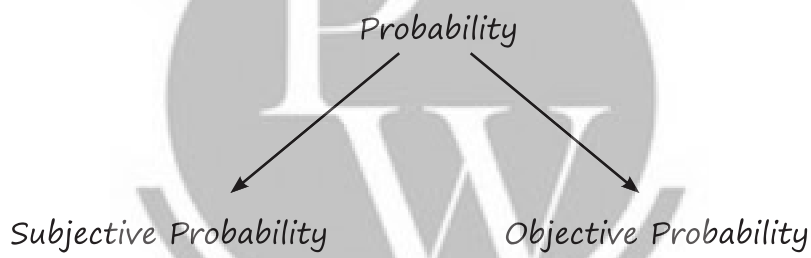
CHAPTER

15 Probability

If we were to consider an India vs. Pakistan cricket match today, predicting the outcome involves probability.

PROBABILITY

- The terms 'Probably' 'in all likelihood', 'chance', 'odds in favor', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability.
- Thus, it is a branch of mathematics that deals with quantifying uncertainty and analyzing the likelihood of events occurring.



SUBJECTIVE PROBABILITY

- Subjective probability is based on an individual's personal beliefs, judgments, and opinions about the likelihood of an event.
- It is often used when there is limited or no historical data available to estimate probabilities objectively.
- Subjective probabilities are influenced by personal experiences, biases, and perceptions.

OBJECTIVE PROBABILITY

- Objective probability, also known as frequentist probability, is based on observed or historical data and the relative frequency of an event occurring in a large number of trials or observations.
- It is considered a more objective and data-driven approach to probability, as it relies on empirical evidence.
- Objective probability can be expressed as a ratio of the number of favorable outcomes to the total number of possible outcomes.

  In order to develop a sound knowledge about probability, it is necessary to get ourselves familiar with a few terms.

- **EXPERIMENT:** An experiment refers to the performance of certain tasks to produce certain results.
- **RANDOM EXPERIMENT**: A random experiment is one in which the results depend solely on chance and cannot be predicted with certainty.
- **EVENTS:** Events are the results or outcomes of a random experiment. Sometimes, it gives a combination of outcomes.
  For example, {HH} represents the event of getting 2 heads.

MUTUALLY EXCLUSIVE EVENTS OR INCOMPATIBLE EVENTS

These are events that cannot occur simultaneously.

E.g. Consider rolling a six-sided die. The events "rolling a 1" and "rolling a 2" are mutually exclusive because it is impossible for both events to occur simultaneously.

- Two events A and B are mutually exclusive if $P(A \cap B) = 0$, meaning there are no common outcomes between A and B.
- Similarly, three events A, B, and C are mutually exclusive if $P(A \cap B \cap C) = 0$, meaning there are no common outcomes among all three events.

EXHAUSTIVE EVENTS

Exhaustive events are events that together cover all possible outcomes of an experiment.

E.g.

1. In a coin toss, there are two exhaustive events: "Getting heads" or "Getting tails".
2. In throwing a die, there are 6 exhaustive events: {1, 2, 3, 4, 5, 6}.
   (a) Two events A and B are exhaustive if $P(A \cup B) = 1$, meaning all possible outcomes are covered by A and B.
   (b) Similarly, three events A, B, and C are exhaustive if $P(A \cup B \cup C) = 1$, meaning all possible outcomes are covered by A, B, and C.

EQUALLY LIKELY EVENTS or Mutually Symmetric Events or Equi-Probable Events: Equally likely events are events that have the same probability of occurring.

E.g. In a fair coin toss, getting heads or tails is an example of equally likely events.

- Three events A, B, and C are equally likely if $P(A) = P(B) = P(C)$, meaning the probabilities of each event are the same.

CLASSICAL DEFINITION OF PROBABILITY OR A PRIOR DEFINITION

FOR FINITE ELEMENTARY EVENTS

Let's consider a random experiment that results in n finite elementary events, which are assumed to be equally likely. If n_A ($\le n$) events are favorable to an event A, then the probability of occurrence of event A is defined as the ratio of the number of events favorable to A to the total number of events. This can be expressed as:

$$P(A) = \frac{n_A}{n} = \frac{\text{Number of events favorable to A}}{\text{Total number of events}}$$

FOR COMPOSITE EVENTS

In the case of composite events that are mutually exclusive, exhaustive, and equally likely, we can consider m (≤ n) such events. If m_A (≤ n_A) represents the number of mutually exclusive, exhaustive, and equally likely events favourable to A, then the probability of event A is given by:

$$P(A) = \frac{m_A}{m} = \frac{\text{Number of mutually exclusive, exhaustive, and equally likely events favorable to A}}{\text{Total number of mutually exclusive, exhaustive, and equally likely events}}$$

E.g. Consider the rolling of a dice once. The sample space S is given by S = {1, 2, 3, 4, 5, 6}.

We define events A, B and C as follows:

A: The event of getting an even number: A = {2, 4, 6}

B: The event of getting an odd number: B = {1, 3, 5}

C: The event of getting a multiple of 3: C = {3, 6}

Calculation of Probabilities

Now, let's apply the classical definition of probability to the given example:

- P(A) is the probability of getting an even number. Since there are 3 even numbers in S and a total of 6 sample points, we have $P(A) = \frac{3}{6} = \frac{1}{2}$
- P(B) is the probability of getting an odd number. There are 3 odd numbers in S, so $P(B) = \frac{3}{6} = \frac{1}{2}$
- P(C) is the probability of getting a multiple of 3. Among the 6 sample points, 2 of them are multiples of 3. Therefore, $P(C) = \frac{2}{6} = \frac{1}{3}$
- Also, since $P(A \cap B) = 0$ since $A \cap B = \{2, 4, 6\} \cap \{1, 3, 5\} = \phi$
  Thus, A and B are mutually exclusive events.

Example 1. A die is thrown, then the probability of getting a prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

Sol. (b) When a die is thrown, then the sample space will be:

Total outcomes = 6

Now, event of getting a prime number is: {2, 3, 5}

i.e., Number of favourable outcomes = 3

Thus, the required probability = $\frac{3}{6} = \frac{1}{2}$

Hence, the correct option is (b).

Example 2. The probability that exactly one head appears in a single throw of two fair coins is **(ICAI)**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

Sol. (b) The sample space of throwing two coins in a single time is given by: {(H, H); (H, T); (T, H); (T, T)}

$\Rightarrow$ Total no. of possible outcomes = 4

Now, the outcomes of appearing exactly one head is (H, T); (T, H)

$\Rightarrow$ Total no. of favorable outcomes = 2

We know that,

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Hence, the correct option is (b) i.e. $\frac{1}{2}$.

Example 3. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is **(ICAI)**

(a) 0.30 (b) 0.70 (c) 0.25 (d) 0.20

Sol. (b) We know that,

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$$

Number of favourable outcomes = 25 + 10 = 35

Total outcomes = 25 + 10 + 15 = 50

Therefore, the required probability $= \frac{35}{50} = \frac{7}{10} = 0.7$

Hence, the correct option is (b) i.e., 0.7

Example 4. Three coins are tossed together. The probability of getting three tails is **(ICAI)**

(a) 5/8 (b) 3/8 (c) 1/8 (d) None of these

Sol. (c) The sample space of tossing 3 coins is given by,

S = { HHH, TTT, HTT, THT, TTH, THH, HTH, HHT}

where, { H $\rightarrow$ Heads, T $\rightarrow$ Tails}

Here, total outcomes = 8

Favorable outcomes = { TTT} = 1

We know that,

$$\text{Probability (P)} = \frac{\text{Favorable Outcomes}}{\text{Total Number of Outcomes}} = \frac{1}{8}.$$

Hence, the correct answer is option (c) i.e., $\frac{1}{8}$.

Example 5. A coin is tossed three times. What is the probability of getting:

(I) 2 tails (II) at least 2 tails

(a) $\frac{3}{8}, \frac{5}{8}$ (b) $\frac{3}{8}, \frac{3}{8}$ (c) $\frac{3}{8}, \frac{1}{8}$ (d) None of these

Sol. (d) Given: A coin is tossed three times

Thus, the events are: {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}

(I) Events of 2 tails: {(HTT), (THT), (TTH)}

Therefore, the required probability $= \frac{3}{8}$

(II) Events of at least 2 tails: {(HTT), (THT), (TTH), (TTT)}

Therefore, the required probability $= \frac{4}{8} = \frac{1}{2}$

Hence, the correct option is (d).

Some key points related to the classical definition of probability:

1. The probability of an event lies between 0 and 1, inclusive. It cannot be negative or greater than 1.
2. The non-occurrence of event A is denoted by A' or A^c or $\overline{A}$ and is known as the complementary event of A. The event A and its complementary event A' form a set of mutually exclusive and exhaustive events.
3. The ratio of the number of favourable events to the number of unfavourable events is known as the odds in favour of event A. Its inverse ratio is known as the odds against event A. i.e. If 'p' be the number of favourable outcomes of an event and 'q' be the number of unfavourable outcomes of the event, then

   Probability $= \frac{p}{p+q}$ where $\frac{p}{q}$ is the odds in favour of event and $\frac{q}{p}$ is the odds against the event.

