Business Mathematics

Ratio, Proportion, Indices and Logarithms

- **Ratio:**

  A Ratio is the Simplest form of two or numbers or we can say that it is a comparison of the sizes of the two or more quantities of the same kind by division.

- **Point of Remember:**
  - If a and b are Two quantities of the same kind, then the fraction a/b is called the ratio of a to b and it is written as a:b. The quantities a and b are called terms of the ratio a is called the first term or antecedent and b is called the second term or consequent.

    For **Example** – 3:2

    3 is called the antecedent. 2 is called the consequent.
  - Both the terms of ratio can be multiplied and divided by the same number (non-zero).
  - Terms of the ratio compared must be in the same unit.
  - Duplicate Ratio of $\frac{a}{b}$ is $\frac{a^2}{b^2}$
  - Triplicate Ratio of $\frac{a}{b}$ is $\frac{a^3}{b^3}$
  - Sub-duplicate Ratio of $\frac{a}{b}$ is $\sqrt{\frac{a}{b}}$
  - Sub-triplicate Ratio of $\frac{a}{b}$ is $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$
  - Inverse Ratio of $\frac{a}{b}$ is $\frac{b}{a}$
  - or Inverse Ratio of $\frac{b}{a}$ is $\frac{a}{b}$
  - Compounded Ratio → It means Multiplication of the ratios.

    For, **Example** Compounded Ratio of a: b and c: d

    $= \frac{a}{b} \times \frac{c}{d} = ac : bd$
  - Continued ratio→ It is the relation (or comparison) between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as a: b: c.
  - A ratio a : b is said to be of greater inequality if a>b and of less inequality if a<b.

- **Proportion:**
  - **Proportion:** It means equality of two Ratios.
  - If a : b = c : d. It can also be written as a : b :: c : d

    $\frac{a}{b} \times \frac{c}{d}$
  - In a : b :: c : d

    a & d are called Extremes

and b & c are called Means

$\frac{a}{b} \times \frac{c}{d} \Rightarrow ad = bc$

- Product of extremes = Product of means

a	b	c	d
↓	↓	↓	↓
First Proportion	Second Proportion	Third Proportion	Fourth Proportion

- If there are three quantities a, b, c of the same kind are said to be in continuous proportion

  $a : b :: b : c$ i.e. $\frac{a}{b} = \frac{b}{c}$ i.e. $b^2 = ac$

- **Componendo:**

  $\frac{a}{b} = \frac{c}{d} \Rightarrow$ by using component $\frac{a+b}{b} = \frac{c+d}{d}$

- **Dividendo:**

  $\frac{a}{b} = \frac{c}{d}$ = by using dividendo $\frac{a-b}{b} = \frac{c-d}{d}$

- **Componendo and Dividendo:**

  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

- **Addendo:**

  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

  Then, $\frac{a+c+e}{b+d+f} = k$

- **Subtrahendo:**

  If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

  Then, $\frac{a-c-e}{b-d-f} = k$

- **Alternendo:**

  If a: b = c: d then a:c = b: d

  $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$

- **Invertendo:**

  If $\frac{a}{b} = \frac{c}{d}$ *then* $\frac{b}{a} = \frac{d}{c}$

Indices:

Point of Remember:

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$
- $a^3 - b^3 = (a - b)\,(a^2 + b^2 + ab)$
- $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

- $a^2 - b^2 = (a + b)(a - b)$
- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $a^0 = 1$
- $a^m = k \Rightarrow = a^{1/m}$
- $(a^m)^n = a^{mn}$
- $a^{-m} = \frac{1}{a^m}$
- $\frac{1}{a^{-m}} = a^m$
- If $a^x = a^y$ then $x = y$
- $x^a = y^a$ then $x = y$
- $\sqrt[m]{a} = (a)^{\frac{1}{m}}$

Logarithm:

Point of Remember:

- $log a + log b = log ab$
- $\log\left(\frac{A}{B}\right) = \log A - \log B$
- $\log_a b^n = n \log_a b$
- $\log_{a^m} b = \frac{1}{m} \log_a b$
- $\log_{a^m} b^n = \frac{n}{m} \log_a b$
- $a^{\log b} = b^{\log a}$
- $\log_a a = 1$
- $\log_a b = \frac{\log_e b}{\log_e a}$
- $\log 1 = 0$
- $\log 10 = 1$

  When $a^x = b$ then $\log_{ab} = x$

  $2^3 = 8$
- $\log_2 8 = 3$
- $a^{\log n} = n$
- $\log_b a \times \log_c b \Rightarrow \log_c a$

  Apply base change formula

  $$\frac{\log a}{\log b} \times \frac{\log b}{\log c} = \frac{\log a}{\log c} = \log_c a$$

Equation

- Equation is defined to be a mathematical statement of equality.
- A Simple equation is one unknown is in the form of $ax + b = 0$

Where a, b are known constants.

A Simple equation has only one root.

- **Elimination method:** In this method, two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknown and then solving for the other unknown.
- **Cross Multiplication method:** let two equations be: -

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

$$x = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1} \qquad y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1}$$

- An equation of the form where $ax^2 + bx + c = 0$ is a variable, and a, b, c are constants with a ≠ 0 is called a quadratic equation or equation of the second degree.
- When b = 0, the equation is called a pure quadratic equation; when b ≠ 0, the equation is called an affected quadratic.
- The roots of a quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The Sum and Product of the roots of the quadratic equation
- Sum of roots = $-\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$
- Product of roots = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$
- To construct a quadratic equation for the equation we have

x^2 −(Sum of roots) x + Product of roots = 0.

- **Conditions for solvability of pair of liner equations:**

S. No.	Pairs of lines	Condition	Graphical representation	Algebric interpretation
1.	$a_1 x + b_1 y + c_1 = 0$ $a_2 x + b_2 y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique Solution (Consistent)
2.	$a_1 x + b_1 y + c_1 = 0$ $a_2 x + b_2 y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many Solution (Consistent)
3.	$a_1 x + b_1 y + c_1 = 0$ $a_2 x + b_2 y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No Solution (Inconsistent)

- **Nature of the roots:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  - If $b^2 - 4ac = 0$ the roots are real and equal.
  - If $b^2 - 4ac > 0$ then the roots are real and unequal (or distinct).
  - If $b^2 - 4ac < 0$ then the roots are imaginary.
  - If $b^2 - 4ac$ is Perfect Square (≠0); the roots are real, rational and unequal (distinct).
  - If $b^2 - 4ac > 0$ but not a perfect square. The roots are real Irrational and Unequal.

- Since $b^2 - 4ac$ discriminates the roots $b^2 - 4ac$ is called discriminant in the equation.
- $ax^2 + bx + c = 0$ as it actually discriminates between the roots.

Special points

- Irrational roots occur in pairs means if one root is $a - \sqrt{b}$ another root will be $a + \sqrt{b}$
- If one root is the reciprocal of another root, then the product of roots will be 1.
- If two roots of the equation are equal but with opposite signs, then the sum of roots will be zero.

Linear Inequality

- Two real numbers or algebraic expressions related by the symbols <, >, ≤ or ≥ form an inequality.
- Equal number added to (subtracted from) both side of the inequality without affecting the sign of inequality.
- Both sides of an inequality can be multiplied or divided by the same positive number (non-zero). But when both sides multiplied or divided by the negative number, then the sign of inequality is reversed.

Time Value of Money

- Simple Interest = $\frac{P \times R \times T}{100}$

  P = Principal, **R** = Rate of Interest, **T** = Time
- Amount = Principal + Interest

  If the Simple Interest, Then

  $A = Principal + \frac{P \times R \times T}{100}$ or $A = P + PiT$

  A = P(1 + *i*T)

  A= Amount, R= Rate of interest, $i = \frac{R}{100}$
- If 'r' is the Simple rate of Interest, then Amount becomes double of itself in $\frac{100}{r}$ years.
- If 'r' is the Simple rate of Interest, then Amount becomes triple of itself in $\frac{200}{r}$ years.
- When the Interest is compounded. The present value P of the amount, A, due at the end of n period at the rate of I per interest period may be obtained by:

  Formula to find the amount:

  A = P(1+ *i*)^n

  A = Amount

  P = Principal

  $i = \frac{Rate\,of\,Interest}{100}$

  n = Total no.of conversions.
- **When Compound Interest is to be find:**

  C.I. = P [(1+*i*)^n−1]

  C.I. = Compound Interest =, **P** = Principal, **n** = Total number of conversions

  $i = \frac{Rate\,of\,Interest}{100}$

- If 'r' is the Compound rate of Interest, then Amount becomes double itself in $\frac{72}{r}$ years.
- If 'r' is the Compound rate of Interest, then it will become triple of itself in $\frac{114}{r}$ years.
- If the difference between S.I. and C.I. is given for 2 years, in such a case P will be

  $$P = \frac{d \times (100)^2}{r^2}$$

  d = difference between Simple Interest and Compound Interest.

