SETS, FUNCTIONS & RELATIONS

9

SET

A set is a well defined collection of objects, whereby, by 'well defined' we mean that there is a rule (or rules) by means of which it is possible to say, without ambiguity, whether a particular object belongs to the collection or not The objects in the set can be anything.

The individual object of the collection or set is called an element or a member of the set Sets are usually denoted by the capital alphabets such as X, Y, Z, etc., and the elements of a set are denoted by small alphabets such as x, y, z, etc.

If X is a set and x is a member of set x,y,z, i.e., x is an element of X, then we express it symbolically as $x \in$ X and read it as 'x belongs to X'. Also $x \notin$ X means that x is not an element of x.

It is customary, in mathematics, to put a vertical line / or / or / through a symbol to indicate the opposite or negative meaning of the symbol.

Example 1. The collection of first six even natural numbers is a set containing the elements, 2, 4, 6, 8, 10, 12.

Example 2. The collection of vowels in the English alphabets is a set containing the elements a, e, i, 0 u.

Example 3. The collection of first four prime natural numbers is a set containing the elements 2, 3, 5, 7.

UNIVERSAL SET

In any mathematical discussion we shall consider all the sets to be the subsets of a given large fixed set, known as Universal Set or Universe of Discourse and is usually denoted by U. i.e., A universal set is the largest non-empty set of which all the sets, under consideration, are subsets.

Example 4. In a plane geometry the universal set consists of all the points in the plane.

Example 5. In any study of human population all the people in the world constitute the universal set.

REPRESENTATION OFSETS

The most common methods of describing a set are :

(i) ROSTER METHOD (Tabular Form)

(ii) SET-BUILDER METHOD

Roster Method. In this method a set is described by listing all its elements, separated by commas (,) with in the braces { }.

Example 6. Let S be the set of all vowels of the alphabets] then elements of S are a, e, i, o, u. We can write it as : **S = (a, e, i, o, u)**

Here the elements are separated by commas and are then enclosed by brackets { }. This is known as **Roster Method** of representation of a set of **Tabular Form** of a set.

Note. Roster Method is used only when the number of elements in a set is finite.

Example 7. If A is a set of all prime numbers less than 13 then A = {2, 3, 5, 7, 11}.

Example 8. The set of all odd natural numbers can be described as {1, 3, 5, 7, 9 ...}. Here the dots (...) stand for 'and so on'.

Set Builder Method. In this method a set' is described by stating a property P(x) which is satisfied by all its elements. In such case set is described by **S = { x P (x) holds}** or S = **{ x P (x) holds}** or **S = (x I x has the property P (x)}**

The symbol '/ 'or' : 'is read as 'such that' :

Example 9. The set A = (1, 2, 3, 4, 5, & 7, 8, 9, 10 can be written as

A = {x/x $\leq$ 101}

Here P (x) = x is a natural number less than or equal to 10.

Example 10. The set N of all natural numbers 1, 2, 3, 4, 5, 6, ... can be written as

N = {x / x is a natural number}

Here P (x) = x is a natural number

Example 11. The set A of all real numbers lying between - 2 and 1 can be written as **A = {x / x $\in$ R and - 2 < x < 1}**

Here P (x) = x is a real number and x lies between - 2 and 1.

FINITE AND INFINITE SETS

11.4.1 Finite Set. A set is said to be afinite set if the number of distinct elements in it is

finite, For Example. let S be the set of all vowels, i.e., S = (a, e i, o u}. Clearly S is a finite :

Example 12. Each one of the following set is a finite set

(i) The set of odd natural numbers less than 50

(ii) The set of prime numbers less than 100.

11.4.2 Infinite Set. If the number or elements in a set is not finite then the set is known as an infinite set.

Example 13.

(i)I the set of all integers is an infinite set;

(ii)N, the set of all natural numbers is an infinite set N = 1 2, 3, ...}

(iii)The set of all points is a plane is an infinite set.

(iv)The set of all lines in a plane is an infinite set.

ORDER OF A FINITE SET OR CARDINAL NUMBER

The number of distinct elements in a finite set is called the order of the set. If X is a finite set having m distinct elements, then the order of x is m. Symbolically,

0(X) = m, m is also called the cardinal number of the set x. It is also written as n(X) = m.

Example 14. Let S = {1 3, 5, 7, 9, 11} be a set, then its order is 6. or the cardinal number of the set S is 6

EQUIVALENT SETS

Two sets A and B are said to be equivalent sets if their cardinal numbers are same, i.e.,

n (A) = n (B).

SUBSET

A set x is said to be a subset of the set Y if every element of the set x is also an elemEnt of the set Y, i.e., $\forall \ x \in X \Rightarrow x \in Y$ We expressit by saying that x is contained in Y or Y contains X. We use the symbol C to denote the fact that the 'set x is a subset of Y and we write it as

(i) X $\subseteq$ Y, to be read as X is contained in Y.

(ii) Y $\supseteq$ X, to be read as Y contains X.

More specifically, X is a subset of Y if, $\forall \ x \in X \Rightarrow x \in Y$.

If X $\subseteq$ Y, then Y is called the superset of X.

Example 15.

(i) If A = {1, 2, 3, 4, 5} and B = {3, 5} then B is a subset of A.

(ii) If P be the set of all parallelograms and T is the set of all squares in a plane, then T is a subset of P i.e., T $\subseteq$ P.

Remarks. If X is not a subset of Y we write it as x $\not\subset$ Y or Y $\not\subset$ X, where Z $\not\subset$ Y = there is at least one element x $\in$ X such that x $\notin$ Y.

Proper Subset. A set X is called a proper subset of the set Y if

(i) X $\subset$ Y; (ii) Y $\not\subset$ X

and we denote it by X $\subset$ Y, i.e. X is said to be a proper subset of Y if every element of X belongs to Y but there is at least one element of Y which is not in X.

Example 16.

(i)The set N of natural numbers is a proper subset of the set I, the set of all integers.

(ii) Q $\subset$ R, i.e., the set of rational numbers is a proper subset of real numbers. A 'set which is nOt a proper subset is called improper subset

EQUAL SETS

Two sets A and B are said to be equal if every element of A is an element of B and every element of B is an element of A'ánd it is denoted by A = B

i.e. $\forall x \in A \Rightarrow x \in B$ and $\forall y \in B \Rightarrow y \in A$

Thus A = B $\Rightarrow$ A $\subseteq$ B and B $\subseteq$ A

Example 17.

(i) Let S = {x / x is a vowel} and T = {a, e, i, o, u}, then S = T.

(ii) Let S = {x | x is a solution of x^2 - x - 2 = 0} and T = { -1, 2 }.

We know that the solutions of x^2 - x - 2 = 0 are -1 and 2.

$\therefore$ S = {-1, 2}. Thus S = T.

NULL SET OR EMPTY SET

A set having no element is called a null set or a void set or an empty set and is denoted by ϕ.
Example 18.

(i) The set S = {x / x is an integer and x^2 = 2} is, clearly, a null set as there is no integer whose square is 2.

(ii) If X is the set of all real solutions of the equation x^2 + 1 = 0, then trivially X = ϕ, for there is no real number which satisfies the equation x^2 + 1 = 0.

Thus we notice from the above examples, that in the case of an empty set, the defining óperty for the membership of a set is such that it is not satisfied 'by any of its members.

SINGLETON SET

A set consisting of only one element as its member, is known as singleton set

Example 19. (i) {1} is a singleton set whose only element is unity.

{ϕ}, it is a set whose only element is a null set, therefore, {ϕ} is singleton.

Remarks. The set ϕ contains no element whereas the set {0} contains only one element, viz 'zero'.

COUNTABLE AND UNCOUNTABLE SETS

A set whose elements can be put in one-to-one correspondence with the elements of the set N, the set of positive integers is called countable or enumerable set otherwise it is Matching sets.

When a one-to-one correspondence exists between two sets, they are said to be **matching sets.**

Example 20. The set T = {a, e, i, o, u} is a countable set as we can write it :

a	e	i	o	u
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
1	2	3	4	5

Thus T is a countable set.

Comparable Sets. Two sets A and B are said to be comparable if either A $\subset$ B or B $\subset$ A. If neither A $\subset$ B nor B $\subset$ A, then A and B are called non-comparable sets.

