

STATISTICS

- \* Statistics is a branch of mathematics
- \* Statistics is not for individuals. <sup>or</sup> ~~or~~
- \* Statistics is aggregate (group) of facts.
- \* Statistics word is originated in 1749 by Gaott Ericd

Acmenwall  
\* Statistics word is suggested by some words.

(i) Latin word  $\rightarrow$  Status

(ii) Italian word  $\rightarrow$  Statista

(iii) German word  $\rightarrow$  Statistik

(iv) French word  $\rightarrow$  Statistique

↓  
French word  $\leftarrow$  Statistics

Definition  $\Rightarrow$

Statistics deals with collection classification analysis's presentation and interpretation of numerical Data.   
 (Decision making)

Definition  $\rightarrow$  2 Sense

- (i) Singular sense  $\rightarrow$  Data is defined in terms of collection classification presentation of data.
- (ii) Plural sense

\* Data is defined in terms of Quantitative and Qualitative data.

Measures of central tendency (Meat)  $\Rightarrow$  1<sup>st</sup> order averages

$$A.M \Rightarrow 2 + 5 + 6 + 8 + 9 \dots$$

$$G.M \Rightarrow 2 \times 5 \times 6 \dots$$

$$H.M \Rightarrow \frac{1}{2} \quad \frac{1}{5} \quad \frac{1}{7} \dots$$

$$2 \quad 5 \quad (6) \quad 8 \quad 9$$

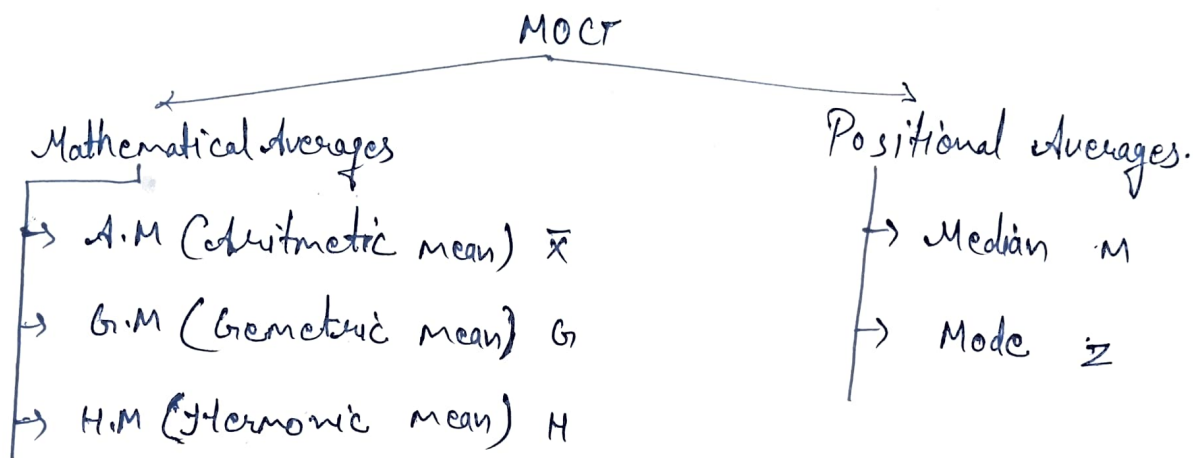
Median

→ MOCT measures central location of observation.

→ MOCT is also known as averages.

→ In this tendency there is a single value which can be regarded as representative value of data set (Observations)

MOCT (Averages) are two types :->



A.M for individuals series

①  $\bar{x} = \frac{\text{Sum of observations}}{\text{number of observations}}$

②  $\bar{x} = \frac{\sum x}{n}$

③  $\bar{x} \Rightarrow \frac{\text{1st value} + \text{last value}}{2}$  (if difference is same)

④ Assumed mean method

$$\bar{x} = A + \frac{\sum dx}{n}$$

$\sum dx \rightarrow$  deviation

$$\sum dx = \sum (x - A)$$

$$\sum dx = \sum (x - \bar{x}) = 0$$

Question :->  $\frac{x}{2} \quad \frac{x}{3} \quad \frac{x}{4} \quad \frac{x}{5}$

A.M  $\Rightarrow$  40 find  $x$ .

