RPIL

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Ratios

- Ratio is a comparison of two similar attributes in same units.

Ratio		
Multiplier	as a bridging element	Comparison

- Types of ratios : (a : b)

1. Duplicate $a^2 : b^2$
2. Sub dupl. $\sqrt{a} : \sqrt{b}$
3. Triplicate $a^3 : b^3$
4. Sub Tripl. $\sqrt[3]{a} : \sqrt[3]{b}$
5. Compound $(a : b, c : d) \dashrightarrow a \times c : b \times d$
6. Continued $\dashrightarrow a : b : c$ $(a : b, b : c)$
7. Inverse $\dashrightarrow b : a$

Proportion

- If two ratios are equal they are said to be in proportion.
- Each pair of ratio should have same units.

Proportion		
Mean Proportion	Third Proportion	Fourth Proportion

$b^2 = \sqrt{ac}$ | $\frac{a}{b} = \frac{b}{c}$

- Product of means= Prod of extremes
- Properties of Proportion {a:b}

1. Invertendo $\frac{b}{a} = \frac{d}{c}$
2. Alternendo $\frac{a}{c} = \frac{b}{d}$
3. Componendo $\frac{a+b}{b} = \frac{c+d}{d}$
4. Dividendo $\frac{a-b}{c} = \frac{c-d}{d}$
5. Componedo & dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Indices

- It is a power game..
- Properties :

1. $\sqrt[b]{a} = a^{\frac{1}{b}}$
2. $a^b a^c = a^{b+c}$
3. $\frac{a^b}{a^c} = a^{b-c}$
4. $(a \times b)^c = a^c \times b^c$
   $(a+b)^c \neq a^c + b^c$
   $(a-b)^c \neq a^c - b^c$
5. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$
6. $a^b = c \dashrightarrow a = c^{\frac{1}{b}}$
7. $a^b = a^c$ then Base same
   $\boxed{b = c}$ power equate
8. Power – same base – equate
   $a^b = c^b \dashrightarrow a = c$
9. $\frac{1}{a^{-b}} = a^b, \frac{1}{a^b} = a^{-b}$
10. $(a)^0 = 1.$

Logarithms

$a^b = c$
$\downarrow$
$\log_a^c = b$

- Always assume base to be 10.
- Log a + log b = log a×b
- log a – log b = $\log\frac{a}{b}$
- log 1 = 0
- m log n = $\log n^m$
- $\log_a^b = \frac{1}{\log_a^b}$
- $a^{\log^a x} = x$
- Shortcut :

  Type no.

  Type $\sqrt{}$ 19 times

  Type -1 × 227695

Champions वाला Chart

Equations

General Form

1 variable $ax + b = 0$
2 Variables $ax + by + c = 0$
3 Variables $ax + by + cz + d = 0$

To get unique solutions
No. of equations = No. of Variables

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ **(Infinite Solution)**

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ **(Unique Solution)**

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ **(No Solution)**

Quadratic Equations

General Form

- $ax^2 + bx + c = 0$
- *if* α & β *are roots then,*

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Sum of roots $(\alpha + \beta) = -\frac{b}{a}$

Product of roots $(\alpha\beta) = \frac{c}{a}$

The roots can be found out using,

$$x = \frac{-b \pm \sqrt{b^2 - 4ca}}{2a}$$

Nature of Roots depends on $D = b^2 - 4ac$

(a) $D<0$ *Roots are imaginary*

(b) $D=0$ *Real and equal*

(c) $D>0$ *and perfect square number real, distinct (unequal) and Rational*

(d) $D>0$ *and NOT a perfect square real distinct and Irratinal*

Cubic Equation

General Form

$ax^3 + bx^2 + cx + d = 0$

Sum of roots $(\alpha + \beta + \gamma) = \frac{-b}{a}$

Product of roots $(\alpha\beta\gamma) = \frac{-d}{a}$

Linear In-equations

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Making the Inequation

- Use logics
- Signs of inequality
  $\geq, \leq, >, <$
- Knock out in objective questions:
  Right values – satisfy
  Wrong values – not satisfy

Solving the inequation

Graphical Method***

S.01 Ignore the sign of inequality
Put x = 0 & calculate y , point (0, y)
Put y = 0 & calculate x , point (x, 0)

S.02 Plot the points from S.01 on the graph,
Draw straight lines.

S.03

If line passes through (0, 0)	If it does not pass through (0, 0),
Put any point in the form (x, 0)	Put (0, 0)
If satisfy –Shade towards	If satisfy –Shade towards
If not –Shade against	If not – Shade against

S.04 Shade the common region ; common area for all the inequations.

- Feasible
  - Bounded
  - Unbounded
- Infeasible

Algebra

- Divide / Multiply with a negative no. ; change sign of inequality.
- Always change both sides
- Brackets

( )	[ ]	{ }
Open	Closed	Curly
(2, 3)	[2, 3]	{2, 3}
↓	2✓	2✓
2×	3✓	3✓
3×	2-3×	2-3×
2-3		

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Time Value Of Money

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Simple Interest

- S.I. = $\frac{pxrxt}{100}$
- A = P + S.I.

1 S. I. is not बेवफा !
S.I. is always calculated on principal.

2 S.I. is constant for every year.

3 If Q. is चुपकी assume it to be of S.I.

4 Nature of r, t should be same

5 Time Scale में + P होगा

Compound Interest

- Interest on Interest.
- C.I. ≥ S. I.
- For the first period
  C.I. = S.I.
- $A = P \times (1+i)^n$
- C.I. = A – P
- Nature of r & t is always same. Always focus on factor.
- Higher the compounding higher the amount.
- Time scale में × होगा
- WDV = H.V. × (1 – i)
- Doubling Period formula
  $T = 0.35 + \frac{69}{r}$
- Tripling Period formula
  $T = 0.35 + \frac{111.111}{r}$

Effective Interest

E.I. = { एक साल का factor – 1 } × 100

- Always assume
  t = 1 year
- Nature of r & t should be same.

