

Time value of money (FORMULAS)

Date

Simple Interest -

1. $SI = \frac{P \times R \times t}{100}$

2. Amount = $P(1 + \frac{Rt}{100})$, $P + \frac{P \times R \times t}{100}$, $P + SI$

P = principal Value
R = rate of Interest
t = time (in yrs)

Ex- The sum required to earn a monthly interest of ₹1200 at 18% p.a SI is:

Sol. $1200 = P \times \frac{18}{100} \times \frac{1}{12} = \boxed{80000} = P$ (Ans)

Compound Interest -

1. $CI = P[(1+i)^n - 1]$, A - P

2. Amount = $P(1+i)^n$

P = principal Value

3. Dep = $A = P(1+i)^n$

n = t x no copy

Ex- The CI on ₹40000 at 10% p.a for 1 year when the interest is payable quarterly is :- $i = \frac{10}{4}\%$, $n = 1 \times 4 = 4$

$i = \frac{R}{no\ copy}$

no copy

Sol. $CI = 40000 [(1 + \frac{10}{4}\%)^4 - 1] = 4152.51$ (Ans)

Effective interest rate -

$ERR = [(1+i)^n - 1]$

Ex- The ERR corresponding to nominal rate of 7% convertible quarterly is =

$E = [(1 + \frac{7}{4}\%)^4 - 1] \times 100 = 7.18\%$ (Ans)

(FV) - Kuch nhi likha → Regular
Immediately, → Date
Starting today, →
now

ANNUITY

Future Value

Present Value

SINGLE:

$FV = CF(1+i)^n$

CF = cash flow

SINGLE:

$PV = \frac{CF}{(1+i)^n}$, (CF = Cash flow)

REGULAR:

$FVAR = Ai \times FVAF(n, i)$

$= Ai \times \left\{ \frac{[(1+i)^n - 1]}{i} \right\}$

REGULAR:

$PVAR = Ai \times PVAF(n, i)$

$= Ai \times \left[\frac{1}{i} \times \left\{ 1 - \frac{1}{(1+i)^n} \right\} \right]$

Ai = Annuity
 $PVAF$ = present value annuity factor
 $PVAF = \frac{1 - \frac{1}{(1+i)^n}}{i}$

Ai = Annuity, $FVAF$ = Future value annuity factor

DUE:

$FVAD = Ai \times FVAF(n, i) \times (1+i)$
 $= Ai \times \left\{ \frac{[(1+i)^n - 1]}{i} \right\} \times (1+i)$

DUE:

$PVAD = [Ai \times PVAF(n-1, i)] + Ai$

Perpetuity -

1. Present value, $PVP = \frac{Ai}{i}$ 2. Growing, $PVGP = \frac{Ai}{i-g}$ $g = \text{growth rate}$
 $i < g \rightarrow \text{not possible}$

Real rate of Return -

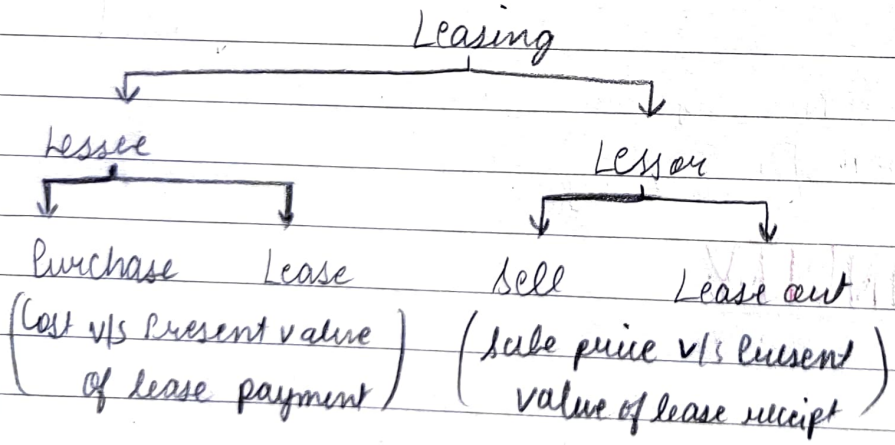
$RRR = \text{Nominal rate of Return} - \text{Rate of Inflation}$

