Chapter 1

RATIO AND PROPORTION , INDICES , LOGARITHMS

TOPIC : RATIO

BY : SHIVANI SHARMA

55; 1/4 1, 155; 14 1, 14 M/4 QUANTITATIVE APTITUDE ROMANOVA 7 years of experience in teaching **Mathematics** Gold Medalist in M.Sc. and B.Sc. Maths Was #1 Maths Faculty in Magnet Brains Teaches Maths in CA foundation and **K-12** Taught 30000+ students with highest score as 100



RATIO

• A ratio is a comparison of two quantities of the same kind and of same units by division .



RATIO

• Ratio exists only between quantities of the same kind.

Illustration

• There is no ratio between the weight of one child and the age of another child.



• Quantities to be compared (by division) must be in the same units.

Illustration

Ratio between 3 kg & 5 kg = 3/5



Illustration

(i) Ratio between 150 gm and 2 kg



(ii) Ratio between 25 minutes and 45 seconds

RATIO

- If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as a : b.
- The quantities a and b are called the terms of the ratio .
- a is called the first term or antecedent and b is called the second term or consequent.

$$\mathbf{a} : \mathbf{b} = \frac{\mathbf{a}}{\mathbf{b}}$$
 consequent



Que 2 The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is

(a) 16

- (b) 60
- (c) 22
- (d) 20

- Both terms of a ratio can be multiplied or divided by the same (non-zero) number.
- Usually a ratio is expressed in lowest terms (or simplest form)

Illustration

* 12:16 = 12/16 = 3/4 = 3:4

The order of the terms in a ratio is important.

Illustration

3:4 is not same as 4:3.

To compare two ratios ,convert them into equivalent like fractions

Illustration To find which ratio is greater _____.

$$2\frac{1}{3}: 3\frac{1}{3}; 3.6: 4.8$$

• If a quantity increases or decreases in the ratio a : b then

new quantity = b/a of the original quantity.

Illustration VII Rounaq weighs 56.7 kg. If he reduces his weight in the ratio 7 : 6, find his new weight.

Example 1 Simplify the ratio 1/3 : 1/8 : 1/6

INVERSE RATIO

- One ratio is the inverse of another if their product is 1.
- Thus a : b is the inverse of b: a and vice-versa.



Quel The inverse ratio of 11 : 15 is

- (a) 15:11
- (b) $\sqrt{11} : \sqrt{15}$
- (c) 121:225
- (d) None of these



Que 3 The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is

(a) 5

(b) √5

(c) 7

PROPERTIES OF RATIO

• The ratio compounded of the two ratios a : b and c : d is ac: bd.

For example

compound ratio of 3 : 4 and 5 : 7 is 15 : 28.

Compound ratio of 2:3, 5:7 and 4:9 is 40:189.



Que 4 The ratio compounded of 2:3, 9:4, 5:6 and 8:10 is

- (a) 1:1
- (b) 1:5
- (c) 3:8
- (d) none of these

PROPERTIES OF RATIO

For example

• A ratio compounded of itself is called its duplicate ratio.

 $a^2:b^2$ is the duplicate ratio of a:b. $a^3:b^3$ is the triplicate ratio of a:b

duplicate ratio of 2:3 is 4:9.

Triplicate ratio of 2:3 is 8:27.

PROPERTIES OF RATIO

\sqrt{a} : \sqrt{b} is the Sub-duplicate ratio of a:b. $\sqrt[3]{a}$: $\sqrt[3]{b}$ is the Sub-triplicate ratio of a:b.

For example

Sub-duplicate ratio of 4:9 is 2:3

Sub-triplicate ratio of 8:27 is 2:3



Que 5 The duplicate ratio of 3 : 4 is

- (a) √3:2
- (b) 4:3
- (c) 9:16



Que 6 The sub-duplicate ratio of 25 : 36 is

(a) 6:5

(b) 36:25

(c) 50:72

(d) 5:6



Que 7 The triplicate ratio of 2 : 3 is

(a) 8:27

(b) 6:9

(c) 3:2



Que 8 The sub-triplicate ratio of 8 : 27 is

(a) 27:8

(b) 24:81

(c) 2:3



Que9 The ratio compounded of 4:9 and the duplicate ratio of 3: 4 is

- (a) 1:4
- (b) 1:3
- (c) 3:1
- (d) none of these



Que10 The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is

(a) 2:7

- (b)7:2
- (c) 2 : 21



Quell The ratio compounded of 4:5, 9:7, the triplicate ratio of 3:4, and the triplicate ratio of 2:3 is

