RATIO | PROPORTION | LOGS & INDICES

1. Ratio is a Fraction used for comparison of 2 or more quantities which are

of same type	Expressed in same unit of measurement

Example : If Vinod's weight is 90 kgs and Anand's weight is 60 kgs then we can say that : Ratio of weights is 90:60

2. Weights of Vinod, Anand are in the ratio of 90 : 60 = 3 : 2

Simplest form of 90:60

Also we can write that, weights of Anand, Vinod are in the ratio of 60 : 90 = 2 : 3

Ratio can expressed without any unit of measurement

3. Ratio can be expressed like $2:3$, $9:11$, $\frac{9}{11}$, $\frac{12}{31}$, $5:7:9$

It means Ratios are UNIT-FREE

4. The ratio 30 : 60 can also be written as 1 : 2 in its simplest form

5. Ratio is generally expressed in its Simplest form

simplest form of 12 : 18 is 2 : 3

6. a : b can also be written as $\frac{a}{b} = \frac{am}{bm} = ak : bk = ax : bx = ay : by = \frac{a}{x} : \frac{b}{x}$ provide $m, k, x, y \neq 0$

7. All the terms of a ratio can be multiplied OR divided by same non-zero number.

$x : y = 3x : 3y = 10x : 10y = \frac{x}{12} : \frac{y}{12} = \frac{x/m}{y/m} = xm : ym$ provided $m \neq 0$

8. In the ratio x : y

x = First term = Antecedent

y = Second term = Consequent

In the ratio 9:8 9: Antecedent 8: consequent

9.

In the ratio a:b If	then Ratio is known as	Examples
a > b i.e. Antecedent > Consequent	Ratio of Greater Inequality	9:7, 11:3, 101:89, 35:34, 39:21
a < b i.e. Antecedent < Consequent	Ratio of Lesser Inequality	5:11, 35:38, 41:43, 93:523, 581:1000
a = b i.e. Antecedent = Consequent	Ratio of Equality	8:8, 5:5, 19:19, 1:1 38:38

Simplest form of Ratio of equality is always 1 : 1

10. Find simplest form of ratio of quantities
(3 hours 5 mins 8 secs), (8 hours 10 mins 40 secs)

→

$$\frac{3 \text{ hrs } 5 \text{ mins } 8 \text{ secs}}{8 \text{ hrs } 10 \text{ mins } 40 \text{ secs}} = \frac{(3\times60\times60)+(5\times60)+8}{(8\times60\times60)+(10\times60)+40}$$

$$= \left[\frac{10800+300+8 \text{ seconds}}{28800+600+40 \text{ seconds}}\right] = \frac{11108}{29440} = \frac{2777}{7360} = 2777:7360$$

11. Ratio can be expressed without unit of measurement
(i.e. Ratio are unit-free)

12.

Quantities	Ratio in simplest form
5 hrs, 5 mins	$\frac{(5\times60) \text{ mins}}{5 \text{ mins}} = 60:1$
5 hrs 10 mins, 10 hrs 15 mins PIS Note : 1 hour = 60 mins 1 min = 60 secs	$\frac{(5\times60)+10}{(10\times60)+15} = \frac{310}{615} = \frac{62}{123}$ $= 62:123$
8 feets 6 inches, 12 feets 8 inches PIS Note : 1 foot = 12 inches	$\frac{(8\times12+6) \text{ inches}}{(12\times12+8) \text{ inches}}$ $= \frac{102 \text{ inches}}{152 \text{ inches}} = \frac{51}{76} = 51:76$
8 meters 10 cms, 10 meters 18cms PIS Note : 1 km = 1000 meters 1 Meter = 100 cms = 1000 mms 1 cm = 10 mms	$\frac{(8\times100+10) \text{ cms}}{(10\times100+18) \text{ cms}} = \frac{810 \text{ cms}}{1018 \text{ cms}}$ $= 810:1018 = 405:509$

5 mins 33 secs, 8 mins 45 secs	$\frac{(5\times60)+33 \text{ secs}}{(8\times60)+45 \text{ secs}} = \frac{333 \text{ secs}}{525 \text{ secs}}$ $= 111:175$
8GB, 512 MB 1024 bytes = 1 kB 1024 kB = 1 MB 1024 MB = 1 GB 1024 GB = 1 TB	$\frac{8\times1024 \text{ MB}}{512 \text{ MB}}$ $= 8192:512$ $= 16:1$
8.5 kgs, 880 gms 1 kg = 1000 gms 1 gm = 1000 mgs	$\frac{8.50\times1000 \text{ gms}}{880 \text{ gms}}$ $= \frac{8500 \text{ gms}}{880 \text{ gms}} = \frac{850}{88} = 425:44$
8.8 kms, 44000 mms	$\frac{8.80\times1000\times100\times10 \text{ mms}}{44,000 \text{ mms}}$ $= \frac{88,00,000}{44,000} = 200:1$

13. Order of terms in a ratio is important

$a:b \neq b:a$ $\quad 3:7 \neq 7:3$

14. If we interchange the position of antecedent & consequent, we get Inverse Ratio.

15. Inverse Ratio of 5:7 is 7:5

& Pls remember $5:7 \neq 7:5$

16. Inverse Ratio of Inverse Ratio of a : b is a:b

Inverse Ratio of Inverse Ratio of 13:28 is 13:28

Did You Know ?

17. Inverse Ratio of Ratio of Greater inequality is Ratio of Lesser inequality and vice versa.

18. Inverse Ratio of Ratio of Equality is Ratio of Equality only.

19. Duplicate Ratio : . Duplicate ratio of $m:n$ is $m^2:n^2$

. Duplicate ratio of $1:9$ is $1:81$

. Duplicate ratio of $\sqrt{3}:15$ is $3:225 = 1:75$

. Duplicate ratio of $a^2:b^3$ is $(a^2)^2:(b^3)^2 = a^4:b^6$

20. Triplicate Ratio : (Triplicate ratio of $a:b$) $= a^3:b^3$

Triplicate ratio of $2:5$ is $2^3:5^3 = 8:125$

21. Sub-duplicate Ratio : sub-duplicate ratio of $81:625$ is $9:25$

. Sub-duplicate ratio of $5:11$ is $\sqrt{5}:\sqrt{11} = 5^{1/2}:11^{1/2}$

22. Sub-triplicate Ratio : sub-triplicate ratio of $8:125$ is $2:5$

sub-triplicate ratio of $m:n$ is $\sqrt[3]{m}:\sqrt[3]{n}$ $= m^{1/3}:n^{1/3}$

$\sqrt[m]{a} = (a)^{1/m}$

Fill in the blanks

A Ratio	It's				
	Duplicate Ratio	Triplicate Ratio	Sub-duplicate Ratio	Sub-triplicate Ratio	Inverse Ratio
$p:q$	$p^2:q^2$	$p^3:q^3$	$\sqrt{p}:\sqrt{q}$	$\sqrt[3]{p}:\sqrt[3]{q}$	$q:p$
$3:11$	$9:121$	$27:1331$	$3^{1/2}:11^{1/2}$	$3^{1/3}:11^{1/3}$	$11:3$
$a^2:b^3$	$(a^2)^2:(b^3)^2$ $= a^4:b^6$	$(a^2)^3:(b^3)^3$ $= a^6:b^9$	$(a^2)^{1/2}:(b^3)^{1/2}$ $= a:b^{3/2}$	$(a^2)^{1/3}:(b^3)^{1/3}$ $= a^{2/3}:b$	$b^3:a^2$
$a^7:p^{10}$	$(a^7)^2:(p^{10})^2$ $= a^{14}:p^{20}$	$(a^7)^3:(p^{10})^3$ $= a^{21}:p^{30}$	$(a^7)^{1/2}:(p^{10})^{1/2}$ $= a^{7/2}:p^5$	$(a^7)^{1/3}:(p^{10})^{1/3}$ $= a^{7/3}:p^{10/3}$	$p^{10}:a^7$
$3^{1/3}:8^{1/7}$	$(3^{1/3})^2:(8^{1/7})^2$ $= 3^{2/3}:8^{2/7}$	$(3^{1/3})^3:(8^{1/7})^3$ $= 3:8^{3/7}$	$(3^{1/3})^{1/2}:(8^{1/7})^{1/2}$ $= 3^{1/6}:8^{1/14}$	$(3^{1/3})^{1/3}:(8^{1/7})^{1/3}$ $= 3^{1/9}:8^{1/21}$ $= \sqrt[9]{3}:\sqrt[21]{8}$	$8^{\frac{1}{7}}:3^{\frac{1}{3}}$ $= \sqrt[7]{8}:\sqrt[3]{3}$

23. Compounded Ratio or Ratio Compounded

Compounded ratio of 2 : 3, x : y, m : n, p : q, e : f is

Compounded Ratio = (2xmpe) : (3ynqf)

= (product of antecedents) : (product of consequents)

24. Find Ratio Compounded of 3 : 7, 8 : 3, 5 : 11, 22 : 5, 14 : 17

→

$$\text{Compounded Ratio (or Ratio compounded)} = (3\times8\times5\times22\times14) : (7\times3\times11\times5\times17)$$

$$= \frac{3\times8\times5\times\overset{2}{22}\times\overset{2}{14}}{7\times3\times11\times5\times17} = \frac{8\times2\times2}{17}$$

$$= \frac{32}{17} = 32:17$$

25. Find compounded ratio of 8 : 13, 39 : 5, 25 : 18, 27 : 15, 30 : 8, 16 : 5, 50 : 2

→

$$\text{Compounded ratio} = \left(\frac{8\times\overset{3}{39}\times25\times\overset{3}{27}\times\overset{2}{30}\times16\times50}{13\times5\times18\times15\times8\times5\times2}\right)$$

$$= \frac{3\times25\times3\times16}{1} = 3600:1$$

- Duplicate ratio of 9 : 13 is

ⓐ 9 : 13 ⓑ 82 : 130 ⓒ 162 : 338 ⓓ None of these

Dupli. ratio of 9 : 13 = $9^2 : 13^2$ = 81 : 169 = 162 : 338

26. Find compounded ratio 5 : 7, 14 : 3, 9 : 2, 6 : 15,
Duplicate Ratio of 5 : 2, Triplicate Ratio of 4 : 5,
Sub-duplicate ratio of 25 : 49, Sub-triplicate Ratio of 343 : 64,
inverse ratio of 2 : 9, Duplicate Ratio of 3 : 2

→ 5:7, 14:3, 9:2, 6:15, 25:4, 64:125, 5:7, 7:4, 9:2, 9:4

$$\text{Compounded Ratio} = \frac{5 \times 14 \times 9 \times 6 \times 25 \times 64 \times 5 \times 7 \times 9 \times 9}{7 \times 3 \times 2 \times 15 \times 4 \times 125 \times 7 \times 4 \times 2 \times 4}$$

(cancelling: 14 with 7 and 2; 6 with 3 and 2; 25 with 15 and 5; 64 with 4 and 4; 5 with 5; 9 with 9 and 3; 15 with 3 and 5; 125 with 25 and 5; 4 with 4; 2 with 2; 4 with 4)

$$= \frac{9 \times 9 \times 3}{1} = \frac{243}{1} = 243 : 1$$

27. Find duplicate ratio of sub-triplicate ratio of inverse ratio of 125 : 343

→ = Duplicate ratio of sub tripli. ratio of inverse ratio of 125:343
= Dupli. ratio of sub tripli. ratio of 343:125
= Dupli. ratio of 7:5
= 49:25

28. Find Inverse Ratio of Duplicate Ratio of Triplicate Ratio of 2 : 5

→ = Inverse ratio of Dupli. ratio of Tripli. ratio of 2:5
= Inverse ratio of Dupli. ratio of 8:125
= Inverse ratio of 64:15625
= 15625 : 64

29. Find Duplicate Ratio of Inverse Ratio of Sub-duplicate ratio of 625 : 225

→ = Dupli. ratio of Inverse ratio of 25:15
= Dupli. ratio of 15:25
= 225:625 = 9:25

30. Find duplicate ratio of Sub-triplicate ratio of Inverse ratio of 10 : 3

→ = Dupli. ratio of subtriplicate ratio of 3 : 10

= Dupli. ratio of $(3)^{1/3} : (10)^{1/3}$

= $(3^{1/3})^2 : (10^{1/3})^2$

= $3^{2/3} : 10^{2/3}$

31. Find sub-triplicate ratio of sub-duplicate ratio of inverse ratio of 8 : 11

→ = Sub-triplicate ratio of sub-dupli. ratio of 11 : 8

= Sub-tripli. ratio of $11^{1/2} : 8^{1/2}$

= $(11^{1/2})^{1/3} : (8^{1/2})^{1/3} = 11^{1/6} : 8^{1/6} = 11^{1/6} : (2^3)^{1/6} = 11^{\frac{1}{6}} : 2^{\frac{1}{2}}$

$= \sqrt[6]{11} : \sqrt[6]{8}$ $= \sqrt[6]{11} : \sqrt{2}$

32. Find Inverse ratio of Duplicate ratio of Triplicate ratio of a : b

→ = Inverse ratio of Dupli. ratio of $a^3 : b^3$

= Inverse ratio of $a^6 : b^6$

= $b^6 : a^6 = \left(\frac{b^6}{a^6}\right)$

33. Find Duplicate ratio of sub-duplicate ratio of 8 : 35

→ Dupli. ratio of $\sqrt{8} : \sqrt{35}$

= 8 : 35

34. **Please Note**

1. Duplicate ratio of sub-duplicate ratio of 13 : 18 is 13 : 18

2. Sub-duplicate ratio of duplicate ratio of m : n is m : n

3. Triplicate ratio of sub-triplicate ratio of a : b is a : b

4. Sub-triplicate ratio of triplicate ratio of p : q is p : q

5. Inverse ratio of Inverse ratio of 35 : 88 is 35 : 88

35. Find inverse ratio of duplicate ratio of sub-triplicate ratio of sub-triplicate ratio of m : n

→

= Inverse of Duplicate of subtripli. of $m^{1/3} : n^{1/3}$

= Inverse of Duplicate of $m^{1/9} : n^{1/9}$

= Inverse of $(m^{1/9})^2 : (n^{1/9})^2$

= Inverse of $m^{2/9} : n^{2/9}$

= $n^{2/9} : m^{2/9} = (n^2)^{1/9} : (m^2)^{1/9} = \sqrt[9]{n^2} : \sqrt[9]{m^2}$

36. Find compounded ratio of 8 : 3, 6 : 11, 5 : 16, 48 : 15

→

Compounded Ratio = $\dfrac{\text{Product of antecedents}}{\text{Product of all consequents}}$

$= \dfrac{8 \times \overset{2}{\cancel{6}} \times \cancel{5} \times \cancel{48}}{\cancel{3} \times 11 \times \cancel{16} \times \cancel{15}\ \cancel{5}} = \dfrac{16}{11} = 16 : 11$

37. Find compounded ratio of a : b and a : b

→ $a^2 : b^2$

38. Find compounded ratio of a : b and $a^2 : b^2$

→ $a \cdot a^2 : b \cdot b^2 = a^3 : b^3$

39. Find compounded ratio of 8 : 11 & 11 : 8

→ $8 \times 11 : 11 \times 8 = 88 : 88 = 1 : 1$

40. **Please Note :**

1. If a ratio is compounded with itself then result is its Duplicate ratio

2. If a ratio is compounded with its duplicate ratio then result is its Triplicate ratio

3. If a ratio is compounded with its inverse ratio then result is Ratio of Equality

41. **Some Calculator Tricks :**

$\sqrt{64} = (64)^{1/2} = 8$

$\sqrt[4]{5000} = (5000)^{1/4} = \left(5000^{1/2}\right)^{1/2} = 8.408964153253$

$\sqrt[8]{256} = (256)^{1/8} = \left[\left(256^{1/2}\right)^{1/2}\right]^{1/2} = 2$

$\sqrt[16]{20821}$ = Take 20821 on calculator & press $\sqrt{}$ button 4 times = 1.861685640 64

$\sqrt[32]{85926.30}$ = Take 85926.30 on calculator & press $\sqrt{}$ button 5 times = 1.426236152 99

42.

If I enter a number in calculator and press $\sqrt{\ }$ button	I will get
Once	2nd root
Twice	4th root
Thrice	8th root
Four Times	16th root
Five Times	32nd root
Six Times	64th root

43. 1.1085^{18} = Take 1.1085 on calculator then press × then press =, =, =, = ... till step count comes 19

= 6.386200 56833

$2^5 = 32$

1.18632^{65} = Take 1.18632 then × then = till step count 66 = 66545.7602712

1.156321^{49} = 1232.65696746

2.85^{18} = 153888965.491

1.2351^{41} = 5751.577 68902

1.0096^{93} = 2.43158130991

2.2283^{16} = 369469.475819

3^{12} = 531441

6^8 = 1679616

1.2258^{46} = 11676.2230502

$\sqrt[64]{1,90,800}$ = 1.209230345

$\sqrt[8]{859686}$ = 5.5181370 4047

$\sqrt[32]{1.50632196}$ = 1.012884 51453

44.

$\sqrt[5]{17}$

$= (17)^{\frac{1}{5}}$

$= 17^{0.20}$

Here $a = 17$, $b = 0.20$

$= 1.762616666858$

cross-check

$(1.762616666858)^5$

$= 17$ (approx)

How to find the answer of a^b on calculator? (when b is in fractions)

1. Take 'a' on calculator
2. press $\sqrt{}$ 12 times
3. Deduct 1
4. Multiply by 'b'
5. Add 1
6. press '$x =$' 12 times
7. You will get answer of a^b on calculator screen.

