MIND MAP

RATIO AND PROPORTION

RITU JINDAL

Ratio

a:b → Consequent (Second term)
→ Antecedent (first term)

Features of Ratio

- Fraction → $\frac{a}{b}$ → No unit
- Same kind → Eg. Height Both a, b
- Same Unit → Eg kg. Both a, b
- Order → a:b ≠ b:a

Types of Ratio

- Compound Ratio → a:b = c:d, ac : bd
- Inverse Ratio → a:b = b:a
- Duplicate Ratio → a:b ⇒ $a^2:b^2$
- Triplicate Ratio → a:b = $a^3:b^3$
- Sub-duplicate → a:b = $\sqrt{a}:\sqrt{b}$
- Sub-triplicate → a:b ⇒ $\sqrt[3]{a}:\sqrt[3]{b}$
- Continued Ratio → a:b ; b:c, a:b:c

Continued Ratio

- a:b b:c then a: b:c → 11 21 22
- a:b, b:c and c:d then a: b: c:d → 111 211 221 222

Types of Questions

- Eg. 495 → 4:7 11 → 45 (180 315 495) 45
- Based on Types of Ratio
- Ratio Given Relation find out
- Relation Given Ratio find out (GBC)
- Ratio Given Add or Sub New Ratio
  - Given → given ratio ke sath k
  - Not Given → Add or Sub → x
- Other
  - Comparision → Convert into decimal
  - No of Coins Convert into ₹
  - Same Power Direct apply
  - Different Power Proper method

Proportion ⇒

a:b = c:d, $\frac{a}{b} \times \frac{c}{d}$, ad=bc
Product of Extremes = Product of Means

Continued Proportion

a:b = b:c
$\frac{a}{b} \times \frac{b}{c}$, $b^2 = ac$
b ↓ Mean

Properties of Proportion

- Invertendo
- Componendo
- Dividendo
- Addendo
- Subtrahendo
- Alternendo
- Componendo and Dividendo

Variation

- Direct → $\frac{a}{b} = \frac{c}{d}$, ad=bc
- Inverse → ab = cd

MIND MAP

INDICES

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Laws of Indices

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn} = a^{nm} = (a^n)^m$
- $(ab)^n = a^n \cdot b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $a^0 = 1$
- $a^x = a^y$ then $x = y$
- $x^a = y^a$ then $x = y$

Laws of Surds

- $\sqrt[n]{a} = a^{1/n}$
- $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $(\sqrt[n]{a})^n = a$

- $a^{-m} = \frac{1}{a^m}$
- $a^x = b$ then $a = b^{1/x}$

Some Basic Results

- $(a+b)^2 = a^2+b^2+2ab$
- $(a-b)^2 = a^2+b^2-2ab$
- $(a+b)^3 = a^3+b^3+3ab(a+b)$
- $(a-b)^3 = a^3-b^3-3ab(a-b)$
- $a^2-b^2 = (a+b)(a-b)$
- $(a^3+b^3) = (a+b)(a^2+b^2-ab)$
- $(a^3-b^3) = (a-b)(a^2+b^2+ab)$
- $a^3+b^3+c^3-3abc = (a+b+c)$
- $(a^2+b^2+c^2-ab-bc-ca)$
- $a+b+c=0$ iff $a^3+b^3+c^3 = 3abc$

Miscellaneous

- # $\sqrt{6+\sqrt{6+\sqrt{6+\cdots\infty}}} = 2 \times 3$
  - +ve $-(n+1)$
  - -ve $-(n)$
- # $a+b+c=0$

Types of Questions

- # Numerical → Prime factor Combine one by one
- # Variable → Combine one by one
- # Numerical + Variable

  - # Power given first simplify Power
  - # = sign given Base Same Power Same
  - # $\sqrt{}$ Hatane ke liye Squaring
  - # $\sqrt[3]{}$ Hatane ke liye Cubic

More than Two Values Equal

Relation Given

Power ki multiply Base ki Power ke reciprocal se

- × + = as
- ÷ − = it is
- 1 0

Relation Not given

Maximum Relation Highest Value ⇓ Maximum Prime factor kauke

Formula Based

- $a = x^{1/3} + x^{-1/3}$
- $a^3 - 3a = \frac{x^2+1}{x}$

or

- $a = x^{1/3} - x^{-1/3}$
- $a^3 + 3a = \frac{x^2-1}{x}$

Circular

- $\frac{\sin\theta}{\cos\theta} = \tan\theta$
- $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Square Root

Eg. $\sqrt{5+2\sqrt{6}}$

$\sqrt{3} + \sqrt{2}$ (6)

MIND-MAP
LOGARITHM

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Laws of Logarithm

- $\log_a mn = \log_a m + \log_a n$
- $\log_a \frac{m}{n} = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$
- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a b = \frac{\log b}{\log a} = \frac{\log_c b}{\log_c a}$
- $a^{\log_a b} = b$
- $a^{\log b} = b^{\log a}$

$\log_a b = x$
$a^x = b$

Types of Questions

Single Log
Convert into Exponential form

More than One log with + or − sign
Convert into Single log
$\log m + \log n = \log mn$
$\log m - \log n = \log \frac{m}{n}$

Single log but No benefit to Convert into Exponential
Expand
× +
÷ −

More than one log without + or − sign
- = given → Left side Convert into Exponential
- ≠ given → Right side Prime factor Power Aage

Some More Types

Different Base
$\log_{a^n} b^m = \frac{m}{n} \log_a b$

Divide me log
$\frac{1}{\log_a b} = \log_b a$

Circular
$\log_{\boxed{A}} \log_{\boxed{B}} \log_{\boxed{C}} \boxed{A} = 1$

More than one log with = sign
Multiply ⇒ 1
Add or Sub ⇒ 0

Calci Tricks

$\log_{10} m$
m √ 19 time −1 × 227695 =

$\log_{2\sqrt{7}} 21952$
2 × 7 √ x = = = ⋯ □

$\text{Antilog}_{10} m$
$m \div 227695$ +1 = (x=) 19 times

MIND MAP

EQUATION

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Linear Equation

- One Variable
- Two Variable
  - Elimination
  - GBC
    - Substitution
  - Cross Multiplication
- Three Variable

Types of Lines

- Intersecting
- Parallel
- Coincide

Conditions

- Irrational and Imaginary Roots occur in Pair
- Reciprocal Roots $c = a$
- Roots Equal but opposite in sign $b = 0$

Any Information About Roots

- Sum of Roots $\alpha + \beta = -b/a$
- Product of Roots $\alpha\beta = \frac{c}{a}$

Cubic Equation

- GBC
  - Trial & Error
  - Synthetic Division

Quadratic Equation

- Factorization Method
- formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Shortcut
- GBC

Qudratic

- Roots find out
- Formation of Equation
  - $x^2 - Sx + P = 0$
- Nature of Roots
  - $D = b^2 - 4ac$
    - Real
      - $D \geq 0$
        - Equal
          - $D = 0$
        - Unequal
          - $D > 0$
            - Perfect Square
              - Rational
            - Not a Perfect Square
              - Irrational
    - imaginary
      - $D < 0$

Symmetric Function

- Equation Given
  - $\alpha + \beta = -\frac{b}{a}$ $\alpha\beta = \frac{c}{a}$
- Relation Given
- Formation of New Eqn.
- Convert into $\alpha\beta$, $\alpha + \beta$ form

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

MIND MAP

TIME VALUE OF MONEY

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Interest

Simple Interest

$S.I = \frac{P \times r \times t}{100}$

$P = \frac{S.I \times 100}{r \times t}$

$r = \frac{S.I \times 100}{P \times t}$

$t = \frac{S.I \times 100}{P \times r}$

Difference only Two Values

$S_1 - S_2 = \frac{(P_1 - P_2) \times r \times t}{100}$

$S_1 - S_2 = \frac{P \times (r_1 - r_2) \times t}{100}$

$S_1 - S_2 = \frac{P \times r \times (t_1 - t_2)}{100}$

Simple Interest Types of Questions

Amount n times

$\frac{n \times 100}{r} = t$ or $\frac{(n-1) \times 100}{t} = r$

S.I. n times

$\frac{(n) \times 100}{r} = t$ or $\frac{(n) \times 100}{t} = r$

n_1 times in t_1 years, n_2 times in t_2 years

$\frac{n_1 - 1}{n_2 - 1} = \frac{t_1}{t_2}$

Divide Sum of Money Into Parts

$\frac{1}{r_1 t_1} : \frac{1}{r_2 t_2}$: Answer. S.I. Same

Amount for two Periods

1 year S.I. 1000 [5000 : 6000] 4 : 4 / 5 : 6

Conversion

Year into Month

Month into Year

Days into Month

Month into Days

C.I.

Divide Sum of Money into parts Amount Same

$\frac{1}{(1+i_1)^{n_1}} : \frac{1}{(1+i_2)^{n_2}}$

n_1 times t_1 years n_2 times ?

Eg. 2 times 4 years 64 times = ?

$2^6 \rightarrow 6 \times 4 = 24$

Compound Interest

Yearly: $A = P(1+i)^n$

Half Yearly: $\frac{i}{2}, 2n$

Quarterly: $\frac{i}{4}, 4n$

Monthly: $\frac{i}{12}, 12n$

Types of Questions

Sum Double: $\frac{69}{r} + .35$ or $\frac{72}{r}$

Sum Triple: $\frac{111.444}{r} + .35$ or $\frac{114}{r}$

More than 3 years Proper

Amount for Two Periods

Application of C.I.

Population: $FV = IV(1+i)^n$

Depreciation: $FV = IV(1-i)^n$

Effective Rate: $r = [(1+i)^n - 1]$

Difference between C.I and S.I

gap 2 years: Sum = diff $\left(\frac{100}{r}\right)^2$

gap 3 years: Sum = diff $\left(\frac{100}{r}\right)^2 \left(\frac{100}{300+r}\right)$

gap More than 3 years: C.I - S.I = $P((1+i)^n - 1 - it)$

MIND MAP

ANNUITY

RITU JINDAL

Future Value

- **Ordinary**
  - $FV = A\left[\frac{(1+i)^n - 1}{i}\right]$
- **Immediate or Due**
  - $FV = A\left[\frac{(1+i)^n - 1}{i}\right](1+i)$

Present Value

- **Ordinary**
  - $P.V = -A\left[\frac{(1+i)^{-n} - 1}{i}\right]$
- **Immediate or Due**
  - $P.V = -A\left[\frac{(1+i)^{-n} - 1}{i}\right](1+i)$

Compound Interest

$A = P(1+i)^n$

$P = \frac{A}{(1+i)^n}$ or

$i = \left(\frac{A}{P}\right)^{\frac{1}{n}} - 1$ or

for $n \Rightarrow \left(\frac{A}{P}\right) = (1+i)^n$

either GBC or taking log

Application of Annuity

- **Sinking Fund**
  - F.V.
- **Leasing**
  - P.V
- **Capital Expenditure**
  - P.V
- **Valuation of Bond**
  - P.V
- **Perpetuity**
  - **forever**
    - $= \frac{R}{i}$
  - **Growing Perpetuity**
    - $\frac{R}{i-g}$

Net Present Value

- $N > 0$ → Accept
- $N = 0$ → Depend
- $N < 0$ → Reject

NPV → P.V of Cash inflow − P.V. of Cash outflow

Compound Annual Growth Rate

$$\text{Rate} = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

MIND MAP

MEASURES OF CENTRAL TENDENCY

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Arithmetic Mean

Individual Series

- **Direct**: $\bar{X} = \frac{\Sigma x}{n}$
- **Shortcut**: $\bar{X} = A + \frac{\Sigma d}{n}$, $d_i = x_i - A$
- **Step Deviation**: $\bar{X} = A + \frac{\Sigma u}{n} \times h$, $u = \frac{x_i - A}{h}$

Discrete Series

- **Direct**: $\bar{X} = \frac{\Sigma fx}{\Sigma f}$
- **Shortcut**: $\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$, $d = x_i - A$
- **Step Deviation**: $\bar{X} = A + \frac{\Sigma fu}{\Sigma f} \times h$, $u_i = \frac{x_i - A}{h}$

Correcting AM

$$\frac{\text{Incorrect Sum} - \text{Incorrect } x + \text{Correct } x}{n}$$

Important Points

Best Measure
Most Commonly Used

Continuous Series

- **Direct**: $\bar{X} = \frac{\Sigma fm}{\Sigma f}$, m = mid Point
- **Shortcut**: $\bar{X} = A + \frac{\Sigma fd}{\Sigma fi}$, $d_i = m_i - A$
- **Step Deviation**: $\bar{X} = A + \frac{\Sigma fu}{\Sigma f} \times h$, $u_i = \frac{m_i - A}{h}$

Some Important formulae

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Combined A.M.

$$\bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$$

$$\bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2 + n_3 \bar{X}_3}{n_1 + n_2 + n_3}$$

MIND MAP

MEASURES OF CENTRAL TENDENCY

Rita Jindal

Median

Individual Series

Arrange Data in ascending or Descending order

- **Odd** → $\left(\frac{n+1}{2}\right)^{th}$ item
- **Even** → $\frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$

Discrete Series

Arrange Data in ascending or Descending order

Find out Cumulative frequency

$\left(\frac{n+1}{2}\right)^{th}$ obs.