Three types of Questions

E.I. ✓	C.I. ✓	Compare
C.I. = ?	E.I. = ?	

Annuity

F.v. P.v.

F.V. = I× $\left\{\frac{(1+i)^n - 1}{i}\right\}$ I÷ factor = n times GT

- Due : Starting from today ⇢
  ans. × (1+i)
- Bulk amount

आज आया गया P.v. | कल आया गया F.v.

- 3 conditions for annuity.
  - Fixed Time Interval
  - Fixed Installments
  - Regular Payments

Permutation & Combination

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Events :
And ⇢ ×
Or ⇢ +

P & C

$n \geq r$

$0! = 1$

n, r ⇢ positive integers

Permutation

It is all about arrangement, order matters.

$^{n}p_r = \frac{n!}{n-r!}$

$^{n}p_r = n_{c_r} \times r!$

n = no. of objects

r = objects taken at a time

Restriction

जहाँ restriction हो पहले उसे भरो then focus on the rest.

Always Together

- Always make a box
- Count box as one element & permutate. Also arrange inside the box.
- Never together
  - For 2 elements: Total – always together
  - For > 2 elements: use logic

Or

$(n-1)(n-2)!$

Repetition

- If objects are repeated then, जितनी बार एक object repeat उतने factorial से divide करो

Circular

- $(n-1)!$ ways.
- Necklace etc.

$\frac{1}{2}(n-1)!$

Division

- $\frac{\text{Total objects!}}{\text{group ka!}}$
- अगर group same तो उतने ! से divide.
- For distn × person !

Combination

It is about selection order does not matter

$^{n}c_r = \frac{n!}{(n-r)! \times r!} = \frac{n_{p_r}}{r!}$

Properties

1. $n_{c_r} = n_{c_{n-r}}$
2. PasCal's law
   $n_{c_r} + n_{c_{r-1}} = n+1_{c_r}$
3. $n_{c_1} = 1$
4. $n_{c_0} = 1$
5. $n_{c_{r_1}} = n_{c_{r_2}}$
   $r_1 + r_2 = n$

Geometry

- Diagonals
  $= n_{c_r} - n$
- Δ can not be made from collinear points.

All or None

2^n

Taking all or any no. of object i.e. r at a time

Champions वाला Chart

Sequence & Series

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AP / GP

Arithmetic Progression

- It is about adding the constant no. to the first term & again.
- Every no. is A.M. of its previous & succeeding no.
- First Term = a common difference = d
- Variety -1 series : given value of term = ?
  $T_n = a + (n-1)d$
- Variety -2 series : given value = given n=?
- Variety – 3 series : given sum =?
- $S_n = \frac{n}{2}\{a+r\}$ or $\frac{n}{2}\{2a+(n-r)d\}$
- Variety -4 series : given sum = given no. ?
- Variety – 5 if two non consecutive term are given;
  $d = \frac{T_m - T_n}{m-n}$
- Variety – 6 Insertion of A.M. 's between two no.'s results in A.P.
- Variety – 7 Sum's machine =given Term = ?

Geometric Progression

- It is about multiplying the constant no. again.
- Every no. is GM of its previous & succeeding terms.
- First term -a common ratio = r
- Variety no. 1 n = given Tn = ?
  $T_n = ar^{n-1}$
- Variety – 2 n = ? Tn = ?
- Variety – 3 Series = given sum = ?
  $= s_n = \frac{ax(r^n-1)}{r-1}$ (r >1) $= s_n \frac{a(1-r^n)}{1-r}$
  If (r < 1)
- Variety – 4 sum = given n=?
- Variety – 5 calculation of r in two non consecutive;
  $r = \left(\frac{T_m}{T_n}\right)^{\frac{1}{m-n}}$
- Variety – 6 Insertion of GM's
- Variety – 7 Sum of infinity series.
  $S_\infty = \frac{a}{1-r}$

Sets, Function & Relations

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Sets

Basics

"It is a well defined group of distinct objects."

Expression
- Roaster (Listing)
- Set Builder

- Cardinal no. = no. of elements in a set

No. of subsets = 2^n

Proper subsets = 2^n-1

Types

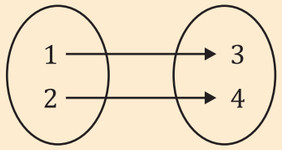
1. Universal Set : contains all the objects.
2. Subset : every element of A is in B. $A \subset B$
3. Superset : every element of A is in B. $B \supset A$
4. Null Set : $\varnothing$, 0 element.
5. Equal Set : Every element of A is in B & vice versa.
6. Equivalent Set : $n(A) = n(B)$
7. Power Set : Set of all subsets.

Relations

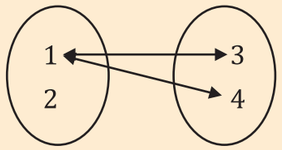
Basics

Every subset of a Cartesian product of $A \times B$ is called relation.

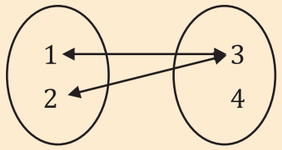
One to One



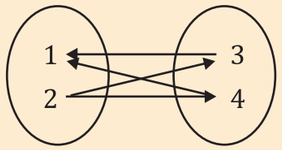
One to Many



Many to One



Many to Many



Types

- Reflexive
  $A=\{1,2,3\}$
  $R=\{(1,1)(2,2)(3,3)\}$
  all $a, a \in R$
- Symmetric
  $A=\{1,2,3\}$
  $R=\{(1,2)(2,1)$
  $(2,3)(3,2)\}$
  $a,b \in R$ then $b,a \in R$
- Transitive
  $A=\{1,2,3\}$
  $R=\{(1,2)(2,3)(1,3)\}$
  $a,b \in R$ & $b,c \in R$ then,
  $a,c \in R$
- S ✓ R ✓ T ✓
  = Equivalence

Function

Basics

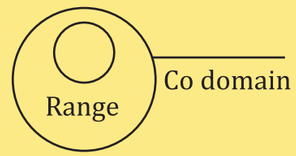
"Every R is not F but every F is a R."

