

# CHANAKYA 2.0

*For CA Foundation*

**Central Tendency**

**QUANTITATIVE APTITUDE**

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**TOPICS TO  
BE  
COVERED**

01

**Mean Median Mode**

02

**Geometric Mean**

03

**Harmonic Mean**

04

**PYQ'S**





## Characteristic Of An Ideal Central Tendency

- Easy calculation.
- Easy to understand
- Should be Based on all observation.
- It should be properly defined
- mathematical properties.
- least affected by extreme values.



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# Arithmetic Mean

Mean (Average)

Denoted by  $\bar{x}$  or  $\mu$   
for sample for population

$$A.M(\bar{x}) = \frac{\text{Sum of all observation}}{\text{Total no. of observation}}$$





$$\bar{x} = \frac{\sum x_i}{N}$$

marks: 6, 2, 8, 9, 7

mean marks ( $\bar{x}$ )

$$= \frac{6 + 2 + 8 + 9 + 7}{5}$$

$$= \frac{32}{5} = 6.4$$

marks ( $x_i$ )

$$x_1 = 6$$

$$x_2 = 2$$

$$x_3 = 8$$

$$x_4 = 9$$

$$x_5 = 7$$

$$\sum x_i = 32$$





Data

Un grouped Data

Grouped Data

Continuous series.

Individual Series

Discrete Series

$x_i$
/
/
/
/
/
/

$x_i$	$f_i$
/	/
/	/
/	/
/	/
/	/

Class interval	$f_i$
0-2	6
2-4	8
4-6	10
6-8	12









Savings

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weight : 62.1, 32.6

Continuous Series

$$\frac{30+35}{2} = \frac{65}{2} = 32.5$$

Weight (C.I.)	( $f_i$ ) No. of Students	(midvalue) $x_i$
30-35	5	32.5
35-40	7	37.5
40-45	8	42.5
45-50	2	47.5
50-55	3	52.5
$N = \sum f_i = 25$		





# Arithmetic mean ( $\bar{x}$ ) $\Rightarrow$ methods

Direct method

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$\bar{x} = \frac{\sum x_i^o}{N}$$

Assume mean

$$d_i = x_i - A$$

$$\bar{x} = A + \frac{\sum f_i d_i}{N}$$

Shortcut method

Step Deviation

$$u_i = \frac{x_i - A}{h}$$

$$\bar{x} = A + \left[ \frac{\sum f_i u_i}{N} \right] \times h$$





The following table gives the monthly income of 10 families in a city:

Income: 7800 7600 6900 7500 8400 9200 11000 8100 10500 9500  
( $x_i$ )

find mean income

~~A.~~ 8650

B. 8600

C. 8750

D. 8250

$$\begin{aligned}\bar{X} &= \frac{\sum x_i}{N} \\ &= \frac{86500}{10} \\ &= 8650\end{aligned}$$

Ind p v inlay  
Series



Calculate the arithmetic mean from the following data:

X:	10	15	20	25	30
F:	2	5	4	9	5

Discrete

A. 20

B. 22

C. 25

D. 26

$x_i$	$f_i$	$f_i x_i$
10	2	20
15	5	75
20	4	80
25	9	225
30	5	150
<b>N = 25</b>		<b>550</b>

$$\begin{aligned} \bar{X} &= \frac{\sum f_i x_i}{N} \\ &= \frac{550}{25} \\ &= 22 \end{aligned}$$



Compute the arithmetic mean from the following data:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students: ( $f_i$ )	8	12	10	8	3	2	7

A. 28

B. 25

C. 29

D. 32

$C \pm$	$f_i$	$x_i^0$	$f_i x_i^0$
0-10	8	5	40
10-20	12	15	180
20-30	10	25	250
30-40	8	35	280
40-50	3	45	135
50-60	2	55	110
60-70	7	65	455
	<u>50</u>		<u>1450</u>

Direct method

$$\bar{x} = \frac{\sum f_i x_i^0}{N}$$

$$= \frac{1450}{50}$$

$$= 29$$





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$C \pm$	$f_i$	$x_i$	$d_i = x_i - A$	$f_i d_i$
0-10	8	5	$5 - 35 = -30$	-240
10-20	12	15	$15 - 35 = -20$	-240
20-30	10	25	-10	-100
30-40	8	35 = A	0	00
40-50	3	45	10	30
50-60	2	55	20	40
60-70	7	65	30	210
	50			-300

Assumed mean

$$\bar{X} = A + \frac{\sum f_i d_i}{N}$$

$$= 35 + \frac{-300}{50}$$

$$= 35 - 6$$

$$= 29$$





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$C_i$	$f_i$	$x_i^o$	$U_i^o = \frac{x_i^o - A}{h}$	$f_i U_i^o$
0-10	8	5	-3	-24
10-20	12	15	-2	-24
20-30	10	25	-1	-10
30-40	8	35=A	0	0
40-50	3	45	1	3
50-60	2	55	2	4
60-70	7	65	3	21
	50			-30

$$\begin{aligned} \bar{x} &= A + \frac{\sum f_i U_i^o}{N} \times h \\ &= 35 + \frac{(-30)}{50} \times 10 \\ &= 35 - 6 \\ &= 29 \end{aligned}$$





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$C_i$	$f_i$	$x_i^o$	$f_i x_i^o$
1-4	6	2.5	15
4-7	8	5.5	44
7-10	2	8.5	17
10-13	9	11.5	103.5
13-16	5	14.5	72.5
	30		252

Direct method

$$\bar{x} = \frac{\sum f_i x_i^o}{N} = \frac{252}{30} = 8.4$$

$x_i$	$f_i$	$U_i$	$f_i U_i$
2.5	6	-2	-12
5.5	8	-1	-8
8.5 = A	2	0	0
11.5	9	1	9
14.5	5	2	10
	N=30		-1

$$\begin{aligned} \bar{x} &= A + \frac{\sum f_i U_i}{N} \times h \\ &= 8.5 + \frac{(-1)}{30} \times 3 \\ &= 8.5 - \frac{1}{10} \\ &= 8.5 - 0.1 = 8.4 \end{aligned}$$





For the data given below, find the missing frequency if the arithmetic mean is 28.

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
No. of students:	12	18	27	$f = ?$	17	6

A. 15

B. 20

C. 22

D. 26

$x_i$	$f_i$	$f_i x_i$
5	12	60
15	18	270
25	27	675
35	$f$	$35f$
45	17	765
55	6	330
$N = 80 + f$		$2100 + 35f$

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$28 = \frac{2100 + 35f}{80 + f}$$

$$\Rightarrow 2240 + 28f = 2100 + 35f$$

$$\Rightarrow 2240 - 2100 = 35f - 28f$$

$$\Rightarrow 140 = 7f$$

$$\Rightarrow \boxed{f = 20}$$





The average marks of 80 students were found to be 40. Later on it was discovered that a score of 54 was misread as 84. Find the correct mean of 80 students.

- A. 28
- B. 39.625**
- C. 38
- D. None

$N = 80$   
 $\bar{x} = 40$

$\bar{x} = \frac{\sum x_i}{n}$   
 $40 = \frac{\sum x_i}{80}$

wrong  $\sum x_i = 3200$   
 correct  $\sum x_i = 3200 - 84 + 54 = 3170$

$x_i$
54
<del>84</del>
<hr/>
<del>3200</del>

correct mean =  $\frac{3170}{80}$   
 $= 39.625$

$40 - \frac{30}{80}$   
 $= 40 - 0.375$   
 $= 39.625$





The arithmetic mean of the marks obtained by 50 students was calculated as 44. It was later discovered that a score of 36 was misread as 56. Find the correct value of the arithmetic mean of the marks obtained by the students.

- A. 43.6
- B. 44.4
- C. 43.4
- D. None

$$N = 50$$

$$\bar{x} = 44$$

$$\frac{\sum x_i}{N}$$

$$\frac{36}{\cancel{56}}$$

Correct mean

$$= \frac{2180}{50}$$

$$= 43.6$$

$$\frac{\sum x_i}{N} = 44$$

$$\frac{\sum x_i}{50} = 44$$

Wrong  $\sum x_i = 2200$

Correct  $\sum x_i = 2200 - 56 + 36 = 2180$

$$44 - \frac{20}{50}$$

$$44 - 0.4$$

$$43.6$$







Class 12<sup>th</sup> has three sections A, B and C with 50, 60, 50 students respectively. The mean marks for the three sections were determined as 73, 60 and 55 respectively. Find the mean marks of all three sections put together.