Continuous Series

Arrange Data in Ascending or Descending order

Find out Cumulative frequency

$M = l + \frac{\frac{N}{2} - cf}{f} \times h$

Properties of Median

- $Y = a + bX$, $Y_{me} = a + bX_{me}$
- $\Sigma |x_i - \text{median}| = \text{minimum}$

For open-end classification $\Rightarrow$ Median is best

Emperical Relation

Mode = 3 median − 2 Mean

Mean − Mode = 3 (Mean − Median)

Mode

- **Individual** → Highest frequency
- **Discrete** → Highest frequency
- **Continuous** → $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Property of Mode → $Y = a + bx$, $Y_{mo} = a + bX_{mo}$

Partition Values

- Quartile
- Decile
- Percentile

Arrange Data Into Ascending Order Only

for Individual and Discrete

$Q_k = k\left(\frac{n+1}{4}\right)^{th}$ $D_k = k\left(\frac{n+1}{10}\right)^{th}$ $P_k = k\left(\frac{n+1}{100}\right)^{th}$

for Continuous Series

$Q_k = l + \frac{k\left(\frac{N}{4}\right) - cf}{f} \times h$

$D_k = l + \frac{k\left(\frac{N}{10}\right) - cf}{f} \times h$

$P_k = l + \frac{k\left(\frac{N}{100}\right) - cf}{f} \times h$

MIND MAP

MEASURES OF CENTRAL TENDENCY

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Geometric Mean

- **Individual**: $(x_1 \cdot x_2 \cdots x_n)^{\frac{1}{n}}$
- **Discrete**: $(x_1^{f_1} \cdot x_2^{f_2} \cdots x_n^{f_n})^{\frac{1}{\Sigma f_i}}$
- **Continuous**: $(m_1^{f_1} \cdot m_2^{f_2} \cdots m_n^{f_n})^{\frac{1}{\Sigma f_i}}$

Harmonic Mean

- **Individual**: $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$
- **Discrete**: $\frac{\Sigma f}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}}$
- **Continuous**: $\frac{\Sigma f}{\frac{f_1}{m_1} + \frac{f_2}{m_2} + \cdots + \frac{f_n}{m_n}}$, m_i = mid-value

Combined H.M.

$$H = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

Relationship Between A.M G.M. H.M

(i) $A \geq G \geq H$ (No indication)
$A > G > H$ (Unequal)
$A = G = H$ (Equal)

(ii) $A = \frac{a+b}{2}$ $G = \sqrt{ab}$
$H = \frac{2ab}{a+b}$

(iii) $G^2 = AH$

G.M. Logarithmic formulae

$$\text{Antilog } \frac{\Sigma \log x}{N}$$

Combined G.M.

$$(x_{11} x_{12} \cdots x_{1n_1} x_{21} x_{22} \cdots x_{2n_2})^{\frac{1}{n_1+n_2}}$$

Ratio Aug

- G.M.
- H.M.

Aug. Rate, Ratio, Percentage → G.M.
Aug. Rate and Ratio → H.M.
Aug. Speed or Aug Cost → H.M.

MIND MAP

MEASURES OF DISPERSION

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Range

- **Formula** ⇓ $L - S$
  - For Continuous Series Convert Inclusive Series into Exclusive
- **Coefficient of Range** ⇓ $\frac{L-S}{L+S} \times 100$
- **Property** ⇓ $Y = a + bx$, $R_y = |b| R_x$

For Open End Classification ⇓ O.D. is Best Measure of Dispersion

Quartile Deviation or Semi-inter quartile Range

- **Formula** ⇓ $\frac{Q_3 - Q_1}{2}$, Q_1 = lower quartile, Q_3 = Upper quartile
- **Coefficient of QD** ⇓ $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$ or $\frac{QD}{\text{median}} \times 100$
- **Property** ⇓ $Y = a + bx$, $QD_y = |b| QD_x$

Mean Deviation

- **Formula**
  - Individual ⇓ $\frac{\Sigma |x_i - A|}{n}$
  - Discrete ⇓ $\frac{\Sigma |x_i - A| f_i}{\Sigma f_i}$
  - Continuous ⇓ $\frac{\Sigma |m_i - A| f_i}{\Sigma f_i}$, m_i = Mid-point
- **Coefficient of M.D** ⇓ $\frac{\text{M.D. about A}}{A} \times 100$, A = Mean or Median or Mode
- **Property** ⇓ $Y = a + bx$, $MD_y = |b| MD_x$ ⇓ Mean Deviation is minimum when deviation taken from median

MIND MAP

MEASURES OF DISPERSION

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Standard Deviation

Individual Series

$$\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$$

$$\sqrt{\frac{\Sigma x_i^2}{n}-(\bar{x})^2}$$

Discrete Series

$$\sqrt{\frac{\Sigma f_i(x_i-\bar{x})^2}{\Sigma f_i}}$$

$$\sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2}$$

or

$$\sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i}-(\bar{x})^2}$$

Continuous Series

$$\sqrt{\frac{\Sigma f_i(m_i-\bar{x})^2}{\Sigma f_i}}$$

$$\sqrt{\frac{\Sigma f_i m_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i m_i}{\Sigma f_i}\right)^2}$$

or

$$\sqrt{\frac{\Sigma f_i m_i^2}{\Sigma f_i}-(\bar{x})^2}$$

Shortcut Method

$$\sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

Step Deviation Method

$$\sqrt{\frac{\Sigma f_i u_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)^2}\times h$$

Variance = $S.D^2$

$C.V = \frac{\sigma}{\bar{x}}\times 100$

$Y = a+bx$

$SD_y = |b| SD_x$

Best Measure of Dispersion

⇓

S.D.

S.D. of 'n' natural No.

$$\sqrt{\frac{n^2-1}{12}}$$

SD of 2 Item = $\frac{|a-b|}{2}$

Combined S.D.

$$S=\sqrt{\frac{n_1S_1^2+n_1S_2^2+n_1d_1^2+n_2d_2^2}{n_1+n_2}}$$

$d_1 = \bar{x}_1-\bar{x}$

$d_2 = \bar{x}_2-\bar{x}$

$$\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$$

Correct S.D.

⇓

find Summation terms
Correct them
Put in formula

Relation

MD : MD : SD
10 : 12 : 15

⇓

MD < MD < SD

MIND MAP

SEQUENCE AND SERIES

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Arithmetic Progression
⇓
Common difference

Calci $a_n = a + (n-1)d$

- $a_2 = ?$ $a_4 = ?$ $a_n =$ or general A.P. a and d Calci
- Sum of 3 terms, 4 terms, 5 terms: $a-d, a, a+d$ or GBC
- A.M
  - Single → $\frac{a+b}{2}$
  - More than One → $d = \frac{b-a}{n+1}$, n = No of A.M's or Calci

Calci $S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}[a+a_n]$

⇒ Sum of n A.M's = n · Single A.M. = $n \cdot \frac{(a+b)}{2}$

Geometric Progression
⇓
Common Ratio

Calci $a_n = aR^{n-1}$

- $a_2 = ?$ $a_4 = ?$ $a_n =$ or General G.P. a or R. GBC Calci
- Product of 3 terms, 4 terms: $\frac{a}{R}, a, aR$ or GBC
- G.M
  - Single → $\sqrt{ab}$
  - More than One → $R = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, n = No of G.M's or Calci.

Calci $S_n = \frac{a(R^n-1)}{R-1}$

$S_\infty = \frac{a}{1-R}$ $R<1$

⇒ Product of n G.M's = $(\text{Single GM})^n = (ab)^{n/2}$

Some Important Formulae

$1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1+3+5+\cdots+ = n^2$

$2+4+6+\cdots+ = n(n+1)$

- $S_n = Pn^2 + Qn$ → $a = P+Q$, $d = 2P$
- $a_n = Pn + Q$ → $a = P+Q$, $d = P$

MIND MAP

PERMUTATION AND COMBINATION

RITU JINDAL

⇒ factorial

Permutation

Arrangement = Order matter

Types of Questions

- n things r places: ${}^{n}P_{r} = \frac{n!}{(n-r)!}$, $n \geq r$ (P.N.A.)
- n things n places: ${}^{n}C_{n} = \frac{n!}{(n-n)!} = n!$
- n things p alike, q alike... ⇒ $\frac{n!}{p!q!\cdots}$
- Repetition allowed: n^{n} To Compute - Sony 100 buses

Restricted Permutation

- n things r at a time. One particular thing always occur ⇒ $r \cdot {}^{n-1}P_{r-1}$
- n things r at a time. One particular thing Never Occur ⇒ ${}^{n-1}P_{r}$
- n things taken all at a time m specified things always Come together ⇒ $(n-m+1)! \cdot m!$
- n things taken all at a time m specified things Never Come together ⇒ $n! - m!(n-m+1)!$

Circular Permutation ⇒ $(n-1)!$

- No change in clockwise and anticlockwise Direction
- They Don't have Same Neighbourhood in Any two occassion
- Necklace or Garland ⇒ $\frac{(n-1)!}{2}$

Fundamental Principle of Counting

- Addition Theorem: m+n ways Or
- Multiplication Theorem: m×n ways And

Main Problems

- # Word Problems
- # Sitting Arrangement
- # Problems on Digit

MIND MAP

PERMUTATION AND COMBINATION

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COMBINATION

Selection or Order Doesn't matter

Types of Questions

- n things Taken r at a time: $nC_r = \frac{n!}{r!(n-r)!} = \frac{nP_r}{r!}$
- n things Taken all at a time: $n_n = \frac{n!}{n!(n-n)!} = 1$
- n things taken r at a time 1 Particular always occur: $n-1C_{r-1}$
- n things Taken r at a time 1 Particular Never occur: $n-1C_r$

Some Important Results

- $nC_r = nC_{n-r}$
- $nC_x = nC_y$, $x = y$ or $x + y = n$
- $nC_r + nC_{r-1} = n+1C_r$
- $nC_1 + nC_2 + \dots nC_n = 2^n - 1$ → One or More, Atleast One, Some or All

Geometrical Problems

- Straight Line
  - Simple → nC_2
  - Collinear → $nC_2 - mC_2 + 1$
- Triangle
  - Simple → nC_3
  - Collinear → $nC_3 - mC_3$
- Parallelogram → $mC_2 \, nC_2$
- No. of Diagonals → $nC_2 - n$ or $\frac{n(n-3)}{2}$
- No of Chords → nC_2

Some Important Points

Selected digits given Repetition Not allowed

Balls → Colour
- Permutation → Identical
- Combination → Different

Books → Subject → Different

MIND MAP
CORRELATION AND REGRESSION

RITU JINDAL

Correlation

- Positive
- Negative
- $-1 \le \mu \le 1$
- Scatter Diagram
  - Positive Slope
  - Negative Slope
- Karl Pearson Correlation
  - $\frac{Cov(x,y)}{\sigma_x \sigma_y}$
  - $\frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$ ($x = dx$, $y = dy$)
  - $\frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$
  - $\frac{\Sigma xy}{n\sigma_x\sigma_y}$
- Spearman's Rank Corr.
  - Simple: $1 - \frac{6\Sigma d_i^2}{n(n^2-1)}$
  - Tied Rank: $1 - \frac{6\left[\Sigma d_i^2 + \Sigma\frac{(t_j^3 - t_j)}{12}\right]}{n(n^2-1)}$
- Coefficient of Concurrent deviation: $\mu_c = \pm\sqrt{\pm\frac{(2c-m)}{m}}$

Regression

- Line Y on X: $y - \bar{y} = b_{yx}(x - \bar{x})$
  - $b_{yx} = \mu\frac{\sigma_y}{\sigma_x}$
- Line X on Y: $x - \bar{x} = b_{xy}(y - \bar{y})$
  - $b_{xy} = \mu\frac{\sigma_x}{\sigma_y}$

Main Questions

- Point of Intersection = Mean
- Identity Line
- Find μ
- Eqn of Line

Important Formulae

- S.E: $\frac{1-\mu^2}{\sqrt{n}}$
- P.E.: $\frac{2}{3} \times \frac{1-\mu^2}{\sqrt{n}}$ or $.6745 = \frac{2}{3}$
- Coefficient of Determination: μ^2
- Coeff of Non-Determination: $1 - \mu^2$

Properties

- Correlation
  - Independent of change of origin & scale.
  - $\mu_{xy} = \frac{bd}{|b||d|}$ same
- $\mu = \pm\sqrt{b_{yx} b_{xy}}$
- Regression
  - Independent of change of origin but dependent on scale
  - $b_{yx} = \frac{q}{p} b_{uv}$, $u = \frac{x-a}{p}$
  - $b_{xy} = \frac{p}{q} b_{uv}$ $v = \frac{y-c}{q}$

- $\mu = 0$ Regression lines are Perpendicular
- $\mu = \pm 1$ Regression lines Coincide

MIND MAP

STATISTICAL DESCRIPTION OF DATA

RITU JINDAL

PART-1

Introduction

- Latin → Status
- Italian → Statista
- German → Statistik
- French → Statistique

Meaning

- Singular → method
- Plural → Data

Application of Statistics

- Economics
- Business Management
- Industry
- Commerce

Collection of Data

- Primary → Collected for the first time
- Secondary → Already Collected

Primary Data

- Interview
  - Personal → Best, Costly, Time Consuming, Natural Calamities
  - Indirect → Not reliable, Rail Accident
  - Telephonic → Quick and Non expensive, Non-response Maximum
- Mailed Questionnaire method → Wide area, Non Response Low
- Observation → Best, Time Consuming, Laborious, Cover Small Area
- Questionnaire Sent by Enumerator → More Reliable, Costly and Time Consuming

Secondary Data

- International Sources
- Government Sources
- Private and Quasi gut Sources
- Unpublished Sources

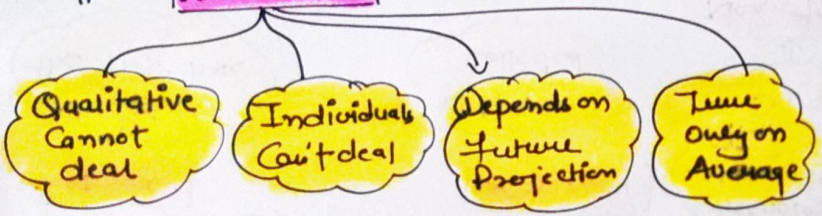
Classification (Organisation) of Data

- Chronological or temporal or Time Series Data → Non-frequency group
- Geographical or Spatial Series Data → Non-frequency group
- Qualitative or Qualitial Data → frequency group
- Quantitative or Cardinal Data → Nofrequency group

Qualitative or Attribute. Eg. Nationality, Drinking habit, Beauty, Intelligence

Lovely Italian Girl friend

Limitations



MIND MAP

STATISTICAL DESCRIPTION OF DATA

RITU JINDAL

PART-2

Mode of Presentation of Data

- Textual
  - Paragraph or No of Paragraphs
- Tabular
  - Logical and Systematic Arrangement
- Diagrammatic
  - Provided by charts, Diagrams & Picture

Line Diagram

- Logarithmic or Ratio chart
  - Wide range of fluctuation
- Multiple line chart
  - Two or more related series Same Unit
- Multiple Axis chart
  - Two or More related series Different units

Parts of Table

- Caption
  - Upper Part Columns and Sub-Columns
- Box-head
  - Entire upper Part - Including Columns and Sub Columns Unit of Measurement
- Stub
  - left Part Provide description of Rows
- Body
  - Main Part Contain Numerical figures
- Footnotes & Sources
  - Bottom Part of the table

Bar Diagram

- Horizontal Bar diag.
  - Qualitative Data or Data Varying Over space
- Vertical Bar diag
  - Quantitative or Time-Series Data
- Multiple or Grouped
  - Used to Compare related Series
- Component or Sub-divided
  - Representing Data Divided into No of Components
- Divided or Percentage
  - Compare different Components and their Relation to whole

Types of Diagrams

- Line Diagram or Historiagram
- Bar Diagram
- Pie chart

Scrutiny of Data

- Accuracy
- Consistency

Columns of Table

- Variable
  - Eg. height Age
- Tally
  - Block of five
- Frequency
  - No of time an obs occurs

Types of Diagrams

- One Dimensional
  - Bar
- Two Dimensional
  - Pie Rectangle
- Three Dimensional
  - Cube

MIND MAP

STATISTICAL DESCRIPTION OF DATA

RITU JINDAL
Part-3

Graph of Frequency Distribution

Histogram or Area Diagram

- Containing set of Rectangles
  - width → class Interval
  - height → frequency
- Frequency Density
  - If class-intervals are Unequal
- Mode can be Calculated
- Class-Interval should be in exclusive form

Frequency Polygon

- Joining mid-points of top horizontal side of rectangles in a Histogram
- Can be drawn
  - (i) By pre-fixing Histogram
  - (ii) Direct

Cumulative Frequency Curve or Ogive

- less than ogive
- More than ogive
- Point of Intersection is Median.