$$\frac{\frac{x}{2} + \frac{x}{3} + \frac{x}{4} + \frac{x}{5}}{4} \Rightarrow 40$$

$$\frac{x}{2} + \frac{x}{3} + \frac{x}{4} + \frac{x}{5} \Rightarrow 160$$

$$\frac{30x + 20x + 15x + 12x}{60} = 160$$

$$\frac{77x}{60} = 160$$

$$77x \Rightarrow 9600 \quad (\text{160} \times 60)$$

$$x \Rightarrow \frac{9600}{77}$$

$$x \Rightarrow 124.6 \text{ Ans}$$

\* A.M for discrete series  $\Rightarrow$

Q.  $x$  |  $f$  |  $fx$

frequency.

variable

$x_1$	$f_1$	
$x_2$	$f_2$	
$x_3$	$f_3$	
$\vdots$	$\vdots$	
$x_n$	$f_n$	
	$N$	

$\bar{x} \Rightarrow \frac{\sum fx}{\sum f}$

or

$\bar{x} = \frac{\sum fx}{N}$

$N = \sum f$

Q.  $x$  |  $f$  |  $fx$

1	1	
2	2	
3	3	
4	4	
5	5	
$\vdots$	$\vdots$	
16	16	
	16	

$$\bar{x} \Rightarrow \frac{2n+1}{3}$$

$$\bar{x} \Rightarrow \frac{2 \times 16 + 1}{3}$$

$$\bar{x} \Rightarrow \frac{32+1}{3}$$

$$\bar{x} \Rightarrow 11 \text{ Ans}$$

Q.  $x$  |  $f$  |  $fx$

2	2	4
4	3	12
8	3	24
16	2	32
	10	72

~~$$\bar{x} \Rightarrow \frac{72}{10}$$~~

$$\bar{x} \Rightarrow \frac{72}{10} \Rightarrow 7.2$$

\* A.M for continuous series  $\Rightarrow$

Grouped frequency distribution

Exclusive classification

$$\bar{x} \Rightarrow \frac{\sum fx}{\sum f}$$

$\bar{x} \Rightarrow$  Mid value of class.

$x$	$f$	$x$	$fx$
8-16	2	12	24
16-24	3	20	60
24-32	4	28	112
32-40	2	36	72
40-48	5	44	220
	16		488

$$\bar{x} \Rightarrow \frac{\sum fx}{\sum f}$$

$$\bar{x} \Rightarrow \frac{488}{16}$$

$$\bar{x} \Rightarrow 30.5 \text{ Ans}$$

a. Combined A.M

Average of 5 pencil is 30 Rs.

Average of 10 copy is 40 Rs.

Average of total quantity

$$I \quad n_1, n_2$$

$$5 \rightarrow 30 \Rightarrow 150$$

$$10 \rightarrow 40 \Rightarrow 400$$

$$II \quad x_1, x_2$$

$$\bar{x} \Rightarrow \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\bar{x} \Rightarrow \frac{5 \times 30 + 10 \times 40}{5 + 10} \Rightarrow \frac{150 + 400}{15}$$

$$\bar{x} \Rightarrow 36.66 \text{ Ans}$$

Aug. of 5 quantity in 100 Rs.

Aug. of next 5 is 80 Rs.

Aug. of next 5 is 50 Rs.

Aug. of all.

$$\bar{x} \Rightarrow n, x$$

$$5 \rightarrow 100 \Rightarrow 500$$

$$5 \rightarrow 80 \Rightarrow 400$$

$$5 \rightarrow 50 \Rightarrow \frac{250}{1150}$$

$$\Rightarrow \frac{1150}{15}$$

$$\Rightarrow 76.66 \text{ Ans}$$

Aug. of 20 students is 400

Aug. of 12 of them is 200

Aug. of remaining students

$$20 \rightarrow 400 \Rightarrow 8000$$

$$12 \rightarrow 200 \Rightarrow \frac{2400}{5600}$$

$$\Rightarrow \frac{5600}{8}$$

$$\Rightarrow 700 \text{ Ans}$$

Average of 30 students is 100

4 students joined the class.

Average increased by 5 find avg. of joined students

$$34 \rightarrow 105 \Rightarrow 3570$$

$$30 \rightarrow 100 \Rightarrow \frac{3000}{570}$$

$$\Rightarrow \frac{570}{4}$$

$$\Rightarrow 142.5 \text{ Ans}$$

Q. 50 students avg. 5850

But mistake two values are wrongly taken as 4500 and 5400 instead of 7800 and 8000 find correct mean.

$$\begin{array}{r} 50 \rightarrow 5850 \Rightarrow 292500 \\ - 4500 \\ - 5400 \\ + 7800 \\ + 8000 \\ \hline 298400 \end{array}$$

$$\Rightarrow \frac{298400}{50}$$

$$\Rightarrow 5968 \text{ Ans}$$

$y \Rightarrow a + bx \rightarrow$  slope  
Intercept  $\swarrow$  Change of origin.  
 $\searrow$  Change of scale

Change of origin  $\rightarrow$  add. sub + each.

