Chapter 1

RATIO | PROPORTION LOGS & INDICES

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1. What is Ratio?

→ Ratio is a fraction used for comparison of 2 or more quantities, which are of same type, expressed in the same unit of measurement.

2. Find simplest form of 3.50 : 8.75.

→ Generally ratio is expressed in its simplest form

$$\frac{3.50}{8.75} = \frac{350}{875} = \frac{14}{35} = \frac{2}{5} = 2:5$$

All the terms of the ratio can be multiplied OR divided by same nonzero number

3. 5:7 can also be written as :

→ $\frac{5}{7} = \frac{50}{70} = \frac{5/8}{7/8} = \frac{1}{7/5} = \frac{0.50}{0.70} = \frac{500}{700} = \frac{10}{14} = \frac{25}{35} = \frac{5m}{7m}$ where $m \neq 0$

4.

Ratio	It's	Answer
5:7	Duplicate Ratio	$5^2 : 7^2 = 25:49$
8:3	Triplicate Ratio	$8^3 : 3^3 = 512:27$
11: 19	Inverse Ratio	19:11
64:625	Sub-Duplicate Ratio	$\sqrt{64} : \sqrt{625} = 8:25$
125:27	Sub-Triplicate Ratio	$\sqrt[3]{125} : \sqrt[3]{27} = 5:3$

First term : Antecedent Second term : consequent

5. Find compounded ratio of 5:7, a:b, x:y, 9:8

→ Compounded Ratio = (product of antecedent : product of consequents)

$= (5 \times a \times x \times 9) : (7 \times b \times y \times 8)$

$= 45ax : 56by$

6. 3 : 8 : 9 : 11 is a

→ continued Ratio

7. Ratio of 3 or more terms is known as

→ Continued Ratio.

8. Ratio is unit free. All the Ratios are unit free means Ratio can be expressed without unit of measurement

9. First term of the ratio = Antecedent

Second term of the ratio = Consequent

10. Find the ratio of 3kg : 35,000 grams

→ 3,000 gms : 35,000 gms = 3 : 35

11. a:b can also be written as (ak : bk) or $\left(\frac{a}{k}\ \frac{b}{k}\right)$ provided $k \neq 0$

12. The order of the terms in a ratio is important. $a:b \neq b:a$

13. Find simplest form of $2\frac{1}{3} : 3\frac{2}{3}$

→ $= \frac{2\frac{1}{3}}{3\frac{2}{3}} = \frac{\frac{7}{3}}{\frac{11}{3}} = \frac{7}{11} = 7:11$

14.

In the Ratio	then a:b is called as	examples
a:b If		
a > b	Ratio of Greater inequality	7:5, 8:3, 101:71
a < b	Ratio of lesser inequality	5:11, 2:35, 36:101
a = b	Ratio of Equality	8:8, 13:13, 1:1

simplest form of Ratio of Equality is always 1:1

15. Ratio exists only when 2 or more quantities are of same kind.

OR same type. There can't be Ratio of age & height.

16. Find simplest form of $\frac{1}{3}:\frac{1}{8}:\frac{1}{10}$

→ $\frac{1}{3} : \frac{1}{8} : \frac{1}{10} = 80:30:24 = 40:15:12$

17. Find simplest form of $\frac{3}{5}:\frac{2}{3}:\frac{8}{5}$

→ $\frac{3}{5} : \frac{2}{3} : \frac{8}{5} = \left(15\times\frac{3}{5} : 15\times\frac{2}{3} : 15\times\frac{8}{5}\right) = 9:10:24$

18. Ratios are unit - free

→ This statement is True as Ratio can be expressed without unit of measurement.

19. If a:b = 2:3; b:c = 4:7; c:d = 8:1. Find a:b:c:d, a:d, b:d

→ a:b = 2:3 b:c = 4:7 a:b:c = 8:12:21 = 64:96:168
a:b = 8:12, b:c = 12:21 c:d = 8:1 = 168:21

a:b:c:d = 64:96:168:21
a:d = 64:21
b:d = 96:21 = 32:7

20. If Quantity increase or decreases in the ratio a:b then new quantity = b of original quantity = a

∴ **New quantity = original quantity x mutiplying ratio**

My income = x, it changed in the ratio of 10:13
⟹ then
New Income = $x \times \frac{13}{10}$

where multiplying ratio = Reciprocal of given ratio = Inverse ratio of given ratio

original quantity = New quantity x Given ratio

21. Population of a city is x then it changes in the ratio of p:q then find new population

→ New population = $\left(\text{old population} \times \text{Multiplying ratio}\right) = x \times \frac{q}{p} = \left(\frac{qx}{p}\right)$

22. Inverse ratio of Inverse ratio of a:b is = a:b

Duplicate ratio of sub duplicate ratio of p:q is = p:q

Triplicate ratio of sub triplicate ratio of m:n is = m:n

Sub triplicate ratio Triplicate ratio of x:y is = x:y

Sub duplicate ratio of duplicate ratio of u:v = u:v

23. Find Duplicate ratio of Inverse ratio of 5:7

→ Dupli. ratio of 7:5
$= 7^2 : 5^2 = 49:25$

24. Find Triplicate ratio of sub duplicate ratio of 25:49

→ Triplicate ratio 5:7
$= 5^3 : 7^3 = 125 : 343$

25. Find compounded ratio of Duplicate ratio of 2:3, Triplicate ratio of 9:4, Sub duplicate ratio of 81:64, sub duplicate ratio of 512:27

→ $4:9$, $729:64$, $9:8$, $\sqrt{512}:\sqrt{27}$

Compounded Ratio $= \frac{4 \times 729 \times 9 \times \sqrt{512}}{9 \times 64 \times 8 \times \sqrt{27}} = \frac{729 \times \sqrt{512}}{128 \times \sqrt{27}} = \frac{729 \times 16\sqrt{2}}{128 \times 3\sqrt{3}}$

$= \frac{243}{8} \times \frac{16\sqrt{2}}{3\sqrt{3}} = 243\sqrt{2} : 8\sqrt{3}$

26. When 4 quantities a,b,c,d are said to be in proportion?

→ When $a:b = c:d$ then a,b,c,d are said to be in proportion.