Frequency Curve

- Bell Shaped
- Ush aped
- J-sh aped
- Mixed

Series

Inclusive Series

- Upper and lower Both limit included

Exclusive Series

- Upper limit Excluded

Class Limit

- End values of class

Class Boundaries or Actual class limit

- Lower class limit
- Upper class limit

Lower class Boundaries

- LCL − $\frac{1}{2}$ the gap

Upper class Boundaries

- UCL + $\frac{1}{2}$ the gap

Class-Mark or Mid Point or Mid value

- $\frac{\text{LCL+UCL}}{2}$ or $\frac{\text{LCB+UCB}}{2}$

Relative frequency

- $\frac{\text{Class frequency}}{\text{Total frequency}}$

No of classes

- $\frac{\text{Range}}{\text{Class Size}}$

Percentage frequency

- $\frac{\text{Class frequency}}{\text{Total frequency}} \times 100$

Frequency Density

- $\frac{\text{freq. of that class Interval}}{\text{class length}}$

Cumulative frequency Distribution

- less than type
- More than type

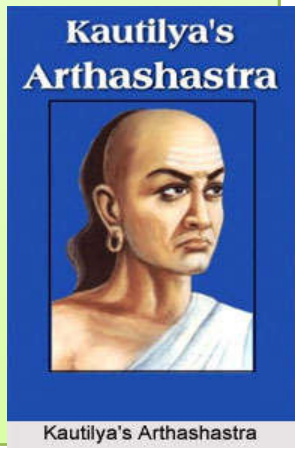
STATISTICAL DESCRIPTION OF DATA

Introduction of Statistics

- The word statistics has been derived from the word.
- Latin – Status
- Italian – Statista
- German - Statistik
- French - Statistique

HISTORY OF STATISTICS

- Kautilya
- famous book ‘Arthashastra’
- during Chandragupta’s reign
- in the fourth century B.C
- Ain-i-Akbari written by Abu Fazl
- . Referring to Egypt,
- the first census was conducted by the Pharaoh during 300 B.C. to 2000 B.C.



Meaning (Definition) of statistics

- i) Singular Sense : - (Method)
- ii) Plural Sense: - (Data)

(i) Singular Sense:Scientific method that is employed for collecting; analysing and presenting data, Leading finally to drawing statistical inferences

(ii) Plural Sense:- Data qualitative as well as quantitative.

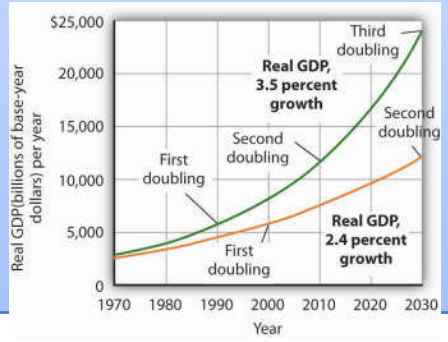
Use of statistics

- public services,
- defence
- banking,
- insurance sector
- tourism and hospitality,
- police and military etc

Application of Statistics:-

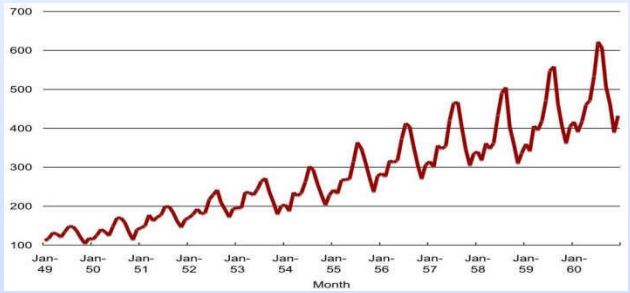
- (i) Economics
- (ii) Business Managements
- (iii)Industry
- (iv) Commerce

Reliance
Industrial Infrastructure
Limited



ECONOMICS

- Time Series Analysis,
- Index Numbers,
- Demand Analysis



- Conducting socio-economic surveys and analysing the data derived from it
- Regression analysis,
- for making future projection of demand of goods, sales, prices, quantities etc.

BUSINESS MANAGEMENT

Statistics in Commerce and Industry

- Data on previous sales
- raw materials,
- wages and salaries,
- products of identical nature of other factories etc
- collected, analysed and experts are consulted in order to maximise profits

Limitations of Statistics

- 1. Do not study qualitative phenomenon such as Beauty, Honesty, poverty etc.
- 2.It deals with groups and not with individuals.
- 3. Statistical laws are not exact. Statistical results are true only on average.
- 4.It can be missued.

VARIABLE

DISCRETE

- a variable assumes a finite or a countably infinite number of isolated values, it is known as a discrete variable.

Examples

- number of petals in a flower,
- the number of misprints a book contains
- the number of road accidents in a particular locality and so on.

CONTINUOUS

- it can assume any value from a given interval. Examples
- height,
- weight,
- sale
- , profit .

COLLECTION OF DATA

- (a) Primary;

  The data which are collected for the first time by an investigator or agency are known as primary data

- (b) Secondary

  if the data, as being already collected, are used by a different person or agency

Collection of Primary Data

- (i) Interview method;
- (ii) Mailed questionnaire method;
- (iii) Observation method;
- (iv) Questionnaries filled and sent by enumerators

Interview method

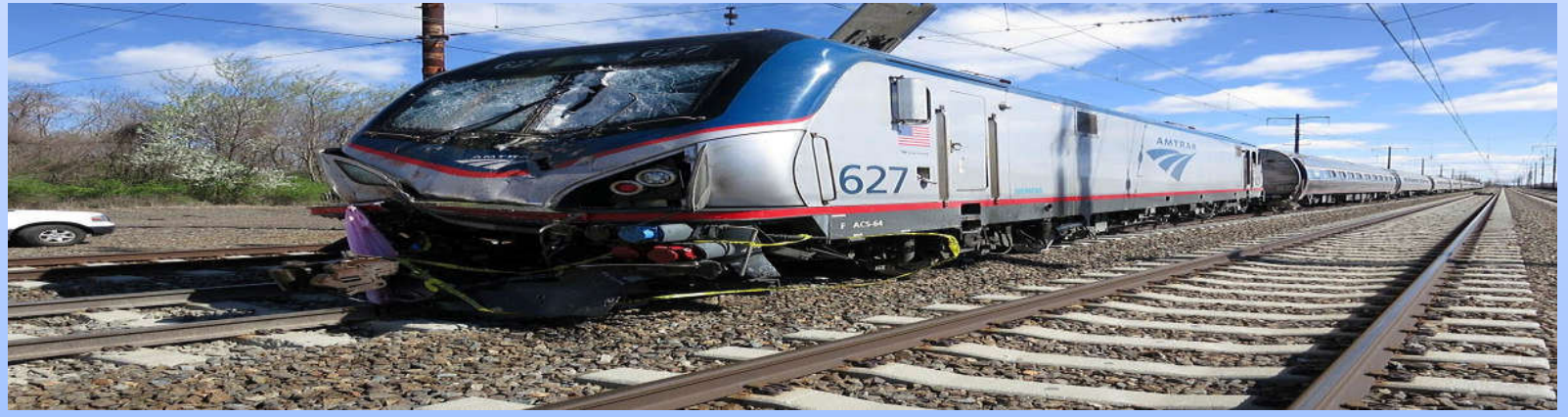
- (a) Personal Interview method,
- (b) Indirect Interview method
- (c) Telephone Interview method

personal interview method

- the investigator meets the respondents directly and collects the required information.
- In case of a natural calamity like a super cyclone
- or an earthquake or an epidemic like plague,

Indirect Interview method

- If there are some practical problems in reaching the respondents directly,
- as in the case of a rail accident
- investigator collects the necessary information from the persons associated with the problems



Telephone interview method

the relevant information can be gathered by the researcher himself by contacting the interviewee over the phone.

Mailed questionnaire method

- framing a well-drafted and soundly-sequenced questionnaire covering all the important aspects of the problem under consideration and sending

1. Have you heard of this test (through a family member or friend?)		
a) Yes		
b) No		
2. Have you done this test before?		
a) Yes		
b) No		
3. What is your biggest fear regarding this test?		
a)	Shortness of breath	
b)	Asphyxia	
c)	Death	
d)	Results of the test	

Rev Port Pneumol. 2012;18:226–32

observation method

- data are collected, as in the case of obtaining the data on the height and weight of a group of students, by direct observation or using instrument.

Questionnaries filled and sent by enumerators.

- Enumerators collects information directly
- by interviewing the persons having information
- Question are explained and hence data is collected.

COMPARISON

Types of interview/ parameters	Personal interview	Indirect interview	Telephonic interview
accuracy	high	medium	low
coverage	low	low	maximum
Non response	low	low	maximum

Best method Personal Interview

Sources of Secondary Data

- (a) International sources like

  WHO, ILO, IMF, World Bank etc.
- (b) Government sources like

  Statistical Abstract by CSO,

  Indian Agricultural Statistics by the Ministry of Food and Agriculture .
- (c) Private and quasi-government sources like

  ISI, ICAR, NCERT etc.
- (d) Unpublished sources of various research institutes, researchers etc.

Scrutiny of Data

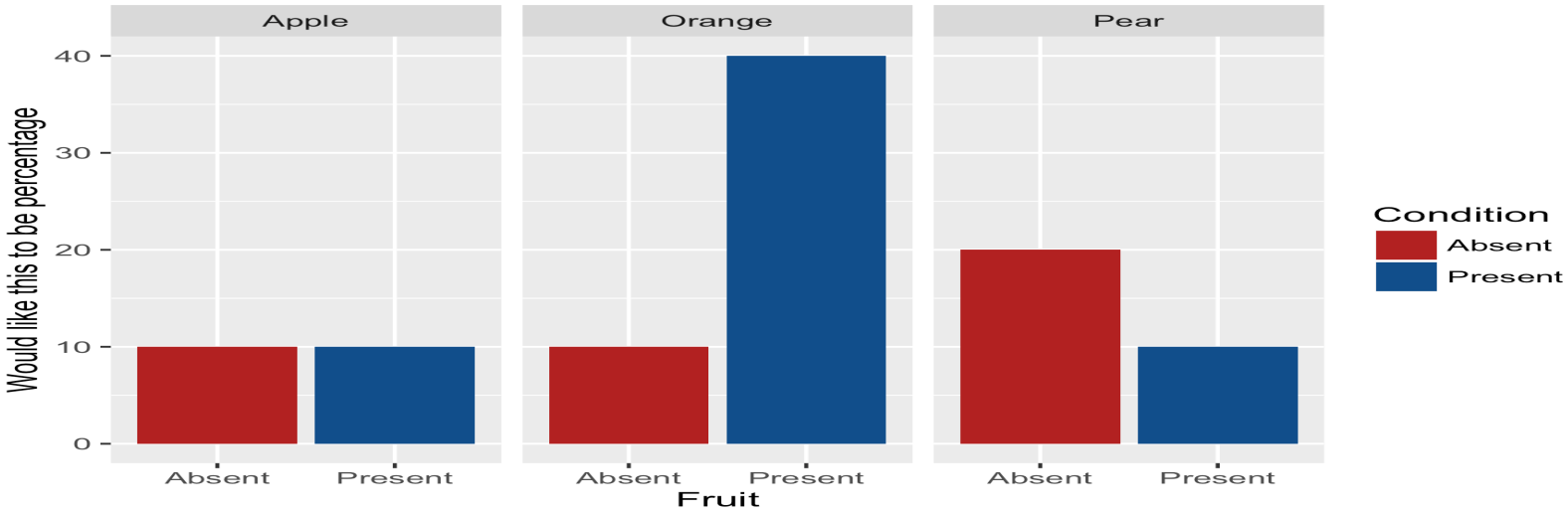
- check whether the data under consideration are accurate as well as consistence.
- Errors in data may creep in while writing or copying the answer on the part of the enumerator. A keen observer can easily detect that type of error.
- Again, there may be two or more series of figures which are in some way or other related to each other.
- If the data for all the series are provided, they may be checked for internal consistency

Check accuracy

- Consistency
- Date of birth :- 12/3/2005
- Age :- 50
- Accuracy
- Weight :- 120
- But very slim

Classification or Organisation of Data

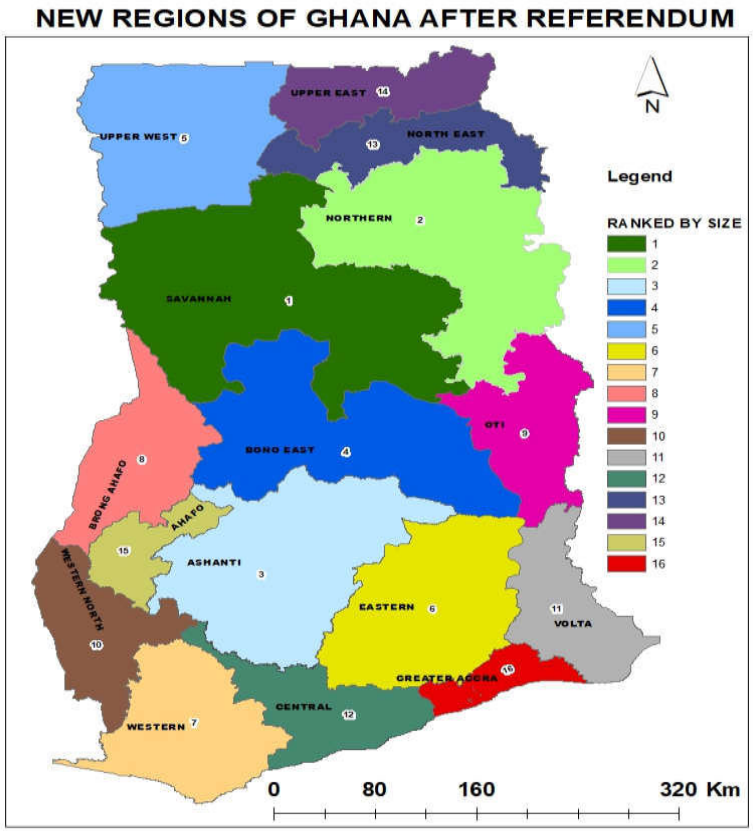
- (i) Chronological or Temporal or Time Series Data;
- (ii) Geographical or Spatial Series Data;
- (iii) Qualitative or Ordinal Data;
- (iv) Quantitative or Cardinal Data.



