Mathematics of Finance

By Anurag Chauhan

Finance : Art & science of management of money

Interest : cost of using other's money

Mr. X (Rich) ⇓ Lender

Mr. Y (Poor) ⇓ Borrower

Risk Taker ⇓

Reducing His Liquidity ⇓

Time value of money ⇓

opportunity cost

wants Return

wants Reward

wants compensation for mantaining same purchasing power

wants atleast same profit

Because of all these

"Interest is charged"

Some important terms

→ **Principal** = Borrowed money (Invested money)

→ Amount or (Accumulated Amount) = Principle + Interest

Types of Interest

Simple interest

Compound interest

Simple Interest

↓

No Interest on Interest

eg Mr. X Borrowed ₹10,000 @ 10% S.I. for 3 years.

Principle =	10,000
+ Interest for 1st year	+ (1000)
Amount after 1st year	11000
+ Interest for IInd year	+ (1000)

Amount after IInd year 12000

+ Interest for IIIrd year + (1000)

Amount after IIIrd year 13000

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Principle = ₹10,000

Interest for 3 year = 3000

Amount after 3 year = 13000

$\boxed{S.I = P \cdot r \cdot t}$

P = Principle

r = Rate of interest in Decimals

t = Time

Now Amount = P + I

= P + Prt

$\boxed{\text{Amount} = P[1 + rt]}$

In simple interest
For Double amount
$$r = \frac{1}{t} \;\&\; t = \frac{1}{r}$$

For Triple amount
$$r = \frac{2}{t} \;\&\; t = \frac{2}{r}$$

In simple interest
when two time & two amounts
are given

t_1	t_2
A_1	A_2

Then $r = \dfrac{A_2 - A_1}{A_1 t_2 - A_2 t_1}$

In simple interest
when two rates & two
amounts are given

r_1 r_2

A_1 A_2

Then

$$t = \frac{A_2 - A_1}{A_1 r_2 - A_2 r_1}$$

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Compound Interest

⇓

Interest on Interest is also calculated

⇓

Mr. X Borrowed ₹ 200,000 @ 20% for 3 years

Principle = ₹ 200,000
Int for Ist year = + 40,000

Amount after Ist year 2,40,000
Int. for IInd year = + 48000

Amount after 3 year 2,88,000
Int for IIIrd year + 57600

Amount after 3 year 345,600

Principle = 2 00,000

Amount = 3,45,600

Interest = A − P

= 1,45,600

$$\underline{\text{In compound Interest}}$$

$$\text{Amount} = P\left[1 + \frac{r}{m}\right]^{t \times m}$$

m = No. of conversion Period in a year

for Annually ⇒ m = 1

for semiannually ⇒ m = 2

for Quarterly ⇒ m = 4

for monthly ⇒ m = 12

for Daily ⇒ m = 365

अगर Direct C.I. find करना हो तो

$$C.I. = A - P$$

$$= P\left[1+\frac{r}{m}\right]^{t\times m} - P$$

$$C.I. = P\left\{\left(1+\frac{r}{m}\right)^{t\times m} - 1\right\}$$

$$\boxed{C.I. = P\left\{\left(1+\frac{r}{m}\right)^{t\times m} - 1\right\}}$$

Calculator Tricks

#1> Square of Any Number

$\boxed{x^2}$ → Press $\boxed{\times}$ एक बार

→ Press $\boxed{=}$ एक बार

#2> Cube of Any Number