45. $\sqrt[3]{80} = 80^{1/3} = 80^{0.3333333} = 4.311114448146$

cross-check : $4.311114448146^3 = 80$

$25180^{0.70} = 1207.485\,46831$

$48125^{1.20} = 414350.295255$

$\sqrt[10]{28500} = (28500)^{1/10} = 28500^{0.10} = 2.792459\,19177$

$(1,02,800)^{1/20} = (1,02,800)^{0.05} = 1.782113\,26241$

$(56,800)^{1/25} = (56,800)^{0.04} = 1.550308\,33411$

46. If a : b = 3 : 5, b : c = 5 : 8. Find a : b : c, a : c, c : a

→ $a:b = 3:5$

$b:c = 5:8$

$\therefore \quad a:b:c = 3:5:8$

$a:c = 3:8$

$c:a = 8:3$

47. If a : b = 5 : 11, b : c = 2 : 7. Find a : b : c, a : c, c : a

→ $a:b = 5:11 = 10:22$

$b:c = 2:7 = 22:77$

$\therefore \quad a:b:c = 10:22:77$

$a:c = 10:77$ & $c:a = 77:10$

48. If a : b = 3 : 8, b : c = 5 : 13. Find a : b : c, a : c, c : a

→ $a:b = 3:8 = 15:40$

$b:c = 5:13 = 40:104$

$\therefore \quad a:b:c = 15:40:104$

$a:c = 15:104$

$c:a = 104:15$

49. If a : b = 3 : 10, b : c = 4 : 5, c : d = 8 : 13
Find a : b : c : d, a : d, b : d, a : c

→ $a:b = 3:10 = 12:40$

$b:c = 4:5 = 40:50$

$\therefore \quad a:b:c = 12:40:50 = 96:320:400$

$c:d = 8:13 = 400:650$

$\therefore a:d = 48:325$

$b:d = 160:325 = 32:65$

$a:c = 48:200 = 6:25$

$a:b:c:d = 96:320:400:650 = 48:160:200:325$

50. Find duplicate ratio of subtriplicate ratio of sub-duplicate ratio of inverse ratio of $\sqrt[5]{18} : \sqrt[9]{35}$

→

$= \text{Dupli. of subtripli. of sub-dupli. of } 35^{1/9} : 18^{1/5}$

$= \text{Dupli. of subtripli. of } (35)^{1/18} : (18)^{1/10}$

$= \text{Dupli. of } (35^{1/18})^{1/3} : (18^{1/10})^{1/3}$

$= \text{Dupli. of } (35)^{1/54} : (18)^{1/30}$

$= (35^{1/54})^2 : (18^{1/30})^2$

$= 35^{2/54} : 18^{2/30} = 35^{1/27} : 18^{1/15} = \sqrt[27]{35} : \sqrt[15]{18}$

52. $\sqrt[20]{285798632598} = (285798632598)^{1/20}$

$= 3.754556099 38$

$\sqrt[16]{826398562281} = 5.55\ 6794\ 64007$

$(1.126391)^{58} = 995.\ 341154466$

$\sqrt[32]{82619835} = 1.767701315 44$

$(2289365288)^{0.20} = 75.142721436 4$

$(1.1119381)^{38} = 56.371\ 9725703$

53. If a : b = 2 : 3

b : c = 5 : 4

c : d = 7 : 9

Find a : b : c : d, a : d, b : d, a : c

→ a : b = 2 : 3 = 10 : 15

b : c = 5 : 4 = 15 : 12

∴ a : b : c = 10 : 15 : 12 = 70 : 105 : 84

c : d = 7 : 9 = 84 : 108

∴ a : b : c : d = 70 : 105 : 84 : 108

a : d = 70 : 108 = 35 : 54

b : d = 105 : 108 = 35 : 36

a : c = 70 : 84 = 5 : 6

54. a : b = 2 : 3

a : c = 3 : 8

a : d = 5 : 11 Find a : b : c : d

→ a : b = 2 : 3 = 30 : 45

a : c = 3 : 8 = 30 : 80

a : d = 5 : 11 = 30 : 66

∴ a : b : c : d = 30 : 45 : 80 : 66

55. If a : c = 9 : 8

b : d = 5 : 4

a : d = 2 : 7 Find a : b : c : d

→ a : c = 9 : 8 = 18 : 16

a : d = 2 : 7 = 18 : 63

a : c : d = 18 : 16 : 63 = 72 : 64 : 252

b : d = 5 : 4 = 315 : 252

a : b : c : d = 72 : 315 : 64 : 252

56. a : d = 2 : 17

c : b = 15 : 4

c : d = 2 : 9

Find a : b : c : d

→ a : d = 2 : 17 = 18 : 153

c : d = 2 : 9 = 34 : 153

a : c : d = 18 : 34 : 153 = 270 : 510 : 2295

c : b = 15 : 4 = 510 : 136

a : b : c : d = 270 : 136 : 510 : 2295

57. p : q = 3 : 11

r : s = 5 : 6

p : r = 4 : 7 Find p : q : r : s

→

p : q = 3 : 11 = 12 : 44

p : r = 4 : 7 = 12 : 21

∴ p : q : r = 12 : 44 : 21 = 60 : 220 : 105

r : s = 5 : 6 = 105 : 126

∴ p : q : r : s = 60 : 220 : 105 : 126

Q. If a : b = 2 : 3, b : c = 2 : 3, c : d = 2 : 3, d : e = 2 : 3

Find a : b : c : d : e

A. a : b = 2 : 3 = 4 : 6

b : c = 2 : 3 = 6 : 9

a : b : c = 4 : 6 : 9 = 8 : 12 : 18

c : d = 2 : 3 = 18 : 27

∴ a : b : c : d = 8 : 12 : 18 : 27 = 16 : 24 : 36 : 54

d : e = 2 : 3 = 54 : 81

∴ a : b : c : d : e = 16 : 24 : 36 : 54 : 81

58.

Ratio	Simplest form of the ratio
30 : 20	3 : 2
80 : 64	40 : 32 = 20 : 16 = 10 : 8 = 5 : 4
1.50 : 2.50	15 : 25 = 3 : 5
8 : 9	8 : 9
2.75 : 8.25	275 : 825 = 11 : 33 = 1 : 3
8.26 : 3.30	826 : 330 = 413 : 165
$\frac{8}{3} : \frac{9}{3}$	8 : 9
$\frac{3}{8} : \frac{11}{3}$	$\frac{3}{8} : \frac{11}{3} = 9 : 88$ OR $\frac{\frac{3}{8}}{\frac{11}{3}} = \frac{3}{8} \times \frac{3}{11} = \frac{9}{88} = 9 : 88$
$2\frac{1}{3} : 8\frac{2}{3}$	$2\frac{1}{3} : 8\frac{2}{3}$ $\frac{7}{3} : \frac{26}{3} = 7 : 26$
$7\frac{2}{7} : 19\frac{3}{7}$	$7\frac{2}{7} : 19\frac{3}{7} = \frac{51}{7} : \frac{136}{7} = 51 : 136 = 3 : 8$
$8\frac{2}{9} : 5\frac{3}{7}$	$\left(\frac{8\frac{2}{9}}{5\frac{3}{7}}\right) = \left(\frac{74/9}{38/7}\right) = \frac{74}{9} \times \frac{7}{38} = \frac{518}{342} = \frac{259}{171}$
$\sqrt{5} : \sqrt{8}$	$\sqrt{5} : \sqrt{8}$
$\sqrt{25} : \sqrt{81}$	5 : 9
$11\frac{2}{7} : 15\frac{2}{3}$	$\frac{11\frac{2}{7}}{15\frac{2}{3}} = \frac{\frac{79}{7}}{\frac{47}{3}} = \frac{79}{7} \times \frac{3}{47} = \frac{237}{329}$

59. Multiplying Ratio

Mr A's income in year 2023 is ₹ 5,00,000. In 2024 his income changed in the ratio of 25 : 27. Find his new income?

⟹

Years	2023	2024
Income	5,00,000	?
Ratio	25	27

$? = \text{New Income} = \frac{27 \times 500{,}000}{25} = 5{,}40{,}000$

$\text{New Income} = 5{,}00{,}000 \times \frac{27}{25}$

$\text{New Income} = \text{Old Income} \times \left(\text{Inverse ratio of } 25:27\right)$

$\text{New Income} = \text{old Income} \times \text{Multiplying Ratio}$

New quantity = old quantity × multiplying ratio

60. India's population in 2011 census is 140 crores. In 2021 population has changed in the ratio of 7 : 9. Find new population.

➡ New population = old population × Multiplying ratio

where Multiplying Ratio = Inverse of given ratio

$\therefore$ New population = $140 \text{ crores} \times \frac{9}{7} = 180 \text{ crores}$

New quantity = old quantity × multiplying ratio

old quantity = New quantity × Given ratio

61. Vinod's weight is x kgs, his weight changes in the ratio of 5 : 7
Find his new weight?

→ New weight = old weight × Multiplying ratio

$= x \times \frac{7}{5} = \frac{7x}{5}$

old weight	New weight
5	7
x	?

$? = x \times \frac{7}{5}$

New weight = old weight × Multi. Ratio

62. Anand's income in 2028 year is ₹ 50 crores. In comparison with year 2027 his income has changed in the ratio of 9 : 10.
Find his income for year 2027

→

Old Income = New Income × Given ratio

Anand's Income for the Year 2027 $= 50 \text{ crores} \times \frac{9}{10} = 45 \text{ crores}$

Old Income 2027	New Income 2028
9	10
?	50 crores

63. 2 quantities are in the ratio of 3 : 4. If 6 is added to both terms ratio becomes 4 : 5. Find old quantities.

→ Let those 2 quantities be $3x$ & $4x$

$$\frac{3x+6}{4x+6} = \frac{4}{5}$$

$$5(3x+6) = 4(4x+6)$$

$$15x+30 = 16x+24$$

$$6 = x$$

∴ old quantities : $3x = 18$, $4x = 24$

64. 2 quantities are in the ratio of 8 : 9. If 4 is added to 1st term & 7 is deducted from second term, ratio becomes 36 : 29. Find 2 quantities.

→ Let those 2 quantities be $8m, 9m$

$$\frac{8m+4}{9m-7} = \frac{36}{29}$$

$$29(8m+4) = 36(9m-7)$$

$$232m + 116 = 324m - 252$$

$$368 = 92m$$

$$\boxed{m = 4}$$

∴ Those quantities are : $8m = 8 \times 4 = 32$, $9m = 9 \times 4 = 36$

66. Present age of father and son is in the ratio of 16 : 1. After 8 years their age will be in the ratio of 4 : 1. Find the sum of their present age.

→ Let present age of Father $= 16x$, son $= x$

$$\frac{16x+8}{x+8} = \frac{4}{1}$$

$$16x+8 = 4x+32$$

$$12x = 24$$

∴ $x = 2$

∴ present age of Father = $16x = 16 \times 2 = 32$

son = $x = 2$

∴ sum of their present age = $32 + 2 = 34$ years

67. 2 quantities are in the ratio of 7 : 8, if 2 is subtracted from 1st and 12 is added to 2nd term, ratio becomes 2 : 3. Find quantities.

→ Let those 2 quantities be $7x$ & $8x$

$$\frac{7x-2}{8x+12} = \frac{2}{3}$$

$$21x - 6 = 16x + 24$$

$$5x = 30$$

$$x = 6$$

∴ Those quantities are

$7x = 7 \times 6 = 42$

$8x = 8 \times 6 = 48$

68. 3 quantities are in the ratio of 2 : 3 : 5. If 4, 6, 0 is added to 3 terms resp. then ratio becomes 2 : 3 : 4. Find sum of original quantities.

→ Let those 3 quantities be : $2y, 3y, 5y$

$$(2y+4) : (3y+6) : (5y+0) = 2:3:4$$

$$\left(\frac{2y+4}{3y+6}\right) = \frac{2}{3} \qquad \frac{3y+6}{5y+0} = \frac{3}{4}$$

$$6y + 12 = 6y + 12 \qquad 12y + 24 = 15y$$

$$0 = 0 \qquad 24 = 3y$$

$$y = 8$$

∴ Those original quantities are : 16, 24, 40

Sum of original quantities = $16 + 24 + 40 = 80$

69. A mixture contains milk, water in the ratio of 3 : 8 on adding 5 litres of milk the ratio becomes 1 : 2. Find milk quantity, water quantity in litres in original mixture.

→

5 Litres of milk

Milk : water
$3x : 8x$

$$\frac{3x+5}{8x} = \frac{1}{2}$$

$$6x + 10 = 8x$$

$$10 = 2x$$

$$\therefore x = 5$$

∴ Original mixture contains : milk $= 3x = 15$ litres

water $= 8x = 40$ litres

70. There are 14 litres of mixture in 1st container where ratio of milk, water is 2 : 5. In 30 litres mixture of 2nd container ratio of water, milk is 8 : 7. If mixture of these containers is put in 3rd container. Find ratio of milk, water quantity in 3rd container.

→

Milk : water
$2x : 5x$
4 litres : 10 litres

container – 1

Milk : water
$7y : 8y$
14 litres : 16 litres

container – 2

Milk : water $= \left(\frac{18 \text{ litres}}{26 \text{ litres}}\right)$

$= 9 : 13$

Milk = 4 + 14
water = 10 + 16

container – 3

71. Distribute ₹ 600 in the ratio of 2 : 3 : 7

→ Let 3 quantities be $2x, 3x, 7x$

$2x + 3x + 7x = 600$

$12x = 600$

$x = 50$

∴ 3 shares of ₹600 are : $2x = ₹100$, $3x = ₹150$, $7x = ₹350$

72. Distribute ₹ 5,00,000 among A, B, C in the ratio of 8 : 3 : 19

→

A's share : $5,00,000 \times \frac{8}{30} = 1,33,333.33333$

B's share : $5,00,000 \times \frac{3}{30} = 50,000$

C's share : $5,00,000 \times \frac{19}{30} = 3,16,666.66666$

₹ 5,00,000

73. Distribute ₹ 80,000 among A, B, C in the ratio of

$\frac{1}{3} : \frac{2}{3} : \frac{1}{2}$

$\frac{1}{3} : \frac{2}{3} : \frac{1}{2}$

$= 2 : 4 : 3$

→

A's share : $\frac{2}{9} \times 80,000 = 17,777.7777$

B's share : $\frac{4}{9} \times 80,000 = 35,555.5555$

C's share : $\frac{3}{9} \times 80,000 = 26,666.6666$

₹80,000/-

(OR)

$A : B : C = \frac{1}{3} : \frac{2}{3} : \frac{1}{2} = \frac{x}{3} : \frac{2x}{3} : \frac{x}{2}$

$\frac{x}{3} + \frac{2x}{3} + \frac{x}{2} = 80,000$

A's share $= x/3 = 17,777.77777$

$2x + 4x + 3x = 4,80,000$

$9x = 4,80,000$

B's share $= 2x/3 = 35,555.55555$

$x = 53,333.333$

C's share $= x/2 = 26,666.666666$

74. Divide ₹ 600 among A, B, C, D in the ratio of $2\frac{1}{3}, 8\frac{2}{3}, 5\frac{1}{3}, 10\frac{2}{3}$

$2\frac{1}{3} : 8\frac{2}{3} : 5\frac{1}{3} : 10\frac{2}{3}$

$= \frac{7}{3} : \frac{26}{3} : \frac{16}{3} : \frac{32}{3}$

$= 7 : 26 : 16 : 32$

₹

A's share : $\frac{7}{81} \times ₹600 = 51.85185$

B's share : $\frac{26}{81} \times ₹600 = 192.5926$

C's share : $\frac{16}{81} \times ₹600 = 118.5185$

D's share : $\frac{32}{81} \times ₹600 = 237.037$

PROPORTIONS

75. When a, b, c, d are said to be in proportion?

a, b, c, d are said to be in proportion if

(Ratio of 1st to 2nd term) = (Ratio of 3rd to 4th term)

i.e. $a : b = c : d$

i.e. $\frac{a}{b} = \frac{c}{d}$

i.e. $ad = bc$

ie. product of extremes = product of means

5, 15, 200, 600 are in proportion as $5 : 15 = 200 : 600 = 1 : 3$

(OR) $5 \times 600 = 15 \times 200 = 3000$

76. If p : q = r : s
OR (ps) = (qr)
then p, q, r, s are said to be in proportion

77. 18, 30, 2k, 900 are in proportion. Find the value of k.

→ product of extremes = product of means

$18 \times 900 = 30 \times 2k$

$18 \times 900 = 60k$

$k = 270$

OR $18 : 30 = 2k : 900$

$\frac{18}{30} = \frac{2k}{900}$

$18 \times 900 = 30 \times 2k$

$\therefore k = 270$

78. When a, b, c, d are said to be in continued proportion?

→

If $a:b = b:c = c:d$ then 4 terms a, b, c, d are said to be in continued proportion.

① 20, 30, 18, 27 Here $20:30 = 18:27 \neq 30:18$

20, 30, 18, 27 are in proportion but not in continued proportion

② 8, 12, 18, 27 Here $8:12 = 12:18 = 18:27 = 2:3$

$\therefore$ 8, 12, 18, 27 are in proportion as well as continued proportion.

Did You Know ?

If 4 quantities are in continued proportion then they must be in proprotion; whereas

If 4 quantities are in proportion then it is not necessary that they are in continued proportion.

79.

4 Quantities	Whether quantities are in	
	Proportion?	Continued Proportion?
8, 12, 18, 27	Yes	Yes
100, 200, 500, 1000	Yes	No
a, ab, abc, abcd	NO	NO
1, 3, 5, 7	NO	NO
18, 20, 90, 100	Yes	NO
5, 10, 20, 40	Yes	Yes
20, 2, 40, 4	Yes	NO
a, ax, ax^2, ax^3	Yes	Yes
k, k^3, k^5, k^7	Yes	Yes
a, ax, b, bx	Yes	No

80. p, q, r, s are said to be in

Proportion

when $p:q = r:s$

i.e. $p/q = r/s$

i.e. $ps = qr$

Continued Proportion

when $p:q = q:r = r:s$

i.e. $\frac{p}{q} = \frac{q}{r} = \frac{r}{s}$

81. When a, b, c are said to be in Proportion OR Continued Proportion?

→ Three terms are said to be in proportion & continued prop if (Ratio of 1st to 2nd term) = (Ratio of 2nd to 3rd term)

If $a:b = b:c$ i.e. $\frac{a}{b} = \frac{b}{c}$ i.e. $b^2 = ac$

then a, b, c are said to be in propⁿ & conti. propⁿ.

