

CHANAKYA 2.0

For CA Foundation

Quantitative Aptitude

Ratio Proportion Log & Indices

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TOPICS

to be covered

- 1 Ratio
- 2 Proportions
- 3 Indices
- 4 Logarithm





Ratio



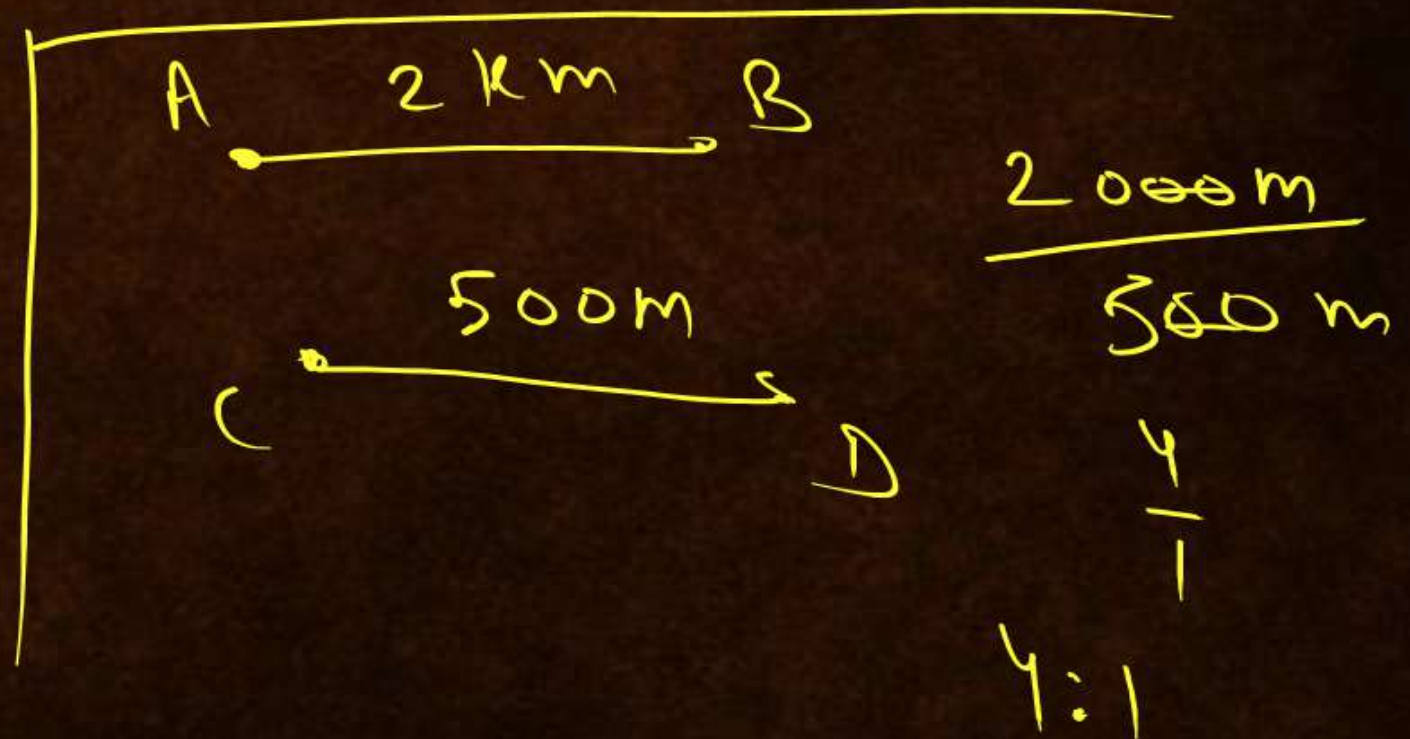
$$\begin{array}{r} \Downarrow \\ 3 \overline{) 15} \\ \underline{9} \\ 6 \\ \underline{6} \\ 0 \end{array}$$

Division of two numbers

Comparison of two or more quantities of same kind & in same units

Height
Weight
Salary

$$\begin{array}{r} 4 \\ 8 \overline{) 40} \\ \underline{8} \\ 0 \\ \underline{0} \\ 0 \end{array} = \frac{4}{5} = 4:5$$





Ratio of a to $b = \frac{a}{b}$ or $a : b$

↓ ↓
first second
term term
(Antecedent) (consequent)

Ratio of number of Boys
to numbers of girls in a class
is = $3 : 2$

eg Boys = 24 & Girls = 16
Total students = $24 + 16 = 40$

$$\frac{\text{Boys}}{\text{Girls}} = \frac{24}{16} = \frac{3}{2} \text{ or } 3:2$$



eg. Let there are 80 students in a class
 Ratio of No. of Boys to no. of girls is 4:1
 find no. of Boys & no. of girls.

Sol.



$$\text{Boys} = \frac{16}{80} \times \frac{4}{5} = 64$$

$$\text{Girls} = 80 \times \frac{1}{5} = 16$$

Or

B:G = 4:1
 let B = 4x & G = x

nm

Total students = 80

$$4x + x = 80$$

$$5x = 80$$

$$x = \frac{80}{5}$$

$$x = 16$$

$x = 16$

$$\begin{aligned} B &= 4x \\ &= 4 \times 16 \\ &= 64 \end{aligned}$$

$$\begin{aligned} G &= x \\ &= 16 \end{aligned}$$



If weight of Arjun is 80kg
& he reduces the weight in 5:4.
find new weight?

Sol.

$$\frac{\text{old weight}}{\text{new weight}} = \frac{5}{4}$$

$$\frac{80\text{kg}}{x\text{kg}} = \frac{5}{4}$$

$$80 \times 4 = 5x$$

$$\frac{80 \times 4}{5} = x$$

$x = 64\text{kg}$

80	10	5
64	8	4



Ratio of a to b = a : b

Both terms can be multiplied or Divided
by any number

$$\begin{array}{r} 3 \\ 9 \\ \hline 18 \\ \hline 24 \\ \hline 12 \\ 4 \end{array}$$

$$\text{or } \begin{array}{r} 3 \quad 4 \\ \hline 18 : 24 \end{array}$$

$$\frac{3}{4} \quad \text{or} \quad 3 : 4$$

$$\begin{array}{l} \text{eg } 24 : 28 \\ = 12 : 14 \\ = 6 : 7 \end{array}$$

$$\begin{array}{l} \text{eg } \frac{1}{3} : 4 \\ = 1 : 12 \end{array}$$



$$\frac{\frac{1}{3} : 4}{\left(\frac{4}{1}\right)}$$

$$= \frac{1}{3} \times \frac{1}{4}$$

$$= \frac{1}{12}$$

$$1 : 12$$

$$= \frac{1}{3} : 4$$

$$= 1 : 12$$

$$\frac{\frac{2}{5} : \frac{3}{5}}{= 20 \times \frac{2}{5} : 20 \times \frac{3}{5}}$$

$$= 8 : 15$$

$$a : b$$

$$= \frac{a}{b}$$

eg

$$\frac{\frac{2}{5} : \frac{3}{5}}{2 : \frac{15}{5}}$$

$$8 : 15$$

$$\frac{\frac{2}{5} : \frac{3}{5}}{\frac{10}{5} \times \frac{3}{5}}$$

$$\frac{2}{5} : \frac{3}{5} = 8 : 15$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$$

$$\frac{2}{5} : \frac{3}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{2}{5} : \frac{3}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$2 + 2 \times 5 = 12$$

$$= 20$$



Lesser Inequality

$$\frac{a}{b} \text{ when } a < b$$

eg $\frac{2}{5} = 0.4 < 1$

Greater Inequality

$$\frac{a}{b} \text{ when } a > b$$

eg $\frac{8}{5} = 1.6 > 1$

Equality



$$\frac{a}{b} \text{ when } a = b$$

eg $\frac{2}{2} = \frac{1}{1} \quad 1:1$

eg $\frac{3}{3} = \frac{1}{1} = 1:1$



eg

2:3 or 5:8

which ratio is greater

Sol.

$$\frac{2}{3} = 0.666 \dots$$

$$\frac{5}{8} = 0.625$$

$$\frac{2}{3} > \frac{5}{8}$$



A & B

Salary: 12000 & 15000

Ratio \Rightarrow 12000 : 15000

12 : 15

4 : 5

$$\begin{array}{r} 4 \\ \cancel{12000} \\ \hline 15000 \\ 5 \end{array}$$

Continued Ratio



Ratio of more than two quantities of same kind & in same units

§ Salary

12000, 14000 & 18000

12000 : 14000 : 18000

12 : 14 : 18

6 : 7 : 9



Inverse of $a:b = b:a$

eg inverse of $4:7$
 $= 7:4$



#

Ratio compounded of $a:b$ & $c:d$

$$= \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$= ac:bd$$

g

Ratio compounded of $2:3$, $4:5$ & $6:7$

$$= \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$$

$$= \frac{16}{105} = \frac{16}{35}$$



Duplicate Ratio of $a : b = a^2 : b^2$

Triplicate Ratio of $a : b = a^3 : b^3$

Sub Duplicate Ratio of $a : b = \sqrt{a} : \sqrt{b} = (a)^{1/2} : (b)^{1/2}$

Sub Triplicate Ratio of $a : b = (a)^{1/3} : (b)^{1/3}$

eg Duplicate Ratio
of $2 : 5$
 $= (2)^2 : 5^2$
 $= 4 : 25$

eg Triplicate Ratio
of $2 : 3$
 $= (2)^3 : (3)^3$
 $= 8 : 27$

eg Sub Duplicate Ratio
of $16 : 36$
 $= 4 : 6$
 $= 2 : 3$

eg Sub Triplicate Ratio
of $8 : 125$
 $= (8)^{1/3} : (125)^{1/3}$
 $= 2 : 5$



$(x)^{\frac{1}{n}} = ?$

\Downarrow

$\rightarrow \sqrt{\quad}$ 12 times

$\rightarrow -$ 1

$\rightarrow \div n$

$\rightarrow + 1$

$\rightarrow \boxed{x =}$ 12 times

$$(125)^{\frac{1}{3}} = 5$$



Commensurable quantities



If Ratio of two quantities can be expressed as a Rational no.

eg $\frac{2}{3}$, $\frac{5}{8}$, $\frac{6.1}{2} = \frac{61}{20}$

Incommensurable quantities



Otherwise

eg $\frac{\sqrt{2}}{8}$

eg $\frac{3}{\sqrt{5}}$

eg $\frac{7}{\pi}$



Q

$$A : B = 2 : 3$$
$$B : C = 3 : 7$$
$$A : C = ?$$
$$A : B : C = ?$$

← Sol.

$$A : B = 2 : 3$$
$$B : C = 3 : 7$$

$$A : B : C = 2 : 3 : 7$$

$$A : C = 2 : 7$$

$$\frac{A}{\cancel{B}} \times \frac{\cancel{B}}{C} = \frac{2}{3} \times \frac{\cancel{3}}{7}$$
$$\frac{A}{C} = \frac{2}{7}$$

$$A : C = 2 : 7$$



Q

$$A : B = 2 : 3$$

$$B : C = 4 : 5$$

$$A : C = ?$$

$$A : B : C = ?$$

Sol.

$$\frac{A}{B} = \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$$

$$\frac{B}{C} = \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$$

$$A : B : C = 8 : 12 : 15$$

$$A : C = 8 : 15$$

$$\frac{A}{B} \times \frac{B}{C} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

$$A : C = 8 : 15$$

$$A : B = 2 : 3$$

$$B : C = 4 : 5$$

The diagram shows a box containing the numbers 3 and 4. An arrow points from the 3 in the first ratio to the 4 in the second ratio, and another arrow points from the 4 in the second ratio to the 5 in the second ratio.

$$A : B : C = 8 : 12 : 15$$



Q.