  r = rate of interest
- If the difference between Simple Interest and Compound Interest is given for 3 years, in such a case, 'P' will be

  $$P = \frac{d \times (100)^3}{r^2 (r + 300)}$$
- If a sum of money deposited in a bank becomes in , years, & A_2 in years, then amount deposit initially.

  $$= \frac{A_2 t_1 - t_2 A_1}{t_1 - t_2}$$ ← **Simple Interest**
- If the sum of money becomes 'n' times in 't' years. Then it will become 'M' times in $\left(\frac{m-1}{n-1}\right) \times t$ years from simple interest.
- **Effective Rate of interest:** To compute Rate of Interest Compounded Annually;

  $E = (1+i)^n - 1$

  $i = \frac{Rate\ of\ Interest}{100}$, n = *total number of conversions*

  E = *Effective Rate of Interest*
- **Annuity:** Regular Payment at Regular Interval

Annuity

Ordinary Annuity/ Regular annuity	**Annuity Due / Annuity Immediate**
First Payment is made at the end of the period.	First Payment at the beginning of period.
Example – *Rent Paid, Repayment of loan.*	Example – *Recurring deposit, Insurance Premium.*

	Future Value	Present Value
Ordinary Annuity/ Regular Annuity	$R\left[\frac{(1+i)^n - 1}{i}\right]$	$R\left[\frac{1-(1+i)^{-n}}{i}\right]$
Annuity Due / Annuity Immediate	$R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$	$R\left[\frac{1-(1+i)^{-(n-1)}}{i}\right]+R$

R = Annuity, $i = \frac{Rate\ of\ Interest}{100}$

n = *Total number of conversions*

- **Sinking Fund:** It is a fund created for a specified purpose by way of periodic payment over a time period at a special Rate of Interest. Interest is compounded at the end of every period. The amount of sinking fund deposit is computed from

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Amount of Sinking Fund = Future value.

- **Investment Decision:**

Benefits from Asset > Cost of Asset → Buy the Asset

Benefit from Assets < Cost of Asset → Take it on lease

- **Benefits form Asset:**

Benefits Form Asset

- **Reduction in Cost of Production**
- **Increase in Revenue (Sales)**

Step I → Present Value of Benefits from Asset

$$P.V = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

R = Annuity, $i = \frac{Rate\ of\ Interest}{100}$

n = *total number of conversions*

Step II→ Cost of Asset → Given in the question

Step III → Step I > Step II (Buy it)

Step I < Step II (Ignore the Proposal)

Step I = Step II (Indifferent)

- **Bond:** A bond is a debt security in which the Issuer owes the holder debt and is obliged to Repay the Principal and interest. Bonds are generally issued for a fixed term, generally longer than one year.

- **Valuation of Bond:**

Step 1 → Find Interest Receivable

= Face Value × Coupon Rate

Step 2→ Find Present value of Interest receivable

$$= R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

R = Interest Receivables, $i = \frac{Required\ Rate\ of\ Return}{100}$

Step 3 → Find Present value of Redemption Value

$$P.V. = \frac{A}{(1+i)^n}$$

P.V. = Present Value

A = Future Value

Step 4→ Step 2 + Step 3

In step 4 value of Bond is find

- **Perpetuity:** It is an annuity in which the Periodic payments or Receipts begins on a fixed date and continue Indefinitely or perpetually.

- **Present Value** = $\frac{R}{i}$

R → Annuity Per Month/Half-yearly/Yearly

$i \rightarrow$ Interest Per Month/Half-yearly/Yearly

- **Growing Perpetuity:** A Stream of cash flows that grow at a constant rate forever is known as growing perpetuity.

  Present Value Annuity = $\frac{R}{i-g}$

  R → Annuity Per Month/Half-yearly/Yearly

  $g \rightarrow$ Growing rate Per Month/Half-yearly/Yearly

  i and g should be in the same quantities.

- **P.V Factor** = $\frac{1}{(1+i)^n}$

- **Net present Value:**
  - **Net present Value:** Present Value of Cash Inflow – Present Value of Cash Outflow.
  - **Steps to Calculating Net Present Value are;**
    - Determine the Net Cash Inflow in each year of the Investment.
    - Select rate of return or discounted rate, or weighted average cost of capital.
    - Find the discount factor for each year based on the desired rate of return selected.
    - Determine the present values of net Cash flows by multiplying the cash flows by respective discount factors of the Respective period called Present Value of Cash flows.
    - Total the Amount of all P.V's. of Cash flows.
    - **Decision Rule:**
      - **N.P.V.** > 0[Accept the proposal]
      - **N.P.V.** < 0 [Reject the proposal]

- **Nominal Rate of Interest** ⇒ Nominal Rate of Interest = Real Interest Rate + Inflation.
  - **CAGR:** Compound Annuity Growth Rate is an important factor is business valuation, and particularly used in growth Industries to compare the growth rates of two investments. CAGR is often used to describe the growth over a period of time of some elements of the business for Example *(Revenue, units delivered, Registered users etc.)*
  - **CAGR** = × $\left[\left[\frac{V(t_n)}{V(t_0)}\right]^{\frac{1}{t_n-t_0}} - 1\right] 100$
  - **V (t_n)** = Value at the End period.
  - **V (t_0)** = Value at the Beginning period.

- **Present Value of Deferred Annuity:**
  - First find the present value of an ordinary consisting of payment of ₹R each made at the end of each of m payment intervals, together with payments of ₹R each made at the end of each of n additional payment intervals. The present value of this annuity is

    $$P_1 = R\left[\frac{1-(1+i)^{-m-n}}{i}\right]$$

    Next, find the present value of ordinary annuity consisting of payments of ₹R each made at the end of each of m payment intervals. The present value of this annuity is:

    $$P_2 = R\left[\frac{1-(1+i)^{-m}}{i}\right]$$

- Hence the present value of the deferred annuity is:

$$P = P_1 - P_2 = R\left[\frac{1-(1+i)^{-n}}{i}\right] - R\left[\frac{1-(1+i)^{-m}}{i}\right]$$

Permutation and Combination

- **Combinations:** The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or Arrangement is not Important, are called combination.

  Number of combinations of n different things taken r at a time.

  Denoted by ^nC_r or C (n, r)

  n = Total Items

  r = Items to be Selected

  $$^nC_r = \frac{n!}{(n-r)!r!}$$

- **Permutation:** The ways of arranging or Selecting Smaller or equal number of persons or collection of objects with due regard being paid to the order of arrangement or selection, are called permutation.

  Number of Permutation when r objects are chosen out of n different objects. Denoted by ^nP_r or (n, r)

  n = Total Items

  r = Items to be Selected and arranged

  $$^nP_r = \frac{n!}{(n-r)!}$$

- **Factorial:**

  The factorial n, written as Or ⌊n, represents the Product of all Integers from 1 to n both inclusive.

  To make the notations meaningful. When n = 0, the desire 0! The define 0! or ⌊0 = 1

  Thus, $n! = n\,(n-1)(n-2)$..............3.2.1

- **Circular Permutation:** The number of Circular permutations of n different things chosen at a time is ((n - 1)!)

  - The number of ways of arrangement n persons along a round table so that no person has the same two neighbors is $\frac{1}{2}$(n-1)!
  - In Forming a necklace or a garland there is no distinction between a clockwise and anti-clockwise direction, because we can simply turn it over. So that Clockwise become anticlockwise and vice-versa. So required number of arrangements is $\frac{1}{2}$(n-1)!

- **Restricted Permutation:**

  Number of Permutations of n distinct objects taken r at a time. When a particular object is not taken in any arrangement is $n - 1_{Pr}$

  $$^{n-1}P_r = \frac{(n-1)!}{(n-1-r)!}$$

  Number of Permutations of r objects out of n distinct objects when a Particular object is always included in any arrangement is r.

  $^{(n-1)}P_{(r-1)}$

  $$^{(n-1)}P_{(r-1)} = \frac{(n-1)!}{n-1-r+1} = \frac{(n-1)!}{(n-r)!}$$

- **Properties of ^nC_r**
  - $^nC_r = {}^nC_{n-r}$
  - $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$
  - $^nC_0 = \frac{n!}{(n-0)!0!} = \frac{n!}{n!} = 1$
  - $^nC_n = \frac{n!}{(n-n)!n!} = \frac{n!}{n!\,0!} = 1$ $[\because 0! = 1]$
- Permutation when some of the things are alike, taken all at a time

  $P = \frac{n!}{n_1!n_2!n_3!}$
- The number of permutations of n things taken 'r' at time. When each thing may be repeated r items in any arrangement is n^r.
- Combination of n different things taking some or all n things at a time.

