

RATIO | PROPORTION | LOGS & INDICES



1. Ratio is a **Fraction** used for comparison of 2 or more quantities which are

of same type

Expressed in same unit of measurement

Example : If Vinod's weight is 90 kgs and Anand's weight is 60 kgs then we can say that : **Ratio of weights is 90:60**

2. Weights of Vinod, Anand are in the ratio of $90 : 60 = 3 : 2$
Simplest form of 90:60

Also we can write that, weights of Anand, Vinod are in the ratio of $60 : 90 = 2 : 3$

Ratio can be expressed without any unit of measurement

3. Ratio can be expressed like $2 : 3$, $9:11$, $\frac{9}{11}$, $\frac{12}{31}$, $5:7:9$
It means Ratios are UNIT-FREE

4. The ratio $30 : 60$ can also be written as $1 : 2$ in its simplest form

5. Ratio is generally expressed in its simplest form
simplest form of $12 : 18$ is $2 : 3$

6. $a : b$ can also be written as $\frac{a}{b} = \frac{am}{bm} = ak : bk = ax : bx = ay : by$
 $= \frac{a}{x} : \frac{b}{x}$ provide $m, k, x, y \neq 0$

7. All the terms of a ratio can be multiplied OR divided by same non-zero number.

$$x : y = 3x : 3y = 10x : 10y = \frac{x}{12} : \frac{y}{12} = \frac{x/m}{y/m} \text{ provided } m \neq 0$$

8. In the ratio $x : y$

x = First term = Antecedent

y = Second term = Consequent

In the ratio $9:8$ 9: Antecedent 8: consequent

9.

In the ratio a:b If	then Ratio is known as	Examples
$a > b$ i.e. Antecedent > Consequent	Ratio of Greater Inequality	9:7, 11:3, 101:89, 35:34, 39:21
$a < b$ i.e. Antecedent < Consequent	Ratio of Lesser Inequality	5:11, 35:38, 41:43, 93:523, 581:1000
$a = b$ i.e. Antecedent = Consequent	Ratio of Equality	8:8, 5:5, 19:19, 1:1 38:38

Simplest form of Ratio of equality

is always $1:1$



10. Find simplest form of ratio of quantities
(3 hours 5 mins 8 secs), (8 hours 10 mins 40 secs)



$$\frac{3 \text{ hrs } 5 \text{ mins } 8 \text{ secs}}{8 \text{ hrs } 10 \text{ mins } 40 \text{ secs}} = \frac{(3 \times 60 \times 60) + (5 \times 60) + 8}{(8 \times 60 \times 60) + (10 \times 60) + 40}$$

$$= \left[\frac{10800 + 300 + 8 \text{ seconds}}{28800 + 600 + 40 \text{ seconds}} \right] = \frac{11108}{29440} = \frac{2777}{7360} = 2777 : 7360$$

11. Ratio can be expressed without unit of measurement
(i.e. Ratio are unit-free)

12.

Quantities	Ratio in simplest form
5 hrs, 5 mins	$\frac{(5 \times 60) \text{ mins}}{5 \text{ mins}} = 60 : 1$
5 hrs 10 mins, 10 hrs 15 mins PIS Note: 1 hour = 60 mins 1 min = 60 Secs	$\frac{(5 \times 60) + 10}{(10 \times 60) + 15} = \frac{310}{615} = \frac{62}{123}$ $= 62 : 123$
8 feet 6 inches, 12 feet 8 inches PIS Note: 1 foot = 12 inches	$\frac{(8 \times 12 + 6) \text{ inches}}{(12 \times 12 + 8) \text{ inches}} = \frac{102 \text{ inches}}{152 \text{ inches}} = \frac{51}{76} = 51 : 76$
8 meters 10 cms, 10 meters 18cms PIS Note: 1 km = 1000 meters 1 Meter = 100 cms = 1000 mms 1 cm = 10 mms	$\frac{(8 \times 100 + 10) \text{ cms}}{(10 \times 100 + 18) \text{ cms}} = \frac{810 \text{ cms}}{1018 \text{ cms}}$ $= 810 : 1018 = 405 : 509$

5 mins 33 secs, 8 mins 45 secs

$$\frac{(5 \times 60) + 33 \text{ secs}}{(8 \times 60) + 45 \text{ secs}} = \frac{333 \text{ secs}}{525 \text{ secs}}$$

$$= 111:175$$

8GB, 512 MB

1024 bytes = 1 KB

1024 KB = 1 MB

1024 MB = 1 GB

1024 GB = 1 TB

$$\frac{8 \times 1024 \text{ MB}}{512 \text{ MB}}$$

$$= 8192:512$$

$$= 16:1$$

8.5 kgs, 880 gms

1 kg = 1000 gms

1 gm = 1000 mgs

$$\frac{8.50 \times 1000 \text{ gms}}{880 \text{ gms}}$$

$$= \frac{8500 \text{ gms}}{880 \text{ gms}} = \frac{850}{88} = 425:44$$

8.8 kms, 44000 mms

$$\frac{8.80 \times 1000 \times 100 \times 10 \text{ mms}}{44,000 \text{ mms}}$$

$$= \frac{88,00,000}{44,000} = 200:1$$

13. Order of terms in a ratio is important

$$a:b \neq b:a \quad 3:7 \neq 7:3$$

14. If we interchange the position of antecedent & consequent,
we get Inverse Ratio.

15. Inverse Ratio of 5:7 is **7:5**

& Pls remember $5:7 \neq 7:5$

16. Inverse Ratio of Inverse Ratio of $a:b$ is **$a:b$**

Inverse Ratio of Inverse Ratio of 13:28 is 13:28

Did You Know ?

17. Inverse Ratio of Ratio of Greater inequality is Ratio of Lesser inequality and vice versa.
18. Inverse Ratio of Ratio of Equality is Ratio of Equality only.

19. Duplicate Ratio : Duplicate ratio of $m:n$ is $m^2:n^2$
- Duplicate ratio of $1:9$ is $1:81$
 - Duplicate ratio of $\sqrt{3}:15$ is $3:225 = 1:75$
 - Duplicate ratio of $a^2:b^3$ is $(a^2)^2:(b^3)^2 = a^4:b^6$

20. Triplicate Ratio : (Triplicate ratio of $a:b$) = $a^3:b^3$
- Triplicate ratio of $2:5$ is $2^3:5^3 = 8:125$

21. Sub-duplicate Ratio : sub-duplicate ratio of $81:625$ is $9:25$
- sub-duplicate ratio of $5:11$ is $\sqrt{5}:\sqrt{11} = 5^{1/2}:11^{1/2}$

22. Sub-triplicate Ratio : sub-triplicate ratio of $8:125$ is $2:5$
- sub-triplicate ratio of $m:n$ is $\sqrt[3]{m}:\sqrt[3]{n}$

Fill in the blanks

$= m^{1/3}:n^{1/3}$

$\sqrt[m]{a} = (a)^{1/m}$

A Ratio	It's				
	Duplicate Ratio	Triplicate Ratio	Sub-duplicate Ratio	Sub-triplicate Ratio	Inverse Ratio
$p:q$	$p^2:q^2$	$p^3:q^3$	$\sqrt{p}:\sqrt{q}$	$\sqrt[3]{p}:\sqrt[3]{q}$	$q:p$
$3:11$	$9:121$	$27:1331$	$3^{1/2}:11^{1/2}$	$3^{1/3}:11^{1/3}$	$11:3$
$a^2:b^3$	$(a^2)^2:(b^3)^2$ $= a^4:b^6$	$(a^2)^3:(b^3)^3$ $= a^6:b^9$	$(a^2)^{1/2}:(b^3)^{1/2}$ $= a:b^{3/2}$	$(a^2)^{1/3}:(b^3)^{1/3}$ $= a^{2/3}:b$	$b^3:a^2$
$a^7:p^{10}$	$(a^7)^2:(p^{10})^2$ $= a^{14}:p^{20}$	$(a^7)^3:(p^{10})^3$ $= a^{21}:p^{30}$	$(a^7)^{1/2}:(p^{10})^{1/2}$ $= a^{7/2}:p^5$	$(a^7)^{1/3}:(p^{10})^{1/3}$ $= a^{7/3}:p^{10/3}$	$p^{10}:a^7$
$3^{1/3}:8^{1/7}$	$(3^{1/3})^2:(8^{1/7})^2$ $= 3^{2/3}:8^{2/7}$	$(3^{1/3})^3:(8^{1/7})^3$ $= 3:8^{3/7}$	$(3^{1/3})^{1/2}:(8^{1/7})^{1/2}$ $= 3^{1/6}:8^{1/14}$	$(3^{1/3})^{1/3}:(8^{1/7})^{1/3}$ $= 3^{1/9}:8^{1/21}$ $= \sqrt[9]{3}:\sqrt[21]{8}$	$8^{1/7}:3^{1/3}$ $= \sqrt[7]{8}:\sqrt[3]{3}$

23. **Compounded Ratio or Ratio Compounded**

Compounded ratio of $2 : 3, x : y, m : n, p : q, e : f$ is

Compounded Ratio = $(2xmpqe) : (3ynqf)$
 = (product of antecedents) : (product of consequents)

24. Find Ratio Compounded of $3 : 7, 8 : 3, 5 : 11, 22 : 5, 14 : 17$



$$\left[\begin{array}{l} \text{Compounded Ratio} \\ \text{OR} \\ \text{Ratio compounded} \end{array} \right] = \frac{(3 \times 8 \times 5 \times 22 \times 14)}{(7 \times 3 \times 11 \times 5 \times 17)}$$

$$= \frac{(\cancel{3} \times 8 \times \cancel{5} \times \overset{2}{\cancel{2}} \times \overset{2}{\cancel{14}})}{(\cancel{7} \times \cancel{3} \times \cancel{11} \times \cancel{5} \times 17)}$$

$$= \frac{8 \times 2 \times 2}{17}$$

$$= \frac{32}{17} = 32 : 17$$

25. Find compounded ratio of $8 : 13, 39 : 5, 25 : 18, 27 : 15, 30 : 8, 16 : 5, 50 : 2$



$$\text{Compounded ratio} = \frac{(8 \times \overset{3}{\cancel{39}} \times 25 \times \overset{3}{\cancel{27}} \times \overset{2}{\cancel{30}} \times 16 \times \cancel{50})}{(\cancel{13} \times \cancel{5} \times \cancel{18} \times \cancel{15} \times \cancel{8} \times \cancel{5} \times \cancel{2})}$$

$$= \frac{3 \times 25 \times 3 \times 16}{1} = 3600 : 1$$

• Duplicate ratio of $9 : 13$ is

- (a) $9 : 13$ (b) $82 : 130$ (c) $162 : 338$ (d) None of these

Dupli. ratio of $9 : 13 = 9^2 : 13^2 = 81 : 169 = 162 : 338$



26. Find compounded ratio 5 : 7, 14 : 3, 9 : 2, 6 : 15,

Duplicate Ratio of 5 : 2, Triplicate Ratio of 4 : 5,

Sub-duplicate ratio of 25 : 49, Sub-triplicate Ratio of 343 : 64,

inverse ratio of 2 : 9, Duplicate Ratio of 3 : 2

→ 5:7, 14:3, 9:2, 6:15, 25:4, 64:125, 5:7, 7:4, 9:2, 9:4

$$\begin{aligned} \text{Compounded Ratio} &= \frac{5 \times 14 \times 9 \times 6 \times 25 \times 64 \times 5 \times 7 \times 9 \times 9}{7 \times 3 \times 2 \times 15 \times 4 \times 125 \times 7 \times 4 \times 2 \times 4} \\ &= \frac{9 \times 9 \times 3}{1} = \frac{243}{1} = 243 : 1 \end{aligned}$$

27. Find duplicate ratio of sub-triplicate ratio of inverse ratio of 125 : 343

→ = Duplicate ratio of sub tripli. ratio of inverse ratio of 125:343
 = Dupli. ratio of sub tripli. ratio of 343:125
 = Dupli. ratio of 7:5
 = 49:25

28. Find Inverse Ratio of Duplicate Ratio of Triplicate Ratio of 2 : 5

→ = Inverse ratio of Dupli. ratio of Tripli. ratio of 2:5
 = Inverse ratio of Dupli. ratio of 8:125
 = Inverse ratio of 64:15625
 = 15625:64

29. Find Duplicate Ratio of Inverse Ratio of Sub-duplicate ratio of 625 : 225

→ = Dupli. ratio of Inverse ratio of 25:15
 = Dupli. ratio of 15:25
 = 225:625 = 9:25

30. Find duplicate ratio of Sub-triplicate ratio of
Inverse ratio of 10 : 3



$$= \text{Dupli. ratio of subtriplicate ratio of } 3:10$$

$$= \text{Dupli. ratio of } (3)^{1/3} : (10)^{1/3}$$

$$= (3^{1/3})^2 : (10^{1/3})^2$$

$$= 3^{2/3} : 10^{2/3}$$

31. Find sub-triplicate ratio of sub-duplicate ratio of inverse
ratio of 8 : 11



$$= \text{sub-triplicate ratio of sub-dupli. ratio of } 11:8$$

$$= \text{sub-tripli. ratio of } 11^{1/2} : 8^{1/2}$$

$$= (11^{1/2})^{1/3} : (8^{1/2})^{1/3} = 11^{1/6} : 8^{1/6} = 11^{1/6} : (2^3)^{1/6} = 11^{1/6} : 2^{1/2}$$

$$= \sqrt[6]{11} : \sqrt[6]{8}$$

$$= \sqrt[6]{11} : \sqrt{2}$$

32. Find Inverse ratio of Duplicate ratio of Triplicate ratio of
a : b



$$= \text{Inverse ratio of Dupli. ratio of } a^3:b^3$$

$$= \text{Inverse ratio of } a^6:b^6$$

$$= b^6 : a^6 = \left(\frac{b^6}{a^6}\right)$$

33. Find Duplicate ratio of sub-duplicate ratio of 8 : 35



$$\text{Dupli. ratio of } \sqrt{8} : \sqrt{35}$$

$$= 8 : 35$$

34. **Please Note**

1. Duplicate ratio of sub-duplicate ratio of 13 : 18 is **13 : 18**
2. Sub-duplicate ratio of duplicate ratio of m : n is **m : n**
3. Triplicate ratio of sub-triplicate ratio of a : b is **a : b**
4. Sub-triplicate ratio of triplicate ratio of p : q is **p : q**
5. Inverse ratio of Inverse ratio of 35 : 88 is **35 : 88**

35. Find inverse ratio of duplicate ratio of sub-triplicate ratio of sub-triplicate ratio of m : n



$$\begin{aligned}
 &= \text{Inverse of Duplicate of subtripli. of } m^{1/3} : n^{1/3} \\
 &= \text{Inverse of Duplicate of } m^{1/9} : n^{1/9} \\
 &= \text{Inverse of } (m^{1/9})^2 : (n^{1/9})^2 \\
 &= \text{Inverse of } m^{2/9} : n^{2/9} \\
 &= n^{2/9} : m^{2/9} = (n^2)^{1/9} : (m^2)^{1/9} = \sqrt[9]{n^2} : \sqrt[9]{m^2}
 \end{aligned}$$

36. Find compounded ratio of 8 : 3, 6 : 11, 5 : 16, 48 : 15



$$\begin{aligned}
 \text{Compounded Ratio} &= \frac{\text{Product of antecedents}}{\text{Product of all consequents}} \\
 &= \frac{8 \times \cancel{6}^2 \times \cancel{5} \times \cancel{48}}{\cancel{3} \times 11 \times \cancel{16} \times \cancel{15} \cancel{3}} = \frac{16}{11} = 16 : 11
 \end{aligned}$$

37. Find compounded ratio of $a : b$ and $a : b$

→ $a^2 : b^2$

38. Find compounded ratio of $a : b$ and $a^2 : b^2$

→ $a \cdot a^2 : b \cdot b^2 = a^3 : b^3$

39. Find compounded ratio of $8 : 11$ & $11 : 8$

→ $8 \times 11 : 11 \times 8 = 88 : 88 = 1 : 1$

40. **Please Note :**

1. If a ratio is compounded with itself then result is its

Duplicate ratio

2. If a ratio is compounded with its duplicate ratio then result is its

Triplicate ratio

3. If a ratio is compounded with its inverse ratio then result is

Ratio of Equality

41. **Some Calculator Tricks :**

$$\sqrt{64} = (64)^{1/2} = 8$$

$$\sqrt[4]{5000} = (5000)^{1/4} = (5000^{1/2})^{1/2} = 8.40896415253$$

$$\sqrt[8]{256} = (256)^{1/8} = \left[(256^{1/2})^{1/2} \right]^{1/2} = 2$$

$$\sqrt[16]{20821} = \text{Take } 20821 \text{ on calculator \& press } \sqrt{\text{ button 4 times}} = 1.86168564064$$

$$\sqrt[32]{85926.30} = \text{Take } 85926.30 \text{ on calculator \& press } \sqrt{\text{ button 5 times}} = 1.42623615299$$



42.

If I enter a number in calculator and press $\sqrt{\quad}$ button	I will get
Once	2 nd root
Twice	4 th root
Thrice	8 th root
Four Times	16 th root
Five Times	32 nd root
Six Times	64 th root

43. $1.1085^{18} =$ Take 1.1085 on calculator
 then press \times
 then press $=, =, =, = \dots$ till step count comes 19
 $= 6.38620056833$

$2^5 = 32$

$1.18632^{65} =$ Take 1.18632 then \times then $=$ till step count 66 = 66545.