Chronological or Temporal or Time Series Data

- When the data are classified in respect of successive time points or intervals, they are known as time series data.
- The number of students appeared for CA final for the last twenty years,
- the production of a factory per month from 2000 to 2015 etc. are examples of time series data.

Geographical or Spatial Series Data



- Data arranged region wise are known as geographical data.
- If we arrange the students appeared for CA final in the year 2015 in accordance with different states, then we come across Geographical Data.

Qualitative or Ordinal Data

- Data classified in respect of an attribute are referred to as qualitative data
- Examples
- Data on nationality,
- gender,
- smoking habit of a group of individuals

Quantitative or Cardinal Data

- When the data are classified in respect of a variable,say
- height,
- weight,
- profits,
- salaries etc.

DATA

frequency data

- qualitative
- quantitative data

non-frequency data.

- time series data
- geographical data

Mode of Presentation of Data

- (a) Textual presentation;
- (b) Tabular presentation or Tabulation;
- (c) Diagrammatic representation.

Textual presentation

- This method comprises presenting data with the help of a paragraph or a number of paragraphs.
- The official report of an enquiry commission is usually made by textual presentation.
- 'In 2009, out of a total of five thousand workers of Roy Enamel Factory, four thousand and two hundred were members of a Trade Union. The number of female workers was twenty per cent of the total workers out of which thirty per cent were members of the Trade Union.
- In 2010, the number of workers belonging to the trade union was increased by twenty per cent as compared to 2009 of which four thousand and two hundred were male. The number of workers not belonging to trade union was nine hundred and fifty of which four hundred and fifty were females.'

Tabular presentation or Tabulation

Main parts of a table - (Five Parts):

(a) Caption:- upper part of the table, describing the columns and sub-columns, if any.

(b) Box-head:- The entire upper part of the table which includes columns and sub-column numbers, unit of measurement along with caption.

(c) Stub:- The left part of the table providing the description of the rows.

(d) Body:- Main part of the table that contains the numerical figures.

(e) Footnotes & Sources: - Should be shown at the bottom part of the table.

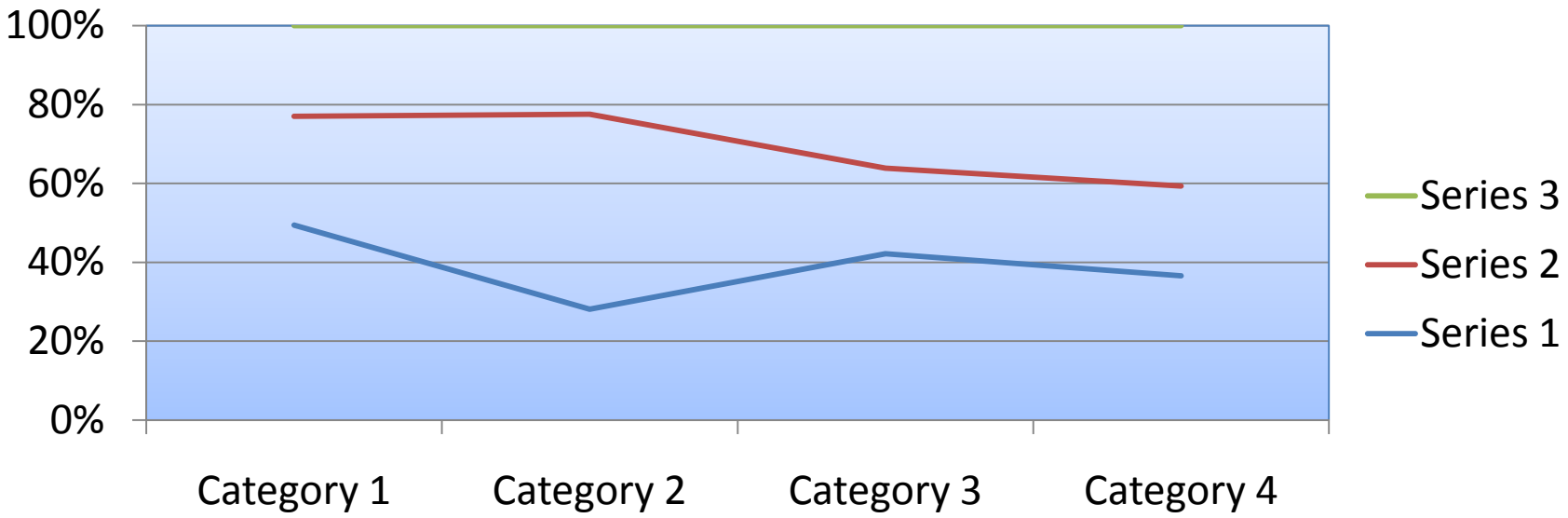
Why tabulation

- (i) It facilitates comparison between rows and columns.
- (ii) Complicated data can also be represented using tabulation.
- (iii) It is a must for diagrammatic representation.
- (iv) Without tabulation, statistical analysis of data is not possible.

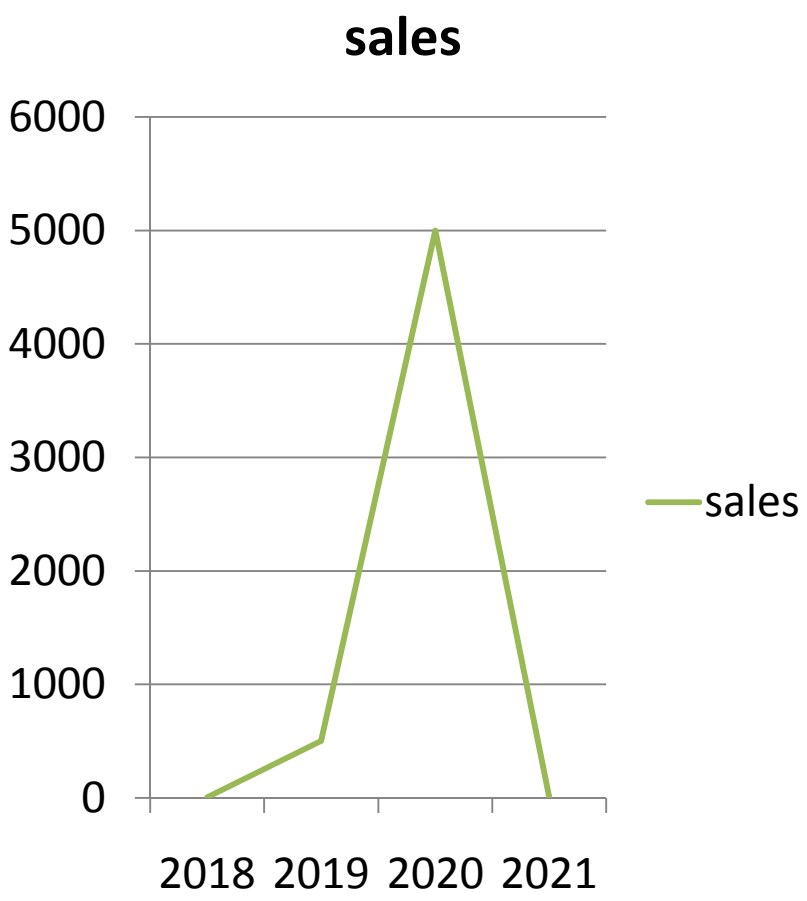
Diagrammatic representation of data

- Line diagram or Historiagram;
- Bar diagram
- Pie chart

Line diagram or Historiagram

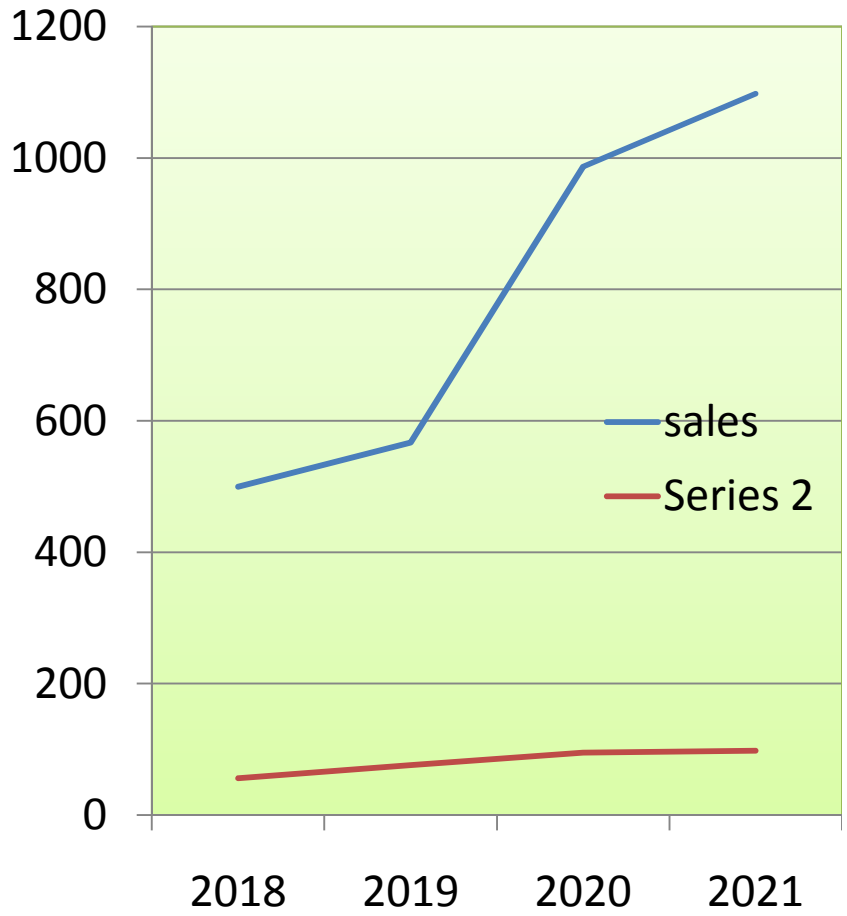


logarithmic or ratio chart



Years	Sales
2018	5
2019	500
2020	5000
2021	500000

Multiple line chart



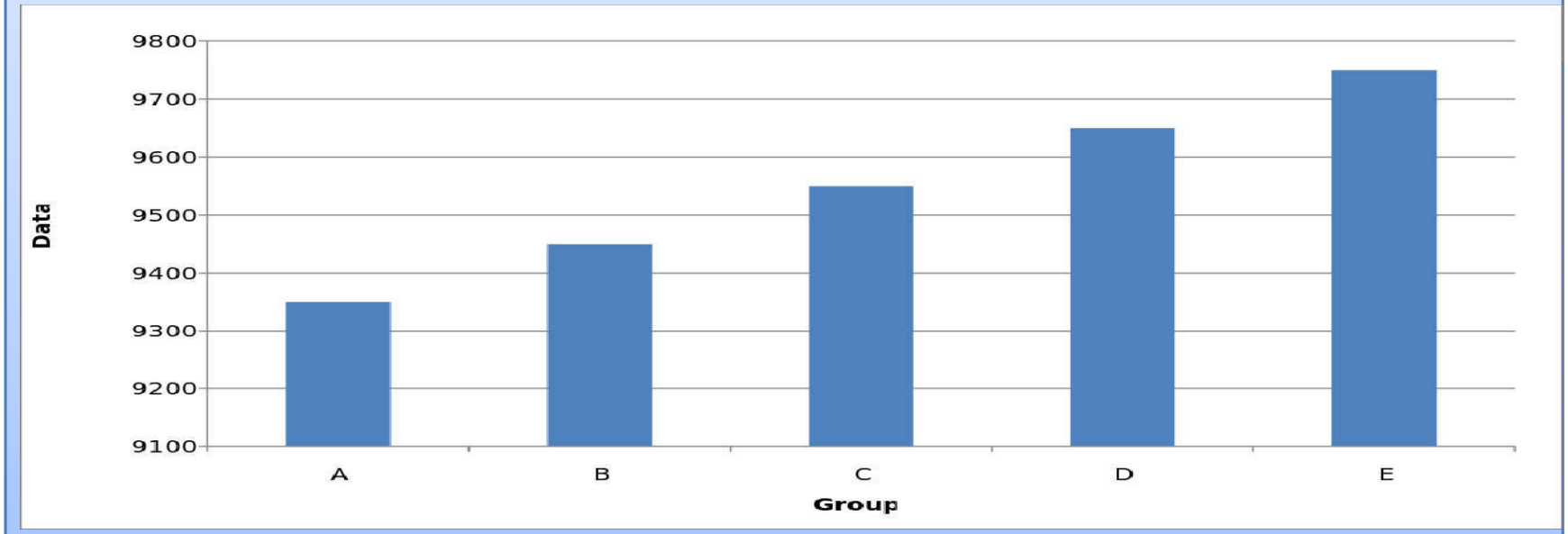
years	Sales (in crores)	Profit (in crores)
2018	500	56
2019	567	76
2020	987	95
2021	1098	98