Example 6. If $P(A) = \frac{5}{9}$, then the odds against the event A is **(ICAI)**

(a) 5 : 9 (b) 5 : 4 (c) 4 : 5 (d) 5 : 14

Sol. (c) We know that,

Probability of the event is given by $\frac{p}{p+q}$

where $\frac{p}{q}$ are the odds in favor of an event and $\frac{q}{p}$ are the odds against an event.

Given, $P(A) = \frac{5}{9}$

$\Rightarrow \frac{p}{p+q} = \frac{5}{9}$

$\Rightarrow 9p = 5p + 5q$

$\Rightarrow 4p = 5q$

$\Rightarrow \frac{q}{p} = \frac{4}{5}$

Hence, the correct option is (c).

Example 7. Three unbiased coins are tossed simultaneously, then the probability of getting at least 2 heads is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

Sol. (b) Here, the sample space

= {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

Now, the number of favourable outcomes ={HHT, HTH, THH, HHH}

Thus, the required probability $= \frac{4}{8} = \frac{1}{2}$

Hence, the correct option is (b).

Example 8. A dice is rolled twice. What is the probability of getting a difference of 5 points?

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{9}$ (d) None of these

Sol. (a) A dice is rolled twice

Then the total number of outcomes,

n(S) = {(1, 1); (1, 2); (1, 3); (1, 4); (1, 5); (1, 6)

(2, 1); (2, 2); (2, 3); (2, 4); (2, 5); (2, 6)

(3, 1); (3, 2); (3, 3); (3, 4); (3, 5); (3, 6)

(4, 1); (4, 2); (4, 3); (4, 4); (4, 5); (4, 6)

(5, 1); (5, 2); (5, 3); (5, 4); (5, 5); (5, 6)

(6, 1); (6, 2); (6, 3); (6, 4); (6, 5); (6, 6)}

The favourable outcomes of getting a difference of 5 points are {(1, 6); (6,1)}

We know that,

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total numberof outcomes}}$$

Then, p (getting difference of 5 points) $= \frac{n(A)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

Hence, the correct option is (a).

Example 9. Find the probability that a four digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{24}$ (d) None of these

Sol. (b) iven digits: 2, 5, 6, 7

Here, these 4 numbers can be form a 4 digit number in 4! = 24 ways

For a four digit number to be divisible by 4, its last two digits should also be divisible by 4.

So, the possible last digits: 52, 56, 72, 76.

Thus, if we have the last two digits as 52, so the 1st two places of the four digit number can be filled up using the remaining 2 digits i.e., in 2! or 2 ways.

i.e., 6752 and 7652

Thus there are 2 four digit numbers that end with 52.

Similarly, the number of four digit numbers that are divisible by 4 is 4 × 2 = 8.

We know that,

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Thus, the required probability $= \frac{8}{24} = \frac{1}{3}$

Hence, the correct option is (b).

Example 10. A committee of 6 members is to be formed from a group comprising 6 gentlemen and 4 ladies. What is the probability that the committee would comprise:

(I) 3 ladies (II) at least 3 ladies

(a) $\frac{8}{21}, \frac{19}{42}$ (b) $\frac{24}{110}, \frac{19}{42}$ (c) $\frac{1}{21}, \frac{5}{21}$ (d) None of these

Sol. (a) Given that,

Number of gentlemen = 6

Number of ladies = 4

Total members = 6 + 4 = 10

(I) The committee has 3 ladies:

Total ways of selection of 6 members

$$= {}^{10}C_6 = \frac{10!}{6! \times 4!} = \frac{10 \times 9 \times 8 \times 7}{4!} = 10 \times 3 \times 7 = 210$$

Since, the committee should have 3 ladies thus the number of gentlemen would be 3.

Thus, the possible ways $= {}^6C_3 \times {}^4C_3 = \frac{6!}{3! \times 3!} \times \frac{4!}{3! \times 1!} = 5 \times 4 \times 4 = 80$

The required probability $= \frac{80}{210} = \frac{8}{21}$

(II) At least 3 ladies:

Total ways of selection of 3 or more ladies = "3 ladies and 3 gentlemen" or "4 ladies and 2 gentlemen"

$= ({}^6C_3 \times {}^4C_3) + ({}^6C_2 \times {}^4C_4) = (20 \times 4) + (15 \times 1) = 80 + 15 = 95$

The required probability $= \frac{95}{210} = \frac{19}{42}$

Hence the correct answer is option (a).

PRACTICE QUESTIONS (PART A)

1. A coin is tossed twice, what is the probability that at least one tail occurs?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) 1

2. Two broad divisions of probability are **(ICAI)**

(a) Subjective probability and objective probability

(b) Deductive probability and non–deductive probability

(c) Statistical probability and Mathematical probability

(d) None of these

3. A die is thrown, then the probability of getting a number greater than or equal to 3 is

(a) $\frac{4}{5}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

4. A bag contain 6 white and 5 black balls. One ball is drawn. The probability that it is white is

(a) $\frac{5}{11}$ (b) 1 (c) $\frac{6}{11}$ (d) $\frac{1}{11}$

5. Two dice are thrown simultaneously, then the probability that sum on the faces is exactly 5 is

(a) $\frac{1}{9}$ (b) $\frac{}{36}$ (c) $\frac{1}{4}$ (d) None of these

6. A bag contains 5 white, 6 red and 7 green balls. 3 balls are drawn at random. Find the probability that balls are 2 white and 1 green.

(a) $\frac{35}{136}$ (b) $\frac{1}{6}$ (c) $\frac{35}{408}$ (d) None of these

7. Three unbiased coins are tossed simultaneously, then the probability of getting at most 2 tails is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$

8. A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise 2 ladies?

(a) $\frac{37}{210}$ (b) $\frac{24}{110}$ (c) $\frac{140}{429}$ (d) None of these

9. Three events A, B and C are mutually exclusive, exhaustive and equally likely. What is the probability of the complementary event of (A' – B) ?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) None of these

10 Two dice are thrown simultaneously. What is the probability that the sum of the numbers rolled is a prime number?

(a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) 1

11. 20 Books are placed at random in a shelf. Find the probability that a particular pair of books is always together.

(a) 5 (b) $\frac{1}{10}$ (c) $\frac{1}{10!}$ (d) None of these

Answer Key

1. (c) **2.** (a) **3.** (d) **4.** (c) **5.** (a) **6.** (c) **7.** (c) **8.** (c) **9.** (b) **10.** (a) **11.** (b)

THIS CLASSICAL DEFINITION OF PROBABILITY HAS THE FOLLOWING DEMERITS OR LIMITATIONS

1. It is applicable only when the total number of events is finite
2. It can be used only when the events are equally likely or equi-probable. This assumption is made well before the experiment is performed.
3. This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance. In the field of uncertainty or where no prior knowledge is provided, this definition is inapplicable.

RELATIVE FREQUENCY DEFINITION OF PROBABILITY

KEY POINTERS TO STUDY

- Concept of Relative frequency was first developed by the British mathematicians in connection with the survival probability of a group of people.
- If a random experiment is performed by repeating "n times" (under an identical set of condition) then the probability of an event A can be defined as limiting value of the ratio of event occurrence (f_A) to number of times experiment is being repeated (n)

  i.e. $P(A) = \lim_{n \to \infty} \frac{f_A}{n}$

  This statistical definition is applicable if the above limit exists and tends to a finite value.

Example 11. The following data relate to the distribution of salary of a group of employee:

Salary (thousand ₹)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of workers	12	23	24	32	19	11	8

If an employee is selected at random from the entire group of employees, what is the probability that

(I) His Salary would be less than ₹30,000?
(a) 0 (b) 0.5 (c) 3 (d) 0.7

(II) His Salary would be less than ₹60,000?
(a) 0.3 (b) 0.457 (c) 0.8 (d) 0.44

(III) His Salary would be more than ₹100,000?
(a) 0 (b) 0.4 (c) 0.2 (d) 0.55

(IV) His salary would be between ₹40,000 and ₹80,000?
(a) 0.66 (b) 0.759 (c) 0.99 (d) 0.1

Sol. (I) (a) According to the given data there are no workers with a salary less than ₹30,000, so the probability is 0.

Hence, the correct answer is option (a).

(II) (b) Total number of employees = (12 + 23 + 24 + 32 + 19 + 11 + 8) = 129

There are (12 + 23 + 24) workers whose salary is less than ₹60,000.

Thus, Probability $= \frac{12+23+24}{129} = \frac{59}{129} = 0.457$

Hence, the correct answer is option (b).

(III) (a) According to the given data,

There are no workers with salaries more than ₹1,00,000, so the probability is 0.

Hence, the correct answer is option (a).

(IV) (b) There are (23 + 24 + 32 + 19) workers whose salary is between ₹40,000 and ₹80,000.

Thus, Probability $= \frac{23+24+32+19}{129} = \frac{98}{129} = 0.759$

Hence, the correct answer is option (b).

