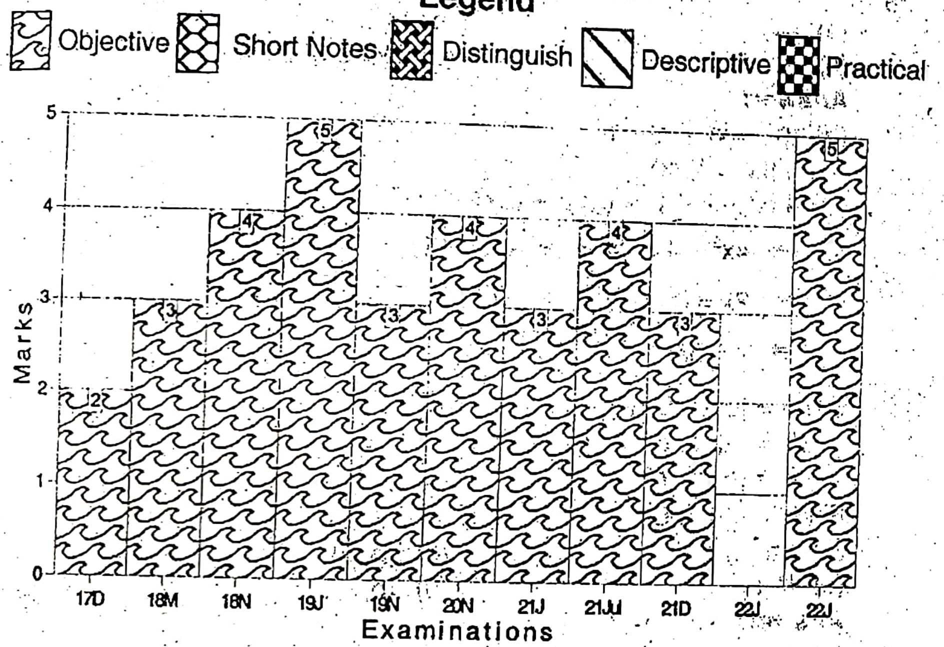
CHAPTER 7

SETS, RELATIONS AND FUNCTIONS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions



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PAST YEAR QUESTIONS AND ANSWERS

2008 - FEBRUARY

[1] Out of total 150 students, 45 passed in Accounts, 30 in Economics and 50 in Maths, 30 in both Accounts and Maths, 32 in both Maths and Economics, 35 in both Accounts and Economics, 25 students passed in all the three subjects. Find the numbers who passed at least in any one of the subjects :

(a) 63 (b) 53

(c) 73 (d) None (1 mark)

Answer:

(b) $n(A) = 45$

$n(M) = 50$

$n(E) = 30$

$n(A \cap M) = 30$

$n(M \cap E) = 32$

$n(A \cap E) = 35$

$n(A \cap M \cap E) = 25$

$n(A \cup M \cup E) = n(A) + n(M) + n(E) - n(A \cap M)$
$- n(A \cap E) - n(M \cap E) + n(A \cap M \cap E)$
$= 45 + 50 + 30 - 30 - 35 - 32 + 25$

$n(A \cup M \cup E) = 53$

2008 - JUNE

[2] If $f(x) = \frac{2+x}{2-x}$, then $f^{-1}(x)$:

(a) $\frac{2(x-1)}{x+1}$ (b) $\frac{2(x+1)}{x-1}$

(c) $\frac{x+1}{x-1}$ (d) $\frac{x-1}{x+1}$ (1 mark)

Answer:

(a) Let $f(x) = y$

$\frac{2+x}{2-x} = y$

$2 + x = 2y - xy$

$x + xy = 2y - 2$

$x(1 + y) = 2(y - 1)$

$x = \frac{2(y-1)}{(y+1)}$

$f^{-1}(y) = \frac{2(y-1)}{y+1}$

Therefore, $f^{-1}(x) = \frac{2(x-1)}{(x+1)}$

2008 - DECEMBER

[3] If A = {1, 2, 3, 4, }
B = {2, 4, 6, 8,}
f (1) = 2, f (2) = 4, f (3) = 6 and
f (4) = 8, And $f: A \to B$ then f^{-1} is :

(a) {(2,1), (4, 2), (6, 3), (8, 4)}
(b) {(1, 2), (2, 4), (3, 6), (4, 8)}
(c) {(1, 4), (2, 2), (3, 6), (4, 8)}
(d) None of these (1 mark)

Answer:

(a) A = {1, 2, 3, 4}, B = {2, 4, 6, 8}
When $f : A \to B$, f = {(1, 2), (2, 4), (3, 6), (4, 8)}
f^{-1} implies $f : B \to A$
f^{-1} = {(2, 1), (4, 2), (6, 3), (8, 4)}

[4] If $f(x) = x^2 + x - 1$ and $4f(x) = f(2x)$ then find 'x'.

(a) 4/3 (b) 3/2
(c) – 3/4 (d) None of these (1 mark)

Answer:

(b) $f(x) = x^2 + x - 1$

$4 f(x) = f(2x)$

$4 [x^2 + x - 1] = (2x)^2 + (2x) - 1$

$\Rightarrow 4x^2 + 4x - 4 = 4x^2 + 2x - 1$

$\Rightarrow 2x = 3$

$\Rightarrow x = 3/2$

[5] If A = {p, q, r, s}
B = {q, s, t}
C = {m, q, n}
Find C – (A ∧ B)

(a) {m, n} (b) {p, q}
(c) {r, s} (d) {p, r} (1 mark)

Answer:

(a) A = {p, q, r, s}
B = {q, s, t}
$A \cap B$ = {q, s}
C = {m, q, n}
$C - (A \cap B)$ = {m, n}

2009 - DECEMBER

[6] $X = \{x, y, w, z\}$, $y = \{1,2,3,4\}$
$H = \{(x, 1), (y, 2), (y, 3), (z, 4), (x, 4)\}$
(a) H is a function from X to Y
(b) H is not a function from X to Y
(c) H is a relation from Y to X
(d) None of the above (1 mark)

Answer:

(b) Any relation from X to Y in which no two different ordered pairs have the same first element is called a FUNCTION.
Therefore, in the given question, H is NOT a function from X to Y because the different ordered pairs of H have the same first element.

[7] Given the function $f(x) = (2x + 3)$, then the value of $f(2x) - 2f(x) + 3$ will be :
(a) 3 (b) 2
(c) 1 (d) 0 (1 mark)

(d) $f(x) = 2x + 3$
$f(2x) - 2f(x) + 3$
$= [2(2x) + 3] - [2(2x + 3)] + 3$
$= 4x + 3 - 4x - 6 + 3$
$= 4x - 4x + 6 - 6.$
$= 0.$

[8] If $f(x) = 2x + h$ then find $f(x + h) - 2f(x)$
(a) $h - 2x$ (b) $2x - h$
(c) $2x + h$ (d) None of these (1 mark)

Answer:

(a) $f(x) = 2x + h$
$f(x + h) - 2f(x)$
$= [2(x + h) + h] - [2(2x + h)]$
$= 2x + 2h + h - 4x - 2h$
$= -2x + h$
$= h - 2x.$

2010 - JUNE

[9] If $A = \{x : x^2 - 3x + 2 = 0\}$,
$B = \{x : x^2 + 4x - 12 = 0\}$, then
B – A is Equal to
(a) $\{-6\}$ (b) $\{1\}$
(c) $\{1,2\}$ (d) $\{2, -6\}$ (1 mark)

Answer:

