Central Tendency

Meaning		Central Tendency is the tendency of a given set of observations to cluster around a single central or middle value and the single value that represents the given set of observations is described as a measure of central tendency or, location, or average.
Arithmetic Mean	Definition	the sum of all the observations divided by the number of observations
	Formula for discrete distribution	$\bar{x} = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$ or $\frac{\Sigma x}{n}$
	Formula for frequency distribution	$\bar{x} = \frac{\Sigma fx}{N}$ $N = \Sigma f$, x = mid-point in case of grouped frequency distribution
	Deviation Method	$\bar{x} = A + \frac{\Sigma fd}{N} \times C$, where $d = \frac{(x-A)}{C}$ A = assumed mean, C = class length
	Properties	→ If all the observations are constant, AM is also constant → the algebraic sum of deviations of a set of observations from their AM is zero → AM is affected both due to change of origin and scale → Combined Mean: $\bar{x}_c = \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$
Median (one of the partition values)	Definition	the middle-most value when the observations are arranged either in an ascending order or a descending order of magnitude
	For Discrete Distribution	Step 1: Arrange data in ascending (or descending) order Step 2: Use the formula $\left[\frac{n+1}{2}\right]^{th}$ term
	For Frequency Distribution *(refer example 15.1.6 – Page 15.8 Study Mat)*	**Step 1**: Prepare a less than type cumulative frequency distribution with Class boundaries as base. **Step 2:** Calculate N/2 and check between which class boundaries it falls. Mark LCB as l_1 and l_2 and corresponding cumulative FD as N_l and N_u **Step 3**: Apply the below formula $Me = l_1 + \left[\frac{\frac{N}{2} - N_l}{N_u - N_l}\right] \times Class\ length$
	Properties	→ Median is affected by both change of origin and scale → For a set of observations, the sum of absolute deviations is minimum, when the deviations are taken from the median.

Partition Values

Meaning	values dividing a given set of observations into a number of equal parts
Median	Median is also a quartile that divides the set of observations into two equal parts.

Quartiles

Number of equal parts	Four (4)
Number of Quartiles	Three (3)
Denoted by	Q_1, Q_2, Q_3

Deciles

Number of equal parts	Ten (10)
Number of Deciles	Nine (9)
Denoted by	$D_1, D_2, D_3, \dots, D_9$

Percentiles

Number of equal parts	Hundred (100)
Number of Percentiles	Ninety Nine (99)
Denoted by	$P_1, P_2, P_3, \dots, P_{99}$

How to calculate Partition Values

P^{th} Quartile	$(n+1)P^{th} term,$ $here\ p = \frac{1}{4}, \frac{2}{4}, \frac{3}{4},$
P^{th} Decile	$(n+1)P^{th} term,$ $here\ p = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \dots, \frac{9}{10}$
P^{th} Percentile	$(n+1)P^{th} term,$ $here\ p = \frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \dots, \frac{99}{100}$

Mode

Definition	Mode is the value that occurs the maximum number of times.
Type of Mode	A distribution can be uni-modal, bi-modal or multi-modal
For Frequency Distribution	$Mode = l_1 + \left[\frac{f_0 - f_{-l}}{2f_0 - f_{-l} - f_1}\right] \times Class\ length$ Here, $f_0 = frequency\ of\ the\ modal\ class,$ $f_{-1} = frequency\ of\ pre-modal\ class,$ $f_1 = frequency\ of\ the\ post\ modal\ class$

Empirical Relationship

For a moderately skewed distribution,

$$Mean - Mode = 3 \times (Mean - Median)$$

Geometric Mean

Definition	For a given set of n positive observations, the geometric mean is defined as the n^{th} root of the product of the observations
Formula	$G = (x_1 \times x_2 \times x_3 \dots \times x_n)^{1/n}$
Properties	→ $\log G = \frac{1}{n} \Sigma \log x$ → If all observations are constant GM is also constant → $GM\ of\ xy = GM\ of\ x \times GM\ of\ y$ → $GM\ of\ \frac{x}{y} = \frac{GM\ of x}{GM\ of\ y}$

Harmonic Mean	Definition	For a given set of non-zero observations, harmonic mean is defined as the reciprocal of the AM of the reciprocals of the observation
	Formula	$H = \frac{n}{\Sigma(1/x)}$
	Properties	→ If all observations are constant HM is also constant → Combined HM: $\bar{x}_c = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$
When to use GM and HM	In case of rates like speed, hours per day, etc.	HM is used
	In case of % and ratios	GM is used
Relationship between AM, GM and HM	General	AM ≥ GM ≥ HM
	When all the observations are same	AM = GM = HM
	When all the observations are distinct	AM > GM > HM
Ideal Measure of Central Tendency	Best Measure – Overall	AM
	Best Measure for Open End Class	Median
	Based on all observations	AM, GM, HM
	Based on 50% values	Median
	Not affected by Sampling fluctuations	Median
	Rigidly defined, easy to comprehend	AM, Median, GM, HM
	No Mathematical Property	Mode

Dispersion

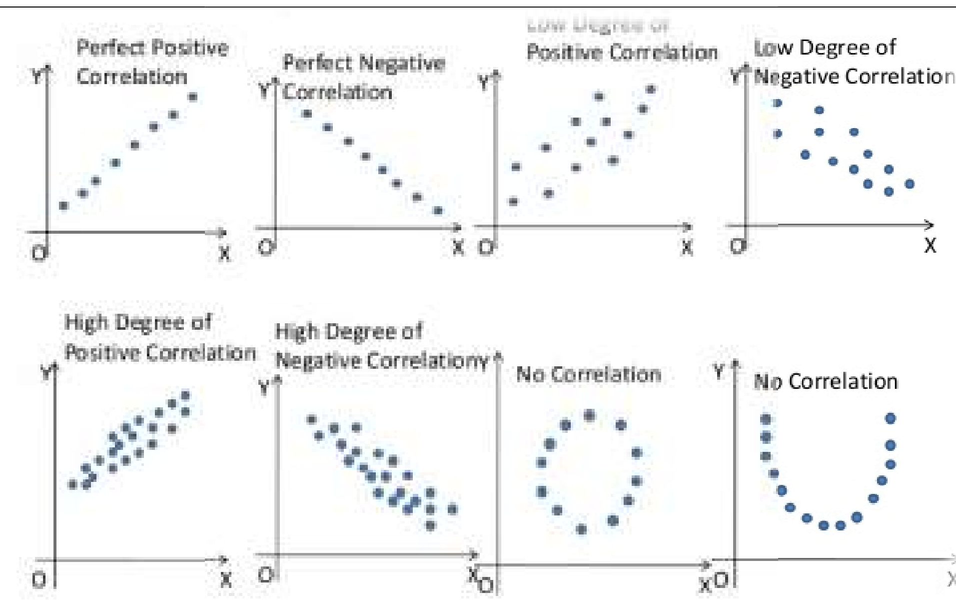
Definition		Dispersion for a given set of observations may be defined as the amount of deviation of the observations, usually, from an appropriate measure of central tendency
Types of Dispersion	Absolute Measures of Dispersion	These are with units and not useful for comparison of two variables with different units. Example: Range, Mean Deviation, Standard Deviation, Quartile Deviation
	Relative Measures of Dispersion	These are unit free measures and useful for comparison of two variables with different units. Example: Coefficient of Range, Coefficient of Mean Deviation, Coefficient of variation, Coefficient of Quartile Deviation
Range	Definition	Difference between the largest and smallest of observations.
	Formula	Range = L – S
	Relative Measure	Coefficient of Range = $\frac{L-S}{L+S} \times 100$
	Properties	→ No effect of change of origin but affected by change of scale in the magnitude (ignore sign).
Mean Deviation	Definition	Mean deviation is defined as the arithmetic mean of the absolute deviations of the observations from an appropriate measure of central tendency
	Formula	$MD_A = \frac{1}{n}\Sigma\lvert x-A\rvert$ Here A is mean or median as given in question
	Relative Measure	$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Deviation about A}}{\text{A}} \times 100$
	Properties	→ No effect of change of origin but affected by change of scale in the magnitude (ignore sign).
Standard Deviation	Definition	It is defined as the root mean square deviation when the deviations are taken from the AM of the observations
	Formula	$SD_x \text{ or } \sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{\text{n}}} \text{ or } \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$
	Relative Measure	$\text{Coefficient of Variation} = \frac{\text{SD}}{\text{AM}} \times 100$
	Standard Result	For any two numbers, a and b: $\text{SD} = \frac{\lvert a-b\rvert}{2}$ SD of first n natural numbers: $\sqrt{\frac{(n^2-1)}{12}}$
	Properties of SD	→ If all the observations are constant, SD is Zero → No effect of change of origin but affected by change of scale in the magnitude (ignore sign) → Combined SD $= \sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2 + n_1 d_1{}^2 + n_2 d_2{}^2}{n_1+n_2}}$

Quartile Deviation	Definition	It is defined as the semi-inter quartile range
	Formula	$Q_d = \frac{Q_3 - Q_1}{2}$
	Relative Measure	Coefficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

Ideal Measure of Dispersion

Best Measure – Overall	SD
Best Measure for Open End Class	QD
Quickest to compute	Range
Not based on all observations	Range
Difficult to comprehend and less Mathematical	Mean Deviation
Rigidly defined, easy to comprehend	Mean Deviation, SD, QD
Not affected by Sampling fluctuations	QD

