CHAPTER

5

Basic Concepts of Permutations and Combinations

Permutation and combination are essential concepts that help solve complex counting problems. For instance, let's consider the scenario of arranging members of the Lok Sabha in different seating arrangements.

FUNDAMENTAL PRINCIPLE OF COUNTING

- **Multiplication Rule**: If one task can be done in m ways and another task which is independent of the first task can be done in n ways, after the first task has been performed, then the number of possible ways in which both the tasks can be done simultaneously is $m \times n$.

  E.g.: Let's say Task A is choosing a shirt from a collection of 5 different shirts, and Task B is selecting a pair of pants from a collection of 3 different pants. Using the Multiplication Rule:

  Number of ways to choose a shirt = 5

  Number of ways to select a pair of pants = 3

  Total number of ways to choose a shirt and pants simultaneously = $5 \times 3 = 15$

Example 1. Raghav has 3 different types of shirts and 2 different types of trousers. Whenever he goes out, he likes to wear a shirt and a trouser. In how many ways can he decide what to wear?

(a) 5 (b) 4 (c) 6 (d) 8

Sol. (c) Given, Number of shirts = 3

Number of trousers = 2

Total ways = $3 \times 2 = 6$

Hence, the correct option is (c).

- **Addition Rule**: If one task can be done in m ways and another task which is independent of the first task can be done in n ways, then the total number of ways either of them can perform is $m + n$.

  E.g.: Task A: Choosing a dessert from a menu with 4 options.

  Task B: Selecting a drink from a menu with 3 options.

  Number of ways to choose a dessert = 4

  Number of ways to select a drink = 3

  Total number of ways to either choose a dessert or select a drink = $4 + 3 = 7$

Example 2. Rani has 3 different types of shoes and 2 different types of sandals. Whenever she goes out, she likes to wear either a shoe or a sandal. In how many ways can she decide what to wear?

(a) 3 (b) 2 (c) 5 (d) 6

Sol. (c) Given, Number of shoes = 3

Number of sandals = 2

Since, she likes to wear either a shoe or a sandal, thus

Total required ways = 3 + 2 = 5

Hence, the correct option is (c).

What we understand is that, if "And" comes in a statement use MULTIPLICATION RULE, when "Or" comes in a statement use ADDITION RULE.

Example 3. A room has 6 doors. In how many ways can a man enter the room through one door and come out through a different door?

(a) 30 (b) 36 (c) 64 (d) 80

Sol. (a) Given, Total doors = 6

If entered from one door then there are 6 options available and for exit, there are 5 doors.

Total ways = $5 \times 6 = 30$

Hence, the correct option is (a).

Example 4. In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy or 1 girl to represent the class in a function. In how many ways can the teacher make this selection?

(a) 41 (b) 224 (c) 278 (d) 378

Sol. (a) Given, Total boys = 27

Total girls = 14

Since, the teacher wants to select 1 boy or 1 girl as class representative, thus

Total required ways = 27 + 14 = 41

Hence, the correct option is (a).

Example 5. Given 4 flags of different colors, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

(a) 5 (b) 4 (c) 12 (d) 8

Sol. (c) Given, There are 4 flags of different colors.

Since, a signal requires the use of 2 flags one below the other.

Thus, the selection of the first signal can be done in 4 ways and that of the second can be done in 3 ways.

Using, multiplication rule,

Total required signals = $4 \times 3 = 12$

Hence, the correct option is (c).

Example 6. In a monthly test, the teacher decides that there will be three questions, one from each of Exercise 7, 8 and 9 of the textbooks. If there are 12 questions in Exercise 7, 18 in Exercise 8 and 9 in Exercise 9, in how many ways can three questions be selected?

(a) 1214 (b) 1234 (c) 6000 (d) 1944

Sol. (d) ***Given:*** Number of questions in Exercise 7 = 12,

Number of questions in Exercise 8 = 18,

Number of questions in Exercise 9 = 9

Total ways of selecting three questions = $12 \times 18 \times 9 = 1944$

Hence, the correct option is (d).

Example 7. In how many ways can 5 letters be posted in 4 letter boxes?

(a) 120 (b) 1024 (c) 625 (d) None of these

Sol. (b) Total number of letters = 5

Total number of letter boxes = 4

$\therefore$ For each letter, there are 4 options (letter boxes) where it can be posted.

Thus, total number of ways = $4 \times 4 \times 4 \times 4 \times 4 = 1024$

Hence, the correct option is (b).

Example 8. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, when

(I) the repetition of the letters is not allowed.

(II) the repetition of the letters is allowed.

(a) 16, 186 (b) 24, 256 (c) 24, 172 (d) None of these

Sol. (b) Given word = ROSE

Total letters = 4

(I) repetition of letters is not allowed = $4 \times 3 \times 2 \times 1 = 24$

(II) repetition of letters is allowed = $4 \times 4 \times 4 \times 4 = 4^4 = 256$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART A)

1. There are 10 trains running between Calcutta and Delhi. The number of ways in which a person can go from Calcutta to Delhi and return by a different train is

   (a) 99 (b) 90 (c) 80 (d) None of these

2. You have four different colors of socks, and you want to wear one sock of each color. In how many ways can you choose which socks to wear?

   (a) 8 ways (b) 12 ways (c) 16 ways (d) 24 ways

3. A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to 'A' by any mode other than the one earlier. The number of different ways, the entire journey can be complete is

   (a) 110 (b) 10^{10} (c) 9^5 (d) 10^9

4. There are ten flights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different flight is

(a) 90 (b) 95 (c) 80 (d) 78

5. Consider the word "TRAIN". How many distinct 5-letter words can be formed from its letters without repeating any letter?

(a) 120 (b) 24 (c) 360 (d) 30

6. How many 3 digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated?

(a) 55 (b) 75 (c) 65 (d) 36

7. In how many ways can 3 letters be posted in 4 letter boxes?

(a) 24 (b) 27 (c) 64 (d) None of these

Answer Key

1. (b) **2.** (b) **3.** (a) **4.** (a) **5.** (a) **6.** (b) **7.** (c)

THE FACTORIAL

If we take a number 'n', then n! or ∠n is called the factorial of 'n' and the value of n! is equal to multiplication of 1 to n i.e. $n! = 1 \times 2 \times 3 \times 4 \times \ldots \times n$ or we can write, $n! = n(n-1)(n-2) \ldots 3 \times 2 \times 1$

Note: $0! = 1$

Example 9. Find the value of

(I) 8! (II) 5! (III) $\frac{10!}{4!}$

(a) 5040, 720, 75600 (b) 20,160, 120, 151200

(c) 40,320, 120, 151200 (d) 40,320, 520, 151100

Sol. (c) We know that,

$n! = n(n-1)(n-2) \ldots 3 \times 2 \times 1$

(I) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$

(II) $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

(III) $\frac{10!}{4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 1{,}51{,}200$

Hence, the correct option is (c).