82. If ratio of 1st term to 2nd term is equal to ratio of 2nd term to 3rd term then 3 quantities are said to be in proportion.

If 3 quantities are in proⁿ then they are in conti. proⁿ also.

83. Quantities a, b, c, d, e, f, g, h are said to be in continued proportion.

If a : b = b : c = c : d = d : e = e : f = f : g = g : h

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 are in continued proⁿ

84.

Quantities are in Proportion	Value of k
a, b, 13, k	product of extremes = product of means $ak = 13b$ $k = (13b/a)$
2a, 3b, 8k, 100	$2a \times 100 = 3b \times 8k$ $\therefore k = \frac{200a}{24b} = \left(\frac{25a}{3b}\right)$
18, 20, 25, -3k	$18 \times -3k = 20 \times 25$ $k = \frac{500}{-54} = -250/27$ $k = -9.259259259$
5, 12, 15, 8k	$5 \times 8k = 12 \times 15$ $k = \frac{180}{40} = \frac{9}{2} = 4.50$
5m, -2k, $\frac{8}{3}$, $\frac{7}{33}$	$5m \times \frac{7}{33} = -2k \times \frac{8}{3}$ $k = 5m \times \frac{7}{33} \times \frac{3}{8} \times \frac{1}{-2} = \frac{-105m}{528} = \frac{-35m}{176}$
18, k, 40.50	$k^2 = 18 \times 40.50 = 729$ $\therefore k = 27$

85. Find 4th proportional to 18, 26, 90

→ Let 4th proportional to 18, 26, 90 be 'm'

∴ 18, 26, 90, m are in proportion.

∴ $18m = 26 \times 90$

$m = 130$

∴ 4th proportional to 18, 26, 90 is 130.

86. Find 4th proportional to $\frac{2}{3}, \frac{8}{7}, \frac{9}{11}$

→ Let 4th proportional to $\frac{2}{3}, \frac{8}{7}, \frac{9}{11}$ be x

∴ product of extremes = product of means

$\frac{2}{3} \times x = \frac{8}{7} \times \frac{9}{11}$

$\therefore x = \frac{8}{7} \times \frac{9}{11} \times \frac{3}{2} = \frac{216}{154} = \frac{108}{77}$

∴ 4th proportional to $\frac{2}{3}, \frac{8}{7}, \frac{9}{11}$ is $\frac{108}{77}$.

87. Find third proportional to 12, 40

→ Let third proportional to 12, 40 be m

∴ $\frac{12}{40} = \frac{40}{m}$ ∴ $12m = 1600$

$m = \frac{1600}{12} = \frac{400}{3} = 133.33333$

$= 133\frac{1}{3}$

∴ $12m = 40 \times 40$

88. Find Mean proportional to 12, 100

→ Let mean proportional to 12, 100 be m

∴ 12, m, 100 must be in proportion.

∴ $m^2 = 1200$

$m = \sqrt{1200} = 34.641016513$

Properties of Proportion

89. If a, b, c, d are in proportion then $\frac{a}{b} = \frac{c}{d}$

then

→ ① If $\frac{a}{b} = \frac{c}{d}$ then

$\left(\frac{b}{a}\right) = \left(\frac{d}{c}\right)$ ..... Invertendo

If $\left(\frac{3}{5} = \frac{p}{q}\right)$ then as per Invertendo $\left(\frac{q}{p} = \frac{5}{3}\right)$

---

② If $\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right)$

Adding '1' on both sides

$\left(\frac{a}{b}\right) + 1 = \left(\frac{c}{d}\right) + 1$

$\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$ ..... (Componendo)

If $\frac{3}{7} = \frac{12}{28}$ then $\frac{3+7}{7} = \frac{12+28}{28} = \frac{10}{7}$

---

③ If $\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right)$

subtracting '1' from both sides

$\frac{a}{b} - 1 = \frac{c}{d} - 1$

then $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$ ..... (Dividendo)

If $\frac{70}{10} = \frac{14}{2}$ then $\frac{70-10}{10} = \frac{14-2}{2} = \frac{6}{1}$

④ If $\frac{a}{b} = \frac{c}{d}$ then

$\frac{a+b}{b} = \frac{c+d}{d}$ ..... ① $\quad \frac{a-b}{b} = \frac{c-d}{d}$ ..... ②

eqn ① ÷ eqn ②

$= \left(\frac{\frac{a+b}{b}}{\frac{a-b}{b}}\right) = \left(\frac{\frac{c+d}{d}}{\frac{c-d}{d}}\right)$

$\left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$ ..... (componendo & Dividendo)

---

⑤ If $\frac{a}{b} = \frac{c}{d}$ then

$ad = bc$

then $\frac{a}{c} = \frac{b}{d}$ ..... (Alternendo)

If $\frac{3}{30} = \frac{8}{80}$ then $\frac{3}{8} = \frac{30}{80}$

---

⑥ If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \left(\frac{a+c}{b+d}\right) = \left(\frac{c+a}{d+b}\right)$

If $\frac{p}{q} = \frac{r}{s}$ then $\frac{p}{q} = \frac{r}{s} = \frac{p+r}{q+s}$ ----- Addendo

$\frac{3}{11} = \frac{30}{110} = \left(\frac{3+30}{11+110}\right) = \left(\frac{33}{121}\right)$ $\quad \frac{5}{10} = \frac{90}{180} = \frac{5+90}{10+180}$

---

⑦ If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \left(\frac{a-c}{b-d}\right) = \left(\frac{c-a}{d-b}\right)$

..... Subtrahendo

$\frac{18}{45} = \frac{2}{5} = \frac{18-2}{45-5} = \frac{16}{40}$, $\frac{88}{55} = \frac{8}{5} = \frac{88-8}{55-5} = \frac{80}{50}$

90. If p, q, r, s are in proportion i.e. $\left(\frac{p}{q}\right)=\left(\frac{r}{s}\right)$ then

1. $\left(\frac{q}{p}\right)=\left(\frac{s}{r}\right)$ ---------- Invertendo

2. $\left(\frac{p}{q}\right)=\left(\frac{r}{s}\right)=\frac{p+r}{q+s}$ ---------- Addendo

3. $\left(\frac{p}{q}\right)=\left(\frac{r}{s}\right)=\frac{p-r}{q-s}=\frac{r-p}{s-q}$ ---------- Subtrahendo

4. $\left(\frac{p}{r}\right)=\left(\frac{q}{s}\right)$ ---------- Alternendo

5. $\left(\frac{p+q}{q}\right)=\left(\frac{r+s}{s}\right)$ ---------- Componendo

6. $\left(\frac{p-q}{q}\right)=\left(\frac{r-s}{s}\right)$ ---------- Dividendo

7. $\left(\frac{p+q}{p-q}\right)=\left(\frac{r+s}{r-s}\right)$ ---------- Componendo & Dividendo

$\frac{36}{84}=\frac{3}{7}=\frac{36+3}{84+7}=\frac{36-3}{84-7}=\frac{3}{7}=\frac{39}{91}=\frac{33}{77}$ .... Addendo & subtrahendo

91. If $\frac{a}{b}=\frac{c}{d}$ then $\left(\frac{a}{b}\right)=\left(\frac{c}{d}\right)=\left(\frac{a+c}{b+d}\right)$ ..... Addendo

& $\left(\frac{a}{b}\right)=\left(\frac{c}{d}\right)=\left(\frac{a-c}{b-d}\right)$ ..... Subtrahendo

$\therefore \frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}=\frac{a-c}{b-d}$ Addendo & Subtrahendo

92. If $\left(\frac{a}{7}\right) = \left(\frac{b}{8}\right) = \left(\frac{c}{13}\right)$ Find value of $\left(\frac{10a+11b-3c}{2b}\right) = ?$

$\frac{a}{7} = \frac{b}{8} = \frac{c}{13}$ ------ Given

$\frac{10a}{70} = \frac{11b}{88} = \frac{3c}{39} = \frac{10a+11b-3c}{70+88-39}$ ..... Addendo & subtrahendo

$\frac{10a+11b-3c}{119} = \frac{2b}{8\times 2}$

$\therefore \frac{10a+11b-3c}{2b} = \frac{119}{16}$

OR $\frac{a}{7} = \frac{b}{8} = \frac{c}{13} = x \quad \therefore a=7x,\ b=8x,\ c=13x$

$\frac{10a+11b-3c}{2b} = \frac{10(7x)+11(8x)-3(13x)}{2(8x)}$

$= \left(\frac{70x+88x-39x}{16x}\right) = \frac{119x}{16x} = \left(119/16\right)$

93. If $\frac{a}{20} = \frac{b}{13}$ Find value of $\left(\frac{7a+12b}{3b}\right)$

$\frac{a}{20} = \frac{b}{13} = \frac{7a}{140} = \frac{12b}{156} = \frac{7a+12b}{296}$

$\frac{7a+12b}{296} = \frac{3b}{39} \quad \therefore \frac{7a+12b}{3b} = \left(\frac{296}{39}\right)$

OR $\frac{a}{20} = \frac{b}{13} = k \quad \therefore a=20k,\ b=13k$

$\left(\frac{7a+12b}{3b}\right) = \frac{7\times 20k+12\times 13k}{3\times 13k} = \frac{296k}{39k} = \left(296/39\right)$

94. If $\frac{a}{30}=\frac{b}{60}=\frac{c}{147}$ then Find value of $\left(\frac{3a+2b-c}{b}\right)$

$$\frac{a}{30}=\frac{b}{60}=\frac{c}{147}$$

$$\frac{3a}{90}=\frac{2b}{120}=\frac{c}{147}=\frac{3a+2b-c}{90+120-147}$$

$$\frac{3a+2b-c}{63}=\frac{b}{60}$$

$$\therefore \frac{3a+2b-c}{b}=\frac{63}{60}=\frac{21}{20}$$

$$\frac{a}{30}=\frac{b}{60}=\frac{c}{147}=m$$

$$\therefore a=30m,\ b=60m,\ c=147m$$

$$\left(\frac{3a+2b-c}{b}\right)$$

$$=\frac{3(30m)+2(60m)-147m}{60m}$$

$$=\frac{63m}{60m}=\frac{63}{60}=\left(\frac{21}{20}\right)$$

95. If $\frac{a}{3}=\frac{b}{8}=\frac{c}{5}$ Find value of $\left(\frac{12a+8b-2c}{7c}\right)$

$$\frac{a}{3}=\frac{b}{8}=\frac{c}{5}=\frac{12a}{36}=\frac{8b}{64}=\frac{2c}{10}=\frac{12a+8b-2c}{36+64-10}$$

$$\frac{12a+8b-2c}{36+64-10}=\frac{7c}{7\times5}$$

$$\frac{12a+8b-2c}{7c}=\frac{90}{35}=\frac{18}{7}$$

OR $a=3m,\ b=8m,\ c=5m$

$$\frac{12a+8b-2c}{7c}=\frac{12(3m)+8(8m)-2(5m)}{7\times5m}$$

$$=\frac{90m}{35m}=\frac{90}{35}=\left(\frac{18}{7}\right)$$

96. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \left(\frac{a+c}{b+d}\right) = \frac{e}{f} = \left(\frac{a+c-e}{b+d-f}\right)$

$= \left(\frac{a-c}{b-d}\right) = \left(\frac{e}{f}\right) = \left(\frac{a-c-e}{b-d-f}\right)$

$\frac{100}{25} = \frac{16}{4} = \frac{20}{5} = \left(\frac{100+16}{25+4}\right) = \left(\frac{100-16}{25-4}\right) = \frac{20}{5} = \left(\frac{100-16-20}{25-4-5}\right) = \frac{64}{16} = \frac{4}{1}$

97. If $\frac{a}{8} = \frac{b}{20} = \frac{c}{35}$ then $\frac{a}{8} = \frac{3a+2b-c}{9}$ True / False

→ $\frac{a}{8} = \frac{b}{20} = \frac{c}{35} = m$

$\therefore \quad a = 8m, \; b = 20m, \; c = 35m$

L. H.S. $= \frac{a}{8} = \frac{8m}{8} = m$

R.H.S. $= \frac{3a+2b-c}{9} = \frac{3(8m)+2(20m)-35m}{9}$

$= \frac{24m+40m-35m}{9} = \frac{29m}{9}$

AS L. H. S. $\neq$ R.H.S.

Given statement is False

98. If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a^3+c^3}{b^3+d^3}=\frac{(a+c)^3}{(b+d)^3}$

Correct / incorrect

$\frac{a}{b}=\frac{c}{d}$ ●●●●●●●●●●●●●●●●●●●● Given

Taking cube on both sides

$\left(\frac{a}{b}\right)^3=\left(\frac{c}{d}\right)^3$

$\frac{a^3}{b^3}=\frac{c^3}{d^3}=\frac{a^3+c^3}{b^3+d^3}$ ..... ①

$\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$

Taking cube on both sides

$\left(\frac{a}{b}\right)^3=\left(\frac{c}{d}\right)^3=\left(\frac{a+c}{b+d}\right)^3$

$\frac{a^3}{b^3}=\frac{c^3}{d^3}=\frac{(a+c)^3}{(b+d)^3}$ ..... ②

From Eq^{n}s ① & ②

$$\frac{a^3+c^3}{b^3+d^3}=\frac{(a+c)^3}{(b+d)^3}$$

∴ Given statement is correct

$\frac{a}{b}=\frac{c}{d}$ L.H.S. $=\frac{a^3+c^3}{b^3+d^3}=\frac{5^3+10^3}{11^3+22^3}=\frac{1125}{11979}=\frac{125}{1331}$

$\frac{5}{11}=\frac{10}{22}$ R.H.S. $=\frac{(a+c)^3}{(b+d)^3}=\frac{(5+10)^3}{(11+22)^3}=\frac{3375}{35937}=\frac{125}{1331}$

99. Some Imp formulae from School

1. $(a+b)^2 = (a+b)(a+b)$
   $= a^2+ab+ba+b^2$
   $(a+b)^2 = a^2+b^2+2ab$

2. $(a-b)^2 = (a-b)(a-b) = a^2-ab-ab+b^2 = a^2+b^2-2ab$

3. $(a^2+b^2) = (a+b)^2-2ab = (a-b)^2+2ab$

4. $(a+b)^3 = (a+b)(a+b)(a+b)$
   $= (a^2+b^2+2ab)(a+b)$
   $= a^3+a^2b+ab^2+b^3+2a^2b+2ab^2$
   $(a+b)^3 = a^3+b^3+3a^2b+3ab^2 = a^3+b^3+3ab(a+b)$

5. $(a^3+b^3) = (a+b)^3-3ab(a+b)$

6. $(a-b)^3 = (a-b)(a-b)(a-b)$
   $= (a-b)(a^2+b^2-2ab)$
   $= a^3+ab^2-2a^2b-a^2b-b^3+2ab^2$
   $= a^3-b^3+3ab^2-3a^2b$
   $= a^3-b^3+3ab(b-a) = a^3-b^3-3ab(a-b)$

7. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

8. $(a-b+c)^2 = a^2+b^2+c^2-2ab-2bc+2ac$

9. $(a-b-c)^2 = a^2+b^2+c^2-2ab+2bc-2ac$

10. $(a^2-b^2) = (a+b)(a-b)$

11. $(a-b)(a^2+ab+b^2) = a^3+a^2b+ab^2-a^2b-ab^2-b^3$
    $= a^3-b^3$
    $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

12. $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

13. $(a-b)^2 = (a+b)^2-4ab$

14. $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

100. Find 4th proportional to $\frac{8}{11}, \frac{3}{22}, \frac{3m}{19}$

→ Let 4th proportional be k

$\therefore \frac{8}{11}, \frac{3}{22}, \frac{3m}{19}, k$ are in proportion

$\therefore$ product of extremes = product of means

$$\frac{8k}{11} = \frac{9m}{418}$$

$$\therefore k = \frac{9m}{418} \times \frac{11}{8} = \frac{99m}{3344} = \frac{9m}{304}$$

$\therefore$ 4th proportional to $\frac{8}{11}, \frac{3}{22}, \frac{3m}{19}$ is $\frac{9m}{304}$

101. Find mean proportional to 62.50, 10

→ Let mean proportional be x

$\therefore$ 62.50, x, 10 are in proportion

$\therefore$ (square of middle term) = product of extremes

$$x^2 = 62.50 \times 10 = 625$$

$$\therefore x = 25$$

$\therefore$ Mean proportional to 62.50 & 10 is 25.