Change of scale  $\rightarrow x \div$

Q. A.M  $\Rightarrow$  10 each data added by 6 New A.M

$$\bar{x} \Rightarrow 10 + 6$$

$$\bar{x} \Rightarrow 16 \text{ Ans}$$

Q. A.M - 5 each data  $\times$  by 4 then  $\div$  10 New A.M

$$\bar{x} \Rightarrow \frac{5 \times 4}{10} \Rightarrow \frac{20}{10}$$

$$\bar{x} \Rightarrow 2 \text{ Ans}$$

Q. A.M  $\Rightarrow \frac{\bar{x}}{10}$  each data  $\div$  by 10 then increased by  $\alpha$

$$\frac{\bar{x}}{10} + \alpha \Rightarrow \frac{\bar{x} + 10\alpha}{10}$$

Q.  $y \Rightarrow 2 + 5x$   $x$  mean is 8  $y$  mean?

$$\bar{y} \Rightarrow 2 + 5x$$

$$\bar{y} \Rightarrow 2 + 5 \times 8$$

$$\bar{y} \Rightarrow 42 \text{ Ans}$$

Q.  $2x + 3y - 7 \Rightarrow 0$   $x$  A.M is  $\neq y$  A.M

$$2(1) + 3y - 7 \Rightarrow 0$$

$$2 + 3y - 7 \Rightarrow 0$$

$$3y - 5 \Rightarrow 0$$

$$3y \Rightarrow 5$$

$$\boxed{\bar{y} \Rightarrow \frac{5}{3}}$$

Q.  $x$  A.M is 50 find A.M of  $\frac{3x - 100}{5}$

$$\bar{y} = \frac{3x - 100}{5}$$

$$\bar{y} \Rightarrow \frac{150 - 100}{5}$$

$$\bar{y} \Rightarrow \frac{50}{5}$$

$$\bar{y} \Rightarrow 10$$

Q.  $x$  Mean is  $b$  find mean of  $\frac{x - b}{a}$

$$\frac{x - b}{a} \Rightarrow \frac{b - b}{a} \Rightarrow \frac{0}{a} \Rightarrow 0 \text{ Ans}$$

- A.M is the mathematical Averages.
- A.M is based on all observations.
- A.M can be + or - or 0
- For equal observation  $K, K, K, K \rightarrow A.M \Rightarrow K$
- A.M is the best measure of central tendency.
- A.M is rigidly defined everywhere.
- A.M is stable measure of C.T
- A.M cannot be determined graphically.
- A.M is highly affected by presence of extreme observations.

→ Change of origin ✓

→ Change of scale ✓

→ Linear Equation Property

$$y = a + b \cdot x$$

$$\bar{y} \Rightarrow a + b \cdot \bar{x}$$

Median



Q. 6, 16, 10, 50, 40, 90, 15 find median?

$n = 7$   
6, 10, 15, 16, 40, 50, 90

odd

$M \Rightarrow 16$

(Middle value of numbers).

Q. 9, 50, 40, 30, 100, 11, 17, 44 find median?

$n = 8$

9, 11, 17, 30, 40, 44, 50, 100

even  $\Rightarrow \frac{I + II}{2}$

$M \Rightarrow \frac{30 + 40}{2}$

$M \Rightarrow 35$  Ans

Q.  $a-8$   $a+4$   $a+\frac{1}{2}$   $a$   $a-1$   $a+3$   $a-\frac{1}{2}$  find median?

$$\left( a-8 \quad a-1 \quad a-\frac{1}{2} \right) \quad a \quad \left( a+\frac{1}{2} \quad a+3 \quad a+4 \right)$$

$M \Rightarrow a$  Ans

Q.  $a+\frac{1}{2}$ ,  $a-1$ ,  $a+4$ ,  $a+\frac{3}{2}$ ,  $a+7$ ,  $a-5$ ,  $a$ ,  $a+6$ ,  $a+2$ ,  $a-3$   
find median?

