

Chapter: Theory of production & cost.

* ISO quants:

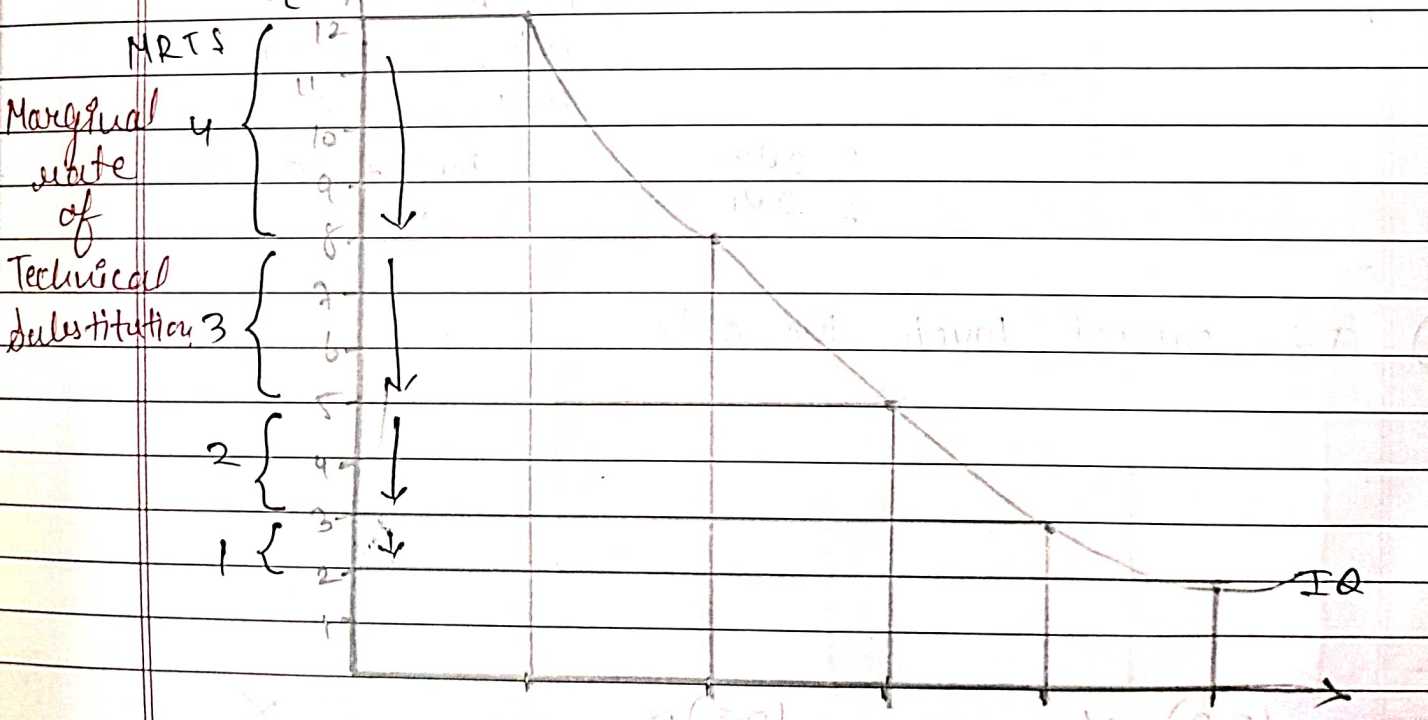
→ ISO quants is also known as production indifference curve, equal product curves, Iso product curves.

→ An Isoquant represents various combination of two input (Capital & labour) that given some level of output.

→ All the points on 1 isoquant give the same level of output using two factors of prodⁿ.

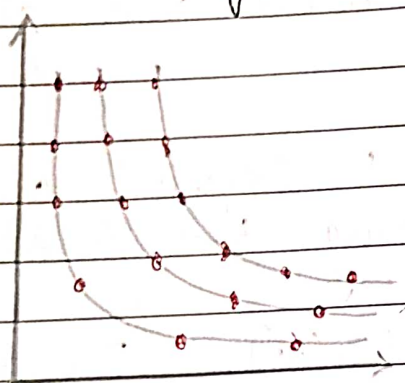
$$MRTS = \frac{\Delta y}{\Delta x}$$

Combination	Factor (x)	Factor (y)	MRTS $\frac{\Delta y}{\Delta x}$
A	1	12	-
B	2	8	4
C	3	5	3
D	4	3	2
E	5	2	1



* Properties of Isoquant:

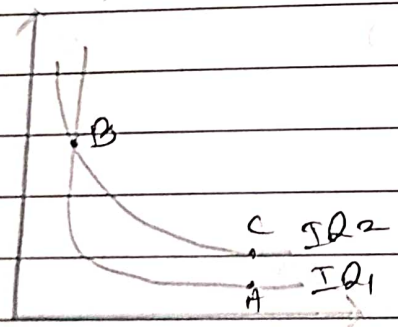
- 1) IQ curve is a downward sloping curve (inverse relation)
- 2) IQ are convex to origin (MRTS ↓)
- 3) Higher the IQ, higher the level of output.