$\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right)$

$(ad = bc)$ (product of extremes) = (product of means)

27. When 4 quantities a,b,c,d are said to be in continued proportion?

→ when $a:b = b:c = c:d$ then a,b,c,d are said to be in continued proportion

If 4 quantities are in continued proportion then they are in proportion also but if they are in proportion then it is not necessary that they are in continued proportion.

28.

4 Quantities	Whether 4 Quantities are in	
	Continued Proportion?	Proportion?
2,6,18,54	Yes	Yes
3,8,12,32	No	Yes
8,24,96,288	No	Yes
8,5,80,45	No	No
4,6,9,13.50	Yes	Yes

29. When 3 quantities a,b,c are said to be in proportion?

→ • When $a:b = b:c$ then a,b,c are said to be in proportion as well as continued proportion

• 5,20,80 are in proportion as well as continued proportion as $5:20 = 20:80 = 1:4$

30. If a,b,c,d are in proportion i.e. $\frac{a}{b}=\frac{c}{d}$ then

Invertendo : $\left(\frac{b}{a}\right)=\left(\frac{d}{c}\right)$

Componendo : $\left(\frac{a+b}{b}\right)=\left(\frac{c+d}{d}\right)$

Dividendo : $\left(\frac{a-b}{b}\right)=\left(\frac{c-d}{d}\right)$

Componendo and Dividendo : $\left(\frac{a+b}{a-b}\right)=\left(\frac{c+d}{c-d}\right)$

Alternendo : $\left(\frac{a}{c}\right)=\left(\frac{b}{d}\right)$

Addendo : $\left(\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}\right)$

Subtrahendo : $\left(\frac{a}{b}=\frac{c}{d}=\frac{a-c}{b-d}=\frac{c-a}{d-b}\right)$

$$\frac{a}{b}=\frac{c}{d}=\left(\frac{a+c}{b+d}\right)=\left(\frac{a-c}{b-d}\right)$$

31. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=\frac{i}{j}=k$, then

→

As per addendo $k=\frac{a+c+e+g+i}{b+d+f+h+j}$

Addendo of

As per subtrahendo $k=\frac{3a+5c-8e-28g+18i}{3b+5d-8f-28h+18j}=\frac{a}{b}=\frac{c}{d}=k$

32. If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$ then, Find value of $\left(\frac{4a+2b-3c}{5b}\right)$

→ $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=m$

$\therefore a=3m,\ b=4m,\ c=7m$

$\frac{4a+2b-3c}{5b}=\frac{4(3m)+2(4m)-3(7m)}{5(4m)}$

$=\frac{-m}{20m}=-1/20$

33. Find Fourth Proportional to 8, 12, 20

→ 8, 12, 20, $\boxed{x}$

then product of extremes = product of means

$8x=12\times20 \quad \therefore x=30$

34. Find mean proportional to 9, 25

→ 9, $\boxed{x}$, 25

$\therefore$ square of middle term = product of extremes

$x^2=9\times25=225$

$\therefore x=15$

35.

4 Quantities in Proportion	Value of k = ?
8, 9, k, 63	$504=9k \quad \therefore k=56$
58, -3k, 28, 85	$4930=-84k \quad \therefore k=-58.690476$
36, 60, 2k, 98	$120k=3528 \quad \therefore k=29.40$
-3k, 86, 25, 63	$-189k=2150 \quad \therefore k=-11.3756$

36. Rules of Indices

1. $a^m \times a^n = (a)^{m+n}$
2. $\frac{a^m}{a^n} = (a)^{m-n}$
3. $(a^m)^n = (a)^{mn}$
4. $a^{-m} = 1/a^m$
5. $(a.b)^m = a^m . b^m$
6. $\left(\frac{a}{b}\right)^m = \left(\frac{a^m}{b^m}\right) = a^m . b^{-m}$
7. $a^{1/m} = \sqrt[m]{a}$
8. $[(a^m)^n]^p = (a)^{mnp}$
9. $(a^{m/n}) = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$
10. If $a^x = a^y$; then $x = y$
11. If $a^m = b^m$; then $a = b$
12. $(a)^0 = 1$, $a^1 = a$

37. $2x^{1/2} \times 3x^{-1} = ?$ If $x = 4$

$\rightarrow 2x^{1/2} . 3x^{-1} = 2(4)^{1/2} \times 3 \times (4)^{-1}$

$= 2 \times 2 \times 3 \times \frac{1}{4} = 3$

38. $\frac{6ab^2c^3}{2a^2bc^8} = 3a^{1-2} \times b^{2-1} \times c^{3-8}$

$= 3a^{-1} . b^1 . c^{-5} = \left(\frac{3b}{a.c^5}\right)$

39. $\frac{64 \times \sqrt[3]{128}}{\sqrt[5]{512}} = \frac{2^6 \times (2^7)^{1/3}}{(2^9)^{1/5}} = \frac{2^6 \times 2^{7/3}}{2^{9/5}} = (2)^{6+\frac{7}{3}-\frac{9}{5}}$

$= (2)^{\frac{90}{15}+\frac{35}{15}-\frac{27}{15}} = (2)^{98/15} = \sqrt[15]{2^{98}}$

40. $\left(\frac{4x^{-1}}{x^{-1/3}}\right) = 4 \times (x)^{-1-(-\frac{1}{3})} = 4 . (x)^{-1+\frac{1}{3}} = 4 . x^{-2/3}$

$= \left(4 / x^{2/3}\right)$

41. $\frac{2a^{1/2} \times a^{2/3} \times a^{-7/3}}{9a^{-5/3} \times a^{3/2}} = ?$ If $a = 4$

$= \frac{2}{9} \times (a)^{\frac{1}{2}+\frac{2}{3}-\frac{7}{3}+\frac{5}{3}-\frac{3}{2}}$

$= \frac{2}{9} \times (a)^{\frac{3}{6}+\frac{4}{6}-\frac{14}{6}+\frac{10}{6}-\frac{9}{6}}$

