MM-14-7

Chap-05

Permutation & Combination

- **Principal of Counting:**

1. Multiplication Rule → When we have to do 2 work together so do the multiplication of the ways by which we can do both work.

when '&', 'and' is used.

2. Additional Rule: When we have to do any one work from 2.

when 'or' is used.

The factorial: If we take a number 'n', then n! or Ln is called as factorial of 'n' and the value of n! is equal to multiplication of 1 to n

e.g. → $8! \rightarrow 8\times7\times6\times5\times4\times3\times2\times1 \rightarrow 40320$

$\rightarrow n! = n(n-1)\times n(n-2)\times n(n-3)\times \text{-------}$

Remmember $0! \rightarrow 1$ and $1! \rightarrow 1$

e.g. → $10!/4! = 10\times9\times8\times7\times6\times5 = 151200$
$9!/5! = 9\times8\times7\times6 = 3024$

Permutation = Selection & Arrangement
Combination = Only Selection

Permutation:

determines the no. of possible arrangement in set when the order of arrangement matters

$\boxed{{}^nP_r}$

total n hai usme se r ko arrange krna h

$$\frac{n!}{(n-r)!}$$

and if we take n thing at time

$$\frac{n!}{(n-n)!} \Rightarrow \frac{n!}{0}$$

Condition of ${}^nP_r$ is $n \geq r$

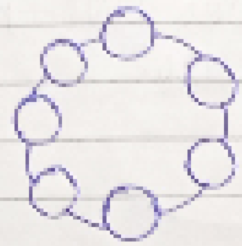
1!	1
2!	2
3!	6
4!	24
5!	120
6!	720
7!	5040
8!	40320
9!	362880
10!	3628800

FACTORIAL

If you get 0 on the left most side so it will not be considered.

Circular Permutation:

Means Circular arrangement



Condition → 1st person can sit only in one way

- If n person are arranged at n circular places → $(n-1)!$
- In a circular permutation, arrangement is such that clockwise and anticlockwise result is same.

$$\frac{1}{2}(n-1)!$$

Permutation with restriction:

Theorem 1 → Number of permutation of n distinct objects taken r at a time when a particular object is not taken in any arrangement

$${}^{n-1}P_r$$

Theorem 2 → Number of permutation of r object out of n distinct object when a particular object is always included in any arrangement

$${}^{n-1}P_{r-1}$$

★ $\sum_{r=1}^{n} r \cdot {}^{r}P_r = (n+1)! - 1$

Combination:

The no. of ways in which selection is done where order does not matter can be calculated as:

$${}^{n}C_r = \frac{n!}{(n-r)!\, r!}$$

$${}^{n}C_n = \frac{n!}{n!(n-n)!} = \frac{1}{0!} = \frac{1}{1} = 1$$

$${}^{n}C_0 = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \times n!} = \frac{1}{1} = 1$$

note that $0 \le r \le n$, then only ${}^{n}C_r$ exist and will always be greater than 0

IMP rule ⇒ ${}^{n}C_r = {}^{n}C_{n-r}$

${}^{n}C_r$ → $\frac{n \text{ se ulti ginti } (r \text{ tak})}{r \text{ se ulti ginti } 1 \text{ tak}}$

Card's concept

♡ → 13	Red 26	
◇ → 13		
♣ → 13	Black 26	
♠ → 13		
52		

For finding $r!$ = $r! = \frac{{}^{n}P_r}{{}^{n}C_r}$

Some imp result to taken care:

1. $^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$

eg→ $^{8}C_4 + {}^{8}C_3 = {}^{8+1}C_4 = {}^{9}C_4$

2. $^{n}P_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$

eg→ $^{8}P_4 + 4\,{}^{8}P_3 = {}^{9}P_4$

Permutation when some of the things are alike, taken all at a time:

$$\boxed{n = \frac{n!}{n_1!\,n_2!\,n_3!}}$$

e.g.→ 9 balls are there. 2 are of green and 3 are of yellow and 4 of red. arrange them

→ $\frac{9!}{2!\,3!\,4!}$

Permutation when each thing may be repeated once, twice... upto times in any arrangement:

n things m se r ko arrange krna hai and repitation is allowed

no. of arrangement → $\boxed{n^r}$

e.g→ $n = 6$ → 1, 2, 3, 4, 5, 6

$r = 3$ _ _ _ → $6 \times 6 \times 6 = 6^3 = 216$

Permutation when each thing may be repeated once, twice ... upto times in any arrangement

$$\boxed{{}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + {}^{n}C_4 - - - - {}^{n}C_n = 2^n - 1}$$

Permutation when each things may be repeated once, twice ... upto times in any arrangement.

No. of Selection:

n

n_1 are alike | n_2 are alike | n_3 are alike

(n_1+1) (n_2+1) (n_3+1) -1 → when n other is selected

- If we have to select the combination such that r_1 things to selected from n_1 and r_2 thing to selected from n_2

$${}^{n_1}C_{r_1} \;\&\; {}^{n_2}C_{r_2}$$

e.g→ 9 thing to be divided in 3 groups contain 2, 3, 4 thing.

9 | 7 | 4

2 | 3 | 4

$${}^{9}C_2 \times {}^{7}C_3 \times {}^{4}C_4$$