Multiple axis chart

years	Sales (in crores)	No.of employes
2018	500	3000
2019	678	4567
2020	908	8765
2021	1098	9087

Bar diagram

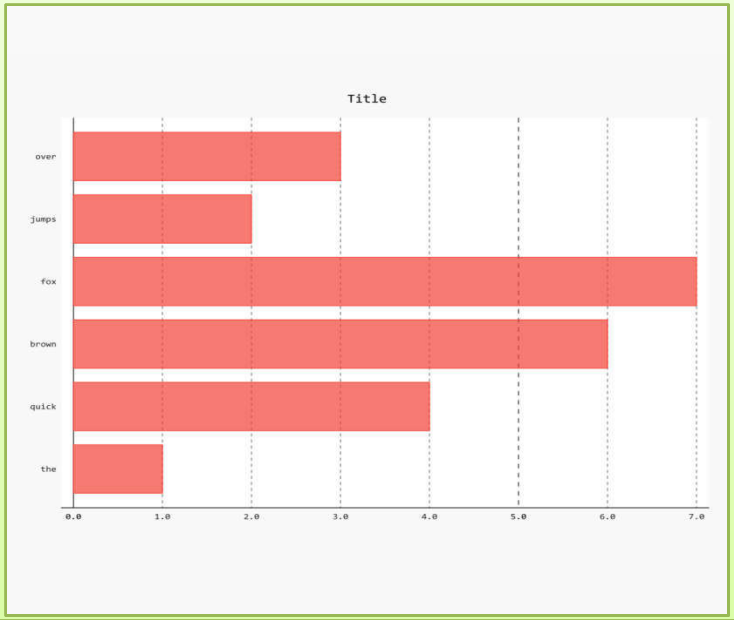
- rectangles of equal width and usually of varying lengths are drawn either horizontally or vertically.



Types of bar diagram

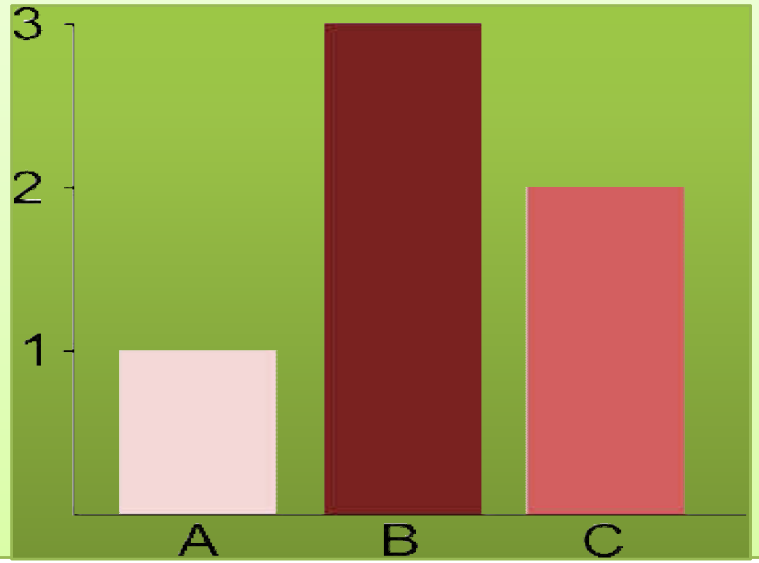
Horizontal Bar diagram

- used for qualitative data or data varying over space

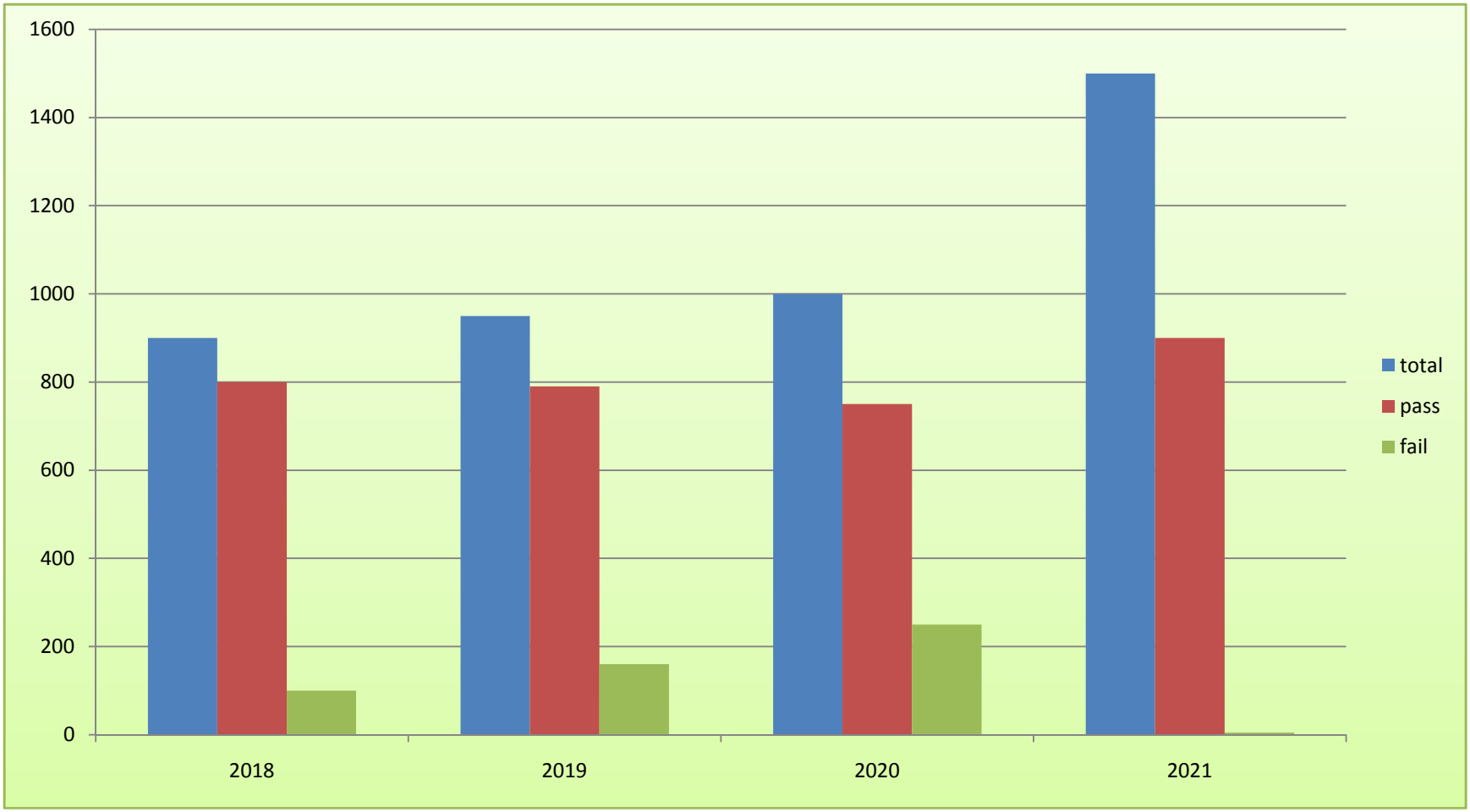


Vertical Bar diagram

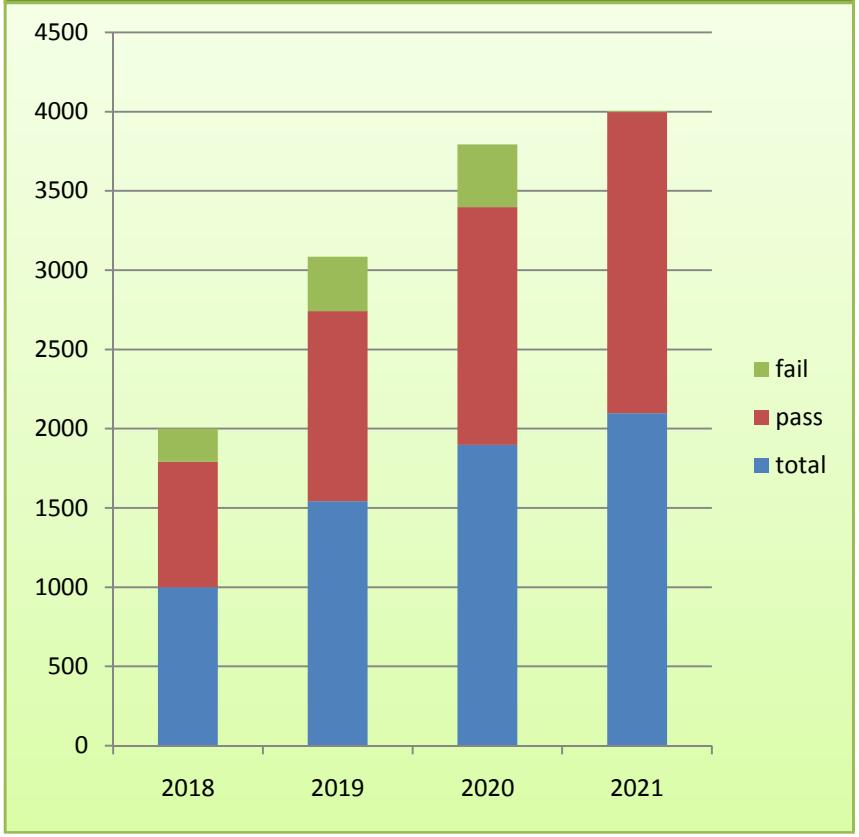
- is associated with quantitative data or time series data



Multiple or Grouped Bar Diagrams:



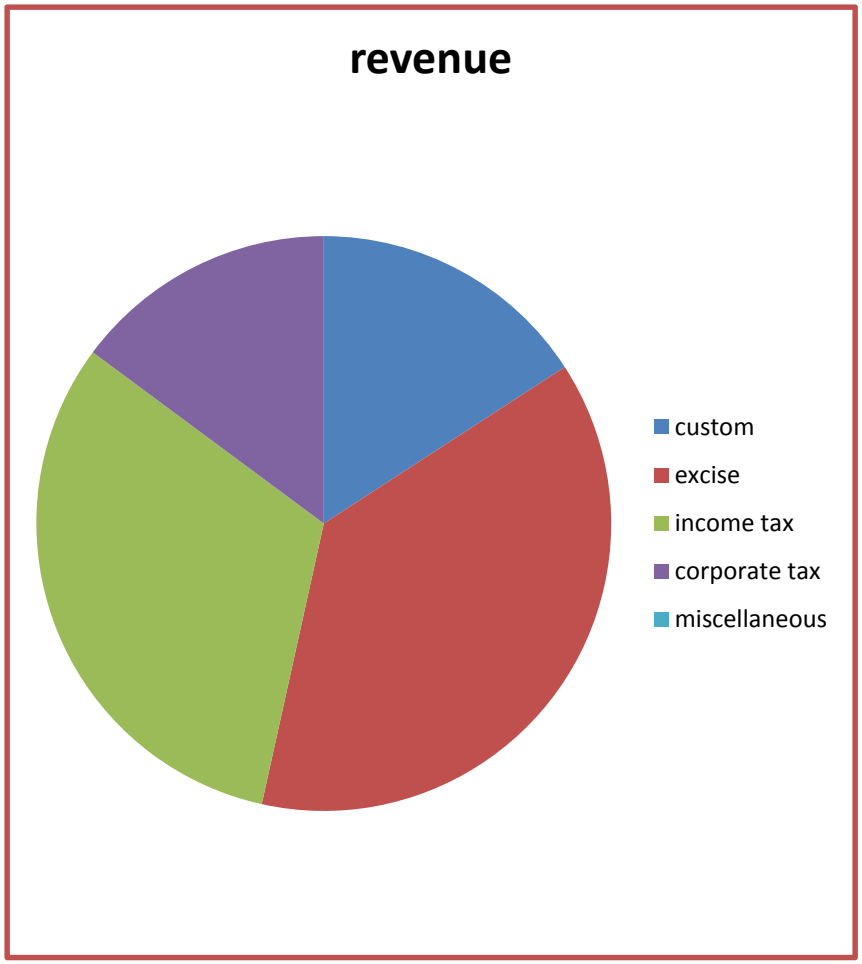
Component or Sub-divided Bar Diagrams



Divided Bar Charts or Percentage Bar Diagrams

year	total	pass	fail
2018	1000	790	210
2019	1587	1200	387
2020	1879	1500	379
2021	2089	1900	189

Pie Chart



Source	Revenue
Customs	80
Excise	190
Income tax	160
Corporate tax	75
miscellaneous	35

FREQUENCY DISTRIBUTION

- A frequency distribution may be defined as
- a tabular representation of statistical data,
- usually in an ascending order,
- relating to a measurable characteristic
- according to individual
- or a group of values of the characteristic under study

frequency distribution

simple Frequency Distribution

- When tabulation is done in respect of a discrete random variable, it is known as Discrete or Ungrouped or simple Frequency Distribution
- Example
- The distribution of the number of car accidents in Delhi during 12 months of the year 2005

Grouped Frequency Distribution

- in case the characteristic under consideration is a continuous variable, such a classification is termed as Grouped Frequency Distribution.
- Example
- distribution of heights of the students of St. Xavier's College for the year 2004

Example

Following are the records of babies born in a nursing home in Bangalore during a week

(B denoting Boy and G for Girl) :

B G G B G G B G G B B B G B B G B B B B G B B G G G G B B G

Construct a frequency distribution according to gender.