102. Find fourth proportional to -4, -6, -9

→ Let 4th proportional to x

$\therefore -4, -6, -9, x$ are in proportion

$$\therefore -4 \times x = -6 \times -9$$

$$x = \frac{54}{-4} = -27/2 = -13.50$$

$\therefore$ 4th proportional to $-4, -6, -9$ is -13.50

Logarithms (Logs)

103. Log of a number consists of 2 parts
- Integer Part = Characteristic
- Fractional / Decimal Part = Mantissa

- Log x = (Characteristic of x + Mantissa of x)

- Log x
  - characteristic of x (Integer part)
    - If $x \geq 1$: (No. of digits before decimal point -1)
    - If $0 < x < 1$: (No. of zeros immediately after decimal point $+1$) BAR
  - Mantissa of x (Fractional part): From Log-table

Log 283.6 = characteristic of 283.6 + mantissa of 283.6
= (3−1) + 0.4527
Log 283.6 = 2 + 0.4527 = 2.4527

Log 28.36 = 1 + 0.4527 = 1.4527

Log 2.836 = 0 + 0.4527 = 0.4527

Log 7 = Log 7.000 = 0 + 0.8451 = 0.8451

104.

x	Characteristic of x	Mantissa of x	Log x
786.89	3-1 = 2	0.8959	2.8959
78.33	2-1 = 1	0.8940	1.8940
2358.00	4-1 = 3	0.3726	3.3726
29.98	2-1 = 1	0.4769	1.4769
711.68	3-1 = 2	0.8523	2.8523
859.63	3-1 = 2	0.9343	2.9343
10.00	2-1 = 1	0.000	1.0000
100.00	3-1 = 2	0.000	2
939.8	3-1 = 2	0.9731	2.9731
2,35,288.00	6-1 = 5	0.3717	5.3717
8.000	1-1 = 0	0.9031	0.9031
893.6	3-1 = 2	0.9512	2.9512
28.11	2-1 = 1	0.4489	1.4489
636363.3	6-1 = 5	0.8038	5.8038
211.93	3-1 = 2	0.3261	2.3261
71.77	2-1 = 1	0.8559	1.8559
8.123	1-1 = 0	0.9098	0.9098
11.263	2-1 = 1	0.0515	1.0515
1.093	1-1 = 0	0.0386	0.0386
3.861	1-1 = 0	0.5867	0.5867
6.000	1-1 = 0	0.7782	0.7782
5.000	1-1 = 0	0.6990	0.6990
7,00,000.00	6-1 = 5	0.8451	5.8451
70,000	5-1 = 4	0.8451	4.8451
7000	4-1 = 3	0.8451	3.8451
700	3-1 = 2	0.8451	2.8451

x	Characteristic of x	Mantissa of x	Log x
70	$2-1=1$	0.8451	1.8451
7.000	$1-1=0$	0.8451	0.8451
0.70	$\bar{1}$	0.8451	$\bar{1}.8451 = -1+0.8451 = -0.1549$
0.070	$\bar{2}$	0.8451	$\bar{2}.8451 = -2+0.8451 = -1.1549$
0.0070	$\bar{3}$	0.8451	$\bar{3}.8451 = -3+0.8451 = -2.1549$
0.0008263	$\bar{4}$	0.9172	$\bar{4}.9172 = -4+0.9172 = -3.0828$
8.009	$1-1=0$	0.9036	0.9036

105.

1. Log 89.53 = $1 + 0.9519 = 1.9519$
2. Log 63.33 = $1 + 0.8016 = 1.8016$
3. Log 998.6 = $2 + 0.9994 = 2.9994$
4. Log 0.3963 = $\bar{1} + 0.5980 = \bar{1}.5980 = -1+0.5980 = -0.4020$
5. Log 0.000005822 = $\bar{6} + 0.7650 = \bar{6}.7650 = -6+0.7650 = -5.2350$
6. Log 0.009981 = $\bar{3} + 0.9991 = -3+0.9991 = -2.0009$

106. Find $\text{Log}_8 23$ = Log of 23 to the base 8.

$= \text{Log}_8 23 = \frac{\text{Log}23}{\text{Log}8}$

$= \left(\frac{1.3617}{0.9031}\right)$

$= 1.507806444446$

$\text{Log}_b a = \left(\frac{\text{Log}a}{\text{Log}b}\right)$

107. $\text{Log}_{50} 2086 = \frac{\text{Log}2086}{\text{Log}50} = \frac{3.3194}{1.6990}$

$= 1.953737449264$

108. $\text{Log}_b a = \frac{\text{Log}a}{\text{Log}b}$ $\quad \text{Log}_n m = \frac{\text{Log}m}{\text{Log}n}$

$\text{Log}_{81} 1086 = \frac{\text{Log}1086}{\text{Log}81} = \frac{3.0359}{1.9085} = 1.590725700081$

109. $\text{Log}_{10} 81 = \frac{\text{Log}81}{\text{Log}10} = \frac{1.9085}{1.0000} = 1.9085$

$\text{Log}_{10} 50 = \text{Log}50/\text{Log}10 = 1.6990/1 = 1.6990$

10 → Common Base of Logs

e → Exponential factor = 2.7183 = Natural Base

110. $\text{Log}_{10} 28 = \text{Log}28/\text{Log}10 = 1.4472/1 = 1.4472$

$\text{Log}_{10} 100 = \text{Log}100/\text{Log}10 = 2/1 = 2.0000$

$\text{Log}_{10} 1000 = \text{Log}1000/\text{Log}10 = 3/1 = 3$

If $\text{Log}_b a = k$; then $b^k = a$	if $\text{Log}_z x = m$ then $z^m = x$

111. $\text{Log}_{10} 8 = 0.9031$ $\quad 10^{0.9031} = 8$

If $\text{Log}_q p = m$; then $q^m = p$

$\text{Log}_8 81 = \frac{\text{Log}81}{\text{Log}8} = \frac{1.9085}{0.9031} = 2.1133 \quad \therefore 8^{2.1133} =$

112.

A. Log 2.8381 = 688.9
A. Log 3.8381 = 6889.00
A. Log 1.3379 = 21.77
A. Log 3.2263 = 1684.00
A. Log 5.8893 = 775000.00
A. Log 6.1193 = 1316000.00
A. Log 2.1156 = 130.5
A. Log 0.0891 = 1.227
A. Log 0.0063 = 1.015
A. Log 5.89993 = 794100
A. Log 7.0909 = 12330000
A. Log 10.8122 = 6489×10^7

$\text{Log}5 = 0.6990$

A. Log 0.6990 = 5.000

A. Log 0.6990 = 5

A. log (Log 5) = 5

$\therefore$ A. log (log k) = k

Log 5 = 0.6990

Log (A. log 0.6990) = 0.6990

Log (A. log m) = m

113. $\text{Log}_{10} 234 = 2.3692$

Crosscheck : $10^{2.3692} = 234$

If $\text{Log}_y x = m$ then $y^m = x$

114. If $\text{Log}_z x = y$; then $z^y = x$

Log 55.332 = 1.7429
A. Log 7.8122 = 64890000
A. Log 10.1198 = $13170000000 = 1317 \times 10^7$
Log 2386579 = 6.3779

115. Log 5 = 0.6990 , Log 2 = 0.3010 , Log 10 = 1.000

$1 = 0.6990 + 0.3010$

$\text{Log}10 = \text{Log}5 + \text{Log}2$

$\text{Log}(5 \times 2) = \text{Log}5 + \text{Log}2$

$\therefore \text{Log}(m \times n)$

$= \text{Log}m + \text{Log}n$

116. Log (xyz) = $\text{Log}x + \text{Log}y + \text{Log}z$

Log (abcd) = $\text{Log}a + \text{Log}b + \text{Log}c + \text{Log}d$

117. Find Value of m; if $m = 8 \times 5$

$m = 8 \times 5$

Taking Log on both sides

$\text{Log}m = \text{Log}(8 \times 5)$

we know $\text{Log}(ab) = \text{Log}a + \text{Log}b$

$\therefore \text{Log}m = \text{Log}8 + \text{Log}5 = 0.9031 + 0.6990$

$\therefore \text{Log}m = 1.6021$

Taking A.Log on both sides

$A.\log(\text{Log}m) = A.\text{Log}(1.6021) = 40.00$

$m = 40$

118. Log 23 = 1.3617 , A.Log (1.3617) = 23.00

Log 100 = 2.0000 , A.Log 2.0000 = 100.00

$A.Log(Log\ 25) = A.Log(1.3979) = 25.00$

$Log(A.Log\ 2.1111) = Log(129.1) = 2.1109$

A.Log (log m) = m
Log (A.Log k) = k = A.Log (log k)

$Log_2\ 16 = ?$

$Log_2 16$

$= \frac{Log\ 16}{Log\ 2}$

$= \left(\frac{1.2041}{0.3010}\right)$

$= 4$

Log m = ?
Enter m in calculator
$\sqrt{}$ 15 times
Deduct 1
Multiply by 14230.9635

$Log_2 16 = m$

$\therefore 2^m = 16$

$\therefore 2^m = 2^4$

$\therefore m = 4$

$Log_2 16 = 4$

$Log_2 16$

$= \frac{Log\ 16}{Log\ 2}$

$= \frac{Log\ 2^4}{Log 2}$

$= \frac{4.Log 2}{Log 2}$

$= 4$

$Log_2 16$

$= \frac{Log\ 16}{Log\ 2}$

$= \left(\frac{1.204170907 56}{0.301033120 99}\right)$

$= 4$

119.

Log 29

Take : 29 on calculator

$\sqrt{\ }$: 15 times

Deduct : 1

Multiply by 14230.9635

↓

You will get answer
on calculator

Log 29
= 1.462473 14138

Log x = ?

Enter : x

$\sqrt{\ }$ 15 times

Deduct : 1

Multiply by 14230.9635

↓

You will get answer

120. Find (By using calculator)

Log 22.22 = 1.346807 85472

Log 819.9 = 2.914059 45338

Log 234234 = 5.370 6637312

Log 81998 = 4.914652 28688

Log 35.86 = 1.554695 33494

Log 0.00008122 = (−4.089749 93369)

Log 0.5693 = −0.244656 87446

Log 8.007 = 0.903498 39919

A.Log 2.5122 = 325.070 72328

A.Log 0.8163 = 6.550 528 45147

A.Log 0.20 = 1.584 887 63077

A.Log 5.2081 = 161118.85027

121. Log 22.22

By using Log-table

= characteristic of x + Mantissa of x

= (2 − 1) + 0.3468

= 1 + 0.3468

= 1.3468

By using Calculator

Enter 22.22

$\sqrt{}$ 15 times

Deduct 1

Multiply by 14230.9635

= 1.3468078547 2

122. **How to find Log x on calculator?**

$\Rightarrow$ Enter x

$\sqrt{}$ 15 times

Deduct 1

Multiply by 14230.9635

How to find A.Log (y) on calculator?

$\Rightarrow$

Enter y

Divide by 14230.9635

Add 1

$\sqrt{}$ 15 times

$X =$

How to find a^b on calculator?

$\Rightarrow$ Enter : a

$\sqrt{}$ 12 times

Deduct 1

Multiply by : b

Add 1

$\sqrt{}$ 12 times

$X =$

- Log 2 = 0.3010 33 12099
- A·Log(0.3010 33 12099)

  = 2.0000

123. Please solve the following by using calculator.

1) Log 589.3 = 2.770606360 46

2) Log 863.6 = 2.936615815 15

3) Log 0.000008153 = −5.087736655 7

4) Log 813692 = 5.911688247 58

5) A.Log 2.3691 = 233.831154327

6) A.Log 0.8886 = 7.736987431 32

7) $85^{1.36}$ = 420.240316097

124.

1) Log x = (Characteristic of x) + (Mantissa of x)

Integer Part : characteristic

Fractional Part : Mantissa

2) A.Log (Log k) = k = Log (A.log k)

A.Log (Log 83) = 83, Log (A.log 5.2381) = 5.2381

3) Common Base : 10 as $\text{Log}_{10} 10 = 1$

Natural Base : e where e = exponential Factor
= 2.7183 (approx)

4) If $\text{Log}_b a = k$, then $b^k = a$

$\text{Log}_{10} 100 = 2$, $\text{Log}_{10} 1000 = 3$, $\text{Log}_{10} 1 = 0$ ∴ $\text{Log}_a 1 = 0$ provided $a \neq 1$

5) $\text{Log}_b a$ = Log a / Log b

6) Log (a x b) = Log a + Log b

7) $\text{Log}_m (xyz)$ = $\text{Log}_m x + \text{Log}_m y + \text{Log}_m z$

125. $\text{Log}(2)^5 = \text{Log}(2\times2\times2\times2\times2) = \text{Log}2 + \text{Log}2 + \text{Log}2 + \text{Log}2 + \text{Log}2$

$\text{Log}(2)^5 = 5.\text{Log}2$

$\boxed{\text{Log}(2)^5 = 5.\text{Log}2}$

$\text{Log}(a)^b = b.\text{Log}a$

$\text{Log}\, m^n = n.\text{Log}m$

$\text{Log}\, x^{-13} = -13.\text{Log}x$

126. $\dfrac{2^2}{2^5} \quad \dfrac{\cancel{2}\times\cancel{2}}{\cancel{2}\times\cancel{2}\times2\times2\times2} = 2^{2-5} = 2^{-3} = \dfrac{1}{2^3}$

$2^{2-5} = \dfrac{1}{2^3} = 2^{-3}$

$8^{10} = \left(\dfrac{1}{8^{-10}}\right)$

$100^{-3} = \left(\dfrac{1}{100^3}\right)$

$\dfrac{64}{64} = \dfrac{2^6}{2^6} =$

$a^{-m} = \dfrac{1}{a^m}$ OR $a^m = \dfrac{1}{a^{-m}}$

OR $a^m \times a^{-m} = 1 = a^{m+(-m)} = a^0$

127. $\text{Log}\left(\dfrac{a}{b}\right) = \text{Log}\left(a\times\dfrac{1}{b}\right) = \text{Log}\left(a\times\dfrac{1}{b^1}\right)$

$= \text{Log}(a\times b^{-1}) = \text{Log}a + \text{Log}b^{-1}$

$= \text{Log}a + (-1.\text{Log}b) = \text{Log}a - \text{Log}b$

$\boxed{\text{Log}\left(\dfrac{a}{b}\right) = \text{Log}a - \text{Log}b}$

128. $\text{Log}\left(\dfrac{xy}{z}\right) = \text{Log}(xy) - \text{Log}z$

$= \text{Log}x + \text{Log}y - \text{Log}z$

129. $\text{Log}\left(\dfrac{ab}{cd}\right) = \text{Log}(ab) - \text{Log}(cd)$

$= (\text{Log}a + \text{Log}b) - (\text{Log}c + \text{Log}d)$

$= \text{Log}a + \text{Log}b - \text{Log}c - \text{Log}d$

130. $\text{Log } a^{-m} = -m \cdot \text{Log } a$

$\text{Log } x^{z} = z \cdot \text{Log } x$

$\boxed{\text{Log}_y x \times \text{Log}_z y = \text{Log}_z x}$

$\text{Log}_b a \times \text{Log}_c b = \frac{\text{Log } a}{\text{Log } b} \times \frac{\text{Log } b}{\text{Log } c} = \text{Log}_c a$

$\text{Log}_a a = 1$ provided $a \neq 1$

$\left(\frac{\text{Any Number}}{0}\right)$ = undefined = Not defined

$\left(\frac{0}{\text{Any Number}}\right) = 0 = \text{Zero}$

$\text{Log}_{81} 1024 \times \text{Log}_{\sqrt[7]{64}} (\sqrt{3})^8$

$= \frac{\text{Log } 1024}{\text{Log } 81} \times \frac{\text{Log } (3^{1/2})^8}{\text{Log } (64)^{1/7}}$

$= \frac{\text{Log } 2^{10}}{\text{Log } 3^4} \times \frac{\text{Log } 3^4}{\text{Log } (2^6)^{1/7}}$

$= \frac{10 \cdot \text{Log } 2}{4 \cdot \text{Log } 3} \times \frac{4 \cdot \text{Log } 3}{\frac{6}{7} \text{Log } 2} = 10 \times \frac{7}{6} = \frac{70}{6} = \left(35/3\right)$

131. **Summary of Logs**

1. Log x = characteristic of x + Mantissa of x

2. Integer part : characteristic

   Fractional part : Mantissa

3. [redacted] = (Log x / Log y) = $\text{Log}_y x$

4. Common base : 10

   Natural base : e = Exponential Factor

   = 2.7183 (approx.)

5. A.log (log y) = y = Log (A.log y)

6. If $\text{Log}_m a = n$ then $m^n = a$

7. If $x^y = z$ then $\text{Log}_x z = y$

8. $\text{Log}_{10} 10 = 1$, $\text{Log}_{10} 100 = 2$

   $\text{Log}_{10} 1 = 0$, $\text{Log}_{10} 1000 = 3$, $\text{Log}_{10} 10{,}000 = 4$

9. $\text{Log}_a 1 = 0$ provided $a \neq 1$

10. Log (abc) = Log a + Log b + Log c

11. $\text{Log}_m (xy)$ = $\text{Log}_m x + \text{Log}_m y$

12. $\text{Log}\left(\frac{a}{b}\right)$ = Log a - Log b

13. $\text{Log } a^b$ = b . Log a

14. $\text{Log } a^{-b}$ = - b . Log a

15. $\text{Log}_b a \times \text{Log}_a b = 1$

16. $\text{Log}_b a \times \text{Log}_c b = \text{Log}_c a$

17. $\text{Log}\left(\frac{ab}{c}\right)$

    = Log a + Log b - Log c

18. $\text{Log}_z x^y$

    $= \frac{\text{Log } x^y}{\text{Log } z} = \frac{y . \text{Log } x}{\text{Log } z}$

19. If $m^n = j$

    then $\text{Log}_m j = n$

20. $\text{Log}\left(\frac{xy}{mn}\right)$

    = (Log x + Log y - Log m - Log n)

132. $\text{Log}_{16}\ 64 = ?$

$$= \frac{\text{Log } 64}{\text{Log } 16} = \frac{\text{Log } 4^3}{\text{Log } 4^2} = \frac{3.\text{Log } 4}{2.\text{Log } 4} = 3/2$$

$$= 1.50$$

133. $\text{Log}_{\sqrt[4]{27}}\ 243 =$

$$= \frac{\text{Log } 243}{\text{Log } \sqrt[4]{27}} = \frac{\text{Log } 3^5}{\text{Log } (3^3)^{1/4}} = \frac{5.\text{Log } 3}{\frac{3}{4}.\text{Log } 3}$$

$$= 5 \times \frac{4}{3} = \frac{20}{3} = 6.66666666 = 6\frac{2}{3}$$

134. $\text{Log}_{\sqrt[4]{243}} \sqrt[7]{81} =$

$$= \frac{\text{Log } \sqrt[7]{81}}{\text{Log } \sqrt[4]{243}} = \frac{\text{Log } (3^4)^{1/7}}{\text{Log } (3^5)^{1/4}} = \frac{\frac{4}{7}\text{Log } 3}{\frac{5}{4}\text{Log } 3}$$

$$= \frac{4}{7} \times \frac{4}{5} = \left(\frac{16}{35}\right) = 0.457142857\overline{14}$$

135. $\text{Log}_{\sqrt[9]{121}} \sqrt[5]{11} =$

$$= \frac{\text{Log } \sqrt[5]{11}}{\text{Log } \sqrt[9]{121}} = \frac{\text{Log } (11)^{1/5}}{\text{Log } (11^2)^{1/9}} = \frac{\frac{1}{5}.\text{Log } 11}{\frac{2}{9}.\text{Log } 11} = \frac{1}{5} \times \frac{9}{2}$$

$$= 9/10 = 0.90$$

136. $\text{Log}_{(7/8)} x = 3$. Find the value of x.

