol. Option b)

$r_R = 1 - \frac{6\sum D^2}{N(N^2-1)}$

$\Rightarrow 0.4 = 1 - \frac{6\sum D^2}{6\,(6^2-1)}$

$\Rightarrow 0.6 = \frac{6\sum D^2}{6\,(6^2-1)}$

$\Rightarrow \sum D^2 = 0.6 \times 35 = 21$

$\therefore$ rectified $\sum D^2 = 21 + 4^2 - 3^2$

$= 21 + 16 - 9 = 28$

$\therefore$ Rectified $r_R = 1 - \frac{6\times28}{6\,(6^2-1)}$

$= 1 - \frac{28}{35}$

$= 1 - 0.8 = \mathbf{0.2}$

Q.14) The value of Spearman's rank correlation coefficient of a certain number of observations was to be $\frac{2}{3}$. The sum of the squares of differences between the corresponding ranks was 55. Find the number of Pairs.

a) 10 **b)** 12 **c)** 11 **d)** None of these

Sol. Option a)

Given, r = $\frac{2}{3}$, $\sum D^2 = 55$

$\therefore r = 1 - \frac{6\sum D^2}{N(N^2-1)}$

$\Rightarrow \frac{2}{3} = 1 - \frac{6(55)}{N(N^2-1)}$

$\Rightarrow \frac{2}{3} - 1 = -\frac{330}{N(N^2-1)}$

$\Rightarrow -\frac{1}{3} = -\frac{330}{N(N^2-1)}$

$\Rightarrow N(N^2-1) = 990$

Check from the option

$\Rightarrow 10(10^2-1) = 990$

$\therefore \boldsymbol{N = 10}$

Q.15) Following are the marks of 10 students in Botany and Zoology:

Serial No.:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Marks in Botany:	58	43	50	19	28	24	77	34	29	75
Marks in Zoology:	62	63	79	56	65	54	70	59	55	69

The coefficient of rank correlation between marks in Botany and Zoology is

a) 0.65 **b)** 0.70 **c)** 0.72 **d)** 0.75

Sol. Option c)

S. No.	x	y	R_x	R_y	d	d^2
1	58	62	3	6	−3	9
2	43	63	5	5	0	0
3	50	79	4	1	3	9
4	19	56	10	8	2	4
5	28	65	8	4	4	16
6	24	54	9	10	−1	1
7	77	70	1	2	−1	1
8	34	59	6	7	−1	1
9	29	55	7	9	−2	4
10	75	69	2	3	−1	1
						46

$$r_R = 1 - \frac{6\sum D^2}{N(N^2-1)}$$

$$r_R = 1 - \frac{6\times 46}{10\times 99} = \mathbf{0.72}$$

Q.16) What is the value of Rank correlation coefficient between the following marks in Physicsand Chemistry:

Roll No.:	**1**	**2**	**3**	**4**	**5**	**6**
Marks in Physics:	25	30	46	30	55	80
Marks in Chemistry:	30	25	50	40	50	78

a) 0.782 **b)** 0.696 **c)** 0.932 **d)** 0.857

Sol. Option d)

S. No.	x	y	R_x	R_y	$d = R_x - R_y$	d^2
1	25	30	6	5	1	1
2	30	25	4.5	6	−1.5	2.25
3	46	50	3	2.5	0.5	0.25
4	30	40	4.5	4	0.5	0.25
5	55	50	2	2.5	-0.5	0.25
6	80	78	1	1	0	0

$$r_R = 1 - \frac{6\left[\frac{\sum D^2+\sum(m_1{}^3-m_1)+\sum(m_2{}^3-m_2)+------+\sum(m_n{}^3-m_n)}{12}\right]}{N(N^2-1)}$$

$$= \frac{\sum(m_1{}^3-m_1)+\sum(m_2{}^3-m_2)}{12}$$

$$= \frac{(2^3-2)+(2^3-2)}{12} = \frac{6+6}{12} = 1$$

$$r_R = 1 - \frac{6(4+1)}{6\times 35} = \mathbf{0.857}$$

Q.17) For 10 pairs of observations, number of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation?

a) $\sqrt{0.2}$ **b)** 1/3 **c)** -1/3 **d)** $-\sqrt{0.2}$

Sol. Option c)

Given $c = 4, n = 10,$

$So, m = n - 1 = 10 - 1 = 9$

$2c - m = 2 \times 4 - 9 = -1\,(-ve)$

$$\because r_c = \pm\sqrt{\frac{\pm(2c-m)}{m}}$$

$$\because r_c = -\sqrt{\frac{-(2 \times 4 - 9)}{9}}$$

$$\Rightarrow r_c = \frac{(-1)}{3} = -\frac{1}{3}$$

Q.18) The coefficient of concurrent deviation for p pairs of observations was found to be $\sqrt{1/3}$ If the number of concurrent deviations was found to be 6, then the value of p is.

a) 10 **b)** 9 **c)** 8 **d)** None of these

Sol. Option a)

$$r_c = \pm\sqrt{\pm\frac{(2\,c-m)}{m}}$$

$n = p \text{ and } m = n - 1$

$m = p - 1$

$$\therefore \frac{1}{\sqrt{3}} = \sqrt{\frac{2\times6-(p-1)}{(p-1)}}$$

$$\therefore \frac{1}{3} = \frac{12-p+1}{p-1}$$

$\Rightarrow p - 1 = 39 - 3p$

$\Rightarrow 4p = 40$

$\therefore \boldsymbol{p = 10}$

Q.19) What is the coefficient of concurrent deviations for the following data:

Year:	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**
Price:	35	38	40	33	45	48	49	52
Demand:	36	35	31	36	30	29	27	24

a) -1 **b)** 0.43 **c)** 0.5 **d)** $\sqrt{2}$

Sol. Option a)

Year	Price	Sign. of Deviation of Price	Demand	Sign. of Deviation of Demand	Product of Deviation
1996	35		36		
1997	38	+	35	–	–
1998	40	+	31	–	–
1999	33	–	36	+	–
2000	45	+	30	–	–
2001	48	+	29	–	–
2002	49	+	27	–	–
2003	52	+	24	–	–

$m = n - 1$
$m = 8 - 1 \Rightarrow m = 7$
$= c = 0$

$$r_c = \pm\sqrt{\pm\frac{(2c-m)}{m}} \Rightarrow r = \pm\sqrt{\pm\frac{(0-7)}{7}}$$

$$= -\sqrt{\frac{-7}{7}} = -1$$

Q.20) If x and y are independent, the value of regression coefficient b_{yx} is equal to

a) 1 **b)** 0 **c)** ∞ **d)** any positive value

Sol. Option b)

If x and y are independent, the value of regression coefficient b_{yx} is equal to 0.

Q.21) If each value of x is divided by 2 and of y is multiplied by 2. Then the new b_{yx} is

a) Same as b_{yx} **b)** Twice of b_{yx} **c)** Four time of b_{yx} **d)** Eight times of b_{yx}

Sol. Option c)

If each value of x is divided by 2 and of y is multiplied by 2. Then the new b_{yx} is four time of b_{yx}.

$$= b_{vu} = \frac{2}{\frac{1}{2}} b_{yx}$$

$$= b_{vu} = 4b_{yx}$$

Q.22) If $8y–10x = 30$ is the regression line of y on x and the coefficient of correlation between x and y is 0.85, what is the value of the regression coefficient of x on y?

a) 0.578 **b)** 0.45 **c)** 0.6 **d)** None of these

Sol. Option a)

$\because$ Line of regression y on x be
$8y - 10x = 30$
$\Rightarrow y = \frac{5}{4}x + \frac{15}{4}$
$\therefore b_{yx} = \frac{5}{4} = 1.25$
$r^2 = b_{xy} \times b_{yx}$
$\Rightarrow (0.85)^2 = b_{xy} \times 1.25$
$\therefore b_{xy} = \frac{0.7225}{1.25} = \mathbf{0.578}$

Q.23) The coefficient of regression of Y on X is $b_{yx} = 1.2$. If $u = \frac{x-100}{2}$ and $v = \frac{y-200}{3}$ find b_{vu}.

a) 0.9 **b)** 0.8 **c)** 0.7 **d)** None of these

Sol. Option b)

Given, $b_{yx} = 1.2$
$u = \frac{x-100}{2}$
and $v = \frac{y-200}{3}$

$$b_{vu} = \frac{\frac{1}{3}}{\frac{1}{2}} b_{yx}$$

$$\therefore b_{vu} = \frac{2}{3} b_{yx} = \frac{2}{3} \times 1.2 = \mathbf{0.8}$$

Q.24) In a partially destroyed laboratory record of an analysis of regression data, the following data are legible: Variance of $x = 9$. Regression equations $8x - 10y + 66 = 0$ and $40x - 18y = 214$. Find the mean values of X and Y.

a) 17 & 14 **b)** 13 & 17 **c)** 10 & 15 **d)** 15 & 10

Sol. Option b)

The two regression equations are:

$8x - 10y + 66 = 0$

i.e., $8x - 10y = -66$ ...(i)

$40x - 18y = 214$ ...(ii)

By $5 \times (i) - (ii)$, we get

$= 40x - 50y = -30$

$= 40x - 18y = 14$

$\underline{\quad - \quad + \quad -}$

$-32y = -544$

$\therefore y = 17$

Substituting $y = 17$ in (i), we get,

$8x - 10 \times 17 = -66$

$\therefore x = 13$

Since the point of intersection of two regression line is $(\bar{x}, \bar{y})$

$\bar{x}$ = mean value of $x = 13$

$\bar{y}$ = mean value of $y = 17$

Q.25) The coefficient of correlation between ages of husbands and wives in a community was found to be 0.8, the average of the husband's age was 25 years and that of wives' age was 22 years. Their standard deviations were 4 and 5 respectively. Find the expected age of the husband when wife's age is 18 years, with the help of regression equation.

a) 20 **b)** 22

c) 24.22 **d)** 22.44

Sol. Option d)

Let ages of husbands and wives be denoted by X and Y respectively.

Given, $\bar{X} = 25$, $\bar{Y} = 22$, $\sigma_x = 4$, $\sigma_y = 5$ and $r = 0.8$

$b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.8 \times \frac{4}{5} = 0.64$

Regression line of X on Y:

$x - \bar{X} = b_{xy}(y - \bar{Y})$

$\Rightarrow x - 25 = 0.64(y - 22)$

$\Rightarrow x = 0.64y + 10.92$

The expected age of husband when wife's age is 18 years is obtained by substituting y = 18

$\therefore 0.64 \times 18 + 10.92 = \mathbf{22.44}$ **years**

Q.26) Given the regression equations as $3x + y = 13$ *and* $2x + 5y = 20$, which one is the regression equation of y on x?

a) 1st equation **b)** 2nd equation **c)** both (a) and (b) **d)** None of these

Sol. Option b)

$3x + y = 13$______(I)

$2x + 5y = 20$________(II)

Let the line of regression x *on* y be 1st equation

$\therefore x = \frac{-1}{3}y + \frac{13}{3}$

$\therefore b_{xy} = {}^{-1}/_{3}$

and line of regression y on x be

$2x + 5y = 20 \Rightarrow y = \frac{-2}{5}x + 4$

$\therefore b_{yx} = {}^{-2}/_{5}$

Now $r = \pm\sqrt{b_{xy} \times b_{yx}}$

$\therefore r = -\sqrt{\left(\frac{-1}{3}\right)\left({}^{-2}/_{5}\right)} = -\sqrt{\frac{2}{15}} > -1$

Which is correct

Hence the line of regression y on x be $\mathbf{2x + 5y = 20}$

Q.27) If $4y - 5x = 15$ is the regression line of y on x and the coefficient of correlation between x and y is 0.75, what is the value of the regression coefficient of x on y?

a) 0.45 **b)** 0.9375 **c)** 0.6 **d)** None of these

Sol. Option a)

$\because$ Line of regression y on x be

$4y - 5x = 15 \Rightarrow y = \frac{5}{4}x + \frac{15}{4}$

$\therefore b_{yx} = {}^{5}/_{4} = 1.25$

$r^2 = b_{xy} \times b_{yx}$

$\Rightarrow (0.75)^2 = b_{xy} \times 1.25$

$\Rightarrow b_{xy} = \frac{0.5625}{1.25} = \mathbf{0.45}$

Q.28) If the regression coefficient of y on x, the coefficient of correlation between x and y and variance of y are $-3/4$, $\sqrt{\frac{3}{2}}$ and 4 respectively, what is the variance of x?

a) $2\sqrt{3/2}$ **b)** 32/3 **c)** 4/3 **d)** 4

Sol. Option b)

$b_{yx} = r\frac{\sigma_y}{\sigma_x} \Rightarrow \frac{-3}{4} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{4}}{\sigma_x}$

$\Rightarrow \sigma_x = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{2}{-3} \times 4$

$\Rightarrow \sigma_x^2 = \left(\frac{-4\sqrt{2}}{\sqrt{3}}\right)^2 = \frac{32}{3}$

Q.29) If $y = 3x + 4$ is the regression line of y on x and the arithmetic mean of x is -1, what is the arithmetic mean of y?

a) 1 **b)** -1 **c)** 7 **d)** None of these

Sol. Option a)

The line of regression *y on x be*

$\Rightarrow y = 3x + 4$

$\therefore\ \bar{Y} = 3\bar{X} + 4$

$= 3 \times (-1) + 4$

$= -3 + 4 = 1$

$\therefore\ \bar{Y} = 1$

Q.30) Given below the information about the capital employed and profit earned by a companyover the last twenty-five years:

	Mean	**SD**
Capital employed (0000 `)	62	5
Profit earned (000 `)	25	6

Correlation coefficient between capital employed and profit = 0.92. The sum of the Regression coefficients for the above data would be:

a) 1.871 **b)** 2.358 **c)** 1.968 **d)** 2.346

Sol. Option a)

Mean of $x = 62$ *Mean of* $y = 25$

$\sigma_x = 5$ $\sigma_y = 6$ $r = 0.92$

$\Rightarrow b_{yx} = r \times \frac{\sigma_y}{\sigma_x} \Rightarrow b_{yx} = 0.92 \times \frac{6}{5}$

$\Rightarrow b_{yx} = 1.104$

$\Rightarrow b_{xy} = r \times \frac{\sigma_x}{\sigma_y} \Rightarrow b_{xy} = 0.92 \times \frac{5}{6}$

$\Rightarrow b_{xy} = 0.767$

Sum of regression coefficient $= \mathbf{0.767 + 1.104 = 1.871}$

Q.31) The two lines of regression are given by

$8x + 10y = 25$ *and* $16x + 5y = 12$ respectively.

If the variance of x is 25, what i.e., the standard deviation of y?

a) 16 **b)** 8 **c)** 64 **d)** 4

Sol. Option b)

$8x + 10y = 25, \quad 16x + 5y = 12$

Assume it is y on x $\Rightarrow 10y = 25 - 8x$

$\Rightarrow y = \frac{25}{10} - \frac{8}{10}x$

$\Rightarrow y = \frac{5}{2} - \frac{4}{5}x$

Assume it is x on y $\Rightarrow 16x = 12 - 5y$

$\Rightarrow x = \frac{12}{16} - \frac{5}{16}y$

$\Rightarrow x = 0.75 - \frac{5}{16}y$

$\Rightarrow r = \pm\sqrt{b_{yx} \times b_{xy}}$

$\Rightarrow r = -\sqrt{\frac{4}{5} \times \frac{5}{16}}$

$\Rightarrow r = -0.5$

$\Rightarrow b_{yx} = r\frac{\sigma_y}{\sigma_x} \Rightarrow \frac{-4}{5} = 0.5 \times \frac{\sigma_y}{5} \Rightarrow \boldsymbol{\sigma_y = 8}$

Q.32) If $u = 2x + 5$ and $v = -3y - 6$ and regression coefficient of y on x is 2.4, what is the regression coefficient of v on u?

a) 3.6 **b)** -3.6 **c)** 2.4 **d)** -2.4

Sol. Option b)

$b_{yx} = 2.4$

$u = 2x + 5 \therefore Scale\ u = 2$

$v = -3y - 6 \therefore Scale\ v = -3$

$b_{vu} = \frac{Scale\ v}{Scale\ u}\ b_{yx}$

$= \frac{-3}{2} \times 2.4 = \mathbf{-3.6}$

Q.33) If r = 0.6 then the coefficient of non-determination is:

a) 0.4 **b)** -0.6 **c)** 0.36 **d)** 0.64

Sol. Option d)

Co-efficient of non-determination

$= 1 - r^2 = 1 - (0.6)^2 = \mathbf{0.64}$

Q.34) If the coefficient of correlation between two variables is -0 .9, then the coefficient of determination is

a) 0.9 **b)** 0.81 **c)** 0.1 **d)** 0.19

Sol. Option b)

Coefficient of determination is $r^2 = (-0.9)^2 = \mathbf{0.81}$

Q.35) If the coefficient of correlation between two variables is 0.7 then the percentage of variation unaccounted for is

a) 70% **b)** 49% **c)** 30% **d)** 51%

Sol. Option d)

Given, $r = 0.7$

Percentage of variation accounted for = $r^2 = (0.7)^2 = 0.49$

∴ Percentage of variation unaccounted for = $1 - 0.49 \times 100 = \mathbf{51\%}$

Q.36) In a bivariate data: $\sigma_x = 11, r = 0.60$, then the standard error of estimate of X on Y is given by:

a) 7.24 **b)** 7.4 **c)** 8.8 **d)** 7.5

Sol. Option c)

Standard error of estimates of X on Y

$= \sigma_x(1 - r^2)^{1/2} = 11 \times (1 - (0.6)^2)^{1/2}$

$= 11 \times 0.8 = \mathbf{8.8}$

Q.37) If the slope of the regression line is calculated to be 5.5 and the intercept 15 then the value of Y when x is 6 is

a) 88 **b)** 48 **c)** 18 **d)** 78

Sol. Option b)

The regression line equation:

$$Y = a + bx \qquad [\therefore a = intercept\ and\ b = slope]$$
$$Y = 15 + 5.5x$$
$$Y = 15 + 5.5(6) = 15 + 33 = \mathbf{48}$$

Q.38) For a number of towns, the coefficient of rank correlation between the people living below the poverty line and increase of population is 0.50. If the sum of squares of the differences in ranks awarded to these factors is 82.50, find the number of towns.

a) 10 **b)** 20 **c)** 30 **d)** 40

Sol. Option a)

As given $r = 0.50, \sum d^2 = 82.50$.

Thus $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$

$$0.50 = 1 - \frac{6\times 82.50}{n(n^2-1)}$$
$$= n(n^2-1) = 990$$
$$= n(n^2-1) = 10(10^2-1)$$

$\therefore$ **n=10** as n must be a positive integer.

Q.39) Find the coefficient of correlation r: when us probable error is 0.2 and the number of pairs of items is 9.

a) 0.194 **b)** 0.332 **c)** 0.227 **d)** 0.442

Sol. Option b)

Here P.E. = 0.2, n = 9, Also P.E. = $0.6745 \times \frac{(1-r^2)}{\sqrt{n}}$

$$\Rightarrow 0.2 = 0.6745 \times \frac{(1-r^2)}{\sqrt{9}}$$
$$= 0.2 = \frac{0.6745 - 0.6745\,r^2}{3}$$
$$\Rightarrow 0.6745 - 0.6745\,r^2 = 0.2 \times 3$$
$$\Rightarrow 0.6745\,r^2 = 0.6745 - 0.6$$
$$\Rightarrow 0.6745\,r^2 = 0.6745 - 0.6$$
$$\Rightarrow r^2 = \frac{0.0745}{0.6745} \Rightarrow r = \sqrt{\frac{0.0745}{0.6745}}$$
$$\Rightarrow r = \sqrt{0.1105} = \mathbf{0.332}$$

Q.40) The following table gives the age of bikes of a certain make and annual maintenance costs. Obtain the regression equation for costs related to age:

Age of bikes	4	6	8	10
Cost of maintenance	20	25	30	35

a) y = 10.75 + 3.5 x **b)** y = 10 + 2.5 x **c)** x = 30.5 + 2.5y **d)** None of these

Sol. Option b)

x	y	x^2	y^2	xy
4	20	16	400	80
6	25	36	625	150
8	30	64	900	240
10	35	100	1225	350
$\sum x = 28$	$\sum y = 110$	$\sum x^2 = 216$	$\sum y^2 = 3150$	$\sum xy = 820$

$\bar{X} = \frac{\sum x}{n} = \frac{28}{4} = 7$

$\bar{Y} = \frac{\sum y}{n} = \frac{110}{4} = 27.5$

$b_{yx} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{4 \times 820 - (28)(110)}{4 \times 216 - (28)^2}$

$= \frac{3280 - 3080}{864 - 784} = \frac{200}{80} = 2.5$

Regression equation of y on x is given by

$y - \overline{Y} = b_{yx}(x - \overline{X})$

$\Rightarrow y - 27.5 = 2.5x - 2.5 \times 7$

$\therefore \mathbf{y = 10 + 2.5\,x}$

Q.41) A simple random sample of size 36 is drawn from a finite population consisting of 101 units. If the population Standard Deviation is 12.6, find the Standard Error of sample mean when the sample is drawn with replacement.

a) 2.1
b) 1.69
c) 2.23
d) None of these

Sol. Option a)

Standard Error of mean = $\frac{\sigma}{\sqrt{n}}$

$= \frac{12.6}{\sqrt{36}} = \frac{12.6}{6}$

$\therefore$ Standard error of mean = **2.1**

Q.42) In fitting of a regression of y on x to a bivariate distribution consisting of 9 observations, the explained and unexplained variations were computed as 24 and 36 respectively. Find the coefficient of determination and the standard error of estimate of Y on X.

a) 0.2 & 4
b) 0.1 & 3
c) 0.4 & 2
d) 0.3 & 1

Sol. Option c)

Given, n = 9, explained variation = 24 and unexplained variation = 36

$\therefore$ Total variation = Explained Variation + Unexplained Variation = 24 + 36 = 60

(i) Coefficient of determination (r^2) is given by

$$r^2 = \frac{\textit{Explained Variation}}{\textit{Total Variation}} = \frac{24}{60} = \mathbf{0.4}$$

(ii) Standard error of estimate of Y on X is given by

$$S_{yx} = \sqrt{\frac{\textit{Unexpalined Variation}}{n}} = \sqrt{\frac{36}{9}} = \mathbf{2}$$

Probability

Q.1) If P(A) = 5/9, then the odds against the event A is

a) 5 : 9 **b)** 5 : 4 **c)** 4 : 5 **d)** 5 : 14

Sol. Option c)

$\because P(A) = \frac{5}{9}$

$\therefore P(A') = 1 - P(A) = 1 - \frac{5}{9} = {}^{4}/_{9}$

∴ P (odd against the event A)

$= \frac{P(A')}{P(A)} = \frac{{}^{4}/_{9}}{{}^{5}/_{9}} = {}^{4}/_{5}$

Q.2) If A denotes that a student reads in a school and B denotes that he plays cricket, then

a) $P(A \cup B) = 1$ **b)** $P(A \cap B) = 1$ **c)** $P(A \cap B) = 0$ **d)** P(A) = P(B).