AXIOMATIC OR MODERN DEFINITION OF PROBABILITY

The axiomatic or modern definition of probability states that for a sample space S and an event A defined on S, the probability of A, denoted as P(A), is determined by the following axioms:

1. The probability of any event A is always greater than or equal to zero, i.e., $P(A) \geq 0$ for every $A \subseteq S$ (subset).
2. The probability of the entire sample space S is equal to 1, i.e., P(S) = 1.
3. For any sequence of mutually exclusive events $A_1, A_2, A_3, \ldots$, the probability of their union is equal to the sum of their individual probabilities, i.e.,

   $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$

ADDITION THEOREMS OR THEOREMS ON TOTAL PROBABILITY

- **THEOREM 1:** For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B. i.e. $P(A \cup B)$ or $P(A + B) = P(A) + P(B)$ .... or P(A or B) whenever A and B are mutually exclusive.

  E.g. Let's consider the events A and B representing the outcomes of rolling a fair six-sided die. If A is the event of getting an even number (2, 4 or 6), and B is the event of getting an odd number (1, 3 or 5), then A and B are mutually exclusive. The probability of either getting an even number or an odd number is given by:

  $$P(A \cup B) = P(A) + P(B) = \frac{3}{6} + \frac{3}{6} = 1$$

  This means that the probability of getting either an even number or an odd number when rolling the die is 1, which is the total probability of the entire sample space.

Example 12. If A and B are two mutually exclusive events such that $P(A \cup B) = \frac{2}{3}$, $P(A) = \frac{2}{5}$, then P(B) =

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

Sol. (a) Given: $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$

Since, A and B are two mutually exclusive events thus

$P(A \cup B) = P(A) + P(B)$

$\Rightarrow \frac{2}{3} = \frac{2}{5} + P(B)$

$\Rightarrow P(B) = \frac{2}{3} - \frac{2}{5}$

$\Rightarrow P(B) = \frac{10-6}{15} = \frac{4}{15}$

Hence, the correct option is (a) i.e. $\frac{4}{15}$.

Example 13. A number is selected from the first 20 natural numbers. What is the probability that it would be divisible by 3 or 8?

(a) 0 (b) 0.4 (c) 0.33 (d) 0.75

Sol. (b) Total observations: 1, 2, 3, ... 20

Now, numbers divisible by 3 = 3, 6, 9, 12, 15, 18

Numbers divisible by 8 = 8, 16

Since, the events that the numbers would be divisible by 3 or 8 is a mutually exclusive events, thus probability that the numbers would be divisible by 3 or 8 is given by:

P(3 or 8) = P(3) + P(8)

$= \frac{6}{20} + \frac{2}{20} = \frac{8}{20}$

= 0.4

Hence, the correct answer is option (b).

- **THEOREM 2**: For any k (≥ 2) mutually exclusive events $A_1, A_2, A_3 ..., A_k$ then probability that at least one of them occurs is given by the sum of the individual probabilities of the K events. i.e. $P(A_1 \cup A_2 \cup A_3 \cup .... \cup A_k) = P(A_1) + P(A_2) + ... + P(A_k)$
- **THEOREM 3:** For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

  i.e., $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 14. A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 3 or 7?

(a) 0 (b) 0.426 (c) 0.33 (d) 0.75

Sol. (b) Given, A number is selected at random from the first 1000 natural numbers.

According to the question, n(S) = 1000

Number of multiples of 7 in first 1000 natural numbers,

$P(A) = \frac{1000}{7} = 142\,(approx)$

Number of multiples of 3 in first 1000 natural numbers,

$P(B) = \frac{1000}{3} = 333\,(approx)$

And, numbers divisible by 7 and 3 are

$P(A \cap B) = \frac{1000}{(3 \times 7)} = 47\,(approx)$

Therefore, the probability that the number so selected would be a multiple of 7 or 11

$= P(A) + P(B) - P(A \cap B)$

$= \frac{142}{1000} + \frac{333}{1000} - \frac{47}{1000} = \frac{142 + 333 - 47}{1000} = 0.426$

Hence, the correct answer is option (b).

- **THEOREM 4:** For any three events A, B and C, the probability that at least one of the events occurs is given by: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Example 15. There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 1 (b) 0.9 (c) 0.5 (d) None of these

Sol (b) According to the given information, we have

$P(A) = 0.80, P(B) = 0.60, P(C) = 0.50,$

$P(A \cap B) = 0.46, P(B \cap C) = 0.32, P(A \cap C) = 0.48$ and

$P(A \cap B \cap C) = 0.26$

Now, the probability that atleast one of them survives another 5 years is given by $P(A \cup B \cup C)$

On substituting the values, we get

$P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26$

0.90

Therefore, the required probability is 0.9.

Hence, the correct option is (b).

Example 16. Which of the following pairs of events are mutually exclusive?

(a) A: The team wins the football match.
B: The team lost the football match.

(b) A: The card drawn is a heart.
B: The card drawn is a red card.

(c) A: Anita is 20 years old.
B: She is a great dancer.

(d) A: The dice shows an even number.
B: The dice shows a prime number.

Sol (a) We know that,

Mutually exclusive events are those events that do not occur at the same time.

We know that,

Mutually exclusive events are those events that do not occur at the same time.

Clearly, option (a) i.e., the team wins the football match and the team loses the football match.

Hence, the correct answer is option (a).

Example 17. If two events A and B, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$ then find $P(A \cap B)$

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Sol. (b) Given: $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$

We know that,

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{2}{3}$

$\Rightarrow P(A \cap B) = \frac{3+2-4}{6}$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

Hence, the correct option is (b) i.e. $\frac{1}{6}$.

Example 18. If A and B are mutually exclusive events, then **(ICAI)**

(a) $P(A) = P(A - B)$ (b) $P(B) = P(A - B)$
(c) $P(A) = P(A \cap B)$ (d) $P(B) = P(A \cap B)$

Sol. (a) Given, A and B are mutually exclusive events.

Thus, $A \cap B = 0$

We know that,

$P(A - B) = P(A) - P(A \cap B)$

$\Rightarrow P(A - B) = P(A)$ [$\because P(A \cap B) = 0$]

Hence, the correct option is (a) i.e. $P(A) = P(A - B)$.

Example 19. A coin is tossed thrice. What is the probability of getting 2 or more tails?

(a) 0 (b) 0.4 (c) 0.5 (d) 0.75

Sol. (c) Total cases: {TTT, HHH, HTT, THT, TTH, HHT, HTH, THH} = 8

Cases of getting 2 or more tails i.e., 2 tails or 3 tails

For 2 tails, favourable outcomes = {HTT, THT, TTH}

for 3 tails, favourable outcomes = {TTT}

Therefore, the required Probability = P(2 tails) + P(3 tails)

$\frac{3}{8}+\frac{1}{8}=\frac{4}{8}=0.5$

Hence, the correct option is (c).

Example 20. A certain problem has odds of 5 to 2 against A solving it, and odds of 3 to 1 in favor of B solving it. What is the probability of the problem being solved if both A and B attempt it?

(a) $\frac{13}{14}$ (b) $\frac{15}{28}$ (c) $\frac{9}{14}$ (d) None of these

Sol. (a) Given, Odds against A solving a certain problem are 5 to 2 and Odds in favour of B solving the problem are 3 to 1.

Then, $P(A) = \frac{5}{5+2} = \frac{5}{7}$ (Probability to solve the problem)

$P(A)' = 1 - P(A) = 1 - \frac{5}{7} = \frac{2}{7}$

$P(B) = \frac{3}{3+1} = \frac{3}{4}$ (Probability to solve the problem)

$P(B)' = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$

Required Probability will be given as,

$\Rightarrow 1 - [P(A)' \times P(B)']$

$= 1 - \left[\frac{2}{7} \times \frac{1}{4}\right] = 1 - \left(\frac{2}{28}\right) = \frac{28-2}{28} = \frac{26}{28} = \frac{13}{14}$

Hence, the correct answer is option (a) i.e., $\frac{13}{14}$.

PRACTICE QUESTIONS (PART B)

1. A class consists of 10 boys and 20 girls of which half of boys and half the girls have blue eyes. Find the probability that a student chosen at random is a boy and has blue eyes.

   (a) $\frac{1}{6}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) None of these

2. An event that can be split into further events is known as **(ICAI)**

   (a) Complex event (b) Mixed event

   (c) Simple event (d) Composite event

3. Which of the following pairs of events are mutually exclusive? **(ICAI)**

(a) A: The student reads in a school. B: He studies Philosophy.

(b) A: Raju was born in India. B: He is a fine Engineer.

(c) A: Ruma is 16 years old. B: She is a good singer.

(d) A: Peter is under 15 years of age. B: Peter is a voter of Kolkata.