→ If $\text{Log}_b a = k$ then $b^k = a$

$\therefore \left(\frac{7}{8}\right)^3 = x \qquad x = \frac{7^3}{8^3} = \left(\frac{343}{512}\right)$

137. $\text{Log}_{\sqrt{2}} m = \frac{8}{11}$ Find the value of m.

→ If $\text{Log}_b a = k$ then $b^k = a$

$\therefore (\sqrt{2})^{8/11} = m$

$m = (2^{1/2})^{8/11} = (2)^{\frac{8}{22}} = (2)^{\frac{4}{11}} = \sqrt[11]{2^4} = \sqrt[11]{16}$

138. $\text{Log}_n m \times \text{Log}_p n \times \text{Log}_m p =$

→ $= \frac{\text{Log } m}{\text{Log } n} \times \frac{\text{Log } n}{\text{Log } p} \times \frac{\text{Log } p}{\text{Log } m} = 1.00$

139. $\text{Log}_{\sqrt[3]{8}} x = 13$. Find the value of x.

→ $\text{Log}_2 x = 13$

$\therefore x = 2^{13} = 8192$

140. $\text{Log}_{\sqrt[4]{27}} 81 \times \text{Log}_{\sqrt[7]{16}} \sqrt{512}$

→ $= \frac{\text{Log } 81}{\text{Log } (27)^{1/4}} \times \frac{\text{Log } \sqrt{512}}{\text{Log } \sqrt[7]{16}}$

$= \frac{\text{Log } 3^4}{\text{Log } (3^3)^{1/4}} \times \frac{\text{Log } (2^9)^{1/2}}{\text{Log } (2^4)^{1/7}}$

$= \frac{4 \cdot \text{Log } 3}{\frac{3}{4} \text{Log } 3} \times \frac{\frac{9}{2} \cdot \text{Log } 2}{\frac{4}{7} \text{Log } 2}$

$= \frac{2}{1} \times \frac{4}{3} \times \frac{9}{2} \times \frac{7}{4}^{3}$

$= 42$

141. Basic Rules of Indices

1) a. $2^3 \times 2^5 = (2\times2\times2)\times(2\times2\times2\times2\times2) = 2^8 = 2^{3+5}$

$2^3 \times 2^5 = 2^{3+5} = 2^8$ $\quad\therefore\quad x^m \times x^n = x^{m+n}$

b) $5^3 \times 5^4 = (5\times5\times5)\times(5\times5\times5\times5) = 5^7$

$5^3 \times 5^4 = 5^{3+4} = 5^7$

$$x^p \times x^q = x^{(p+q)}$$

$\therefore a^m \times a^n = (a)^{m+n}$

2. $\frac{3^8}{3^4} = \frac{3\times3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3} = 3^4 = (3)^{8-4}$

$\frac{a^m}{a^n} = (a)^{m-n}$ $\qquad$ $\frac{x^z}{x^m} = (x)^{z-m}$

3. $(2^3)^4 = 2^3\times2^3\times2^3\times2^3 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$

$2\times2\times2$

$= 2^{12} = (2)^{3\times4}$

$(a^m)^n = a^{mn}$

4. $[(a^m)^n]^q = (a)^{mnq}$

5. $a^x \times a^y \times a^z = a^{(x+y+z)}$

6. $\sqrt[m]{a} = a^{\frac{1}{m}}$

$\sqrt[3]{8} = (8)^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3\times\frac{1}{3}} = 2^1 = 2$

$\sqrt[5]{243} = (243)^{1/5} = (3^5)^{1/5} = 3^{5\times\frac{1}{5}} = 3^1 = 3$

7. $\frac{2^3}{2^6} = 2^{3-6} = 2^{-3}$ ..... ① $\quad \frac{2^3}{2^6} = \frac{2\times2\times2}{2\times2\times2\times2\times2\times2} = \frac{1}{2^3}$ ..... ②

$a^{-m} = \frac{1}{a^m}$ OR $a^m = \frac{1}{a^{-m}}$ OR $a^m \times a^{-m} = 1$ $\quad\therefore\quad 2^{-3} = \frac{1}{2^3}$

$x^{-y} = \frac{1}{x^y}$ $\qquad$ $x^{-k} = \frac{1}{x^k}$ $\quad$ & $\quad 2^3 = \frac{1}{2^{-3}}$

8. $\frac{32}{32} = \frac{2^5}{2^5} = (2)^{5-5} = 2^0 = 1$

$\frac{100}{100} = \frac{10^2}{10^2} = 10^{2-2} = 10^0 = 1$

$\therefore$ (Any Number)$^0 = 1$

9. If $a^x = a^y$ then $x = y$

10. If $a^x = b^x$ then $a = b$

11. $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$

$8^{\frac{5}{7}} = \sqrt[7]{8^5}$

12. $a^1 = a, a^0 = 1$

- If $2^m = 2^5$ $\therefore$ $m = 5$
- If $8^2 = m^2$ then $m = 8$
- $13^{8/36} = \sqrt[36]{13^8}$

13. $6^4 = (2 \times 3)^4 = 1296$

$6^4 = (2\times3)^4 = 2^4 \times 3^4 = 16 \times 81 = 1296$

$\therefore$ $(2\times3)^4 = 2^4 \times 3^4$ $\therefore$ $\boxed{(ab)^m = a^m \times b^m}$

14. $\left(\frac{6}{2}\right)^3 = 27$

$\left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3} = \frac{216}{8} = 27$

$\therefore$ $\left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3}$ $\therefore$ $\left(\frac{a}{b}\right)^m = \left(\frac{a^m}{b^m}\right)$

15. $\left(\frac{ab}{c}\right)^m = \frac{(ab)^m}{c^m} = \frac{a^m \times b^m}{c^m} = a^m \cdot b^m \cdot c^{-m}$

① $2^8 \times 2^{11} = 2^{8+11} = 2^{19}$

② $\left(\frac{8^{33}}{8^{91}}\right) = 8^{33-91} = 8^{-58} = \left(\frac{1}{8^{58}}\right)$

③ $\frac{10^{\frac{1}{3}} \times 10^{\frac{1}{6}}}{10^{\frac{1}{9}}} = (10)^{\frac{1}{3}+\frac{1}{6}-\frac{1}{9}} = (10)^{\frac{18}{54}+\frac{9}{54}-\frac{6}{54}}$

$= (10)^{21/54} = (10)^{7/18} = \sqrt[18]{10^7}$

142. Rules of Indices

1. $a^m \times a^n =$ $(a)^{m+n}$

2. $a^x \times a^y \times a^z =$ $(a)^{x+y+z}$

3. $\frac{a^m}{a^n} =$ $(a)^{m-n}$ $\quad$ $\frac{b^m}{b^n} = (b)^{m-n}$

4. $(a \times b)^x =$ $a^x \cdot b^x$ $\quad$ $(xyz)^k =$ $x^k \cdot y^k \cdot z^k$

5. $\left(\frac{a}{b}\right)^m =$ $\frac{a^m}{b^m} = a^m \cdot b^{-m}$ $\quad$ $\left(\frac{xy}{z}\right)^m = \frac{x^m \cdot y^m}{z^m} = x^m \cdot y^m \cdot z^{-m}$

6. $(a^x)^y =$ $a^{xy} = (a^y)^x$ $\quad$ $\{(m^p)^q\}^r =$ $(m)^{pqr}$

7. $(a^{1/m}) =$ $\sqrt[m]{a}$

8. $a^{-m} =$ $\frac{1}{a^m}$ OR $a^x = \frac{1}{a^{-x}}$ $\quad$ OR $a^p \times a^{-p} = 1 = a^{p+(-p)} = a^0$

$a^p \times \frac{1}{a^p} = 1$

9. $a^1 = a, a^0 = 1$

10. $a^{x/y} = \sqrt[y]{a^x}$ $\quad$ $x^{m/n} = \sqrt[n]{x^m} = (x^m)^{1/n}$

11. $a^{-1/m} = \frac{1}{a^{1/m}} = \frac{1}{\sqrt[m]{a}}$ $\quad$ $28^{9/8} = \sqrt[8]{28^9}$

12. If $a^x = a^y$ then $x = y$ $\quad$ $36^{x/y} = \sqrt[y]{36^x}$

13. If $a^x = b^x$ then $a = b$

14. $\frac{a^b \times a^c}{a^d} =$ $(a)^{b+c-d}$ $\quad$ 16. $\sqrt[m]{\frac{a}{b}} =$ $(a/b)^{1/m}$

15. $\frac{a^b \times a^c}{a^e \times a^f} =$ $(a)^{b+c-e-f}$ $\quad$ $= \left(\frac{a^{1/m}}{b^{1/m}}\right) = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$

143. $2^3 \times (16)^8 \times (\sqrt{2})^9 =$

$= 2^3 \times (2^4)^8 \times (2^{\frac{1}{2}})^9 = 2^3 \times 2^{32} \times 2^{\frac{9}{2}}$

$= (2)^{3+32+4.50} = (2)^{39.50} = (2)^{79/2} = \sqrt{2^{79}}$

144. $3^8 \times 3^{-12} \times (\sqrt{3})^{16} \times (\sqrt[3]{3})^{18} =$

$= 3^8 \times 3^{-12} \times (3^{1/2})^{16} \times (3^{1/3})^{18}$

$= 3^8 \times 3^{-12} \times 3^8 \times 3^6$

$= (3)^{8-12+8+6} = (3)^{10} = 59049$

145. $x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$

$= (x)^{a-b+b-c+c-a} = x^0 = 1$

Please remember : (Any Number)^0 = 1.00

146. $(x^{a-b})^{a^2+ab+b^2} \cdot (x^{b-c})^{b^2+bc+c^2} \cdot (x^{c-a})^{c^2+ac+a^2}$

$= (x)^{(a-b)(a^2+ab+b^2)} \cdot (x)^{(b-c)(b^2+bc+c^2)} \cdot (x)^{(c-a)(c^2+ac+a^2)}$

$= (x)^{a^3-b^3} \cdot (x)^{b^3-c^3} \cdot (x)^{c^3-a^3} = (x)^{a^3-b^3+b^3-c^3+c^3-a^3} = (x)^0$

$= 1$

147. $\left[\frac{(\sqrt{2})^9 \times 16^{-5/2} \times 64^{-7/2}}{32^{11/2}}\right] = \frac{(2^{\frac{1}{2}})^9 \times (2^4)^{-5/2} \times (2^6)^{-7/2}}{(2^5)^{11/2}}$

$= \frac{2^{9/2} \times 2^{-20/2} \times 2^{-42/2}}{2^{55/2}} = (2)^{\frac{9}{2}-\frac{20}{2}-\frac{42}{2}-\frac{55}{2}} = (2)^{-108/2} = (2)^{-54} = \frac{1}{2^{54}}$

148. $\sqrt[9]{\left[\sqrt[3]{4}\right]^{1/8}}$

$= \left\{\left[(4)^{1/3}\right]^{1/8}\right\}^{1/9} = (4)^{\frac{1}{3}\times\frac{1}{8}\times\frac{1}{9}} = (4)^{\frac{1}{216}} = (2^2)^{\frac{1}{216}}$

$= (2)^{2\times\frac{1}{216}} = 2^{\frac{1}{108}} = \sqrt[108]{2}$

149. $\left[\frac{8^{1/4}\times 32^{1/8}}{\sqrt[16]{1024}}\right]^9$

$= \left[\frac{(2^3)^{1/4}\times(2^5)^{1/8}}{(2^{10})^{1/16}}\right]^9 = \left(\frac{2^{3/4}\times 2^{5/8}}{2^{5/8}}\right)^9 = \left(2^{\frac{3}{4}}\right)^9 = 2^{\frac{27}{4}} = (2^{27})^{\frac{1}{4}}$

$= \sqrt[4]{2^{27}}$

150. $\sqrt[16]{\left[\frac{\sqrt[7]{3}\times\sqrt[14]{81}\times(27)^{-9}}{243^{8/7}}\right]^{42}}$

$= \left[\frac{3^{\frac{1}{7}}\times(3^4)^{\frac{1}{14}}\times(3^3)^{-9}}{(3^5)^{8/7}}\right]^{42\times\frac{1}{16}} = \left[(3)^{\frac{1}{7}+\frac{2}{7}-\frac{189}{7}-\frac{40}{7}}\right]^{\frac{21}{8}}$

$= \left[\frac{3^{\frac{1}{7}}\times 3^{\frac{2}{7}}\times 3^{-27}}{3^{40/7}}\right]^{\frac{42}{16}} = (3)^{\frac{-226}{7}\times\frac{21}{8}}$

$= (3)^{\frac{-4746}{56}} = (3)^{-339/4}$

$= \frac{1}{(3)^{339/4}} = \frac{1}{\sqrt[4]{3^{339}}}$

151. $\left[\sqrt[3]{4^{8}}\right]^{\frac{11}{2}} \times \left[\sqrt[9]{32^{-8}}\right]^{\frac{17}{2}}$

$= \left\{\left[(2^2)^{\frac{1}{3}}\right]^{8}\right\}^{\frac{11}{2}} \times \left\{\left[(2^5)^{1/9}\right]^{-8}\right\}^{\frac{17}{2}}$

$= (2)^{2 \times \frac{1}{3} \times 8 \times \frac{11}{2}} \times (2)^{5 \times \frac{1}{9} \times -8 \times \frac{17}{2}}$

$= (2)^{\frac{88}{3}} \times (2)^{\frac{-680}{18}} = (2)^{\frac{528}{18}} \times (2)^{\frac{-680}{18}} = (2)^{\frac{528}{18} - \frac{680}{18}}$

$= (2)^{-\frac{152}{18}} = (2)^{-76/9} = \sqrt[9]{2^{-76}} = \frac{1}{2^{76/9}} = \left(\frac{1}{2}\right)^{76/9}$

152. $\text{Log}_{\sqrt[5]{64}} \sqrt[8]{512} =$

$= \frac{\text{Log}\sqrt[8]{512}}{\text{Log}\sqrt[5]{64}} = \frac{\text{Log}(2^9)^{1/8}}{\text{Log}(2^6)^{1/5}}$

$= \frac{\text{Log}(2)^{9/8}}{\text{Log}(2)^{6/5}} = \frac{\frac{9}{8}\text{Log}2}{\frac{6}{5}\text{Log}2} = \frac{9}{8} \times \frac{5}{6} = \frac{45}{48} = \frac{15}{16}$

153. $\text{Log}_{\sqrt[8]{81}} \sqrt[18]{729} =$

$= \frac{\text{Log}\sqrt[18]{729}}{\text{Log}\sqrt[8]{81}} = \frac{\text{Log}(3^6)^{1/18}}{\text{Log}(3^4)^{1/8}} = \frac{\frac{6}{18}\text{Log}3}{\frac{4}{8}\text{Log}3}$

$= \left(\frac{\frac{1}{3}}{\frac{1}{2}}\right) = \frac{1}{3} \times \frac{2}{1} = \left(\frac{2}{3}\right)$

154. $\left[\dfrac{\text{Log}\sqrt[3]{16} \times \text{Log}\sqrt[9]{2187}}{\text{Log}\left(\sqrt{3}\right)^{19} \times \text{Log}\sqrt[15]{1024}}\right]$

$$= \frac{\text{Log}\left(2^4\right)^{1/3} \times \text{Log}\left(3^7\right)^{1/9}}{\text{Log}\left(3^{1/2}\right)^{19} \times \text{Log}\left(2^{10}\right)^{1/15}}$$

$$= \frac{\frac{4}{3}\text{Log}2 \times \frac{7}{9}\text{Log}3}{\frac{19}{2}\text{Log}3 \times \frac{10}{15}\text{Log}2} = \frac{4}{3} \times \frac{7}{9} \times \frac{2}{19} \times \frac{3}{2} = \left(\frac{28}{171}\right)$$

155. If Log 2 = x, Log 3 = y. Find Log (1.20)

a. 2x + 3y - 1 b. 2x + y - 1 c. x + y + 3 d. None

$\Longrightarrow$ $\text{Log}(1.20) = \text{Log}\left(\frac{12}{10}\right) = \text{Log}\,12 - \text{Log}\,10$

$= \text{Log}(2\times2\times3) - \text{Log}\,10 = \text{Log}\,2 + \text{Log}\,2 + \text{Log}\,3 - \text{Log}\,10$

$= x + x + y - 1 = 2x + y - 1$

Answer to question

$= \text{Log}\,4 + \text{Log}\,3 - \text{Log}\,10$

$2x + y - 1 = 2\,\text{Log}\,2 + \text{Log}\,3 - \text{Log}\,10 = \text{Log}\left(\frac{4\times3}{10}\right) = \text{Log}\,1.20$

156. If Log 2 = x, Log 3 = y. Find Log (7.20)

$\text{Log}(7.20) = \text{Log}\left(\frac{72}{10}\right) = \text{Log}\,72 - \text{Log}\,10 = \text{Log}(9\times8) - 1$

$= \text{Log}\,9 + \text{Log}\,8 - 1 = \text{Log}\,3^2 + \text{Log}\,2^3 - 1 = 2.\text{Log}\,3 + 3.\text{Log}\,2 - 1$

$= 2y + 3x - 1 = 3x + 2y - 1$

157. If $\text{Log}_{3/2}\, x = -5$. Find the value of x.

$\text{Log}_{3/2}\, x = -5 \quad \therefore \left(\frac{3}{2}\right)^{-5} = x \quad \therefore x = \left(\frac{2}{3}\right)^{5} = \frac{32}{243}$

If $\log_b a = k$ then $b^k = a$

$$\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$$

158. $\log_x 16 = \log_9 64$. Find the value of x.

$$\log_x 16 = \log_9 64$$

$$\frac{\log 16}{\log x} = \frac{\log 64}{\log 9}$$

$$\frac{4.\log 2}{\log x} = \frac{6.\log 2}{\log 9}$$

$$6.\log x = 4.\log 9$$

$$3.\log x = 2.\log 9 = \log 81$$

$$\log x = \frac{1}{3}\log 81 = \log(81)^{1/3} = \log(3)^{4/3}$$

$$\log x = \log(3)^{4/3}$$

$$x = 3^{4/3} \quad \text{OR} \quad x = \sqrt[3]{3^4} = \sqrt[3]{81}$$

159. $\log_2 \log_2 \log_2 16 = ?$

$$= \log_2 \log_2 (\log_2 16)$$

$$= \log_2 \log_2 \left(\frac{4.\log 2}{\log 2}\right)$$

$$= \log_2 (\log_2 4) = \log_2 \left(\frac{2.\log 2}{\log 2}\right) = \log_2 2 = 1.00$$

160. $\log_x 9 = 8$. Find value of $x\sqrt{x}$

$\log_x 9 = 8$

$\therefore \quad x^8 = 9$

Taking 8th root on both sides

$(x^8)^{1/8} = (9)^{1/8}$

$x = (3^2)^{1/8} = 3^{1/4}$

$x \cdot \sqrt{x}$

$= x^1 \cdot x^{\frac{1}{2}}$

$= x^{1+\frac{1}{2}}$

$= x^{3/2}$

$= (3^{1/4})^{3/2}$

$= 3^{3/8} = \sqrt[8]{3^3}$

$= \sqrt[8]{27}$

161. $\log_x 27 = 81$. Find value of $\sqrt[3]{x} \times \sqrt[5]{x}$

$\log_x 27 = 81$

$\therefore \quad x^{81} = 27$

$x = (27)^{1/81}$

$x = (3^3)^{1/81}$

$x = (3)^{1/27}$

$\sqrt[3]{x} \times \sqrt[5]{x}$

$= x^{\frac{1}{3}} \cdot x^{\frac{1}{5}} = (x)^{\frac{1}{3}+\frac{1}{5}}$

$= (x)^{\frac{8}{15}} = \left(3^{\frac{1}{27}}\right)^{\frac{8}{15}}$

$= (3)^{\frac{8}{405}} = \sqrt[405]{3^8}$

Question

$\log_x 27 = 4$. Find value of $x\sqrt{x}$.