(a) 4:512

(b) 3:32

(c) 1:12



Que 12 If a : b = 3 : 4, the value of (2a+3b) : (3a+4b) is

- (a) 54:25
- (b) 8:25
- (c) 17:24
- (d) 18:25



Que 13 Two numbers are in the ratio 2 : 3. If 4 be subtracted from each, they are in the ratio 3 : 5. The numbers are (a) (16, 24) (b) (4, 6) (c) (2, 3) (d) none of these



Que 14 The angles of a triangle are in ratio 2 : 7 : 11. The angles are (a) (20°, 70°, 90°) (b) (30°, 70°, 80°) (c) (18°, 63°, 99°) (d) none of these



Que 15 Division of ₹ 324 between X and Y is in the ratio 11:7. X & Y would get Rupees (a) (204, 120) (b) (200, 124) (c) (180, 144) (180, 144)



Que 16 Anand earns ₹ 80 in 7 hours and Promode ₹ 90 in 12 hours. The ratio of their earnings is

(a) 32:21

(b) 23:12

(c) 8:9



Que 17 The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are (a) (200, 305) (b) (185, 290) (c) (245, 350) (d) none of these



Que 19 If x : y = 3 : 4, the value of $x^2y + xy^2$: $x^3 + y^3$ is
(a) 13:12
(b) 12:13
(c) 21:31
(d) none of these


Que 20 If p : q is the sub-duplicate ratio of $p-x^2$: $q-x^2$ then x^2 is



(d) none of these



Que 21 If 2s : 3t is the duplicate ratio of 2s-p : 3t-p then (a) p² = 6st (b) p = 6st (c) 2p = 3st (d) none of these



Que 22 If p : q = 2 : 3 and x : y = 4 : 5, then the value of 5px + 3qy : 10px + 4qy is (a) 71:82 (b) 27:28 (c) 17:28 (d) none of these



Que 23 The number which when subtracted from each of the terms of the ratio 19:31 reducing it to 1:4 is

(a) 15

(b) 5

(c)1

(d) none of these

PROPERTIES OF RATIO

• If the ratio of two similar quantities can be expressed as a ratio of two

integers, the quantities are said to be commensurable.

• otherwise, they are said to be incommensurable.

EXAMPLE

• $\sqrt{3}$: $\sqrt{2}$ cannot be expressed as the ratio of two integers and therefore, $\sqrt{3}$ and $\sqrt{2}$ are incommensurable quantities.

PROPERTIES OF RATIO

- Continued Ratio is the relation (or comparison) between the magnitudes of three or more quantities of the same kind.
- The continued ratio of three similar quantities a, b, c is written as a : b : c.

Illustration The continued ratio of ₹ 200, ₹ 400 and ₹ 600 is

₹200 : ₹400 : ₹600 = 1:2:3.



Que 18 P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22:27

(b) 27:22

(c) 32:33

(d) none of these

Example 2 The ratio of the number of boys to the number of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2 : 3.

Example 1 The monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenditures are in the ratio 7 : 9. If each saves ₹ 50 per month, find their monthly incomes.

(a) ₹ 400 , ₹ 500
(b) ₹800 , ₹ 1000
(c) ₹40 , ₹ 50

(d) none of these

Example 2 The ratio of the prices of two houses was 16 : 23. Two years later when the price of the first has increased by 10% and that of the second by ₹ 477, the ratio of the prices becomes 11 : 20. Find the original prices of the two houses.

- (a) ₹848, ₹1219
- (b) ₹828, ₹1000
- (c) ₹848, ₹1229
- (d) none of these

Example 3 Find in what ratio will the total wages of the workers of a factory be increased or decreased if there be a reduction in the number of workers in the ratio 15 : 11 and increment in their wages in the ratio 22 : 25



Que 24 Daily earnings of two persons are in the ratio 4:5 and their daily expenses are in the ratio 7:9. If each saves ₹ 50 per day, their daily earnings in ₹ are

- (a) (40, 50)
- (b) (50, 40)
- (c) (400, 500)
- (d) none of these



Que 25 The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms in 5 hours, the speed of the first train is (a) 10 Km/hr (b) 50 Km/hr (c) 70 Km/hr (d) none of these

Que 1. If p/q = -2/3, then the value of (2p + q) / (2p - q)is: (a) 1 (b) -1/7 (c) 1/7 (d) 7



Que 2. What must be added to each term of the ratio 49:68, so that it becomes 3:4?