A. 62

B. 62.5

C. 63

D. None

$$\begin{array}{ccc} & A & B & C \\ N_1 = & 50 & N_2 = 60 & N_3 = 50 \\ \bar{x}_1 = & 73, & \bar{x}_2 = 60 & \bar{x}_3 = 55 \end{array}$$

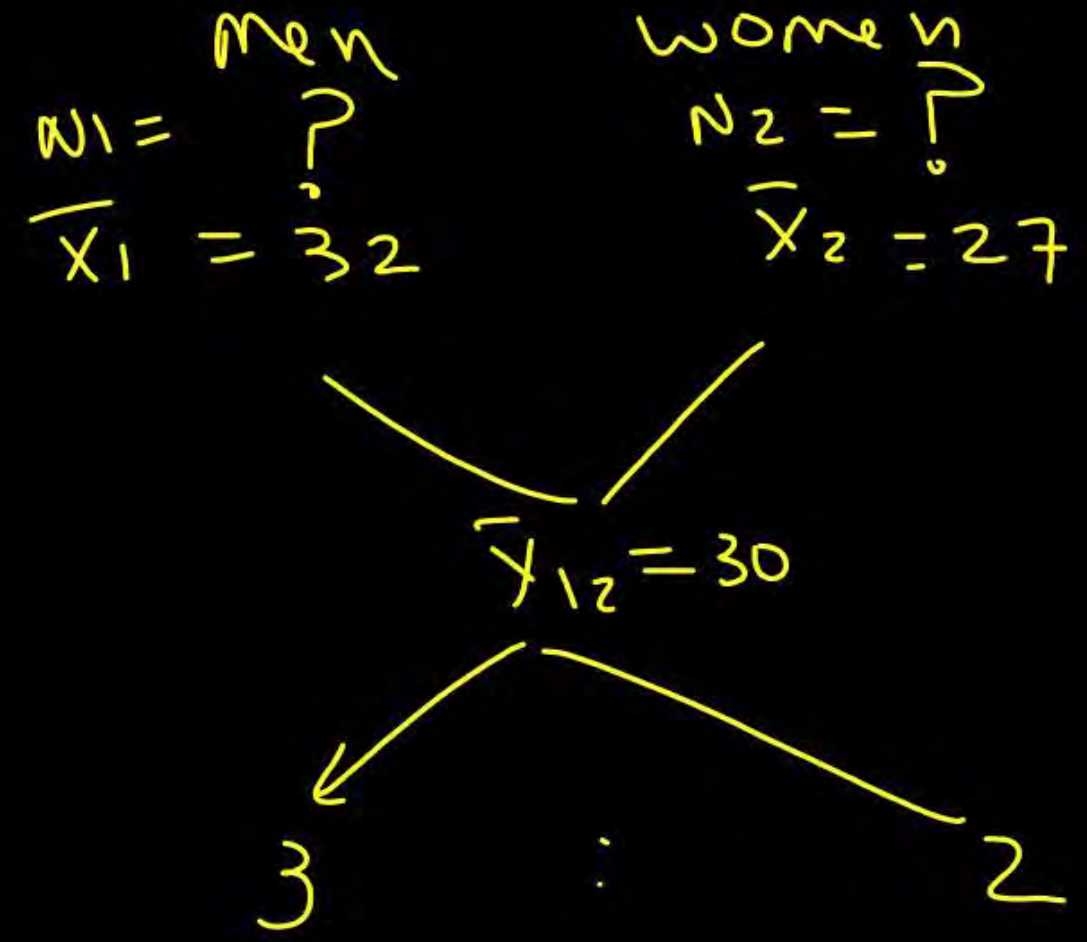
$$\begin{aligned} \bar{x}_{123} &= \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2 + N_3 \bar{x}_3}{N_1 + N_2 + N_3} \\ &= \frac{50(73) + (60)(60) + 50(55)}{50 + 60 + 50} \\ &= \frac{10,000}{160} = 62.5 \end{aligned}$$





The mean age of a combined group of men and women is 30 years. If the mean age of the group of men is 32 and that of the group women is 27, find the percentage of men

- A. 60%
- B. 40
- C. 35
- D. None



$N_1 : N_2 = 3 : 2$

men = 3  
 women = 2

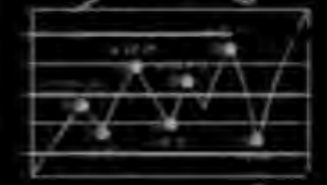
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men =  $\frac{3}{5} \times 100$   
 = 60%





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# Weighted Arithmetic Mean

Salary ( $x_i$ )	weight ( $w_i$ )	$w_i x_i$
8000	70	560000
10000	20	200000
40000	10	400000
	100	1,160,000

weighted mean

$$= \frac{\sum w_i x_i}{\sum w_i}$$

$$= \frac{1,160,000}{100}$$

$$= 11600$$





Calculate weighted mean from the following data:

Value:	10	12	15	18	20
Weight:	2	5	12	4	7

- A. 15.7
- B. 16.2
- C. 14.87
- D. 18.22

$x_i$	$w_i$	$w_i x_i$
10	2	20
12	5	60
15	12	180
18	4	72
20	7	140
30		472

Weighted mean

$$= \frac{\sum w_i x_i}{\sum w_i}$$

$$= \frac{472}{30} = 15.73$$





# # Change of origin

⇓  
when a fix number  $k$  is added or subtracted from each observation.

$$y_i = x_i + 10$$

$$\begin{array}{r} y_i \\ 12 \\ 15 \\ 18 \end{array}$$

$$\bar{y} = \frac{45}{3} = 15$$

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$$\begin{array}{r} x_i \\ 2 \\ 5 \\ 8 \end{array}$$

---

$$\bar{x} = \frac{15}{3} = 5$$

# If  $\bar{x}$  is the mean of some observations &  $k$  is added to each observation then new mean will be  $\bar{x} + k$





eg  $\bar{x} = 15$   
 If  $y_i = x_i + 25$   
 find  $\bar{y}$

Sol.  
 $\bar{y} = \bar{x} + 25$   
 $= 15 + 25$   
 $= 40$

eg  $\bar{x} = 30$   
 If 5 is subtracted from each item - find new mean.

Sol.  
 $\bar{x} = 30$   
 $y_i = x_i - 5$   
 $\bar{y} = ?$   
 $\bar{y} = 30 - 5$   
 $= 25$



# # Change of scale

when each item is multiplied  
or divided by some scalar 'k'

# If mean of some observation  
is  $\bar{x}$  & each element  
is multiplied by  $k$   
then new mean =  $k \bar{x}$

g

$$\frac{x_i}{3} = \frac{12}{3} = 4$$

$$y_i = 2x_i$$

$y_i$
2
8
5
150
11
24
3
8





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Qe

$$\bar{x} = 10$$

$$y_i = 6x_i$$

find

$$\bar{y} = ?$$

Sol.

$$\bar{y} = 6\bar{x}$$

$$= 6 \times 10$$

$$= 60$$

Q11

$$\bar{x} = 25$$

$$y_i = 3x_i + 10$$

find  $\bar{y} = ?$

Sol.

$$\bar{y} = 3\bar{x} + 10$$

$$= 3 \times 25 + 10$$

$$= 75 + 10$$

$$= 85$$





# Properties of A.M.

#1] If each element is constant 'k' then  $A.M. = k$

eg

$$\frac{\begin{matrix} x_i \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{matrix}}{\quad} = \frac{10}{5} = 2$$

#2] Combined mean

$$= \bar{x}_{12}$$

$$= \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

#3]  $\sum (x_i - \bar{x}) = 0$

$$\sum (k - \bar{x}) = 0$$





#

Sum of deviations from arithmetic mean is always zero.

$$\sum (x_i - \bar{x}) = 0$$

eg

$x_i$	$d_i = x_i - \bar{x} = x_i - 8$
1	$1 - 8 = -7$
2	$2 - 8 = -6$
5	$5 - 8 = -3$
24	$24 - 8 = 16$
$\bar{x} = \frac{32}{4} = 8$	0





# Sum of the Square of Deviation  
 is minimum only when deviations  
 are taken from Arithmetic mean.

i.e.  $\sum (x_i - A)^2$  is minimum

when  $A = \text{Arithmetic mean} (7)$

eg

$x_i$	$x_i - 5$	$x_i - 4$	$x_i - 6$	$x_i - 7$	$(x_i - 5)^2$	$(x_i - 4)^2$	$(x_i - 6)^2$	$(x_i - 7)^2$
1	-4	-3	-5	-6	16	9	25	36
5	0	1	-1	-2	0	1	1	4
12	7	8	6	5	49	64	36	25
					65	74	62	65

$$\bar{x} = \frac{18}{3} = 6$$

minimum





Sum of the square of deviation is minimum when taken from \_\_\_\_\_

A. MEAN

B. MEDIAN

C. MODE

D. None







If relationship between  $x$  and  $y$  is  $y=2x+10$  & mean of  $x$  is 25 ,then mean of  $y$  will be = ?

$$y = 2x + 10$$
$$\bar{x} = 25$$
$$\bar{y} = 2\bar{x} + 10$$
$$= 2 \times 25 + 10$$
$$= 60$$

A. 50

B. 35

C. 60

D. None





Which of the following statements is true?

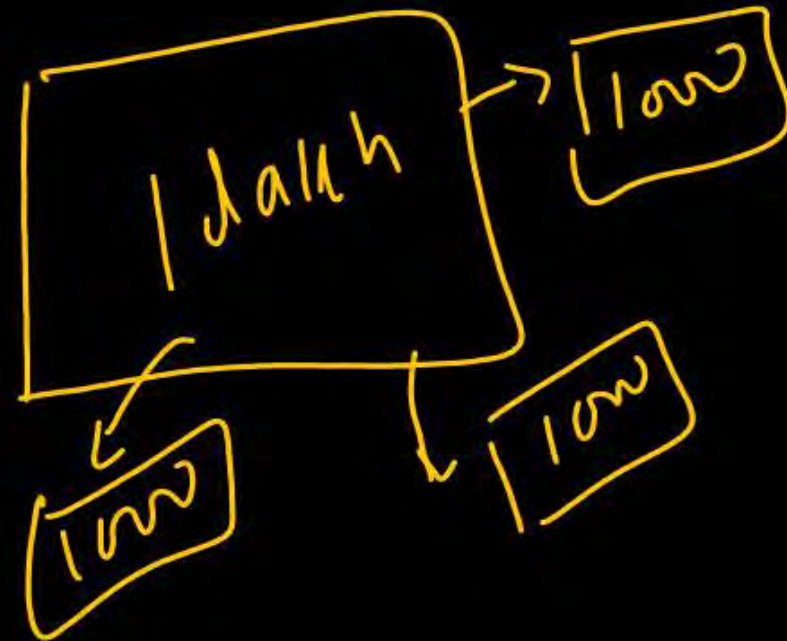
- (a) ✓ Usually mean is the best measure of central tendency
- (b) Usually median is the best measure of central tendency
- (c) Usually mode is the best measure of central tendency
- (d) Normally GM is the best measure of central tendency



Which of the following statements is wrong?