$$\left( a-5, a-3, a-1, a \right), a+\frac{1}{2}, a+\frac{3}{2}, \left( a+2, a+4, a+6, a+7 \right)$$

$$M \Rightarrow \frac{a+\frac{1}{2} + a+\frac{3}{2}}{2}$$

$$M \Rightarrow \frac{2a+2}{2} = \frac{2(a+1)}{2}$$

$M \Rightarrow a+1$  Ans

Q. 10 12 14  $x+2$   $x+4$  40 50 60  
median  $\Rightarrow$  30  
find  $x$

$$M \Rightarrow \frac{x+2 + x+4}{2}$$

$$30 \Rightarrow \frac{2x+6}{2}$$

$$2x+6 \Rightarrow 30 \times 2$$

$$2x \Rightarrow 60-6$$

$$x \Rightarrow \frac{54}{2}$$

$$x \Rightarrow 27$$

$$x+2 \Rightarrow 27+2 \Rightarrow 29$$

$$x+4 \Rightarrow 27+4 \Rightarrow 31$$



Q.  $\frac{x}{4}$   $\frac{x}{2}$   $\frac{x}{5}$   $\frac{x}{3}$  Median  $\Rightarrow$  40 find  $x$ .

$$\frac{x}{5} \cdot \frac{x}{4} \frac{x}{3} \frac{x}{2}$$

$$40 \Rightarrow \frac{\frac{x}{4} + \frac{x}{3}}{2}$$

$$2 \times 40 \Rightarrow \frac{3x + 4x}{12}$$

$$3x + 4x \Rightarrow 80 \times 12$$

$$7x \Rightarrow 960$$

$$x \Rightarrow \frac{960}{7}$$

$$x \Rightarrow 137.142 \text{ Ans}$$

Q. In a class of 11 students 3 all failed  
 Marks of passed candidates all.

90, 84, 33, 56, 61, 99, 80, 74 had median of class

(A, B, C, 33, 56), 61, (74, 80, 84, 90, 99)

$$M \Rightarrow 61 \text{ Ans}$$

\* Median for discrete series  $M \Rightarrow \frac{N+1}{2}$  value

X	f	C.F
5	3	3
15	4	7
40	5	12
60	7	19
90	9	28
92	8	36
N $\Rightarrow$ 36		

$$M \Rightarrow \frac{n+1}{2}$$

$$M \Rightarrow \frac{36+1}{2}$$

$$M \Rightarrow 18.5$$

$$M \Rightarrow 60 \text{ Ans}$$

Search in C.F  
 $N \Rightarrow 36$

C.F  $\Rightarrow$  Cumulative frequency

Q.  $\frac{3}{\frac{1}{3} + \frac{1}{6} + \frac{1}{12}}$   
use calculator trick.

$$1 \div 3 \quad M+$$

$$1 \div 6 \quad M+$$

$$1 \div 12 \quad M+$$

MRC

$$\div 3$$

$$\div = 5.14 \text{ Ans}$$

Q.  $\frac{4}{\frac{1}{6} + \frac{1}{8} + \frac{1}{12} + \frac{1}{36}}$

$$1 \div 6 \quad M+$$

$$1 \div 8 \quad M+$$

$$1 \div 12 \quad M+$$

$$1 \div 36 \quad M+$$

MRC

$$\div 4$$

$$\div = 9.93 \text{ Ans}$$

Q.  $\frac{10}{\frac{2}{2} + \frac{3}{4} + \frac{3}{8} + \frac{2}{16}}$

$$2 \div 2 \quad M+$$

$$3 \div 4 \quad M+$$

$$3 \div 8 \quad M+$$

$$2 \div 16 \quad M+$$

MRC

$$\div 10 \div = 4.44 \text{ Ans}$$

Q.  $\frac{140}{\sqrt{351 \times 352}}$

$$351 \times 352$$

$$= \sqrt{\quad} \div 140 \div =$$

$$\Rightarrow 0.39$$

\* Median for continuous series  $\frac{N}{2}$  search in c.f median class.

$$M \Rightarrow \frac{L_1 + \frac{N}{2} - C}{f} \times i$$

$L_1 \Rightarrow$  Lower class limit

$N \Rightarrow$   $\Sigma f$

$C \Rightarrow$  c.f. of pre median class.

$f \Rightarrow$  frequency of median class.