Example 21. Let A = {3, 6, 9} and B = {3, 6, 9, 12, 15} then A and B are **comparable** as A $\subset$ B.

Example 22. A = {x : x is an even integer} and B = (x : x is an odd integer} Then A and B are non-comparable as A $\not\subset$ B and B $\not\subset$ A.

DISJOINT SETS

Two sets A and B are said to be disjoint sets if they have no element in common, i.e., their intersection is a null set, i.e., if A $\cap$ B = ϕ.

Thus two sets A and B are said to be disjoint sets if A $\cap$ B = ϕ.

The sets X = {x I x is an even natural number} and Y = {y I y is an odd natural number} are disjoint as X $\cap$ Y = ϕ.

FAMILY OF SETS OR SET OF SETS

If the elements of a set are sets themselves, then this set is called the family of sets or a class of sets or a set of sets.

(i) The set {{ϕ}, {2}, {1}, {1, 2}} is a family of sets because its members are the sets (ϕ), {2}, {1}, {1, 2}.

(ii) {ϕ} is also a family of sets as its only element is the set ϕ.

Power Set

It is the set of all possible subsets of the given set A and is denoted by P (A), i.e., P(A) = {B / B $\subseteq$ A}. Thus a power set is a family of sets.

For Example. Let A = {2,3,4} 3, 4} then, P(A) = {{ ϕ } {2}, {3}, {4}, {3, 4} {2, 4}, {2, 3}, {2, 3, 4}}.

Theorem 1. If A is a finite set consisting of n elements, then P(A) contains 2^n elements.

Proof. Let us consider all the subsets of set A.

The number of subsets having no element is ${}^nC_0$.

The number of subsets having one element is ${}^nC_1$.

The number of subsets having two elements is ${}^nC_2$.

... ...

... ...

The number of subsets having n elements is ${}^nC_n$.

Adding all these, we get the number of elements in

P(A) = $[{}^nC_0 + {}^nC_1 + ... + {}^nC_n = (1 + 1)^n = 2^n$. [See Binomial Theoremi]

Hence the power set P(A) has 2^n elements if A contains n elements, where P(A) is the 'power set of A.

BASIC OPERATIONS OF SETS

The main operations in the set theory are union, intersection, difference and complement.

Union of Two Sets. Let A and B be two given sets. Then the union of the two sets A and B is the set of all those elements x such that x belongs to A or x belongs to B or x belongs to both A and B, and is denoted by A $\cup$ B, to be read as 'A union B' or

A $\cup$ B.

i.e., A $\cup$ B = {x I x $\in$ A or x $\in$ B or x $\in$ A as well as x $\in$ B}.

Note 1. It is clear from the definition of union of two sets A and B, that A $\cup$ B = B $\cup$ A.

Note 2. The sets A and B are always subsets of A $\cup$ B, i.e A $\subseteq$ A $\cup$ B and B $\subseteq$ A $\cup$ B

Note 3. If $A_1, A_2, A_3, ... A_n$ is a finite family of sets, then their Union of denoted by $\bigcup_{i=1}^{n} A_i$

where $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup ... \cup A_n$.

Example 23. (i) Let A = {a, b, c} and B = {b, c, d}, then

A $\cup$ B = (x I x $\in$ A or x $\in$ B or x $\in$ both A and B} = {a, b, c, d}.

Example 24. Let A {1, 3, 2, 1, 3, 5} and B = {4, 6, 5, 1, 5, 4}

then A $\cup$ B = { x / x $\in$ A or x $\in$ B or x $\in$ both A and B}

= {1, 2, 3, 4, 5, 6}.

Example 25. Let A = {x : x = 2n, n $\in$ Z}, B = {x : :x = 2n + 1, n $\in$ Z}, then

A $\cup$ B = {x : x is even integer} $\cup$ {x is an odd integer}

= {x : x is an integer] = [x : x $\in$ Z}

Intersectioin of Two Sets. Let A and B be two sets. Then the intersection of two sets A and B is the set of all those elements which belong to A as well as to B and is denoted by A $\cap$ B, to be read as A intersection B or A cap B.

i.e., A $\cap$ B = {x I x A and x $\in$ B}.

Therefore, by the intersection of two sets A and B we mean the set of alt those elements which are common to both A and B.

Clearly, x $\in$ A $\cap$ B $\Leftrightarrow$ x $\in$ Aand x $\in$ B.

Note 1. If A and B are two sets, then A $\cap$ B $\subseteq$ A and A $\cap$ B $\subseteq$ B, i.e., the intersection of two sets A and B is a subset of A as well as of B.

Note 2. The intersection of a finite family of sets A_1, A_2, A_3, A_4 .... A_n is denoted by

$$\bigcap_{i=1}^{n} A_i$$

where, $\bigcap_{i=1}^{n} A_i$ = A_1, $\cap$ A_2, $\cap$ A_3, $\cap$ A_4 $\cap$ .... $\cap$ A_n.

Example 26. Let R = {2, 4; 6, ...} and S = {3, 6, 9,...}, then R $\cap$ S = { 6, 12, 18, 24, ...}

Example 27. Let A = {a, b, 1}, B = {b. c, d, 1}, then A $\cap$ B = {b, 1}.

Example28. ifA { x : x = 3n n $\in$ Z} and B = { x : x = 5n, n $\in$ Z], then find A $\cap$ B.

Solution. Let x $\in$ A $\cap$ B $\Leftrightarrow$ (x = 3n, n $\in$ Z) and (x = 5n $\in$ Z)

$\Leftrightarrow$ (x is a multiple of 3) and (x is a multiple of 5)

$\Leftrightarrow$ x is amultiple of 15 x =15n, n $\in$ Z

Hence A $\cap$ B = {x : x = 15n n $\in$ Z}

DIFFERENCE OF TWO SETS

Let A and B be, two sets, then the difference of the sets A and B is the set of all those elements x such that x bqlongs to A and x does not belong to B and is denoted by A B to be read as 'A difference or simply 'A minus B'

i.e., A - B = { x : x $\in$ A and x $\notin$ B}.

Example 29. Let A = {1, 3, 5, 7, 9, 11) and B = {3, 5, 7, 8} then

A - B = {1, 9, 11}, where as B - A = {8}

Obviously A - B = B - A

COMPLEMENT OF A SET

Let U be a universal set and A $\subset$ U then the complement of the set A with respect to the universal set U is the difference of the universal set U and the set A. It is denoted by A^C or A' be read as A complement.

i.e., A^c = U - A = {x / x $\in$ U and x $\notin$ A} or A^c = {x I x $\notin$ A}.

Another Definition. If A and B are two given Sets such that A $\subseteq$ B, then the complement of A in B or complement of A with respect to B is the set of all those elements of B which does not belong to A.

i.e., B - A = (x / x $\in$ B and X $\notin$ A}

e.g.,If we take the set B = {x / x $\in$ N} and A = {y / y is an odd number and y N}, then A^c = {z / z is an even number and z $\in$ N

Note 1. We notice that in order to define the complement of a set A with respect to another set B, it is necessary that A $\subset$ B whereas the difference can be defined for any two sets A and B.

Note 2. A - B = A $\cap$ B^c if for any two sets A and B.

Now A - B = {x I x $\in$ A and x $\in$ B} = {x I x $\in$ A and x $\in$ B^c} = A $\cap$ B^c.

Example 30. Let N, the set of all natural numbers, be taken as the universal set and let A = [x / x is an even number and x $\in$ N), then

A^c = [y / y is an odd number and y $\in$ N).

SYMMETRIC DIFFERENCE

The symmetric difference of two sets A and B is the set (A — B) u (B — A) and is denoted by A Δ B

i.e., $$A \Delta B = (A - B) \cup (B - A)$$

Example31. Let A =(1. 2, 3] and B = {3 4, 5 then A - b {1,2 2) and B- A =(4,5),

then A Δ B = (A — B) $\cup$ (B — A) = {1 2) $\cup$ (4, 5) = 1, 2 4, 5).

PRINCIPLE OF DUALITY

The principle of duality states all laws of algebra remain true if we interchange union and intersection, and also the universal and the null set.

eg.. if the theorem is $(A \cup B) \cup (A \cup B^c) = A$,

then its dual will be obtained by interchanging union and intersection i.e., dual will be

$$(A \cap B) \cup (A \cap B^c) = A$$

Theorem 2. If U is the universal set and A $\subset$ U, then $(A^C)^C = A$.