- (a) Mean is rigidly defined (correct)
- ~~(b)~~ Mean is not affected due to sampling fluctuations
- (c) Mean has some mathematical properties (correct)
- (d) All these X

mean is affected due to sampling fluctuations





Find the missing frequency from the following distribution , given that the median is 24.

Class interval :	0-10	10-20	20-30	30-40	40-50
Frequency:	5	25	-	18	7

A. 20

B. 25

C. 28

D. None





For open-end classification, which of the following is the best measure of central tendency?

(a) AM

(b) GM

(c) Median

(d) Mode





# Median



- A positional average (middle most value)
- It represents 50%
- A number which divides entire series in two equal parts





# # Individual series / Discrete series

Total no. of observation (n)





marks: 12, 18, 5, 2, 9, 18, 17  
 find median marks.

Sol:

- ✓  $x_i$
- ✓ 2
- ✓ 5
- ✓ 9
- ✓ 12 → median
- 17
- 18
- 18

$N = 7$  (odd)

median =  $\left(\frac{7+1}{2}\right)^{\text{th}}$  term  
 = 4<sup>th</sup> term  
 = 12





Ex

marks: 2, 3, 8, 12, 14, 16, 18, 18, 19, 20  
 find median marks

Sol:

$$N = 10 \text{ (even)}$$

$$\begin{aligned} \text{median} &= \frac{\left(\frac{N}{2}\right)^{\text{th}} + \left(\frac{N}{2} + 1\right)^{\text{th}}}{2} \\ &= \frac{\left(\frac{10}{2}\right)^{\text{th}} + \left(\frac{10}{2} + 1\right)^{\text{th}}}{2} \\ &= \frac{5^{\text{th}} + 6^{\text{th}}}{2} = \frac{14 + 16}{2} = 15 \end{aligned}$$

or

$$\begin{aligned} \text{median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} \\ &= \left(\frac{10+1}{2}\right)^{\text{th}} \\ &= (5.5)^{\text{th}} \\ &= 5^{\text{th}} + 0.5(6^{\text{th}} - 5^{\text{th}}) \\ &= 14 + 0.5(16 - 14) \\ &= 14 + 0.5(2) \\ &= 14 + 1 = 15 \end{aligned}$$





g

$x_i$
10
18
22
35
46
50

find median ?

sol:  $N = 6$

$$\text{median} = \left( \frac{n+1}{2} \right)^{\text{th}}$$

$$= \left( \frac{6+1}{2} \right)^{\text{th}}$$

$$= (3.5)^{\text{th}} \text{ term}$$

$$= 3^{\text{rd}} + 0.5(4^{\text{th}} - 3^{\text{rd}})$$

$$= 22 + 0.5(35 - 22)$$

$$= 22 + 0.5(13)$$

$$= 22 + 6.5 = 28.5$$





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# Discrete Series



(xi)	(fi)
Salary	no of employees
10,000	3
15,000	2
<b>20,000</b>	4
25,000	2
N = 11	

(Cumulative frequency)

C. f.
(1, 2, 3)
3
5 (4, 5)
9 (6, 7, 8, 9)
11 (10, 11)

$N = 11$

$$\text{Median} = \left( \frac{11+1}{2} \right)^{\text{th}} \text{ term}$$

$$= 6^{\text{th}} \text{ term}$$

$$= 20,000$$

find median salary

10,000, 10,000, 10,000, 15,000, 15,000, **20,000**, 20,000, 20,000, 20,000, 25,000, 25,000



Q

$x_i$	$f_i$	(C. f.)
Marks	No. of students	
1	4	4 (1-4)
2	6	10 (5-10)
3	5	15 (11-15)
4	7	22 (16-22)
5	2	24 (23-24)
	$N = 24$	

find median marks

$$\begin{aligned}
 \text{median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \\
 &= \left(\frac{24+1}{2}\right)^{\text{th}} \\
 &= (12.5)^{\text{th}} \\
 &= 12^{\text{th}} + 0.5(13^{\text{th}} - 12^{\text{th}}) \\
 &= 3 + 0.5(3 - 3) \\
 &= 3
 \end{aligned}$$

Sol. 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 5, 5





Find the value of median from the following data:

88 72 33 29 70 86 54 91 61 57

A. 61

B. 70

C. 65.5

D. None





Find the value of median from the following data:

8      3      11      7      12      6      9

**A.** 8

**B.** 7

**C.** 9

**D.** None



Calculate median from the following data:

X:	10	15	20	25	30
F:	2	5	4	9	11

A. 20

B. 25

C. 18

D. 26

$x_i$	$f_i$	$cf$
10	2	2 (1-2)
15	5	7 (3-7)
20	4	11 (8-11)
25	9	20 (12-20)
30	11	31 (21-31)
<hr/>		$N=31$

$$\begin{aligned} \text{median} &= \left( \frac{31+1}{2} \right)^{th} \\ &= 16^{th} \\ &= 25 \end{aligned}$$



# Median in continuous series

- find  $\frac{N}{2}$
- locate  $\frac{N}{2}$  in cf
- Select median class

$$M_e = l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h$$

$l$  = Lower limit of median class

cf = Cumulative frequency preceding to median class

$f$  ⇒ frequency median class

$h$  ⇒ class length







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<u>CI</u>	<u>fi</u>	<u>cf</u>	
0-4	1	1	x
4-8	2	3	x
8-12	1	4	x
12-16	8	12	
16-20	4	16	
	16		

median class

find median



sol:  $N = 16$   
 $\frac{N}{2} = 8$

median class = 12-16  
 $l = 12$

$$\begin{aligned} \text{median} &= l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h \\ &= 12 + \left\{ \frac{8 - 4}{8} \right\} \times 4 \\ &= 12 + \frac{4 \times 4}{8} \\ &= 12 + 2 = 14 \end{aligned}$$



Marks:	0-10	10-20	20-30	30-40	40-50	50-60
No. of students :	12	16	18	25	20	9

Calculate the median marks

- A. 31
- B. 31.4
- C. 31.6
- D. 32

CI	$f_i$	Cf
0-10	12	12
10-20	16	28
20-30	18	46
30-40	25	71
40-50	20	91
50-60	9	100

$N = 100$

median class

$$\begin{aligned} \frac{N}{2} &= \frac{100}{2} = 50 \\ M_e &= l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h \\ &= 30 + \left\{ \frac{50 - 46}{25} \right\} \times 10 \\ &= 30 + \frac{4}{25} \times 10 \\ &= 31.6 \end{aligned}$$





<u>CI</u>	<u>f<sub>i</sub></u>
5-10	4
11-16	3
17-22	6
23-28	2

find median

Inclusive class interval

<u>CI</u>	<u>f<sub>i</sub></u>	<u>cf</u>
4.5 - 10.5	4	4
10.5 - 16.5	3	7
16.5 - 22.5	6	13
22.5 - 28.5	2	15

N = 15

$$\frac{N}{2} = \frac{15}{2} = 7.5$$

$$\begin{aligned}
 m_e &= l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h \\
 &= 16.5 + \left\{ \frac{7.5 - 7}{6} \right\} \times 6 = 16.5 + 0.5 \\
 &= 17
 \end{aligned}$$





Find the missing frequency from the following distribution, given that the median is 24.

Class interval :	0-10	10-20	20-30	30-40	40-50
Frequency:	5	25	$f = ?$	18	7

$$8f = (f - 5)10$$

- A. 20 ~~X~~
- B. 25 ✓
- C. 28
- D. None

Ci	fi	Cf
0-10	5	5
10-20	25	30
20-30	f	30 + f
30-40	18	48 + f
40-50	7	55 + f
		$N = 55 + f$

$N = 55 + f$   
 med class = 20-30  
 median = 24

$$Me = l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h$$

$$24 = 20 + \left\{ \frac{55 + f}{2} - 30 \right\} \times 10$$

$$4 = \left( \frac{55 + f - 60}{2f} \right) \times 10$$

$$8f = 10f - 50$$

$$-2f = -50$$

$$f = 25$$





$$\sum (x_i - \bar{x}) = 0$$

$$\sum (x_i - A)^2 \text{ is minimum when } A = \bar{x}$$

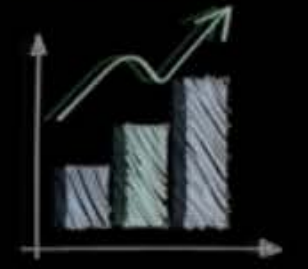


# Sum of absolute deviation is minimum when deviations are taken from median.

$$\sum |x_i - m| \text{ is minimum}$$

or

$$\sum (x_i - A) \text{ is minimum when } A = \text{Median}$$





$x_i$	$x_i - \bar{x}$	$x_i - m$	$ x_i - \bar{x} $	$ x_i - m $
1	-2	-1	2	1
2	-1	0	1	0
6	3	4	3	4
			6	5

$\bar{x} = \frac{9}{3} = 3$

$m_e = 2$

5  
↓ minimum





	mean	median	mode
Change of origin	✓	✓	✓
Change of scale	✓	✓	✓

$$y = a + bx$$

$$\text{mean of } \bar{y} = a + b(\bar{x})$$

$$\text{median of } y = a + b(\text{median of } x)$$





If relationship between  $x$  and  $y$  is  $y=3x-2$  & median of  $x$  is 10, then median of  $y$  will be = ?

- A. 30
- B. 8
- C. 28
- D. None

$$y = 3x - 2$$

$$\begin{aligned} \text{Median of } y &= 3(10) - 2 \\ &= 28 \end{aligned}$$







Sum of the absolute deviation is minimum when taken from \_\_\_\_\_

A. MEAN

B. MEDIAN

C. MODE

D. None

$\sum |x_i - m|$   
Minimum



For open-end classification, which of the following is the best measure of central tendency?