Sinking Fund -

$SF = a \frac{(1+i)^n - 1}{i}$

Difference b/w CI & SI -

1. For 2 years = $P \left(\frac{R}{100} \right)^2$
 2. For n years = $P \left[\left(1 + \frac{R}{100} \right)^n - 1 - \frac{R \times n}{100} \right]$



1. No. of possible subset of any set = 2^n

2. No. of proper subset of any set = $2^n - 1$

3. If $n(A) = n(B)$ then A, B

↓
cardinal No.

4. De Morgan's law = $(P \cap Q)' = P' \cup Q'$

$$(P \cup Q)' = P' \cap Q'$$

5. Two sets formula = $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

6. Three sets formula -

$$= n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) +$$

$$n(A \cap B \cap C)$$

Types of Relation

Reflexive

$(a, a)(b, b)(c, c)$

- is equal to
- is same as

Symmetric

$(a, b) = (b, a)$

- is reciprocal of

Transitive

$(a, b)(b, c) = (a, c)$

- is smaller than

Equivalence

includes all three

$(R + S + T)$

1) Quadratic Equation - $ax^2 + bx + c = 0$

2) Formula - $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3) Sum of Roots - $\alpha + \beta = \frac{-b}{a}$

4) Product of Roots - $\alpha\beta = \frac{c}{a}$

5) Construction of a Quadratic Equation = $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
= $x^2 - \text{sum of roots (x)} + \text{product of Roots}$

6) Discriminant - If $b^2 - 4ac \geq 0$

If $b^2 - 4ac < 0$

↓
roots are unreal or imaginary

↓
roots are real

$b^2 - 4ac = 0$
↓
roots are real & equal ($\alpha = \beta$)

$b^2 - 4ac > 0$
↓
roots are real & unequal ($\alpha \neq \beta$)

↓
 $b^2 - 4ac$ is a perfect sq
roots are real, unequal & rational also

↓
 $b^2 - 4ac$ is not a perfect sq - roots are real, unequal & irrational

7) difference b/w root use equation } $(a+b)^2 - (a-b)^2 = 4ab$

8) Identity $(a^2 + b^2) = (a+b)^2 - 2ab$

(H-5) Permutations & Combinations

OR \rightarrow + Plus
AND \rightarrow X Product

$${}^n P_r = \frac{n!}{(n-r)!} \quad \left\{ \begin{array}{l} n \geq r \\ n \rightarrow +ve \text{ integer} \end{array} \right.$$

Permutations (arrangement)

direct (if all persons/objects are involved)

After selection

Particular case of theorem ($n=r$)

$${}^n P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! \quad \{0! = 1\}$$

Special formula

$$(n+1)! - n! = n \cdot n!$$

ex- $6! - 5! = 5 \times 5!$

$720 - 120 = 5 \times 120$

$600 = 600$

Circular permutation (Part-II)

- No person has the same two neighbours
- different forms of necklaces/ garlands.

$$\frac{1}{2} (n-1)!$$

(Part I) $\rightarrow (n-1)!$

Steps to do sum of all n digit numbers:-

1. Find the number of numbers that can be formed (no. of possible no.s)
2. Find the repetition value of each digit
 Repetition of each digit = $\frac{\text{Value of step 1}}{\text{no. of different-digits}}$
3. Find the sum of digits
4. Sum of digits \times repetition
5. Multiply value of step by IIII, III etc in case of four digit no.s & three digit no.s respectively.