Sol. Option c)

A & B are mutually exclusive events ∴ $\boldsymbol{P(A \cap B) = 0}$

Q.3) Which of the following set of function define a probability space on S = {a_1, a_2, a_3}

a) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{2}, P(a_3) = \frac{1}{4}$

b) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{6}, P(a_3) = \frac{1}{2}$

c) $P(a_1) = P(a_2) = \frac{2}{3}, P(a_3) = \frac{1}{4}$

d) None

Sol. Option b)

$\boldsymbol{P(a_1) + P(a_2) + P(a_3) = 1}$

Option (a)

$\frac{1}{3} + \frac{1}{2} + \frac{1}{4} = \frac{4+6+3}{12} = \frac{13}{12} = 1$ (×)

Option (b)

$\frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{2+1+3}{6} = \frac{6}{6}\ 1$ (√)

Q.4) For any two events A and B,

a) P(A) + P(B) > P(A ⍰ B)

b) P(A) + P(B) < P(A ⍰ B)

c) P(A) + P(B) ≥P(A ⍰ B)

d) P(A) x P(B) ≤ P(A ⍰ B)

Sol. Option c)

$\because P(A) \geq P(A \cap B), P(B) \geq P(A \cap B)$

$\therefore P(A) + P(B) > P(A \cap B)$

[When A & B both are not impossible events]

$P(A) + P(B) = P(A \cap B)$

[When A & B, both are impossible events]

d) 5 : 14

cricket, then

d) P(A) = P(B).

a₂, a₃}

Q.5) For any two events A and B,

a) P(A–B) = P(A) – P(B)

b) P(A–B) = P(A) – P(A ⍰ B)

c) P(A–B) = P(B) – P(A ⍰ B)

d) P(B–A) = P(B) + P(A ⍰ B).

Sol. Option b)

$$P\,(A - B) = P(A \cap B') = \boldsymbol{P(A) - P(A \cap B)}$$

Q.6) The classical definition of probability is based on the feasibility at subdividing the possibleoutcomes of the experiments into

a) Mutually exclusive and exhaustive

b) Mutually exclusive and equally likely

c) Exhaustive and equally likely

d) Mutually exclusive, exhaustive and equally likely cases.

Sol. Option d)

Mutually exclusive, exhaustive and equal likely cases.

Q.7) The probability of occurrence of at least one of the 2 events A and B (which may not be mutually exclusive) is given by

a) P(A+B)= P(A)– P(B)

b) P(A+B)= P(A)+ P(B)– P(AB)

c) P(A+B)= P(A)– P(B)+ P(AB)

d) P(A+B)= P(A)+P (B)

Sol. Option b)

$$P(A \cup B) = P\,(A) + P\,(B) - P\,(A \cap B)$$
$$\Rightarrow P\,(A + B) = P\,(A) + \,P(B) - P\,(AB)$$

Q.8) The conditional probability of an event B on the assumption that another event A has actually occurred is given by

a) P(B/A)= P(AB)/P(A)

b) P(A/B)= P(AB)/ P(A)

c) P(B/A)= P(AB)

d) P(A/B)= P(AB)/ P(A)P(B)

Sol. Option a)

$$P\left({}^{B}\!/_{A}\right) = \frac{P\,(A \cap B)}{P\,(A)} \Rightarrow P\,\left(\frac{B}{A}\right) = \frac{\boldsymbol{P\,(AB)}}{\boldsymbol{P\,(A)}}$$

Q.9) If A is an event and A^C its complementary event then

a) $P(A)=P(A^C)-1$ **b)** $P(A^C)=1-P(A)$ **c)** $P(A)=1 + P(A^C)$ **d)** None

Sol. Option b)

$$P\,(A') = \boldsymbol{1 - P\,(A)}$$

Q.10) If P(A)= 1/5, P(B)= 1/2 and A and B are mutually exclusive then P(AB) is

a) 7/10 **b)** 3/10 **c)** 1/5 **d)** None

Sol. Option d)

$\because$ $A\&B$ are mutually exclusive

$$\therefore P\,(A \cap B) = 0 \Rightarrow P\,(AB) = \boldsymbol{0}$$

Q.11) The number of conditions to be satisfied by three events A, B and C for complete independence is

a) 2 **b)** 3 **c)** 0 **d)** any number

Sol. Option c)

For complete independence of A, B & C

$P(A \cap B) = 0, P(B \cap C) = 0, P(C \cap A) = 0$

& $P(A \cap B \cap C) = \mathbf{0}$

Q.12) If x and y are independent, then

a) $E(xy) = E(x) \times E(y)$

b) $E(xy) = E(x) + E(y)$

c) $E(x - y) = E(x) + E(y)$

d) $E(x - y) = E(x) + x\,E(y)$

Sol. Option a)

$E(x\,y) = \mathbf{E(x) \times E(y)}$

Q.13) If a random variable x assumes the values x_1, x_2, x_3, x_4 with corresponding probabilitiesp_1, p_2, p_3, p_4 then the expected value of x is

a) $p_1 + p_2 + p_3 + p_4$

b) $x_1 p_1 + x_2 p_3 + x_3 p_2 + x_4 p_4$

c) $p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$

d) None of these

Sol. Option c)

$E(x) = \mathbf{x_1 p_1 + x_2 p_3 + x_3 p_2 + x_4 p_4}$

Q.14) If two random variables x and y are related by y = 2 – 3 x, then the SD of y is given by

a) -3 × SD of x

b) 3 × SD of x

c) 9 × SD of x

d) 2 × SD of x

Sol. Option b)

$\mathbf{3 \times S.D\ of\ x}$

Q.15) If $P(A) = p$ and $P(B) = q$, then

a) P(A/B) ≤ p/q

b) P(A/B)≤ p/q

c) P(A/B) ≤ q/p

d) None of these

Sol. Option a)

$P(A) = p \text{ and } P(B) = q$

$\therefore P(A/_B) = \frac{P(A \cap B)}{P(B)} \leq \frac{P(A)}{P(B)}$

$\Rightarrow P(A/_B) \leq \frac{p}{q}$

Q.16) If P (A) = a, P (B) = b and P(A∩ B) = c then the expression of P (A'∩ B') in terms of a, b and c is

a) 1 – a – b – c

b) a + b – c

c) 1 + a – b – c

d) 1 – a – b + c

Sol. Option d)

$P(A) = a, P(B) = b, P(A \cap B) = C$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = a + b - c$

Now, $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - (a + b - c) = \mathbf{1 - a - b + c}$

Q.17) Sum of all probabilities of mutually exclusive and exhaustive events is equal to

a) 0

b) $\frac{1}{2}$

c) $\frac{3}{4}$

d) 1

Sol. Option d)

Sum of all probabilities of mutually exclusive and exhaustive events is equal to 1

Q.18) Let P be a probability function on S = $\{x_1, x_2, x_3\}$, $P(x_1) = \frac{1}{4}$ *and* $P(x_3) = \frac{1}{3}$ *then* $P(x_2)$ is equal to

a) 5/12 **b)** 7/12 **c)** 3/4 **d)** None

Sol. Option a)

$P\,(x_1) = {}^1/_4\,, P\,(x_3) = {}^1/_3\,, P\,(x_2) = ?$

$\because\ P(x_1) + P\,(x_2) + P\,(x_3) = 1$

$\Rightarrow \frac{1}{4} + P\,(x_2) + \frac{1}{3} = 1$

$\Rightarrow P\,(x_2) = 1 - \left(\frac{1}{4} + \frac{1}{3}\right)$

$P\,(x_2) = 1 - \frac{7}{12} = \mathbf{{}^5/_{12}}$

Q.19) If $P\,(\bar{A} \cup \bar{B}) = {}^5/_6$, P(A) = ½ and $P\,(\bar{B}) = {}^2/_3$, what is P (A ▯ B)?

a) 1/3 **b)** 5/6 **c)** 2/3 **d)** 4/9

Sol. Option c)

$P\,(\bar{A} \cup \bar{B}) = {}^5/_6 \Rightarrow P(\overline{A \cup B}) = {}^5/_6$

$\therefore P(A \cap B) = 1 - {}^5/_6 = {}^1/_6$

$P(A) = {}^1/_2$

$P\,(\bar{B}) = {}^2/_3\ \therefore P(B) = 1 - {}^2/_3 = {}^1/_3$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6}$

$= {}^2/_3$

Q.20) What is the probability that an ordinary year has 53 Sundays?

a) 1/7 **b)** 2/7 **c)** 3/7 **d)** None

Sol. Option a)

An ordinary year has 365 days

There are 52 weeks and 1 day left

Total outcome = 7 (Monday, Tuesday, Wednesday, Friday, Saturday, and Sunday)

Favourable outcomes = 1

The required probability = **1/7**

Q.21) A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A), given that P(B)=3/2 P(A) and P(C) = ½ P(B).

a) $\frac{4}{13}$ **b)** $\frac{3}{4}$ **c)** $\frac{3}{5}$ **d)** $\frac{3}{25}$

Sol. Option a)

Given that P(B)=3/2 P(A)

and P(C) = ½ P(B)

P(C) = ½ (3/2 P(A))

= P(C) = $\frac{3}{4}$ P(A)

Mutually exclusive and exhaustive = P(AUBUC) = 1

= P(AUBUC) = P(A)+P(B)+P(C) = 1

= P(A)+ $\frac{3}{2}$ P(A)+ $\frac{3}{4}$ P(A)=1

= $\frac{13}{4}$ P(A)=1

=P(A)= $\frac{4}{13}$

Q.22) If the probability of a horse A winning a race is 1/6 and the probability of a horse B winning the same race is 1/4, what is the probability that one of the horses will win

a) 5/12 **b)** 7/12 **c)** 1/12 **d)** None

Sol. Option a)

$P\ (A) = {}^{1}/_{6},\ P\ (B) = {}^{1}/_{4}$

P (one of the horse win) = $\frac{1}{6}+\frac{1}{4}=\frac{2+3}{12}={}^{5}/_{12}$

Q.23) If an unbiased coin is tossed twice, the probability of obtaining at least one tail is

a) 0.25 **b)** 0.50 **c)** 0.75 **d)** 1.00

Sol. Option c)

$S = \{HH,\ HT,\ TH,\ TT\}$

∴ P (Obtaining at least one tail) = $\frac{3}{4} = \mathbf{0.75}$

Q.24) Three coins are tossed together. The probability of getting exactly two heads is

a) 5/8 **b)** 3/8 **c)** 1/8 **d)** None

Sol. Option b)

$n\ (S) = 2 \times 2 \times 2 = 8$

Favourable outcomes = {(HHT), (THH), ((HTH)}

$n\ (E) = 3 \quad \therefore P\ (E) = {}^{3}/_{8}$

Q.25) The chance of getting 7 or 11 in a throw of 2 dice is

a) 7/9 **b)** 5/9 **c)** 2/9 **d)** None

Sol. Option c)

$n\ (S) = 6 \times 6 = 36$

Favourable outcomes = {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (5, 6), (6, 5)}

$n\ (E) = 6 + 2 = 8$

$P\ (E) = {}^{8}/_{36} = {}^{2}/_{9}$

Q.26) Two dice are thrown together. The probability that 'the event the difference of numbers shown is 2' is

a) 2/9 **b)** 5/9 **c)** 4/9 **d)** 7/9

Sol. Option a)

$n(S) = 6 \times 6 = 36,\ n(E) = 8\ \{(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)\}$

$P(E) = \frac{8}{36} = \frac{2}{9}$

Q.27) Two dice with face marked 1, 2, 3, 4, 5, 6 are thrown simultaneously and the points on the dice are multiplied together. The probability that product is 12 is

a) 1/9 **b)** 5/36 **c)** 12/36 **d)** None

Sol. Option a)

$n(S) = 6 \times 6 = 36$

$n(E) = 4 \quad [\because E = \{(2,6), (6,2), (4,3), (3,4)\}]$

$\therefore P(E) = \frac{4}{36} = \frac{1}{9}$

Q.28) Let A and B be the events with P(A)= 1/3, P(B) = 1/4 and P(AB)= 1/12 then P(A/B) isequal to

a) $\frac{1}{3}$ **b)** $\frac{1}{4}$ **c)** $\frac{3}{4}$ **d)** $\frac{2}{3}$

Sol. Option a)

$P(A/B) = \frac{P(AB)}{P(B)} = \frac{1/12}{1/4} = \frac{1}{3}$

Q.29) The odds in favour of one student passing a test are 3:7.The odds against another student passing at are 3:5.The probability that both pass is

a) $\frac{7}{16}$ **b)** $\frac{21}{80}$ **c)** $\frac{9}{80}$ **d)** $\frac{3}{16}$

Sol. Option d)

Let $A \rightarrow$ One student passing a test
$B \rightarrow$ Another student passing a test

$\frac{P(A)}{P(\bar{A})} = \frac{3}{7} \quad \therefore P(A) = \frac{3}{10}$

$\frac{P(\bar{B})}{P(B)} = \frac{3}{5} \quad \Rightarrow P(B) = \frac{5}{8}$

$\therefore P(A \cap B) = P(A) \times P(B) = \frac{3}{10} \times \frac{5}{8} = \frac{3}{16}$

Q.30) A traffic census show that out of 1000 vehicles passing a junction point on a highway 600 turned to the right. The probability of an automobile turning the right is

a) 2/5 **b)** 3/5 **c)** 4/5 **d)** None

Sol. Option b)

$n(S) = 1000$

$n(E) = 600$

$P(E) = \frac{600}{1000} = 3/5$

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ability of a horse B winning the same
win

d) None

least one tail is

vo heads is

d) None

d) None

Q.31) If x and y are random variables having expected values as 4.5 and 2.5 respectively, thenthe expected value of $(x - y)$ is

a) 2 **b)** 7 **c)** 6 **d)** 0

Sol. Option a)

$E\,(x-y) = E(x) - E(y)$

$= 4.5 - 2.5 = 2$

Q.32) The table below shows the history of 1000 men:

Life (in years) :	60	70	80	90
No. survived :	1000	500	100	60

The probability that a man will survived to age 90 is

a) 60/1000 **b)** 160/1000 **c)** 660/1000 **d)** None

Sol. Option a)

$n\,(S) = 1000$

$n\,(E) = 60$

$P\,(E) = \frac{60}{1000}$

Q.33) If probability of drawing a spade from a well-shuffled pack of playing cards is $\frac{1}{4}$ then the probability that of the card drawn from a well-shuffled pack of playing cards is 'not a spade' is

a) 1 **b)** $\frac{1}{2}$ **c)** $\frac{1}{4}$ **d)** $\frac{3}{4}$

Sol. Option d)

Let $A \rightarrow$ Spade

$P\,(A) = {}^1/_4$

$\therefore P\,(A') = 1 - P(A) = 1 - {}^1/_4 = {}^3/_4$

Q.34) A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?

a) 1/3 **b)** 2/3 **c)** 13/15 **d)** 3/15

Sol. Option c)

Required probability

$= 1 - \text{P (Both defective)}$

$= 1 - \frac{8_{C_2} \times 2_{C_2}}{10_{C_4}}$

$= 1 - \frac{8\times7}{2\times1} \times 1 \times \frac{4\times3\times2\times1}{10\times9\times8\times7}$

$= 1 - \frac{2}{15} = \frac{13}{15}$

Q.35) Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(AB) = \frac{1}{4}$, the value of P(A+B) is

a) $\frac{3}{4}$ **b)** $\frac{7}{12}$ **c)** $\frac{5}{6}$ **d)** $\frac{31}{6}$

Sol. Option b)

$$P(A) = {}^{1}/_{2}, P(B) = \frac{1}{3}, P(AB) = {}^{1}/_{4}$$

$$\therefore P(A+B) = P(A) + P(B) - P(AB)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6+4-3}{12} = \frac{7}{12}$$

Q.36) If events A and B are independent and P(A)= 2/3 , P(B)= 3/5 then P(A+B)is equal to

a) $\frac{13}{15}$ **b)** $\frac{6}{15}$ **c)** $\frac{1}{15}$ **d)** None

Sol. Option a)

$$P(AB) = P(A) \times P(B) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

$$\therefore P(A+B) = P(A) + P(B) - P(AB) = \frac{2}{3} + \frac{3}{5} - \frac{2}{5}$$

$$= \frac{10+9-6}{15} = \frac{13}{15}$$

Q.37) A bag contains 15 one-rupee coins, 25 two-rupee coins and 10 five-rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is

a) 0.30 **b)** 0.70 **c)** 0.25 **d)** 0.20

Sol. Option b)

P (not selecting a one-rupee coin) = 1 − P (Selecting a one-rupee coin)

$$= 1 - \frac{15}{50} = \frac{35}{50} = \frac{7}{10} = \mathbf{0.70}$$

Q.38) What is the probability of having at least one 'six' from 3 throws of a perfect die?

a) 5/6 **b)** $(5/6)^3$ **c)** $1-(1/6)^3$ **d)** $1-(5/6)^3$

Sol. Option d)

P (having at least one 'six')= 1 − P (having no six)

$$= 1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \mathbf{1} - \left(\frac{5}{6}\right)^3$$

Q.39) Following are the wages of 8 workers in rupees:

50, 62, 40, 70, 45, 56, 32, 45

If one of the workers is selected at random, what is the probability that his wage would be lower than the average wage?

a) 0.625 **b)** 0.50 **c)** 0.375 **d)** 0.450

Sol. Option b)

Wages are 50, 62, 40, 70, 45, 56, 32, 45

$$\bar{X} = \frac{50+62+40+70+45+56+32+45}{8}$$

$$= \frac{400}{8} = 50$$

P (Wages lower than average)

$$= \frac{4}{8} = \frac{1}{2} = \mathbf{0.50}$$

Q.40) Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

a) 0.325 **b)** 0.40 **c)** 0.925 **d)** 0.075

Sol. Option b)

Required Probability

$$= \frac{30}{100} \times \frac{75}{100} + \frac{70}{100} \times \frac{25}{100}$$

$$= \frac{3}{10} \times \frac{3}{4} + \frac{7}{10} \times \frac{1}{4} = \frac{9+7}{40}$$

$$= \frac{16}{40} = \mathbf{0.4}$$

Q.41) Two dice are thrown at a time. The probability that the numbers shown are equal is

a) $\frac{2}{6}$ **b)** $\frac{5}{6}$ **c)** $\frac{1}{6}$ **d)** None

Sol. Option c)

$n(S) = 6 \times 6 = 36$

$n(E) = 6$ $[\because \mathrm{E} = \{(1,1),\ (2,2),\ (3,3),\ (4,4),\ (5,5),\ (6,6)\}]$

$P(E) = \frac{6}{36} = {}^{1}/_{6}$

Q.42) A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random,what is the probability that the number of the ball will be a multiple of 5 or 6 ?

a) 0.30 **b)** 0.25 **c)** 0.20 **d)** 1/3

Sol. Option d)

$n(S) = 12$

$n(E) = 4\ (5, 6, 10, 12)$

$P(E) = \frac{n(E)}{n(S)} = \frac{4}{12} = {}^{1}/_{3}$

Q.43) A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made (i) with replacement (ii) without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively

a) $\frac{6}{321}$ *and* $\frac{3}{926}$ **b)** $\frac{1}{20}$ *and* $\frac{1}{30}$ **c)** $\frac{35}{144}$ *and* $\frac{35}{108}$ **d)** $\frac{7}{968}$ *and* $\frac{35}{1848}$

Sol. Option d)

(i) $\frac{{}^{5}C_{3}}{{}^{12}C_{3}} \times \frac{{}^{7}C_{3}}{{}^{12}C_{3}} = \frac{\frac{5\times4}{2}}{\frac{12\times11\times10}{3\times2}} \times \frac{\frac{7\times6\times5}{3\times2}}{\frac{12\times11\times10}{3\times2}}$

$= \frac{10}{220} \times \frac{35}{220} = \frac{\mathbf{7}}{\mathbf{968}}$

(ii) $\frac{{}^{5}C_{3}}{{}^{12}C_{3}} \times \frac{{}^{7}C_{3}}{{}^{9}C_{3}}$ ($\because$ Without replacement remain ball =12-3=9)

$$= \frac{\frac{5\times4}{2}}{\frac{12\times11\times10}{3\times2}} \times \frac{\frac{7\times6\times5}{3\times2}}{\frac{9\times8\times7}{3\times2}} = \frac{10}{220} \times \frac{35}{84}$$

$$= \frac{1}{22} \times \frac{35}{84} = \frac{\mathbf{55}}{\mathbf{1848}}$$

Q.44) A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and the second 3 red balls is

a) 5/223 **b)** 6/257 **c)** 7/429 **d)** 3/548

Sol. Option c)

Required Probability $= \frac{{}^{5}C_3}{{}^{13}C_3} \times \frac{{}^{8}C_3}{{}^{10}C_3}$

$$= \frac{\frac{5\times4}{2}}{\frac{13\times12\times11}{3\times2}} \times \frac{\frac{8\times7\times6}{3\times2}}{\frac{10\times9\times8}{3\times2}} = \frac{10}{13\times22} \times \frac{56}{120} = \frac{10\times7}{13\times22\times15}$$

$$= \frac{7}{429}$$

Q.45) There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of samecolour?

a) 89/729 **b)** 97/729 **c)** 82/729 **d)** 23/32

Sol. Option a)

$R \rightarrow Red, W \rightarrow white, B \rightarrow Blue$

Required Probability = P(RRR) + P(WWW) + P(BBB)

$$= \frac{5}{18} \times \frac{4}{18} \times \frac{3}{9} + \frac{7}{18} \times \frac{8}{18} \times \frac{4}{9} + \frac{6}{18} \times \frac{6}{18} \times \frac{2}{9}$$

$$= \frac{60+224+72}{18\times18\times9}$$

$$= \frac{356}{18\times18\times9} = \frac{89}{729}$$

Q.46) There are two urns. The first urn contains 3 red and 5 white balls whereas the second urn contains 4 red and 6 white balls. A ball is taken at random from the first urn and is transferred to the second urn. Now another ball is selected at random from the secondurn. The probability that the second ball would be red is

a) 7/20 **b)** 35/88 **c)** 17/52 **d)** 3/20

Sol. Option b)

Let $A \rightarrow 1^{st}$ transferred ball is red

$B \rightarrow 1^{ST}$ transferred ball is white

$E \rightarrow 2^{nd}$ ball is red

$$P(E) = P(A) \times P(E/_A) + P(B) \times P(E/_B)$$

$$= \frac{3}{8} \times \frac{5}{11} + \frac{5}{8} \times \frac{4}{11} = \frac{15+20}{88} = \frac{35}{88}$$

Q.47) A family has 2 children. The probability that both of them are boys if it is known that one of them is a boy

a) 1 **b)** 1/2 **c)** 3/4 **d)** None

Sol. Option d)

$S = \{BB,\ BG,\ GB,\ GG\}$

Total outcomes = {BB, BG, GB}, Favourable outcomes = {BB}

$P = \frac{1}{3}$

Q.48) What is the probability that 4 children selected at random would have different birthdays?

a) $\frac{364\times363\times362}{(365)^3}$ **b)** $\frac{6\times5\times4}{7^3}$ **c)** 1/365 **d)** $(1/7)^3$

Sol. Option a)

$n(S) = (365)^4$

$n(E) = 365 \times 364 \times 363 \times 362$

$\therefore P(E) = \frac{365\times364\times363\times362}{(365)^4}$

$= \frac{364\times363\times362}{(365)^3}$

Q.49) In a class 40 % students read Mathematics, 25 % Biology and 15 % both Mathematics and Biology. One student is select at random. The probability that he reads Mathematics if it isknown that he reads Biology is

a) 2/5 **b)** 3/5 **c)** 4/5 **d)** None

Sol. Option b)

Let $M \to$ Mathematics & B $\to$ Biology

$P(M) = \frac{40}{100}, P(B) = {}^{25}/_{100}, P(M \cap B) = {}^{15}/_{100}$

$\therefore P\left({}^{M}/_{B}\right) = \frac{P(M\cap B)}{P(B)} = \frac{{}^{15}/_{100}}{{}^{25}/_{100}} = \frac{15}{25} = {}^{3}/_{5}$

Q.50) For a group of students, 30 %, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probabilitythat he passed in Physics if it is known that he failed in Chemistry?

a) 1/2 **b)** 1/3 **c)** 1/4 **d)** 1/6

Sol. Option a)

$A \to$ Failed in Physics

$B \to$ Failed in chemistry

$P(A) = \frac{30}{100}$

$P(B) = \frac{40}{100}$

$P(A \cup B) = \frac{50}{100}$

$P\left({}^{A'}/_{B}\right) = \frac{P(A'\cap B)}{P(B)} = \frac{P(B)-P(A\cap B)}{P(B)}$

$= \frac{P(A\cup B)-P(A)}{P(B)} = \frac{\frac{50}{100}-\frac{30}{100}}{{}^{40}/_{100}}$

$= \frac{20}{100} \times \frac{100}{40} = {}^{1}/_{2}$

Q.51) A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

a) 4/15 **b)** 7/8 **c)** 8/15 **d)** 11/15

Sol. Option d)

$$P(A) = {}^1/_3, P(B) = {}^1/_5, P(C) = {}^1/_2$$

$$\therefore P(A \cap B) = \frac{1}{3} \times \frac{1}{5} = {}^1/_{15}, P(B \cap C) = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

$$P(A \cap C) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$P(A \cap B \cap C) = \frac{1}{3} \times \frac{1}{5} \times \frac{1}{2} = \frac{1}{30}$$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{1}{3} + \frac{1}{5} + \frac{1}{2} - \frac{1}{15} - \frac{1}{10} - \frac{1}{6} + \frac{1}{30}$$

$$= \frac{10+6+15-2-3-5+1}{30} = \frac{22}{30} = \frac{11}{15}$$

Q.52) Find the probability that in a random arrangement of the letters of the word SOCIAL, vowel comes together?

a) 1/8 **b)** 1/5 **c)** 1/4 **d)** 1/3

Sol. Option b)

Total outcomes = 6! = 720

Favourable outcomes = internal arrangement and external arrangement

$= {}^3C_3 \times 3! \times 4!$

The required probability = $\frac{3! \times 4!}{6!} = \frac{1}{5}$

Q.53) If nine persons are seated on a round table, what is the probability that two friends will be a neighbour?

a) 1/6 **b)** 1/7 **c)** 1/4 **d)** 1/5

Sol. Option c)

Total arrangement of sitting in round table = (9-1)! = 8!