CORRELATION

<table>
<tr><td>Bi-Variate Data</td><td>When data are collected on two discrete variables simultaneously, they are known as Bi-Variate data</td></tr>
<tr><td>Bi-Variate Distribution</td><td>Distribution of Bi-Variate data is called as Bivariate Distribution</td></tr>
<tr><td>Bi-Variate Frequency Distribution</td><td>
<table>
<tr><td>Meaning</td><td>Frequency distribution involving two discrete variables.</td></tr>
<tr><td>Marginal Distribution</td><td>If we make a separate distribution from bi-variate frequency distribution where we take aggregate of only one variable at a time. Total no. of marginal distributions = 2</td></tr>
<tr><td>Conditional Distribution</td><td>If we make a separate distribution from bi-variate frequency distribution where we take one variable related one class interval of another variable. Total no. of conditional distributions = m + n (m = no. of rows, n = no. of columns)</td></tr>
</table>
</td></tr>
<tr><td>Correlation</td><td>While studying two variables at the same time, if it is found that the change in one variable leads to change in the other variable either directly or inversely, then the two variables are known to be associated or correlated.
<table>
<tr><td>Positive Correlation</td><td>If two variables move in the same direction</td></tr>
<tr><td>Negative Correlation</td><td>If two variables move in the opposite direction</td></tr>
<tr><td>No Correlation</td><td>If no change due to each other</td></tr>
</table>
</td></tr>
<tr><td>Measure of Correlation</td><td>A measurement or formula that represents the nature/ direction and/or magnitude of correlation.
<table>
<tr><th>Method</th><th>Helps in obtaining</th></tr>
<tr><td>Scatter Diagram</td><td>Only direction of correlation</td></tr>
<tr><td>Karl Pearson's Product moment correlation coefficient</td><td>Direction as well as strength of correlation. Best Method – Most accurate</td></tr>
<tr><td>Spearman's rank correlation co-efficient</td><td>Direction as well as strength of correlation. Useful for attributes.</td></tr>
<tr><td>Co-efficient of concurrent deviations</td><td>Direction as well as strength of correlation. Only preferred for direction and not magnitude. Quickest method.</td></tr>
</table>
</td></tr>
<tr><td>Scatter Diagram</td><td>
</td></tr>
</table>

Topic	Item	Details
Karl Pearson's Product moment correlation coefficient	Defined as	the ratio of covariance between the two variables to the product of the standard deviations of the two variables
	Main Formula	$r_{xy} = \frac{Cov(x,y)}{\sigma_x.\sigma_y}$
	Formula for Covariance	$Cov(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$ or $\frac{\Sigma xy}{n} - \bar{x}.\bar{y}$
	Formula for Standard Deviation σ_x or σ_y	$\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$ or $\sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$
	Properties	→ It is a unit-free measurement → Value of r lies from -1 to +1 both inclusive → Change of origin or Scale Change of Origin: No impact Change of Scale: No impact of value, but if change of scale of both variables are of different sign then sign r will also change
Spearman's Rank Correlation coefficient	Applied to	find the level of agreement (or disagreement) between two judges so far as assessing a qualitative characteristic is concerned
	Main Formula	$r_R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$, here d means difference in ranks
	Adjustment Value in case of Tie Rank	$\Sigma \frac{(t^3-t)}{12}$ here t is a tie length and we need to do summation of all ties
	Formula in case of Tie length	$r_R = 1 - \frac{6(\Sigma d^2 + \text{value of adjustment})}{n(n^2-1)}$
Co-efficient of concurrent deviations	Use	A very simple and casual method of finding correlation when we are not serious about the magnitude of the two variables
	Steps in this method	This method involves in attaching a positive sign for a x-value (except the first) if this value is more than the previous value and assigning a negative value if this value is less than the previous value. Applies to both variable and then these signs are compared. If signs match – pair is counted as concurrent deviation.
	Formula	$r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$ Here, *m = total no. of deviations (it is one less than total no. of pairs under observation i.e m=n-1), c = no. of concurrent deviations, r_c also lies between -1 and 1 incl.*

REGRESSION

Regression Analysis	Estimation of one variable for a given value of another variable on the basis of an average mathematical relationship between the two variables	
Estimation of Y (when it is dependent on X)	Line	Regression line of Y on X
	Regression Coefficient	Regression Coefficient of Y on X denoted by $\boldsymbol{b_{yx}}$
	Form	$\boldsymbol{Y - \bar{Y} = b_{yx}\,(X - \bar{X})}$, $\bar{X}$ *and* $\bar{Y}$ *are means of X series and Y series*
Estimation of X (when it is dependent on Y)	Line	Regression line of X on Y
	Regression Coefficient	Regression Coefficient of X on Y denoted by $\boldsymbol{b_{xy}}$
	Form	$\boldsymbol{X - \bar{X} = b_{xy}\,(Y - \bar{Y})}$, $\bar{X}$ *and* $\bar{Y}$ *are means of X series and Y series*
Important Theory Points	When linear relationship exists between two variables (i.e. correlation is perfect, $r_{xy} = -1\ or + 1$)	The linear equation so arrived can be used both ways for Y on X and X on Y. It means regression lines are identical.
	When no linear relationship exist between two variables	In that case, we need to estimate the regression lines with the help of Method of Least Squares
	To derive regression line of y on x	The minimisation of vertical distances in the scatter diagram is to be done
	To derive regression line of x on y	The minimisation of horizontal distances in the scatter diagram is to be done
Regression Coefficient	Defined as the ratio of	$\frac{\text{Covariance between two variables}}{\text{Variance of Independent variable}}$
	Regression Coefficient of Y on X	$b_{yx} = r.\frac{\sigma_y}{\sigma_x}$ or $b_{yx} = \frac{Cov(x,y)}{{\sigma_x}^2}$
	Regression Coefficient of X on Y	$b_{xy} = r.\frac{\sigma_x}{\sigma_y}$ or $b_{xy} = \frac{Cov(x,y)}{{\sigma_y}^2}$
	$\boldsymbol{r}$ used here is Karl Pearson's Correlation Coefficient	
Properties of Regression lines and coefficient	Change of origin	The regression coefficients remain unchanged
	Change of scale	: If original pair is X, Y and modified pair is U, V where U = $\frac{X-m}{p}$ and V = $\frac{Y-n}{q}$, then $b_{vu} = b_{yx}\frac{q}{p},\ b_{uv} = b_{xy}\ \frac{p}{q}$
	Intersection of two regression lines	Two regression (if not identical) will intersect at the point $(\bar{x}, \bar{y})$ [means]
	Relation between correlation and regression coefficients	$r = \pm\sqrt{\pm b_{xy} \times b_{yx}}$; b_{xy} , b_{yx} *and* r all will have same sign

Coefficient of Determination	Coefficient of Determination	r^2 (square of correlation coefficient)
	Interpretation of value of r^2	It explains the percentage of variation in dependent variable due to variation in independent variable
	Example: if $r_{xy} = 0.8, then\ r^2 = 0.64$	It means 64% of variation in ***x*** is due to variation in **y** and remaining 36% due to other factors. It shows the reliability of correlation coefficient.
Probable Error	Formula	Probable Error [P.E] = $^2/_3$ × Standard Error [S.E.]
	Standard Error	$\frac{1-r^2}{\sqrt{n}}$
	Use	Probable Error is used to test the reliability of ***r***
	Test	If r is less than PE: The value of r is not significant. Not reliable If r is greater than six times of PE: The value of r is significant and there is evidence of correlation

TIME VALUE OF MONEY

Basics

→ The sum of money received in future is less valuable than it is today

→ Rs. 100 Note given today is more valuable than Rs. 100 note given a year later due to various reasons:

Risk Factor	Risk that payer will not give money
Liquidity Preference	Cash given today will be immediately available for spending, hence more valuable
Inflation	In general, as the time goes on purchasing power of the money gets reduced
Opportunity Cost	Cash given today could be invested to a better investment that could appreciate its value

Partied involved in Financial Transaction

Name of Parties	Treatment of Interest
Lender	Income
Borrower	Expense
Investor	Income
Investee	Expense

Simple Interest

Formula	$S.I. = \frac{P.r.t}{100}$
P	Principal means amount of money invested or loan taken
r	Rate of simple interest per annum
t	Time of loan / investment in years
Accumulated Amount under SI	Amount under SI = Principal + Simple Interest *(amount is also called as Balance)*

Compound Interest vs. Simple Interest

Simple Interest	Compound Interest
→ Interest earned is withdrawn every time it is earned → No re-investment of interest earned in earlier periods → Amount includes Principal and Interest on that Principal	→ Interest earned is not withdrawn till maturity → Re-investment of interest earned will be done → Amount includes Principal and Interest on that Principal and interest on interest earned in the earlier periods

Effective Rate of Interest

Meaning	The rate of interest stated in question does not always mean that effectively interest charged/ received will be same % when compared at annual level. Effectiveness depends on Compounding.
Higher the compounding for a rate of interest	Higher the effective rate for the year
Formula	$E = [(1 + i)^n - 1]$
n	here n means no. of periods in one years considering the compounding

Compound Interest

Compounding Frequency and Conversion Periods	It means no. of times interest is compounded in a year or no. of conversions in a year. Compounding means calculation of interest by bank. For e.g. (see table below) While calculating compound interest, we need to adjust interest rate and time period using compounding frequency.

Conversion Period	Compounding Frequency
Yearly	1
Half-yearly	2
Quarterly	4
Monthly	12
Daily	365

Formula for Accumulated Amount of CI	$\boldsymbol{A = P(1+i)^n}$
A	Accumulated amount as per CI
P	Principal means amount of money invested or loan taken
i	Interest rate (adjusted as per compounding) e.g. If rate of interest given is *r=10%* and if compounding is half-yearly, $i = \frac{10\%}{2} = 5\% = 0.05$
n	It means no. of periods (not necessarily no. of years). It depends on type of compounding. E.g. if compounding is quarterly and *t = 2 years,* it means we will have 2×4 = 8 no. of periods. *n=8*
Shortcut in calculator to calculate amount	Example: P=1000, *i* = 10%, n=3 then *Calculator Steps*: *Write P i.e* [1000] *then press* [+] [10] [%] [+] [10] [%] [+] [10] [%] *(three times because n=3)*
Direct Formula of Amount in Calculator	Example: P=1000, *i* = 10% = 0.1, n=3 then *Calculator Steps*: [1 + 0.1] [×] [=] [=] *(first equal will be considered as power 2, second as 3 and so on)* [×] 1000 (Principal)
How to calculate CI?	$A = P + CI \Rightarrow CI = A - P$ $CI = P(1+i)^n - P$ $CI = P[(1+i)^n - 1]$

Annuity

Definition	→ Sequence of periodic payments (installment) → Same amount → Regularly → For a specified period of time
Annuity Regular	Installment commencing from the end of the period
Annuity Due	Installment commencing from the beginning of the period

Future Value	Future value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest.
Present Value	Present value is today's value of tomorrow's money discounted at the interest rate.