- (a) AM
- (b) GM
- (c) Median
- (d) Mode



30-36  
20-35  
15-35

<u>C</u>	<u>X<sub>i</sub></u>
X less than 35	X
30-35 ✓	
35-40 ✓	
40-45 ✓	
45-50 ✓	
50-55 ✓	
more than 55	X





The presence of extreme observations does not affect

- (a) AM
- (b) Median
- (c) Mode
- (d) Any of these.





Q

<u>Marks</u>	<u>no of Students</u>	<u>CF</u>
less than 4	2	
less than 8	5	
less than 12	12	
" " 16	14	
" " 20	15	

<u>Sol:</u>	<u>CF</u>	<u>f<sub>i</sub></u>
	0-4	2
	4-8	3
	8-12	7
	12-16	2
	16-20	1

find median marks.

Sol:

2, 3, 6, 7, 7, 9, 9, 10, 11, 11, 11, 11, 13, 15, 18





eg

Marks  
less than

	cf		
	no of Students	CI	fi
2	6	0-2	6
4	10	2-4	4
6	12	4-6	2
8	18	6-8	6
10	20	8-10	2
12	21	10-12	1
14	25	12-14	4





g

<u>marks</u>	<u>cf</u> <u>No of students</u>	<u>CI</u>	<u>fi</u>
more than 0	23	0-5	3
more than 5	20	5-10	6
more than 10	14	10-15	2
" 15	12	15-20	7
" 20	5	20-25	3
" 25	2	25-30	2

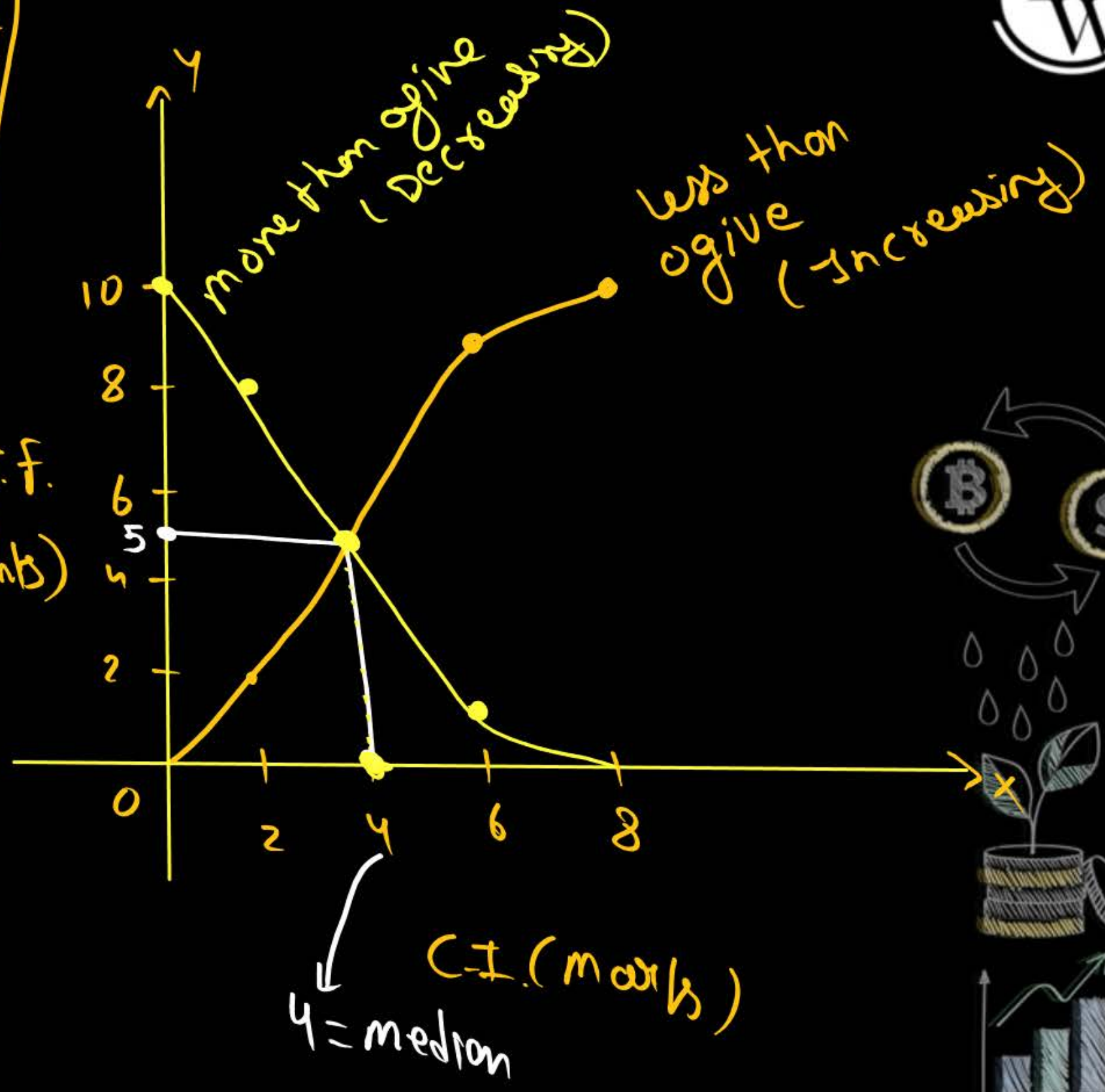




marks CI	$f_i$	$x_i$ less than	$y_i$ Cf	more than	Cf
0-2	2	2	2	0	10
2-4	3	4	5	2	8
4-6	4	6	9	4	5
6-8	1	8	10	6	1
10					

$N = 10$   
 $\frac{N}{2} = 5$

median  
 graphically  $\Rightarrow$  4





Savings



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median  
↓



50%

↓  
single value





# Quartiles



$Q_1$   $Q_2$  &  $Q_3$  are three Quartile  
which represent 25%, 50% & 75% of series

Divide entire series in 4 parts





# Individual series / Discrete series

$$Q_1 = \left( \frac{N+1}{4} \right)^{\text{th}} \text{ term}$$

$$Q_3 = \left[ 3 \left( \frac{N+1}{4} \right) \right]^{\text{th}} \text{ term}$$

## Continuous series

→ locate  $\frac{N}{4}$  &  $\frac{3N}{4}$  in cf  
& select  $Q_1$  class &  $Q_3$  class

$$\rightarrow Q_1 = l + \left\{ \frac{\frac{N}{4} - cf}{f} \right\} \times h \quad \& \quad Q_3 = l + \left\{ \frac{\frac{3N}{4} - cf}{f} \right\} \times h$$



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g

$x_i$
✓ 10
✓ 12
✓ 15 → $Q_1$
20
35
45 → $Q_2$
65
75
80 → $Q_3$
90
98

$n = 11$

$$Q_1 = \left( \frac{n+1}{4} \right)^{th}$$

$$= \left( \frac{11+1}{4} \right)^{th}$$

$$= 3^{rd} \text{ term}$$

$Q_1 = 15$

$$Q_3 = \left[ 3 \left( \frac{n+1}{4} \right) \right]^{th}$$

$$= \left[ 3 \left( \frac{11+1}{4} \right) \right]^{th}$$

$$= 9^{th}$$

$Q_3 = 80$

$Q_2 = \text{median}$

$$= \left( \frac{n+1}{2} \right)^{th} = 6^{th} = 45$$





Q

$x_i$	$f_i$	cf
1	3	3 (1-3)
2	2	5 (4-5)
3	5	10 (6-10)
4	6	16 (11-16)
5	8	24 (17-24)
6	2	26
7	4	30
N=30		

now

$$Q_1 = \left( \frac{30+1}{4} \right)^{th}$$

$$= (7.75)^{th}$$

$$= 7^{th} + 0.75(8^{th} - 7^{th})$$

$$= 3 + 0.75(3 - 3)$$

$$= 3$$

now

$$Q_3 = \left[ 3 \left( \frac{30+1}{4} \right) \right]^{th}$$

$$= (23.25)^{th}$$

$$= 23^{th} + 0.25(24^{th} - 23^{th})$$

$$= 5 + 0.25(5 - 5) = 5$$

find  $Q_1$  &  $Q_3$

1, 1, 1, 2, 2, 3, 3, 3, 3, 3, 4, 4

4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5





g	$C \pm$	$f_i$	$Cf$
	0-8	2	2
$Q_1$ class	8-16	6	8
	16-24	8	16
$Q_3$ class	24-32	4	20
	32-40	2	22
	$N = 22$		

$$N = 22$$

$$\frac{N}{4} = \frac{22}{4} = 5.5$$

$$\frac{3N}{4} = \frac{3 \times 22}{4} = 16.5$$

now

$$Q_1 = l + \left\{ \frac{\frac{N}{4} - cf}{f} \right\} \times h$$

$$= 8 + \left\{ \frac{5.5 - 2}{6} \right\} \times 8$$

$$= 12.67$$

now

$$Q_3 = l + \left\{ \frac{\frac{3N}{4} - cf}{f} \right\} \times h$$

$$= 24 + \left\{ \frac{16.5 - 16}{4} \right\} \times 8$$

$$= 25$$





# Decile

⇓  
Divide entire series in 10 parts.