Proof In order to prove the above result we need to prove that

(a) $(A)^c \subseteq A$ and (b) $A \subseteq (A^c)^c$

Now $x \subset (A^c)^c \Leftrightarrow x \notin A^c$ Thus xe $x \in (A^c)^c \Leftrightarrow x \in A$

$(A^c)^c \subseteq A$ and $(A^c)^C \supseteq A \Rightarrow (A^c)^c = A$

DE-MORGAN'S LAWS

Theorem 3. Let U be the universal set and A $\subset$ U, B $\subset$ U, then

(i) $(A \cup B)^c = A^c \cap B^c$, i.e the complement of union of two sets A and B is equal té intersection of their complements.

(ii) $(A \cap B)^c = A^c \cup B^c$ u if, i.e., the complement of the intersectiOn of two sets is equal to the union of their complements.

ALGEBRA OF SETS

We have introduced the notion of a set and discussed some basic operations in sets, namely intersection, union, difference, etc., which enable us to combine sets so as to give a new set. These basic operations play an important role in the development of set theory. We know that there are certain fundamental laws in the algebra of numbers, where addition and multiplication are the two basic operations. Similarly; -there are certain fundamental laws in set theory with respect to the basic operations of u (union), n (intersection) and (difference).

I Commutative Laws. If A and B are any two sets, then

(i) $A \cup B = B \cup A$

and (ii) $A \cap B = B \cap A$

II Associative Laws. If A, B and C are any three sets then associative law is said to hold good. -

(i) $A \cup (B \cup C) = (A \cap B) \cap C$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

III. Distributive Laws. If A, B and C are three sets and $\cup$ and $\cap$ are operations, then

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

IV. Idempotent Laws. If A is any set, then

(i) $A \cup A = A$

(ii) $A \cap A = A$

V. Identity Laws. If A is any set and U is the universal set and $\varnothing$ is the null set, then

(i) $A \cup \phi = A$

(ii) $A \cap U = A$

VI. Complement Laws. If A is any set and A^C OR A' is the complement of the set A, then

(i) $A \cap A^c = \phi$

(ii) $A \cup A^c = U$

VII. De Morgan's Laws. If A and B are any two sets, then

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(A \cap B)^c = A^c \cup B^c$

VENN DIAGRAM

A Swiss mathematician Euler, first of all, gave an idea to represent a set by points in a closed curve. Later on British mathematician Venn put this idea to practice. It is this fact that the diagrams drawn to represent sets are called Venn - Euler diagrams or simply Venn-diagrams. These diagrams are very useful for the beginners to understand the set theoretic ideas. In Venn-diagrams, the Universal set U is represented by points within a rectangle. The subsets A, B, ... of the Universal set U are represented by points in a closed curves (usually circles) within the rectangle.

Subsets. The subsets are represented by the portion of the region within the region and this portion is generally enclosed by the circle around the elements.

For example the subset A = {a, b, c, d} is represented by the circle enclosing a, b, c, d in -

Fig. 11.1.

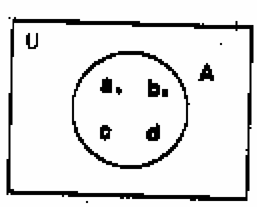

Fig. 11.1

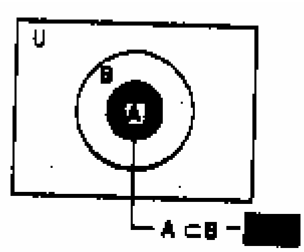

Fig. 11.2

Proper Subset. If A is a proper subset of B, then the circle representing A is drawn inside the circle representing B [see Fig. 11.2].

Example 32. Let A be set of all players and B be the set of all Cricket players of Dyal Singh College. This possibility is represented by saying that B is proper subset of A, i.e., B C A and this is represented by Venn diagram as the circle B inside the circle A (See Fig. 11.3).

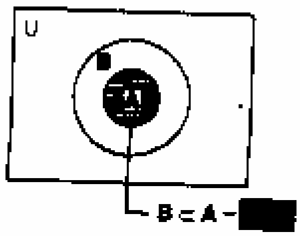

Fig. 11.3

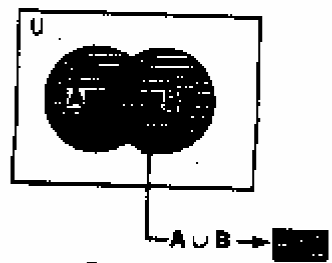

Fig. 11.4

Union. The union of two sets A and B is the shaded region in Fig. 11.4.

Intersection. The intersection of the two sets A and B is given by shaded portion in Fig. 11.5.

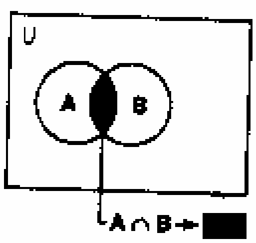

Fig. 11.5

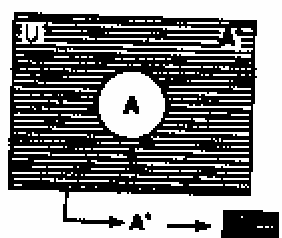

Fig. 11.6

Complement of A Set. Let U be the universal set. Let A be a subset of U. Then the complement A of with respect to U is the shaded region in the Fig. 11.6. -

Example 33. Let the universal set be

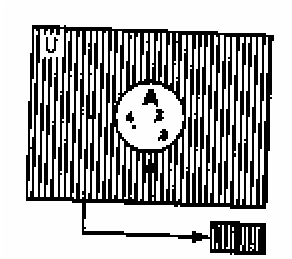

Fig. 11.7

U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and A = {2, 3, 4}.

Now 1, 5, 6, 7, 8, 9 are members of U but not of A, whereas 2, 3, 4 are members of both the sets A and U. Thus the members 1,5,6,7, 8,9, can form another set known as complement of A and is denoted by A^c The shaded portion in Fig. 11.7 is A^c.

Difference of Two Sets. Let A and B be the two subsets of the universal set U. Then the difference A — B is the set of all those elements which do not belong to B. In the Venn-diagram A - B is the shaded region in the figure 11.8.

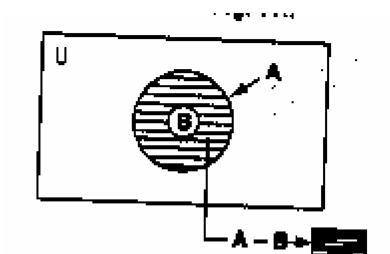

Fig. 11.8

Similarly, the set B A is given by the shaded portion in the figure 11.9.

Symmetric Difference. Let A and B be two subsets of the universal set U. Then the symmetric differencal $A \Delta B = (A - B) \cup (B - A$ is the shaded is the shaded portion in the figure 11.10.

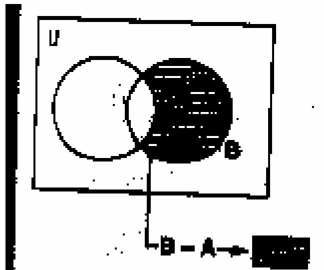

Fig. 11.9

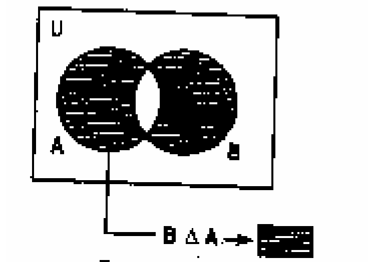

Fig. 11.10

SOME RESULTS ON NUMBER OF ELEMENTS IN SETS

Theorem 4. If A and B are two finite sets, then

(i) n $(A \cup B)$ = n (A) +n (B)—n $(A \cap B)$, whenAandBare not disjoint sets

(ii) n $(A \cup B)$ = n (A) + n (B), when A and B are disjoint sets

(iii) n (A — B) = n (A) - n $(A \cap B)$, i.e., n (A) = n (A — B) + n $(A \cap B)$

(iv) n $(A \Delta B)$ = Number of elements which belongs to exactly one of A or B

$\therefore$ n (A Δ B) = n (A) + n (B) - 2n (A n B)

(v) n $(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B)$

(vi) n $(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$

Theorem 5. If A, B and C are three finite sets, then

(i) n $(A \cup B \cup C) = n(A) + n(B) + (C) - n(A \cap B) - n(A \cap C)$

$$- n(B \cap C) + n(A \cap B \cap C)$$

(ii) Number of elements in exactly two of the sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(iii) Number of elements in exactly are of the sets A, B, and C

= n (A) + n (B) + n (C) — 2n $(A \cap B)$ - 2n $(B \cap C)$ -2n $(A \cap C)$ + 3n $(A \cap B \cap C)$

Example 34. The combined membership of Mathematics AssOciation and Science Club is 122 What is the membership of Science Club if 50 are known to be the members of Mathematics Association and 28 are members of both the organisations ?