$$A : B = a : b$$

$$B : C = c : d$$

$$C : D = e : f$$

$$A : B : C : D = ?$$

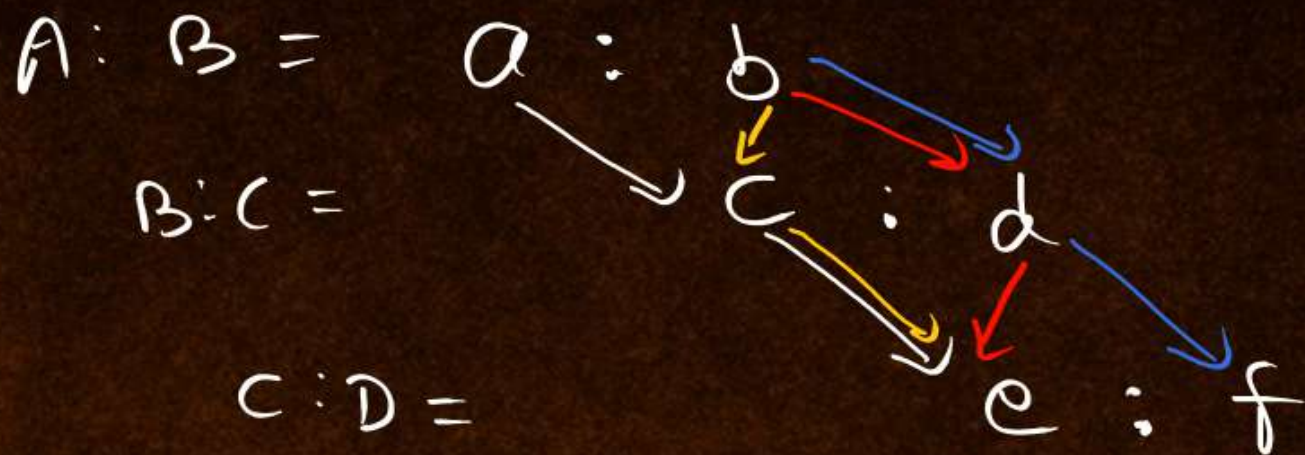
Sol.

$$\frac{A}{B} = \frac{a}{b} \cdot \frac{c}{c} \cdot \frac{e}{e}$$

$$\frac{B}{C} = \frac{c}{d} \cdot \frac{b}{b} \cdot \frac{e}{e}$$

$$\frac{C}{D} = \frac{e}{f} \cdot \frac{d}{d} \cdot \frac{b}{b}$$

$$A : B : C : D = ace : bce : bde : bdf$$



$$A : B : C : D = ace : bce : bde : bdf$$



eg

$$A : B = 2 : 3$$
$$B : C = 4 : 5$$
$$C : D = 2 : 1$$

$$\frac{A}{B} + \frac{B}{C} + \frac{C}{D} = \frac{2}{3} + \frac{4}{5} + \frac{2}{1}$$
$$\frac{A}{D} = \frac{16}{15}$$



$$16 : 24 : 30 : 15$$

2 : 3 4 : 5 2 : 1



QUESTION - ①

The Ratio of height 170 cm and weight 60 kg of a person is

- A 17:6
- B 17cm:60kg
- C Not a ratio
- D None

$$\frac{170 \text{ cm}}{60 \text{ kg}}$$

The Ratio of 25 hours and 45 minutes is

Time

Time

A 5:9

B 100:3

C Not a ratio

D None

$$\begin{array}{r} 25 \times \overset{4}{60} \text{ minutes} \\ \hline 45 \text{ minutes} \\ 3 \\ \hline = \frac{100}{3} \end{array}$$

Which Ratio out of the following is greater ?

A $3\frac{1}{4} : 5\frac{1}{3} \Rightarrow \frac{3.25}{5.33} = 0.609$

B $6\frac{1}{2} : 8\frac{1}{5} \Rightarrow \frac{6.5}{8.2} = 0.792$

C $3.9 : 5.4 \Rightarrow \frac{3.9}{5.4} = 0.722$

D None

$$3\frac{1}{4} = \frac{13}{4}$$
$$= 3 + \frac{1}{4}$$
$$=$$

If $A:B=2:3$, $B:C=5:7$ Then $A:B:C = ?$

- A** 10:15:21
 $\underbrace{2:3}$ $\underbrace{5:7}$
- B** 6:9:14
 $\underbrace{2:3}$ $\underbrace{9:14}$
- C** 4:6:21
 $\underbrace{2:3}$ $\underbrace{2:7}$
- D** None

$$2 : \underbrace{3}_{5} : 7$$
$$10 : 15 : 21$$

If $A:B=2:3$, $B:C=4:5$ & $C:D=6:7$, Then $A:D = ?$

A 2:7

$$\frac{A}{B} + \frac{B}{C} + \frac{C}{D} = \frac{2}{3} + \frac{4}{5} + \frac{6}{7} \quad 2$$

B 7:8

$$\frac{A}{D} = \frac{16}{35}$$

C 16:35

D 4:13

$$A : B : C : D = ?$$

P, Q & R are three cities. The ratio of average temperature between P and Q is 11:12 and that between P and R is 9:8. Find ratio between Q and R

A 22:27

B 27:32

C 32:33

D None

$$\frac{P}{Q} = \frac{11}{12} \quad \& \quad \frac{P}{R} = \frac{9}{8}$$

$$\frac{Q}{P} \times \frac{P}{R} = \frac{12}{11} \times \frac{9}{8}$$

$$\frac{Q}{R} = \frac{27}{22}$$

$$\begin{array}{l} P \\ P \end{array} : Q = \begin{array}{l} 11 \\ 9 \end{array} : \begin{array}{l} 12 \\ 8 \end{array}$$

$$P : Q : R = 99 : 108 : 88$$

$$Q : R = 108 : 88$$

$$27 : 22$$

If $A:B=3:4$, Then the value of $(2a+3b):(3a+4b)$

A 54:25

B 8:25

C 17:24

D 18:25

$$a : b = 3 : 4$$

$$\frac{2a + 3b}{3a + 4b}$$

$$= \frac{2(3) + 3(4)}{3(3) + 4(4)}$$

$$= \frac{6 + 12}{9 + 16} = \frac{18}{25}$$

If $x:y=1:2$, Then the value of $(x^2y + xy^2):(x^3 + y^3)$

A 2:3

$$x:y = 1:2$$

B 3:22

$$\frac{x^2y + xy^2}{x^3 + y^3}$$

C 1:8

D None

Two numbers are in the ratio 2:3, If 4 is subtracted from each, they are in the ratio 3:5, The Numbers are

A 16, 24

B 4, 6

C 2, 3

D None

$$a : b = 2 : 3$$

$$\text{w } a = 2x \text{ \& } b = 3x$$

$$\frac{12}{20} = \frac{3}{5}$$

$$\frac{2x-4}{3x-4} = \frac{3}{5}$$

$$5(2x-4) = 3(3x-4)$$

$$10x-20 = 9x-12$$

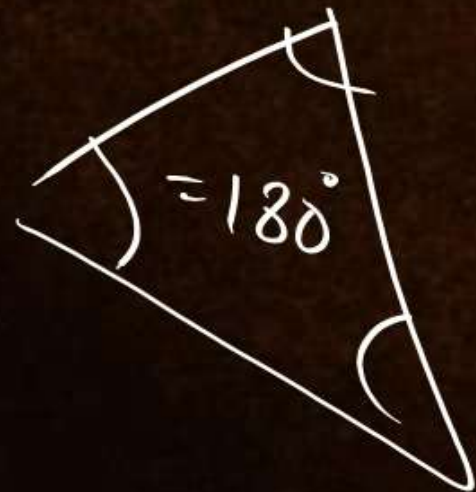
$$10x-9x = -12+20$$

$$x = 8$$

$$\begin{array}{l} 2x \quad 4 \quad 3x \\ 16 \quad \quad 24 \end{array}$$

The angles of a triangle are in the ratio 2:7:11, The angles are

- A $20^\circ, 70^\circ, 90^\circ = 2:7:9$
- B $30^\circ, 70^\circ, 80^\circ = 3:7:8$
- C $18^\circ, 63^\circ, 99^\circ = 2:7:11$
- D None



$$2x + 7x + 11x = 180$$

$$20x = 180$$

$$x = 9$$

$$18, 63, 99$$

Anand earns Rs 80 in 7 hours and Pramod Rs 90 in 12 hours
The Ratio Of Their earning is

A 2:3

B 3:22

C 1:8

D None

Anand
7 h \rightarrow ₹80
1 h \rightarrow ₹ $\left(\frac{80}{7}\right)$

Pramod
12 h \rightarrow ₹90
1 h \rightarrow ₹ $\left(\frac{90}{12}\right) = \frac{30}{4} = ₹\frac{15}{2}$

$$\frac{\frac{80}{7}}{\frac{15}{2}} = \frac{80}{7} \times \frac{2}{15} = \frac{32}{21}$$

If $p:q$ is the sub duplicate ratio of $p - x^2 : q - x^2$ then x^2 is

A $\frac{p}{p+q}$

B $\frac{q}{p+q}$

C $\frac{pq}{p+q}$

D None

$$\frac{p}{q} = \frac{\sqrt{p-x^2}}{\sqrt{q-x^2}}$$

Squaring both side

$$\frac{p^2}{q^2} = \frac{p-x^2}{q-x^2}$$

$$p^2 q - p^2 x^2 = q^2 p - q^2 x^2$$

$$q^2 x^2 - p^2 x^2 = q^2 p - p^2 q$$

$$x^2 (q^2 - p^2) = pq (q - p)$$

$$\therefore a^2 - b^2 = (a+b)(a-b)$$

$$x^2 (q+p)(q-p) = pq (q-p)$$

$$x^2 = \frac{pq}{q+p}$$

If $2s:3t$ is duplicate ratio of $2s-p:3t-p$, Then

A $p^2 = 6st$

$$\frac{2s}{3t} = \left(\frac{2s-p}{3t-p} \right)^2$$

$$\left[\because (a-b)^2 = a^2 + b^2 - 2ab \right]$$

B $p = 6st$

$$\frac{2s}{3t} = \frac{4s^2 + p^2 - 2(2s)p}{9t^2 + p^2 - 2(3t)p}$$

C $2p = 3st$

$$2s(9t^2 + p^2 - 6tp) = 3t(4s^2 + p^2 - 4sp)$$

D None

$$18st^2 + 2p^2s - 12p^2t = 12t^2s + 3tp^2 - 12p^2t$$

$$2p^2s - 3tp^2 = 12t^2s - 18st^2$$

$$p^2(2s - 3t) = 6ts(2s - 3t) \Rightarrow p^2 = 6st$$

The ratio of the number of boys to numbers of girls in a school of 720 students is 3:5 .If 18 new girls are admitted in the school ,Find how many new boys may be admitted so that the ratio of boys to girls may change to 2:3

- A 35
- B 42
- C 48
- D None

$$\begin{array}{l} 720 \\ \swarrow \quad \searrow \\ B: 4 = 3:5 \\ B = 720 \times \frac{3}{8} = 270 \\ G = 720 \times \frac{5}{8} = 450 \end{array}$$

$$\begin{aligned} \frac{B}{G} &= \frac{270+x}{450+18} = \frac{2}{3} \\ \frac{270+x}{468} &= \frac{2}{3} = 0.666 \\ \Rightarrow 270+x &= \frac{2}{3} \times 468 \\ &= 270+x = 312 \\ &\boxed{x = 42} \end{aligned}$$



g A, B & C can finish a work in 4, 5 & 6 Days alone.

in how many Days they can finish it together?

Sol.