  $(2^n - 1)$

  In symbol = $\sum^n_{r=1} n_{cr} \Rightarrow 2^n - 1$
- Shortcuts
  - Arrangement of n items when Particular 2 items never Come together

    $= (n-1)!(n-2)$
  - Number of Straight lines from n points of Which 'm' are Collinear

    $= {}^nC_2 - {}^mC_2 + 1$

    Number of Triangle from n points of which 'm' are Collinear

    $= {}^nC_3 - {}^mC_3$
  - Number of diagonals to be formed from n Points

    $= \frac{n(n-3)}{2}$
  - If a family of m || lines are Intersected by family of n || lines then number of parallelograms will be = $\frac{m.n(m-1)(n-1)}{4}$

Sequence and Series

- An expression of the form $a_1 + a_2 + a_3 + \_\_\_\_\_\_\_ a_n$. Which is the sum of the elements of the sequence (a_n) is called a series. If the series contains a finite number of elements, it is called finite series, otherwise, called an Infinite series.
- Arithmetic Progression (A.P.): A sequence $a_1, a_2, a_3, \_\_\_\_\_\_\_ a_n$ is called an Arithmetic Progression (A.P.). When $a_3 - a_2 = a_2 - a_1 = a_n - a_{n-1}$. That Means A.P. is a sequence in which each term is obtained by adding a constant to the preceding term. This constant 'd' is called the common difference of the A.P. If 3 numbers a, b, c is in A.P.

  Then,

  b – a = c – b or a + c = 2b. b is called the arithmetic mean between a and c.

  a = First Term, **d** = Common difference
- **Formula to find the n^{th} term** = $a_n = a + (n-1)d$
- Sum of first n natural number = $\frac{n(n+1)}{2}$

- Sum of square of first n natural number is = $\frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first, n natural number = $\left[\frac{n(n+1)}{2}\right]^2$
- Sum of the first n odd numbers = $S = n^2$
- Formula to find the sum of n natural numbers = $\frac{n}{2}$ $(a + l)$ or $\frac{n}{2}$ $[2a + [n - 1]d]$
- Sum of the first n odd numbers = $S = n^2$
- **Shortcut**
  - If P^{th} of an A.P. is given as . th term of an A.P. is given as P. Then r^{th} term of A.P. will be:

    $a_p = q,\ a_q = p$

    $a_r = p + q - r$
  - If P^{th} term of an A.P. is given as q. q^{th} term of an A.P. is given as P. then $(p+q)^{th}$ term will be:

    $a_p = q,\ a_q = p$. The $^{a}p + q = 0$
  - If sum of P terms of an A.P. is given as q. The sum of q terms of an A.P. is P. then sum of $(p+q)^{th}$ terms will be:

    $S_p = q, \qquad S_q = p$

    $S_{p+q} = -(p+q)$
  - If p^{th} term of an A.P. is given as $\frac{1}{q}$. q^{th} term of an A.P. is $\frac{1}{p}$. the sum of $(p+q)^{th}$ terms will be:

    $a_p = \frac{1}{q}, \quad a_q = \frac{1}{p}, \quad = S_{pq} = \frac{1}{2}\ (pq + 1)$
  - If Ratio of sum of n terms of two series is given in the question, then Ratio of term of two series will be → Replace 'n' from '2n-1'
  - If Ratio of n^{th} term of two A.P. Series is given then ratio of sum of 'n' terms of A.P. Series will be be → Replace n from $\left[\frac{n+1}{2}\right]$.
- **Geometric Progression:**

  If in a sequence of terms each term is constant multiple of the preceding term, then the sequence is called a Geometric Progression (G.P.). The constant multiplier is called common Ratio.

  $\Rightarrow r = \frac{any\ term}{Preceeding\ term} = \frac{t_n}{t_{n-1}}$

  For Example $\Rightarrow r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$

  r = Common Ratio
- **Formula to find the n^{th} term of G.P. = ar^{n-1}**

  a = First term, r = Common Ratio

  n = Number of terms.
- **Formula to find the sum of n terms of a G.P.**

  $S_n = \frac{a(1-r^n)}{1-r}$ When $r < 1$

  $S_n = \frac{a(r^n-1)}{r-1}$ When $r > 1$

- **Formula to find the sum of G.P of an Infinite series.**

  $S_\infty = \frac{a}{1-r}, r < 1$

- A.M. of a and b is = $\frac{(a+b)}{2}$.
- If a, b, c is in G.P. We get $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$, b is called the geometric mean between a & c.

Sets, Relations and Functions

- Sets: Collection of well-defined distinct objects.
- Singleton Set: A Set containing one element is called singleton set.
- Equal Set: Two Set A & B are said to be equal, written as A=B. If every element of A is in B and every element of B is in A.
- Non-empty Set: A set has at least one element is called non-empty set. Thus, the set {0} is non-empty set. It has one element 0.
- Equivalent set: Two finite sets A & B are said to be equivalent If n (A) = n(B)

  All equal sets are equivalent sets. but all equivalent sets are not equal.
- Null Set/ Void Set/ Empty Set: Set having zero element
- Sub-set: If all the elements of set are present in another set, then such set is sub-set of another Set.
- Power Set: The collections of all possible subsets of a given set A is called the power set of A, to denoted by P(A).
- Every set is a subset of itself
- ϕ is the subset of all sets.
- No. of subsets in set = 2^n, n = no. of elements
- Proper subset: All subsets excluding same or identical one is called proper sub-set.

  Proper subset = $2^n - 1$
- If A and B are two sets then;

  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$, as $n(A \cap B) = 0$
- For three Sets P, Q and R

  $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(P \cap R) - n(Q \cap R) + n(P \cap Q \cap R)$

  When P, Q and R are disjoint sets

  $n(P \cup Q \cup R) = n(P) + n(Q) + n(R)$
- Cartesian Product of sets: If A and B are two non-empty sets then the set of all ordered pair (a, b) Such that a belongs to A and b belongs to B, is called the cartesian product of A and B, to be donated by A × B.

  Thus, $A \times B = \{(a, b): a \in A \text{ and } b \in B\}$

  If $A = \phi$ or $B = \phi \Longrightarrow$ we define $A \times B = \phi$
- Relations and function: Any Subset of the product set X.Y is said to define a relation from X to Y in which no two different ordered pairs have the same first element is called a function.

  Let A and B be two non-empty sets. Then, a rule or correspondence f which associates to each element X of A, a unique element, denoted by f (x) of B is called a function or mapping from A to B and we write F: A→B

- The element $f(x)$ of B is called the Image of x, while x is called the Pre-image of $f(x)$

  Let f: A→B, then A is called the domain of F, while B is called Co-domain of 'f'.
- **One- one Function:** Let f: A→B. If different elements in A have different images in B, then f is said to be one-one or an Injective or mapping.
- **Onto or surjective function:** Let f: A→B If every element in B has atleast one pre-image in A, then f is said to be an onto function.
- **Bijection function:** A one-one and onto function is said to be bijective. Bijective function is also known as one-to-one correspondence.
- **Identity function:** Let A be a non-empty Set. Then, the function I defined by

  I : A→A: I (x) for all X ∈ A is called an Identity function on A.
- **Inverse function:** Let f be a one-one onto function from A to B. let y be an arbitrary element of B. Thus, f being onto, there exists an element x in A Such that $f(x) = y$.
- **Different types of relations:**

  Let S= {a, b, c, d_ _ _ _} be any set then the relation R is subset of the product S×S.

  If R contains all orders pain of the form (a, a) in S×S, then R is called Reflexive, In, a reflexive relation 'a' is related to itself.

  - If (a, b)∈R ⇒ (b, a) ∈R for every a, b∈s then R is called Symmetric.
  - If (a, b) ∈R and (b, c) ∈ R ⇒ (a, c) ⇒R for every a, b, c ∈ S then R is called Transitive.
- A relation which is reflexive, Symmetric and transitive is called equivalence relation.
- **Domain & Range of a relation:** If R is a relation from A to B then set of all first Co-ordinates of elements of R is called domain of R, while the set of all second co-ordinates of elements of R is called the Range of R.