$i \Rightarrow$  Class length.  $UCB - LCB$

$x$	$f$	c.f
8-16	3	3
16-24	4	7
24-32	5	12
32-40	7	19
40-48	9	28
48-56	8	36
	36	

$$M \Rightarrow \frac{L_1 + \frac{N}{2} - C}{f} \times i$$

$$M \Rightarrow 32 + \frac{18 - 12}{7} \times 8$$

$$M \Rightarrow 32 + 6.85$$

$$M \Rightarrow 38.85 \text{ Ans.}$$

Q. Median of  $x$  is  $2$ ,  $3x + 2y - 10 = 0$   $y$  median.

$$6x + 2y - 10 = 0$$

$$2y = 4$$

$$y = 2$$

Q.  $x$  median is 3 median of  $3x - 4$

$$y = 3x - 4$$

$$y \Rightarrow 3 \times 3 - 4$$

$$y \Rightarrow 5 \text{ Ans}$$

Q.  $M \Rightarrow 20$  each data  $x$  by 4 New

$$\Rightarrow 20 \times 4$$

$$\Rightarrow 80 \text{ Ans}$$

- i) Median is a Positional average of central tendency.
- ii) Median is not based on all observations.
- iii) Median is based on central 50% value of observation.
- iv) For equal observation.  $K K K K$ ,  $M = K$
- v) Median is + or - or 0
- vi) Median can be determined graphically using ogive curve.
- vii) Median is not affected by presence of extreme values.
- viii) Median is appropriate for open end classification.

Change of origin ✓

Change of scale ✓

Linear equation property ✓

$$y = a + bx \quad (y_m \Rightarrow a + b \cdot x_m)$$

$$y \Rightarrow 2 + 3x \quad x \text{ median is } 4$$

$$y_m \Rightarrow 2 + 3 \times 4$$

$$y_m \Rightarrow 2 + 12$$

$$y_m \Rightarrow 14 \text{ Ans}$$

## Mode

$\Rightarrow$  Highest frequency observation.

Q. 5, 7, 17, 19, 51, 17, 51, 17

$\Rightarrow$  17  $\in$  Unimodal data set.

Q. 6, 11, 50, 50, 11, 11, 9, 13, 50, 55

$\Rightarrow$  11 and 50 Bimodal data set.

(2 more multimodal data set).

Discrete series.

x	f
4	5
50	15
60	11
16	15
9	6
21	9

$\Rightarrow$  50 and 16  
Bimodal

x	f
5	3
15	17
11	6
9	9
19	15

$\Rightarrow$  15  
Unimodal

Skewed

5	20
7	50
8	50
10	80
7	

Symmetrical distribution

(In which mean, mode, median are equal are called symmetrical distribution.)

(In which mean, mode, median, are not equal are called skewed distribution.)

\* Mode for Continuous series

Highest frequency  $\Rightarrow$  Modal class.

$$\Rightarrow l_i + \frac{f_i - f_0}{2f_i - f_0 - f_2} \times i$$

$2i \Rightarrow$  Lower class limit.

$F_i \Rightarrow$  Frequency of modal class.

$F_0 \Rightarrow$  Frequency of pre modal class

$F_2 \Rightarrow$  Frequency of post modal class.

$i \Rightarrow$  Class length.

$x$	$f$	$x$	$f$	
0-20	5	0-30	12	$F_0$
20-30	7	30-60	17	$F_i$
30-60	17	60-90	8	$F_2$
60-80	6			
80-90	2			

$$z \Rightarrow 2i + \frac{F_i - F_0}{2F_i - F_0 - F_2} \times i$$

$$z \Rightarrow 30 + \frac{17 - 12}{2 \times 17 - 12 - 8} \times 30$$

$$z \Rightarrow 30 + \frac{150}{14}$$

$$z \Rightarrow 40.71 \text{ Ans}$$

Mode is a positional measure of central tendency

Mode is not based on all observations.

Mode can be determined graphically using Histogram curve

Mode is fashionable measure of C.T  $\rightarrow$  scale of woollen clothes

Mode is not uniquely defined everywhere

$K, K, K, K$ . mode =  $K$

Mode can be + or - or 0

Change of origin  $\checkmark$

Change of scale  $\checkmark$

Linear equation  $\checkmark$

# Relation between mean, median, mode

$\bar{x} = M = Z$  Symmetrical Distribution  $y = 0$   
 $\bar{x} \neq M \neq Z$  Skewed Distribution  $y > 0$   $y < 0$   
 $\bar{x} < M < Z$  Negative skewed  
 $\bar{x} > M > Z$  Positive skewed.

