Chapter - 7 Sets, Relations and Functions (3/4 Marks)

- Notes

If we consider a collection of objects given in such a way that it is possible to tell beyond that whether a given object is in the collection under consideration or not then such a collection of objects is called well defined collection of objects.

Sets

A set is a collection of well defined distinct objects.

- Sets are generally denoted by capital letters.
- The elements of sets are generally denoted by small letters.
- Each object of the set is called an element or member of the set.
- Examples -

(1) $A = \{a, e, i, o, u\}$

2) $B = \{1, 2, 3, 4, 5\}$

Representations of Sets

Roster or Braces form
or
Tabular form

Set Builder form
or
Algebraic method
or
Rule method
or
property method

1. Roster Form

It is the method in which we list all the elements separating them by commas and enclosing these in curly braces.

For eg - (1) $\{2, 4, 6, 8, 10\}$

(2) $\{1, 3, 5, 7, 9, 11, 13, 15\}$

• Note -

(a) In this method, the order of elements does not matter.

(b) while ~~th~~ writing the set in roster form all the elements are generally taken as distinct

2. Set - Builder Method

In this method we figure out the common

property which is there in every element of set and not there with any element outside the set.

For eg - (1) $A = \{x : x$ is a letter in the word SAMPURNA$\}$

(2) $B = \{x : x$ is an integer $x^2 \leq 9\}$

★.. Few important and widely used Sets

N - Set of all natural numbers

W - Set of whole numbers

Z - Set of integers

R - Set of real numbers

Z^+ - Set of positive integers

R^+ - Set of positive real numbers

Q - Set of rational numbers

Q^+ - Set of positive rational numbers.

★.. Some Special sets

(a) Finite and Infinite Sets

When the number of elements are countable it is called finite Set.
For eg - $\{1, 4, 9, 16, 25\}$

When the number of elements are uncountable it is called infinite set.
For eg - $\{1, 2, 3, 4, 5 \text{-------}\}$

(b) Empty and Non-empty Sets

The sets which contains no elements are called Empty set / Null Set / Void Set.
Usually denoted by $\{\ \}$ or ϕ
For eg - Set of prime numbers between 32 and 36.

The set which contains atleast one element is called non-empty Set. For eg -
$\{1\}, \{2, 9\}, \{6, 5, 3, 8\}$

(c) Equal and Equivalent Set

Two sets A and B are said to be equal if every element of A is in B and every element of B is in A.

For eg. $A = \{1, 2, 4\}$ and $B = \{1, 2, 4\}$
then $A = B$

Two sets are said to be equivalents if $n(A) = n(B)$. For eg - $\{1, 2, 3\}$ & $\{3, 9, 6\}$

Date ———
Page ———

Remark - Equal sets are equivalent but equivalent sets need not be equal.

(d) Singleton Set

The set containing only one element. Eg = $\{2\}$

(e) Universal Set

The set containing all the elements under consideration in a particular problem is called universal set.

(f) Disjoint Sets

When two sets have no element in common they are called disjoint sets.

★... Subsets

Set A will be the subset of B if every element of A is also an element of B or we can say $A \subset B$ if, whenever $a \in B$ then $a \in B$.

(a) Proper subset and Super Set

If $A \subset B$ but $B \not\subset A \Rightarrow A \neq B$
then A is a proper subset of B.
and B is the super set.
Remark - ϕ has no proper subset.

For eg - {3} is a proper subset of {2, 3, 5}

(b) Power set

The collection of all the possible subsets of a given set A is called the power set of A. It is denoted by P(A)

For eg - If A = {1, 2}
then, P(A) = { {1}, {2}, {1, 2}, { } }.

★... If a ~~set~~ has n elements then the number of subsets is 2^n.

★★... The number of proper subset is $2^n - 1$.

★★★... The number of distinct elements contained in finite set is called its cardinal number

• Subset of the set of Real Numbers

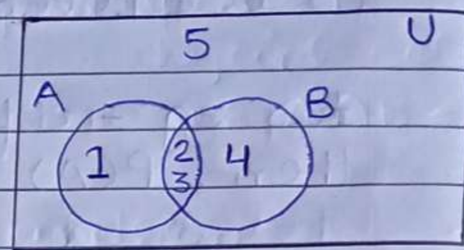
$N \subset Z \subset Q \subset R$

N - Set of natural no.s
Z - Set of integers
Q - Set of rational no.s
R - Set of real no.s

• Venn Diagram

These diagrams consists of rectangles and circles inside it. Universal set is a rectangle and other sets are circles.