Solution. Let M be the set of Mathematics Association arid S be the set of Science Club.

We are given that n $(M \cup S)$ = 122, n (M) = 50; n $(M \cap S)$ = 28.

Also we know that n $(M \cup S)$ = n (M) - n $(M \cap S)$

$\therefore$ 122 = 50 + n (S) - 28 $\Rightarrow$ n (s) = 122 - 22 = 100.

Example 35. In a class containing 50 students, 15 play Tennis,: 20 play Cricket and 20 play Hockey, 3 play Tennis and Cricket, 6 play Cricket and Hockey, and 6 play Tennis and Hockey. 7. play no game at alt How many play Cricket, Tennis and Hockey ?

Solution. Let sets of Tennis, Cricket and Hockey players be denoted by T, C and H respectively, then n (U) = 50, n (T) = 15, n (C) = 20, n (H) = 20, n $(T \cap C)$ = 3, n $(C \cap H)$ = 6, n (T n H) =5 n (T $\cap$ C $\cap$ H)' = 7. We have to find n (T $\cap$ C $\cap$ H).

Now, n (T $\cup$ C $\cup$ H) = n (T) + n (C) + n (H) n (T $\cap$ C) n (C $\cap$ H)

- n (T $\cap$ H) + n (T $\cap$ C $\cap$ H).

$\therefore$ n (T $\cup$ C $\cup$ H) = 15 +20 + 20 - 3 - 6 -5 + n (T $\cap$C $\cap$ H)

or n (T $\cup$ C $\cup$ H) = 41 + n (T $\cap$ C $\cap$ H)

But n (T $\cup$ C $\cup$ H)' = n (U) - n (T $\cup$ C $\cup$ H)

7 = 50 - [41 + n (T $\cap$ C $\cap$ H)] $\Rightarrow$ n (T $\cap$ C $\cap$ H) = 2.

Example 36. A company studies the product preferences of 300 consumers. It was found that 226 liked product X; SI liked product Y and 54 liked product Z; 21 liked products X and Y; 54 liked products X and Z; 39 liked products Y and Z and 9 liked all the three products. Prove that the study results are not correct. (Assuming that each consumer likes at least one of the three products.)

Solution. Let the set of consumers having likings for the product X, product Y and product Z respectively be denoted by X, Y and Z. Then we have

n (X) = 266, n (Y) = 51, n (Z) = 54, n (X $\cap$ y) = 21, n (X $\cap$ Z) = 54, n (V $\cap$ Z) = 39,

n (X $\cap$ Y $\cap$ Z) = 9 and n (X $\cup$ V $\cup$ Z) = 300.

We know that n(X $\cup$ Y $\cup$ Z) = n (X) + n (Y) + n (Z) - n (X $\cap$ Z) - n (X $\cap$ Y) -n (Y $\cap$ Z)+ n (X $\cap$ Y $\cap$ Z).

L.H.S = 256 + 51 + 54 - 21 - 54 - 39 + 9 = 226.

R.H.S. = 300 (as given) But 226 $\neq$ 300 $\Rightarrow$ L.H.S. $\neq$ R.H.S.

Hence the study results are not correct. -

Example 37. If A. M. S. are three groups consisting of Accountants, Management Consultants and Sales Managers respectively, then test the validity of the statement "Some Accountants are Management Consultants : all Management Consultants are Sales Managers, therefore some Accountants are Sales Managers."

Solution. Some Accountants are Management Consultant $\Rightarrow A \cap M \neq \phi$. ...(1)

All Management Consultants are Sales Manager $\Rightarrow M \cap S' = \phi$. ...(2)

Now (1) can be written as $(A \cap M \cap S) \cup (A \cap M \cap S') \neq \phi$. ...(3)

Again (2) can be written as : $(A \cap M \cap S) \cup (A \cap M \cap S') = \phi$. ...(4)

Now (4) $\Rightarrow A \cap M \cap S' = \phi$. ...(5)

But (3) and (5) $\Rightarrow A \cap M \cap S \neq \phi \Rightarrow A \cap S \neq \phi$.

$\Rightarrow$ The class of Accountants who are Sales Manager is not an empty.

Hence "Some Accountants are Sales Manager" is a valid statement.

<u>Relation</u> : Let A, B be any two non-empty sets, then every subset of A × B define a relation from A to B and every relation from A to B is a subset of A × B

If R is a relation from A to B and if $(a, b) \in R$ then we write aRb and say that "a" is related to b" it is denoted by R a - b

$R = \{(a, b) / a \in A; b \in B\}$

Note : Let the number of elements of A and B be m and n respectively then the number of elements of A × B is mn

Therefore the numebr of elements of the powerset of A × B is 2^{mn} Hence the number of different relation from A to B is 2^{mn}

<u>Domain and Range of a relation</u> : If R is a relation from A to B, then the set of al first co-ordinates of elements of R is called the domain of R while the set of all second co-ordinates of elements of R is called the range of R

Dom (R) = $\{a/(a, b) \in R\}$

Range of R = $\{b / (a, b) \in R\}$

<u>Some particular Types of relation :</u>

(1) Void relation : Since $\phi \subset A \times A$, it follws that ϕ is a relation on A, called the empty or void relation

e.g. Let A = {3, 5} B = {7, 11} Let $R = \{(a, b)/ a \in A, b \in B \; a - b \text{ is odd}\}$

Since none of the numbers (3 – 7), (3 – 11), (5 – 7), (5 –11) is an odd number, R is an empty relation.

Universal Relation : Since $A \times A \subseteq A \times B$ it follows that $A \times A$ is a relation on A, called the universal relation.

e.g. Let A = {1, 2, 3} then

$R = A \times A = \{(1, 1)\ (1, 2)\ (1, 3)\ (2, 1)\ (2, 2)\ (2, 3)\ (3, 1)\ (3, 2)\ (3,)\}$ is a universal relation in A.

Identify Relation : The relation $I_A = \{(a, a) / a \in A\}$ is called the identify relation on A

e.g. If A = {1, 2, 3} then the identify relation on A is given by $I_A = \{(1, 1)\ (2, 2)\ (3, 3)\}$

Inverse Relation : If R is a relation on A, then the relation R^{-1} defined by

$R^{-1} = \{(b, a)/ (a, b) \in R\}$ is called an inverse relation on A

Clearly dom (R^{-1}) = Range (R) and

range (R^{-1}) = dom (R)

e.g. Let A = {2, 3, 4} B = {2, 3, 4} and R = {(x, y)/x - y = 1}

be a relation from A to B

i.e. R = {(3, 2) (2, 3) (4, 3) (3, 4)}

then R^{-1} = {(2, 3) (3, 2) (3, 4) (4, 3)}

Reflexive Relations : Let R be a relation in a set A Then R is called a reflexive relation if $(a, a) \in R$

for all $a \in A$

Thus R is reflexive if aRa holds for all $a \in A$

e.g. Let A = {1, 2, 3, 4} then

(i) The relation R_1 = {(1, 1) (2, 4) (3, 3) (4, 1) (4, 4)} in A is not reflexive since $2 \in A$ but $(2, 2) \notin R1$

(ii) The relation R_2 = {(1, 1) (2, 2) (3, 3) (4, 4)} is reflexive.

Symmetric Relations : Let R be a relation in a set A

Then R is said to be symmetric relation if (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$R

e.g. Let L be the set of all straight lines in a plane, The relation R in L defined by "x is parallel to y" is symmetric relations.