$IQ_3 > IQ_2 > IQ_1$

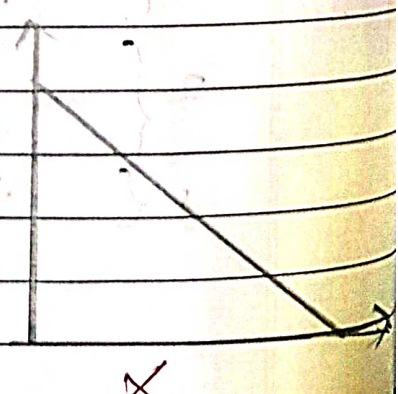
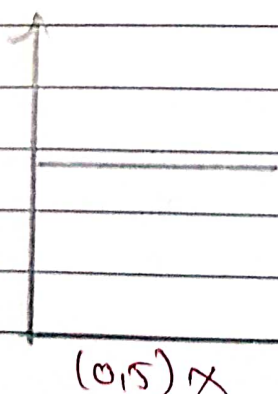
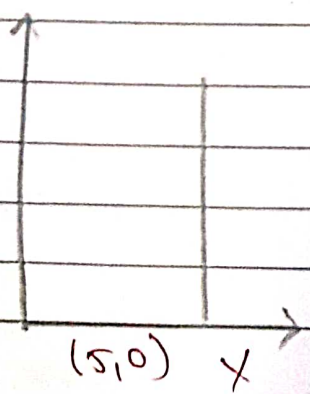
Isoquant map.

- 4) IQ cannot intersect each other.



@ $IQ_1 \rightarrow A = B$
 @ $IQ_2 \rightarrow B = C$
 @ $IQ_3 \rightarrow A = C \therefore IQ_1 = IQ_2$
 but $A \neq C, IQ_1 \neq IQ_2$

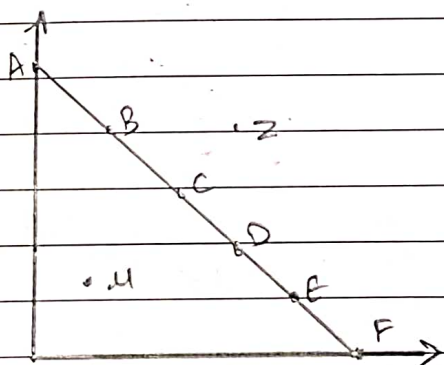
- 5) IQ cannot touch the axis.



* Iso-cost lines:

- 1) Iso cost lines is also called as equal-cost line, prize line, outlay lines & factory lines.
- 2) Iso cost lines show various combinations of two factor inputs which the firm can purchase with a given outlay & given prices of 2 inputs.

Combination	factor(x)	factor(y)
A	0	5
B	2	4
C	4	3
D	6	2
E	8	1
F	10	0



$$M = P_x \cdot Q_x + P_y \cdot Q_y$$

$$P_x = \frac{M}{Q_x} \text{ (assuming } y=0)$$

$$P_y = \frac{M}{Q_y} \text{ (assuming } x=0)$$

$$\begin{aligned} \text{Cost} &= 50/- \\ P_x &= 5/- \\ P_y &= 10/- \end{aligned}$$

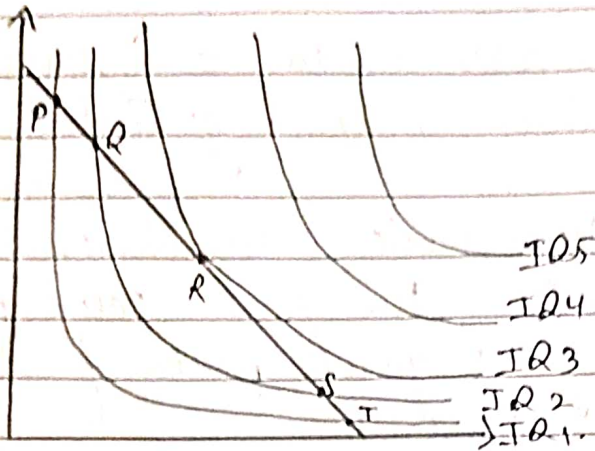
$$MRTS = \frac{\Delta Y}{\Delta X} = \frac{1}{2}$$

$$\frac{P_x}{P_y} = \frac{5}{10} = \frac{1}{2}$$

$$MRTS = \frac{\Delta Y}{\Delta X} = \frac{P_x}{P_y}$$

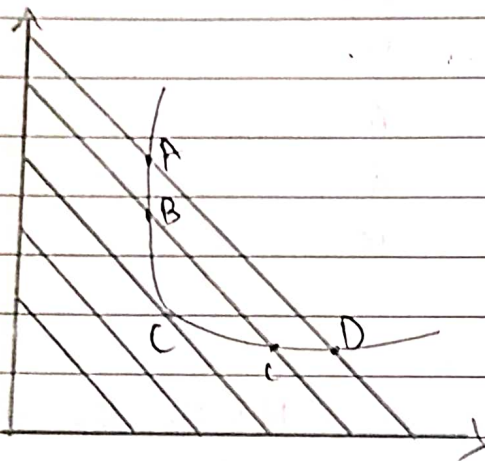
$$MRTS = \frac{P_x}{P_y}$$

* Producer equilibrium



- Ⓐ IQ1, IQ5 → Y
- Ⓑ IQ1 → P & T
- Ⓒ IQ2 → Q & S
- Ⓓ IQ3 → R

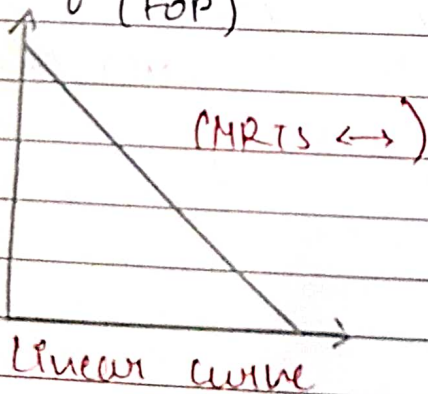
cost → given
output → maximise.