Formula - (Theorem)

$$\left[{}^{n-1} P_r \right] + \left[r \cdot {}^{n-1} P_{r-1} \right] = {}^n P_r \text{ - Total permutations}$$

↓
One particular thing always included

↓
One particular thing always excluded

Combinations

$$\cdot {}^n C_r = \frac{{}^n P_r}{r!} = \frac{n!}{(n-r)! \times r!}$$

$$\cdot {}^n C_0 = 1 \text{ choose none of them}$$

$$\cdot {}^n C_n = 1 \text{ choose all of them}$$

select 4 out of 5 = select 1 out of 5
 select 3 out of 5 = select 2 out of 5

$$\cdot \text{Complementary theorem} = {}^n C_r = {}^n C_{n-r}$$

Picking r thing to select = Picking $(n-r)$ things to select

$$\cdot \text{no. of lines can be formed using given points} = {}^n C_2$$

• Combination of n different thing taking n thing at a time

one or more

$$2^n - 1$$

zero or more

$$2^n$$

(1) No. of straight lines with the given n points = ${}^n C_2$ (as we need to select 2 pts to make a line)

(2) no. of triangles with the given n points = ${}^n C_3$ (as we need to select 3 pts to make a triangle)

(3) No. of straight lines with the given n points = ${}^n C_3 - {}^m C_{2+1}$
 where m points are collinear

(4) no. of triangle with the given n points = ${}^n C_3 - {}^m C_3$
 where m points are collinear

(5) No. of || gm with the given one set of m parallel lines and another set of n || lines = ${}^m C_2 \times {}^n C_2$ (lines)
 selecting 2 lines from each set of

(6) No. of diagonals with n sides = ${}^n C_2 - n$

CHAPTER - 15 Probability

Concepts -

Total no. of possible events = p^q
where p = no. of events in one trial
 q = no. of trials

Addition Theorems :-

- 1) For two mutually exclusive events = $P(A \cup B) = P(A) + P(B)$
- 2) For any n no. of mutually exclusively events = $P(A_1 \cup A_2 \cup A_3 \cup A_4 \dots \cup A_n) = P(A_1) + P(A_2) + P(A_3) + P(A_4) \dots P(A_n)$
- 3) For any two events (in general) = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- 4) For any three events $A, B, C = P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional Probability -

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A'/B) = \frac{P(A' \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A/B') = \frac{P(A \cap B')}{P(B')}$$

$$P(B/A') = \frac{P(B \cap A')}{P(A')}$$

Compound Theorem :-

For Independent $\rightarrow P(B/A) = P(B)$

$$P(A/B) = P(A)$$

Joint Probability $\rightarrow P(A \cap B) = P(A) \times P(B)$

Independents	v/s Mutually exclusive
when occurrence of one event does not influence occurrence of other.	<ul style="list-style-type: none">• events that do not occur simultaneously• occurrence of one implies non-occurrence of other• one event influence others which implies they are <u>dependent</u>.

CH-14 Central Tendency and Dispersion

Arithmetic Mean -

1. Discrete observations $\rightarrow \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + \dots + x_n}{n} = \frac{\sum x}{n}$
2. Frequency Distribution $\rightarrow \bar{x} = \frac{\sum fx}{n}$ } grouped, $x = \text{individual}$
ungrouped, $x = \text{mid-pt}$
3. Assumed mean / step deviation $\rightarrow \bar{x} = A + \frac{\sum fd}{N} \times C$ } a = Assumed mean, C = class length
D = $\frac{x-A}{C}$
4. Combined AM $\rightarrow \bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

Median -

$$1. Me = l_1 + \left[\frac{\frac{n}{2} - n_1}{n_u - n_1} \right] \times C$$

}

$\left. \begin{array}{l} l_1 = \text{lcb of median class, } C = \text{class length} \\ n_u = \text{cf of median class,} \\ n_1 = \text{pre-median cf of median class} \end{array} \right\}$

- i) Find cf = cumulative freq.
- ii) Calculate $n/2$ and median class
- iii) Apply formula

How to find partition value?