Example 10. Find x if $\frac{1}{6!} + \frac{1}{5!} = \frac{x}{7!}$

(a) 49 (b) 81 (c) 88 (d) 91

Sol. (a) Given,

$$\frac{1}{6!} + \frac{1}{5!} = \frac{x}{7!} \Rightarrow \frac{1}{6!} + \frac{6}{6 \times 5!} = \frac{x}{7!} \Rightarrow \frac{1}{6!} + \frac{6}{6!} = \frac{x}{7!}$$

$$\Rightarrow \frac{1+6}{6!} = \frac{x}{7!} \Rightarrow \frac{7}{6!} = \frac{x}{7!} \Rightarrow x = \frac{7! \times 7}{6!}$$

$\Rightarrow x = 7 \times 7 = 49$

Hence, the answer is option (a).

Example 11. Find the value of n if $(n + 1)! = 30(n - 1)!$

(a) 6 (b) 5 (c) 7 (d) None of these

Sol. (b) Given, $(n + 1)! = 30(n - 1)!$

$\Rightarrow (n + 1) \times n \times (n - 1)! = 30(n - 1)!$

$\Rightarrow (n + 1) \times n = 30$

$\Rightarrow (n + 1) \times n = 6 \times 5 \Rightarrow n + 1 = 6$

$\Rightarrow n = 5$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART B)

1. Find the value of 6!

   (a) 720 (b) 360 (c) 120 (d) None of these

2. The value of $\frac{9!}{5!}$ is

   (a) 362,880 (b) 15,120 (c) 3024 (d) None of these

3. The value of $\frac{5!}{10!}$ is

   (a) $\frac{1}{30240}$ (b) 30240 (c) 120 (d) 36,28,800

4. Find n if $(n + 2)! = 12(n)!$

   (a) 4 (b) 3 (c) 6 (d) None of these

5. Find the value of x if $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$.

   (a) 11 (b) 121 (c) 150 (d) None of these

Answer Key

1. (a) 2. (c) 3. (a) 4. (a) 5. (b)

LET US UNDERSTAND THE TWO WORDS

1. **PERMUTATION:** In P**ermutation**, we consider the arrangement of objects in a specific order. The order of the arrangement matters.

   For example, let's consider the football team selection scenario from your class of 30 students. The coach wants to select 11 students for the football team and arrange them in specific positions: forward player, midfielder, backward player, and goalkeeper.

   In this case, the order of selection and arrangement of the players is important.

2. **COMBINATION:** In **Combination**, we consider the selection of objects without any specific order. The order of the selection does not matter.

   Continuing with the football team example, if we only want to select 11 students for the team without considering their specific positions, we are dealing with combinations. Here, the order in which the students are selected does not matter.

   In case of Permutation: We do arrangements, and we say order matters.

   In case of Combination: We do selection, and we say order doesn't matter.

PERMUTATIONS

A permutation determines the number of possible arrangements in a set when the order of the arrangements matters. It can be calculated using the formula:

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

where n is the total number of objects and r is the number of objects to be arranged.

E.g., if we have three different boxes: one Yellow, one Green, and one Red and we want to arrange them on a table, the number of possible arrangements (permutations) would be:

$$^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 6$$

In the scenario where all n students are winners, the permutation formula becomes:

$$^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

Example 12. Evaluate each of the following:

(I) $^{4}P_{2}$ (II) $^{7}P_{3}$ (III) $^{10}P_{6}$

Sol. (I) $^{4}P_{2} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4 \times 3 \times 2!}{2!} = 4 \times 3 = 12$

(II) $^{7}P_{3} = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \times 6 \times 5 \times 4!}{4!} = 210$

(III) $^{10}P_{6} = \frac{10!}{(10-6)!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} = 151200$

Example 13. In $^{n}P_{r}$, the restriction is **(ICAI)**

(a) $n > r$ (b) $n \geq r$ (c) $n \leq r$ (d) None of these

Sol. (b) $^{n}P_{r}$ stands for the permutation of n objects taken r at a time. In other words, it is the number of ways in which r objects can be selected from n distinct objects, where the order of selection matters.

The restriction in $^{n}P_{r}$ is that we can select only r objects out of n objects, and the order of selection matters. This means that we cannot select more than r objects or less than r objects, and we have to select them in a specific order.

Therefore, the correct answer is option 'b', which states that the restriction in ${}^nP_r$ is $n \geq r$. This means that we need at least r objects to select from a total of n objects.

Hence, the answer is option (b).

Example 14. Justify 0! = 1.

Sol. We know that,

$n! = n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1$

Let us consider for n = 3

$3! = 3 \times 2 \times 1$

$$\Rightarrow \frac{3!}{3} = 2!$$

$$\text{or } 2! = \frac{3!}{3}$$

In general, $(n-1)! = \dfrac{n!}{n}$

Put n = 1 in above equation, we get

$$(1-1)! = \frac{1!}{1} \Rightarrow 0! = \frac{1}{1} \Rightarrow 0! = 1$$

Example 15. If ${}^nP_4 = 12 \times {}^nP_2$, then n is equal to **(ICAI)**

(a) –1 (b) 6 (c) 5 (d) None of these

Sol. (b) **Detailed method:**

Given: ${}^nP_4 = 12 \times {}^nP_2$

$$\Rightarrow \frac{n!}{(n-4)!} = 12\frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{[n(n-1)(n-2)(n-3)(n-4)!]}{(n-4)!} = 12\frac{n(n-1)(n-2)!}{(n-2)!}$$

$\Rightarrow n(n-1)(n-2)(n-3) = 12n(n-1)$

$\Rightarrow n(n-1)[(n-2)(n-3) - 12] = 0$

$\Rightarrow n(n-1)(n^2 - 5n + 6 - 12) = 0$

$\Rightarrow n(n-1)(n^2 - 5n - 6) = 0$

$\Rightarrow n(n-1)[n^2 - (6-1)n - 6] = 0$

$\Rightarrow n(n-1)[n^2 - 6n + n - 6] = 0$

$\Rightarrow n(n-1)[n(n-6) + 1(n-6)] = 0$

$\Rightarrow n(n-1)(n-6)(n+1) = 0$

$\Rightarrow n = 0 \text{ or } n - 1 = 0 \text{ or } n - 6 = 0 \text{ or } n + 1 = 0$

$\Rightarrow n = 0 \text{ or } n = 1 \text{ or } n = 6 \text{ or } n = -1$

Since, n cannot be negative.

