› If a quantity increases or decreases in the ratio $a : b$ then

$$\text{new quantity} = \frac{b}{a} \times \text{original quantity}$$

The fraction by which the original quantity is multiplied to get a new quantity is called the **factor multiplying ratio**.

- **Inverse Ratio:** One ratio is the inverse of another if their **product is 1**. Thus b : a is the inverse of **a : b** and *vice-versa*.

- The ratio **compounded** of the two ratios a : b and c : d is ac : bd.
- Compounding two or more ratios means **multiplying them**.

- A ratio compounded of itself is called its duplicate ratio.

$a^2 : b^2$ is the **duplicate ratio** of a:b

$a^3 : b^3$ is the **triplicate ratio** of a:b

$\sqrt{a} : \sqrt{b}$ is the **sub-duplicate ratio** of a:b

$\sqrt[3]{a} : \sqrt[3]{b}$ is the **sub-triplicate ratio** of a:b

- **Continued Ratio:** is the relation or comparison between the magnitudes of **three or more** quantities of same kind.
- The continued ratio of three similar quantities a, b, c can be written as **a:b:c**

› **Cross Product Rule:** If a : b = c : d are in proportion then **ad = bc**

Product of extremes = Product of means

› **Continuous Proportion:** Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if **a : b = b : c**

$$\frac{a}{b} = \frac{b}{c} \qquad b^2 = ac$$

here, a = first proportional, c = third proportional and b is mean proportional (because b is GM of a and c)

› Invertendo

If a : b = c : d, then

$$b : a = d : c$$

› Alternendo

If a : b = c : d, then

$$a : c = b : d$$

› Componendo

If a : b = c : d, then

$$a + b : b = c + d : d$$

› Dividendo

If a : b = c : d, then

$$a - b : b = c - d : d$$

› Componendo and Dividendo

If a : b = c : d, then

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\frac{a-b}{a+b} = \frac{c-d}{c+d}$$

› Addendo

If a:b = c:d = e:f = ... = k

then $$\frac{a+c+e+\ldots}{b+d+f+\ldots}=k$$

› Subtrahendo

If a:b = c:d = e:f = ... = k

then $$\frac{a-c-e+\ldots}{b-d-f+\ldots}=k$$

Indices – Standard Results

› Any base raised to the power zero is defined to be 1

$$a^0 = 1$$

› Roots can also be expressed in the form of power.

$$\sqrt[r]{a} = a^{\frac{1}{r}}$$

Law 1

$$a^m \times a^n = a^{m+n}$$

- If two or more terms with same base are multiplied, we can make them one term having the same base and power as sum of all powers.

Law 2

$$\frac{a^m}{a^n} = a^{m-n}$$

- If two or more terms with same base are in division, we can make them one term having the same base and power as difference of power.

Law 3

$$\left(a^{m}\right)^{n} = a^{m \times n}$$

- If a term having power is raised to another power, we can do product of powers to simplify the expression

Law 4

$$(a \times b)^n = a^n \times b^n$$

- If a product of two or more terms is raised to power, we can split the two terms with same individual power to each one of them.

Calculator Trick for Power

Base $\boxed{\times}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$

Calculator Trick for Reciprocal

$\boxed{\div} \quad \boxed{=}$

Calculator Trick for any root

$$\text{Base}\ \boxed{\sqrt{}}\ \boxed{\sqrt{}}\ \boxed{\sqrt{}} \ldots 12\ times\ \boxed{-1}\ \boxed{\div}\ \boxed{n}$$

$$\boxed{+1}\ \boxed{\times =}\ \boxed{\times =}\ \boxed{\times =}\ \ldots 12\ times$$

Calculator Trick for any power (including non integer)

Base $\boxed{\sqrt{}}\ \boxed{\sqrt{}}\ \boxed{\sqrt{}}\ldots 12\ times\ \boxed{-1}\ \boxed{\times}\ \boxed{n}$

$\boxed{+1}\ \boxed{\times =}\ \boxed{\times =}\ \ \boxed{\times =}\ \ldots$

Log Conditions

- The logarithm of a number to a given base is the **index or the power** to which the **base must be raised** to **produce** the **number**, i.e. to make it equal to the given number.