4. If an unbiased coin is tossed once, then the two events Head and Tail are

(a) Mutually exclusive (b) Exhaustive

(c) Equally likely (d) All these (a), (b) and (c)

5. If $P(A) = \frac{1}{3}$, then the odds against the event A is

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 2 : 3

6. If a number is selected at random from the first 50 natural numbers, what will be the probability that the selected number is a multiple of 3 and 4? **(Dec 2022)**

(a) 5/50 (b) 2/25 (c) 3/50 (d) 4/25

7. If A and B are events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, then $P(A \cup B) =$

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{1}{2}$ (d) None of these

8. The probability that an applicant for an Accountant's job has a B.Com. degree is 0.75. The probability that they have knowledge of Tally is 0.40. The probability that they have a B.Com. degree or knowledge of Tally is 0.30. Out of 1000 applicants, how many would be B.Com. experts in Tally?

(a) 0 (b) 850 (c) 1150 (d) 750

9. A, B, C are three mutually independent with probabilities 0.3, 0.2 and 0.4 respectively. What is $P(A \cap B \cap C)$? **(ICAI)**

(a) 0.400 (b) 0.240 (c) 0.024 (d) 0.500

10. There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years is 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 1.00 (b) 0.28 (c) 0.45 (d) 0.90

11. For any two events A and B,

(a) $P(A) + P(B) > P(A \cap B)$ (b) $P(A) + P(B) < P(A \cap B)$

(c) $P(A) + P(B) \geq P(A \cap B)$ (d) $P(A) \times P(B) \leq P(A \cap B)$

12. A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6? **(ICAI)**

(a) 0.30 (b) 0.25 (c) 0.20 (d) 13

Answer Key

1. (a) **2.** (d) **3.** (b) **4.** (d) **5.** (a) **6.** (b) **7.** (b) **8.** (b) **9.** (c) **10.** (d) **11.** (c) **12.** (d)

CONDITIONAL PROBABILITY AND COMPOUND THEOREM OF PROBABILITY

COMPOUND PROBABILITY OR JOINT PROBABILITY

The probability of an event, discussed so far, is technically known as unconditional or marginal probability. But if there are two or more events occurring simultaneously, how to calculate the probability.

The probability of occurrence of two events A and B simultaneously is known as the Compound Probability or Joint Probability of the events A and B and is denoted by $P(A \cup B)$.

In a similar manner, the probability of simultaneous occurrence of k events $A_1, A_2, A_3 \ldots, A_k$, is denoted by $P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k)$

CONDITIONAL PROBABILITY

Let A and B be two events and S be the sample space, then the probability of event B given that event A has already occurred is called the conditional probability of B given A.

It is denoted be $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

E.g., Let two unbiased coins be tossed then Sample space, S = {HH, HT, TH, TT}

Now,

Let A be the event of getting atleast one head = {HH, HT, TH}

B be the event of getting both head = {HH}

then, $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$

If it is known that A has already happened, then it is sure that TT cannot occur.

Thus, $P\left(\frac{B}{A}\right) = \frac{1}{3}$

Also, $\frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Similarly, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Example 21. If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{6}$, then $P\left(\frac{A}{B}\right)$ is

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (d) $\frac{1}{2}$ (d) $\frac{1}{8}$

Sol. (b) Given, $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{6}$

Thus, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{4}{18} = \frac{2}{9}$$

Hence, the correct option is (b).

Example 22. If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, then $P\left(\frac{B}{A}\right)$ is **(Dec 2022)**

(a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Sol. (c) Given, $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{11}{12} = \frac{1}{3} + \frac{3}{4} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{3} + \frac{3}{4} - \frac{11}{12}$$

$$\Rightarrow P(A \cap B) = \frac{4+9-11}{12}$$

$$\Rightarrow P(A \cap B) = \frac{2}{12} = \frac{1}{6}$$

Thus, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$

Hence, the correct option is (c).

Example 23. If $P(\bar{A} \cup \bar{B}) = \frac{5}{6}, P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, what is $P(A \cup B)$? **(ICAI)**

(a) 1 (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

Sol. (a) Given, $P(\bar{A} \cup \bar{B}) = \frac{5}{6}, P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$

We know that,

$P(\bar{A} \cup \bar{B}) = P(\bar{A} \cap \bar{B}) = 1 - P(A \cap B)$

$$\Rightarrow \frac{5}{6} = 1 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 1 - \frac{5}{6} = \frac{1}{6}$$

We also know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{7}{6} - \frac{1}{6} = 1$

Hence, the correct option is (a) i.e. 1.

INDEPENDENT EVENTS

Independent events are events in which the occurrence or outcome of one event does not affect the probability of the other event. The probability of the second event happening is the same whether the first event has occurred or not.

E.g.: Coin Flips Suppose you flip a fair coin twice. The outcome of the first coin flip (e.g., getting heads) has no influence on the outcome of the second coin flip. The probability of getting heads on the second flip remains $\frac{1}{2}$, regardless of whether you got heads or tails on the first flip. These events are independent.

Thus, if A and B are two independent events, then

$P\left(\frac{A}{B}\right) = P(A)$ and $P\left(\frac{B}{A}\right) = P(B)$

If A and B are independent events, then

$P \cap (A \cap B) = P(A) \times P(B)$

Similarly,

- $P(A \cap C) = P(A) \times P(C)$
- $P(B \cap C) = P(B) \times P(C)$
- $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

  If events A and B are independent, the pairs A and B', A' and B, and A' and B' are also independent, where A' represents the complement of event A and B' represents the complement of event B.

Example 24. For any two events A_1, A_2 : let $P(A_1) = \frac{2}{3}, P(A_2) = \frac{3}{8}, P(A_1 \cap A_2) = \frac{1}{4}$, then A_1, A_2 are

(a) Mutually Exclusive but not independent events

(b) Mutually Exclusive and independent events

(c) Independent but not mutually exclusive

(d) None of these

Sol. (c) Given, $P(A_1) = \frac{2}{3}, P(A_2) = \frac{3}{8}$ and $P(A_1 \cap A_2) = \frac{1}{4}$

As we know, for mutually exclusive events

$P(A_1 \cap A_2) = 0$

On the contrary, given in the question,

$P(A_1 \cap A_2) = \frac{1}{4} \neq 0$

Thus, they are not mutually exclusive events.

Now, for independent events: $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$

Put the values and check for independent events,

$\Rightarrow \frac{1}{4} = \frac{2}{3} \times \frac{3}{8}$

$= \frac{6}{24} \Rightarrow \frac{1}{4} = \frac{1}{4}$ (verified)

Thus, they are independent events however they are not mutually exclusive events.

Hence, the correct answer is option (c) i.e., Independent but not mutually exclusive.

Example 25. If events A and B are given to be independent such that $P(A) = 0.2$, $P(A \cup B) = 0.6$, then $P(B)$ is

(a) 0.4 (b) 0.5 (c) 0.7 (d) None of these

Sol. (b) Given, A and B are independent events such that $P(A) = 0.2$, $P(A \cup B) = 0.6$

We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\Rightarrow 0.6 = 0.2 + P(B) - 0.2 \times P(B)$

$\Rightarrow 0.4 = 0.8 \times P(B)$

$\Rightarrow P(B) = \frac{0.4}{0.8} = \frac{1}{2} = 0.5$

Hence, the correct option is (b).

Example 26. A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) None of these

Sol. (b) Let A, B and C be the events such that $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

Clearly, A, B and c are independent events.

Thus, the probability that the problem will be solved = 1 − P(problem is not solved)

$= 1 - P(A')P(B')P(C')$

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

Hence, the correct option is (b).

Example 27. Ronaldo is known as one of the football players to hit 4 goals out of 10 shots whereas Messi is known to hit 5 goals out of 11 shots. What is the probability that the target would be hit once they both have hit for the penalty shootout?

(a) 0 (b) 0.67 (c) 0.33 (d) 0.75

Sol. (b) Probability of Ronaldo hitting the goal, P(A) = 0.4

Probability of Messi hitting the goal, $P(B) = \frac{5}{11} = 0.45$

Thus, P(A') = 1 – P(A) and P(B') = 1 – P(B)

Probability that the target would be hit once they both have hit for the penalty shootout

$= P(A)\, P(B') + P(A') \cdot P(B) + P(A) \,.\, P(B)$

$= 0.4(1 - 0.45) + (1 - 0.4)0.45 + (0.4)(0.45)$

$= 0.22 + 0.27 + 0.18 = 0.67$

Hence, the correct answer is option (b).

DEPENDENT EVENTS

Dependent events are events in which the outcome of one event does affect the probability of the other event. The probability of the second event happening is influenced by the outcome of the first event.

Example: Drawing Cards

Suppose you have a deck of 52 playing cards. If you draw a card and do not replace it before drawing the next card, these events are dependent. For example, if you draw an Ace of Spades as the first card, there are now only 51 cards left in the deck, and the probability of drawing another Ace of Spades as the second card is $\frac{1}{51}$, not $\frac{1}{52}$ as it was initially. The probabilities change because the events are dependent on each other.

THEOREMS OF COMPOUND PROBABILITY

- **Theorem 5:** For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e.,

  $$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right), \text{ provided } P(A) > 0$$

- **Theorem 6:** For any three events A, B and C, the probability that they occur jointly is given by

  $$P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right), \text{ provided } P(A \cap B) > 0$$

Example 28. If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are

(a) Independent (b) Dependent

(c) Not equally likely (d) Not exhaustive

Sol. (b) We know that,

For two events A and B to be dependent, happening of one affect the happening of other i.e., $P(A \cap B) \neq P(A) \times P(B)$

Hence, the correct option is (b).