- (a) 3
- (b) 5
- (c) 8
- (d) 9



Que 3. The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40

(b) 25, 24

(c) 40, 60

(d) 50, 70



Que 4. If A : B = 2 : 5, then (10A + 3B) : (5A + 2B) is equal to:

- (a) 7:4
- (b) 7:3
- (c) 6:5
- (d)7:9



Que 5. The ratio Compounded of 4:5 and sub-duplicate of "a": 9 is 8:15. Then Value of "a" is:

(a) 2

(b) 3

(c) 4

(d) 5



Que 6. The triplicate ratio of 4 : 5 is: (a) 125 : 64 (b) 16 : 25 (c) 64 : 125 (d) 120 : 46



Que 7. Find three numbers in the ratio 1 : 2 : 3, so that the sum of their squares is equal to 504 (a) 6, 12, 18 (b) 3, 6, 9 (c) 4, 8, 12 (d) 5, 10, 15



Que 8. If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be:

(a) 5:8

(b) 8:5

(c) 5:3

(d) 3:5



Que 9. A person has assets worth ₹1,48,200. He wish to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this assets, the share of his son will be:

(a) ₹24,700

(b) ₹49,400

(c) ₹74,100

(d) ₹ 37,050



Que 10. X, Y, Z together starts a business. If X invests 3 times as much as Y invests and Y invests two third of what Z invests, then the ratio of capitals of X, Y, Z is:

(a) 3:9:2
(b) 6:3:2
(c) 3:6:2
(d) 6:2:3



Que 11. There are total 23 coins of ₹1, ₹2 and ₹5 in a bag. If their value is ₹43 and the ratio of coins of ₹1 and ₹2 is 3:2. Then the number of coins of ₹1 is:

(a) 12

(b) 5

(c) 10

(d) 14



Que 12. If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is: (a) 24 : 35 (b) 8 : 15 (c) 16 : 35 (d) 7 : 15



Que 13. The ratio of the number of ₹ 5 coins and ₹ 10 coins is 8 : 15. If the value of ₹ 5 coins is ₹ 360, then the number of ₹ 10 coins will be:

(a) 72

(b) 120

(c) 135

(d) 185



PYQs





PYQs

Que 15. If x : y : z = 7 : 4 : 11 then (x + y + z)/z is:

(a) 2

(b) 3

(c) 4

(d) 5



Que 16. The price of scooter and moped are in the ratio 7 : 9. The price of moped is ₹ 1,600 more than that of scooter. Then the price of moped is: (a) ₹ 7,200 (b) ₹ 5,600 (c) ₹ 800

(d) ₹ 700



Que 17. The ratio of number of boys and the number of girls in a school is found to be 15 : 32. How many boys and equal number of girls should be added to bring the ratio to 2/3 ?

(a) 19

(b) 20

(c) 23

(d) 27



Que 18. In a certain business A and B received profit in a certain ratio B and C received profits in the same ratio. If A gets ₹ 1600 and C gets ₹ 2500 then how much does B get?

(a) ₹ 2,000

(b) ₹ 2,500

(c) ₹ 1,000

(d) ₹1,500

Ans:a

Que 19. The salaries of A, B and C are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 3:3:10
(b) 10:11:20
(c) 23:33:60
(d) Cannot be determined

Ans:c

Que 20. A bag contains 105 coins containing some 50 paise, and 25 paise coins. The ratio of the number of these coins is 4 : 3. The total value (in ₹) in the bag is

(a) 43.25

(b) 41.25

(c) 39.25

(d) 35.25



SIMPLE POWER

CALCULATOR TRICKS



SIMPLE POWER

CALCULATOR TRICKS

34

4⁵

24

64

PROPORTION

- An equality of two ratios is called a proportion.
- Four quantities a, b, c, d are said to be in proportion

if , $\mathbf{a} : \mathbf{b} = \mathbf{c} : \mathbf{d}$ (also written as $\mathbf{a} : \mathbf{b} : : \mathbf{c} : \mathbf{d}$) i.e.

if a/b = c/d

If ad = bc. (CROSS PRODUCT RULE)
- The quantities a, b, c, d are called terms of the proportion;
 - a, b, c and d are called its first, second, third and fourth terms respectively.
 - They are also called "first proportional ", " second proportional ",

" third proportional ", and " fourth proportional " respectively.