Total work = 120 units

$$\begin{array}{r} 120 \\ \hline 30 + 24 + 20 \\ = \frac{120}{74} = 1.62 \text{ Days} \end{array}$$



Proportion

(समान्यता)

Equality of two Ratios

$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b :: c : d$$

Income

12000 & 15000

Weight

64 kg & 80 kg

$$\frac{12000}{15000} = \frac{64}{80}$$

CA



$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

$$a : b :: c : d$$

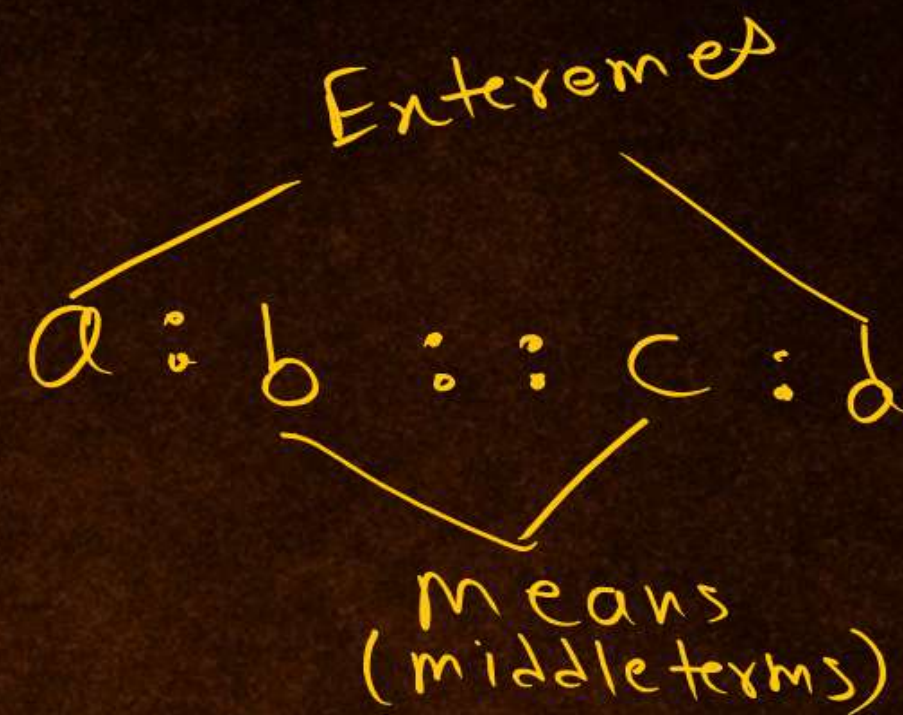
first term

second term

third term

fourth term

a, b, c & d are called terms





Q.

find value of x

$$\frac{2}{3} : x :: 4 : \frac{5}{2}$$

Sol.

$$\frac{\left(\frac{2}{3}\right)}{\left(\frac{x}{1}\right)} = \frac{(4)}{\left(\frac{5}{2}\right)}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{x} = 4 \times \frac{2}{5}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{4} \times \frac{5}{2} = x$$

$$\Rightarrow \boxed{\frac{5}{12} = x}$$

or

$$\frac{2}{3} : x = 4 : \frac{5}{2}$$

$$\frac{2}{3} \times \frac{5}{2} = x \times 4$$

$$\frac{5}{3} = 4x$$

$$\boxed{\frac{5}{12} = x}$$



$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Product of extremes = Product of means

⇔
cross multiplication rule



$$\frac{a}{b} = \frac{b}{c}$$

$$a : b :: b : c$$

$$a : b = b : c$$

Continuous Proportion

a, b & c are in
Continuous Proportion

Then $\frac{a}{b} = \frac{b}{c}$

a & c are first
& third proportional

b is mean proportional

$a \times c = b^2$

QUESTION



Find the value of x if $\frac{10}{3} : x :: \frac{5}{2} : \frac{5}{4}$

- A** $\frac{5}{3}$
- B** $\frac{3}{5}$
- C** $\frac{5}{6}$
- D** None

$$\frac{10}{3} : x = \frac{5}{2} : \frac{5}{4}$$

$$\frac{\cancel{10}^5}{3} \times x = \frac{5}{\cancel{2}_4} \times \frac{5}{4}$$

$$\frac{5}{3} = x$$

QUESTION



The fourth proportional to 4,6,8 is

- A** 12 ✓✓
- B** 32
- C** 48
- D** None

$$4 : 6 = 8 : x$$

$$4x = 6 \times 8$$

$$x = \frac{48}{4}$$

$$x = 12$$

QUESTION



The third proportional to 12, 18 is

- A** 24
- B** 27 ✓✓
- C** 48
- D** None

$$12, 18, x$$

$$\frac{12}{18} = \frac{18}{x}$$

$$12x = (18)^2$$

$$x = 27$$

QUESTION



The mean proportional to 25, 81 is

A 40

B 50

C 45

D None

$$25, x, 81$$

$$\frac{25}{x} = \frac{x}{81}$$

$$25 + 81 = x^2$$

$$2025 = x^2$$

$$\sqrt{2025} = x$$

$$45 = x$$



Continus Proportion

$$\frac{a}{b} = \frac{b}{c}$$

Continwed Proportion

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \dots$$



$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$

CROSS multiplication
Rule

$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

Both side Reciprocal
(Invert)

$$\frac{b}{a} = \frac{d}{c}$$

$$b : a = d : c$$

If $a : b = c : d$
then $b : a = d : c$

Invertendo



$$\left\{ \begin{array}{l} a : b = c : d \\ \frac{a}{b} = \frac{c}{d} \\ ad = bc \\ \frac{a}{c} = \frac{b}{d} \\ a : c = b : d \end{array} \right.$$

#

$$\left\{ \begin{array}{l} \text{if } a : b = c : d \\ \text{then } a : c = b : d \end{array} \right.$$

Alternendo



sf $a:b = c:d$

$$\frac{a}{b} = \frac{c}{d}$$

Add one

$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$

$$\frac{a+b}{b} = \frac{c+d}{d}$$

sf $\frac{a}{b} = \frac{c}{d}$

then $\frac{a+b}{b} = \frac{c+d}{d}$

componendo

eg $\frac{x}{2} = \frac{y}{5}$

Apply componendo

$$\frac{x+2}{2} = \frac{y+5}{5}$$



if $\frac{a}{b} = \frac{c}{d}$
then $\frac{a+b}{b} = \frac{c+d}{d}$
componendo

if $\frac{a}{b} = \frac{c}{d}$
then $\frac{a-b}{b} = \frac{c-d}{d}$
Dividendo

if $\frac{a}{b} = \frac{c}{d}$
then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
Componendo & Dividendo

eg $\frac{x}{2} = \frac{y}{3}$
Apply comp. & Dividendo
 $\frac{x+2}{x-2} = \frac{y+3}{y-3}$



$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{a+c+e+\dots}{b+d+f+\dots} = k$$

Addendo

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a-c-e}{b-d-f}$$

Subtrahendo

eg

$$\frac{12}{6} = \frac{8}{4} = 2$$

$$\frac{12+8}{6+4} = 2$$

QUESTION



$A = \frac{B}{2} = \frac{C}{5}$, Then A:B:C is

A 3:5:2

B 2:5:3

C 1:2:5

D None

$$\frac{A}{1} = \frac{B}{2} = \frac{C}{5} = k$$

$$\frac{A}{1} = k$$
$$A = k$$

$$\frac{B}{2} = k$$
$$B = 2k$$

$$\frac{C}{5} = k$$
$$C = 5k$$

$$A : B : C = k : 2k : 5k$$
$$1 : 2 : 5$$

QUESTION



$\frac{A}{3} = \frac{B}{4} = \frac{C}{7}$, Then $\frac{A+B+C}{C}$ is

- A** 1
- B** 3
- C** 2
- D** None

$$\frac{A}{3} = \frac{B}{4} = \frac{C}{7} = k$$

$$\frac{A}{3} = k \quad \frac{B}{4} = k \quad \frac{C}{7} = k$$

$$A = 3k \quad B = 4k \quad C = 7k$$

$$\frac{A+B+C}{C}$$

$$= \frac{3k + 4k + 7k}{7k}$$

$$= \frac{14k}{7k}$$

$$= 2$$

QUESTION

If $a:b = 4:1$, Then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is

- A $\frac{5}{2}$
- B 4
- C 5
- D None

$$\frac{a}{b} = 4$$

$$\frac{a}{b} = \frac{4}{1}$$

$$\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$$

$$= \sqrt{\frac{4}{1}} + \sqrt{\frac{1}{4}}$$

$$= 2 + \frac{1}{2}$$

$$= \frac{5}{2}$$

QUESTION



If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = k$

Then $(b - c)x + (c - a)y + (a - b)z = ?$

A $\frac{51}{2}$

B 0

C 5

D None

$x = k(b+c-a)$

now $(b-c)x + (c-a)y$

$= k(b-c)[(b+c)-a]$

$= k[(b-c)(b+c) - (b-c)a]$

$= k[\cancel{b^2} - \cancel{c^2} - a\cancel{b} + a\cancel{c}] + k[\cancel{c^2} - \cancel{a^2} - \cancel{b}c + \cancel{a}b]$

$+ k[\cancel{a^2} - \cancel{b^2} - a\cancel{c} + \cancel{b}c]$

cyclic order
 $(a-b) + (b-c) + (c-a)$
 $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)$
 $a(b-c) + b(c-a) + c(a-b)$
 $-(a-b)c$

QUESTION



The sum of the ages of 3 persons is 150 years.
10 years ago their ages were in the ratio 7:8:9.
 Their present ages are

A 45, 50, 55

B 40, 60, 50
 30, 50, 40

C 35, 45, 70

D None

10 years Ago
 Ages: $7x, 8x$ & $9x$
 $7x + 8x + 9x = 120$
 $24x = 120$
 $x = 5$
 Present age
 $45, 50, 55$
 $35, 40, 45$
 $7:8:9$

QUESTION

The ratio between the speeds of two train is 3:8 .If the second train runs 400 kms in 5 hours , the speed of the first train is

A 30 km/h

B 50 km/h

C 70 km/h

D None

$$d = 400 \text{ km}$$

$$t = 5 \text{ hours}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{400}{5}$$