Mean - mode = 3(mean - median).

$$\bar{x} - Z = 3(\bar{x} - M)$$

$$\bar{x} - Z = 3\bar{x} - 3M$$

$$3M - Z = 2\bar{x}$$

$$Z = 3M - 2\bar{x}$$

Q.  $\bar{x} = 25$

$M = 25$

$Z = 25$

Symmetrical method

Q.  $\bar{x} = 40$

$M = 20$

$Z = 10$

Positive skewed

Q. mean = 32.40

median = 44.60

find mode

$$Z = 3M - 2\bar{x}$$

$$Z = 3 \times 44.60 - 2 \times 32.40$$

$$Z = 133.8 - 64.8$$

$$Z = 69 \text{ Ans}$$

Q. Mean - mode = 10

Mean - mode = 20 median?

$$\bar{x} - Z = 10$$

$$\bar{x} + Z = 20$$

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$$2\bar{x} = 30$$

$$\bar{x} = 15$$

$$15 - Z = 10$$

$$Z = 5$$

$$Z = 3M - 2\bar{x}$$

$$5 = 3M - 30$$

$$3M = 35$$

$$M = \frac{35}{3}$$

$$M = 11.66 \text{ Ans}$$

## Harmonic Mean.

$$H.M \Rightarrow \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$

Q. 5, 10, 15, 20, 25 H.M?

Ans H.M  $\Rightarrow \frac{5}{\frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{1}{20} + \frac{1}{25}}$

$\Rightarrow 10.94$  Ans

Q. 4, -5, -6, 2

H.M. cannot be determined.

Q. 4, 5, 0, 6

H.M  $\Rightarrow \frac{4}{\frac{1}{4} + \frac{1}{5} + \frac{1}{0} + \frac{1}{6}}$  H.M. cannot be determined.

\* H.M for discrete series.

x	f
$x_1$	$f_1$
$x_2$	$f_2$
$x_3$	$f_3$
$\vdots$	$\vdots$
$x_n$	$f_n$

$$H.M \Rightarrow \frac{N}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \frac{f_3}{x_3} + \dots + \frac{f_n}{x_n}}$$

$$H.M \Rightarrow \frac{10}{\frac{2}{2} + \frac{3}{4} + \frac{3}{8} + \frac{2}{16}}$$

H.M  $\Rightarrow 4.44$  Ans



- H.M is a mathematical average.
- H.M is based on all observation.
- H.M is always positive
- If any observations is not negative or 0. H.M cannot be determined
- For equal observation  $K, K, K \Rightarrow K \rightarrow H.M \Rightarrow K$ .
- H.M is reciprocal of average of reciprocal of observation.
- Change of origin  $\cdot x$
- Change of scale  $\_$
- Linear equation property  $x$ .
- H.M is used to calculate average speed.

### Geometric mean

$$G.M \Rightarrow (x_1 \times x_2 \times x_3 \dots x_n)^{\frac{1}{n}}$$

<p>Q. 4 9</p> <p>G.M <math>\Rightarrow (4 \times 9)^{\frac{1}{2}}</math></p> <p style="margin-left: 40px;"><math>36^{\frac{1}{2}}</math></p> <p>G.M <math>\Rightarrow 6</math>.</p>	G.M	<p>Q. 6 8 12 36</p> <p>G.M <math>\Rightarrow (6 \times 8 \times 12 \times 36)^{\frac{1}{4}}</math></p> <p style="margin-left: 40px;"><math>\Rightarrow (20736)^{\frac{1}{4}}</math></p> <p style="margin-left: 40px;"><math>\Rightarrow 12^{\frac{1}{4}}</math></p> <p>G.M <math>\Rightarrow 12</math></p>	G.M	<p>Q. 3 6 12</p> <p><math>(3 \times 6 \times 12)^{\frac{1}{3}}</math></p> <p><math>(216)^{\frac{1}{3}}</math></p> <p><math>(6)^3</math></p> <p>G.M <math>\Rightarrow 6</math></p>	G.M
				10 12 14 11	5 6 7 8

Q. 2 4 8 16 32 64 find G.M.

$$\begin{aligned}
 G.M &\Rightarrow (2 \times 4 \times 8 \times 16 \times 32 \times 64)^{\frac{1}{6}} \\
 &\Rightarrow (2 \times 2^2 \times 2^3 \times 2^4 \times 2^5 \times 2^6)^{\frac{1}{6}} \\
 &\Rightarrow (2^{21})^{\frac{1}{6}} \\
 &\Rightarrow 2^{\frac{21}{6}} \\
 &\Rightarrow 2^{\frac{7}{2}} \quad \text{Ans}
 \end{aligned}$$