'No two ordered pairs should have same first element.'

Domain = pre image

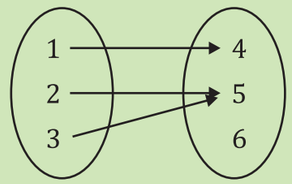
↓

Range = Image

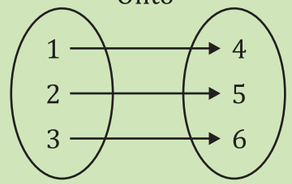


Types

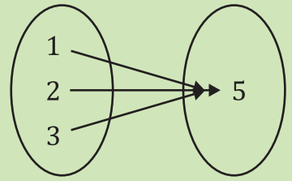
Into



Onto

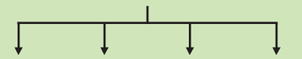


Constant



Inverse
$x \to y$
$y \to x$

Composite



K.O.

Champions वाला Chart

Differential Calculus

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Six Basic Rules of Differentiation

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \log_e a$

$\frac{d}{dx}(constant) = 0$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\frac{d}{dx}(Logx) = \frac{1}{x}$

Note: $\frac{d}{dx}\{cf(x)\} = cf'(x)$ *c being constant.*

For Two Functions

$h(x)=f(x) \pm g(x)$
(Sum/Difference of function)

$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

$h(x)=f(x).g(x)$
(Product of functions)

$\frac{d}{dx}\{h(x)\} = f(x)\frac{d}{dx}\{g(x)\} + g(x)\frac{d}{dx}\{f(x)\}$

$h(x) = \frac{f(x)}{g(x)}$
(Quotient of function)

$\frac{d}{dx}\{h(x)\} = \frac{g(x)\frac{d}{dx}\{f(x)\} - f(x)\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$

Application of Differentiation

Average cost (AC or $\bar{C}$) = $\frac{Total\ Cost}{Out\ Put} = \frac{C(X)}{X}$

Average variable cost (AVC) = $\frac{Variable\ Cost}{Out\ Put} = \frac{V(x)}{x}$

Average Fixed Cost (AFC) = $\frac{Fixed\ Cost}{Out\ Put} = \frac{F(x)}{x}$

Marginal Cost: *If C(x) the total cost producing x units then the increase in cost in producing one more unit is called marginal cost at an output level of x units and is given as* $\frac{dC}{dx}$

Revenue Function: *Revenue, R(x), gives the total money obtained (Total turnover) by selling x units of a product. If x units are sold at 'P per unit, then R(x) = P.X*

Marginal Revenue: It is the rate of change in revenue per unit change in output. If R is the revenue and x is the output, then $MR = \frac{dR}{dx}$

Profit function: *Profit P(x), the difference of between total revenue R(x) and total Cost C (x).*
P(X) = R(x) - C(x)

Marginal Profit: *It is rate of change in profit per unit change in dP output i.e.* $\frac{dP}{dx}$

Slope of Curve: *If y is any function then* $\frac{dy}{dx}$ *represent the slope of tangent to the curve.*

Intergal Calculus

6 Basic Rules of Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int dx = x + c, since \int 1dx = \int x^0 dx = \frac{x^1}{1} = x$$

$$\int e^x dx = e^x + c$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + c$$

$$\int \frac{dx}{x} = logx + c$$

$$\int a^x dx = \frac{a^x}{\log_e a} + c$$

Integration By parts

$$\int uvdx = u\int v\, dx - \int \left[\frac{d(u)}{dx}\int vdx\right]dx$$

$$\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$$

$$\int \frac{f'(x)}{f(x)}dx = logf(x) + c$$

Definite Integration

$$\int_a^b f(x)dx = F(b) - F(a)$$

b' is called the upper limit and 'a' the lower limit of integration.

Important Properties of Definite Intergal

$$\int_a^b f(x)dx = \int_b^a f(x)dx$$

$$\int_a^b f(x)dx = \int_b^c f(x)dx + \int_c^b f(x)\,dx, a < c < b$$

$$\int_o^b f(x)dx = \int_o^a f(a-x)dx$$

$$when f(x) = f(a+x) = \int_o^{na} f(x)dx = n\int_0^a f(x)dx$$

$$\int_{-a}^a f(x)dx = 2\int_o^a f(x)dx \quad if\ f(-x) = f(x)$$

$$\int_{-a}^a f(x)dx = 0 \quad if\ f(-x) = -f(x)$$

Number Series & Coding-Decoding

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Learn by Heart

Squares

$1^2 = 1, 2^2 = 4, 3^2 = 9$-----*upto* $25^2 = 625$

Cubes

$1^3 = 1, 2^3 = 8, 3^3 = 27$----- *upto* $15^3 = 3375$

What is "Method of Difference"?

Questions: 2, 5, 12, 27, 54, 97,?