Favourable outcomes = Internal arrangement × external arrangement

= 2!× 7!

The required probability = $\frac{2! \times 7!}{8!} = \frac{1}{4}$

Theoretical Distribution

Q.1) The probability mass function of binomial distribution is given by
a) $f(x) = p^x q^{n-x}$ **b)** $f(x) = {}^nc_x p^x q^{n-x}$ **c)** $f(x) = {}^nc_x q^x p^{n-x}$ **d)** $f(x) = {}^nc_x p^{n-x} q^x$

Sol. Option b)
The probability of mass function is given by $f(x) = {}^nc_x p^x q^{n-x}$

Q.2) The mean of a binomial distribution with parameter n and p is
a) n (1– p). **b)** np (1 – p) **c)** np **d)** $\sqrt{np(1-p)}$

Sol. Option c)
The mean of binomial distribution when parameters n and p is np

Q.3) The variance of a binomial distribution with parameters n and p is
a) $np^2 (1-p)$ **b)** $\sqrt{np(1-p)}$ **c)** nq (1 – q). **d)** $n^2p^2 (1-p)^2$

Sol. Option c)
The variance of binomial distribution is when parameters n and p is npq or $nq(1-q)$

Q.4) The maximum value of the variance of a binomial distribution with parameters n and p is
a) n/2 **b)** n/4 **c)** np(1-p) **d)** 2n

Sol. Option b)
The maximum value of the variance of a binomial distribution with parameters n and p is n/4.

Q.5) A binomial distribution with parameters n and p can be approximated by a Poissondistribution with parameter m = np is
a) $n \to \infty$ **b)** $p \to 0$
c) $n \to \infty$ and $p \to 0$ **d)** $n \to \infty$ and $p \to 0$ so that np remains finite

Sol. Option d)
$n \to \infty$ and $p \to 0$ so that np remains finite.

Q.6) The normal curve is
a) Positively skewed **b)** Negatively skewed **c)** Symmetrical. **d)** All these

Sol. Option c)
The normal curve is symmetrical.

Q.7) Area of the normal curve
a) Between $-\infty$ to μ is 0.50 **b)** Between μ to ∞ is 0.50.
c) Between ∞ to μ is 0.05 **d)** Both (a) and (b).

Sol. Option d)
Area of the normal curve is $-\infty$ to μ is 0.50 and μ to ∞ is 0.50.

Q.8) The mean deviation about median of a standard normal variate is
a) 0.675 σ **b)** 0.675 **c)** 0.80 σ **d)** 0.80

Sol. Option d)
The mean deviation about median of a standard normal variate is **0.80 σ.**

Q.9) A discrete random variable x follows uniform distribution and takes only the values 6, 8, 11, 12, 17. The probability of P $(x = 8)$ is

a) 1/5 **b)** 3/5 **c)** 2/8 **d)** 3/8

Sol. Option a)

$P\,(x=8)=\frac{n\,(E)}{n\,(S)}={}^{1}/_{5}$

Q.10) A discrete random variable x follows uniform distribution and takes the values 6, 8, 11, 12, 17 The probability of $P(x \leq 12)$ is

a) 3/5 **b)** 4/5 **c)** 1/5 **d)** None

Sol. Option b)

$P\,(x\leq 12)=\frac{n\,(E)}{n\,(S)}={}^{4}/_{5}$

Q.11) The Number of points in a single throw of an unbiased dice has frequency function

a) f(x)=1/4 **b)** f(x)= 1/5 **c)** f(x) = 1/6 **d)** None

Sol. Option c)

$f\,(x)={}^{1}/_{6}$

Q.12) In a discrete random variable x follows uniform distribution and assumes only the values 8 , 9, 11, 15, 18, 20. Then $P(x = 9)$ is

a) 2/6 **b)** 1/7 **c)** 1/5 **d)** 1/6

Sol. Option d)

$P\,(x=9)=\frac{n\,(E)}{n\,(E)}=\frac{1}{6}$

Q.13) If X ~ B (n, p), what would be the greatest value of the variance of x when n = 16?

a) 2 **b)** 4 **c)** 8 **d)** $\sqrt{5}$

Sol. Option b)

When the variance is the greatest

$\therefore p=q={}^{1}/_{2}$

$\sigma^2=npq=16\times\frac{1}{2}\times\frac{1}{2}=4$

Q.14) In Binomial distribution if n = 4 and p = 1/3 then the value of variance is

a) 8/3 **b)** 8/9 **c)** 4/3 **d)** None

Sol. Option b)

$n=4,\ p={}^{1}/_{3}\quad \therefore q=1-p=1-{}^{1}/_{3}={}^{2}/_{3}$

$\therefore\ \sigma^2=npq=4\times\frac{1}{3}\times\frac{2}{3}={}^{8}/_{9}$

Q.15) If is a Binomial distribution mean = 20, S.D.= 4 then n is equal to

a) 80 **b)** 100 **c)** 90 **d)** None

Sol. Option b)

$np=20\ \ \&\ \sqrt{npq}=4\implies npq=16$

$\therefore q=\frac{16}{20}={}^{4}/_{5}$

$\therefore p=1-{}^{4}/_{5}={}^{1}/_{5}$

$\therefore n=20\times 5=100$

Q.16) What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously?

a) 0.50 **b)** 0.25 **c)** 0.3125 **d)** 0.6575

Sol. Option c)

$n = 6$

$p = {}^1/_2, q = {}^1/_2$

$P\,(x = 3) = 6_{C_3} p^3\, q^3$

$= \frac{6\times5\times4}{3\times2\times1} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3$

$= 20 \times \frac{1}{8} \times \frac{1}{8} = \frac{5}{16} = \mathbf{0.3125}$

Q.17) What is the probability of making 3 correct guesses in 5 True – False answer type questions?

a) 0.3125 **b)** 0.5676 **c)** 0.6875 **d)** 0.4325

Sol. Option a)

n = 5

$p = {}^1/_2$

$q = {}^1/_2$

$P\,(X = 3)\ = 5_{C_3}\, p^3 q^2 = \frac{5\times4}{2\times1} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

$= 10 \times \frac{1}{8} \times \frac{1}{4} = \frac{5}{16} = \mathbf{0.3125}$

Q.18) If X ~ P (m) and its coefficient of variation is 50, what is the probability that X wouldassume only non-zero values?

a) 0.018 **b)** 0.982 **c)** 0.989 **d)** 0.976

Sol. Option b)

$C.V. = 50\ \Rightarrow \frac{\sigma}{\mu} \times 100 = 50$

$\Rightarrow \frac{\sigma}{\mu} = \frac{1}{2}$

$\Rightarrow \mu = 2\,\sigma$

$\Rightarrow m = 2\sqrt{m}\ \Rightarrow \sqrt{m} = 2$

($\because$ mean = m & variance = m)

$\Rightarrow m = 4$

P (non-zero)

$= 1 - P\,(X = 0)$

$= 1 - e^{-m} = 1 - e^{-4}$

$= 1 - 0.018 = \mathbf{0.982}$

Q.19) If 1 per cent of an airline's flights suffer a minor equipment failure in an aircraft, what isthe probability that there will be exactly two such failures in the next 100 such flights?

a) 0.50 **b)** 0.184 **c)** 0.265 **d)** 0.256

Sol. Option b)

$n = 100,\ p = \frac{1}{100}$

$m = \mu = np = 100 \times \frac{1}{100} = 1$

$P\,(X = 2) = \frac{e^{-m} \times m^2}{2\,!} = \frac{e^{-1} \times 1^2}{2} = \frac{e^{-1}}{2}$

$= \frac{1}{2} \times 0.36788 = 0.18394 = \mathbf{0.184}$

Q.20) If the two quartiles of N (μ, σ^2) are 14.6 and 25.4 respectively, what is the standard deviation of the distribution?

a) 9 **b)** 6 **c)** 10 **d)** 8

Sol. Option d)

$Q_1 = 14.6 \quad \Rightarrow \mu - 0.675\sigma = 14.6$_____$(I)$

$Q_3 = 25.4 \quad \Rightarrow \mu + 0.675\sigma = 25.4$_____$(II)$

From $[(II - (I)]\ 2 \times 0.675\sigma = 10.8$

$\Rightarrow \sigma = \frac{10.8}{2 \times 0.675} = \mathbf{8}$

Q.21) If the quartile deviation of a normal curve is 4.05, then its mean deviation is

a) 5.26 **b)** 6.24 **c)** 4.24 **d)** 4.86

Sol. Option d)

$4SD = 5MD = 6Q.D$

$Q.D = 4.05$

$\Rightarrow Q.D = \frac{4}{6} \times \sigma = 4.05$

$\Rightarrow \sigma = \frac{4.05 \times 6}{4} = 6.075$

$\therefore M.D. = 0.8 \times \sigma = 0.8 \times 6.075$

$= \mathbf{4.86}$

Q.22) If the first quartile and mean deviation about median of a normal distribution are 13.25 and8 respectively, then the mode of the distribution is

a) 20 **b)** 10 **c)** 15 **d)** 12

Sol. Option a)

$Q_1 = 13.25 \quad \Rightarrow \mu - 0.675\sigma = 13.25$_____$(I)$

$M.D. = 8 \quad \Rightarrow 0.8\,\sigma = 8 \Rightarrow \sigma = 10$

$\therefore \mu = 13.25 + 0.675 \times 10 = 20$

$\therefore$ Mode = Mean = $\mu = \mathbf{20}$

Q.23) If X and Y are 2 independent normal variables with mean as 10 and 12 and SD as 3 and 4,then (X+Y) is normally distributed with

a) Mean = 22 and SD = 7
b) Mean = 22 and SD = 25
c) Mean = 22 and SD = 5
d) Mean = 22 and SD = 49

Sol. Option c)

Required mean $= \mu_1 + \mu_2 = 10 + 12 = \mathbf{22}$

$$\& \; SD = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = \mathbf{5}$$

Q.24) x is a binomial variable such that 2 P(x = 2) = P(x = 3) and mean of x is known to be10/3. What would be the probability that x assumes at most the value 2?

a) 16/81
b) 17/81
c) 47/243
d) 46/243

Sol. Option b)

$$= \frac{10}{3} \Rightarrow np = \frac{10}{3} \_\_\_(I)$$

Also $2P\,(x = 2) = P\,(x = 3)$

$$\Rightarrow 2 \times n_{C_2} p^2\, q^{n-2} = n_{C_3}\, p^3\, q^{n-3}$$

$$\Rightarrow p = \frac{2\; n_{C_2}}{n_{C_3}} q$$

$$\Rightarrow p = 2 \times \frac{n\,!}{2\,! \times (n-2)!} \times \frac{3\,! \times (n-3)!}{n\,!}\; q$$

$$\Rightarrow p = \frac{2 \times 3 \times 2\,! \times (n-3)!\; q}{2\,! \times (n-2)\;(n-3)!}$$

$$\Rightarrow (n-2)p = 6q$$

$$\Rightarrow np - 2p = 6q$$

$$\Rightarrow \frac{10}{3} - 2p = 6(1-p) = 6 - \frac{10}{3} = 6p - 2p$$

$$\Rightarrow 4p = \frac{18-10}{3}$$

$$\Rightarrow 4p = \frac{8}{3}$$

$$\Rightarrow p = {}^{2}/_{3} \quad \therefore q = 1 - {}^{2}/_{3} = {}^{1}/_{3}$$

$$\therefore n \times \frac{2}{3} = \frac{10}{3} \Rightarrow n = 5$$

$$\therefore P\,(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= 5_{C_0}\, p^0\, q^5 + 5_{C_1}\, p\, q^4 + 5_{C_2} p^2\, q^3$$

$$= \left(\frac{1}{3}\right)^5 + 5 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^4 + 10 \times \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3$$

$$= \frac{\mathbf{1+10+40}}{\mathbf{243}} = \frac{\mathbf{51}}{\mathbf{243}} = \frac{\mathbf{17}}{\mathbf{81}}$$

Q.25) Assuming that one-third of the population is tea drinkers and each of 1000 enumerators takes a sample of 8 individuals to find out whether they are tea drinkers or not, how manyenumerators are expected to report that five or more people are tea drinkers?

a) 100 **b)** 95 **c)** 88 **d)** 90

Sol. Option c)

$n = 8$ and $p = 1/3$

$q = 1 - 1/3 = 2/3$

$P(X \geq 5) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$

$= 8_{C_5}\, p^5 q^3 + 8_{C_6}\, p^6 q^2 + 8_{C_7}\, p^7 q + 8_{C_8}\, p^8 q^0$

$= 56 \times \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^3 + 28 \times \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^2 + 8 \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right) + 1 \times \left(\frac{1}{3}\right)^8 \times 1$

$= \frac{56\times8+28\times4+8\times2+1}{3^8}$

$= \frac{577}{6561}$

∴ Required number of enumerators

$= \frac{577}{6561} \times 1000 = \mathbf{88}$

Q.26) In 10 independent rolling of a biased dice, the probability that an even number will appear5 times is twice the probability that an even number will appear 4 times. What is the probability that an even number will appear twice when the dice is rolled 8 times?

a) 0.0304 **b)** 0.1243 **c)** 0.2315 **d)** 0.1926

Sol. Option a)

When $n = 10$, $p = \frac{3}{6} = 1/2$ $\therefore q = 1 - 1/2 = 1/2$

$P(X = 5) = 2\ P(X = 4)$

$\Rightarrow 10_{C_5}\, p^5 q^5 = 2 \times 10_{C_4}\, p^4 q^6$

$\Rightarrow p = \frac{2\times10_{C_4}}{10_{C_5}}\, q = 2 \times \frac{10\,!}{4\,!\times6\,!} \times \frac{5\,!\times5\,!}{10!} \times q$

$\Rightarrow p = \frac{2\times5}{6}\, q = 5/3\, q$

$\Rightarrow \frac{p}{q} = 5/3$

$\therefore p = \frac{5}{8}$ & $q = 3/8$

Now,

If $n = 8,\ p = 5/8$ & $q = 3/8$

The $P(X = 2) = 8_{C_2}\, p^2 q^6$

$= 28 \times \left(\frac{5}{8}\right)^2 \left(\frac{3}{8}\right)^6 = \mathbf{0.0304}$ **(approx.)**

Q.27) If X follows normal distribution with ⍰ = 50 and ⍰ = 10, what is the value of P(x ⍰ 60 / x > 50)?

a) 0.8413 **b)** 0.6826 **c)** 0.1587 **d)** 0.7256

Sol. Option b)

When $X = 50$ $then$ $z = \frac{X-\mu}{\sigma} = \frac{50-50}{10} = 0$

When $X = 60$ $then$ $z = \frac{60-50}{10} = 1$

$\therefore P\ (X \leq 60/x > 50)\ = P\ (Z \leq 1/z > 0)$

$= \frac{P\ (0 \leq z \leq 1)}{P\ (z>0)} = \frac{P\ (0 \leq z \leq 1)}{P\ (z>0)}$

$= \frac{0.3413}{0.5} = \mathbf{0.6826}$

Q.28) A renowned hospital usually admits 200 patients every day. One per cent patients, on an average, require special room facilities. On one particular morning, it was found that only one special room is available. What is the probability that more than 3 patients would require special room facilities?

a) 0.1427 **b)** 0.1732 **c)** 0.2235 **d)** 0.3450

Sol. Option a)

$m = np = 200 \times \frac{1}{100} = 2$

$P\ (X > 3) = 1 - \ [P\ (X = 0) + P\ (X = 1) + P\ (X = 2) + P\ (X = 3)]$

$= 1 - \left[e^{-m} + \frac{e^{-m}.m}{1\,!} + \frac{e^{-m}.m^2}{2\,!} + \frac{e^{-m}.m^3}{3\,!}\right]$

$= 1 - e^{-2}\left[1 + 2 + \frac{2^2}{2} + \frac{2^3}{6}\right]$

$= 1 - e^{-2}\ (1 + 2 + 2 + 1.3333)$

$= 1 - 0.13536 \times 6.3333$

$= \mathbf{0.1427}$ **(approx)**

Q.29) The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with an average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accidents in a year?

a) 162 **b)** 180 **c)** 201 **d)** 190

Sol. Option a)

$m = 2$

$P\ (X \geq 3) = 1 - [P\ (X = 0) + P\ (X = 1) + P\ (X = 2)]$

$= 1 - \left(e^{-m} + \frac{e^{-m} \times m}{1\,!} + \frac{e^{-m}.m^2}{2\,!}\right)$

$= 1 - e^{-m}\left(1 + m + \frac{m^2}{2}\right)$

$= 1 - e^{-2}\ (1 + 2 + 2)$

$= 1 - 0.13536 \times 5 = 1 - 0.6768$

$= 0.3232$

$\therefore$ Required nos. of drivers

$= 500 \times 0.3232$

$= 161.6 \approx \mathbf{162}$

Q.30) In a sample of 800 students, the mean weight and standard deviation of weight are found to be 50 kg and 20 kg respectively. On the assumption of normality, what is the number of students weighing between 46 Kg and 62 Kg? Given area of the standard normal curve between $z = 0$ to $z = 0.20 = 0.0793$ and area between $z = 0$ to $z = 0.60 = 0.2257$.

a) 250 **b)** 244 **c)** 240 **d)** 260

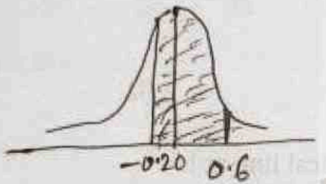
Sol. Option b)

$\mu = 50, \sigma = 20$

$z_1 = \frac{46-50}{20} = \frac{-4}{20} = -0.20$

$z_2 = \frac{62-50}{20} = \frac{12}{20} = 0.60$

$P\,(z_1 \le z \le z_2) = P\,(-0.20 \le z \le 0.60)$



$= P\,(0 \le z \le 0.20) + P\,(0 \le z \le 0.6)$

$[\because P\,(-0.20 \le z \le 0) = P(0 \le z \le 0.20)]$

$= 0.0793 + 0.2257$

$= 0.305$

$\therefore$ Required Number student weighing between 46 kg and 62 kg

$= 0.305 \times 800 = \mathbf{244}$

Q.31) For a normal distribution with mean as 500 and SD as 120, what is the value of k so that the interval [500, k] covers 40.32 per cent area of the normal curve? Given ⍰ (1.30) = 0.9032.

a) 740 **b)** 750 **c)** 656 **d)** 800

Sol. Option c)

$\mu = 500, \quad \sigma = 120$

$P\,(500 \le x \le k) = 40.32\%$

$\Rightarrow P\left(0 \le Z \le \frac{k-500}{120}\right) = 0.4032$

$\Rightarrow P\left(z \le \frac{k-500}{120} = 0.5 + .4032\right)$

$\Rightarrow P\left(z \le \frac{k-500}{120} = 0.9032\right)$

$= Z = 1.30$

$\Rightarrow P\left(z \le \frac{k-500}{120}\right) = P\,(z \le 1.30)$

$\Rightarrow \frac{k-500}{120} = 1.30$

$\Rightarrow k = 500 + 1.30 \times 120$

$= 500 + 156 = \mathbf{656}$

Q.32) In Standard Normal distribution
a) Mean=1, S.D=0 **b)** Mean=1, S.D=1 **c)** Mean=0, S.D = 1 **d)** Mean=0, S.D=0

Sol. Option c)
Mean = 0 & S.D. = 1

Q.33) The probability distribution of x is given below:

Value of x :	1	0	Total
Probability :	p	1-p	1

Mean is equal to
a) p **b)** $1-p$ **c)** 0 **d)** 1

Sol. Option a)
Mean $= E(X) = 1 \times p + 0(1-p) = p + 0 = \boldsymbol{p}$

Q.34) In continuous probability distribution P ($x \le$ t) means
a) Area under the probability curve to the left of the vertical line at t.
b) Area under the probability curve to the right of the vertical line at t.
c) Both
d) None

Sol. Option a)
Area under the probability curve to the left of the vertical line at t.

Q.35) The total area under the normal curve at $\mu - \sigma$ *and* $\mu + \sigma$ is:
a) 68.5% **b)** 68%
c) 95% **d)** 95.5%

Sol. Option b)

The total area under the normal curve at $\mu - \sigma$ *and* $\mu + \sigma$ is **68%.**

Q.36) The ratio of QD:MD:SD in a normal distribution is:
a) 6:5:4 **b)** 10:12:15 **c)** 12:10:15 **d)** 4:5:6

Sol. Option b)

The ratio of QD:MD:SD in normal distribution is **10:12:15.**

Q.37) The interval $\mu - 2\sigma$ *and* $\mu + 2\sigma$ cover
a) 68% of area of a normal distribution **b)** 95.5% of area of a normal distribution
c) 99.73% of area of a normal distribution **d)** 96.5% of area of a normal distribution

Sol. Option b)

The interval $\mu - 2\sigma$ *and* $\mu + 2\sigma$ cover **95.5%** of area of a normal distribution.

Q.38) What is the first quartile of x having the following probability density function?

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}} \quad -\infty < x < \infty$$

a) 5.95 **b)** 4.05 **c)** 6 **d)** 10

Sol. Option a)

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}} \text{ ....1}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ ....2}$$

Compare equation 1 and 2

$\sigma^2 = 36 \Rightarrow \sigma = 6$

$\mu = 10$

$Q_1 = \mu - 0.675\sigma$

$Q_1 = 10 - 0.675 \times 6 = 10 - 4.05 = \mathbf{5.95}$

Q.39) A random variable x has the following probability distribution:

x	1	2	3	4
P(x)	k	2k	3k	4k

Then, variance is:

a) 1 **b)** 3 **c)** 2 **d)** 4

Sol. Option a)

x	P(x)	P(x) × x	P(x) × x^2
1	k	k	k
2	2k	4k	8k
3	3k	9k	27k
4	4k	16k	64k
	$\sum P(x) = 10k$	$\sum x \times P(x) = 30k$	$\sum x^2 \times P(x) = 100k$

Total probability = 1

$= k + 2k + 3k + 4k = 1$

$= 10k = 1$

$= k = \frac{1}{10}$

Mean = $\sum x \times P(x) = 30k = \frac{30}{10} = 3$

Variance = $\sum x^2 \times P(x) - (\sum x \times P(x))^2$

$= 100k - 900\,k^2$

$= \frac{100}{10} - \frac{900}{100}$

$= 10 - 9 = \mathbf{1}$

Index Number

Q.1) Factor Reversal test is satisfied by
a) Fisher's Ideal Index **b)** Laspeyres Index **c)** Paasches Index **d)** None of these

Sol. Option a)
Factor reversal test is satisfied by fisher's index number.

