Chapter - 18

Index Numbers

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* Index number are convenient device for measuring relative changes of difference from time to time or from place to place.

Ex → Wholesale price index, NIFTY 50, Consumer price index, SENSEX etc.

* Index time series is a list of index number for two or more periods of time, where each index number employs the same base year.

→ price Index number - movement in price between base year and other periods.

Ex →

	2020	2023
Potato (1 kg)	20	35

→ Quantity Index number - movement in Quantity level between two periods.

Ex → Quantity - 5 kg. in 2020
6 kg in 2021

Price x Quantity

→ $\boxed{\text{Value}}$ Index number - movement in value level between two periods.

Ex →

	2020	2023
potato (5 kg)	20 x 5	35 x 5

* Relatives are derived because absolute numbers measured in some appropriate unit, are often of little importance and meaningless in themselves.

- price Relative = $\boxed{\dfrac{P_n \rightarrow \text{current year price}}{P_0 \rightarrow \text{Base year price}}}$

- Quantity Relative = $\boxed{\dfrac{Q_n \rightarrow \text{current year Quantity}}{Q_0 \rightarrow \text{Base year Quantity}}}$

- Value Relative = $\boxed{\dfrac{V_n}{V_0} = \dfrac{P_n Q_n}{P_0 Q_0}}$

- Link Relative = $\boxed{\dfrac{P_1}{P_0}, \dfrac{P_2}{P_1}, \dfrac{P_3}{P_2}, \ldots, \dfrac{P_n}{P_{n-1}}}$

(same can be created for Quantity also)

- Chain Relative = $\boxed{\dfrac{P_1}{P_0}, \dfrac{P_2}{P_0}, \dfrac{P_3}{P_0}, \ldots, \dfrac{P_n}{P_0}}$

Methods

- Simple Average
  - Aggregative
  - Relative
- weighted Average
  - Aggregative
    - ① Laspeyres' index
    - ② Passche's index
    - ③ Marshell-Edgeworth index
    - ④ Fischer's index
  - Relative

* Simple Aggregative method

> price index is expressed as total of commodity prices in a given year as a percentage of total of commodity prices in base year.

$$\text{Simple Aggregative Price Index} \Rightarrow \frac{\Sigma P_n}{\Sigma P_0} \times 100$$

Ex:-

Commodity	2010	2015	2020
Milk (Per kg)	50 $P_0(1)$	60 $P_1(1)$	70 $P_2(1)$
Atta (Per kg)	10 $P_0(2)$	12 $P_1(2)$	15 $P_2(2)$
Potato (Per kg)	20 $P_0(3)$	30 $P_1(3)$	30 $P_2(3)$
	$\Sigma P_0 = 80$	$\Sigma P_1 = 102$	$\Sigma P_2 = 115$

Index number (2010) = $\frac{80}{80} \times 100 = 100$

Index number (2015) = $\frac{102}{80} \times 100 = 127.5$

Index number (2020) = $\frac{115}{80} \times 100 = 143.75$

Merit → Easy to compute

Demerit → (1) Commodity with higher price have greater influence.

(2) price quotation become the concealded which have no logical significance.

(3) If units of price are changed index will also change.

* Simple Average of Relatives

> we invert the actual price for each variable into percentage of the base period. These percentage are called Relatives.

formulae ⇒ $\dfrac{\sum \frac{P_n}{P_0}}{N} \times 100$

Ex :-

Commodity	2010	2015	2020
milk (per kg)	50 P_0	60 P_1	70 P_2
Atta (per kg)	10 P_0	12 P_1	15 P_2
Potato (per kg)	20 P_0	30 P_1	30 P_2

Commodity	2010	2015	2020
milk	$\frac{50}{50} \times 100 = 100$	$\frac{60}{50} \times 100 = 120$	$\frac{70}{50} \times 100 = 140$
Atta	$\frac{10}{10} \times 100 = 100$	$\frac{12}{10} \times 100 = 120$	$\frac{15}{10} \times 100 = 150$
Potato	$\frac{20}{20} \times 100 = 100$	$\frac{30}{20} \times 100 = 150$	$\frac{30}{20} \times 100 = 150$

$\sum \frac{P_n}{P_0}$	300	390	440
$\frac{\sum \frac{P_n}{P_0}}{N}$	$\frac{300}{3} = 100$	$\frac{390}{3} = 130$	$\frac{440}{3} = 146.67$

Merits :- One big adventure of price relative is that they are pure numbers.
Price index computed from relative remain the same regardless of the units by which the price Quoted.

Demerits :- It gives equal importance to each relative.
This defect can be remeted by introduction of an appropriate weighing system.

* Weighted Average Index

> we weigh the price of each commodity by suitable factor. often take as the quantity or value weight sold during the base year or given year or an average of some years.

> Here indices are shown as %

Method

- Laspeyres' Index
- passche's Index
- marshall - Edgeworth Index
- fisher's Index

	P_0 2016 Q_0		P_n 2022 Q_n	
Commodity	Price	Quantity	Price	Quantity
A	20	60	25	55
B	30	50	28	50

Weightage Quantity

- Base year Qnty → Lasperey's
- Current year Qnty → Paasche's

Lasperey's

$$\boxed{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100}$$

Paasche's

$$\boxed{\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n} \times 100}$$

$$\Rightarrow \frac{25\times 60 + 28\times 50}{20\times 60 + 30\times 50} \times 100$$

$$\Rightarrow \frac{2900}{2700} \times 100$$

$$\Rightarrow 107.407$$

$$\Rightarrow \frac{25\times 55 + 28\times 50}{20\times 55 + 30\times 50} \times 100$$

$$\Rightarrow \frac{2775}{2600} \times 100$$

$$\Rightarrow 106.73$$

3) Marshall Edgeworth Index →

$$\Rightarrow \boxed{\frac{\Sigma P_n (Q_0 + Q_n)}{\Sigma P_0 (Q_0 + Q_n)} \times 100}$$

