

ALL FORMULAS

CA FOUNDATION MATHEMATICS

(112 formulas)



CA. PRANAV POPAT

- **Chartered Accountant by Qualification**
- **Educator Dil Se** ♥
- **Qualified all CA levels in very first attempt**
- **My Aim is to remove Maths Phobia from commerce background students and make Stats and Maths as their strength to crack CA Exam**
- **Educator at Unacademy for CA Foundation Maths, LR and Stats and CA Intermediate Cost and Management Accounting**

› If a quantity increases or decreases in the ratio $a:b$ then

$$\text{new quantity} = \frac{b}{a} \times \text{original quantity}$$

The fraction by which the original quantity is multiplied to get a new quantity is called the **factor multiplying ratio**.

- › **Inverse Ratio:** One ratio is the inverse of another if their product is 1. Thus $b : a$ is the inverse of $a : b$ and *vice-versa*.

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- › The ratio **compounded** of the two ratios $a : b$ and $c : d$ is $ac : bd$.
- › Compounding two or more ratios means **multiplying** them.

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- › A ratio compounded of itself is called its duplicate ratio.

$$a^2 : b^2$$

is the duplicate ratio of a:b

$$a^3 : b^3$$

is the triplicate ratio of a:b

$$\sqrt{a} : \sqrt{b}$$

is the sub-duplicate ratio of a:b

$$\sqrt[3]{a} : \sqrt[3]{b}$$

is the sub-triplicate ratio of a:b

- › **Continued Ratio:** is the relation or comparison between the magnitudes of **three or more** quantities of same kind.
- › The continued ratio of three similar quantities a, b, c can be written as **a:b:c**

- › **Cross Product Rule:** If $a : b = c : d$ are in proportion then $ad = bc$

Product of extremes = Product of means

- › **Continuous Proportion:** Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if $a : b = b : c$

$$\frac{a}{b} = \frac{b}{c}$$

$$b^2 = ac$$

here, a = first proportional, c = third proportional and b is mean proportional (because b is GM of a and c)

› Invertendo

If $a : b = c : d$, then

$$b : a = d : c$$

› Alternendo

If $a : b = c : d$, then

$$a : c = b : d$$

› Componendo

If $a : b = c : d$, then

$$a + b : b = c + d : d$$

› Dividendo

If $a : b = c : d$, then

$$a - b : b = c - d : d$$

› Componendo and Dividendo

If $a : b = c : d$, then

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\frac{a-b}{a+b} = \frac{c-d}{c+d}$$

› Addendo

If $a:b = c:d = e:f = \dots = k$

then $\frac{a + c + e + \dots}{b + d + f + \dots} = k$

› Subtrahendo

If $a:b = c:d = e:f = \dots = k$

then
$$\frac{a - c - e + \dots}{b - d - f + \dots} = k$$

Indices – Standard Results

- › Any base raised to the power zero is defined to be 1

$$a^0 = 1$$

- › Roots can also be expressed in the form of power.

$$\sqrt[r]{a} = a^{\frac{1}{r}}$$

Law 1

$$a^m \times a^n = a^{m+n}$$

- › If two or more terms with same base are multiplied, we can make them one term having the same base and power as sum of all powers.

Law 2

$$\frac{a^m}{a^n} = a^{m-n}$$

- › If two or more terms with same base are in division, we can make them one term having the same base and power as difference of power.

Law 3

$$\left(a^m\right)^n = a^{m \times n}$$

- › If a term having power is raised to another power, we can do product of powers to simplify the expression

Law 4

$$(a \times b)^n = a^n \times b^n$$

- › If a product of two or more terms is raised to power, we can split the two terms with same individual power to each one of them.

Calculator Trick for Power

Base

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Calculator Trick for any root

Base $\sqrt{\quad} \sqrt{\quad} \sqrt{\quad} \dots$ 12 *times* -1 \div n
 $+1$ $\times =$ $\times =$ $\times =$... 12 *times*

Calculator Trick for any power (including non integer)

Base $\sqrt{\quad} \sqrt{\quad} \sqrt{\quad} \dots$ 12 *times* -1 \times n
 $+1$ $\times =$ $\times =$ $\times =$...