1st Rank (Position) $\rightarrow [(n+1)p]^{th}$ term $\rightarrow n = \text{no. of observations}$

2nd Value obtain

P = depends on type of PV

PV	Q ₁	Q ₂	Q ₃	D ₁	D ₉	P ₁	P ₃₉	Me
value of P	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{10}$	$\frac{9}{10}$	$\frac{1}{100}$	$\frac{39}{100}$	$\frac{1}{2}$

Mode -

1. Discrete \rightarrow Highest / Max freq
2. Frequency Dis $\rightarrow Mo = l_1 + \left[\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \right] \times C$ } f₀ = freq of modal class
f₁ = freq of post-modal class
f₋₁ = freq of pre-modal class

GM = for avg of ratios like Percentage

HM = For avg of ratios like rate of change of cost per unit, speed, hours per day, etc

AM = for all other averages

Relationship b/w mean / Mode / Median -

1) Symmetric Distribution \rightarrow Mean = median = mode

2) Asymmetric / skewed distribution \rightarrow Mean - mode = 3(mean - median)
OR Mode = 3 median - 2 mean

Skewness 1 - Positive (Mean > median > mode)

2 - Symmetric (mean = median = mode)

3 - Negative (mean < median < mode)

Geometric Mean - (only for +ve observations)
(product of observations)^{1/n}

Ex ① GM of 36 & 9
 $\sqrt{36 \times 9} (n=2)$
18

② GM of 8, 5, 6, 4
 $\sqrt[3]{8 \times 5 \times 6 \times 4} (n=4)$
12.1648

Harmonic Mean -

1) Discrete observations = $H = \frac{n}{\sum(1/x)}$

2) Simple / grouped frequency = $H = \frac{N}{\sum(f/x)}$

3) Combined HM = $\bar{x}_c = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$

AM / HM / GM

If que mention

If ques silent

For only TWO obs

$$AM \geq GM \geq HM$$

$$AM \times HM = (GM)^2$$

Obs identical or same

Obs are unique different

$$AM = GM = HM$$

$$AM > GM > HM$$

DISPERSION

1. Range -

Discrete observation

L-S

Grouped freq. Distribution

L-S

{ L = Largest }
{ S = Smallest }

{ L = UCB of last class interval }
{ S = LCB of first class interval }

$$\text{Coefficient of Range} = \frac{L-S}{L+S} \times 100$$

1) Change of origin \rightarrow no effect

2) Change of scale \rightarrow value - affected
 \rightarrow Sign - no effect

2. Mean Deviation -

Frequency Distribution

$$MD_A = \frac{1}{N} \sum f |x-A|$$

Discrete

$$MD_A = \frac{1}{N} \sum |x-A|$$

{ A = appropriate measure of central tendency (mean, mode, median) }

$$\text{Coefficient of MD} = \frac{\text{Mean Deviation of A}}{A} \times 100$$

3. Standard Deviation -

Discrete observations

$$(i) \sigma_x = SD_x = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

frequency distribution

$$(i) \sigma_x = SD_x = \sqrt{\frac{\sum f(x-\bar{x})^2}{n}}$$

$$(ii) \sigma_x = SD_x = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$(ii) \sigma_x = SD_x = \sqrt{\frac{\sum fx^2}{N} - \bar{x}^2}$$

For two numbers

$$S = \frac{\text{Range}}{2}$$

For first n natural numbers

$$S = \sqrt{\frac{n^2-1}{12}}$$

Coefficient of variation = $\frac{SD}{AM} \times 100$

Ex Variance of $x = 49$ & $2y + 3x = 10$. Find variance of $y = ?$

Ans $SD_x = \sqrt{49} = 7$

$var y = (10.5)^2 = 110.25$

$SD_y = \frac{3}{2} SD_x$

$= \frac{3 \times 7}{2}$

$= 10.5$

Combined SD =

$$SD_c = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

- n_1 = no. of obs first set
- n_2 = no. of obs second set
- s_1 = SD of 1st set
- s_2 = SD of 2nd set
- d_1 = $\bar{x}_1 - \bar{x}_c$ [\bar{x}_c = combined mean]
- d_2 = $\bar{x}_2 - \bar{x}_c$ [\bar{x}_c = combined mean]