⇓  
Total Deciles = 9 (D<sub>1</sub>, D<sub>2</sub>, ..., D<sub>9</sub>)



⇓  
5<sup>th</sup> Decile  
D<sub>5</sub> = median





## Individual series / Discrete

$$D_1 = \left(\frac{N+1}{10}\right)^{th}, \quad D_3 = \left[3\left(\frac{N+1}{10}\right)\right]^{th}, \quad D_8 = \left[8\left(\frac{N+1}{10}\right)\right]^{th}$$

## Continuous series

for  $D_1$

→ locate  $\frac{N}{10}$  in cf  
& select  $D_1$  class

$$\rightarrow D_1 = l + \left\{ \frac{\frac{N}{10} - cf}{f} \right\} \times h$$

for  $D_6$

→ locate  $6\frac{N}{10}$  in cf  
→ select  $D_6$  class

$$\rightarrow D_6 = l + \left\{ \frac{6\frac{N}{10} - cf}{f} \right\} \times h$$









# Mode

↓  
 observation with  
 highest frequency

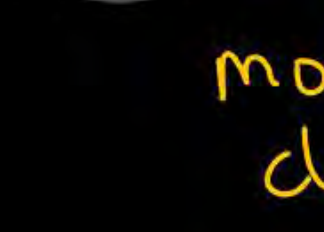
eg marks : 2, 9, 8, 7, 6, 6, 5, 7, 8  
 7, 9, 7, 3, 4, 7, 5, 6, 7  
 mode = 7

marks ( $x_i$ )	No of students ( $f_i$ )
1	5
2	3
3	4
4	12
5	7

→ Highest frequency

mode = 4





marks	$f_i$
0-4	6
4-8	3
8-12	9 $\rightarrow f_0$
12-16	12 $\rightarrow f_1$
16-20	10 $\rightarrow f_2$
20-24	4

modal class

find mode

$$\text{mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h$$

$$= 12 + \left\{ \frac{12 - 9}{2(12) - 9 - 10} \right\} \times 4$$

$$= 12 + \frac{(3) \times 4}{5}$$

$$= 12 + 2.4$$

$$= 14.4$$





g

C I	f <sub>i</sub>
0-2	4
2-6	6
6-8	8
8-12	2

find mode

Sol:

C I	f <sub>i</sub>
0-2	4
2-4	3
4-6	3 → f <sub>0</sub>
6-8	8 → f <sub>1</sub>
8-10	1 → f <sub>2</sub>
10-12	1

modal class

$$\text{mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h$$

$$= 6 + \left\{ \frac{8 - 3}{16 - 3 - 1} \right\} \times 2$$

$$= 6 + \frac{5}{12} \times 2$$

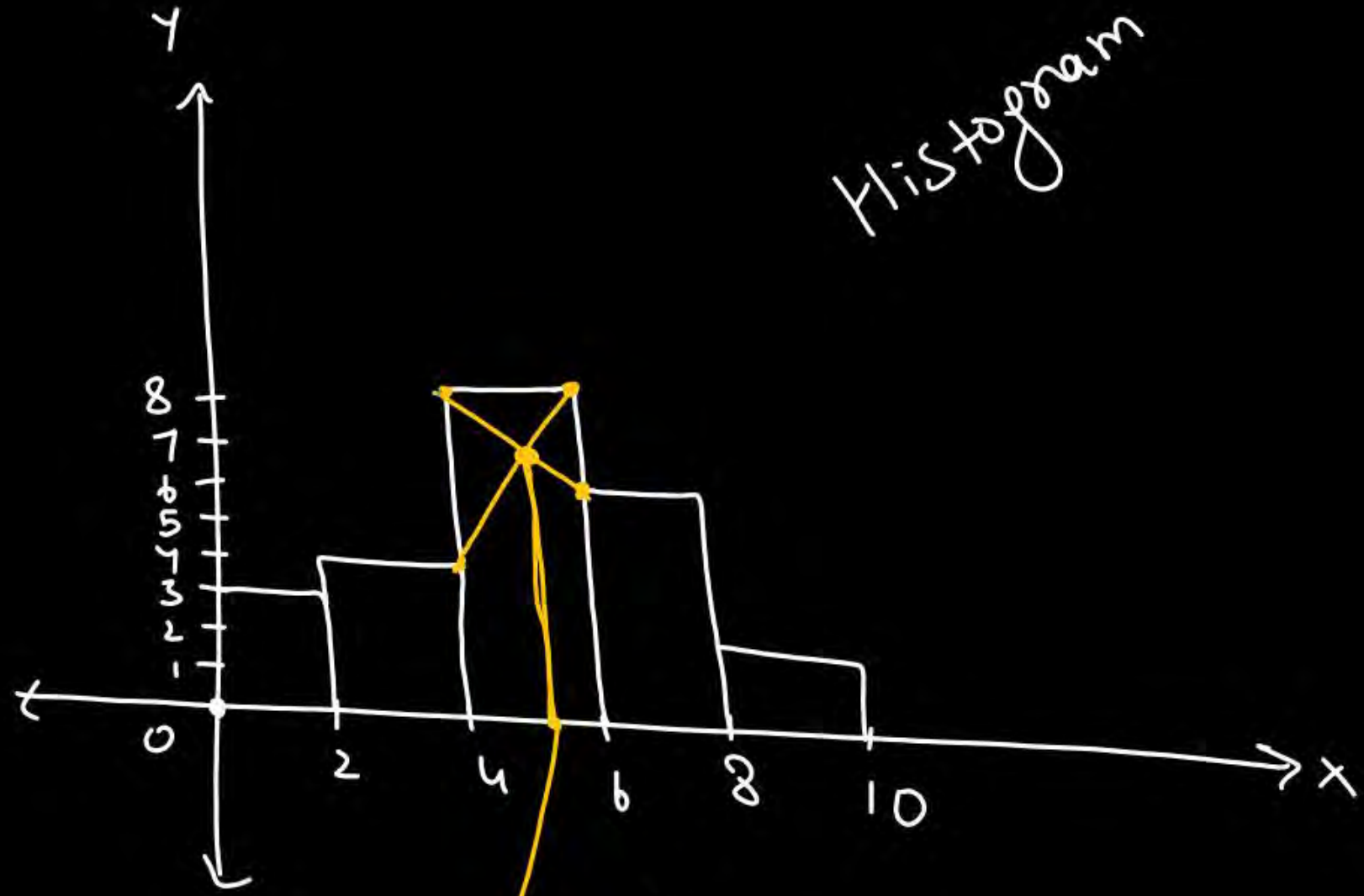
$$= 6.83$$





eg

CI	$f_i$
0-2	3
2-4	4
4-6	8
6-8	6
8-10	2



5.2  
mode



If  $x$  and  $y$  are related by  $x - y - 10 = 0$  and mode of  $X$  is known to be  $23$ , then the mode of  $y$  is

A. 10

B. 12

C. 13

D. None

$$x - y - 10 = 0 \quad \& \quad \text{mode of } x = 23$$

$$x - 10 = y$$

$$y = x - 10$$

$$\begin{aligned} \text{mode of } y &= 23 - 10 \\ &= 13 \end{aligned}$$



# Relation b/w Mean, median & mode

$$\# \quad 3 \text{ median} = \text{mode} + 2 \text{ mean}$$

$\&$   
 Mean = 20  
 mode = 15  
 median = ?

Sol.  
 $3 \text{ Median} = 15 + 2(20)$   
 $3 \text{ median} = 55$   
 $\text{median} = \frac{55}{3} = 18.33$

$\#$  Diff b/w mode & mean = 3 [Diff b/w median & mean]

$$\# \quad \text{mode} - \text{mean} = 3(\text{median} - \text{mean})$$

$$\# \quad \text{mean} - \text{mode} = 3(\text{mean} - \text{median})$$



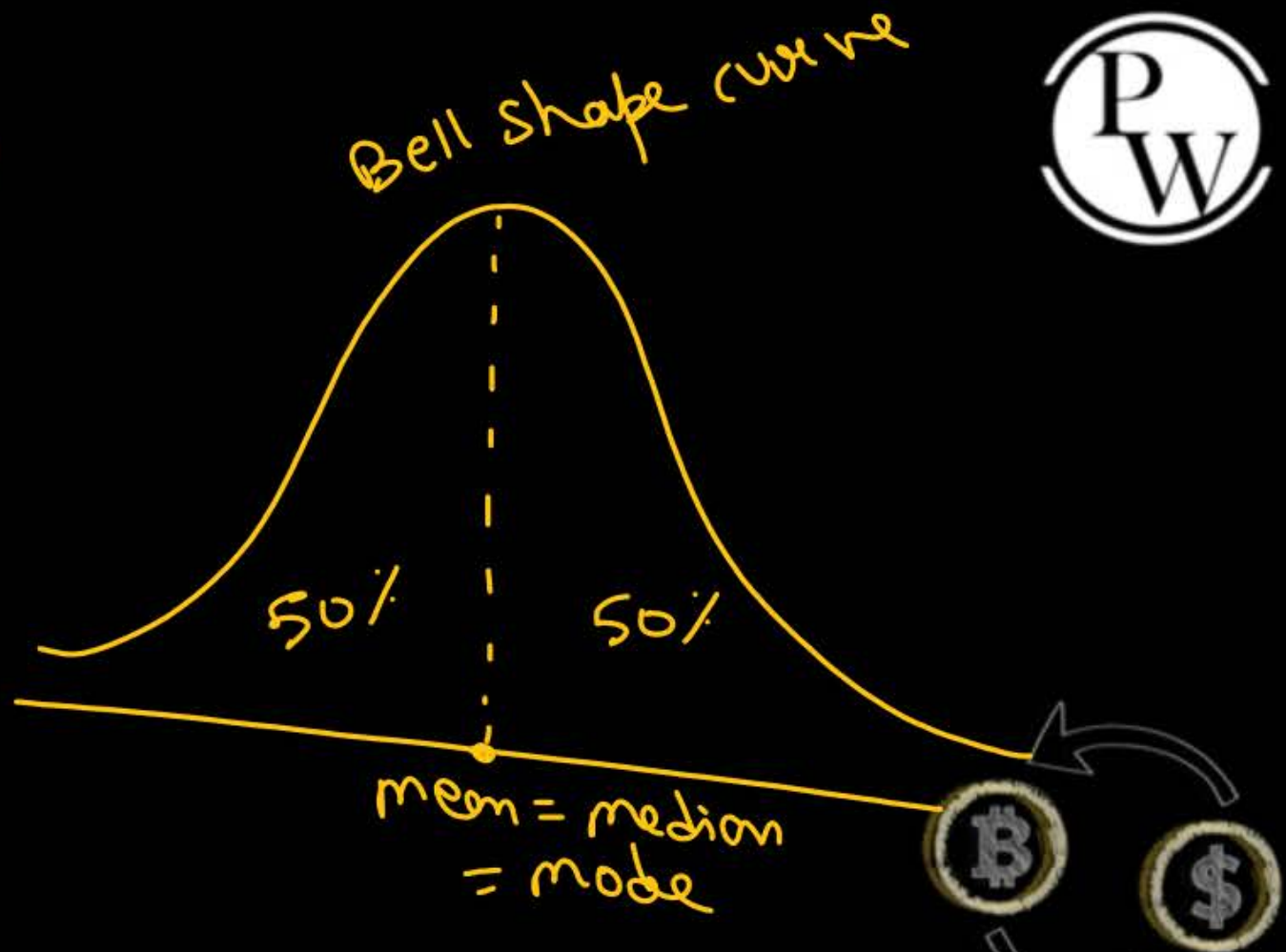


#

Symmetrical Distribution



mean = median = mode



#

non symmetrical Distribution



mean ≠ median ≠ mode

mean < median < mode

mean > median > mode





# Geometric Mean

(Any Rate of increase, decrease  
Any of % , Any Depreciation  
Any of Ratios)



