

Correlation and Regression

The tendency of simultaneous variation between two variable x and y is known as correlation means that whenever increment or decrement in one variable x produce any change in another variable y then x and y are said to be correlated.

→ Correlation can be two types

- ① Positive correlation ② Negative correlation.

$$r > 0$$

$$r < 0$$

① Positive When the variable moves in same direction means

$$\begin{array}{cc} x \uparrow & y \uparrow \\ x \downarrow & y \downarrow \end{array} \quad \text{---} \oplus$$

Ex - Profit and investment
Crop and Rainfall
Temperature and sale of cold drinks
Temperature and electricity bill
Age and premium amount.
Height and weight.

② Negative :-> When the variable moves in opposite directions

$$\begin{array}{cc} x \uparrow & y \downarrow \\ x \downarrow & y \uparrow \end{array} \quad \text{---} \ominus$$

Ex => Price and demand
Profit of Insurance company and number of claims.

Note :-> When movement in one variable doesn't produce any change in another variable then x and y are said to be uncorrelated No. correlation $r = 0$

* Spurious correlation :-

When there is no causal relation.

ex → shoe size and Intelligence

Note :-> The amount nature of correlation is dependent on the value of correlation coefficient which is denoted by r . The value of r is always between -1 to $+1$.

$-1 \leq r \leq +1$ both inclusive

$r = +1 \Rightarrow$ Perfect positive

$r = -1 \Rightarrow$ Perfect negative

$r = 0$ No correlated.

0 to 0.25 -0.25 to 0	→	Low degree (Poor)
0.25 to 0.75 -0.25 to -0.75]	Moderate
0.75 to +1 -0.75 to -1	→	High degree (good)

* Methods of find r .

Qualitative Point ① Spearman Rank correlation coefficient.

Attribute sign ② Concurrent deviation C.C.

Quickest ③

only linear Correlation. ④ Karl Pearson C.C.

and nature both.

Linear

Non-linear

Curvilinear

④ Scatter Diagram

only nature of correlation.

① Spearman's Rank correlation coefficient (r_s)

$x \Rightarrow \text{Rank} = R_x$

$y \Rightarrow \text{Rank} = R_y$

$R_x - R_y = d_i$

$\sum d_i = 0$ Sum of difference of Rank is 0

$\sum d_i^2 \neq 0$ Sum of square of difference of Rank $\sum d_i^2$

$$r = \frac{\sum d_i^2}{n(n^2-1)}$$

Q.

Rank x	Rank y	d_i	d_i^2
7	1	6	36
1	6	-5	25
6	7	-1	1
2	5	-3	9
5	2	3	9
3	4	-1	1
4	3	-1	1
		0	82

Sol $r = \frac{\sum d_i^2}{n(n^2-1)}$

$r = \frac{6 \times 82}{7 \times 48}$

$r = 1 - \frac{492}{336}$

$r = \frac{336 - 492}{336} = -0.46$

$r = -0.46$

Q

R_x	R_y
1	6
2	5
3	4
4	3
5	2
6	1

Q.

R_x	R_y
1	1
2	2
3	3
4	4
5	5
6	6

When ranks are in reverse order
 $r = -1$

When ranks are in same order
 $r = +1$

x	y	R_x	R_y	d_i	d_i^2
18	23	7	5	2	4
90	54	1	4	-3	9
55	61	3	3	0	0
60	72	2	2	0	0
25	81	5	1	4	16
40	19	4	7	-3	9
22	20	6	6	0	0
11	11	8	8	0	0
10	4	9	9	0	0
				0	38