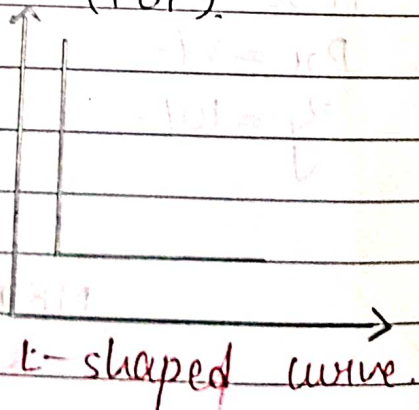
output - given.
cost - minimise.

* Exception to IQ:

(i) perfect substitute (FOP)



(ii) perfect complementary (FOP)



Production

short run

(at least 1 factor of prodⁿ is fixed.)
or

some factor of prodⁿ are fixed & some are variable.

↓
law of return to factors /
law of variable proportions.

long run

(all factors of prodⁿ are variable)

↓
law of returns to scale

→ $TP = \sum MP_n$

→ $MP_n = TP_n - TP_{(n-1)}$
or

$MP_n = \frac{\Delta TP}{\Delta QVF}$

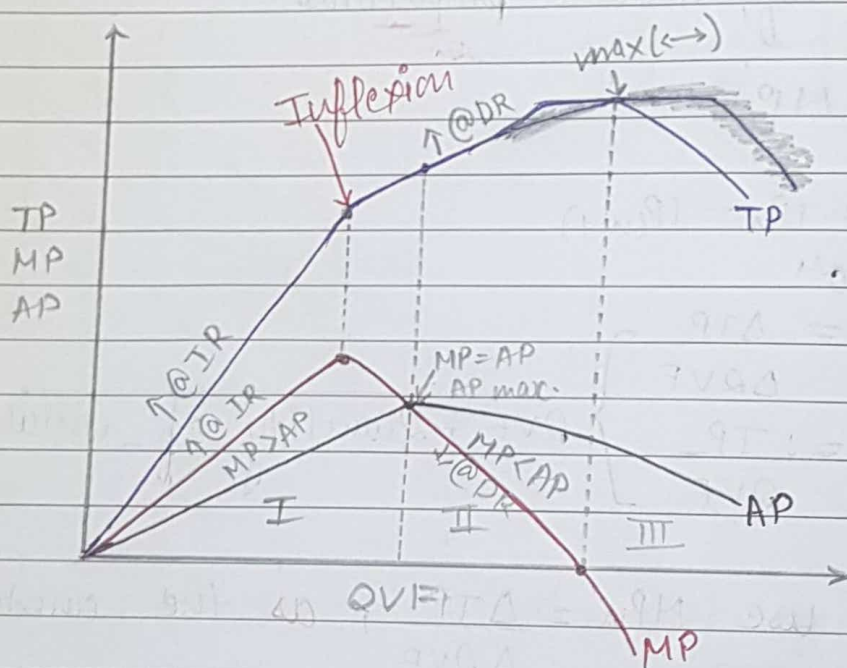
$AP = \frac{TP}{QVF}$

} $QVF = \text{Quantity of variable factor.}$

NOTE:

Always use $MP_n = \frac{\Delta TP}{\Delta QVF}$, as the another formula is applicable only when there is some change in units. i.e. 1, 2, 3, ... etc.

QVF	TP		MP	AP	
0	0		20	20	IMR (MP ↑) (MP > AP) → MP = AP
1	20	@ inc. rate	30	25	
2	50		40	30	
3	90		30	30	DMR (MP ↓) (MP < AP)
4	120	@ dim. rate	15	27	
5	135		10	24	
6	145		02	21	
7	147		01	18.5	NMR (MP = -ve)
8	148	↓ decr.	00	16	
9	148		-02	14.6	
10	146				



	TP	MP	AP
MP > AP	I	↑@IR	↑
MP = AP	II	↑@DR	↓
MP < AP	III	↓	↓

NOTES: Short run:

NOTE:1 In production, short run & long run is defined on the basis of factors of production & not on basis of time period.

NOTE:2 This law is also known as:

- (i) law of returns to factor
- (ii) law of variable proportion
- (iii) law of non-proportional output.
- (iv) law of diminishing marginal returns (DMR)

NOTE:3 The law is further divided into three stages:

- (i) Increasing Marginal returns (IMR) - MP \uparrow
- (ii) Diminishing Marginal returns (DMR) - MP \downarrow
- (iii) Negative Marginal returns (NMR) - MP (-ve)

NOTE:4 This law has been designed & framed keeping in mind the marginal product.

Stages	TP	MP	AP
I	\uparrow @ inc. rate	\uparrow @ inc. rate	\uparrow
II	\uparrow @ dec. rate	\downarrow @ inc. rate	\downarrow
III	\downarrow	-ve	\downarrow

NOTE:6 Assumptions:

- The technology remains unchanged.
- There must be some inputs where quantity is kept fixed.
- Law does not apply where factors are used in fixed proportion.
- Only physical input & output are considered.
- All the units of ^{variable} factors are homogenous.
- Law of variable proportion has three stages.

NOTE:7

When $MP > AP$, MP rises, AP also rises but MP rises at an increasing rate as compared to AP - Therefore MP curve is completely above AP. (MP may rise or fall but MP curve is completely above AP).

NOTE:8

When $MP = AP$, AP does not change, AP remains constant. AP remains the same, & AP is minimum.

when,

$MP = AP$, \Rightarrow AP is maximized.

NOTE:9

When $MP < AP$, MP falls, AP falls but MP falls at a faster rate. Therefore, MP curve is completely below the AP curve.

NOTE:10

MP curve cuts the AP curve from above at the maximum points of AP curve.