$$v_2 = 80 \text{ km/h}$$

$$\frac{v_1}{v_2} = \frac{3}{8}$$

$$\frac{v_1}{80} = \frac{3}{8}$$

$$v_1 = 30 \text{ km/h}$$



Hotel
⇓
Biryani

₹ 40/kg
5 kg

₹ 200

₹ 20/kg
15 kg

₹ 300

Total cost = ₹ 500
Total quantity = 20 kg

Avg. cost = $\frac{500}{20}$
= ₹ 25/kg

$\frac{5}{15} = \frac{1}{3}$



₹ 40/kg

₹ 20/kg

x kg

y kg

Avg cost = ₹ 25/kg

$$\text{Avg cost} = \frac{\text{Total cost}}{\text{Total quantity}}$$

$$25 = \frac{40x + 20y}{x + y}$$

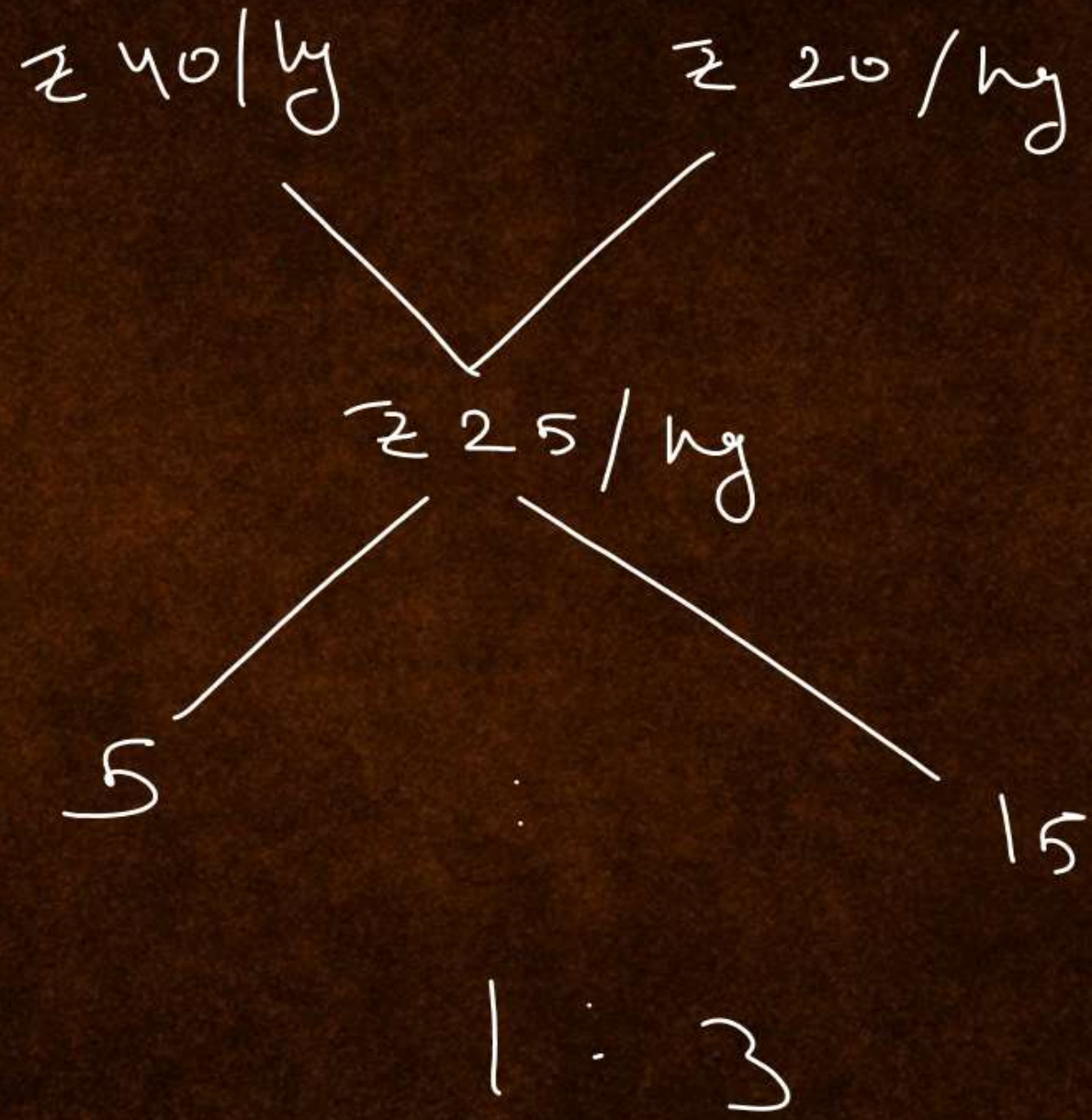
$$25x + 25y = 40x + 20y$$

$$5y = 15x$$

$$\frac{y}{x} = \frac{15}{5} = 3$$

$$\frac{x}{y} = \frac{1}{3} \quad 1:3$$

$$\frac{Q_1}{Q_2} = ?$$



QUESTION



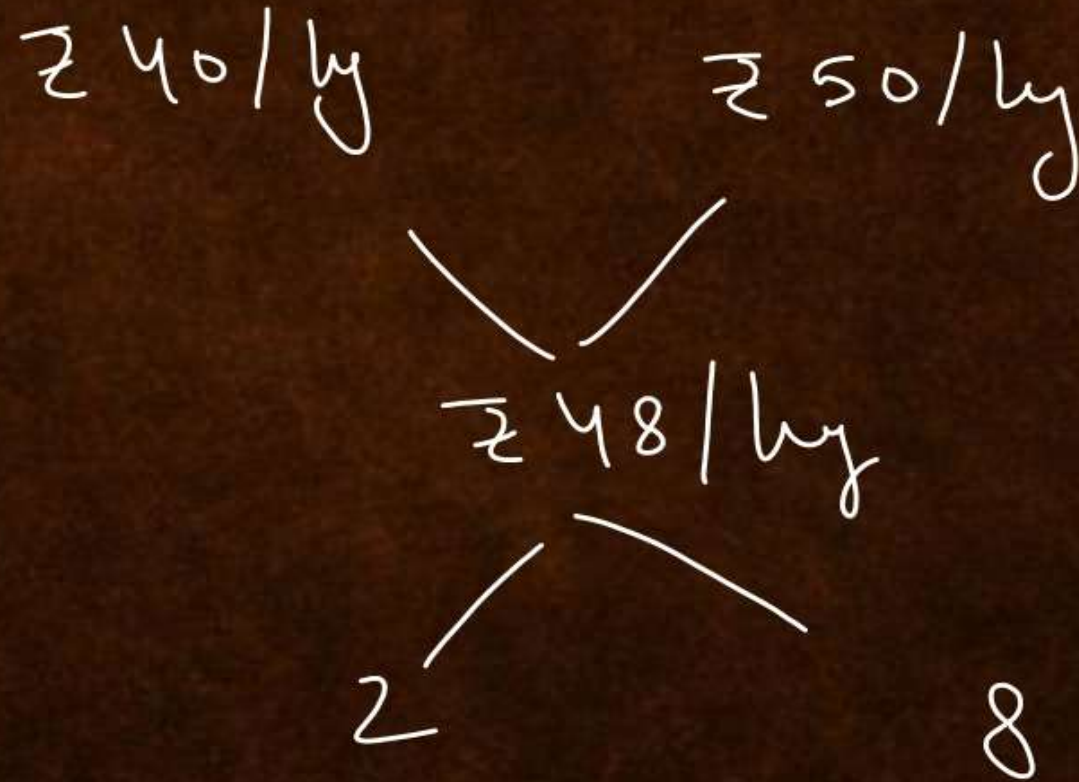
One kind Of rice costs Rs 40/Kg & other kind of rice costs Rs 50/kg , In what proportion they should be mixed so that average cost of mixture is Rs 48/Kg

A 1:2

B 3:4

C 1:4

D 4:1



2 : 8
1 : 4

QUESTION



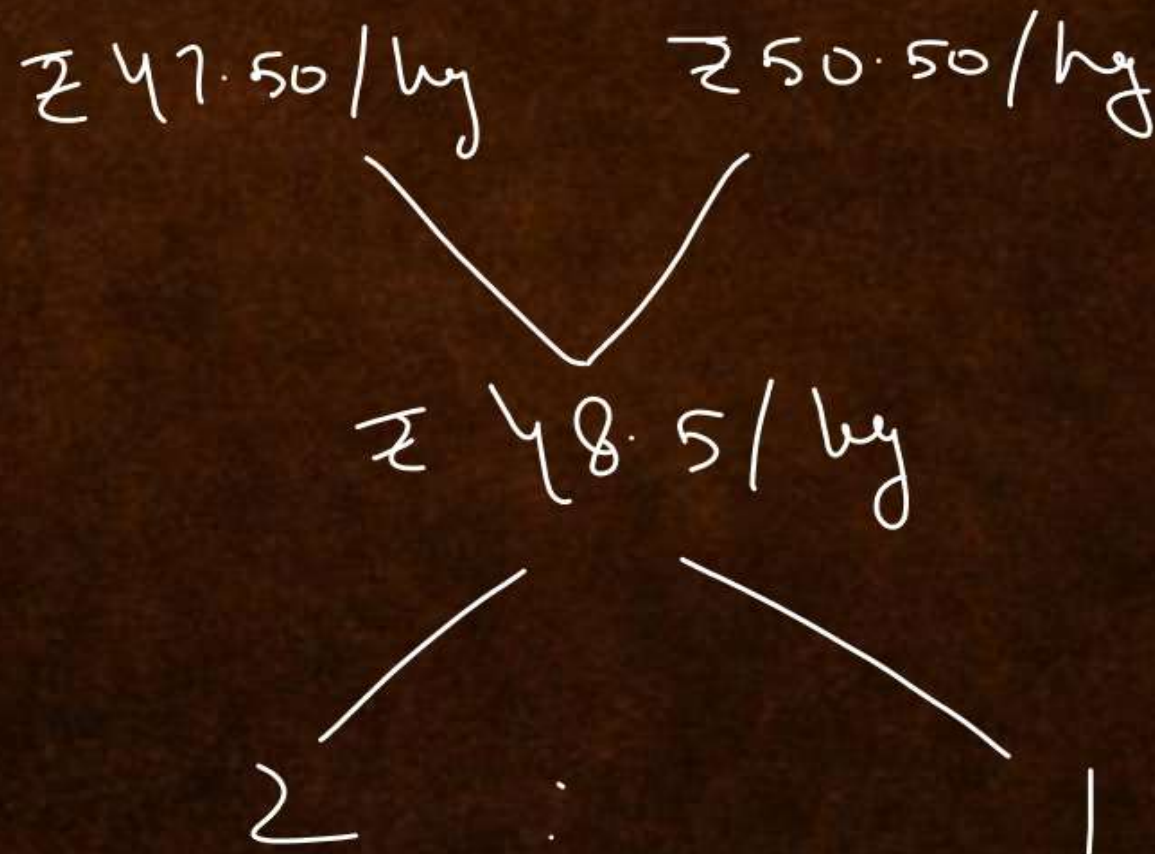
One kind Of tea costs Rs 47.50/Kg & other kind of tea costs Rs 50.50/kg , In what proportion they should be mixed so that seller can sell it for Rs 60.625/kg and earns a profit of 20% on the selling price

A 1:2

B 3:4

C 2:1

D 4:1



$$\begin{aligned} \text{S.P.} &= ₹ 60.625 \\ \text{Profit} &= \frac{12.125}{\text{Cost}} \\ &= \frac{12.125}{₹ 48.5/kg} \end{aligned}$$

QUESTION

$$\text{If } \frac{x+y}{x-y} = \frac{4}{3}$$

Then $x:y = ?$

A 7:1 ✓✓

B 1:7

C 3.5 : 1

D 1 : 3.5

$$\frac{x+y}{x-y} = \frac{4}{3}$$

$$3x + 3y = 4x - 4y$$

$$3x - 4x = -4y - 3y$$

$$-x = -7y$$

$$x = 7y$$

$$\frac{x}{y} = \frac{7}{1}$$

$$\frac{4+3}{4-3} = \frac{7}{1}$$

QUESTION



If $x/y = z/w$, implies $y/x = w/z$, then the process is called

- A** Componendo
- B** Dividendo
- C** Alternendo
- D** Invertendo ✓✓

$$\frac{x}{y} = \frac{z}{w} \implies \frac{y}{x} = \frac{w}{z}$$

QUESTION



If $\frac{a}{4} = \frac{b}{5}$ then

- A** $\frac{a+4}{a-4} = \frac{b-5}{b+5}$ ✗
- B** $\frac{a+4}{a-4} = \frac{b+5}{b-5}$ ✓✓
- C** $\frac{a-4}{a+4} = \frac{b+5}{b-5}$ ✗
- D** none of these

$$\frac{a}{4} = \frac{b}{5}$$
$$\frac{a+4}{a-4} = \frac{b+5}{b-5}$$

QUESTION

CA

If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be:

A 5:8

B 8:5

C 7:8

D 8:7

P
75

Q
100

R
120

$$\frac{R}{P} = \frac{120}{75} = \frac{8}{5}$$



INDICES

CA

$$2 \times 2 \times 2 \times 2 \times 2 = (2)^5 = 32$$

$$3 \times 3 \times 3 \times 3 = (3)^4 = 81$$

If a is multiply with itself (n) times

$$a \times a \times a \times a \dots n \text{ times} = (a)^n$$

$a = \text{Base}$

$n = \text{Power}$
(Exponent)
(Index)

$$2 + 2 + 2 + 2 + 2 = 2(5) = 10$$

$$3 + 3 + 3 + \dots \text{ 12 times} = 3(12) \\ = 36$$



$$(4)^3 = 4 \times 4 \times 4 = 64$$

$$(5)^4 = 5 \times 5 \times 5 \times 5 = 625$$

$$(2)^{12} = 4096$$

$$(1.1)^5 = 1.61051$$



$a^n = a \times a \times a \times \dots$ n times

eg $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$a^m \cdot a^n = a^{m+n}$

eg $(3)^4 \cdot (3)^7 = (3)^{11}$ | eg $x^2 \cdot y^3 \cdot x^5 \cdot y^{3/2} = (x)^{2+5} \cdot (y)^{3+3/2}$

$= (x)^7 \cdot (y)^{9/2}$



$\frac{a^m}{a^n} = (a)^{m-n}$

eg

$$\frac{x^{10}}{x^3} = (x)^{10-3} = (x)^7$$

eg

$$\frac{x^2 y^3 x^3}{y^2 x^{1/2}} = (x)^{2+3-\frac{1}{2}} (y)^{3-2} = (x)^{4.5} (y)^1 = (x)^{9/2} (y)^1$$



$a^n = \frac{1}{a^{-n}}$

eg $x^3 = \frac{1}{x^{-3}}$

eg $y^{1/2} = \frac{1}{(y)^{-1/2}}$

$\frac{1}{a^n} = a^{-n}$

eg $\frac{1}{x^2} = x^{-2}$

eg $x^{-1} = \frac{1}{x^1}$

$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

eg $\frac{x^6}{x^4} = x^{6-4} = x^2$ or $\frac{x^6}{x^4} = \frac{1}{x^{4-6}} = \frac{1}{x^{-2}}$