Since a straight line l_1 is parallel to a straight line l_2 then is also parallel to l1

Thus (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$R

Transitive Relations : Let R be a relation in a set A then R is said to be transitive relation if (a, b) $\in$ R and (b, c) $\in$ R = (a, c) $\in$ R

e.g. Let L be the set of all straight lines in a plane and R be the relation in L defined by "x" is parallel to y" If l_1 is parallel to l_2, l_2 is parallel to l_3 then l_1 is parallel to l_3 Thus (a, b) $\in$R, (b, c) $\in$R $\Rightarrow$ (a, c) $\in$ R

Equivalence Relation : Let R be a relation in a set A

Then R is an equivalence relation in A if

(i) R is reflexive i.e. for all a $\in$ R (a, a) $\in$ R

(ii) R is symmetnic i.e. (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R for all a, b $\in$ A

(iii) R is transitive i.e. (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R

for all a, b, c $\in$ A

e.g. The example of an equivalence relation is that of "equality"

For any elements in any set

(i) a= a i.e. reflexive

(ii) a = b = b = a i.e. symmetnc

(iii) a = b and b = c $\Rightarrow$ a = c i.e. transitive.

CLASS WORK + HOME WORK - 1

Set Theroy :

1. The number of subsets of the sets {6, 8, 11} is:

(a) 9 (b) 6 (c) 8 (d) None of these

2. If the universal set E = {x / x is a positive integer < 25), A = {2, 6, 8, 14, 22}, B = {4, 8, 10, 14}, then :

(a) $(A \cap B)' = A' \cup B'$ (b) $(A \cap B)' = A' \cap B'$

(c) $(A' \cap B)' = \phi$ (d) None of these

3. If the set P has 3 elements, Q has four elements and R has two elements, then the set P x Q x R contains :

(a) 9 elements (b) 20 elements (c) 24 elements (d) None of these

4. If A and B are two sets, then (A $\cup$ B) $\cap$ = B equals

(a) B (b) A (c) A $\cap$ B (d) ϕ

5. Let A = { x / x is a multiple of 2} and B = { x / x is a multjple of 3}. Then A $\cap$ Bis given by :

(a) {3, 6, 9, ...} (b) {2, 4, 6, 8, ... } (c) {6, 12, 18, ... } (d) None of these

6. If A = {1,2,3,4}, B = {2, 4, 5, 6) and C = {1, 3, 4, 6, 8}, then the set (A $\cap$ B) $\cup$ (A $\cap$ C) equal to

(a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5, 6}

(c) {2, 4, 5, 6} (d) None of these

7. Let A = {1, 3, 5, 7, 8, 9} and B= {3, 5, 8), thenA Δ B is :

(a) {1,7, 9} (b) {3, 5, 8} (c) ϕ (d) None of these

8. If A = (1, 3, 5, 7, 11, 13, 15, 17}, B = {2, 4, 6 ...18} and N is the universal set, then A' $\cup$ [(A $\cup$ B) $\cap$ B'] is equal to:

(a) N (b) A' (c) B (d) None of these

9. If X and Y are two sets, then (Y $\cup$ X)' $\cap$ X equals :

(a) ϕ (b) x (c) y (d) None of these

10. If A & B are two sets and U is the universal set such that n (U) = 700, n (A) = 200, n (B) = 300 and n (A $\cap$ B) = 100, then n (A' $\cap$ B') is equal to :

(a) 200 (b) 300 (c) 400 (d) 600

11. A and B are two disjoint sets containing 3 and 6 elements respectively. The number of elements in A $\cup$ B is :

(a) 3 (b) 6 (c) 2 (d) 9

12. If a $\in$ N such that aN = {ax : x $\in$ N}. The set 3N $\cap$ 7N is equal to :

(a) 7 N (b) 3 N (c) 21 N (d) None of these.

13. For any natural number a, we define aN = (ax : x € N}. If b, c, d € N such that bN $\cap$ cN = dN, then d is equal to:

(a) b (b) c (c) bc (d) None of these

DIRECTION : For Q.No. 14 to Q.no.18.

U = {1, 2, ...9} to be the universal set, A = {1, 2, 3, 4} and B = {2, 4, 6, 8}. Then answer the following questions

14. The A $\cup$ B is equal to :

(a) {1, 2, 3,4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {5, 7,9}

15. The set equal to set A $\cap$ B is :

(a) {1,2,3,4,6,8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) (5,7,9)

16. The set equal to set A' is :

(a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {5,7,9}

17. The set (A $\cup$ B)' is :

(a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {5, 7, 9}.

18. The set (A $\cap$ B)' is equal to :

(a) {1, 2, 3, 4, 6, 8} (b) {2, 4}

(c) {5, 6, 7, 8, 9} (d) {1, 3, 5, 6, 7, 8, 9}.

19. Then A x (B $\cup$ C) is equal to

(a) {(2, 4), (2,5), (2,6), (3,4), (3, 5), (3, 6)}

(b) {(2, 5), (3,5)}

(c) {(2, 4), (2, 5), (3, 4), (3, 5) (4, 5), (4, 6), (5,5) (5, 6)}

(d) None of these.

20. The set A x (B $\cap$ C) is equal to

(a) {(2,4), (2,5), (2,6), (3,4), (3, 5), (3, 6)}

(b) {(2,5), (3,5)}

(c) {(2, 4), (2, 5), (3, 4), (3, 5) (4, 5),
(4, 6), (5,5) (5, 6)}

(d) None of these.

21. The set (A x B) $\cup$ (B x C) is equal to

(a) {(2,4), (2, 5), (2, 6), (3,4), (3, 5), (3, 6)}

(b) {(2, 5), (3,5)}

(c) {(2, 4), (2, 5), (3, 4), (3, 5) (4,5), (4, 6), (5, 5) (5, 6)}

(d) None of these.

22. Given A {2, 3}, B = {4, 5}, C = {5, 6}, then A x (B $\cap$ C) is equal to

(a) {(2, 5), (3, 5)} (b) {(5, 2), (5, 3)}

(c) {(2, 3), (5, 5)} (d) None of these.

23. After qualifying out of 400 professionals, 112 joined industry, 120 started practice and 160 joined as service. There were 32 who were in both practice and service, 40 in both practice and industry and 20 in both industry and service. There were 12 who did all the three. How many could not get any of these?

(a) 88 (b) 244 (c) 122 (d) None of these.

24. In a town of 20,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then the number of families which buy A only is :

(a) 6600 (b) 6300 (c) 5600 (d) 600

25. In the above questions(24), the number of families which buy none of A, B and C is :

(a) 6000 (b) 8000 (c) 4000 (d) None of these

26. If A has 32 elements, B has 42 elements and A u B has 62 elements find the number of elements in A $\cap$ B.

(a) 10 (b) 12 (c) -12 (d) None of these

27. A town has a total population of 50000 persons and of them 28000 read, "Gujarat Samachar" and 23000 read "Sandesh" while 4000 read both the papers. Indicate how many read neither "Gujarat Samachar" nor "Sandesh"?

(a) 300 (b) 1300 (c) 3000 (d) None of these

28. If A = {x / x^2 - 17x + 60 = 0}

B = {x / x^2 - 7x+ 12=0} find A $\cup$ B and.

(a) {3, 4, 5, 12} (b) {-3, 4, 5, -12} {3}

(c) {3, -4, 5, 12} {12} (d) None of these.

29. In a college, there are at least 500 girls and of them 300 have taken Economics and 250 have taken Mathematic. How many of them have taken both the subjects ?