(a) $A = \{x : x^2 - 3x + 2 = 0\}$
$x^2 - 3x + 2 = 0$
$x^2 - 2x - x + 2 = 0$
$(x - 1)(x - 2) = 0$
$x = 1, 2$
$A = \{1, 2\}$
$B = \{x : x^2 + 4x - 12 = 0\}$
$x^2 + 4x - 12 = 0$
$x^2 + 6x - 2x - 12 = 0$
$(x - 2)(x + 6) = 0$
$x = 2, -6$
$B = \{2, -6\}$
B – A = All elements present in B but not in A = $\{-6\}$

[10] If $F : A \to R$ is a real valued function defined by $f(x) = \frac{1}{x}$, then A = ________.
(a) R (b) $R - \{1\}$
(c) $R - \{0\}$ (d) $R - N$ (1 mark)

Answer:

(c) $f : A \to R$
$f(x) = \frac{1}{x}$
If $x = 0$
f(x) will be undefined
$\therefore$ $A = R - \{0\}$

[11] In the set N of all natural numbers the relation R defined by a R b "if and only if, a divide b", then the relation R is :

(a) Partial order relation (b) Equivalence relation
(c) Symmetric relation (d) None of these (1 mark)

Answer:

(a) For a function to be a partial order Relation, it should be
(1) Reflexive
(2) Antisymmetric and
(3) Transitive
a divides b satisfies the above 3 relations as follows :

(1) a/a ∴ Reflexive
(2) a/b and b/a ∴ a = b ∴ Antisymmetric
(3) a/b, b/c ∴ a/c ∴ Transitive

a/b is not a symmetric function and hence, not an equivalence relation.

2010 - DECEMBER

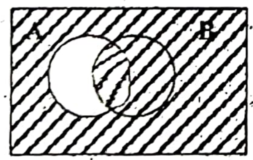
[12] For any two sets A and B, $A \cap (A' \cup B)$ = ______, where A' represent the compliment of the set A

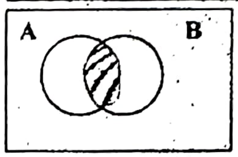
(a) $A \cap B$ (b) $A \cup B$
(c) $A' \cup B$ (d) None of these (1 mark)

Answer:

(a) $A \cap (A' \cup B)$

$\because A' \cup B$ =





$\therefore A \cap (A' \cup B)$

$= A \cap B$

[13] If $f : R \rightarrow R$, $f(x) = x + 1$,
$g : R \rightarrow R$ $g(x) = x^2+1$
then fog(-2) equals to

(a) 6 (b) 5
(c) -2 (d) None (1 mark)

Answer:

(a) $f(x) = x + 1$
$g(x) = x^2 + 1$
$fog(-2) = f[g(-2)] = f(5)$ [$\because g(-2) = 5$]
$f(5) = 5 + 1 = 6$

[14] If $A \subset B$, then which one of the following is true

(a) $A \cap B = B$ (b) $A \cup B = B$
(c) $A \cap B = A^1$ (d) $A \cap B = \varphi$ (1 mark)

Answer:

(a) $A \subset B$
$A \cap B = B$ (as A is a subset of B)

[15] If $f(x - 1) = x^2 - 4x + 8$, then $f(x + 1)$ = ______

(a) x^2+8 (b) x^2+7
(c) x^2+4 (d) $x^2 - 4x$ (1 mark)

Answer:

(c) $f(x - 1) = x^2 - 4x + 8$
$= (x^2 - 2x + 1) - 2x + 7$
$= (x - 1)^2 - 2x + 2 + 7 - 2$
hence, $f(x - 1) = (x - 1)^2 - 2(x - 1) + 5$
$\therefore f(x + 1) = (x + 1)^2 - 2(x + 1) + 5$
$= x^2 + 2x + 1 - 2x - 2 + 5$
$= x^2 + 6 - 2$
$= x^2 + 4$

2011 - JUNE

[16] There are 40 students, 30 of them passed in English, 25 of them passed in Maths and 15 of them passed in both. Assuming that every Student has passed at least in one subject. How many student's passed in English only but not in Maths.

(a) 15 (b) 20
(c) 10 (d) 25 (1 mark)

Answer:

(a) Given :

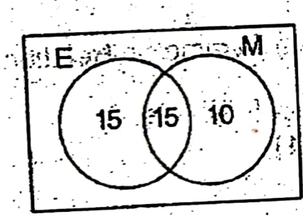
Total No. of Students $n(E \cup N) = 40$

No. of Students passed in Eng. $n(E) = 30$

No. of Students passed in Maths $n(M) = 25$

No. of Students passed in both $n(E \cap M) = 15$

Therefore, required to Find : n(only E) = ?



$\therefore$ n(only E) = $n(E) - n(E \cap M)$

$= 30 - 15$

$= 15$

[17] If A = {± 2, ± 3}, B = {1,4,9} and F = {(2, 4), (– 2, 4), (3, 9), (– 3, 4)} then 'F' is defined as :

(a) One to one function from A into B.
(b) One to one function from A onto B.
(c) Many to one function from A onto B.
(d) Many to one function from A into B. (1 mark)

Answer:

(d) This is a many one function since multiple elements in Set A have the same image in Set B. Also, this is an into function because the element "1" in Set B doesn't even have a single pre-image in Set A. Therefore, it is many one into function.

[18] If $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$ Find *fog* ?

(a) x (b) $\frac{1}{x}$
(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$ (1 mark)

Answer:

(a) Given : $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$

$\therefore fog(x) = f\{g(x)\}$

$= f\left\{\frac{x}{\sqrt{1-x^2}}\right\}$

$$= \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\left(\frac{x^2}{\sqrt{1-x^2}}\right)}}$$

$= x$

2011 - DECEMBER

[19] f(x) = 3+x, for – 3< x <0 and 3 – 2x for 0< x <3, then Value of f(2) will be

(a) – 1 (b) 1
(c) 3 (d) 5 (1 mark)

f(x) = 3 + x −3 < x < 0
3 − 2x 0 < x < 3

Answer:

(a) $f(x) = 3 + x$ if $-3 < x < 0$

$= 3 - 2x$ if $0 < x < 3$

2 Lies $0 < x < 3$

Then

$f(x) = 3 - 2x$

$f(2) = 3 - 2 \times 2 = 3 - 4 = -1$

[20] If A = (1, 2, 3, 4, 5), B = (2, 4) and C = (1, 3, 5) then (A – C) × B is

(a) {(2, 2), (2, 4), (4, 2), (4, 4), (5, 2), (5, 4)}
(b) {(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)}
(c) {(2, 2), (4, 2), (4, 4), (4, 5)}
(d) {(2, 2), (2, 4), (4, 2), (4, 4)} (1 mark)

Answer :

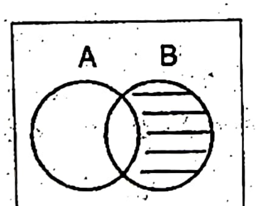
(d) (A – C) = {1, 2, 3, 4, 5} – {1, 3, 5,} = {2, 4}
(A – C) × B = {2, 4,} × {2, 4} = {(2, 2,), (2, 4), (4, 2), (4, 4)

[21] For any two sets A and B the set (AUB')' is Equal to (where' denotes compliment of the set)

(a) B – A
(b) A – B
(c) A' – B'
(d) B' – A' (1 mark)

Answer:

(a) (A∪B')' = B – A



2012 - JUNE

[22] The number of proper sub set of the set {3, 4, 5, 6, 7} is

(a) 32 (b) 31
(c) 30 (d) 25 (1 mark)

Answer:

(b) Given set A = {3, 4, 5, 6, 7}

Cardinal No n(A) = 5

No. of proper subset $= 2^n - 1$

$= 2^5 - 1$

$= 32 - 1$

$= 31$

[23] On the set of lines, being perpendicular is a ________ relation.