Therefore, $n \neq -1$

If n = 0 or 1, then in those cases (n – 2) and (n – 3) are not positive integers.

Thus, $n = 6$ is the correct answer.

Go by choices:

Given: ${}^nP_4 = 12 \times {}^nP_2$

We know that,

For ${}^nP_r$, $n \geq r$

Also, n and r are positive.

Thus, n cannot be −1.

For option (b): n = 6

$$\Rightarrow \text{LHS: } {}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$$

$$\Rightarrow \text{RHS: } 12 \times {}^6P_2 = 12 \times \frac{6!}{(6-2)!} = 12 \times \frac{6!}{4!} = 12 \times 6 \times 5 = 360$$

Clearly, LHS = RHS

Therefore, the required value of n is 6.

Hence, the answer is option (b).

Example 16. If ${}^5P_r = 60$, then the value of r is **(ICAI)**

(a) 3 (b) 2 (c) 4 (d) None of these

Sol. (a) $\Rightarrow \frac{5!}{(5-r)!} = 60 \Rightarrow \frac{5 \times 4 \times 3 \times 2}{(5-r)!} = 60 \Rightarrow \frac{120}{(5-r)!} = 60$

$\Rightarrow (5 - r)! = 2$

Since, $(2)! = 2$

Therefore, $(5 - r) = 2$

$r = 5 - 2 = 3$

Hence, the answer is option (a).

Example 17. If ${}^{n_1+n_2}P_2 = 132$ and ${}^{n_1-n_2}P_2 = 30$, then **(ICAI)**

(a) $n_1 = 6, n_2 = 6$ (b) $n_1 = 10, n_2 = 2$

(c) $n_1 = 9, n_2 = 3$ (d) None of these

Sol. (c) Given: ${}^{n_1+n_2}P_2 = 132 \Rightarrow \frac{(n_1+n_2)!}{(n_1+n_2-2)!} = 132$ ...(i)

Also ${}^{n_1-n_2}P_2 = 30$

$$\Rightarrow \frac{(n_1-n_2)!}{(n_1-n_2-2)!} = 30 \quad ...(ii)$$

Go by choices:

For option (c): $n_1 = 9, n_2 = 3$

$$\Rightarrow \frac{(9-3)!}{(9-3-2)!} = 30 \quad (eq.\ ii)$$

$$\Rightarrow \frac{(6)!}{(4)!} = 30 \Rightarrow \frac{6 \times 5 \times 4!}{(4)!} = 30$$

$\Rightarrow 30 = 30$

$\Rightarrow$ LHS = RHS

Therefore, the required values are $n_1 = 9$, $n_2 = 3$.

Hence, the answer is option (c).

Example 17. How many 4 letter words can be formed from 'COMPUTER'?

(a) 1223 (b) 1680 (c) 7880 (d) 7200

Sol. (b) The word" COMPUTER" has 8 different letters and we have to form words using any 4 letters from the 8 given letters. Therefore, total numbers of words that can be formed:

$$^8P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$$

Hence, the answer is option (b).

Example 18. The number of arrangements of the letters in the word `FAILURE', so that vowels are always coming together is **(ICAI)**

(a) 576 (b) 575 (c) 570 (d) None of these

Sol. (a) Given word: FAILURE

Since, all the 4 vowels should be coming together thus we will assume all 4 vowels as one letter i.e. letters would be F, L, R and AEIU

Now, the possible arrangement of above 4 letters = 4!

But vowels can also rearrange their positions (AEIU, AIUE, AUIE, etc.)

Total possible arrangement of vowels = 4!

Therefore, total arrangements will be $4! \times 4! = 24 \times 24 = 576$

Hence, option (a) is correct.

Example 19. 10 examination papers are arranged in such a way that the best and worst papers never come together. The number of arrangements is **(ICAI)**

(a) $9 \times 8!$ (b) $10!$ (c) $8 \times 9!$ (d) None of these

Sol. (c) Number of ways in which 10 papers can be arranged is 10!

When worst and best paper come together come together consider them as one paper

We will have only 9 papers, so 9 papers can be arranged in 9! ways

And two papers can arrange themself in 2! ways

So, the number of arrangements when the worst and best don't come together

$= 10! - 9! \times 2!$

$= 10 \times 9! - 9! \times 2!$

$= 9! \times (10 - 2) = 8 \times 9!$

Hence, option (c) is correct.

Example 20. 3-digit numbers to be formed out of the figures 0, 1, 2, 3, 4, 7, 8, 9 (no digit is repeated) then number of such numbers is

(a) 336 (b) 294 (c) 1050 (d) None of these

Sol. (b) Given digits: 0, 1, 2, 3, 4, 7, 8, 9

We can't use 0 for the first place, so there are 7 ways to fill the first place.

Now, second place can be filled in 7 ways since at second place 0 can be filled.

Now, for third place there are 6 ways to fill it.

Thus, there are $7 \times 7 \times 6 = 294$ ways to form a 3–digit number using the given digits.

Hence, the correct option is (b).

Example 21. If 19 states teams are participating in a national singing contest then the number of ways the first, second and third positions may be won is

(a) 5814 (b) 93024 (c) 342 (d) None of these

Sol. (a) Given: Number of teams participating = 19

So, number of ways first, second and third positions may be won is $^{19}P_3$

$\Rightarrow \dfrac{19!}{(19-3)!} = \dfrac{19!}{16!}$

$\Rightarrow 19 \times 18 \times 17 = 5814$

Hence, the answer is option (a).

Example 22. In how many ways among 8 students – (5 boys and 3 girls), can school select School Prefect, Head boy and Head Girl, if no member can hold two positions and each boy and girl is eligible for School Prefect too?