$= \frac{2}{9}(a)^{-1} = \left(\frac{2}{9a}\right) = \frac{2}{9 \times 4} = \frac{1}{18}$

42. $\dfrac{(a^m \times a^n \times a^p)}{a^x} =$ $(a)^{m+n+p-x}$

43. $\sqrt[6]{a^{4b} \cdot x^6 \ (a^{2/3} \cdot x^{-1})^{-b}} = ?$

$= \left[a^{4b} \cdot x^6 \cdot a^{-2b/3} \cdot x^b\right]^{1/6}$

$= \left[a^{4b-\frac{2b}{3}} \cdot x^{6+b}\right]^{1/6}$

$= \left[a^{\frac{10b}{3}} \cdot x^6 \cdot x^b\right]^{1/6}$

$= a^{\frac{10b}{18}} \cdot x^{6\times\frac{1}{6}} \cdot x^{\frac{b}{6}}$

$= a^{\frac{5b}{9}} \cdot x^1 \cdot x^{b/6} = a^{\frac{5b}{9}} \cdot x^{\left(\frac{6+b}{6}\right)}$

44. $(\sqrt{9})^7 \times (\sqrt{3})^{-5} = 3^k$ then k = ?

$3^7 \times (3^{\frac{1}{2}})^{-5} = 3^k$

$3^7 \times 3^{-2.50} = 3^k$

$\therefore 3^{4.50} = 3^k$

$\therefore k = 4.50 = (9/2)$

45. $\dfrac{2^5}{2^5} = (2)^{5-5} = (2)^0 = 1$

$(\text{Any number})^0 = 1.00$

46. $\left(\dfrac{81x^4}{y^{-8}}\right)^{1/4} = \dfrac{81^{\frac{1}{4}} \times (x^4)^{1/4}}{(y^{-8})^{1/4}} = \dfrac{3 \times x}{y^{-2}} = 3xy^2$

47. $\left\{\dfrac{(3^3)^2 \times (4^2)^3 \times (5^3)^2}{(3^2)^3 \times (4^3)^2 \times (5^2)^3}\right\} = \dfrac{3^6 \times 4^6 \times 5^6}{3^6 \times 4^6 \times 5^6} = 1$

48. $y^{a-b} \cdot y^{b-c} \cdot y^{c-a} = ?$

$(y)^{a-b+b-c+c-a} = y^0 = 1$

49. $\left[1-\left\{1-(1-x^2)^{-1}\right\}^{-1}\right]^{-1/2} = \left[1-\left\{1-\dfrac{1}{1-x^2}\right\}^{-1}\right]^{-1/2} = \left[1-\left\{\dfrac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1+\dfrac{1-x^2}{x^2}\right]^{-1/2} = \left[\dfrac{x^2+1-x^2}{x^2}\right]^{-1/2} = (x^{-2})^{-1/2} = x^1 = x$

50. $\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$

$= \left[(x^n)^{\frac{n^2-1}{n}}\right]^{\frac{1}{n+1}} = (x)^{n \times \frac{(n-1)(n+1)}{n} \times \frac{1}{(n+1)}}$

$= (x)^{n-1} = \left(\frac{x^n}{x}\right)$

51. If $a^x = b$, $b^y = c$, $c^z = a$ then xyz = ?

$a^x = b$

$(c^z)^x = b$

$(c)^{xz} = b$

$(b^y)^{xz} = b$

$(b)^{xyz} = b^1$

$\therefore xyz = 1$

OR

$a^x = b$

$x \cdot \log a = \log b$

$x = \frac{\log b}{\log a}$

$xyz = \frac{\log b}{\log a} \times \frac{\log c}{\log b} \times \frac{\log a}{\log c}$

$= 1.00$

52. $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \cdot \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \cdot \left(\frac{x^c}{x^a}\right)^{(c^2+ac+a^2)} = ?$

$= (x^{a-b})^{a^2+ab+b^2} \cdot (x^{b-c})^{b^2+bc+c^2} \cdot (x^{c-a})^{c^2+ac+a^2}$

$= x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3} = (x)^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0$

$= 1.00$

53. Log of number consist of 2 parts

Integer Part = characteristic

Fractional Part = decimal part = Mantissa

example $\quad \log 20 = 1 + 0.3010 = 1.3010$

1 ↓ characteristic; 0.3010 ↓ Mantissa

If $\text{Log}_b a = x$ then $b^x = a$ → super imp. rule

54. Log x = characteristic of x + Mantissa of x

$\text{Log}_b^a = (\text{Log } a / \text{Log } b)$

[redacted] $= \text{Log}_m ab = \text{Log}_m a + \text{Log}_m b$

[redacted] $= \text{Log}_m (a/b) = \text{Log}_m a - \text{Log}_m b$

If $\log b^a = k$; then $b^k = a$

If $x^y = z$; then $\text{Log}_x z = y$

$\text{Log } (a)^{-b} = -b.\text{Log } a$

$\text{Log}\left(\frac{ab}{c}\right) = \text{Log } a + \text{Log } b - \text{Log } c$

$\text{Log}\left(\frac{a}{xy}\right) = \text{Log } a - \text{Log } x - \text{Log } y$

$\text{Log}\left(\frac{ab}{cd}\right) = \text{Log } a + \text{Log } a - \text{Log } c - \text{Log } d$

A.log (log x) = x

Log (A.log x) = x provided $a \neq 1$

$\text{Log } a^a = \text{Log}_a a = \text{Log } a / \text{Log } a = 1$

$\text{Log } b^a \times \text{Log } c^b = \text{Log}_b a \times \text{Log}_c b$

$\text{Log}_{10}^{10} = \text{Log}_{10} 10 = 1 = \text{Log}_c a$

$\text{Log}_{10}^{100} = 2$

$\text{Log}_{10}^{1000} = 3$

$\text{Log}_m^{abc} = \text{Log}_m a + \text{Log}_m b + \text{Log}_m c$

55. $\dfrac{\text{Log}_3 8}{\text{Log}_9 16 \times \text{Log}_4 10} = \dfrac{\text{Log } 8 / \text{Log } 3}{\text{Log } 16 / \text{Log } 9 \times \text{Log } 10 / \text{Log } 4}$