(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these. (1 mark)

Answer:

(b) A set of lines, being perpendicular is a Symmetric Relation

[24] The range of the function f : N ⟶ N; $f(x) = (-1)^{x-1}$, is

(a) {0, - 1} (b) {1, - 1}
(c) {1, 0} (d) {1, 0, - 1} (1 mark)

Answer:

(b) Given $f(x) = (-1)^{x-1}$

x = 1 $f(1) = (-1)^{1-1} = 1$ f = N ⇒ N

x = 2 $f(2) = (-1)^{2-1} = -1$

x = 3 $f(3) = (-1)^{3-1} = 1$

x = 4 $f(4) = (-1)^{4-1} = -1$

Range of function = {1, - 1}

[25] The minimum value of the function $x^2 - 6x + 10$ is ________

(a) 1 (b) 2
(c) 3 (d) 10 (1 mark)

x = 1 ⇒ 1 – 6 + 10 = 5 x = 10
x = 2 ⇒ 4 – 12 + 10 = 2 100 – 60 + 10
x = 3 9 – 18 + 10 = 1 = 50

Answer:

(a) Let $x^2 - 6x + 10 = y$

$x^2 - 6x + 10 - y = 0$

$x^2 - 6x + (10 - y) = 0$

$ax^2 + bx + c = 0$

we get

$a = 1, b = -6, c = (10 - y)$

For Real

$D \geq 0$

$b^2 - 4ac \geq 0$

$(-6)^2 - 4 \times 1 \times (10 - y) \geq 0$

$36 - 40 + 4y \geq 0$

$4y \geq 4$

$y \geq 1$

$y = \{1, 2, 3, \ldots\ldots\ldots\ldots \infty)$

Minimum value of function = 1

2012 - DECEMBER

[26] For a group of 200 persons, 100 are interested in music, 70 in photography and 40 in swimming, Further more 40 are interested in both music and photography, 30 in both music and swimming, 20 in photography and swimming and 10 in all the three. How many are interested in photography but not in music and swimming?

(a) 30 (b) 15

(c) 25 (d) 20 (1 mark)

Answer:

(d) Let Photography → P

Music → M

Swimming → S

$n(P \cup M \cup S) = 200, n(m) = 100, n(p) = 70$

$n(S) = 40, n(M \cap P) = 40, n(M \cap S) = 30, n(P \cap S) = 20$

$n(P \cap M \cap S) = 10$

$n(P \cap \overline{M} \cap \overline{S}) = n(P) - n(P \cap M) - n(P \cap S) + n(P \cap M \cap S)$

$= 70 - 40 - 20 + 10$

$= 80 - 60$

$= 20$

[27] If $f: R \rightarrow R$ is a function, defined by $f(x) = 10x - 7$, if $g(x) = f^{-1}(x)$, then g(x) is equal to

(a) $\frac{1}{10x - 7}$ (b) $\frac{1}{10x + 7}$

(c) $\frac{x + 7}{10}$ (d) $\frac{x - 7}{10}$ (1 mark)

Answer:

(c) If $f: R \rightarrow R$ is a function defined by

$f(x) = 10x - 7$ Let $y = f(x)$

$y = 10x - 7$ $x = f^{-1}(y)$ ......(1)

$10x = y + 7$

$x = \frac{y + 7}{10}$

$f^{-1}(y) = \frac{y + 7}{10}$ [from eq (1)]

$f^{-1}(x) = \frac{x + 7}{10}$

The value of $g(x) = f^{-1}(x)$

$= \frac{x + 7}{10}$

[28] The number of elements in range of constant function is

(a) One (b) Zero

(c) Infinite (d) Indetermined (1 mark)

Answer:

(a) The range set of a constant function is a singleton set. Therefore, the number of elements in the range set of a constant function is one.

2013 - JUNE

[29] Let A = {1, 2, 3}, then the relation R = {(1, 1), (2, 3), (2, 2), (3, 3), (1, 2)} is:

(a) Symmetric (b) Transitive
(c) Reflexive (d) Equivalence (1 mark)

Answer:

(c) If A = {1, 2, 3} then

R = {(1, 1) (2, 3) (2, 2) (3, 3) (1, 2)}

Here, R = {(1, 1) (2, 2) (3, 3)} shows reflexive

[30] If $f(x) = x + 2$, $g(x) = 7^x$, than $g\,o\,f(x)$ = ________

(a) $7^x . x + 2 . 7^x$ (b) $7^x + 2$
(c) $49\,(7^x)$ (d) None of these (1 mark)

Answer:

(c) If $f(x) = x + 2$, $g(x) = 7^x$ than

$g\,o\,f(x) = g\{f(x)\}$
$= g\{x + 2\}$
$= 7^{x+2}$
$= 7^x . 7^2$
$= 7^x . (49)$
$= 49 . (7^x)$

[31] If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to:

(a) $f(x)$ (b) $2f(x)$
(c) $3f(x)$ (d) $-f(x)$ (1 mark)

Answer:

(b) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then

$$f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right]$$

$$= \log\left[\frac{\frac{1+x^2+2x}{(1+x^2)}}{\frac{1+x^2-2x}{(1+x^2)}}\right]$$

$$= \log\left[\frac{(1+x)^2}{(1-x)^2}\right]$$

$$= \log\left[\frac{1+x}{1-x}\right]^2$$

$$= 2\log\left[\frac{1+x}{1-x}\right]$$

$$= 2f(x)$$

2013 - DECEMBER

[32] If $f(x) = (a - x^n)^{1/n}$, a > 0 and 'n' is a positive integer, then $f(f(x))$ = ________.

(a) x (b) a
(c) $x^{1/n}$ (d) $a^{1/n}$ (1 mark)

Answer:

(a) If $f(x) = (a - x^n)^{1/n}$, a > 0

$f\{f(x)\} = f\{(a - x^n)^{1/n}\}$
$= \{a - (a - x^n)^{1/n.n}\}^{1/n}$
$= \{a - a + x^n\}^{1/n}$
$= x^{n.1/n}$
$= x$

[33] Of the 200 candidates who were interviewed for a position at a call centre, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone, 40 of them had both a two-wheeler and a credit card, 30 had both a credit card and a mobile phone, 60 had both a two-wheeler and a mobile phone, and 10 had all three. How many candidates had none of the three?

(a) 0 (b) 20
(c) 10 (d) 18 (1 mark)

Answer:

(c) A ⇒ Two wheeler candidate
B ⇒ Credit card candidate
C ⇒ Mobile phone candidate

Given $n(A) = 100$, $n(B) = 70$, $n(C) = 140$
$n(A \cap B) = 40$, $n(B \cap C) = 30$, $n(C \cap A) = 60$
$n(A \cap B \cap C) = 10$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 100 + 70 + 140 - 40 - 30 - 60 + 10$
$= 320 - 130$
$= 190$

No. of candidate who had none of the three
$= 200 - 190$
$= 10$

[34] If $f(x) = \frac{-x^2 - 25}{x - 5}$, then $f(5)$ is

(a) 0
(b) 1
(c) 10
(d) not defined (1 mark)

Answer:

(d) If $f(x) = \frac{x^2 - 25}{x - 5}$

$f(5) = \frac{(5)^2 - 25}{5 - 5} = \frac{0}{0}$ = *does not exist* = *not defined*

2014 - June

[35] Let A={1,2,3} and B={6,4,7}. Then, the relation R={(2,4), (3,6)} will be:

(a) Function from A to B
(b) Function from B to A
(c) Both A and B
(d) Not a function (1 mark)

Answer:

(d) Since the element "1" of Set A does not have an image in Set B, therefore, this relation is not a function.