(a) 90 (b) 150 (c) 60 (d) None of these

Sol. (a) Given,

Total students = 8

Number of boys = 5

Number of girls = 3

Now, if a girl is selected as a prefect, then,

Number of ways of choosing head girl and a girl prefect = $^3P_2 = 6$

Number of ways of choosing head boy is $^5P_1 = 5$

Similarly, if a boy is selected as a prefect, then

Number of ways of selecting head girl = $^3P_1 = 3$

Number of ways of choosing head boy and a boy prefect is $^5P_2 = 20$

Therefore, total no. of ways = $6 \times 5 + 3 \times 20$

$= 30 + 60 = 90$

Therefore, there are a total 90 required ways.

Hence, the correct option is (a).

Example 23. The number of ways the letters of the word `TRIANGLE' to be arranged so that the word 'ANGLE' will be always present is

(a) 20 (b) 60 (c) 24 (d) 32

Sol. (c) Given: In the word 'TRIANGLE' the word 'ANGLE' should always be present.

Thus, consider 'ANGLE' as one letter so the letters will be T, R, I and 'ANGLE'.

Therefore, there are 4 letters which can be arranged in 4! i.e. 24 ways.

Hence, option (c) is correct.

PRACTICE QUESTIONS (PART C)

1. Evaluate $^5P_3$

(a) 30 (b) 60 (c) 120 (d) None of these

2. How many numbers divisible by 5 of 6 digits can be made from the digit 2, 3, 4, 5, 6, 7?

(a) 120 (b) 600 (c) 240 (d) None of these

3. Find the value of n if $^{n-1}P_3 : {}^nP_4 = 1 : 9$.

(a) 3 (b) 9 (c) 10 (d) None of these

4. In how many different ways can five persons stand in a line for a group photograph?

(a) 120 (b) 60 (c) 20 (d) None of these

5. If there are 6 books on Accounts, 3 on Business Mathematics and 2 on Economics. In how many ways can we place them if the books on the same subject are to be together ?

(a) 5184 (b) 8,640 (c) 25,920 (d) 51,840

6. The Number of ways in which the letters of the word 'DOGMATIC' can be arranged is :

(a) 40319 (b) 40320 (c) 40321 (d) None of these

7. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

(a) 40320 (b) 5040 (c) 6720 (d) None of these

8. The total number of ways in which six '+' and four '–' sign can be arranged in a line such that no two '–' sign occur together is **(ICAI)**

(a) $\frac{7!}{3!}$ (b) $\frac{6!\times7!}{3!}$ (c) 35 (d) None of these

9. How many numbers of seven digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits being repeated are not divisible by 5?

(a) 4320 (b) 4690 (c) 3900 (d) 3890

10. In how many ways can the letters of the word 'STRANGE' be arranged so that the vowels come together?

(a) 120 (b) 720 (c) 1440 (d) None of these

Answer Key									
1. (b)	2. (a)	3. (b)	4. (a)	5. (d)	6. (b)	7. (a)	8. (c)	9. (a)	10. (c)

CIRCULAR PERMUTATION

Circular Permutation refers to arrangements where the objects or individuals are arranged in a circular manner. Unlike linear permutation, where the arrangement is in a straight line, circular permutation involves arranging objects in a circular form.

DIFFERENCE BETWEEN LINEAR PERMUTATION AND CIRCULAR PERMUTATION

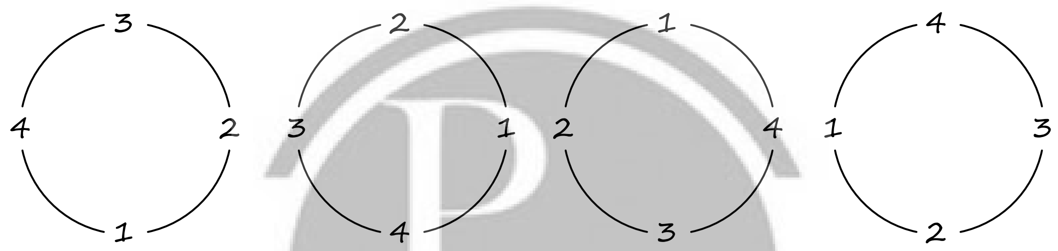
Let's consider two scenarios: the sitting arrangement of a langar (community meal) and the sitting arrangement of a circular table party.

In the case of a langar, where people sit in a line, it follows the concept of linear permutation. Each person has a distinct seat, and the order in which they sit matters. For example, if there are 4 people, the number of linear permutations would be 4!.

On the other hand, in the case of a circular table party, the arrangement is circular. The seats are arranged in a circle, and the order in which individuals sit becomes significant. Circular permutation takes into account that the arrangement repeats after one full revolution. To calculate the number of circular permutations, we use the formula $(n - 1)!$, where n represents the number of objects or individuals.

For example, if there are 4 people at the circular table, the number of circular permutations would be 3!.

Let's say there are 4 people sitting in circular arrangement as shown in the below figure:



Clearly, all the four arrangements are similar to each other.

i.e., these 4-people permutations equal to one in circular.

Thus, n ordinary permutations are equal to one permutation.

Hence, there are ${}^nP_n / n$ ways in which n things can be arranged in circular permutations which is equal to $(n - 1)!$.

Example 24. The number of ways in which 6 boys form a ring is

(a) 100 (b) 110 (c) 120 (d) None of these

Sol. (c) We know that, the number of circular permutations of n different things chosen at a time $(n - 1)!$

Thus, the number of ways in which 6 boys form a ring is $(6 - 1)! = 120$

Hence, option (c) is correct i.e., 120.

Example 25. The number of ways in which 7 boys sit around a table so that two particular boys may sit together is **(ICAI)**

(a) 240 (b) 200 (c) 120 (d) None of these

Sol. (a) Consider two boys as one, so we have 6 boys.

We know that, the number of circular permutation of n different things chosen at a time $(n - 1)!$.

So, the number of ways in which 6 boys form a ring is $(6 - 1)! = 120$

But two boys can arrange themselves in 2! ways.

So, the number of ways in which 7 boys sit around a table so that two particular boys may sit together is

$2! \times 120 = 240$.

Hence, option (a) is correct i.e., 240.

Example 23. 3 ladies and 3 gents can be seated at a round table so that any two and only two of the ladies sit together. The number of ways is **(ICAI)**

(a) 70 (b) 27 (c) 72 (d) None of these

Sol. (c) We have 3 ladies and 3 gents.