$a^m \cdot a^n = a^{m+n}$

$a^{m+n} = a^m \cdot a^n$

$x^{n+2} = x^n \cdot x^2$ | $(x)^{m+n-1} = x^m \cdot x^{n-1}$

~~$0^0 = 1$
 $0^1 = 0$
 $0^2 = 0$
 $0^3 = 0$
 $0^4 = 0$
 $0^5 = 0$
 $0^6 = 0$
 $0^7 = 0$
 $0^8 = 0$
 $0^9 = 0$
 $0^{10} = 0$
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 $0^{95} = 0$
 $0^{96} = 0$
 $0^{97} = 0$
 $0^{98} = 0$
 $0^{99} = 0$
 $0^{100} = 0$~~

$a^0 = 1$

where $a \neq 0$

$2^0 = 1$
 $(100)^0 = 1$
 $\left(\frac{2}{5}\right)^0 = 1$
 $(-5)^0 = 1$

$2^0 = 2^{1-1} = 2^1 \cdot 2^{-1} = 2 \cdot \frac{1}{2} = 1$

$0^0 = \text{Not Defined}$

#

$$(a^m)^n = (a)^{m \times n}$$

eg

$$(2^5)^2 = (2)^{10}$$

eg

$$(x^y)^3 = (x)^{3y}$$

#

$$(a \cdot b \cdot c \cdot d)^n = a^n \cdot b^n \cdot c^n \cdot d^n$$

eg

$$(x^2 y^3 z^4)^{10}$$

$$= (x^2)^{10} (y^3)^{10} (z^4)^{10}$$

$$= (x)^{20} (y)^{30} (z)^{40}$$



$(a^m)^n = a^{mn}$

$a^{m^n} \neq a^{mn}$

$(2^3)^4$
 $= (2)^{12}$
 8^4

$\neq 2^{3^4}$
 $= (2)^{81}$

$3 \times 3 \times 3 \times 3$



If $a^m = a^n$
then $m = n$

g

$2^x = 32$
find value of x

Sol:

$$(2)^x = (2)^5$$

$$2x = 5$$

$$x = 2.5$$

2	3	2
2	16	
2	8	
2	4	
2	2	
		1



Square root

↓
√x = (x)^{1/2}

eg √36
= √(2×2×3×3)
= 2×3
= 6

eg

√200
= √(2×2×2×5×5)
= 2×5×√2
= 10√2

2	2	5
2	1	5
2	5	0
5	2	5
5	5	1



Cube Root

$$\sqrt[3]{x} = (x)^{1/3}$$

eg $\sqrt[3]{216}$

$$= \sqrt[3]{\underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{3 \times 3 \times 3}_{3^3}}$$

$$= 2 \times 3$$

$$= 6$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

4th Root

$$\sqrt[4]{x} = (x)^{1/4}$$

nth Root

$$\sqrt[n]{x} = (x)^{1/n}$$



$$\begin{aligned} & (81)^{1/2} \\ = & (9^2)^{1/2} \\ = & 9^{2 \times \frac{1}{2}} \\ = & 9 \end{aligned}$$

$$\begin{aligned} & (8)^{1/3} \\ = & (2^3)^{1/3} \\ = & 2 \end{aligned}$$

Calculator

$$x^{1/n} = ?$$

→ $\sqrt{\quad}$ 12 times

→ $-$

→ $\frac{\circ}{\circ}$ 5

→ $+$

→ $X =$ 12 times



$$(x)^{1/a} = y$$
$$\left(x^{1/a}\right)^a = (y)^a$$
$$x = y^a$$

eg

$$x^{1/3} = y$$

then $x = y^3$

eg

$$x^{1/2} = y$$
$$x^3 = y^2$$



$\left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}$

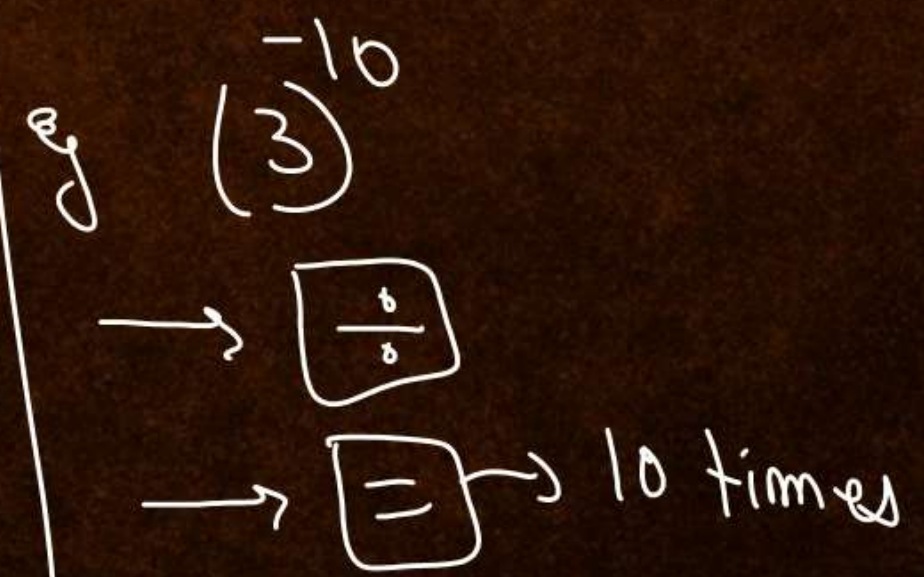
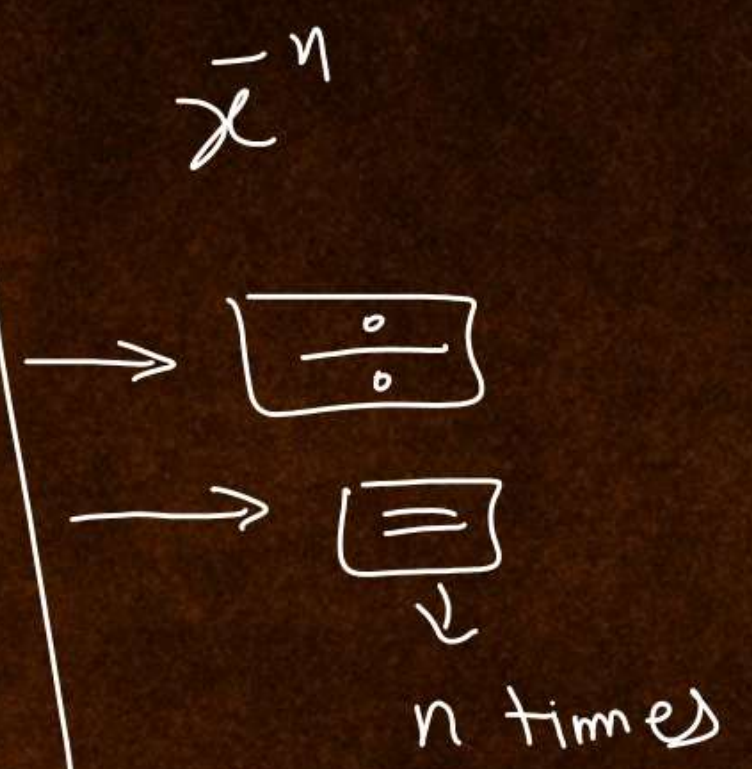
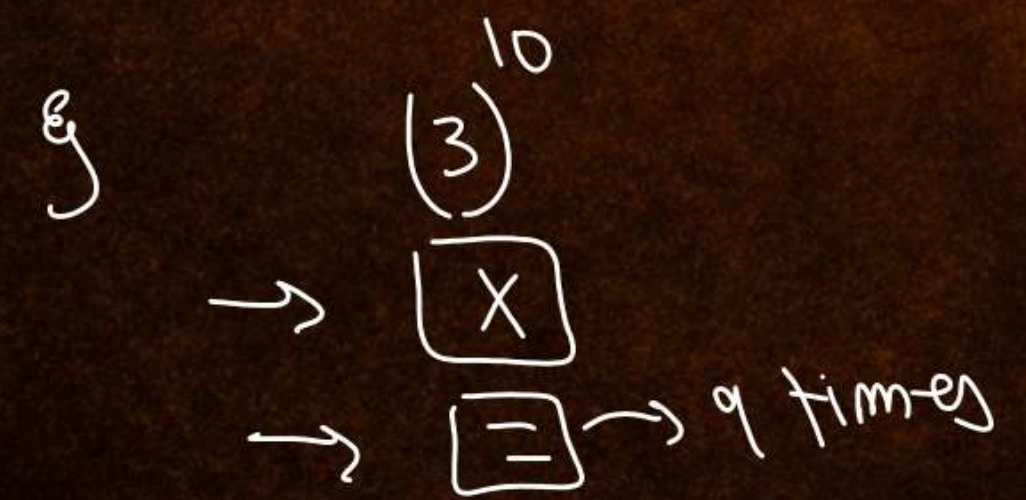
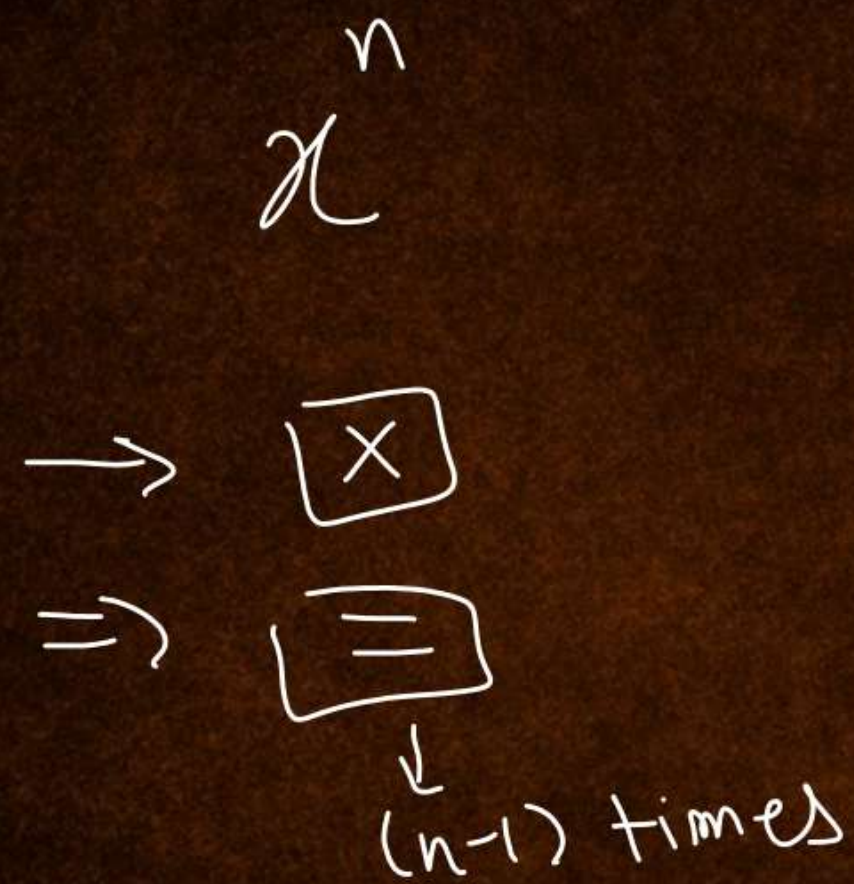
$\left(\frac{3}{5}\right)^2 = \left(\frac{5}{3}\right)^{-2}$

$(2)^8 = 256$

$(2)^{-8} = \frac{1}{(2)^8} = 0.00390625$

$(3)^{-4} = \frac{1}{(3)^4} = 0.01234$

Reciprocal
 $\frac{1}{x} =$





#

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

#

$$a^2 + b^2 = (a+b)^2 - 2ab$$

#

$$a^2 + b^2 = (a-b)^2 + 2ab$$

#

$$(a+b)^2 - (a-b)^2 = 4ab$$



$$\# \quad \{ a^2 - b^2 = (a+b)(a-b) \}$$

$$\text{eg} \quad x^2 - 4^2 = (x+4)(x-4)$$

$$\text{eg} \quad x^2 - 1 = x^2 - (1)^2 \\ = (x+1)(x-1)$$

$$\# (a+b)^3 = a^3 + b^3 + 3ab(a+b) \rightarrow a^3 + \underbrace{3a^2b + 3ab^2}_{\text{}} + b^3$$



$$\# (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\# a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$\# a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$



$$\# (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$\# a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$\# \text{ If } a+b+c=0 \\ \text{ then } a^3 + b^3 + c^3 = 3abc$$



$$\begin{aligned} \text{eg } (4)^3 + (5)^3 + (-9)^3 & \\ &= 3(4)(5)(-9) \\ &= -540 \end{aligned}$$

$$\begin{aligned} 4^3 &= 64 \\ 5^3 &= 125 \\ (-9)^3 &= -729 \\ \hline & -540 \end{aligned}$$

$$\text{eg } (p-q)^3 + (q-r)^3 + (r-p)^3 = ?$$

a) pqr
 b) $3pqr$
 c) $(p-q)(q-r)(r-p)$
 d) None

sol. $a = p-q, b = q-r, c = r-p$
 $a+b+c = p-q+q-r+r-p = 0$

$$\begin{aligned} a^3 + b^3 + c^3 &= 3abc \\ &= (p-q)^3 + (q-r)^3 + (r-p)^3 \\ &= 3(p-q)(q-r)(r-p) \end{aligned}$$

QUESTION



The value of $\left(\frac{2p^2q^3}{3xy}\right)^0$ where $p, q, x, y \neq 0$ is equal to

- A 0
- B 1
- C 2
- D None

$a^0 = 1$ where $a \neq 0$

QUESTION



Simplify $4x^{1/3} 2x^{-1}$ if $x = 4$?

