Permutation & Combination - SUN

Permutation & Combination (4-6 m)

Mathematical Principle of counting →

* If there are m ways to do one task & n ways to do another.

Agar dono ek sath karna toh → $m \times n$ ways to do both the tasks simultaneously.
(and case)

Agar dono mai se ek karna hai → $m + n$ ways.
(or case)

* Factorial ($\lfloor 7$ or $7!$) → $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$0! = 1 \quad 1! = 1$

as $n! = n \times \underbrace{(n-1) \times (n-2) \cdots \times 1}$

$(n-1)! = (n-1) \times (n-2) \cdots 1$

$\therefore \; n! = n \times (n-1)!$

* Permutation — Selection with arrangement → order matters
* Combination — Selection without arrangement → order doesn't matter

* Permutation — $3 \times 2 \rightarrow {}^3P_2 = \dfrac{3!}{(3-2)!} = \dfrac{3!}{1}$

3 mai se 2 ko nikal ke arrange

Calc. trick → ${}^nP_r$ → n se lehe r no. of terms multiply

NOTE → • ${}^nP_n = n!$ • ${}^nP_0 = 1$
• ${}^nP_1 = n$ • if $2! = (5-n)!$
then $2 = 5-n$

In Linear arrangement, for n things & n places → possible arrangements $= {}^nP_n = n!$

* When units can stick together → Ex- vowels in 'FAILURE'

Condition 1 → when order of vowels can change sticking.

Step 1 → Consider all units as one → F L R AIUE
(1) (2) (3) (4)

Step 2 → find the possible arrangement in these 4 then multiply it with the possible arrangement in (4)

i.e. $^4P_4 \times {}^4P_4$

Condition 2 → when order does not change
As there is only one possibility in the sticking unit.

So, the possible arrangement of the 1, 2, 3 & 4 i.e. $^4P_4$ will be the answer.

* Condn = Condition
* P = Possibilities.

* Can't Stick together →
For n units with 2 unit not together

Never come together = Normal P without any condition (n!) − Come together P (n−1)!(2!)

opposite of condn given in q

NOTE → Always clear the most plot twisting condn

* Circular Permutation → For n things & n places

$P = (n-1)! \leftarrow 1 \times {}^{n-1}P_{n-1} \leftarrow$ Cuz it doesn't matter where the 1st person sit / 1st unit is placed

• For not together → we come together P as $1 \times {}^{n-2}P_{n-2}$ ($^2P_2$)
and Normal P as $(n-1)!$
↓
−1 for 2 units sticking together

* Agar units mai kai fark naho → $P = \frac{(n-1)!}{2}$ (no. of unit)
(In circular)

* Theorem 1 → no. of units → n, no. of obj. can be taken by n → r
if one unit didn't take r, then
↳ it can be more

$$P = {}^{n-1}P_r$$

* Theorem 2 → no. of units → n, no. of obj. can be taken by n → r
if ek obj. toh hoga hi Let it be ex i mean x

$$P = \underset{{}^rP_1}{r} \times {}^{n-1}P_{r-1}$$

cuz ek fix hai

- $\sum_{r=1}^{9} r \cdot {}^rP_r = 1(1!) + 2(2!) \dots 9(9!)$
$= 10! - 1! = 10! - 1$

$$\sum_{r=1}^{n} r \cdot {}^rP_r = (n+1)! - 1$$

- Sum of n digits without repetitions — $(n-1)!$ (Sum of digits) * (111.. n times)

* Combination → ${}^nC_r = \frac{n!}{(n-r)!(r!)} = \frac{{}^nP_r}{{}^rP_r}$ $\underbrace{\text{Chose \& Arrange}}_{\text{Permutation}}$ (Combination)

${}^nC_n = 1$, ${}^nC_0 = 1$, $n \geq r \geq 0$ always.

***** ${}^nC_r = {}^nC_{n-r}$

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$${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$$

$${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$$

* Permutation when → repetition

1) In n things, n_1 are alike of one kind, n_2 of one kind ....

$$\frac{n!}{n_1!\,n_2!}$$

Ex → MISSISSIPPI, here I, S, P are identical
I = 4, S = 4, P = 2

∴ Possible arrangements = $\frac{11!}{4!\,4!\,2!}$

2) n things can be arranged in r places with repetition →

$$n^r$$

Ex → No. of w in which 6 letters be posted to 4 letter box → 6^4 ×
4^6 ✓ → ~~coz letters can be repeated~~

l_1 l_2 l_3 l_4 l_5 l_6
4 4 4 4 4 4 = 4^6

3) Combination of different things taking some or all of n things at a time → kabhi 10 mai se 1, 10 mai se 2, ... 10 mai se 10.

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 \dots {}^nC_n = 2^n - 1 \rightarrow 1 \text{ for } {}^nC_0$$

this is ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \dots {}^{10}C_{10} = 2^{10} - 1$

Sub. in RHS,

Kyuki na choose karne wala scene nai hai wrong no.

4) Combination of n things taken some or all at a time with similarity (combo of 1 & 3), n_1 things are alike, n_2 are alike, etc.

Yeh specific n_1 se kuch nai liya

$$(n_1+1)(n_2+1)(n_3+1)-1$$

sab n_1, n_2 & n_3 se kuch bhi nai liya.

$$= \frac{^{n_1}P_1}{1!} + \frac{^{2}P_2}{2!} \cdots \frac{^{n_1}P_{n_1}}{n!}$$

coz identical.

Ex- 10 donuts, 6 waffles & 8 pastries, different ways you can take.

$$(10+1)(6+1)(8+1)-1 = 692$$

$$\frac{^1P_1}{1!} + \frac{^2P_2}{2!} + \frac{^3P_3}{3!} + \frac{^4P_4}{4!} + \frac{^5P_5}{5!} + \frac{^6P_6}{6!}$$

$$\Rightarrow 1+1+1+1+1+1 = 6$$

5) Combination when r_1 from n_1 & r_2 from n_2 ... things are selected.

$$^{n_1}C_{r_1} \times {^{n_2}C_{r_2}}$$

Ex→ Ways in which 9 can be divided into group of 2, 4, 3 things.

$$^9C_2 \times {^7C_4} \times {^3C_3}$$

Here n after every group division is decreasing.

NOTE → If koi shape wala q hai toh sides ka khayal rakhna

Imp. Questions → Lecture 2 → Q2 & 3
" 3 → Q13
Lecture 4 → Q1 & 9
" 5 → Q5 & 8 & 21.