$\Downarrow$   
 $n^{\text{th}}$  Root of the product of 'n' observations

$$GM = \left[ x_1 \times x_2 \times x_3 \times \dots \times x_n \right]^{\frac{1}{n}}$$

eg  
GM of 2 & 8  
 $= (2 \times 8)^{\frac{1}{2}}$   
 $= (16)^{\frac{1}{2}} = 4$

eg  
GM of 2, 4 & 8  
 $= (2 \times 4 \times 8)^{\frac{1}{3}}$   
 $= (64)^{\frac{1}{3}} = 4$







9 find hrm of 2, 3, 5 & 8

$$= (2 \times 3 \times 5 \times 8)^{1/4}$$
$$= (240)^{1/4}$$
$$= 3.9386$$



$(x)^{1/n} = ?$

$\rightarrow \sqrt{\quad}$  12 times

$\rightarrow \frac{\quad}{n}$

$\rightarrow + 1$

$\rightarrow \boxed{x = \quad}$  12 times





9

$x_i$	$f_i$
2	3
3	2
4	1
5	2

find G.M

Sol:

$$\begin{aligned} G.M &= (2 \times 2 \times 2 \times 3 \times 3 \times 4 \times 5 \times 5)^{1/8} \\ &= (7200)^{1/8} \\ &= 3.0382 \end{aligned}$$





eg

$CI$	$f_i$	$x_i$
0-4	3	2
4-8	2	6
8-12	5	10

find  $GM$

Sol.

$$GM = \left( 2 + 2 + 2 \times 6 + 6 \times 10 \times 10 \times 10 \times 10 \times 10 \right)^{1/10}$$
$$= 5.589$$



Find geometric mean of:

3,6,24 and 48

A. 28

B. 32

C. 36

D. None

$$\begin{aligned}
 \text{GM} &= (3 \times 6 \times 24 \times 48)^{\frac{1}{4}} \\
 &= (20736)^{\frac{1}{4}} = (20736)^{\frac{1}{2} + \frac{1}{2}} \\
 &= \underline{12.0271} \qquad \qquad \qquad = 12
 \end{aligned}$$





# If all observations are same (let say  $k$ )  
then  $\ln m = k$

eg  $\ln m$  of 2, 2, 2, 2

$$= (2 + 2 + 2 + 2)^{1/4}$$
$$= (4)^{1/4}$$
$$= (2)$$





$$GM = G = (x_1 \times x_2 \times x_3 \times \dots \times x_n)^{1/n}$$

$$\log G = \log [x_1 x_2 x_3 \dots x_n]^{1/n}$$

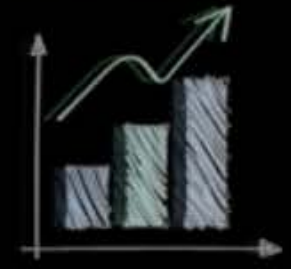
$$= \frac{1}{n} \log [x_1 x_2 x_3 \dots x_n]$$

$$\log G = \frac{\log x_1 + \log x_2 + \log x_3 + \dots + \log x_n}{n}$$

$$\# \quad \log G = \frac{\sum \log x_i}{N} \Rightarrow G = AL \left[ \frac{\sum \log x_i}{N} \right]$$

$$\log(x^n) = n \log x$$

$$\log(xy) = \log x + \log y$$





$$\# \quad \text{GM of } (xy) = \text{GM of } x \times \text{GM of } y$$

$$\# \quad \text{GM of } \left(\frac{x}{y}\right) = \frac{\text{GM of } x}{\text{GM of } y}$$

e.g.	$\frac{x}{2}$	$\frac{y}{3}$	$\frac{xy}{6}$	$\frac{x}{\frac{8}{2}}$
	8	27	216	$\frac{2}{\frac{8}{27}}$

$$\text{GM of } x = (2 \times 8)^{\frac{1}{2}} = 4$$

$$\text{GM of } y = (3 \times 27)^{\frac{1}{2}} = 9$$

$$\text{GM of } (xy) = (6 \times 216)^{\frac{1}{2}} = 36$$

$$\text{GM of } x \times \text{GM of } y = 4 \times 9 = 36$$

$$\text{GM of } \left(\frac{x}{y}\right) = \left(\frac{2}{3} \times \frac{8}{27}\right)^{\frac{1}{2}} = 0.4444$$

$$\frac{\text{GM of } x}{\text{GM of } y} = \frac{4}{9} = 0.444444$$



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If GM of x is 10 and GM of y is 15, then the GM of xy is

- A. 150
- B. 1.5
- C.  $\text{Log}(150)$
- D. None

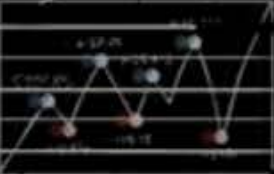
$$\begin{aligned} \ln(xy) &= \ln x + \ln y \\ &= 10 + 15 \\ &= 150 \end{aligned}$$







Savings



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Q If GM of 2 items is 4 &  
 GM of other 3 items is 8  
 find combined GM of all 5 items.

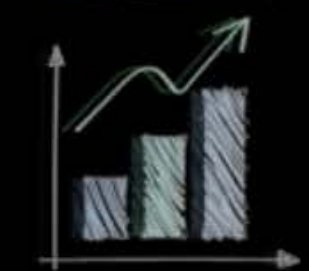
Sol.

$$\left[ \begin{aligned} (a \times b)^{1/2} &= 4 \\ a \times b &= (4)^2 = 16 \\ (p + q + r)^{1/3} &= 8 \\ p + q + r &= (8)^3 = 512 \end{aligned} \right.$$

$$\left. \begin{aligned} \text{Combined GM} &= (a + b + p + q + r)^{1/5} \\ &= (16 + 512)^{1/5} \\ &= (8192)^{1/5} \\ &= (6.07) \end{aligned} \right\}$$

$$\begin{aligned} \text{Combined GM} &= \left[ G_1^{N_1} \times G_2^{N_2} \right]^{1/(N_1 + N_2)} \end{aligned}$$

$$\begin{aligned} \text{Combined GM} &= \left[ 4^2 \times 8^3 \right]^{1/(2+3)} \\ &= (8192)^{1/5} \end{aligned}$$





Savings



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$$(2 \times 8)^{1/2} = \sqrt{16} = 4$$

$$\left[ (2) \times (-8) \right]^{1/2} = \sqrt{-16} = \text{NOT DEFINED}$$





Any speed work time

# Harmonic Mean (H.M.)

(H.M.)

"Hm is the reciprocal of the average of reciprocal of 'n' items"

$$HM = \frac{1}{\left[ \frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}{N} \right]}$$

$$HM = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$





eg Hm of 2 & 4

$$\begin{aligned} \text{Hm} &= \frac{2}{\frac{1}{2} + \frac{1}{4}} \\ &= \frac{2}{0.5 + 0.25} \\ &= 2.67 \end{aligned}$$

$$\begin{aligned} &= \frac{2}{\frac{2+1}{4}} \\ &= \frac{2 \times 4}{3} \\ &= \frac{8}{3} \\ &= 2.67 \end{aligned}$$





g Hm of 2, 3 & 5

$$Hm = \frac{3}{\frac{1}{2} + \frac{1}{3} + \frac{1}{5}}$$

$$= \frac{3}{\left( \frac{15 + 10 + 6}{30} \right)}$$

$$= \frac{90}{31} = 2.90$$

$$= \frac{3}{0.5 + 0.3333 + 0.20}$$

$$= \frac{3}{1.03333} = 2.90$$

$$\frac{3}{1.03333}$$





Square

⇓  
 $x =$

Reciprocal

⇓  
 $\frac{\quad}{\quad} =$





Find the harmonic mean of 5 numbers 4, 5, 6, 10 and 12.

A. 2.5

B. 6.25

C. 1.50

D. None

$$\begin{aligned} Hm &= \frac{5}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{10} + \frac{1}{12}} \\ &= \frac{5}{0.80} \\ &= 6.25 \end{aligned}$$





Savings



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$$Hm = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

$$Hm = \frac{N}{\sum \left( \frac{f_i}{x_i} \right)}$$







g

$x_i$	$f_i$	$\frac{f_i}{n_i}$
1	2	2
2	5	2.50
3	6	2
4	2	0.50
$N=15$		7

$$Hm = \frac{15}{\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$$

find H.m.