Q. 8 24 40 find G.M.

$$\begin{aligned}
 \text{G.M} &\Rightarrow (8 \times 24 \times 40)^{\frac{1}{3}} && \text{G.M} \\
 &\Rightarrow (8 \times 8 \times 3 \times 8 \times 5)^{\frac{1}{3}} && \sqrt[3]{144} \\
 &\Rightarrow (8^3)^{\frac{1}{3}} \times (15)^{\frac{1}{3}} && \sqrt[3]{15} \\
 &\Rightarrow 8 \times \sqrt[3]{15} && \sqrt[3]{15} \checkmark \\
 &\Rightarrow 8\sqrt[3]{15} \text{ Ans}
 \end{aligned}$$

Q. G.M for discrete series.

x	f
$x_1$	$f_1$
$x_2$	$f_2$
$x_3$	$f_3$
$\vdots$	$\vdots$
$x_n$	$f_n$
N	

$$\text{G.M} \Rightarrow (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3})^{\frac{1}{N}}$$

x	f
2	2
4	3
8	3
16	2
10	

$$\begin{aligned}
 \text{G.M} &\Rightarrow (2^2 \times 4^3 \times 8^3 \times 16^2)^{\frac{1}{10}} && \text{G.M.} \\
 &\Rightarrow (2^2 \times (2^2)^3 \times (2^3)^3 \times (2^4)^2)^{\frac{1}{10}} && \sqrt{2} \\
 &\Rightarrow (2^2 \times 2^6 \times 2^9 \times 2^8)^{\frac{1}{10}} && \sqrt[3]{2} \checkmark \\
 &\Rightarrow (2^{25})^{\frac{1}{10}} && \sqrt{2} \\
 &\Rightarrow 2^{\frac{25}{10}} && 5\sqrt{2} \\
 &\Rightarrow 2^{\frac{5}{2}} = 2 \times \frac{1}{2} && 3\sqrt{2} \\
 &\Rightarrow 4\sqrt{2} \text{ Ans}
 \end{aligned}$$

Q. Rate of Return (Bank)

100%, 200%, 400% average rate.

$$\begin{aligned}
 \text{G.M} &\Rightarrow (100 \times 200 \times 400)^{\frac{1}{3}} \\
 &\Rightarrow (800000)^{\frac{1}{3}} \\
 &\Rightarrow 200^{\frac{1}{3}} \text{ Ans}
 \end{aligned}$$

- G.M is a mathematical average
- G.M is based on all observations.
- If any observation is positive and negative both G.M cannot be determined.
- For equal observation.  $K \ K \ K \ G.M \Rightarrow \frac{1}{K}$

abc  $G.M \Rightarrow K$  then  $\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{1}{c} \quad G.M \Rightarrow \frac{1}{K}$

- G.M of  $(x, y) = G.M$  of  $n \times G.M$  of  $y$
- Change of origin  $x$
- Change of scale ✓
- Linear equation  $x$
- G.M is difficult measure of central tendency.

### Partition values

Median	Quartiles	Deciles	Percentiles
$\frac{50\% \quad 50\%}{M}$ Two equal parts $M \Rightarrow \frac{n+1}{2}$ value Discrete $M \Rightarrow \frac{N+2}{2}$ value Search. c.f	$\frac{25\% \quad 25\% \quad 25\% \quad 25\%}{Q_1 \quad Q_2 \quad Q_3}$ $Q_k \Rightarrow k \left( \frac{n+1}{4} \right)$ value Discrete $Q_k \Rightarrow k \left( \frac{N+1}{4} \right)$ value Search c.f	$\frac{10 \text{ parts}}{D_1 \quad \dots \quad D_9}$ $D \Rightarrow k \left( \frac{n+1}{10} \right)$ value Discrete $D_k \Rightarrow k \left( \frac{N+1}{10} \right)$ Search. c.f	$\frac{100 \text{ parts}}{P_1 \quad \dots \quad P_{99}}$ $P = k \left( \frac{n+1}{100} \right)$ value Discrete $P_k = k \left( \frac{N+1}{100} \right)$ Search c.f

Q. 16 12 11 9 100 120 130 90 54 23  
 9 11 12 16 23 54 90 100 120 130  
 $P_7$

Q<sub>1</sub> ⇒  $1 \frac{(n+1)}{4}$  value

$1 \frac{(11)}{4} ⇒ 2.75$  value

2<sup>nd</sup> + .75 (3<sup>rd</sup> - 2<sup>nd</sup>)