Example 29. In a group of 15 males and 10 females, 5 males and 7 females are service holders. What is the probability that a person selected at random from the group is not a service holder given that the selected person is a female?

(a) 0 (b) 0.5 (c) 0.3 (d) 0.75

Sol. (c) Given, Total number of males = 15

Total number of females = 10

Number of female service holders = 7

Number of male service holders = 5

Let A be the event that a person selected is female and B be the event that a person is not a service holder.

To find: $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

Probability that the person is not a service holder and a female:

$$P(B \cap A) = \frac{3}{15 + 10} = \frac{3}{25} = 0.12$$

Probability that the person is female:

$$P(A) = \frac{10}{25} = 0.4$$

Therefore, the required probability $= \frac{0.12}{0.4} = 0.3.$

Hence, the correct answer is option (c).

Example 30. A card is drawn from a pack of 52 cards, the card drawn is a red card. What is the probability of its being a card of diamond?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{5}$

Sol. (b) Let A be the event of drawing a red card and B be the event of drawing a diamond.

Thus, $P(A) = \frac{26}{52}$

Now, probability that the card is red and card of diamond is: $P(A \cap B) = \frac{13}{52}$

Now, probability of its being a card of diamond given that it is red card is:

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P\left(\frac{B}{A}\right) = \frac{\frac{13}{52}}{\frac{26}{52}} = \frac{13}{52} \times \frac{52}{26} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

Hence, the correct answer is option (b).

Example 31. A pair of dice is thrown together and the sum of points of the two dice is noted to be 9. What is the probability that one of the two dice has shown the point 4?

(a) 0 (b) 0.5 (c) 0.33 (d) 0.75

Sol. (b) Since, sum of points of the two dice is 9

Here, Possible ways to get 9 as a sum = (4, 5); (5, 4); (3, 6); (6, 3) i.e., total outcomes = 4

Number of favourable outcomes (one of the two dice has shown the point 4) = 2

Therefore, probability $= \frac{\ }{4} = 0.5$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART C)

1. If A and B are two events such that P(A) = 0.6, P(B) = 0.3 and $P(A \cap B) = 0.2$, then $P\left(\frac{A}{B}\right) =$

   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) None of these

2. If P(B) = 0.5 and $P(A \cap B) = 0.32$, then $P\left(\frac{A}{B}\right) =$

   (a) $\frac{25}{16}$ (b) $\frac{16}{25}$ (c) $\frac{7}{16}$ (d) None of these

3. Events A and B are given to be independent such that P(A) = 0.40 and $P(A \cup B) = 0.70$, then P(B) =

   (a) 0.2 (b) 0.4 (c) 0.5 (d) 1

4. In connection with a random experiment, it is found that

   $P(A) = \frac{1}{3}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{4}$

   Evaluate the following probabilities: **(ICAI)**

   (I) P(A/B) (II) P(B/A) (III) P(A'/ B)
   (IV) P(A/ B') (V) P(A'/ B')

5. The probability of the occurrence of a number greater than 2 in a throw of a die if it is known that only even numbers can occur is

   (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) None of these

Answer Key

1. (a) 2. (b) 3. (c) 4. (I) 1318 (II) 1320 (III) 518 (IV) 712 (V) 512 5. (c)

Example 32. *Given that for two events A and B,* $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{3}$ *and* $P(A \cup B) = \frac{3}{5}$, *what is* $P\left(\frac{A}{B}\right)$?

(a) 0.655 (b) $\frac{13}{60}$ (c) $\frac{60}{31}$ (d) 0.775

Sol. (d)

Example 33. *If* $P(A \cap B) = 0$, *then the two events A and B are*

(a) Mutually exclusive (b) Exhaustive

(c) Equally likely (d) Independent

Sol. (a) If $P(A \cap B) = 0$, it means the probability of the intersection of events A and B is 0, which implies that events A and B have no common outcomes.

In such a case, the events A and B are considered mutually exclusive.

Hence, the correct option is (a).

Example 34. *If A, B and C are mutually exclusive, independent and exhaustive events then what is the probability that they occur simultaneously?*

(a) 1 (b) 0.50

(c) 0 (d) any value between 0 and 1

Sol. (c) If events A, B, and C are mutually exclusive, it means that they cannot occur simultaneously. This implies that the probability of all three events occurring simultaneously is 0.

Hence, the correct option is (c).

Example 35. *If for two independent events A and B,* $P(A \cup B) = \frac{2}{3}$ *and* $P(A) = \frac{2}{5}$, *what is* $P(B)$?

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

Sol. (a) We know that,

$P(B) = P(A \cup B) - P(A) + P(A \cap B)$

Here, $P(A \cap B)$ is 0 as they are independent events.

Thus, $P(B) = \frac{2}{3} - \frac{2}{5} + 0$

Hence, the correct answer is option (a).

Example 36. *For three events A, B and C, the probability that only A occur is*

(a) $P(A)$ (b) $P(A \cup B \cup C)$

(c) $P(A' \cap B \cap C)$ (d) $P(A \cap B' \cap C')$

Sol. (d) We know that,

For sets A, B and C the probability of occurrence of event A only is given by:

$P(A \cap B' \cap C')$

Hence, the correct option is (d).

Example 37. The probability that is leap year has 53 Monday is: **(Dec 2022)**

(a) $\frac{1}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{5}$

Sol. (c) We know that,

In a year there are 52 weeks, thus in a leap year there are 52 weeks and 2 odd days.

The two odd days can be:

(Saturday, Sunday),

(Sunday, Monday),

(Monday, Tuesday),

(Tuesday, Wednesday),

(Wednesday, Thursday),

(Thursday, Friday),

(Friday, Saturday)

So, there are 7 possibilities out of which 2 have Monday.

Therefore, Probability of 2 Monday = $\frac{2}{7}$

Hence, the correct answer is option (c).

Example 38. There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of the same colour?

(a) $\frac{89}{729}$ (b) $\frac{97}{729}$ (c) $\frac{82}{729}$ (d) $\frac{23}{32}$

Sol. (a) Given, There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 4 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

Here,

The probability of selection of red ball from box 1 is $\frac{5}{5+7+6} = \frac{5}{18}$

The probability of selection of red ball from box 2 is $\frac{4}{4+8+6} = \frac{4}{18}$

The probability of selection of red ball from box 3 is $\frac{3}{3+4+2} = \frac{3}{9}$

Now,

The probability of selection of white ball from box $\frac{7}{18}$

The probability of selection of white ball from box 2 is $\frac{8}{18}$

The probability of selection of white ball from box 3 is $\frac{4}{9}$

Now,

The probability of selection of blue ball from box 1 is $\frac{6}{18}$

The probability of selection of blue ball from box 2 is $\frac{6}{18}$

The probability of selection of blue ball from box 3 is $\frac{2}{9}$

Now,

By using multiplication theorem of probability we get, the required probability as

$$\Rightarrow \frac{5}{18} \times \frac{4}{18} \times \frac{3}{9} + \frac{7}{18} \times \frac{8}{18} \times \frac{4}{9} + \frac{6}{18} \times \frac{6}{18} \times \frac{2}{9}$$

$$\Rightarrow \frac{89}{729}$$

Hence, the final option is (a).

Example 39. A problem in probability was given to three CA students A, B and C whose chances of solving it are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{2}$ respectively. What is the probability that the problem would be solved?

(a) $\frac{4}{15}$ (b) $\frac{7}{8}$ (c) $\frac{8}{15}$ (d) $\frac{11}{15}$

Sol. (d) Given: Probability was given to three CA students A, B and C whose chances of solving it are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{2}$ respectively.

i.e., $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(C) = \frac{1}{2}$

Thus,

$$P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(\bar{B}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P(\bar{C}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Thus, the probability that the problem would not be solved if none of them solved the problem is given by $P(\bar{A} \cap \bar{B} \cap \bar{C})$

Since, they are independent events thus $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = \frac{2}{3} \times \frac{4}{5} \times \frac{1}{2} = \frac{4}{15}$

Therefore, the probability that the problem would be solved

=1 – Probability (problem not solved)

$= 1 - \frac{4}{15} = \frac{11}{15}$

Hence, the correct answer is option (d).

Example 40. For a group of subjects 30%, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry? **(ICAI)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Sol. (a) Let A be the event that a student fails in Physics.

Let B be the event that a student fails in Chemistry.

According to the question,

Then, P(A) = 0.30, P(B) = 0.40

Also, the probability of failing in at least one of the subjects is 0.50

$\Rightarrow P(A \cup B) = 0.50$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = 0.50$

$\Rightarrow 0.30 + 0.40 - P(A \cap B) = 0.50$

$\Rightarrow P(A \cap B) = 0.70 - 0.50$

$\Rightarrow P(A \cap B) = 0.20$

Now, the probability that if student pass in Physics then he will fail in chemistry is denoted by $P\left(\frac{\bar{A}}{B}\right)$

We know that,

$P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A} \cap B)}{P(B)}$

Here, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

= 0.40 – 0.20 = 0.20

$\Rightarrow P\left(\frac{\bar{A}}{B}\right) = \frac{0.20}{0.40} = \frac{1}{2}$

Hence, the correct answer is option (a) i.e., $\frac{1}{2}$.