Differentiation and Integration

- **Basic formulas of differentiation:**
  - $\frac{d}{dx}(x^n) = nx^{n-1}$
  - $\frac{d}{dx}$ (constant) = 0
  - $\frac{d}{dx}(e^x) = e^x$
  - $\frac{d}{dx}(a^x) = a^x \log_e a$
  - $\frac{d}{dx}(e^{ax}) = a.e^{ax}$
  - $\frac{d}{dx}(\log x) = \frac{1}{x}$
  - $\frac{d}{dx}(uv) = \frac{d}{dx}(u).v + \frac{d}{dx}(v).u$ **(Product Rule)**
  - $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}v}{v^2}$ **(Quotient Rule)**
- **Application Related from cost function:**
  - TC = TVC + TFC
  - AC = $\frac{TC}{Output}$, AVC = $\frac{TVC}{Output}$, AFC = $\frac{TFC}{Output}$

- $MC = \frac{d}{dx}(TC)$
- $MC = \frac{d}{dx}(TVC)$

■ **Basic Formulas of Integration:**

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
- $\int dx = x + c$
- $\int e^x \, dx = e^x + C$
- $\int e^{ax} \, dx = \frac{e^{ax}}{a} + C$
- $\int \frac{dx}{x} = \log x + C$
- $\int a^x \, dx = \frac{a^x}{\log_e a} + C$

■ **Some Important formulas of Integration:**

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + C$
- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int e^x [f(x) + f'(x)] \, dx = e^x f(x) + C$
- $\int \sqrt{x^2 + a^2} \, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| + C$
- $\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int \frac{f'(x)}{f(x)} \, dx = \log f(x) + C$

■ **Integration by parts** $\int uv \, dx = u \int v \, dx - \int \left(\frac{d}{dx}(u) \int v \, dx\right) dx$

■ **Important Properties of definite Integrals**

- $\int_a^b f(x)dx = \int_a^b f(t)\, dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)\, dt$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, $a < C < b$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

  When $f(x) = f(a+x) \Rightarrow \int_0^{na} f(x)dx = n\int_0^a f(x)dx$
- $\int_{-a}^a f(x)\, dx = 2\int_0^a f(x)dx$ if $f(-x) = f(x)$
- $f(-x) = -f(x) = 0$

Important Questions of Mathematics (ICAI)

Ratio, Proportion, Indices and Logarithms	
Examples	Example-2 (Page - 1.3), (Page - 1.5) Example-3 (Page - 1.5)
Exercise- 1A	2, 11, 16, 19, 20, 21, 23, 24 Example -3 (Page - 1.11)
Exercise- 1B	6, 7, 10, 14, 24, 28, 30 Example -9 (Page - 1.18)
Exercise- 1C	6, 8, 11, 13, 17, 18, 21, 22, 27, 30
Exercise- 1D	8, 10, 15, 16, 19, 21, 22
Equation	
Exercise- (A)	2, 4, 5, 8
Exercise- (B)	4, 8, 11
Exercise- (C)	2, 5, 6, 9
Exercise- (D)	6, 7, 9
Exercise- (E)	2, 5, 10, 11
Exercise- (F)	2, 3, 11
Exercise- (G)	4, 7, 10
Exercise- (H)	2, 4, 8
Exercise- (I)	5, 6, 8, 10
Time Value of Money	
Example	6, 8, 9, 16, 18, 20, 24, 33, 37
Exercise- 4A	7, 8, 9, 10
Exercise- 4B	4, 7, 8, 12, 13
Exercise- 4C	3, 9, 10, 11, 13
Exercise- 4D	5, 7, 8
Additional Question Bank	2, 5, 7, 9, 11, 12, 13, 14, 15
Permutation and Combination	
Exercise- 5A	5, 8, 14, 16, 20, 23
Examples	Example-8 (Page-9.12), Example-10 (Page-9.13), Example-6 (Page-5.25)
Exercise- 5B	3, 5, 7, 10, 17, 18, 19
Exercise- 5C	9, 10, 12, 17, 19, 21, 22
Exercise- 5D	1, 5, 12, 14, 19
Sequence & Series	
Exercise- 6A	5, 8, 9, 14, 18, 19, 22, 24
Examples	Example-5 (Page-6.13)
Exercise- 6B	6, 7, 9, 10, 14, 16, 17, 22, 23, 24
Exercise- 6C	2, 4, 8, 14, 15, 19, 25, 27
Additional Question Bank	12, 14, 24, 32, 41, 59, 63, 73, 85, 93, 99

Sets Relations & Functions	
Exercise- 7A	2, 5, 8, 13, 17, 18, 20, 24, 32
Exercise- 7B	8, 9, 11, 12, 16, 19, 20
Exercise- 7C	1, 3, 5, 8, 12, 16, 21
Differentiation and Integration Calculus	
Examples	Example-1 (Page-8.15), Example-2 (Page-8.16), Example-4 (Page-8.17)
Exercise- 8A	2, 5, 7, 11, 13, 15, 18, 19, 21, 23, 27, 28, 33, 37, 46, 49, 50
Exercise- 8B	3, 6, 8, 11, 13, 16, 21, 23, 28, 33, 35, 39, 42, 45, 47

Important Questions of Mathematics (Perfect Practice)

Ratio, Proportion, Indices and Logarithms
2, 4, 8, 10, 13, 14, 18, 19, 22, 23, 26, 29, 34, 37, 40, 45, 48, 51, 56, 60, 66, 70, 73, 76, 80, 84, 91, 92
Equation
4, 5, 6, 8, 20, 22, 23, 25, 29, 32
Time Value of Money
4, 7, 10, 11, 18, 20, 22, 23, 26, 32, 34, 39, 40, 42, 47, 52, 56, 62, 64, 67
Permutation and Combination
4, 6, 12, 14, 16, 18, 22, 28, 29, 31, 33, 40, 41, 42, 44, 47, 52, 53, 56, 57, 59, 64
Sequence & Series
4, 8, 9, 10, 13, 16, 20, 21, 25, 27, 29, 32, 42, 47, 50, 54, 56
Sets Relations & Functions
6, 10, 15, 17, 18, 20, 24, 30, 32
Differentiation and Integration Calculus
3, 8, 10 ,11, 18, 26, 27, 29, 30, 39, 40, 43, 48, 56

Business Statistics

Statistical Description of Data

- Statistics come from Latin word 'status', Italian word 'Statista', German word 'Statistik' and French word 'Statisque'.
- Statistics can be defined in a singular and plural sense. In a plural sense, it means the data collected is qualitative and quantitative. While in the singular, it refers to the methods applied to these data.
- Statistics applied in economics, Business Management and Commerce and Industry.
- Attribute is a Qualitative characteristic. Variable refers to quantitative characteristics. Discrete, if finite or infinite and continuous if it assumes a value in a given interval.
- Data can be primary or secondary. Primary data, if directly collected from the source, is collected at the first source. If data has been obtained not from the primary source, then it is secondary data.
- **Methods of collecting Primary method:**
  - Interview method;
  - Mailed questionnaire method;
  - Observation method;
  - Questionnaires were filled and sent by enumerators.
- **Interview Method:** Personal interview, indirect interview & telephone interview method.
- **Sources of Secondary data:** International sources, government sources, private and quasi-government, unpublished source.
- The data collected should be scrutinise since the statistical analyses are made only on the basis of data; it is necessary to check whether the data under consideration are accurate as well as consistence.
- Mailed questionnaire method is the method of data collection that covers the widest area.
- Internal consistency of the collected data can be checked when a number of related series are given.
- Classification refers to the process of Arranging data. It makes data more relevant, precise and condensed, make it comparable, and serve as the base for analysis.
- Data can be frequency data and non-frequency data. Time series is an example of non-frequency data.
- **Presentation of data:**

  Textual Presentation: Presenting data in Paragraph, it is simple. However, Non-Statistical Preferred.

  Tabular Presentation: Presentation in Tables.

  Table has 4 components caption, box head, stub and body; the caption is the upper part of the table describing Column and Sub-columns. Box head is the entire upper part of the table; the stub is the left part of the table providing the description of rows. The body is the main part of the table that contains the numerical figures.
- **Diagrammatic Presentation:**
  - **Line diagram or Historiagram,** graph shows the relationship between two variables, ogives.

- Bar diagram (horizontal for qualitative data & vertical for quantitative data).
- Divided Bar charts or percentage bar diagrams for comparing and relating different components of a variable.
- Pie chart circular diagrams are two dimensional.

- **Frequency distribution** is a tabular representation of statistical data; when made in respect of discrete series, it is discrete distribution and when it relates to continuous data, it is called grouped frequency distribution.
- **Hidden trend**, if any, in the data can be noticed in diagrammatic representation.
- The chart that uses the logarithm of the variable is known as the ratio chart.
- Multiple line chart is applied for two or more related time series when the variables are expressed in the same unit.
- Multiple axis line chart is considered when there is more than one time series, and the units of the variables are different.
- Mutually exclusive series is for continuous series. Mutually inclusive series are for discrete series.
- Relative frequency lies between 0 & 1. Frequency density corresponding to a class interval is a ratio of class frequency/class length.
- Class limit means the upper and lower limit of class interval.
- Class boundary refers to the actual class limit of an interval. They are included in class intervals.
- **Graphical Representation of frequency distribution:**
  - **Historiagram helps in comparison of frequencies, calculations of mode:** It is also an area diagram; classes are overlapping. The width of all classes is equal.
  - **Frequency Polygon:** Meant for single frequency distribution, all its classes, have equal width.
  - **Ogives or cumulative frequency graph for cumulative distribution:** used for quartile, median etc.
  - **Frequency curve** is a smooth curve for which area is a limiting factor of frequency polygon and historiagram
    - Bell-shaped – Most used, **Example** where bell shape curve is used – *Profits of a company.*
    - U Shapes;
    - J Shapes;
    - Mixed Curve.