$$\Rightarrow 1 - \frac{6 \times 38}{9 \times (81-1)}$$

$$\Rightarrow 1 - \frac{228}{720}$$

$$\Rightarrow \frac{720 - 228}{720}$$

$$\Rightarrow +0.68$$

Q. Sum of square of difference of Rank is 66 (Ed²)
 $n = 8$ Find r .

Sol. $r = 1 - \frac{6 \times 66}{8 \times 63}$

$$\Rightarrow 1 - \frac{396}{504}$$

$$\Rightarrow \frac{504 - 396}{504}$$

$$\Rightarrow 0.21 \text{ Ans}$$

Q. Rank correlation was 0.4

$$6 \sum d_i^2 = 990 \times 0.6 \Rightarrow 10$$

$\sum d_i^2 \Rightarrow \frac{990 \times 0.6}{6}$ by mistake one rank difference

is wrongly taken \rightarrow instead of 5

Wrong = $0.4 = 1 - \frac{\sum d_i^2}{10 \times 99}$

$$\frac{6 \sum d_i^2}{990} = 1 - 0.4$$

$$\sum d_i^2 \Rightarrow \begin{array}{r} 99 \\ - 49 \\ + 45 \end{array}$$

$$\sum d_i^2 = 75$$

$$1 - \frac{6 \times 45}{990}$$

$$1 - \frac{450}{990} = \frac{990 - 450}{990}$$

$$\Rightarrow 0.54 \text{ Ans}$$

Q. Rank correlation 0.52

$$n = 8$$

mistake 5 instead of 4

$$0.52 = 1 - \frac{6d_i^2}{8 \times 63}$$

$$\frac{6d_i^2}{504} = 0.48$$

$$d_i^2 = \frac{40.32}{6}$$

$$1 - \frac{6 \times 24.32}{504}$$

$$\Rightarrow \frac{504 - 145.92}{504}$$

$$\Rightarrow 0.71 \text{ Ans}$$

★ observation Repeat \Rightarrow

$$\rightarrow \text{tied length } m = \frac{8m^3 - m}{12}$$

$$r = \frac{1 - \frac{6 \left[8d_i^2 + \frac{8m^3 - m}{12} \right]}{n(n^2 - 1)}}{1}$$

Q.

x	y	R _x	R _y	d _i	d _i ²
15	12	9	8	2	4
40	16	6	7	-1	1
60	19	4	6	-2	4
40	22	6	4.5	1.5	2.25
90	50	1	2	-1	1

Hied $40 = 3 = M_1$
 Hied $85 = 2 = M_2$
 $23 = 2 = M_3$

85	82	2.5	2	1.5	2.25
20	23	.8	4.5	3.5	12.25
40	40	6	2	2	9
85	10	2.5	9	-6.5	42.25
11	5	10	10	0	0

$$\frac{\sum M^3 - M}{12}$$

$$\frac{3^3 - 3}{12} + \frac{2^2 - 2}{12} + \frac{2^2 - 2}{15}$$

$$15 + 3 \Rightarrow 18$$

$$2 + 0.5 + 0.5$$

$$\Rightarrow 3$$

$$1 - \frac{6 \times 18}{10 \times 93}$$

$$\Rightarrow 1 - 0.472$$

$$\Rightarrow 0.52$$

Q. Sum of square of diff of rank is 66. $n=8$
 Hied = 4, 3, 2
 $\sum di^2 = 6$

$$\text{Sol. Hied} = \frac{4^2 - 4}{12} + \frac{3^2 - 3}{12} + \frac{2^2 - 2}{12}$$

$$\Rightarrow 5 + 2 + 0.5$$

$$\Rightarrow 7.5$$

$$\text{So. } 1 - \frac{6 \times 75}{8 \times 63}$$

$$\text{So. } \Rightarrow 1 - 0.875$$

$$\Rightarrow 0.125$$

* Concurrent deviation. \Rightarrow

$$n \rightarrow \text{sign} = S_x$$

$$y \rightarrow \text{sign} = S_y$$

$$\text{Product} \Rightarrow S_x \times S_y$$

C = number of + sign

C = number of Concurrent deviation.