[36] In a class of 50 students, 35 opted for Mathematics and 37 opted for Commerce. The number of such students who opted for both Mathematics and Commerce are:

(a) 13
(b) 15
(c) 22
(d) 28 (1mark)

Answer:

(c) Given $n(m \cup c) = 50$
$n(m) = 35$
$n(c) = 37$
$n(m \cup c) = n(m) + n(c) - n(m \cap c)$
$50 = 35 + 37 - n(m \cap c)$
$n(m \cap c) = 35 + 37 - 50$
$= 72 - 50$
$n(m \cap c) = 22$

[37] The range of {(1,0), (2,0), (3,0), (4,0), (0,0)} is:

(a) {1,2,3,4,0}
(b) {0}
(c) {1,2,3,4}
(d) None of these (1 mark)

Answer:

(b) The Range of {(1, 0), (2, 0), (3, 0), (4, 0), (5, 0)}
= {0}

2014 - December

[38] Let N be the set of all Natural numbers; E be the set of all even natural numbers then the function

$f: N \rightarrow E$ defined as $f(x) = 2x + x \in N$ is:

(a) One-one into
(b) One-one onto
(c) Many-one into
(d) Many-one onto (1 mark)

Answer:

(b) N = Set of all Natural No.
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ....}

E = Set of all Even No.
= {2, 4, 6, 8, 10, ...}

$f: N \rightarrow E$

$f(x) = 2x$

If $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$ So $f(x)$ is one-one

at $f(x) = 2x$

$y = 2x$

$x = \frac{y}{2}$

Then Range of f = Even No. (E)

So $f(x)$ is onto

Hence, $f(x)$ is one-one onto.

[39] If A = {2, 3}, B = {4, 5}, C = {5, 6}, then A x (B ∩ C) = ______

(a) {(5, 2), (5, 3)} (b) {(2, 5), (3, 5)}
(c) {(2, 4), (3, 5)} (d) {(3, 5), (2, 6)} (1 mark)

Answer:

(b) A = {2, 3}, B = {4, 5}, C = {5, 6}

B ∩ C = {5}

A x (B ∩ C) = {2, 3} x {5}
= {(2, 5), (3, 5)}

[40] If S = {1, 2, 3} then the relation {(1, 1), (2, 2), (1, 2), (2, 1)} is symmetric and

(a) Reflexive but not transitive
(b) Reflexive as well as transitive
(c) Transitive but not reflexive
(d) Neither transitive nor reflexive (1 mark)

Answer:

(c) If S = {1, 2, 3} then
The Relation {(1, 1), (2, 2), (1, 2), (2, 1)} is symmetric and **transitive but not reflexive.**

[41] If $f(x) = \frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)} =$ ______

(a) x/y (b) y/x
(c) -x/y (d) -y/x (1 mark)

Answer:

(c) If $f(x) = \frac{x}{x-1}$

$$f(x/y) = \frac{x/y}{\frac{x}{y}-1} = \frac{\frac{x}{y}}{\frac{x-y}{y}} = \frac{x}{x-y}$$

$$f(y/x) = \frac{y/x}{\frac{y}{x}-1} = \frac{\frac{y}{x}}{\frac{y-x}{x}} = \frac{y}{y-x}$$

$$\frac{f(x/y)}{f(y/x)} = \frac{x/(x-y)}{y/(y-x)} = \frac{x}{(x-y)} \cdot \frac{(y-x)}{y}$$

$$= \frac{-x(x-y)}{y(x-y)}$$

$$= \frac{-x}{y}$$

2015 - JUNE

[42] If N be the set of all natural numbers and E be the set of all even natural numbers then the function $f: N \rightarrow E$, such that $f(x) = 2x$ for all $x \in N$ is

(a) one-one onto (b) one-one into
(c) many-one onto (d) constant (1 mark)

Answer:

(a) $N = \{1, 2, 3, 4, \ldots\ldots\ldots \infty\}$

$E = \{2, 4, 6, 8, \ldots\ldots\ldots \infty\}$

$f: N \rightarrow E$

$f(x) = 2x$

$f(1) = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(3) = 2 \times 3 = 6$

(i) Range of function (R) = E

(ii) $f(x_1) = f(x_2)$ then function is one-one onto

2015 - DECEMBER

[43] If $A = \{x, y, z\}$, $B = \{a, b, c, d\}$, then which of the following relation from the set A to set B is a function?

(a) {(x, a), (x, b), (y, c), (z, d)}
(b) {(x, a), (y, b), (z, d)}
(c) {(x, c), (z, b), (z, c)}
(d) {a, z), (b, y), (c, z), (d, x)} (1 mark)

Answer:

(b) if

$A = \{x, y, z\}$

$B = \{a, b, c, d\}$

$A \times B = \{x, y, z\} \times \{a, b, c, d\}$

$= \{(x, a)\ (x, b)\ (x, c)\ (x, d)\ (y, a)\ (y, b)\ (y, c)\ (y, d)\ (z, a)\ (z, b)\ (z, c)\ (z, d)\}$

Then {(x, a), (y, b), (z, d)} is a functions.

[44] In a class of 80 students, 35% students can play only cricket, 45% students can play only table tennis and the remaining students can play both the games. In all how many students can play cricket?

(a) 55 (b) 44
(c) 36 (d) 28 (1 mark)

Answer:

(b) Total students in the class = 80

$n(A \cup B) = 80$

Let no. of students who play both Table Tennis and Cricket = x

i.e. $n(A \cap B) = x$

No. of person who play only Cricket

$n(A \cap \bar{B}) = 80 \times \frac{35}{100} = 28$

$n(A \cap \bar{B}) = 28$

$n(A) - n(A \cap B) = 28$

$n(A) - x = 28$

$n(A) = 28 + x$

No. of students who play only Table Tennis

$n(B \cap \bar{A}) = 45\%$ of 80

$= \frac{45}{100} \times 80$

$n(B \cap \bar{A}) = 36$

$n(B) - n(A \cap B) = 36$

$n(B) - x = 36$

$(B) = (36 + x)$

We know that,

$n(A \cup B) = n(A) + n(B) + n(B) - n(A \cap B)$

$80 = 28 + x + 36 + x - x$

$80 = 64 + x$

$n = 80 - 64$

$n = 16$

i.e. $n(A \cap B) = 16$

No. of students who play cricket

$n(A) = 28 + x$

$= 28 + 16 = 44$

[45] If $f(x) = 2x + 2$ and $g(x) = x^2$, then the value of fog (4) is:

(a) 18 (b) 22

(c) 34 (d) 128 (1 mark)

Answer:

(c) $f(x) = 2x + 2$ and $g(x) = x^2$

$fog(x) = f\{g(x)\}$

$= f\{x^2\}$

$= 2x^2 + 2$

then

$fog(4) = 2(4)^2 + 2$

$= 2 \times 16 + 2$

$= 32 + 2$

$= 34$

2016 - JUNE

[46] If set $A = \left\{x : \frac{x}{2} \in Z, \ 0 \leq x \leq 10\right\}$,

$B = \{x : x$ is one digit prime number$\}$

and $C = \left\{x : \frac{x}{3} \in N, \ x \leq 12\right\}$

then $A \cap (B \cap C)$ is equal to –

(a) φ (b) Set A

(c) Set B (d) Set C (1 mark)

Answer:

(a) If $A = \left\{x : \frac{x}{2} \in Z, \ 0 \leq x \leq 10\right\}$

$A = \left\{\frac{0}{2}, \frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \frac{10}{2}\right\} = \{0, 1, 2, 3, 4, 5\}$

$B = \{x : x$ is one digit prime number$\}$

$= \{2, 3, 5, 7\}$

and $C = \left\{x : \frac{x}{3} \in N, \ x \leq 12\right\}$

$= \left\{\frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{12}{3}\right\}$

$= \{1, 2, 3, 4\}$

$B \cap C = \{2, 3\}$

$A \cap (B \cap C) = \{0, 1, 2, 3, 4, 5\} \cap \{2, 3\}$

$= \{2, 3\}$

[47] Let A be the set of squares of natural numbers and let $x \in A$, $y \in A$ t

(a) $X + Y \in A$ (b) $X - Y \in A$

(c) $\frac{X}{Y} \in A$ (d) $xy \in A$ (1 m

Answer:

(d) Let A be the set of square of Natural No.

