CH 1

Ratio Basics

Definition

A ratio is a comparison of the size of two or more quantities of the same kind by division

Terms

=> a and b.

-> a => 1^{st} term / antecedent

-> b => 2^{nd} term / consequent

Simplest form of ratio

* Both terms of ratio can be multiplied or divided by the same (non-zero) number
* Usually, a ratio is expressed in lowest terms.

Order of terms

The order of the terms in a ratio is important.

Quantities of same kind

-> Ratio exists only between quanties of same kind.

Quantities of same unit

Quantities to be compared must be in the same units.

Equivalent like fractions

To compare two ratios, convert them into equivalent like fractions i.e. ratios with same denominator.

Increase or decrease of quantity by ratios

→ If a quantity increases or decreases in the ratio a : b, then new quantity is $\frac{b}{a}$ times of original quantity

→ The fraction by which the original quantity is multiplied (i.e. $\frac{b}{a}$) to get a new quantity is called the factor multiplying ratio.

Properties of Ratio

Inverse ratio

* One ratio is the inverse of another if their product is 1

* Thus b : a is the inverse of a : b and vice-versa

Compounding

* The ratio compounded of two ratios $a:b$ and $c:d$ is $ac:bd$
* compounding two or more ratios means multiplying them.

Duplicate ratio :- a ratio compounded itself.

* $a^2:b^2$ is the duplicate ratio of $a:b$

Sub-duplicate ratio :- $\sqrt{a}:\sqrt{b}$ is the sub-duplicate ratio of $a:b$

Triplicate ratio :- $a^3:b^3$ is the triplicate ratio of $a:b$

Sub-triplicate ratio :- $\sqrt[3]{a}:\sqrt[3]{b}$ is the sub-triplicate ratio of $a:b$.

Commensurable

If the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are said to be commensurable.

Continued ratio

* It is the relation or comparison between the magnitudes of three or more quantities of same kind.

* eg: $a : b : c$.

Proportion Basics

Definition

* An equality of two ratios is called a proportion.

* Four quantities a, b, c, d are said to be in proportion if $a : b = c : d$ or $a : b :: c : d$

Terms

* The quantities a, b, c, d are called terms of the proportional

* first and fourth terms are called extremes

* second and third ~~or~~ terms are called means

Cross product rule

* If $a : b = c : d$ are in proportion then
  $ad = bc$

* Product of extremes = Product of means.

Continued Proportion

Three quantities a, b, c of the same kind are said to be in continuous proportion if $a:b = b:c$

$$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$$

a => first proportional

c => third proportional

b => mean proportional (because b is GM of a & c).

* If $a:b = c:d$, then;

Properties of Proportion	
Invertendo	$b:a = d:c$
Alternendo	$a:c = b:d$
Componendo	$a+b : b = c+d : d$
Dividendo	$a-b : b = c-d : d$

Componendo and dividendo	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ OR $\frac{a-b}{a+b} = \frac{c-d}{c+d}$
Addendo (if $a:b=c:d=e:f=\ldots k$)	$\frac{a+c+e+\ldots}{b+d+f+\ldots} = k$
Subtrahendo (if $a:b=c:d=e:f\ldots k$)	$\frac{a-c-e-\ldots}{b-d-f-\ldots} = k$

Indices Basics

Base

Number which is raised to some power is called Base

Power

Number of times base is multiplied by itself.

Index

Entire number including base and power is index

Example

Power

Index $\leftarrow$ $3^4 = 81$

Base

Result

Standard results

* Any base raised to the power zero is defined to be 1

$$a^0 = 1$$

* Roots can also be expressed in the form of power

$$\sqrt[r]{a} = a^{1/r}$$

Power shifting Punch

If $6^3 = x \implies 6 = x^{1/3}$

If $5^{3/2} = y \implies 5 = y^{2/3}$

Law of indices

Law 1	If two or more terms with same base are multiplied, we can make them one term having the same base and powers as sum of all powers $a^m \times a^n = a^{m+n}$

Law 2.	If two or more term with same base are in division, we can make them one term having the same base and power as difference of power $$\frac{a^m}{a^m} = a^{m-n}$$
Law 3	If a term having power is raised to another power, we can ~~split the two term with same~~ do product of powers to simplify the expression $$(a^m)^n = a^{m \times n}$$
Law 4	If a product of two or more terms is raised to power, we can split the two terms with same individual power to each of one of them. $$(a \times b)^n = a^n \times b^n$$

Logarithm Basics

Meaning

The logrithm of a number to a given base is the index or the power to which the base must be raised to produce the number

Mathematical explanation of log

If $a^x = n$ then $\log_a n = x$

Condition under logarithm function

* Log can be calculated only for positive number.

* Base should be positive and not equal to 1

$$n > 0, \quad a > 0, \quad a \neq 1$$

Standard results

* Log of a number with same base as number is equal to 1

$$\log_a a = 1$$

* Log of 1 (one) for any base is equal 0

$$\log_a 1 = 0$$

Laws of logarithm

Law 1	$\log_a mn = \log_a m + \log_a n$
Law 2	$\log_a \frac{m}{n} = \log_a m - \log_a n$

Law 3	$\log_a m^n = n \log_a m$
Change of Bare theorem	$\log_b m = \dfrac{\log m}{\log b} = \dfrac{\log_a m}{\log_a b}$
Special relation	$\log_b a \times \log_a b = 1$