Q.2) $\frac{\text{Sum of all commodity prices in the current year}}{\text{Sum of all commodity prices in the base year is}} \times 100$
a) Relative Price Index **b)** Simple Aggregative Price Index **c)** Both (a) and (b) **d)** None of these

Sol. Option b)

Simple aggregative price index $= \frac{\text{Sum of all commodity prices in the current year}}{\text{Sum of all commodity prices in the base year is}} \times 100$

Q.3) Chain index is equal to

a) $\frac{\text{link relative of current year} \times \text{chain index of the current year}}{100}$

b) $\frac{\text{link relative of previous year} \times \text{chain index of the current year}}{100}$

c) $\frac{\text{link relative of current year} \times \text{chain index of the previous year}}{100}$

d) $\frac{\text{link relative of previous year} \times \text{chain index of the previous year}}{100}$

Sol. Option c)

Chain index is equal to $\frac{\text{Link relative of current year} \times \text{chain index of previous year}}{100}$

Q.4) Fisher's Ideal Formula for calculating index numbers satisfies the_____tests
a) Unit Test **b)** Factor Reversal Test **c)** Both (a) and (b) **d)** None of these

Sol. Option c)
Fisher's ideal formula for calculating index numbers satisfies the unit test and factor reversal test.

Q.5) Laspeyre's and Paasche's method__________time reversal test
a) satisfy **b)** do not satisfy **c)** are **d)** are not

Sol. Option b)
Laspeyre's and Paasche's method do not satisfy time reversal test.

Q.6) Theoretically, G.M. is the best average in the construction of index numbers but in practice,mostly the A.M. is used

a) false **b)** true **c)** both **d)** None

Sol. Option b)
Theoretically G.M is the best average in constructing of index numbers, but in practice, mostly the A.M is used.

Q.7) ________ is concerned with the measurement of price changes over a period of years,when it is desirable to shift the base.

a) Unit Test **b)** Circular Test **c)** Time Reversal Test **d)** None of these

Sol. Option b)

Circular test is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base.

Q.8) The formula for conversion to current value

a) Deflated value = $\frac{\text{Price Index of the current year}}{\text{previous value}}$

b) Deflated value = $\frac{\text{current value}}{\text{Price Index of the current year}}$

c) Deflated value = $\frac{\text{Price Index of the previous year}}{\text{previous value}}$

d) Deflated value = $\frac{\text{Price Index of the previous year}}{\text{previous value}}$

Sol. Option b)

The formula for conversion to current value – deflated value = $\frac{\text{current value}}{\text{price index of current year}}$

Q.9) Shifted price Index = $\frac{\textit{Original Price Index} \times 100}{\textit{Price Index of the year on which it has to be shifted}}$

a) True **b)** false **c)** both **d)** None

Sol. Option a)

Shifted price index = $\frac{\text{original price} \times 100}{\text{price index of the year on which it has to be shifted}}$

Q.10) Price-relative is expressed in term of

a) $P = \frac{P_n}{P_0}$ **b)** $P = \frac{P_0}{P_n}$ **c)** $P = \frac{P_n}{P_0} \times 100$ **d)** $P = \frac{P_0}{P_n} \times 100$

Sol. Option c)

Price relative is expressed in term of $p = \frac{p_n}{p_0} \times 100$.

Q.11) Paasehe's index number is expressed in terms of:

a) $\frac{\Sigma P_n q_n}{\Sigma P_o q_n}$ **b)** $\frac{\Sigma P_o q_n}{\Sigma P_n q_n}$ **c)** $\frac{\Sigma P_n q_n}{\Sigma P_o q_n} \times 100$ **d)** $\frac{\Sigma P_n q_n}{\Sigma P_o q_o} \times 100$

Sol. Option c)

Paasche index number is expressed in terms of $\frac{\Sigma P_n q_n}{\Sigma P_o q_n} \times 100$.

Q.12) Cost of Living Index number (C. L. I.) is expressed in terms of:

a) $\frac{\Sigma P_n q_o}{\Sigma P_o q_o} \times 100$ **b)** $\frac{\Sigma P_n q_n}{\Sigma P_o q_o}$ **c)** $\frac{\Sigma P_o q_n}{\Sigma P_n q_n} \times 100$ **d)** None of these

Sol. Option a)

Cost of living index number is expressed in terms of $\frac{\Sigma P_n q_o}{\Sigma P_o q_o} \times 100$.

Q.13) If the ratio between Laspeyre's index number and Paasche's Index number is 28 : 27. Then the missing figure in the following table P is :

Commodity	**Base Year**		**Current Year**	
	Price	**Quantity**	**Price**	**Quantity**
X	L	10	2	5
Y	L	5	P	2

a) 7 **b)** 4 **c)** 3 **d)** 9

Sol. Option b)

	P_0	Q_0	P_1	Q_1	$P_0\,Q_0$	$P_1\,Q_0$	$P_0\,Q_1$	$P_1\,Q_1$
X	L	10	2	5	10L	20	5L	10
Y	L	5	P	2	5L	5P	2L	2P
Total					15L	20+5P	7L	10+2P

Now,

$$\frac{\text{Laspeyre's Index number}}{\text{Pasche's Index number}} = \frac{28}{27}$$

$$\Rightarrow \frac{\frac{\sum P_1\,Q_0}{\sum P_0\,Q_0} \times 100}{\frac{\sum P_1\,Q_1}{\sum P_0\,Q_1} \times 100} = \frac{28}{27}$$

$$\Rightarrow \frac{20+5\,P}{15L} \times \frac{7\,L}{10+2P} = \frac{28}{27}$$

$$\Rightarrow 9(20+5P) = 20(10+2P)$$

$$\Rightarrow 180 + 45P = 200 + 40P$$

$$\Rightarrow 5\,P = 20 \quad \Rightarrow P = \frac{20}{5} = \mathbf{4}$$

Q.14) If the index number of prices at a place in 1994 is 250 with 1984 as base year, then the prices have increased on average by

a) 250% **b)** 150% **c)** 350% **d)** None of these

Sol. Option b)

$$\text{Index number} = \frac{P_n}{P_0} \times 100$$

$$\Rightarrow 250 \;= \frac{P_n}{P_0} \times 100$$

$$\Rightarrow P_n = 2.5\,P_0$$

$$\therefore \textit{Increased \%} = \frac{P_n - P_0}{P_0} \times 100$$

$$= \frac{(2.5-1)\,P_0}{P_0} \times 100$$

$$= 1.5 \times 100 = \mathbf{150\%}$$

Q.15) Marshall-edge worth Index formula after interchange of p and q is expressed in terms of :

a) $\frac{\sum q_n(P_o+P_n)}{\sum q_o(P_o+P_n)} \times 100$ **b)** $\frac{\sum P_n(q_o+q_n)}{\sum P_o(q_o+q_n)}$ **c)** $\frac{\sum P_0(q_o+q_n)}{\sum P_n(P_o+P_n)}$ **d)** None of these

Sol. Option a)
Marshall-edge worth Index

$$= \frac{\sum P_n (q_0+q_n)}{\sum P_0 (q_0+q_n)} \times 100$$

∴ After interchanging p & q then Marshall-edge worth Index

$$= \frac{\sum q_n (P_0+P_n)}{\sum q_0 (P_0+P_n)} \times 100$$

Q.16) If $\sum p_n\, q_n = 249$, $\sum p_0\, q_0 = 150$, Paasche's Index Number = 150 and Drobiseh and Bowely'sIndex number = 145, then the Fisher's Ideal Index Number is
a) 75 **b)** 60 **c)** 145.97 **d)** 144.91

Sol. Option d)
Drobiseh and Bolwely's Index number

$$= \frac{1}{2}\,(L+P) \; [\because \text{L = Laspeyre's Index no. P = Paasche's Index no.}]$$

$$\Rightarrow 145 = \frac{1}{2}\,(L+150) \Rightarrow L = 290 - 150 = 140$$

∴ Fisher's Ideal Index number

$$= \sqrt{L \times P} \; = \sqrt{140 \times 150}$$

$$= \mathbf{144.91\ (approx)}$$

Q.17) If $\sum p_0\, q_0 = 3500$, $\sum p_n\, q_0 = 3850$, then the Cost of living Index (C.L.I.) for 1950 w. r. to base 1960 is
a) 110 **b)** 90 **c)** 100 **d)** None of these

Sol. Option a)

$$C.L.I. = \frac{\sum p_n\, q_0}{\sum p_0\, q_0} \times 100 = \frac{3850}{3500} \times 100 = \mathbf{110}$$

Q.18) In 1980, the net monthly income of the employee was ₹ 800 p. m. The consumer priceindex number was 160 in 1980. It rises to 200 in 1984. If he has to be rightly compensated. The additional D. A. to be paid to the employee is
a) ₹ 175 **b)** ₹ 185 **c)** ₹ 200 **d)** ₹ 125

Sol. Option c)
Monthly Income in the year 1984

$$= \frac{200}{160} \times 800 = ₹1000$$

∴ D.A. to be paid to the Employee = 1000 − 800 = ₹ 200

Q.19) From the following data with 1966 as base year

Commodity	Quantity Units	Values (₹)
A	100	500
B	80	320
C	60	150
D	30	360

The price per unit of commodity A in 1966 is
a) ₹ 5 **b)** ₹ 6 **c)** ₹ 4 **d)** ₹ 12

Sol. Option a)

The Price per unit of commodity A

$$= \frac{Values}{Quantity\ units} = \frac{500}{100} = ₹\ 5$$

Q.20) The index number in whole sale prices is 152 for August 1999 compared to August 1998. During the year there is net increase in prices of whole sale commodities to the extent of

a) 45% **b)** 35% **c)** 52% **d)** 48%

Sol. Option c)

Increase in prices $= \frac{P_1 - P_0}{P_0} \times 100$

$$= \frac{152\ P_0 - 100\ P_0}{100\ P_0} \times 100$$

$$\left[\because 152 = \frac{P_1}{P_0} \times 100 \quad \Rightarrow P_1 = \frac{152\ P_0}{100}\right]$$

$$= \frac{52\ P_0}{P_0}\ \% = \mathbf{52\%}$$

Q.21) If the price of all commodities in a place have increased 1.25 times in comparison to the base period prices, then the index number of prices for the place is now

a) 100 **b)** 125 **c)** 225 **d)** None of these

Sol. Option c)

Index number $= \frac{P_1}{P_0} \times 100$

$$= \frac{P_0\ \frac{(100+125)}{100}}{P_0} \times 100$$

$$= \frac{225}{100} \times 100 \ = \mathbf{225}$$

Q.22) If the 1970 index with base 1965 is 200 and 1965 index with base 1960 is 150, the index 1970 on base 1960 will be;

a) 700 **b)** 300 **c)** 500 **d)** 600

Sol. Option b)

$$P_{02} = \frac{P_{01} \times P_{12}}{100} = \frac{150 \times 200}{100} = \mathbf{300}$$

Q.23) Time Reversal Test is represented symbolically by:

a) $P_{01} \times P_{10}$ **b)** $P_{01} \times P_{10} = 1$ **c)** $P_{01} \times P_{10}$ 1 **d)** None of these

Sol. Option b)

Time Reversal Test

$$\boldsymbol{P_{01} \times P_{10} = 1}$$

Q.24) If the price of all commodities in a place has increased 20% in Comparison to the base period prices, then the index number of prices for the place is now ________.

a) 100 **b)** 120 **c)** 20 **d)** 150

Sol. Option b)

Index No. of current year = 100 + 20 = **120**

Q.25) The factor reversal test is as represented symbolically is:

a) $P_{01} \times Q_{10} = V$ **b)** $P_{01} \times Q_{10} = V_{01}$ **c)** $P_{01} \times Q_{01} = V_{01}$ **d)** None

Sol. Option c)

The factor reversal test represented by: $\boldsymbol{P_{01} \times Q_{01} = V_{01}}$

Q.26) The price of a commodity increases from ₹ 5 per unit in 1990 to ₹ 7.50 per unit in 1995 and the quantity consumed decreases from 120 units in 1990 to 90 units in 1995. The price and quantity in 1995 are 150% and 75% respectively of the corresponding price and quantity in 1990. Therefore, the product of the price ratio and quantity ratio is:

a) 1.8 **b)** 1.125 **c)** 1.75 **d)** None of these

Sol. Option b)

Price ratio $= \frac{7.5}{5} = 1.5$

Quantity ratio $= \frac{90}{120} = 0.75$

∴ Required Product = 1.5 × 0.75 = **1.125**

Q.27) Consumer price index number goes up from 110 to 200 and the Salary of a worker is also raised from ₹ 325 to ₹ 500. Therefore, in real terms, to maintain his previous standard of living he should get an additional amount of:

a) ₹ 85 **b)** ₹ 90.91 **c)** ₹ 98.25 **d)** None of these

Sol. Option b)

Worker salary should increase to $= \frac{200}{110} \times 325 =$ ₹ 590.91 (approx)

∴ Required additional amount

= 590.91 − 500 = **₹90.91**

Q.28) The average price of certain commodities in 1980 was ₹ 60 and the average price of the same commodities in 1982 was ₹ 120. Therefore, the increase in 1982 on the basis of 1980 was 100%. The decrease in 1980 with 1982 as base is: using 1982, comment on the above statement is :

a) The price in 1980 decreases by 60% using 1982 as base
b) The price in 1980 decreases by 50% using 1982 as base
c) The price in 1980 decreases by 90% using 1982 as base
d) None of these

Sol. Option b)

Decrease in Price on the basis of 1982

$$= \frac{120 - 60}{120} \times 100$$

$$= \frac{60}{120} \times 100 = \mathbf{50\%}$$

The price in 1980 decreases by 50% using 1982 as base.

Q.29) In 1976 the average price of a commodity was 20% more than that in 1975 but 20% less than that in 1974 and more over it was 50% more than that in 1977. The price relatives using 1975 as base year (1975 price relative = 100) then the reduce data is:

a) 80,75 **b)** 150,80 **c)** 75,125 **d)** None of these

Sol. Option b)

Price relative of 1976 $= 100 \times \frac{(100+20)}{100} = 120$

Price relative of 1974 $= 120 \times \frac{100}{100-20}$

$= 120 \times \frac{100}{80} = \mathbf{150}$

Price relative of 1977 $= 120 \times \frac{100}{100+50}$

$= 120 \times \frac{100}{150} = \mathbf{80}$

Q.30) The prices of a commodity in the years 1975 and 1980 were 25 and 30 respectively, taking 1975 as base year the price relative is:

a) 120 **b)** 135 **c)** 122 **d)** None of these

Sol. Option a)

Required Price relative $= \frac{P_1}{P_0} \times 100$

$= \frac{30}{25} \times 100 = \mathbf{120}$

Q.31) Net monthly salary of an employee was ₹ 3000 in 1980. The consumer price index number in 1985 is 250 with 1980 as base year. If the has to be rightly compensated then, 7th dearness allowances to be paid to the employee is:

a) ₹ 4,800 **b)** ₹ 4,700 **c)** ₹ 4,500 **d)** None of these

Sol. Option c)

Salary in 1985 $= \frac{250}{100} \times 3000 =$ ₹ 7500

∴ Required dearness allowances

$= 7500 - 3000 =$ **₹ 4500**

Q.32) When the cost of Tobacco was increased by 50%, a certain hardened smoker, who maintained his formal scale of consumption, said that the rise had increased his cost of living by 5%. Before the change in price, the percentage of his cost of living was due to buying Tobacco is

a) 15% **b)** 8% **c)** 10% **d)** None of these

Sol. Option c)

Let the cost of Tobacco initially be ₹ 100 then Increased cost of Tobacco

∴ $100 \times \frac{100+50}{100} =$ ₹ 150

∴ Increase in Price Tobacco

= (150-100) = ₹ 50

∵ ₹ 50 is the 5% of Index number

∴ ₹ 100 *is the* $\frac{5}{50} \times 100$ *Index number*

= 10%

Q.33) If the price index for the year, say 1960 be 110.3 and the price index for the year, say 1950 be 98.4, then the purchasing power of money (Rupees) of 1950 in 1960 is

a) ₹ 1.12 **b)** ₹ 1.25 **c)** ₹ 1.37 **d)** None of these

Sol. Option a)
Purchasing Power of money of 1950

$in\ 1960 = \frac{110.3}{98.4} = ₹\ \mathbf{1.12\ (approx.)}$

Q.34) The consumer price Index for April 1985 was 125. The food price index was 120 and other items index was 135. The percentage of the total weight given to food index is

a) 66.67% **b)** 68.28% **c)** 90.25% **d)** None of these

Sol. Option a)
Let the weight of food Index be x & other be y

$125\,(x + y) = 120x + 135y$

$\Rightarrow 125\,x + 125\,y = 120x + 135y$

$\Rightarrow 5\,x = 10\,y \ \Rightarrow x = 2y$

$\therefore$ Required % $= \frac{x}{x+y} \times 100$

$= \frac{2y}{3y} \times 100 = \mathbf{66.67\%}$ (approx.)

Q.35) From the following data for the 5 groups combined

Group	Weight	Index Number
Food	35	425
Cloth	15	235
Power & Fuel	20	215
Rent & Rates	8	115
Miscellaneous	22	150

a) 270 **b)** 269.2 **c)** 268.5 **d)** 272.5

Sol. Option b)

Group	Weight	Index Number	Weighted I. No.
Food	35	425	14875
Cloth	15	235	3525
Power & Fuel	20	215	4300
Rent & Rates	8	115	920
Miscellaneous	22	150	3300
	100		**26920**

Group weight index number = Weighted Index/Total Weight
= 26920/100
= 269.2
Group weight index number = **269.2**

Q.36) Consumer price index is commonly known as;

a) Chain Based Index **b)** Ideal Index

c) Wholesale price index **d)** Cost of living index

Sol. Option d)
Cost of living index

Q.37) If Laspeyre's index number is 90 and Paasche's index number is 160 then Fisher's index number will ___________.

a) 144 **b)** 120 **c)** 125 **d)** None of these

Sol. Option b)

Fisher's index No. = $\sqrt{\text{Laspeyre's I. No.} \times \text{Paasche's I. No.}}$

Fisher's index No. = $\sqrt{90 \times 160} = \mathbf{120}$

Q.38) If the old series is connected with the new series of an index number, it is known as

a) Forward splicing **b)** Joint
c) Backward splicing **d)** None of these

Sol. Option a)

If the old series is connected with the new series of index number it is knows as forward splicing.

Q.39) The Paasche's index number is based on

a) Base year quantity **b)** Current year Quantity
c) Base year price **d)** Current year price

Sol. Option b)

The Paasche's index number is based on current year Quantity

Q.40) A worker earned ₹ 14000 per month in 2010. The Cost of the Living index increased by 65% between 2010 and 2020. How much extra income should the worker have earned in 2020 so that he could buy the exact quantities as in 2010?

a) ₹ 23000 **b)** ₹ 23100 **c)** ₹ 9100 **d)** ₹ 5100

Sol. Option c)

A worker earned Rs. 14000 per month in 2010.

The Cost of Living index 165 between 2010 and 2020.

Worker should earned in 2020

$= \frac{165}{100} \times 14000 = ₹\ 23100$

Extra earning = 23100 – 14000 = **₹ 9100**

Q.41) The following table gives the Cost of living index number for 2018 and 2016 as the base for different commodity groups:

Food	410
Clothing	470
Fuel and light	320
Rent	450
Miscellaneous	150

With their weights in order in the ratio 15: 1: 2: 3: 4.

Obtain the overall Cost of living index number. Suppose a person was earning Rs. 14000 in 2016. What should be his salary in 2018 to maintain the same standard of living.

a) 16000 **b)** 64000 **c)** 32470 **d)** 51576

Sol. Option d)

Items	Index(P)	Weight (W)	WP
Food	410	$15x$	$6150\,x$
Clothing	470	$1\,x$	$470\,x$
Fuel and light	320	$2\,x$	$640\,x$
Rent	450	$3\,x$	$1350\,x$
Miscellaneous	150	$4\,x$	$600\,x$
		$25\,x$	$9210\,x$

Overall cost of living $= \frac{\Sigma PW}{\Sigma P} = \frac{9210x}{25x} = 368.4$

If a person was earning Rs. 14000 in 2016, then in order to maintain the same standard of living as in 2018 his salary in 2018 should be:

$$= \frac{368.4}{100} \times 14000 = \mathbf{51576}$$

Q.42) An index is at 100 in 2010. It rises 14% in 2011, falls 16% in 2012, falls 14% in 2013 and rises 13% in 2014. calculate the index number for 2014 with the base 2012

a) 86 **b)** 100 **c)** 97.18 **d)** 114

Sol. Option c)

Year	Old index (base 2010=100)	F.B.I. number
2010	100	$\frac{100}{95.76} \times 100 = 104.43$
2011	$\frac{114}{100} \times 100 = 114$	$\frac{114}{95.76} \times 100 = 119.04$
2012	$\frac{84}{100} \times 114 = 95.76$	$\frac{95.76}{95.76} \times 100 = 100$
2013	$\frac{86}{100} \times 95.76 = 82.35$	$\frac{82.35}{95.76} \times 100 = 86$
2014	$\frac{113}{100} \times 82.35 = 93.06$	$\frac{93.06}{95.76} \times 100 = \mathbf{97.18}$

Q.43) The time reversal test is satisfied when:

a) $P_{01} \times P_{10} = 1$ **b)** $P_{01} \times P_{10} = 0$

c) $P_{01} \times P_{10} > 1$**d)** $P_{01} \times P_{10} < 1$

Sol. Option a)

Time reversal test should be satisfied: $\mathbf{P_{01} \times P_{10} = 1}$

Q.44) The circular test is satisfied when:

a) $P_{01} \times P_{12} \times P_{20} = 1$
b) $P_{01} \times P_{12} \times P_{20} = 0$
c) $P_{01} \times P_{12} \times P_{10} > 1$
d)$P_{01} \times P_{12} \times P_{10} < 1$

Sol. Option a)

The circular test will be satisfied if: $\mathbf{P_{01} \times P_{12} \times P_{20} = 1}$

Q.45) From the following data for the 5 groups combined

Group	Weight	Index Number
Food	35	425
Cloth	15	235
Power & Fuel	20	215
Rent & Rates	8	115
Miscellaneous	22	150

The general Index number is

a) 270 **b)** 269.2 **c)** 268.5 **d)** 272.5

Sol. Option b) General Index number

$$= \frac{35\times425+15\times235+20\times215+8\times115+22\times150}{35+15+20+8+22}$$

$$= \frac{14875+3525+4300+920+3300}{100}$$

= 269.2

Q.46) Cost of Living Index (C.L.I.) numbers are also used to find real wages by the process of

a) Deflating of Index number. **b)** Splicing of Index number

c) Base shifting **d)** None of these

Sol. Option a) Cost of living index numbers are also used to find real wages by the process of deflating of index number.