$A = \{1^2, 2^2, 3^2, 4^2, \ldots\ldots\ldots\ldots\ldots \infty\}$

$A = \{1, 4, 9, 16, \ldots\ldots\ldots\ldots\ldots\ldots\}$

If $x \in A$, $y \in A$ then $xy \in A$

[48] The domain (D) and range (R) of the function $f(x) = 2 - |x + 1|$ is

(a) D = Real numbers, R = $(2, \infty)$

(b) D = Integers, R = $(0, 2)$

(c) D = Integers, R = $(-\infty, \infty)$

(d) D = Real numbers, R = $(-\infty, 2)$ (1 m

Answer:

(d) Given function

$f(x) = 2 - \{(x + 1)\}$

Domain = Real Number

and $f(x) = 2 - [x + 1]$

$y = 2 - [x + 1]$

$[x + 1] = 2 - y$

$\pm (x + 1) = 2 - y$

+ve sign taking

$x + 1 = 2 - y$

$x = 2 - y - 1$

$x = 1 - y$

– ve sign

$-(x + 1) = 2 - y$

$x + 1 = -2 + y$

$x = y - 2 - 1$

$x = y - 3$

So Range $= [-\infty, 2]$

Domain = Real No, Range $= (-\infty, 2)$

2016 - DECEMBER

[49] If R is the set of all real numbers, then the function f: R→R defined by $f(x) = 2^x$

(a) one-one onto (b) one-one into

(c) many-one into (d) many-one onto (1 mark)

Answer:

(b) $f(x) = 2^x$

$f(x_1) = 2^{x_1}$ and $f(x_2) = 2^{x_2}$

Now, $f(x_1) = f(x_2)$

$2^{x_1} = 2^{x_2} \Rightarrow x_1 = x_2$

so, $f(x) = 2^x$ is one-one

and

$f(x) = 2^x$

$y = 2^x$

$\log y = \log 2^x$

$\log y = x \log 2$

$x = \log_2 y$ [log is not valid value if y is negative]

So, range of function ≠ B so it is into function.

[50] The inverse function f^{-1} of $f(x) = 100x$ is:

(a) $\frac{x}{100}$ (b) $\frac{1}{100x}$

(c) $\frac{1}{x}$ (d) None of these (1 mark)

Answer:

(a) Given $f(x) = 100x$

$y = 100x$

$x = \frac{y}{100}$

$f^{-1}(y) = \frac{y}{100}$

$f^{-1}(x) = \frac{x}{100}$

[51] The number of subsets of the set formed by the word Allahabad is:

(a) 128 (b) 16

(c) 32 (d) 64 (1 mark)

Answer:

(c) A = Set of the letter of the word 'ALLAHABAD'

= {A,L,H,B,D}

n(A) = 5

No. of subset $= 2^n$

$= 2^5$

$= 32$

2017 - JUNE

[52] The range of function f defined by $f(x) = \frac{x}{x^2+1}$ is:

(a) $\{x: \frac{-1}{2} < x < \frac{1}{2}\}$ (b) $\{x: \frac{-1}{2} \le x < \frac{1}{2}\}$

(c) $\{x: \frac{-1}{2} \le x \le \frac{1}{2}\}$ (d) $\{x: x > \frac{1}{2} \text{ or } x < \frac{-1}{2}\}$ (1 mark)

Answer:

(c) $f(x) = \frac{x}{x^2+1}$

$y = \frac{x}{x^2+1}$

$yx^2 + y = x$

$yx^2 - x + y = 0$

$a = y, b = -1, c = y$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times y \times y}}{2.y}$

$x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$1 - 4y^2 \geq 0$

$1 \geq 4y^2$

$\frac{1}{4} \geq y^2$

$\pm\frac{1}{2} \geq y$

Range → $\left\{x: -\frac{1}{2} \leq x \leq \frac{1}{2}\right\}$

[53] In a group of students 80 can speak Hindi, 60 can speak English and 40 can speak English and Hindi both, then number of students is:

(a) 100 (b) 140

(c) 180 (d) 60 (1 mark)

Answer:

(a) A = Hindi, B = English

$n(A) = 80, n(B) = 60, n(A \cap B) = 40$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 80 + 60 - 40$

$= 140 - 40$

$= 100$

[54] If $f(x) = \frac{x+1}{x}$ and $g(x) = \frac{1}{1-x}$ then (fog) (x) is equal to:

(a) $x - 1$ (b) x

(c) $1 - x$ (d) $-x$ (1 mark)

Answer:

(b) Given $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{1-x}$

$\log(x) = f\{g(x)\}$

$= f\left\{\frac{1}{1-x}\right\}$

$= \frac{\frac{1}{(1-x)} - \frac{1}{1}}{\frac{1}{(1-x)}} = \frac{\frac{1-1+x}{(1-x)}}{\frac{1}{(1-x)}} = \frac{x}{1} = x$

2017 - DECEMBER

[55] If $f(x) = \frac{x+1}{x+2}$, then $f\left\{f\left(\frac{1}{x}\right)\right\} =$ ______.

(a) $\frac{2x+3}{3x+5}$ (b) $\frac{2x+5}{3x+2}$

(c) $\frac{3x+2}{5x+3}$ (d) $\frac{5x+2}{2x+3}$

Answer:

(c) Given $f(x) = \frac{x+1}{x+2}$

$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}+1}{\frac{1}{x}+2} = \frac{1+x}{1+2x}$

$f\left\{f\left(\frac{1}{x}\right)\right\} = f\left\{\frac{1+x}{1+2x}\right\}$

$= \frac{\frac{1+x}{1+2x}+1}{\frac{1+x}{1+2x}+2}$

$= \frac{\frac{1+x+1+2x}{(1+2x)}}{\frac{1+x+2+4x}{(1+2x)}} = \frac{(3x+2)}{(5x+3)}$

[56] In a class of 35 students, 24 like to play cricket and 16 like to play football. Also each student likes to play at least one of the two games. How many students like to play both cricket and football?

(a) 5 (b) 11
(c) 19 (d) 8 (1 mark)

Answer:

(a) Let A → Cricket
B → Football
n(A) = 24, n(B) = 16, n(A∪B) = 35
n(A∩B) = ?
n(A∪B) = n(A) + n(B) – n(A∩B)
35 = 24 + 16 – n(A ∩ B)
n(A∩B) = 24 + 16 – 35
= 5

2018 - MAY

[57] Let N be the set of all natural numbers; E be the set of all even natural numbers then the function;
F:N ⇒ E defined as f(x) = 2x–V×∈N is =

(a) One-one-into (b) Many-one-into
(c) One-one onto (d) Many-one-onto (1 mark)

Answer:

(c) Given
N = {1, 2, 3, 4,5,6 .... ∞}
E = {2, 4, 6, 8, ... ∞}
f: N → E
f(x) = 2x V X ∈ N
f(1) = 2 × 1 = 2
f(2) = 2 × 2 = 4
f(3) = 2 × 3 = 6

Range of function = {2,4,6,.......}
= E
and $f(x_1) = f(x_2)$
$2x_1 = 2x_2 = x_2$
So *f*(x) function is one-one and one to.