Mod →2 5 12 27 54 97 *160*

3 7 15 27 43 *63*

4 8 12 16 20

Ans. → 160

Alphabet Position Chart

1	2	3	4	5	6	7	8	9	10	11	12	13
A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N
26	25	24	23	22	21	20	19	18	17	16	15	14

Thumb Rule of Position

Alphabet Forward Position +
Alphabet Backward Position = 27

Example: Position of "I"

Forward Position = 9.
Backward Position = 27 - 9 = 18

What is N Backward Position?
27 - 14 = 13

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Direction Test

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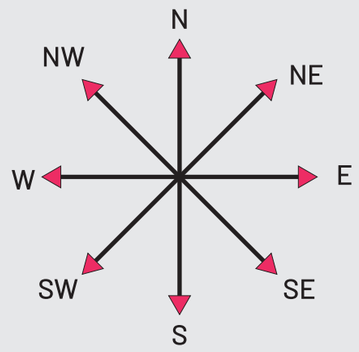
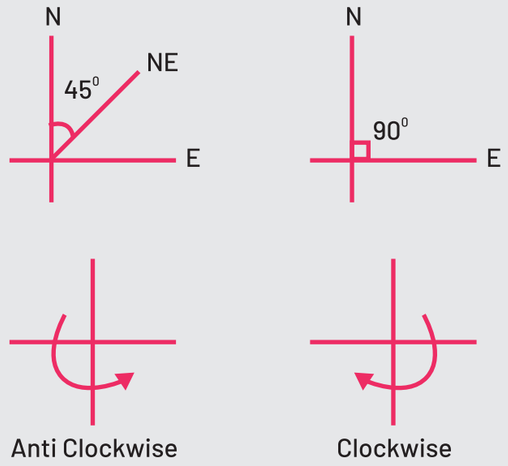
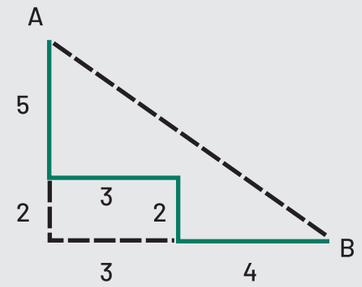
Basic 8 Directions



Angles & Direction



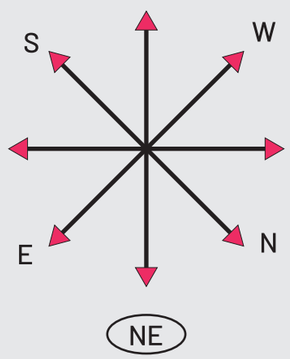
How to Calculate Distance?



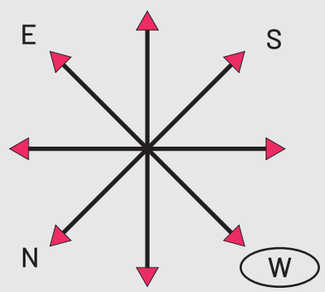
$$AB = \sqrt{(5+2)^2 + (3+4)^2}$$

$$B = \sqrt{49+49} = 7\sqrt{2}$$

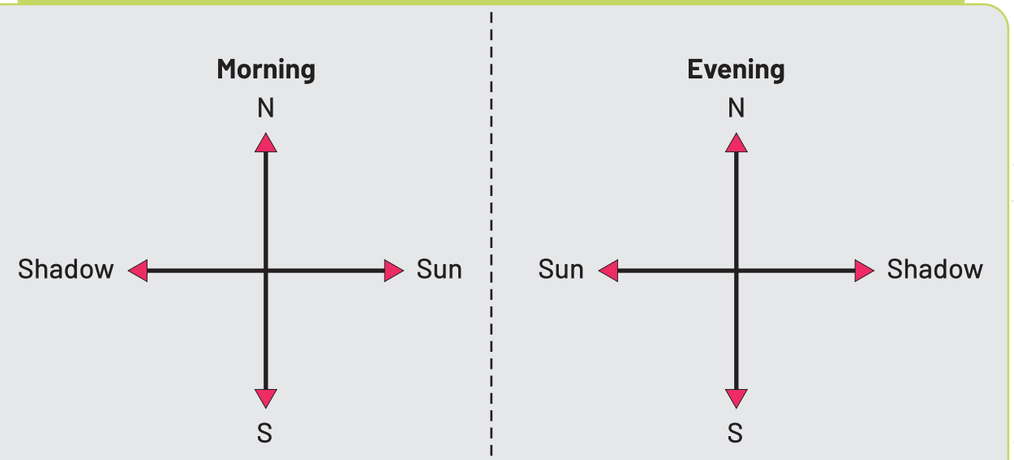
If South Becomes NE



If SE Becomes West



Shadows base questions



Champions वाला Chart

Seating Arrangement

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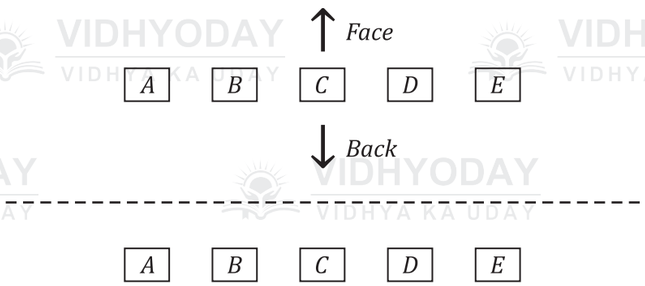
In Linear Arrangement People Sit Facing North



In the above arrangement

→ B & A are to the left of C.

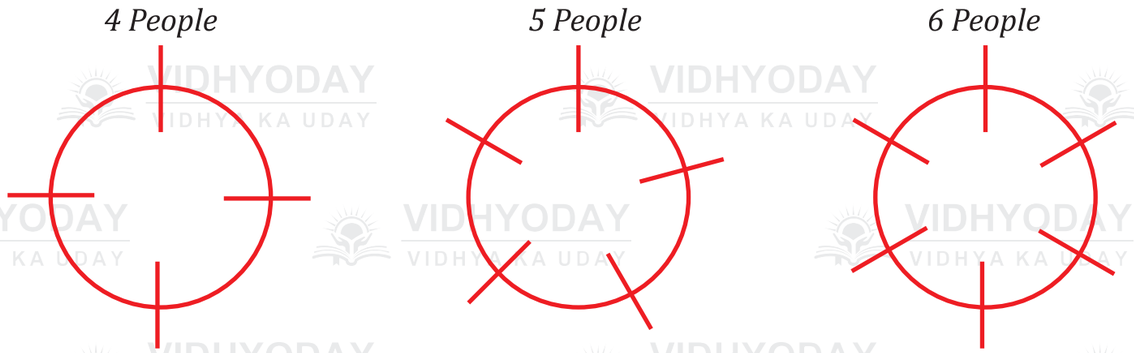
→ D & E are to the right of C.