Log Conditions

- › The logarithm of a number to a given base is the index or the power to which the **base must be raised to produce the number**, i.e. to make it equal to the given number.

$$3^4 = 81$$

$$\log_3 81 = 4$$

- › If $a^x = n$ then $\log_a n = x$

- › *Conditions:*

- *Number should be positive*
- *Base should be positive*
- *Base cannot be equal to one*

$$n > 0, a > 0, a \neq 1$$

Standard Results of Log

- › Log of a number with same base as number is equal to 1

$$\log_a a = 1$$

- › Log of 1 (one) for any base is equal to zero

$$\log_a 1 = 0$$

Law 1

- › Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base

$$\log_a mn = \log_a m + \log_a n$$

Law 2

- › The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

Law 3

- › Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base.

$$\log_a m^n = n \log_a m$$

Change of Base Theorem

- › If the logarithm of a number to any base is given, then the logarithm of the same number to any other base can be determined from the following relation

$$\log_b m = \frac{\log m}{\log b} = \frac{\log_a m}{\log_a b}$$

$$\log_b a \times \log_a b = 1$$

Base of Log

› Common Log's Base

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› Natural Log's Base

e

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Quadratic Equation

- › Equation having **degree = 2** is called as Quadratic Equation
- › QE will have two roots/ solutions usually denoted by α, β
- › Equation Format $ax^2 + bx + c = 0$

where,
 a is coefficient of x^2
 b is coefficient of x
 c is constant
 $a \neq 0$

Solution of Quadratic Equation

$$ax^2 + bx + c = 0$$

› Formula to calculate roots:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where,
 a is coefficient of x^2
 b is coefficient of x
 c is constant
 $a \neq 0$

Sum and Product of Roots of QE

$$ax^2 + bx + c = 0$$

› *Sum of roots*

$$\alpha + \beta = -\frac{b}{a}$$

› *Product of roots*

$$\alpha\beta = \frac{c}{a}$$

› Construction of Quadratic Equation

If sum of roots and product of roots are given, equation can be constructed in the below manner:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

› Concept of discriminant – to get nature of roots

Discriminant of QE is the mathematical expression which is used to understand nature of roots of QE, it is expressed as below:

$$b^2 - 4ac$$

Condition	Nature of Roots
$b^2 - 4ac = 0$	<i>Real and Equal</i>
$b^2 - 4ac < 0$	<i>Imaginary</i>
$b^2 - 4ac > 0$	<i>Real and Unequal</i>
$b^2 - 4ac > 0$ and a perfect square	<i>Real, Unequal and Rational</i>
$b^2 - 4ac > 0$ & not a perfect square	<i>Real, Unequal and Irrational</i>

› Conjugate Pairs

- If one root of the equation is

$$m + \sqrt{n}$$

- The other one is surely

$$m - \sqrt{n}$$

- This pair is called as conjugate pairs

Simple Equation

- › Equation of one degree and having one unknown variable is simple.
- › A simple equation has only one root.
- › *Form of Equation:*

$$ax + b = 0$$

where,

a is coefficient of *x*

b is constant

$a \neq 0$

- › Solution Method – Direct basic algebra

Simultaneous Linear Equations (*two unknowns*)

- › Here we always deal with two equations as it consist of 2 unknowns
- › Form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where,

a is coefficient of x

b is coefficient of y

c is constant

a ≠ 0

Methods of Solution Simultaneous Linear Equations

- › **Elimination Method:** In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.
- › **Substitution Method:** equation is written in the form of one variable in LHS and that value is substituted in other equation.
- › **Cross Multiplication Method:** Formula based method

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Cubic Equation

› Form:

$$ax^3 + bx^2 + cx + d = 0$$

where,

a is coefficient of x^3

b is coefficient of x^2

c is coefficient of x

d is constant

$a \neq 0$

› Method of solution: Trial and Error

Addition/Subtraction of Matrices

› Property

– Commutative Law:

$$A + B = B + A$$

– Associative Law: $(A + B) + C = A + (B + C)$

– Distributive Law: $k(A + B) = kA + kB$

Multiplication of Matrices

› Condition

- The product $A \times B$ of two matrices A and B is defined only if the number of columns in Matrix A is equal to the number of rows in Matrix B .

$$A_{m \times n} \times B_{n \times p} = AB_{m \times p}$$

Determinant – 2x2

› If a square matrix of order 2 x 2 is given

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\det A = a_{11} \times a_{22} - a_{12} \times a_{21}$$