- A** 2
- B** $2^{1/3}$
- C** $2^{5/3}$
- D** None

$$\begin{aligned}
 & 4(x)^{1/3} \times 2(x)^{-1} \\
 & 8 \times (x)^{1/3-1} \\
 & 8(x)^{-2/3} \\
 & = 8 \times (4)^{-2/3} \\
 & = (2)^3 \times (2^2)^{-2/3} \\
 & = (2)^3 (2)^{-4/3} \\
 & = (2)^{3-4/3} \\
 & = (2)^{5/3}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{3} - 1 \\
 & \frac{1}{3} - \frac{3}{3} \\
 & \frac{1-3}{3} \\
 & \frac{-2}{3}
 \end{aligned}$$

$$2 + \left(-\frac{2}{3}\right)$$

QUESTION

$x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to

- A** x
- B** 0
- C** 1
- D** *None*

$$\begin{aligned} & (x)^{\cancel{a-b} + \cancel{b-c} + \cancel{c-a}} \\ &= x^0 \\ &= 1 \end{aligned}$$

$$(a-b) + (b-c) + (c-a) = 0$$

$$\begin{aligned} & (x)^{a-b} \times x^{b-c} \times x^{c-a} \\ &= x^0 = 1 \end{aligned}$$

QUESTION



The value of $(125)^{\frac{2}{3}} \times (625)^{\frac{3}{4}}$ is

A 150

B 625

C 3125

D None

$$\begin{aligned} & \Downarrow \\ & (5^3)^{\frac{2}{3}} \times (5^4)^{\frac{3}{4}} \\ & = 5^2 \times 5^3 \\ & = 5^5 \end{aligned}$$

$$\begin{aligned} 5^3 &= 125 \\ 5^4 &= 625 \end{aligned}$$

QUESTION



$$\left[(x^n)^{n - \frac{1}{n}} \right]^{\frac{1}{n+1}}$$

A x^n

B x^{n+1}

C x^{n-1}

D None

$$\begin{aligned}
 &= \left[(x^n)^{n - \frac{1}{n}} \right]^{\frac{1}{n+1}} \\
 &= (x^n)^{\frac{n^2 - 1}{n} \times \frac{1}{n+1}} \\
 &= (x^n)^{\frac{(n^2 - 1) \times 1}{(n+1)}} \\
 &= (x^n)^{\frac{(n+1)(n-1) \times 1}{(n+1)}} \\
 &= (x^n)^{n-1} \\
 &= x^{n(n-1)} \\
 &= x^{n^2 - n}
 \end{aligned}$$

$$\begin{aligned}
 n^2 - 1 &= (n+1)(n-1) \\
 &= n^2 - 1
 \end{aligned}$$

QUESTION



The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

A 1

B 0

C 2

D None

$$\left(x^{a-b}\right)^{(a+b)}$$

$$= \left(x\right)^{a^2-b^2} \cdot \left(x\right)^{b^2-c^2} \cdot \left(x\right)^{c^2-a^2}$$

$$= \left(x\right)^{\cancel{a^2} - \cancel{b^2} + \cancel{b^2} - \cancel{c^2} + \cancel{c^2} - \cancel{a^2}}$$

$$= x^0 = 1$$



QUESTION

If $a^3 - b^3 = (a-b) \cdot (a^2 + ab + b^2)$, then the simplified form of

$$\left[\frac{x^l}{x^m} \right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n} \right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l} \right]^{l^2+ln+n^2}$$

A 1

$$(x^{l-m})^{l^2+lm+m^2}$$

B 0

$$= (x)^{l^3-m^3} \cdot (x)^{m^3-n^3} \cdot (x)^{n^3-l^3}$$

C x

D None

$$= (x)^0 = 1$$

QUESTION



On simplification

$$\frac{1}{1+a^{m-n}+a^{m-p}} + \frac{1}{1+a^{n-m}+a^{n-p}} + \frac{1}{1+a^{p-m}+a^{p-n}}$$

$$= \frac{a^{-m} + a^{-n} + a^{-p}}{a^{-m} + a^{-n} + a^{-p}} = 1$$

A 1

B 0

C a

D 1/a

Let $m=n=p=0$

$$\frac{1}{1+a^0+a^0}$$

$$\frac{1}{1+1+1} + \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$= 1$$

$$\frac{1}{1+a^{m-n}+a^{m-p}}$$

$$\frac{1}{1+a^m a^{-n} + a^m a^{-p}}$$

Divide Num & Den by a^m

$$\frac{a^{-m}}{a^{-m} + a^{-n} + a^{-p}} + \frac{a^{-n}}{a^{-n} + a^{-m} + a^{-p}} + \frac{a^{-p}}{a^{-p} + a^{-m} + a^{-n}}$$



$$\frac{2}{3} + \frac{5}{3} - \frac{7}{3}$$
$$= \frac{2+5-7}{3}$$
$$= \frac{0}{3}$$
$$= 0$$



QUESTION

If $2^x = 3^y = 6^{-z}$ Then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

- A** 1
- B** 0
- C** 2
- D** None

$$2^x = 3^y = 6^{-z} = k$$

$2^x = k$	$3^y = k$	$6^{-z} = k$
$2 = (k)^{1/x}$	$3 = (k)^{1/y}$	$6 = (k)^{-1/z}$

or $6 = 2 \times 3$

$$(k)^{-1/z} = (k)^{1/x} \cdot (k)^{1/y}$$

$$(k)^{-1/z} = (k)^{\frac{1}{x} + \frac{1}{y}}$$

$$-\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$$

$$0 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$6 = 2 \times 3$$

$$\Rightarrow$$

$$-\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$$

$$0 = \frac{1}{z} + \frac{1}{x} + \frac{1}{y}$$



eg If $2^a = 3^b = 12^c$

then value of $\frac{1}{c} - \frac{1}{b} - \frac{2}{a} = ?$

- a) 1
- b) 0
- c) -1
- d) None

Sol.

$$12 = 2 \times 2 \times 3$$

$$\frac{1}{c} = \frac{1}{a} + \frac{1}{a} + \frac{1}{b}$$

$$\frac{1}{c} - \frac{1}{b} - \frac{2}{a} = 0$$



Q 56

then

$$3^{x^2} = 5^{y^2} = (375)^{-z^2} = k$$

$$\frac{1}{x^2} + \frac{1}{z^2} = ?$$

$$3^{x^2} = k$$

$$3 = (k)^{1/x^2}$$

$$5 = (k)^{1/y^2}$$

$$375 = (k)^{-1/z^2}$$

$$375 = 5 \times 5 \times 5 \times 3$$

$$(k)^{-1/z^2} = (k)^{1/y^2} (k)^{1/y^2} (k)^{1/y^2} (k)^{1/x^2}$$

$$-\frac{1}{z^2} = (k)^{3/y^2 + 1/x^2}$$

$$-\frac{1}{z^2} = \frac{3}{y^2} + \frac{1}{x^2}$$

$$-\frac{3}{y^2} = \frac{1}{x^2} + \frac{1}{z^2}$$

Sol:

$$375 = 3 \times 5 \times 5 \times 5$$

$$-\frac{1}{z^2} = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{y^2} + \frac{1}{y^2}$$

$$-\frac{1}{z^2} = \frac{1}{x^2} + \frac{3}{y^2}$$

$$-\frac{3}{y^2} = \frac{1}{x^2} + \frac{1}{z^2}$$

a) $-\frac{1}{y^2}$

b) $-\frac{2}{y^2}$

c) $-\frac{3}{y^2}$

d) None

QUESTION



If $x = 4^{1/3} + 4^{-1/3}$ Then the value of $x^3 - 3x = ?$

$$x^3 - 3x = 4 + \frac{1}{4} = \frac{17}{4}$$

A 16/5

B 17/4

C 4

D None

$$x = 4^{1/3} + 4^{-1/3}$$

$$x^3 = \left(4^{1/3} + 4^{-1/3}\right)^3$$

$$[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

$$x^3 = \left(4^{1/3}\right)^3 + \left(4^{-1/3}\right)^3 + 3\left(4^{1/3}\right)\left(4^{-1/3}\right)\left[4^{1/3} + 4^{-1/3}\right]$$

$$x^3 = 4 + 4^{-1} + 3(1)x$$

$$x^3 - 3x = 4 + \frac{1}{4} = \frac{17}{4}$$

If $x = p^{1/3} + p^{-1/3}$
then $x^3 - 3x = p + \frac{1}{p}$

If $x = p^{1/3} - p^{-1/3}$
then $x^3 + 3x = p - \frac{1}{p}$

QUESTION



If $x = 5^{1/3} + 5^{-1/3}$ Then the value of $5x^3 - 15x = ?$

A 25

B 26 ✓✓

C 27

D None

$$x = 5^{1/3} + 5^{-1/3}$$

$$x^3 - 3x = 5 + \frac{1}{5}$$

$$x^3 - 3x = \frac{26}{5}$$

$$5x^3 - 15x = 26$$

QUESTION

If $x = p^{1/3} - p^{-1/3}$ Then the value of $x^3 + 3x = ?$

- A** $p + \frac{1}{p}$
- B** $p - \frac{1}{p}$
- C** $p + 1$
- D** None

$$x = p^{1/3} - p^{-1/3}$$

$$x^3 + 3x = p - \frac{1}{p}$$

QUESTION



If $a^p = b$, $b^q = c$ & $c^r = a$, Then the value of $pqr = ?$

- A** 0
- B** 1
- C** -1
- D** None

$$\begin{aligned} a^p &= b \\ b^q &= c \\ c^r &= a \end{aligned}$$

$$a^p b^q c^r = b^q c^r a^p$$

$$\begin{aligned} p &= 1 \\ q &= 1 \\ r &= 1 \end{aligned}$$

Now

$$\begin{aligned} pqr &= (1)(1)(1) \\ &= 1 \end{aligned}$$

$$\begin{aligned} a^p &= b \\ a^{pq} &= b^q \end{aligned}$$

$$a^{pq} = c$$

$$a^{pqr} = c^r$$

$$a^{pqr} = a$$

$$pqr = 1$$

QUESTION



If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of $p + q + r$ is

A 0

B 1

C 1/2

D None

$$x^{1/p} = y^{1/q} = z^{1/r} = k$$

$$k^0 = 1$$

$$x^{1/p} = k$$

$$x = k^p$$

$$y = k^q$$

$$z = k^r$$

now

$$xyz = 1$$

$$k^p k^q k^r = 1$$

$$k^{p+q+r} = 1 = k^0$$

$$p + q + r = 0$$

QUESTION

If $a = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$ then the value of $[a(a-14)]^2$ is

A 14

B 7

C 2

D 1

$$a = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}}$$

$$= \sqrt{\frac{(7+4\sqrt{3})^2}{(7)^2 - (4\sqrt{3})^2}}$$

$$= \sqrt{\frac{(7+4\sqrt{3})^2}{49-48}}$$

$$a = 7+4\sqrt{3}$$

$$(4\sqrt{3})^2 = 16 \times 3 = 48$$

$$a = 4\sqrt{3} + 7$$

∴

$$[a(a-14)]^2$$

$$= [(4\sqrt{3}+7)(4\sqrt{3}+7-14)]^2$$

$$= [(4\sqrt{3}+7)(4\sqrt{3}-7)]^2$$

$$= [(4\sqrt{3})^2 - 7^2]^2$$

$$= [48 - 49]^2$$

$$= (-1)^2 = 1$$





Logarithm



$$a^n = x$$

Diagram illustrating the components of a power equation: a is labeled as the **Base**, n is labeled as the **Power**, and x is labeled as the **Value**.