$$n = \text{Pairs}$$

$$M = n - 1$$

$$\Rightarrow r_c = \sqrt{\frac{2c - M}{M}}$$

$++ \Rightarrow +$ $= - \Rightarrow -$
 $-- \Rightarrow +$ $- + \Rightarrow -$
 $= = \Rightarrow +$ $+ - \Rightarrow -$
 $= + \Rightarrow -$

Q.

x	y	Σx	Σy	p
90	20	x	x	x
15	27	-	-	+
54	26	+	+	+
16	54	+	+	-
25	11	+	-	-
26	60	+	+	+
20	15	-	-	+
29	61	+	+	+
11	14	-	-	+

$c = 6$ (+ total)

$n = 9$

$M = 8$

$$r = \sqrt{\frac{2 \times 6 - 8}{8}}$$

$$r = \sqrt{\frac{12 - 8}{8}}$$

$$r = \sqrt{\frac{4}{8}}$$

$r = 0.7071$

x	y	Σx	Σy	p
15	13	x	x	x
24	10	+	-	-
90	100	+	+	+
11	23	-	-	+
99	54	+	+	+
16	61	-	+	-
100	21	+	-	-
100	21	=	=	+
24	54	-	+	-
25	10	+	-	-
25	4	=	-	-

$c = 4$

$n = 11$

$M = 10$

$$r = \sqrt{\frac{2 \times 4 - 10}{10}}$$

$$r = \sqrt{\frac{8 - 10}{10}}$$

$$r = \sqrt{\frac{-2}{10}}$$

$r = -0.4472$

Q. Number of concurrent deviation $\Rightarrow 10$

Pair 12

Find 8

$$C \Rightarrow 10$$

$$n = 12$$

$$m = 11$$

$$\sqrt{\frac{2 \times 10 - 11}{11}} \Rightarrow \sqrt{\frac{9}{11}} \Rightarrow 0.9045$$

$$Q. r_c = \frac{1}{\sqrt{3}}$$

Number of concurrent deviation $\Rightarrow 6$

Pair = P

Find = P

Solve

$$\frac{1}{\sqrt{3}} = \sqrt{\frac{2C - M}{m}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{2 \times 6 - (P-1)}}{2-1}$$

$$\frac{1}{\sqrt{3}} = \frac{12 - P + 1}{2-1}$$

$$33 - 3P = P - 1$$

$$40 = 4P$$

$$P \Rightarrow 10 \text{ Ans}$$

* Karl Pearson correlation

Product moment correlation coefficient

'Table give'

$$r = \frac{n \sum xy - \sum x - \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{2 \sum y^2 - (\sum y)^2}}$$

x	y	xy	x ²	y ²
2	2	4	4	4
4	3	12	16	9
8	3	24	64	9
16	2	32	250	4
<u>30</u>	<u>10</u>	<u>72</u>	<u>340</u>	<u>26</u>

$$r \Rightarrow \frac{4 \times 72 - 30 \times 10}{\sqrt{4 \times 340 - 900} \sqrt{4 \times 26 - 100}}$$

$$r \Rightarrow \frac{-12}{\sqrt{460} \sqrt{4}}$$

$$r \Rightarrow 0.27$$

Q.10
(Book)

$$n = 11$$

$$\sum dx \sum dy = 3942$$

$$\sum dx = 13$$

$$\sum dy = 42$$

$$\sum dx^2 = 2667$$

$$\sum dy^2 = 6964$$

Sol

$$\frac{11 \times 3942 - 13 \times 42}{\sqrt{11 \times 2667 - (13)^2} \sqrt{11 \times 6964 - (42)^2}}$$

$$\frac{43372 - 546}{\sqrt{29337 - 169} \sqrt{76604 - 1764}}$$

$$\frac{42827}{\sqrt{29168} \times 74840}$$

$$r \Rightarrow 0.92 \text{ Ans}$$

Q.