$= \dfrac{3.\text{Log } 2}{\text{Log } 3} \times \dfrac{2.\text{Log } 3}{4.\text{Log } 2} \times \dfrac{2.\text{Log } 2}{1} = 3.\text{Log } 2 = \text{Log } (2)^3$

$= \text{Log } 8$

56. **Log x (where x > 0)**

(Log exists only for positive numbers)

Characteristic of x + (Fractional part) **Mantissa of x**

(Integer part)

If x ≥ 1: (No. of digits before decimal point − 1)

1 > x > 0: (No. of zeros immediately after decimal point + 1) BAR

Log 2 = 0.3010

Log 0.00000020 = $\bar{7}.3010$ = −7 + 0.3010 = −6.699

Log 0.000023 = $\bar{5}.3617$ = −5 + 0.3617 = −4.6383

Log 23.00 = (2 − 1) + 0.3617 = 1.3617

57.

x	Characteristic of x
56.81	$2-1=1$
583.2	$3-1=2$
81.93	$2-1=1$
5.81	$1-1=0$
13.00	$2-1=1$
0.008126	$(2+1)$ Bar $=\bar{3}$
0.5826	$\bar{1}$
8.5926	$1-1=0$

58. How to find Log x on calculator?

→ Enter x
√ 15 times
Deduct 1
Multiply by 14230.9635

① Log 28
= 1.4472 3163716
② Log 81.97
= 1.913783671 53

59. How to find A.log y on calculator?

→ Enter 'y'
Divide by 14230.9635
Add 1
$X_1 =$ 15 times

① A.Log 1.44723163716
= 28
② A.Log 1.913783671 53
= 81.97

60. How to find a^b on calculator? (Particularly when b is in fractions)

→ Enter a
√ 12 times
deduct 1
Mutiply by 'b'
Add 1
$X_1 =$ 12 times

61. Common base of Logs is : 10

Natural base of Logs is : e where e = exponential factor = 2.7183 (approx)

$\text{Log}_{10} 10 = 1$, $\text{Log}_{10} 2 = \text{Log}2/1$

62. $\text{Log}_{\sqrt{2}} 64 =$ By using $\text{Log}_b a = \text{Log}a / \text{Log}b$

$$= \frac{\text{Log } 64}{\text{Log}\sqrt{2}} = \frac{\text{Log } 2^6}{\text{Log } 2^{1/2}} = \frac{6 \cdot \text{Log}2}{\frac{1}{2}\text{Log}2} = 6 \times 2 = 12$$

63. $\text{Log}_2 \text{Log}_2 \text{Log}_2 16 = \text{Log}_2\left[\text{Log}_2\left(\text{Log}_2 16\right)\right]$

$= \text{Log}_2\left[\text{Log}_2(4)\right] = \text{Log}_2 2 = 1.00$

64. $\text{Log}_9 (1/3) = m$ — By using if $\text{Log}_x a = y$ then $x^y = a$

$\therefore (9^m) = \frac{1}{3} \quad 3^{2m} = 3^{-1} \quad \therefore 2m = -1 \quad \therefore m = -1/2$

65. $\text{Log}_{16} 32^{-8} = \dfrac{\text{Log}(32)^{-8}}{\text{Log}(16)} = \dfrac{\text{Log}(2^5)^{-8}}{\text{Log}(2^4)} = \dfrac{-40 \cdot \text{Log}2}{4 \cdot \text{Log}2}$

$= -10$

66. Log x = (m + n) ; Log y = (m - n); then $\text{Log}\left(\dfrac{10x}{y^2}\right) =$

$= \text{Log}\,10 + \text{Log}x - \text{Log}y^2$

$= 1 + m+n - 2(m-n)$

$= 1 + m+n-2m+2n = 1-m+3n = 3n-m+1$

67. 2 Log 5 + Log 8 - (1/2) Log 4 =

$= \text{Log}25 + \text{Log}8 - \text{Log}2 = \text{Log}\left(\dfrac{25 \times 8}{2}\right) = \text{Log}_{10} 100$

$= 2.00$

68. $\sqrt[4]{729 \times \sqrt[3]{9^{-1} \times 27^{-4/3}}} = ?$

$= \left[3^6 \times \left(\frac{1}{3^2} \times 3^{-4}\right)^{1/3}\right]^{1/4} = \left[3^6 \times (3^{-6})^{1/3}\right]^{1/4} = \left[3^6 \times 3^{-2}\right]^{1/4} = (3^4)^{1/4}$

$= 3^1 = 3$

69. $\text{Log}_{2\sqrt{2}} 64 = ?$

$= \dfrac{\text{Log}64}{\text{Log}2\sqrt{2}} = \dfrac{\text{Log}(2)^6}{\text{Log}(2)^{3/2}} = \dfrac{6 \cdot \text{Log}2}{\frac{3}{2}\text{Log}2} = 6 \times \frac{2}{3} = 4.00$

70. Find 4th proportional to $\frac{2}{3}, \frac{3}{7}, 4$.

→ $\frac{2}{3}, \frac{3}{7}, 4, x$ product of extremes = product of means

$\frac{2x}{3} = \frac{12}{7}$ $\therefore x = \frac{12}{7} \times \frac{3}{2} = \frac{36}{14} = \frac{18}{7}$

71. If $2^x = 3^y = 6^{-z}$; then $(1/x) + (1/y) + (1/z) = ?$

→ $2^x = 3^y = 6^{-z} = k$

$x \cdot \log 2 = y \cdot \log 3 = -z \cdot \log 6 = \log k$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

$= \frac{\log 2}{\log k} + \frac{\log 3}{\log k} + \frac{-\log 6}{\log k}$

$= \frac{\log\left(\frac{2 \times 3}{6}\right)}{\log k} = 0$

72. Find in what ratio will the total wages of the workers of a factory be increased or decreased if there is reduction in no. of workers in the ratio of 17:12 and increment in wage rate per worker in the ratio of 24:29