Cashflow

Single cash flow	Meaning	Payment / Receipt one time at the beginning. No other payment/ receipt till maturity
	Formula of Future Value	$FV = PV\ (1+i)^n$
	Formula for Present Value	$PV = \frac{FV}{(1+i)^n}$
	Remark	Both the above formulas are similar to formula of Amount of compound interest. Principal is taken as PV and Amount is taken FV
Future value of Annuity	Formula for FV of Annuity Regular	$FVA = A_I \times [FVAF(n,i)]$ $FVA = A_I\left[\frac{(1+i)^n - 1}{i}\right]$ A_I = amount of installment or Annuity
	Formula for FV of Annuity Due	$FVA\ Due = FVA \times (1+i)$ Calculate FVA regular normally and then multiply it by $(1+i)$
Present Value of Annuity	Formula for PV of Annuity Regular	$PVA = A_I \times [PVAF(n,i)]$ $\boldsymbol{PVA = A_I\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$ or $\boldsymbol{PVA = \frac{A_I}{i}\left[1 - \frac{1}{(1+i)^n}\right]}$ A_I = amount of installment or Annuity
	Formula for PV of Annuity Due	PVA Regular for one shorter period + Initial Cashflow
	Calculator Trick of PVAF (Present Value Annuity Factor)	$\boxed{1+i}\ \boxed{\div}\ \boxed{=}\ \boxed{=}$ …… *n times* $\boxed{\text{GT}}$

Applications of Time Value of Money

Particulars	Application	Remark
Sinking Fund	Future Value of Annuity is the amount which is required in future and annuity amounts are the regular savings required for creation of fund	Sinking fund means a fund created for specific purpose where a big amount of money is required at any specific point in future. An annuity is set aside and invested so that it will mature on that specific date giving the required amount.
Leasing	Present Value of Annuity (Lease Rentals) are compared with Asset Cash down price	Lessor: Owner of Asset, who gives asset on rent. Lease Rentals are income for Lessor Lessee: User of the asset who has taken asset on rent. Lease Rentals are expense for Lessee
Capital Expenditure or Investment Decision	Present value of savings and benefits are compared with purchase value of asset, to decide whether asset to purchase or not	Capital Expenditure: Expenditure on capital assets in anticipation of future benefits Future Benefits: Contribution from sales and other benefits or savings derived from a capital investment
Valuation of Bond	Present value of interest income and maturity value is compared with the issue price of bond	Bond: It is a debt security. Type of loan taken by company from public. Like debentures Face Value: Value written on the document of bond. This value is used to calculate Interest Amount Issue Price: Actual payment made to purchase the bond Maturity value: Amount to be received on redemption or maturity of bond

Perpetuity

Meaning	An annuity that continues till infinite period of time is called as Perpetuity.
Formula Perpetuity	Present Value of Perpetuity = $\frac{A_I}{i}$
Formula Growing Perpetuity	Present Value of Growing Perpetuity = $\frac{A_I}{(i-g)}$ *g is constant growth rate*

Net Present Value	NPV = Present Value of Cash Inflows – Present Value of Cash Outflows If NPV ≥ 0, accept the proposal, If NPV < 0, reject the proposal
Nominal Rate of Return	Real Rate of Return = Nominal Rate of Return – Rate of Inflation
CAGR	Compounded Annual Growth rate is the interest rate we used in Compound Interest. It is used to see returns on investment on yearly basis

RATIO

Meaning of Ratio	Division of two quantities a and b of same units. Denoted by a:b
Inverse Ratio	b:a is inverse ratio of a:b
Compound Ratio	Compound ratio of a:b and c:d is ac:bd
Duplicate Ratio	Duplicate ratio of a:b is $a^2:b^2$
Sub-duplicate Ratio	Duplicate ratio of a:b is $\sqrt[2]{a}:\sqrt[2]{b}$
Triplicate Ratio	Triplicate ratio of a:b is $a^3:b^3$
Sub-triplicate Ratio	Triplicate ratio of a:b is $\sqrt[3]{a}:\sqrt[3]{b}$
Commensurate	If ratio can be expressed in the form of integers
Incommensurate	If ratio cannot be expressed in the form of integers
Continued Ratio	Ratio of three or more quantities e.g. a:b:c

PROPORTION

Proportion	a,b,c,d are in proportion if a:b = c:d [it is an equality of two ratios]
Term/ Proportional	first = a, second = b, third =c, fourth = d
Mean Proportional	In a continued proportion a:b=b:c, b^2=ac, b is called mean proportional
Cross Product Rule	If a:b=c:d, then ad = bc
Invertendo	If a:b=c:d, then b:a = d:c
Alternendo	If a:b=c:d, then a:c = b:d
Componendo	If a:b=c:d, then (a+b):b = (c+d):d
Dividendo	If a:b=c:d, then (a−b):b = (c−d):d
Componendo and Dividendo	If a:b=c:d, then (a+b):(a−b) = (c+d):(c−d) or (a−b):(a+b) = (c−d):(c+d)
Addendo	If a:b = c:d = e:f = ……. = k, then also (a+c+e+……):(b+d+f+….) = k

INDICES

Index / Indices	Here in 4^2, 4 is base and 2 is power or index. Plural of index is indices
Basic 1	$a^0 = 1$, any number raised to power zero equals to 1
Basic 2	$\sqrt{a} = a^{1/2}$, $\sqrt[3]{a} = a^{1/3}$
Law 1	$a^m \times a^n = a^{(m+n)}$
Law 2	$a^m / a^n = a^{(m-n)}$
Law 3	$a^{(m)^n} = a^{m\times n} = (a^m)^n$
Law 4	$(ab)^n = a^n b^n$

LOG

Basic	If 2^4=16 [2 is base, 4 is power], then $\log_2 16 = 4$ (i.e log of 16 base 2)
How to remember?	2 should be raised to what power so that it becomes 16 2 ka kitna power karne wo 16 ho jaye, ans is 4
Standard Result	$\log_a a = 1, \log_a 1 = 0$
Law 1	$\log_a(mn) = \log_a m + \log_a n$
Law 2	$\log_a(\frac{m}{n}) = \log_a m - \log_a n$
Law 3	$\log_a m^n = n \log_a m$
Change of Base	$\log_b m = \frac{\log_a m}{\log_a b}$

EQUATIONS - BASICS

Equation Means	mathematical statement of equality
Identity Equation	If equality is true for all the values of variable, ex. $2x + 3 = x + x + 3$
Conditional Equation	If the equality is true for certain value of the variable ex. $2x + 1 = 3$
Solution or Root	It is the value of variable that satisfies the equation
Degree	Highest power of variable in equation

SIMPLE EQUATION

Type	Linear equation with one unknown	Linear equation with two unknowns	Quadratic Equation	Cubic Equation
Form	$ax + b = 0$, where a and b are constants	$ax + by + c = 0$ a,b,c are constants	$ax^2 + bx + c = 0$ a,b,c are constants with a≠0	$ax^3 + bx^2 + cx + d = 0$
Degree	1 (One)	1	2	3
Roots	1 (One)	1 each for both	2 (α, β)	3
Remarks	NA	Need minimum two equations to get roots	Trial Error/ Formula based	Trial and Error
Methods for solution	NA	1. Elimination 2. Cross Multiplication	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	NA

LINEAR EQUATIONS WITH TWO UNKNOWNS

Elimination	Eliminate one variable by algebraic operations on given equations, and then calculate the value of variable that remains. Using this value, find out the value of other root.
Cross Multiplication	$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ Solution is given by: $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

QUADRATIC EQUATION

<table>
<tr><td>Formula</td><td>$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</td></tr>
<tr><td>Sum of Roots</td><td>$\alpha + \beta = -\frac{b}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$</td></tr>
<tr><td>Product of Roots</td><td>$\alpha \times \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$</td></tr>
<tr><td>How to construct a quadratic equation</td><td>$x^2 - (\text{sum of roots: } \alpha + \beta)x + \text{Product of Roots: } \alpha \times \beta = 0$</td></tr>
<tr><td>Nature of Roots</td><td>
<table>
<tr><th>Condition</th><th>Nature of Roots</th></tr>
<tr><td>$b^2 - ac = 0$</td><td>Real and Equal (α=β)</td></tr>
<tr><td>$b^2 - ac > 0$</td><td>Real and Unequal</td></tr>
<tr><td>$b^2 - ac < 0$</td><td>Imaginary</td></tr>
<tr><td>$b^2 - ac$ is a perfect square</td><td>Real, Unequal and Rational</td></tr>
<tr><td>$b^2 - ac > 0$ but not perfect square</td><td>Real, Unequal and Irrational</td></tr>
</table>
</td></tr>
<tr><td>Irrational Roots</td><td>If one root is $(m + \sqrt{n})$, then other root will be $(m - \sqrt{n})$</td></tr>
</table>