$$3^4 = 81 \quad \log_3 81 = 4$$

- *If* $a^x = n$ *then* $\log_a n = x$

- *Conditions:*
  - *Number should be positive*
  - *Base should be positive*
  - *Base cannot be equal to zero*

$$n > 0, a > 0, a \neq 1$$

Standard Results of Log

- Log of a number with same base as number is equal to 1

$$\log_a a = 1$$

- Log of 1 (one) for any base is equal to zero

$$\log_a 1 = 0$$

Law 1

- Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base

$$\log_a mn = \log_a m + \log_a n$$

Law 2

› The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

Law 3

- Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base.

$$\log_a m^n = n \log_a m$$

Change of Base Theorem

› If the logarithm of a number to any base is given, then the logarithm of the same number to any other base can be determined from the following relation

$$\log_b m = \frac{\log m}{\log b} = \frac{\log_a m}{\log_a b}$$

$$\log_b a \times \log_a b = 1$$

Base of Log

- Common Log's Base

$$10$$

- Natural Log's Base

$$e$$

Quadratic Equation

- Equation having **degree = 2** is called as Quadratic Equation
- QE will have two roots/ solutions usually denoted by α, β
- Equation **Format** $ax^2 + bx + c = 0$

where,
a is coefficient of x^2
b is coefficient of x
c is constant
$a \neq 0$

Solution of Quadratic Equation

$$ax^2 + bx + c = 0$$

- Formula to calculate roots:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where,
a is coefficient of x^2
b is coefficient of x
c is constant
$a \neq 0$

Sum and Product of Roots of QE

$$ax^2 + bx + c = 0$$

- *Sum of roots* $\alpha + \beta = -\dfrac{b}{a}$

- *Product of roots* $\alpha\beta = \dfrac{c}{a}$

› Construction of Quadratic Equation

If sum of roots and product of roots are given, equation can be constructed in the below manner:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

› Concept of discriminant – to get nature of roots

Discriminant of QE is the mathematical expression which is used to understand nature of roots of QE, it is expressed as below:

$$b^2 - 4ac$$

Condition	*Nature of Roots*
$b^2 - 4ac = 0$	*Real and Equal*
$b^2 - 4ac < 0$	*Imaginary*
$b^2 - 4ac > 0$	*Real and Unequal*
$b^2 - 4ac > 0$ *and a perfect square*	*Real, Unequal and Rational*
$b^2 - 4ac > 0$ *& not a perfect square*	*Real, Unequal and Irrational*

Conjugate Pairs

- *If one root of the equation is*

$$m+\sqrt{n}$$

- *The other one is surely*

$$m-\sqrt{n}$$

- *This pair is called as conjugate pairs*

Simple Equation

- Equation of one degree and having one unknown variable is simple.
- A simple equation has only one root.
- *Form of Equation*:

$$ax + b = 0$$

where,
a is coefficient of x
b is constant
$a \neq 0$

- Solution Method – Direct basic algebra

Simultaneous Linear Equations *(two unknowns)*

- Here we always deal with two equations as it consist of 2 unknowns
- Form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where,
a is coefficient of x
b is coefficient of y
c is constant
$a \neq 0$

Methods of Solution Simultaneous Linear Equations

- **Elimination Method:** In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.
- **Substitution Method:** equation is written in the form of one variable in LHS and that value is substituted in other equation.
- **Cross Multiplication Method:** Formula based method

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Cubic Equation

› **Form:**

$$ax^3 + bx^2 + cx + d = 0$$

where,
a is coefficient of x^3
b is coefficient of x^2
c is coefficient of x
d is constant
$a \neq 0$