NOTE:11

As per classical economist stage-I ends at the max. point of ~~MP/AP~~ curve.

NOTE:12

As per modern economist stage-I ends at the max. point of AP.

NOTE:13

As soon as the producer enters the stage III, its time for the producer to change its technology.

NOTE:14

Stage I & II are called as stages of economic nonsense or economic absurdity.

→ Stage II is called as stage of operation. (∵ TP is max.)

NOTE:15 From the point at which TP starts to rise but at a diminishing rate, corresponding to the maximum point of MP curve is called as point of inflexion.

NOTE:16 Mathematically marginal is the subtraction of total.
 → Marginal is the difference of total.
 → Marginal is the change in total.
 → Marginal is the ~~change~~ slope of total.

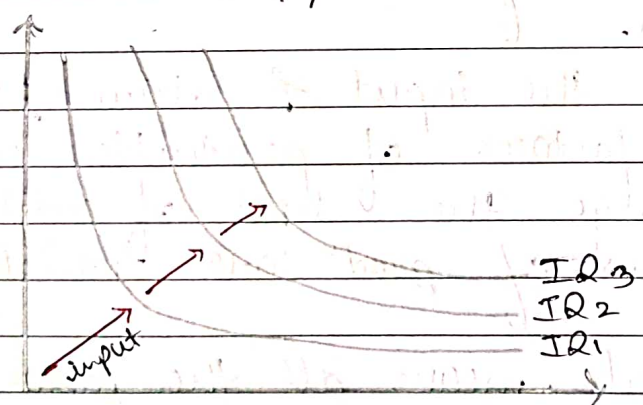
* Long run: Returns to scale (RTS)
 (all the factors of prodⁿ are variable.)

→ $\frac{\Delta P}{P}$ → production / product output.

→ $\frac{\Delta F}{F}$ → factor input.

Case: I

50% ↑ output > 10% ↑ input.

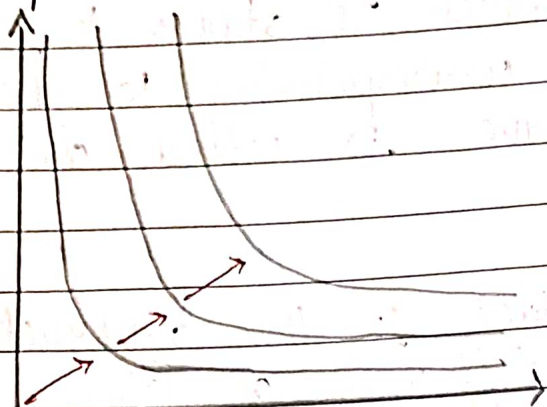


IRTS
 (E > D)

@ increasing returns to scale.

Case: II

10% ↑ output = 10% ↑ input

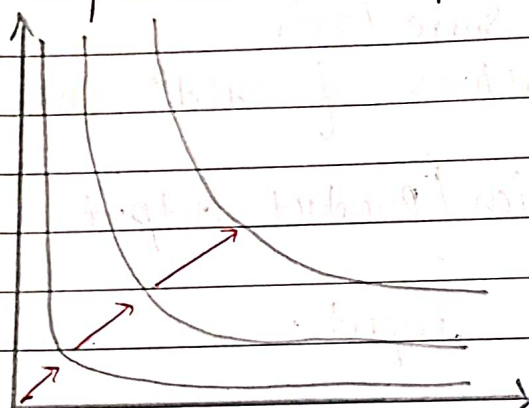


CRTS
($E=D$)

@ constant returns to scale.

Case: III

5% ↑ output < 10% ↑ input



DRTS
($E < D$)

@ decreasing returns to scale.

NOTE: 1

Long run production function is called as law of returns to scale.

This law shows the input & output behaviour when all the factors of production can be changed to change the level of output. (i.e. the all the factors of prodⁿ are variable).

NOTE: 2

A change in scale means all the factors of prodⁿ can be increased or decreased in the same proportion. (i.e. FOP can be changed in already established proportion).

IMP

NOTE:3

The law of returns to scale is greater further divided into

- (i) Increasing returns to scale (IRTS) $E > D$
- (ii) Constant returns to scale (CRTS) $E = D$
- (iii) Decreasing returns to scale (DRTS) $E < D$

NOTE:4

Isoquant represents output whereas the gap b/w isoquant represents input.

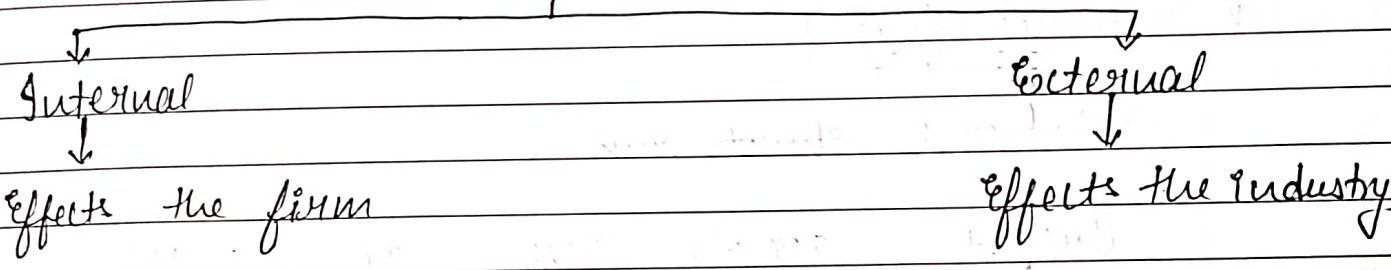
NOTE:5

If input is 10% & output received is 5%, it means that output has fallen down?