MATRICES

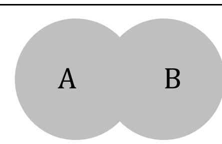
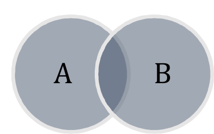
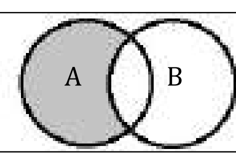
Matrix	A rectangular array of numbers (real/complex) with m rows and n columns
Order of Matrix	Order is m × n where m= no. of rows and n = no. of columns
Row Matrix	Matrix having only one row $\begin{bmatrix}1 & 4 & 2\end{bmatrix}$
Column Matrix	Matrix having only one column $\begin{bmatrix}1\\4\\2\end{bmatrix}$
Zero/ Null Matrix	If all the elements of matrix (any order) are zero $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
Square Matrix	If in a matrix, no. of columns = no. of rows $\begin{bmatrix}1 & 3\\9 & 2\end{bmatrix}$
Rectangular Matrix	If in a matrix, no. of columns ≠ no. of rows $\begin{bmatrix}1 & 3 & 2\\9 & 2 & 5\end{bmatrix}$
Leading Diagonal	Diagonal elements starting from top left to bottom right
Diagonal Matrix	A square matrix where all the elements except leading diagonal elements are zero. $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$
Scalar Matrix	A diagonal square matrix where all the leading elements are equal $\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$
Unit Matrix	A scalar matrix whose leading diagonal elements are equal to 1 $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
Upper Triangle Matrix	A matrix whose all the elements below the leading diagonal are zero $\begin{bmatrix}3 & 4 & 5\\0 & 1 & 9\\0 & 0 & 5\end{bmatrix}$
Lower Triangle Matrix	A matrix whose all the elements above the leading diagonal are zero $\begin{bmatrix}3 & 0 & 0\\5 & 1 & 0\\2 & 8 & 5\end{bmatrix}$
Sub Matrix	The matrix obtained by deleting one or more rows or columns or both of a matrix is called its sub matrix.
Equal Matrices	Two matrices are are equal matrices if order of both is same and corresponding elements are same
Addition/ Subtraction	All the corresponding elements will be added/ subtracted to make a new matrix. (only possible when both matrix are of same order)
Properties of Addition/ Subtraction	**a**. A+B = B+A [Commutative], **b**. (A+B)+C = A+(B+C) [Associative], **c**. k(A+B) = kA + kB, k is constant
Multiplication	Multiplication of two matrices is possible only when no. of columns of first matrix = no. of rows of second matrix. *[To understand how to do multiplication – refer page 2.40 Example 3]*
Properties of Multiplication	**a.** In general, A×B ≠ B×A, **b**. (A×B)×C = A×(B×C) if defined, **c.** A(B+C) = AB + AC also, (A+B)C= AC+BC, **d.** if AB = AC then B≠C in general, **e.** A×O = O [O means null matrix], **f.** A×I = IA = O [I means Unit Matrix],
Transpose of a Matrix	A matrix obtained by changing rows and columns of a matrix **A** is called as Transpose Matrix of **A**. It is denoted by - **A^T or A'**

Properties of Transpose	a. A = (A')' ; b. (A+B)' = A' + B' ; c. (KA)'=K.A' ; d. (AB)'=B'×A'
Symmetric Matrix	If after transposing also there is no change in matrix. A'=A
Skew Symmetric	If after transposing a matrix, it becomes its negative. A'=−A

DETERMINANTS

Determinants	It is a valuation of a matrix using some rules. It only applies for square matrix
Denote	It is denoted by **det A** or **| A |** or **Δ**
2 × 2 Matrix	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$
3 × 3 Matrix	$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3-b_3c_2)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)$
Minor	M_{ij}=Minor of the element located in i^{th} row and j^{th} column. It is equal to determinant of sub matrix obtained after i^{th} row and j^{th} column
Cofactor	$C_{ij} = (-1)^{i+j} M_{ij}$
3 × 3 Formula using Cofactors	$a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$
Properties	a. Δ remains unaltered if its rows or columns are interchanged. b. Δ change its sign if two rows or columns interchanges c. If any two rows or columns of a determinant are identical, then Δ =0 d. If each element of matrix is multiplied by constant k , Δ will also get multiplied by k e. If some or all of the elements of a row or column of a determinant are expressed as sum, then Δ is expressed as sum of Δs f. Δ will remain same if equi-multiple of any row or column is added to each element of any row or column
Singular Matrix	if det A = 0, then singular matrix otherwise non-singular matrix
Adjoint Matrix	Adjoint of A Matrix is the transpose of the Cofactor Matrix
Inverse Matrix	If A is a square matrix, and det A ≠ 0 (non-singular), then $\mathbf{A^{-1} = \frac{1}{\|A\|} \times Adj.\ A}$
Cramer's rule to find solution of linear eq. in 3 variables	$x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$, provided Δ≠0 [$\Delta x$ means determinant of matrix by replacing first column of matrix with RHS values of equations] See Example
Properties of Cramer's	a. If Δ≠0, the system has unique solution b. If Δ=0 and atleast one of $\Delta x, \Delta y, \Delta z$ ≠0, then system has no solution and it is inconsistent c. If Δ=0 and all of $\Delta x, \Delta y, \Delta z$ ≠0, then system may or may not have solution,. If it has solution, equations are dependent and there will be infinite no. of solutions. If it doesn't have solution, equations are inconsistent.

SET

<table>
<tr><td>Set means</td><td>Collection of well-defined distinct objects. It is usually denoted by capital letter</td></tr>
<tr><td>Element</td><td>Each object of set is called as element. It is usually denoted by small letter</td></tr>
<tr><td>Braces Form</td><td>When set shown as a list of elements within braces { } e.g. A = {1,3,5,7}</td></tr>
<tr><td>Descriptive Form</td><td>Set can be presented in statement form e.g. A = set of first four odd numbers</td></tr>
<tr><td>Set-Builder or Algebraic form</td><td>Here Set is written in the algebraic form in this format – $\{x: x$ satisfies some properties or rule}. The method of writing this form is called as Property or Rule method</td></tr>
<tr><td>Belongs to</td><td>It is denoted by '∈', a ∈ A means that element a is one of the element of Set A. ∉ used for do not belongs to.</td></tr>
<tr><td>Subset</td><td>Set A is a subset of Set B if all the elements of Set A also exist in Set B. It is denoted as - A⊂B</td></tr>
<tr><td>Proper Subset</td><td>A is a proper subset of B if A is a subset of B and A≠B</td></tr>
<tr><td>Improper Subset</td><td>Two equal sets are improper subsets of each other</td></tr>
<tr><td>Null Set</td><td>A set having no elements is called as Null or Empty Set. It is denoted by ϕ</td></tr>
<tr><td>No. of subsets</td><td>Formula: no. of subsets = 2^n, no. of proper subsets = 2^n-1</td></tr>
<tr><td>Intersection
denoted by [A∩B]</td><td>Intersection set of A and B is a set that contains common elements between both of the sets</td></tr>
<tr><td>Union
denoted by [A∪B]</td><td>Union set of A and B is a set that contains all the elements contained in both the sets without repeating the common elements</td></tr>
<tr><td>Universal Set</td><td>The set which contains all the elements under consideration in a particular problem is called the universal set generally denoted by S</td></tr>
<tr><td>Complement Set</td><td>A complement set of set P is a set that contains all the elements contained in the universe other than elements of P. It is denoted by P' or P^c</td></tr>
<tr><td>Set (A-B)</td><td>A-B is a set that contains elements of A other than those which are common in A and B. [A-B = A - A∩B]</td></tr>
<tr><td>De Morgan's Law</td><td>1. (P∪Q)' = P'∩Q'
2. (P∩Q)' = P'∪Q'</td></tr>
<tr><td>Venn Diagrams</td><td>
<table>
<tr><td>Universal Set</td><td>
</td></tr>
<tr><td>Union Set A∪B</td><td>
</td></tr>
<tr><td>Intersection Set A∩B</td><td>
</td></tr>
<tr><td>Set A-B</td><td>
</td></tr>
</table>
</td></tr>
<tr><td>2 sets – Formula</td><td>n(A∪B) = n(A) + n(B) – n(A∩B)</td></tr>
<tr><td>3 sets – Formula</td><td>n(A∪B∪C) = n(A) + n(B) + n(C) – n(A∩B) –n(B∩C) –n(C∩A) + n(A∩B∩C)</td></tr>
</table>

<table>
<tr><td>Venn Diagram related some basics</td><td>
<table>
<tr><td>A or B , atleast A or B, either A or B</td><td>$A \cup B$</td></tr>
<tr><td>A and B, Both A and B</td><td>$A \cap B$</td></tr>
<tr><td>Only A means</td><td>$A - B$</td></tr>
<tr><td>Only B means</td><td>$B - A$</td></tr>
<tr><td>Neither A nor B</td><td>$(A \cup B)'$</td></tr>
</table>
</td></tr>
<tr><td>Cardinal Number</td><td>No. of distinct elements contained in a finite Set A is called Cardinal Number.
For Set A = {4,6,8,3}, cardinal no. n(A) = 4</td></tr>
<tr><td>Equivalent Set</td><td>Two sets A and B are equivalent sets if n(A) = n(B)</td></tr>
<tr><td>Power Set</td><td>Collection of all possible subsets of a given set A is called Power set of Set A. It is denoted by P(A)</td></tr>
<tr><td>Ordered Pair</td><td>Pair of two elements both taken from different Sets. E.g. if $a \in A$ and $b \in B$ then ordered pair is (a,b) where first element will always from A and second always from B in every pair</td></tr>
<tr><td>Product of Sets</td><td>Also called as Cartesian Product. If A and B are two non-empty sets, then set of all the ordered pairs such that $a \in A$ and $b \in B$ is called as Product Set. It is denoted by A×B. [A×B = {(a:b): a∈A and b∈B}]</td></tr>
<tr><td>Why Product?</td><td>n(A×B) = n(A) × n(B) i.e. cardinal no. of product set is equal to product of cardinal no. of each set</td></tr>
</table>

FUNCTION

<table>
<tr><td>Relation</td><td>Any subset of product set is called A×B is said to define relation from A to B. It's any collection of ordered pairs taken from a product set.</td></tr>
<tr><td>Function (set based definition)</td><td>A relation where no ordered pairs have same first elements is called Function. First element of the ordered should not be repeated in the relation set. (a,b) all a should be unique for different values of b</td></tr>
<tr><td>Function (non set based definition)</td><td>A rule which associate all elements of A to B is called function from A to B. It is denoted by $f: A \rightarrow B$ or $f(x)$ of B</td></tr>
<tr><td>Image, Pre-image</td><td>$f(x)$ is called the image of x and x is called the pre-image of $f(x)$
Pre-image is input and Image is output</td></tr>
<tr><td>Domain, Co-domain, Range</td><td>Let $f: A \rightarrow B$, then A is called domain of f and B is called the co-domain of f.
Set of all the images (contained in B) of pre-images taken from A is called Range. Domain is a set of all pre-images and Range is a set of all images. Also Range is a subset of Co-domain.</td></tr>
<tr><td>Types of Functions</td><td>
<table>
<tr><td>One-One Function</td><td>Let $f: A \rightarrow B$, if different elements in A have different images in B then f is one-one or injective function or one-one mapping</td></tr>
<tr><td>Onto Function</td><td>Let $f: A \rightarrow B$, if every element in B has at least one pre-image in A, then f is an onto or surjective function</td></tr>
<tr><td>Into Function</td><td>Let $f: A \rightarrow B$, if even a single element in B is not having pre-image in A, then it is said to be into function</td></tr>
<tr><td>Bijection Function</td><td>If a function is both one-one and onto it is called as Bijection Function</td></tr>
<tr><td>Identity Function</td><td>If domain and co-domain are same then function is identity function Let $f: A \rightarrow A$ and $f(x) = x$</td></tr>
<tr><td>Constant Function</td><td>If all pre-images in A will have a single constant value in B then the function is constant function</td></tr>
</table>
</td></tr>
<tr><td>Equal Function</td><td>Two functions f and g are said to be equal function if both have same domain and same range</td></tr>
<tr><td>Inverse Function</td><td>Let $f: A \rightarrow B$, is a one-one and onto function. Every value of x (preimage)will give unique image $f(x)$ using f. If there is a function that takes value of images as input and gives pre-images as output, such function is called inverse</td></tr>
</table>

	function. It is denoted as $f^{-1}: B \rightarrow A$.
Composite Function	A function of function is called composite function. Example: if f and g are functions, then $f[g(x)]$ and $g[f(x)]$ are composite functions. Also called as fog or gof