- First and fourth terms are called extremes (or extreme terms).
- Second and third terms are called means (or middle terms).

CROSS PRODUCT RULE

If a : b = c : d are in proportion then
a/b = c/d



ad = bc

• i.e. product of extremes = product of means.

Note:

- In a ratio a : b, both quantities must be of the same kind while in a proportion a : b = c : d, all the four quantities need not be of the same type.
- The first two quantities should be of the same kind and last two quantities should be of the same kind.

Illustration ₹ 6 : ₹ 8 = 12 toffees : 16 toffees are in a proportion.

Here 1st two quantities are of same kind and last two are of same kind



Example 2 Find the value of x if 10/3 : x : : 5/2 : 5/4.

Example 3 Find the fourth proportional to 2/3, 3/7, 4.



Que. 1 The fourth proportional to 4, 6, 8 is

(a) 12

(b) 32

(c) 48



Que. 4 The number which has the same ratio to 26 that 6 has to 13 is

(a) 11

(b) 10

(c) 21



Que. 5 The fourth proportional to 2a, a², c is (a) ac/2 (b) ac (c) 2/ac (d) None of these



Que. 6 If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is

(a) 6/5

(b) 5/6

(c) 15/2



• Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if

a:b=b:c

i.e. a/b = b/c $b^2 = ac$ $b = \sqrt{ac}$ here , a = first proportional c = third proportional b = mean proportional

CONTINUOUS PROPORTION

Example4 Find the third proportion to 2.4 kg, 9.6 kg.

CONTINUOUS PROPORTION

Example 5 Find the mean proportion between 1.25 and 1.8

CONTINUED PROPORTION

When three or more numbers are related such that a/b= b/c = c/d = d/ethe numbers a, b,c,d,and e are said to be in continued proportion .



Que. 2 The third proportional to 12, 18 is

(a) 24

(b) 27

(c) 36



Que. 3 The mean proportional between 25, 81 is(a) 40(b) 50(c) 45(d) None of these



Que. 7 The mean proportional between 12x² and 27y² is

(a) 18xy

(b) 81xy

(c) 8xy



Que. 23 12, 16, *, 20 are in proportion. Then * is

(a) 25

(b) 14

(c) 15



Que. 24 4, *, 9, 13 1/2 are in proportion. Then * is

(a) 6

(b) 8

(c) 9



Que. 10 If p/q = r/s = 2.5/1.5, the value of ps : qr is (a) 3/5 (b) 1 : 1 (c) 5/3



Que. 8 If A = B/2 = C/5, then A : B : C is (a) 3 : 5 : 2 (b) 2 : 5 : 3 (c) 1 : 2 : 5 (d) None of these



Que. 9 If a/3 = b/4 = c/7, then a + b + c/c is (a) 1 (b) 3 (c) 2



Que. 11 If x : y = z : w = 2.5 : 1.5, the value of (x + z)/(y + w) is

(a) 1

(b) 3/5

(c) 5/3



Que. 13 If A : B = 3 : 2 and B : C = 3 : 5, then A : B : C is

(a) 9:6:10

(b) 6:9:10

(c) 10:9:6



Que. 14 If x/2 = y/3 = z/7, then the value of (2x - 5y + 4z)/2y is (a) 6/23 (b) 23/6 (c) 3/2 (d) 17/6



Que. 15 If x : y = 2 : 3, y : z = 4 : 3 then x : y : z is (a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) 8 : 12 : 9



Que. 16 Division of ₹ 750 into 3 parts in the ratio 4 : 5 : 6 is

(a) (200, 250, 300)

(b) (250, 250, 250)

(c) (350, 250, 150)



Que. 18 The numbers 14, 16, 35, 42 are not in proportion. The fourth term for which they will be in proportion is

(a) 45

(b) 40

(c) 32



Que. 25 The mean proportional between 1.4 gms and 5.6 gms is (a) 28 gms (b) 2.8 gms (c) 3.2 gms



Que. 26 If
$$\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$$
 then $\frac{a+b+c}{c}$ is
(a) 4
(b) 2
(c) 7