$$\text{Cov} = 40$$

$$a_x = 4$$

$$a_y = 5$$

find r

$$r = \frac{\text{Cov}}{a_x \times a_y}$$

$$r = \frac{40}{4 \times 5} = \frac{40}{20} \Rightarrow 2$$

(a)

0.2

(b)

2 x

(c)

0.5

(d)

none ✓

Q. $r = 0.5$
 $Cov = 20$
 $var(x) = 81$
 $var(y) = 7$

$$r = \frac{Cov}{a_x \times a_y}$$

$$0.5 = \frac{20}{9 \times a_y}$$

$$a_y = \frac{20}{0.5 \times 9}$$

$$a_y = \frac{20}{4.5}$$

$$a_y = 4.44$$

$$V_y = 19.75 \text{ Ans}$$

Q. Sum of Product of deviation = 42075 find r .

$$\sum dx dy = 42075 \quad n = 450$$

$$Cov \Rightarrow \frac{\sum dx dy}{n} \rightarrow \begin{matrix} var(x) = 64 \\ var(y) = 163 \end{matrix}$$

Sol $\Rightarrow \frac{42075}{450} \Rightarrow 93.5$

$$r \Rightarrow \frac{93.5}{8 \times 13}$$

$$r \Rightarrow .89 \text{ Ans}$$

Q. $Cov = 20$
 SD of x is 6 SD of y

$$r = \frac{Cov}{a_x \times a_y}$$

$$r = \frac{20}{6 \times 5}$$

(r is d. and less than 0)

option \Rightarrow

(a) More than 2

(b) More than 4

(c) More than 3

(d) None of these

* Properties

Correlation coefficient - (r)

- ① C.C is a unit free measure
- ② Change of origin \times
- ③ Change of scale \times
- ④ C.C does not depend on unit
- ⑤ C.C is pure real number
- ⑥ Linear equation property \rightarrow

$$u = \frac{x-a}{b} \quad v = \frac{y-c}{d}$$

Correlation coefficient between x and y is r .

Correlation between u and v is also $\pm r$.

Q $\hat{u} = \frac{\hat{x}-7}{4} \quad \hat{v} = \frac{\hat{y}-2}{2} \quad r_{xy} = 0.90$

Sol $\begin{matrix} ++ \\ ++ \end{matrix}$

then $r_{uv} = 0.90$ (No change).

$$u = -2x + 5$$

$$v = -3y + 7$$

$$r_{xy} = 0.64$$

$\begin{matrix} ++ \\ -- \end{matrix}$ No change

So, $r_{uv} = 0.64$

* Sign to measures.

$++$	$++$	$--$	$+-$	$++$	$--$	$++$
$++$	$--$	$++$	$+-$	$+-$	$+-$	$-+$
No	No	No	No	change	change	change.

Q.

u	v	x	y
3	7	-3	-4
4	2	-5	-2
5	9	-6	-7
6	4	-2	-2

Sol $\begin{matrix} ++ \\ -- \end{matrix}$

$$r_{uv} = 0.64$$

$$r_{xy} = 0.64$$

Q.

x	y	u	v
4	2	4	-3
5	6	5	-2
7	3	2	-1

Sol So $\begin{matrix} + + \\ + - \end{matrix}$
 $\Rightarrow -r$

\rightarrow Same Pattern.