RELATION

<table>
<tr><td>Relations</td><td colspan="2">Any subset of product set is called A×B is said to define relation from A to B. It’s any collection of ordered pairs taken from a product set.</td></tr>
<tr><td>Domain and Range</td><td colspan="2">If R is a relation from A to B, then set of all first elements of ordered pairs is domain and set of all second elements of ordered pairs is range.</td></tr>
<tr><td rowspan="3">Types of Relation</td><td>Reflexive</td><td>If S is a universal set, S = {a,b,c …} then R is a relation from S to S. If this R contains all the ordered pairs in the form (a,a) in S×S, then it is a reflexive relation</td></tr>
<tr><td>Symmetric</td><td>If (a,b) ∊ R, then if (b,a) ∊ R then R is called Symmetric</td></tr>
<tr><td>Transitive</td><td>If (a,b) ∊ R and also (b,c) ∊ R, then if (a,c) ∊ R such relation is Transitive. [if in a relation only (a,b) is present but (b,c) is not present we will consider it as transitive relation]</td></tr>
<tr><td>Equivalence Relation</td><td colspan="2">If a relation is Reflexive, Transitive and Symmetric as well, then it is called as Equivalence Relation</td></tr>
</table>

Permutations and Combinations

Fundamental Principles of Counting	Multiplication Rule AND → Multiply: If one thing can be done in 'm' ways and when it has been done, another thing can be done in 'n' different ways then the total number of ways of doing **both the things simultaneously = m × n** Addition Rule OR → Add: If two alternative jobs can be done in 'm' and 'n' way respectively then **either of the two jobs** can be done in **(m+n) ways**
Factorial	It is written as n! or $n\rfloor = n(n-1)(n-2)\ldots 3\times 2\times 1$ 0! = 1, 1! = 1, 2! = 2×1, 3! = 3×2×1, 4! = 4×3×2×1
Permutations means	It is the ways of **arranging or selecting** things from a group of things with due regard being paid **to order** of the arrangement or selection.
Basic Example 1	**Arranging** three persons A,B,C for a group photograph can be done as {ABC, ACB, BAC, BCA, CAB, CBA}, thus total no. of ways is 6
Basic Example 2	**Selecting** two persons as Winner and Runner-up for a contest having 4 participants P,Q,R,S can be done as {PQ, PR, PS, QP, QR, QS, RP, RQ, RS, SP, SQ, SR}, thus total no. of ways is 12 (here in the set of arrangement first element is winner and second is runner up)
Theorem for Permutations	The number of permutations of n things chosen r at a time is given by $^{n}P_{r} = \frac{n!}{n-r!}$ *or* $n(n-1)(n-2)\ldots(n-r+1)$
Basic Example 3	$^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5\times4\times3\times2\times1}{2\times1} = 5\times4\times3 = 60$ Or simply here r = 3, so do reverse multiplication of 5 up to three terms so it will be 5×4×3 = 60
Use of Theorem	We are able to find no. of ways manually also *(as done in Basic Example 1 and 2)* but that is easy for lower values of n and r. When there is a higher value of n, manually creating the set of arrangements will be tedious which requires the need of this theorem. Check Basic Example 1 and Example 2 using theorem
Why 0! = 1	$^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ also, $^{n}P_{n} = n!$, thus $\frac{n!}{0!} = n!$, $0! = \frac{n!}{n!} = 1$
Special Formula	$(n+1)! - n! = n.n!$ (for proof – refer Example 10 Study Mat Page 5.6)

Question Patterns with remarks

Type	Remark
Calculate No. of words using letters of a particular word	Simple $^{n}P_{r}$ Note: Meaning of words has no relevance
Group Photograph	$^{n}P_{n}$
Rank Awards first, second, third etc.	$^{n}P_{r}$ here r is no. of ranks
Theorem based questions, calculation of n or r with the given data	Directly apply theorem
Selection of different unique designations/ positions from a group of persons	$^{n}P_{r}$ here r is no. of unique designations/ positions

<table>
<tr><td>Circular Permutations</td><td>Above discussion was relevant for things that are arranged in a row. However when the things are arranged in a circle, the permutation is termed as circular.</td></tr>
<tr><td>Theorem: Circular Permutations</td><td>The number of circular permutations of n different things chosen all at a time is (n–1)!</td></tr>
<tr><td>Standard Results</td><td>
<table>
<tr><td>number of ways of arranging n persons along a round table so that no person has the same two neighbors is</td><td>$\frac{1}{2}(n-1)!$</td></tr>
<tr><td>the number of necklaces formed with n beads of different colors</td><td>$\frac{1}{2}(n-1)!$</td></tr>
</table>
</td></tr>
<tr><td>Permutation with Restrictions
Note: These two theorems are useful for formula based questions. For practical questions we will use logic. (explained in example)</td><td>
<table>
<tr><td>Theorem 1</td><td>Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is $^{(n-1)}P_r$</td></tr>
<tr><td>Theorem 2</td><td>Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is $^{(n-1)}P_{(r-1)}$</td></tr>
</table>
</td></tr>
<tr><td>Some tips useful while solving problems having restrictions</td><td>
<table>
<tr><th>Requirement of Que.</th><th>Tips</th></tr>
<tr><td>Calculate permutation when two or more objects are always together</td><td>In that case consider that group of objects as 1 object for the purpose of ^nP_r formula, then multiply factorial of no. of objects in the group</td></tr>
<tr><td>Calculate permutation when two or more objects will never come together</td><td>Step 1: Calculate the no. of ways without restriction using ^nP_r
Step 2: Calculate Permutation of 2 or more thing always together (as per above point)
Step 3: Result of Step 1 – Result of Step 2</td></tr>
<tr><td>When there are two types of objects and ask is to calculate the ways in which no two objects of one the category will be together</td><td>In that case, that particular group of objects can be arranged in the alternate places as a neighbor of each object of other category
Refer Example 10 Study Mat Page 5.13SS</td></tr>
</table>
</td></tr>
<tr><td>Standard Results</td><td>
<table>
<tr><td>Permutations when some of the things are alike, taken all at a time</td><td>$p = \frac{n!}{n_1! \times n_2! \times n_3!}$</td></tr>
<tr><td>Permutations when each thing may be repeated once, twice, upto r times in any arrangement.</td><td>n^r</td></tr>
</table>
</td></tr>
</table>

<table>
<tr><td>Combinations</td><td>The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations. It is just a GROUPING</td></tr>
<tr><td>Basic Example 1</td><td>Grouping of two persons out of three persons A,B,C for a group photograph can be done as {AB, BC, AC}, thus total no. of ways is 3. Here AB and BA are same group and will be counted once only, even though the sequence is not same. Sequence has no relevance while finding combinations.</td></tr>
<tr><td>Basic Example 2</td><td>Selection of persons for a committee of 2 out of total 4 applicants P,Q,R,S can be done in {PQ, QR, RS, PS, PR, QS} – total 6 ways. Here we used combinations because in the committee of two there is no designations all are same so sequence of selection does not matter.</td></tr>
<tr><td>Theorem of Combinations</td><td>$^{n}C_{r} = \frac{n!}{r!(n-r)!}$ or $^{n}C_{r} = \frac{^{n}P_{r}}{r!}$</td></tr>
<tr><td>Standard Results</td><td>$^{n}C_{0} = 1$, $^{n}C_{n} = 1$</td></tr>
<tr><td>Complimentary Combinations</td><td>$^{n}C_{r} = {}^{n}C_{(n-r)}$ example: $^{5}C_{3} = {}^{5}C_{2}$</td></tr>
<tr><td>Special Formulas</td><td>$\boldsymbol{^{n+1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r-1}}$
<u>Memorize:</u>
Combination of (n+1) things when one thing is always included [$^{n}C_{r}$]+
Combination of (n+1) things when one thing is always excluded [$^{n}C_{r-1}$]</td></tr>
<tr><td>Permutation Special formula</td><td>$\boldsymbol{^{n}P_{r} = {}^{n-1}P_{r} + r.{}^{n-1}P_{r-1}}$
Memorize in the same way as above</td></tr>
<tr><td>Standard Results</td><td><table><tr><td>Combinations of n different things taking some or all of n things at a time</td><td>$\boldsymbol{2^{n} - 1}$ [1 is subtracted because we are removing all rejection case]</td></tr></table></td></tr>
<tr><td>Question Patterns with remarks</td><td><table>
<tr><th>Type</th><th>Remark</th></tr>
<tr><td>Different pocker hands in a pack of cards</td><td>When we play Poker, Teen Patti etc. only group of 5 cards, sequence in which it is picked does not matter hence we take combinations</td></tr>
<tr><td>Formation of triangles when vertices (corner points) are given</td><td>We need three vertices to make a triangle. Now with group of three points to make a triangle and sequence of points does not matter, hence will use combination. Example: Using eight points how many triangle can be formed - $^{8}C_{3} = 56$</td></tr>
<tr><td>No. of ways of invitation</td><td>Here also sequence does not matter, hence will use combination</td></tr>
<tr><td>Selection of color balls from box</td><td>Here combination is used assuming that balls are of identical color</td></tr>
<tr><td>No. of ways of forming words from n letter taking few letters and the letter are not unique</td><td>Refer Example 6 – Page 5.25 Study Mat</td></tr>
<tr><td>Number of diagonals of a polygon</td><td>$^{n}C_{2} - n$, here n means no. of side of polygon (refer Q.10 Exercise 5C)</td></tr>
</table></td></tr>
</table>