Sol.  $Hm = \frac{N}{\sum \left( \frac{f_i}{n_i} \right)} = \frac{15}{7} = 2.14$





eg

$x_i$	$f_i$	$\frac{f_i}{x_i}$
5	10	2
10	20	2
15	45	3
20	15	0.75
$N = 90$ HM		7.75

$$= \frac{N}{\sum \left( \frac{f_i}{x_i} \right)} = \frac{90}{7.75} = 11.61$$





g

$C_i$	$f_i$	$x_i$	$\frac{f_i}{x_i}$
0-2	3	1	3
2-4	8	3	2.67
4-6	5	5	1
6-8	4	7	0.57

$$H_m = \frac{N}{\sum \left( \frac{f_i}{x_i} \right)}$$





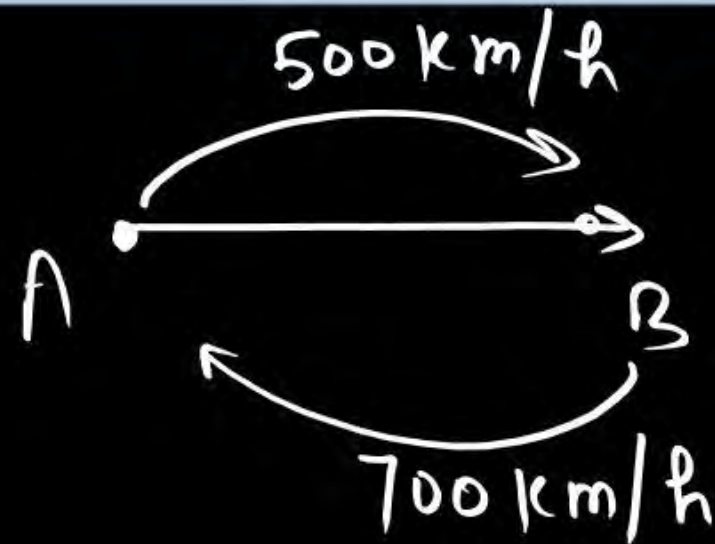
An aeroplane flies from A to B at the rate of 500 km/hour and comes back from B to A at the rate of 700 km/hour. The average speed of the aeroplane is

A. 100

B. 583.33

C.  $100\sqrt{35}$

D. 620



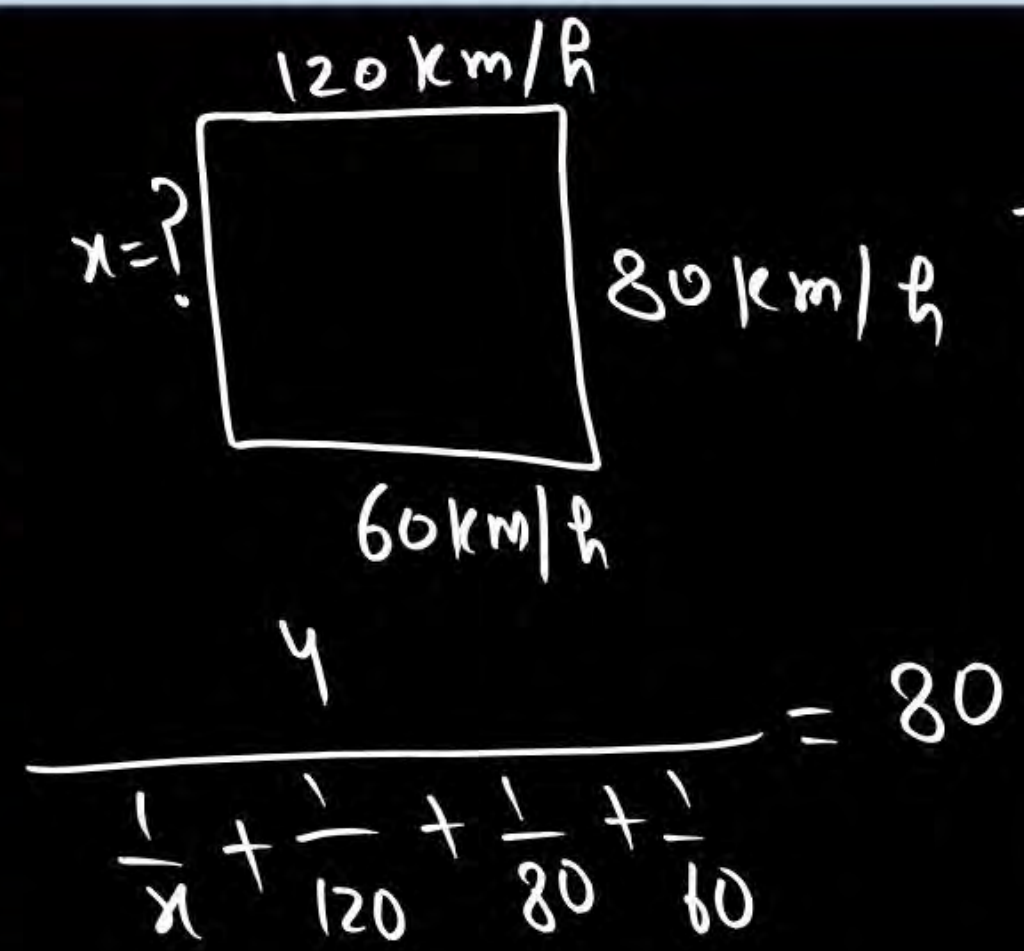
$$\begin{aligned} \text{Avg speed} &= \frac{2 \times \text{Hm} (500 \& 700)}{\frac{1}{500} + \frac{1}{700}} \\ &= \frac{2}{\frac{1}{500} + \frac{1}{700}} = 583.33 \end{aligned}$$





A taxi travelled around a square at an overall speed of 80 kmph. If it travelled at an average speed of 60 kmph on the first side, at 80 kmph on the second side and at 120 kmph on the third side, what must have been the average speed on the fourth side?

- A. 60
- B. 80
- C. 90
- D. None



Avg speed of all 4 sides = 80 km/h

$$\frac{4}{80} = \frac{1}{x} + \frac{1}{120} + \frac{1}{80} + \frac{1}{60}$$

$$\frac{1}{20} - \frac{1}{120} - \frac{1}{80} - \frac{1}{60} = \frac{1}{x}$$

$$0.05 - 0.0083 - 0.0125 - 0.0166 = \frac{1}{x}$$

$$0.0125 = \frac{1}{x}$$

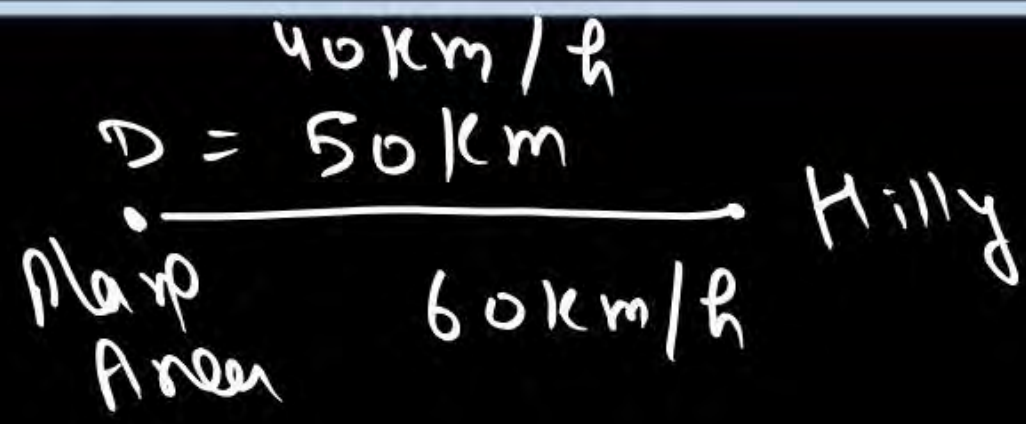
$$x = \frac{1}{0.0125} = 80 \text{ km/h}$$





A car travels 50 km from plain to a hill station at a speed of 40 kmph. It then makes the return trip at a speed of 60 kmph. Find the average speed per hour by using appropriate average

- A. 48
- B. 50
- C. 54
- D. None



$$Hm = \frac{2}{\frac{1}{40} + \frac{1}{60}} = \frac{2}{\frac{3+2}{120}} = \frac{240}{5} = 48 \text{ km/h}$$





# If all observations are same (let say  $k$ )  
Then  $Hm = k$

g  $Hm$  of  $2, 2, 2, 2 = \frac{4}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{4}{2} = 2$

# Combined  $Hm = \frac{N_1 + N_2}{\left(\frac{N_1}{H_1}\right) + \left(\frac{N_2}{H_2}\right)}$



Savings

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The harmonic mean for 40 observations of group 1 data is 520 and for 50 observations of group 2 data is 680. What is the combined harmonic mean?

- A. 450.60
- B. 598.20**
- C. 800
- D. None

$$N_1 = 40 \quad | \quad N_2 = 50$$

$$H_1 = 520 \quad | \quad H_2 = 680$$

$$H_m = \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$$

$$= \frac{40 + 50}{\frac{40}{520} + \frac{50}{680}} = \frac{90}{0.15045} = 598.19$$







eg Hm of 3 items is 5  
 Hm of 2 items is 10  
 find combined Hm ?