Quartile Deviation = $QD_k = \frac{Q_3 - Q_1}{2}$

best measure for open end classification

Coefficient = $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

Correlation and Regression

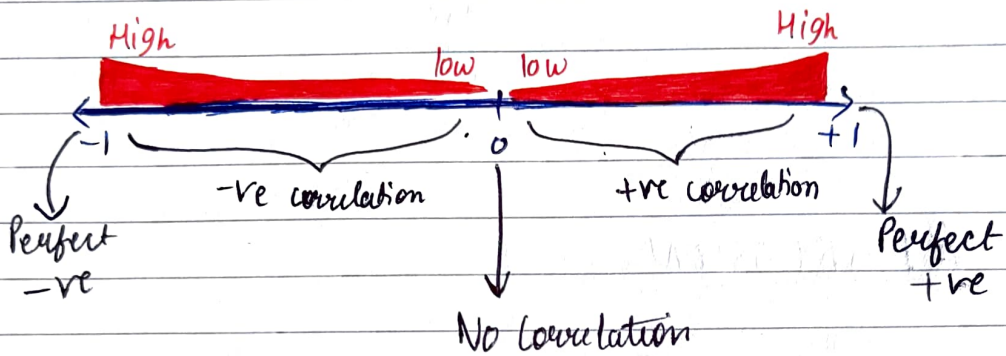
Bivariate Data-

No. of Marginal distribution in bivariate data = 2

No. of Conditional Distributions = m+n

any (Karl's Pearson's Product moment Correlation coefficient)

$$-1 \leq r \leq 1$$



Variance of x
(univariate)

$$\frac{\sum (x - \bar{x})^2}{n} = \frac{\sum [(x - \bar{x})(x - \bar{x})]}{n}$$

OR

$$\frac{\sum x^2}{n} - \bar{x}^2 = \frac{\sum (x \cdot x)}{n} - \bar{x} \cdot \bar{x} \rightarrow \frac{\sum xy - \bar{x} \cdot \bar{y}}{n}$$

Covariance of x, y
(bivariate)

$$\frac{\sum [(x - \bar{x})(y - \bar{y})]}{n}$$

$$R_{xy} =$$

Covariance(x, y)

$$SD_x \cdot SD_y$$

$$\downarrow \frac{\sum x^2}{n} - (\bar{x})^2 \downarrow$$

$$\sqrt{\frac{\sum y^2}{n} - (\bar{y})^2}$$

Spearman's Rank Correlation Coefficient

$$R_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

d = difference in Ranks

In case of tie in Rank:

$$R_s = 1 - \frac{6 \left[\sum d^2 + \frac{t^3 - t}{12} \right]}{n(n^2 - 1)}$$

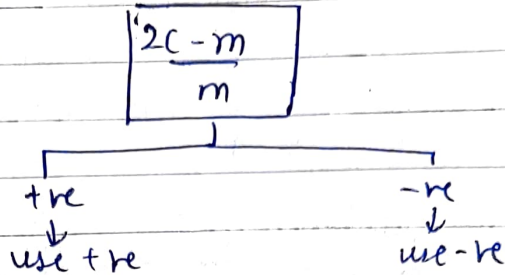
$$A = \text{adjustment value} = \frac{t^3 - t}{12}$$

t = tie length
(no. of obs tied)