[58] In a town of 20,000 families it was found that 40% families buy newspaper. A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C if 2% families buy all the three newspapers, then the number of families which buy A only is:

(a) 6600 (b) 6300
(c) 5600 (d) 600 (1 mark)

Answer:

(a) Total Families n(u) = 20000
No. of families who buy Newspapers 'A'
n(A) = 40% of 20000 = 8000
No. of families who buy Newspaper 'B'
n(B) = 20% of 20000 = 4000
No. of families who buy Newspaper 'C'
n(C) = 10% of 20000 = 2000
No. of families who buy Newspapers A & B
n (A ∩ B) = 5% of 20000 = 1000
No. of families who buy Newspapers B & C
n (B ∩ C) = 3% of 20000 = 600
No. of families who buy Newspapers C & A
n (C ∩ A) = 4% of 20000 = 800
No. of families who buy all newspapers
n (A ∩ B ∩ C) = 2% of 20000 = 400
No. of families which buy Newspapers 'A' only
$= n\left(A \cap \bar{B} \cap \bar{C}\right)$

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 8000 - 1000 - 800 + 400$

$= 6600$

[59] The numbers of proper sub set of the set {3,4,5,6,7} is:

(a) 32 (b) 31

(c) 30 (d) 25 (1 mark)

Answer:

(b) Given

A = {3, 4, 5, 6, 7}

n(A) = 5

No. of proper subset $= 2^n - 1$

$= 2^5 - 1$

$= 32 - 1$

$= 31$

2018 - NOVEMBER

[60] A is {1,2,3,4} and B is {1,4,9,16,25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:

(a) {1,2,3,4} (b) {1,4,9,16}

(c) {1,4,9,16,25} (d) None of these (1 mark)

Answer:

(b) Given A = {1,2,3,4}

B = {1,4,9,16,25}

If f : A→ B and $f(x) = x^2$

$f(1) = (1)^2 = 1$

$f(2) = (2)^2 = 4$

$f(3) = (3)^2 = 9$

$f(4) = (4)^2 = 16$

Range of f = {1,4,9,16}

[61] If A = {1,2} and B = {3,4}. Determine the number of relations from A and B:

(a) 3

(b) 16

(c) 5

(d) 6 (1 mark)

Answer:

(b) Given A = {1,2}

B = {3,4}

A×B = {1,2} × {3,4}

= {(1,3),(1,4) (2,3) (2,4)}

n(A×B) = 4

No. of relation from A and B $= 2^n$

$= 2^4$

$= 16$

or

A liter Shortcut

A = {1,2} , n (A) = 2

B = {3,4} , n (B) = 2

No. of Relation from A and B $= 2^{m \times n}$

$= 2^{2 \times 2}$

$= 2^4 = 16$

[62] If A = {1,2,3,4,5,6,7} and B = {2,4,6,8}. Cardinal number of A - B is:

(a) 4

(b) 3

(c) 9

(d) 7 (1 mark)

Answer:

(a) Given A = {1,2,3,4,5,6,7}

B = {2,4,6,8}

A-B = {1,2,3,4,5,6,7} – {2,4,6,8}

= {1,3,5,7}

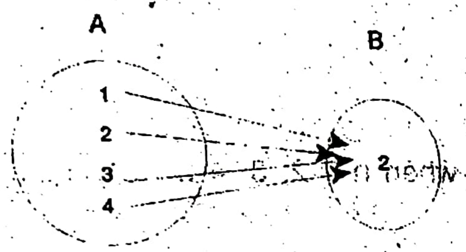
n (A–B) = 4

[63] Identity the function from the following:

(a) {(1,1), (1,2), (1,3)} (b) {(1,1), (2,1), (2,3)}
(c) {(1,2), (2,2), (3,2),(4,2)} (d) None of these (1 mark)

Answer:

(c) {(1,2) (2,2) (3,2) (4,2)} is the function.



Many one function

2019 - JUNE

[64] If A = { 1, 2, 3, 4, 5, 6, 7, 8, 9}
B = {1, 3, 4, 5, 7, 8}; C = {2, 6, 8} then find (A – B) ∪ C

(a) {2, 6}
(b) {2, 6, 8}
(c) {2, 6, 8, 9}
(d) None (1 mark)

Answer:

(c) A = {1, 2, 3, 4, 5, 6, 7, 8, 9}
B = {1, 3, 4, 5, 7, 8}, C = {2, 6, 8}
Then
A – B = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {1, 3, 4, 5, 7, 8}
= {2, 6, 9}
(A – B) ∪ C = {2, 6, 9} ∪ {2, 6, 8}
= {2, 6, 8, 9}

[65] A = {1 2 3 4 .....10} a relation on A, R = {(x, y)/ x + y = 10, x ∈ A, Y ∈ A, x ≥ Y} then domain of R^{-1} is

(a) {1, 2, 3, 4, 5}
(b) {0, 3, 5, 7, 9}
(c) {1, 2, 4, 5, 6, 7}
(d) None (1 mark)

Answer:

(a) Given, A= {1,2,3,4,............10}
R = {x, y): x + y = 10, x∈A, y∈A, x≥y}
⇒ R = {(5, 5), (6, 4), (7, 3), (8, 2), (9, 1)}
R^{-1} = {5, 5) (4, 6) (3, 7) (2, 8) (1, 9)}
Domain of R^{-1} = {5, 4, 3, 2, 1}

[66] The no. of subsets of the set {3, 4, 5} is :

(a) 4
(b) 8
(c) 16
(d) 32 (1 mark)

Answer:

(b) Here, A = {3, 4, 5}
n(A) = 3
No of Subset $= 2^n$
$= 2^3$
= 8

[67] If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) go f(3) = 3
(b) go f(-3) = 9
(c) go f(9) = 3
(d) go f(-9) = 3 (1 mark)

Answer:

(a) Given, $f(x) = x^2$ and $g(x) = \sqrt{x}$
fog (x) = f {g (x)}
$= f\{\sqrt{x}\}$
$= (\sqrt{x})^2$

fog (x) = x

and gof (x) = g {f (x)}

= g {x^2}

= $\sqrt{x^2}$

gof (x) = x

gof (3) = 3

[68] If A = {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(a) R_1 = {(a, p), (b, q), (c, s)}

(b) R_2 = {(p, a), (b, r), (d, s)}

(c) R_3 = {(b, p), (c, s), (b, r)}

(d) R_4 = {(a, p), (b, r), (c, q), (d, s)} (1 mark)

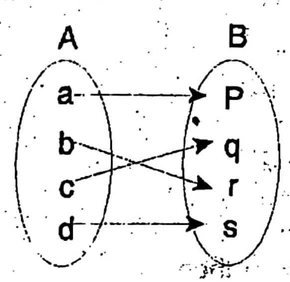
Answer:

(d) If A = {a, b, c, d}

B = {p, q, r, s}

R_4 = {(a, p), (b, r), (c, q), (d, s)}

is a function from A to B



2019 - NOVEMBER

[69] $(A^T)^T$ = ?

(a) A

(b) A^T

(c) $A^T \cdot A^T$

(d) A^{2T}

(1 mark)

Answer:

(a) $(A^T)^T = A$

Example A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$

So, $(A^T)^T = A$

[70] f(n) = f(n − 1) + f(n − 2) when n = 2, 3, 4 .... f(0) = 0, f(1) = 1 then f(7) = ?

(a) 3

(b) 5

(c) 8

(d) 13

(1 mark)

Answer:

(d) f(n) = f(n-1) + f(n-2)

f(2) = f(1) + f(0) = 1 + 0 = 1 = f(2)

f(3) = f(2) + f(1) = 1 + 1 = 2 = f(3)

f(4) = f(3) + f(2) = 2 + 1 = 3

Similarly,

f(7) = f(6) + f(5)

f(7) = [f(5) + f(4)] + [f (4) + f(3)]

f(7) = [f(4) + f(3) + f(4)] + (f(4) + f(3)]

f(7) = [3 + 2 + 3] + [3 + 2]

f(7) = 13

[71] $f(x) = \frac{x+1}{x}$ find $f^{-1}(x)$.