$\boxed{x^3}$ → Press $\boxed{\times}$ एक बार

→ Press $\boxed{=}$ दो बार

#3> For $(x)^n = ?$

→ Press $\boxed{\times}$ one time

→ Press $\boxed{=}$ $(n-1)$ times

Examples $(2)^{10}$

→ $\boxed{\times}$ one time

→ $\boxed{=}$ 9 times

#4] Reciprocal of Any Number

$\left(\frac{1}{n}\right)$ → $\boxed{\div}$ one time

→ $\boxed{=}$ one time

eg $\frac{1}{2} = 0.5$

→ 2 on the screen

→ Press $\boxed{\div}$

→ Press $\boxed{=}$

#5] $(x)^{1/n} = ?$

→ Press $\boxed{\sqrt{}}$ 12 times

→ Subract 1 $\boxed{\text{Press } \boxed{-}}$

→ Dyvide by n

→ Add 1 $\boxed{\text{Press } \boxed{+}}$

→ Press $\boxed{\boxed{\times} \text{ then } \boxed{=}}$ → 12 times

eg $(8)^{1/3}$ ⇒ 8 on the screen

→ Press √ √ √ √ ... 12 times

→ -1

→ $\div 3$

→ $+1$

→ $\times =$
$\times =$

x =
x =
x =
⋮
x =
12 times

eg for $(2)^{4.5}$

→ 2 on the screen

→ Press √√√√ ... 12 times

→ −1

→ multiply by 4.5

→ +1

→ x =
x =
x =
x =
⋮
12 times

#6] $\log_{10}(x) = ?$

→ $\sqrt{}$ 19 times

→ -1

→ multiply by 227695

⇓

Remember Property of log

⇓

$$\log(x)^n = n\log x$$

eg

$$\log(1\cdot5)^{10} = 10\log(1\cdot5)$$

#7] Antilog $(x) = ?$

- $\rightarrow \div 227695$
- $\rightarrow +1$
- $\rightarrow \boxed{x=}$ 19 times

eg

$P = 100$

$A = 200$

$r = 10\%$ Annually

find time

<u>Sol:</u>

$$A = P\left[1 + \frac{r}{m}\right]^{t \times m}$$

$$\cancel{200} = \cancel{100}\left[1 + \frac{0.10}{1}\right]^{t \times 1}$$

$$2 = (1.10)^{t}$$

Here power is variable

So we have to use Log

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Put log Both side

$$\log 2 = \log (1.10)^t$$

$$\log 2 = t \log (1.10)$$

$$t = \frac{\log 2}{\log (1.10)}$$

now use log Trick

$$t = \frac{0.3010}{0.0414}$$

$t = 7.27$ years

eg

$P = 100$

$A = 300$

$t = 8$ years

Find compound Rate of interest.

Sol:

$$A = P\left[1+\frac{r}{m}\right]^{t\times m}$$

$$300 = 100\left[1+r\right]^{8}$$

$$(3)^{1} = \underbrace{(1+r)^{8}}$$

Here Base is variable, so transfer Power 8 to left side

$$(3)^{1/8} = 1+r$$

$$(3)^{1/8} - 1 = r$$

$$r = \underbrace{(3)^{1/8}}_{\Downarrow} - 1$$

calculator trick

- → √ 12 times
- → −1
- → ÷8
- → +1
- → $\boxed{\times =}$ 12 times

$$r = 1.1472 - 1$$

$$r = 0.1472$$

or

14.72%

Depriciation

$\Downarrow$

Decrease in value of Asset

$\Downarrow$

C = cost

d = Rate of Depriciation

t = Time period for which asset is used

(scrap value) $S.V.$ = value of Asset after 't' years

$$\boxed{S.V. = C\left[1 - d\right]^{t}}$$

Nominal Rate

⇓

Any Rate which is compounded

→ monthly

→ quarterly

→ semiannually

→ Daily

(Isme interest kam Dikhta But interest Jaada milta hai)

Effective Rate

⇓

Any Annual compounding Rate

⇓

(Isme interest Jitna Dikhta hai, utna hi milta hai)