Frequency Distribution of a Variable

- Find the largest and smallest observations and obtain the difference between them, known as Range, in case of a continuous variable.
- Form a number of classes depending on the number of isolated values assumed by a discrete variable. In case of a continuous variable, find the number of class intervals using the relation.
- No. of class Interval X class length = Range.
- Present the class or class interval in a table known as frequency distribution table.
- Apply ‘tally mark’ i.e. a stroke against the occurrence of a particulars value in a class or class interval.
- Count the tally marks and present these numbers in the next column, known as frequency column, and finally check whether the total of all these class frequencies tally with the totalnumber of observations.

Example

- A review of the first 30 pages of a statistics book reveals the following printing mistakes:

- 0 4 2 1 1 3 3 1 2 3 2 5 6 0 2
- 3 2 2 3 3 4 0 5 6 1 0 1 0 4 4

- Make a frequency distribution of printing mistakes.

Difference between class limit and class boundary

Class Limit (CL)	Class Boundary (CB)
• Corresponding to a class interval, the class limits may be defined as the minimum value and the maximum value the class interval may contain. The minimum value is known as the lower • class limit (LCL) and the maximum value is known as the upper class limit (UCL).	• Class boundaries may be defined as the actual class limit of a class interval. • For mutually inclusive classification • For mutually exclusive classification

Mid-point or Mid-value or class mark

The central value of the class interval is called the mid point or mid-value or class mark.

Mid Point or class mark =

LCL+UCL/ 2

or

LCB+UCB/2

Width or size of a class interval

- The width of a class interval may be defined as the difference between the UCB and the LCB of that class interval.

Cumulative Frequency

- The cumulative frequency corresponding to a value for a discrete variable and corresponding to a class boundary for a continuous variable may be defined as
- the number of observations less than the value or less than or equal to the class boundary.

Frequency density of a class interval

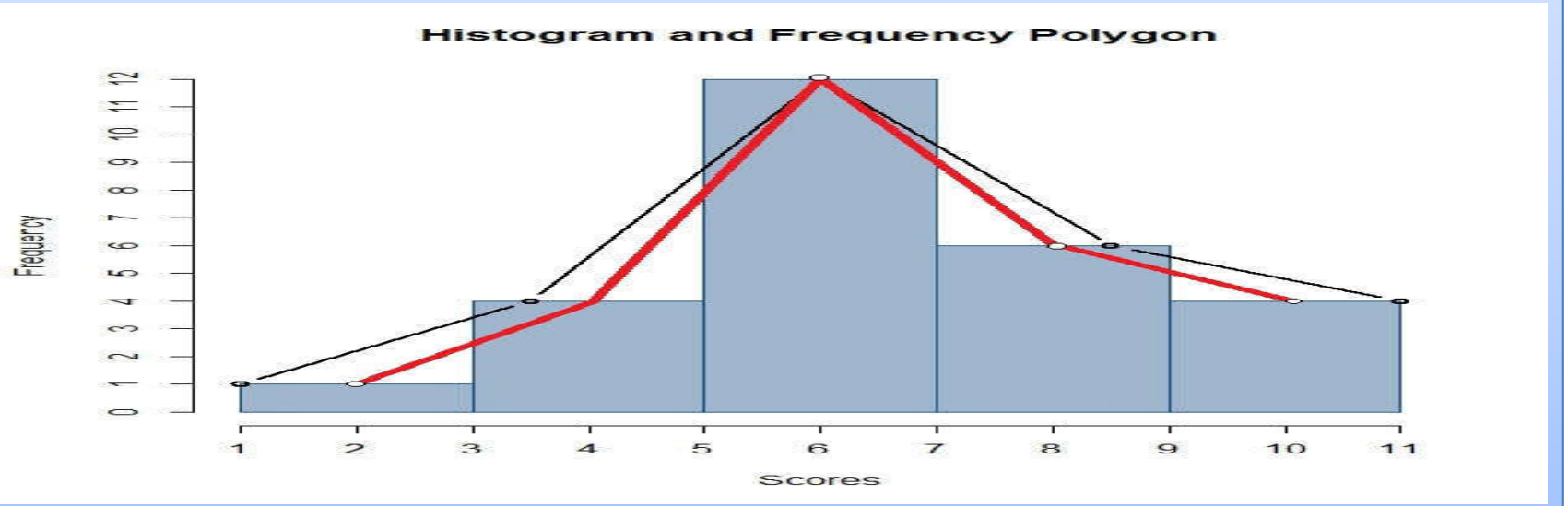
- It may be defined as the ratio of the frequency of that class interval to the corresponding class length

Relative frequency and percentage frequency of a class interval

- Relative frequency of a class interval may be defined as the ratio of the class frequency to the total frequency.
- Percentage frequency of a class interval may be defined as the ratio of class frequency to the total frequency, expressed as a percentage

GRAPHICAL REPRESENTATION OF A FREQUENCY DISTRIBUTION

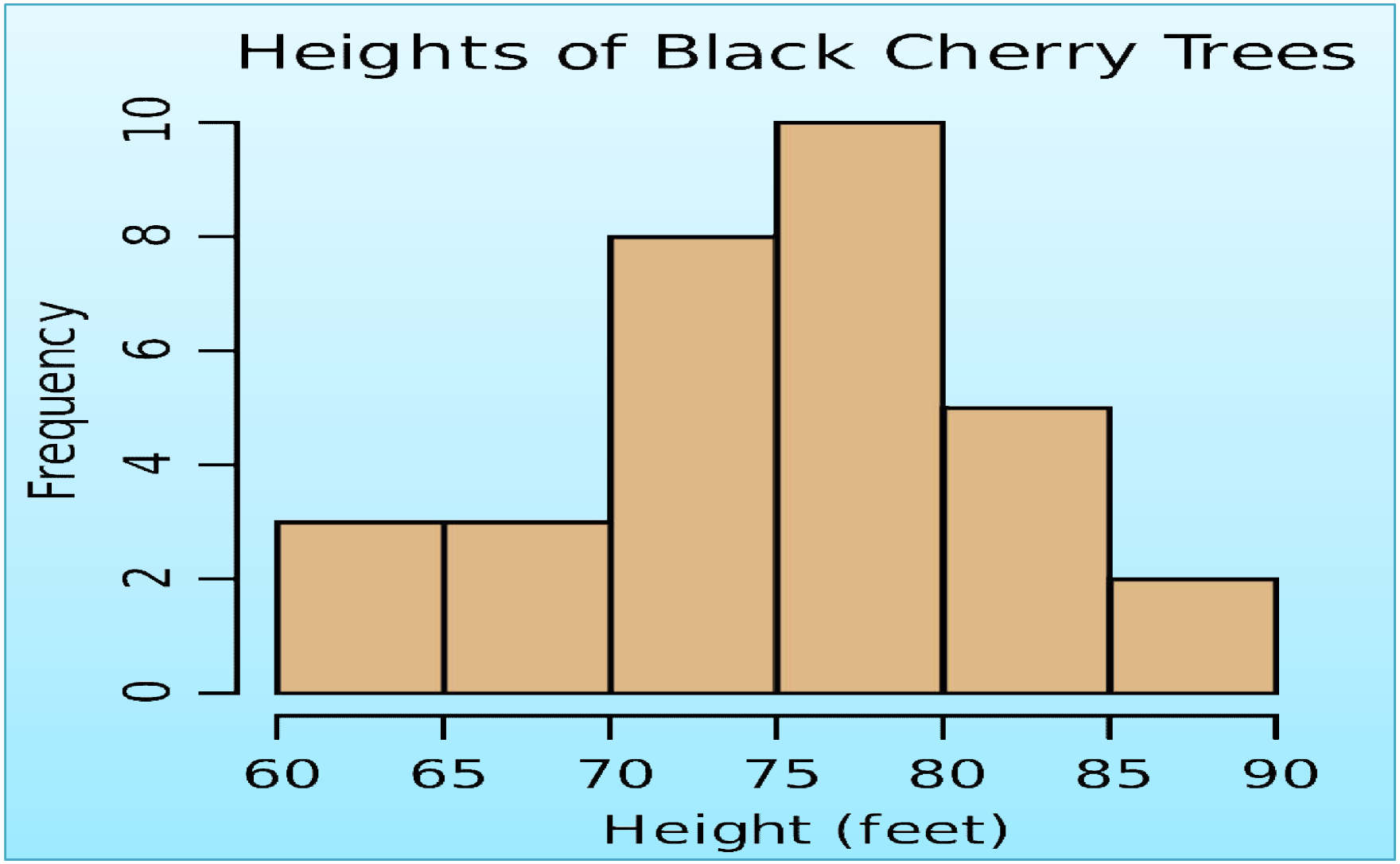
- (i) Histogram or Area diagram;
- (ii) Frequency Polygon;
- (iii) Ogives or cumulative Frequency graphs.



Histogram or Area diagram

- A comparison among the frequencies for different class intervals is possible in this mode of diagrammatic representation.
- In order to draw a histogram,
- the class limits are first converted to the corresponding class boundaries
- and a series of adjacent rectangles, one against each class interval, with the class interval as base or breadth
- and the frequency or frequency density usually when the class intervals are not uniform as length or altitude, is erected.

histogram


Heights of Black Cherry Trees
Frequency
0
2
4
6
8
10
60
65
70
75
80
85
90
Height (feet)

Histogram (Area diagram)

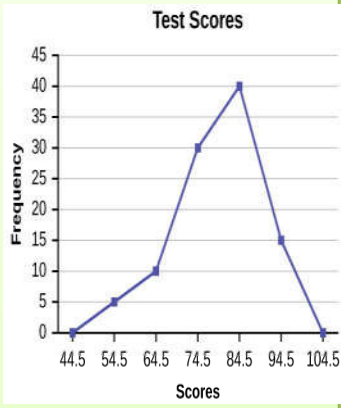
- A Histogram is a graph containing a set of rectangles,
- each being constructed to represent the size of the class interval by its width
- and the frequency in each class-interval by its height.
- The area of each rectangle is proportional to the frequency in the respective class-interval and the total area of the histogram is proportional to the total frequency.

Important points

- When the class-intervals are unequal the heights of rectangles are made proportional not to the class frequencies, but to the frequency densities.
- In construction of histogram the class intervals should be in exclusive form.
- We can find mode graphically by histogram.

Frequency Polygon

- If we mark the mid-points of the top horizontal sides of the rectangles in a histogram
- and join them by straight lines,
- the figure so formed is called a frequency polygon.
- A frequency polygon is useful in comparing two or more frequency distribution.

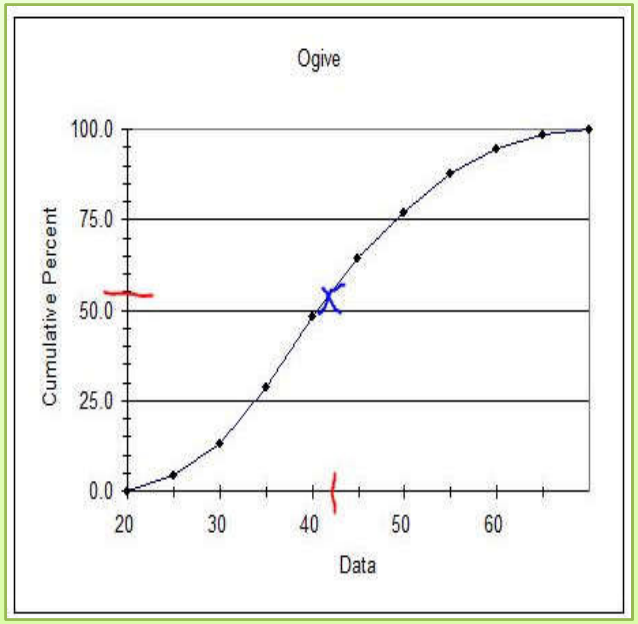


Frequency polygon can be drawn in two ways:

- (a) By preparing histogram first.
- (b) Direct method.
- Usually frequency polygon is meant for single frequency distribution.
- However, we also apply it for grouped frequency distribution provided the width of the class intervals remains the same.

Ogives or Cumulative Frequency Graph

- A graph which represents the data of a cumulative frequency distribution is called ogive curve.
- Ogive is a line diagram



TYPES OF OGIVES

Less than ogive

- If the cumulative frequencies are plotted at the upper limit of the class interval, it is a less than ogive.

More than ogive

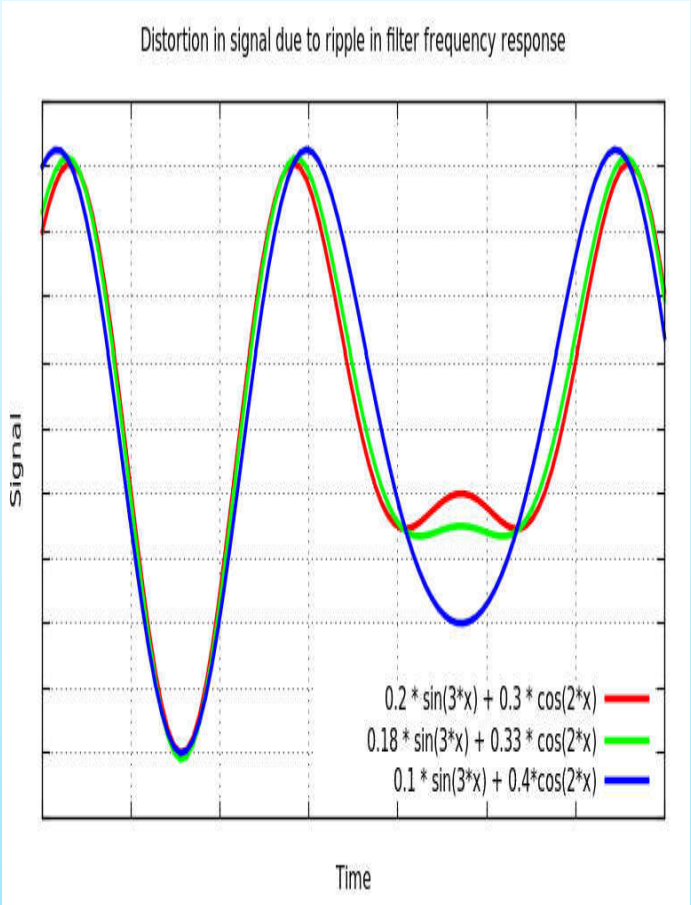
- Cumulative frequencies are plotted against the lower class boundaries of the respective class, intervals.