Ans → NO, this means the output has increased but at a Diminishing rate.

NOTE:6

Economies & Diseconomies of Scale:
(Advantages & Disadvantages)



internal:

- eg: → managerial,
- financial,
- risk bearing, &
- commercial.

externals:

- eg: → cheaper raw material & capital equipment.
- Development of skilled labour.
- Transportation & marketing facility
- & growth of ancillary industry.

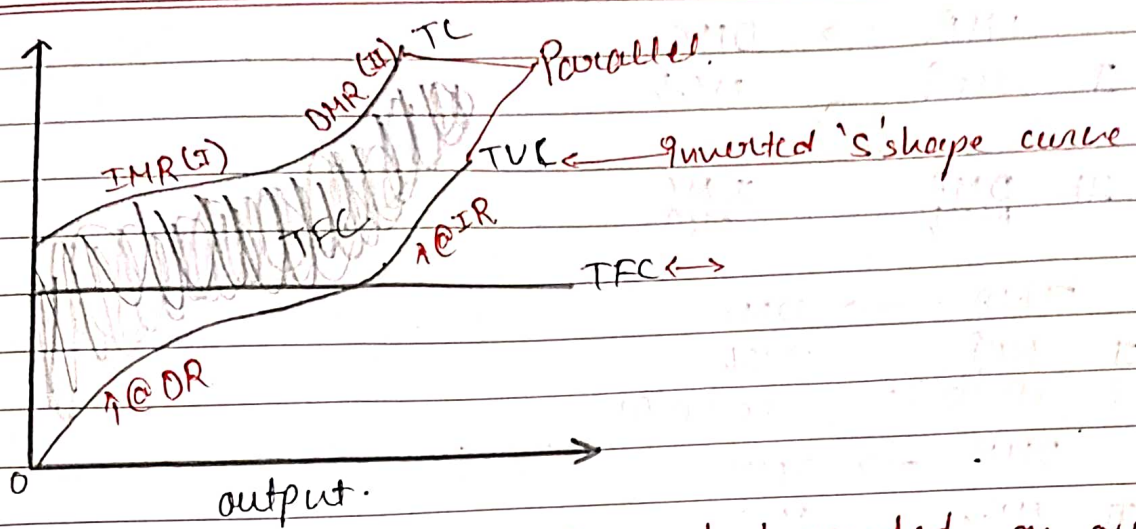
NOTE: 7

	Short run	Long run
POD		
law →	law of returns to factor.	Returns to scale.
FOP →	At least one factor is fixed.	All the factors are variable.
experiencal/planned	Short run is experienced.	Long run is planned.
Stages →	IMR } cost DMR } NMR }	IRTS } economies/diseconomies CRTS } DRTS }
FOP →	$P = f(\underbrace{a, b, c, d}_{\text{variable FOP}}, \underbrace{e, F, \bar{T}}_{\text{fixed FOP}})$	$P = f(\underbrace{a, b, c, d, \dots}_{\text{variable FOP}})$

* Theory of cost:

→ Total cost - short run.

Output	TPC ↔	TVC	TC	MC
0	1000	0	1000	-
1	1000	2000	3000	2000
2	1000	3500	4500	1500
3	1000	4500	5500	1000
4	1000	9000	10000	4500
5	1000	18000	9000	9000



NOTE:1

Output	TFC	→ TFC is not depended on output.
0	≠ 0 ↔	eg: rent, insurance premium,
↑	↔	int. on loan.
↓	↔	

NOTE:2

Output	TVC	→ TVC is depended on output.
0	0	eg: wages, raw materials, etc.
↑	↑	
↓	↓	

NOTE:3

$TC = TFC + TVC$	→ $TC = ATC \times Q$
$TFC = TC - TVC$	→ $TFC = AFC \times Q$
$TVC = TC - TFC$	→ $TVC = AVC \times Q$

$MC_n = TC_n - TC_{n-1}$

$MC_n = \frac{\Delta TC}{\Delta Q}$

$MC_n = \frac{\Delta TVC}{\Delta Q}$

NOTE:4

When output = 0

$TC = TFC$

$TFC \neq 0$

$TC = TFC$

NOTE:5 IMR \rightarrow DMC
 I MP \uparrow MC \downarrow
 (I)
 II DMR
 MP \downarrow MC \uparrow

NOTE:6 IMR \rightarrow DMC
 I MP \uparrow MC \downarrow
 TP \uparrow @JR TC \uparrow @DR
 DMR \rightarrow IMR
 II MP \downarrow MC \uparrow
 TP \downarrow @DR TC \downarrow @JR

NOTE:7 Initially TC & TVC curve rise with but at a diminishing rate. Because, of stage-I of prodⁿ, (IMR)
 \rightarrow After a particular point, TC & TVC continue to rise but at an increasing rate. Because of stage-II of prodⁿ. (DMR).

NOTE:8 \rightarrow The gap b/w TC & TVC is TFC, TFC is constant.
 \therefore TC & TVC curve are parallel to each other.