Measure of Central Tendency

- Average are two types. It can be Mathematical, or it can be positional average.
- **Mathematical Average can find by:**
  - Arithmetical mean
  - Geometrical mean
  - Harmonic mean
- **Positional Average can find by:**
  - Median
  - Mode

- Measure of central tendency for a set of observations measures the central location of the observations.
- The best Measure of Central tendency usually is the Arithmetic Mean. It is rigidly defined, based on all observations, easy to comprehend and simple to calculate. However, it has one drawback that is very much affected by sampling fluctuations, and it is not used in the case of open-end classification.
- Median is also rigidly defined and easy to compute. But it is not based on all the observations, and it is a positional average, so mathematical formulae cannot be applied.
- While computing the A.M from a grouped for frequency distribution, we assume that all the values of a class are equal to the Mid-value of that class.
- For open-end classification, Median is the best measure of central tendency as it is not much affected by sampling fluctuation.
- The presence of extreme observations does not affect the median as it is not affected by sampling fluctuation.
- The arithmetic mean is appropriate if the values have the same units, whereas the geometric mean is appropriate if the values have different units.
- The Harmonic mean is appropriate if the data values are ratios of two variables with different measures, called rates. That's why geometric mean and harmonic mean are considered for finding the average rates.
- **Relationship between mean, median and Mode for moderately as distribution:**
  - **Mean – Mode** = 3 (Mean - Median)
  - **Mode** = 3 Median – 2 Mean (Imperial relationship)
- Weighted Averages are considered when all the observations are not of equal importance.
- **Relationship between A.M., G.M card H.M:**
  - A.M ≥ G.M ≥ H.M
  - But A.M = G.M = H.M (when the observations are same and when there are distinct positive observations then relationship between A.M, G.M and H.M is A.M > G.M > H.M.
  - When the observations are both positive and negative, then the geometric mean cannot be used because the root can't be negative.
  - A.M, G.M and H.M Posses Mathematical Property.
- Mode and Median does not Possess Mathematical Property.
- **Arithmetic Mean:** Sum of all observation divided by no. of observations.
  - **Individual Series** = $\frac{\Sigma x}{N}$
  - **Discrete Series/ Frequency distribution** = $\frac{\Sigma f_x}{\Sigma f}$
  - **Continuous Series/Grouped Frequency distribution** A + $\frac{\Sigma fd'}{\Sigma f} \times C$
    - **A** = Assumed mean
    - **d'** = $\frac{X - A}{C}$
    - **C** = Class Interval
    - **X** = Mid-point of class interval
- If all the observations are constant 'K', then the mean will also be K.
- If the deviations in the series are taken from the mean, then the sum of deviations from the mean will be zero.

[deviation - A particular number is subtracted from all items of series].

- Mean affected by both changes in origin and change in scale.
- If $Y = a + b$ then $\bar{Y} = a + b\bar{X}$. Here a is the change in origin and b is the change in scale.
- $\bar{Y}$ = Mean of y series, $\bar{X}$ = Mean of series x
- Combined mean:

$$\bar{X}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

- Mean, first n natural number = $\frac{n+1}{2}$
- Mean, first n odd natural number = n
- Mean of square of first n natural number is = $\frac{(n+1)(2n+1)}{6}$
- Mean of cubes of first, n natural number = $n\left[\frac{(n+1)}{2}\right]^2$
- **Median:**
    - It is a positional average, divided a series into two parts, not affected by extremes.
    - **Median** = $\left(\frac{n+1}{2}\right)^{th}$ in the case of an odd number, and in the case of an even number, it is a simple average of two middle values.
    - For grouped frequency = $L_1 + \frac{\frac{n}{2} - CF_p}{f} \times i$

      L_1 = lower limit

      n = Total frequency

      CF_p = Previous cumulative frequency from the median class

      f = Frequency in median class
    - For moderately skewed distribution, **y** = a + bx.
    - If absolute deviations are taken from the median, the sum of absolute deviations will be minimum.
- **Quartiles, deciles and Percentile:**
    - It is also positional average quartiles, deciles, and percentiles divide equation into 4, 10 and 100 parts respectively. There are 3, 9, 99 Quartile, decile and percentile.
    - $Q_2 = D_5 = P_{50}$ = Median
    - **For quartiles** = $L_1 + \frac{k\left(\frac{N}{4}\right) - CF_p}{f} \times i$
    - **For deciles** = $L_1 + \frac{k\left(\frac{N}{10}\right) - CF_p}{f} \times i$
    - **For percentile** = $L_1 + \frac{k\left(\frac{N}{100}\right) - CF_p}{f} \times i$

Mode:

- It is the most popular measure of central tendency; there are cases when mode remains undefined.
- Highest value in the series. It is not uniquely defined; it does not exist if all the observations are equal. It represents a number which has repeatedly been most of the time.
- **Mode** = $L_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$
- When it is difficult to compute mode with the formula, it can be calculated using an equation.
- **Mode** = 3 Median – 2 Mean.
- For Moderately skewed distribution on y = a + bx.
- Graphically it can be calculated by Histogram.
- Mode is affected by sampling fluctuations.

Geometric Mean:

- It is the n[th] root of the product of the observation.
- For individual series G.M = $(x_1 \times x_2 \times x_3 \times \_\_\_\_\_\_ x_n)^{\frac{1}{n}}$
- For grouped frequency G.M = $(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \_\_\_\_\_\_ x_n^{f_n})^{\frac{1}{n}}$
- Where $\frac{1}{n} = \frac{1}{\Sigma f}$
- If all the observation of a series is K, then G.M is also K.
- Geometric mean of the product of two variables is the product of their G. M's, i.e. $Z = xy$ then G.M of Z = (G.M of x) × (G.M of y).
- The ratio of two variables is the ratio of their G.M.
- It can be calculated if all the observations have a positive sign and none of them is 0.
- It is used for calculating the growth rate of the population.
- It is rigidly defined, difficult to comprehend for computing.

Harmonic Mean:

- It is reciprocal of the A.M of the reciprocal of the observation.
- **H.M** = $\frac{N}{\Sigma\left(\frac{1}{x}\right)}$ [for individual series]
- **H.M** = $\frac{N}{\Sigma\left(\frac{f_i}{x_i}\right)}$
- If all the observation is K, then the Harmonic is also K.
- **Combined Harmonic Mean** = $\frac{n_1 + n_2}{\frac{n_1}{h_1} + \frac{n_2}{h_2}}$
- It is used for calculation of the average of prices, used for an average of speed.

Weighted Average are as follows:

- **Weighted A. M** = $\frac{\Sigma Wx}{\Sigma W}$
- **Weighted G.M.** = Antilog $\frac{\Sigma W \log x}{\Sigma W}$

- **Weighted H.M** = $\frac{\Sigma W}{\Sigma\left(\frac{W_i}{x_i}\right)}$
- Weighted Averages are used when all the observations are not of equal values.
- For two value A.M × H.M = GM^2

Measure of Dispersion

- Measure of dispersion represents the Scatterness of the series.
- **It is broadly classified into:**
  - Absolute Measures of dispersion
  - Relative Measures of dispersion
- **Absolute Measures of dispersion are:**

  a) Range; **b)** Mean deviation; **c)** Standard deviation; **d)** Quartile deviation.
- **Relative Measures of dispersion are:**

  a) Coefficient of Range; **b)** Coefficient of Mean deviation; **c)** Coefficient of variation; **d)** Coefficient of Quartile deviation.
- Absolute measures of dispersion are unit passed, whereas the Relative measures of dispersion are unit free.
- For comparing two or more distributions, relative measures of dispersion are considered, not absolute measures of dispersion are considered.
- Absolute measures of dispersion are easy to compute. Whereas relative measures of dispersion are difficult to compute and comprehend.
- Standard deviation is the most useful measure of dispersion.
- Quartile deviation is not affected by the presence of extreme observations.
- Mean deviation is based on absolute deviations only.
- Quartile deviation is the most appropriate measure of dispersion for open-end classification.
- Standard deviation is considered for finding a pooled measure of dispersion after combining several groups.
- For any two numbers, the standard deviation is always half of the range.
- **Range:**
  - **Range =** Largest – Smallest *[L – S]*
  - **Coefficient of Range =** $\frac{L-S}{L+S}\times 100$ It is affected by the presence of extremes, just based on two observations. Affected by change of scale, not origin.
  - If **equation** $y = a + bx$, the $R_y = |b| \times R_x$
- **Mean Deviation:**
  - It is defined as the arithmetic mean of the absolute deviations of the observations from Mean, Median or Mode.
  - For ungrouped frequency, Mean deviation = $\frac{\Sigma|X-\bar{X}|}{N}$
  - For grouped frequency, Mean deviation = $\frac{\Sigma f|X-\bar{X}|}{\Sigma f}$