Frequency Curve

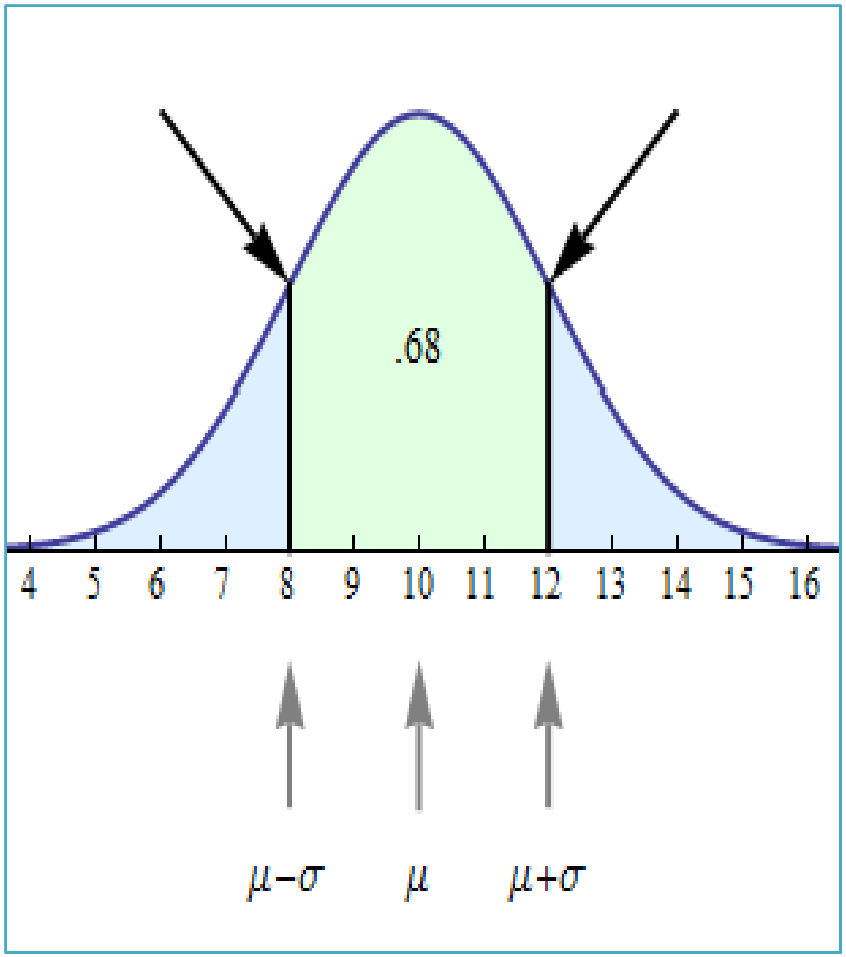
- A frequency curve is drawn by smoothing the frequency polygon.
- It is smoothed in such a way that the sharp turns are avoided.
- A frequency curve can be regard as a limiting form of frequency polygon or histogram.

types of frequency curves

- (a) Bell-shaped curve;
- (b) U-shaped curve;
- (c) J-shaped curve;
- (d) Mixed curve.

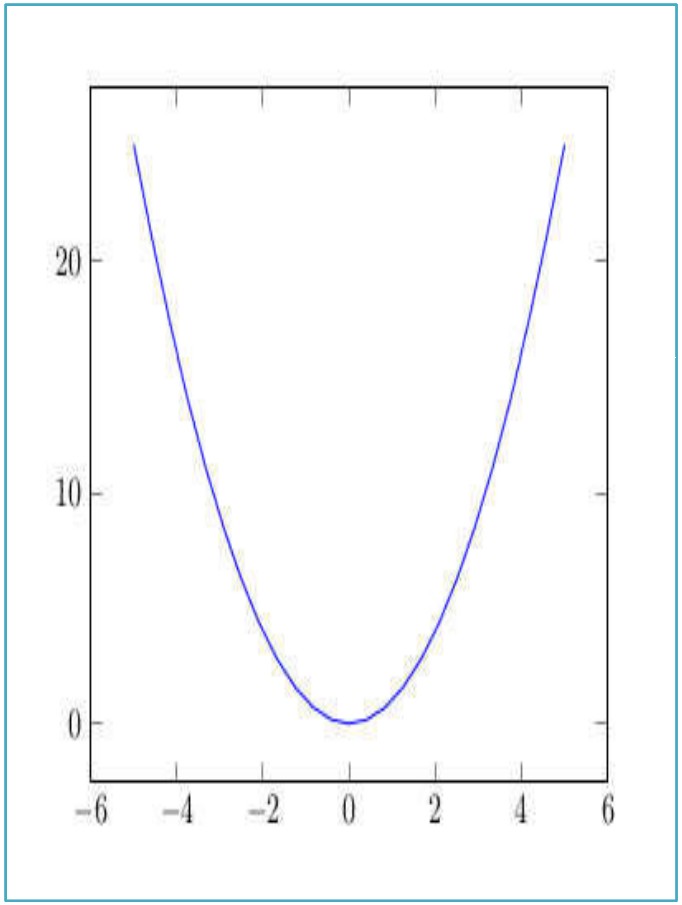


Bell Shaped Curves



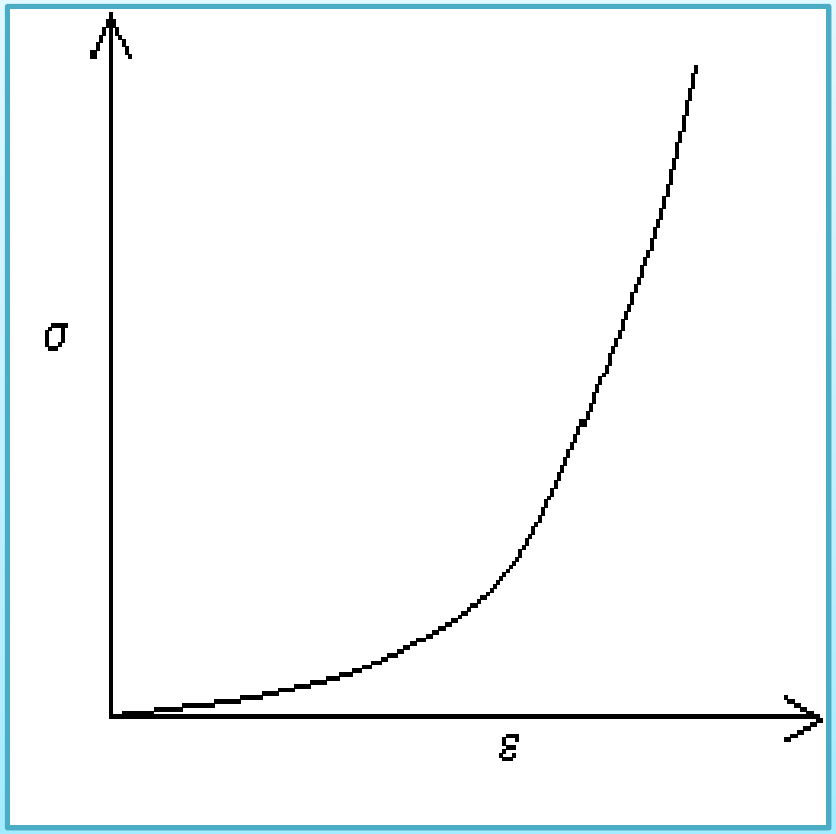
- Most of the commonly used distributions are bell shaped curves.
- The distribution of height, marks, profit etc. belongs to this category.
- On a bell shaped curves, the frequency starting from a rather low value gradually reaches the maximum value
- and then gradually decreases to reach its lowest
- value at the other extremity frequency is maximum at central part.

U-Shaped Curve



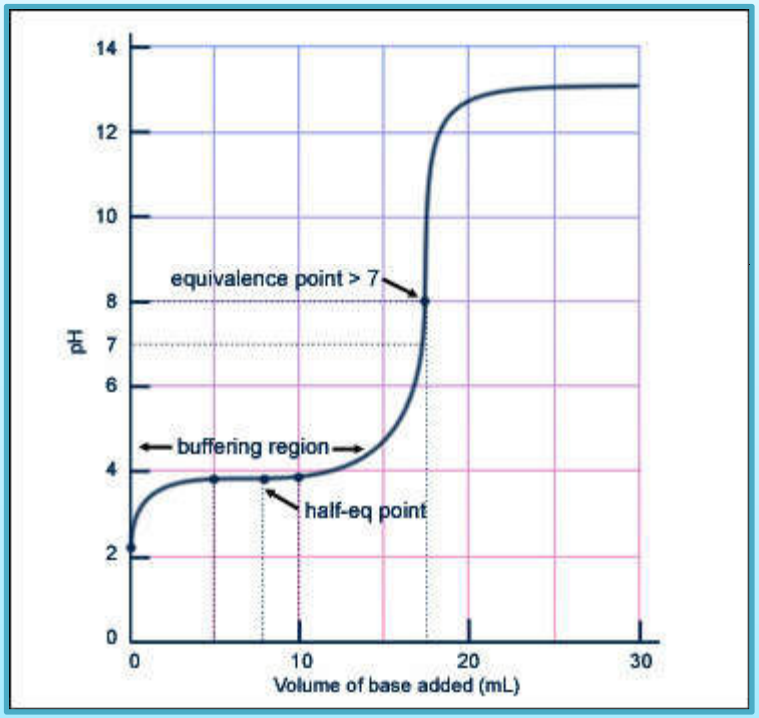
- Frequency is minimum near the central part and the frequency reaches maximum at the two extremities.
- The distribution of Kolkata bound commuters belongs to this type of curve
- as there are maximum number of commuters during the peak hours in the morning and in the evening.

J-shaped curve



- J-shaped curve starts with a minimum frequency and then gradually reaches its maximum frequency at the other extremity.
- The distribution of commuters coming to Kolkata from the early morning hour to peak morning hour follows such a distribution

Mixed Curve



- Combination of above curve is known as mixed curves.

False Base Line

- The false base line graph technique is useful from two point of views:
- (i) To magnify the minor fluctuation in time series data.
- (ii) To economic the space.

MIND MAP

SET RELATIONS AND FUNCTION

RITU JINDAL

SET

Well defined Collection of objects

Method of representing a set

- Roster form or Tabular form → Eg: {a, e, i, o, u}
- Set Builder form or Rule method → Eg: {x / x is a vowel}

Types of Set

- Unit Set → {5}
- Null Set → ϕ or {}
- Equivalent Set → {1,2,3} = {a,b,c}
- Equal Set → {1,2,3} = {2,3,1}
- Finite Set → Vowels
- Infinite Set → Natural No's

Subset

- Subset → A ⊆ B, Every element of A also an element of B
- Proper Subset → A ⊂ B, A ≠ B
- Proper Subset → A set of All possible Subset of a set

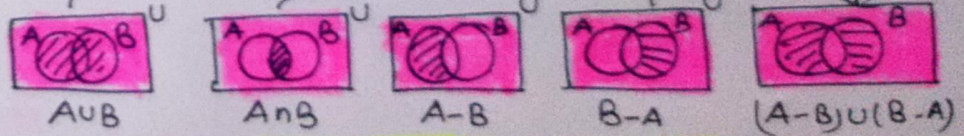
Fundamental Operations on Set

- Union → A ∪ B
- Intersection → A and B
- Compliment → $A^c = A' =$ All Element Except A
- Difference → A − B: A only; B − A: B only
- Symmetric Difference → A − B ∪ (B − A)

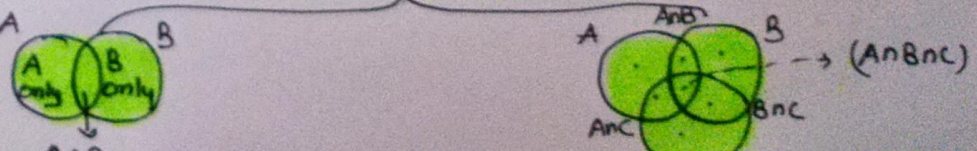
Application Based on Set

- 2 Values → $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- 3 Values → $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Venn Diagram



To Solve Numerical Problems



MIND MAP

SET RELATIONS AND FUNCTIONS

RITU JINDAL

Relation

Cartesian Product

$A=\{1,2\}$ $B=\{3,4\}$

$A\times B=\{(1,3)(1,4)(2,3)(2,4)\}$

$(1,3)\neq(3,1)$

Types of Relations

- Universal
- Void
- Identity
- Inverse

Types of Relations

- Symmetric: If aRb then bRa
- Reflexive: aRa $\forall a\in A$
- Transitive: If aRb, bRc then aRc
- Anti-Symmetric: aRb then bRa iff a=b

- Equivalence
- P.o Relation

- Domain: first element
- Range: Second element

Function

Definition

$x=\{1,2,3\}$ $y=\{4,5\}$

Every Element $x\in X$ ∃ a unique $y\in Y$

Inverse find

$y=f(x)$

$x=f^{-1}(y)$

$y^{-1}(x)$

Value Put

Eg $y=x^2+1$

$f(3)=3^2+1$

$ff(x)=f(x^2+1)$

$(x^2+1)^2+1$

Composite f^n

Eg. $f(x)=x^2+1$

$g(x)=x+1$

$fog(x)=f(g(x))$

$gof(x)=gf(x)$

Types

One One

Onto

Domain & Range

D- first element

R- Second

Types of function

- One-One
  - Into
  - Onto
- Onto
- Many One
  - Into
  - Onto
- Into

- Domain
  - # first element
  - # observation
  - # Value of x for which y defined
  - y undefined
    - $\frac{Nr}{Dr}$, Dr = 0
    - $\sqrt{\text{term}}$, -ve value
- Range
  - # Second element
  - # observation
  - # Value of y for which x defined
  - Same.
  - $y=f(x)$, $x=f(y)$, Now check x defined or Not

MIND MAP
PROBABILITY

RITU JINDAL

Two Broad Division

- Subjective
- Objective
  - Classical: $P(A) = \frac{\text{favourable}}{\text{Total}}$
  - Statistical: $P(A) = \lim_{n \to \infty} \frac{f_A}{n}$
  - Modern: $0 \le P(A) \le 1$, $P(S) = 1$, $P(A_1 \cup A_2 \cdots \cup) = P(A_1) + \cdots$

Odds

- Odds in favour: $m : n$, $P(A) = \frac{m}{m+n}$
- Odds Against: $m : n$, $P(A) = \frac{n}{m+n}$

Atleastone → 1 − No

Addition Theorem

- Mutually Exclusive: $P(A \cup B) = P(A) + P(B)$
- Non-Mutually Exclusive: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Theorem

- Dependent: $P(A \cap B) = P(A)\,P(B/A)$ or $P(A \cap B) = P(B)\,P(A/B)$
- Independent: $P(A \cap B) = P(A) \cdot P(B)$

Given, observe, Notice, known if, found

Three Prob.