(a) 40 (b) 50 (c) -50 (d) None of these

30. Let the universal set U = $\{x / 3 \leq x \leq 13, x \in N\}$

A = $\{ y / 2 \angle y \angle 7, y \in N\}$

B = {3, 5, 7, 9}, find A'

(i) { 7, 8, 9, 10, 11, 12, 13} (ii) {3, 6, 8, 9, 10, 11, 12, 13}

(iii) {4, 5, 6, 10, 11, 12, 13} (iv) None of these

31. Let the universal set U = $\{x / 3 \leq x \leq 13, x \in N\}$

A = $\{ y / 2 \angle y \angle 7, y \in N\}$

B = {3, 5, 7, 9}, find B'

(i) {4, 5, 6, 7, 8, 9, 10 11, 12, 13} (ii) {4, 6, 8, 10, 11, 12, 13}

(iii) {5, 6, 7, 11, 12, 13} (iv) None of these

32. Let the universal set U = $\{x / 3 \leq x \leq 13, x \in N\}$

A = $\{ y / 2 \angle y \angle 7, y \in N\}$

B = {3, 5, 7, 9}, find $(A \cup B)'$

(i) {8, 10, 11, 12, 13} (ii) {3, 4, 5, 11, 12, 13}

(iii) {3, 4, 5, 8, 9, 10, 11, 12, 13} (iv) None of these

33. If U = (a, b, c, 1, 2, 3) is a universal set, A = (a, b, c}, B = (1,2,3), C = (a, 1,2), D = (a, b, 3) then find the following sets $(A — B) \cap (B — A)$

(i) $\varnothing$ (ii) {a, b, 1}

(iii) {1, 2, 3} (v) None of these

34. If U = (a, b, c, 1, 2, 3) is a universal set, A = (a, b, c}, B = (1,2,3), C = (a, 1,2), D = (a, b, 3) then find the following sets $(C \cup D)'$

(i) {a} (ii) {b} (iii) {c} (iv) None of these

35. If U = (a, b, c, 1, 2, 3) is a universal set, A = (a, b, c}, B = (1,2,3), C = (a, 1,2), D = (a, b, 3) then find the following sets $(A — D) \cap (A \cap D)$.

(i) {1, 2, 3} (ii) {a, b, 2} (iii) $\varnothing$ (iv) None of these

36. If A = {2, 3} then find A^2

(i) (2, 2) (2,3) (3, 2) (3,3) (ii) {(4, 9)}

(iii) {(2, 2) (3, 3) } (iv) None of these

37. If the universal set is X = { x / x $\in$ N, 1 $\leq$ x $\leq$ 12] and A= {1,9,10], B = {3,4,6, 11, 12},C = {2,5,6} are subsets of X, find the sets A $\cup$ (B $\cap$ C)

(i) {1, 6, 9, 10} (ii) {1, 6} (iii) {1, 6, 10} (iv) None of these

38. If the universal set is X = { x / x $\in$ N, 1 $\leq$ x $\leq$ 12] and A= {1,9,10], B = {3,4,6, 11, 12},C = (2,5,6) are subsets of X, find the sets (A $\cup$ B) $\cap$ (A $\cup$ C).

(i) {1, 6, 9, 10) (ii) {1, 2, 5, 6} (iii) {1, 2, 5, 6, 9, 10} (iv) None of these

39. If A = {2,4}, B = {2,4,6} find A x B

(i) {(2,2), (2,4), (2,6), (4,2), (4,4,) ,(4,6)} (ii) {(2,2), (2,4), (4,2), (4,4)}

(iii) {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)} (iv) None of the above

40. If A = {2,4}, B = {2,4,6} find A x A

(i) {(2,2), (2,4), (2,6), (4,2), (4,4,) ,(4,6)} (ii) {(2,2), (2,4), (4,2), (4,4)}

(iii) {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)} (iv) None of the above

41. If A = {2,4}, B = {2,4,6} find B x B.

(i) {(2,2), (2,4), (2,6), (4,2), (4,4,) ,(4,6)} (ii) {(2,2), (2,4), (4,2), (4,4)}

(iii) {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)} (iv) None of the above

42. There are 120 students in a class of a college. 30 students do not have interest in games and they do not take part in the games. 64 students of the class play foot-ball and 36 students play hockey. How many students play both the games?

(i) 5 (ii) 10 (iii) -5 (iv) None of these

43. There are 120 students in a class of a college. 30 students do not have interest in games and they do not take part in the games. 64 students of the class play foot-ball and 36 students play hockey. How many students play only foot-ball?

(i) 45 (ii) 54 (iii) 34 (iv) None of these

44. There are 120 students in a class of a college. 30 students do not have interest in games and they do not take part in the games. 64 students of the class play foot-ball and 36 students play hockey. How many students play only hockey?

(i) 62 (ii) 16 (iii) 36 (iv) None of these

45. In a class, 85 students pass in at least one of the three subjects Mathematics, Statistics and Economics. The number of students passing in each of these three subjects is same. Also the number of students passing in both Mathematics and Statistics is 20; passing in both Statistics and Economics is 25 and passing in both Economics and Mathematics is 35. The number of students passing in all the three subjects is 15. Then find the number of students passing in each of the three subjects.

(i) 50 (ii) 100 (iii) 25 (iv) None of these

46. A class of 100 students appeared for F. Y. B.B.A. examination. Out of 100 students, 40 passed in Mathematics, 36 passed in Management, 60 passed in Accountancy, 8 students passed in Mathematics and Management, 17 passed in Management and Accountancy 16 passed in Mathematics and Accountancy 5 passed in all the three subjects find : How many students passed exactly in one subject?

(i) 69 (ii) 136 (iii) 56 (iv) None of these.

Function :

47. If f(x) = x + 3, g(x) = x^2, then f(x) g(x) is :

(a) $(x + 3)^2$ (b) $x^2 + 3$ (c) $x^3 + 3x^2$ (d) None of these.

48. The range of the function f (x) = log (1 + x) for the domain of real values of x when x $\in$ [0, 9] is

(a) (0, -1) (b) (0, 1, 2) (c) [0, 1] (d) None of these

49. If f(x) = x + 3, g(x) = x^2, then fog(x) is :

(a) $x^2 + 3$ (b) $x^2 + x + 3$ (c) $(x + 3)^2$ (d) None of these.

50. If f(x) = 1/(1 - x), then f(-1) is :

(a) 0 (b) 1/2 (c) 0 (d) None of these.

51. The domain of {(1, 7), (2, 6)} is :

(a) (1, 6) (b) (7, 6) (c) {1, 2} (d) (6, 7)

52. The range of the function f: A $\rightarrow$ R, f(x) = x^2 + 1, where A = {-1, 0, 2,4} is :

(a) {1, 2, 5, 17} (b) {2, 5,17} (c) {5, 17} (d) None of these.

53. The range of the functiong :A $\rightarrow$ N, g(x) = 2x, where A = {x $\in$ N, x $\leq$ 5) is :

(a) {1, 2, 3, ..., 10) (b) {2, 4, 6, 8, 10}

(c) {1, 3, 5, 7} (d) None of these

54. Let f : R $\rightarrow$ R be a function given by f(x) = $x^2 + 1$, then f^{-1} (-5) is equal to

(a) {- 5} (b) ϕ (c) {2. 3} (d) None of these

55. The domain for which the functions f(x) = $2x^2 - 1$ and g(x) = 1 - 3x are equal is :

(a) {- 2, 1/2} (b) {1/2, 2} (c) { - 2, -1/2} (d) None of these

56. Let f = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function defined as f(x) = ax + b for some integers a and b. Then the value of (a, b) is :

(a) (2, -1) (b) (1,2) (c) (1, -2) (d) (-1, -2)

57. Let f: R $\rightarrow$ R, f(x) = 2^x, then the range off is :

(a) N (b) R^+ (c) R (d) 1

58. If f(x) = x + 3, g(x) = x^2, then go f(x) is equal to

(a) $(x + 3)^2$ (b) $x^2 + 3$ (c) $x^2(x + 3)$ (d) None of these

59. Let A = {a, b}. Set of subsets of A is called power set of A denoted by P(A). Now n(P(A)) is

(a) 2 (b) 4 (c) 3 (d) None of these

60. If f R$\rightarrow$ R, f(x) = x^3 + 2 for all, then f is :

(a) One-One onto (b) Onto (c) Into (d) None of these

61. If f : A $\rightarrow$ B, f(x) = x^2, A = {-1, 1, -2, 2}, and B = {1,4, 9, 16}, then f is :

(a) One-One (b) Many-one Into (c) Into (d) None of these

62. 1ff: Z $\rightarrow$ Z,f(x) = x^2 + x for all x $\in$ z, thenf is :

(a) Many-one (b) One-One (c) Onto (d) None of these

63. If f :Z$\rightarrow$Z, f(x) = 3x + 2 for all x $\in$ Z, then f is :

(a) Onto (surjective) (b) One-One (c) Injective (d) None of these

64. If f : R$\rightarrow$R, f(x) = $3x^3 + 5$ for all x $\in$ R, then f is :

(a) Into (b) One-One onto (bijections)

(c) One-One into (d) None of these

65. If f : R$\rightarrow$R; f(x) = x^2, and g : R $\rightarrow$ R; g(x) = 2x + 1, then fog is :

(a) $2x^2 + 1$ (b) $(2x + 1)^2$ (c) $4x^2 + 1$ (d) None of these

66. If the function f and g are given by f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}, then gof is:

(a) {(1, 3), (3, 1), (4, 3)} (b) {(2, 5), (5, 2), (1, 5)}

(c) {(1, 3), (2, 5), 3, 5} (d) None of these

67. If f: R $\rightarrow$ R, f(x) = x^2 - 3x + 2, then f(f(x)) =

(a) $x^4 + 10x^2 - 3x$ (b) $x^4 - 6x^3 + 10x^2 - 3x$

(c) $x^4 + 6x^4 - 10x^2 + 3x$ (d) None of these.