Sol:

$$H_1 = H_1 = 5$$

$$\frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} = 5$$

$$\frac{3}{5} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

$$H_2 = 10$$

$$\frac{2}{\frac{1}{x} + \frac{1}{y}} = 10$$

$$\frac{2}{10} = \frac{1}{x} + \frac{1}{y}$$

Combined Hm

$$= \frac{5}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{x} + \frac{1}{y}}$$

$$= \frac{5}{\frac{3}{5} + \frac{2}{10}}$$

$$Hm = \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$$

$$= \frac{3 + 2}{\frac{3}{5} + \frac{2}{10}}$$





If all items are same  
 $AM = HM = GM$

g

2, 2, 2, 2

$AM = 2$

$GM = 2$

$HM = 2$



If all observations are different

2 & 8

$AM = \frac{2+8}{2} = 5$

$GM = \sqrt{2 \times 8} = 4$

$HM = \frac{2}{\frac{1}{2} + \frac{1}{8}} = \frac{2+8}{5} = 3.2$

#  $AM > GM > HM$





$$\# \quad \{ \quad Am \geq hm \geq Hm \quad \}$$

$$\# \quad \{ \quad \text{for only two numbers } a \ \& \ b \\ Am \times Hm = hm^2 \quad \}$$



eg for two numbers  
 $Hm = 5$   
 $Am = 20$   
 $hm = ?$

Sol:  $hm^2 = Am \times Hm$   
 $= 20 \times 5$   
 $hm^2 = 100$   
 $hm = 10$





$$\# \text{ Weighted AM} = \frac{\sum w_i x_i}{\sum w_i}$$

$$\# \text{ Weighted HM} = \frac{\sum w_i}{\sum \left( \frac{w_i}{x_i} \right)}$$

$$\# \text{ Weighted GM} = \text{A.L.} \left[ \frac{\sum w_i (\log x_i)}{\sum w_i} \right]$$





$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n(n+1)(2n+1)}{6}$$



$$\text{A.M. of first 'n' natural no} = \frac{n+1}{2}$$

eg Am of first 5 natural no.

$$= \frac{(1+2+3+4+5)}{5}$$

$$= \frac{15}{5}$$

$$= 3$$

OR  $\frac{5+1}{2}$

$$= 3$$





The A.M. and G.M. of two numbers are 15 and 12 respectively.  
Find the H.M. of the numbers

A. 2

B. 15

C. 20

D. 9.6 ✓✓

$$\begin{aligned} \text{Am} &= 15 \\ \text{Gm} &= 12 \\ \text{Hm} &= ? \end{aligned}$$

$$\begin{aligned} \text{Am} \times \text{Hm} &= \text{Gm}^2 \\ 15 \times \text{Hm} &= 12^2 \\ \text{Hm} &= \frac{144}{15} \\ &= 9.6 \end{aligned}$$





The median of  $x, x/2, x/3, x/5$  is 10

Then the value of  $x$

(2009)

A. 24

B. 8

C. 32

D. 16

$$x, \frac{x}{2}, \frac{x}{3}, \frac{x}{5}$$

$$\text{median} = \frac{\left(\frac{x}{2}\right) + \left(\frac{x}{3}\right)}{2}$$

$$2 \text{ median} = \frac{x}{2} + \frac{x}{3}$$

$$2 \times 10 = \frac{3x + 2x}{6}$$

$$120 = 5x$$

$$x = 24$$







If the A and G are the AM & GM of two unequal quantities then

(2009)

A.  $A < G$

B.  $A > G$  ✓

C.  $A \leq G$

D.  $A \geq G$

$$A > S > H$$





Harmonic mean of  $1, 1/2, 1/3, 1/4, \dots, 1/n$  is

(2010)

A.  $1/(n+1)$

B.  $2/(n+1)$

C.  $\frac{n+1}{2}$

D.  $\frac{1}{n-1}$

$$H.M = \frac{n}{1+2+3+\dots+n}$$

$$= \frac{1}{\frac{1}{3+1}}$$

$$= \frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$= \frac{2}{\left[ \frac{\cancel{n}(n+1)}{2} \right]}$$

$$= \frac{2}{n+1}$$

$$H.M = \frac{3}{1+2+3}$$

$$= \frac{3}{6} = \frac{1}{2}$$

$n=3$





The difference between mode and mean is 63, then the difference between median and mean will be ? (2011)

A. 63

B. 21 ✓

C. 31.5

D. None

$$\text{mode} - \text{mean} = 3(\text{median} - \text{mean})$$

$$63 = 3(\text{med} - \text{mean})$$

$$\frac{63}{3} = \text{med} - \text{mean}$$

21





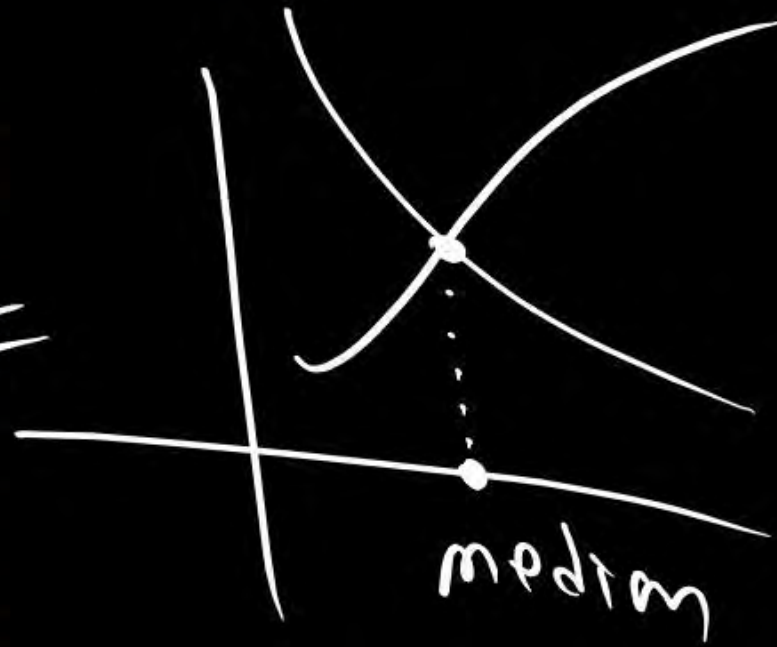
The intersection point of less than ogive and more than ogive gives the \_\_\_ of a frequency distribution. (2012)

A. mean

B. median

C. Mode

D. None





The harmonic mean of two numbers is 4, their A.M. A, and G.M. G. satisfy the relation  $2A + G^2 = 27$ . (2014)

A.  $1,3 \times 2$

B.  $9,5 \times 7$

~~C.  $6,3 \ll 4,5$~~

D.  $12,7 \times$

$Hm = 4$

$Am = A$

$Gm = G$

$Gm^2 = Am \times Hm$

$G^2 = A \times 4$

$G^2 = 4A$

$2A + G^2 = 27$

$2A + 4A = 27$

$6A = 27$

$A = 4.5$

$\frac{9+3}{2} = 4.5$

$9+3=9$





The average of 10 observations is 14.4. If the average of first 4 observations is 16.5. The average of remaining 6 observations is: (2016)

- A. 13.6
- B. 12.8
- C. 13
- D. None

$$\begin{array}{l|l} N_1 = 4 & N_2 = 6 \\ \hline \bar{x}_1 = 16.5 & \bar{x}_2 = ? \end{array}$$

$$\bar{x}_{12} = 14.4$$
$$\frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2} = 14.4$$

$$\frac{16.5(4) + 6(\bar{x}_2)}{10} = 14.4$$

$$66 + 6\bar{x}_2 = 144$$
$$\bar{x}_2 = 13$$





When all observation have same frequency then which central tendency does not exist ? (2017)

- A. Mean
- B. Median
- C. *Mode*
- D. Gm

$x_i$	$f_i$
2	10
3	10



Relation between mean median and mode ? (2018)



$$3 \text{ median} = \text{mode} + 2 \text{ mean}$$

~~A.~~  $\text{Mean} - \text{median} = \text{Mode} - \text{mean}$

~~B.~~  $\text{Mean} - \text{Mode} = 2(\text{Mode} - \text{median})$

~~C.~~  $\text{Mode} - \text{Mean} = 3(\text{Median} - \text{Mean})$

~~D.~~  $\text{Mode} - \text{Mean} = 3(\text{Mean} - \text{Median})$







Which of the following is a positional average ?

(2019)

A. Mean

B. Median

C. Mode

D. Gm

50%





The AM and GM of two observations are 30 and 24  
The two numbers are ? (2019)

~~A.~~ 12,24

B. 12,48

~~C.~~ 30,30

~~D.~~ 20,40

$$Am = 30 \text{ \& } Gm = 24$$

$$\frac{a+b}{2} = 30$$

$$a+b = 60$$

$$\sqrt{a \times b} = 24$$

$$a \times b = 24^2 = 576$$





50<sup>th</sup> percentile is ?

(2020)

A. Median

B. Mean

C. *HM*

D. GM

$Q_2 = 850 = D_5 = \text{median}$





There are  $n$  numbers. When 50 is subtracted from each of these number the sum of the numbers so obtained is -10. When 46 is subtracted from each of the original  $n$  numbers, then the sum of numbers, So obtained is 70. What is the mean of the original  $n$  numbers? (2021)

$$\bar{x} = ?$$

(2021)

A. 56.8

B. 25.7

C. 49.5

D. 53.8

$$\sum (x_i - 50) = -10 \quad \& \quad \sum (x_i - 46) = 70$$

$$\sum x_i - 50n = -10$$

$$\sum x_i = 50n - 10 \quad \text{--- (1)}$$

$$\sum x_i - 46n = 70$$

$$\sum x_i = 46n + 70 \quad \text{--- (2)}$$





$$50n - 10 = 46n + 70$$

$$4n = 80$$

$$n = 20$$

Now

$$\sum x_i = 50n - 10$$

$$= 50(20) - 10$$

$$= 1000 - 10$$

$$\sum x_i = 990$$

Now

$$\bar{X} = \frac{\sum x_i}{N} = \frac{990}{20} = 49.5$$



$$\sum (2x_i) = 2 \sum x_i$$

$$\sum (2) = 2n$$

$$= 2n$$

$$\sum (3) = 3n$$

$$= 3n$$

$$\sum (1) = n$$

$$= n$$





**T H A N K  
Y O U**

**KEEP REVISING  
&  
STAY MOTIVATED !!**