→ B is immediate left of C.

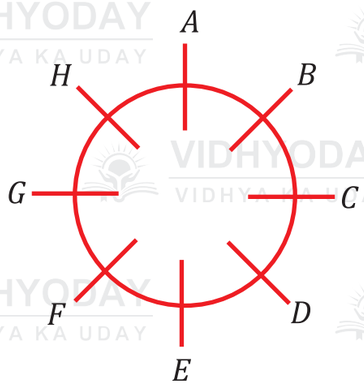
→ There are two persons between A & D.

→ D is third to the right of A.

Circular Arrangement



Note : Spacing between any two person should be same.



A is Diametrically opposite to E, H to D ..... and so on

G is second to left of E.

G is third to right of B.

Blood Relations

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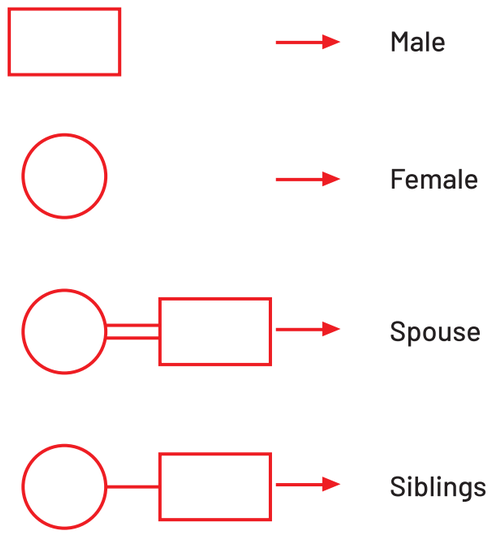
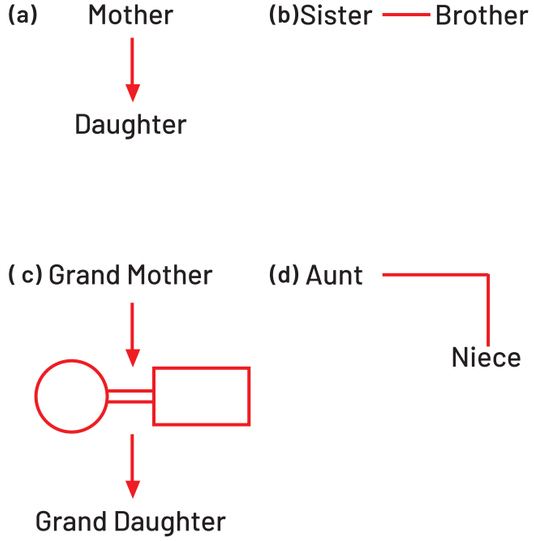
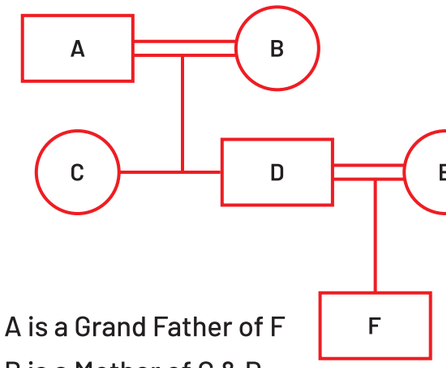
Gender Representation



Different Relationship Levels



Sample Family Tree



A is a Grand Father of F
B is a Mother of C & D
C is a Sister-in-Law of E
F is a Nephew of C
A is a Father-in-Law of E
C & D are siblings
D & E are Spouse

Statistical Distribution of Data

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Meaning, History's Limitations

Meaning

- **Singular** {Data Science}
- **Plural** {Collection of data}

- History
  1. French – Statistique
  2. Italian – Statista
  3. German – Statistic
  4. Latin – Status

- Limitations
  1. Aggregation
  2. Certain assumptions
  3. Random sampling
  4. Quantitative Data

Collection of Data

Primary
- ✓ First time
- ✓ Accurate
- ✓ Tailor made
- ✓ Reliable

- **Interview**
  - Direct (one to one)
  - Telephone (surveys)
  - Indirect (accidents)
- **Mail Questionnaire (Speak Asia)**
- **Enumerators** जनगणा
- **Observation** ठेकेदार

Secondary
- ✓ Second time
- ✓ Less accurate
- ✓ Not customised
- ✓ Less reliable

- **International Reports (WHO)**
- **Government Reports (NSO)**
- **Quasi-Government (NCERT)**

Representation of Data

Textual
- ✓ Unpopular
- ✓ Difficult to interpret
- ✓ Time consuming

Tabular
- ✓ Preferred Method
- ✓ Represented in table format
- ✓ Parts of table:
  - Table no.
  - Stub
  - Heading
  - Sub-heading
  - Caption
  - Body of the table
  - Footnote