Determinant – 3x3

› If a square matrix of order 3 x 3 is given

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\det A = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Minors and Cofactors

- › Minor of the element of a determinant is the determinant of M_{ij} by deleting i^{th} row and j^{th} column in which element is existing.

$$C_{ij} = (-1)^{i+j} M_{ij}$$

Inverse of Matrix

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$$A^{-1} = \frac{1}{\det A} \times \text{adj. } A$$

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Cramer's Rule

$$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta}, z = \frac{\Delta z}{\Delta}$$

Simple Interest

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$$SI = \frac{P.r.t}{100}$$

P = principal value

r = rate of interest per annum

t = time period in years

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Simple Interest

› *Amount as per SI*

$$A = P + SI = P + \frac{P.r.t}{100} = P\left(1 + \frac{rt}{100}\right)$$

Conversion Period

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

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Compound Interest Amount

- Calculation of Accumulated Amount under CI denoted by A

$$A = P(1 + i)^n$$

where,

P = Initial Principal

i = adjusted interest rate

n = no. of periods

$$i = \frac{r\%}{nocppy}$$

$$n = t \times nocppy$$

Compound Interest Amount by Trick

› Calculator Tricks for Amount as per CI

- Example: $P = 1000, i = 10\%, n = 3$ then

Calculator Steps to obtain A:

1000	+	10	%	+	10	%	+	10	%
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Compound Interest

› Formula for Compound Interest

- Calculation of Compound Interest Value denoted by CI

$$CI = P[(1 + i)^n - 1]$$

- where,

P = Initial Principal

i = adjusted interest rate

n = no. of periods

$$i = \frac{r\%}{nocppy}$$

$$n = t \times nocppy$$

Effective Rate of Interest

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$$E = [(1 + i)^n - 1]$$

where,

i = adjusted interest rate

n = no. of periods in a year

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Future Value – Single Cashflow

$$FV = CF(1 + i)^n$$

where,

CF = Single Cashflow of which *FV* is to be calculated

i = adjusted interest rate

n = no. of periods

Future Value – Annuity Regular

$$FVAR = A_i \times FVAF(n, i)$$

Future Value Annuity Factor:

It is a multiplier for Annuity Value to obtain Final Future Value

$$FVAR = A_i \times \left\{ \frac{[(1+i)^n - 1]}{i} \right\}$$

where,

FVAR = Future Value of Annuity Regular

A_i = Annuity Value (Installment)

FVAF = Future Value Annuity Factor

i = adjusted interest rate

n = no. of periods

Future Value – Annuity Due

› Formula:

$$FVAD = A_i \times FVAF(n, i) \times (1 + i)$$

$$FVAD = A_i \times \left\{ \frac{[(1 + i)^n - 1]}{i} \right\} \times (1 + i)$$

where,

FVAD = Future Value of Annuity Due

A_i = Annuity Value (Installment)

FVAF = Future Value Annuity Factor

i = adjusted interest rate

n = no. of periods

Future Value Annuity Factor:

It is a multiplier for Annuity Value to obtain Final Future Value

Present Value – Single Cashflow

$$PV = \frac{CF}{(1+i)^n}$$

where,

CF = Single Cashflow for which *PV* is to be calculated

i = adjusted interest rate

n = no. of periods

Compounding and Discounting Factor

› Compounding

- Finding Future Value of any Cashflow
- *Compounding Factor:*

$$\times (1 + i)^n$$

› Discounting

- Finding Present Value of any Cashflow
- *Discounting Factor:*

$$\times \frac{1}{(1 + i)^n}$$

Present Value – Annuity Regular

$$PVAR = A_i \times PVAF(n, i)$$

$$PVAR = A_i \times \left[\frac{1}{i} \times \left\{ 1 - \frac{1}{(1+i)^n} \right\} \right]$$

Present Value Annuity Factor:

It is a multiplier for Annuity Value to obtain Final Present Value

where,

PVAR = Present Value of Annuity Regular

A_i = Annuity Value (Installment)

PVAF = Present Value Annuity Factor

i = adjusted interest rate

n = no. of periods

Calculator trick of PVAF

$$\boxed{1 + i} \boxed{\div} \boxed{=} \boxed{=} \dots n - \text{times} \boxed{GT}$$