11 + .75

⇒ 11.75 Ans

Q<sub>7</sub> ⇒  $7 + \frac{(n+1)}{10}$

⇒  $\frac{7 \times 11}{10} ⇒ 7.7$  value

⇒ 7<sup>th</sup> + .7 (8<sup>th</sup> - 7<sup>th</sup>)

⇒ 90 + .7 (10)

⇒ 90 + 7

⇒ 97 Ans

Q. 16, 25, 34, 11, 50, 44, 100, 90, 120, 130

$P_{82}$

11, 16, 25, 34, 44, 50, 90, 100, 120, 130

$P_{82} ⇒ \frac{82(n+1)}{100}$  value

$\frac{82 \times 11}{100} ⇒ 9.02$  value

⇒ 120 + 0.2

⇒ 120.2 Ans

9<sup>th</sup> + 0.02 (10<sup>th</sup> - 9<sup>th</sup>)

120 + 0.02 × 10

120 + 0.2

x	f	n	f	C.F
5	3	5	3	3
11	4	11	5	8
15	5	15	4	12
16	4	16	4	16
20	7	20	7	23
30	8	30	9	32
50	8	50	8	40

Q<sub>6</sub>

$Q_6 = \frac{6(N+1)}{10}$  value

$Q_6 ⇒ \frac{6(40+1)}{10}$

$Q_6 ⇒ \frac{6 \times 41}{10}$

$Q_6 ⇒ 24.6$

$Q_6 ⇒ 30$

Combined A.M

$$\frac{n_1}{x_1} \quad \frac{n_2}{x_2}$$

$$\bar{x}_T = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

B	G
40	60
Aug. 50	30

$$\bar{x}_T = \frac{40 \times 50 + 60 \times 30}{100}$$

$$\Rightarrow \frac{2000 + 1800}{100}$$

$$\Rightarrow 38 \text{ Ans}$$

Combined G.M

$$n_1 \quad n_2$$

$$a_1 \quad a_2$$

Combined

$$G.M \Rightarrow (a_1^{n_1} \times a_2^{n_2})^{\frac{1}{n_1+n_2}}$$

B	G
3	5
G.M 4	3

$$\text{Combined } (4^3 \times 3^5)^{\frac{1}{8}}$$

$$\Rightarrow (64 \times 243)^{\frac{1}{8}}$$

$$\Rightarrow (15552)^{\frac{1}{8}}$$

$$\Rightarrow 3.34 \text{ Ans}$$

Combined H.M

$$n_1 \quad n_2$$

$$h_1 \quad h_2$$

Combined H.M

$$\frac{n_1 + n_2}{\frac{n_1}{M_1} + \frac{n_2}{M_2}}$$

B	G
13	15
H.M 65	75

$$\text{Combined} = \frac{28}{\frac{13}{65} + \frac{15}{75}}$$

$$\Rightarrow 70.0 \text{ Ans}$$

\* Relation between A.M G.M H.M

→ for unequal observation  $A.M > G.M > H.M$

→ for equal observation  $A.M = G.M = H.M$

→ Nothing is mentioned  $A.M \geq G.M \geq H.M$

$$G.M^2 = A.M \times H.M$$

$$A.M = 15$$

$$H.M = 15$$

$$G.M = 15$$

$$A.M = 64$$

$$H.M = 16$$

$$G.M = ?$$

$$G.M^2 = A.M \times H.M$$

$$G.M^2 = 16 \times 64$$

$$G.M = \sqrt{16 \times 64}$$

$$G.M = 4 \times 8$$

$$G.M = 32$$

Q Girl Boys.  
 $n_1$   $n_2$   
 Average 30 (low) 80 (high)  
 combined avg.  $\Rightarrow 60$

$$\bar{x}_T = \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$$

$$\frac{60}{T} = \frac{n_1 \times 30 + n_2 \times 80}{n_1 + n_2}$$

$$60 n_1 + 60 n_2 = 30 n_1 + 80 n_2$$

$$30 n_1 = 20 n_2$$

$$3 = 2 \frac{n_2}{n_1}$$

$$\frac{n_2}{n_1} = \frac{3}{2}$$

$$\frac{n_1}{n_2} = \frac{2}{3}$$

Ratio निकालना है  
 $5A = 5B = 3C$   
 $15 = 18 = 30$   
 $3 : 6 : 10$

Q.  $1 + 2 + 3 + \dots + 65$

$$\frac{n(n+1)}{2} \Rightarrow \frac{65(65+1)}{2}$$

$$\frac{65 \times 66}{2} = 2145$$

Formula.

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$