$1.156321^{49} = 1232.65696746$ 7602712

$2.85^{18} = 153888965.491$

$1.2351^{41} = 5751.57768902$

$1.0096^{93} = 2.43158130991$

$2.2283^{16} = 369469.475819$

$3^{12} = 531441$

$6^8 = 1679616$

$1.2258^{46} = 11676.2230502$

$64\sqrt{1,90,800} = 1.209230345$

$8\sqrt{859686} = 5.5181370$
4047

$32\sqrt{1.50632196} = 1.01288451453$



44.

$$\sqrt[5]{17}$$

$$= (17)^{\frac{1}{5}}$$

$$= 17^{0.20}$$

Here $a=17, b=0.20$

$$= 1.76261666858$$

cross-check

$$(1.76261666858)^5$$

$$= 17 \text{ (approx)}$$

How to find the answer of a^b on calculator?
(when b is in fractions)



① Take 'a' on calculator

② press $\sqrt{\quad}$ 12 times

③ Deduct 1

④ Multiply by 'b'

⑤ Add 1

⑥ press 'x=' 12 times

⑦ You will get answer of a^b on calculator screen.

$$45. \sqrt[3]{80} = 80^{1/3} = 80^{0.3333333} = 4.3111448146$$

$$\text{cross-check : } 4.3111448146^3 = 80$$

$$25180^{0.70} = 1207.48546831$$

$$48125^{1.20} = 414350.295255$$

$$\sqrt[10]{28500} = (28500)^{1/10} = 28500^{0.10} = 2.79245919177$$

$$(1,02,800)^{1/20} = (1,02,800)^{0.05} = 1.78211326241$$

$$(56,800)^{1/25} = (56,800)^{0.04} = 1.55030833411$$

46. If $a : b = 3 : 5$, $b : c = 5 : 8$. Find $a : b : c$, $a : c$, $c : a$



$$a : b = 3 : 5$$

$$a : c = 3 : 8$$

$$b : c = 5 : 8$$

$$c : a = 8 : 3$$

$$\therefore a : b : c = 3 : 5 : 8$$

47. If $a : b = 5 : 11$, $b : c = 2 : 7$. Find $a : b : c$, $a : c$, $c : a$



$$a : b = 5 : 11 = 10 : 22$$

$$b : c = 2 : 7 = 22 : 77$$

$$\therefore a : b : c = 10 : 22 : 77$$

$$a : c = 10 : 77 \quad \& \quad c : a = 77 : 10$$

48. If $a : b = 3 : 8$, $b : c = 5 : 13$. Find $a : b : c$, $a : c$, $c : a$



$$a : b = 3 : 8 = 15 : 40$$

$$b : c = 5 : 13 = 40 : 104$$

$$\therefore a : b : c = 15 : 40 : 104$$

$$a : c = 15 : 104$$

$$c : a = 104 : 15$$

49. If $a : b = 3 : 10$, $b : c = 4 : 5$, $c : d = 8 : 13$

Find $a : b : c : d$, $a : d$, $b : d$, $a : c$



$$a : b = 3 : 10 = 12 : 40$$

$$b : c = 4 : 5 = 40 : 50$$

$$\therefore a : b : c = 12 : 40 : 50 = 96 : 320 : 400$$

$$c : d = 8 : 13 = 400 : 650$$

$$\therefore a : d = 48 : 325$$

$$b : d = 160 : 325$$

$$= 32 : 65$$

$$a : c = 48 : 200$$

$$= 6 : 25$$

$$a : b : c : d = 96 : 320 : 400 : 650 = 48 : 160 : 200 : 325$$

50. Find duplicate ratio of subtriplicate ratio of sub-duplicate ratio of inverse ratio of $\sqrt[5]{18} : \sqrt[9]{35}$



$$\begin{aligned}
 &= \text{Dupli. of subtripli. of sub-dupli. of } 35^{1/9} : 18^{1/5} \\
 &= \text{Dupli. of subtripli. of } (35)^{1/18} : (18)^{1/10} \\
 &= \text{Dupli. of } (35^{1/18})^{1/3} : (18^{1/10})^{1/3} \\
 &= \text{Dupli. of } (35)^{1/54} : (18)^{1/30} \\
 &= (35^{1/54})^2 : (18^{1/30})^2 \\
 &= 35^{2/54} : 18^{2/30} = 35^{1/27} : 18^{1/15} = \sqrt[27]{35} : \sqrt[15]{18}
 \end{aligned}$$

$$\begin{aligned}
 52. \quad \sqrt[20]{285798632598} &= (285798632598)^{1/20} \\
 &= 3.75455609938
 \end{aligned}$$

$$\sqrt[16]{826398562281} = 5.55679464007$$

$$(1.126391)^{58} = 995.341154466$$

$$\sqrt[32]{82619835} = 1.76770131544$$

$$(2289365288)^{0.20} = 75.1427214364$$

$$(1.1119381)^{38} = 56.3719725703$$

53. If $a : b = 2 : 3$

$b : c = 5 : 4$

$c : d = 7 : 9$

Find $a : b : c : d$, $a : d$, $b : d$, $a : c$



$a : b = 2 : 3 = 10 : 15$

$b : c = 5 : 4 = 15 : 12$

$\therefore a : b : c = 10 : 15 : 12 = 70 : 105 : 84$

$c : d = 7 : 9 = 84 : 108$

$\therefore a : b : c : d = 70 : 105 : 84 : 108$

$a : d = 70 : 108 = 35 : 54$

$b : d = 105 : 108 = 35 : 36$

$a : c = 70 : 84 = 5 : 6$

54. $a : b = 2 : 3$

$a : c = 3 : 8$

$a : d = 5 : 11$

Find $a : b : c : d$



$a : b = 2 : 3 = 30 : 45$

$a : c = 3 : 8 = 30 : 80$

$a : d = 5 : 11 = 30 : 66$

$\therefore a : b : c : d$
 $= 30 : 45 : 80 : 66$

55. If $a : c = 9 : 8$

$b : d = 5 : 4$

$a : d = 2 : 7$

Find $a : b : c : d$



$a : c = 9 : 8 = 18 : 16$

$a : d = 2 : 7 = 18 : 63$

$a : b : c : d = 72 : 315 : 64 : 252$

$a : c : d = 18 : 16 : 63$
 $= 72 : 64 : 252$

$b : d = 5 : 4$
 $= 315 : 252$

56. $a:d = 2:17$

$c:b = 15:4$

$c:d = 2:9$

Find $a:b:c:d$



$a:d = 2:17 = 18:153$

$c:d = 2:9 = 34:153$

$a:c:d = 18:34:153 = 270:510:2295$

$c:b = 15:4 = 510:136$

$a:b:c:d = 270:136:510:2295$

57. $p:q = 3:11$

$r:s = 5:6$

$p:r = 4:7$

Find $p:q:r:s$



$p:q = 3:11 = 12:44$

$p:r = 4:7 = 12:21$

$\therefore p:q:r = 12:44:21 = 60:220:105$

$r:s = 5:6 = 105:126$

$\therefore p:q:r:s = 60:220:105:126$

Q. If $a:b = 2:3$, $b:c = 2:3$, $c:d = 2:3$, $d:e = 2:3$

Find $a:b:c:d:e$

A. $a:b = 2:3 = 4:6$

$b:c = 2:3 = 6:9$

$a:b:c = 4:6:9 = 8:12:18$

$c:d = 2:3 = 18:27$

$\therefore a:b:c:d = 8:12:18:27$

$= 16:24:36:54$

$d:e = 2:3 = 54:81$

$\therefore a:b:c:d:e = 16:24:36:54:81$

58.

Ratio	Simplest form of the ratio
30 : 20	3 : 2
80 : 64	40 : 32 = 20 : 16 = 10 : 8 = 5 : 4
1.50 : 2.50	15 : 25 = 3 : 5
8 : 9	8 : 9
2.75 : 8.25	275 : 825 = 11 : 33 = 1 : 3
8.26 : 3.30	826 : 330 = 413 : 165
$\frac{8}{3} : \frac{9}{3}$	8 : 9
$\frac{3}{8} : \frac{11}{3}$	$\frac{3}{8} : \frac{11}{3} = 9 : 88$ (OR) $\frac{\frac{3}{8}}{\frac{11}{3}} = \frac{3}{8} \times \frac{3}{11} = \frac{9}{88} = 9 : 88$
$2\frac{1}{3} : 8\frac{2}{3}$	$2\frac{1}{3} : 8\frac{2}{3}$ $\frac{7}{3} : \frac{26}{3} = 7 : 26$
$7\frac{2}{7} : 19\frac{3}{7}$	$7\frac{2}{7} : 19\frac{3}{7} = \frac{51}{7} : \frac{136}{7} = 51 : 136$ $= 3 : 8$
$8\frac{2}{9} : 5\frac{3}{7}$	$\left(\frac{8\frac{2}{9}}{5\frac{3}{7}}\right) = \left(\frac{74/9}{38/7}\right) = \frac{74}{9} \times \frac{7}{38} = \frac{518}{342} = \frac{259}{171}$
$\sqrt{5} : \sqrt{8}$	$\sqrt{5} : \sqrt{8}$
$\sqrt{25} : \sqrt{81}$	5 : 9
$11\frac{2}{7} : 15\frac{2}{3}$	$\frac{11\frac{2}{7}}{15\frac{2}{3}} = \frac{79}{7} \div \frac{47}{3} = \frac{79}{7} \times \frac{3}{47} = \frac{237}{329}$

59. Multiplying Ratio

Mr A's income in year 2023 is ₹ 5,00,000. In 2024 his income changed in the ratio of 25 : 27. Find his new income?



Years	2023	2024
Income	5,00,000	?
Ratio	25	27

$$? = \text{New Income} = \frac{27 \times 5,00,000}{25} = 5,40,000$$

$$\text{New Income} = 5,00,000 \times \frac{27}{25}$$

$$\text{New Income} = \text{old Income} \times \left(\begin{array}{l} \text{Inverse ratio} \\ \text{of 25:27} \end{array} \right)$$

$$\text{New Income} = \text{old Income} \times \text{Multiplying Ratio}$$

$$\text{New quantity} = \text{old quantity} \times \text{Multiplying ratio}$$

60. India's population in 2011 census is 140 crores. In 2021 population has changed in the ratio of 7 : 9. Find new population.

$$\rightarrow \text{New population} = \text{old population} \times \text{Multiplying ratio}$$

where Multiplying Ratio = Inverse of given ratio

$$\therefore \text{New population} = 140 \text{ crores} \times \frac{9}{7} = 180 \text{ crores}$$

$$\text{New quantity} = \text{old quantity} \times \text{multiplying ratio}$$

$$\text{old quantity} = \text{New quantity} \times \text{Given ratio}$$

61. Vinod's weight is x kgs, his weight changes in the ratio of 5 : 7

Find his new weight?

→
$$\text{New weight} = \text{old weight} \times \text{Multiplying ratio}$$

$$= x \times \frac{7}{5} = \frac{7x}{5}$$

old weight	new weight
5	7
x	?

$$? = x \times \frac{7}{5}$$

$$\text{New weight} = \text{old weight} \times \text{Multi. Ratio}$$

62. Anand's income in 2028 year is ₹ 50 crores. In comparison with year 2027 his income has changed in the ratio of 9 : 10.

Find his income for year 2027



$$\text{old Income} = \text{New Income} \times \text{Given ratio}$$

Anand's Income for the year 2027 = $50 \text{ crores} \times \frac{9}{10} = 45 \text{ crores}$

Old Income	New Income
2027	2028
9	10
?	50 crores

63. 2 quantities are in the ratio of 3 : 4. If 6 is added to both terms ratio becomes 4 : 5. Find old quantities.

→ Let those 2 quantities be $3x$ & $4x$

$$\frac{3x + 6}{4x + 6} = \frac{4}{5}$$

$$15x + 30 = 16x + 24$$

$$6 = x$$

$$5(3x + 6) = 4(4x + 6) \quad \therefore \text{old quantities : } \begin{array}{l} 3x = 18 \\ 4x = 24 \end{array}$$

64. 2 quantities are in the ratio of 8 : 9. If 4 is added to 1st term & 7 is deducted from second term, ratio becomes 36 : 29. Find 2 quantities.

→ Let those 2 quantities be $8m$, $9m$

$$\frac{8m + 4}{9m - 7} = \frac{36}{29}$$

$$29(8m + 4) = 36(9m - 7)$$

$$232m + 116 = 324m - 252$$

$$368 = 92m$$

$$m = 4$$

$$\therefore \text{Those quantities are : } \begin{array}{l} 8m = 8 \times 4 = 32 \\ 9m = 9 \times 4 = 36 \end{array}$$

66. Present age of father and son is in the ratio of 16 : 1. After 8 years their age will be in the ratio of 4 : 1. Find the sum of their present age.

→ Let Present age of Father = $16x$, son = x

$$\frac{16x + 8}{x + 8} = \frac{4}{1} \quad \therefore 16x + 8 = 4x + 32 \quad \therefore x = 2$$

$$12x = 24$$

$$\therefore \text{present age of Father} = 16x = 16 \times 2 = 32$$
$$\text{son} = x = 2$$

$$\therefore \text{sum of their present age} = 32 + 2 = 34 \text{ years}$$

67. 2 quantities are in the ratio of 7 : 8, if 2 is subtracted from 1st and 12 is added to 2nd term, ratio becomes 2 : 3. Find quantities.

→ Let those 2 quantities be $7x$ & $8x$

$$\frac{7x - 2}{8x + 12} = \frac{2}{3}$$

\therefore Those quantities are

$$21x - 6 = 16x + 24$$

$$7x = 7 \times 6 = 42$$

$$5x = 30$$

$$8x = 8 \times 6 = 48$$

$$x = 6$$

68. 3 quantities are in the ratio of 2 : 3 : 5. If 4, 6, 0 is added to 3 terms resp. then ratio becomes 2 : 3 : 4. Find sum of original quantities.

→ Let those 3 quantities be : $2y, 3y, 5y$

$$(2y + 4) : (3y + 6) : (5y + 0) = 2 : 3 : 4$$

$$\left(\frac{2y + 4}{3y + 6}\right) = \frac{2}{3}$$

$$\frac{3y + 6}{5y + 0} = \frac{3}{4}$$

$$6y + 12 = 6y + 12$$

$$12y + 24 = 15y$$

$$0 = 0$$

$$24 = 3y$$

$$y = 8$$

\therefore Those original quantities are : 16, 24, 40

$$\text{Sum of original quantities} = 16 + 24 + 40 = 80$$

69. A mixture contains milk, water in the ratio of 3 : 8 on adding 5 litres of milk the ratio becomes 1 : 2. Find milk quantity, water quantity in litres in original mixture.

→

Milk : water
 $3x : 8x$

5 Litres of milk

$$\frac{3x + 5}{8x} = \frac{1}{2}$$

$$6x + 10 = 8x$$

$$10 = 2x$$

$$\therefore x = 5$$

\therefore Original mixture contains : milk = $3x = 15$ litres
water = $8x = 40$ litres

70. There are 14 litres of mixture in 1st container where ratio of milk, water is 2 : 5. In 30 litres mixture of 2nd container ratio of water, milk is 8 : 7. If mixture of these containers is put in 3rd container. Find ratio of milk, water quantity in 3rd container.

→

<p>Milk : water $2x : 5x$ 4 litres : 10 litres</p>	<p>Milk : water $7y : 8y$ 14 litres : 16 litres</p>	
container-1	container-2	
<p>Milk : water = $\left(\frac{18 \text{ litres}}{26 \text{ litres}} \right)$ = 9 : 13</p>		<p>Milk = 4 + 14 Water = 10 + 16</p>
		container-3

71. Distribute ₹ 600 in the ratio of 2 : 3 : 7

→ Let 3 quantities be $2x, 3x, 7x$

$$2x + 3x + 7x = 600$$

$$12x = 600$$

$$x = 50$$

∴ 3 shares of ₹ 600 are : $2x = ₹ 100$
 $3x = ₹ 150$
 $7x = ₹ 350$

72. Distribute ₹ 5,00,000 among A, B, C in the ratio of 8 : 3 : 19

→

$$A's \text{ share} : 5,00,000 \times \frac{8}{30} = 1,33,333.33333$$

$$B's \text{ share} : 5,00,000 \times \frac{3}{30} = 50,000$$

$$C's \text{ share} : 5,00,000 \times \frac{19}{30} = 3,16,666.66666$$

₹ 5,00,000

73. Distribute ₹ 80,000 among A, B, C in the ratio of

$$\frac{1}{3} : \frac{2}{3} : \frac{1}{2}$$

$$\frac{1}{3} : \frac{2}{3} : \frac{1}{2}$$

$$A's \text{ share} : \frac{2}{9} \times 80,000 = 17,777.7777$$

$$B's \text{ share} : \frac{4}{9} \times 80,000 = 35,555.5555$$

$$C's \text{ share} : \frac{3}{9} \times 80,000 = 26,666.6666$$

$$= 2 : 4 : 3$$

(OR)

$$A : B : C = \frac{1}{3} : \frac{2}{3} : \frac{1}{2} = \frac{x}{3} : \frac{2x}{3} : \frac{x}{2} \quad \frac{x}{3} + \frac{2x}{3} + \frac{x}{2} = 80,000$$

$$A's \text{ share} = \frac{x}{3} = 17,777.77777 \quad 2x + 4x + 3x = 4,80,000$$

$$9x = 4,80,000$$

$$B's \text{ share} = \frac{2x}{3} = 35,555.55555 \quad x = 53,333.333$$

$$C's \text{ share} = \frac{x}{2} = 26,666.66666$$

74. Divide ₹ 600 among A, B, C, D in the ratio of

$$2\frac{1}{3}, 8\frac{2}{3}, 5\frac{1}{3}, 10\frac{2}{3}$$

₹



$$\begin{aligned} &2\frac{1}{3} : 8\frac{2}{3} : 5\frac{1}{3} : 10\frac{2}{3} \\ &= \frac{7}{3} : \frac{26}{3} : \frac{16}{3} : \frac{32}{3} \\ &= 7 : 26 : 16 : 32 \end{aligned}$$

$$\text{A's share} : \frac{7}{81} \times ₹600 = 51.85185$$

$$\text{B's share} : \frac{26}{81} \times ₹600 = 192.5926$$

$$\text{C's share} : \frac{16}{81} \times ₹600 = 118.5185$$

$$\text{D's share} : \frac{32}{81} \times ₹600 = 237.037$$

PROPORTIONS

75. When a, b, c, d are said to be in proportion?



a, b, c, d are said to be in proportion if

$$\left(\text{Ratio of 1st to 2nd term} \right) = \left(\text{Ratio of 3rd to 4th term} \right)$$

i.e. $a:b = c:d$

i.e. $\frac{a}{b} = \frac{c}{d}$

i.e. $ad = bc$

i.e. product of extremes = product of means

5, 15, 200, 600 are in proportion as $5:15 = 200:600 = 1:3$
 (OR) $5 \times 600 = 15 \times 200 = 3000$



76. If $p : q = r : s$
OR $(ps) = (qr)$
then p, q, r, s are said to be in proportion

77. 18, 30, $2k$, 900 are in proportion. Find the value of k .

→ product of extremes = product of means (OR) $18 : 30 = 2k : 900$
 $18 \times 900 = 30 \times 2k$ $\frac{18}{30} = \frac{2k}{900}$
 $18 \times 900 = 60k$ $18 \times 900 = 30 \times 2k$
 $k = 270$ $\therefore k = 270$

78. When a, b, c, d are said to be in continued proportion?

→ If $a : b = b : c = c : d$ then 4 terms a, b, c, d are said to be in continued proportion.

① 20, 30, 18, 27 Here $20 : 30 = 18 : 27 \neq 30 : 18$

20, 30, 18, 27 are in proportion but not in continued proportion

② 8, 12, 18, 27 Here $8 : 12 = 12 : 18 = 18 : 27 = 2 : 3$

\therefore 8, 12, 18, 27 are in proportion as well as continued proportion.

Did You Know ?

If 4 quantities are in continued proportion then they must be in proportion; whereas

If 4 quantities are in proportion then it is not necessary that they are in continued proportion.

79.

4 Quantities	Whether quantities are in	
	Proportion?	Continued Proportion?
8, 12, 18, 27	Yes	Yes
100, 200, 500, 1000	Yes	No
a, ab, abc, abcd	NO	NO
1, 3, 5, 7	NO	NO
18, 20, 90, 100	Yes	NO
5, 10, 20, 40	Yes	Yes
20, 2, 40, 4	Yes	NO
a, ax, ax ² , ax ³	Yes	Yes
k, k ³ , k ⁵ , k ⁷	Yes	Yes
a, ax, b, bx	Yes	No

80.

p, q, r, s are said to be in

Proportion

When $p:q = r:s$
 i.e. $\frac{p}{q} = \frac{r}{s}$
 i.e. $ps = qr$

Continued Proportion

When $p:q = q:r = r:s$
 i.e. $\frac{p}{q} = \frac{q}{r} = \frac{r}{s}$

81. When a, b, c are said to be in Proportion OR Continued

Proportion?

Three terms are said to be in proportion & continued prop if (Ratio of 1st to 2nd term) = (Ratio of 2nd to 3rd term)

If $a:b = b:c$ i.e. $\frac{a}{b} = \frac{b}{c}$ i.e. $b^2 = ac$

then a, b, c are said to be in propⁿ & conti. propⁿ.

82. If ratio of 1st term to 2nd term is equal to ratio of 2nd term to 3rd term then 3 quantities are said to be in proportion.

If 3 quantities are in propⁿ then they are in conti. propⁿ also.