§ $2^5 = 32$

§ $(3)^2 = 9$

$$\log_a(x) = n$$

Diagram illustrating the components of a logarithm equation: a is labeled as the **Base**, x is labeled as the **Value**, and n is labeled as the **Exponent**, **Index**, or **Power**.



eg

$$\log_2(4) = 2 \rightarrow \text{power}$$

Base Value

$$2^? = 4$$

$$2^2 = 4$$

power

eg

$$\log_2(16) = 4$$

Base

$$2^? = 16 = 2^4$$



eg

$$\log_3(9) = 2$$

$$3^? = 9 = 3^2$$

eg

$$\log_5(625) = 4$$

$$5^? = 625$$

$$5^1 = 5, 5^2 = 25, 5^3 = 125, 5^4 = 625$$

eg

$$\log_{10}(100) = 2$$

Base \swarrow \searrow value
 $10^? = 100$
 \searrow power

eg

$$\log_{10}(10) = 1$$

$$10^? = 10$$

eg

$$\log_{10}(1) = 0$$

$$10^? = 1 = 10^0$$



Ex

$$\log_2 (512) = 8 + 1 = 9$$

$$2^? = 512$$
$$2^? = (2)^9$$

Ex

$$\log_{(\sqrt{3})} (243) = 9 + 1 = 10$$



$$\log_a(x) = y$$

where

$$a > 0$$

$$a \neq 1$$

$$x > 0$$

y can be any Real no.

$$\int \log_2(0) = \text{Not Defined}$$

$$\int \log_2(-2) = \text{Not Defined}$$



Logarithm

Natural log

Base = $e = 2.7183$

$\log_e(x)$

Calculus

Common log

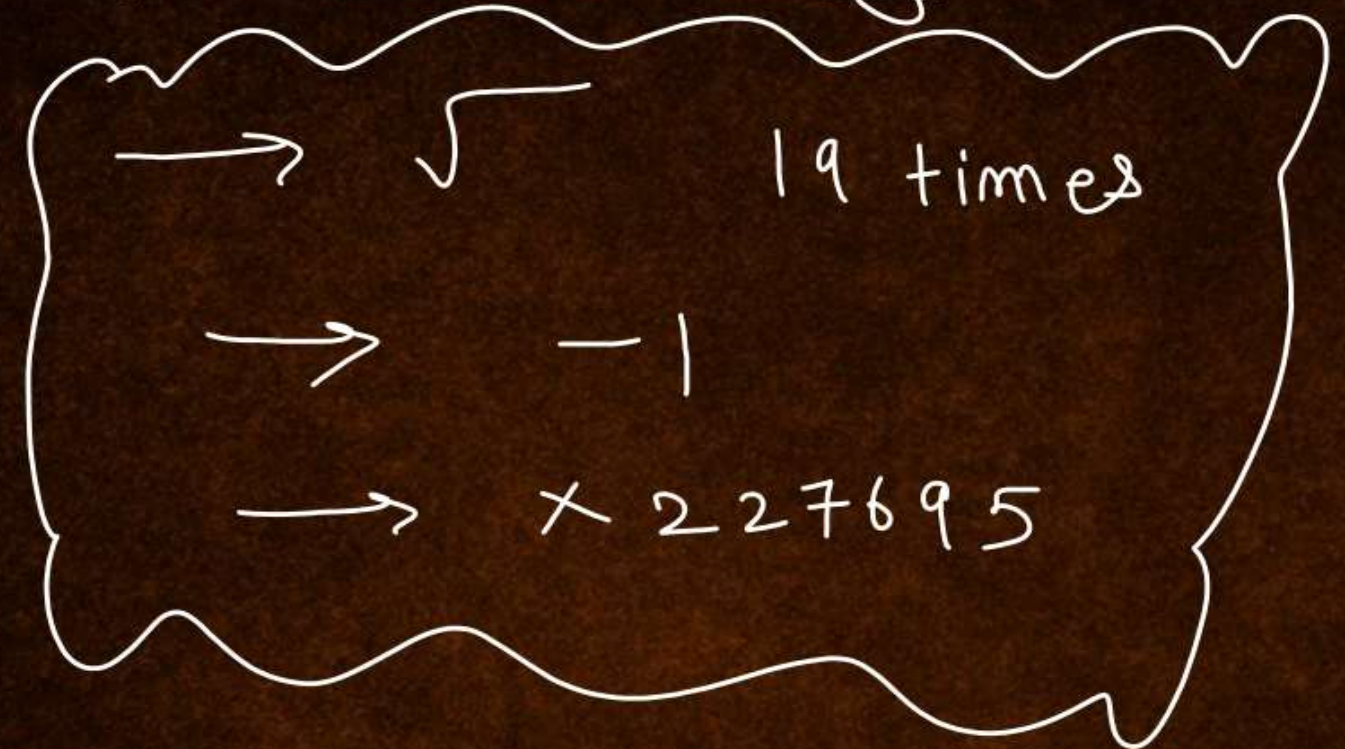
Base = 10

$\log_{10}(x) = \log(x)$

Calculations simplification



Common log



eg

$$\log_{10}(100) = 2.0000$$

eg

$$\log(2) = 0.3010$$

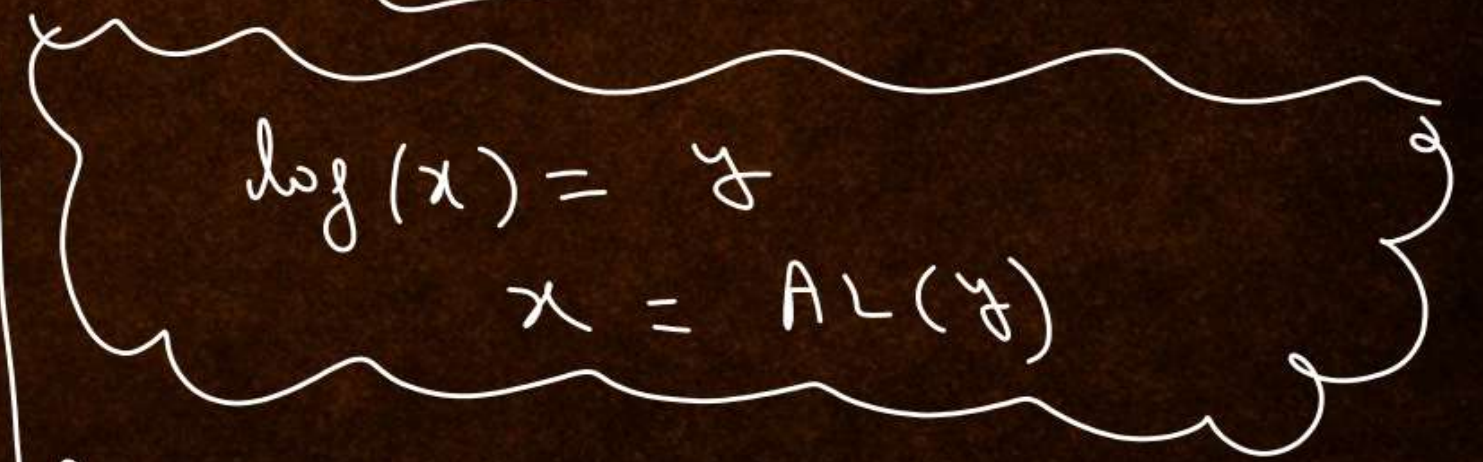
$$\log(30) = 1.4771$$

Antilog

$$\rightarrow \div 227695$$

$$\rightarrow +1$$

$$\rightarrow \boxed{X = \quad} \text{ 19 times}$$



$$\rightarrow AL(2) = 100$$

$$\rightarrow AL(1.4771) = 30$$



QUESTION

Value of $\log_2 16$

- A** 3
- B** 4 ✓✓
- C** 5
- D** 8

$\log_2(16) = 4$
 $2^? = 16 = 2^4$

QUESTION



Value of $\log_{1.5} 5.0625$

$$= 3 + 1 \\ = 4$$

- A 3
- B 4
- C 5
- D None

QUESTION



Value of $\log_2 0.0625$

$= -4$

A 3 X

B 4 X

C 5 X

D 8 X

(E) -4

$2^1 = 2$
 $2^2 = 4$
 $2^3 = 8$
 $2^4 = 16$

$2^{-1} = \frac{1}{2} = 0.5$
 $2^{-2} = \frac{1}{4} = 0.25$
 $2^{-3} = \frac{1}{8} = 0.125$
 $2^{-4} = 0.0625$

$2^x = 0.0625 = (2)^{-4}$

$2^4 = 16$

$2^{-4} = 0.0625$

QUESTION

Value of $\log_5 \left(\frac{1}{3125} \right)$

A 3 X

B 4 X

C 5 X

D 8 X

(e) -5

$$\log_5 \left(\frac{1}{3125} \right)$$

$$\log_5 (0.00032)$$

$$= -5$$



$$\log_2(32) = 5$$

Base → Value → Power

$$32 = 2^5$$

eg

$$\log_4(64) = 3$$

Base → Value → Power

$$64 = 4^3$$

$$4 \times 4 \times 4 = 4^3 = 64$$

#

$$\text{If } \log_a(x) = y$$
$$x = a^y$$

QUESTION

Value of a if $\log_a(\sqrt{2}) = \frac{1}{6}$

A 3

B 4

C 5

D 8

$$\log_a(\sqrt{2}) = \frac{1}{6}$$

$$\sqrt{2} = (a)^{\frac{1}{6}}$$

$$(2)^{\frac{1}{2}} = (a)^{\frac{1}{6}}$$

$$(2)^{\frac{1}{2} \times 6} = a$$

$$(2)^3 = a$$

$$a = 8$$

QUESTION



Find the log of 1728 to base $2\sqrt{3}$

A 3

B 4

C 5

D 6

$$\log_{2\sqrt{3}}(1728) = 5 + 1 = 6$$

$$(2\sqrt{3})^? = 1728 = (2\sqrt{3})^6$$

Properties of Log

$$\Rightarrow \log_a(1) = 0$$

$$\Rightarrow \log_a(a) = 1$$

$$\Rightarrow \log(xy) = \log x + \log y$$

$$\begin{aligned} \text{eg } \log(1 \cdot 2 \cdot 3) \\ &= \log 1 + \log 2 + \log 3 \\ &= \log 2 + \log 3 \end{aligned}$$

$$\begin{aligned} \text{eg } \log[x(x+1)(x+2)] \\ &= \log x + \log(x+1) + \log(x+2) \end{aligned}$$

$$\text{eg } \log(x) + \log(2x) = \log[x(2x)] = \log(2x^2)$$



$$4 > \log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

$$f \log\left(\frac{2}{5}\right) = \log 2 - \log 5$$

$$g \log\left(\frac{x \times y}{z}\right) \\ = \log(x) + \log(y) - \log(z)$$

Note
⇓

$$\log(x+y) \neq \log x + \log y$$

$$\log(x-y) \neq \log x - \log y$$

$$\log\left(\frac{1}{x}\right) = \log 1 - \log x \\ = 0 - \log x$$

$$\log\left(\frac{1}{x}\right) = -\log x$$



5 >

$$\log(x^n) = n \log x$$

eg

$$\log(x^2) = 2 \log x$$

$$\log(x^2 \cdot y^3)$$

$$= \log x^2 + \log y^3$$

$$= 2 \log x + 3 \log y$$

eg

$$2 \log(\sqrt{x})$$

$$= \log(\sqrt{x})^2$$

$$= \log(x)$$



$$6) \log_a(x) = \frac{\log_m(x)}{\log_m(a)}$$

$$\text{eg } \log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)}$$

$$\text{eg } \log_3(35) = \frac{\log_{10}(35)}{\log_{10}(3)}$$

$$\text{eg } \log_2 3 \times \log_5 2 \times \log_3 5 = \frac{\log 3}{\log 2} \times \frac{\log 2}{\log 5} \times \frac{\log 5}{\log 3}$$