Example 41. An article consists of two parts A and B. The manufacturing process of each part is such that probability of defect in A is 0.08 and that B is 0.05. What is the probability that the assembled product will not have any defect?

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

Sol. (d) Given, Probability of defect in A, P(A) = 0.08

Probability of defect in B, P(B) = 0.05

Now, Probability that part A will have no defect,

$P(A)' = 1 - 0.08 = 0.92$

Now, Probability that part B will have no defect,

$P(B)' = 1 - 0.05 = 0.95$

Now, Products are non defective will be given as,

$P(A' \cap B') = P(A)' \times P(B)'$

$\Rightarrow 0.92 \times 0.95$

$\Rightarrow 0.874$

Hence, the correct answer is option (d) i.e., 0.874.

PRACTICE QUESTIONS (PART D)

1. If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are
   (a) Independent (b) Dependent
   (c) Not equally likely (d) Not exhaustive
2. If two events A and B are independent, then
   (a) They can be mutually exclusive
   (b) They cannot be mutually exclusive
   (c) They cannot be exhaustive
   (d) Both (b) and (c)
3. If $P\left(\frac{A}{B}\right) = P(A)$, then
   (a) A is independent of B (b) B is independent of A
   (c) B is dependent of A (d) Both (a) and (b).
4. If $P(A - B) = P(B - A)$, then the two events A and B satisfy the condition
   (a) $P(A) = P(B)$ (b) $P(A) + P(B) = 1$
   (c) $P(A \cap B) = 0$ (d) $P(A \cup B) = 1$
5. If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, then $P\left(\frac{B}{A}\right)$ is **(Dec 2022)**
   (a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$
6. If $P(\bar{A} \cup \bar{B}) = \frac{5}{6}$, $P(A) = \frac{1}{2}$ and $P(\bar{B}) = \frac{2}{3}$, what is $P(A \cup B)$?
   (a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

7 If two unbiased dice are rolled, what is the probability of getting points neither 6 nor 9?

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.80

8. A machine is made of two parts A and B. The manufacturing process of each part is such that probability of defective in part A is 0.08 and that B is 0.05. What is the probability that the assembled part will not have any defect? **(Dec 2022)**

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

9 If $P(\bar{A} \cup \bar{B}) = \frac{5}{6}, P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, what is $P(A \cup B)$

(a) 1 (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

10. The probability of winning of a person is $\frac{6}{11}$ and at a result he gets ₹77. The expectation of this person is

(a) ₹35 (b) ₹42 (c) ₹58 (d) None of these

11. In a class 40% students read Mathematics, 25% Biology and 15% both Mathematics and Biology. One student is selected at random. The probability that he reads Mathematics if it is known that he reads Biology is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) None of these

Answer Key

1. (b) **2.** (b) **3.** (a) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (d) **9.** (a) **10.** (b) **11.** (b)

RANDOM VARIABLE - PROBABILITY DISTRIBUTION

A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.

A random variable is denoted by a capital letter.

E.g.: Consider the experiment of tossing a coin three times. Let X represent the number of heads obtained. In this case, the sample space is given by:

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

The different values of X would be:

X = 0 (no heads)

X = 1 (one head)

X = 2 (two heads)

X = 3 (three heads)

TYPE OF RANDOM VARIABLE

Discrete Random Variable	Continuous Random Variable
A random variable from the discrete sample space is called a discrete random variable. For example, consider the experiment of tossing a coin three times. Let X represent the number of heads obtained. In this case, the sample space is given by: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} The different values of X would be: X = 0 (no heads) X = 1 (one head) X = 2 (two heads) X = 3 (three heads)	A random variable from the continuous sample space is called a discrete random variable. For example, consider the experiment of measuring the height of a randomly selected person. Let X represent the height in centimeters. The values of X would be any real number within a certain range, such as 150 cm to 200 cm. Since the height can take on any value within the interval, X is a continuous random variable

PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE

It may be defined as a statement where we take different values for random variables with their corresponding probabilities. Writing it mathematically, if a random variable X assumes n finite values $X_1, X_2, X_3, \ldots, X_n$ with corresponding probabilities $P_1, P_2, P_3, \ldots, P_n$ such that

1. $P_i \geq 0$ (for every i)
2. $\Sigma P_i = 1$ (over all i)

then the probability distribution of the random variable X is given by

X:	X_1	X_2	X_3	...	X_n	Total
P:	P_1	P_2	P_3	...	P_n	1

For example, if an unbiased coin is tossed three times and if X denotes the number of heads then, as we have already discussed, X is a random variable and its probability distribution is given by:

Probability Distribution of X (Number of heads when a coin is tossed thrice)

X:	0	1	2	3	Total
P:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

PROBABILITY MASS FUNCTION (PMF) OF X

If a function f(X) exists which defines the probability (P) as a function of X, where X is discrete random variable, where f(X) satisfies the below given condition:

1. $f(X) \geq 0$ for every X
2. $\Sigma f(X) = 1$

where, f(X) is given by $f(X) = P(X = x)$

Example 42. Which of the following set of function define a probability space on $S = \{a_1, a_2, a_3\}$?

(a) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{2}, P(a_3) = \frac{1}{4}$

(b) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{6}, P(a_3) = \frac{1}{6}$

(c) $P(a_1) = \frac{2}{3}, P(a_2) = \frac{2}{3}, P(a_3) = \frac{1}{4}$

(d) None of these

Sol. (b) Given, $S = \{a_1, a_2, a_3\}$

As we know that, the sum of all the probabilities is equal to 1.

For option (a):

$$\Rightarrow \frac{1}{3} + \frac{1}{2} + \frac{1}{4} = \frac{13}{12} \neq 1$$

Here, the sum of probabilities is not equal to 1.

For option (b):

$$\Rightarrow \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{6}{6} = 1$$

Therefore, the sum of probabilities is equal to 1.

For option (c):

$$\Rightarrow \frac{2}{3} + \frac{2}{3} + \frac{1}{4} = \frac{19}{12} \neq 1$$

Here, the sum of probabilities is not equal to 1.

Hence, the correct option is (b).

Example 43. Let P be a probability function on $S = \{X_1, X_2, X_3\}$ if $P(X_1) = \frac{1}{4}, P(X_3) = \frac{1}{3}$ then $P(X_2)$ is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{3}{4}$ (d) None of these

Sol. (a) Given, $P(X_1) = \frac{1}{4}$ and $P(X_3) = \frac{1}{3}$

We know that, the sum of all probabilities of all the elements of the sample space is 1.

Thus, $P(X_1) + P(X_2) + P(X_3) = 1$

$$\Rightarrow \frac{1}{4} + P(X_2) + \frac{1}{3} = 1$$

$$\Rightarrow P(X_2) = 1 - \frac{7}{12} = \frac{5}{12}$$

$$\Rightarrow P(X_2) = \frac{5}{12}$$

Hence, the correct option is (a) i.e. $\frac{5}{12}$.

Example 44. The probability distribution of a random variable is as follows:

x	1	2	3	4	5
P	3k	2k	3k	k	k

Find the value of k and P (x ≤ 3).

(a) $\frac{1}{10}, 0.2$ (b) $\frac{1}{10}, 0.5$ (c) $\frac{1}{5}, 1.5$ (d) None of these

Sol. (b) As it is given as probability distribution function,

$\Rightarrow \Sigma P = 1$

$\Rightarrow 10k = 1$

$\Rightarrow k = \frac{1}{10}$

Now, $P(x < 3) = P(x = 1) + P(x = 2)$

$= 3k + 2k = 5k$

$= 5\left(\frac{1}{10}\right) = \frac{1}{2} = 0.5$

Hence, the correct option is (a) i.e., 0.5.

Example 45. A random variable X taking values 0, 1, 2 has the following probability distribution for some number k.

$P(X) = k$ if $X = 0$

$= 2k$ if $X = 1$

$= 3k$ if $X = 2$

Find the value of k.

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) None of these

Sol. (c) As it is given as probability distribution function,

$\Rightarrow \Sigma P(X) = 1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) = 1$

$\Rightarrow k + 2k + 3k = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

Therefore, the value of k is $\frac{1}{6}$.

Hence, the correct option is (c).

Example 46. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2 + k$

Find the value of k.

(a) 10 (b) −1 (c) $\frac{1}{10}$ (d) None of these

Sol. (c) We know that,

$\Sigma P(X) = 1$

$\Rightarrow 0 + 2k + 3k + k + 2k + k^2 + 7k^2 + 2k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$

$\Rightarrow 10k(k + 1) - (k + 1) = 0$

$\Rightarrow (10k - 1)(k + 1) = 0$

$\Rightarrow 10k - 1 = 0, k + 1 = 0$

$\Rightarrow k = \frac{1}{10}, -1$

Since, k cannot be negative.