x	y	U	V
5	6	3	4
-4	-3	-5	-4
-7	-2	-1	-1
6	7	2	3

0 not effect

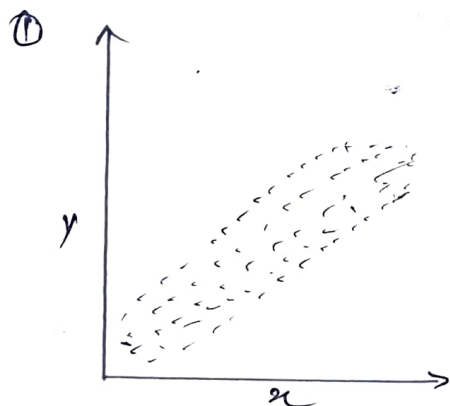
$$r_{xy} = 0.22$$

So No change

$$r_{uv} = 0.22$$

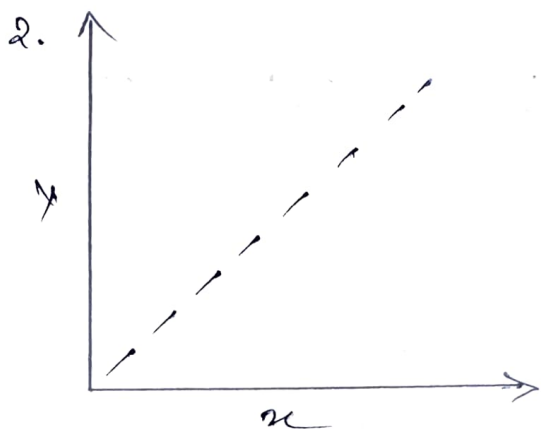
But the negative UV row -3 than the answer become $\Rightarrow -0.22$.

* (4) Scatter Diagram or DOT Diagram \Rightarrow



If the Plotted points moves in direction of lower left corner to upper right corner (scattered) there will be positive correlation.
 $r > 0$

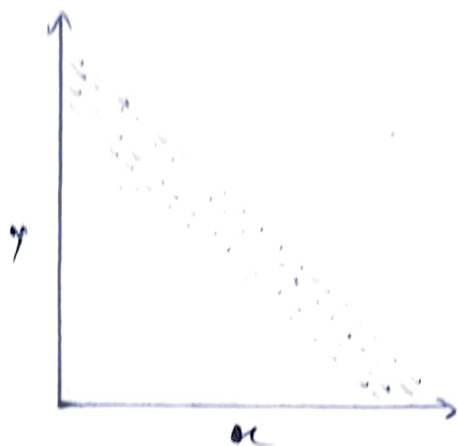
Now lines
 or
Custodines.



If the Plotted points moves in direction of lower left to upper right uniform of straight line there will be perfect positive correlation $r = +1$

Linear correlation.

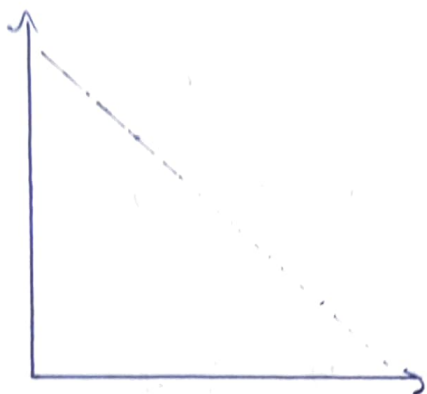
3.



If the plotted points move from upper left to lower right and they are dispersed there will be negative correlation

$$r < 0$$

4.



Upper left to lower right in form of straight line perfect negative. $r = -1$

Regression

Regression is concerned to make a mathematical relation between x and y it also predicts the value of one variable with respect to other variable.

For example: \rightarrow In profit and investment. Profit depend on investment so it is expressed as regression line or equation of Profit on investment.

So there are two straight line in Regression.

y on x regression

$$y = a + bx$$

$b = b_{yx}$ \Rightarrow regression coefficient of y on x

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$b_{yx} = \frac{\text{Cov}}{\sigma_x^2}$$