83. Quantities a, b, c, d, e, f, g, h are said to be in continued proportion.

If $a:b = b:c = c:d = d:e = e:f = f:g = g:h$
2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 are in continued propⁿ

84. Quantities are in Proportion	Value of k
a, b, 13, k	product of extremes = product of means $ak = 13b$ $k = (13b/a)$
2a, 3b, 8k, 100	$2a \times 100 = 3b \times 8k$ $\therefore k = \frac{200a}{24b} = \left(\frac{25a}{3b}\right)$
18, 20, 25, -3k	$18 \times -3k = 20 \times 25$ $k = \frac{500}{-54} = -250/27$ $k = -9.25925925925$
5, 12, 15, 8k	$5 \times 8k = 12 \times 15$ $k = \frac{180}{40} = \frac{9}{2} = 4.50$
5m, -2k, $\frac{8}{3}$, $\frac{7}{33}$	$5m \times \frac{7}{33} = -2k \times \frac{8}{3}$ $k = 5m \times \frac{7}{33} \times \frac{3}{8} \times \frac{1}{-2} = \frac{-105m}{528} = \frac{-35m}{176}$
18, k, 40.50	$k^2 = 18 \times 40.50 = 729$ $\therefore k = 27$

85. Find 4th proportional to 18, 26, 90

→ Let 4th proportional to 18, 26, 90 be 'm'
 \therefore 18, 26, 90, m are in proportion.
 $\therefore 18m = 26 \times 90$
 $m = 130$
 \therefore 4th proportional to 18, 26, 90 is 130.

86. Find 4th proportional to $\frac{2}{3}, \frac{8}{7}, \frac{9}{11}$

→ Let 4th proportional to $\frac{2}{3}, \frac{8}{7}, \frac{9}{11}$ be x
 \therefore product of extremes = product of means
 $\frac{2}{3} \times x = \frac{8}{7} \times \frac{9}{11}$
 $\therefore x = \frac{8}{7} \times \frac{9}{11} \times \frac{3}{2} = \frac{216}{154} = \frac{108}{77}$
 \therefore 4th proportional to $\frac{2}{3}, \frac{8}{7}, \frac{9}{11}$ is $\frac{108}{77}$.

87. Find third proportional to 12, 40

→ Let third proportional to 12, 40 be m
 $\therefore \frac{12}{40} = \frac{40}{m} \quad \therefore 12m = 1600$
 $m = \frac{1600}{12} = \frac{400}{3} = 133.33333$
 $\therefore 12m = 40 \times 40 = 133\frac{1}{3}$

88. Find Mean proportional to 12, 100

→ Let mean proportional to 12, 100 be m
 $\therefore 12, m, 100$ must be in proportion.
 $\therefore m^2 = 1200$
 $m = \sqrt{1200} = 34.6410161513$



Properties of Proportion

89. If a, b, c, d are in proportion then $\frac{a}{b} = \frac{c}{d}$

then

→ ① If $\frac{a}{b} = \frac{c}{d}$ then

$$\left(\frac{b}{a}\right) = \left(\frac{d}{c}\right) \dots\dots\dots \text{Invertendo}$$

If $\left(\frac{3}{5} = \frac{P}{9}\right)$ then as per Invertendo $\left(\frac{9}{P} = \frac{5}{3}\right)$

② If $\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right)$

Adding '1' on both sides

$$\left(\frac{a}{b}\right) + 1 = \left(\frac{c}{d}\right) + 1$$

$$\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right) \dots\dots\dots \text{(Componendo)}$$

If $\frac{3}{7} = \frac{12}{28}$ then $\frac{3+7}{7} = \frac{12+28}{28} = \frac{10}{7}$

③ If $\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right)$

subtracting '1' from both sides

$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$

then $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right) \dots\dots\dots \text{(Dividendo)}$

If $\frac{70}{10} = \frac{14}{2}$ then $\frac{70-10}{10} = \frac{14-2}{2} = \frac{6}{1}$

④ If $\frac{a}{b} = \frac{c}{d}$ then

$$\frac{a+b}{b} = \frac{c+d}{d} \dots\dots ①$$

$$\frac{a-b}{b} = \frac{c-d}{d} \dots\dots ②$$

eqn ① \div eqn ②

$$= \left(\frac{\frac{a+b}{b}}{\frac{a-b}{b}} \right) = \left(\frac{\frac{c+d}{d}}{\frac{c-d}{d}} \right)$$

$$\left(\frac{a+b}{a-b} \right) = \left(\frac{c+d}{c-d} \right) \dots\dots \text{(componendo \& Dividendo)}$$

⑤ If $\frac{a}{b} = \frac{c}{d}$ then

$$ad = bc$$

then $\frac{a}{c} = \frac{b}{d} \dots\dots \text{(Alternendo)}$

If $\frac{3}{30} = \frac{8}{80}$ then $\frac{3}{8} = \frac{30}{80}$

⑥ If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \left(\frac{a+c}{b+d} \right) = \left(\frac{c+a}{d+b} \right)$

If $\frac{p}{q} = \frac{r}{s}$ then $\frac{p}{q} = \frac{r}{s} = \frac{p+r}{q+s} \dots\dots \text{Addendo}$

$$\frac{3}{11} = \frac{30}{110} = \left(\frac{3+30}{11+110} \right) = \left(\frac{33}{121} \right)$$

$$\frac{5}{10} = \frac{90}{180} = \frac{5+90}{10+180}$$

⑦ If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \left(\frac{a-c}{b-d} \right) = \left(\frac{c-a}{d-b} \right)$

$\dots\dots \text{Subtrahendo}$

$$\frac{18}{45} = \frac{2}{5} = \frac{18-2}{45-5} = \frac{16}{40}$$

$$\frac{88}{55} = \frac{8}{5} = \frac{88-8}{55-5} = \frac{80}{50}$$

90. If p, q, r, s are in proportion i.e.

$\left(\frac{p}{q}\right) = \left(\frac{r}{s}\right)$ then

1. $\left(\frac{q}{p}\right) = \left(\frac{s}{r}\right)$ Invertendo

2. $\left(\frac{p}{q}\right) = \left(\frac{r}{s}\right) = \frac{p+r}{q+s}$ Addendo

3. $\left(\frac{p}{q}\right) = \left(\frac{r}{s}\right) = \frac{p-r}{q-s} = \frac{r-p}{s-q}$ Subtrahendo

4. $\left(\frac{p}{r}\right) = \left(\frac{q}{s}\right)$ Alternendo

5. $\left(\frac{p+q}{q}\right) = \left(\frac{r+s}{s}\right)$ Componendo

6. $\left(\frac{p-q}{q}\right) = \left(\frac{r-s}{s}\right)$ Dividendo

7. $\left(\frac{p+q}{p-q}\right) = \left(\frac{r+s}{r-s}\right)$ Componendo & Dividendo

$\frac{36}{84} = \frac{3}{7} = \frac{36+3}{84+7} = \frac{36-3}{84-7} = \frac{3}{7} = \frac{39}{91} = \frac{33}{77}$ Addendo & subtrahendo

91. If $\frac{a}{b} = \frac{c}{d}$ then $\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right) = \left(\frac{a+c}{b+d}\right)$ Addendo
 & $\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right) = \left(\frac{a-c}{b-d}\right)$ Subtrahendo
 $\therefore \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$ Addendo & Subtrahendo



92. If $\left(\frac{a}{7}\right) = \left(\frac{b}{8}\right) = \left(\frac{c}{13}\right)$ Find value of $\left(\frac{10a+11b-3c}{2b}\right) = ?$

→ $\frac{a}{7} = \frac{b}{8} = \frac{c}{13}$ ----- Given

$\frac{10a}{70} = \frac{11b}{88} = \frac{3c}{39} = \frac{10a+11b-3c}{70+88-39}$ Addendo & Subtrahendo

$\frac{10a+11b-3c}{119} = \frac{2b}{8 \times 2}$

∴ $\frac{10a+11b-3c}{2b} = \frac{119}{16}$

(OR) $\frac{a}{7} = \frac{b}{8} = \frac{c}{13} = x$ ∴ $a=7x, b=8x, c=13x$

$\frac{10a+11b-3c}{2b} = \frac{10(7x) + 11(8x) - 3(13x)}{2(8x)}$

$= \left(\frac{70x + 88x - 39x}{16x}\right) = \frac{119x}{16x} = \left(\frac{119}{16}\right)$

93. If $\frac{a}{20} = \frac{b}{13}$ Find value of $\left(\frac{7a+12b}{3b}\right)$

→ $\frac{a}{20} = \frac{b}{13} = \frac{7a}{140} = \frac{12b}{156} = \frac{7a+12b}{296}$

$\frac{7a+12b}{296} = \frac{3b}{39}$ ∴ $\frac{7a+12b}{3b} = \left(\frac{296}{39}\right)$

(OR) $\frac{a}{20} = \frac{b}{13} = k$ ∴ $a=20k, b=13k$

$\left(\frac{7a+12b}{3b}\right) = \frac{7 \times 20k + 12 \times 13k}{3 \times 13k} = \frac{296k}{39k} = \left(\frac{296}{39}\right)$

94. If $\frac{a}{30} = \frac{b}{60} = \frac{c}{147}$ then Find value of $\left(\frac{3a + 2b - c}{b}\right)$

$$\frac{a}{30} = \frac{b}{60} = \frac{c}{147}$$

$$\frac{3a}{90} = \frac{2b}{120} = \frac{c}{147} = \frac{3a+2b-c}{90+120-147}$$

$$\frac{3a+2b-c}{63} = \frac{b}{60}$$

$$\therefore \frac{3a+2b-c}{b} = \frac{63}{60} = \frac{21}{20}$$

$$\frac{a}{30} = \frac{b}{60} = \frac{c}{147} = m$$

$$\therefore a = 30m, b = 60m, c = 147m$$

$$\left(\frac{3a+2b-c}{b}\right)$$

$$= \frac{3(30m) + 2(60m) - 147m}{60m}$$

$$= \frac{63m}{60m} = \frac{63}{60} = \left(\frac{21}{20}\right)$$

95. If $\frac{a}{3} = \frac{b}{8} = \frac{c}{5}$ Find value of $\left(\frac{12a + 8b - 2c}{7c}\right)$

$$\rightarrow \frac{a}{3} = \frac{b}{8} = \frac{c}{5} = \frac{12a}{36} = \frac{8b}{64} = \frac{2c}{10} = \frac{12a + 8b - 2c}{36 + 64 - 10}$$

$$\frac{12a + 8b - 2c}{36 + 64 - 10} = \frac{7c}{7 \times 5}$$

$$\frac{12a + 8b - 2c}{7c} = \frac{90}{35} = \frac{18}{7}$$

(OR) $a = 3m, b = 8m, c = 5m$

$$\frac{12a + 8b - 2c}{7c} = \frac{12(3m) + 8(8m) - 2(5m)}{7 \times 5m}$$

$$= \frac{90m}{35m} = \frac{90}{35} = \left(\frac{18}{7}\right)$$

$$96. \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{(a+c)}{(b+d)} = \frac{e}{f} = \frac{(a+c-e)}{(b+d-f)}$$

$$= \frac{(a-c)}{(b-d)} = \frac{e}{f} = \frac{(a-c-e)}{(b-d-f)}$$

$$\frac{100}{25} = \frac{16}{4} = \frac{20}{5} = \frac{(100+16)}{(25+4)} = \frac{(100-16)}{(25-4)} = \frac{20}{5} = \frac{(100-16-20)}{(25-4-5)} = \frac{64}{16} = \frac{4}{1}$$

97. If $\frac{a}{8} = \frac{b}{20} = \frac{c}{35}$ then $\frac{a}{8} = \frac{3a+2b-c}{9}$

True / False

→ $\frac{a}{8} = \frac{b}{20} = \frac{c}{35} = m$

∴ $a = 8m, b = 20m, c = 35m$

L.H.S. = $\frac{a}{8} = \frac{8m}{8} = m$

R.H.S. = $\frac{3a+2b-c}{9} = \frac{3(8m) + 2(20m) - 35m}{9}$

$$= \frac{24m + 40m - 35m}{9} = \frac{29m}{9}$$

AS L.H.S. \neq R.H.S.

Given statement is False

98. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a^3 + c^3}{b^3 + d^3} = \frac{(a+c)^3}{(b+d)^3}$

Correct / incorrect



$\frac{a}{b} = \frac{c}{d}$ Given

Taking cube on both sides

$\left(\frac{a}{b}\right)^3 = \left(\frac{c}{d}\right)^3$

$\frac{a^3}{b^3} = \frac{c^3}{d^3} = \frac{a^3 + c^3}{b^3 + d^3}$ ①

$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

Taking cube on both sides

$\left(\frac{a}{b}\right)^3 = \left(\frac{c}{d}\right)^3 = \left(\frac{a+c}{b+d}\right)^3$

$\frac{a^3}{b^3} = \frac{c^3}{d^3} = \frac{(a+c)^3}{(b+d)^3}$ ②

From Eqⁿs ① & ②

$\frac{a^3 + c^3}{b^3 + d^3} = \frac{(a+c)^3}{(b+d)^3}$

∴ Given statement is correct

$\frac{a}{b} = \frac{c}{d}$ L.H.S. = $\frac{a^3 + c^3}{b^3 + d^3} = \frac{5^3 + 10^3}{11^3 + 22^3} = \frac{1125}{11979} = \frac{125}{1331}$

$\frac{5}{11} = \frac{10}{22}$ R.H.S. = $\frac{(a+c)^3}{(b+d)^3} = \frac{(5+10)^3}{(11+22)^3} = \frac{3375}{35937} = \frac{125}{1331}$



99. Some Imp formulae from School

$$1. (a + b)^2 = (a+b)(a+b)$$

$$= a^2 + ab + ba + b^2$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$2. (a - b)^2 = (a-b)(a-b) = a^2 - ab - ab + b^2 = a^2 + b^2 - 2ab$$

$$3. (a^2 + b^2) = (a+b)^2 - 2ab = (a-b)^2 + 2ab$$

$$4. (a + b)^3 = (a+b)(a+b)(a+b)$$

$$= (a^2 + b^2 + 2ab)(a+b)$$

$$= a^3 + a^2b + ab^2 + b^3 + 2a^2b + 2ab^2$$

$$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 = a^3 + b^3 + 3ab(a+b)$$

$$5. (a^3 + b^3) = (a+b)^3 - 3ab(a+b)$$

$$6. (a - b)^3 = (a-b)(a-b)(a-b)$$

$$= (a-b)(a^2 + b^2 - 2ab)$$

$$= a^3 + ab^2 - 2a^2b - a^2b - b^3 + 2ab^2$$

$$= a^3 - b^3 + 3ab^2 - 3a^2b$$

$$= a^3 - b^3 + 3ab(b-a) = a^3 - b^3 - 3ab(a-b)$$

$$7. (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$8. (a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ac$$

$$9. (a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ac$$

$$10. (a^2 - b^2) = (a+b)(a-b)$$

$$11. (a - b)(a^2 + ab + b^2) = a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

$$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$12. (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$13. (a - b)^2 = (a+b)^2 - 4ab$$

$$14. (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$



100. Find 4th proportional to $\frac{8}{11}, \frac{3}{22}, \frac{3m}{19}$

→ Let 4th proportional be k

$\therefore \frac{8}{11}, \frac{3}{22}, \frac{3m}{19}, k$ are in proportion

\therefore product of extremes = product of means

$$\frac{8k}{11} = \frac{9m}{418}$$

$$\therefore k = \frac{9m}{418} \times \frac{11}{8} = \frac{99m}{3344} = \frac{9m}{304}$$

\therefore 4th proportional to $\frac{8}{11}, \frac{3}{22}, \frac{3m}{19}$ is $\frac{9m}{304}$

101. Find mean proportional to 62.50, 10

→ Let mean proportional be x

$\therefore 62.50, x, 10$ are in proportion

\therefore (square of middle term) = product of extremes

$$x^2 = 62.50 \times 10 = 625$$

$$\therefore x = 25$$

\therefore Mean proportional to 62.50 & 10 is 25.

102. Find fourth proportional to -4, -6, -9

→ Let 4th proportional to x

$\therefore -4, -6, -9, x$ are in proportion

$$\therefore -4 \times x = -6 \times -9$$

$$x = \frac{54}{-4} = -\frac{27}{2} = -13.50$$

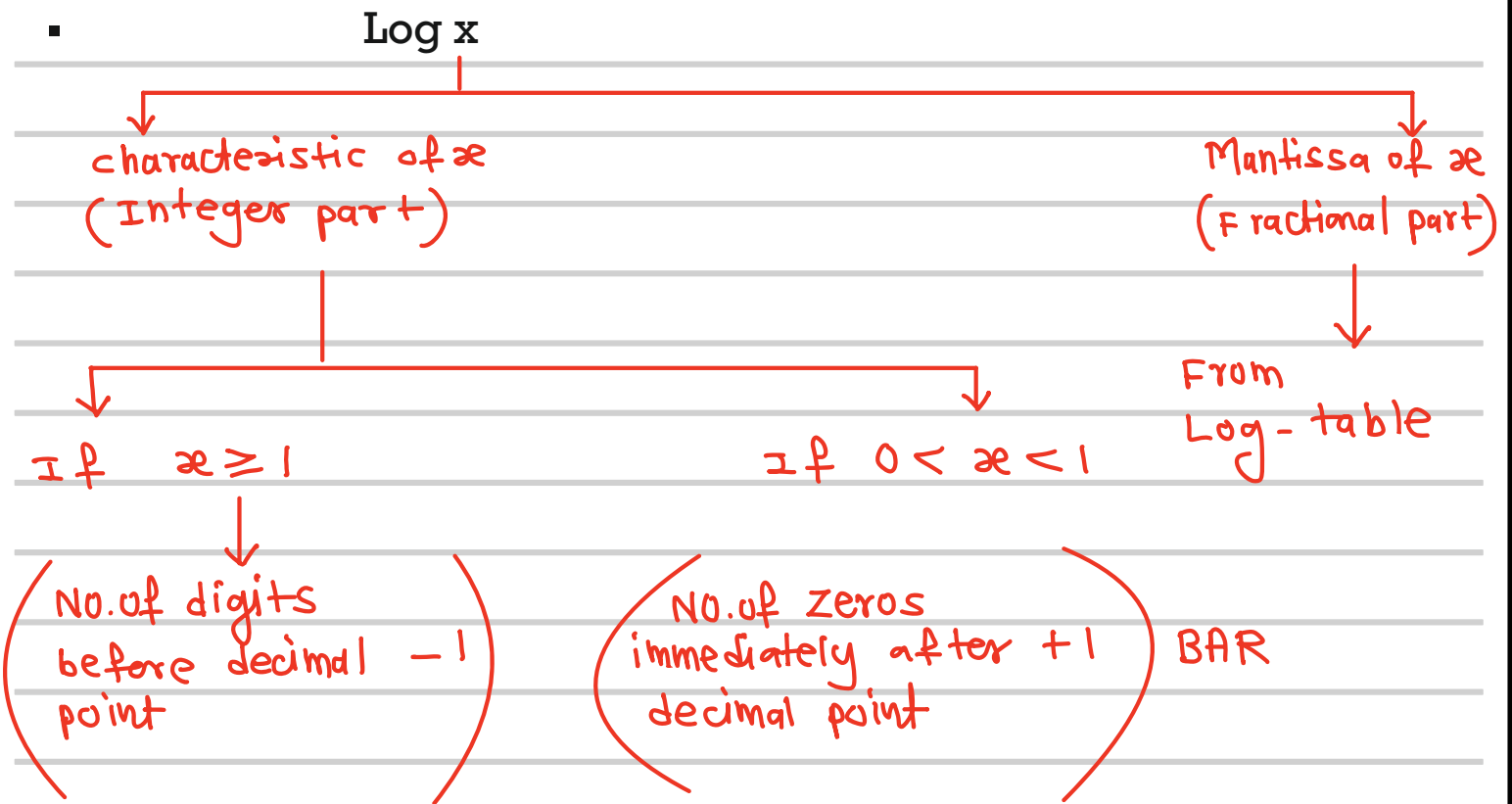
\therefore 4th proportional to -4, -6, -9 is -13.50