(a) 1/(x − 1)

(b) 1/(y − 1)

(c) $\frac{1}{y} - 1$

(d) x

(1 mark)

Answer:

(a) $f(x) = \frac{x+1}{x}$ — Equation (1)

Let $f(x) = y$

$x = f^{-1}(y)$

Further Solving — Equation (1)

$y = \frac{x+1}{x}$

$xy = x + 1$

$xy - x = 1$

$x(y-1) = 1$

$x = \frac{1}{(y-1)}$

$f^{-1}(y) = \frac{1}{(y-1)}$

$f^{-1}(x) = \frac{1}{(x-1)}$

2020 - NOVEMBER

[72] Two finite sets respectively have x and y number of elements. The total number of subsets of the first is 56 more than the total number of subsets of the second. The value of x and y respectively.

(a) 6 and 3

(b) 4 and 2

(c) 2 and 4

(d) 3 and 6 (1 mark)

Answer:

(a) Let A and B are two set

Given $n(A) = x$ and $n(B) = y$

No. of subset of $A = 2^x$ and No. of subset of $B = 2^y$

According the question

$2^x = 2^y + 56$ 1

Option (a) is satisfied eq (1) so

$x = 6, \quad y = 3$

[73] The number of items in the set A is 40; in the set B is 32; in the set C is 50; in both A and B is 4, in both A and C is 5; in both B and C is 7 in all the sets 2. How many are in only one set?

(a) 110

(b) 65

(c) 108

(d) 84 (1 mark)

Answer:

(c) Given: $n(A) = 40$ $\quad n(A \cap B) = 4$

$n(B) = 32$ $\quad n(B \cap C) = 7$

$n(C) = 50$ $\quad n(C \cap A) = 5$

$n(A \cap B \cap C) = 2$

$n(A \cup B \cup C) = ?$

We know that:

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 40 + 32 + 50 - 4 - 7 - 5 + 2$

$= 124 - 16$

$= 108$

[74] The set of cubes of the natural number is:

(a) A null set

(b) A finite set

(c) An infinite set

(d) A finite set of three numbers (1 mark)

Answer:

(c) The set of cubes of the Natural Number is Infinite Set.

∵ because Natural Number is Infinite.

[75] The inverse function f^{-1} of $f(y) = 3y$ is:

(a) 1/3y

(b) y/3

(c) -3y

(d) 1/y (1 mark)

Answer:

(b) Given f(y) = 3y

Let f (y) = x → y = $f^{-1}(x)$

x = 3y

$y = \frac{x}{3}$

$f^{-1}(x) = \frac{x}{3}$

$f^{-1}(y) = \left(\frac{y}{3}\right)$

2021 - JANUARY

[76] The set of cubes of natural number is

(a) Null set

(b) A finite set

(c) An infinite set

(d) Singleton Set (1 mark)

Answer:

(c) The set of cubes of Natural Number is **an infinite set** because Natural Number is Infinite.

[77] In the set of all straight lines on a plane which of the following is Not 'TRUE'?

(a) Parallel to an equivalence relation

(b) Perpendicular to is a symmetric relation

(c) Perpendicular to is an equivalence relation

(d) Parallel to a reflexive relation (1 mark)

Answer:

(c) 'Perpendicular to' is an equivalence relation' which is not true.

[78] Let F : R → R be defined by

$$f(x) = \begin{cases} 2x & \text{for } x>3 \\ x^2 & \text{for } 1<x\leq 3 \\ 3x & \text{for } x\leq 1 \end{cases}$$

The value of f(-1) + f(2) + f(4) is

(a) 9

(b) 14

(c) 5

(d) 6 (1 mark)

Answer:

(a) Here f: R → R

$$f(x) = \begin{cases} 2x \text{ for } x > 3 \\ x^2 \text{ for } 1\leq x\leq 3 \\ 3x \text{ for } x\leq 1 \end{cases}$$

∵ (-1) lies x ≤ 1 then f (x) = 3x

f(-1) = 3 x (-1) = -3

Now 2 lies b/w 1≤ x ≤ 3 then f(x) = x^2

f(2) = $(2)^2$ = 4

and 4 lies b/w n > 3 then f(x) = 2x

f(4) = 2 x 4 = 8

Now f(-1) + f(2) + f(4)

= -3 + 4 + 8

= 9

[79] The number of integers from 1 to 100 which are neither divisible by 3 nor by 5 nor by 7 is

(a) 67

(b) 55

(c) 45

(d) 33 (1 mark)

Answer:

(c) Numbers which is divisible by 3 from 1 to 100 are 3, 6, 9,---------------99

Here a = 3, d = 6-3 = 3, l = 99

$n = \frac{l-a+d}{d} = \frac{99-3+3}{3} = 33$

let n = n (a) = 33

Number which is divisible by 5 from 10 to 100 are 5, 10, 15---------------100

$a = 5, d = 10 - 5 = 5, l = 100$

$n = \frac{l - a + d}{d} = \frac{100 - 5 + 5}{5} = 20$

it $n = n(B) = 20$

Number which is divisible by 7 from 1 to 100
are 7, 14, 21 ----------98

$a = 7, d = 14 - 7 = 7, l = 98$

$n = \frac{l - a + d}{d} = \frac{98 - 7 + 7}{7} = 14$

let $n = n(C) = 14$

Number which is divisible by 3 and 5 (=15)

15, 30, 45, 60, 75, 90

$n(A \cap B) = 6$

Number which is divisible by 5 and 7 (=35)
are 35, 70

$n(B \cap C) = 2$

Number which is divisible by 7 and 3 (= 21)
are 21, 42, 63, 84

$n(C \cap A) = 4$

Nos. which is divisible by 3, 5 and 7 (=105)

$n(A \cap B \cap C) = 0$

Nos. which are not divisible by 3, nor by 5, nor (7)

$= n(\bar{A} \cap \bar{B} \cap \bar{C})$

$= n(\overline{A \cup B \cup C})$

$= n(u) - n(A \cup B \cup C)$

$= n(u) - [n(A) + n(B) + n(c) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)]$

$= 100 - [33 + 20 + 14 - 6 - 2 - 4 + 0$

$= 100 - [67 - 12]$

$= 100 - 55$

$\boxed{= 45}$

2021 - JULY

[80] The range of the function F defined by $f(x) = \sqrt{16 - x^2}$ is
(a) [-4, 0]
(b) [-4, 4]
(c) [0, 4]
(d) [+4, 4] (1 mark)

Answer:

(b) Here $f(x) = \sqrt{16 - x^2}$

$y = \sqrt{16 - x^2}$

on squaring both side

$y^2 = 16 - x^2$

$x^2 = 16 - y^2$

$x = \sqrt{16 - y^2}$

$16 - y2 \geq 0$

$16 \geq y^2$

$\pm 4 \geq y$

Range of function = [– 4, 4]

[81] Let A = R-{3} and B = R -{1}. Let f(x) → B defined by $f(x) = \frac{x-2}{x-3}$. What is the value of $f^{-1}\left(\frac{1}{2}\right)$?
(a) 2/3
(b) 3/4
(c) 1
(d) -1 (1 mark)

Answer:

(c) Given $f(x) = \frac{x-2}{x-3}$

Let $y = \frac{x-2}{x-3}$

$\Rightarrow y(x - 3) = x - 2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow xy - x = 3y - 2$

$\Rightarrow x(y - 1) = 3y - 2$

$\Rightarrow x = \frac{3y - 2}{y - 1}$

Therefore, $f^{-1}(x) = \frac{3x - 2}{x - 1}$

Therefore, $f^{-1}\left(\frac{1}{3}\right) = \frac{\left(3\times\frac{1}{2}\right) - 2}{\frac{1}{2} - 1} = 1$

[82] If $F(x) = x^2 - 1$ and $g(x) = |2x + 3|$, then Fog (3) - g.of (-3) = ?