$$b_{yx} = \frac{\sum x \cdot ay}{\sum x^2}$$

x on y regression

$$x = a + by$$

$b = b_{xy}$ regression coefficient of x on y

$$x - \bar{x} = b_{xy} (y - \bar{y})$$

$$b_{xy} = \frac{\text{Cov}}{\sigma_y^2}$$

$$b_{xy} = \frac{\sum x \cdot ay}{\sum y^2}$$

$$r = \sqrt{b_{yx} \cdot b_{xy}}$$

$$b_{yx} + b_{xy} + r +$$

$$b_{yx} - b_{xy} - r -$$

$$b_{yx} + b_{xy} - r \times$$

$$b_{yx} > 1 \quad b_{xy} < 1$$

$$b_{yx} < 1 \quad b_{xy} > 1$$

$$Q. \bar{x} = 53.2$$

$$\bar{y} = 27.9$$

$$b_{yx} = -1.5$$

$$b_{xy} = -0.2$$

find y on x line

Sol $y - \bar{y} = b_{yx}(x - \bar{x})$

$$y - 27.9 = -1.5(x - 53.2)$$

$$y - 27.9 = -1.5x + 79.8$$

$$y - 27.9 + 79.8 = 1.5x$$

$$y - 107.7 = 1.5x$$

$$y = 107.7 - 1.5x$$

$$Q. x = 6$$

$$y = ?$$

find y on x line

$$y = 107.7 - 1.5 \times 6$$

$$y = 17.7$$

$$Q. \bar{x} = 10$$

$$\bar{y} = 20$$

$$b_{yx} = 0.4$$

$$b_{xy} = 0.5$$

x on y line

Sol $x - \bar{x} = b_{xy}(y - \bar{y})$

$$x - 10 = 0.5(y - 20)$$

$$x - 10 = 0.5y - 10$$

$$x - 10 + 10 = 0.5y$$

$$x = 0.5y$$

$$Q. y = 30$$

$$x = ?$$

$$x = 0.5 \times 30$$

$$x = 15$$

$$Q. b_{yx} = 0.6$$

$$b_{xy} = 0.4$$

$$r = ?$$

Sol $r = \sqrt{0.6 \times 0.4}$

$$r = 0.48$$

$$Q. b_{yx} = -0.25$$

$$b_{xy} = -0.40$$

$$r = \sqrt{0.25 \times 0.40}$$

$$r = -0.31$$

$$Q. x = 0.5$$

$$b_{xy} = 0.20$$

$$b_{yx} = ?$$

Sol $0.5 = \sqrt{0.20 \times b_{yx}}$

$$0.5 \times 0.5 = 0.20 \times b_{yx}$$

$$\frac{0.5 \times 0.5}{0.20} = b_{yx}$$

$$b_{yx} = 1.25$$

$$Q. b_{yx} = 0.75$$

$$r = 0.50$$

$$xy = 4$$

$$ax = ?$$

$$Q. \text{reg coeff } y \text{ on } x (b_{yx}) = 0.75$$

$$\text{coeff. of correlation } (r) = 0.50$$

$$y = 50$$

$$(xy) = 4$$

SD of $x = 2$

(σ_x) = ?

$$b_{yx} = \frac{\sigma_x \cdot \sigma_y}{\sigma_x^2}$$

$$0.75 = \frac{0.5 \times 4}{\sigma_y}$$

$$0.75 \times \sigma_y = 0.5 \times 4$$

$$\sigma_y = \frac{0.5 \times 4}{0.75}$$

$$\sigma_y = 2.66$$

Q. $\text{Cov} = 40$

$\sigma_x = 12$

$b_{yx} = ?$

Sol $b_{yx} = \frac{\text{Cov}}{\sigma_x^2} = \frac{40}{144} \Rightarrow 0.27$

Q. $r = -\frac{\sqrt{3}}{2}$

$$b_{yx} = \frac{-3}{4}$$

$$\text{var}(x) = 16$$

$$\text{var}(y) = ?$$

Sol. $\text{Cov} = \frac{\sigma_x \cdot \sigma_y}{r}$

$$\frac{-3}{4} = \frac{-\frac{\sqrt{3}}{2} \times \sigma_y}{4}$$

$$-3 = \frac{-\sqrt{3}}{2} \times \sigma_y$$

$$\frac{-6}{\sqrt{3}} = \sigma_y$$

$$\text{var}(y) = \left(\frac{6}{\sqrt{3}}\right)^2 \Rightarrow \frac{36}{3} \Rightarrow 12$$

$\text{var}(y) = 12$ Ans

Q. x

σ_x

avg. 40

SD 5.60

$r = 0.48$

y

Statics

50

6.30

$\sigma_y = 30$

marks $n = 30$

Statics = ?