Q.47) From the following data

Commodity	Base Price	Current Price
Rice	35	42
Wheat	30	35
Pulse	40	38
Fish	107	120

Simple Aggregative Index is:

a) 115.8 **b)** 110.8 **c)** 112.5 **d)** 113.4

Sol. Option b)

Commodity	P_0	P_1
Rice	35	42
Wheat	30	35
Pulse	40	38
Fish	107	120
Total	212	235

$$\therefore \text{Simple Aggregative Index} = \frac{\sum P_1}{\sum P_0} \times 100$$

$$= \frac{235}{212} \times 100$$

$$= \mathbf{110.849\ (approx)}$$

Q.48) Given below are the data on prices of some consumer goods and the weights attached to the various items. Compute price index number for the year 1985 (Base 1984 = 100)

Items	Unit	1984	1985	Weight
Wheat	Kg.	0.50	0.75	2
Milk	Litre	0.60	0.75	5
Egg	Dozen	2.00	2.40	4
Sugar	Kg.	1.80	2.10	8
Shoes	Pair	8.00	10.00	1

Then weighted average of price Relative Index is :

a) 121.08 **b)** 123.3 **c)** 124.53 **d)** 124.52

Sol. Option a)

Item	P_0	P_1	W_0	$P_0 W_0$	$P_1\ W_0$
Wheat	0.50	0.75	2	1.00	1.50
Milk	0.60	0.75	5	3.00	3.75
Egg	2.00	2.40	4	8.00	9.60
Sugar	1.80	2.10	8	14.40	16.80
Shoes	8.00	10.00	1	8.00	10.00
Total				34.40	41.65

∴ A weighted average of price Relative

$$\text{Index} = \frac{\Sigma P_1 W_0}{\Sigma P_0 W_0} \times 100 = \frac{41.65}{34.40} \times 100$$

= 121.08 (approx)

Q.50) From the following data

Group	A	B	C	D	E	F
Group Index	120	132	98	115	108	95
Weight	6	3	4	2	1	4

The general Index I is given by :

a) 111.3 **b)** 113.45 **c)** 117.25 **d)** 114.75

Sol. Option a)

Group	P_0	q_0	$P_0\ q_0$
A	120	6	720
B	132	3	396
C	98	4	392
D	115	2	230
E	108	1	108
F	95	4	380
		20	2226

$$\therefore I = \frac{\Sigma P_0\, q_0}{\Sigma q_0} = \frac{2226}{20} = \mathbf{111.3}$$

Q.51) The price of a commodity increases from ₹ 5 per unit in 1990 to ₹ 7.50 per unit in 1995 and the quantity consumed decreases from 120 units in 1990 to 90 units in 1995. The price and quantity in 1995 are 150% and 75% respectively of the corresponding price and quantity in 1990. Therefore, the product of the price ratio and quantity ratio is:

a) 1.8 **b)** 1.125 **c)** 1.75 **d)** None of these.

Sol. Option b) Price ratio = $\frac{7.5}{5} = 1.5$

Quantity ratio = $\frac{90}{120} = 0.75$

∴ Required Product = $1.5 \times 0.75 =$ **1.125**

Q.52) Test whether the index number due to Walsh given by:

$$I = \frac{\sum P_1\sqrt{Q_0Q_1}}{\sum P_0\sqrt{Q_0Q_1}} \times 100 \text{ Satisfies by: -}$$

- **a)** Time reversal Test.
- **b)** Factor reversal Test.
- **c)** Circular Test.
- **d)** None of these.

Sol. Option a)

$I = \frac{\sum P_1\sqrt{Q_0Q_1}}{\sum P_0\sqrt{Q_0Q_1}} \times 100$ Satisfies by Time reversal test.

Q.53) The number of tests of adequacy is

a) 2 **b)** 5 **c)** 3 **d)** 4

Sol. Option d)

The number of tests of adequacy are

1) Unit Test
2) Circular Test
3) Time reversal test
4) Factor reversal test

Q. 54) The index number is not a special type of averages

a) False **b)** True
c) Both (a) and (b) **d)** None of these

Sol. Option a)

The index number is a special type of averages.

Q.55) Fisher index formula does not satisfy_______ test.

a) Unit Test **b)** Circular Test
c) Time reversal test **d)** None of these

Sol. Option b)

Fisher index formula does not satisfy circular test.

Q.56) When the price or quantity of all goods are charging in the same ratio then Laspeyre's and Paasche's index number will be

a) Equal **b)** unequal
c) Either (a) or (b) **d)** None of these

Sol. Option a)

Laspeyre's index number = $\frac{\sum P_n q_0}{\sum P_o q_0} \times 100$

Paasches's index number $\frac{\sum P_n q_n}{\sum P_o q_n} \times 100$

When the price or quantity of all goods are charging in the same ratio then Laspeyre's and Paasche's index number will be equal.

Q.57) Between 1990 and 2000 the price of a commodity increased by 60% when the production decreased by 30% by what percentage did the value index of production of commodity change in 2000 with respect of its value 1990?

a) 10% **b)** 15%

c) 12% **d)** None of these

Sol. Option c)

Value of the commodity at the base on 1990 is 100

Value of the commodity on 2000 is $= 160 \times (100 - 30)\% = 112$

Change in the value of the commodity is $112 - 100 = 12\%$

Logical Reasoning

Number Series

Q.1) 4832, 5840, 6848, 7856, ?

a) 8864 **b)** 8815 **c)** 8846 **d)** 8887

Sol. Option a)
4832, 5840, 6848, 7856, ?

Let the no. be x

$5840 - 4832 = 1008,\ 6848 - 5840 = 1008,\ 7856 - 6848 = 1008$

$\therefore x - 7856 = 1008 \ \Rightarrow x = 8864$

Q.2) 165, 195, 255, 285, ?, 375

a) 345 **b)** 390 **c)** 335 **d)** 395

Sol. Option a)
165, 195, 255, 285, ?, 375

Let the number be x

Here $195 - 165 = 30,\ 255 - 195 = 60,\ 285 - 255 = 30,\ \therefore x - 285 = 60$

$\Rightarrow x = 345$

Q.3) Look at this series 2, 1, (1/2), (1/4), ... What number should come next?

a) (1/3) **b)** (1/8) **c)** (2/8) **d)** (1/16)

Sol. Option b)
This is a simple division series; each number is one-half of the previous number.
In other terms to say, the number is divided by 2 successively to get the next result.
4/2 = 2
2/2 = 1
1/2 = 1/2
(1/2)/2 = 1/4
(1/4)/2= 1/8 and so on.

Q.4) Look at this series 53, 53, 40, 40, 27, 27, ... What number should come next?

a) 12 **b)** 14 **c)** 27 **d)** 53

Sol. Option b)
In this series, each number is repeated, then 13 is subtracted to arrive at the next number.

Q.5) Priya is 2 times older than Riya. 3 years ago the sum of their current ages will be 66. What is the present age of Priya?

a) 20 **b) 40** **c) 60** **d)** None of these

Sol. Option b)
Assume that Riya's age is x
It is given that the Priya is twice as old as Riya, then, Priya's age will be 2 x
After 3 years,
Riya's age = x + 3
Priya's age = 2 x +3
Their total sum is 66

Hence, the equation becomes,

$= x + 3 + 2x + 3 = 66$

$= 3x + 6 = 66$

$= 3x = 60$

$= x = \frac{60}{3} = 20$

So, the current age of Riya is 20 and that of Priya is 40.

Q.6) A monkey climbs 30 feet at the beginning of each hour and rests for a while when he slips back 20 feet before he again starts climbing at the beginning of the next hour. If he begins his ascent at 8.00 a.m., at what will he first touch a flag at 120 feet from the ground?

a) 8.00p.m. **b)** 6.00p.m. **c)** 7.00p.m. **d)** None of these

Sol. Option c)

Let ascent of the monkey in 1 hour = (30- 20) = 10 feet.

So, the monkey ascends 90 feet in 9 hours i.e., 5 p.m.

Clearly, in the next 1 hour i.e., till 6 p.m.

The monkey ascends the remaining 30 feet to touch the flag in the next one hour

Q.7) 3, 12, 8, 19, 13, 32, 18, 42, 23, 52

a) 12 **b)** 19 **c)** 42 **d)** 32

Sol. Option b)

The sequence is a combination of following two series:

i)3, 8, 13, 18, 23 and ii) 12, 19, 32, 42, 52

The pattern in i) has a constant difference of +5, and the pattern in ii) has a constant difference of +10.

So, in ii) 19 is wrong and should be replaced by (12+10) i.e. 22

Q.8) Please choose one correct answer;

1|?|5|?|9|11

a) 2, 6 **b)** 3, 7 **c)** 2, 8 **d)** 3, 6

Sol. Option b)

There are two items missing. The only two visible adjacent items are 9 &11.

The difference between them is 2.

In addition, the difference between 5 and 9 is 4 as well as between 1 and 5.

That is, in both cases the difference between the 1st and 3rd item and the 3rd and 5th item is 4.

We can conclude that the missing numbers in the series should have a difference of 2 between the items adjacent to them on either side.

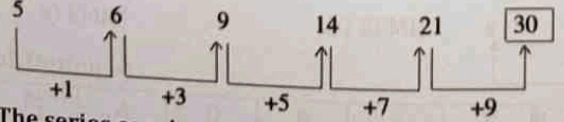
The numbers 3 & 7 complete the series.

The answer is 3,7

Q.9) 5, 6, 9, 14, 21, ?

a) 26 **b)** 28 **c)** 30 **d)** 34

Sol. Option c)

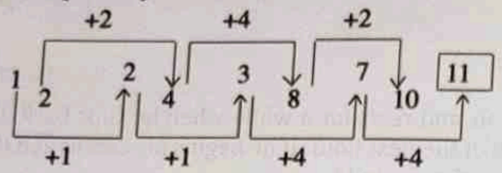


The series consists of a pattern of addition of consecutive odd numbers.

Q.10) 1, 2, 2, 4, 3, 8, 7, 10, ?

a) 9 **b)** 8 **c)** 11 **d)** 13

Sol. Option c)



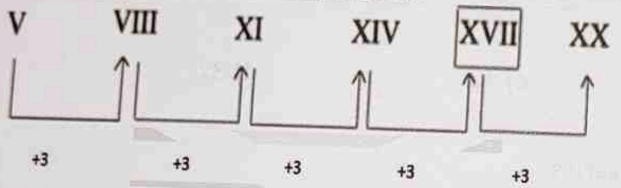
This series contains two separate series. In first series, 1 is added in first two steps and 4 is added in third and fourth steps. In second series, 2 and 4 is added alternately.

Q.11) Look at this series: V, VIII, XI, XIV, _____, XX, ... What number should fill the blank?

a) V **b)** XI **c)** XX **d)** XVII

Sol. Option d)

Pattern of the series is as shown below

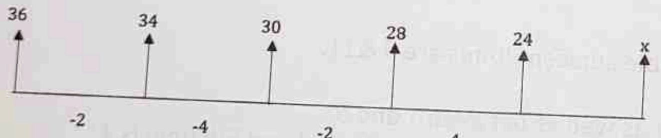


This is an arithmetic series in Roman numerals; each no. is 3 more than the previous one. Thus, the missing number will be the Roman equivalent of 20 - 3 = 17, i.e., XVII.

Q.12) Look at this series: 36, 34, 30, 28, 24, ... What number should come next?

a) 20 **b)** 22 **c)** 23 **d)** 26

Sol. Option b)



$= x = 24 - 2 = 22$

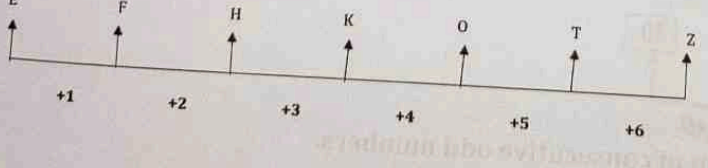
Q.13) What are next two letters in the given series?

E F H K O ? ?

a) TZ **b)** SY **c)** SX **d)** RW

Sol. Option a)

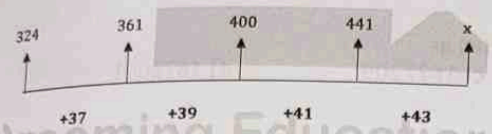
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z



Q.14) 324, 361, 400, 441, ?

a) 484 **b)** 488 **c)** 482 **d)** 486

Sol. Option a)



= x = 441 + 43 = 484

Q.15) What will come at the place of question mark?
4, 7, 12, 19, 28, ?

a) 49 **b)** 36 **c)** 30 **d)** 39

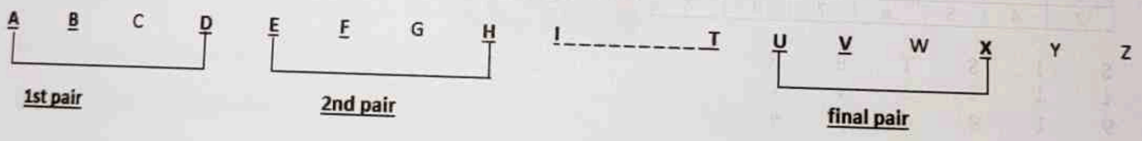
Sol. Option d)

First term: 4
Second term: 4 + 3 = 7
Third term: 7 + 5 = 12
Fourth term: 12 + 7 = 19
Fifth term: 19 + 9 = 28.
Therefore, Next Terms: 28 + 11 = 39

Q.16) Find the missing element in the series given below:
ABD, EFH, IJL, MNP, QRT, ?

a) XYZ **b)** VWY **c)** WXZ **d)** UVX

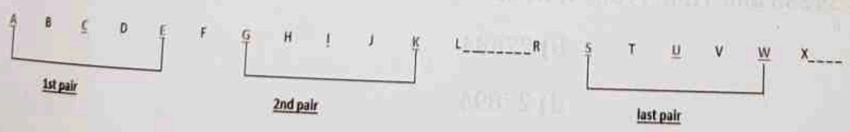
Sol. Option d)



Q.17) Find the missing element in the series given below:
ACE, ?, MOQ, SUW

a) GIK **b)** FHJ **c)** GHJ **d)** GIL

Sol. Option a)



Q.18) In a certain language, MADRAS is coded NBESBT, how DELHI is coded in that code?

a) EMMJ **b)** EFMIJ **c)** EMFIJ **d)** JIFEM

Sol. Option b)

M	A	D	R	A	S
↓	↓	↓	↓	↓	↓
N	B	E	S	B	T

∴ + 1 in the letters.

D	E	L	H	I
↓	↓	↓	↓	↓
E	F	M	I	J

Q.19) If RED is coded as 6720 then GREEN would be coded as

a) 9207716 **b)** 167129 **c)** 1677209 **d)** 1672091

Sol. Option c)

R	E	D
↓ +2	↓ +2	↓ +2
20	7	6

Write the code in reverse order as 6720

G	R	E	E	N
↓ +2	↓ +2	↓ +2	↓ +2	↓ +2
9	20	7	7	16

Write the code in reverse order as 1677209

Q.20) If BROTHER is coded 2456784, SISTER coded as 919684, what is coded for BORBERS?

a) 2542849 **b)** 2542898 **c)** 2454889 **d)** 2524889

Sol. Option a)

B	R	O	T	H	E	R
↓	↓	↓	↓	↓	↓	↓
2	4	5	6	7	8	4

S	I	S	T	E	R
↓	↓	↓	↓	↓	↓
9	1	9	6	8	4

∴

B	O	R	B	E	R	S
↓	↓	↓	↓	↓	↓	↓
2	5	4	2	8	4	9

Q.21) If CLOCK is coded as 34235 and TIME is 8679, what will be code of MOTEL?

a) 72894

b) 77684

c) 72964

d) 27894

Sol. Option a)

C	L	O	C	K
↓	↓	↓	↓	↓
3	4	2	3	5

and

T	I	M	E
↓	↓	↓	↓
8	6	7	9

∴

M	O	T	E	L
↓	↓	↓	↓	↓
7	2	8	9	4

Q.22) In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

a) 2
b) 5
c) 8
d) 3

Sol. Option c) 256 means → you are good

637 means → we are bad

358 means → good and bad

Clearly, **6** for **are**, **5** for **good** and **3** for **bad**

∴ **and** is **8**

Q.23) Which of the following is odd one?

a) CEHL
b) KMPT
c) OQTX
d) NPSV

Sol. Option d)

C	+2	E	+3	H	+4	L
K	+2	M	+3	P	+4	T
O	+2	Q	+3	T	+4	X
N	+2	P	+3	S	+3	V

Here, all are in same pattern except NPSV.

Q.24) If PLAY is coded as 8123 and RHYME is coded as 49367. What will be code of MALE?

a) 6217
b) 6198
c) 6395
d) 6285

Sol. Option a)

$$\begin{matrix} P & L & A & Y \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 8 & 1 & 2 & 3 \end{matrix} \quad \text{and} \quad \begin{matrix} R & H & Y & M & E \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 9 & 3 & 6 & 7 \end{matrix}$$

$$\therefore \quad \begin{matrix} M & A & L & E \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 6 & 2 & 1 & 7 \end{matrix}$$

Q.25) If HONEY is coded as JQPGA, which word is code as VCTIGVU?

a) CARPETS
b) TRAPETS
c) TARGETS
d) UMBRELU

Sol. Option c)

$$\because \begin{tikzcd} H \arrow[d, "+2"'] & O \arrow[d, "+2"'] & N \arrow[d, "+2"'] & E \arrow[d, "+2"'] & Y \arrow[d, "+2"'] \\ J & Q & P & G & A \end{tikzcd}$$

∴ VCTIGVU is the code of the word whose each letter will be 2 letter before the letter of given code. *i.e.,* TARGETS.

Q.26) $- - \ aba \ - - \ ba - ab$

a) *abbba*
b) *abbab*
c) *baabb*
d) *bbaba*

Sol. Option b) $- - \ aba \ - - ba - ab$

Here the series is ab, ab, ab, ab, ab, ab

∴ Missing letters = $abbab$

Q.27) If PALAM could be given the code number 43, what code number can be given to SANTACRUZ?

a) 123
b) 85
c) 120
d) 125

Sol. Option a) ∵ PALAM = 16 + 1 + 12 + 1 + 13 = 43

∴ SANTACRUZ = 19 + 1 + 14 + 20 + 1 + 3 + 18 + 21 + 26 = 123

Q.28) If A=1, FAT = 27, FAITH = ?

a) 44
b) 45
c) 46
d) 36

Sol. Option a) *Here* $A = 1$

$FAT = 27 = 6 + 1 + 20$

$\therefore FAITH = 6 + 1 + 9 + 20 + 8 = 44$

Q.29) Find odd one:

16, 25, 36, 73, 144, 196, 225

a) 36
b) 73
c) 196
d) 225

Sol. Option b)
16, 25, 36, 73, 144, 196, 225

All nos. are perfect square no. except 73.

Q.30) Find odd man out of the following:

1, 3, 5, 7, 11, 13, 17.

a) 1
b) 11
c) 7
d) 13

Sol. Option a) ∵ 1, 3, 5, 7, 11, 13, 17

Here, all are Prime number except 1

Q.31) Find odd man out of the following:

15, 25, 37, 49

a) 15 **b)** 25
c) 37 **d)** 49

Sol. Option c) $\because$ 15, 25, 37, 49

Here, all are composite number except 37 which is prime number.

Q.32) Find odd man out of the following:

8, 28, 343, 125

a) 8 **b)** 28
c) 343 **d)** 125

Sol. Option b)

8, 28, 343, 125

Here $8 = 2^3,\ 28 = 2^2 \times 7;\ \ 343 = 7^3,\ 125 = 5^3$

$\therefore$ Except 28, all are cube of a number.

Q.33) Find odd man out of the following:

295, 381, 552, 729

a) 295 **b)** 381
c) 552 **d)** 729

Sol. Option c) $\because$ 295, 381, 552, 729

Here, all are odd number except 552 which is even.

Q.34) Find odd man out of the following series:

7, 9, 13, 17, 19

a) 7 **b)** 9
c) 19 **d)** 13

Sol. Option b) $\because$ 7, 9, 13, 17, 19

Here, all are prime number except 9 which is composite number, also perfect square.

Q. 35) 165, 195, 255, 285, ?, 375

a) 345 **b)** 390 **c)** 335 **d)** 395

Sol. Option a) 165, 195, 255, 285, ?, 375

Let the number be x

Here $195 - 165 = 30,\ 255 - 195 = 60,\ 285 - 255 = 30,\ \therefore x - 285 = 60$

$\Rightarrow \boldsymbol{x = 345}$

Q. 36) 7, 26, 63, 124, 215, ?, 511

a) 342 **b)** 343 **c)** 441 **d)** 421

Sol. Option a) 7, 26, 63, 124, 215, ?, 511

Let the number be x

$7 = 2^3 - 1,\ 26 = 3^3 - 1,\ 63 = 4^3 - 1,\ 124 = 5^3 - 1,\ 215 = 6^3 - 1$

$\therefore x = 7^3 - 1 = 343 - 1 = 342\ \&\ 511 = 8^3 - 1$

∴ Required number = **342**

Q. 37) 3, 7, 15, 31, ?, 127

a) 62 **b)** 63 **c)** 64 **d)** 65

Sol. Option b) 3, 7, 15, 31, ?, 127

Let the number be x

$7 = 2 \times 3 + 1,\ 15 = 2 \times 7 + 1,\ 31 = 2 \times 15 + 1$

$\therefore x = 2 \times 31 + 1 = 63$

Also $127 = 2x + 1 \Rightarrow 2x = 126 \Rightarrow \boldsymbol{x = 63}$

Q. 38) 8, 28, 116, 584, ?

a) 1752 **b)** 3502

c) 3504 **d)** 3508

Sol. Option d) 8, 28, 116, 584, ?

Let the no. be x

Here $28 = 8 \times +4,\ 116 = 28 \times 4 + 4,\ 584 = 116 \times 5 + 4$

$\therefore x = 584 \times 6 + 4$

$= 3504 + 4 = \mathbf{3508}$

Q. 39) 6, 13, 28, 59, ?

a) 122 **b)** 114

c) 113 **d)** 112

Sol. Option a) 6, 13, 28, 59, ?

Let the no. be x

Here $13 = 6 \times 2 + 1,\ 28 = 13 \times 2 + 2,\ 59 = 28 \times 2 + 3$

$\therefore x = 59 \times 2 + 4 = \mathbf{122}$

Q. 40) 2, 7, 27, 107, 427, ?

a) 1707 **b)** 4027

c) 4207 **d)** 1207

Sol. Option a) 2, 7, 27, 107, 427, ?

Let the no. be x

Here $7 = 2 \times 4 - 1,\ 27 = 7 \times 4 - 1,\ 107 = 27 \times 4 - 1,\ 427 = 107 \times 4 - 1$

$\therefore x = 427 \times 4 - 1 = 1708 - 1 = \mathbf{1707}$

Q. 41) 5, 2, 7, 9, 16, 25, 41, ?

a) 65	**b)** 66
c) 67	**d)** 68

Sol. Option b) 5, 2, 7, 9, 16, 25, 41, ? (In the end 25 is written in the question is wrong)

Let the no. be x

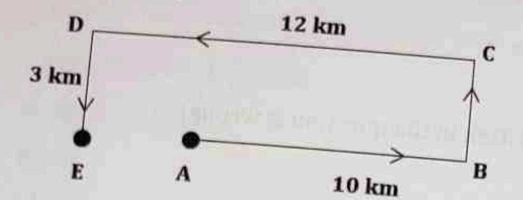
Here $7 = 5 + 2,\ 9 = 2 + 7,\ 16 = 7 + 9,\ 28 = 9 + 16,\ 41 = 16 + 25$

$\therefore x = 25 + 41 = \mathbf{66}$

Q.1) Arun started from point A and walked for 10 km East to point B, then turned to North andwalked for 3 kms to point C and then turned West and walked for 12 kms to point D, then againturned South and walked for 3 kms to point E. In which direction is he from his startpoint?

a) East
b) South
c) West
d) North

Sol. Option c)

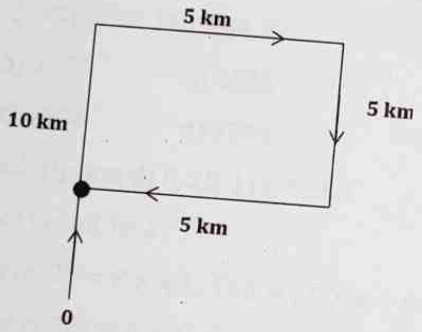


∴ Arun is in the West direction from his starting point.