Nominal Rate can be converted to Effective Rate

$\Downarrow$

$$r_e = \left[1+\frac{r}{m}\right]^m - 1$$

<u>Example</u>

8% Semi annually is nominal Rate

$\Downarrow$

It can be converted in Annually

$$r_e = \left[1+\frac{r}{m}\right]^m - 1$$

$$= \left[1+\frac{0.08}{2}\right]^2 - 1$$

$$= (1.04)^2 - 1$$

$$= 1.0816 - 1$$

$$= 0.0816$$

or

8.16% Effective Rate

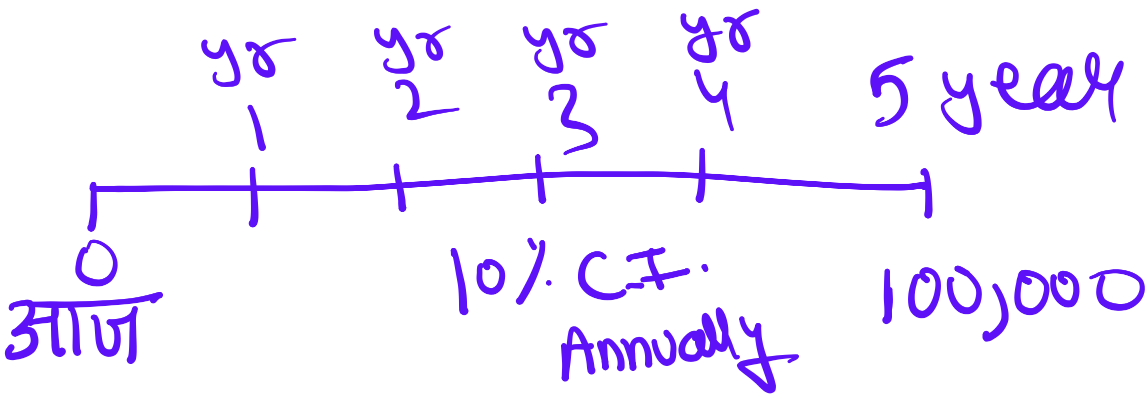
8% SemiAnnually = 8.16% Annually

Both are same

Present value of an Amount which is to be Recieved in future

$$\text{Present value} = \frac{\text{Future value}}{\left[1 + \frac{r}{m}\right]^{t \times m}}$$

eg



5 years ke baad ₹ 100,000 milenge, lekin wo Paise agar Aaj milenge to ₹ 100,000 nahi milega, thoda kam milega

$$P.V. = \frac{100{,}000}{\left[1+\frac{0.10}{1}\right]^5}$$

$$= \frac{100{,}000}{1.61051}$$

$$P.V. = 62{,}092$$

आज Bus इतना ही मिलेगा

Annuity

→ Sequence of payments
(or Reciepts)

→ same payment
(5000, 5000, 5000, 5000)

→ same time interval
b/w two payments
(3m, 3m, 3m, 3m, ...)

eg → LIC premium
→ House Loan EMI

Two Type of Annuities

Regular Annuity
(ordinary)
⇓
when regular payment are made at the end of period
⇓
eg House loan Emi
(Har mahine ke end mein paise Bank se katenge

Immediate
(Annuity Due)
⇓
when regular payments are made in beginning of each period
⇓
eg LIC premium har year ki starting mein Bank se katega

Regular Annuity

⇒ Future value : when benefit of many periodic investments is recieved in future

$$F.V. = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

R = Regular periodic payment

$$i = \frac{r}{m}$$

$n = t \times m$ (Total payments)

Q You start investing ₹ 5000 every year in a Recurring Deposit which gives 10% C.I. How much will you recieve after 15 year?

Sol: Here you are investing 5000 every year (Assuming in the end)