$y = ?$

Sol $y - \bar{y} = b_{yx} (x - \bar{x})$

$$y - 50 = 0.54 (30 - 40)$$

$$y - 50 = -5.4$$

$$y = 50 - 5.4$$

$$\Rightarrow 44.6$$

$$b_{yx} = \frac{\sigma_x \cdot \sigma_y}{\sigma_x^2}$$

$$b_{yx} = \frac{0.48 \times 6.30}{5.60}$$

$$b_{yx} = 0.54$$

Q. Company

A

B

SD

10

12

Avg.

30

20

Sum of Product of dev $\Rightarrow 42075$

$n = 450$

Share A price $\Rightarrow 60$

$x = 6$

Share B Price = ?

$y = ?$

Sol $y - \bar{y} = b_{yx} (x - \bar{x})$

$$y - 20 = 0.935 (60 - 30)$$

$$y = 48.05$$

$$\text{Edmody} = 42075$$

$$\text{Cov} = \frac{\text{Edmody}}{n}$$

$$\text{Cov} = \frac{42075}{450}$$

$$\text{Cov} = 93.5$$

$$r_{yx} = \frac{\text{Cov}}{a_x^2} \Rightarrow \frac{93.5}{100}$$

$$r_{yx} = 0.935$$

$$\textcircled{1} \quad \begin{aligned} a_1 x + b_1 y + c_1 &\Rightarrow 0 \\ a_2 x + b_2 y + c_2 &\Rightarrow 0 \end{aligned}$$

$$\frac{a_1 b_2}{a_2 b_1} < 1$$

So y on x I
 x on y II

$$r = -\frac{a_1 b_2}{a_2 b_1}$$

$$\textcircled{2} \quad \frac{a_1 b_2}{a_2 b_1} > 1$$

y on x II
 x on y I

$$r = -\frac{a_2 - b_1}{a_1 - b_2}$$

$$\text{Q.} \quad \begin{aligned} 3x + 2y + 9 &= 0 \\ 14x + 5y + 2 &= 0 \end{aligned}$$

Sol $\frac{15}{28} < 1$ which y on x I
 x on y II

$$-\frac{\sqrt{15}}{28}$$

$$\Rightarrow -0.72$$

$$\Rightarrow y = a + bx \quad b_{yx} =$$

$$3x + 2y + 9 = 0$$

$$\rightarrow 9 - 3x = 2y$$

$$2y = -3x - 9$$

$$y = \frac{-3x - 9}{2} \quad b_{yx} = \frac{-3}{2}$$

$$\Rightarrow x = a + by$$

$$14x + 5y + 2 = 0$$

$$2 + 5y = 14x$$

$$y = \frac{14x - 2}{5}$$

$$\text{Q.} \quad 2x + 3y - 7 = 0$$

$$4x - 5y - 2 = 0$$

+ - None of these

y on x (x)

$$\text{Q.} \quad 4x + 5y - 6 = 0$$

$$3x + 12y - 9 = 0$$

y on x II

x on y I

$$\frac{48}{15} > 1 \quad r = -\frac{\sqrt{15}}{48}$$

$$r = -0.55$$

$$\text{Q.} \quad 4x + 5y + 6 = 0 \text{ reg. line}$$

Sol one line so sign \oplus
than value of r is $= -1$

$$\text{Q.} \quad 4x - 5y + 6 = 0 \text{ reg. line}$$

Sol one line so sign \oplus

than value of r is $+1$

$$r = +1 \quad \text{or} \rightarrow -1$$

$$2x + 3y - 7 = 0$$

$$4x - 5y + 2 = 0$$

then $r = 0$

Perpendicular line

Q. $3x + 2y + 9 = 0$
 $12x + 7y + 5 = 0$

$$\frac{21}{24} < 1 \quad \begin{matrix} \text{y on x I} \\ \text{x on y II} \end{matrix}$$