Q.2) A man is facing East, then he turns left and goes for 10 km, then turns right and goes for 5 km then goes for 5 km to the South and from there for 5 km to West. In which direction is he from his original place?

a) East
b) West
c) North
d) South

Sol. Option c)



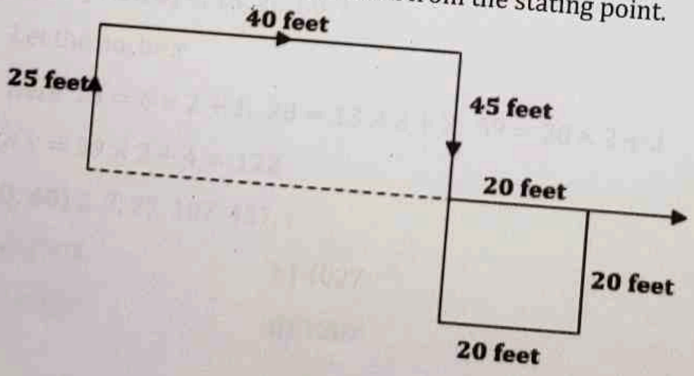
∴ Required direction of final position with original position in the North.

Q.3) A child walks 25 feet towards North, turns right and walks 40 feet, turns right again and walks 45 feet. He then turns left and walks 20 feet. He turns left again walks 20 feet. Finally, he turns to his left to walks another 20 feet. In which direction is the child from his starting point?

a) North
b) South
c) West
d) East

Sol. Option d)

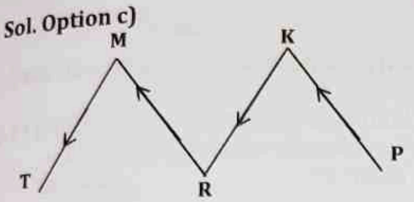
∴ The child is in the East direction from the stating point.



Q.4) K is a place which is located 2 kms away in the north-west direction from the capital P. R is another place that is located 2 kms away in the south-west direction from K. M is anotherplace and that is located 2 kms away in the north-west direction from R. T is yet another place that is located 2 kms away in the south-west direction from M. In which direction is T located in relation to P?

a) South-west **b)** North-west **c)** West **d)** North

Sol. Option c)

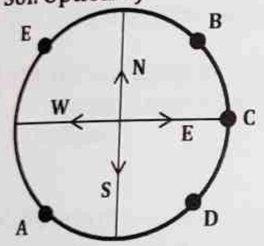


∴ T is in the West direction in relation to P.

Q.5) Five boys A, B, C, D, and E, are sitting in a park in a circle. A is facing South-West, D isfacing South-East, B and E are right opposite to A and D respectively and C is sitting equidistant between D and B. Which direction is C facing?

a) West **b)** South **c)** North **d)** East

Sol. Option d)

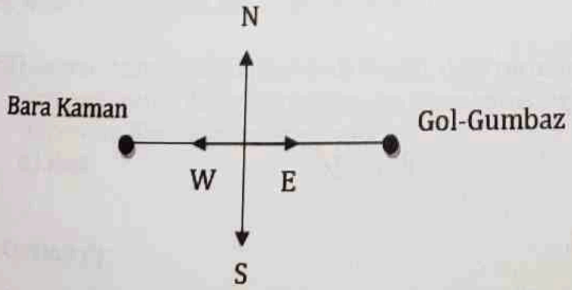


∴ C is facing in the East direction.

Q.6) Daily in the morning the shadow of Gol Gumbaz falls on Bara Kaman and in the evening the shadow of Bara Kaman falls on Gol Gumbaz exactly. So, in which direction is Gol Gumbaz to Bara Kaman?

a) Eastern side **b)** Western side **c)** Northern side **d)** Southern side

Sol. Option a)

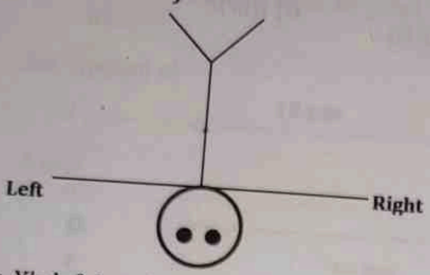


∴ Gol-Gumbaz is in the Eastern side of the Bara Kaman.

Q.7) If X stands on his head fairy face towards south, to which direction will his left-hand point?

a) East **b)** West **c)** North **d)** South

Sol. Option b)

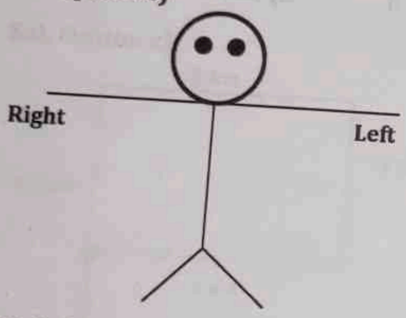


∴ X's left hand is in the West direction.

Q.8) If A stands on his head with his face towards north. In which direction will his left-hand point?

a) North-East **b)** North **c)** East **d)** North-West

Sol. Option c)

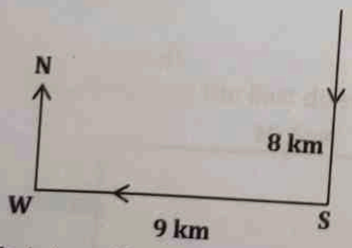


∴ A's left hand is in the East direction.

Q.9) A car travelling from south covers a distance of 8 kms, then turns right and runs another 9 kms and again turns to the right and was stopped. Which direction does it face now?

a) South **b)** North **c)** North **d)** East

Sol. Option a)

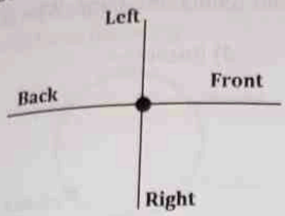


He is traveling in north direction to his face in south.

Q.10) Ram is standing with my right-hand extended side-ways towards South. Towards which direction will his back be?

a) North **b)** West **c)** East **d)** South

Sol. Option b)

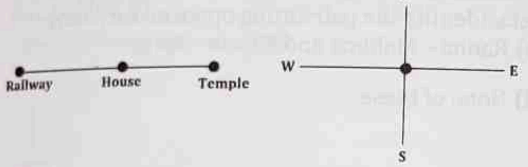


∴ Back side of Ram is in the West direction.

Q.11) If Mohan sees the rising sun behind the temple and the setting sun behind the railway station from his house, what is the direction of the temple from the railway station?

a) South **b)** North **c)** East **d)** West

Sol. Option c)

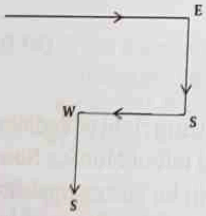


∴ The temple is in the east direction from the railway station.

Q.12) You are going straight, first eastwards, then turn to the right, then right again, then left. In which direction would you be going now?

a) East **b)** West **c)** South **d)** East

Sol. Option c)

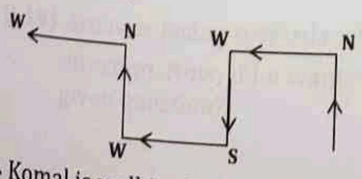


∴ He would be going to the South direction.

Q.13) Kamal starts walking towards North, then turns left and cover some distance, then he turns towards left and walks. After some time, he turns to his right and then turns right finally he turn left. In which direction Kamal is walking now

a) East **b)** South **c)** West **d)** South-East

Sol. Option c)

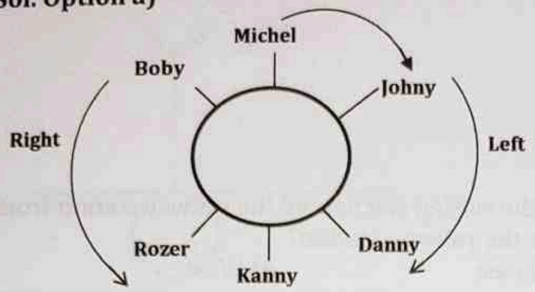


∴ Komal is walking in the west direction.

Q.14) Six friends are playing cards sitting around a circle and facing the centre. Kanny is sitting to the left of Danny, Michel is sitting between Body and Johny. Rozer is between Kanny and boby. Who is sitting to the left of Michel?

a) Johny **b)** Boby **c)** Kanny **d)** Rozer

Sol. Option a)



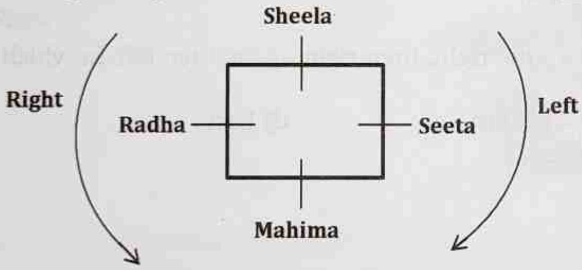
∴ Johny is in the left side of the Michel.

Q.15) Radha, Sheela and Mahima and Seeta are sitting around a rectangular table. Radha is sitting to the right of Sheela. Mahima is sitting to the left of Seeta. Identify the pair sitting opposite each other.

a) Radha – Seeta and Sheela -Mahima **b)** Radha – Mahima and Sheela - Seeta

c) Radha – Sheela and Mahima - Seeta **d)** None of these

Sol. Option a)



∴ Radha – Seeta and Sheela – Mahima

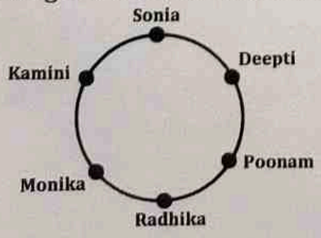
Q.16) Six girls are setting in a circle. Sonia in sitting opposite to Radhika. Poonam is sitting right of Radhika but left of Deepti. Monika is sitting left of Radhika. Kamini is sitting right of Sonia and left of Monika. Now, Deepti and Kamini, Monika and Radhika mutually exchange their positions. Who will be sitting opposite to Sonia?

a) Monika is sitting opposite to Sonia. **b)** Kamini is sitting opposite to Sonia

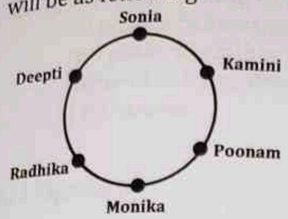
c) Deepti is sitting opposite to Sonia **d)** None of these

Sol. Option a

The given information, initial arrangement will be as following



Now, Deepti and Kamini, Monika and Radhika mutually exchange their positions. So, final arrangement will be as following.

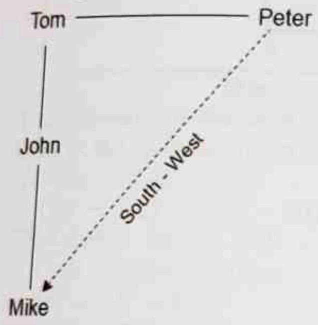


∴ Monika is sitting opposite to Sonia.

Q.17) Peter is in the East of Tom and Tom is in the North of John. Mike is in the South of John then in which direction of Peter is Mike?

a) South-East **b)** South-West **c)** South **d)** North-East

Sol. Option b)



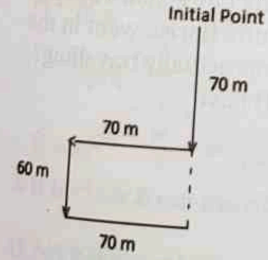
∴ Mike is in the direction of South-West.

Q.18) From a certain point, Smriti walks 70 m towards the south. Then, she turns to her right & starts walking straight for another 70m. Then, again turning to her left he walks for 60 m. She then turns to her left & walks for 70 m. How far is she from the starting point?

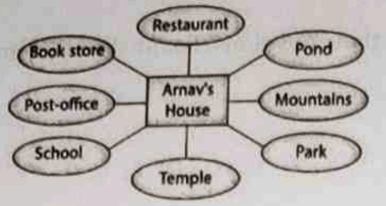
a) 120m **b)** 135m **c)** 140m **d)** 130m

Sol. Option d)

∴ **130 m, 70 + 60 = 130 m**



Q.19) Arnav is facing towards north. He makes $\frac{3}{4}$| turn to his right and then $\frac{1}{2}$| turn in anti-clockwise direction. Now, if he wants to face towards the bookstore, then what turn will he make. From the given question?



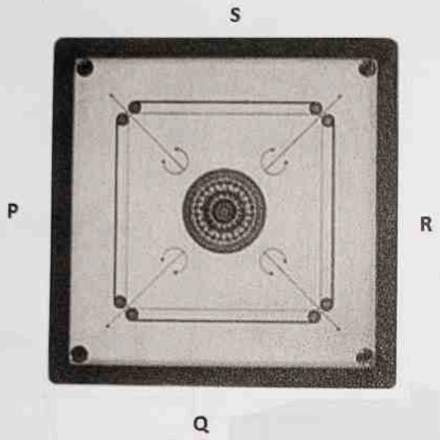
a) $\frac{1}{2}$ | turn anticlockwise
b) $\frac{5}{8}$ | turn clockwise
c) $\frac{3}{4}$ | turn towards right
d) $\frac{5}{8}$ | turn towards left

Sol. Option b)

Arnav is facing towards, north i.e., Arnav is facing towards restaurant. After Making $\frac{3}{4}$ | turn to his right and. $\frac{1}{2}$ | turn in anti-clockwise direction he will face towards.

Mountains. Now, if he want to face towards bookstore he will have to make. $\frac{5}{8}$ | turn clockwise.

Q.20)



P, Q, R, and S are playing a game of carrom. P, R, and S, Q are partners. S is to the right of R who is facing west. Then Q is facing?

a) North **b)** South **c)** East **d)** West

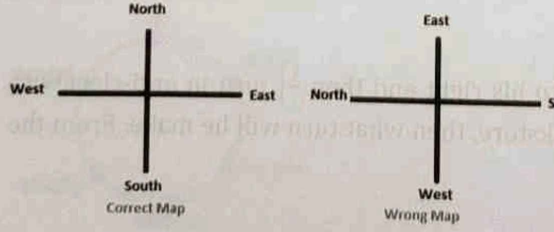
Sol. Option a)

Since R is facing West and P is the partner of R, therefore, P is facing East. Also, S is to the right of R, so S will be facing South and Q is the partner of S. Therefore, Q will face North.

Q.21) At a crossing, there was a direction pole, which was showing all the four correct directions. But due to the wind, it turns in such a manner that now West pointer is showing South. Harish went in the wrong direction thinking that he was travelling East. In what direction he was actually travelling?

a) South **b)** North **c)** West **d)** East

Sol. Option b)

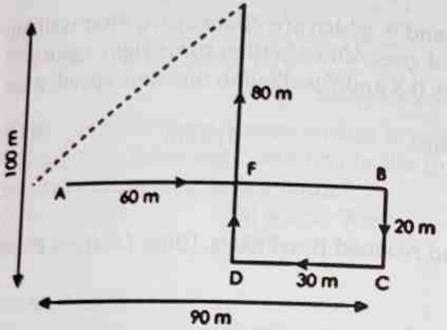


∴ Harish was actually travelling in the north direction.

Q.22) A child is looking for his father. He went 90 metres in the East before turning to his right. He went 20 meters before turning to his right again to look for his father at his uncle's place 30 metres from this point. His father was not there. From here he went 100 metres to the North before meeting his father in a street. What is the smallest distance between the starting point and his father's position?

a) 80 metres **b)** 100 metres **c)** 140 metres **d)** 260 metres

Sol. Option b)



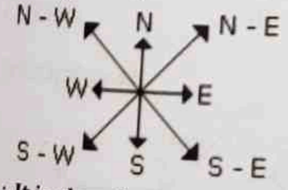
The movement of the child from A to E is as shown in fig.
Clearly, the child meets his father at E.

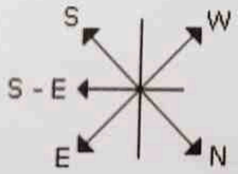
$Now, AF = (AB - FB)$
$= (AB - DC) = (90 - 30)\,m = 60\,m.$
$EF = (DE - DF) = (DE - BC)$
$= (100 - 20)m = 80\,m$
$= AE^2 = AF^2 + EF^2$
$= (60)^2 + (80)^2$
$= 3600 + 6400$
$= AE^2 = 10000$
$= AE = 100\,m$

Q.23) If South-East becomes North, North-East becomes West and so on. What will West become?

a) North-East **b)** North-West **c)** South-East **d)** South-West

Sol. Option c)



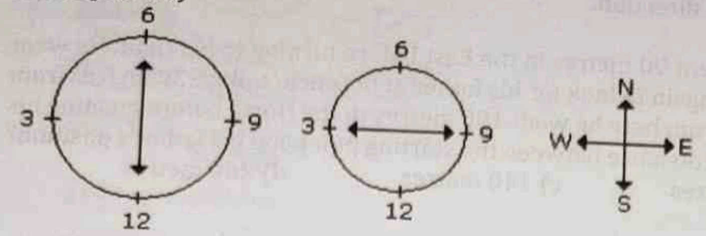


∴ It is clear from the diagrams that new name of West will become South-East.

Q.24) Rahul put his timepiece on the table in such a way that at 6 P.M. hour hand points to North. In which direction the minute hand will point at 9.15 P.M.?

a) South-East **b)** South **c)** North **d)** West

Sol. Option d)



∴ **At 9.15 P.M., the minute hand will point towards west.**

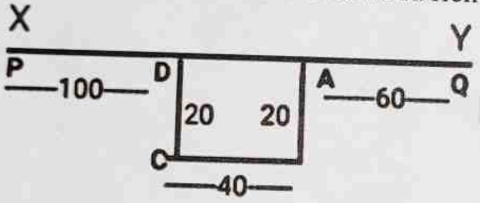
Q.25) X and Y start moving towards each other from points A and B, which are 200m apart. After walking 60m, Y turns left and goes 20m, then he turns right and goes 40m. He then turns right again and comes back to the road on which he had started walking. If X and Y walk with the same speed, what is the distance between them?

a) 20m **b)** 30m **c)** 40m **d)** 50m

Sol. Option c)

It is clear from the diagram given, Y covered 140m space and reached D, which is 100m far from P and also from Q.

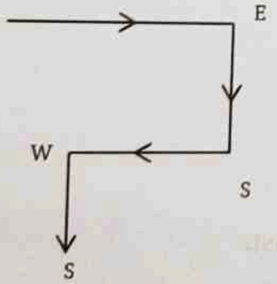
X also covered 140m and reached A. Hence, the distance DA = 40m.



Q.26) A man started to walk East. After moving a distance,. He turned to his right. After moving a distance, he turned to his again. After moving a little he turned in the end to his left. In which direction was he going now?

a) North **b)** South
c) East **d)** West

Sol. Option b)

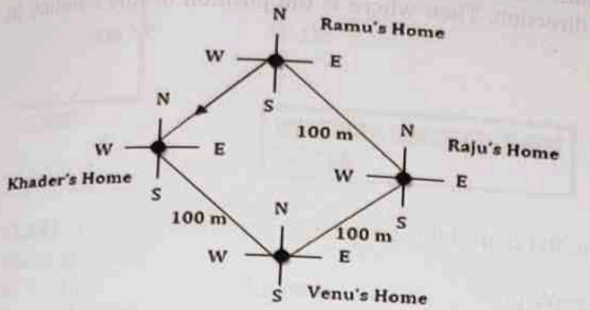


Q.27) Raju is Ramu's neighbour and he stay 100 metres away towards southeast. Venu is Raju's neighbour and he stay 100 metres away towards south east. Khadar is Venu's neighbour and he stays 100 metres away towards, north-west. Then where is the position of Khader's home in relation to Ramu's?

a) South-East **b)** South-West
c) North-West **d)** East

Sol. Option b)

∴ Position of Khadar's home is on relation of Ramu's home is south west

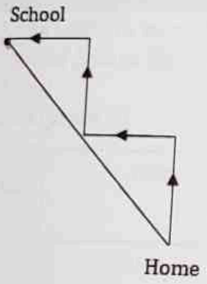


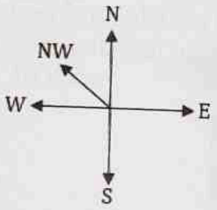
Q.28) From her home Prerna wishes to go to school. From home she goes towards North and then turns left and then turns right, and finally she turns left and reaches school. In which direction her school is situated with respect to her home?

a) North-East
b) North-West
c) South-East
d) South-West

Sol. Option b)

∴ Required direction is North-west

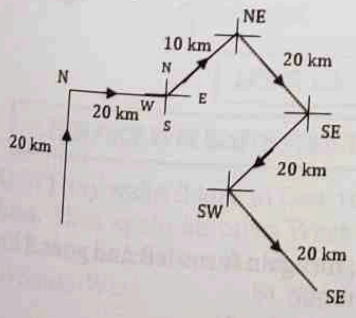




Q.29) Raju facing north and moves 20 kms, then he turned to his right and moves 20 kms and then he moves 10 kms in norths-east, then he turned to his right and moves 20 kms and then he turned to his right and moves 20 kms and again he turned to his left and moves 20 kms. Now in which direction Raju is facing?

a) South-East
b) South-West
c) North
d) North-East

Sol. Option a)

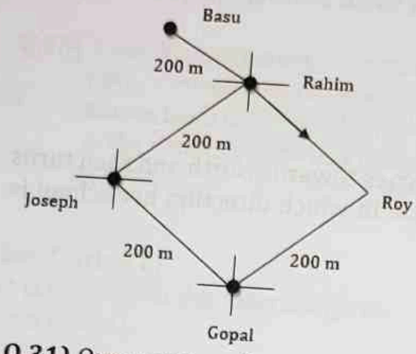


Q.30) Babu is Rahim's neighbour and his house is 200 meters away in the north-west direction. Joseph is Rahim's neighbour and his house is located 200 meter away in the south-west direction. Gopal is Joseph's neightbour and he stay 200 meters way in the south-east direction. Roy is Gopal's neighbour and his house is located 200 meters ways in the north-east direction. Then where is the position of Roy's house in relation of Bahu's?

a) South-East **b)** South-West
c) North **d)** North-East

Sol. Option a)

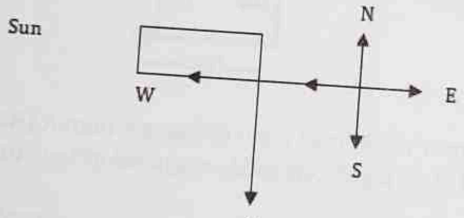
∴ Required direction is south-east



Q.31) One evening, Raja started to walk toward the Sun. After walking a while, he turned to his right and again to his right. After walking a while, he again turned right. In which direction is he facing?

a) South b) East c) West d) North

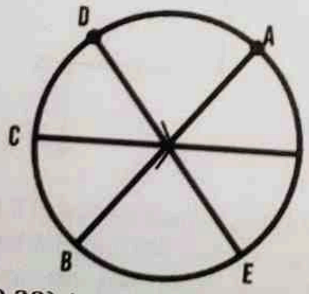
Sol. Option a)



Q.32) Five boys A, B, C, D and E are sitting in a park in a circle. A is facing South-West, D is facing South-East, B and E are right opposite A and D respectively and C is equidistant between D and B. Which direction is C facing?

a) West **b)** South
c) North **d)** East

Sol. Option d)



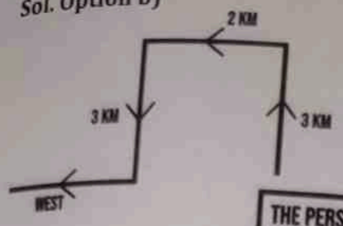
WHEN A,B,C D AND E FACING TOWARDS CENTRE OF THE CIRCLE

C IS FACING IN EAST DIRECTION

Q.33) A walks 3 kms northward and then he turns left and goes 2 kms. He again turns left and goes 3 kms. He turns right and walks straight. In which direction is he walking now?

a) East b) West c) North d) South

Sol. Option b)

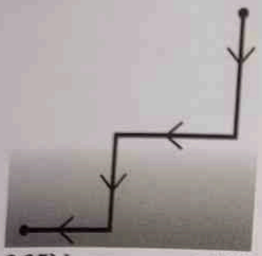


THE PERSON IS NOW WALKING IS WEST DIRECTION

Q.34) A walks southearts, then turns right, then left and then right. In which direction is he from the starting point?

a) South b) East c) West d) North

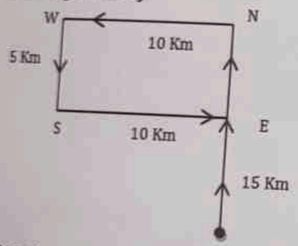
Sol. Option a)



Q.35) Laxman went 15 kms to North then he turned West and covered 10 kms. Then he turned south and covered 5 kms. Finally turning to East he covered 10 kms. In which direction he is from his house?

a) East b) West c) North d) South

Sol. Option c)



Q.36) Lakshmi walked 2 kms north from her house and took a turn to left and continued to walk another one kilometre and finally she turned left and reached the school. Which direction is she facing now?

a) West b) North c) South d) North

Sol. Option c)

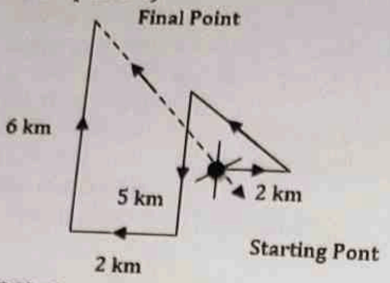
SHE IS WALKING IN SOUTH

∴ HER FACE IS IN SOUTH DIRECTION.