Graphical
- Line chart
  - ✓ 2-D
    - Continous variable
    - Discrete variable
  - Bar chart
  - ✓ 2-D
  - ✓ 1-D
  - Pie chart
  - ✓ Angle

$$= \frac{\text{Amount of segment}}{\text{Total amount}} \times 360^\circ$$

Analysis of Data

Misc. terms & Formulae

No. of class Intervals

✓ No. of class Intervals

$$= \frac{\text{Max. - Min.}}{\text{Class Length}}$$

✓ % of relative frequency

$$= \frac{\text{Class frequency}}{\text{Total frequency}}$$

✓ Frequency Density

$$= \frac{\text{Class frequency}}{\text{C.L.}}$$

Champions वाला Chart

Central Tendency

लवारिस Property

- Δ of origin ✓
- Δ of scale ✓
- Δ of sign ✓

- AM ≥ GM ≥ HM
- GM = $\sqrt{AM \times HM}$

Relationship :
Mode : 3md - 2$\bar{X}$
$m_0 - \bar{X} = 3(md - \bar{X})$

Quantitative Average

Positional Average

AM

- Average formula = $\frac{\text{sum}}{\text{no}}$

  $\frac{\sum x}{n}, \frac{\sum fx}{\sum f}, \frac{\sum fm}{\sum f},$

- **Properties**

1. A.M. is the most popular measure of CT.
2. Sum of deviations from A.M. is always 0.
   $\sum X - \bar{X} = 0$
3. Combined A.M. can be calculated.
   $\bar{X}_{12} = \frac{\bar{X}, n, + \bar{X}_2 n_2}{n_1 + n_2}$
4. Mean can be calculated using assumed mean formula
   $\bar{X} = A + \frac{\sum d}{n}$
5. A.M. can not be represented graphically.
6. $\sum (X - \bar{X})^2$ = minimum

GM

- GM is best measure of CT for ratios & percentages.
- Formula Individual
  $(axbac \ldots)\frac{1}{n}$

Discrete

$\left(X\frac{f^1}{-}xX\frac{f^2}{2} \ldots X\frac{fn}{n}\right)\frac{1}{\sum n}$

Continuos

$(M_1{}^{f1} x M_2{}^{f2} \ldots)^{\frac{1}{\sum f}}$

HM

- Used for variables having reciprocal relationship
- Formula

Individual

$HM = \frac{n}{\frac{1}{X1} + \frac{1}{X2} \ldots\ldots \frac{1}{Xn}}$

Discrete

$HM = \frac{\sum f}{\frac{f1}{X1} + \frac{f2}{X2} \ldots\ldots \frac{fn}{Xn}}$

$HM = \frac{\sum f}{\frac{f1}{m1} + \frac{f2}{m2} \ldots\ldots \frac{fn}{m_n}}$

- आवन जावन Q is imp
- HM is the reciprocal of AM
- Combined HM

$= \frac{n1 + n2}{\frac{n1}{HM1} + \frac{n2}{HM2}}$

MEDIAN

- Individual
  इधर से काटो, उधर से काटो, बीच में जो बचा वो median
- Discrete

  S.01 $\frac{N}{2}$

  S.01 $\frac{N}{2}$ को Locate करो in C.F.

  S.03 आगे वाला is median
- Continuos

  S.01 follow discrete

  S.02 $M_d = l_1 + \left\{\frac{\frac{N}{2} - C}{F}\right\} xH$
- Md is not affected by extremities of the observations
- Sum of absolute deviation from median is minimum.

  $\sum |x - xmd| =$ minimum
- Calculated through Ogive.
- Partition Values

  $\text{Value} = \left[\text{order}X\left\{\frac{n+1}{4/10/100}\right\}\right]_{th}$ term

  Quartiles *Deciles* *Percentiles*
- Best for open' end classification

MODE

- Individual
  Most repeated no.
- Discrete
  No. with highest frequency
- Continuos
  Find out model class & use.
  Formula :
  $MO = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
- It is not uniquely defined.
- Calculated using Histogram.
- Mode
  Unimodel Bimodel Multimodel

Measures of Dispersion

[Measures of Dispersion] "Second order of averages"

लवारिस Property

- Δ of origin ×
- Δ of scale ✓
- Δ of sign ×

Relationship Between MD, QD & S.D.

4 S.D. = 5 MD = 6QD {4,5,6 करके समुन्दर में कूदो}

Absolute MOD

Mean Deviation

$$M.D. = \frac{\sum|X - \bar{X}|}{n}$$

Or

$$\frac{\sum|X - Xm|}{n}$$

Or

$$\frac{\sum|X - Xmo|}{n}$$

- If frequencies of all observations are same, count them only once,
- M.D. from Median is minimum.

Range

Max Value – Min Value = Range

* easiest & quickest to calculate dispersion,

Quartile Deviation

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

* It is best suited for open end classification as it is based on middle 50% values
* Also called as semi inter quartile range.

Standard Deviation

$$S.D. = \sqrt{\frac{\sum(X - \bar{X})^2}{n}}$$

$$S.D. = \sqrt{\frac{\sum X^2}{n} - \left(\bar{X}\right)^2}$$

S.D. = (sigma)

$S.D^2$ = variance

$\sqrt{\text{variance}}$ = S.D.

- For first n natural no.'s

$$S.D. = \sqrt{\frac{n^2 - 1}{12}}$$

If frequencies of all observations are same, count them once only.

S.D. between 2 no.'s

$$= \frac{|a - b|}{2}$$

- Combined S.D.

$$= \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

$d_1 = \bar{X}_{12} - \bar{X}_1$

$d_2 = \bar{X}_{12} - \bar{X}_2$

Relative MOD

M.D.

Coeff. Of M.D.

$$= \frac{M.D.}{\bar{x}\,|x_{md}|x_{mo}} \times 100$$

Range

Coeff of Range

$$= \frac{Max - Min}{Max + Min} \times 100$$

Q.D.

Coeff of Q.D.

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Coeff of Q.D.

$$= \frac{Q.D.}{M_d} \times 100$$

Variation

Coeff of variation

$$= \frac{S.D.}{\bar{x}} \times 100$$

* It is used to measure consistency

Decision rule : Lower the better.