We can select 2 ladies out of 3 in ${}^3P_2 = 6$ ways

Also, only two ladies sitting together means the remaining lady can't sit adjacent to selected, which leaves us with the option that selected ladies are surrounded by gents.

Now, 4 seats remain and the remaining lady can't take the adjacent one, which leaves 2 ways for her.

Gents can be arranged in 3! ways on the remaining seats.

So, the total number of ways $= 6 \times 2 \times 3! = 72$

Hence, option (c) is correct.

Example 24. 5 persons are sitting in a round table in such way that tallest person is always on the right–side of the shortest person; the number of such arrangements is **(ICAI)**

(a) 6 (b) 8 (c) 24 (d) None of these

Sol. (a) Assume the tallest and shortest person as one.

Now, we have a total of 4 persons.

We know that, the number of circular permutations of n different things chosen at a time $(n - 1)!$.

Therefore, the number of such arrangements is $(4 - 1)! = 3! = 6$

Hence, option (a) is correct i.e., 6.

Example 25. If 50 different jewels can be set to form a necklace then the number of ways is

(a) $\frac{50!}{2}$ (b) $\frac{49!}{2}$ (c) 49! (d) None of these **(ICAI)**

Sol. (b) We know that,

Since, in forming a necklace or a garland there is no difference between a clockwise and anti-clockwise direction because we can simply turn it over so that clockwise becomes anti clockwise and vice versa.

Thus, number of necklaces that can be formed with n beads of different colours is $\frac{(n-1)!}{2}$

So, if 50 different jewels can be set to form a necklace then the number of ways is $\frac{(50-1)!}{2} = \frac{49!}{2}$.

Hence, option (b) is correct i.e. $\frac{49!}{2}$.

PERMUTATION WITH RESTRICTIONS

Theorem 1: Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_r$.

Example 26. How many ways 5 glasses of Coca – cola can be served to 10 people if one says that he does not drink coca cola?

(a) 10000 (b) 27506 (c) 6290 (d) 15120

Sol. (d) Since Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_r$

So, here n = 10, r = 5

Total ways : ${}^{9}P_5$

$\Rightarrow \frac{9!}{4!} = 9 \times 8 \times 7 \times 6 \times 5$

$\Rightarrow 15120$

Hence, the correct option is (d).

Theorem 2: Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is: $r \cdot {}^{n-1}P_{r-1}$

Example 27. The number of arrangements of 6 different things taken 3 at a time in which one particular thing always occurs is :

(a) 60 (b) 25 (c) 30 (d) None of these

Sol. (a) We know that, number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is $r \cdot {}^{n-1}P_{r-1}$.

So, the number of arrangements of 6 different things taken 3 at a time in which one particular thing always occurs is: $3 \times {}^{6-1}P_{3-1} = {}^{5}P_2 \times 3$

$\Rightarrow 5 \times 4 \times 3 = 60$

Hence, option (a) is correct.

Example 28. How many four digit numbers greater than 5000 can be formed out of the digits 0, 1, 2, 3, 5, 7, 8, 9 if no digit is repeated in any number?

(a) 330 (b) 840 (c) 460 (d) None of these

Sol. (b) Given digits: 0, 1, 2, 3, 5, 7, 8, 9

Four digits number greater than 5000 that can be formed out of the given digits can begin with 5, 7, 8 or 9

Thus, possible ways for thousands place = 4

So, rest 3 digits can be chosen in $7 \times 6 \times 5$

Therefore, the required ways:

$\Rightarrow 4 \times 7 \times 6 \times 5 = 840$

Hence, option (b) is correct.

Example 29. The sum of all 4-digit number containing the digits 2, 4, 6, 8, without repetitions is

(a) 1,33,330 (b) 1,22,220 (c) 2,13,330 (d) 1,33,320

Sol. (d) (**Hint**, Formula: $(n-1)! \times$ Sum of digits $\times$ (11111 ... n times)

Here, $n = 4$

Thus, $(n-1)! = (4-1)! = 3! = 6$

Sum of digits = 2 + 4 + 6 + 8 = 20

Therefore, the required sum = $6 \times 20 \times 1111$ = 133,320

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART D)

1. The number of ways in which 7 girls form a ring is

   (a) 120 (b) 710 (c) 720 (d) None of these

2. How many arrangements can be made out of the letters of the word 'RAJESH', the vowels never beings separated?

   (a) 120 (b) 240 (c) 360 (d) None of these

3. How can we arrange 10 people on a circular table such that two people do not want to sit together?

   (a) $7 \times 8!$ (b) $10 \times 9!$ (c) $10!$ (d) None of these

4. There are 6 varieties of Rice, 3 of Breads and 2 of sweets. In how many ways can these be placed on the table if the same types of dishes to be are to be together?

   (a) 15,000 (b) 20,450 (c) 51,840 (d) None of these

5. How many different numbers can be formed by using any five out of six digits 1, 2, 3, 4, 5, 6, no digit being repeated in any number?

   (a) 1 (b) 36 (c) 120 (d) 720

6. A group of 8 students is forming a circle for a game. In how many ways can they arrange themselves if rotations of the same arrangement are considered the same?

   (a) 5760 (b) 1680 (c) 5040 (d) 720

7. The number of numbers lying between 100 and 1000 can be formed with the digits 1, 2, 3, 4, 5, 6, 7 is **(ICAI)**

   (a) 210 (b) 200 (c) 110 (d) None of these

8. The number of arrangements in which the letters of the word `TABLE' be arranged so that the words thus formed begin with T and do not end with E is

   (a) 18 (b) 120 (c) 24 (d) None of these

9. Find the sum of all the digit numbers that can be formed with the digits 3,4,5,and 6.

   (a) 119988 (b) 11988 (c) 191988 (d) None of these

10. The number of arrangements of 10 diffrent things taken 4 at a time in which one particular thing always occurs is

    (a) 2015 (b) 2016 (c) 2014 (d) None of these

11. The value of $\sum_{r=1}^{9} r\,.\,^{r}P_{r}$ is

(a) 10! (b) 10! – 1 (c) 11! (d) 9! – 1

12. The number of 4 digit numbers greater than 5,000 can be formed out of the digit 3, 4, 5, 6 and 7 (No. digit is repeated).The number of such is

(a) 72 (b) 27 (c) 70 (d) None of these

Answer Key

1. (c)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)								

COMBINATION

The number of ways in which selection is done where order does not matter can be calculated as $^{n}C_{r}$ where $^{n}C_{r} = \dfrac{n!}{r! \times (n-r)!}$.