Que. 27 Two numbers are in the ratio 3 : 4; if 6 be added to each number of the ratio, then the new ratio will be 4 : 5, then the numbers are

(a) 14, 20

(b) 17, 19

(c) 18 and 24

EXERCISE 1(B)

Que. 29 If
$$a:b = 4:1$$
 then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is

(b) 4

(c) 5

1. If a:b=c:d, then b:a=d:c





2. If a:b=c:d, then a:c=b:d OR d:b=c:a



3. If a:b=c:d, then a+b:b=c+d:d

COMPONENDO

c+da+0

4. If a:b=c:d, then a-b:b=c-d:d

DIVIDENDO

c-dС \boldsymbol{a} \boldsymbol{a}

COMPONENDO AND DIVIDENDO

5.If a:b=c:d, then a+b:a-b=c+d:c-d

c+da
PROPERTIES OF PROPORTION

ADDENDO

6. If a : b= c : d = e : f =, then each of these ratios is equal

If
$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$
 then each of these ratios is equal to $\frac{a+c+e}{b+d+f}$.

$$\frac{\frac{a}{b}}{b+d+f...}, \frac{\frac{c}{d}}{b+d+f...}, \frac{\frac{c}{d}}{b+d+f...}, \frac{\frac{e}{f}}{b+d+f...}, \frac{\frac{a+c+e...}{b+d+f...}}{b+d+f...}$$

PROPERTIES OF PROPORTION

SUBTRAHENDO

7. If a : b= c : d = e : f =, then each of these ratios is equal







Example1 If a : b = c : d= 2.5 : 1.5, what are the values of ad : bc and

a + c : b + d?

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Example 2

If
$$\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$$
, then prove that $\frac{a+b+c}{c}$

Que. 19 If x/y = z/w, implies y/x = w/z, then the process is called

- (a) Dividendo
- (b) Componendo
- (c) Alternendo
- (d) None of these

EXERCISE 1(B)

Que. 20 If p/q = r/s = p - r/q - s, the process is called

(a) Subtrahendo

(b) Addendo

(c) Invertendo

Que. 21 If a/b = c/d, implies (a + b)/(a – b) = (c + d)/(c – d), the process is called

(a) Componendo

(b) Dividendo

(c) Componendo and Dividendo

Que. 22 If u/v = w/p, then (u - v)/(u + v) = (w - p)/(w + p). The process is called

(a) Invertendo

(b) Alternendo

(c) Addendo



Que. 28 If
$$\frac{a}{4} = \frac{b}{5}$$
 then
(a) $\frac{a+4}{a-4} = \frac{b-5}{b+5}$
(b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$
(c) $\frac{a-4}{a+4} = \frac{b+5}{b-5}$



Que. 17 The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are (a) (45, 50, 55) (b) (40, 60, 50) (c) (35, 45, 70) (d) None of these



Que. 12 If (5x - 3y)/(5y - 3x) = 3/4, the value of x : y is (a) 2 : 9 (b) 7 : 2 (c) 7 : 9

EXERCISE 1(B)

Que. 30 If
$$\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$$
 then $(b-c)x + (c-a)y + (a-b)z$ is
(a) 1
(b) 0
(c) 5

Example3 A dealer mixes tea costing ₹ 6.92 per kg. with tea costing ₹ 7.77 per kg and sells the mixture at ₹ 8.80 per kg and earns a profit of 171/2 % on his sale price. In what proportion does he 2 mix them? A dealer mixes rice costing ₹ 13.84 per . with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per kg . So, he earns a profit of 14.6 % on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3:7

(b) 5:7

(c) 7:9

(d) 9:11

Ans:a

Fourth proportional to x, 2x, (x + 1) is:

- (a) (x + 2)
- (b) (x-2)
- (c) (2x+2)
- (d) (2x 2)



Which of the numbers are not in proportion?

- (a) 6, 8, 5, 7
- (b) 7, 3, 14,6
- (c) 18, 27, 12, 18
- (d) 8, 6, 12, 9



The ratio of third proportion of 12,30 to the mean proportion of 9,25

is:

- (a) 2:1
- (b) 5:1
- (c) 7:15
- (d) 3:5

Ans:b

What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers in proportion?

(a) 2

(b) 4

(c) 8

(d) None of these.