Total wages have changed in the ratio of 34:29

→

	old	new
No. of workers	x	$x \times \frac{12}{17}$
wage rate per worker	y	$y \times \frac{29}{24}$
Total wages	$x \cdot y$	$xy \times \frac{12}{17} \times \frac{29}{24} = \frac{29xy}{34} = \frac{29}{34} \cdot xy$

73. What least number must be added to each one of 6, 14, 18, 38 to make them in proportion

a. 5 **b. 3** ~~**c. 2**~~ **d. 4**

$(6+x), (14+x), (18+x), (38+x)$ are in prop when

$\therefore$ 8, 16, 20, 40 $x = 2$

74. The incomes of X and Y are in the ratio of 3:2 and their expenditures are in the ratio of 5:3. If each saves ₹ 1500 then incomes of X and Y resp. are :

→

	Income	saving	exp
X	$3x$	1500	$3x - 1500$
Y	$2x$	1500	$2x - 1500$

$\therefore \frac{3x - 1500}{2x - 1500} = \frac{5}{3}$

$\therefore 9x - 4500 = 10x - 7500 \therefore x = 3000$

$\therefore$ Incomes are ₹9000, ₹6000

75. In a sugar solution of 300 gms, the proportion of sugar is 40%. How much sugar should be added to make it 50%

→ In 300 gms : sugar = 120 gms

$50\%(300 + x) = 120 + x$

$150 + 0.50x = 120 + x$

$30 = 0.50x$

$\therefore x = 60$ gms

76. A mixture contains milk and water in the ratio of 5:1. On adding 5 litres of water, the ratio of milk to water becomes 5:2. The quantity of milk in the original mixture is :

→ Milk = $5x$, water = x

$$\frac{5x}{x+5} = \frac{5}{2}$$

$10x = 5x + 25$

$5x = 25$ $\therefore x = 5$ Litres

$\therefore$ qty of milk in origin. mixture = 25 litres

77. If the denominator of a fraction exceed the numerator by 8. If numerator and denominator are both increased by 5, then fraction becomes 3/5. Find the original fraction.

→ Fraction : $\frac{x}{x+8}$

$$\frac{x+5}{x+13} = \frac{3}{5}$$

$5x + 25 = 3x + 39$

$2x = 14$ $x = 7$

$\therefore$ Fraction is $= \frac{7}{15}$

78. If $\text{Log}_{3/2} x = 3$, Find value of x

→ $\text{Log}_{3/2} x = 3$

$\therefore \left(\frac{3}{2}\right)^3 = x$

$\therefore x = \frac{27}{8}$

If $\text{Log}_p d = m$ then $p^m = d$

79. $\text{Log}_{(1/9)} 243 = x$. Find x

→ $\therefore \left(\frac{1}{9}\right)^x = 243$

$\left(\frac{1}{3^2}\right)^x = 3^5$

$\therefore (3^{-2})^x = 3^5$

$3^{-2x} = 3^5$

$\therefore -2x = 5$ $\therefore x = -5/2$

80. $\text{Log } x^3 - 2 \text{ Log } x - 2 = 0$. Find x

→ $3 \text{Log } x - 2 \text{Log } x = 2$

$\text{Log}_{10} x = 2$

$\therefore x = 10^2 = 100$

81. $\text{Log}_a 3 = 2$, $\text{Log}_b 8 = 3$ then $\text{Log}_b a$ = ?

→ $\text{Log}_a 3 = 2$ $\text{Log}_b 8 = 3$

$\therefore a^2 = 3$ $b^3 = 8$

$a = \sqrt{3}$ $b = 2$

$\text{Log}_b a$

$= \frac{\text{Log } a}{\text{Log } b} = \frac{\text{Log } \sqrt{3}}{\text{Log } 2} = \frac{2 . \text{Log } \sqrt{3}}{2 . \text{Log } 2}$

$= \text{Log } 3 / \text{Log } 4 = \text{Log}_4 3$

82. If 2 Log a + 3 Log b - 2 = 0 then a^2b^3 = ?

→ $\text{Log}\, a^2 + \text{Log}\, b^3 = 2$

$\text{Log}_{10}(a^2b^3) = 2$ $\quad \therefore\ a^2b^3 = 10^2 = 100$

83. $\text{Log}_2[\text{Log}_2\{\text{Log}_3(\text{Log}_3 27^3)\}]$

→ $\text{Log}_2\left[\text{Log}_2\left\{\text{Log}_3\ 9\right)\right\}\Big]$

$= \text{Log}_2\left[\text{Log}_2(2\ )\right] = \text{Log}_2 1 = \text{Zero}$

84. 2 numbers are in the ratio of 3:4. If 6 is added to each term then the new ratio will be 4:5 then the numbers are

→ $3x, 4x$ $\quad \frac{3x+6}{4x+6} = \frac{4}{5}$ $\quad \therefore x = 6$

$15x + 30 = 16x + 24$ $\quad \therefore$ Numbers are 18, 24

85. The sub-duplicate ratio of 1250:50 is :

→ Sub-doplicate ratio of 1250 : 50

= Sub-doplicate ratio of 25 : 1

= 5 : 1

86. Dhrish earns ₹ 2,780 in 7 hrs and Vinod earns ₹ 990 in 12 hrs. Ratio of their earning per hour is :

→ Ratio of earning per hour $= \frac{2780/7}{990/12} = \frac{2780}{7} \times \frac{12^{4}}{\cancel{990}_{33}} = \frac{1112}{231}$

87. P, Q, R are 3 cities. The ratio of avg. temp. of P, Q is 11:12 and that of P, R is 9:8. Find the ratio of avg temp. of Q:R.