Example 30. $^{12}C_{8} =$

(a) 215 (b) 495 (c) 745 (d) None of these

Sol. (b) We know that, $^{n}C_{r} = \dfrac{n!}{r! \times (n-r)!}$

Thus, $^{12}C_{8} = \dfrac{12!}{8! \times (12-8)!}$

$= \dfrac{12 \times 11 \times 10 \times 9 \times 8!}{4 \times 3 \times 2 \times 1 \times 8!}$

$= \dfrac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$

So, the value of $^{12}C_{8}$ is 495.

Hence, option (b) is correct i.e., 495.

Example 31. In how many ways can I select 5 cards from a pack of 52?

(a) 2598960 (b) 2598830 (c) 2600480 (d) None of these

Sol. (a) Total number of ways 5 cards can be selected from 52 pack of cards is :

$\Rightarrow {}^{52}C_{5} = \dfrac{52!}{5!(52-5)!}$

$\Rightarrow \dfrac{52!}{5! \times 47!} = \dfrac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2} = 2598960$

Hence, the correct option is (a).

Example 32. The number of straight lines obtained by joining 16 points on a plane, no three of them being on the same line is

(a) 120 (b) 110 (c) 210 (d) None of these

Sol. (a) Total points = 16

We know, to form a straight line, 2 points are required.

Since, no three points are in same line thus the number of straight line that can be formed from 16 points

$= {}^{16}C_2 = \dfrac{16!}{14! \times 2!} = 8 \times 15 = 120$

Therefore, the required number of straight lines is 120.

Hence, the correct option is (a).

Example 33. Out of 7 boys and 4 girls, a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is

(a) 429 (b) 439 (c) 419 (d) 441

Sol. (d) Given,

Number of boys = 7

Number of girls = 4

Number of teams with at least one girl = Total number of teams - Number of teams with no girl = ${}^{11}C_5 - {}^{7}C_5$

$= \dfrac{11!}{6! \times 5!} - \dfrac{7!}{2! \times 5!} = 462 - 21 = 441$

Hence, the correct option is (d) i.e. 441.

Example 34. If there are 40 guests in a party. If each guest takes a shake hand with all the remaining guests. Then the total number of hands shake is

(a) 780 (b) 840 (c) 1560 (d) 1600

Sol. (a) No. of guests = 40

As we know for the shake hands, 2 persons are required.

Thus, total number of hand shakes = ${}^{40}C_2$

$= \dfrac{40!}{(40-2)! \times 2!} = \dfrac{40!}{38! \times 2!} = \dfrac{40 \times 39}{2} = 780$

Hence, the correct answer is option (a) i.e. 780.

Example 35. The number of diagonals in a decagon is **(ICAI)**

(a) 30 (b) 35 (c) 45 (d) None of these

Sol. (b) We know that, decagon have 10 sides.

The number of diagonals in a polygon having n sides is ${}^{n}C_2 - n$ or $\dfrac{1}{2}n(n-3)$

Here, n = 10

Thus, number of diagonals

$= \dfrac{1}{2} \times 10 \times (10-3) = 5 \times 7 = 35$

Therefore, the number of diagonals in decagon are 35.

Hence, option (b) is correct i.e., 35.

PRACTICE QUESTIONS (PART E)

1. A CA needs three accountants and ten men apply. In how many ways can these selections take place?

   (a) 6,04,800 (b) 36,28,800 (c) 6 (d) 120

2. Army General wishes to simultaneously promote 3 of its 5 Captains to Majors. In how many ways can these promotions take place?

   (a) 10 (b) 20 (b) 40 (d) None of these

3. Find the value of n in $4 \cdot {}^nC_2 = {}^{n+2}C_3$

   (a) 2 (b) 7 (c) Both (a) and (b) (d) None of these

4. If ${}^8P_r = 6720$ and ${}^8C_r = 56$; find the value of r.

   (a) 2 (b) 3 (c) 4 (d) None of these

5. A committee of 7 members is to be chosen from 6 CA, 4 CS and 5 CS. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants?

   (a) 2570 (b) 1200 (c) 3570 (d) None of these

6. Out of 6 boys & 4 girls, find the number of ways for selecting 5 members for a committee in which there are exactly two girls?

   (a) 120 (b) 1440 (c) 720 (d) 71

Answer Key					
1. (d)	2. (a)	3. (c)	4. (b)	5. (c)	6. (a)

SOME RESULTS OF COMBINATION TO TAKEN CARE

- Formula: ${}^nC_r = \dfrac{n!}{r! \times (n-r)!}$
- If r = n then, ${}^nC_r = {}^nC_n = 1$
- Similarly, if r = 0, then ${}^nC_r = {}^nC_0 = 1$
- Also, note that $0 \le r \le n$, then only ${}^nC_r$ exists. Similarly, ${}^nC_{n-r}$ exist only when $0 \le n - r \le n$
- ${}^nC_r = {}^nC_{n-r}$

Two more important results:

1. ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$
2. ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

Example 36. The value of ${}^{11}C_2 + {}^{11}C_3$ is

(a) 210 (b) 200 (c) 220 (d) None of these

Sol. (c) Since ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$^{11}C_2 + {}^{11}C_3 = \frac{11!}{2!(11-2)!} + \frac{11!}{3!(11-3)!}$$

$$\Rightarrow \frac{11!}{2! \times 9!} + \frac{11!}{3! \times 8!} = \frac{11 \times 10}{2} + \frac{11 \times 10 \times 9}{3 \times 2}$$

$$\Rightarrow \frac{110}{2}(1+3) = 110 \times 2 = 220$$

Hence, the correct option is (c).

PERMUTATIONS WHEN SOME OF THE THINGS ARE ALIKE, TAKEN ALL AT A TIME

Let us take the case, where there are n things in which n_1 things are alike of one kind, n_2 things alike of second kind and n_3 things are alike of third kind. Then the number of possible permutations when all n things taken at a time is: $P = \frac{n!}{n_1!n_2!n_3!}$

PERMUTATIONS WHEN EACH THING MAY BE REPEATED ONCE, TWICE, ... UPTO TIMES IN ANY ARRANGEMENT

The number of permutations of n things taken r at time when each thing may be repeated r times in any arrangement = n^r.