Ans:a

A vessel contained a solution of acid and water in which water was 64 % . Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30 % acid, the quantity (in litres) of the solution, in the beginning in the vessel, was

(a) 12

(b) 36

(c) 24

(d) 2



A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1,000 more than D, what is B 's share?

(a) ₹ 2,000

(b) ₹1,500

(c) ₹2,500

(d) ₹ 1,000

Ans:a

INDICES

- The word "Indices" is the plural of " index .
- When a number is expressed in the form of **Q**ⁿ

a is called the base ,

and n is called the index / exponent / power



For any real number a

- $a^n = a x a x a$to n factors.
- **Example:** 3⁴ = 3 × 3 × 3 × 3

•
$$a^{-n} = \frac{1}{a^n}$$

• Example: $2^{-5} = 1/2^5$
• Example: $1/2^{-5} = 2^5$



For example :

 $3^4 \times 3^5 = 3^9$

LAW OF INDICES





For example

 $2^7/2^4 = 2^3$







For example

 $(2^4)^3 = 2^{12}$

LAW OF INDICES

For Example

$$(ab)^n = a^n b^n$$

$$(2 \times 3)^3 = 2^3 \times 3^3$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

For Example

$$(5/3)^4 = 5^4/3^4$$

LAW OF INDICES



$$a^{m/n} = (a^m)^{1/n}$$
, $a^{m/n} = n\sqrt{a^m} = (n\sqrt{a})^m$

For Example

$$\sqrt{9} = 9^{1/2}$$

$$5\sqrt{9^2} = 9^{2/5}$$

REMARK

- If $a^x = a^y$, then x = y
- If $x^{\alpha} = y^{\alpha}$, then x = y
- If $a^m = k$, then $a = k^{1/m}$

•
$$\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$$

Example1 Simplify $2 x^{1/2} 3x^{-1}$ if x = 4



Example2 Simplify 6ab²c³ × 4b⁻²c⁻³d





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Example 5 Simplify $(x^{a}.y^{-b})^{3}.(x^{3}y^{2})^{-a}$



Example 6
$$\sqrt[6]{a^{4b}x^6} \cdot (a^{2/3}x^{-1})^{-b}$$



Example 7 Find x , $x\sqrt{x} = (x\sqrt{x})^x$



Example 8 Find the value of k from $(\sqrt{9})^{-7} \times (\sqrt{3})^{-5} = 3^k$


Que. 1 4x^{-1/4} is expressed as

(a) $-4x^{1/4}$

(b) x^{-1}

(c) $4/x^{1/4}$



<mark>Que. 2</mark> The value of 8^{1/3} is

(a) 32

(b) 4

(c) 2

Que. 3 The value of 2 × (32)^{1/5} is (a) 2 (b) 10 (c) 4

Que. 4 The value of 4/(32)^{1/5} is

(a) 8

(b) 2

(c) 4



<mark>Que. 5</mark> The value of (8/27)^{1/3} is

(a) 2/3

(b) 3/2

(c) 2/9

Que. 6 The value of 2(256)^{-1/8} is

(a) 1

(b) 2

(c) 1/2

Que. 7 2^{1/2} . 4^{3/4} is equal to

(a) a fraction

(b) a positive integer

(c) a negative integer



Que. 8 $\left(\frac{81x^4}{y^8}\right)^{\frac{1}{4}}$ has simplified value equal to

(a) xy² (b) x²y (c) 9xy²

<mark>Que. 9</mark> x^{a-b} × x^{b-c} × x^{c-a} is equal to

(a) x

(b)1

(c) 0





(a) x

(b)1

(c) 0



Que. 11 { $(3^3)^2 \times (4^2)^3 \times (5^3)^2$ } / { $(3^2)^3 \times (4^3)^2 \times (5^2)^3$ } is

(a) 3/4

(b) 4/5

(c) 4/7

(d) 1



Que. 12 Which is True ?