⇓

Benefit will be recieved after 15 years in Future

$$\text{Future value} = 5000\left\{\frac{(1+0.10)^{15}-1}{0.10}\right\}$$

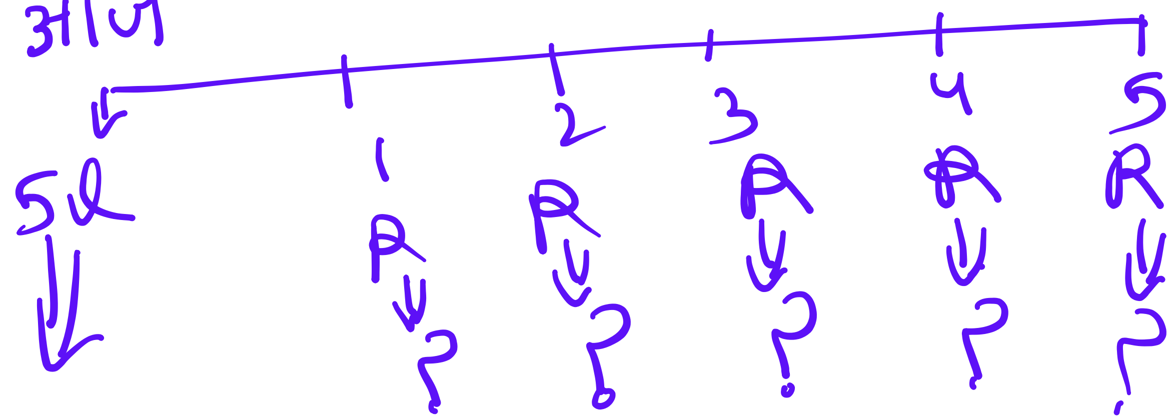
$= ₹ 158,862$

⇒ <u>Present value</u> : when some monetary benefit is recieved today then it repaid in installments.

$$P.V. = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$$

Eg Mr. X took a loan of Rs 500,000 which is to be repaid in 5 yearly installments. If Rate of interest is 6% compounded Annually. find the value of each installment.

Sol: आज



आगे Benefit

le रहा

$\Downarrow$

P.V.

$$500000 = R\left\{\frac{1-(1+0.06)^{-5}}{0.06}\right\}$$

$$500000 = R\,(4.2123)$$

$$R = \frac{500000}{4.2123}$$

$$\boxed{R = ₹\ 1,18,700}$$

<u>Calculator</u>

$(x)^{-n} \Rightarrow$ Press $\boxed{\div}$

Press $\boxed{=}$ 'n' times

eg $(1.06)^{-5} = 0.7472$

Press $\boxed{\div}$

Press $\boxed{=}$ 5 times

Immediate Annuity

$$F.V. = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

$$P.V. = R\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$$

Perpetuity

⇓

Infinite Period Annuity

Ordinary Perpetuity

$$P.V. = \frac{R}{i}$$

Immediate Perpetuity

$$P.V. = \frac{R}{i}(1+i)$$

or

$$\frac{R}{i} + R$$

Q. How much you should invest now to recieve ₹ 10,000 at the end of every year for indefinite period if rate of discount is 8% C.I. Annually?

Sol:

0	1	2	3	4	... ∞
?	10,000	10,000	10,000	10,000	...

P.V = ?

$$P.V. = \frac{R}{i} = \frac{10,000}{0.08} = 1,25,000$$

Q How much you should invest now to recieve ₹ 10,000 in the beginning of every year starting from today for indefinite period if rate of Discount is 8% C.I. Ann

Sol:

$$P.V. = \frac{R}{i}(1+i)$$

$$= \frac{10,000}{0.08}(1+0.08)$$

$$= 1,35,000$$

Growing Perpetuity

⇓

when regular payment is Recieved with growth for infinite period

Regular

⇓

$$P.V. = \frac{R}{i-g}$$

Immediate

⇓

$$P.V. = \frac{R(1+i)}{i-g}$$

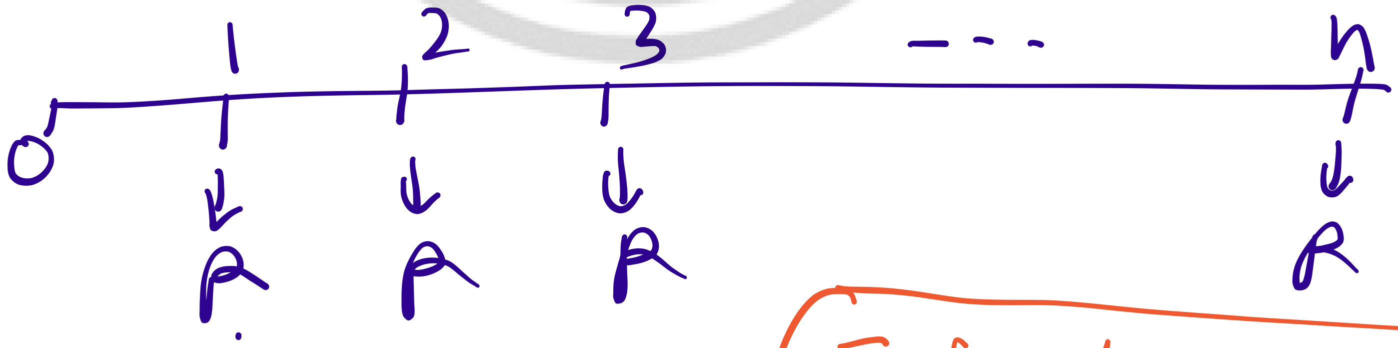
Sinking Fund

⇓

Saving money out of Profits so that you can purchase an Asset in future or you can payoff a liability in Future

⇓

Saved amount is invested in which interest is earned



R = Saving

F.V. of Regular Annuity will be used

In this case benefit is Recieved in Future

Eg Suppose you have to payoff a Liability after 10 years for that you required ₹ 500000 after 10 years. How much you should invest in a sinking fund which gives 12% p.a. compounded Annually so that you can fulfill your future needs?

Sol:

0   1   2   3   - - -   10 yr

   R   R   R - - -   ⇓ 5L

Annuity

$$F.V. = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

$$500000 = R\left\{\frac{(1.12)^{10} - 1}{0.12}\right\}$$

$$500000 = R(17.54)$$

R = 28506

Net Present value

⇓

used for taking investment decisions

⇓

NPV = Present value of all cash inflow — Present value of all cash outflow

If NPV ≥ 0 Accept Proposal

If NPV < 0 Reject Proposal

Q In a project ₹ 50,000 spend today. It generated ₹ 30,000 after one year & 60,000 after 4 year. Find NPV of Project if discount Rate is 10% Annually.

Sol:

0	1 yr.	4 year
(50,000)	30,000	60,000

$$NPV = \frac{30000}{(1 \cdot 10)^1} + \frac{60,000}{(1 \cdot 10)^2} - 50,000$$

$$= 27272.72 + 49586.77 - 50,000$$

$$= 26859.49$$

Leasing

⇓

Taking Assent on Rent (For Long Period)

Owner of Asset	User of Asset
⇓	⇓
Lesser	Lessee
मालिक	किरायेदार

In this type of question Purchase value of Asset will be given.

But Problem is we dont

know how much should be the Annual Rent?

$\Downarrow$

use P.V. of Regular Annuity to find Reasonable Rent

$\Downarrow$

$$\text{Cost of Asset} = \text{Rent}\left\{\frac{1-(1+i)^{-n}}{i}\right\}$$

eg Mr. X can purchase a machine for ₹ 50,000 Today or he can take the machine on lease for 7 year in ₹ 9000 Annual Rent. which option is better?

Purchasing or Leasing if money is worth 10% Annually.

Sol: In this question find reasonable rent & then compare it with actual Rent (i.e. 9000)

$$50{,}000 = \text{Rent}\left\{\frac{1-(1+i)^{-n}}{i}\right\}$$

$$50{,}000 = R\left\{\frac{1-(1.10)^{-7}}{0.10}\right\}$$

$$50{,}000 = R(4.8684)$$

$$R = 10{,}270$$

Reasonable Rent is ₹ 10,270

But he is getting the machine on lease in much more cheaper Price (i.e. 9000)