- Combinations of n different things taking some or all of n things at a time = $2^n - 1$.
- Combination of n things taken some or all at a time when n_1 things are alike of one kind, n_2 things are alike of second kind & n_3 things are alike of third kind:
  $= (n_1 + 1) \times (n_2 + 1) \times (n_3 + 1) - 1$
- If we have to select the combination such that r_1 things to be selected from n_1 and r_2 things to be selected from n_2 then, total selections are: ${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$

Example 37. How many different permutations are possible from the letters of the word 'MATHEMATICS'?

(a) 11! (b) $\frac{11!}{2! \times 2! \times 2!}$ (c) $\frac{11!}{2! \times 2!}$ (d) None of these

Sol. (b) The word given is 'MATHEMATICS'

Total number of letters in 'MATHEMATICS' is 11.

Number of 'T' = 2

Number of 'M' = 2

Number of 'A' = 2

Therefore number of arrangements $= \frac{11!}{2! \times 2! \times 2!}$

Hence, option (b) is correct.

Example 38. Rajesh is planning Christmas party, in how many ways he can invite his 11 friends for the party?

(a) 11! (b) 2048 (c) 2047 (d) None of these

Sol. (c) We know that,

Combinations of n different things taking some or all of n things at a time = $2^n - 1$

Thus, the number of ways Rajesh can invite his 11 friends = $2^n - 1 = 2048 - 1 = 2047$

Hence, the correct option is (c).

Example 39. By how many different ways you can take 10 Donuts, 6 Waffles and 8 pastries from your pantry for the picnic?

(a) 692 (b) 693 (c) 480 (d) None of these

Sol. (a) Given,

Number of Donuts (n_1) = 10

Number of Waffles (n_2) = 6

Number of Pastries (n_3) = 8

Therefore, the required ways = $(n_1 + 1) \times (n_2 + 1) \times (n_3 + 1) - 1$

$= (10 + 1) \times (6 + 1) \times (8 + 1) - 1$

$= 11 \times 7 \times 9 - 1 = 693 - 1 = 692$

Hence, the correct option is (a).

Example 40. The number of ways in which 9 things can be divided into twice groups containing 2, 3, and 4 things respectively is **(ICAI)**

(a) 1250 (b) 1260 (c) 1200 (d) None of these

Sol. (b) Total number of things = 9

The first group can be formed by choosing 2 things out of 9 things

Number of ways = ${}^9C_2$

Now, second group can be formed by choosing 3 things out of 7

Number of ways = ${}^7C_3$

Now, second group can be formed by choosing 4 things out of 4

Number of ways = ${}^4C_4$

Therefore, total number of ways = ${}^9C_2 \times {}^7C_3 \times {}^4C_4$ = 1260 ways.

Hence, option (b) is correct.

Example 41. The number of ways a person can contribute to a fund out of 1 ten-rupee note, 1 five-rupee note, 1 two-rupee and 1 one-rupee note is

(a) 15 (b) 25 (c) 10 (d) None of these

Sol. (a) A person can contribute to a fund out of 1 ten rupees note, 1 five-rupees, 1 two-rupee and 1 one rupee note.

As a person has 4 notes, he can contribute either one note or two notes or three notes.

The number of different contributions can be

$= 2 \times 2 \times 2 \times 2 - 1 = 16 - 1 = 15$

Hence, option (a) is correct.

Example 42. ${}^{(n-1)}P_r + r \,.\, {}^{(n-1)}P_{(r-1)}$ is equal to

(a) ${}^nC_r$ (b) $\dfrac{n}{r!(n-r)!}$ (c) ${}^nP_r$ (d) None of these

Sol. (c) By property, we have

${}^{(n-1)}P_r + r \,.\, {}^{(n-1)}P_{(r-1)} = {}^nP_r$

Hence, option (c) is correct.

Example 43. ${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n$ equals

(a) $2^n - 1$ (b) 2^n (c) $2n + 1$ (d) None of these

Sol. (a) We know that,

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots + {}^nC_nx^n$

Now, put x = 1 in above equation, we get

$(1 + x)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n$

${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$

Hence, option (a) is correct i.e., $2^n - 1$.

Example 44. The number of ways in which 12 students can be equally divided into three groups is

(a) 5775 (b) 7575 (c) 7755 (d) None of these

Sol. (a) Given: Total students = 12

Since, 12 students are equally divided into three groups thus

$$\text{Possible ways} = \frac{{}^{12}C_4 \times {}^8C_4 \times {}^4C_4}{3!}$$

$$= \frac{12!}{(4!)^3(3!)} = 5775$$

Hence, the correct option is (a).

Example 45. The Number of ways in which 15 mangoes can be equally divided among 2 students is

(a) $\dfrac{15}{(5!)^4}$ (b) $\dfrac{15}{(5!)^3}$ (c) $\dfrac{15}{(5!)^2}$ (d) None of these

Sol. (b) Given, Total number of mangoes = 15

Number of students = 3

$$\Rightarrow \frac{15}{3} = 5 \text{ mangoes}$$

So, the required number of ways in which 15 mangoes may be equally distributed among 3 students is $= \dfrac{15!}{5! \times 5! \times 5!} = \dfrac{15!}{(5!)^3}$

Hence, option (b) is correct.

PRACTICE QUESTIONS (PART F)

1. The letters of the word 'BOOKKEEPER' are rearranged to form different arrangements. How many different arrangements can be formed considering the repetitions of the letters?