#7] $\log_a(b) \times \log_b(a) = 1$
or

$$\log_a(b) = \frac{1}{\log_b(a)}$$

eg $\frac{1}{\log_8(2)}$
 $= \log_2(8)$
 $= 3$

eg $\log_2(5) \times \log_5(2)$
 $= \frac{\cancel{\log 5}}{\cancel{\log 2}} \times \frac{\cancel{\log 2}}{\cancel{\log 5}}$
 $= 1$



8]

$$\log_{a^b}(x) = \frac{1}{b} \log_a(x) = \log_a(x)^{1/b}$$

$$\begin{aligned} \text{eg } \log_{2^3}(100) \\ = \frac{1}{3} \log_2(100) \end{aligned}$$

$$\begin{aligned} \text{eg } \log_{5^2}(25) \\ = \frac{1}{2} \log_5(25) \end{aligned}$$

$$\begin{aligned} \text{eg } \log_{a^2}(x) \\ = \frac{1}{2} \log_a(x) = \log_a(x)^{1/2} \end{aligned}$$



#

$$\log_{(a^b)} (x)^{\frac{1}{b}} = \frac{1}{b} \log_a (x)$$

a)

$$a^{\log_a(x)} = x$$

ε

$$\begin{aligned} & (2)^{\log_2(16)} \\ &= (2)^4 \\ &= 16 \end{aligned}$$

ε

$$3^{\log_3(x)} = x$$

ε

$$\begin{aligned} & 9^{\log_3(x)} \\ &= (3^2)^{\log_3(x)} \\ &= (3)^{2 \log_3(x)} \\ &= (3)^{\log_3(x^2)} \\ &= x^2 \end{aligned}$$



QUESTION



Value of $16 \log_{10} \left(\frac{64}{60} \right) + 12 \log_{10} \left(\frac{50}{48} \right) + 7 \log_{10} \left(\frac{81}{80} \right) + \log_{10} 2$

A 0

B 1

C -1

D 2

$$16 \log \left(\frac{64}{60} \right) + 12 \log \left(\frac{50}{48} \right) + 7 \log \left(\frac{81}{80} \right) + \log 2$$

$$\log \left(\frac{64}{60} \right)^{16} + \log \left(\frac{50}{48} \right)^{12} + \log \left(\frac{81}{80} \right)^7 + \log 2$$

$$= \log \left[\left(\frac{64}{60} \right)^{16} \times \left(\frac{50}{48} \right)^{12} \times \left(\frac{81}{80} \right)^7 \times 2 \right]$$

$$= \log [2.8084 \times 1.6326 \times 1.0908 \times 2]$$

$$= \log_{10} (10) = 1$$

$$n \log x = \log x^n$$

$$\log x + \log y = \log (xy)$$

QUESTION

If $\log_8 x + \log_4 x + \log_2 x = 11$
Then the value of $x =$

A 2

B 4

C 8

D 64

$$\log_8 x + \log_4 x + \log_2 x = 11$$

$$\log_{2^3}(x) + \log_{2^2}(x) + \log_2(x) = 11$$

$$\frac{1}{3} \log_2 x + \frac{1}{2} \log_2 x + \log_2 x = 11$$

$$\log_2 x \left[\frac{1}{3} + \frac{1}{2} + 1 \right] = 11$$

$$\log_2 x \left[\frac{2+3+6}{6} \right] = 11$$

$$6 + 3 + 2 = 11$$

CA

$$\log_{a^b}(x) = \frac{1}{b} \log_a x$$

$$\log_2(x) \times \frac{6}{6} = 6$$

$$\log_2(x) = 6$$

$$x = 2^6$$

$$x = 64$$

QUESTION



If $\log_3[\log_4(\log_2 x)] = 0$, then the value of 'x' will be

- A** 4
- B** 8
- C** 16
- D** 32

$$\log_3 [\log_4 (\log_2 x)] = 0$$

$$\log_4 [\log_2 x] = 3^0 = 1$$

$$\log_2 (x) = 4^1 = 4$$

$$x = 2^4$$

$$x = 16$$

$$\log_a (x) = y$$
$$x = a^y$$



QUESTION

. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, then the value of $\log 24$ is :

- A** 1.0791
- B** 1.7323
- C** 1.3801
- D** None

$$\begin{aligned} &= \log(8 \times 3) \\ &= \log(2^3 \times 3) \\ &= \log 2^3 + \log 3 \\ &= 3 \log 2 + \log 3 \\ &= \end{aligned}$$



QUESTION

The value of $\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}$

A 1

B 3

C 5

D 8

$$= \log_{60} (3) + \log_{60} (4) + \log_{60} (5)$$

$$= \log_{60} (3 \times 4 \times 5)$$

$$= \log_{60} (60)$$

$$= 1$$

$$\frac{1}{\log_a (b)} = \log_b (a)$$

QUESTION

The value of $\frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ac} + \frac{1}{1+\log_c ab}$

- A** 0
- B** 1
- C** abc
- D** $a+b+c$

$$\frac{1}{\log_a a + \log_a (bc)} + \frac{1}{\log_b b + \log_b ac} + \frac{1}{\log_c c + \log_c (ab)}$$

$$\frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$$

$$\log_{abc}(a) + \log_{abc}(b) + \log_{abc}(c)$$

$$= \log_{abc}(abc) = 1$$

QUESTION



Value of $\frac{\log_3 8}{\log_9 16 \log_4 10}$

- A $3 \log_{10} 2$
- B $\log_{10} 2$
- C $3 \log_2 10$
- D None

$$\frac{\log_3(8)}{\log_9 16 \times \log_4 10}$$

$$= \frac{\log_{10} 8}{\log_{10} 3}$$

$$\frac{\frac{\log_{10} 16}{\log_{10} 9} \times \frac{\log_{10} 10}{\log_{10} 4}}$$

$$= \frac{\log(2)^3}{\log 3}$$

$$\frac{\frac{\log(4)^2}{\log(3)^2} \times \frac{1}{\log 4}}{\log 3}$$

$$= \frac{3 \log 2}{\log 3}$$

$$\frac{\frac{2 \log 4}{2 \log 3} \times \frac{1}{\log 4}}{\log 3}$$

$$= 3 \log 2$$

$$= \frac{\log_a(x)}{\log_b a}$$

QUESTION



Find the number whose logarithm is -2.4678

A 0.003405

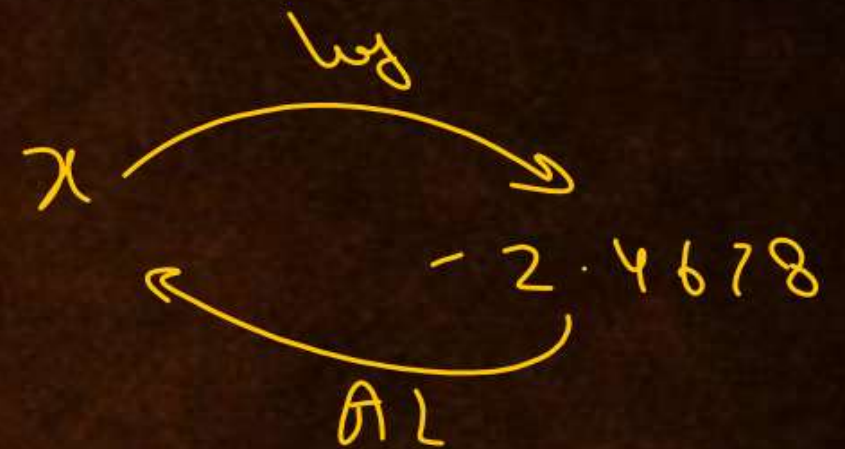
B 4.003205

C 0.001405

D 0.003719

$$\log(x) = -2.4678$$

$$x = \text{Ant}(-2.4678)$$



$$\div 227695$$

+1

$x \approx$ 19 times

QUESTION

$$400 - 280 = 120$$

A group of 400 soldiers had a provision for 31 days. After 28 days 280 person were called back . Find the number of days for which the remaining ration will be sufficient

A 3

$$\text{Total Ration} = 1 \times 400 \times 31 = 12,400 \text{ units}$$

B 6

Ration consumed in

$$28 \text{ Days} = 1 \times 28 \times 400 = 11,200 \text{ units}$$

C 8

$$\text{Remaining Ration} = \frac{1200 \text{ unit}}{1}$$

D 10

$$\begin{aligned} \text{No of Days} &= \frac{1200 \text{ unit}}{120} \\ &= 10 \text{ Days} \end{aligned}$$

$$1 \times 120 \times x = 1200$$

$$x = 10$$

QUESTION



If $\log 3 = x$ and $\log 4 = y$, then the value of $\log 120$

A $x - y + 1$

B $x + y + 1$

C $x + y - 1$

D $2x + y - 1$

$$\log(3) = x$$

$$\log(4) = y$$

$$\log(120)$$

$$= \log(12 \times 10)$$

$$= \log(4 \times 3 \times 10)$$

$$= \log_{10} 4 + \log_{10} 3 + \log_{10} 10$$

$$y + x + 1$$

QUESTION



If $\log 3 = x$ and $\log 4 = y$, then the value of $\log 120$

- A** $x - y + 1$
- B** $x + y + 1$
- C** $x + y - 1$
- D** $2x + y - 1$

QUESTION



$$\log_5 \left(1 + \frac{1}{5} \right) + \log_5 \left(1 + \frac{1}{6} \right) + \log_5 \left(1 + \frac{1}{7} \right) \dots \log_5 \left(1 + \frac{1}{624} \right) = ?$$

A 2

B 3

C 5

D 0

$$\log_5 \left(\frac{6}{5} \right) + \log_5 \left(\frac{7}{6} \right) + \log_5 \left(\frac{8}{7} \right) + \dots + \log_5 \left(\frac{625}{624} \right)$$

$$= \log_5 \left[\frac{6}{5} \times \frac{7}{6} \times \frac{8}{7} \times \dots \times \frac{625}{624} \right]$$

$$= \log_5 \left(\frac{1}{5} \times 625 \right)$$

$$= \log_5 (125) = 3$$



Character & Mantissa

$$\log(200) = 2.301$$

$$= 2 + 0.301$$

integral part

fractional part

200.0000

character

Mantissa

it can be any integer like -2, -1, 0, 1, 2, 3

a positive number between 0 & 1



eg $\log(25)$
 $= 1.3979$
 $= \underbrace{1}_{\text{character}} + \underbrace{0.3979}_{\text{Mantissa}}$

\checkmark 25.00

eg $\log(2.5)$
 $= 0.3979$
 $= \underbrace{0}_{\text{character}} + \underbrace{0.3979}_{\text{Mantissa}}$



$$\text{Eg } \log(0.02)$$
$$= -1.699$$

$$= -1 - 0.699$$

$$= \underbrace{-1 - 1}_{-2} - 0.699 + 1$$

$$= -2 + 0.301$$

Character

Mantissa

$$\text{Eg } \log(0.0003)$$

$$= -3.5228$$

$$= \underbrace{-1 - 3}_{-4} - 0.5228 + 1$$

-4 Character
 $+0.4772$ Mantissa



267.32

26.73

2.673

0.2673

0.02673

0.002673

0.0002673

eheraten

$$3-1=2$$

$$2-1=1$$

$$1-1=0$$

-1

-2

-3

-4

QUESTION



What is Character in $\log(0.004594) = ?$

$$\log(0.\underline{00}4594)$$

A -1

B -2

C -3

D -4



QUESTION

What is mantissa in $\log(0.004594) = ?$

- A** 0.6622
- B** 0.6242
- C** 0.3377
- D** 0.0307

$$\begin{aligned} & -2.3377 \\ & \underbrace{-1 - 2} \quad \underbrace{-0.3377 + 1} \\ & -3 \quad + 0.6623 \end{aligned}$$



THANK YOU