Logarithms (Logs)

103. Log of a number consists of 2 parts

- Integer Part = **Characteristic**
- Fractional / Decimal Part = **Mantissa**

▪ $\text{Log } x = (\text{Characteristic of } x + \text{Mantissa of } x)$



$\text{Log } 283.6 = \text{characteristic of } 283.6 + \text{mantissa of } 283.6$
 $= (3-1) + 0.4527$
 $\text{Log } 283.6 = 2 + 0.4527 = 2.4527$

$\text{Log } 28.36 = 1 + 0.4527 = 1.4527$

$\text{Log } 2.836 = 0 + 0.4527 = 0.4527$

$\text{Log } 7 = \text{Log } 7.000 = 0 + 0.8451 = 0.8451$



104.

x	Characteristic of x	Mantissa of x	Log x
786.89	$3-1=2$	0.8959	2.8959
78.33	$2-1=1$	0.8940	1.8940
2358.00	$4-1=3$	0.3726	3.3726
29.98	$2-1=1$	0.4769	1.4769
711.68	$3-1=2$	0.8523	2.8523
859.63	$3-1=2$	0.9343	2.9343
10.00	$2-1=1$	0.000	1.0000
100.00	$3-1=2$	0.000	2
939.8	$3-1=2$	0.9731	2.9731
2,35,288.00	$6-1=5$	0.3717	5.3717
8.000	$1-1=0$	0.9031	0.9031
893.6	$3-1=2$	0.9512	2.9512
28.11	$2-1=1$	0.4489	1.4489
636363.3	$6-1=5$	0.8038	5.8038
211.93	$3-1=2$	0.3261	2.3261
71.77	$2-1=1$	0.8559	1.8559
8.123	$1-1=0$	0.9098	0.9098
11.263	$2-1=1$	0.0515	1.0515
1.093	$1-1=0$	0.0386	0.0386
3.861	$1-1=0$	0.5867	0.5867
6.000	$1-1=0$	0.7782	0.7782
5.000	$1-1=0$	0.6990	0.6990
7,00,000.00	$6-1=5$	0.8451	5.8451
70,000	$5-1=4$	0.8451	4.8451
7000	$4-1=3$	0.8451	3.8451
700	$3-1=2$	0.8451	2.8451

x	Characteristic of x	Mantissa of x	Log x
70	$2 - 1 = 1$	0.8451	1.8451
7.000	$1 - 1 = 0$	0.8451	0.8451
0.70	$\bar{1}$	0.8451	$\bar{1}.8451$ $= -1 + 0.8451$ $= -0.1549$
0.070	$\bar{2}$	0.8451	$\bar{2}.8451$ $= -2 + 0.8451$ $= -1.1549$
0.0070	$\bar{3}$	0.8451	$\bar{3}.8451$ $= -3 + 0.8451$ $= -2.1549$
0.0008263	$\bar{4}$	0.9172	$\bar{4}.9172$ $= -4 + 0.9172$ $= -3.0828$
8.009	$1 - 1 = 0$	0.9036	0.9036

105.

$$1. \text{Log } 89.53 = 1 + 0.9519 = 1.9519$$

$$2. \text{Log } 63.33 = 1 + 0.8016 = 1.8016$$

$$3. \text{Log } 998.6 = 2 + 0.9994 = 2.9994$$

$$4. \text{Log } 0.3963 = \bar{1} + 0.5980 = \bar{1}.5980 = -1 + 0.5980$$

$$= -0.4020$$

$$5. \text{Log } 0.000005822 = \bar{6} + 0.7650 = \bar{6}.7650$$

$$= -6 + 0.7650 = -5.2350$$

$$6. \text{Log } 0.009981 = \bar{3} + 0.9991 = -3 + 0.9991$$

$$= -2.0009$$



Ratio, Proportion, Logs & Indices



106. Find $\text{Log}_8 23 = \text{Log of } 23 \text{ to the base } 8.$

$$\begin{aligned} &= \text{Log}_8 23 = \frac{\text{Log } 23}{\text{Log } 8} \\ &= \frac{(1.3617)}{(0.9031)} \\ &= 1.5078064446 \end{aligned}$$

$$\begin{aligned} &\text{Log}_b a \\ &= \left(\frac{\text{Log } a}{\text{Log } b} \right) \end{aligned}$$

$$\begin{aligned} 107. \text{Log}_{50} 2086 &= \frac{\text{Log } 2086}{\text{Log } 50} = \frac{3.3194}{1.6990} \\ &= 1.95373749264 \end{aligned}$$

$$108. \text{Log}_b a = \frac{\text{Log } a}{\text{Log } b} \quad \text{Log}_n m = \frac{\text{Log } m}{\text{Log } n}$$

$$\text{Log}_{\sqrt{81}} 1086 = \frac{\text{Log } 1086}{\text{Log } 81} = \frac{3.0359}{1.9085} = 1.59072570081$$

$$109. \text{Log}_{10} 81 = \frac{\text{Log } 81}{\text{Log } 10} = \frac{1.9085}{1.0000} = 1.9085$$

$$\text{Log}_{10} 50 = \text{Log } 50 / \text{Log } 10 = 1.6990 / 1 = 1.6990$$

10 \rightarrow Common Base of Logs

e \rightarrow Exponential factor = 2.7183 = Natural Base

$$110. \text{Log}_{10} 28 = \text{Log } 28 / \text{Log } 10 = 1.4472 / 1 = 1.4472$$

$$\text{Log}_{10} 100 = \text{Log } 100 / \text{Log } 10 = 2 / 1 = 2.0000$$

$$\text{Log}_{10} 1000 = \text{Log } 1000 / \text{Log } 10 = 3 / 1 = 3$$

If $\text{Log}_b a = k$; then $b^k = a$ | if $\text{Log}_z x = m$ then $z^m = x$

Ratio, Proportion, Logs & Indices



111. $\text{Log}_{10} 8 = 0.9031$ $10^{0.9031} = 8$

If $\text{Log}_q p = m$; then $q^m = p$

$\text{Log}_8 81 = \frac{\text{Log} 81}{\text{Log} 8} = \frac{1.9085}{0.9031} = 2.1133 \therefore 8^{2.1133} =$

112. A. $\text{Log} 2.8381 = 688.9$

A. $\text{Log} 3.8381 = 6889.000$

A. $\text{Log} 1.3379 = 21.77$

A. $\text{Log} 3.2263 = 1684.00$

A. $\text{Log} 5.8893 = 775000.000$

A. $\text{Log} 6.1193 = 1316000.000$

A. $\text{Log} 2.1156 = 130.5$

A. $\text{Log} 0.0891 = 1.227$

A. $\text{Log} 0.0063 = 1.015$

A. $\text{Log} 5.89993 = 794100$

A. $\text{Log} 7.0909 = 12330000$

A. $\text{Log} 10.8122 = 6489 \times 10^7$

$\text{Log} 5 = 0.6990$

A. $\text{Log} 0.6990 = 5.000$

A. $\text{Log} 0.6990 = 5$

A. $\text{log} (\text{Log} 5) = 5$

$\therefore \text{A. log} (\text{log } k) = k$

$\text{Log} 5 = 0.6990$

$\text{Log} (\text{A. log } 0.6990) = 0.6990$

$\text{Log} (\text{A. log } m) = m$

113. $\text{Log}_{10} 234 = 2.3692$

Crosscheck : $10^{2.3692} = 234$

If $\text{Log}_y x = m$ then $y^m = x$

114. If $\text{Log}_z x = y$; then $z^y = x$

$\text{Log} 55.332 = 1.7429$

A. $\text{Log} 7.8122 = 64890000$

A. $\text{Log} 10.1198 = 131700000000 = 1317 \times 10^7$

$\text{Log} 2386579 = 6.3779$



115. $\text{Log } 5 = 0.6990$, $\text{Log } 2 = 0.3010$, $\text{Log } 10 = 1.0000$

$$1 = 0.6990 + 0.3010$$

$$\text{Log } 10 = \text{Log } 5 + \text{Log } 2$$

$$\text{Log}(5 \times 2) = \text{Log } 5 + \text{Log } 2$$

$$\therefore \text{Log}(m \times n)$$

$$= \text{Log } m + \text{Log } n$$

116. $\text{Log}(xyz) = \text{Log } x + \text{Log } y + \text{Log } z$

$$\text{Log}(abcd) = \text{Log } a + \text{Log } b + \text{Log } c + \text{Log } d$$

117. Find Value of m; if $m = 8 \times 5$



$$m = 8 \times 5$$

Taking Log on both sides

$$\text{Log } m = \text{Log}(8 \times 5)$$

we know $\text{Log}(ab) = \text{Log } a + \text{Log } b$

$$\therefore \text{Log } m = \text{Log } 8 + \text{Log } 5 = 0.9031 + 0.6990$$

$$\therefore \text{Log } m = 1.6021$$

Taking $A \cdot \text{Log}$ on both sides

$$A \cdot \text{Log}(\text{Log } m) = A \cdot \text{Log}(1.6021) = 40.00$$

$$m = 40$$

118. $\text{Log } 23 = 1.3617$, $\text{A.Log } (1.3617) = 23.00$
 $\text{Log } 100 = 2.0000$, $\text{A.Log } 2.0000 = 100.00$

$\text{A.Log } (\text{Log } 25) = \text{A.Log } (1.3979) = 25.00$

$\text{Log } (\text{A.Log } 2.1111) = \text{Log } (129.1) = 2.1109$

$\text{A.Log } (\log m) = m$

$\text{Log } (\text{A.Log } k) = k = \text{A.Log } (\log k)$

$\text{Log}_2 16 = ?$

$\text{Log}_2 16$
 $= \frac{\text{Log } 16}{\text{Log } 2}$
 $= \left(\frac{1.2041}{0.3010} \right)$
 $= 4$

$\text{Log}_2 16 = m$
 $\therefore 2^m = 16$
 $\therefore 2^m = 2^4$
 $\therefore m = 4$
 $\text{Log}_2 16 = 4$

$\text{Log}_2 16$
 $= \frac{\text{Log } 16}{\text{Log } 2}$
 $= \frac{\text{Log } 2^4}{\text{Log } 2}$
 $= \frac{4 \cdot \text{Log } 2}{\text{Log } 2}$
 $= 4$

$\text{Log}_2 16$
 $= \frac{\text{Log } 16}{\text{Log } 2}$
 $= \left(\frac{1.20417090756}{0.30103312099} \right)$
 $= 4$

$\text{Log } m = ?$
 Enter m in calculator
 $\sqrt{\quad}$ 15 times
 Deduct 1
 Multiply by 14230.9635

119.

Log 29

Take : 29 on calculator

$\sqrt{\quad}$: 15 times

Deduct : 1

Multiply by 14230.9635



You will get answer
on calculator

Log 29

= 1.46247314138

Log $x = ?$

Enter : x

$\sqrt{\quad}$ 15 times

Deduct : 1

Multiply by 14230.9635



You will get answer

120. Find (By using calculator)

Log 22.22 = 1.34680785472

Log 819.9 = 2.91405945338

Log 234234 = 5.3706637312

Log 81998 = 4.91465228688

Log 35.86 = 1.55469533494

Log 0.00008122 = (-4.08974993369)

Log 0.5693 = -0.24465687446

Log 8.007 = 0.90349839919

A.Log 2.5122 = 325.07072328

A.Log 0.8163 = 6.55052845147

A.Log 0.20 = 1.58488763077

A.Log 5.2081 = 16118.85027



121. Log 22.22

By using Log-table

= characteristic of x + Mantissa of x

= (2-1) + 0.3468

= 1 + 0.3468

= 1.3468

By using Calculator

Enter 22.22

$\sqrt{\quad}$ 15 times

Deduct 1

Multiply by 14230.9635

= 1.34680785472

122. How to find Log x on calculator?

\Rightarrow Enter x
 $\sqrt{\quad}$ 15 times
 Deduct 1
 Multiply by 14230.9635

How to find A.Log (y) on calculator?

\Rightarrow
 Enter y
 Divide by 14230.9635
 Add 1
 $x =$ 15 times

How to find a^b on calculator?

\Rightarrow Enter : a
 $\sqrt{\quad}$ 12 times
 Deduct 1
 Multiply by : b
 Add 1
 $x =$ 12 times

• $\text{Log } 2 = 0.30103312099$
 • $\text{A.Log}(0.30103312099)$
 = 2.0000

123. Please solve the following by using calculator.

1) $\text{Log } 589.3 = 2.77060636046$

2) $\text{Log } 863.6 = 2.93661581515$

3) $\text{Log } 0.000008153 = -5.08777366557$

4) $\text{Log } 813692 = 5.91168824758$

5) $\text{A.Log } 2.3691 = 233.831154327$

6) $\text{A.Log } 0.8886 = 7.73698743132$

7) $85^{1.36} = 420.240316097$

124.

1) $\text{Log } x = (\text{Characteristic of } x) + (\text{Mantissa of } x)$

Integer Part : characteristic

Fractional Part : Mantissa

2) $\text{A.Log } (\text{Log } k) = k = \text{Log } (\text{A.log } k)$

$\text{A.Log } (\text{Log } 83) = 83, \text{ Log } (\text{A.log } 5.2381) = 5.2381$

3) Common Base : 10 as $\text{Log}_{10} 10 = 1$

Natural Base : e where e = exponential Factor
= 2.7183 (approx)

4) If $\text{Log}_b a = k$, then $b^k = a$

$\text{Log}_{10} 100 = 2, \text{Log}_{10} 1000 = 3, \text{Log}_{10} 1 = 0 \therefore \text{Log } a = 0$
provided $a \neq 1$

5) $\text{Log}_b a = \text{Log } a / \text{Log } b$

6) $\text{Log } (a \times b) = \text{Log } a + \text{Log } b$

7) $\text{Log}_m (xyz) = \text{Log}_m x + \text{Log}_m y + \text{Log}_m z$

Ratio, Proportion, Logs & Indices

math

$a^2 + b^2 = c^2$

$$125. \text{Log}(2)^5 = \text{Log}(2 \times 2 \times 2 \times 2 \times 2) = \text{Log}2 + \text{Log}2 + \text{Log}2 + \text{Log}2 + \text{Log}2$$

$$\text{Log}(2)^5 = 5 \cdot \text{Log}2$$

$$\text{Log}(2)^5 = 5 \cdot \text{Log}2$$

$$\text{Log}(a)^b = b \cdot \text{Log}a$$

$$\text{Log} m^n = n \cdot \text{Log}m$$

$$\text{Log} x^{-13} = -13 \cdot \text{Log}x$$

$$126. \frac{2^2}{2^5} = \frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 2 \times 2 \times 2} = 2^{2-5} = 2^{-3} = \frac{1}{2^3}$$

$$2^{2-5} = \frac{1}{2^3} = 2^{-3}$$

$$8^{10} = \left(\frac{1}{8^{-10}}\right)$$

$$100^{-3} = \left(\frac{1}{100^3}\right)$$

$$\frac{64}{64} = \frac{2^6}{2^6} =$$

$$a^{-m} = \frac{1}{a^m} \quad \text{OR} \quad a^m = \frac{1}{a^{-m}}$$

$$\text{OR} \quad a^m \times a^{-m} = 1 = a^{m+(-m)} = a^0$$

$$127. \text{Log}\left(\frac{a}{b}\right) = \text{Log}\left(a \times \frac{1}{b}\right) = \text{Log}\left(a \times b^{-1}\right)$$

$$= \text{Log}(a \times b^{-1}) = \text{Log}a + \text{Log}b^{-1}$$

$$= \text{Log}a + (-1 \cdot \text{Log}b) = \text{Log}a - \text{Log}b$$

$$\text{Log}\left(\frac{a}{b}\right) = \text{Log}a - \text{Log}b$$

$$128. \text{Log}\left(\frac{xy}{z}\right) = \text{Log}(xy) - \text{Log}z$$

$$= \text{Log}x + \text{Log}y - \text{Log}z$$

$$129. \text{Log}\left(\frac{ab}{cd}\right) = \text{Log}(ab) - \text{Log}(cd)$$

$$= (\text{Log}a + \text{Log}b) - (\text{Log}c + \text{Log}d)$$

$$= \text{Log}a + \text{Log}b - \text{Log}c - \text{Log}d$$

Ratio, Proportion, Logs & Indices

math

$a^2 + b^2 = c^2$

$$130. \text{Log } a^{-m} = -m \cdot \text{Log } a$$
$$\text{Log } x^z = z \cdot \text{Log } x$$

$$\text{Log}_y x \times \text{Log}_z y = \text{Log}_z x$$

$$\text{Log}_b a \times \text{Log}_c b = \frac{\text{Log } a}{\text{Log } b} \times \frac{\text{Log } b}{\text{Log } c} = \text{Log}_c a$$

$$\text{Log}_a a = 1 \quad \text{provided } a \neq 1$$

$$\left(\frac{\text{Any Number}}{0} \right) = \text{undefined} = \text{Not defined}$$

$$\left(\frac{0}{\text{Any Number}} \right) = 0 = \text{Zero}$$

$$\text{Log}_{\sqrt[3]{81}} 1024 \times \text{Log}_{\sqrt[7]{64}} (\sqrt{3})^8$$

$$= \frac{\text{Log } 1024}{\text{Log } 81} \times \frac{\text{Log } (3^{1/2})^8}{\text{Log } (64)^{1/7}}$$

$$= \frac{\text{Log } 2^{10}}{\text{Log } 3^4} \times \frac{\text{Log } 3^4}{\text{Log } (2^6)^{1/7}}$$

$$= \frac{10 \cdot \text{Log } 2}{4 \cdot \text{Log } 3} \times \frac{4 \cdot \text{Log } 3}{6 \cdot \text{Log } 2} = 10 \times \frac{7}{6} = \frac{70}{6} = \left(\frac{35}{3} \right)$$

131.