› **Method of solution:** Trial and Error

Simple Interest

$$SI = \frac{P.r.t}{100}$$

P = principal value
r = rate of interest per annum
t = time period in years

Simple Interest

- *Amount as per SI*

$$A = P + SI = P + \frac{P.r.t}{100} = P(1 + \frac{rt}{100})$$

Conversion Period

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

Compound Interest Amount

- Calculation of Accumulated Amount under CI denoted by A

$$A = P(1+i)^n$$

where,

P = Initial Principal
i = adjusted interest rate
n = no. of periods

$$i = \frac{r\%}{nocppy}$$

$$n = t \times noccpy$$

Compound Interest Amount by Trick

› Calculator Tricks for Amount as per CI

– Example: *P= 1000, i = 10%, n = 3* then

Calculator Steps to obtain A:

$\boxed{1000}\ \boxed{+}\ \boxed{10}\ \boxed{\%}\ \boxed{+}\ \boxed{10}\ \boxed{\%}\ \boxed{+}\ \boxed{10}\ \boxed{\%}$

Compound Interest

› **Formula for Compound Interest**

– Calculation of Compound Interest Value denoted by CI

$$CI = P[(1+i)^n - 1]$$

– where,

P = *Initial Principal*
i = *adjusted interest rate*
n = *no. of periods*

$$i = \frac{r\%}{nocppy}$$

$$n = t \times noccpy$$

Effective Rate of Interest

$$E = [(1+i)^n - 1]$$

where,

i = adjusted interest rate

n = no. of periods in a year

Future Value – Single Cashflow

$$FV = CF(1+i)^n$$

where,

CF = Single Cashflow of which FV is to be calculated
i = adjusted interest rate
n = no. of periods

Future Value – Annuity Regular

$$FVAR = A_i \times FVAF(n, i)$$

Future Value Annuity Factor:
It is a multiplier for Annuity Value to obtain Final Future Value

$$FVAR = A_i \times \left\{ \frac{[(1+i)^n - 1]}{i} \right\}$$

where,

FVAR = Future Value of Annuity Regular
A_i = Annuity Value (Installment)
FVAF = Future Value Annuity Factor
i = adjusted interest rate
n = no. of periods

Future Value – Annuity Due

› **Formula:**

$$FVAD = A_i \times FVAF(n,i) \times (1+i)$$

$$FVAD = A_i \times \left\{ \frac{[(1+i)^n - 1]}{i} \right\} \times (1+i)$$

where,

FVAD = *Future Value of Annuity Due*
A_i = *Annuity Value (Installment)*
FVAF = *Future Value Annuity Factor*
i = *adjusted interest rate*
n = *no. of periods*

Future Value Annuity Factor:
It is a multiplier for Annuity Value to obtain Final Future Value

Present Value – Single Cashflow

$$PV = \frac{CF}{(1+i)^n}$$

where,

CF = Single Cashflow for which PV is to be calculated
i = adjusted interest rate
n = no. of periods

Compounding and Discounting Factor

- **Compounding**
  - Finding Future Value of any Cashflow
  - *Compounding Factor*: $$\times(1+i)^n$$
- **Discounting**
  - Finding Present Value of any Cashflow
  - *Discounting Factor:* $$\times\frac{1}{(1+i)^n}$$

Present Value – Annuity Regular

$$PVAR = A_i \times PVAF(n,i)$$

$$PVAR = A_i \times \left[\frac{1}{i} \times \left\{ 1 - \frac{1}{(1+i)^n} \right\} \right]$$

Present Value Annuity Factor:
It is a multiplier for Annuity Value to obtain Final Present Value

where,

PVAR = *Present Value of Annuity Regular*
A_i = *Annuity Value (Installment)*
PVAF = *Present Value Annuity Factor*
i = *adjusted interest rate*
n = *no. of periods*

Calculator trick of PVAF

$$\boxed{1+i}\ \boxed{\div}\ \boxed{=}\ \boxed{=}\ldots n-times\ \boxed{GT}$$

Present Value – Annuity Due

$$PVAD = \left[A_i \times PVAF\{(n-1), i\} \right] + A_i$$

where,

PVAD = *Present Value of Annuity Due*
A_i = *Annuity Value (Installment)*
PVAF = *Present Value Annuity Factor*
i = *adjusted interest rate*
n = *no. of periods*
n-1 = *one lesser period*