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Present Value – Annuity Due

$$PVAD = \left[A_i \times PVAF \{ (n-1), i \} \right] + A_i$$

where,

PVAD = Present Value of Annuity Due

A_i = Annuity Value (Installment)

PVAF = Present Value Annuity Factor

i = adjusted interest rate

n = no. of periods

n-1 = one lesser period

Perpetuity

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$$PVP = \frac{A_i}{i}$$

where,

PVP = Present Value of Perpetuity

A_i = Annuity Value (Installment)

i = adjusted interest rate

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Growing Perpetuity

$$PVGP = \frac{A_i}{i - g}$$

where,

PVGP = Present Value of Growing Perpetuity

A_i = Annuity Value (Installment)

i = adjusted interest rate

g = growth rate

Net Present Value

- › Formula
 - $\text{NPV} = \text{Present Value of Cash Inflows} - \text{Present Value of Cash Outflows}$
- › Decision Base:
 - If $\text{NPV} \geq 0$, accept the proposal, If $\text{NPV} < 0$, reject the proposal

Real Rate of Return

› Meaning:

- The real interest rate is named so to show what a lender or investor receives in real terms after inflation is factored in.

› Formula:

- Real Rate of Return = Nominal Rate of Return – Rate of Inflation

CAGR

- › Compounded Annual Growth rate is the interest rate we used in Compound Interest.
- › It is used to see returns on investment on yearly basis

Rules of Counting

› Multiplication Rule

- If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways then total number of ways of doing both things simultaneously is $(m \times n)$ ways

› Addition Rule

- If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in $(m + n)$ ways

Word Used	Use
OR	+ Plus
AND	\times Product

Factorial

- › $n! = n(n - 1)(n - 2) \dots 3.2.1$
- › $n! = 1.2.3 \dots (n - 2)(n - 1)n$
- › $n! = n(n - 1)!$
- › $n! = n(n - 1)(n - 2)!$
- › $0! = 1$

Factorial Values

Value of n	Value of n!
1	1
2	2
3	6
4	24
5	120
6	720
7	5040

Value of n	Value of n!
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	871178291200

Theorem of Permutations

Number of Permutations when r objects are chosen out of n different objects

$${}^n P_r = \frac{n!}{(n-r)!}$$

Few Observations:

$$n \geq r$$

n is a positive integer

Particular Case of theorem ($n = r$)

Number of Permutations when n objects are chosen out of n different objects

$${}^n P_n = n!$$

Special Formula (Must Remember)

$$(n + 1)! - n! = n \cdot n!$$

Circular Permutations

› Theorem:

- The number of circular permutations of n different things chosen at a time is $(n-1)!$
- Note: this theorem applies only when we choose all of n things

Circular Permutations (Type II)

- › number of ways of arranging n persons along a closed curve so that no person has the same two neighbours is

$$\frac{1}{2} (n - 1)!$$

- › Same formula will apply if ask is to find number of different forms of necklaces/ garlands

Permutation with Restriction : Theorem 1

- › Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is

$${}^{n-1}P_r$$

Permutations with Restrictions : Theorem 2

- › Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is

$$r \cdot {}^{n-1}P_{r-1}$$

Relation between restriction theorem

$${}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = {}^n P_r$$

One particular thing
always included

One particular thing
always excluded



Total
Permutations

No. of ways when things are never together

Ways of Never Together =

Total ways – Ways of always together

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Theorem of Combinations

Number of Combinations when r objects are chosen out of n different objects

$${}^n C_r = \frac{n!}{(n-r)! r!}$$

Few Observations:

- › $n \geq r$
- › n is a positive integer

Linkage of PNC Theorems

$${}^n C_r = \frac{{}^n P_r}{r!}$$

Few Observations:

- › $n \geq r$
- › n is a positive integer

Special Result of Combinations

$${}^n C_0 = 1$$

$${}^n C_n = 1$$

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Complimentary Combinations

$${}^n C_r = {}^n C_{n-r}$$

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Special Formula of Combination

$${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$$

Combinations of one or more

Combinations of n different things taking **one or more** out of n things at a time

$$2^n - 1$$

Geometry in PNC

Particulars	Tips to Solve
No. of Straight Lines with the given n points	${}^n C_2$ 2 is used as we need to select two points to make a line
No. of Triangles with the given n points	${}^n C_3$ 3 is used as we need to select two points to make a line
Adjustment of Collinear Points	If there are collinear points in any problem, no. of lines or triangles formed using those points should be deducted from total no. of lines/ triangles
No. of Parallelogram with the given one set of m parallel lines and another set of n parallel lines	${}^n C_2 \times {}^m C_2$ Selecting 2 lines from each set of parallel lines
No. of Diagonals	${}^n C_2 - n$