Summary of Logs

- | | |
|---|---|
| 1. $\text{Log } x = \text{characteristic of } x + \text{Mantissa of } x$ | 12. $\text{Log} \left(\frac{a}{b} \right) = \text{Log } a - \text{Log } b$ |
| 2. Integer part : <u>characteristic</u>
Fractional part : <u>Mantissa</u> | 13. $\text{Log } a^b = b \cdot \text{Log } a$ |
| 3. $\log_x y = (\text{Log } x / \text{Log } y) = \text{Log}_y x$ | 14. $\text{Log } a^{-b} = -b \cdot \text{Log } a$ |
| 4. Common base : <u>10</u>
Natural base : <u>e = Exponential Factor</u>
$= 2.7183$ (approx.) | 15. $\text{Log}_b a \times \text{Log}_a b = 1$ |
| 5. $A \cdot \log (\log y) = y = \text{Log} (A \cdot \log y)$ | 16. $\text{Log}_b a \times \text{Log}_c b = \text{Log}_c a$ |
| 6. If $\text{Log}_m a = n$ then $m^n = a$ | 17. $\text{Log} \left(\frac{ab}{c} \right) = \text{Log } a + \text{Log } b - \text{Log } c$ |
| 7. If $x^y = z$ then $\text{Log}_x z = y$ | 18. $\text{Log}_z x^y = \frac{\text{Log } x^y}{\text{Log } z} = \frac{y \cdot \text{Log } x}{\text{Log } z}$ |
| 8. $\text{Log}_{10} 10 = 1, \text{Log}_{10} 100 = 2$
$\text{Log}_{10} 1 = 0, \text{Log}_{10} 1000 = 3, \text{Log}_{10} 10,000 = 4$ | 19. If $m^n = j$
then $\text{Log}_m j = n$ |
| 9. $\text{Log}_a 1 = 0$ provided $a \neq 1$ | 20. $\text{Log} \left(\frac{xy}{mn} \right) = (\text{Log } x + \text{Log } y - \text{Log } m - \text{Log } n)$ |
| 10. $\text{Log} (abc) = \text{Log } a + \text{Log } b + \text{Log } c$ | |
| 11. $\text{Log}_m (xy) = \text{Log}_m x + \text{Log}_m y$ | |



132. $\text{Log}_{16} 64 = ?$

$$\begin{aligned} &\rightarrow = \frac{\text{Log } 64}{\text{Log } 16} = \frac{\text{Log } 4^3}{\text{Log } 4^2} = \frac{3 \cdot \text{Log } 4}{2 \cdot \text{Log } 4} = \frac{3}{2} \\ &= 1.50 \end{aligned}$$

133. $\text{Log}_{\sqrt[4]{27}} 243 =$

$$\begin{aligned} &\rightarrow = \frac{\text{Log } 243}{\text{Log } \sqrt[4]{27}} = \frac{\text{Log } 3^5}{\text{Log } (3^3)^{1/4}} = \frac{5 \cdot \text{Log } 3}{\frac{3}{4} \cdot \text{Log } 3} \\ &= 5 \times \frac{4}{3} = \frac{20}{3} = 6.666666666 = 6\frac{2}{3} \end{aligned}$$

134. $\text{Log}_{\sqrt[4]{243}} \sqrt[7]{81} =$

$$\begin{aligned} &\rightarrow = \frac{\text{Log } \sqrt[7]{81}}{\text{Log } \sqrt[4]{243}} = \frac{\text{Log } (3^4)^{1/7}}{\text{Log } (3^5)^{1/4}} = \frac{\frac{4}{7} \text{Log } 3}{\frac{5}{4} \text{Log } 3} \\ &= \frac{4}{7} \times \frac{4}{5} = \left(\frac{16}{35}\right) = 0.45714285714 \end{aligned}$$

135. $\text{Log}_{\sqrt[9]{121}} \sqrt[5]{11} =$

$$\begin{aligned} &\rightarrow = \frac{\text{Log } \sqrt[5]{11}}{\text{Log } \sqrt[9]{121}} = \frac{\text{Log } (11)^{1/5}}{\text{Log } (11^2)^{1/9}} = \frac{\frac{1}{5} \cdot \text{Log } 11}{\frac{2}{9} \cdot \text{Log } 11} = \frac{1}{5} \times \frac{9}{2} \\ &= \frac{9}{10} = 0.90 \end{aligned}$$

136. $\text{Log}_{(7/8)} x = 3$. Find the value of x.



If $\text{Log}_b a = k$ then $b^k = a$

$$\therefore \left(\frac{7}{8}\right)^3 = x \quad x = \frac{7^3}{8^3} = \left(\frac{343}{512}\right)$$

137. $\text{Log}_{\sqrt{2}} m = \frac{8}{11}$ Find the value of m.



If $\text{Log}_b a = k$ then $b^k = a$

$$\therefore (\sqrt{2})^{8/11} = m$$

$$m = (2^{1/2})^{8/11} = (2)^{\frac{8}{22}} = (2)^{\frac{4}{11}} = \sqrt[11]{2^4} = \sqrt[11]{16}$$

138. $\text{Log}_n m \times \text{Log}_p n \times \text{Log}_m p =$



$$= \frac{\text{Log } m}{\text{Log } n} \times \frac{\text{Log } n}{\text{Log } p} \times \frac{\text{Log } p}{\text{Log } m} = 1.00$$

139. $\text{Log}_{\sqrt[3]{8}} x = 13$. Find the value of x.



$$\text{Log}_2 x = 13$$

$$\therefore x = 2^{13} = 8192$$

140. $\text{Log}_{\sqrt[4]{27}} 81 \times \text{Log}_{\sqrt[4]{16}} \sqrt{512}$



$$= \frac{\text{Log } 81}{\text{Log } (27)^{1/4}} \times \frac{\text{Log } \sqrt{512}}{\text{Log } \sqrt[4]{16}}$$

$$= \frac{\text{Log } 3^4}{\text{Log } (3^3)^{1/4}} \times \frac{\text{Log } (2^9)^{1/2}}{\text{Log } (2^4)^{1/4}}$$

$$= \frac{4 \cdot \text{Log } 3}{\frac{3}{4} \text{Log } 3} \times \frac{\frac{9}{2} \cdot \text{Log } 2}{\frac{4}{7} \text{Log } 2} = 4 \times \frac{4}{3} \times \frac{9}{2} \times \frac{7}{4} = 42$$

141. Basic Rules of Indices

1) a. $2^3 \times 2^5 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) = 2^8 = 2^{3+5}$

$2^3 \times 2^5 = 2^{3+5} = 2^8 \quad \therefore x^m \times x^n = x^{m+n}$

b) $5^3 \times 5^4 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^7$

$5^3 \times 5^4 = 5^{3+4} = 5^7$

$x^p \times x^q = x^{(p+q)}$

$\therefore a^m \times a^n = (a)^{m+n}$

2. $\frac{3^8}{3^4} = \frac{\cancel{3 \times 3 \times 3 \times 3} \times 3 \times 3 \times 3 \times 3}{\cancel{3 \times 3 \times 3 \times 3}} = 3^4 = (3)^{8-4}$

$\frac{a^m}{a^n} = (a)^{m-n}$

$\frac{x^z}{x^m} = (x)^{z-m}$

3. $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
 $= 2^{12} = (2)^{3 \times 4}$

$(a^m)^n = a^{mn}$

4. $[(a^m)^n]^q = (a)^{mnq}$

5. $a^x \times a^y \times a^z = a^{(x+y+z)}$

6. $\sqrt[m]{a} = a^{\frac{1}{m}}$

$\sqrt[3]{8} = (8)^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3 \times \frac{1}{3}} = 2^1 = 2$

$\sqrt[5]{243} = (243)^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3^{5 \times \frac{1}{5}} = 3^1 = 3$

7. $\frac{2^3}{2^6} = 2^{3-6} = 2^{-3} \dots \textcircled{1} \quad \frac{2^3}{2^6} = \frac{\cancel{2 \times 2 \times 2}}{\cancel{2 \times 2 \times 2} \times 2 \times 2 \times 2} = \frac{1}{2^3} \dots \textcircled{2}$

$a^{-m} = \frac{1}{a^m}$

OR

$a^m = \frac{1}{a^{-m}}$

OR

$a^m \times a^{-m} = 1$

$\therefore 2^{-3} = \frac{1}{2^3}$

$x^{-y} = \frac{1}{x^y}$

$x^{-k} = \frac{1}{x^k}$

$\& 2^3 = \frac{1}{2^{-3}}$

Ratio, Proportion, Logs & Indices

math

$a^2 + b^2 = c^2$

$$\frac{100}{100} = \frac{10^2}{10^2} = 10^{2-2} = 10^0 = 1$$

$$8. \frac{32}{32} = \frac{2^5}{2^5} = (2)^{5-5} = 2^0 = 1$$

9. If $a^x = a^y$ then $x = y$

$$\therefore (\text{Any number})^0 = 1$$

10. If $a^x = b^x$ then $a = b$

$$11. a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

$$8^{\frac{5}{7}} = \sqrt[7]{8^5}$$

- If $2^m = 2^5 \quad \therefore m = 5$
- If $8^2 = m^2$ then $m = 8$

$$12. a^1 = a, a^0 = 1$$

- $13^{\frac{8}{36}} = \sqrt[36]{13^8}$

$$13. 6^4 = (2 \times 3)^4 = 1296$$

$$6^4 = (2 \times 3)^4 = 2^4 \times 3^4 = 16 \times 81 = 1296$$

$$\therefore (2 \times 3)^4 = 2^4 \times 3^4 \quad \therefore \boxed{(ab)^m = a^m \times b^m}$$

$$14. \left(\frac{6}{2}\right)^3 = 27$$

$$15. \left(\frac{ab}{c}\right)^m = \frac{(ab)^m}{c^m}$$

$$\left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3} = \frac{216}{8} = 27$$

$$= \frac{a^m \times b^m}{c^m}$$

$$\therefore \left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3} \quad \therefore \left(\frac{a}{b}\right)^m = \left(\frac{a^m}{b^m}\right)$$

$$= a^m \cdot b^m \cdot c^{-m}$$

$$\textcircled{1} 2^8 \times 2^{11} = 2^{8+11} = 2^{19}$$

$$\textcircled{2} \left(\frac{8^{33}}{8^{91}}\right) = 8^{33-91} = 8^{-58}$$

$$= \left(\frac{1}{8^{58}}\right)$$

$$\textcircled{3} \frac{10^{\frac{1}{3}} \times 10^{\frac{1}{6}}}{10^{\frac{1}{9}}} = (10)^{\frac{1}{3} + \frac{1}{6} - \frac{1}{9}}$$

$$= (10)^{\frac{18}{54} + \frac{9}{54} - \frac{6}{54}}$$

$$= (10)^{\frac{21}{54}} = (10)^{\frac{7}{18}} = \sqrt[18]{10^7}$$

142. Rules of Indices

1. $a^m \times a^n = (a)^{m+n}$

2. $a^x \times a^y \times a^z = (a)^{x+y+z}$

3. $\frac{a^m}{a^n} = (a)^{m-n}$

$\frac{b^m}{b^n} = (b)^{m-n}$

4. $(a \times b)^x = a^x \cdot b^x$

$(xyz)^k = x^k \cdot y^k \cdot z^k$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} = a^m \cdot b^{-m}$

$\left(\frac{xy}{z}\right)^m = \frac{x^m \cdot y^m}{z^m} = x^m \cdot y^m \cdot z^{-m}$

6. $(a^x)^y = a^{xy} = (a^y)^x$

$\left((m^p)^q\right)^r = (m)^{pqr}$

7. $(a^{1/m}) = \sqrt[m]{a}$

8. $a^{-m} = \frac{1}{a^m}$ OR $a^x = \frac{1}{a^{-x}}$

OR $a^p \times a^{-p} = 1 = a^{p+(-p)} = a^0$
 $a^p \times \frac{1}{a^p} = 1$

9. $a^1 = a, a^0 = 1$

10. $a^{x/y} = \sqrt[y]{a^x}$

$x^{m/n} = \sqrt[n]{x^m} = (x^m)^{1/n}$

11. $a^{-1/m} = \frac{1}{a^{1/m}} = \frac{1}{\sqrt[m]{a}}$

$28^{9/8} = \sqrt[8]{28^9}$

12. If $a^x = a^y$ then $x = y$

13. If $a^x = b^x$ then $a = b$

14. $\frac{a^b \times a^c}{a^d} = (a)^{b+c-d}$

15. $\frac{a^b \times a^c}{a^e \times a^f} = (a)^{b+c-e-f}$

16. $\sqrt[m]{\frac{a}{b}} = (a/b)^{1/m}$

$= \left(\frac{a^{1/m}}{b^{1/m}}\right) = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$

143. $2^3 \times (16)^8 \times (\sqrt{2})^9 =$

$\rightarrow = 2^3 \times (2^4)^8 \times (2^{\frac{1}{2}})^9 = 2^3 \times 2^{32} \times 2^{\frac{9}{2}}$
 $= (2)^{3+32+4.50} = (2)^{39.50} = (2)^{79/2} = \sqrt{2^{79}}$

144. $3^8 \times 3^{-12} \times (\sqrt{3})^{16} \times (\sqrt[3]{3})^{18} =$

$\rightarrow = 3^8 \times 3^{-12} \times (3^{1/2})^{16} \times (3^{1/3})^{18}$
 $= 3^8 \times 3^{-12} \times 3^8 \times 3^6$
 $= (3)^{8-12+8+6} = (3)^{10} = 59049$

145. $x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$

$\rightarrow = (x)^{\cancel{a-b} + \cancel{b-c} + \cancel{c-a}} = x^0 = 1$
 Please remember : (Any Number)⁰ = 1.00

146. $(x^{a-b})^{a^2+ab+b^2} \cdot (x^{b-c})^{b^2+bc+c^2} \cdot (x^{c-a})^{c^2+ac+a^2}$

$\rightarrow = (x)^{(a-b)(a^2+ab+b^2)} \cdot (x)^{(b-c)(b^2+bc+c^2)} \cdot (x)^{(c-a)(c^2+ac+a^2)}$
 $= (x)^{a^3-b^3} \cdot (x)^{b^3-c^3} \cdot (x)^{c^3-a^3} = (x)^{\cancel{a^3-b^3} + \cancel{b^3-c^3} + \cancel{c^3-a^3}} = (x)^0 = 1$

147. $\left[\frac{(\sqrt{2})^9 \times 16^{-5/2} \times 64^{-7/2}}{32^{11/2}} \right] = \frac{(2^{\frac{1}{2}})^9 \times (2^4)^{-5/2} \times (2^6)^{-7/2}}{(2^5)^{11/2}}$
 $= \frac{2^{9/2} \times 2^{-20/2} \times 2^{-42/2}}{2^{55/2}} = (2)^{\frac{9}{2} - \frac{20}{2} - \frac{42}{2} - \frac{55}{2}} = (2)^{-108/2} = (2)^{-54} = \frac{1}{2^{54}}$

148. $\sqrt[9]{[3\sqrt{4}]^{1/8}}$

$$\begin{aligned} \rightarrow & \left\{ \left[(4)^{1/3} \right]^{1/8} \right\}^{1/9} = (4)^{\frac{1}{3} \times \frac{1}{8} \times \frac{1}{9}} = (4)^{\frac{1}{216}} = (2^2)^{\frac{1}{216}} \\ & = (2)^{2 \times \frac{1}{216}} = 2^{\frac{1}{108}} = \sqrt[108]{2} \end{aligned}$$

149. $\left[\frac{8^{1/4} \times 32^{1/8}}{\sqrt[16]{1024}} \right]^9$

$$\begin{aligned} \rightarrow & \left[\frac{(2^3)^{1/4} \times (2^5)^{1/8}}{(2^{10})^{1/16}} \right]^9 = \left(\frac{2^{3/4} \times 2^{5/8}}{2^{5/8}} \right)^9 = \left(2^{3/4} \right)^9 = 2^{\frac{27}{4}} = (2^{27})^{1/4} \\ & = \sqrt[4]{2^{27}} \end{aligned}$$

150. $\sqrt[16]{\left[\frac{\sqrt[7]{3} \times \sqrt[14]{81} \times (27)^{-9}}{243^{8/7}} \right]^{42}}$

$$\begin{aligned} \rightarrow & \left[\frac{3^{\frac{1}{7}} \times (3^4)^{\frac{1}{14}} \times (3^3)^{-9}}{(3^5)^{8/7}} \right]^{42 \times \frac{1}{16}} = \left[(3)^{\frac{1}{7} + \frac{2}{7} - \frac{189}{7} - \frac{40}{7}} \right]^{\frac{21}{8}} \\ & = (3)^{\frac{-226}{7} \times \frac{21}{8}} \\ & = (3)^{\frac{-4746}{56}} = (3)^{-339/4} \\ & = \frac{1}{(3)^{339/4}} = \frac{1}{\sqrt[4]{3^{339}}} \end{aligned}$$

151. $\left[\sqrt[3]{4^8} \right]^{\frac{11}{2}} \times \left[\sqrt[9]{32^{-8}} \right]^{\frac{17}{2}}$

\rightarrow

$$= \left\{ \left[(2^2)^{\frac{1}{3}} \right]^8 \right\}^{\frac{11}{2}} \times \left\{ \left[(2^5)^{\frac{1}{9}} \right]^{-8} \right\}^{\frac{17}{2}}$$

$$= (2)^{2 \times \frac{1}{3} \times 8 \times \frac{11}{2}} \times (2)^{5 \times \frac{1}{9} \times -8 \times \frac{17}{2}}$$

$$= (2)^{\frac{88}{3}} \times (2)^{\frac{-680}{18}} = (2)^{\frac{528}{18}} \times (2)^{\frac{-680}{18}} = (2)^{\frac{528}{18} - \frac{680}{18}}$$

$$= (2)^{\frac{-152}{18}} = (2)^{-\frac{76}{9}} = \sqrt[9]{2^{-76}} = \frac{1}{2^{76/9}} = \left(\frac{1}{2}\right)^{76/9}$$

152. $\text{Log} \frac{\sqrt[8]{512}}{\sqrt[5]{64}} =$

\rightarrow

$$= \frac{\text{Log} \sqrt[8]{512}}{\text{Log} \sqrt[5]{64}} = \frac{\text{Log} (2^9)^{1/8}}{\text{Log} (2^6)^{1/5}}$$

$$= \frac{\text{Log} (2)^{9/8}}{\text{Log} (2)^{6/5}} = \frac{\frac{9}{8} \text{Log} 2}{\frac{6}{5} \text{Log} 2} = \frac{9}{8} \times \frac{5}{6} = \frac{45}{48} = \frac{15}{16}$$

153. $\text{Log} \frac{\sqrt[18]{729}}{\sqrt[8]{81}} =$

\rightarrow

$$= \frac{\text{Log} \sqrt[18]{729}}{\text{Log} \sqrt[8]{81}} = \frac{\text{Log} (3^6)^{1/18}}{\text{Log} (3^4)^{1/8}} = \frac{\frac{6}{18} \text{Log} 3}{\frac{4}{8} \text{Log} 3}$$

$$= \left(\frac{\frac{1}{3}}{\frac{1}{2}}\right) = \frac{1}{3} \times \frac{2}{1} = \left(\frac{2}{3}\right)$$



154. $\left[\frac{\text{Log } \sqrt[3]{16} \times \text{Log } \sqrt[9]{2187}}{\text{Log } (\sqrt{3})^{19} \times \text{Log } \sqrt[15]{1024}} \right]$

$$= \frac{\text{Log } (2^4)^{1/3} \times \text{Log } (3^7)^{1/9}}{\text{Log } (3^{1/2})^{19} \times \text{Log } (2^{10})^{1/15}}$$

$$= \frac{\frac{4}{3} \text{Log} 2 \times \frac{7}{9} \text{Log} 3}{\frac{19}{2} \text{Log} 3 \times \frac{10}{15} \text{Log} 2} = \frac{4}{3} \times \frac{7}{9} \times \frac{2}{19} \times \frac{3}{2} = \left(\frac{28}{171} \right)$$

155. If $\text{Log } 2 = x$, $\text{Log } 3 = y$. Find $\text{Log } (1.20)$

- a. $2x + 3y - 1$ ~~b. $2x + y - 1$~~ c. $x + y + 3$ d. None

$$\implies \text{Log } (1.20) = \text{Log } \left(\frac{12}{10} \right) = \text{Log } 12 - \text{Log } 10$$

$$= \text{Log } (2 \times 2 \times 3) - \text{Log } 10 = \text{Log } 2 + \text{Log } 2 + \text{Log } 3 - \text{Log } 10$$

$$= x + x + y - 1 = 2x + y - 1$$

Answer to question

$$2x + y - 1 = 2 \text{Log } 2 + \text{Log } 3 - \text{Log } 10 = \text{Log } \left(\frac{4 \times 3}{10} \right) = \text{Log } 1.20$$