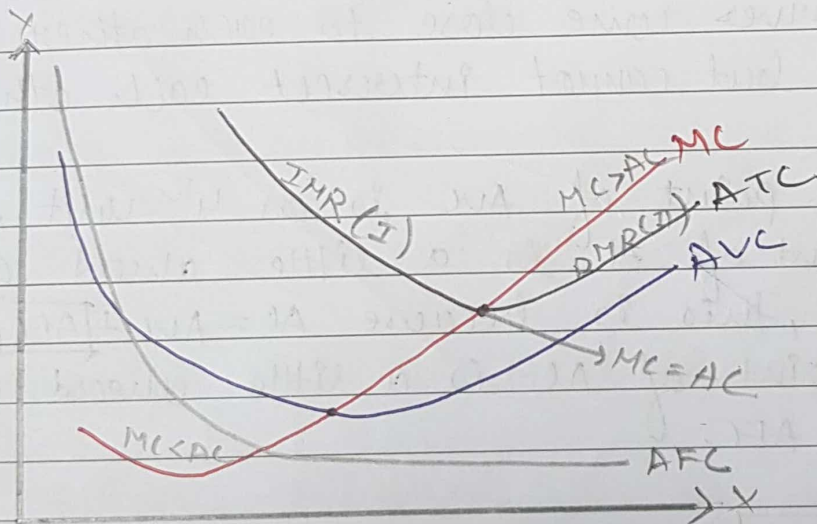
NOTE:9 TC curve takes the shape of TVC curve because, TFC is constant and the rise in TC is purely because of TVC.

NOTE:10 Marginal cost is closely connected with the concept of TVC. Because, TFC is constant.
 And MC is the change in TC, change in TC is due to change in TVC.
 \rightarrow Marginal cost is either the slope of TC or TVC.

* Average cost:

Q	TFC	TVC	TC	MC	AFC	AVC	ATC
0	100	0	100	—	100	0	100
1	100	25	125	25	100	25	125
2	100	40	140	15	50	20	70
3	100	50	150	10	33.33	16.67	50
4	100	60	160	10	25	15	40
5	100	80	180	20	20	16	36
6	100	120	220	40	16.67	2	38.67
7	100	200	300	80	14.29	28.57	42.86

$ATC = AFC + AVC$, $ATC = TC/Q$
 $AFC = ATC - AVC$, $AFC = TFC/Q$
 $AVC = ATC - AFC$, $AVC = TVC/Q$



$MC < AC$
 $MC = AC$
 $MC > AC$

$IMR (I)$ $MP \uparrow$ $MC \downarrow$ $AC \downarrow$
 $DMR (II)$ $MP \downarrow$ $MC \uparrow$ $AC \uparrow$

NOTE: (1) AFC curve is a downward sloping rectangular hyperbolic curve.
MC, AC & AVC curve are U-shaped curve.

(2) AFC curve cannot touch the axis because if AFC touches the axis then $AFC=0$, $TFC=0$ which is not possible.

(3) We know that TFC can never be zero therefore AFC cannot be zero. Therefore AFC cannot touch the axis.

(3) The gap b/w AC & AVC is AFC, AFC is diminishing. Therefore, the gap b/w AC & AVC also diminishes.

(4) AC & AVC curves cannot intersect each other since $AFC \neq 0$.

(5) AC & AVC curves come close to each other far approach each other but cannot intersect each other.

(6) The minimum point of AVC is on 4th unit, the minimum point of AC is a little ahead of AVC curve on 5th unit, this is because $AC = AVC + AFC$. The minimum point of AC is a little ahead of AVC because of AFC.

(7) When $MC < AC$; MC falls, AC also falls but MC falls at a faster rate. Therefore MC is completely below AC (when $MC < AC$, MC may fall, rise or constant).

- (8) When $AC = MC$ / AC is constant
 AC remains the same.
 AC does not same
 AC is minimum.
 When $MC = AC$, then AC is minimised, this point is called as **productive efficiency**.
- (9) When $MC > AC$ / MC rises, AC also rises but MC rises at a faster rate therefore MC is completely above AC.