4) Fischer's Index →

$$\Rightarrow \sqrt{\text{Layspere} \times \text{paasche}}$$

$$\Rightarrow \boxed{\sqrt{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}}}$$

5) Bowley's Index →

$$\Rightarrow \frac{L + P}{2}$$

$$\Rightarrow \boxed{\frac{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} + \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}}{2}}$$

* Weighted Average of Relative method

Formulae $\Rightarrow \dfrac{\Sigma \frac{P_n}{\cancel{P_0}} \times (\cancel{P_0} Q_0)}{\Sigma P_0 Q_0} = \boxed{\dfrac{\Sigma P_n Q_0}{\Sigma P_0 Q_0}}$

Same as ~~Lays~~ Laspeyer's Index

> To overcome the disadvantages of simple average of relative method we can use weighted average of relative method.

Link relative = $\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \dots, \frac{P_n}{P_{n-1}}$

Chain relative = $\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \dots, \frac{P_n}{P_0}$

Chain Index Number →

$$= \boxed{\frac{\text{Link relative of current year} \times \text{Chain relative of previous year}}{100}}$$

Ex →

year	Price	link relative	Chain relative	Chain Indices
1991	50	100	100	100
1992	60	$\frac{60}{50} \times 100 = 120$	$\frac{60}{50} \times 100 = 120$	$\frac{120 \times 100}{100} = 100$
1993	62	$\frac{62}{60} \times 100 = 103.33$	$\frac{62}{50} \times 100 = 124$	$\frac{103.33 \times 120}{100} = 124$
1994	65	$\frac{65}{62} \times 100 = 104.83$	$\frac{65}{50} \times 100 = 130$	$\frac{104.83 \times 124}{100} = 130$

* Value indices

Value index equals the sum of values of a given year divided by sum of values of base year.

$$\frac{\Sigma V_n}{\Sigma V_0} = \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_0}$$

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* Limitations of Index number

- Changes of error due to Sampling
- It gives broad trend not real picture
- Due to many methods at time it create confusion.

* Usefulness of Index numbers

- Index number are very useful in defleting (eg. nominal wages into real)
- framing suitable policies in economics and business
- They reveal trend & tendencies in making important conclusions

* Defleted Value

$$\text{Defleted Value} = \frac{\text{Current value}}{\text{Price index of current year}}$$

$$\text{Defleted value} = \text{Current Value} \times \frac{\text{Base Price}}{\text{current Price}}$$

* Shifted price Index → Base year must relative recent.

$$\boxed{\frac{\text{Original price index}}{\text{Price index of the year on which it has to be Shifted}} \times 100}$$

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Example → Deflated Value

Cost of living		Salary	
2015	2018	2015	2018
97.5	115	19500	?

$$\Rightarrow \frac{19500}{97.5} \times 115 = \boxed{23000}$$

Example → Shifted price Index

Year	Original price index	Shifted Price index as base 1990
1983	100	$\frac{100}{130} \times 100 = 76.92$
1984	104	$\frac{104}{130} \times 100 = 80$
1985	106	$\frac{106}{130} \times 100 = 81.53$
1986	107	$\frac{107}{130} \times 100 = 82.30$
1987	110	$\frac{110}{130} \times 100 = 84.61$
1888	117	$\frac{117}{130} \times 100 = 90$
1989	125	$\frac{125}{130} \times 100 = 96.15$
1990	130	100
1991	140	$\frac{140}{130} \times 100 = 107.69$

* Spilicing → Two index covering different bases may be Combined into single series by splicing.

Ex :-

Year	Old price Index [1990 = 100]	Revised price Index [1995 = 100]	Spliced price Index [1995 = 100]
1990	100	$\frac{100}{114.2} \times 100$	87.56
1991	102.3	$\frac{100}{114.2} \times 102.3 =$	89.57
1992	105.3	$\frac{105.3 \times 100}{114.2} =$	92.2
1993	107.6	$100/114.2 \times 107.6 =$	94.22
1994	$\boxed{119.4}$	$\frac{100}{114.2} \times 119.4 =$	97.98
1995	$\boxed{114.2}$ →	$\boxed{100}$ →	100
1996		102.5 →	102.5

[formulae same as shifted price index]

Test of Adequacy

- Unit test
- Time Reversal test
- factor Reversal test
- Circular test

① Unit test - This test require that the formulae should be independent of unit in which or for which price and Quantities are quoted.

Except Simple Aggregate (unweighted) index all other formulae satisfy the test.

② Time Reversal test - It is a test to determine whether a given method will work both ways in a time, forward and backward.

→ Two Indices should be reciprocal to each other.

Symbolically > $\boxed{P_{01} \times P_{10} = 1}$

→ Laspeyres & Paasche's method do not satisfy this test but Fisher's ideal formulae satisfies.

③ Factor Reversal test → This hold when price index and Quantity index Should be equal to the corresponding Value Index.

Symbolically > $\boxed{P_{01} \times Q_{01} = V_{01}}$

Only fisher's index satisfy factor reversal test.

④ Circular test → This property enable us to adjust the index value from period to period without referring each time to original base.

→ The test of this shiftability of base is called circular test

→ This test not met by Laspeyres or fisher or paasche's index.

→ But met by Simple geometric mean of price relative and weighted aggregative with fixed weight.

⑤ Cost of Living Index (CLI) (General Index)

→ CLI is defined as weighted AM of index number of few groups of basic necessities. (food, cloth, House rent).

→ Example of CLI = WPI, CPI etc.