(a) $2^0 > (1/2)^0$

(b) $2^0 < (1/2)^0$

(c) $2^0 = (1/2)^0$



Que. 13 If $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1, then the value of p + q + r is

(a) 1

(b) 0

(c) 1/2



Que. 14 The value of y^{a-b} × y^{b-c} × y^{c-a} × y^{-a-b} is

 $(a) y^{a+b}$

(b) y

(c)1

(d) 1/y^{a+b}



Que. 15 The True option is

(a) $x^{2/3} = \sqrt[3]{x^2}$

(b) $x^{2/3} = \sqrt{x^3}$

(c) $x^{2/3} > \sqrt[3]{x^2}$

(d) $x^{2/3} < \sqrt[3]{x^2}$



Que. 16 The simplified value of 16x⁻³y² × 8⁻¹x³y⁻² is

(a) 2xy

(b) xy/2

(c) 2



Que. 17 The value of (8/27)^{-1/3} × (32/243)^{-1/5} is

(a) 9/4

(b) 4/9

(c) 2/3



Que. 18 The value of
$$\begin{cases} \frac{(x+y)^{2/3} \times (x-y)^{2/3}}{\sqrt{x+y} \times \sqrt{(x-y)^3}} \end{cases}^6 \text{ is} \\ (a) (x+y)^2 \\ (b) (x-y) \\ (c) x+y \\ (d) \text{ None of these} \end{cases}$$



Que. 19 Simplified value of $(125)^{2/3} \times \sqrt{25} \times \sqrt[3]{5^3} \times 5^{1/2}$ is

(a) 5

(b) 1/5

(c)1



Que. 20 $[{(2)^{1/2}.(4)^{3/4}.(8)^{5/6}.(16)^{7/8}.(32)^{9/10}}^4]^{3/25}$ is

(a) A fraction

(b) an integer

(c)1



Que. 21 [1-{1-(1-x²)⁻¹}⁻¹]^{-1/2} is equal to

(a) x

(b) 1/x

(c)1



Que. 22
$$\left[\left(x^n \right)^{n-\frac{1}{n}} \right]^{\frac{1}{n+1}}$$
 is equal to
(a) x^n

(b) xⁿ⁺¹

(c) xⁿ⁻¹

Que. 23 If $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$, then the simplified form of

$$\left[\frac{\mathbf{x}^{l}}{\mathbf{x}^{m}}\right]^{l^{2}+lm+m^{2}} \times \left[\frac{\mathbf{x}^{m}}{\mathbf{x}^{n}}\right]^{m^{2}+mn+n^{2}} \times \left[\frac{\mathbf{x}^{n}}{\mathbf{x}^{l}}\right]^{l^{2}+lm+n^{2}}$$

(a) 0

(b)1

(c) x



Que. 24 Using $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$ tick the correct of these when x = $p^{1/3} - p^{-1/3}$

(a) $x^3 + 3x = p + 1/p$

(b) $x^3 + 3x = p - 1/p$

(c) $x^3 + 3x = p + 1$

Que. 25 On simplification, 1/(1 + a^{m-n} + a^{m-p}) + 1/(1 + a^{n-m} + a^{n-p}) + 1/(1 + a^{p-m} + a^{p-n}) is equal to

(a) 0

(b) a

(c)1

(d) 1/a



<mark>Que. 26</mark> The value of (a) 1 (b) 0

$$\left(\frac{x^{a}}{x^{b}}\right)^{a+b} \times \left(\frac{x^{b}}{x^{c}}\right)^{b+c} \times \left(\frac{x^{c}}{x^{a}}\right)^{c+a}$$

Que. 27 If x = 3^{1/3} + 3^{-1/3}, then 3x³ - 9x is

(a) 15

(b) 10

(c) 12

<mark>Que. 28</mark> If a^x = b, b^y = c, c^z = a, then xyz is

(a) 1

(b) 2

(c) 3



Que. 29 The value of
$$\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$$
 (a) 1

(b) 0

(c) -1

Que. 30 If
$$2^{x} = 3^{y} = 6^{-z}$$
, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

(a) 1

(b) 0

(c) 2

Que. Find the value of a from the following:

$$\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

(a) 11

(b) 13

(c) 15

(d) 17

Ans:d





The value of
$$\left(1 - \sqrt[3]{0.027} \left(\frac{5}{6}\right) \left(\frac{1}{2}\right)^2\right)$$
 is:

(a) 11/16 (b) 13/16 (c) 15/16 (d) 1

Ans:c

By simplifying (2 a³ b⁴)⁶/ (4a³ b)² × (a² b²), the answer will be:

(a) 4a² b³

(b) 4a⁶ b⁴

(c) 4a¹⁰ b¹⁰

(d) 4a¹⁰ b²⁰

Ans:d

If
$$\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$$
 then the value of $\left(\frac{a+b+c}{3}\right)^3$ is equal to:

(a) abc

(b) 9abc

(c) 1/abc

(d) 1/9abc

Ans:a



Value of
$$\left[9^{n+\frac{1}{4}} \cdot \frac{\sqrt{3.3^{n}}}{3 \cdot \sqrt{3^{-n}}}\right]^{\frac{1}{n}}$$

(a) 9
(b) 27
(c) 81
(d) 3

Ans:b
PYQs

If
$$2^{x^2} = 3^{y^2} = 12^{2^2}$$
 then
(a) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$
(b) $\frac{1}{x^2} + \frac{2}{y^2} = \frac{1}{z^2}$
(c) $\frac{2}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$

(d) None of the above

Ans:c

PYQs

If x = 3^{1/3} + 3^{-1/3} then find the value of 3x³-9x (a) 3

(b) 9

(C) 12

(d) 10

Ans:d



If $a^x = n$

then

x = log_an

CONDITIONS

- n>0
- a > 0
- a≠1

logarithm of n to the base a is x



2⁴ = **1**6

10³ = 1000

 $5^{-3} = 1/125$

 $2^3 = 8$

 $4 = \log_{2} 16$

 $3 = \log_{10} 1000$

 $-3 = \log_{5}(1/125)$

 $3 = \log_2 8$

<mark>Que. 2</mark> log₂8 is equal to

(a) 2

(b) 8

(c) 3



Que. 5 The value of log 0.0001 to the base 0.1 is

(a) -4

(b) 4

(c) 1/4

Que. 7 log $\sqrt{2}$ 64 is equal to

(a) 12

(b) 6

(c)1



<mark>Que. 8</mark> log_{2√3} 1728 is equal to

(a) 2√3

(b) 2

(c) 6

Que. 9 log (1/81) to the base 9 is equal to

(a) 2

(b) 1/2

(c) -2



Que. 10 log 0.0625 to the base 2 is equal to

(a) 4

(b) 5

(c)1

Que. 13 The value of log $\frac{1}{3}$ to the base 9 is

(a) - 1/2

(b) 1/2

(c)1

PROPERTIES OF LOGARITHM

• $\log_{a} 1 = 0$

 $\cdot \log_a a = 1$

PROPERTIES OF LOGARITHM

$\log_a m + \log_a n = \log_a (mn)$

Que. 1 log 6 + log 5 is expressed as (a) log 11 (b) log 30 (c) log 5/6 (d) None of these

Que. 4 log (1 × 2 × 3) is equal to

```
(a) log 1 + log 2 + log 3
```

(b) log 3

(c) log 2

Que. 11 Given log2 = 0.3010 and log3 = 0.4771 the value of log 6 is

(a) 0.9030

(b) 0.9542

(c) 0.7781

Que. 14 If $\log x + \log y = \log (x+y)$, y can be expressed as

(a) x-1

(b) x

(c) x/(x-1)

Que. 17 Given that log₁₀2 = x and log₁₀3 = y, the value of log₁₀60

is expressed as

(a) x - y + 1

(b) x + y + 1

(c) x - y - 1

PROPERTIES OF LOGARITHM

 $\log_a m - \log_a n = \log_a \frac{m}{n}$



Que. 3 log (32/4) is equal to

(a) log 32/log 4

(b) log 32 – log 4

(c) 2³

<mark>Que. 18</mark> Given that log₁₀2 = x, log₁₀3 = y, then log₁₀1.2 is expressed in terms of x and y as

(a) x + 2y - 1

(b) x + y - 1

(c) 2x + y - 1

PROPERTIES OF LOGARITHM

$\log_a m^n = n \log_a m$

Example : log₁₀100⁵



Que.6 If 2 log $x = 4 \log 3$, the x is equal to

(a) 3

(b) 9

(c) 2





(a) 0

(b) 2

(c)1

Que. 15 The value of log₂ [log₂ {log₃ (log₃27³)}] is equal to

(a) 1

(b) 2

(c) 0

Que. 19 Given that log x = m + n and log y = m – n, the value of

 $\log(10x/y^2)$ is expressed in terms of m and n as

(a) 1 – m + 3n

(b) m – 1 + 3n

(c) m + 3n + 1

Que. 20 The simplified value of 2 log₁₀5 + log₁₀8 – 1/2 log₁₀4 is

(a) 1/2

(b) 4

(c) 2