Probability

Know about Probability		→ First use of Probability was made 300 years back in Europe by a group of mathematicians to enhance their chances of winning in gambling → It is a full-fledged subject and become an integral part of statistics → Theories of Testing Hypothesis and Estimation are based on probability
Types	Subjective Probability	Dependent on personal judgment, useful in decision making. It is out scope of our syllabus
	Objective Probability	This is based on Mathematical Rules and not judgment based. We will study this section in our chapter.
Random Experiment	Experiment	A performance that produces certain results
	Random Experiment	An experiment is defined to be random if the results of the experiment depend on chance only.
	Examples	Tossing a coin, throwing a dice, drawing cards from a pack
Events		The **results or outcomes** of a **random experiment** are known as events
Types of Events	**Based on Combination of Events**	
	Simple or Elementary	If the event cannot be decomposed into further events
	Composite or Compound	An event that can be decomposed into two or more simple events
	Based on nature of occurrence (applicable for set of events)	
	Mutually Exclusive or Incompatible Events	A set of events A1, A2 ... is said to be mutually exclusive if they cannot occur simultaneously. Occurrence of one implies non occurrence of other.
	Exhaustive Events	A set of events A1, A2 ... is said to be exhaustive if one of these must necessarily occur on a random experiment
	Equally Likely or Equi-Probable Events or Mutually Symmetric	If it is evident that from the set of events, none of the events is expected to occur more frequently than others.
Classical Definition of Probability		Also called Prior Definition of Probability, this formula is Event (Result) Based. It is given by Bernoulli and Laplace. $P(A) = \frac{\text{no. of events favorable to A}}{\text{total number of events}}$
More about Classical Probability	Demerits or Limitations	→ Applicable only when events are finite and are equally likely → Limited application of this definition like in tossing coin, throwing dice, cards etc.
	Other Notes	→ $0 \leq P(A) \leq 1$, P(A) = 1 means Sure Event, P(A) = 0 means impossible event → Probability of non-occurrence of an event A is denoted by P(A') or P($\bar{A}$) is called as complimentary event of A. P(A') = 1−P(A)
	Odds in Favor of an Event	$\frac{\text{no. of favorable events}}{\text{no. of unfavorable events}}$
	Odds Against an Event	$\frac{\text{no. of unfavorable events}}{\text{no. of favorable events}}$

Special Formula	If an experiment results in p outcomes and if it is repeated q times then $\text{Total no. of outcomes} = p^q$
Terms used in 52 Cards Deck	Suits (four): Spades - ♠ Hearts - ♡ Diamond - ♢ Clubs - ♣ Ranks (13): A (Ace), K (King), Q (Queen), J (Jack), 10, 9, 8, 7, 6, 5, 4, 3, 2
Relative Frequency Definition of Probability	$\text{Relative Frequency} = \frac{\text{no.of times the event occured during experimental trials}}{\text{total no.of trials}} = \frac{f_A}{n}$ Probability by this method is defined as $P(A) = \lim_{n\to\infty} \frac{f_A}{n}$ (Relative Frequency on infinite no. of trials is equal to probability)
Set Based Probability	Sample Space (denoted by **S** or **Ω**-omega): a non-empty set containing all the elementary events of a random experiment as sample points Event A: Event which is under consideration for probability calculations is defined as a non empty subset of Set S (Sample Space) Probability Formula: $P(A) = \frac{\text{no. of sample points in A}}{\text{no. of sample points in S}} = \frac{n(A)}{n(S)}$
Axiomatic Or Modern Definition of Probability	This definition is also based on Sets Concepts. Here Probability is not a simple ratio like above, but can be said as function P defined on S known as Probability Measure. P(A) is defined as the probability of A as per this function only if below conditions are satisfied: Condition 1: $P(A) \geq 0$, for every $A \subseteq S$ Condition 2: $P(S) = 1$ Condition 3: For any sequence of mutually exclusive events A1, A2, A3... $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$
Addition Theorems	Theorem 1: $P(A \cup B) = P(A + B) = P(A\ or\ B) = P(A) + P(B)$ If A and B are mutually exclusive events Theorem 2: For set of mutually exclusive events $A_1, A_2, A_3, \ldots$ $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$ Theorem 3: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ For any two events A and B Theorem 4: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
Expected Frequency	No. of sample points n(S) × P(A)
Conditional Probability or Compound Theorem	Dependent Events: If occurrence of one event is influenced by occurrence of another event, then two events are dependent. Independent Events: Two events are said to be independent if occurrence of one event do not influence the occurrence of other. Probability in case of Dependent Events A and B: Conditional Probability of B/A: means probability of event B given that event A has already been occurred $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$, provided $P(A) > 0$ Similarly, Conditional Probability of A/B: $P\left(\frac{A}{B}\right) = \frac{P(B \cap A)}{P(B)}$, provided $P(B) > 0$ **Compound Theorem: P(A∩B) = P(B) × P(A/B) or P(A∩B) = P(A) × P(B/A)**

	Probability in case of Independent Events	Since there is no dependency, Conditional Probability = Normal Probability i.e. P(B/A) = P(B) and P(A/B) = P(A) Here, P(A∩B)= P(A) × P(B) And for three events, A, B, C P(A∩B∩C) = P(A) × P(B) × P(C) Also, if A and B are independent, then below are also independent : A and B', A' and B, A' and B'
Random Variable: Probability Distribution	Random Variable	It is a function defined on Sample Space of a random experiment that can take any value (Real Number)
	Discrete Random Variable	RV that can take only discrete values. RV on a discrete sample space
	Continuous Random Variable	RV that can take any values within an interval. [infinite no. of sample points in a sample space]
	Probability Distribution	If is defined as the statement/ table that shows no. of different value taken by Random Variable and their corresponding probabilities
	Conditions of Probability Dist.	If X (a random variable) takes n finite values like $X_1, X_2, X_3, \dots, X_n$ and probabilities are $P_1, P_2, P_3, \dots, P_n$ then, $P_i \geq 0$ for every i and $\sum P_i = 1$
Expected Value	Expected Value	It is defined as the sum of products of different values taken by Random Variable and corresponding probabilities. $E(x) = \sum p_i\, x_i$ (this formula is similar to AM of frequency distribution)
	Mean of Probability Distribution	Since this is mean, we can say that Expected value is equal to arithmetic mean of probability distribution. Here mean is denoted by μ, hence $\mu = E(x) = \sum p_i\, x_i$
	Variance of Probability Distribution	$V(x) = \sigma^2 = E(x-\mu)^2 = E(x)^2 - \mu^2$
	Properties of E.V.	→ E.V. of a constant is constant → $E(x+y) = E(x) + E(y)$ → $E(k.x) = E(x).k$ → $E(x.y) = E(x).E(y)$

Theoretical Distribution

Binomial Distribution (bi-parametric discrete probability distribution)	Bernoulli's Trial	→ Each trial is associated with two mutually exclusive and exhaustive outcomes [one is success and other one is failure] → Trials are independent → Probability of success (p) and failure ($q=1-p$) will remain unchanged throughout the process → No. of trials is a positive integer
	Binomial Variable	It is a discrete random variable X that follows binomial distribution and is denoted by $\boldsymbol{X \sim B(n, p)}$
	Probability Mass Function	$\boldsymbol{f(x) = P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}}$ for $x = 0,1,2,3,\ldots,n$ and $f(x) = 0$ if x is otherwise
	Mean	$\boldsymbol{\mu = np}$
	Variance	$\boldsymbol{\sigma^2 = npq}$, also Variance is always less than mean, maximum value of variance is $^{n}/_{4}$
	Mode	Calculate $(n+1)p$, if the resulting value is integer then Bi-modal: $\boldsymbol{\mu_0} = (n+1)p$ *and* $[(n+1)p - 1]$ If the resulting value is non-integer then Uni-modal: $\boldsymbol{\mu_0}$ = largest integer contained in $(n+1)p$
	Additive Property	If X and Y are two independent variables such that $\boldsymbol{X \sim B(n_1, p)}$ and $\boldsymbol{Y \sim B(n_2, p)}$, then $\boldsymbol{(X+Y) \sim B(n_1 + n_2, p)}$
Poisson Distribution (uni-parametric discrete probability distribution)	History	Simeon Denis Poisson of France introduced this distribution way back in the year 1837
	Conditions	It is a limiting form of Binomial Distribution, where n→∞, p→0. It is also a discrete distribution
	Poisson Variable	It is a discrete random variable that follows Poisson Distribution denoted as $\boldsymbol{X \sim P(m)}$
	Probability Mass Function	$\boldsymbol{f(x) = P(X = x) = \frac{(e^{-m}.m^{x})}{x!}}$ for $x = 0, 1, 2, \ldots \infty$
	Mean	$\boldsymbol{\mu = m}$
	Variance	$\boldsymbol{\sigma^2 = m}$
	Mode	Calculate m, if the resulting value is integer then Bi-modal: $\boldsymbol{\mu_0 = m}$ *and* $[\boldsymbol{m - 1}]$ If the resulting value is non-integer then Uni-modal: $\boldsymbol{\mu_0}$ = largest integer contained in $\boldsymbol{m}$
	Additive Property	If X and Y are two independent variables such that $\boldsymbol{X \sim P(m_1)}$ and $\boldsymbol{Y \sim P(m_2)}$, then $\boldsymbol{(X+Y) \sim P(m_1 + m_2)}$
Normal Distribution (bi-parametric continuous probability distribution)	Basics	Various Mathematical experiments have proved that most of the continuous random variables will follow normal distribution. It is universally accepted distribution.
	Probability Density Function	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-(\frac{x-\mu}{\sigma})^2 \times \frac{1}{2}}$ It is defined for $-\infty < x < \infty$