68. If f, g : R$\rightarrow$R, f(x) = x^2 + 3x + 1, g (x) = 2x -3, for all x $\in$ R, then fog is :

(a) $4x^2 - 6x + 1$ (b) $4x^2 + 6x - 1$

(c) $4x^2 + 6x + 1$ (d) None of these

69. If g : R$\rightarrow$R, g(x) = 2x - 3, for all x $\in$ R, then gog is :

(a) 4x + 9 (b) 4x - 9

(c) 4x + 7 (d) None of these

70. If f : R $\rightarrow$R, f(x) = 2x + 7 then the inverse off is :

(a) $f^{-1}(x) = (x - 7)/2$ (b) $f^{-1}(x) = (x + 7)/2$

(c) $f^{1}(x) = (x - 3)/2$ (d) None of these

71. If f : R$\rightarrow$R is a bijection given by f(x) = x^3 + 3, then f^{-1} (x) is :

(a) $f^{-1}(x) = (x - 3)^{1/3}$ (b) $f^{-1}(x) = (x - 3)^{-1/3}$

(c) $f^{-1}(x) = (x + 3)^{1/3}$ (d) None of these

72. Let f: R $\rightarrow$ R be such that f (x) = 2^x, then f(x + y) equals

(a) f(x) + f(y) (b) f(x) . f(y)

(c) f(x) ÷ f(y) (d) None of these

Relation :

73. The range of the relation {(x, y) : X $\in$ N, y $\in$N, and x + y = 10) is :

(a) {1,2,3,4,5,6,7, 8,9} (b) {9, 8, 7, 6, 5, 4, 3,2, 1}

(c) {1, 2, 3, 4, 5, 7} (d) None of these

74. The domain of there relation {(x, y) : y = $|x-1|$, x $\in$ Z and $|x| \leq 3$} is :

(a) {-3, -2, -1, 0, 1, 2, 3} (b) {4, 3, 2, 1, 0}

(c) {6, 1, 2, 3, 4} (d) None of these.

75. The range of the relation R, where R = {(x, x^3); x is a prime number less than 10) is :

(a) {2, 3, 5, 7} (b) {8, 27, 125, 343}

(c) {8, 27, 125, 243} (d) None of these

76. If f is relation from set A = {2, 3, 5} to set B = {2, 3, 6, 8, 10} defined by xfy $\Leftrightarrow$ x divides y, then f =

(a) {(2, 6), (2, 8), (2, 10), (3, 6), (5, 10)}

(b) {(2, 2), (2, 6), (2, 8), (2, 10), (3, 6), (3, 3), (5, 10)}

(c) $\{(2, 2), (2, 6), (2, 8), (2, 10), (3, 6), (5, 10)\}$

(d) none of these.

77. $A = \{1, 2, 3\}$ $R = \{(1, 1), (2, 2), (3, 3)\} \subseteq A \times A$. Then R is :

(a) not reflexive but transitive and symmetric

(b) not transitive but reflexive and symmetric

(c) an equivalence relation

(d) not an equivalence relation

78. If $A = \{x / x \in N, 1 \le x \le 8\}$ and $R = \{(x, y) : x \in A \text{ and } x + 2y = 9\}$ is a relation in A, then dom R^{-1} =

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 3, 5, 7\}$ (c) $\{1, 2, 3, 5\}$ (d) $\{1, 2, 3, ...8\}$.

79. $A = \{1, 2, 3\}$. $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$ is a relation which is

(a) reflexive, symmetric, transitive

(b) reflexive, symmetric but not transitive

(c) symmetric, not reflexive and not transitive

(d) reflexive and transitive but not symmetric.

80. If R is the relation "less than" from $A = \{1, 3, 4, 5\}$ to $B = \{2, 3, 5\}$, then number of elements in $R^{-1} oR$ is :

(a) 7 (b) 6 (c) 8 (d) 9

81. $A = \{1, 2, 3\}$ $R = \{1, 1), (2, 2), (3, 3), (1, 2), (2,1), (2, 3), (3, 2)\} \subseteq A \times A$, the R is :

(a) reflexive, transitive but not symmetric

(b) transitive, reflexive and symmetric

(c) reflexive, symmetric nut not transitive

(d) reflexive, but not symmetric and not transitive

82. The relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow x - y$ divisible by n is :

(a) reflexive, transitive and not symmetric

(b) not reflexive, transitive and not symmetric

(c) reflexive, not symmetric and not transitive

(d) an equivalence relation

83. The domain of the relation (x, y) : x, y $\in$ N, and x + y = 10} is :

(a) {1, 2, 3, 4, 5, 6, 7, 8, 9} (b) {9, 8, 7, 6, 5, 4, 3, 2, 1}

(c) {7, 6, 5, 4, 3, 2} (d) none of these

84. The range of the relation {(x, y) : y = $|x-1|$; x $\in$ Z, and $|x| \leq 3$} is :

(a) {4, 3, 2, 1, 0} (b) {0, 1, 2, 3, 4}

(c) {1, 2, 3, 4} (d) none of these

85. Let A = {1, 2, 3, ..., 13, 14}, R = {x, y) : 3x - y = 0, x, y $\in$ A}, then co-domain of R is

(a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

(b) {3, 6, 9, 12}

(c) {12, 9, 6, 3}

(d) none of these

86. The domain of the relation R, where R = {(x, x^3) : x is a prime number less than 10} is :

(a) {2, 4, 6, 8} (b) {2, 3, 5, 7}

(c) {2, 3, 5, 7, 9} (c) none of these

87. The domain of the relation R defined by R = {(x, x + 5) : x $\in$ {0, 1, 2, 3, 4, 5} } is :

(a) {0, 1, 2, 3, 4, 5} (b) {5, 4, 3, 2, 1, 0}

(c) {5, 6, 7, 8, 9, 10} (d) none of these

88. The range of the relation R, where R = {(x, x + 5) : x $\in$ {0, 1, 2, 3, 4, 5}} is :

(a) {0, 1, 2, 3, 4, 5} (b) {5, 6, 7, 8, 9, 1}

(c) {1, 9, 8, 7, 6, 5} (d) none of these

89. The domain of the relation R, where R = {(-3, 1), (-1, 1), (1, 0), (3, 0)} is :

(a) {-3, -1, 1, 3} (b) { -1, -3, 1, 3}

(c) {0, 1} (d) none of these

90. The range of the relation R, where

R = {(x, y) : x is a multiple of 3 and y is a multiple of 5} is :

(a) {3n, : n $\in$ Z} (b) {5n, : n $\in$ Z}

(c) {15n, n $\in$ Z} (d) none of these

91. The range of the relation R, where R = {(x, x3) : x is a prime number less than 15} is :

(a) {2, 3, 5, 7, 11, 13} (b) {4, 9, 25, 49, 121, 169}

(c) {7, 9, 25, 49, 121, 169, 225} (d) none of these

92. The co-domain of the relation R, where R = {x, y) : y = x + 1}, and R : A $\rightarrow$ A, where A = {1, 2, 3, 4, 5, 6} is :

(a) {1, 2, 3, 4, 5, 6} (b) {1, 2, 5, 9, 10}

(c) {1, 2, 3, 4, 5} (d) none of these

93. Let A = {1, 2} and B = {3, 4}. The number of relation from A into B is :

(a) 16 (b) 12

(c) 4 (d) 8

94. The range of the relation R, where R = {(4x + 3, 1 - x) : $x \leq 4$, $x \in N$} is :

(a) {0, -1, -2, -3} (b) {2, 5, 10, 17, 26}

(c) {7, 11, 15, 19} (c) none of these

95. The range of the relation R, where R = {1 + x, 1 + x^2) : $x \in 5$, $x \in N$} is :

(a) {2, 3, 4, 5, 6} (b) {2, 5, 10, 17, 26}

(c) {2, 3, 4, 6, 10} (d) none of these

96. The domain of the relation R, where R = {(1 + x, 1 + x^2) : $x \leq 5$, $x \in N$} is :

(a) {2, 3, 4, 5, 6} (b) {1, 2, 3, 4, 5}

(c) {3, 4, 5, 6, 7} (d) none of these.