Co-efficient of Concurrent deviation →

$$r_c = \pm \sqrt{\pm \left(\frac{2c-m}{m} \right)}$$

$c =$ no. of concurrent data
 $m =$ no. of pairs compared $(n-1)$



REGRESSION

Imperfect Correlation

Calculate y
 when x is given

↓
 Reg eq of Y on X

↓
 minimize vertical
 Distances

Y on X

$$y - \bar{y} = (b_{yx})(x - \bar{x})$$

↓
 slope of line Y on X

$$b_{yx} = \frac{\text{Covariance of } x, y}{\text{Variance of } x}$$

Calculate x
 when y is given

↓
 Reg eq of X on Y

↓
 minimize horizontal
 Distances

X on Y

$$x - \bar{x} = (b_{xy})(y - \bar{y})$$

↓
 slope of line X on Y

$$b_{xy} = \frac{\text{Covariance of } x, y}{\text{Variance of } y}$$

If x is given in ques,

- ① $b_{yx} = r \times \frac{SD_y}{SD_x}$ ② $b_{xy} = r \times \frac{SD_x}{SD_y}$
 $r =$ correlation coefficient (kare b Pearson)

Change of Scale

① $b_{uv} = b_{ny} \times$ change of scale of x

change of scale of y

② $b_{vu} = b_{yx} \times$ change of scale of y
change of scale of x

} both effect of sign scale & effect of value scale

Portable Error (PE) $\rightarrow 0.6745 \times \frac{1-r^2}{\sqrt{n}}$ { r = correlation coefficient
 n = no. of pairs of observation }

limits of correlation = $r \pm PE$

$r < PE \rightarrow$ No correlation

$-1 < r < 1 \rightarrow$ PE can never be -ve

$r > 6PE \rightarrow$ presence of correlation is certain

If x & y are non-linear but related $r = 0$

If x & y are not related at all $r = 0$

If $r = 0$, then there is no linear correlation

but you cannot confirm that there is no correlation

Review of Correlation & Regression :-

1) Coefficient of determination = r^2

2) Coefficient of non-determination = $1 - r^2$

INDEX NUMBERS

Simple Aggregate method
 Formula = $\frac{\sum P_n}{\sum P_0} \times 100$

Merits - easy to compute
 = does not satisfy
 UNIT TEST

Simple Aggregate Relatives
 Formula = $\frac{\sum \frac{P_n}{P_0}}{n}$

Merits = pure numbers

Weighted Aggregate Index

Laspeyres's Index

↓ (L)

$$\frac{\sum P_n \times q_0}{\sum P_0 \times q_0} \times 100$$

(u.k.a Weighted Avg of Relative method)

Pasche's method

↓ (P)

$$\frac{\sum P_n \times q_n}{\sum P_0 \times q_n} \times 100$$

Marshall - Edgeworth Index

↓ (M)

$$\frac{\sum P_n (q_0 + q_n)}{\sum P_0 (q_0 + q_n)} \times 100$$

Fisher's Ideal Method

↓ (F)

$$\sqrt{L \times P}$$

=> GM of L & P

Bowley's Index (B)

$$\frac{L+P}{2} = \text{AM of L \& P}$$

Chain Index Number

= $\frac{\text{Link relative of current year} \times \text{Chain Index of previous year}}{100}$

Time Reversal Test → $P_{01} \times P_{10} = 1$
 (only Rember)

Factor Reversal Test → $\frac{P_{01}}{\text{Price test}} \times \frac{Q_{01}}{\text{Quantity Index}} = \frac{V_{01}}{\text{Value Index}}$ (only Fisher satisfy)

$e = 2.71828$

Theoretical Distribution of Data

Types of probability function

Random Variable

discrete
(mass funcⁿ)

Continuous
(density funcⁿ)

Binomial
 $n \cdot p^x \cdot q^{(n-x)}$

Poisson
 $\frac{e^{-m} \cdot m^x}{x!}$

Normal

mean = $n \cdot p$

mean = $\mu = m$

Variance = $n \cdot p \cdot q$
(max value $\frac{n}{4}$)

$\sigma^2 = m$
mode $\left\{ \begin{array}{l} \text{Integer } \rightarrow \begin{cases} \mu_0 = m @ \\ m-1 @ \end{cases} \\ \text{Non-integer } \rightarrow m \end{array} \right.$
Unimodal

mode = Find $(n+1) \cdot p$

Integer
Bimodal

Non-integer
Unimodal

$(n+1)p$ $(n+1)p-1$

largest integer
 $\leq (n+1)p$