156. If $\text{Log } 2 = x$, $\text{Log } 3 = y$. Find $\text{Log } (7.20)$

$$\implies \text{Log } (7.20) = \text{Log } \left(\frac{72}{10} \right) = \text{Log } 72 - \text{Log } 10 = \text{Log } (8 \times 9) - 1$$

$$= \text{Log } 8 + \text{Log } 9 - 1 = \text{Log } 2^3 + \text{Log } 3^2 - 1 = 2 \cdot \text{Log } 3 + 3 \cdot \text{Log } 2 - 1$$

$$= 2y + 3x - 1 = 3x + 2y - 1$$

157. If $\text{Log}_{3/2} x = -5$. Find the value of x .

$$\implies \text{Log}_{3/2} x = -5 \quad \therefore \left(\frac{3}{2} \right)^{-5} = x \quad \therefore x = \left(\frac{2}{3} \right)^5 = \frac{32}{243}$$

$$\text{If } \log_b a = k \text{ then } b^k = a$$

$$\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$$

158. $\log_x 16 = \log_9 64$. Find the value of x.



$$\log_x 16 = \log_9 64$$

$$\frac{\log 16}{\log x} = \frac{\log 64}{\log 9}$$

$$\frac{4 \cdot \cancel{\log 2}}{\log x} = \frac{6 \cdot \cancel{\log 2}}{\log 9}$$

$$6 \cdot \log x = 4 \cdot \log 9$$

$$3 \cdot \log x = 2 \cdot \log 9 = \log 81$$

$$\log x = \frac{1}{3} \log 81 = \log(81)^{1/3} = \log(3)^{4/3}$$

$$\log x = \log(3)^{4/3}$$

$$x = 3^{4/3}$$

$$\text{OR } x = \sqrt[3]{3^4} = \sqrt[3]{81}$$

159. $\log_2 \log_2 \log_2 16 = ?$

$$= \log_2 \log_2 (\log_2 16)$$

$$= \log_2 \log_2 \left(\frac{4 \cdot \cancel{\log 2}}{\cancel{\log 2}}\right)$$

$$= \log_2 (\log_2 4) = \log_2 \left(\frac{2 \cdot \cancel{\log 2}}{\cancel{\log 2}}\right) = \log_2 2 = 1.00$$

160. $\text{Log}_x 9 = 8$. Find value of $x\sqrt{x}$



$$\text{Log}_x 9 = 8$$

$$\therefore x^8 = 9$$

Taking 8th root on both sides

$$(x^8)^{1/8} = (9)^{1/8}$$

$$x = (3^2)^{1/8} = 3^{1/4}$$

$$x \cdot \sqrt{x}$$

$$= x^1 \cdot x^{1/2}$$

$$= x^{1+1/2}$$

$$= x^{3/2}$$

$$= (3^{1/4})^{3/2}$$

$$= 3^{3/8} = \sqrt[8]{3^3}$$

$$= \sqrt[8]{27}$$

161. $\text{Log}_x 27 = 81$. Find value of $\sqrt[3]{x} \times \sqrt[5]{x}$



$$\text{Log}_x 27 = 81$$

$$\therefore x^{81} = 27$$

$$x = (27)^{1/81}$$

$$x = (3^3)^{1/81}$$

$$x = (3)^{1/27}$$

$$\sqrt[3]{x} \times \sqrt[5]{x}$$

$$= x^{1/3} \cdot x^{1/5} = (x)^{1/3+1/5}$$

$$= (x)^{8/15} = (3^{1/27})^{8/15}$$

$$= (3)^{8/405} = \sqrt[405]{3^8}$$

Question

$\text{Log}_x 27 = 4$. Find value of $x\sqrt{x}$.



$$\text{Log}_x 27 = 4$$

$$\therefore x^4 = 27$$

$$x = (27)^{1/4}$$

$$x = (3^3)^{1/4} = 3^{3/4}$$

$$x \cdot \sqrt{x} = 3^{3/4} \times (3^{3/4})^{1/2}$$

$$= 3^{3/4} \times 3^{3/8}$$

$$= (3)^{3/4+3/8} = (3)^{36/32} = (3)^{9/8}$$

162. $\text{Log}_3 x = 4$. Find value of $(\sqrt{x})^x$



$$\begin{aligned} \text{Log}_3 x &= 4 & (\sqrt{x})^x \\ \therefore 3^4 &= x & = (\sqrt{81})^{81} = 9^{81} = (3^2)^{81} \\ x &= 81 & = (3)^{162} \end{aligned}$$

163. $\text{Log}_{\sqrt[7]{2}} (32)^{-5} = m$. Find m



$$m = \left[\frac{\text{Log}(32)^{-5}}{\text{Log} \sqrt[7]{2}} \right] = \left[\frac{\text{Log}(2^5)^{-5}}{\text{Log}(2)^{1/7}} \right] = \left[\frac{\text{Log}(2)^{-25}}{\text{Log}(2)^{1/7}} \right]$$

$$m = \left(\frac{-25 \cdot \cancel{\text{Log} 2}}{\frac{1}{7} \cdot \cancel{\text{Log} 2}} \right) = -25 \times \frac{7}{1} = -175$$

$\therefore m = -175$

164. $\text{Log}_{\sqrt[9]{243}} (\sqrt[8]{81})^8 =$



$$\begin{aligned} &= \frac{\text{Log}(\sqrt[8]{81})^8}{\text{Log}(\sqrt[9]{243})} &= \frac{4 \cdot \cancel{\text{Log} 3}}{\frac{5}{9} \cdot \cancel{\text{Log} 3}} \\ &= \frac{\text{Log}(81 \frac{1}{8})^8}{\text{Log}(243)^{1/9}} &= 4 \times \frac{9}{5} = \frac{36}{5} = 7.20 \\ &= \frac{\text{Log}(3^4)}{\text{Log}(3^5)^{1/9}} \end{aligned}$$



165. $5 \text{ Log } x = \frac{1}{2} \text{ Log } \sqrt[5]{30}$. Find the value of x



$$5 \cdot \text{Log } x = \frac{1}{2} \text{ Log } \sqrt[5]{30}$$

$$\text{Log } x^5 = \text{Log } (30^{1/5})^{1/2}$$

$$\text{Log } x^5 = \text{Log } 30^{1/10}$$

$$A \cdot \text{log} (\text{log } x^5) = A \cdot \text{log} (\text{Log } 30^{1/10})$$

$$x^5 = 30^{1/10}$$

$$(x^5)^{1/5} = (30^{1/10})^{1/5}$$

$$x = (30)^{1/50} = \sqrt[50]{30}$$

166. $\frac{\text{Log}_5 16 \times \text{Log}_8 125 \times \text{Log}_7 243}{\text{Log}_{\sqrt{7}} 729}$



$$= \frac{\frac{\text{Log } 16}{\text{Log } 5} \times \frac{\text{Log } 125}{\text{Log } 8} \times \frac{\text{Log } 243}{\text{Log } 7}}{\frac{\text{Log } 729}{\text{Log } \sqrt{7}}}$$

$$= \frac{4 \cdot \cancel{\text{Log } 2} / \cancel{\text{Log } 5}}{\cancel{\text{Log } 2} / \cancel{\text{Log } 2}} \times \frac{3 \cdot \cancel{\text{Log } 5} / \cancel{\text{Log } 2}}{3 \cdot \cancel{\text{Log } 2} / \cancel{\text{Log } 2}} \times \frac{5 \cdot \cancel{\text{Log } 3} / \cancel{\text{Log } 7}}{\cancel{\text{Log } 3} / \cancel{\text{Log } 7}} \times \frac{\frac{1}{2} \cancel{\text{Log } 7}}{\frac{6 \cdot \cancel{\text{Log } 3}}{1} / \cancel{\text{Log } 3}}$$

$$= \left(4 \times 5 \times \frac{1}{2} \times \frac{1}{6} \right) = \frac{20}{12} = \left(\frac{5}{3} \right)$$

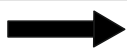


167. $\text{Log}_{\sqrt{5}} x = 3$. Find value of $(x \times \sqrt[5]{x})$



$$\begin{aligned} \text{Log}_{\sqrt{5}} x &= 3 & x \cdot \sqrt[5]{x} \\ \therefore x &= (\sqrt{5})^3 &= 5^{\frac{3}{2}} \cdot (5^{\frac{3}{2}})^{\frac{1}{5}} \\ x &= (5^{\frac{1}{2}})^3 &= 5^{\frac{3}{2}} \cdot 5^{\frac{3}{10}} \\ x &= 5^{\frac{3}{2}} &= 5^{\frac{15}{10}} \cdot 5^{\frac{3}{10}} = (5)^{\frac{15}{10} + \frac{3}{10}} \\ & &= (5)^{\frac{18}{10}} = (5)^{\frac{9}{5}} = \sqrt[5]{5^9} \\ & &= \sqrt[5]{1953125} \end{aligned}$$

168. If $a^2 + b^2 = 50$ & $ab = 20$. Find value of $(a+b)^2$

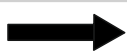


$$\begin{aligned} (a+b)^2 &= (a^2 + b^2) + 2ab \\ &= 50 + (2 \times 20) = 50 + 40 = 90 \end{aligned}$$

Question If $(a+b) = 35$, $ab = 50$ Find $(\frac{1}{a} + \frac{1}{b}) = ?$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{35}{50} = \frac{7}{10}$$

169. $\text{Log}_2 \text{Log}_2 \text{Log}_2 \text{Log}_2 x = 0$. Find x .



$$\begin{aligned} \text{Log}_2 [\text{Log}_2 \text{Log}_2 \text{Log}_2 x] &= 0 \\ \text{Log}_2 (\text{Log}_2 \text{Log}_2 x) &= 2^0 = 1 \\ \text{Log}_2 \text{Log}_2 x &= 2^1 = 2 \\ \text{Log}_2 x &= 2^2 = 4 \\ \therefore \text{Log}_2 x &= 4 \\ \therefore x &= 2^4 \\ x &= 16 \end{aligned}$$

cross-check

$$\begin{aligned} &= \text{Log}_2 \text{Log}_2 \text{Log}_2 \text{Log}_2 16 \\ &= \text{Log}_2 \text{Log}_2 \text{Log}_2 4 \\ &= \text{Log}_2 \text{Log}_2 2 \\ &= \text{Log}_2 1 \\ &= 0 \end{aligned}$$

170. $\text{Log}_a x = a$ then $x = ?$

→ $\text{Log}_a a^a = a \quad \therefore a^a = x$

Q. If $\text{Log}_{27} x = \text{Log}_{81} 3$. Find x

→ $\frac{\text{Log} x}{3 \cdot \text{Log} 3} = \frac{\text{Log} 3}{4 \cdot \text{Log} 3} \quad \therefore \text{Log} x = \frac{1}{4} \cdot 3 \text{Log} 3$
 $\text{Log} x = \text{Log} 3^{3/4} \quad \therefore x = 3^{3/4}$

171. $\text{Log}(1 + 2 + 3) = \text{Log} 1 + \text{Log} 2 + \text{Log} 3$

- a. True b. False

$\text{Log}(1+2+3) = \text{Log} 6 = \text{Log}(1 \times 2 \times 3) = \text{Log} 1 + \text{Log} 2 + \text{Log} 3$

Given statement is correct.

172. $\text{Log}(5 + 6 + 7) = \text{Log} 5 + \text{Log} 6 + \text{Log} 7$

- a. True b. False

L.H.S. = $\text{Log}(5+6+7) = \text{Log} 18$

R.H.S. = $\text{Log} 5 + \text{Log} 6 + \text{Log} 7$

= $\text{Log}(5 \times 6 \times 7) = \text{Log}(210)$

L.H.S. \neq R.H.S.

Given statement is incorrect.

173. $\text{Log}_2 x + \text{Log}_8 x + \text{Log}_{32} x = \frac{23}{5}$

Find value of x.

→ $\frac{\text{Log} x}{\text{Log} 2} + \frac{\text{Log} x}{3 \cdot \text{Log} 2} + \frac{\text{Log} x}{5 \cdot \text{Log} 2} = \frac{23}{5}$

$\frac{15 \text{Log} x}{15 \text{Log} 2} + \frac{5 \cdot \text{Log} x}{15 \cdot \text{Log} 2} + \frac{3 \text{Log} x}{15 \cdot \text{Log} 2} = \frac{23}{5}$

$\frac{15 \cdot \text{Log} x + 5 \cdot \text{Log} x + 3 \cdot \text{Log} x}{15 \cdot \text{Log} 2} = \frac{23}{5}$

$\therefore \frac{23 \text{Log} x}{15 \text{Log} 2} = \frac{23}{5}$

$\frac{\text{Log} x}{\text{Log} 2} = \frac{15}{5} = 3$

$\text{Log} x = 3 \cdot \text{Log} 2$

$\text{Log} x = \text{Log} 8$

$x = 8$

174. $\text{Log } 5 = 0.6990$

$\text{Log } 3 = 0.4771$

Find value of $\text{Log } \left(\frac{50}{3,00,000} \right)$



$$\begin{aligned} \text{Log } \left(\frac{50}{3,00,000} \right) &= \text{Log } 50 - \text{Log } 3,00,000 \\ &= 1.6990 - 5.4771 \\ &= -3.7781 \end{aligned}$$

(OR)

$$\begin{aligned} &\text{Log } \left(\frac{5}{30,000} \right) \\ &= \text{Log } \left(\frac{5}{3 \times 10,000} \right) \\ &= \text{Log } 5 - \text{Log } 3 - \text{Log } 10,000 \\ &= 0.6990 - 0.4771 - 4 \\ &= -3.7781 \end{aligned}$$

175. Find $\text{Log } \left(\frac{0.003}{2000} \right)$ If $\text{Log } 2 = 0.3010, \text{Log } 3 = 0.4771$



$$\begin{aligned} \text{Log } \left(\frac{0.003}{2000} \right) &= \text{Log } 0.003 - \text{Log } 2000 = \bar{3}.4771 - 3.3010 \\ &= (-3 + 0.4771) - 3.3010 = -5.8239 \end{aligned}$$

(OR)

$$\begin{aligned} \text{Log } \left(\frac{0.003}{2000} \right) &= \text{Log } \left(\frac{3}{20,00,000} \right) = \text{Log } 3 - \text{Log } 20,00,000 \\ &= 0.4771 - 6.3010 \\ &= -5.8239 \end{aligned}$$

176. If $2s : 3t$ is duplicate ratio of $(2s-p) : (3t-p)$ then

~~a. $p^2 = 6st$~~

b. $p = 6st$

c. $2p = 3st$

d. None of these

$$\frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$$

$$\frac{2s}{3t} = \frac{4s^2 - 4ps + p^2}{9t^2 - 6pt + p^2}$$

$$2s(gt^2 - 6pt + p^2) = 3t(4s^2 - 4ps + p^2)$$

$$18t^2s - 12pts + 2p^2s = 12s^2t - 12pts + 3tp^2$$

$$18t^2s - 12s^2t = 3tp^2 - 2p^2s$$

$$6st(3t - 2s) = p^2(3t - 2s)$$

$$\therefore p^2 = 6st$$

177. $\frac{A}{3} = \frac{B}{8} = \frac{C}{7}$ Find $A : B : C = ?$

a. 7 : 8 : 3

~~b. 3 : 8 : 7~~

c. 8 : 7 : 3

d. None of these

$$\frac{A}{3} = \frac{B}{8}$$

$$\frac{B}{8} = \frac{C}{7}$$

$$\therefore A : B = 3 : 8 \quad \therefore B : C = 8 : 7$$

$$\therefore A : B : C = 3 : 8 : 7$$

178. Anand earns ₹ 5000 in 6 hours & Vinod earns ₹ 7000 in 8 hours.

Find ratio of their earning per hour.

	Anand	vinod
Earning per hour	$\left(\frac{₹ 5000}{6}\right)$	$\left(\frac{₹ 7000}{8}\right)$
Ratio of earning per hour	$= \frac{5000/6}{7000/8} = \frac{5000}{6} \times \frac{8}{7000} = \frac{40}{42} = 20 : 21$	

179. Monthly income of A, B is in the ratio of 5:6 & monthly expenses are in the ratio of 9:11. If both of them save ₹ 50 p.m.

Find their incomes.

	Monthly income	Monthly saving	Monthly exp
A	5x	50	5x - 50
B	6x	50	6x - 50

$$\frac{5x - 50}{6x - 50} = \frac{9}{11}$$

$$55x - 550 = 54x - 450$$

$x = 100$

\therefore Monthly incomes of A : $5x = 5 \times 100 = ₹ 500$
 B : $6x = 6 \times 100 = ₹ 600$

OR

$$5x - 9y = 50$$

$$6x - 11y = 50$$

$$55x - 99y = 550$$

$$54x - 99y = 450$$

$$\begin{array}{r} - \\ + \\ \hline x = 100 \end{array}$$

\therefore Incomes are

$$A : 5x = ₹ 500$$

$$B : 6x = ₹ 600$$

180. Duplicate ratio of sub-duplicate ratio of 8:9 is

- a. 64:81 b. 9:8 c. 128:162 ~~d. None of these~~

= Duplicate ratio of sub-dupli. ratio of 8:9 = 8:9

181. What least number should be added to each one of 6, 14, 18, 38 to make them in proportion?

- a. 5 b. 3 ~~c. 2~~ d. 4

$(6+2), (14+2), (18+2), (38+2)$ i.e. 8, 16, 20, 40 are in prop.

182. Raja can walk certain distance in 50 days if he takes rest for 9 hrs per day. How long it will take him to walk twice as far if he walks twice as fast and rests twice as long each day?