So He should go for Leasing

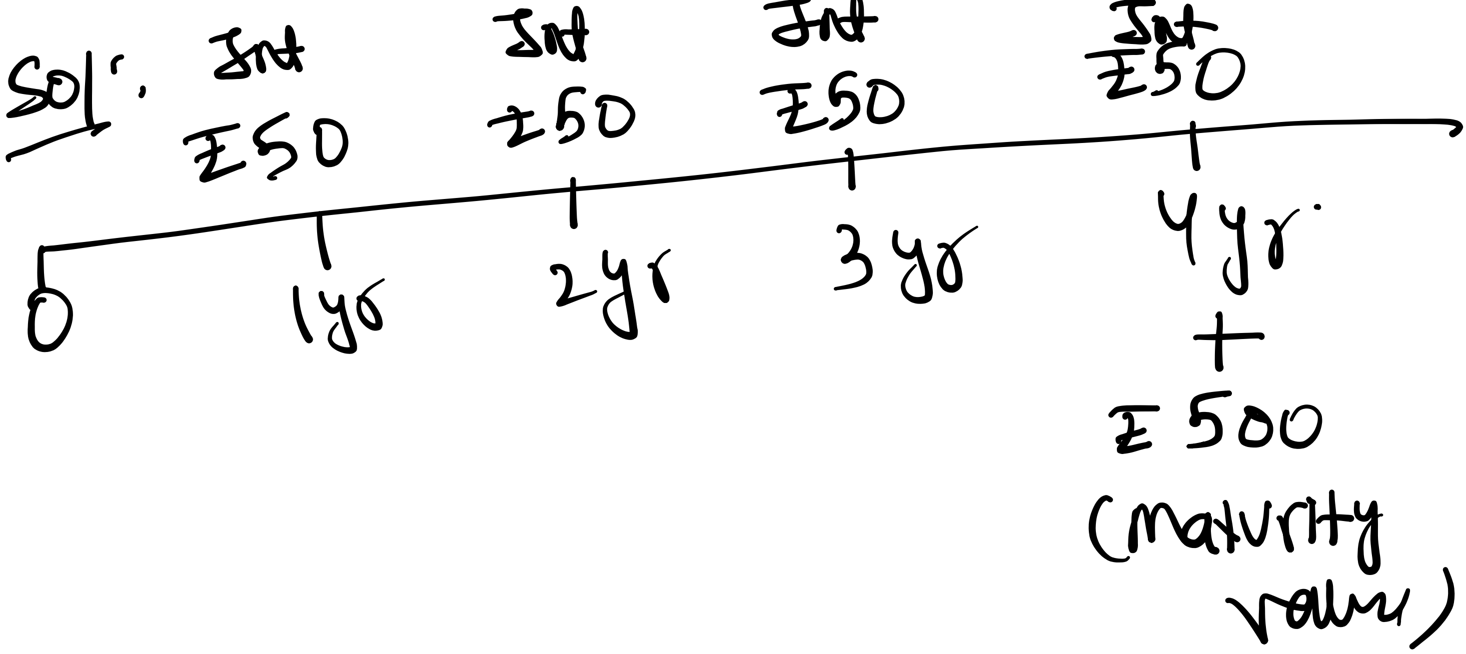
Valuation of Bond

⇓

Price at which Bond should be Purchased = Present value of All Future interest + Present value of Bond's maturity value

eg Bond of ₹ 500 at which interest Rate is 10% & maturity period is 4 year. At what Price it should be purchased if investor wants 15% Return.

Sol:



$$\text{Price of Bond} = \frac{50}{(1.15)^1} + \frac{50}{(1.15)^2} + \frac{50}{(1.15)^3} + \frac{50}{(1.15)^4} + \frac{500}{(1.15)^4}$$

$$= 43.47 + 37.80 + 32.87 + 28.58 + 285.87$$

$$= ₹428.59$$

C.A.G.R.

Compound Annual Growth Rate

Time	2010	2011	2012	2013
Revenue	100	110	140	160

find CAGR

Sol:

$P = 100$

$A = 160$

Time = 2013 − 2010 = 3 years

$$A = P[1+r]^t$$

$$160 = 100\,[1+r]^3$$

$$\frac{160}{100} = (1+r)^3$$

$$\left(\frac{160}{100}\right)^{1/3} = 1+r$$

$$r = \left(\frac{160}{100}\right)^{\frac{1}{3}} - 1$$

Or

Use Direct formula

$$CAGR = \left[\frac{V(t_n)}{V(t_0)}\right]^{\frac{1}{t_n - t_0}} - 1$$

$$= \left(\frac{160}{100}\right)^{\frac{1}{2013-2010}} - 1$$

$$= \left(\frac{160}{100}\right)^{\frac{1}{3}} - 1$$

$$= 1.1696 - 1$$

$$= 0.1696$$

or

16.96%

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