- Coefficient of Mean deviation = $\frac{Mean\ deviation\ from\ Mean}{Mean}$ or

$\frac{Mean\ deviation\ from\ median}{Median}$; or

$\frac{Mean\ deviation\ from\ mode}{Mode}$

Standard deviation:

- It is the best method to calculate dispersion, and it is rigidly defined.
- It is defined as the root mean square deviation when the deviations are taken from the A.M. of the observations.
- For ungrouped frequency distribution, **S. D.** = $\sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$
- For grouped frequency distribution, **S. D.** = $\sqrt{\frac{\Sigma f(X-\bar{X})^2}{\Sigma f}}$
- **Variance =** $(SD)^2$
- Coefficient of variation represents the variation in a series $\frac{S.D.}{AM.}\times 100$
- More C.V. means More dispersion, and less coefficient of variation means more consistent.
- If all the variables of the series are K, then S.D. is zero.
- Standard deviation is affected by the change of scale only, but it is not affected by the change of origin.
- *If* $y = a + bx$ *then,* $S.D_y = |b| \times S.D_x$
- **Combined Standard Deviation =** $\sqrt{\frac{n_1\ \sigma_1^2 + n_2\ \sigma_2^2 + n_1\ d_1^2 + n_2\ d_2^2}{n_1 + n_2}}$
- **Where,**

  $d_1 = \bar{X}_{12} - \bar{X}_1$

  $d_2 = \bar{X}_{12} - \bar{X}_2$

  n_1 = *No. of items in Series* 1.

  n_2 = *No. of items in Series* 2.

  σ_1 = *S. Dof the Series* 1.

  σ_2 = *S. Dof the Series* 2.
- For any, the number S.D. is half of the range. Standard deviation of two numbers a & b $\frac{|a-b|}{2}$
- Standard deviation of the first 'n' natural number is $\sqrt{\frac{n^2-1}{12}}$

Quartile Deviation:

- Quartile deviation or Semi-Quartile deviation = *Q. D.* = $\frac{Q_3 - Q_1}{2}$
- Coefficient of Quartile Deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1}\times 100$
- It is the best measure for open-end distribution, it is not affected by the change of origin, but it is affected by the change scale.
- It is just based on the 50% of the observation
- Inter-Quartile range = $Q_3 - Q_1$
- 4SD = 5MD = 6QD, when the data is normal distributed.

Probability

- Probability means likelihood or something that is likely to happen; two types of probability are subjective and objective.
- Experiment may be described as a performance that produces a certain result. The results or outcomes of a random experiment are known as events. Sometimes events may be a combination of outcomes. The events are of two types.
  - Simple or elementary
  - Composite or Compound
- **Mutually Exclusive:** A set of evenly A and B are known to be mutually exclusive. If nothing is common between them or none of them can occur simultaneously. The occurrence of one event implies the non-occurrence.

  Example $P(A \cap B) = 0$
- **Exhaustive event:** The events $A_1, A_2, A_3,$ .... are known to be exhaustive. If one of the events must occur and the set of events constitute the Universe.
- **Equally likely:** The events are those whose occurrence is equal.
- **Odds in Favour** = $\dfrac{\text{Chances of happening}}{\text{Chances of not happening}}$
- **Odds in Against** = $\dfrac{\text{Chances of not happening}}{\text{Chances of happening}}$
- The Probability of an event lies between 0 and 1, both Inclusive, i.e., $0 \le P(A) \le 1$
- When P (A) = 0, A is known to be an Impossible event, and when P (A) = 1, A is known to be a sure event.
- Non-occurrence of event A is denoted by A^c or A' or Ā It is known as a complimentary event of A. Event A along with its complimentary A' forms a set of mutually exclusive and exhaustive events.

  i.e., $P(A) + P(A') = 1$

  $P(A') = 1 - P(A)$
- **Important Formulas:**
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
  - $P(A - B) = P(A) - P(A \cap B)$
  - $P(A \cap B) = 0$ For mutually exclusive events.
  - $P(A \cup B \cup C) = 1$ For mutually exhaustive events.
  - $P(A \cap B) = P(A) \times P(B)$, For independent events.
  - $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(B)}$, For compound probability or conditional probability.
  - $P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$
  - When is a discrete random variable with probability mass function f (x), then its expected value is given by

    $ll = \sum x f(x)$

Variable = $E(x)^2 - \mu^2$

Where = $E(x)^2 = \sum x^2 f(x)$

- Expectation of the product of a constant of a constant and a random variable is the product of the constant and the expectation of the random variable.

  $E(kx) = k\,E(x)$

- Expectation of the product of two random variables is the product of the expectation of the two random variables. Provided two variables are Independent.

  $E(xy) = E(x) \times E(y)$

  Whenever x and y are Independent

Theoretical Distribution

- A Probability distribution also possesses all the characteristics of an observed distribution. We define population mean (μ), Population median ($\tilde{\mu}$), Population mode (μ_0), Population standard deviation (σ) etc. These characteristics are also known as population parameters. Probability distribution or continuous probability distribution depending upon the random variable.
- Two important discrete probability distributions are **a)** Binomial distribution; **b)** Poisson distribution.
- Normal distribution is an important continuous probability distribution.
- **Binomial distribution:**
  - Discrete Probability Distribution, Invented by Bernoulli.
  - It is used when no. of trials is too large but finite.
  - Every trial is Independent.
  - Events should be Mutually exclusive and exhaustive.
  - Every trial has two outcomes, the occurrence of one will be known as 'success' Indicated by 'P', and the occurrence of another is known as 'failure' represented by q.

    $^nC_r\,(p)^r\,(q)^{n-r}$

    n = Number of trials

    r = Number of success required

    p = Probability of Success in one trial

    q = Probability of failure in one trial

    It is bi-Parametric.
- **Properties of binomial distribution:**
  - ***Mean*** = np
  - ***variance*** = npq
  - σ^2 = npq
  - $\sigma = \sqrt{npq}$
  - Variance is always less than mean
  - Variance will be highest when p = q = 0.5, and it is skewed to the right.
  - It can be bi-Modal or uni-Modal. If (n+1) P is an Integer, then it is bi-Modal where two modes are (n+1) P and (n+1) P-1, and if (n+1) P is a non-integer, then it is Uni-Modal, where the greatest integer in (n+1) P is the mode.

- **Additive Property of Bimodal distribution**

  If X and Y are two independent variables such that

  $(X \sim B)(n_1, P)$ and $(Y \sim B)(n_2, P)$

  Then $(X + Y) \sim B(n_1 + n_2, P)$

Poisson Distribution:

A random variable X is defined to follow the Poisson distribution. It is denoted by If the probability mass function of is given by.

$F(x) = P(X = r) = \frac{e^{-m} \times m^{r}}{r!}$ for r=0,1,2, ... ∞

- It is a discrete random variable invented by Simon Denis Poisson.
- It is Uni-Parametric distribution as it is characterized by only one Parameter m.
- It is used when no. of trials (n) are too large (tends to infinite) and the probability of success is very small (tends to zero)
- Trials are Independent.

Properties of Poisson distribution:

Mean = m $m = np$

Variable = $(\sigma)^2 = m$

$\sigma = \sqrt{m}$

- It can also be Uni-modal or Bi-Modal. If m is an integer, then it is Bi-Modal, Where m and m -1 are modes. And it is Uni-Modal if m is non-Integer. Where greatest Integer in m is the mode
- **Additive Property** = If x and y are two Independent variables such that X ~ $P(m_1)$ and Y~P (m_2) then $(x + y) \sim P(m_1 + m_2)$
- Applications or examples – No. of Printing mistakes per page on large book, no. of the accident on the busy road per minute, no. of demand per minute for a health care.
- It is symmetrical When the mean value is high.