Perpetuity

$$PVP = \frac{A_i}{i}$$

where,

PVP = *Present Value of Perpetuity*
A_i = *Annuity Value (Installment)*
i = *adjusted interest rate*

Growing Perpetuity

$$PVGP = \frac{A_i}{i - g}$$

where,

$PVGP$ = Present Value of Growing Perpetuity
A_i = Annuity Value (Installment)
i = adjusted interest rate
g = growth rate

Net Present Value

- Formula
  - NPV = Present Value of Cash Inflows – Present Value of Cash Outflows
- Decision Base:
  - If NPV $\geq$ 0, accept the proposal, If NPV $<$ 0, reject the proposal

Real Rate of Return

- **Meaning:**
  - The real interest rate is named so to show what a lender or investor receives in real terms after inflation is factored in.
- **Formula:**
  - Real Rate of Return = Nominal Rate of Return – Rate of Inflation

CAGR

- Compounded Annual Growth rate is the interest rate we used in Compound Interest.
- It is used to see returns on investment on yearly basis

Rules of Counting

› Multiplication Rule

– If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n ' different ways then total number of ways of doing both things simultaneously is **(m × n) ways**

› Addition Rule

– It there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in **(m + n) ways**

Word Used	Use
OR	+ Plus
AND	× Product

Factorial

- $n! = n(n-1)(n-2)\ldots 3.2.1$
- $n! = 1.2.3\ldots(n-2)(n-1)n$
- $n! = n(n-1)!$
- $n! = n(n-1)(n-2)!$
- $0! = 1$

Factorial Values

Value of n	Value of n!
1	1
2	2
3	6
4	24
5	120
6	720
7	5040

Value of n	Value of n!
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200

Theorem of Permutations

π

Number of Permutations when r objects are chosen out of n different objects

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Few Observations:

$n \geq r$

n *is a positive integer*

Particular Case of theorem (n = r)

Number of Permutations when n objects are chosen out of n different objects

$$^{n}P_{n} = n!$$

Special Formula (Must Remember)

$$(n+1)! - n! = n.n!$$

Circular Permutations

› Theorem:

– The number of circular permutations of n different things chosen at a time is $(n-1)!$

– Note: this theorem applies only when we choose all of n things

Circular Permutations (Type II)

- number of ways of arranging n persons along a closed curve so that no person has the same two neighbours is

$$\frac{1}{2}(n-1)!$$

- Same formula will apply if ask is to find number of different forms of necklaces/ garlands

Permutation with Restriction : Theorem 1

- Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is

$$^{n-1}P_r$$

Permutations with Restrictions : Theorem 2

› Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is

$$r.\,{}^{n-1}P_{r-1}$$

Relation between restriction theorem

$$^{n-1}P_r + r.\,^{n-1}P_{r-1} = \,^{n}P_r$$

One particular thing always included

+

One particular thing always excluded

→ Total Permutations

No. of ways when things are never together

Ways of Never Together =

Total ways – Ways of always together

Theorem of Combinations

Number of Combinations when r objects are chosen out of n different objects

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Few Observations:

- $n \geq r$
- n is a positive integer

Linkage of PNC Theorems

$$^nC_r = \frac{^nP_r}{r!}$$

Few Observations:

- $n \geq r$
- n is a positive integer

Special Result of Combinations

$$^{n}C_{0} = 1$$

$$^{n}C_{n} = 1$$

Complimentary Combinations

$$^{n}C_{r} = {}^{n}C_{n-r}$$

Special Formula of Combination

$$^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$$

Combinations of one or more

Combinations of n different things taking **one or more** out of n things at a time

$$2^n - 1$$

Geometry in PNC

Particulars	Tips to Solve
No. of Straight Lines with the given n points	${}^{n}C_{2}$ 2 is used as we need to select two points to make a line
No. of Triangles with the given n points	${}^{n}C_{3}$ 3 is used as we need to select two points to make a line
Adjustment of Collinear Points	If there are collinear points in any problem, no. of lines or triangles formed using those points should be deducted from total no. of lines/ triangles
No. of Parallelogram with the given one set of m parallel lines and another set of n parallel lines	${}^{n}C_{2} \times {}^{m}C_{2}$ Selecting 2 lines from each set of parallel lines
No. of Diagonals	${}^{n}C_{2} - n$