→ $P:Q = 11:12 = 99:108$

$P:R = 9:8 = 99:88$

$\therefore\ P:Q:R = 99:108:88$

$\therefore \frac{Q}{R} = \frac{108}{88} = \frac{27}{22} = 27:22$

88. If 2s : 3t is the duplicate ratio of (2s-p) : (3t-p) then

a. $p^2 = 6st$ b. $p = 6st$ c. $2p = 3st$ d. None of these

$\frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$ $\quad \therefore \frac{2s}{3t} = \frac{4s^2 - 4ps + p^2}{9t^2 - 6tp + p^2}$

$18st^2 - 12pts + 2p^2s = 12s^2t - 12pts + 3tp^2$

$18st^2 - 12s^2t = 3tp^2 - 2p^2s \quad \therefore \quad 6st(3t-2s) = p^2(3t-2s)$

$\therefore \quad p^2 = 6st$

89. If A = B/2 = C/5; then A:B:C is :

$\left(\frac{A}{1}\right) = \left(\frac{B}{2}\right) = \left(\frac{C}{5}\right) \quad \therefore \quad A:B:C = 1:2:5$

$A:B = 1:2 \qquad B:C = 2:5$

90. Log 5 = 0.6990, Log 3 = 0.4771 then Log $\left(\frac{50}{300}\right)$ = ?

$\text{Log}\left(\frac{50}{300}\right) = \text{Log}50 - \text{Log}300$

$= 1.6990 - 2.4771$

$= -0.7781$

91. Log 2 = x; Log 3 = y; then Log 60 = ?

$\text{Log}(60) = \text{Log}(2 \times 3 \times 10)$

$= \text{Log}2 + \text{Log}3 + \text{Log}10$

$= x + y + 1$

92. Log $\left(\frac{1}{81}\right)$ to the base 9 is equal to :

$\text{Log}_9(1/81) = \frac{\text{Log}(1/81)}{\text{Log}(9)} = \frac{\text{Log}\,9^{-2}}{\text{Log}\,9} = -2$

93. $\bar{4}.5671 + 7.8253$ = ?

$\bar{4}.5671 + 7.8253$

$= (-4 + 0.5671) + 7.8253$

$= -3.4329 + 7.8253 = 4.3924$

94. $\sqrt[(a+b)]{\frac{x^{a^2}}{x^{b^2}}} \cdot \sqrt[(b+c)]{\frac{x^{b^2}}{x^{c^2}}} \cdot \sqrt[(c+a)]{\frac{x^{c^2}}{x^{a^2}}}$

$= \left(x^{a^2-b^2}\right)^{\frac{1}{a+b}} \times \left(x^{b^2-c^2}\right)^{\frac{1}{(b+c)}} \cdot \left(x^{c^2-a^2}\right)^{\left(\frac{1}{c+a}\right)}$

$= x^{(a-b)} \cdot x^{(b-c)} \cdot x^{(c-a)} = (x)^{a-b+b-c+c-a} = x^0$

$= 1.00$

$\sqrt{5} : \sqrt{11}$ is an incommensurable ratio.

3.50 : 2.50 , 8.116 : 2.2
8.16 : 3.20 , 2 : 5 , $\sqrt{25} : \sqrt{49}$ } commensurable Ratios

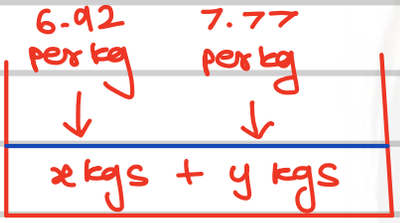
95. What is a commensurable ratio and incommensurable ratio?

→ • If Ratio of 2 or more term can be written in the form of Ratio of integers : commensurable ratio

• If Ratio of 2 or more term can't be written in the form of Ratio of integers : Incommensurable ratio

96. A Dealer mixes tea costing ₹ 6.92 per kg with tea costing ₹ 7.77 per kg and sells the mixture at ₹ 8.80 per kg and earns profit of $17\frac{1}{2}$% on sales price. In what proportion does he mix them ?

a. 2 : 3 ~~b.3: 2~~ c.5 : 2 d. None of these



selling price = 8.80 per kg
profit per kg = 8.80 × 17.50% = 1.54
cost per kg = 8.80 − 1.54 = 7.26 per kg

$6.92x + 7.77y = 7.26(x+y)$
$692x + 777y = 726x + 726y$
$51y = 34x$
$\frac{51}{34} = \frac{x}{y}$ ∴ $\frac{x}{y} = \frac{3}{2}$

97. If x : y = z : w = 8 : 7; then $\left(\frac{x+z}{y+w}\right)$ = ?

→

$\frac{x}{y} = \frac{z}{w} = \frac{8}{7} = \frac{x+z}{y+w}$ as per Addendo

$\therefore \left(\frac{x+z}{y+w}\right) = \frac{8}{7}$

98. If $\left(\frac{5x-3y}{5y-3x}\right) = \frac{3}{4}$ then x : y = ?

→ $\frac{5x-3y}{5y-3x} = \frac{3}{4}$

$20x - 12y = 15y - 9x$
$29x = 27y$

∴ $\frac{x}{y} = \frac{27}{29}$

x : y = 27 : 29

99. Find value of x if $x^2\sqrt{x} = (x\sqrt{x})^x$

→ $x^2 \cdot \sqrt{x} = (x\sqrt{x})^x$

$x^2 \times x^{\frac{1}{2}} = \left(x^1 \times x^{\frac{1}{2}}\right)^x$

$x^{\frac{5}{2}} = x^{\frac{3x}{2}}$

$\therefore \frac{5}{2} = \frac{3x}{2}$

$3x = 5$

$x = 5/3$

100. $\dfrac{(3^3)^2 \times (4^2)^3 \times (5^3)^2}{(3^2)^3 \times (4^3)^2 \times (5^2)^3} = \dfrac{3^6 \times 4^6 \times 5^6}{3^6 \times 4^6 \times 5^6} = 1.00$

$a^m \times a^n = a^{m+n}$	$a^m \times a^{-m} = a^0 = 1$
$(a \times b)^m = a^m \cdot b^m$	$[(ab)^d]^x = a^{bdx}$
$\frac{a^m}{a^n} = (a)^{m-n}$	$\sqrt[m]{x} = x^{\frac{1}{m}}$, $a^{c/d} = \sqrt[d]{a^c}$
$a^{-k} = \frac{1}{a^k}$ OR $a^k = \frac{1}{a^{-k}}$	$\frac{a^m \times a^n \times a^p}{a^x} = a^{m+n+p-x}$