Normal or Gaussian distribution:

- Continuous Probability distribution. Most important and universally accepted. It is based on true parameters μ and σ². Denoted by x~N (μ, σ²).
- If the Probability density function of the random variable X is given by

  $$F(x) = \frac{1}{\sigma\sqrt{2\pi}} - e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

  σ = S.D.; μ = Mean, x = r = No. of success required

  Where μ and σ, are constants, and σ > **0**

Properties of the normal distribution:

- Under normal distribution, mean = median = mode. At mean, the Probability is highest.
- The mean of the normal distribution is given by **μ.**
- Mean deviation from mean = Mean deviation from median = Mean deviation from mode = **0.8** $\sigma = \sigma\sqrt{2}$
- **Quartiles**

  First Quartile = μ – 0.675 σ

  Third Quartile = μ – 0.675 σ

Quartile deviation = $\frac{Q_3 - Q_1}{2} = 0.675\sigma$

- Point of Inflexion = $(\mu - \sigma)$, $(\mu + \sigma)$
- The normal distribution is symmetrical about $x = \mu$. As such, its skewness is zero, i.e., the normal curve is neither inclined to move towards the right (negatively skewed) nor towards the left (positively skewed).
- We note that $(\mu \pm \sigma)$ cover 68.27% area, $(\mu \pm 2\sigma)$ cover 95.45% area, and $(\mu \pm 2\sigma)$ cover 99.73% area,
- If x and y are independent normal variables with meant and standard deviations as and and and respectively, then $(Z = x + y)$ also follows a normal distribution with mean $(\mu_1 + \mu_2)$ and S.D $= \sqrt{\sigma_1^2 + \sigma_2^2}$ Respectively.

Correlation and Regression

- **Correlation** shows association or relation between two variables, whereas regression shows the value of the variable based on other Bivariate data.
  - Bivariate Data are the data collected for two variables irrespective of time.
  - Can be Marginal and conditional distribution.
  - Collected for 2 variables at the same time.
  - For distribution p + q, a number of cells are pq.
  - Some cells may be 0.
  - For p × q, marginal distribution is 2, and conditional are p + q.
- **Correlation:** The change in one variable is Reciprocated by a corresponding change in the other variable either directly or inversely; then the two variables are known to be associated or correlated. Correlation analysis aim is establishing relation between two variables and measuring the extent of relation between two variable.
- **There are two types of correlation:**

  a) Positive correlation **b)** Negative correlation

  The value of correlation is between −1 to 1. -1 means perfect negative, 0 to -1 means negatives, 0 means no correlation, while 0 to 1 means positive and +1 means perfect positive correlation.
- **Scatter diagram:** This is a simple diagrammatic method to establish a correlation between a pair of variables, and it is used for linear and non-linear (Curvilinear) distribution.
  - If the plotted points in a scatter diagram lie from upper left to lower right, then the correlation is negative and vice versa correlation is positive.
  - If all the potted points in a scatter diagram are evenly distributed, then the correlation is zero and plotted points lie on the single line then the correlation is either positive or negative.
- **Karl Pearson's Product moment correlation coefficient:**

$$r = r_{xy} = \frac{(cov.(x, y)}{(\sigma_x \times \sigma_y)}$$

Where $Cov(x, y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{N}$ or $Cov.(x, y) = \frac{\Sigma xy}{n} - \bar{x}\bar{y}$

$$\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}} \text{ or } \sigma_x = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$$

$$\sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{N}} \text{ or } \sigma_y = \sqrt{\frac{\Sigma y^2}{N} - (\bar{y})^2}$$

$$r = \frac{n\Sigma xy - \Sigma x.\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$ (Direct Method)

$$r = \frac{n\Sigma dx\,dy - \Sigma dx \times \Sigma dy}{\sqrt{n\Sigma dx^2 - (\Sigma dx)^2}\sqrt{n\Sigma dy^2 - (\Sigma dy)^2}}$$

- The coefficient of correlation is a unit free measure.
- The coefficient of correlation is unaffected by change of origin or scale, but it changes its sign with the change of sign of variables. If the sign of both variables is the same, r remains the same. While if sign differs r sign also changes.
- The coefficient of correlation always is between – 1 and 1, including both limiting values.
- Spurious correlation means the correlation between two variables has no causal relation.
- Product moment correlation coefficient is considered for finding the nature of correlation and the amount of correlation.
- Product moment correlation coefficient may be defined as the ratio of covariance between the variable to the product of their standard deviations.

Spearman's rank correlation:

For finding the correlation between two attributes we consider it.

$$r_R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

Where r_R denotes rank correlation coefficient, and it lies between –1 and 1 inclusive of these two values. $d = Rx - Ry$. The Rx and Ry are Rankings given to x and y series. Ranks are given in descending order. 1 is given to highest and so on.

In case the same Ranks are received by individuals, then

$$r_R = 1 - \frac{6\left[\Sigma d^2 + \frac{\Sigma m^3 - m}{12}\right]}{n(n^2-1)}$$

m represents the number of repetitions. $(m^3 - m)$ will come under number of times numbers are repeated.

Coefficient of concurrent deviations:

It is the quickest method to find correlation between two variables.

$$r_c = \pm\sqrt{\pm\frac{(2C-M)}{M}}$$

If $(2c - m) > 0$, then we take the positive sign both inside and outside the radical sign, and if $(2c - m) < 0$, we are to consider the negative sign both inside and outside the radical sign.

C = no. of positive signs

m = n – 1

n = no. of observation

Regression:

In regression analysis, we are concerned with the estimation of one variable for a given value of another variable or establishing a mathematical relationship between two variables and predicting the value of the dependent variable for a given value of the independent variable. The method applied for deriving the regression equations is knowns as least square method.

If $y = a + bx$, a and b are regression parameters, regression equation y on x, b_{yx}, methods based on least square. Regression coefficient also represents the shape of regression equations.

$$b_{yx} = \frac{Cov.(x,y)}{s_x^2}, \quad b_{yx} = r.\frac{s_y}{s_x}$$

$$b_{xy} = \frac{Cov.(x,y)}{s_y^2}, \quad b_{xy} = r.\frac{s_x}{s_y}$$

$a = y - bx$ [where y=mean of y, x=mean of x] for solving a and b

$\Sigma_y = na + b\Sigma x$

$\Sigma_{xy} = a\Sigma x + b\Sigma x^2$

Direct Method

$$b_{yx} = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

For solving equation b_{xy} replace x with y.

$$b_{xy} = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

The regression coefficients remain unchanged due to a shift of origin but change due to a shift of scale.

This property states that if the original pair of variables is (x, y) and if they are changed to the pair (u, v) under:

$u = a + bx$

$v = c + dy$

$$b_{uv} = \frac{b}{d} \times b_{xy}$$

$$b_{vu} = \frac{d}{b} \times b_{yx}$$

- The regression equations are interested in their means.
- The coefficient of correlation between two variables is the simple G.M of the two-regression coefficients. The sign of the correlation coefficient would be the common sign of the two-regression coefficients.

  $r = \pm\sqrt{b_{yx} \times b_{xy}}$
- Correlation coefficient measuring a linear relationship between the two variables indicate the amount of variation of one variable accounted for by the other variable. A better measure for this purpose is provided by the square of the correlation coefficient known as the coefficient of determination. This can be interpreted as the ratio between explained variance to total variance.

  $r^2 = \frac{Explained\ variance}{total\ variance}$
- Coefficient of non-determination = $1 - r^2$
- Two regression lines coincide when $r = -1$ or 1. And perpendicular if $r = 0$.
- Product of regression coefficient must be numerically less than 1.

- **Probable error:**
  - It is a method of obtaining the correlation coefficient of population. It is defined as –
  - $P.E = 0.6745 \times \frac{1-r^2}{\sqrt{N}}$
  - S.E = Standard error of correlation coefficient
  - $S.E = \frac{1-r^2}{\sqrt{N}}$
  - The limit of correlation coefficient = r ± P.E.

    = r > 6 P.E. = Presence of correlation is certain.

    = r < P.E. = No evidence of correlation.

Index Number

- It means ratio or average of ratios expressed as a percentage, having two or more time periods; one of the time periods is base period.
- A series of numerical figures which show the relative position is called index number.
- **Issues involved in index numbers:**
  - Selection of data;
  - Base period;
  - Selection of weights (weights play a very important part in the construction of index number);
  - Use of averages (GM is particularly suitable for the construction of index number);
  - Choice of variables;
  - Selection of formula;
- **There are three types of index numbers:**
  - Price index
  - Quantity index
  - Value index
- **Price index numbers:**
  - **Simple Aggregative Price Index** = $\frac{\Sigma P_n}{\Sigma P_0} \times 100$

    Simple Average Relative = $\frac{\Sigma\left(\frac{P_1}{P_0}\right)}{N} \times 100$
  - **Laspeyres Index:** In this index, base year quantity is used as weights, or we can say base year quantities:

    Laspeyres Index = $\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$
  - **Paasche's Index:** In this index, current year quantities are used as weights, or we can say current year quantities:

    Paasche's Index = $\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n} \times 100$
  - The Marshal-Edgeworth index uses this method by taking the average of the base year and the current year.