(10) MC curve cuts the AC curve from below at the minimum point of AC

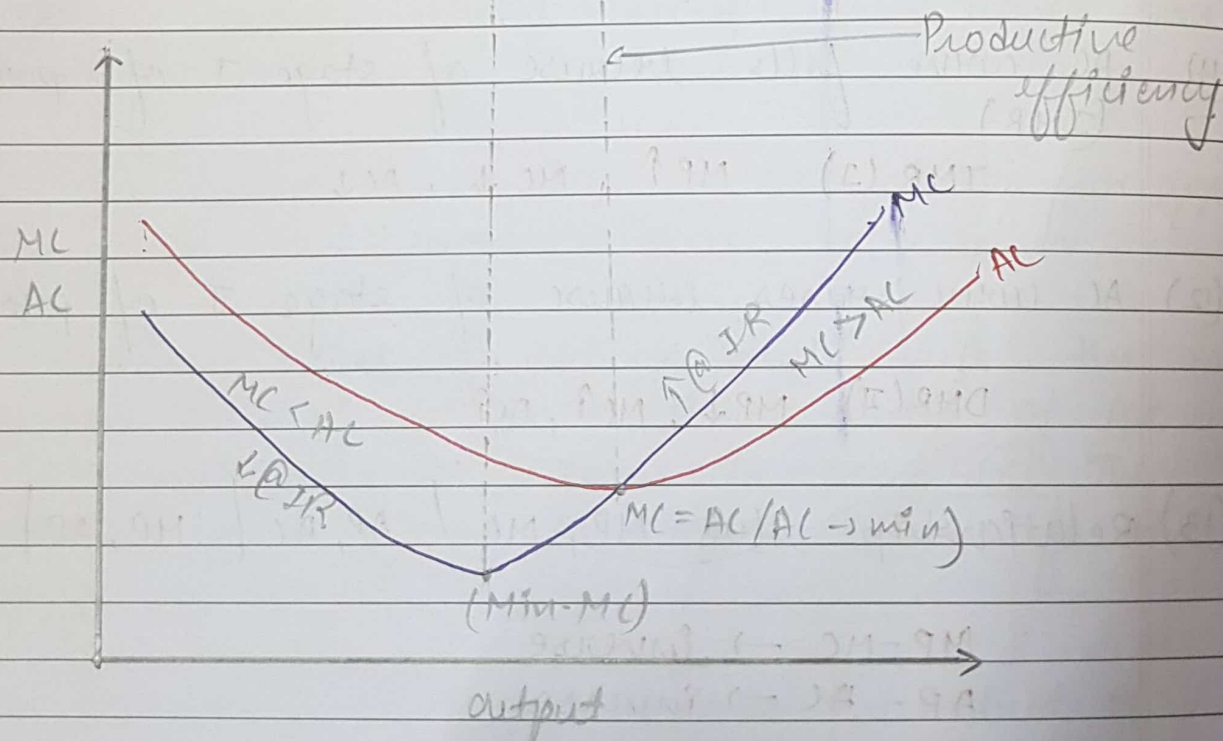
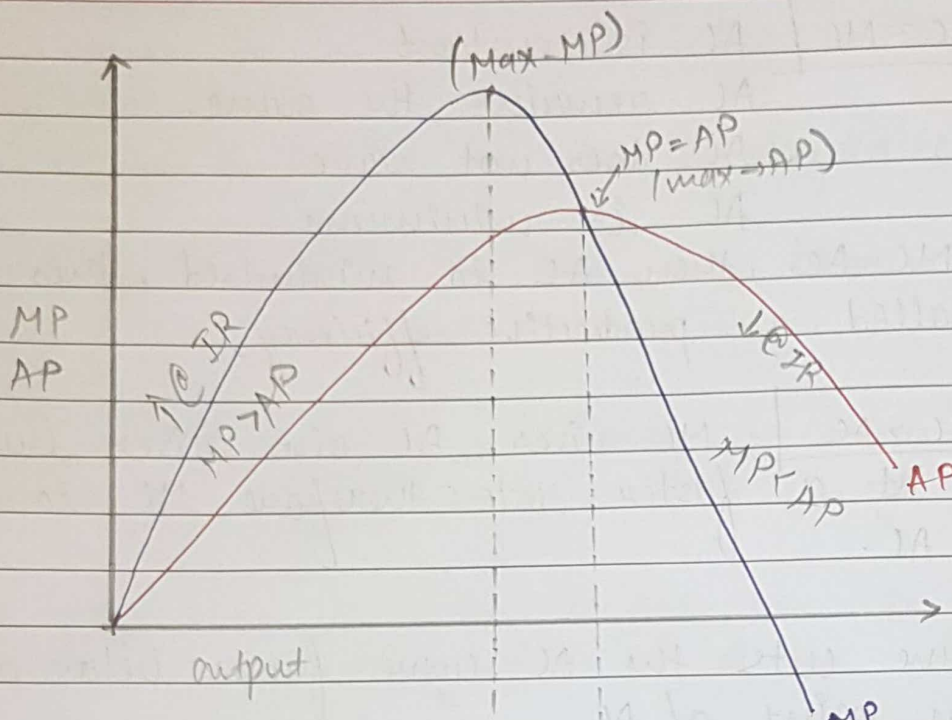
(11) AC curve falls because of stage I of production (IMR)
IMR (I) $MP \uparrow, MC \downarrow, AC \downarrow$

(12) AC curve rises because of stage II of production (DMR)
DMR (II) $MP \downarrow, MC \uparrow, AC \uparrow$

(13) Relationship btw MP, MC / AP, AC / MP, AP / MC, AC

- $MP - MC \rightarrow$ inverse
- $AP - AC \rightarrow$ inverse
- $MP - AP \rightarrow$ direct
- $MC - AC \rightarrow$ direct.

Diagram



long run.

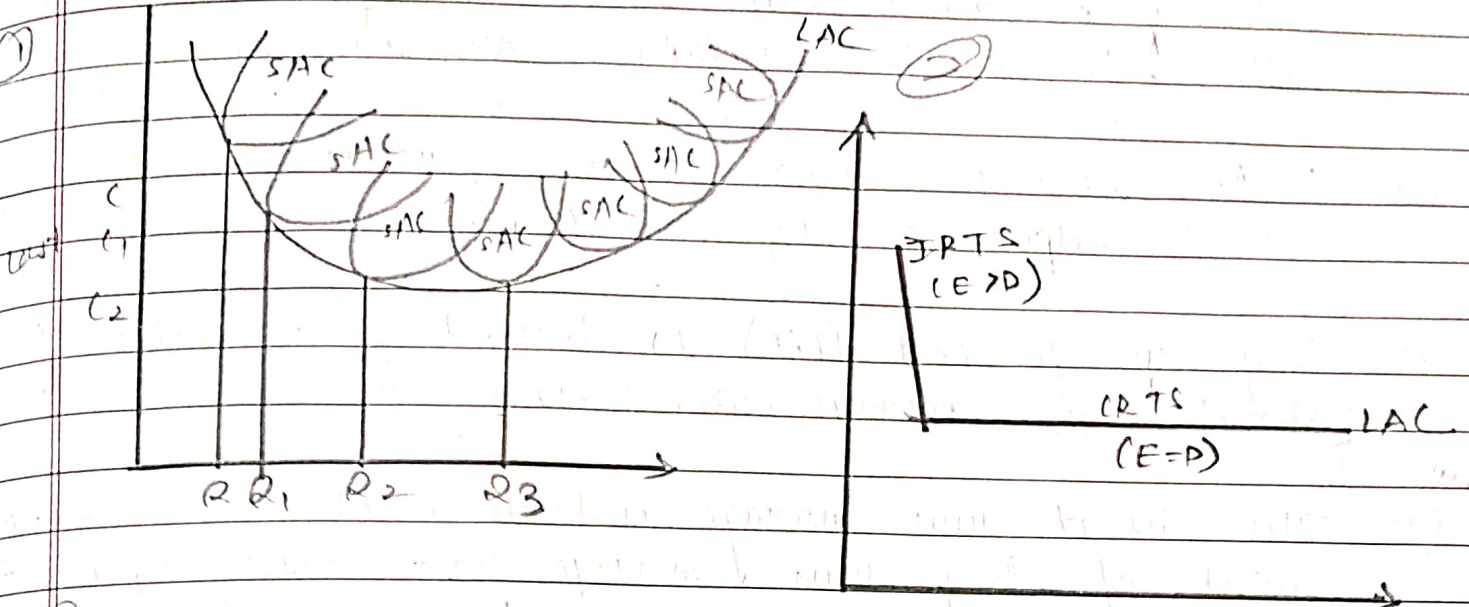
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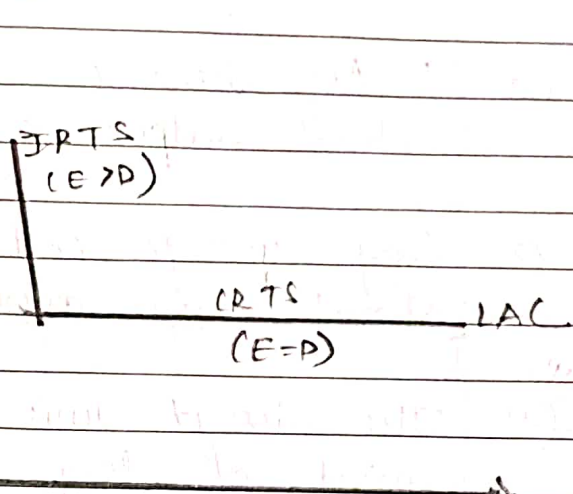
* Long run Cost