Therefore, the value of k is $\frac{1}{10}$.

Hence, the correct option is (c).

PROBABILITY DENSITY FUNCTION (PDF)

When x is a continuous random variable defined over an interval $[\alpha, \beta]$, where $\beta > \alpha$, then x can assume an infinite number of values from its interval. And in such cases we assign intervals of values to probability rather than assigning individual probability to every mass point x.

Then, if a function f(x) exists such that it defines the probability. It will be called as probability density function if it satisfies the below given condition:

1. $f(x) \geq 0$ for $x \in [\alpha, \beta]$
2. and if the probability that x lies between two specified values a and b, where $\alpha \leq a < b \leq \beta$ is given by $\int_a^b f(x)dx$

Example 47. If $f(x)=\begin{cases} cx^2, 0 < x < 1 \\ 0, \text{elsewhere} \end{cases}$, elsewhere is the probability density function of a continuous random variable X, find the value of c.

(a) 3 (b) 2 (c) 1 (d) None of these

Sol. (a) Given function, $f(x)=\begin{cases} cx^2, 0 < x < 1 \\ 0, \text{elsewhere} \end{cases}$ Since, f(x) represents probability density function of a random variable X, then

$\int_0^1 f(x) = 1 \Rightarrow \int_0^1 cx^2 = 1$

$\Rightarrow \left[c\frac{x^3}{3}\right]_0^1 = 1$

$\Rightarrow \frac{c}{3}[(1)^3 - (0)^3] = 1$

$\Rightarrow \frac{c}{3}[1-0] = 1 \Rightarrow c = 3$

Therefore, the value of x is 3.

Hence, the correct option is (a).

EXPECTED VALUE OF A RANDOM VARIABLE

Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

Note: Expected value gives the mean of all values.

Hence, if a random variable x assumes n values $x_1, x_2, x_3, \ldots, x_n$ with corresponding probabilities $p_1, p_2, p_3, \ldots, p_n$, where p_i satisfies

1. $p_i(x_i) \geq 0$ for every X
2. $\Sigma p_i x_i = 1$

then the expected value of x is given by $\mu = E(x) = \Sigma p_i x_i$

Expected Value of x^2 is given by $E(x^2) = \Sigma p_i x_i^2$

In particular expected value of a monotonic function g(x) is given by $E[g(x)] = \Sigma p_i E(x_i)$

Variance of x, to be denoted by, σ^2 is given by

$V(x) = \sigma^2 = E(x - \mu)^2 = E(x^2) - \mu^2$

The positive square root of variance is known as standard deviation and is denoted by σ.

If $y = a + bx$, for two random variables x and y and for a pair of constants a and b, then the mean i.e., expected value of y is given by:

$\mu_y = a + b\mu_x$

and the standard deviation of y is given by:

$\sigma_y = |b| \times \sigma_x$

Now, when x is discrete random variable and f(x) is the probability mass function, then the expected value (i.e. Mean) is given by

$\mu = E(x) = \sum_x xf(x)$

Its variance is given by $\sigma^2 = E(x^2) - \mu^2$, where, $E(x^2) = \sum_x x^2 f(x)$

If x is continuous random variable defined in $(-\infty, \infty)$, then the expected value is given by

$E(x) = \int_{-\infty}^{\infty} xf(x)dx$ and $\sigma^2 = E(x^2) - \mu^2$

where, $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

PROPERTIES OF EXPECTED VALUES

1. Expectation of a constant k is k i.e., if all values of x is equal to k, then the expected value will be equal to $E(k) = k$ for any constant k.
2. Expectation of sum of two random variables is the sum of their expectations i.e., $E(x + y) = E(x) + E(y)$ for any two random variables x and y.

3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.

   i.e., E(kx) = k.E(x) for any constant k.

4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.

   i.e., $E(xy) = E(x) \times E(y)$ where x and y are independent.

Example 48. If x and y are random variables having expected values as 4.5 and 2.5 respectively, then the expected value of (x − y) is **(ICAI)**

(a) 2 (b) 7 (c) 6 (d) 0

Sol. (a) Given, x and y are random variables having expected values as 4.5 and 2.5

$\Rightarrow E(x) = 4.5$ and $E(y) = 2.5$

$\therefore$ Expected value of (x − y)

$= E(x - y) = E(x) - E(y)$

$= 4.5 - 2.5 = 2$

Therefore, the required expected value is 2.

Hence, the correct option is (a) i.e. 2.

Example 49. The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statement is **(ICAI)**

(a) 170 (b) 176 (c) 178 (d) 180

Sol. (c) Given, The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively.

$P(\bar{A}) = 0.2, P(\bar{B}) = 0.3, P(\bar{C}) = 0.1$

So, probability that there is no error will be

$P(A) = 1 - P(\bar{A}) = 1 - 0.2 = 0.8$

$P(B) = 1 - P(\bar{B}) = 1 - 0.3 = 0.7$

$P(C) = 1 - P(\bar{C}) = 1 - 0.1 = 0.9$

If A, B and C prepare 60, 70 and 90 such statements, so the expected correct statement is given as follows:

X:	60	70	90
P(X):	0.8	0.7	0.9

Thus, $E(X) = \Sigma XP(X)$

Expected value $= 60 \times 0.8 + 70 \times 0.7 + 90 \times 0.9 = 48 + 49 + 81 = 178$

Here, the correct answer is option (c) i.e., 178.

Example 50. A wholesaler can make a profit of ₹12,000 or incur a loss of ₹8,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.8 and 0.2 respectively. What is his expected profit?

(a) ₹8,000 (b) ₹12,000 (c) ₹4,000 (d) None of these

Sol. (a) According to the given information,

X:	12,000	8,000
P:	0.8	0.2

Thus, the expected profit is given by:

E(X) = 12,000(0.8) + (−8,000) (0.2)

E(X) = 9,600 − 1,600

E(X) = 8,000

Therefore, his expected profit is ₹8,000.

Hence, the correct option is (a).

Example 51. A number is selected at random from a set containing the first 50 natural numbers and another number is selected at random from another set containing the first 100 natural numbers. What is the expected value of

(I) the sum (II) the product

(a) $76, \frac{5151}{2}$ (b) $66, \frac{8151}{2}$ (c) $50, \frac{9158}{2}$ (d) $45, \frac{5190}{2}$

Sol. (a) According to the given information,

X	1	2	3	4	....	50
P_i	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{50}$	....	$\frac{1}{50}$
P_iX_i	$\frac{1}{50}$	$\frac{2}{50}$	$\frac{3}{50}$	$\frac{4}{50}$	....	$\frac{50}{50}=1$

Y	1	2	3	4	....	100
P_i	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	....	$\frac{1}{100}$
P_iY_i	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{3}{100}$	$\frac{4}{100}$	....	$\frac{100}{100}=1$

(I) Sum $= E(X+Y) = \sum P_iX_i + \sum P_iY_i$

$$= \left\{\frac{1}{50}+\frac{2}{50}+\frac{3}{50}+\ldots\ldots\ldots+\frac{50}{50}\right\}+\left\{\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\ldots\ldots\ldots+\frac{100}{100}\right\}$$

$$= \frac{1+2+3\ldots\ldots+50}{50}+\frac{1+2+3+\ldots\ldots\ldots+100}{100}$$

$$= \frac{\frac{50(50+1)}{2}}{50}+\frac{\frac{100(100+1)}{2}}{100}$$

$$= \frac{51}{2}+\frac{101}{2}=\frac{152}{2}=76$$

(II) Product $= E(XY) = \left(\sum P_i X_i\right)\left(\sum P_i Y_i\right)$

$$= \frac{51}{2} \times \frac{101}{2} = \frac{5151}{2}$$

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART E)

1. A random variable has the following probability distribution:

X: 4 5 7 8 10

P: 0.15 0.20 0.40 0.15 0.10

Find $E[x - E(x)]^2$. Also obtain $v(3x - 4)$.

(a) 3.04, 17.36 (b) 3.04, 27.36

(c) 4.15, 37.65 (d) None of these

2. A box contains 10 tubelights of which 3 are defective. A man selects four tube lights at random. What is the expected number of defective tube lights in his selection?

(a) 1.21 (b) 1.69 (c) 1.71 (d) 1.20

3. Raju draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹500 if he draws a white ball and ₹200 if he draws a red ball. What is his expectation? But here are the twists, for each game he has to pay ₹400 for participating in the game, would he consider it a fair game and participate?