Q.37) Roy walks 2 kms to East, then turns North-West and walks 3 kms. Then he turns South and walks 5 kms. Then again he turns West and walks 2 kms. Finally he turns North and walks 6 kms. In which direction, is he from the starting point?

a) South-West b) South-East c) North-West d) North-East

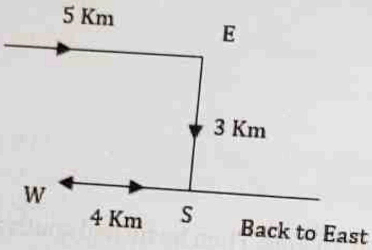
Sol. Option c)



Q.38) Shyam was facing East. He walked 5 kms forward and then after turning to his right walked 3 kms. Again he turned to his right and walked 4 km. After this he turned back. Which direction was he facing at that time?

a) East
b) West
c) North
d) South

Sol. Option a)

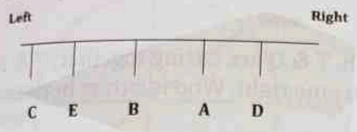


Sitting Arrangements

Q.1) Five boys A, B, C, D and E are sitting in a row A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end?

a) D **b)** A **c)** E **d)** B

Sol. Option c)

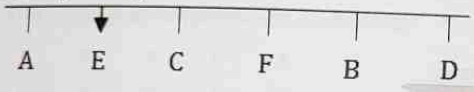


∴ Second from left end is E.

Q.2) Six children A, B, C, D, E and F are standing in a row. B is between F and D. E is between A and C. A does not stand next to either F or D. C does not stand next to D. F is between which of the following pairs of children?

a) B and E **b)** B and C **c)** B and D **d)** B and A

Sol. Option b)

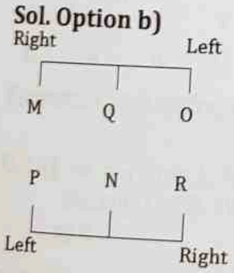


∴ F is between B and C.

Q.3) Six persons M, N, O, P, Q and R are sitting in two row with three persons in each row. Both therow are in front of each other. Q is not at the end of any row. P is second the left of R. O is theneighbour of Q and diagonally opposite to P. N is the neighbour of R. Who is in front of N?

a) R **b)** Q **c)** P **d)** M

Sol. Option b)



∴ The person front of N is Q.

Q.4) Read the following information carefully and answer the questions given below:
i) P,Q, R, S, T, U and V are sitting around a circular table facing the centre, ii) R is next to the left of U and V is second to the left of R. iii) P is sitting third to the left of T. iv) Q is between S and T. Which of the following is false?

a) P is fourth to the right of T.

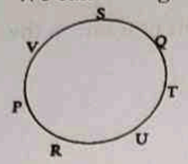
b) U is to the immediate right of R.

c) U is third to the left of S.

d) Q is to the immediate left of S.

Sol. Option c)

We can arrange the persons in the following way.



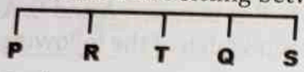
∴ U is third to the left of S is wrong.

Q.5) There are five students P, Q, R, S and T who are sitting on a bench. T & Q are sitting together, T & R are sitting together, P is on the extreme left, Q is second from extreme right. Who is sitting between P &Q?

a) Q & R **b)** R & T **c)** R & P **d)** R & S

Sol. Option b)

∴ R & T are sitting between P & Q.

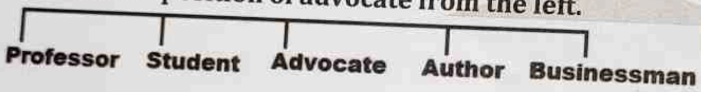


Q.6) Five persons are standing in one line. One of the two persons at extreme end is a professor and the other is a businessman. An advocate is to right of a student. An author is to the left of the businessman. What is the position of advocate from the left?

a) 2nd **b)** 3rd **c)** 4th **d)** 1st

Sol. Option b)

∴ The 3rd position of advocate from the left.

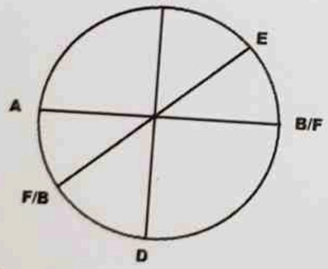


Q.8) A, B, C, D, E and F are sitting in a circular manner facing at the centre. D is between F and B, A is second to the left of D and second to the right of E. Who is facing A?

a) B **b)** F **c)** D **d)** B or F

Sol. Option d)

∴ B or F is facing A.

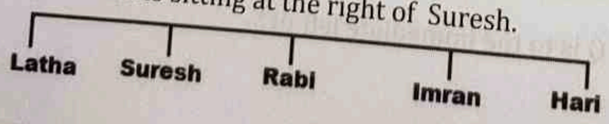


Q.9) Five friends are seated in a bench for a photograph, Imran sits to the immediate right of Rabi, who is not beside Hari. Latha sits to the immediate left of Suresh and is at the corner of the bench. Who among the following are sitting at to the right of the Suresh?

a) Imran **b)** Hari **c)** Rabi **d)** Can't say

Sol. Option c)

∴ Rabi is sitting at the right of Suresh.

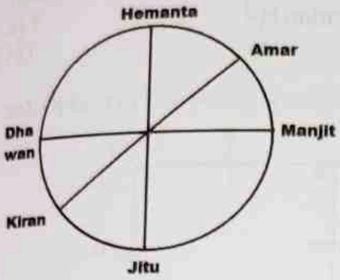


Q.10) Six friends were sitting around a circular table facing at the center Amar, Kiran, Jitu, Hemanta, Dhawan and Manjeet. Jitu is sitting 2 places to the left of Amar and opposite to Kiran. If Dhawan and Manjeet are opposite to each other. Who is sitting left of Jitu?

a) Dhawan **b)** Manjeet **c)** Kiran **d)** Hemanta

Sol. Option c)

∴ Kiran is sitting left of Jitu.



Q.11) A, P, R, X, S and Z are sitting in a row. S and Z are in the center. A and P are at the ends. R is sitting to the left of A. Who is to the right of P?

a) A **b)** X **c)** S **d)** Z

Sol. Option b)

The seating arrangement is as follows:

• • • • • •

P X S Z R A

Therefore, right of P is X.

Q.12) A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D the bench. C is on the second position from the right. A is to the right of B and E. A is sitting ?

a) Between B and D **b)** Between B and C **c)** Between E and D **d)** Between C and E

Sol. Option

• • • • •

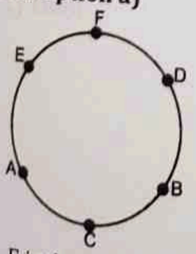
E B A C D

Therefore, A is sitting in between B and C.

Q.13) Six persons A, B, C, D, E & F are standing in a circle. B is between D & C. A is between E & C. F is at the right of D. Who is between A & F?

a) E **b)** C **c)** D **d)** None of these

Sol. Option a)



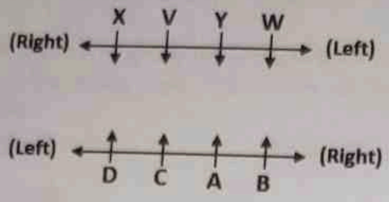
∴ E is the between A and F.

Q.14) There are eight persons sitting in two parallel rows. Four persons A, B, C and D are facing north and other four V, W, X and Y are facing south but not necessarily in the same order. Two persons sit between D and B. The one who faces B sits second to the left of V. V and X are immediate neighbours of each other. Only one person sits between D and A who is immediate neighbour of B. Y is neighbour of W who sit at an extreme end. Who sit opposite of Y?

a) X **b)** V

c) B **d)** A

Sol. Option d) The sitting arrangement on the basis of the given information is



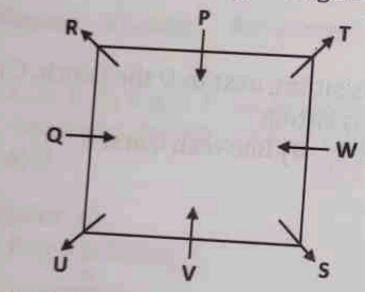
∴ Clearly, A sits opposite to Y.

Q.15) Eight persons are sitting around a square table. Four persons are sitting at middle of the sides of the square and all are facing towards inside. Remaining four are sitting at corners and they face outside. Two persons are sitting between P and U. R who is an immediate neighbour of P, sits opposite to S. T sits 3rd to the right of V. W sits immediate right of T. Q faces W. Who among the following person sit third to the right of Q?

a) S **b)** P

c) U **d)** R

Sol. Option a) The sitting arrangement on the basis of the given information is



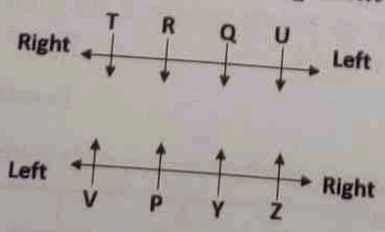
∴ Clearly, S sits third right of Q.

Q.16) Eight friends P, Q, R, T, U, V, Y& Z are sitting in two opposite rows, facing each other. Each row has 4 persons. P is between V and Y and are facing north. Z is opposite to U who is to the immediate left of Q. R is between T and Q. Z is to the immediate right of Y. Which of the following pairs of persons has second person is sitting to the immediate left of the first person?

a) VP **b)** YP

c) UQ **d)** RT

Sol. Option b) The sitting arrangement on the basis of the given information is

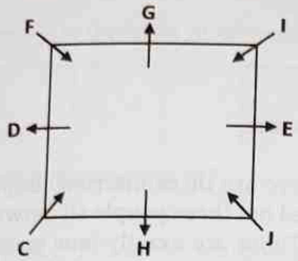


∴ Clearly, as per the fig only option (b) is true.

Q.17) C, D, E, F, G, H, I and J are sitting around a square table in such a way that four of them sit at four corners of the square and face the centre, while four sit in the middle of each of four sides and face outward.
D does not sit at any corners but sits second to the right of H. C sits third to the right of E, who sits second right of G. G and I are immediate neighbours of each other but G does not sit at any of the corners of the table. F is not an immediate neighbour of E and H.
Who among the following is the immediate neighbour of H and E?

a) F **b)** C
c) J **d)** I

Sol. Option c) The sitting arrangement on the basis of the given information is



∴ Clearly, J is the immediate neighbour of H and E.

Q.18) Six plays i.e. A, B, C, D, E and F are to be staged, one on each day from Monday to Saturday such that - A must be staged a day before E. C must not be staged on Tuesday. B must be staged on the day following the day on which F is staged. D must be staged on Friday only and should not be immediately preceded by B. E must not be staged on the last day of the schedule. Which of the following plays immediately follows B?

a) C **b)** A
c) D **d)** E

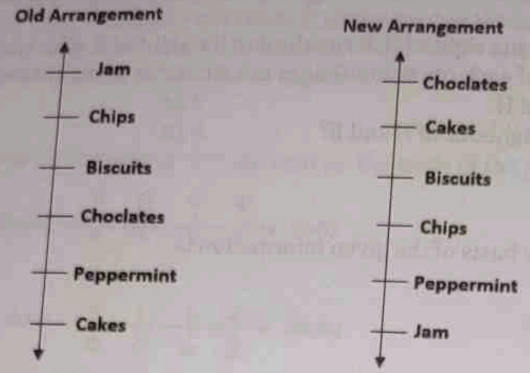
Sol. Option b) As per the below sequence, play A immediately follow play B.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
F	**B**	**A**	**E**	**D**	**C**

Q.19) Biscuits are arranged above the tins of chocolates but below the rows of packets of chips, cakes are at the bottom and the bottles of peppermints are below the chocolates. The topmost row had the display of jam bottles. Shopkeeper makes a small rearrangement where cakes occupy the place of chips, chips moved to chocolates row and chocolates to the topmost row. Where exactly are the cakes placed now? Mention the place from the bottom.

a) Second **b)** Fourth
c) Third **d)** Fifth

Sol. Option d) The sitting arrangement on the basis of the given information is



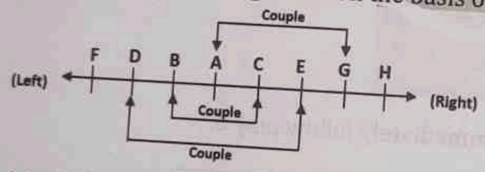
∴ Clearly, as per the new arrangement cakes placed fifth from the bottom.

Q.20) Eight people A, B, C, D, E, F, G and H are sitting in a row. There are three married couples in the group. Any person does not sit with her spouse. D and E, both married but three people sit between them. H is single and she is sitting at a very end. A is not married to D. There are exactly four people sitting between H and B. C who is married to B is an immediate neighbour of A. F is unmarried and he is not sitting on the right of any of the members.

Which of the following is definitely sitting between a married couple?

a) D
b) C
c) B
d) A

Sol. Option d) The sitting arrangement on the basis of the given information is

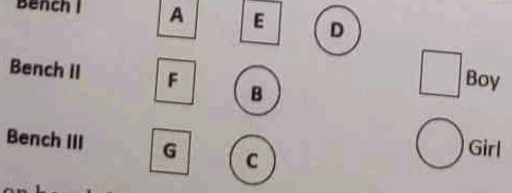


∴ Clearly, A is definitely sitting between a married couple.

Q.21) In a class there are seven students (including boys and girls) A, B, C, D, E, F and G. They sit on three benches I, II and III. Such that at least two students on each bench and at least one girl on each bench. C who is a girl student, does not sit with A, E and D. F the boy student sits with only B. A sit on the bench I with his best friends. G sits on the bench III. E is the brother of C. On which bench there are 3 students?

a) Bench I
b) Bench II
c) Bench III
d) Bench I or II

Sol. Option a) The sitting arrangement on the basis of the given information is



∴ Clearly, on bench I there are three students.

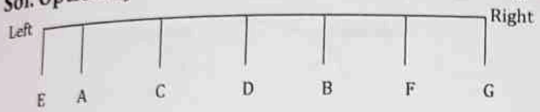
Q.22) A group of seven singers, facing the audience, are standing in line on the stage as follows.

i) D is to the immediate right to C
ii) F is standing beside G.
iii) B is to the immediate left of F
iv) E is to the immediate left of
v) C and B have one person between E and F
vi) A and D have oneperson between them

Who is on the Second extreme right?

a) D
b) F
c) G
d) E

Sol. Option b) On the basis of information standing arrangement of the singers is

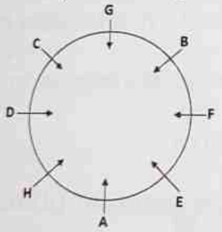


∴ The second extreme right is F

Q.23) A, B, C, D, E, F, G and H are sitting around a circle facing the centre. F is second to the right of A and third to the left of C. B is second to the left of C and fourth to the right of H. D is second to the right of G. In which of the following combinations is the third person sitting in between the first and the second person?

a) BGC
b) EFB
c) DAH
d) AEF

Sol. Option c) The sitting arrangement on the basis of the given information is

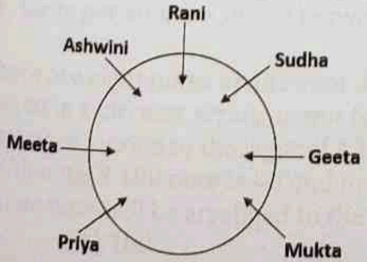


∴ Clearly, In the combination DAH, third person sitting in between the first and the second person.

Q.24) Ashwini, Priya, Sudha, Rani, Meeta, Geeta and Mukta are sitting around a circle facing the centre. Ashwini is third to the left of Mukta and to the immediate right of Rani. Priya is second to the left of Geeta, who is not an immediate neighbour of Meeta.
Which of the following is the correct position of Rani with respect to Mukta?

a) Third to the right
b) Third to the left
c) Fourth to the left
d) Fourth to the right

Sol. Option a) The sitting arrangement on the basis of the given information is



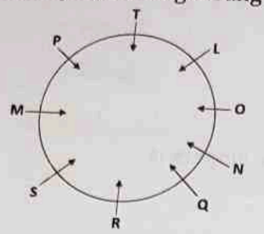
∴ Clearly, Rani's position is third to the right with respect to Mukta.

Q.25) Nine friends L, M, N, O, P, Q, R, S and T are sitting around a circle facing the centre. T sits 5th to the right of R. N is not an immediate neighbour of neither R nor T. M sits between S and P. N sits 4th to the left of P. O sits 2nd to the right of Q. S is not an immediate neighbour of T. Who is immediate left of L?

a) T **b)** P

c) S **d)** O

Sol. Option d) The sitting arrangement on the basis of the given information is



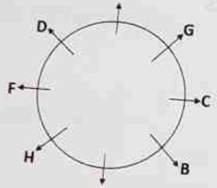
∴ Clearly, immediate left of L is O.

Q.26) Persons A, B, C, D, E, F ,G and H are sitting around a circular table but not necessarily in the same order and are facing away from the center.
G is one of the immediate neighbour of E. There are two persons sitting between A and D when counted from the right of A. B sits second to the right of G. F is an immediate neighbour of D. C sits exactly opposite to the F. Who sits fifth to the left of H?

a) G **b)** E

c) D **d)** C

Sol. Option b) The sitting arrangement on the basis of the given information is



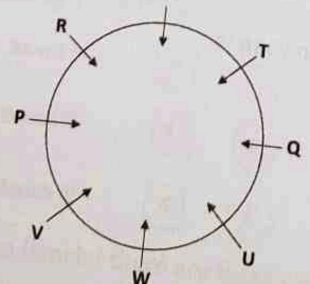
∴ Clearly, E sits fifth to the left of H.

Q.26) P, Q, R, S, T, U, V, W are sitting in a circle facing its center. S is sitting diametrically opposite to W who is sitting in the middle of V and U. T is sitting to the immediate left of S and to the immediate right of Q. P is diametrically opposite to Q and between (middle of) V and R. Who is sitting immediately to right of S.

a) P **b)** R

c) T **d)** V

Sol. Option b) The sitting arrangement on the basis of the given information is



∴ Clearly, R is sitting immediate right of S.

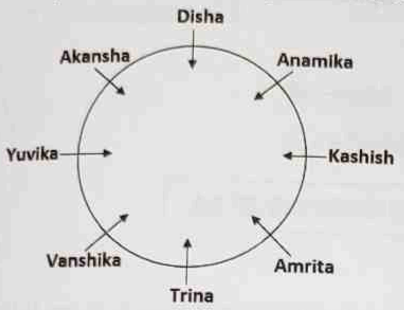
Q.27) Eight friends namely Kashish, Anamika, Amrita, Akansha, Trina, Yuvika, Vanshika and Disha are sitting around a circular table and each of them is facing the centre.

i) Kashish is second to the right of Trina who is the neighbour of Amrita and Vanshika
ii) Akansha is not the neighbour of Kashish
iii) Vanshika is the neighbour of Yuvika
iv) Anamika is not between Akansha and Disha
v) Disha is not between Yuvika and Akansha
vi) Disha does not sit next to Kashish

How many people sitting between Disha and Vanshika (counted anti-clockwise)?

a) Two **b)** One
c) Three **d)** Four

Sol. Option a) The sitting arrangement on the basis of the given information is



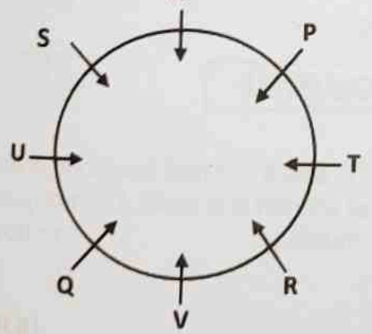
∴ Clearly, two people seated between Disha and Vanshika when counted anti-clockwise.

Q.28) Eight persons P, Q, R, S, T, U, V, W from two families are taking breakfast around a round table. T is sitting second to right of V. In all cases R has same position with respect to S, who is second to left of Q. S is sitting adjacent to W. U is not sitting between V and T. Q is immediate left of V. W is sitting immediate right of P.

How many persons sitting between P and Q when counted anti-clockwise?

a) 2 **b)** 3
c) 4 **d)** None

Sol. Option b) The sitting arrangement on the basis of the given information is

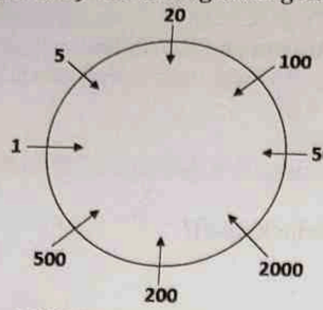


∴ Clearly, three persons are sitting between P and Q when counted anti-clockwise.

Q.29) There are eight notes of different denominations i.e., 1, 5, 20, 50, 100, 200, 500, 2000 rupees which are arranged in a circular arrangement facing towards the center not necessarily in the same order. ₹ 50 note is arranged second to the right of ₹ 200 note. ₹ 20 note is third to the left of ₹ 500 note who is exactly left of ₹ 200 note. ₹ 100 note is second to the left of ₹ 5 note who is in between ₹ 20 and ₹ 1 note. Which rupee note will be arranged to the third to the right of 1 rupee note?

a) 2000 **b)** 100
c) 200 **d)** 20

Sol. Option a) The sitting arrangement on the basis of the given information is



∴ Clearly, ₹ 2000 note will be arranged to the third right of 1 rupee note.