an ay . ratio

$$r_2 = \sqrt{\frac{21}{24}}$$

- 0.93

$$b_{yx} = \frac{ax \cdot ay}{aa}$$

$$-\frac{3}{2}x + \frac{0.93 \cdot ax}{ax}$$

$$b_{yx} =$$

$$2y = -3x - 9$$

$$y = -\frac{3}{2}x - \frac{9}{2}$$

$$b_{yx} = -\frac{3}{2}$$

$$b_{xy} = -\frac{7}{12}$$

Properties:-

- ① b_{yx} and b_{xy} all called reg. coefficients
- ② r_1 , b_{yx} , b_{xy} should be of same sign.
- ③ $b_{yx} + b_{xy} + r$ will be +
- ④ $b_{yx} - b_{xy} - r$ will be -
- ⑤ $b_{yx} + b_{xy} - r$ will not be determined.
- ⑥ $r = \sqrt{b_{yx} \times b_{xy}}$
- ⑦ Correlation coefficient is geometric mean of regression coefficient
- ⑧ $r^2 = b_{yx} \times b_{xy}$
- ⑨ $r^2 \leq 1 \quad r \leq 1$
- ⑩ $b_{yx} \times b_{xy} \leq 1$ The product of reg. coefficient is less than or equal to unity
- ⑪ Reg. coefficient Independent from origin.
- ⑫ Reg. coefficient dependent on change of scale.
- ⑬ A.M \geq G.M $\quad \frac{a+b}{2} \geq \sqrt{ab}$
- ⑭ $\frac{b_{yx} + b_{xy}}{2} \geq \sqrt{b_{yx} \times b_{xy}}$
- ⑮ A.M of reg. coefficient is greater or equal to r .
- ⑯ $b_{yx} > 1 \quad b_{xy} < 1$

If one of the reg. coefficient is more than unity other reg. coefficient should less than unity

(16) $b_{yx} > 1$ $b_{xy} < 1$

(17) $b_{yx} < 1$ $b_{xy} > 1$

(18) Linear equation

$$u = \frac{x-a}{p} \quad v = \frac{y-b}{q}$$

$$b_{yx} = \frac{q}{p} \times b_{vu} \quad b_{xy} = \frac{p}{q} \times b_{uv}$$

$$u = 2x+5 \quad v = 3y+7 \quad b_{yx} = 2.4 \quad b_{vu}$$

$$p = \frac{M}{x} = \frac{1}{2}$$

$$b_{yx} = \frac{q}{p} \times b_{vu}$$

$$q = \frac{V}{y} = \frac{1}{3}$$

$$2.4 = \frac{\frac{1}{3}}{\frac{1}{2}} \times b_{vu}$$

$$1.2 = \frac{1}{3} \times b_{vu}$$

$$b_{vu} \Rightarrow 3.6$$

$r \Rightarrow r^2 =$ Coefficient of determination.

$$r^2 = \frac{\text{Explained variation}}{\text{Total variation}}$$

$r^2 \Rightarrow$ Percentage of variation accounted

$1-r^2 =$ Coefficient of Non-determination.

$$1-r^2 = \frac{\text{Unexplained variation}}{\text{Total variation}}$$

$1-r^2 =$ % of variation unaccounted

\Rightarrow In regression there are two types of Errors.
Errors को Residue भी कहते हैं।

Errors \Rightarrow observed value - estimated value

① Standard Errors

② Probable Errors

$$S.E \Rightarrow \frac{1-r^2}{\sqrt{n}}$$

$$P.E \Rightarrow 0.6745 \times \frac{1-r^2}{\sqrt{n}}$$