(a) $10!$ (b) $\frac{10!}{2!\times 2!\times 2!\times 3!}$

(c) $\frac{10!}{2!\times 3!}$ (d) None of these

2. If 3 books on computer, 3 books on commerce, and 5 books on economics are arranged in such away that the books of same subject are kept together, then the number of ways in which this can be done are

(a) 4320 (b) 35820 (c) 35920 (d) 25920

3. The number of triangle that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line is

(a) 185 (b) 175 (c) 105 (d) 115

4. The number of numbers between 1000 and 10000, which can be formed by the digits 1, 2, 3, 4, 5, 6 without repetition is

(a) 720 (b) 180 (c) 360 (d) 540

5. The number of ways in which 4 person can occupy 9 vacant seats is

(a) 6048 (b) 3024 (c) 1512 (d) 4536

6. How many different permutations are possible from the letters of the word 'CALCULUS'?

(a) 40,320 (b) 20,160 (c) 10,080 (d) 5,040

7. If ${}^{10}C_3 + 2 \,.\, {}^{10}C_4 + {}^{10}C_4 = {}^{n}C_5$, then value of n is

(a) 10 (b) 11 (c) 12 (d) 13

8. The maximum number of points of intersection of 10 circles of will be

(a) 2 (b) 20 (c) 90 (d) 180

9. If ${}^{n+1}C_{r+1} : {}^{n}C_r : {}^{n-1}C_{r-1} = 8 : 3 : 1$, then n is equal to

(a) 22 (b) 16 (c) 10 (d) 15

10. There are 6 men and women in a group , then the number of ways in which a committee of 5 persons can be formed of them , if the committee is to include at least 2 women are

(a) 180 (b) 186 (c) 120 (d) 105

11. If ${}^{n}P_r = 720$ and ${}^{n}C_r =120$, then value of 'r' is

(a) 4 (b) 5 (c) 6 (d) 3

12. A student has three books on computer, three books on economics and five books on commerce. If these books are to be arranged subject wise , then these can be placed on shelf in the number of ways

(a) 25290 (b) 25920 (c) 4230 (d) 4320

13. A person has ten friends of whom six are relatives. If he invites five guests such that three of them are his relatives , then the total number of ways in which he can invite them are

(a) 30 (b) 60 (c) 120 (d) 75

14. 5 men and 4 women to sit in a row in such a manner that the women always occupy the even places. The number of such arrangement will be

(a) 126 (b) 1056 (c) 2080 (d) 2880

15. There are 5 books on English , 4 books on Tamil and 3 books on Hindi . In how many ways can these books be placed on a shell if the books on the same subjects are to the together?

(a) 1,36,800 (b) 1,83,600 (c) 1,03,680 (d) 1,63,800

16. If six times the number of permutations of 'n' items taken 3 at a time is equal to seven times the number of permutation of (n – 1) items taken at a time, then the value of 'n' will be

(a) 7 (b) 9 (c) 13 (d) 21

17. If $^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{98}$, then value of x will be

(a) 999 (b) 998 (c) 997 (d) None of these

18. How many different words can be formed with the letters of the word 'LIBERTY'?

(a) 4050 (b) 5040 (c) 5400 (d) 4500

19. In how many ways can a family consist of 3 children have different birthdays in a leap years?

(a) $^{365}C_{365}$ (b) $^{366}C_3 - 3$ (c) $366 \times 365 \times 364$ (d) $^{366}C_3$

20. If $^{15}C_{3r} = {}^{15}C_{r+3}$, then 'r' is equal is

(a) 2 (b) 3 (c) 4 (d) 5

21. Number of ways of shaking hands in a group of 10 persons shaking hands to each other are

(a) 45 (b) 54 (c) 90 (d) 10

22. If $^{n}P_4 = 20\ (^{n}P_2)$ then the value of 'n' is

(a) –2 (b) 7 (c) –2 and 7 both (d) None of these

23. If $^{n}P_r = 3024$ and $^{n}C_r = 126$, then find n and r **(Dec, 2022)**

(a) 9, 4 (b) 10, 3 (c) 12, 4 (d) 11, 4

24. The number of permutations of the word 'ACCOUNTANT' is

(a) $10! \div (2!)^4$ (b) $10! \div (2!)^3$ (c) $10!$ (d) None of these

25. In how many ways that the crew of an eight be arranged so that if 3 of crew can row only on a stroke side and 2 row on the other side is **(July 2019)**

(a) 1728 (b) 256 (c) 164 (d) 126

26. 5 boys and 3 girls are to be seated together such that no two girls are together

(a) 14, 4000 (b) 2400 (c) 720 (d) None of these **(Dec 2019)**

27. Out of 6 Boys and 4 Girls, find the number of ways of selecting 5 members committee in which there is exactly two girls?

(a) 120 (b) 1440 (c) 720 (d) 71

Answer Key

1. (b)	2. (d)	3. (a)	4. (c)	5. (b)	6. (d)	7. (c)	8. (b)	9. (d)	10. (b)
11. (d)	12. (b)	13. (c)	14. (d)	15. (c)	16. (d)	17. (a)	18. (b)	19. (c)	20. (b)
21. (a)	22. (b)	23. (a)	24. (a)	25. (a)	26. (a)	27. (a)			

SUMMARY

- **Multiplication rule:** If a certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways, then total number of ways of doing both things simultaneously = $m \times n$.
- **Addition rule:** If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in $(m + n)$ ways.
- **Factorial:** The factorial n, written as $n!$ or $\lfloor n$, represents the product of all integers from 1 to n both inclusive i.e. $n! = n(n-1)(n-2) \ldots 3 . 2 . 1$ and note that $0! = 1$.
- **Permutation:** The ways of arranging or selecting a smaller or equal number of objects from a collection of objects with due regard being paid to the order of arrangement or selection, are called permutations.
- The number of permutations of n things chosen r at a time is given by: ${}^nP_r = \dfrac{n!}{(n-r)!}$.
- **Circular Permutation:**

  (i) Arranging n things in circular arrangement is given by: $\dfrac{{}^nP_n}{n} = (n-1)!$ ways in which all the n things can be arranged in a circle.

  (ii) Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_r$.

  (iii) Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is $r . {}^{n-1}P_{r-1}$.
- **Combinations:** The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations given by: ${}^nC_r = \dfrac{n!}{r!(n-r)!}$.
- Permutations when some of the things are alike, taken all at a time is $\dfrac{n!}{n_1! . n_2! . n_3! \ldots}$.
- Permutations of r things out of n when each thing may be repeated once, twice, ...upto r times in any arrangement n^r.
- The total number of ways in which it is possible to form groups by taking some or all of n things $2^n - 1$. The total, number of ways in which it is possible to make groups by taking some or all out of n $(= n_1 + n_2 + n_3 + \ldots)$ things, where n_1 things are alike of one kind and so on, is given by $\{(n_1 + 1)(n_2 + 1)(n_3 + 1) \ldots\} - 1$.
- The combinations of selecting r_1 things from a set having n_1 objects and r_2 things from a set having n_2 objects where combination of r_1 things, r_2 things are independent is given by: ${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$.

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