Champions वाला Chart

Probability

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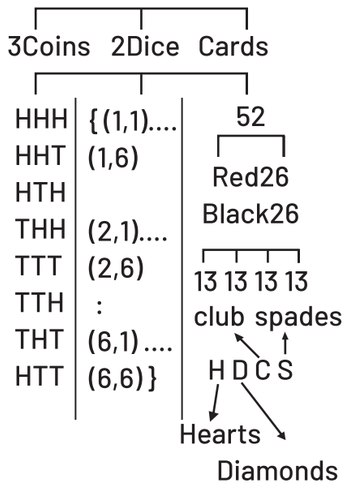
Basics

$P(A)=\frac{Fav.}{Total}$

Odds in favour = m : n

Odds in against = n : m

$P(A)=\frac{m}{m+n}$ $P(\bar{A})=\frac{n}{m+n}$



Terminology

- Exp. = कीड़े करना
- Random Exp = outcome is not know
- Exhaustive = पूरी दुनिया Union = 1
- Equally likely = सब बराबर P(A) = P(B)
- Mutually Exclusive
  मछली = 0
  P(A ∩ B) = 0
- Sure event
  P(A) = 1
- Impossible Event
  P(A) = 0
- Dependent = formula
  P(A∩B) = P(A) × P(B/A)
- Independent
  मछली = P(A) × P(B)

P & C

- Mostly combination (balls, cards, committee, geometry figures etc.)
- $\frac{Fav.}{Total}$
- Fav = with restrictions
- Total = w/o restriction

Exp. Value

- Nothing but weighted avg.
- E(x) = ∑ px
- Sum of probability = 1
  ∑ p = 1
- अगर bracket में x के अलावा expression है तो change x.
- Properties :
  E(x±y) = E(x) ± E(y)
  E(Kx) = KE(x)
  E(x÷y) = E(x) / E(y)
  E(x×y) = E(x) × E(y)
- Variance
  $E[X\text{-}e(X)]^2$

Venn Diagram

1) A ∪ B = A+B-A∩B

2) A∩B
   - Dependent
   - Independent A×B

3) Only A
   A - A∩B

4) Only B
   B - A∩B

5) $\overline{A\cap B}$
   $=\bar{A}\cup\bar{B}=1-A\cup B$
   $\bar{A}\cup\bar{B}=1-P(A\cup B)$

6) $\overline{A\cup B}$
   $\bar{A}\cap\bar{B}=1-P(A\cup B)$

7) A^c = 1-A

8) B^c = 1-B

Conditional Probability

Probability of A when B has already occurred
P(A/B)
$=\frac{P(A\cap B)}{P(B)}$

For e.g.
$P(\bar{A}/\bar{B})$
$=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$
$=\frac{1-P(A\cap B)}{1-P(B)}$

Champions वाला Chart

Theoretical Distribution

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Binomial

Given by James Bournouli
(Discrete Distribution)

Formula : $P(x = r) = {}^nC_r x p^r x q^{n-r}$

Applicability : Future में IPM करेंग

↓ Finite Trials
↓↓↓ Independent Events
↓ Probability
↓ Mutually Exclusive Events

Properties : सप्तऋषि

1. Mean = np
2. Variance = npq
3. Mean = Variance
4. Biparametric B(n, P)
5. Max. Variance = $\frac{n}{4}$
6. Mode = (n+1)xp

Decimal	Integer
Ignore Decimal; Unimodal Case	Bimodal: Ans, Ans-1

7. Combined binomial Distribution
   $P(x+y=r) = {}^{n1+n2}C_r x P^r x Q^{n1+n2-r}$

Poission

Given by Simon Poission
(Discrete Distribution)

Formula $P(x = r) = \frac{e^{-m} x\, m^r}{r!}$

Applicability : $n \to \infty$
$p \to 0$
$np \to$ infinite
{Flight Example}

Properties : Six – Sense

1. mean = np
2. variance = mean
3. Uniparametric
   (only m is parameter)
4. mode = n×p

Decimal	Integer
Ignore Decimal; Unimodal Case	Bimodal: Ans, Ans-1

5. B.D. can be approximated with P.D.
6. Combined Poission Distribution
   $P(x+y=r) = \frac{e^{-(m1+m2)}(m1+m2)^r}{r!}$

Normal

Given by Carl Gauss (Continuous Distribution)
Also called Gaussion distribution

Formula : $P(x) = \frac{1}{\sigma\sqrt{2\pi}} x e^{-\frac{1}{2}\left\{\frac{x-\mu}{\sigma}\right\}^2}$

$z = \left\{\frac{x-\mu}{\sigma}\right\}^2$

Properties

1. It is bi-parametric X-N (μ, σ^2)
2. It is bell shaped curve.
3. It is symmetrical around mean.
4. Standard Results:
   $\mu \pm \sigma = 68.30\%$, $\mu \pm 2\sigma = 95.50\%$, $\mu \pm 3\sigma = 97.70\%$
   Fix Rate free, Shine Life Life, Shine Shine Raven
5. Points of inflexion
   $\mu+\sigma$
   $\mu-\sigma$
6. 4, 5, 6 करके समुदंर में' 'कूदो,
   4.S.D. = 5 M.D. = 6 Q.D.
7. Standard Normal Variate
   $\mu=0$ $\sigma=1$
8. Quartiles
   $Q_1 = \mu - 0.675\sigma$
   $Q_2 = \mu$
   $Q_3 = \mu + 0.675\sigma$

Normal Curve

Champions वाला Chart

Correlation

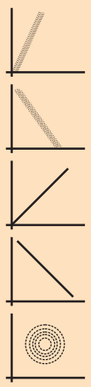
लवारिस Property

- Δ of origin ×
- Δ of scale ×
- Δ of sign ✓

Correlation (Measures the degree of linear relationship between two variables)

Scatter Diagram

- It only tell us the nature of correlation & not degree of correlation
- Five Diagrams



- Correlation of Straight line is always +1 or -1. It depends upon the direction between x & y.
- Equation of a Straight Line is ax + by = c
- Proportion --> decides 1 or not
  direction --> decides + or -

Rank Correlation

$$r_0 = 1 - \frac{6\sum d^2}{n^3 - n}$$

n= no. of observations.
d = difference of ranks

- Sum of difference of ranks is always 0.
- If ranks are exactly opposite then r = -1.
- Even if ranks are reversed, it remains same.