Common Difference of AP

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$$d = t_2 - t_1 = t_3 - t_2 = \dots = t_n - t_{n-1}$$

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General Term of an AP

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$$t_n = a + (n - 1)d$$

where,

a = first term

d = common difference

n = position number of term

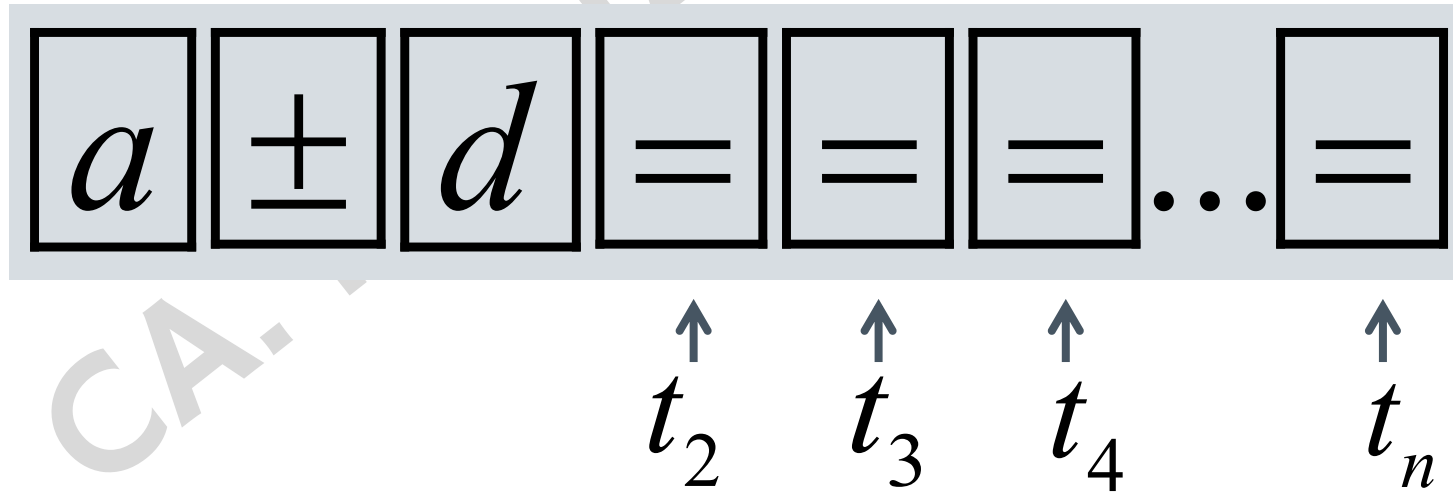
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General Term of an AP

π

$$t_n = a + (n - 1)d$$

Calculator Trick:



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Sum of first n terms of an AP