$\log_x 27 = 4$

$\therefore \quad x^4 = 27$

$x = (27)^{1/4}$

$x = (3^3)^{1/4} = 3^{3/4}$

$x \cdot \sqrt{x}$

$= 3^{\frac{3}{4}} \times \left(3^{\frac{3}{4}}\right)^{\frac{1}{2}}$

$= 3^{\frac{3}{4}} \times 3^{\frac{3}{8}}$

$= (3)^{\frac{3}{4}+\frac{3}{8}} = (3)^{\frac{30}{32}} = (3)^{\frac{9}{8}}$

162. $\text{Log}_3 x = 4$. Find value of $(\sqrt{x})^x$

$\text{Log}_3 x = 4$

$\therefore \quad 3^4 = x$

$x = 81$

$(\sqrt{x})^x$

$= (\sqrt{81})^{81} = 9^{81} = (3^2)^{81}$

$= (3)^{162}$

163. $\text{Log}_{\sqrt[7]{2}} (32)^{-5} = m$. Find m

$$m = \left[\frac{\text{Log}(32)^{-5}}{\text{Log}\sqrt[7]{2}}\right] = \left[\frac{\text{Log}(2^5)^{-5}}{\text{Log}(2)^{1/7}}\right] = \left[\frac{\text{Log}(2)^{-25}}{\text{Log}(2)^{1/7}}\right]$$

$$m = \left(\frac{-25 \cdot \text{Log}2}{\frac{1}{7} \cdot \text{Log}2}\right) = -25 \times \frac{7}{1} = -175$$

$\therefore m = -175$

164. $\text{Log}_{\sqrt[9]{243}} (\sqrt[8]{81})^8 =$

$$= \frac{\text{Log}(\sqrt[8]{81})^8}{\text{Log}(\sqrt[9]{243})}$$

$$= \frac{\text{Log}(81^{1/8})^8}{\text{Log}(243)^{1/9}}$$

$$= \frac{\text{Log}(3^4)}{\text{Log}(3^5)^{1/9}}$$

$$= \frac{4 \cdot \text{Log}3}{\frac{5}{9} \cdot \text{Log}3}$$

$= 4 \times \frac{9}{5} = 36/5 = 7.20$

165. $5 \log x = \frac{1}{2} \log \sqrt[5]{30}$. Find the value of x

$5 \cdot \log x = \frac{1}{2} \log \sqrt[5]{30}$

$\log x^5 = \log (30^{1/5})^{1/2}$

$\log x^5 = \log 30^{1/10}$

$A \cdot \log (\log x^5) = A \cdot \log (\log 30^{1/10})$

$x^5 = 30^{1/10}$

$(x^5)^{1/5} = (30^{1/10})^{1/5}$

$x = (30)^{1/50} = \sqrt[50]{30}$

166. $\dfrac{\log_5 16 \times \log_8 125 \times \log_7 243}{\log_{\sqrt{7}} 729}$

$= \dfrac{\frac{\log 16}{\log 5} \times \frac{\log 125}{\log 8} \times \frac{\log 243}{\log 7}}{\frac{\log 729}{\log \sqrt{7}}}$

$= \dfrac{4 \cdot \log 2}{\log 5} \times \dfrac{3 \cdot \log 5}{3 \cdot \log 2} \times \dfrac{5 \cdot \log 3}{\log 7} \times \dfrac{\frac{1}{2} \log 7}{\frac{6}{1} \cdot \log 3}$

$= \left(4 \times 5 \times \frac{1}{2} \times \frac{1}{6}\right) = \frac{20}{12} = \left(\frac{5}{3}\right)$

167. $\text{Log}_{\sqrt{5}} x = 3$. Find value of $(x \times \sqrt[5]{x})$

$\text{Log}_{\sqrt{5}} x = 3$

$\therefore \quad x = (\sqrt{5})^3$

$x = (5^{1/2})^3$

$x = 5^{3/2}$

$x \cdot \sqrt[5]{x} = 5^{\frac{3}{2}} \cdot \left(5^{\frac{3}{2}}\right)^{\frac{1}{5}}$

$= 5^{\frac{3}{2}} \cdot 5^{\frac{3}{10}}$

$= 5^{\frac{15}{10}} \cdot 5^{\frac{3}{10}} = (5)^{\frac{15}{10}+\frac{3}{10}}$

$= (5)^{18/10} = (5)^{9/5} = \sqrt[5]{5^9}$

$= \sqrt[5]{1953125}$

168. If $a^2 + b^2 = 50$ & $ab = 20$. Find value of $(a+b)^2$

$(a+b)^2 = (a^2 + b^2) + 2ab$

$= 50 + (2 \times 20) = 50 + 40 = 90$

Question If $(a+b) = 35$, $ab = 50$ Find $\left(\frac{1}{a} + \frac{1}{b}\right) = ?$

$\Longrightarrow \quad \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{35}{50} = \frac{7}{10}$

169. $\text{Log}_2 \text{Log}_2 \text{Log}_2 \text{Log}_2 x = 0$. Find x.

$\text{Log}_2 [\text{Log}_2 \text{Log}_2 \text{Log}_2 x] = 0$

$\text{Log}_2 (\text{Log}_2 \text{Log}_2 x) = 2^0 = 1$

$\text{Log}_2 \text{Log}_2 x = 2^1 = 2$

$\text{Log}_2 x = 2^2 = 4$

$\therefore \quad \text{Log}_2 x = 4$

$\therefore \quad x = 2^4$

$x = 16$

cross-check

$= \text{Log}_2 \text{Log}_2 \text{Log}_2 \text{Log}_2 16$

$= \text{Log}_2 \text{Log}_2 \text{Log}_2 4$

$= \text{Log}_2 \text{Log}_2 2$

$= \text{Log}_2 1$

$= 0$

170. $\text{Log}_a x = a$ then $x = ?$

→ $\text{Log}_a x = a \quad \therefore \quad a^a = x$

Q. If $\text{Log}_{27} x = \text{Log}_{81} 3$. Find x

$\Rightarrow \quad \frac{\text{Log}x}{3.\text{Log}3} = \frac{\text{Log}3}{4.\text{Log}3} \quad \therefore \text{Log}x = \frac{1}{4} \cdot 3\,\text{Log}3$

$\text{Log}x = \text{Log}3^{3/4} \quad \therefore x = 3^{3/4}$

171. $\text{Log}(1 + 2 + 3) = \text{Log}1 + \text{Log}2 + \text{Log}3$

~~a. True~~ b. False

$\text{Log}(1+2+3) = \text{Log}6 = \text{Log}(1\times2\times3) = \text{Log}1 + \text{Log}2 + \text{Log}3$

Given statement is correct.

172. $\text{Log}(5 + 6 + 7) = \text{Log}5 + \text{Log}6 + \text{Log}7$

a. True ~~b. False~~

L.H.S. $= \text{Log}(5+6+7) = \text{Log}18$

R.H.S. $= \text{Log}5 + \text{Log}6 + \text{Log}7$

$= \text{Log}(5\times6\times7) = \text{Log}(210)$

L.H.S. $\neq$ R.H.S.

Given statement is incorrect.

173. $\text{Log}_2 x + \text{Log}_8 x + \text{Log}_{32} x = \frac{23}{5}$

Find value of x.

→ $\frac{\text{Log}x}{\text{Log}2} + \frac{\text{Log}x}{3.\text{Log}2} + \frac{\text{Log}x}{5.\text{Log}2} = \frac{23}{5}$

$\frac{15\,\text{Log}x}{15\,\text{Log}2} + \frac{5.\text{Log}x}{15.\text{Log}2} + \frac{3\,\text{Log}x}{15.\text{Log}2} = \frac{23}{5}$

$\frac{15.\text{Log}x + 5.\text{Log}x + 3.\text{Log}x}{15.\text{Log}2} = \frac{23}{5}$

$\therefore \frac{23\,\text{Log}x}{15\,\text{Log}2} = \frac{23}{5}$

$\frac{\text{Log}x}{\text{Log}2} = \frac{15}{5} = 3$

$\text{Log}x = 3.\text{Log}2$

$\text{Log}x = \text{Log}8$

$x = 8$

174. Log 5 = 0.6990

Log 3 = 0.4771

Find value of Log $\left(\frac{50}{3,00,000}\right)$

$\rightarrow$

$\text{Log}\left(\frac{50}{3,00,000}\right) = \text{Log}\,50 - \text{Log}\,3,00,000$

$= 1.6990 - 5.4771$

$= -3.7781$

OR

$\text{Log}\left(\frac{5}{30,000}\right)$

$= \text{Log}\left(\frac{5}{3 \times 10,000}\right)$

$= \text{Log}5 - \text{Log}3 - \text{Log}10,000$

$= 0.6990 - 0.4771 - 4$

$= -3.7781$

175. Find Log $\left(\frac{0.003}{2000}\right)$ If Log2 = 0.3010, Log3 = 0.4771

$\rightarrow$

$\text{Log}\left(\frac{0.003}{2000}\right) = \text{Log}\,0.003 - \text{Log}2000 = \bar{3}.4771 - 3.3010$

$= (-3 + 0.4771) - 3.3010 = -5.8239$

OR

$\text{Log}\left(\frac{0.003}{2000}\right) = \text{Log}\left(\frac{3}{2000,000}\right) = \text{Log}\,3 - \text{Log}\,20,00,000$

$= 0.4771 - 6.3010$

$= -5.8239$

176. If 2s : 3t is duplicate ratio of (2s-p) : (3t-p) then

a. $p^2 = 6st$ b. $p = 6st$ c. $2p = 3st$ d. None of these

$\frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$ $\frac{2s}{3t} = \frac{4s^2 - 4ps + p^2}{9t^2 - 6pt + p^2}$

$2s(9t^2 - 6pt + p^2) = 3t(4s^2 - 4ps + p^2)$

$18t^2s - 12pts + 2p^2s = 12s^2t - 12pts + 3tp^2$

$18t^2s - 12s^2t = 3tp^2 - 2p^2s$

$6st(3t - 2s) = p^2(3t - 2s)$

$\therefore \quad p^2 = 6st$

177. $\frac{A}{3} = \frac{B}{8} = \frac{C}{7}$ Find A : B : C = ?

a. 7 : 8 : 3 b. 3 : 8 : 7 c. 8 : 7 : 3 d. None of these

$\frac{A}{3} = \frac{B}{8}$ $\frac{B}{8} = \frac{C}{7}$

$\therefore$ A : B = 3 : 8 $\therefore$ B : C = 8 : 7

$\therefore$ A : B : C = 3 : 8 : 7

178. Anand earns ₹ 5000 in 6 hours & Vinod earns ₹ 7000 in 8 hours. Find ratio of their earning per hour.

→

	Anand	vinod
Earning per hour	$\left(\frac{₹5000}{6}\right)$	$\left(\frac{₹7000}{8}\right)$

Ratio of earning per hour $= \frac{5000/6}{7000/8} = \frac{5000}{6} \times \frac{8}{7000} = \frac{40}{42}$

$= 20 : 21$

179. Monthly income of A, B is in the ratio of 5:6 & monthly expenses are in the ratio of 9:11. If both of them save ₹ 50 p.m. Find their incomes.

→

	Monthly income	Monthly saving	Monthly exp
A	$5x$	50	$5x - 50$
B	$6x$	50	$6x - 50$

$\frac{5x - 50}{6x - 50} = \frac{9}{11}$

$55x - 550 = 54x - 450$

$x = 100$

∴ Monthly incomes of A : $5x = 5 \times 100 = ₹500$

B : $6x = 6 \times 100 = ₹600$

OR

$5x - 9y = 50$

$6x - 11y = 50$

$55x - 99y = 550$

$54x - 99y = 450$

$x = 100$

∴ Incomes are

A : $5x = ₹500$

B : $6x = ₹600$

180. Duplicate ratio of sub-duplicate ratio of 8:9 is

a. 64:81 b. 9:8 c. 128:162 d. None of these

= Duplicate ratio of sub-dupli. ratio of 8 : 9 = 8 : 9

181. What least number should be added to each one of 6, 14, 18, 38 to make them in proportion?

a. 5 b. 3 c. 2 d. 4

(6+2), (14+2), (18+2), (38+2) i.e 8, 16, 20, 40 are in prop.

182. Raja can walk certain distance in 50 days if he takes rest for 9 hrs per day. How long it will take him to walk twice as far if he walks twice as fast and rests twice as long each day?

a. 125 days b. 50 days c. 100 days d. None

	original	New
walking speed	x kms/hr	$2x$ kms/hr
Rest per day	9 hrs	18 hrs
walking per day	24 − 9 = 15 hrs	24 − 18 = 6 hrs
Dist. per day	$15x$ kms	$6 \times 2x = 12x$ kms
No. of days	50	$\frac{1500x}{12x} = 125$ days ← Answer
Total distance traveled	$15x \times 50 = 750x$	$1500x$

183. 3, x, 27, y are in continued proportion then x : y = ?

a. 9:1 ~~b. 1:9~~ c. 1:27 d. 3:1

3, x, 27, y are in continued prop^n

$\frac{3}{x} = \frac{27}{y}$ $\therefore \frac{3}{27} = \frac{x}{y}$ $\therefore \frac{x}{y} = \frac{1}{9}$

184. $\text{Log}_{1/9}\ 243 = x$. Find x

→ If $\text{Log}_b a = k$ then $b^k = a$

$\therefore \left(\frac{1}{9}\right)^x = 243$ $\quad (3^{-2})^x = 3^5$

$\left(\frac{1}{3^2}\right)^x = 3^5$ $\quad 3^{-2x} = 3^5$

$\therefore -2x = 5$

$x = -5/2 = -2.50$

185. $\text{Log}_2 x + \text{Log}_4 x + \text{Log}_{16} x = \frac{21}{4}$ then x = ?

a. 7 ~~b. 8~~ c. 9 d. 10

$\frac{4\,\text{Log}x}{4\,\text{Log}2} + \frac{2\,\text{Log}x}{4.\text{Log}2} + \frac{\text{Log}x}{4.\text{Log}2} = \frac{21}{4}$

$\frac{7.\,\text{Log}x}{4.\text{Log}2} = \frac{21}{4}$

$\frac{\text{Log}x}{\text{Log}2} = \frac{21}{\cancel{4}} \times \frac{\cancel{4}}{7}$

$\therefore \frac{\text{Log}x}{\text{Log}2} = 3$

$\text{Log}x = 3.\text{Log}2$

$\text{Log}x = \text{Log}2^3$

$\text{Log}x = \text{Log}\,8$

$\therefore x = 8$

186. $\text{Log}_a 3 = 2, \text{Log}_b 8 = 3$; then $\text{Log}_b a$ = ?

a. $\text{Log}_3 2$ b. $\text{Log}_2 3$ c. $\text{Log}_3 4$ ~~d. $\text{Log}_4 3$~~

→ $\text{Log}_a 3 = 2$ $\therefore a^2 = 3$ $\therefore a = 3^{1/2}$

$\log_b 8 = 3 \quad \therefore \quad b^3 = 8 = 2^3 \quad \therefore \quad b = 2$

$\log_b a = \frac{\log a}{\log b} = \frac{\log 3^{\frac{1}{2}}}{\log 2} = \frac{\frac{1}{2}\cdot\log 3}{\log 2} = \frac{\log 3}{2\cdot\log 2}$

$= \frac{\log 3}{\log 2^2} = \frac{\log 3}{\log 4} = \log_4 3$

187. Find x if $\log_x 10 + \log_x 100 + \log_x 1000 = 6$

a. 10 b. 2 c. 4 d. 6

→ $\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

$\therefore \quad 1 + 2 + 3 = 6$

$\boxed{x = 10}$

188. $2 \log a + 3 \log b - 2 = 0$ the $a^2b^3 = ?$

a. 10^4 b. 10^3 ~~c. 10^2~~ d. None of these

→ $2\log a + 3\cdot\log b = 2$

$\log a^2 + \log b^3 = 2$

$\log_{10}(a^2\cdot b^3) = 2$

$\therefore \quad a^2 b^3 = 10^2$

189. $\log_2 [\log_2 \{\log_3(\log_3 27^3)\}] = ?$

a. $\frac{1}{2}$ b. 1 ~~c. 0~~ d. 2

→ $= \log_2 \left[\log_2 \left\{\log_3 \left(\log_3 27^3\right)\right\}\right]$

$= \log_2 \left[\log_2 \left\{\log_3 \left(\frac{9\cdot\log 3}{\log 3}\right)\right\}\right]$

$= \log_2\left[\log_2\{\log_3 9\}\right]$

$= \log_2\left[\log_2 2\right] = \log_2 1 = 0 = \text{Zero}$

190. 2 Log x = 4 Log 4 then x = ?