ANSWER KEYS

Q.	A.	Q.	A.	Q.	A.	Q.	A.	Q.	A.
1		21		41		61		81	
2		22		42		62		82	
3		23		43		63		83	
4		24		44		64		84	
5		25		45		65		85	
6		26		46		66		86	
7		27		47		67		87	
8		28		48		68		88	
9		29		49		69		89	
10		30		50		70		90	
11		31		51		71		91	
12		32		52		72		92	
13		33		53		73		93	
14		34		54		74		94	
15		35		55		75		95	
16		36		56		76		96	
17		37		57		77			
18		38		58		78			
19		39		59		79			
20		40		60		80			

HOME WORK - 2

1. Which of the following pairs of events are mutually exclusive?

(a) A : The student reads in a school. B : He studies Philosophy.

(b) A : Raju was born in India. B : He is a fine Engineer.

(c) A : Ruma is 16 years old. B : She is a good singer.

(d) A : Peter is under 15 years of age. B : Peter is a voter of Kolkata.

2. If y= f(x) = $\frac{ax+b}{ax-a}$ then f(y) is ______ .

(a) $-x$ (b) $2x$ (c) x (d) x^2

3. If g(x) = x–1/x, g(–½) is

(a) 1 (b) 2 (c) 3/2 (d) 3

4. If the set P has 3 elements, Q four and R two then the set P×Q×R contains

(a) 9 elements (b) 20 elements

(c) 24 elements (d) None of these

5. In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is

(a) 6 (b) 7 (c) 1 (d) None of these.

6. If A has 32 elements, B has 42 elements and $A \cup B$ has 62 elements, the number of elements in $A \cap B$ is

(a) 12 (b) 74 (c) 10 (d) None of these

7. $A \cup A'$ is equal to

(a) A. (b) Sample Space. (c) ϕ. (d) None of these.

8. If A has 70 elements, B has 32 elements and $A \cup B$ has 22 elements then A B is

(a) 60 (b) 124 (c) 80 (d) None of these.

9. If A = {1,2,3,5,7} and B= {1,3,6,10,15} then cardinal number of A-B is

(a) 3 (b) –4 (c) 6 (d) None of these

10. If P is a set of natural number then $P \cap P'$ is

(a) $\varnothing$ (b) Sample Space. (c) 0 (d) $(P \cup P')'$

11. If A = {1, 2, 3, 4}, B = {5, 6, 7} then cardinal number of A X B is

(a) 4 (b) 7 (c) 12 (d) None of these

12. The set of squares of positive integers is

(a) A finite set (b) Null set (c) An infinite set (d) None of these

13. If B is any set then $B \cap B$ is

(a) Null Set (b) B (c) Whole set (d) None of these

14. If B is any set then $B \cup B$ is

(a) B (b) Null set (c) Whole set (d) None of these

15. "Is equal to" is a

(a) Symmetric relation (b) Reflexive relation

(c) Transitive relation (d) Equivalence relation

16. If $f(x) = x^2 + 2$, then the given function is

(a) odd function (b) even function

(c) Neither odd nor even function (d) None of these

17. For the function $f(x) = 12^{1+x}$, the domain of real values of x where $0 \leq x \leq 9$ the range is

(a) $12 \leq f(x) \leq 12^{10}$ (b) $0 \leq f(x) \leq 12^{10}$

(c) $0 \leq f(x) \leq 12$ (d) None of these

18. "Is greater than" over the set of real number s is

(a) Transitive relation (b) Symmetric relation

(c) Reflexive relation (d) Equivalence relation

19. If $f(x) = x^2+3x$ then $f(2) - f(4)$ is equal to

(a) –15 (b) –18 (c) 18 (d) 12

20. If $f(x)= \left|\frac{1}{x}\right| - x$, $f\left(\frac{1}{2}\right)$ is

(a) 3/2 (b) 2/3 (c) 1 (d) 0

21. If $f(x+1) = 2x + 7$ then $f(-2)$ is

(a) 1 (b) 2 (c) 3 (d) 4

22. If $f(x) = 2x + 3$ then $f(2x) - 2f(x)+3$ is equal to

(a) 1 (b) 0 (c) –1 (d) None of these

23. If f’ f(x) = $3x^2 + 2$ & f(0) = 0 then find f(2).

(a) 8 (b) 10 (c) 12 (d) None of these

24. If $f(x)=x^2-1$ and g(x) = $\frac{x+1}{2}$ then $\frac{f(3)}{f(3)+g(3)}$ is

(a) 5/4 (b) 4/5 (c) 3/5 (d) 5/3

25. If f(x) = 2x+5 and g(x) = x^2+1 the fog = ?

(a) $2x^2 + 7$ (b) 2x + 1 (c) $x^2 + 5$ (d) None of these

26. If f(x) = $x^3+\frac{1}{x^3}$ then value of f(x) – f (1/x) is equal to

(a) 0 (b) 1 (c) $x^3 + \frac{1}{x^3}$ (d) None of these

27. A function f(x) is an even function if

(a) –f(x) = f(x) (b) f(–x) = f(x) (c) f(–x) = –f(x) (d) None of these

28. If a function in x is defined by f(x) = $\frac{x}{x^2+1}$, X∈ R then f(1/x) = ________

(a) f(x) (b) f(–x) (c) –f(x) (d) 0

29. In a class 30 students, 20 students like maths, 18 like science and 12 like both the subject. Find the number of student who like no subject.

(a) 4 (b) 5 (c) 8 (d) None of these

30. The number of subsets of the set {2, 3, 5} is

(a) 3 (b) 8 (c) 6 (d) None of these.

31. Given A = {2, 3}, B = {4, 5}, C = {5, 6} then A × (BC) is

(a) {(2, 5), (3, 5)} (b) {(5, 2), (5, 3)} (c) {(2, 3), (5, 5)} (d) None of these.

32. A town has a total population of 50,000. Out of it 28,000 read the newspaper X and 23000 read Y while 4000 read both the papers. The number of persons not reading X and Y both is

(a) 2000 (b) 3000 (c) 2500 (d) None of these.

33. The number of subsets of the set {1,2,3,4} is

(a) 13 (b) 12 (c) 16 (d) 15

34. The number of subset of set {2, 4, 6} is

(a) 12 (b) 8 (c) 6 (d) None of these

35. The number of subset of a set containing n element is:

(a) 2n (b) 2^n (c) 2^{-n} (d) None of these

36. The set of cubes of the natural number is

(a) A finite set (b) An infinite set (c) As null set (d) None of these

37. If A = {4,5}, B={2,3}, C={5,6} then AX BÇC is

(a) {(2,5), (3,5)} (b) {(4,2), (4,6)} (c) {(4,3), (4,2) (d) None of these

* * * * *

ANSWER KEYS

1	(d)	11	(c)	21	(a)	31	(a)
2	(c)	12	(c)	22	(b)	32	(b)
3	(d)	13	(b)	23	(c)	33	(c)
4	(c)	14	(a)	24	(b)	34	(b)
5	(b)	15	(d)	25	(a)	35	(b)
6	(a)	16	(b)	26	(a)	36	(b)
7	(b)	17	(a)	27	(b)	37	(d)
8	(c)	18	(a)	28	(a)		
9	(a)	19	(b)	29	(a)		
10	Nil	20	(a)	30	(b)		

* * *