$$S_n = \frac{n}{2} (a + t_n)$$

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

where,

a = first term

d = common difference

n = position number of term

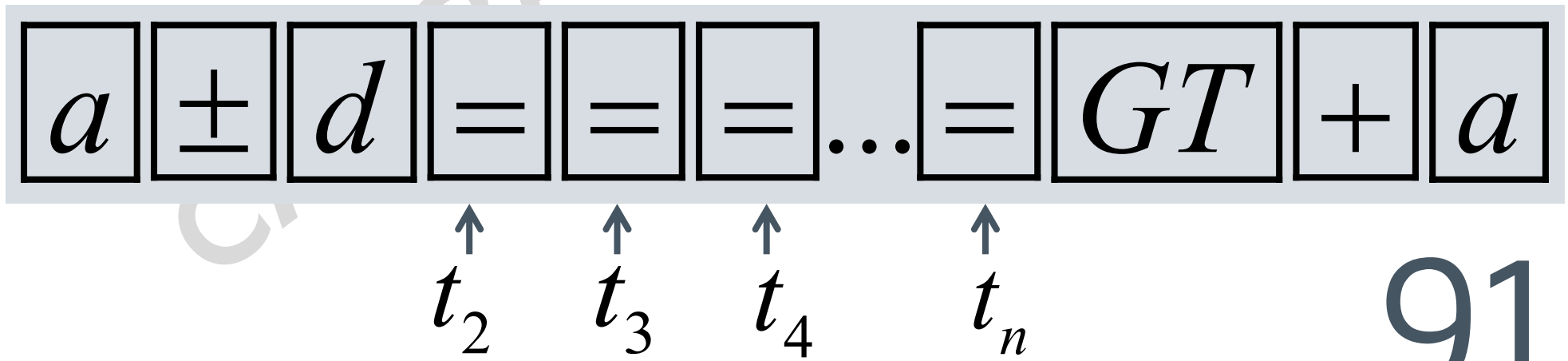
t_n = n th term of AP

Sum of first n terms of an AP

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Calculator Trick



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Sum of first n natural or counting numbers

$$S = \frac{n(n+1)}{2}$$

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Sum of first n odd numbers

$$S = n^2$$

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Sum of the squares of first n natural numbers

$$S = \frac{n(n+1)(2n+1)}{6}$$

Sum of the cubes of first n natural numbers

$$S = \left\{ \frac{n(n+1)}{2} \right\}^2$$

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Common Ratio of GP

π

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}$$

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General Term of an GP

$$t_n = ar^{n-1}$$

where,

a = first term

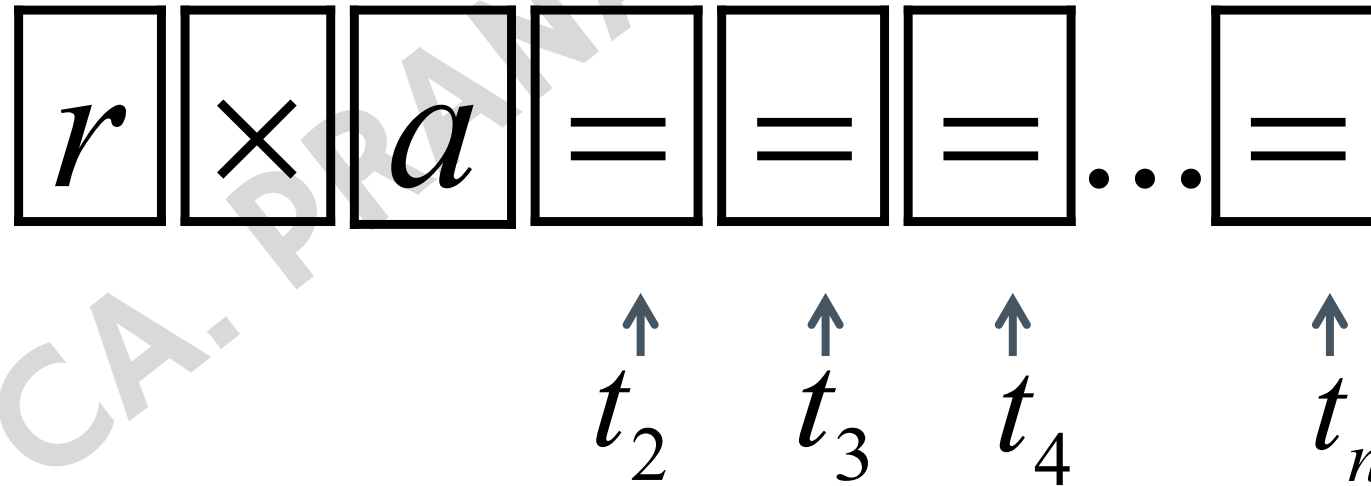
r = common ratio

n = position number of term

General Term of an AP

$$t_n = ar^{n-1}$$

Calculator Trick:



Sum of first n terms of a GP

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Use when $r < 1$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Use when $r > 1$

where,

a = first term

r = common ratio

n = position number of term

Sum of first n terms of a GP

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a(r^n-1)}{r-1}$$

Calculator Trick

r	\times	a	$=$	$=$	$=$	\dots	$=$	GT	$+$	a
			↑	↑	↑		↑			
			t_2	t_3	t_4		t_n			

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Sum of Infinite Geometric Series

$$S_{\infty} = \frac{a}{1-r}$$

Can be used only
if $-1 < r < 1$

where,

a = first term

r = common ratio

n = position number of term

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Subset

- › No. of possible subset of any set

$$\text{Total} = 2^n$$

$$\text{Proper} = 2^n - 1$$

De Morgan's Law

$$\succ (P \cup Q)' = P' \cap Q'$$

$$\succ (P \cap Q)' = P' \cup Q'$$

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2 Set Operations Formulas

› $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

– Proof:

› Example: $A = \{6, 2, 4, 1\}$ $B = \{2, 4, 3\}$

3 Set Operations Formula

$$\triangleright n(A \cup B \cup C) =$$

$$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Composition of Functions

$$\triangleright fog = fog(x) = f[g(x)]$$

$$\triangleright gof = gof(x) = g[f(x)]$$

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Step Method of finding inverse of f

1. Write your function in the form of y
 - $y = f(x)$
2. From above expression, find the value of x
 - $x = \square$
3. Interchange value of x and y, now the RHS is Inverse function
 - $y = \square$

Differentiation Basic Formulas

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$f(x)$	$f'(x)$
$\frac{d}{dx}(x^n)$	nx^{n-1}
$\frac{d}{dx}(e^x)$	e^x
$\frac{d}{dx}(a^x)$	$a^x \log_e a$
$\frac{d}{dx}(\text{constant})$	0
$\frac{d}{dx}(e^{ax})$	ae^{ax}
$\frac{d}{dx}(\log x)$	$\frac{1}{x}$

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Basic Laws of Differentiation

Function	Derivative of the Function
$h(x) = c \cdot f(x)$ where c is a real constant, scalar <i>multiplication of function</i>	$\frac{d}{dx}\{h(x)\} = c \cdot \frac{d}{dx}\{f(x)\}$
$h(x) = f(x) \pm g(x)$ <i>sum/ difference of function</i>	$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}\{f(x)\} \pm \frac{d}{dx}\{g(x)\}$
$h(x) = f(x) \cdot g(x)$ <i>Product of functions</i>	$\frac{d}{dx}\{h(x)\} = f(x) \frac{d}{dx}g(x) + g(x) \frac{d}{dx}f(x)$
$h(x) = \frac{f(x)}{g(x)}$ <i>Quotient of Function</i>	$\frac{d}{dx}\{h(x)\} = \frac{g(x) \frac{d}{dx}f(x) - f(x) \frac{d}{dx}g(x)}{\{g(x)\}^2}$

Cost and Revenue Functions

Cost Function	$y = C(x)$
Average Cost	$A(x) = \frac{C(x)}{x}$
Average Cost is minimum or maximum when	$A'(x) = 0$
Marginal Cost	$M(x) = \frac{dC}{dx}$
Marginal Cost is minimum or maximum when	$M'(x) = 0$
Marginal Revenue	$MR(x) = \frac{dR}{dx}$

Integration – Basic Formulas

i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ (If $n=-1, \frac{x^{n+1}}{n+1} = \frac{1}{0}$ which is not defined)

ii) $\int dx = x + c$, since $\int 1 dx = \int x^0 dx = \frac{x^1}{1} = x + c$

iii) $\int e^x dx = e^x + c$, since $\frac{d}{dx} e^x = e^x$

iv) $\int e^{ax} dx = \frac{e^{ax}}{a} + c$, since $\frac{d}{dx} \left(\frac{e^{ax}}{a} \right) = e^{ax}$

v) $\int \frac{dx}{x} = \log x + c$, since $\frac{d}{dx} \log x = \frac{1}{x}$

vi) $\int a^x dx = a^x / \log_e a + c$, since $\frac{d}{dx} \left(\frac{a^x}{\log_e a} \right) = a^x$

Integration by Parts – ILATE Rule

$$\int u v dx = u \int v dx - \int \left[\frac{d(u)}{dx} \int v dx \right] dx$$

where u and v are two different functions of x

Guidelines for Selecting u and dv :

(There are always exceptions, but these are generally helpful.)

“L-I-A-T-E” Choose ‘ u ’ to be the function that comes first in this list:

- L: Logarithmic Function
- I: Inverse Trig Function
- A: Algebraic Function
- T: Trig Function
- E: Exponential Function