101. Log 5 = 0.6990, Log = 0.4771. Find Log $\left(\dfrac{500}{0.003}\right)$

Log 3 = 0.4771

→ $\text{Log}\left(\frac{500}{0.003}\right) = \text{Log}500 - \text{Log}0.003 = 2.6990 - (\overline{3}.4771)$

$= 2.6990 - (-3 + 0.4771) = 2.6990 - (-2.5229)$

$= 5.2219$

OR $\text{Log}\left(\frac{500}{0.003}\right) = \text{Log}\left(\frac{500000}{3}\right) = \text{Log}500000 - \text{Log}3 = 5.6990 - 0.4771$

$= 5.2219$

102. Log 2 = x , Log 3 = y. Find Log (2.40)

→ $\text{Log}(2.40) = \text{Log}\left(\frac{3 \times 2 \times 2 \times 2}{10}\right)$

$= \text{Log}3 + \text{Log}2 + \text{Log}2 + \text{Log}2 - \text{Log}10 = y + 3x - 1$

$= 3x + y - 1$

Calculator Tricks

1. How to find a^b on calculator. (Mainly when b is a fractions)

Enter 'a'
√ 12 times
Deduct 1
Multiply by 'b'
Add 1
'=' 12 times

Find -

1. $12^{0.35}$ = 2.386645311 97
2. $286^{1.3528}$ = 2099.728501 72
3. $1.0296^{0.3}$ = 1.00 878 947544
4. $878^{1.2896}$ = 6237.66084318
5. $\sqrt[5]{100} = 100^{1/5} = 100^{0.20}$ = 2.5129271555 2

2. How to find Log x on calculator

Enter 'x'
√ 15 times
Deduct 1
Multiply by 14230.9635

Find -

1. Log 35 = 1.544151 8987
2. Log 896.8 = 2.95300 220038
3. Log 0.008671 = -2.06178184957

3. How to find A.Log y on calculator

Enter 'y'
Divide by 14230.9635
Add 1
'=' 15 times

Find -

1. A.Log 2.8935 = 782
2. A.Log 0.08613 = 1.219353290 35
3. A.Log 5.8863 = 767506.389 108
4. A. Log 1.2287 = 16.92960061 2

4.

1.0686^{90} = 392.071510773

1.0296^{56} = 5.121980 23366

1.0811^{61} = 116.36380 3473

1.0686	x =	till step count comes 91
1.0296	x =	till step count comes 57
1.0811	x =	till step count comes 62

5. How to find discounting factor on calculator? (for n^{th} year)

1 ÷ (1+r) then '=' n times

(OR) 1 ÷ (1+r) then press '=' till step count comes (n+2)

6. How to find annuity factor on calculator?

1 ÷ (1+r) then '=' n times and GT

Annuity factor = $\left[\frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots + \frac{1}{(1+r)^n}\right]$

7. 8, 15, 22, 29..... Find t_{28}, t_{38}

→ $t_{28} = 197$

$t_{38} = 267$

8. $5^2 =$ 5 x = Answers : 25

$15^2 =$ 15 x = 225

$28^2 =$ 28 x = 784

9. $\frac{1}{2 \times 2} = 0.25$ ÷ = will give you reciprocal

$\frac{1}{20} = 0.05$

$\frac{1}{5 \times 28} = 0.007142857142$

$\frac{1}{25 \times 4} = 0.01$

10. 3 + 5 = 8

8 + 5 = 13

9 + 5 = 14

10 + 5 = 15

100 + 5 = 105

2086 + 5 = 2091

3 + 5 =
then 8 =
9 =
10 =
100 =
2086 =

11. 100 - 3 = 97

208 - 3 = 205

98 - 3 = 95

63 - 3 = 60

238 - 3 = 235

100 - 3 =
then 208 =
98 =
63 =
238 =

12. 13 x 3 = 39

13 x 5 = 65

13 x 8 = 104

13 x 11 = 143

13 x 20 = 260

13 x 3 =
then 5 =
8 =
11 =
20 =

13. a. (1.20 x 5.36) + (28.96 + 15.92) + (28.11 x 18.63)

= 575 .0013

b. (15.92 x 21.83) + (28.66 x 11.193) - (5.06 x 18.193)

= 576. 2684

14. $5^2 + 8^2 + 9^2 + 13^2 + 16^2$ =

= 595

15. (5 x 9) + (33 x 18) + (28 x 93) - (16 x 6) + 13^2 = 3316

$(1.20 \times 18.19) - (63.81 \times 2.231) + (1.22 \times 1.8193) =$

$= -118.312564$

EXERCISE

1. Log 28.96 = 1.461873 7332

2. A.Log 2.8592 = 722.623831645

3. 1.20868592^{28} = 201.733008327

4. $883.9281^{1.5625}$ = 39960.8104092

5. 68, 74, 80, 86..... Find t_{28}, t_{32}

t_{28} = 230

t_{32} = 254

6.

	x	y	x^2	y^2	xy	x^2y	xy^2
	1.20	8.53					
	9.63	2.58					
	10.61	11.93					
Total	21.44	23.04	206.749	221.7422	161.6587	1594.529 555	1661.481 401

7. $\sqrt{\frac{63581}{8} - 56^2}$ = 69.36 58 777786

8. $\sqrt{(86 \times 93) + (59 \times 81) + (29 \times 63)}$ = ?

= $\sqrt{14604}$ = 120.847010 72

9. $\sqrt[10]{58263}$ = $(58263)^{1/10}$

= 2.999950 85665

10. $\sqrt[15]{56298193} = (56298193)^{1/15} = 3.29\,421\,172539$

11. $\dfrac{8}{\frac{1}{2}+\frac{1}{3}+\frac{1}{9}+\frac{1}{13}+\frac{1}{16}+\frac{1}{18}} = 7.021097046\,49$

12.