    Marshal-Edgeworth Index = $\frac{\Sigma P_n (Q_0 + Q_n)}{\Sigma P_0 (Q_0 + Q_n)} \times 100$

- **Dorbish and Bowley's index number P_{01} = $\frac{L+P}{2}$ (L = Laspeyres and P = Paasche's index number)**
- **Fisher's ideal price index:** This index is the geometric mean of Laspeyres and Paasche's

  Fisher's Index = $\sqrt{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}} \times 100$
- **Weighted Average of Relative method** = $\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$
- **Bowley's Index number** = $\frac{\text{Laspeyres Index number} + \text{Paasche's Index number}}{2}$
- **Chain Index** = $\frac{\text{Link Relative of Current Year} \times \text{Chain Index of Previous Year}}{100}$
- **Link Relative of Current Year** = $\frac{P_1}{P_0} \times 100$
- **Real wage** = $\frac{\textit{Money wage}}{\textit{Price Index}} \times 100$

■ **Quantity Index Numbers:**

- **Simple Aggregate of Quantities** = $\frac{\Sigma Q_n}{\Sigma Q_0} \times 100$
- **Simple Average of Quantity Relatives** = $\frac{\Sigma\left(\frac{Q_1}{Q_0}\right)}{N} \times 100$
- Weighted Aggregate Quantity Indices:
  - **Laspeyres Index** = $\frac{\Sigma Q_n P_0}{\Sigma Q_0 P_0} \times 100$
  - **Paasche's Index** = $\frac{\Sigma Q_n P_n}{\Sigma Q_0 P_n} \times 100$
  - **Fisher's Ideal Index** = $\sqrt{\frac{\Sigma Q_n P_0}{\Sigma Q_0 P_0} \times \frac{\Sigma Q_n P_n}{\Sigma Q_0 P_n}} \times 100$

■ **Value Index Numbers:**

- **Value Index** = $\frac{V_n}{V_0} = \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_0}$
- **Deflated Value (Real Value)** = $\frac{\textit{Current Value}}{\textit{Price index for current year}}$

  OR

  Deflated Value = Current Value × $\frac{\textit{Base Price}}{\textit{Current Price}}$
- **Value** = Price × Quantity
- **Current Value** = $P_n\, Q_n$
- **Shifted Price Index** = $\frac{\textit{Original Price index}}{\textit{Price index of the year on which it has to shifted}} \times 100$

■ **Test of Adequacy:**

- **Unit Test:** This test Requires that the formula should be independent of the unit in which price and quantities are quoted. Except for the simple Aggregative index, all other formulae satisfy this test.
- **Time Reversal Test:** It is a test to determine whether the method will work both ways in time forward and backwards. The test checks the index number should be such that two ratios, the current on the base and base on the current. So, the two indexes should be reciprocals of each other.

$P_{01} \times P_{10} = 1$

Laspeyres and Paasche's do not satisfy this test, but fisher's ideal formula satisfy this test.

This test is necessary to check the consistency of the index number.

- **Factor Reversal Test**: This holds when the product of price index and the quantity index should be equal to the corresponding value index,

  i.e., = $\frac{\Sigma P_1Q_1}{\Sigma P_0Q_0} \times 100$

  Symbolically

  $P_{01} \times Q_{01} = V_{01}$

  Fisher's Index Satisfy Factor Reversal test. Because fisher's index number satisfy the time-reversal and factor reversal test, so, it is called an ideal index number.

- **Circular Test:** It is concerned with the measurement of price changes over a period of the year when it is desirable to shift the base.

  $P_{01} \times P_{12} \times P_{20} = 1$

- It is a simple geometric mean of price relatives.
- Paasche's, Laspeyres and Fisher's index number do not satisfy the test.
- Simple aggregative method and the fixed weight aggregative method meet this test.
- It is an extension of the time reversal test.
- It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base.

Point to Remember:

- Purchasing power of money is the reciprocal of price index number.
- Index numbers for the base period are always taken as 100.
- Index numbers show the percentage changes rather than absolute amounts of change.
- Geometric mean makes the index number time-reversible.
- Circular test is an extension of the time-reversal test.
- Weighted Geometric mean of the relative formula satisfies the time-reversal test.
- The index number is a special type of average.
- The AM of the group of given general index.
- The choice of a suitable base period is the best temporary solution.
- There are 4 tests of adequacy.
- Index number is equal to average of price relative.
- The value at the base time period serves as the standard point of comparison.
- Base period is a point of reference in comparing various data describing individual behaviour.

Important Questions of Logical Reasoning and Statistics (ICAI)

Logical Reasoning

Number Series	
Exercise	1, 3, 6, 7, 8, 9, 10, 14, 15, 17, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 32, 33, 34, 38, 40, 42, 47, 49, 51, 54, 56, 59, 61
Examples	4, 7, 8, 9, 13, 16
Direction Test	
Exercise	3, 4, 7, 10, 11, 12, 13, 16, 17, 19, 20, 22, 24, 25, 27, 31, 33, 37, 39, 42, 46, 48, 50, 53
Example	3
Seating Arrangement	
Exercise	1, 4, 5, 7, 8, 9, 12, 13, 15, 16, 19, 21, 22, 24, 26, 27
Examples	Example - 6, Example - 8, Example - 9, Example - 10, Example - 11, Example - 12
Blood Relations	
Exercise	1, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 23, 25, 30, 31, 32, 36, 37, 39, 42, 46, 47, 50, 54, 55
Examples	5, 8, 10, 13

Important Questions of Logical Reasoning (Perfect Practice)

Number Series
5, 6, 11, 16, 19, 20, 22, 24, 26, 33, 40, 41
Direction Test
3, 4, 5, 8, 13, 15, 18, 20, 22, 24, 26, 28, 31, 33, 34
Seating Arrangement
4, 7, 8, 10, 12, 13, 16, 17, 18, 21, 23, 25, 26, 28
Blood Relations
3, 6, 10, 12, 14, 15, 17, 18, 19, 20, 23, 25, 26

Statistics

Measure of Central Tendency	
Exercise Set-B	1, 4, 5, 6, 7, 10, 11, 12, 13, 14, 19, 20
Examples	15.1.2, 15.1.4, 15.1.5, 15.1.7, 15.1.8, 15.1.11, 15.1.12, 15.1.14, 15.1.16, 15.1.17, 15.1.20
Measure of Dispersion	
Exercise Set-B	2, 4, 6, 7, 9, 10, 12, 16, 20
Examples	15.2.2, 15.2.3, 15.2.6, 15.2.7, 15.2.8, 15.2.9, 15.2.10, 15.2.13, 15.2.14, 15.2.15, 15.2.16, 15.2.17, 15.2.20
Probability	
Exercise Set-B	1, 2, 6, 8, 12, 13, 15, 17, 19, 21
Examples	16.1, 16.4, 16.6, 16.7, 16.8, 16.10, 16.11, 16.12, 16.13, 16.14, 16.15, 16.16, 16.18, 16.19, 16.21, 16.23, 16.27, 16.29, 16.31
Theoretical Distribution	
Exercise Set-B	1, 4, 7, 8, 9, 11, 12, 15, 17, 20, 21, 22

Examples	17.1, 17.2, 17.5, 17.8, 17.9, 17.10, 17.11, 17.12, 17.13, 17.15, 17.18, 17.21, 17.22, 17.23, 17.28, 17.30, 17.32
Correlation and Regression	
Exercise Set-B	1, 2, 5, 6, 7, 9, 12, 14, 16, 17, 18, 21, 23
Examples	18.2, 18.3, 18.6, 18.8, 18.11, 18.13, 18.15, 18.17, 18.18, 18.19, 18.20, 18.21, 18.22
Index Number	
Additional Question	13, 14, 16, 17, 22, 23, 24, 30, 34, 35, 41, 44, 45, 48, 58, 62, 65, 67, 73, 75, 76, 84, 87, 91, 92, 95

Important Questions of Statistics (Perfect Practice)

Measure of Central Tendency
10, 13, 16, 18, 19, 23, 26, 27, 30, 38, 39, 40 ,47 ,50 ,52
Measure of Dispersion
2, 6, 8, 10, 15, 20, 24, 25, 38, 40, 41, 44
Probability
7, 8, 14, 15, 16, 18, 20, 22, 24, 25, 27, 28, 33, 35, 37, 40, 42, 44, 47, 49, 53
Theoretical Distribution
5, 9, 12, 13, 15, 18 19, 23, 27, 33, 36
Correlation and Regression
2, 7, 9, 11, 12, 14, 17, 18, 22, 23, 27, 28, 29, 32, 36, 38
Index Number
11, 12, 13, 14, 17, 18, 20, 27, 31, 32, 34, 43, 44