(a) 71

(b) 61

(c) 41

(d) 51 (1 mark)

Answer:

(b) The function g(x) is a modulus function. It means that it can generate only positive values.

$fog(3) = f[g(3)]$

Let's calculate g(3).

$g(3) = |(2 \times 3) + 3| = |6 + 3| = |9| = 9$

Now, $fog(3) = f[g(3)] = \{g(3)\}^2 - 1$

$\Rightarrow fog(3) = 9^2 - 1 = 81 - 1 = 80$

Similarly, $gof(-3) = g[f(-3)]$

Let's calculate $f(-3)$.

$f(-3) = (-3)^2 - 1 = 9 - 1 = 8$

Now, $gof(-3) = g[f(-3)] = |[2 \times \{f(-3)\}] + 3|$

$gof(-3) = |(2 \times 8) + 3| = |16 + 3| = |19| = 19$

Therefore, $fog(3) - gof(-3) = 80 - 19 = 61$

[83] Let U be the universal set, A and B are the subsets of U. If n(U) = 650, n(A) = 310

$n(A \cap B) = 95$ and $n(B) = 190$, then $n(\bar{A} \cap \bar{B})$ is equal to ($\bar{A}$ and $\bar{B}$ are the complement of A and B respectively):

(a) 400

(b) 200

(c) 300

(d) 245 (1 mark)

Answer:

(d) $n(\bar{A} \cap \bar{B}) = n(A \cup B)' = n(U) - n(A \cup B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 310 + 190 - 95 = 405$

$n(\bar{A} \cap \bar{B}) = n(U) - n(A \cup B) = 650 - 405 = 245$

2021 - DECEMBER

[84] Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics; how many teach only Physics?

(a) 2, 3

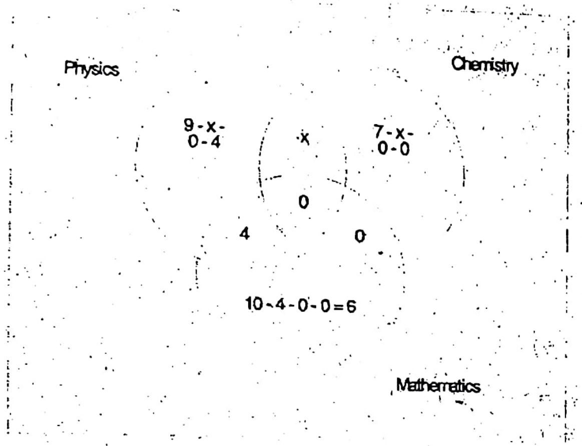
(b) 3, 2

(c) 4, 6

(d) 6, 4 (1 mark)

Answer:

(a) Let the number of teachers teaching both Physics and Chemistry be x.



In the absence of information, it is safe to assume that all the teachers teach at least one of the subjects. Therefore,

$9 - x - 0 - 4 + x + 7 - x - 0 - 0 + 4 + 0 + 0 + 6 = 20$

$\Rightarrow 9 - 4 + 7 + 4 + 6 - x + x - x = 20$

$\Rightarrow 22 - x = 20$

$\Rightarrow x = 22 - 20 = 2$

Therefore, number of teachers teaching both Physics and Chemistry = 2.

Number of teachers teaching only Physics = 9 – 2 – 4 = 3

[85] If a is related to b if and only if the difference in a and b is an even integer. This relation is
(a) symmetric, reflexive but not transitive
(b) symmetric, transitive but not reflexive
(c) transitive, reflexive but not symmetric
(d) equivalence relation (1 mark)

Answer:

(d) 1. **Check for Reflexivity:**
(a) A relation is reflexive if every element has a relation with itself.
(b) In this question, the relation exists only if the difference between the elements is an even integer.
(c) Take, for example, the number 2. Now, for this relation to be a reflexive relation, this element 2 would have to have a relation with itself.
(d) 2 – 2 = 0, which is an even integer.
(e) Therefore, any element can have a relation with itself, and hence, this is a reflexive relation.

2. **Check for Symmetry:**
(a) A relation is symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$.
(b) Take two integers, 2 and 6.
(c) Here, 2 – 6 = – 4, which is an even integer.
(d) Also, 6 – 2 = 4, which is an even integer.
(e) Therefore, $(2, 6) \in R$ and $(6, 2) \in R$.
(f) Therefore, this is a symmetric relation.

3. **Check for Transitivity:**
(a) A relation is transitive if $(a, b) \in R$, and $(b, c) \in R \Rightarrow (a, c) \in R$.
(b) Take the values of a, b, and c to be 2, 6, and 10 respectively.
(c) Now, a = 2; b = 6; c = 10
(d) Clearly, $(a, b) \in R$ as 2 – 6 = – 4, which is an even integer.
(e) Also, $(b, c) \in R$ as 6 – 10 = – 4, which is an even integer.
(f) Also, $(a, c) \in R$ as 2 – 10 = – 8, which is an even integer.
(g) Therefore, this relation is a transitive relation.

Since this relation is a Reflexive, Symmetric, as well as a Transitive Relation, it is an Equivalence Relation.

[86] If $u(x) = \frac{1}{1-x}$, then $u^{-1}(x)$ is:

(a) $\frac{1}{x-1}$ (b) $1-x$

(c) $1 - \frac{1}{x}$ (d) $\frac{1}{x} - 1$ (1 mark)

Answer:

(c) Let $y = u(x)$

Therefore, $y = \frac{1}{1-x}$

$\Rightarrow y(1 - x) = 1$

$\Rightarrow y - xy = 1$

$\Rightarrow y - 1 = xy$

$\Rightarrow xy = y - 1$

$\Rightarrow x = \frac{y-1}{y}$

Now, simply replace x with $u^{-1}(x)$, and y with x, and you'll get the answer.

$u^{-1}(x) = \frac{x-1}{x}$

$\Rightarrow u^{-1}(x) = \frac{x}{x} - \frac{1}{x}$

$\Rightarrow u^{-1}(x) = 1 - \frac{1}{x}$

2022 - JUNE

[87] f(x) = {(2,2) ; (3,3) ; (4,4) ; (5,5) ; (6,6)} be a relation of set A = {2,3,4,5,6}

It is a:

(a) Reflexive and Transitive (b) Reflexive and Symmetric

(c) Reflexive only (d) An equivalence relation

(1 mark)

Answer:

(c) If f(x) = {(2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} be the Relation of A = {2, 3, 4, 5, 6}

It is a Reflexive only.

[88] If $f(y) = \frac{y-1}{y}$, find $f^{-1}(x)$.

(a) $\frac{1}{1-y}$ (b) y

(c) $\frac{y}{y-1}$ (d) $\frac{y}{1-y}$ (1 mark)

Answer:

(a) Given $f(y) = \left(\frac{y-1}{y}\right)$

Let $f(y) = x \rightarrow y = f^{-1}(x)$

$x = \frac{y-1}{y}$

$xy = y - 1$

$xy - y = -1$

$y(x-1) = -1$

$y = \frac{-1}{(x-1)}$

$f^{-1}(x) = \frac{-1}{(x-1)}$

$f^{-1}(y) = \frac{-1}{(y-1)} = \frac{1}{1-y}$

[89] Two finite sets have x and y number of elements. The total number of subsets of first is 56 more than the total number of subsets of second. The value of x and y is:

(a) 6 and 3

(b) 4 and 2

(c) 2 and 4

(d) 3 and 4

(1 mark)