$\sqrt[8]{93} = 1.762220984\,74$

$\sqrt[4]{124} = 3.336\,993\,965\,47$

$\sqrt[16]{28963} = 1.900\,488\,392\,86$

$\sqrt[32]{58231} = 1.409000\ 25029$

$\sqrt[64]{28,63,588} = 1.261505\,72879$

$\sqrt[11]{52,93,211} = (5293211)^{1/11} = 4.095\,419\,842\,6$

$\sqrt[20]{5,85,93,288} = (58593288)^{1/20} = 2.449\,555\,450\,424$

13.

100 x 18% = 18	283 + 3.53% = 292.9899
100 + 18% = 118	18 + 2% = 18.36
200 + 16% = 232	200 - 3% = 194
300 + 12% = 336	300 - 2% = 294
1050 + 16% = 1218	

14. $\left(\frac{3}{5}+\frac{8}{7}+\frac{11}{9}+\frac{25}{8}\right) = 6.090\,079\,365\,07$

15. $15^2 + 8^3 + 3^4 + 18^2 + 2.82^3 + 9.53^4 =$

$= 9412.861644481$

16. $\frac{16}{(2/5)} + \frac{18}{(3/8)} + \frac{19}{(5/7)} + \frac{28}{(3/11)} =$

$= 217.266666666$

$\log x$ = characteristic of x + mantissa of x

Integer part = characteristic

Fractional part = Mantissa

$\log_b a = \frac{\log a}{\log b}$

common base of Logs : 10

Natural base of Logs : e

$\log_k (ab) = \log_k a + \log_k b$

$\log_x (m/n) = \log_x m - \log_x n$

$\log_p (xyz) = \log_p x + \log_p y + \log_p z$

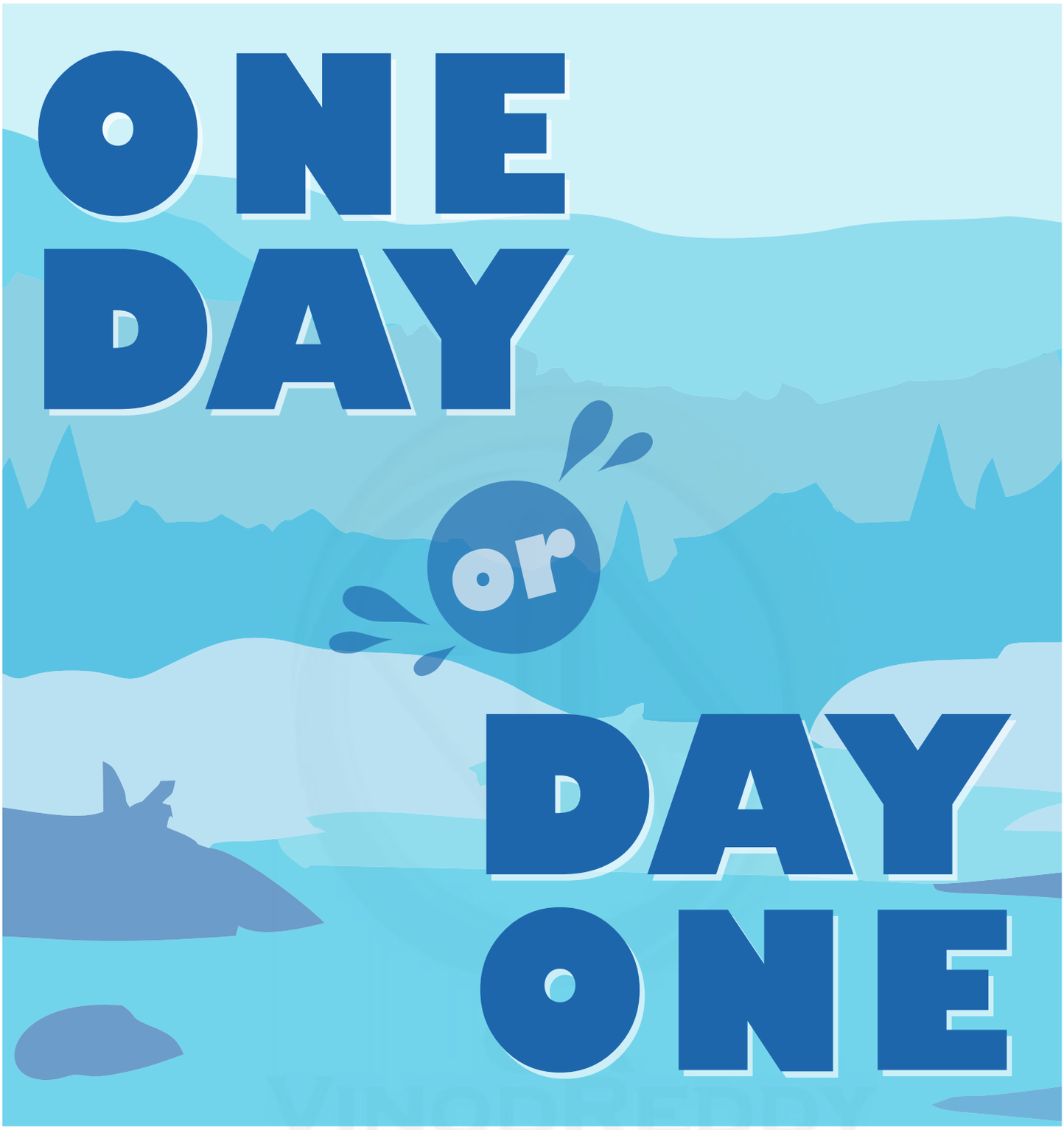
$\log (a)^b = b \cdot \log a$, $\log (a)^{-x} = -x \cdot \log a$

$A \cdot \log (\log x) = x = \log (A \cdot \log x)$

If $\log_x a = y$ then $x^y = a$

$\log_a a = 1.00$, $\log_b a \times \log_a b = 1$, $\log_b a \cdot \log_c b = \log_c a$

$\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, $\log 1 = 0$



YOU DECIDE.

CA VINOD REDDY

CA VINOD REDDY

IN THE END

WE ONLY REGRET

THE CHANCES

WE DIDN'T TAKE

CA VINOD REDDY

HEROS ARE MADE BY
THE PATH THEY CHOOSE
NOT THE POWER
THEY ARE GRACED WITH!