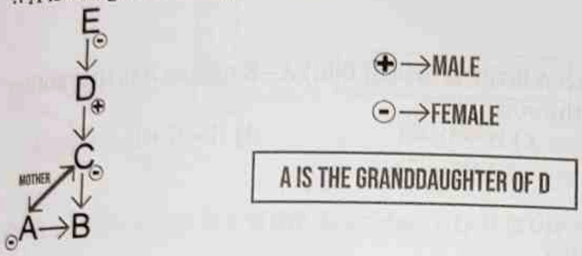
Q.1) A is B's Sister. C is B's Mother. D is C's Father. E is D's Mother. Then how is A related to D?
a) Grandmother **b)** Grandfather **c)** Daughter **d)** Grand-daughter

Sol. Option d)
∵ A is sister of B

∴ A is daughter of C who is mother of B

And D is the father of C

∴ A is the grand-daughter of D

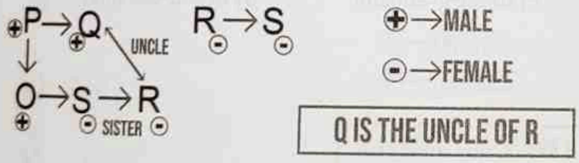


Q.2) P and Q are brothers. R and S are sister. P's son is S's brother. How is Q related to R?
a) Uncle **b)** Brother **c)** Father **d)** Grandfather

Sol. Option a)
Let O is the son of P
Q is uncle of R as Q is brother of P and P's son is brother of S

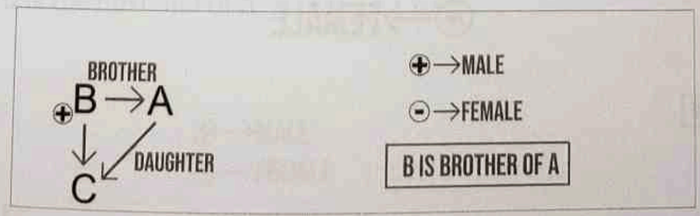
∴ P's son is brother of R

∴ R is the daughter of P.



Q.3) A reads a book and find the name of the author familiar. The author 'B' is the paternal uncle of C.C is the daughter of A. How is B related to A?
a) Brother **b)** Sister **c)** Father **d)** Uncle

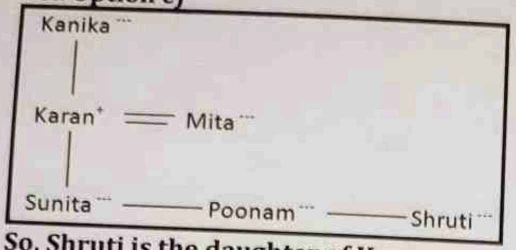
Sol. Option a)
B is brother of A as C is daughter of A and B is parental uncle of C .



Q.4) Sunita is daughter of Karan. Kanika, is the mother of Karan. Mita is the wife of Karan. Poonam & Shruti are daughters of Karan. How is Shruti related to Karan?

a) Mother **b)** Wife **c)** Daughter **d)** Niece

Sol. Option c)



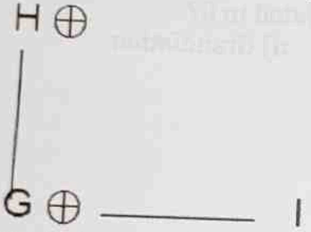
So, Shruti is the daughter of Karan.

Q.5) i) A+B means A is the brother of B ii) A × B means A is the father of B iii) A÷B means A is the mother of B. Which of the following would mean "G is the son of H"?

a) H × I × G **b)** H + G × I **c)** H ÷ G ÷ I **d)** H × G + I

Sol. Option d)

Go by options. In fourth option, our diagram will be like



We don't know the gender of I. So. We will not put any symbol on its side.

Q.6) A is B's brother. C is A's mother. D is C's father. F is A's son. How is F related to D?

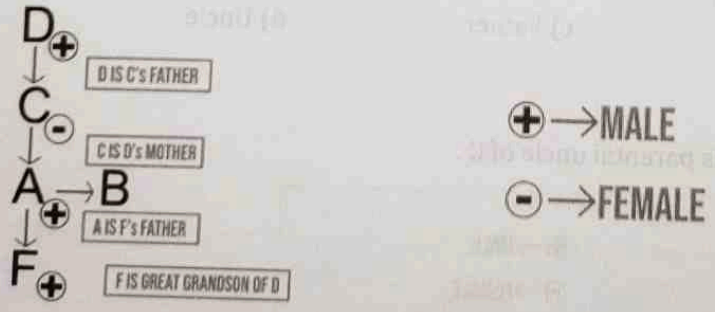
a) Son **b)** Grandson **c)** Great-grandson **d)** Grand-daughter

Sol. Option c)

F is great grandson of D

as F is son of A & C is the mother of A

∴ F is grandson of C and D is the father of C



Q.7) A and B are brothers. E is the daughter of F. F is the wife of B. What is the relation of E to A?

a) Sister **b)** Daughter **c)** Niece **d)** Sister-in-law

Sol. Option c)

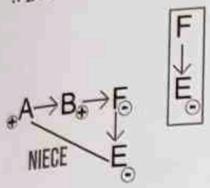
∵ E is the daughter of F

F is the wife of B

∴ E is the daughter of B

A is brother of B

∴ E is the niece of A



E IS THE NIECE OF A

⊕→MALE

⊖→FEMALE

Q.8) Q is the son of P. X is the daughter of Q. R is the aunty (Bua) of X and C is the son of R, then what is C to P?

a) Grandson **b)** ,,,,,,,,,,,,,,Grand-daughter **c)** Daughter **d)** Nephew

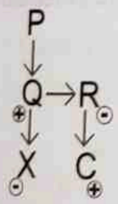
Sol. Option a)

∵ Q is son of P & X is daughter of Q

And R is the Aunt (Bua) of X

∴ R is the daughter of P and C is the son of R

∴ C is the grandson of P.



⊕→MALE

⊖→FEMALE

C IS THE GRANDSON OF P

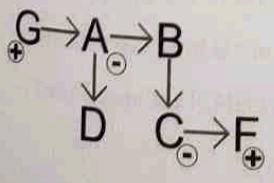
Q.9) A is the mother of D and sister of B. B has daughter a C who is married to F. G is the husband of A. How is G related to D?

a) Uncle **b)** Brother **c)** Father **d)** Grandfather

Sol. Option c)

∵ A is mother of D and G is the husband of A

∴ G is the father of D



⊕→MALE

⊖→FEMALE

G IS THE FATHER OF D

Q.10) P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and V. Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the grand-daughter of the family?

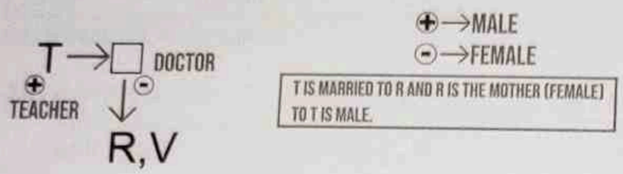
a) She is a lawyer **b)** She is an engineer **c)** She is a student **d)** She is a doctor

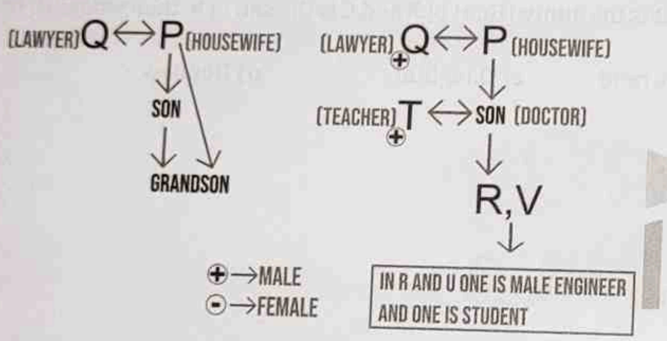
Sol. Option c)

∵ The family has two married couple

∴ Grand-daughter is not a lawyer, not engineer & not a doctor

∴ Grand-daughter will be a student





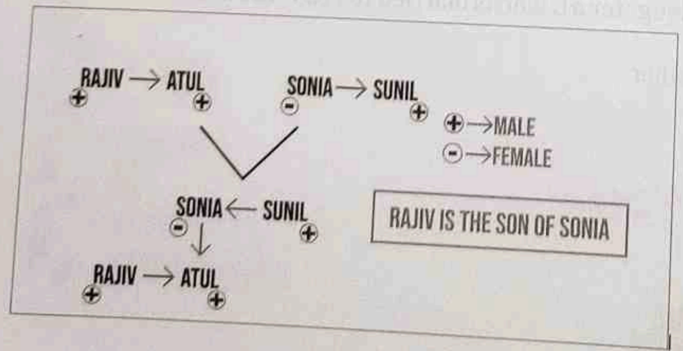
Q.11) Rajiv is the brother of Atul. Sonia is the sister of Sunil. Atul is the son of Sonia. How is Rajiv related to Sonia?

a) Nephew **b)** Son **c)** Brother **d)** Father

Sol. Option b)

∵Rajiv is the bother of Atul and Atul is the son of Sonia

∴ Rajiv is the son of Sonia



couples. T, a teacher is
. P has one son and one
student and one male
er of the family?
) She is a doctor

low is Rajiv related

ther

Q.12) Sita is the niece of Ashok. Ashok's mother is Lakshmi. Kalyani is Lakhshmi's mother. Kalyani's husband is Gopal. Parwathi is the mother-in-law of Gopal. How is Sita related to Gopal?

a) Great grandson's daughter **b)** son **c)** grand-daughter **d)** None of these

Sol. Option c)

∵ Sita is niece of Ashok

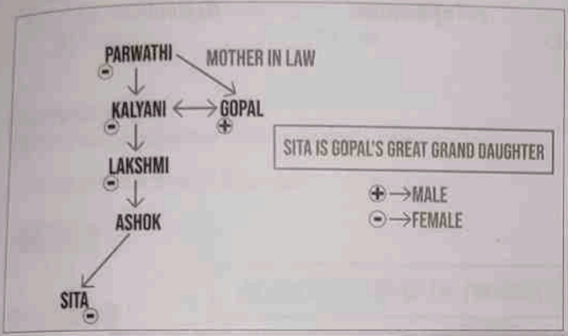
Lakshmi is the mother of Ashok

∴ Sita is the grand-daughter of Lakshmi & Kalyani is the Lakshmi's mother

∴ Sita is the great grand-daughter of Kalyani

& Gopal is the husband of Kalyani

∴ Sita is the great grand-daughter of Gopal



Q.13) Seema is the daughter-in-law of Sudhir and sister-in-law of Ramesh. Mohan is the son of Sudhirand only brother of Ramesh. Find the relation between Seema and Mohan

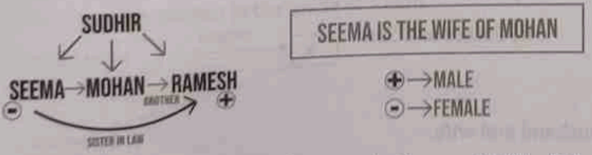
a) Sister-in-law **b)** Aunt **c)** Cousin **d)** Wife

Sol. Option d)

Seema is daughter in law of Sudhir and sister in law of Ramesh

∴ Sudhir is father of Ramesh and Mohan is the only brother of Ramesh

∴ Seema is the wife of Mohan



Q.14) Pointing to a lady in a photograph. Meera said. "Her father's only son's wife is my mother-in-law"How is Meera's husband related to that lady in the photo?

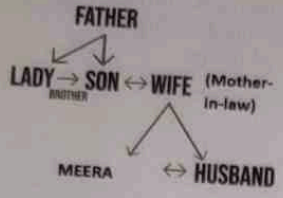
a) Nephew **b)** Uncle **c)** Son **d)** Father

Sol. Option a)

∵The lady's father's only son's wife is the mother in law of Meera

∴ Meera's husband is the lady's father's only son's son

∴ Lady's nephew is Meera's husband.



MEERA'S HUSBAND IS NEPHEW OF THAT LADY

⊕→MALE
⊖→FEMALE

Q.15) Suresh's sister is the wife of Ram. Ram is Rani's brother. Ram's father is Madhur. Sheetal is Ram's grandmother. Reema is Sheetal's daughter-in-law. Rohit is Rani's brother's son. Who is Rohit to Suresh?

a) Brother-in-law **b)** Son **c)** Brother **d)** Nephew

Sol. Option d)
Rohit is Rani's brother's son

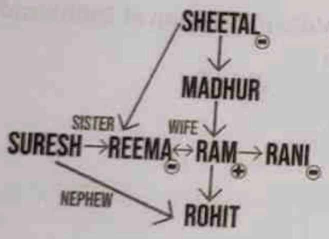
& Ram is the brother of Rani

∴ Rohit is the son of Ram

Suresh's sister is the wife of Ram

∴ Suresh's sister of the mother of Rohit

∴ Rohit is the nephew of Suresh



⊕→MALE
⊖→FEMALE

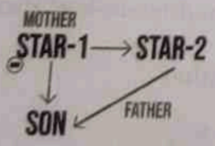
ROHIT IS THE NEPHEW OF SURESH

Q.16) There are 2 film stars. One is the father of the other's son. What is the relationship of the two with each other?

a) Grandfather and Grandson **b)** Grandfather and son

c) Husband and wife **d)** Father and Son

Sol. Option c)
One is the father of other son so they are husband and wife.



⊕→MALE
⊖→FEMALE

STAR-1 AND START-2 ARE HUSBAND AND WIFE

Q.17) Vijay says, Ananda's mother is the only daughter of my mother". How is Ananda relation to Vijay?

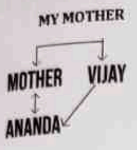
a) Brother **b)** Father **c)** Nephew **d)** Grandfather

Sol. Option c)

∵ Ananda's mother is only daughter of Vijay's mother

∴ Ananda's mother is sister of Vijay

∴ Ananda is nephew of Vijay



ANANDA IS THE NEPHEW OF VIJAY

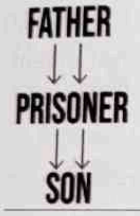
Q.18) A prisoner introduced a boy who came to visit him to the jailor as "Brothers and sisters I have none, he is my father's son's son". Who is the boy.

a) Nephew **b)** Son **c)** Cousin **d)** Uncle

Sol. Option b)

The boy is the Prisoner's father's son's son

∴ The boy is the son of Prisoner



THE BOY IS THE SON OF THE PRISONER

Q.19) Pointing to a person, Aman said, "His only brother is the father of my daughter's father". How is the person related to Aman?

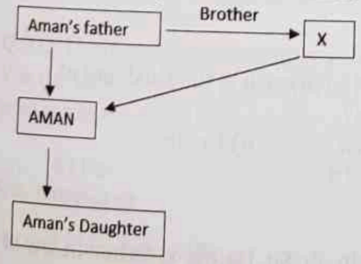
a) Nephew **b)** Father **c)** Uncle **d)** Brother

Sol. Option c)

Father of Aman's daughter's father → Aman's father.

Hence, the person in the brother of Aman's father.

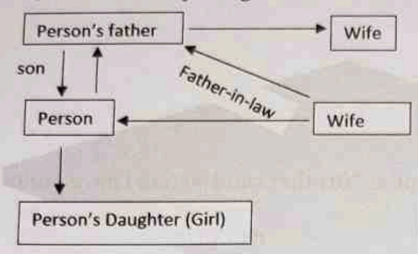
Therefore, the person is the uncle of Aman.



Q.20) Pointing towards a girl, a Person said, "She is the only daughter of the only son of the wife of the father-in-law of my wife". How is the girl related to the Person?

a) Daughter **b)** Wife **c)** Mother **d)** None of these

Sol. Option a)
Father-in-Law of my wife = Father in Law of the Person's Wife = Person's Father
Wife of the Father-in-law of my wife = Wife of the Person's Father = Person's Mother.
Only son of Person's Mother = That Person only (because Person's parents don't have any other child)
So, she is the only daughter of Person as the Person does not have any other Children.

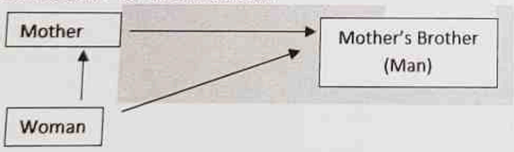


Q.21) A woman walking with a boy meets another woman and on being asked about her relationship with the boy, she says, "My maternal uncle and his maternal uncle's maternal uncle are brothers." How is the boy related to the woman?
a) Son **b)** Nephew **c)** Grandson **d)** Brother-in-law

Sol. Option a)
The boy is the son of the woman. Boy's maternal uncle will be brother of boy's mother. Maternal uncle of mother's brother and maternal uncle of lady are brother means lady is sister of mother's brother i.e., lady is the mother of the boy. So, the boy is woman's son.

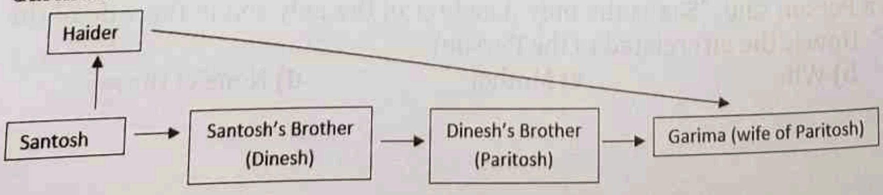
Q.22) A woman introduces a man as the son of the brother of her mother. How is that man related to the woman?
a) Son **b)** Nephew **c)** Cousin **d)** Uncle

Sol. Option c)
Brother of woman's mother is the maternal uncle of the woman and the son of maternal uncle is related as cousin to the woman.



Q.23) Santosh is the brother of Dinesh. Dinesh is the brother of Paritosh. Paritosh is the Husband of Garima. Haider is the father of Santosh. What is Haider to Garima?
a) Brother **b)** Mother **c)** Father-in-law **d)** Uncle

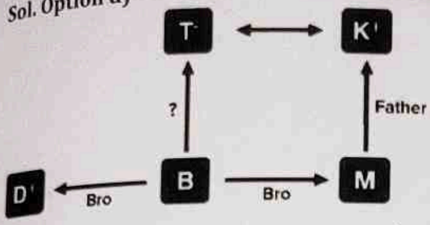
Sol. Option c)
Santosh, Dinesh and Paritosh are siblings and Garima is the wife of Paritosh. So, Haider is father-in law of Garima.



Q.24) D is the brother of B, M is the brother of B, K is the father of M and, T is the wife of K. How B is related to T?

a) Son **b)** Daughter **c)** Both **d)** None of these

Sol. Option d)



As per the given statements, we can say that no gender is assigned to B value.
Following are the conclusion you will get after drawing the Tree.
D, B and M are siblings
K is Father of D, B and M.
T is Mother of D, B and M.
So, the conclusion is B is either Son or Daughter of T.

Q.25) If A + B means A is the mother of B; A - B means A is the brother of B; A % B means A is the father of B and A x B means A is the sister of B, which of the following shows that P is the maternal uncle of Q?

a) Q-N+M×P **b)** P-M+N×Q **c)** P+S×N-Q **d)** Q-S%P

Sol. Option b)
P-M = P is the brother of M.
M+N = M is the mother of N.
N×Q = N is the sister of Q.
Therefore, P is the maternal uncle of Q.

Q.26) In a joint family, there are father, mother, 3 married sons and one unmarried daughter. Of the sons two have 2 daughters each and one has a son. How many female members are there in the family?

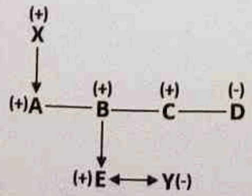
a) 2 **b)** 7 **c)** 9 **d)** 6

Sol. Option c) In the family there is a mother, one unmarried daughter, three married sons which means three wives. Two of the son has two daughter each which means four daughters.
Hence, there are 9 female members in the family.

Q.27) There are seven members is a train. X, A, B, C, D, E and Y. A and B are brothers, C is the uncle of E, Y is the wife of E who is the son of B. X is the father of A. D is the sister of C. How many siblings are present in the family?

a) One **b)** Two **c)** Three **d)** Four

Sol. Option c)



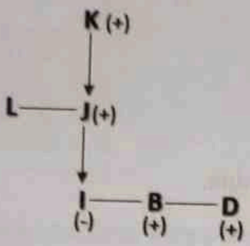
A and B are brothers, C is the uncle of E and E is the son of B. C is the sister of D.
According to the above statement, there are three siblings are present here.

Q.28) I, J, K and L are all distinct individual. I is the daughter of J. J is the son of K and K is the father of L. If B is the son of J and B has one brother, D. Then which of the following statement is true?

I. I is the sister of D.
II. D and B are brother.
III. K is the grandfather of D.

a) Only I **b)** I and II **c)** Only III **d)** All of these

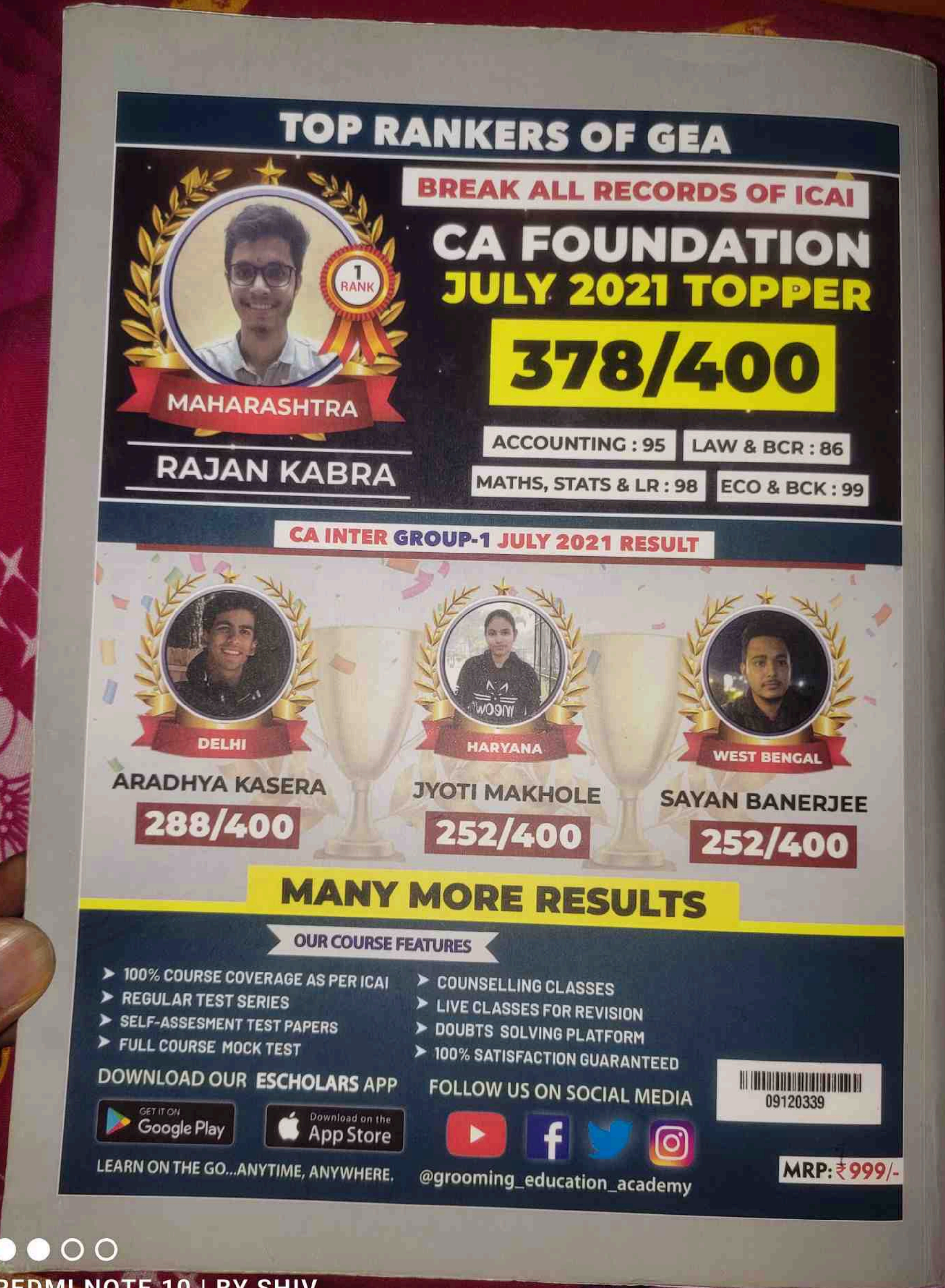
Sol. Option d)



I is the sister of D.
D and B are brother.
K is the grandfather of D.
All these statements are true.

e son of K and K is the father of L,
g statement is true?

d) All of these



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