For eg -



$U = \{1, 2, 3, 4, 5\}$
$A = \{1, 2, 3\}$
$B = \{2, 3, 4\}$

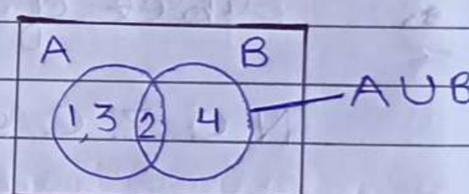
★... Operations on Sets

Union of Sets

$A \cup B$ = A or B

In set builder form = $A \cup B = \{x : x \in A$ or $x \in B\}$

For eg. $A = \{1, 2, 3\}$
$B = \{2, 4\}$
$A \cup B = \{1, 2, 3, 4\}$

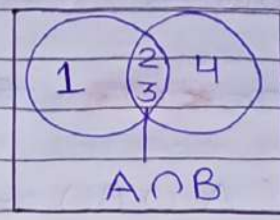


2. Intersection of Sets

only common elements
$A \cap B = \{x : x \in A$ and $x \in B\}$

$A = \{1, 2, 3\}$
$B = \{2, 3, 4\}$

$A \cap B = \{2, 3\}$



★★.. If $A \cap B = \phi$ the set is called the disjoint set.

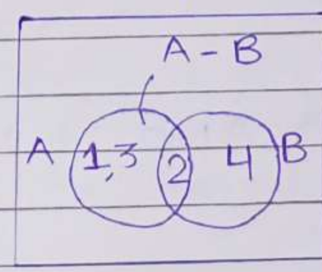
3. Difference of Sets

In set builder form definition of A-B =

$A - B = \{x : x \in A \text{ and } x \notin B\}$

For eg. $A = \{1, 2, 3\}$
$B = \{2, 4\}$

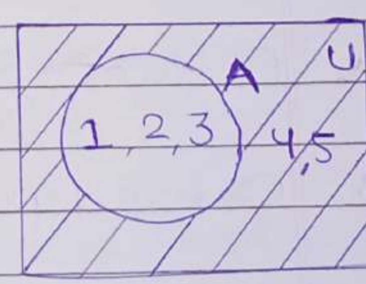
$A - B = \{1, 3\}$



4. Complement of Set

Complement of a set is denoted by A^c and A' and it includes all elements in universal set except of elements in A.

$U = \{1, 2, 3, 4, 5\}$
$A = \{1, 2, 3\}$
$B = \{4, 5\}$



Note -

(A) For two sets A and B -

Case 1 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Case 2 $n(A \cup B) = n(A) + n(B)$ [if A & B are disjoint sets, i.e. $A \cap B = \phi$]

(B) For ~~two~~ three sets A, B and C -

Case 1 $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Case 2 $n(A \cup B \cup C) = n(A) + n(B) + n(C)$ [if A, B and C are disjoint sets]

(C) De Morgan's Law

(i) $(P \cup Q)' = P' \cap Q'$

(ii) $(P \cap Q)' = P' \cup Q'$

★★. Product Sets

(a) Ordered pair

Two elements a and b listed in specified order form an ordered pair denoted by (a, b).

(b) Cartesian Product of Sets

If A and B are two non empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B is called the Cartesian product of A and B to be denoted by $A \times B$

For eg. if $A = \{a, b\}$ and $B = \{p, q\}$
then, $A \times B = \{(a, p), (a, q), (b, p), (b, q)\}$

$B \times A = \{(p, a), (p, b), (q, a), (q, b)\}$

Points to be noted -

(a) If $A = \phi$ or $B = \phi$ then $A \times B = \phi$

(b) $n(A \times B) = n(A) \times n(B)$

(c) ordered pairs $(a, b) \neq (b, a)$

(d) Also, $A \times B \neq B \times A$

(e) But, $n(A \times B) = n(B \times A)$

Relations

Any subset of the product of A and B is called its subset.

If we have two non-void or null or empty A and B then the relation R from Set A to Set B is represented by a R b where a is the set of elements belonging to A and b is the set of elements belonging to Set B.

Relation $\subseteq$ cartesian product

$R : A \to B \subseteq A \times B$

- Domain and Co-domain of Relation

For a relation from Set A to Set B ie a R b,

Domain (R) = $\{a : (a, b) \in R\}$
Codomain (R) = Set B
Range (R) = $\{b : (b, a) \in R\}$

- Types of Relation

1. Reflexive relation

If R contains all the ordered pairs of form (a, a) in $S \times S$ then R is called Reflexive relation
Relation = is equal to
In reflexive relation a is related to itself.

For eg - $A = \{1, 2, 3\}$
then $R = \{(1,1), (2,2), (3,3)\}$ is reflexive.

2. Symmetric Relation

If $(a,b) \in R$, $(b,a) \in R$ for every $a, b \in S$ the R is called the symmetric relation.

3. Transitive Relation

If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$ for every $a, b, c \in S$ then R is called transitive relation.

Equivalence Relation

If a relation is reflexive, symmetric as well as transitive it is called an equivalence relation.

Functions

If we take two sets A and B, then relation f from set A to set B will be function only if every element of A has a unique image in B.

The element $f(x)$ of B is called the image of x while x is called the pre-image of $f(x)$

- Domain, Co-domain and Range of a Function

For a function from set A to set B ie $f(a)=b$ where $a \in A$ and $b \in B$ -

Domain = Set A
Co-Domain = Set B
Range $\leq$ Co-domain [Range is set of y, $y \in B$ & $y = f(x)$]

- Types of Functions

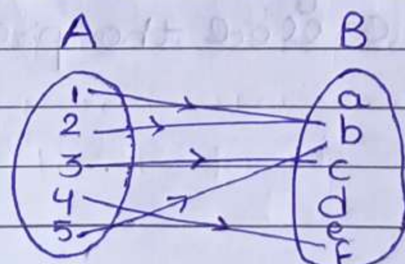
One-One ~~or~~ Injective Function.