Normal Distribution Properties

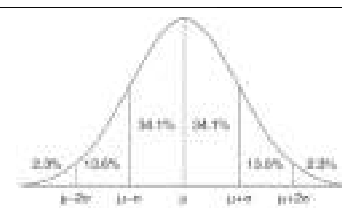
Mean = Median = Mode	μ
Standard Deviation	σ
Mean Deviation	$\sigma \times \sqrt{2/\pi} = 0.8\,\sigma$
Quartile Deviation	$Q_1 = \mu - 0.675\sigma$ and $Q_3 = \mu + 0.675\sigma$
Shape of Normal Curve	Bell Shaped
Normal Variable	$X \sim N(\mu, \sigma^2)$
Additive Property	Only applicable when two different random variables are independent. Assume we have two variables X and Y such that $\boldsymbol{X \sim N(\mu_1, \sigma_1^2)}$, $\boldsymbol{Y \sim N(\mu_2, \sigma_2^2)}$ then $\boldsymbol{X+Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)}$
Normal Curve is symmetrical at	$x = \mu$
Points of Inflexion	$\mu - \sigma$ & $\mu + \sigma$
Ratio between QD:MD:SD	10:12:15

Standard Normal Distribution

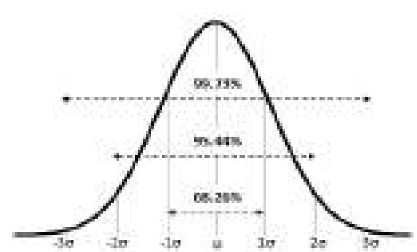
Conditions	Parameter: Mean μ = 0; Standard Deviation σ = 1
Standard Normal Variate	The variable used in this distribution is called as Standard Normal Variate and is denoted by ~~Z~~ *[Striked Z]*
Area from X=-3σ to X=3σ	99.73%
Z Table	This table gives us the probability of values from *X=μ=0* to *X=any value up to 3*
Z Score	$Z = \frac{x - \mu}{\sigma}$
Cumulative Distribution Function	$\phi(x) = P(X \leq x$
Probability Function	$f(\cancel{z}) = \frac{1}{\sqrt{2\pi}} e^{-(\cancel{z})^2 \times \frac{1}{2}}$ for $-\infty < \cancel{z} < \infty$
Mean, Median, Mode	μ=0
SD, Variance	σ=1, σ^2=1
Points of Inflexion	-1, 1
Mean Deviation	0.8
Quartile Deviation	0.675
Probability Function	$f(\cancel{z}) = \frac{1}{\sqrt{2\pi}} e^{-(\cancel{z})^2 \times \frac{1}{2}}$ for $-\infty < \cancel{z} < \infty$

Area under Normal Curve

From	To	Area/Probability
μ	+σ	34.135%
+σ	+2σ	13.59%
+2σ	+3σ	2.14%
3σ	∞	0.135%



From	To	Area/Probability
-σ	+σ	68.3%
-2σ	+2σ	95.5%
-3σ	+3σ	99.7%



SEQUENCE AND SERIES

<table>
<tr><td>Sequence</td><td>An ordered collection of numbers arranged as per some definite rule or pattern. $a_1, a_2, a_3, \dots, a_n$ is a sequence if you are able to identify pattern and there by the value of a_n (n^{th} term)</td></tr>
<tr><td>Examples of Sequence</td><td><table>
<tr><th>Collection</th><th>Ordered</th><th>Rule/ Pattern</th><th>Conclusion</th></tr>
<tr><td>1, 4, 9, 17, 18, ……</td><td>Yes</td><td>No</td><td>Not a sequence</td></tr>
<tr><td>20, 17, 4, 3, 1, ……</td><td>Yes</td><td>No</td><td>Not a sequence</td></tr>
<tr><td>1, 4, 7, 10, 13, ……</td><td>Yes</td><td>Yes +3 on each term</td><td>Yes Sequence</td></tr>
<tr><td>20, 10, 5, 5/2, ……</td><td>Yes</td><td>Yes ÷2 on each term</td><td>Yes Sequence</td></tr>
</table></td></tr>
<tr><td>Terms</td><td>$a_1, a_2, a_3, \dots, a_n$ are called as 1^{st} Term, 2^{nd} Term, 3^{rd} Term…nth term respectively</td></tr>
<tr><td>General Term</td><td>a_n is called as the n^{th} term of the sequence or General Term</td></tr>
<tr><td>Types of sequence</td><td>Finite Sequence – sequence having finite elements $\{a_i\}_{i=1}^{n}$
Infinite Sequence – sequence having infinite elements $\{a_i\}_{i=1}^{\infty}$</td></tr>
<tr><td>Series</td><td>Sum of the elements of the sequence is called as Series. $S_n = \sum_{i=1}^{n} a_i$
$S_n = a_1 + a_2 + a_3 + \cdots + a_n$
$S_1 = a_1$, $S_2 = a_1 + a_2$, $S_3 = a_1 + a_2 + a_3$</td></tr>
<tr><td>Arithmetic Progression (A.P.)</td><td>AP is a sequence in which each next term is obtained by adding a constant ‘d’ to the preceding term. This constant ‘d’ is called as common difference.
Let say a = first term and d = common difference, then AP can be written as –
a, a+d, a+2d, a+3d …. a+(n-1)d</td></tr>
<tr><td>Common Difference ‘d’</td><td>d = any term – preceding term or $\{t_n - t_{n-1}\}$</td></tr>
<tr><td>n^{th} term of an AP</td><td>$t_n = a + (n-1)d$</td></tr>
<tr><td>Insert AMs between two numbers</td><td>If there is a problem to find out AMs between two number, consider it as an AP with first number as first term of AP and other number as last term of AP.
Number of AMs required = no. of terms between first term and last term
Example: If 3 AMs between a and b is asked, form an AP as below:
a, ___ , ___ , ___ , b</td></tr>
<tr><td>Sum of first n terms of an AP</td><td>$S_n = \frac{n(a+t_n)}{2}$ or $S_n = \frac{n}{2}\{2a + (n-1)d\}$</td></tr>
<tr><td>Other Useful Formulas</td><td><table>
<tr><td>Sum of first n natural numbers</td><td>$\frac{n(n+1)}{2}$</td></tr>
<tr><td>Sum of first n odd numbers</td><td>n^2</td></tr>
<tr><td>Sum of squares of first n natural numbers</td><td>$\frac{n(n+1)(2n+1)}{6}$</td></tr>
<tr><td>Sum of cubes of first n natural numbers</td><td>$\left\{\frac{n(n+1)}{2}\right\}^2$</td></tr>
</table></td></tr>
</table>

Geometric	GP is a sequence of terms where each term is a constant multiple of preceding

Progression (G.P.)	term. This constant multiplier is called as common ratio. Let say *a* = first term and *r* = common ratio then GP can be written as $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$
nth term of a GP	$t_n = ar^{(n-1)}$
Common Ratio 'r'	$r = \frac{\text{any term}}{\text{preceding term}} = \frac{t_n}{t_{n-1}}$
Insert GMs between two numbers	If there is a problem to find out GMs between two number, consider it as a GP with first number as first term of GP and other number as last term of GP. Number of GMs required = no. of terms between first term and last term Example: If 3 GMs between a and b is asked, form an GP as below: *a*, ___ , ___ , ___ , *b*
Sum of first n terms of a GP	$S_n = \frac{a(1-r^n)}{(1-r)}$ when r<1, $\frac{a(r^n-1)}{(r-1)}$ when r>1
Sum of infinite GP	$S_\infty = \frac{a}{(1-r)}$ [only possible when r<1]

A. INTRODUCTION TO STATS

Definition

Singular Sense:

- Scientific method that is used for collecting, analyzing and presenting data
- Used to draw statistical inferences
- Inferences means conclusion reached on the basis of evidence and reasoning

Example:

After applying statistical methods we have arrived at a conclusion that in last 5 years crime rate is reduced.

Plural Sense:

- Data qualitative or quantitative collected to do statistical analysis

Example: Based on Cricket Match statistic of this stadium, chasing team wins mostly

History of Stats

- Word Origin
  - ✓ Latin word – Status
  - ✓ Italian word – Statista
  - ✓ German word – statistic
  - ✓ French word – statistique
- Publication:
  - ✓ Koutilya's book Arthashastra
  - ✓ Stat records on Agriculture found in Ain-i-Akbari (author Abu Fezal)
- Census: First ever census done in Egypt (300 years BC to 2000 BC)

Application of Stats

There are various but we will confine to below:

1. Economics: Time Series analysis, index, demand analysis, econometrics, regression analysis
2. Business Management: business decisions rely upon QT
3. Commerce/ Industry: Sales, Purchase, RM, Salary Wages etc. data are analyze for business decisions and policy making

Limitation of Stats:

1. Relevant for aggregate data and not individual data
2. Quantitative data can only be used, however for qualitative – it needs to be converted into quantitative
3. Projections are based on conditions/ assumptions and any change in that will change the projection
4. Sampling based conclusions are used, improper sampling leads to improper results

B. COLLECTION OF DATA

Data and Variable

- Variable = measurable quantity
  - Discrete variable: when a variable assumes a finite or count ably infinite isolated values. Example: no. of petals in a flower, no. of road accident in locality
  - Continuous variable: when a variable assumes any value from the given interval (can also be in decimals, fractions). Example: height, weight, sale, profit
  - Attribute: qualitative characteristics. Example: Gender of a baby, nationality of a person
- Data = quantitative information shown as number. These are of two types:
  - Primary : first time collected by agency/ investigator
  - Secondary: collected data used by different person/ agency

How to collect Primary Data?