$2\log x = 4 \cdot \log 4$

$\log x = 2 \cdot \log 4$

$\log x = \log 4^2$

$\log x = \log 16$ $\quad \therefore \; x = 16$

191. Age of a person is twice the sum of ages of his 2 sons & 5 years ago his age was 3 times the sum of ages of his 2 sons, his present age is _____ years

a. 60 b. 52 c. 51 d. 50

	A person's age	sum of ages of his 2 sons
present	$2x$	x
5 years before	$2x - 5$	$x - 5 - 5 = x - 10$

$2x - 5 = 3 \times (x - 10)$

$2x - 5 = 3x - 30$

$25 = x$

$\therefore x = 25$

$\therefore$ His present age $= 2x = 50$ years

192. Sum of 2 numbers is 45 and the mean proportional between them is 18. The numbers are :

a. 15,30 b. 32,13 ~~c. 36,9~~ d. 25,20

as $36 + 9 = 45$ & $36 \times 9 = 324$

Let numbers are x & y

$x + y = 45$

$x, 18, y$ are in propⁿ $\therefore xy = 18^2 = 324$

193. If $(7p+3q) : (3p-2q) = 2:1$ then $p:q$ = ?

$\frac{7p+3q}{3p-2q} = \frac{2}{1}$

$7p + 3q = 6p - 4q$

$\therefore p = -7q$

$\therefore \frac{p}{q} = \frac{-7}{1} = -7:1$ OR $7:-1$

194. If $\text{Log}_a 23 = b$ then $\Longrightarrow \therefore a^b = 23$

a. $a^{23} = b$ b. $b^a = 23$ c. $b^{23} = a$ d. $a^b = 23$

If $\text{Log}_x a = y$ then $x^y = a$

195. The angles of a triangle are in the ratio of 2 : 29 : 5 then Find the angles :

Let the angles be : $2x, 29x, 5x$

$\therefore 2x + 29x + 5x = 180$

$36x = 180$

$x = 5$

$\therefore$ Angles are $2x = 10°$, $29x = 145°$, $5x = 25°$

196. $a^2 + b^2 = 45$, $ab = 18$. Find $\left(\frac{1}{a}+\frac{1}{b}\right)$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b)^2 = 45 + 2(18) = 81$

$\therefore (a+b) = 9$

$\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{a+b}{ab} = \frac{9}{18} = \frac{1}{2}$

$= 0.50$

197. If $a^2.\text{Log}_3 x = b.\text{Log}_{27} x$ then

a. $a = 3$ ~~b. $3a^2 = b$~~ c. $b^2 = 3a$ d. None of these

$a^2 \cdot \frac{\text{Log}x}{\text{Log}3} = b \cdot \frac{\text{Log}x}{3.\text{Log}3}$

$\therefore a^2 = \frac{b}{3}$

$\therefore 3a^2 = b$

198. Mr. A says to his son '7 years before I was 7 times as old as you were' and 3 years later I shall be 3 times as old as you will be. Find present age of Mr. A's son :

a. 12 years b. 15 years c. 5 years d. 7 years

→

	Mr. A	Mr. A's son
present age	x	y
7 years before	$(x-7)$	$(y-7)$
3 years later	$(x+3)$	$(y+3)$

$x - 7 = 7(y-7)$

$x - 7 = 7y - 49$

$x - 7y = -42$ ..... (1)

$(x+3) = 3(y+3)$

$x + 3 = 3y + 9$

$x - 3y = 6$ ..... (2)

$x - 7y = -42$

$-\ x - 3y = -\ 6$

$-4y = -48 \quad \therefore y = 12$

$\therefore$ present age of Mr. A's son = 12 years

199. One third of one half of three fourth of a number is 60, the number is :

a. 480 b. 520 c. 500 d. None

→

$\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} \times x = 60$

$x = 60 \times 2 \times 4 = 480$

200. $x^2 + y^2 = 14xy$ then $2 \text{ Log } 4 + \text{Log } x + \text{Log } y = ?$

a. $\frac{1}{2}\text{Log}(x+y)$ b. $2 \text{ Log } (x+y)$ c. $\text{Log } (x+y)$ d. None

→

$x^2 + y^2 = 14xy$

$x^2 + y^2 + 2xy = 14xy + 2xy$

$(x+y)^2 = 16xy$

$2\text{Log}4 + \text{Log}x + \text{Log}y$

$= \text{Log}16 + \text{Log}x + \text{Log}y$

$= \text{Log}(16xy)$

$= \text{Log}(x+y)^2 = 2.\text{Log}(x+y)$

201. $(2p^2q^3r^2)^0 = ?$

a. 0 ~~b. 1~~ c. $4p^4q^6r^4$ d. None

→ (Any number)^0 = 1

202. $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1 then (p+q+r) = ?

a. 1 ~~b. 0~~ c. $\frac{1}{2}$ d. None

→ $x^{1/p} = y^{1/q} = z^{1/r} = k$

$x^{1/p} = k \therefore (x^{1/p})^p = k^p \therefore x = k^p$

$y^{1/q} = k \therefore y = k^q$

$z^{1/r} = k \therefore z = k^r$

$xyz = 1$

$k^p \cdot k^q \cdot k^r = 1$

$k^{(p+q+r)} = 1$

$\therefore p+q+r = 0$

203. $a^x = b, b^y = c, c^z = a$; then xyz = ?

~~a. 1~~ b. 2 c. 3 d. None

→ $a^x = b \quad b^y = c \quad c^z = a$

$a = b^{1/x} \quad b = c^{1/y} \quad c = a^{1/z}$

$a = (c^{1/y})^{1/x} = (c)^{\frac{1}{xy}}$

$a^1 = (a^{1/z})^{1/xy} = (a)^{1/xyz}$

$\therefore 1 = \frac{1}{xyz} \quad \therefore xyz = 1$

(OR) $a^x = b$

$x \cdot \log a = \log b$

$\therefore x = \frac{\log b}{\log a}$

$y = \frac{\log c}{\log b}$

$z = \frac{\log a}{\log c}$

$xyz = \frac{\log b}{\log a} \times \frac{\log c}{\log b} \times \frac{\log a}{\log c} = 1.00$

204. $\log_{2\sqrt{3}} 1728 = m$ then m = ?

a. 2 b. 3 c. 4 ~~d. None~~

→ $\log_{2\sqrt{3}} 1728 = m$

$\therefore (2\sqrt{3})^m = 1728 = 64 \times 27 = 2^6 \times 3^3$

$(2\sqrt{3})^m = 2^6 \times [(\sqrt{3})^2]^3 = 2^6 \times \sqrt{3}^6$

$(2\sqrt{3})^m = (2\sqrt{3})^6 \quad \therefore m = 6$

205. $\frac{a^m}{a^{-m}} = ?$

a. 1 b. -1 c. 0 ~~d. None~~

$= \frac{a^m}{a^{-m}} = a^{m-(-m)} = a^{m+m} = a^{2m}$

206. a:b = 2:5, b:c = 15:46, c:d = 92:200 then a:d = ?

a. 2:3 b. 3:192 c. 1:100 d. None

$\frac{a}{d} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{2}{5} \times \frac{15}{46} \times \frac{92}{200}$ So

$= \frac{3}{50} = 3:50$

a:d is compounded ratio of a:b, b:c, c:d

207. $\text{Log}_4(x^2+x) - \text{Log}_4(x+1) = 2$, then x = ?

~~a. 16~~ b. 4 c. 8 d. None

$\text{Log}_4(x^2+x) - \text{Log}_4(x+1) = 2$

$\text{Log}_4\left[\frac{x^2+x}{x+1}\right] = 2$ ..... AS $\text{Log}_m a - \text{Log}_m b = \text{Log}_m(a/b)$

$\text{Log}_4\left[\frac{x(x+1)}{(x+1)}\right] = 2$ $\therefore \text{Log}_4 x = 2$

$\therefore x = 4^2 = 16$

208. Characteristic of 7.128 is

a. 6 b. $\bar{8}$ ~~c. 0~~ d. None

Characteristic of 7.128 = No. of digits before decimal point -1

$= 1 - 1 = 0$

209. $2^x - 2^{x-1} = 4$ then $x^x = ?$

a. 2 b. 1 c. 64 ~~d. 27~~

$2^x - 2^{x-1} = 4$

$2^x - \frac{2^x}{2^1} = 4$

$\therefore 2^x = 4 \times 2 = 8 = 2^3$

$\therefore x = 3$

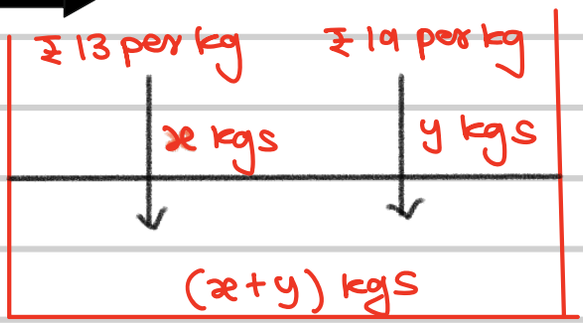
$2^x\left(1-\frac{1}{2}\right) = 4$

$2^x \times \frac{1}{2} = 4$

$x^x = 3^3 = 27$

210. 2 kinds of rice, 1st costs ₹ 13 per kg & 2nd costs ₹ 19 per kg are mixed together. Find in what ratio they are mixed so that mixture costs ₹ 14.20 per kg?

a. 3:1 b. 4:1 c. 3:4 d. 4:3



Total cost of (x+y) kgs = Total cost of Rice

$13x + 19y = 14.20(x+y)$

$13x + 19y = 14.20x + 14.20y$

$4.80y = 1.20x$

$\frac{4.80}{1.20} = \frac{x}{y}$

$\therefore \quad \frac{x}{y} = \frac{48}{12} = 4:1$

211. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$ then $3x^3 - 9x = ?$

a. 15 b. 10 c. 12 d. None

$x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$

we know

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Taking cube on both sides

$x^3 = \left(3^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)^3$

$x^3 = \left(3^{\frac{1}{3}}\right)^3 + \left(3^{-\frac{1}{3}}\right)^3 + 3 \times 3^{\frac{1}{3}} \times 3^{-\frac{1}{3}}\left(3^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)$

$x^3 = 3 + 3^{-1} + 3^{1+\frac{1}{3}-\frac{1}{3}}(x)$

$x^3 = 3 + \frac{1}{3} + (3^1 \times x)$

$3x^3 = 9 + 1 + 9x \quad \therefore \quad 3x^3 - 9x = 10$

212. Commensurable ratio = If ratio of 2 or more terms can be written in the form of **Ratio of Integers** then it is said to be a Commensurable Ratio

213. **Incommensurable Ratio OR Non-Commensurable Ratio**

If ratio of 2 or more terms cannot be written in the form of Ratio of integers then it is said to be a Incommensurable ratio.
OR Non-commensurable

214.

Ratio	Commensurable / Incommensurable Ratio
5 : 18	As it is a ratio of integers, It is a commensurable ratio
8.50 : 3.50	$8.50 : 3.50 = 85 : 35 = 17 : 7$ As 8.50 : 3.50 can be written in the form of Ratio of integers ∴ It is commensurable ratio.
8.15 : 2.221	$8.15 : 2.221 = 8150 : 2221$ As 8.15 : 2.221 can be written in the form of Ratio of integers ∴ It is commensurable ratio.
5.8361 : 3	$5.8361 : 3 = 58361 : 30000$ As 5.8361 : 3 can be written in the form of Ratio of integers ∴ It is commensurable ratio.
$\sqrt{5} : \sqrt{7}$	$2.23606797749\ldots : 2.64575131106\ldots$ $\sqrt{5} : \sqrt{7}$ can not be written in the form of Ratio of integers ∴ $\sqrt{5} : \sqrt{7}$ is incommensurable OR Non-commensurable ratio.

215.

$\sqrt{25} : \sqrt{64}$	It can be written as 5:8 $\therefore$ Commensurable ratio
$\sqrt{50} : \sqrt{450}$	$\frac{\sqrt{50}}{\sqrt{450}} = \frac{\sqrt{50}}{\sqrt{50\times9}} = \frac{\sqrt{50}}{\sqrt{50}\times\sqrt{9}} = \frac{1}{\sqrt{9}} = \frac{1}{3} = 1:3$ $\therefore \sqrt{50} : \sqrt{450}$ is a commensurable ratio
$\sqrt{147} : \sqrt{75}$	$\frac{\sqrt{147}\times\sqrt{3}}{\sqrt{75}\times\sqrt{3}} = \frac{\sqrt{147\times3}}{\sqrt{75\times3}} = \frac{\sqrt{441}}{\sqrt{225}} = \frac{21}{15} = \frac{7}{5}$ (OR) $\frac{\sqrt{147}}{\sqrt{75}} = \frac{\sqrt{49\times3}}{\sqrt{25\times3}} = \frac{\sqrt{49}\times\sqrt{3}}{\sqrt{25}\times\sqrt{3}} = \frac{7}{5} = 7:5$ $\therefore \sqrt{147} : \sqrt{75}$ is a commensurable ratio
$8\frac{1}{3} : 9\frac{2}{3}$	$8\frac{1}{3} : 9\frac{2}{3} = \frac{25}{3} : \frac{29}{3} = 25:29$ $\therefore 8\frac{1}{3} : 9\frac{2}{3}$ is a commensurable ratio
$8\frac{1}{7} : 11\frac{2}{3}$	$8\frac{1}{7} : 11\frac{2}{3} = \frac{57}{7} : \frac{35}{3} = 171:245$ $\therefore 8\frac{1}{7} : 11\frac{2}{3}$ is a commensurable ratio
$\sqrt{5} : \sqrt{10}$	$\frac{\sqrt{5}}{\sqrt{5}\times\sqrt{2}} = 1 : \sqrt{2}$ $\sqrt{5} : \sqrt{10}$ is incommensurable ratio.
$\sqrt{100} : \sqrt{196}$	$\sqrt{100} : \sqrt{196} = 10:14 = 5:7$ $\therefore \sqrt{100} : \sqrt{196}$ is a commensurable ratio.

216. 2 whole numbners whose sum is 100 cannot be in the ratio of _____

a. 16:9 → 64,36 ~~b. 3:4~~ → 42.85,57.15 c. 3:7 → 30,70 d. 4:1 → 80,20

217. Divide ₹61,000 among a,b,c such that a & b get in the ratio of 2:5 and b & c get in the ratio of 3:8.

→ $a:b = 2:5 = 6:15$

$b:c = 3:8 = 15:40$

$\therefore a:b:c = 6:15:40$

a's share $= \frac{6}{61} \times 61000 = ₹6,000$

b's share $= \frac{15}{61} \times 61000 = ₹15,000$

c's share $= \frac{40}{61} \times 61000 = ₹40,000$

218. $\frac{2^{\frac{1}{3}} \times 2^{\frac{1}{5}}}{4^{\frac{2}{3}}} =$

→ $= \left[\frac{2^{\frac{1}{3}} \times 2^{\frac{1}{5}}}{4^{\frac{2}{3}}}\right] = \left[\frac{2^{\frac{1}{3}+\frac{1}{5}}}{(2^2)^{\frac{2}{3}}}\right] = \left[\frac{2^{\frac{8}{15}}}{2^{\frac{4}{3}}}\right] = \left[\frac{2^{\frac{8}{15}}}{2^{\frac{20}{15}}}\right] = (2)^{\frac{8}{15}-\frac{20}{15}}$

$= (2)^{-12/15}$

$= 2^{-4/5}$

219. A & B together can finish the work in 8 days. If A alone can finish the work in 24 days then B alone can finish the work in _____ no. of days.

a. 10 ~~b. 12~~ c. 15 d. None

A's work of 1 day $= \frac{1}{24}$

suppose B alone can complete the work in x days

$\therefore$ B's work of 1 day $= \frac{1}{x}$

$\left(\frac{1}{24} + \frac{1}{x}\right) = \frac{1}{8}$

$\frac{1}{x} = \frac{3}{24} - \frac{1}{24} = \frac{2}{24}$

$\frac{1}{x} = \frac{1}{12} \quad \therefore x = 12$ days

220. $\text{Log}_{2\sqrt{3}}\ 1728 = m$. Find the value of m^2 is

a. 6 b. 36 c. $\sqrt{6}$ d. None of these

$\text{Log}_{2\sqrt{3}}\ 1728 = m$

$\therefore \quad (2\sqrt{3})^m = 1728 = 64 \times 27 = 2^6 \times [(\sqrt{3})^2]^3$

$\therefore \quad (2\sqrt{3})^m = (2\sqrt{3})^6$

$\therefore \quad m = 6 \qquad \therefore \quad m^2 = 6^2 = 36$

221. $2\ \text{Log}\ x = 15.\ \text{Log}\sqrt[7]{2}$. Find the value of x.