Common Difference of AP

$$d = t_2 - t_1 = t_3 - t_2 = \ldots = t_n - t_{n-1}$$

General Term of an AP

$$t_n = a + (n-1)d$$

where,
a = first term
d = common difference
n = position number of term

General Term of an AP

$$t_n = a + (n-1)d$$

Calculator Trick:

$$\boxed{a}\ \boxed{\pm}\ \boxed{d}\ \underset{t_2}{\boxed{=}}\ \underset{t_3}{\boxed{=}}\ \underset{t_4}{\boxed{=}} \ldots \underset{t_n}{\boxed{=}}$$

Sum of first n terms of an AP

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where,
a = first term
d = common difference
n = position number of term
t_n = nth term of AP

Sum of first n terms of an AP

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Calculator Trick

$$\boxed{a}\,\boxed{\pm}\,\boxed{d}\,\underset{t_2}{\boxed{=}}\,\underset{t_3}{\boxed{=}}\,\underset{t_4}{\boxed{=}}\ldots\underset{t_n}{\boxed{=}}\,\boxed{GT}\,\boxed{+}\,\boxed{a}$$

Sum of first n natural or counting numbers

$$S = \frac{n(n+1)}{2}$$

Sum of first n odd numbers

$$\boxed{S = n^2}$$

Sum of the squares of first n natural numbers

$$S = \frac{n(n+1)(2n+1)}{6}$$

Sum of the cubes of first n natural numbers

$$S = \left\{ \frac{n(n+1)}{2} \right\}^2$$

Common Ratio of GP

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}$$

General Term of an GP

$$t_n = ar^{n-1}$$

where,
a = first term
r = common ratio
n = position number of term

General Term of an AP

$$t_n = ar^{n-1}$$

Calculator Trick:

$$\begin{array}{cccccccc} \boxed{r} & \boxed{\times} & \boxed{a} & \boxed{=} & \boxed{=} & \boxed{=} & \cdots & \boxed{=} \\ & & & \uparrow & \uparrow & \uparrow & & \uparrow \\ & & & t_2 & t_3 & t_4 & & t_n \end{array}$$

Sum of first n terms of a GP

$$S_n = \frac{a(1-r^n)}{1-r}$$

Use when r < 1

$$S_n = \frac{a(r^n-1)}{r-1}$$

Use when r > 1

where,
a = first term
r = common ratio
n = position number of term

Sum of first n terms of a GP

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a(r^n-1)}{r-1}$$

Calculator Trick

$$\boxed{r}\ \boxed{\times}\ \boxed{a}\ \underset{\uparrow\, t_2}{\boxed{=}}\ \underset{\uparrow\, t_3}{\boxed{=}}\ \underset{\uparrow\, t_4}{\boxed{=}} \ldots \underset{\uparrow\, t_n}{\boxed{=}}\ \boxed{GT}\ \boxed{+}\ \boxed{a}$$

Sum of Infinite Geometric Series

$$S_\infty = \frac{a}{1-r}$$

Can be used only if $-1 < r < 1$

where,
a = first term
r = common ratio
n = position number of term

Subset

- No. of possible subset of any set

$$2^n - 1$$

De Morgan's Law

- $(P \cup Q)' = P' \cap Q'$

- $(P \cap Q)' = P' \cup Q'$

2 Set Operations Formulas

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

  - Proof:
    - Example: $A = \{6, 2, 4, 1\}$ $B = \{2, 4, 3\}$

3 Set Operations Formula

- $n(A \cup B \cup C) =$

$$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Composition of Functions

- $fog = fog(x) = f[g(x)]$

- $gof = gof(x) = g[f(x)]$

Step Method of finding inverse of f

1. Write your function in the form of y
   - $y = f(x)$
2. From above expression, find the value of x
   - $x = \square$
3. Interchange value of x and y, now the RHS is Inverse function
   - $y = \square$