- Conditional: $P(A/B) = \frac{P(A \cap B)}{P(B)}$ or $P(B/A) = \frac{P(A \cap B)}{P(A)}$
- Total Prob: $P(A) = P(E_1)P(A/E_1) + P(E_2)(P(A/E_2) \cdots P(E_n)\,P(A/E_n)$
- Baye's theo.: $P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{P(A)}$

Probability Distribution

- Mean: $\Sigma P_i x_i$
- Standard Deviation: $\sqrt{P_i x_i^2 - (\Sigma P_i x_i)^2}$
- Variance: $P_i x_i^2 - (\Sigma P_i x_i)^2$

Some Useful formulae

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A - B) = P(A) - P(A \cap B)$

$P(A - B) = P(A \cap B^c)$

$P(B - A) = P(B) - P(A \cap B)$

$P(B - A) = P(B \cap A^c)$

$P(A \cup B)^c = 1 - P(A \cup B)$

$P(A^c \cap B^c) = P(A \cup B)^c$

$P(A \cap B)^c = 1 - P(A \cap B)$

$P(A^c \cup B^c) = P(A \cap B)^c$

Important Points

Nothing mention → Consider without replacement.

With replacement → Independent

Without replacement → Dependent

(If order not given without replacement → Combination

Independent $\Leftrightarrow$ Mutually Exclusive

$P(A) = 1$ Sure event

$P(A) = 0$ Impossible event.

$0 \le P(A) \le 1$; $P(A) + P(A') = 1$

Properties of Expected Values:

$E(x+y) = E(x) + E(y)$

$E(kx) = k\,E(x)$

$E(ax+b) = a\,E(x) + b$

$E(x \cdot y) = E(x) \cdot E(y)$

$E(x-y) = E(x) - E(y)$

$E(k) = k$.

whenever x and y are Independent

MIND MAP

DIFFERENTIAL AND INTEGRAL CALCULUS

RITU JINDAL

DERIVATIVE

Important Formulae

$\frac{d}{dx}(x^n) = n x^{n-1}$

$\frac{d}{dx}(\log x) = \frac{1}{x}$

$\frac{d}{dx}(a^x) = a^x \log a$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

Rate of change of y wrt x $\frac{dy}{dx}$

$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} f'(x)$

$\frac{d}{dx}\log(f(x)) \Rightarrow \frac{1}{f(x)} f'(x)$

$\frac{d}{dx} a^{f(x)} = a^{f(x)} \log f(x)$

$\frac{d}{dx}[c f(x)] = c \frac{d}{dx} f(x)$

$\frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x)$

$\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}} f'(x)$

$\frac{d}{dx}\frac{1}{f(x)} = \frac{-1}{(f(x))^2} f'(x)$

Basic Laws of Differentiation

$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Function

Implicit function

we cannot explicitly.

$\frac{dy}{dx} = -\frac{w.r.t\ x}{w.r.t\ y}$

w.r.t x ⇒ treated y as Cons.

w.r.t y = treated x as Cons

Parametric function

$x = f(t)$ $y = f(t)$

$\frac{dx}{dt} = ?$ $\frac{dy}{dt} = ?$

$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$

Types of Questions

$y = f^{unction}_{unction}$

Taking log both sides

- Term Ek taraf Simple Solve
- Implicit = Implicit Shortcut

Higher Order Derivative

- Higher Order derivative exist if value = 0
- Parametric Second derivative me $\frac{d^2y}{dx^2} = \frac{dt}{dx}$

Gradient
Slope of tangent

⇐

Application Maxima or Minima

$y(x) =$ . .

$f'(x) =$ Now $f'(x) = 0$

Critical points say a, b, c

$f''(x)$

$f''(a) \cdots$ < Maxima

$f''(-) >$ Minima

MIND MAP

DIFFERENTIAL AND INTEGRAL CALCULUS

RITU JINDAL

INTEGRATION

Basic Formulae

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int a^x dx = \frac{a^x}{\log a} + C$$

$$\int \frac{1}{x} dx = \log x + C$$

$$\int e^x dx = e^x + C$$

$$\int 1 \cdot dx = x$$

Reverse Process of derivative

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C$$

$$\int a^{bx+c} dx = \frac{a^{bx+c}}{b \log a} + C$$

$$\int \frac{1}{ax+b} dx = \frac{\log(ax+b)}{a} + C$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$$

Types of Questions

$\int \frac{f'(x)}{f(x)} dx$ → Anywhere $f'(x)$ Ignore Treated f^n as x

Two linear factors $\int \frac{1}{(x-a)(x-b)} dx =$ [A] $\log(x-a)$ + [B] $\log(x-b) + C$

By Parts $\int u \cdot v \, dx = u \int v \, dx - \int \frac{du}{dx} \left(\int v \, dx \right) dx$

Some Standard Formulae

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \frac{x-a}{x+a} + C$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \frac{a+x}{a-x} + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \log \left| x + \sqrt{x^2+a^2} \right| + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \log \left| x + \sqrt{x^2-a^2} \right| + C$$

$$\int \sqrt{x^2+a^2} \, dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2+a^2} \right| + C$$

$$\int \sqrt{x^2-a^2} \, dx = \frac{x}{2} \sqrt{x^2+a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2-a^2} \right| + C$$

Definite Integral

$\int_a^b f(x) dx = F(x) \Big|_a^b$ $F(b) - F(a)$

$\int_a^b \frac{f(x) \, dx}{f(x) + f(a+b-x)}$ $= \frac{b-a}{2}$

$\int_{-a}^{a} f(x) dx = 0$ $f(x)$ odd

$$\int e^x \left[f(x) + f'(x) \right] dx \Rightarrow e^x f(x) + C$$

MIND MAP

INDEX NUMBER

RITU JINDAL

- Price
- Quantity
- Value

Simple

Aggregative

$$\frac{\Sigma P_1}{\Sigma P_0} \times 100$$

Relative

$$\frac{\Sigma \frac{P_1}{P_0} \times 100}{n}$$

Weighted

$$\frac{\Sigma \frac{P_1}{P_0}(P_0 Q_0)}{\Sigma P_0 Q_0} \times 100$$

Relative

Aggregative

Laspeyre

$$\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$$

Base year Quantity

Paasche

Current year Quantity

$$\frac{\Sigma P_1 \cdot Q_1}{\Sigma P_0 \cdot Q_1} \times 100$$

Fisher

G.M of Laspeyre & Paasche

$$\sqrt{I_L \times I_P}$$

Marshall Edgeworth

Base year + current year Quantity

$$\frac{\Sigma P_1 (Q_0 + Q_1)}{\Sigma P_0 (Q_0 + Q_1)} \times 100$$

Bowley & Dorbish

A.M of Laspeyre & Paasche

$$\frac{I_L + I_P}{2}$$

Test of Adequacy

Unit test

Unit free

Time Reversal

$$P_{01} P_{10} = 1$$

Factor Reversal

$$P_{01} Q_{01} = V_{01}$$

Circular

$$I_{01} I_{12} I_{20} = 1$$

Cost of Living Index

$$\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$$

Consumer Price Index

Chain Index

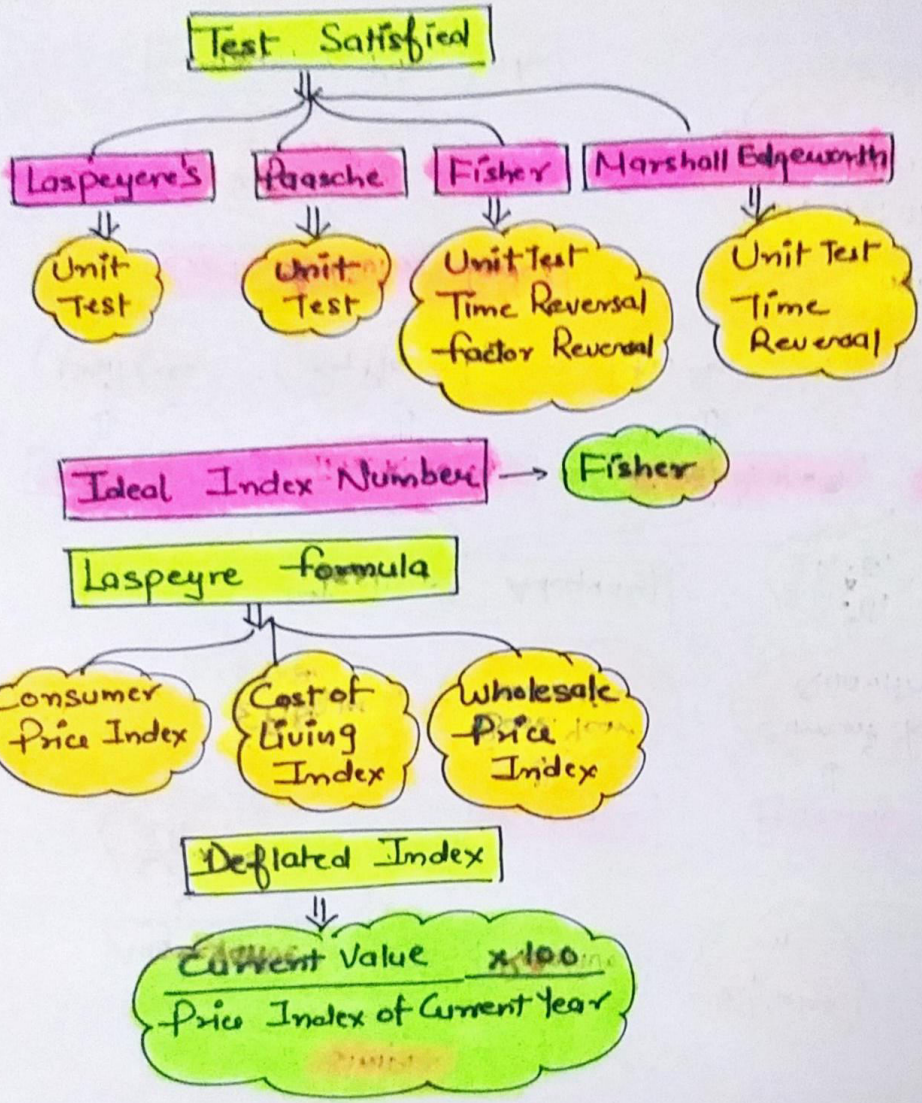
$$\frac{\text{link relative of current year} \times \text{chain index of previous year}}{100}$$

Shifted Price Index

$$\frac{\text{Original price Index} \times 100}{\text{Price Index of year on which it has to be shifted}}$$

Link Relative Index

$$\frac{P_n}{P_{n-1}} \times 100$$


Test Satisfied
Laspeyere's
Paasche
Fisher
Marshall Edgeworth
Unit Test
Unit Test
Unit Test
Time Reversal
factor Reversal
Unit Test
Time Reversal
Ideal Index Number
Fisher
Laspeyre formula
Consumer Price Index
Cost of Living Index
Wholesale Price Index
Deflated Index
Current Value x 100
Price Index of Current Year

MIND MAP

THEORETICAL DISTRIBUTIONS OF DATA

RITU JINDAL

Binomial Distribution

$$P(X=r) = {}^{n}C_r\, p^r q^{n-r}$$
$$r = 0, 1, 2 \cdots n$$

Properties

- Mean = np
- Variance = npq
- S.D = $\sqrt{npq}$
- Mean > Variance
- Maximum Variance at $p = q = .5$
- Maximum Value = $n/4$
- Symmetrical when $p = q = \frac{1}{2}$
- Positively skewed $p < .5$
- Negatively skewed $p > .5$
- Mode = $(n+1)p$
  Integer k $(k-1)$
  Non-Integer Integral
- Uni-modal or Bimodal
- Biparametric, n, p

Poisson Distribution

$$P(X=r) = \frac{e^{-m} \cdot m^r}{r!}$$
$$r = 0, 1, 2 \cdots$$

Properties

- $n \to \infty$ $p \to 0$ $q \to 1$
  $np \to$ finite
- $np = m$ mean
- Variance = m
- S.D = $\sqrt{m}$
- Mean = Variance
- Uni-parametric m
- Always positively skewed
- m Integer k $k-1$
  m Non-Integer → Integral Part Mode
- Uni-modal or Bimodal
- Fitting by method of moments
  (Binomial and Poisson)

Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
$$-\infty < x < \infty$$

Properties

- Symmetrical and Bell shaped
- Symmetrical about Mean
- Mean = Median = Mode
- Unimodal
- Bi-parametric μ, σ^2
- Fitting (i) Ordinate (ii) Area
- Point of Inflexion $\bar{x} \pm \sigma$
- Total Area = 1
- $\mu \pm \sigma$ = 68.27%
  (34.135%) either side
- $\mu \pm 2\sigma$ = 95.45%
  (47.72%) either side
- $\mu \pm 3\sigma$ = 99.73%
  (49.87%) either side
- QD : MD : SD = 10 : 12 : 15
- $Z = \frac{x - \bar{x}}{\sigma}$ $\bar{x} = 0$ $\sigma = 1$
- SNV Point of Inflexion -1 and 1
- $Q_1 = \mu - .675\sigma$; $Q_3 = \mu + .675\sigma$