$2\ \text{Log}\ x = 15.\ \text{Log}(2)^{1/7}$

$\text{Log}\ x = \frac{15}{2}\ \text{Log}\ 2^{1/7}$

$\text{Log}\ x = \text{Log}\left(2^{1/7}\right)^{15/2}$

$\text{Log}\ x = \text{Log}\ (2)^{15/14}$

$\therefore \quad x = 2^{15/14} = \sqrt[14]{2^{15}} = \sqrt[14]{32768}$

222. If $a^x = b, b^y = c, c^z = a$ then find value of $(xyz)^3$

a. 1 b. 0 c. 8 d. None of these

$a^x = b \quad x.\text{Log}\ a = \text{Log}\ b \quad \therefore x = \text{Log}\ b / \text{Log}\ a$

$b^y = c \quad y.\text{Log}\ b = \text{Log}\ c \quad \therefore y = \text{Log}\ c / \text{Log}\ b$

$c^z = a \quad z.\text{Log}\ c = \text{Log}\ a \quad \therefore z = \text{Log}\ a / \text{Log}\ c$

$(xyz)^3 = \left(\frac{\text{Log}\ b}{\text{Log}\ a} \times \frac{\text{Log}\ c}{\text{Log}\ b} \times \frac{\text{Log}\ a}{\text{Log}\ c}\right)^3 = 1^3 = 1$

$a^x = b \quad \therefore (c^z)^x = b \quad c^{xz} = b \quad \therefore (b^y)^{xz} = b \quad \therefore b^{xyz} = b^1$

223. If $2^x - 2^{x-1} = 16$; then find value of $\sqrt{x}$

$2^x - 2^{x-1} = 16$

$2^x - \frac{2^x}{2^1} = 16$

AS $x = 5$

$\sqrt{x} = \sqrt{5}$

$2^x\left(1-\frac{1}{2}\right) = 16$

$2^x \times \frac{1}{2} = 16$

$2^x = 32 = 2^5$

$\therefore x = 5$

224. If Log 20 = 1.3010. Find Log 0.00000020

→ $\text{Log}(0.00000020) = \bar{7}.3010 = -7 + 0.3010 = -6.6990$

(OR) $\text{Log}(0.0000002) = \text{Log}\left(\frac{20}{100000000}\right) = \text{Log}\,20 - \text{Log}\,100000000$

$= 1.3010 - 8 = -6.6990$

225. $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2}) = ?$

a. $a + \frac{1}{a}$ ~~b. $a - \frac{1}{a}$~~ c. $a^2 + \frac{1}{a^2}$ d. None of these

→ $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$

$= (a^{1/4} - a^{-1/4})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$

$= (a^{1/2} - a^{-1/2})(a^{1/2} + a^{-1/2})$

$= a - a^{-1} = \left(a - \frac{1}{a}\right)$

226. If Log (2a-3b) = Log a - Log b then a = ?

~~a. $\frac{3b^2}{2b-1}$~~ b. $\frac{3b}{2b-1}$ c. $\frac{b^2}{2b+1}$ d. $\frac{3b^2}{2b+1}$

→ $\text{Log}(2a-3b) = \text{Log}\,a - \text{Log}\,b$

$\text{Log}(2a-3b) = \text{Log}\left(\frac{a}{b}\right)$

$2a - 3b = \frac{a}{b}$

$b(2a-3b) = a$

$2ab - 3b^2 = a$

$2ab - a = 3b^2$

$a(2b-1) = 3b^2$

$\therefore a = \left(\frac{3b^2}{2b-1}\right)$

227. $\left(\frac{1}{\text{Log}_{ab}abc}\right)+\left(\frac{1}{\text{Log}_{bc}abc}\right)+\left(\frac{1}{\text{Log}_{ac}abc}\right)=?$

a. 0 b. 1 ~~c. 2~~ d. -1

→

$$= \frac{1}{\frac{\text{Log}\,abc}{\text{Log}\,ab}} + \frac{1}{\frac{\text{Log}\,abc}{\text{Log}\,bc}} + \frac{1}{\frac{\text{Log}\,abc}{\text{Log}\,ac}}$$

$$= \frac{\text{Log}\,ab + \text{Log}\,bc + \text{Log}\,ac}{\text{Log}(abc)} = \frac{\text{Log}(ab \times bc \times ac)}{\text{Log}(abc)}$$

$$= \frac{\text{Log}(a^2b^2c^2)}{\text{Log}(abc)} = \frac{\text{Log}(abc)^2}{\text{Log}(abc)} = \frac{2.\text{Log}(abc)}{\text{Log}(abc)} = 2$$

228. ₹ 407 are to divided in the ratio of $\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$ then smallest share is :

→ $\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$

$= 15:12:10$

Smallest share $= \frac{10}{37} \times ₹407$

$= ₹110/-$

229. Third proportional to $(a^2 - b^2)$ and $(a+b)^2$ is

a. $\left(\frac{a+b}{a-b}\right)$ b. $\left(\frac{a-b}{a+b}\right)$ c. $\frac{(a-b)^2}{a+b}$ ~~d. $\frac{(a+b)^3}{(a-b)}$~~

→ Let 3rd proportional be : x

$\therefore$ (a^2-b^2), $(a+b)^2$, x are in proportion

square of middle term = product of extremes

$$[(a+b)^2]^2 = (a^2-b^2) \times x$$

$$(a+b)^4 = (a+b)(a-b) \cdot x$$

$$\therefore x = \frac{(a+b)^4}{(a+b)(a-b)} = \frac{(a+b)^3}{(a-b)}$$

230. If $x = y^a, y = z^b, z = x^c$ then abc = ?

a. 2 b. 1 c. 3 d. 4

$x^1 = y^a = (z^b)^a = z^{ab} = (x^c)^{ab} = x^{abc}$

$\therefore abc = 1$

OR $x = y^a \quad y = z^b \quad z = x^c$

$a = \frac{\log x}{\log y} \quad b = \frac{\log y}{\log z} \quad c = \frac{\log z}{\log x}$

$abc = \frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log x}$

$= 1.00$

231. $\left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right) = ?$

a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. $\frac{2}{3}$ d. $\frac{1}{3}$

$= \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + \frac{2^n}{2^1}}{2^n \times 2^1 - 2^n} = \frac{2^n\left(1 + \frac{1}{2}\right)}{2^n(2-1)}$

$= \frac{3/2}{1} = 3/2$

232. Find x if $x.x^{1/3} = (x^{1/3})^x$

a. 3 b. 4 c. 2 d. 6

$x^1 \cdot x^{\frac{1}{3}} = \left(x^{1/3}\right)^x$

$x^{1+\frac{1}{3}} = x^{x/3}$

$(x)^{\frac{4}{3}} = (x)^{\frac{x}{3}}$

$\therefore \frac{x}{3} = \frac{4}{3}$

$\therefore x = 4$

233. $25^{150} = (25x)^{50}$ then x = ?

a. 5^3 b. 5^2 c. 5^4 d. 5^1

$$25^{150} = 25^{50} \cdot x^{50}$$

$$x^{50} = \left(\frac{25^{150}}{25^{50}}\right) = 25^{100} = (25^2)^{50}$$

$$(x)^{50} = (625)^{50} \quad \therefore \quad x = 625 = 5^4$$

234. Salary of P is 25% lower than Q and salary of R is 20% higher than Q then ratio of salary of R and P will be

a. 5:8 b. 8:5 c. 5:3 d. 3:5

Suppose Q's salary = ₹100

∴ P's salary = ₹75

R's salary = ₹120

Ratio of salary of R & P

= 120 : 75

= 8 : 5

235. If Log 2 = x, Log 3 = y then Log 60 = ?

a. x+y+1 b. x+y-1 c. 2x+3y+1 d. 2x+5y-1

$$\text{Log}(60) = \text{Log}(2 \times 3 \times 10)$$

$$= \text{Log}2 + \text{Log}3 + \text{Log}10$$

$$= x + y + 1$$

BIG JOURNEYS BEGIN WITH SMALL STEPS !

① If $\frac{a}{7} = \frac{b}{8} = \frac{c}{9}$

Find value of $\left(\frac{a+b+c}{c}\right) = ?$

$$\longrightarrow \frac{a}{7} = \frac{b}{8} = \left(\frac{c}{9}\right) = \left(\frac{a+b+c}{7+8+9}\right)$$

$$\therefore \frac{a+b+c}{24} = \frac{c}{9}$$

$$\therefore \frac{a+b+c}{c} = \frac{24}{9} = \frac{8}{3}$$

$$\frac{7}{12} = \frac{70}{120} = \frac{21}{36} = \frac{7+70+21}{12+120+36} = \frac{7+70-21}{12+120-36}$$

$$= \frac{98}{168} \qquad = \frac{56}{96}$$

② If $\left(\frac{a}{16}\right) = \left(\frac{b}{19}\right) = \left(\frac{c}{11}\right)$ Find value of $\left(\frac{2a+9b-8c}{2b}\right)$

$$\Longrightarrow \left(\frac{a}{16}\right) = \left(\frac{b}{19}\right) = \left(\frac{c}{11}\right)$$

$$\frac{2a}{32} = \frac{9b}{171} = \frac{8c}{88} = \left(\frac{2a+9b-8c}{32+171-88}\right) \quad \ldots\ldots\ldots\ldots \text{By using Addendo \& subtrahendo}$$

$$\therefore \frac{2b}{38} = \left(\frac{2a+9b-8c}{115}\right)$$

$$\therefore \quad \frac{2a+gb-8c}{2b} = \frac{115}{38}$$

OR

$$\left(\frac{a}{16}\right)=\left(\frac{b}{19}\right)=\left(\frac{c}{11}\right) = m \qquad \therefore\ a=16m,\ b=19m,\ c=11m$$

$$\frac{2a+gb-8c}{2b} = \frac{2(16m)+g(19m)-8(11m)}{2\times 19m}$$

$$= \frac{32m+171m-88m}{38m} = \frac{115m}{38m}$$

$$= \left(\frac{115}{38}\right)$$

③ $\text{Log}_b a = \dfrac{\text{Log } a}{\text{Log } b}$

④ If $\text{Log}_b a = k$ then $b^k = a$ — Super Imp Rule

$\text{Log}_{10} 100 = 2$ as $10^2 = 100$

$\text{Log}_2 32 = 5$ as $2^5 = 32$

As $8^7 = 2097152$ $\therefore$ $\text{Log}_8 2097152 = 7$

⑤ $A.\text{Log}(\text{Log } m) = m = \text{Log}(A.\log m)$

⑥ $\text{Log } 36.979 = 1.5680$

$\text{Log } 8196 = 3.9136$

$\text{Log } 0.0000002281 = \bar{7}.3581 = -7 + 0.3581$

$= -6.6419$

$A.\text{Log } 3.2291 = 1694$

⑦ $\text{Log } 28 = 1.4472$

$\text{A.log } 1.4472 = 28.00$

$\text{Log } 28 = 1.4472$

$\text{Log}(\text{A.log } 1.4472) = 1.4472$

$\therefore \text{Log}(\text{A.log } m) = m$

$\text{A.Log } 1.4472 = 28$

$\text{A.Log}(\text{Log } 28) = 28$

$\therefore \text{A.Log}(\text{Log } x) = x$

⑧ Find $\text{Log}(\text{A.log } 2)$

$= \text{Log}(\text{A.log } 2.0000)$

$= \text{Log}(100.0)$

$= 2.0000$

$\text{A.Log}(\text{Log } 100)$

$= \text{A.Log}(2.000)$

$= 100.00$

⑨ $\text{Log } 7 = 0.8451$, $\text{Log } 3 = 0.4771$, $\text{Log } 21 = 1.3222$

$1.3222 = 0.8451 + 0.4771$

$\text{Log } 21 = \text{Log } 7 + \text{Log } 3$

$\text{Log}(7 \times 3) = \text{Log } 7 + \text{Log } 3$

$\therefore \text{Log}(a \times b) = \text{Log } a + \text{Log } b$

$$\boxed{\text{Log}_m(ab) = \text{Log}_m a + \text{Log}_m b}$$

⑩ $\text{Log } 4 = 0.6021$ $\text{Log } 5 = 0.6990$ $\text{Log } 20 = 1.3010$

$1.3011 = 0.6021 + 0.6990$

$\text{Log } 20 = \text{Log } 4 + \text{Log } 5$

$\text{Log}(4 \times 5) = \text{Log } 4 + \text{Log } 5$

(11) $\text{Log}[abc] = \text{Log}[(ab) \times c]$

$= \text{Log}(ab) + \text{Log}c$

$= \text{Log}a + \text{Log}b + \text{Log}c$

$\text{Log}_k(pqrs) = \text{Log}_k p + \text{Log}_k q + \text{Log}_k r + \text{Log}_k s$

(12) $\text{Log}(8^5) = \text{Log}(8 \times 8 \times 8 \times 8 \times 8)$

$= \text{Log}8 + \text{Log}8 + \text{Log}8 + \text{Log}8 + \text{Log}8$

$\text{Log}\ 8^5 = 5.\text{Log}8$

$\therefore \text{Log}\ a^b = b.\text{Log}a$

$\text{Log}\ p^x = x.\text{Log}p$

$\text{Log}\ 38^{-9} = -9.\text{Log}38$

$\text{Log}\ 12^{-100} = -100.\text{Log}12$

$\text{Log}\ 33^{35} = 35.\text{Log}33$

(13) $\text{Log}\left(\frac{a}{b}\right) = \text{Log}\left(a \times \frac{1}{b}\right)$

$= \text{Log}(a \times b^{-1})$

$= \text{Log}a + \text{Log}\,b^{-1}$

$= \text{Log}a + (-1.\text{Log}b)$

$\text{Log}(a/b) = \text{Log}a - \text{Log}b$

$\text{Log}(m/n) = \text{Log}m - \text{Log}n$

(14) $m = 8 \times 50$. Find value of m.

$\Longrightarrow$ $m = 8 \times 50$

Taking Log on both sides

$\text{Log}\, m = \text{Log}\,(8 \times 50)$

$= \text{Log}\, 8 + \text{Log}\, 50$

$= 0.9031 + 1.6990$

$\text{Log}\, m = 2.6021$

Taking A.log on both sides

$A.\text{Log}(\log m) = A.\log 2.6021$

$\boxed{m = 400.0}$

(15)

$\text{Log}_b a = \text{Log}\, a / \text{Log}\, b$	$\text{Log}_{10} 10 = 1$, $\text{Log}_{10} 100 = 2$
$\text{Log}(xy) = \text{Log}\, x + \text{Log}\, y$	$\text{Log}_{10} 1000 = 3$, $\text{Log}_{10} 1 = 0$
$\text{Log}\left(\frac{m}{n}\right) = \text{Log}\, m - \text{Log}\, n$	$\text{Log}_a 1 = 0$ provided $a \neq 1$
$\text{Log}\left(\frac{xy}{k}\right) = \text{Log}\, x + \text{Log}\, y - \text{Log}\, k$	If $\text{Log}_b a = k$ then $b^k = a$
$\text{Log}\, a^b = b.\text{Log}\, a$	$A.\log(\log m) = m$
$\text{Log}\, x^{-m} = -m.\text{Log}\, x$	$\text{Log}(A.\log k) = k$

(16) $\frac{0}{\text{Any Number}} = 0 = \text{zero}$, $\frac{\text{Any Number}}{0} = \text{undefined} = \text{Not defined}$

(17) $\log_{\sqrt[7]{512}} 128^8 = \dfrac{\log(128)^8}{\log \sqrt[7]{512}}$

$= \dfrac{\log(2^7)^8}{\log(2^9)^{1/7}} = \dfrac{\log(2)^{56}}{\log(2)^{9/7}} = \dfrac{56 \cdot \log 2}{\frac{9}{7}\log 2}$

$= 56 \times \dfrac{7}{9} = \left(\dfrac{392}{9}\right) = 43.555555$

(18) $\log_{\sqrt[8]{81}} (\sqrt[7]{9})^{13} = \dfrac{\log(\sqrt[7]{9})^{13}}{\log \sqrt[8]{81}}$

$= \dfrac{\log\left[(3^2)^{1/7}\right]^{13}}{\log(3^4)^{1/8}} = \dfrac{\log(3)^{\frac{26}{7}}}{\log(3)^{\frac{1}{2}}} = \dfrac{\frac{26}{7}\log 3}{\frac{1}{2}\log 3}$

$= \dfrac{26}{7} \times \dfrac{2}{1} = \left(\dfrac{52}{7}\right)$

(19) $\log_{\sqrt[9]{32}} (\sqrt[7]{4})^{-5/2} = \dfrac{\log(\sqrt[7]{2^2})^{-5/2}}{\log(32)^{1/9}}$

$= \dfrac{\log\left[(2^2)^{1/7}\right]^{-5/2}}{\log(2^5)^{1/9}} = \dfrac{\log(2)^{-5/7}}{\log(2)^{5/9}} = \dfrac{-\frac{5}{7}\log 2}{\frac{5}{9}\log 2}$

$= -\dfrac{5}{7} \times \dfrac{9}{5} = -9/7$

(20)

$\text{Log}(a \times b) = \text{Log}a + \text{Log}b$	$a^{-m} = \frac{1}{a^m}$
$\text{Log}(a/b) = \text{Log}a - \text{Log}b$	$a^k \times a^{-k} = 1 = a^0$
$a^m \times a^n = a^{m+n}$	$\sqrt[q]{2 \times 35} = \sqrt[q]{2} \times \sqrt[q]{35}$
$\frac{a^m}{a^k} = a^{m-k}$	$a^{-1/k} = \frac{1}{a^{1/k}} = \frac{1}{\sqrt[k]{a}}$
$(a \times b)^m = a^m . b^m$	$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
$\left(\frac{a}{b}\right)^m = \left(\frac{a^m}{b^m}\right)$	$\text{Log}_y x \times \text{Log}_m y = \text{Log}_m x$