Average cost (4 graphs)

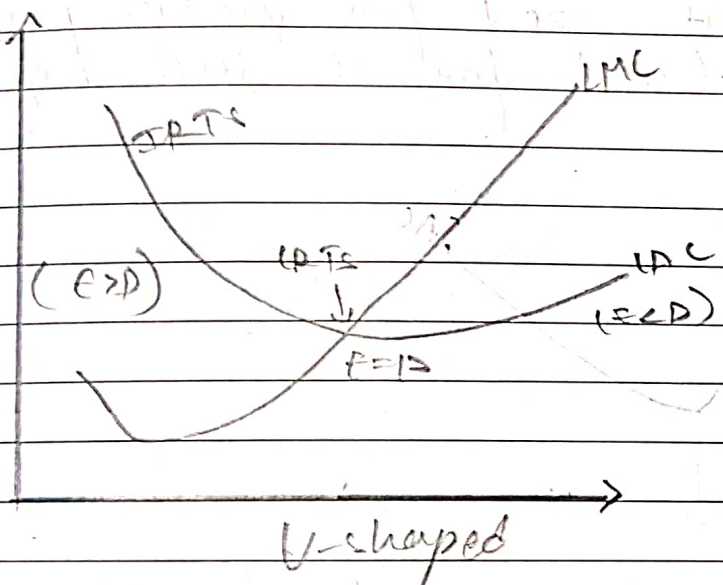
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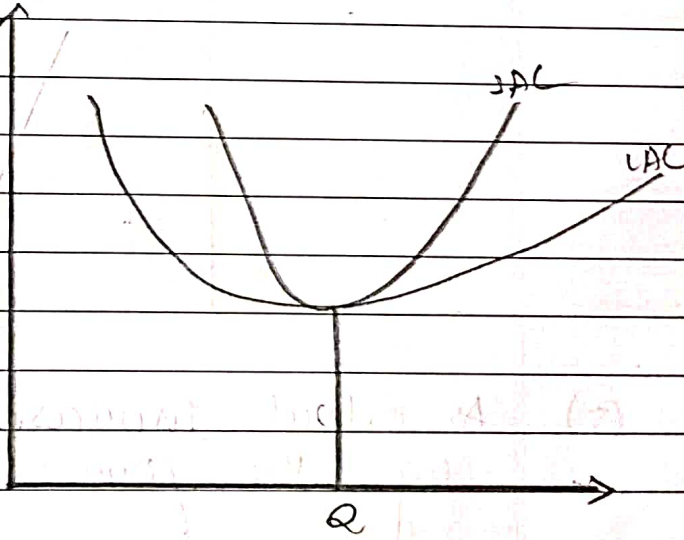
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* Long run cost:

- (1) Long run cost of production is the least possible cost of producing any given level of output when all the individual factors are variable.
- (2) A long run cost curve shows functional relationship b/w output & long run cost.
- (3) Long average cost (LAC) is derived from series of short / small average curve (SAC).

The

- (4) The short run average cost curve at the minimum point of long run average cost curve represent least cost plant size for producing particular level of output & not for all level of output.



- (5) As output increases the amount of capital employed by the firm also increases.
- (6) LAC curve is a tangent to SAC curves.

- (7) LAC curve is also called as planning curve, enveloped-shaped curve, or U shaped curve, whereas SAC curve is called as plant (production) curve.
- (8) The negative slope of LAC (sloping or downward part) is due to IRTS i.e. $E > D$ (economies of scale)
- (9) The positive slope of LAC (rising or upward part) is due to DRTS i.e. $E < D$ (diseconomies of scale)
- (10) ^{historical} Empirical evidence shows that state of technology changes in the long run.
 \therefore According to modern economist LAC curve is a L-shaped curve.