(a) Yes, the game is fair and should participate

(b) No, the game is not fair

(c) Cannot be determined

(d) None of these

4. A dice is thrown repeatedly till a '1' appears. Write down the sample space. Also find the expected number of throws.

(a) 0 (b) 1 (c) 6 (d) None of these

5. A random variable x has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2 + k$

Find:

(I) $P(x \le 2)$ (II) $P(x > 3)$ (III) $P(3 \le x < 6)$

(a) 0.50, 0.40, 0.30 (b) 0.40, 0.40, 0.30

(c) 0.50, 0.40, 0.30 (d) 0.50, 0.40, 0.30

6. If all the values taken by a random variable are equal then

(a) its expected value is zero

(b) its standard deviation is zero

(c) its standard deviation is positive

(d) its standard deviation is a real number

7. If x and y are independent, then
   (a) E(xy) = E(x) x E(y)
   (b) E(xy) = E(x) + E(y)
   (c) E(x − y) = E(x) + E(y)
   (d) E(x − y) = E(x) + x E(y)
8. If two random variables x and y are related by y = 2 − 3x, then the SD of y is given by **(ICAI)**
   (a) −3 × SD of x (b) 3 × SD of x
   (c) 9 × SD of x (d) 2 × SD of x
9. If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1
10. If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is **(ICAI)**
   (a) 1.50 (b) 3 (c) 0.90 (d) 1

Answer Key

1. (b)	2. (d)	3. (a)	4. (c)	5. (d)	6. (b)	7. (a)	8. (b)	9. (a)	10. (c)

PRACTICE QUESTIONS (PART F)

1. When 2 – dice are thrown simultaneously then the probability of getting at least one 5 is
   (a) $\frac{11}{36}$ (b) $\frac{5}{36}$ (c) $\frac{8}{15}$ (d) $\frac{1}{7}$
2. If a coin is tossed 5 times then the probability of getting tail and head occurs alternatively is
   (a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{64}$
3. Two event A and B are such that do not occurs simultaneously then they are called ______ events
   (a) Mutually exhaustive (b) Mutually exclusive
   (c) Mutually independent (d) Equally likely
4. If $Y \geq X$ then mathematical expectation is
   (a) E(X) > E(Y) (b) E(X) ≤ E(Y) (c) E(X) = E(Y) (d) E(X) . E(Y) = 1
5. If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(\bar{A}) + P(\bar{B})$
   (a) 0.3 (b) 0.5 (c) 0.7 (d) 0.9
6. Two different dice are thrown simultaneously, then the probability that the sum of the numbers appearing on the top of dice ids 9 is
   (a) $\frac{8}{9}$ (b) $\frac{1}{9}$ (c) $\frac{7}{9}$ (d) None of these

7. Ram is known to hit a target in 2 out of 3 shots whereas Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that the target would be hit if they both try?

(a) $\frac{9}{11}$ (b) $\frac{3}{11}$ (c) $\frac{10}{33}$ (d) $\frac{6}{11}$

8. A coin is tossed six times, then the probability of obtaining heads and tails alternatively is

(a) $\frac{1}{2}$ (b) $\frac{1}{64}$ (c) $\frac{1}{32}$ (d) $\frac{1}{16}$

9. The Probability that a non-leap year has 53 Wednesday is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{2}{3}$ (d) $\frac{1}{7}$

10. If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B)$ is equal to

(a) $\frac{11}{12}$ (b) $\frac{10}{12}$ (c) $\frac{7}{12}$ (d) $\frac{1}{6}$

11. Sum of all probability mutually exclusive and exhaustive events is equal to

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

12. What is the probability of having at least one 'six' from 3 throws of a perfect die?

(a) $\frac{5}{6}$ (b) $\left(\frac{5}{6}\right)^3$ (c) $1-\left(\frac{1}{6}\right)^3$ (d) $1-\left(\frac{5}{6}\right)^3$

13. If two random variables x and y are related by y = 2 - 3x, then the SD of y is given by

(a) −3 × SD of x (b) 3 × SD of x
(c) 9 × SD of x (d) 2 × SD of x

14. Variance a random variable x is given by

(a) $E(X - \mu)^2$ (b) $E[X - E(X)]^2$ (c) $E(X^2 - \mu)$ (d) (a) or (b)

15. The theorem of compound probability states that for any two events A and B

(a) $P(A \cap B) = P(A) \times P(B/A)$ (b) $P(A \cup B) = P(A) \times P(B/A)$
(c) $P(A \cap B) = P(A) \times P(B)$ (d) $P(A \cup B) = P(A) \times P(B) - P(A \cap B)$

16. The term "chance" and probability are synonyms **(ICAI)**

(a) True (b) False (c) Both (d) None of these

17. Two broad divisions of probability are **(ICAI)**

(a) Subjective probability and objective probability
(b) Deductive probability and mathematical Probability
(c) Statistical probability and mathematical probability
(d) None of these

18. Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, the value of P(A/B) is

(a) ½ (b) 1/6 (c) 2/3 (d) 3/4

19. The Probability distribution of the demand for a commodity is given below:

Demands (x)	5	6	7	8	9	10
Probability [P(x)]	0.05	0.10	0.30	0.40	0.10	0.05

The expected value of demands will be

(a) 7.55 (b) 7.85 (c) 1.25 (d) 8.35

20. If for two mutually exclusive events A and B $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$, then what is the value of P(B)?

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

21. Let A and B are two events with $P(A) = \frac{2}{3}, P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then P(B/A) will be

(a) 7/8 (b) 1/3 (c) 1/8 (d) 8/7

22. If a coin is tossed 10 times, what is the probability of getting at least 7 tails?

(a) 0.1719 (b) 0.3438 (d) 0.5000 (d) 0.8281

23. For two events A,B let $P(A) = \frac{2}{3}, P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{4}$, then A and B are

(a) Mutually exclusive but not independent

(b) Independence but not mutually exclusive

(c) Mutually exclusive and independent

(d) None of these

24. A bag contains 6 white and 5 red balls. One ball is drawn. The probability that it is red is

(a) $\frac{5}{11}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

25. If two unbiased dice are rolled, what is the probability of getting points neither 3 nor 6?

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.80

26. Three coins are tossed together, the probability of getting exactly two head is:

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) None of these

27. A bag contain 15 one rupee coins, 25 two rupees coins and 10 five rupees coins, if a coin is selected at random then probability for not selecting a one rupee coin is:

(a) 0.30 (b) 0.20 (c) 0.25 (d) 0.70

28. If a random variable x assumes the values x_1, x_2, x_3, x_4 with corresponding probabilities p_1, p_2, p_3, p_4; then the expected value of x is **(ICAI)**
(a) $p_1 + p_2 + p_3 + p_4$ (b) $x_1p_1 + x_2p_3 + x_3p_2 + x_4p_4$
(c) $p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$ (d) None of these

29. When two fair dice are thrown, what is the probability of getting a sum that is a multiple of 7?
(a) $\frac{1}{12}$ (b) $\frac{1}{36}$ (c) $\frac{1}{6}$ (d) $\frac{1}{18}$

30. If there are 20 cars, 14 of them are SUVs and 6 of them are sedans, then the probability of randomly selecting 4 cars that include 2 SUVs and 2 sedans is
(a) 0.04 (b) 0.17 (c) 0.28 (d) 0.23

31. Let X be a random variable having following Probability distribution:

x	−3	6	9
P(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find E(X) and E(X^2).
(a) $\frac{11}{2}, \frac{93}{2}$ (b) $\frac{17}{2}, \frac{75}{2}$ (c) $\frac{11}{3}, \frac{97}{3}$ (d) None of these

Answer Key

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (c)
11. (d)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (a)
21. (c)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (c)
31. (a)									

SUMMARY

- **Experiment:** An experiment may be described as a performance that produces certain results.
- **Random Experiment:** An experiment is defined to be random if the results of the experiment depend on chance only.
- **Events:** The results or outcomes of a random experiment are known as events. Sometimes events may be a combination of outcomes.
- **The events are of two types:**
  (i) Simple or Elementary,
  (ii) Composite or Compound.
- **Mutually Exclusive Events or Incompatible Events:** A set of events A_1, A_2, A_3, ... is known to be mutually exclusive if not more than one of them can occur simultaneously.
- **Exhaustive Events:** The events A_1, A_2, A_3, ... are known to form an exhaustive set if one of these events must necessarily occur.

- **Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events:** The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events.
- The probability of occurrence of the event is defined as the ratio of the number of events favorable to A to the total number of events. Denoting this by P(A), we have

$$P(A) = \frac{\text{No. of equally like ly favorable events}}{\text{Total no. of equally likely events}}$$

- The probability of an event lies between 0 and 1, both inclusive i.e., $0 \le P(A) \le 1$.
- When P(A) = 0, A is known to be an impossible event and when P(A) = 1, A is known to be a sure event.
- Non-occurrence of event A is denoted by A' or A^c or $\bar{A}$ and it is known as complimentary event of A. The event A along with its complementary A' forms a set of mutually exclusive and exhaustive events.
- The ratio of no. of favorable events to the no. of unfavorable events is known as odds in favor of the event and its inverse ratio is known as odds against event A.

  i.e. odds in favor of A = $m_A : (m - m_A)$

  and odds in against A = $(m - m_A) : m_A$
- For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.

  i.e. or $P(A + B) = P(A) + P(B)$
- For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

  i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  Conditional Probability: $P(B / A) = \frac{P(A \cap B)}{P(A)}$
- For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e., Compound Probability or Joint Probability Provided.
- A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.
- Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

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