- ~~a. 125 days~~ b. 50 days c. 100 days d. None



	original	New
walking speed	x kms/hr	$2x$ kms/hr
Rest per day	9 hrs	18 hrs
walking per day	$24 - 9$ $= 15$ hrs	$24 - 18$ $= 6$ hrs
Dist. per day	$15x$ kms	$6 \times 2x$ $= 12x$ kms
NO. of days	50	$\frac{1500x}{12x} = 125$ days
Total distance traveled	$15x \times 50$ $= 750x$	$1500x$

← Answer



183. 3, x, 27, y are in continued proportion then $x : y = ?$

a. 9:1

~~b. 1:9~~

c. 1:27

d. 3:1

3, x, 27, y are in continued propⁿ

$$\frac{3}{x} = \frac{27}{y} \quad \therefore \frac{3}{27} = \frac{x}{y} \quad \therefore \frac{x}{y} = \frac{1}{9}$$

184. $\text{Log}_{1/9} 243 = x$. Find x



If $\text{Log}_b a = k$ then $b^k = a$

$$\therefore \left(\frac{1}{9}\right)^x = 243$$

$$(3^{-2})^x = 3^5$$

$$\left(\frac{1}{3^2}\right)^x = 3^5$$

$$3^{-2x} = 3^5$$

$$\therefore -2x = 5$$

$$x = -\frac{5}{2} = -2.50$$

185. $\text{Log}_2 x + \text{Log}_4 x + \text{Log}_{16} x = \frac{21}{4}$ then $x = ?$

a. 7

~~b. 8~~

c. 9

d. 10



$$\frac{4 \text{Log} x}{4 \text{Log} 2} + \frac{2 \text{Log} x}{4 \cdot \text{Log} 2} + \frac{\text{Log} x}{4 \cdot \text{Log} 2} = \frac{21}{4}$$

$$\therefore \frac{\text{Log} x}{\text{Log} 2} = 3$$

$$\text{Log} x = 3 \cdot \text{Log} 2$$

$$\text{Log} x = \text{Log} 2^3$$

$$\text{Log} x = \text{Log} 8$$

$$\therefore x = 8$$

$$\frac{7 \cdot \text{Log} x}{4 \cdot \text{Log} 2} = \frac{21}{4}$$

$$\frac{\text{Log} x}{\text{Log} 2} = \frac{21}{4} \times \frac{4}{7}$$

186. $\text{Log}_a 3 = 2$, $\text{Log}_b 8 = 3$; then $\text{Log}_b a = ?$

a. $\text{Log}_3 2$

b. $\text{Log}_2 3$

c. $\text{Log}_3 4$

~~d. $\text{Log}_4 3$~~



$$\text{Log}_a 3 = 2 \quad \therefore a^2 = 3 \quad \therefore a = 3^{1/2}$$

$$\text{Log}_b 8 = 3 \quad \therefore b^3 = 8 = 2^3 \quad \therefore b = 2$$

$$\text{Log}_b a = \frac{\text{Log} a}{\text{Log} b} = \frac{\text{Log} 3^{\frac{1}{2}}}{\text{Log} 2} = \frac{\frac{1}{2} \cdot \text{Log} 3}{\text{Log} 2} = \frac{\text{Log} 3}{2 \cdot \text{Log} 2}$$

$$= \frac{\text{Log} 3}{\text{Log} 2^2} = \frac{\text{Log} 3}{\text{Log} 4} = \text{Log}_4 3$$

187. Find x if $\text{Log}_x 10 + \text{Log}_x 100 + \text{Log}_x 1000 = 6$

a. 10

b. 2

c. 4

d. 6

→ $\text{Log}_{10} 10 = 1$
 $\text{Log}_{10} 100 = 2$
 $\text{Log}_{10} 1000 = 3$

$\therefore 1 + 2 + 3 = 6$

$x = 10$

188. $2 \text{Log} a + 3 \text{Log} b - 2 = 0$ the $a^2 b^3 = ?$

a. 10^4

b. 10^3

~~c. 10^2~~

d. None of these

→ $2 \text{Log} a + 3 \cdot \text{Log} b = 2$
 $\text{Log} a^2 + \text{Log} b^3 = 2$
 $\text{Log}_{10} (a^2 \cdot b^3) = 2$
 $\therefore a^2 b^3 = 10^2$

189. $\text{Log}_2 [\text{Log}_2 \{\text{Log}_3 (\text{Log}_3 27^3)\}] = ?$

a. $\frac{1}{2}$

b. 1

~~c. 0~~

d. 2

→ $= \text{Log}_2 [\text{Log}_2 \{\text{Log}_3 (\text{Log}_3 27^3)\}]$
 $= \text{Log}_2 [\text{Log}_2 \{\text{Log}_3 (\frac{9 \cdot \text{Log} 3}{\text{Log} 3})\}]$

$$= \text{Log}_2 \left[\text{Log}_2 \left\{ \text{Log}_3 9 \right\} \right]$$

$$= \text{Log}_2 \left[\text{Log}_2 2 \right] = \text{Log}_2 1 = 0 = \text{Zero}$$

190. $2 \text{Log } x = 4 \text{Log } 4$ then $x = ?$



$$2 \text{Log } x = 4 \cdot \text{Log } 4$$

$$\text{Log } x = 2 \cdot \text{Log } 4$$

$$\text{Log } x = \text{Log } 4^2$$

$$\text{Log } x = \text{Log } 16$$

$$\therefore x = 16$$

191. Age of a person is twice the sum of ages of his 2 sons & 5 years ago his age was 3 times the sum of ages of his 2 sons, his present age is _____ years

a. 60

b. 52

c. 51

d. 50



	A person's age	sum of ages of his 2 sons	
present	$2x$	x	$2x - 5 = 3x(x - 10)$
5 Years before	$2x - 5$	$x - 5 - 5 = x - 10$	$2x - 5 = 3x - 30$
			$25 = x$
			$\therefore x = 25$
			\therefore His present age = $2x = 50$ years

192. Sum of 2 numbers is 45 and the mean proportional between them is 18. The numbers are :

a. 15,30

b. 32,13

~~c. 36,9~~

d. 25,20

as $36 + 9 = 45$ & $36 \times 9 = 324$

Ratio, Proportion, Logs & Indices



Let numbers are x & y

$$x + y = 45$$

$$x, 18, y \text{ are in prop}^n \therefore xy = 18^2 = 324$$

193. If $(7p+3q) : (3p-2q) = 2:1$ then $p:q = ?$

$$\begin{aligned} \rightarrow \frac{7p+3q}{3p-2q} &= \frac{2}{1} & \therefore p &= -7q \\ 7p+3q &= 6p-4q & \therefore \frac{p}{q} &= \frac{-7}{1} = -7:1 \text{ OR } 7:-1 \end{aligned}$$

194. If $\text{Log}_a 23 = b$ then $\implies \therefore a^b = 23$

a. $a^{23} = b$

b. $b^a = 23$

c. $b^{23} = a$

d. $a^b = 23$

If $\text{Log}_x a = y$ then $x^y = a$

195. The angles of a triangle are in the ratio of $2 : 29 : 5$ then

Find the angles :

$$\begin{aligned} \rightarrow \text{Let the angles be : } & 2x, 29x, 5x \\ \therefore 2x + 29x + 5x &= 180 & \therefore \text{Angles are } & 2x = 10^\circ \\ 36x &= 180 & & 29x = 145^\circ \\ x &= 5 & & 5x = 25^\circ \end{aligned}$$

196. $a^2 + b^2 = 45, ab = 18$. Find $(\frac{1}{a} + \frac{1}{b})$

$$\begin{aligned} \rightarrow (a+b)^2 &= a^2 + b^2 + 2ab & (\frac{1}{a} + \frac{1}{b}) \\ (a+b)^2 &= 45 + 2(18) = 81 & = \frac{a+b}{ab} = \frac{9}{18} = \frac{1}{2} \\ \therefore (a+b) &= 9 & = 0.50 \end{aligned}$$

197. If $a^2 \cdot \text{Log}_3 x = b \cdot \text{Log}_{27} x$ then

a. $a = 3$

~~b. $3a^2 = b$~~

c. $b^2 = 3a$

d. None of these

$$\begin{aligned} a^2 \cdot \frac{\text{Log} x}{\text{Log} 3} &= b \cdot \frac{\text{Log} x}{3 \cdot \text{Log} 3} & \therefore a^2 &= \frac{b}{3} \\ \therefore 3a^2 &= b \end{aligned}$$

198. Mr. A says to his son '7 years before I was 7 times as old as you were' and 3 years later I shall be 3 times as old as you will be. Find present age of Mr. A's son :

- ~~a. 12 years~~ b. 15 years c. 5 years d. 7 years



	Mr. A	Mr. A's son	$x-7 = 7(y-7)$
			$x-7 = 7y-49$
present age	x	y	$x-7y = -42$ ①
7 years before	$(x-7)$	$(y-7)$	$(x+3) = 3(y+3)$
3 years later	$(x+3)$	$(y+3)$	$x+3 = 3y+9$
			$x-3y = 6$ ②

$$\begin{array}{r} x-7y = -42 \\ -x+3y = 6 \\ \hline -4y = -48 \end{array} \quad \therefore y = 12$$

\therefore present age of Mr. A's son = 12 years

199. One third of one half of three fourth of a number is 60, the number is :

- ~~a. 480~~ b. 520 c. 500 d. None

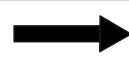


$$\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} \times x = 60$$

$$x = 60 \times 2 \times 4 = 480$$

200. $x^2 + y^2 = 14xy$ then $2 \text{Log } 4 + \text{Log } x + \text{Log } y = ?$

- a. $\frac{1}{2} \text{Log}(x+y)$ ~~b. $2 \text{Log}(x+y)$~~ c. $\text{Log}(x+y)$ d. None



$$\begin{aligned} x^2 + y^2 &= 14xy \\ x^2 + y^2 + 2xy &= 14xy + 2xy \\ (x+y)^2 &= 16xy \end{aligned}$$

$$\begin{aligned} &2 \text{Log } 4 + \text{Log } x + \text{Log } y \\ &= \text{Log } 16 + \text{Log } x + \text{Log } y \\ &= \text{Log } (16xy) \\ &= \text{Log } (x+y)^2 = 2 \cdot \text{Log } (x+y) \end{aligned}$$



Ratio, Proportion, Logs & Indices



201. $(2p^2q^3r^2)^0 = ?$

a. 0

~~b. 1~~

c. $4p^4q^6r^4$

d. None



$(\text{Any number})^0 = 1$

202. $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$ then $(p+q+r) = ?$

a. 1

~~b. 0~~

c. $\frac{1}{2}$

d. None



$x^{1/p} = y^{1/q} = z^{1/r} = k$

$xyz = 1$

$k^p \cdot k^q \cdot k^r = 1$

$x^{1/p} = k \therefore (x^{1/p})^p = k^p \therefore x = k^p$

$k^{(p+q+r)} = 1$

$y^{1/q} = k \therefore y = k^q$

$z^{1/r} = k \therefore z = k^r$

$\therefore p+q+r = 0$

203. $a^x = b, b^y = c, c^z = a$; then $xyz = ?$

~~a. 1~~

b. 2

c. 3

d. None



$a^x = b \quad b^y = c \quad c^z = a$
 $a = b^{1/x} \quad b = c^{1/y} \quad c = a^{1/z}$

$\textcircled{\text{OR}} \quad a^x = b$

$x \cdot \text{Log} a = \text{Log} b$

xyz

$= \frac{\text{Log} b}{\text{Log} a} \times \frac{\text{Log} c}{\text{Log} b} \times \frac{\text{Log} a}{\text{Log} c}$

$a = (c^{1/y})^{1/x} = (c)^{\frac{1}{xy}}$

$a^1 = (a^{1/z})^{1/xy} = (a)^{\frac{1}{xyz}}$

$\therefore x = \frac{\text{Log} b}{\text{Log} a}$

$y = \frac{\text{Log} c}{\text{Log} b}$

$z = \frac{\text{Log} a}{\text{Log} c}$

$= 1 \cdot 0 \cdot 0$

$\therefore 1 = \frac{1}{xyz} \therefore xyz = 1$

204. $\text{Log}_{2\sqrt{3}} 1728 = m$ then $m = ?$

a. 2

b. 3

c. 4

~~d. None~~



$\text{Log}_{2\sqrt{3}} 1728 = m$

$\therefore (2\sqrt{3})^m = 1728 = 64 \times 27 = 2^6 \times 3^3$

$(2\sqrt{3})^m = 2^6 \times [(\sqrt{3})^2]^3 = 2^6 \times 3^6$

$(2\sqrt{3})^m = (2\sqrt{3})^6 \therefore m = 6$

Ratio, Proportion, Logs & Indices



205. $\frac{a^m}{a^{-m}} = ?$

- a. 1 b. -1 c. 0 ~~d. None~~

→ $= \frac{a^m}{a^{-m}} = a^{m - (-m)} = a^{m+m} = a^{2m}$

206. $a:b = 2:5, b:c = 15:46, c:d = 92:200$ then $a:d = ?$

- a. 2:3 b. 3:192 c. 1:100 d. None

→ $\frac{a}{d} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{2}{5} \times \frac{15}{46} \times \frac{92}{200}$
 $= \frac{3}{50} = 3:50$

$a:d$ is compounded ratio of $a:b, b:c, c:d$

207. $\text{Log}_4(x^2+x) - \text{Log}_4(x+1) = 2$, then $x = ?$

- ~~a. 16~~ b. 4 c. 8 d. None

→ $\text{Log}_4(x^2+x) - \text{Log}_4(x+1) = 2$
 $\text{Log}_4 \left[\frac{x^2+x}{x+1} \right] = 2$ AS $\text{Log}_m a - \text{Log}_m b = \text{Log}_m (a/b)$
 $\text{Log}_4 \left[\frac{x(x+1)}{x+1} \right] = 2$ $\therefore \text{Log}_4 x = 2$
 $\therefore x = 4^2 = 16$

208. Characteristic of 7.128 is

- a. 6 b. $\bar{8}$ ~~c. 0~~ d. None

→ characteristic of 7.128 = No. of digits before decimal point - 1
 $= 1 - 1 = 0$

209. $2^x - 2^{x-1} = 4$ then $x^x = ?$

- a. 2 b. 1 c. 64 ~~d. 27~~

→ $2^x - 2^{x-1} = 4$ $\therefore 2^x = 4 \times 2 = 8 = 2^3$
 $2^x - \frac{2^x}{2} = 4$ $\therefore x = 3$

Ratio, Proportion, Logs & Indices



$$2^x \left(1 - \frac{1}{2}\right) = 4$$

$$2^x \times \frac{1}{2} = 4$$

$$x^x = 3^3 = 27$$

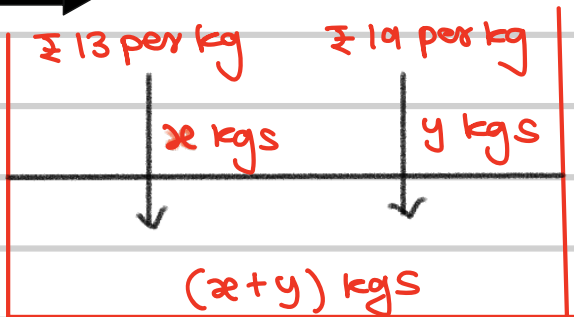
210. 2 kinds of rice, 1st costs ₹ 13 per kg & 2nd costs ₹ 19 per kg are mixed together. Find in what ratio they are mixed so that mixture costs ₹ 14.20 per kg?

a. 3:1

~~b. 4:1~~

c. 3:4

d. 4:3



Total cost of
(x+y) kgs

Total cost of
RICE

$$13x + 19y = 14.20(x+y)$$

$$13x + 19y = 14.20x + 14.20y$$

$$4.80y = 1.20x$$

$$\frac{4.80}{1.20} = \frac{x}{y}$$

$$\therefore \frac{x}{y} = \frac{48}{12} = 4:1$$

211. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$ then $3x^3 - 9x = ?$

a. 15

~~b. 10~~

c. 12

d. None

→ $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$

Taking cube on both sides

$$x^3 = \left(3^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)^3$$

$$x^3 = \left(3^{\frac{1}{3}}\right)^3 + \left(3^{-\frac{1}{3}}\right)^3 + 3 \times 3^{\frac{1}{3}} \times 3^{-\frac{1}{3}} \left(3^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)$$

$$x^3 = 3 + 3^{-1} + 3^{1+\frac{1}{3}-\frac{1}{3}}(x)$$

$$x^3 = 3 + \frac{1}{3} + (3^1 \times x)$$

$$3x^3 = 9 + 1 + 3x$$

$$\therefore 3x^3 - 9x = 10$$

212. Commensurable ratio = If ratio of 2 or more terms can be written in the form of **Ratio of Integers** then it is said to be a Commensurable Ratio

213. Incommensurable Ratio OR Non-Commensurable Ratio

If ratio of 2 or more terms cannot be written in the form of Ratio of integers then it is said to be a Incommensurable ratio.
OR Non-commensurable

214.	Ratio	Commensurable / Incommensurable Ratio
	5 : 18	AS it is a ratio of integers, It is a commensurable ratio
	8.50 : 3.50	$8.50 : 3.50 = 85 : 35 = 17 : 7$ AS 8.50 : 3.50 can be written in the form of Ratio of integers \therefore It is commensurable ratio.
	8.15 : 2.221	$8.15 : 2.221 = 8150 : 2221$ AS 8.15 : 2.221 can be written in the form of Ratio of integers \therefore It is commensurable ratio.
	5.8361 : 3	$5.8361 : 3 = 58361 : 30000$ AS 5.8361 : 3 can be written in the form of Ratio of integers \therefore It is commensurable ratio.
	$\sqrt{5} : \sqrt{7}$	$2.23606797749\dots : 2.64575131106\dots$ $\sqrt{5} : \sqrt{7}$ can not be written in the form of Ratio of integers $\therefore \sqrt{5} : \sqrt{7}$ is incommensurable OR Non-commensurable ratio.

215.

$\sqrt{25} : \sqrt{64}$	It can be written as 5:8 \therefore Commensurable ratio
$\sqrt{50} : \sqrt{450}$	$\frac{\sqrt{50}}{\sqrt{450}} = \frac{\sqrt{50}}{\sqrt{50 \times 9}} = \frac{\cancel{\sqrt{50}}}{\cancel{\sqrt{50}} \times \sqrt{9}} = \frac{1}{\sqrt{9}} = \frac{1}{3} = 1:3$ $\therefore \sqrt{50} : \sqrt{450}$ is a commensurable ratio
$\sqrt{147} : \sqrt{75}$	$\frac{\sqrt{147} \times \sqrt{3}}{\sqrt{75} \times \sqrt{3}} = \frac{\sqrt{147 \times 3}}{\sqrt{75 \times 3}} = \frac{\sqrt{441}}{\sqrt{225}} = \frac{21}{15} = \frac{7}{5}$ (OR) $\frac{\sqrt{147}}{\sqrt{75}} = \frac{\sqrt{49 \times 3}}{\sqrt{25 \times 3}} = \frac{\sqrt{49} \times \sqrt{3}}{\sqrt{25} \times \sqrt{3}} = \frac{7}{5} = 7:5$ $\therefore \sqrt{147} : \sqrt{75}$ is a commensurable ratio
$8\frac{1}{3} : 9\frac{2}{3}$	$8\frac{1}{3} : 9\frac{2}{3} = \frac{25}{3} : \frac{29}{3} = 25:29$ $\therefore 8\frac{1}{3} : 9\frac{2}{3}$ is a commensurable ratio
$8\frac{1}{7} : 11\frac{2}{3}$	$8\frac{1}{7} : 11\frac{2}{3} = \frac{57}{7} : \frac{35}{3} = 171:245$ $\therefore 8\frac{1}{7} : 11\frac{2}{3}$ is a commensurable ratio
$\sqrt{5} : \sqrt{10}$	$\frac{\sqrt{5}}{\sqrt{5 \times \sqrt{2}}} = 1 : \sqrt{2}$ $\sqrt{5} : \sqrt{10}$ is incommensurable ratio.
$\sqrt{100} : \sqrt{196}$	$\sqrt{100} : \sqrt{196} = 10 : 14 = 5:7$ $\therefore \sqrt{100} : \sqrt{196}$ is a commensurable ratio.