Let $f: A \to B$
If different elements in A have different ~~elements~~ images in B then f is said to be one-one or injective function or mapping.

Eg. $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$ $F: A \to B : f(x) = 2x$

2. Many one function

Functions for which we can match more than one element of the ~~set~~ A to the same element of set B are said to be many-one function
Eg.



Note -

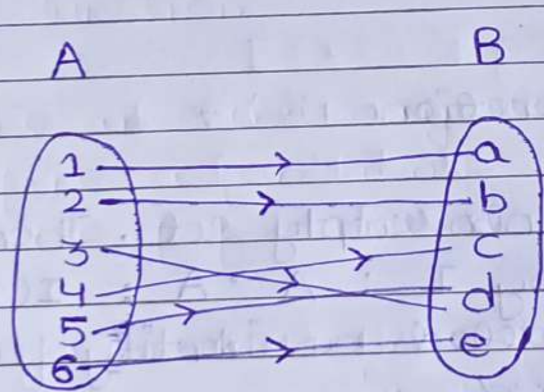
(a) If we have straight line than it is one-one function.

(b) If we have greater than sign from set A to set B then it is many-one function.

(c) If we have less than sign from set A to set B then relation is not a function.

3. Onto or Surjective functions

Let $f: A \to B$
If every element in B has atleast one pre image in A then f is said to be an onto function.
f is onto if and only if range of f $= B$

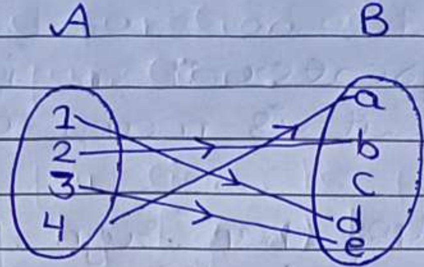


4. Into functions

Let $f : A \to B$

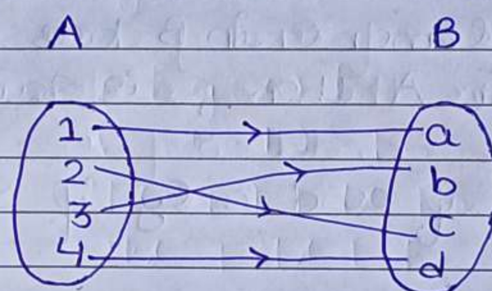
There exists even a single element in B having no pre-image in A then f is said to be an onto function.

Eg.



5. Bijective Function

A one and onto function is bijective.



6. Identity function

Let A be a non empty set. Then the function I defined by $I : A \to A : I(x) = x$ for all $x \in A$ is called an identity function on A.

$y = f(x) = x$

$y = x$

Eg. $A = \{1, 2, 3, 4\}$

$f = \{(1,1), (2,2), (3,3), (4,4)\}$

Date
Page

7. Constant Function

Let $f : A \to B$ defined in such a way that all elements in A have the same image in B then f is said to be a constant function.

$f(x) = C$

Eg. $\Rightarrow$ $f(x) = 3$

$f : A \to A$

$A = \{1, 2, 3, 4\}$

$f = \{(1,3)(2,3), (3,3), (3,4)\}$

8. Equal Function

Two functions f and g are said to be equal if they have the same domain and they satisfy the condition $f(x) = g(x)$ for all x.

9. Inverse Function

Let y be an arbitrary element of B, so we may define a function f which is called as the inverse of f denoted by f^{-1} as follows-

If f be a one-one onto function from A to B

For $f(x) = y$, $f^{-1} : B \to A$

$f : A \to B$, $x \in A$, $y \in B$

$f(x) = y$, this is bijective function.

$f^{-1} = B \to A$ $f^{-1}(y) = x$

10. Composite Function

A composite function is a function that depends on another function. It is created when one function is substituted into another function.

Let two functions be $f(x)$ and $g(x)$

$$f \circ g = f[g(x)]$$

$$g \circ f = g[f(x)]$$

Example - $f(x) = x^2 + 6$
$g(x) = 2x - 1$

$$\begin{aligned}(f \circ g)x &= f[g(x)] \\ &= (2x-1)^2 + 6 \\ &= 4x^2 + 1 + 4x + 6 \\ &= 4x^2 + 4x + 7\end{aligned}$$

$$\begin{aligned}(g \circ f)x &= g[f(x)] \\ &= 2(x^2+6) - 1 \\ &= 2x^2 + 12 - 1 \\ &= 2x^2 + 11\end{aligned}$$

★... Eg. of Inverse function

$f(x) = 2x$, f be a one-one onto function

$f'(x) = 2$ $\quad f^{-1}(y) = x = \frac{y}{2}$

$F(x) = 2x$

$f(x) = y$ $\quad f^{-1}(x) = \frac{x}{2}$

$y = 2x$

$x = y/2$