Concurrent Deviation

$$r_c = \pm\sqrt{\pm\frac{2c - m}{m}}$$

- If there is negative no inside the root, r is going to be negative.

m = no. of observations compared = n – 1.

c = no. of concurrent deviation (+' s)

Karl Pearson

- $r = \frac{Cov(x,y)}{\sigma_x \sigma_y}$
- $r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$
- $Cov(x,y)$

  $= \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{n}$
- Cov (X,Y) desides the nature of correlation -
- $-1 \le r \le +1$
- n is a pure no. (unit free).
- Coefficient of determination = $1 - r^2$ (Unexplained variance)
- $P.E. = \frac{0.675\sqrt{1 - r^2}}{n}$
- Product Moment correlation (nick name)
- Useful for variables having only linear relationship.

Champions वाला Chart

Regression

लवारिस Property

Δ of origin ×
Δ of scale ✓
Δ of sign ✓

Regression

Regression Equations

- Unlike r, it tells us the exact increase in price of y if x is increased or vice versa.
- There are two equations :
  1. Y on x →to calculate y.
  2. X on y → to calculate x.

- $Y \text{ on } x \rightarrow Y - \bar{Y} = byx(X - \bar{X})$
  $X \text{ on } y \rightarrow X - \bar{X} = bxy(Y - \bar{Y})$

- $byx = r\frac{\sigma y}{\sigma x}$ (जो पीछे है वो नीचे है।)

  $bxy = r\frac{\sigma x}{\sigma y}$

Popular Questions :

1 Lines = given slope = ?
2 Lines = given r = ?
3 Slopes = given r = ?
4 Lines = given mean = ?
5 पहचान कौन ?

Properties

1) Regression lines intersect each other at $(\bar{X} - \bar{Y})$i.e. mean. (K.0.)
2) Correlation coefficients is GM of regression coefficient.
   $r = \sqrt{b_{yx} \times b_{xy}}$
3) The product of regression coefficients should be ≤ 1.
   $b_{yx} \times b_{xy} \leq 1$
4) If one coefficients is greater than unity the other should be less than unity.
5) byx, bxy & r are all of same sign.
6) Regression lines are made using least squares deviation method.
7) लावरिस Property : Δ of origin × Δ of scale ✓ Δ of sign ✓
8) r = 0 regression lines are perpendicular, if r ± 1, lines will coincide.
9) पहचान कौन ?
   S.01 Calculate b_{yx} & $b_{xy} \leq 1$
   By assuming one line as
   Y on x & another as x on y.
   S.02 Check if $\sqrt{b_{yx} \times b_{xy} \leq 1}$
   S.03 Yes → assumption is true.
   No → opposite is true.

Champions वाला Chart

Index No.

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CY value, when B.Y value is assumed to be 100. E.g. Sensex (1978-79) Index no. is a pure no.

Simple Method

Aggregative

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$$

Relative

$$I = \frac{\sum I_R}{n}$$

$$I_R = \frac{P_1}{P_0} \times 100$$

Weighted Method

$$\left[\frac{\sum P_1 W}{\sum P_0 W}\right]$$

$$\text{Laspeyer} = \frac{\sum P_1 q_0}{\sum p_0 q_0}$$

(Base year Q.)

$$\text{Passche} = \frac{\sum P_1 q_1}{\sum p_0 q_1}$$

(Current year Q.)
*** (Ideal)
↓

$$\text{Fishers} = \sqrt{\text{La} \times \text{Pa}}$$

$$\text{Dorbish \& Bowley} = \frac{\text{La} \times \text{Pa}}{2}$$

Marshall Edgeworth

$$P01 = \frac{\sum P_1 \left(\frac{q_0 + q_1}{2}\right)}{\sum P_0 \left(\frac{q_0 + q_1}{2}\right)}$$

$$\text{Walsh} \quad \left(\sqrt{P_{01}}\right) = \frac{\sum P_1 \sqrt{q_0 q_1}}{\sum P_0 \sqrt{q_0 q_1}}$$

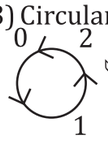
$$\text{Weight} = \sqrt{q_0 q_1}$$

$$(\text{kelly}) = \frac{\sum P_1 q}{\sum P_0 q}$$

Special Points

Test

1) Unit test unit free satisfied by all.
2) Time - Reversal Test $P_{01} \times P_{10} = 1$
   Kelly, MEW, Fishers Simple aggregative Satisfy TRT
3) Circular Test



$P_{01} \times P_{12} \times P_{20} = 1$

Fisher ×
Kelley ✓
aggregative ✓

4) Factor Reversal
$P_{01} \times q_{01} = v_{01}$

Fisher's ✓

$$v_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_0} \times 100$$

Inflation Deflation

(Index No – 100) = Inflation %

Deflated value mean B.Y. dh value

$$= \frac{\text{C.Y.Value}}{\text{C.Y.Index}} \times 100$$

Base Shifting & CBI

$$\text{CBI} = \frac{\text{LR} \times \text{PYCBI}}{100}$$

$$\text{LR} = \frac{\text{CY Price}}{\text{Prev. Price}} \times 100$$

Salary Q.'s

$$\frac{\text{C.Y.Salary}}{\text{C.Y.Index}}$$

$$\text{B.Y.Index}$$

= B.Y. की Salary

$$\frac{\text{B.Y.Salary}}{\text{B.Y.Index}}$$

$$\text{C.Y.Index}$$

= C.Y. की Salary

- Today' salary – should have been = Real gain.
- Should have been –Today's salary = D.A.

Champions वाला Chart