216. 2 whole numbers whose sum is 100 cannot be in the ratio of _____

a. 16:9

~~b. 3:4~~

c. 3:7

d. 4:1

$\rightarrow 64, 36$

$\rightarrow 42.85, 57.15$

$\rightarrow 30, 70$

$\rightarrow 80, 20$

217. Divide ₹ 61,000 among a, b, c such that a & b get in the ratio of 2:5 and b & c get in the ratio of 3:8.

$\rightarrow a:b = 2:5 = 6:15$
 $b:c = 3:8 = 15:40$
 $\therefore a:b:c = 6:15:40$

a's share = $\frac{6}{61} \times 61000 = ₹ 6,000$

b's share = $\frac{15}{61} \times 61000 = ₹ 15,000$

c's share = $\frac{40}{61} \times 61000 = ₹ 40,000$

218. $\frac{2^{\frac{1}{3}} \times 2^{\frac{1}{5}}}{4^{\frac{2}{3}}} =$

$\rightarrow \left[\frac{2^{\frac{1}{3}} \times 2^{\frac{1}{5}}}{4^{\frac{2}{3}}} \right] = \left[\frac{2^{\frac{1}{3} + \frac{1}{5}}}{(2^2)^{\frac{2}{3}}} \right] = \left[\frac{2^{\frac{8}{15}}}{2^{\frac{4}{3}}} \right] = \left[\frac{2^{\frac{8}{15}}}{2^{\frac{20}{15}}} \right] = (2)^{\frac{8}{15} - \frac{20}{15}}$
 $= (2)^{-\frac{12}{15}}$
 $= 2^{-\frac{4}{5}}$

219. A & B together can finish the work in 8 days. If A alone can finish the work in 24 days then B alone can finish the work in _____ no. of days.

a. 10

~~b. 12~~

c. 15

d. None

A's work of 1 day = $\frac{1}{24}$

$\left(\frac{1}{24} + \frac{1}{x} \right) = \frac{1}{8}$

suppose B alone can complete the work in x days

$\frac{1}{x} = \frac{3}{24} - \frac{1}{24} = \frac{2}{24}$

$\therefore B$'s work of 1 day = $\frac{1}{x}$

$\frac{1}{x} = \frac{1}{12} \therefore x = 12$ days

220. $\text{Log}_{2\sqrt{3}} 1728 = m$. Find the value of m^2 is

a. 6

~~b. 36~~

c. $\sqrt{6}$

d. None of these



$$\text{Log}_{2\sqrt{3}} 1728 = m$$

$$\therefore (2\sqrt{3})^m = 1728 = 64 \times 27 = 2^6 \times [(\sqrt{3})^2]^3$$

$$\therefore (2\sqrt{3})^m = (2\sqrt{3})^6$$

$$\therefore m = 6 \quad \therefore m^2 = 6^2 = 36$$

221. $2 \text{Log} x = 15$. $\text{Log} \sqrt[7]{2}$. Find the value of x.



$$2 \text{Log} x = 15 \cdot \text{Log}(2)^{1/7}$$

$$\text{Log} x = \frac{15}{2} \text{Log} 2^{1/7}$$

$$\text{Log} x = \text{Log} (2^{1/7})^{15/2}$$

$$\text{Log} x = \text{Log} (2)^{15/14}$$

$$\therefore x = 2^{15/14} = \sqrt[14]{2^{15}} = \sqrt[14]{32768}$$

222. If $a^x = b$, $b^y = c$, $c^z = a$ then find value of $(xyz)^3$

a. 1

b. 0

c. 8

d. None of these



$$a^x = b \quad x \cdot \text{Log} a = \text{Log} b \quad \therefore x = \text{Log} b / \text{Log} a \quad (xyz)^3$$

$$b^y = c \quad y \cdot \text{Log} b = \text{Log} c \quad \therefore y = \text{Log} c / \text{Log} b = \left(\frac{\text{Log} b}{\text{Log} a} \times \frac{\text{Log} c}{\text{Log} b} \times \frac{\text{Log} a}{\text{Log} c} \right)^3$$

$$c^z = a \quad z \cdot \text{Log} c = \text{Log} a \quad \therefore z = \text{Log} a / \text{Log} c = 1^3 = 1$$

$$a^x = b \quad \therefore (c^z)^x = b \quad c^{xz} = b \quad \therefore (b^y)^{xz} = b \quad \therefore b^{xyz} = b^1$$

223. If $2^x - 2^{x-1} = 16$; then find value of \sqrt{x}



$$2^x - 2^{x-1} = 16$$

$$2^x - \frac{2^x}{2} = 16$$

AS $x = 5$

$$\sqrt{x} = \sqrt{5}$$

Ratio, Proportion, Logs & Indices



$$2^x \left(1 - \frac{1}{2}\right) = 16$$

$$2^x \times \frac{1}{2} = 16$$

$$2^x = 32 = 2^5$$

$$\therefore x = 5$$

224. If $\text{Log } 20 = 1.3010$. Find $\text{Log } 0.00000020$

$$\rightarrow \text{Log } (0.00000020) = \bar{7}.3010 = -7 + 0.3010 = -6.6990$$

$$\textcircled{R} \text{Log}(0.0000002) = \text{Log}\left(\frac{20}{100000000}\right) = \text{Log } 20 - \text{Log } 100000000 = 1.3010 - 8 = -6.6990$$

225. $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2}) = ?$

a. $a + \frac{1}{a}$

~~b. $a - \frac{1}{a}$~~

c. $a^2 + \frac{1}{a^2}$

d. None of these

$$\begin{aligned} &\rightarrow (a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2}) \\ &= (a^{1/4} - a^{-1/4})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2}) \\ &= (a^{1/2} - a^{-1/2})(a^{1/2} + a^{-1/2}) \\ &= a - a^{-1} = \left(a - \frac{1}{a}\right) \end{aligned}$$

226. If $\text{Log } (2a-3b) = \text{Log } a - \text{Log } b$ then $a = ?$

~~a. $\frac{3b^2}{2b-1}$~~

b. $\frac{3b}{2b-1}$

c. $\frac{b^2}{2b+1}$

d. $\frac{3b^2}{2b+1}$

$$\begin{aligned} &\rightarrow \text{Log } (2a-3b) = \text{Log } a - \text{Log } b \\ &\text{Log } (2a-3b) = \text{Log } \left(\frac{a}{b}\right) \\ &2a-3b = \frac{a}{b} \\ &b(2a-3b) = a \end{aligned}$$

$$\begin{aligned} 2ab - 3b^2 &= a \\ 2ab - a &= 3b^2 \\ a(2b-1) &= 3b^2 \\ \therefore a &= \left(\frac{3b^2}{2b-1}\right) \end{aligned}$$

Ratio, Proportion, Logs & Indices



$$227. \left(\frac{1}{\text{Log}_{ab} abc} \right) + \left(\frac{1}{\text{Log}_{bc} abc} \right) + \left(\frac{1}{\text{Log}_{ac} abc} \right) = ?$$

a. 0

b. 1

~~c. 2~~

d. -1



$$= \frac{1}{\frac{\text{Log} abc}{\text{Log} ab}} + \frac{1}{\frac{\text{Log} abc}{\text{Log} bc}} + \frac{1}{\frac{\text{Log} abc}{\text{Log} ac}}$$

$$= \frac{\text{Log} ab + \text{Log} bc + \text{Log} ac}{\text{Log} (abc)} = \frac{\text{Log} (ab \times bc \times ac)}{\text{Log} (abc)}$$

$$= \frac{\text{Log} (a^2 b^2 c^2)}{\text{Log} (abc)} = \frac{\text{Log} (abc)^2}{\text{Log} (abc)} = \frac{2 \cdot \text{Log} (abc)}{\text{Log} (abc)} = 2$$

228. ₹ 407 are to be divided in the ratio of $\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$ then smallest share is :



$$\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$$

$$\text{smallest share} = \frac{10}{37} \times ₹ 407$$

$$= 15 : 12 : 10$$

$$= ₹ 110/-$$

229. Third proportional to $(a^2 - b^2)$ and $(a+b)^2$ is

a. $\frac{(a+b)}{(a-b)}$

b. $\frac{(a-b)}{(a+b)}$

c. $\frac{(a-b)^2}{a+b}$

~~d. $\frac{(a+b)^3}{(a-b)}$~~



Let 3rd proportional be : x

$\therefore (a^2 - b^2), (a+b)^2, x$ are in proportion

square of middle term = product of extremes

$$\left[(a+b)^2 \right]^2 = (a^2 - b^2) \times x$$

$$(a+b)^4 = (a+b)(a-b) \cdot x$$

$$\therefore x = \frac{(a+b)^4}{(a+b)(a-b)} = \frac{(a+b)^3}{(a-b)}$$

230. If $x = y^a, y = z^b, z = x^c$ then $abc = ?$

a. 2

b. 1

c. 3

d. 4

→ $x^1 = y^a = (z^b)^a = z^{ab} = (x^c)^{ab} = x^{abc}$

$\therefore abc = 1$

(OR) $x = y^a \quad y = z^b \quad z = x^c$

$$a = \frac{\text{Log } x}{\text{Log } y} \quad b = \frac{\text{Log } y}{\text{Log } z} \quad c = \frac{\text{Log } z}{\text{Log } x} = \frac{\text{Log } x}{\text{Log } y} \times \frac{\text{Log } y}{\text{Log } z} \times \frac{\text{Log } z}{\text{Log } x} = 1.00$$

231. $\left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right) = ?$

a. $\frac{1}{2}$

~~b. $\frac{3}{2}$~~

c. $\frac{2}{3}$

d. $\frac{1}{3}$

→ $= \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + \frac{2^n}{2}}{2^n \times 2^1 - 2^n} = \frac{2^n \left(1 + \frac{1}{2}\right)}{2^n (2 - 1)}$

$= \frac{3/2}{1} = 3/2$

232. Find x if $x \cdot x^{1/3} = (x^{1/3})^x$

a. 3

~~b. 4~~

c. 2

d. 6

→ $x^1 \cdot x^{1/3} = (x^{1/3})^x \quad \therefore \frac{x}{3} = \frac{x}{3}$

$x^{1+1/3} = x^{x/3} \quad \therefore x = 4$

$(x)^{4/3} = (x)^{x/3}$

233. $25^{150} = (25x)^{50}$ then $x = ?$

a. 5^3

b. 5^2

~~c. 5^4~~

d. 5^1



$$25^{150} = 25^{50} \cdot x^{50}$$

$$x^{50} = \left(\frac{25^{150}}{25^{50}} \right) = 25^{100} = (25^2)^{50}$$

$$(x)^{50} = (625)^{50} \quad \therefore x = 625 = 5^4$$

234. Salary of P is 25% lower than Q and salary of R is 20% higher than Q then ratio of salary of R and P will be

a. 5:8

~~b. 8:5~~

c. 5:3

d. 3:5



suppose Q's salary = ₹ 100

\therefore P's salary = ₹ 75

R's salary = ₹ 120

Ratio of salary of
R & P

= 120 : 75

= 8 : 5

235. If $\text{Log } 2 = x$, $\text{Log } 3 = y$ then $\text{Log } 60 = ?$

~~a. $x+y+1$~~

b. $x+y-1$

c. $2x+3y+1$

d. $2x+5y-1$



$$\text{Log}(60) = \text{Log}(2 \times 3 \times 10)$$

$$= \text{Log } 2 + \text{Log } 3 + \text{Log } 10$$

$$= x + y + 1$$

**BIG JOURNEYS BEGIN WITH
SMALL STEPS !**

① If $\frac{a}{7} = \frac{b}{8} = \frac{c}{9}$

Find value of $\left(\frac{a+b+c}{c}\right) = ?$

$\rightarrow \frac{a}{7} = \frac{b}{8} = \left(\frac{c}{9}\right) = \left(\frac{a+b+c}{7+8+9}\right)$

$\therefore \frac{a+b+c}{24} = \frac{c}{9}$

$\therefore \frac{a+b+c}{c} = \frac{24}{9} = \frac{8}{3}$

$$\frac{7}{12} = \frac{70}{120} = \frac{21}{36} = \frac{7+70+21}{12+120+36} = \frac{7+70-21}{12+120-36}$$

$$= \frac{98}{168} = \frac{56}{96}$$

② If $\left(\frac{a}{16}\right) = \left(\frac{b}{19}\right) = \left(\frac{c}{11}\right)$ Find value of $\left(\frac{2a+9b-8c}{2b}\right)$

$\Rightarrow \left(\frac{a}{16}\right) = \left(\frac{b}{19}\right) = \left(\frac{c}{11}\right)$

$\frac{2a}{32} = \frac{9b}{171} = \frac{8c}{88} = \left(\frac{2a+9b-8c}{32+171-88}\right)$ By using Addendo

$\therefore \frac{2b}{38} = \left(\frac{2a+9b-8c}{115}\right)$ & Subtrahendo

$$\therefore \frac{2a+9b-8c}{2b} = \frac{115}{38}$$

(OR)

$$\left(\frac{a}{16}\right) = \left(\frac{b}{19}\right) = \left(\frac{c}{11}\right) = m \quad \therefore a=16m, b=19m, c=11m$$

$$\begin{aligned} \frac{2a+9b-8c}{2b} &= \frac{2(16m) + 9(19m) - 8(11m)}{2 \times 19m} \\ &= \frac{32m + 171m - 88m}{38m} = \frac{115m}{38m} \\ &= \left(\frac{115}{38}\right) \end{aligned}$$

$$(3) \quad \text{Log}_b a = \frac{\text{Log } a}{\text{Log } b}$$

$$(4) \quad \boxed{\text{If } \text{Log}_b a = k \text{ then } b^k = a} \quad \text{Super Imp Rule}$$

$$\text{Log}_{10} 100 = 2 \quad \text{as } 10^2 = 100$$

$$\text{Log}_2 32 = 5 \quad \text{as } 2^5 = 32$$

$$\text{AS } 8^7 = 2097152 \quad \therefore \text{Log}_8 2097152 = 7$$

$$(5) \quad A \cdot \text{Log}(\text{Log } m) = m = \text{Log}(A \cdot \log m)$$

$$(6) \quad \text{Log } 36.979 = 1.5680$$

$$\text{Log } 8196 = 3.9136$$

$$\begin{aligned} \text{Log } 0.0000002281 &= \bar{7}.3581 = -7 + 0.3581 \\ &= -6.6419 \end{aligned}$$

$$A \cdot \text{Log } 3.2291 = 1694$$

$$\textcircled{7} \quad \text{Log } 28 = 1.4472$$

$$A.\log 1.4472 = 28.00$$

$$A.\text{Log } 1.4472 = 28$$

$$A.\text{Log}(\text{Log } 28) = 28$$

$$\therefore A.\text{Log}(\text{Log } x) = x$$

$$\text{Log } 28 = 1.4472$$

$$\text{Log}(A.\log 1.4472) = 1.4472$$

$$\therefore \text{Log}(A.\log m) = m$$

$$\textcircled{8} \quad \text{Find } \text{Log}(A.\log 2)$$

$$= \text{Log}(A.\log 2.0000)$$

$$= \text{Log}(100.0)$$

$$= 2.0000$$

$$A.\text{Log}(\text{Log } 100)$$

$$= A.\text{Log}(2.0000)$$

$$= 100.00$$

$$\textcircled{9} \quad \text{Log } 7 = 0.8451, \text{Log } 3 = 0.4771, \text{Log } 21 = 1.3222$$

$$1.3222 = 0.8451 + 0.4771$$

$$\text{Log } 21 = \text{Log } 7 + \text{Log } 3$$

$$\text{Log}(7 \times 3) = \text{Log } 7 + \text{Log } 3$$

$$\therefore \text{Log}(a \times b) = \text{Log } a + \text{Log } b$$

$$\boxed{\text{Log}_m(ab) = \text{Log}_m a + \text{Log}_m b}$$

$$\textcircled{10} \quad \text{Log } 4 = 0.6021 \quad \text{Log } 5 = 0.6990 \quad \text{Log } 20 = 1.3010$$

$$1.3011 = 0.6021 + 0.6990$$

$$\text{Log } 20 = \text{Log } 4 + \text{Log } 5$$

$$\text{Log}(4 \times 5) = \text{Log } 4 + \text{Log } 5$$

$$\begin{aligned}\textcircled{11} \quad \text{Log} [abc] &= \text{Log} [(ab) \times c] \\ &= \text{Log} (ab) + \text{Log} c \\ &= \text{Log} a + \text{Log} b + \text{Log} c\end{aligned}$$

$$\text{Log}_k (pqrs) = \text{Log}_k p + \text{Log}_k q + \text{Log}_k r + \text{Log}_k s$$

$$\begin{aligned}\textcircled{12} \quad \text{Log} (8^5) &= \text{Log} (8 \times 8 \times 8 \times 8 \times 8) \\ &= \text{Log} 8 + \text{Log} 8 + \text{Log} 8 + \text{Log} 8 + \text{Log} 8 \\ \text{Log} 8^5 &= 5 \cdot \text{Log} 8\end{aligned}$$

$$\therefore \text{Log} a^b = b \cdot \text{Log} a$$

$$\text{Log} p^x = x \cdot \text{Log} p$$

$$\text{Log} 38^{-9} = -9 \cdot \text{Log} 38$$

$$\text{Log} 12^{-100} = -100 \cdot \text{Log} 12$$

$$\text{Log} 33^{35} = 35 \cdot \text{Log} 33$$

$$\begin{aligned}\textcircled{13} \quad \text{Log} \left(\frac{a}{b}\right) &= \text{Log} \left(a \times \frac{1}{b}\right) \\ &= \text{Log} (a \times b^{-1}) \\ &= \text{Log} a + \text{Log} b^{-1} \\ &= \text{Log} a + (-1 \cdot \text{Log} b)\end{aligned}$$

$$\text{Log} (a/b) = \text{Log} a - \text{Log} b$$

$$\text{Log} (m/n) = \text{Log} m - \text{Log} n$$

$$\begin{aligned} \textcircled{17} \quad \text{Log} \frac{128^8}{\sqrt[7]{512}} &= \frac{\text{Log}(128)^8}{\text{Log} \sqrt[7]{512}} \\ &= \frac{\text{Log}(2^7)^8}{\text{Log}(2^9)^{1/7}} = \frac{\text{Log}(2)^{56}}{\text{Log}(2)^{9/7}} = \frac{56 \cdot \text{Log} 2}{\frac{9}{7} \text{Log} 2} \\ &= 56 \times \frac{7}{9} = \left(\frac{392}{9}\right) = 43.555555 \end{aligned}$$

$$\begin{aligned} \textcircled{18} \quad \text{Log} \frac{(\sqrt[7]{9})^{13}}{\sqrt[8]{81}} &= \frac{\text{Log}(\sqrt[7]{9})^{13}}{\text{Log} \sqrt[8]{81}} \\ &= \frac{\text{Log}[(3^2)^{1/7}]^{13}}{\text{Log}(3^4)^{1/8}} = \frac{\text{Log}(3)^{\frac{26}{7}}}{\text{Log}(3)^{\frac{1}{2}}} = \frac{\frac{26}{7} \text{Log} 3}{\frac{1}{2} \text{Log} 3} \\ &= \frac{26}{7} \times \frac{2}{1} = \left(\frac{52}{7}\right) \end{aligned}$$

$$\begin{aligned} \textcircled{19} \quad \text{Log} \frac{(\sqrt[7]{4})^{-5/2}}{\sqrt[9]{32}} &= \frac{\text{Log}(\sqrt[7]{2^2})^{-5/2}}{\text{Log}(32)^{1/9}} \\ &= \frac{\text{Log}[(2^2)^{1/7}]^{-5/2}}{\text{Log}(2^5)^{1/9}} = \frac{\text{Log}(2)^{-5/7}}{\text{Log}(2)^{5/9}} = \frac{-\frac{5}{7} \text{Log} 2}{\frac{5}{9} \text{Log} 2} \\ &= -\frac{5}{7} \times \frac{9}{5} = -9/7 \end{aligned}$$

(20) $\text{Log}(a \times b) = \text{Log}a + \text{Log}b$	$a^{-m} = \frac{1}{a^m}$
$\text{Log}(a/b) = \text{Log}a - \text{Log}b$	$a^k \times a^{-k} = 1 = a^0$
$a^m \times a^n = a^{m+n}$	$\sqrt[9]{2 \times 35} = \sqrt[9]{2} \times \sqrt[9]{35}$
$\frac{a^m}{a^k} = a^{m-k}$	$a^{-1/k} = \frac{1}{a^{1/k}} = \frac{1}{\sqrt[k]{a}}$
$(a \times b)^m = a^m \cdot b^m$	$(\frac{a}{b})^{-m} = (\frac{b}{a})^m$
$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$\text{Log}_y x \times \text{Log}_m y = \text{Log}_m x$