1. **Interview Method**:
   a. Personal Interview: directly from respondents. Example: Natural Calamity, Door to Door Survey
   b. Indirect Interview: when reaching to person difficult, contact associated persons. Example: Rail accident
   c. Telephone Interview: over phone, quick and non-responsive

Type of Interview/ Parameters	Personal	Indirect	Telephone
Accuracy	High	Low	Low
Coverage	Low	Low	High
Non Response	Low	Low	High

2. **Mailed Questionnaire Method:**
   a. Mailed means by Post or Email
   b. Well drafted + properly sequenced + with guidelines
   c. Non Response is Maximum

3. **Observation Method:**
   a. Data collected by direct observation or using instrument
   b. Example: Height check, Weight check,
   c. Although more accurate but it is time consuming, low coverage and laborious

4. **Questionnaire filled and sent by Enumerators**
   a. Enumerator: Person who directly interact with respondent and fill the questionnaire
   b. Generally used in Surveys

Sources of Secondary Data

1. International sources like World Health Organization (WHO), International Monetary Fund (IMF), International Labor Organization (ILO), World Bank
2. Government Sources – In India – Central Statistics Office (CSO), National Sample Survey Office- NSSO, Regulators – RBI, SEBI, RERA, IRDA
3. Private or Quasi-government sources like Indian Statistical Institute (ISI), Indian Council of Agriculture, NCERT
4. Research Papers and other unpublished sources

Scrutiny of Data

1. Scrutiny – checking accuracy and consistency of data

2. Finding of errors by enumerators while filling or receiving questionnaire
3. Internal consistency check: when two or more series of related data are given check each other
4. Consider enumerators' bias while using data

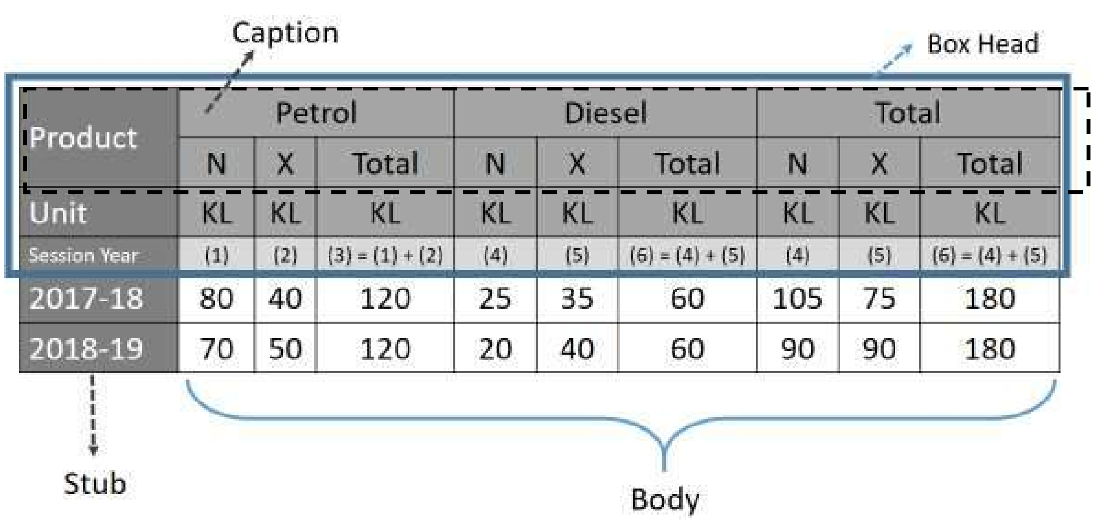
C. PRESENTATION OF DATA

Classification and organization of Data:

- means process of arranging data based on some logic
- there are four types of classification of data
    a. Chronological/ Temporal/ Time Series Data (ex. Profit YoYi.e year on year)
    b. Geographical or Spatial Series Data (ex. Weather in North India and South India)
    c. Qualitative or Ordinal Data (ex. Rating Top 20 songs by Radio Mirchi)
    d. Quantitative or Cardinal Data (no. of left handed batsmen in cricket teams playing CWC19)

Mode of Presentation

1. **Textual:** where text is used in the form of para or sentence. Example: Height of A,B and C is 160cm, 165cm, 175cm respectively
2. **Tabular/ Tabulation:**
    - Data shown in the form of table
    - Some important terms about Table (we will understand by example - next page figure)
    - It is preferred over textual form because
        - Useful in easy comparison
        - Complicated data can be presented
        - Table is must to create a diagram
        - No analysis possible without diagram



Product	Petrol			Diesel			Total		
	N	X	Total	N	X	Total	N	X	Total
Unit	KL	KL	KL	KL	KL	KL	KL	KL	KL
Session Year	(1)	(2)	(3) = (1) + (2)	(4)	(5)	(6) = (4) + (5)	(4)	(5)	(6) = (4) + (5)
2017-18	80	40	120	25	35	60	105	75	180
2018-19	70	50	120	20	40	60	90	90	180

3. **Diagrammatic representation of data**
    - Can be helpful for layman (without having much knowledge of numbers)
    - Hidden trend can be traced

- Table is more accurate than diagrams
- Types of Diagram below:

Line Diagram/ Histogram:

- plotting points in graph and join them to make a line
- used generally for time series (variable y is plotted against time t)
- for wide fluctuation, log chart or ratio chart is used (log y is plotted against t)
- for two or more series of same unit – multiple line chart is used
- for two or more series of distinct unit – multiple axes chart is used
- Refer Material for Diagram

Bar Diagram

- Bar means rectangle of same width and of varying length drawn horizontally or vertically
- For comparable series – multiple or grouped bar diagrams can be used
- For data divided into multiple components – subdivided or component bar diagrams
- For relative comparison to whole, percentage bar diagrams or divided bar diagrams

Pie Chart

- Used for circular presentation of relative data (% of whole)
- Summation of values of all components/segments are equated to 360 Degree (total angle of circle)
- Segment angle = $\frac{\text{segment value x } 360°}{\text{total value}}$

D. FREQUENCY DISTRIBUTION

What is Frequency Distribution?
Frequency means number of times a particular observation is repeated. This applies to both variable and attribute. It is shown in tabular form with class interval or the observation in one column and its frequency in the other.
These are of two types

- Ungrouped/ Simple Frequency Distribution
- Grouped Frequency Distribution

Important Terms

1. **Mutually exclusive classification or Overlapping Classification**: This is usually applicable for continuous variable. An observation as UCL is excluded from the class interval and taken in the class where it is LCL.
   Example: in the below class interval where will the observation 20 fall?

Class	Class where 20 will fall
10-20	No – excluded
20-30	Yes
30-40	No

2. **Mutually inclusive classification or Non Overlapping Classification**: This is usually applicable to discrete variable. All observation including UCL and LCL will be taken in the same class interval as there is no confusion.
   Example:

Class	Class where 20 will fall
10-19	No
20-29	Yes
30-39	No

3. **Class Limit:** for a class interval CL is the minimum and maximum value the class interval may contain. Minimum = Lower Class Interval (LCL) and Maximum = Upper Class Interval (UCL)
   Example:

Class	Type	LCL	UCL	Class	Type	LCL	UCL
10-19	Mutually Inclusive	10	19	10-20	Mutually Exclusive	10	20
20-29	Mutually Inclusive	20	29	20-30	Mutually Exclusive	20	30
30-39	Mutually Inclusive	30	39	30-40	Mutually Exclusive	30	40

4. **Class Boundary:** These are actual class limits of a class interval
   a. **For Mutually Exclusive / Overlapping :** Class Boundary = Class Limit
   LCL = LCB, UCL = UCB
   b. **For Mutually Inclusive / Non Overlapping:** Mid of the two class limits
   LCB = LCL – D/2, UCB = UCL + D/2
   Example:

Class	Type	LCL	UCL	LCB	UCB	Class	Type	LCL	UCL	LCB	UCB
10-19	Mutually Inclusive	10	19	9.5	19.5	10-20	Mutually Exclusive	10	20	10	20
20-29	Mutually Inclusive	20	29	19.5	29.5	20-30	Mutually Exclusive	20	30	20	30
30-39	Mutually Inclusive	30	39	29.5	39.5	30-40	Mutually Exclusive	30	40	30	40

5. **Mid Point/ Mid Value of Class / Class Mark**

$$\frac{\textbf{LCL+UCL}}{\textbf{2}} \textbf{ or } \frac{\textbf{LCB+UCB}}{\textbf{2}}$$

6. **Width / Size of Class Interval**
   UCB – LCB

7. **Cumulative Frequency**

Class	Frequency	Less than type CF	More than type CF
10-20	5	5	18
20-30	2	7	13
30-40	8	15	11
40-50	3	18	3
Total	18		

8. **Frequency Density**

$$\frac{\textbf{Frequency of class}}{\textbf{Class length of that class}}$$

9. **Relative Frequency or % Frequency**

$$\frac{\textbf{Frequency of class}}{\textbf{Total Frequency of table}}$$

Class	Frequency	Class Length	Frequency Density	Relative Frequency	Percent Frequency
10-20	5	10	0.5	5/18	27.7%
20-30	2	10	0.2	2/18	11.11%
30-40	8	10	0.8	8/18	44.44%
40-50	3	10	0.3	3/18	16.67%
Total	18				

Graphical Presentation of Frequency Distribution

1. **Histogram/ Area Diagram** [refer study material page 14.20 for diagram]
    a. It is a convenient way to represent FD
    b. Comparison between frequency of two different classes possible
    c. It is useful to calculate mode also
    d. Steps to create
        - Covert CL into CB and plot in x axis
        - Form rectangles taking class interval as base (x axis)
        - And frequency as length (y axis) | Use frequency density in case of uneven length

2. **Frequency Polygon**
    a. Usually preferable for ungrouped frequency distribution
    b. Can be used for grouped also but only if class lengths are even
    c. Steps to create
        - Plot (x_i, f_i) where x_i = class value (in case of ungrouped), mid value (in case of grouped) and f_i = frequency
        - Join all plotted points to make line segments which eventually will become a polygon (a shape with multiple number of line segments)

3. **Ogives/ Cumulative Frequency Graph**
    a. Create a table where cumulative frequency is mapped against each CB (Class Boundary) and make a curve by plotting and joining points by line segments. (curve is called Ogive)
    b. This graph can be made by both type of Cumulative Frequency and called as Less than Ogive or More than Ogive
    c. It can be used for calculating quartiles also
    d. If we plot both ogives in same graph, perpendicular line drawn from their intersection towards x axis is cutting axis at Median

4. **Frequency Curve**
    a. It is a limiting form of Area Diagram (Histogram) or frequency polygon
    b. It is obtained by drawing smooth and free hand curve though the mid points
    c. These are of below four types:
        - Bell Shaped
        - U-Shaped
        - J-Shaped
        - Combination of Curves as Mixed Curve