CHAPTER

1

Ratio and Proportion, Indices, Logarithm

RATIO

DEFINITION OF RATIO

A ratio is a comparison between two or more quantities of the same kind, where by "same kind" we means that they are similar in nature (like gold is comparable to silver but not to apple).

If a and b are two quantities of the same kind (in same units, E.g. – Kg and gram are two different units of weights).

It is represented by a : b

Then, fraction of a and b is called the ratio of a and b which is written as $\frac{a}{b}$

- The quantities a and b are called **terms** of the ratio.
  E.g.: In 3: 4, 3 and 4 are terms of ratio.
- a is called the **first term** or **antecedent**.
  E.g.: In 3 : 4 , 3 is first term or antecedent.
- b is called the **second term** or **consequent.**
  E.g.: In 3 : 4 , 4 is second term or consequent.

Example 1. The ratio of two quantities is 3 : 4. What is antecedent and what is consequent?

(a) 3, 4 (b) 4, 3 (c) 3, 3 (d) None of these

Sol. (a) As per the definition,
The term '3' is the antecedent and the term '4' is the consequent.
Hence, the correct option is (a).

Example 2. The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is

(a) 16 (b) 60 (c) 22 (d) 20

Sol. (d) Given: Ratio of two quantities = 3 : 4
Let 'x' be the consequent of the ratio whose antecedent is 15.
According to the problem,

$$\frac{3}{4} = \frac{15}{x}$$

On cross-multiplication, we get

$3x = 4 \times 15$

$3x = 60$

Dividing both sides by 3, we get

$x = \frac{60}{3} = 20$

Alternate Solution:

Since, $\frac{3}{4} = \frac{15}{x}$ ...(i)

Now, for option (a): 16

On substituting x = 16 in eq (i), we have

$\frac{3}{4} = \frac{15}{16}$, which is not true

For option (b): 60

On substituting x = 60 in eq (i), we have

$\frac{3}{4} = \frac{15}{60}$

$\frac{3}{4} = \frac{1}{4}$, which is not true

For option (c): 22

On substituting x = 22 in eq (i), we have

$\frac{3}{4} = \frac{15}{22}$, which is not true

For option (d): 20

On substituting x = 20 in eq (i), we have

$\frac{3}{4} = \frac{15}{20}$

$\frac{3}{4} = \frac{3}{4}$, which is true

Therefore, when the antecedent is 15 in a ratio of 3 : 4, the consequent is 20.

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART A)

1. The antecedent and consequent in the ratio 5 : 16 is ___ and ___ respectively.
   (a) 5, 5 (b) 16, 5 (c) 5, 16 (d) 16, 16
2. The ratio of two quantities is 2 : 5. If the antecedent is less than the consequent by Rs. 9000, the consequent will be .....
   (a) 3000 (b) 6000 (c) 9000 (d) 15000
3. The ratio of two quantities is 5 : 8. If the consequent is is 40, then the antecedent is
   (a) 25 (b) 40 (c) 5 (d) 8

Answer Key

1. (c) 2. (d) 3. (a)

KEY CONCEPTS REGARDING RATIOS

- Both terms of a ratio can be multiplied or divided by the same (non-zero) number.
  E.g.: If we have a ratio of 2 : 3, we can multiply both terms by 2 to get 4 : 6 and do the similar with division.
- Usually, a ratio is expressed in lowest terms (or simplest form).
  E.g.: If we have a ratio of 6 : 8, we can simplify it to 3 : 4 by dividing both terms by their greatest common divisor, which is 2.
- The order of the terms in a ratio is important.
  E.g.: The ratio 2 : 3 is different from the ratio 3 : 2. They both represent different quantities.

- Ratio exists only between quantities of the same kind.
  E.g.: We can compare the ratio of apples to oranges, but not apples to minutes.
- Quantities to be compared (by division) must be in the same units.
  E.g.: If we want to compare the ratio of the lengths of two objects, one measured in inches and the other in centimeters, we need to convert them to the same unit (e.g., both in inches or both in centimeters) before dividing them.
- To compare two ratios, convert them into equivalent like fractions.
  E.g.: If we have the ratio 2 : 3 and the ratio 4 : 5, we can convert them into equivalent fractions by converting to a common denominator. The ratios become $\frac{10}{15}$ and $\frac{12}{15}$, respectively, and we can then compare them easily.

Example 3. Which ratio is greater?

I. $3\frac{1}{2} : 4\frac{1}{2}$ or 2.5 : 4.5

(a) $3\frac{1}{2} : 4\frac{1}{2}$ (b) 2.5 : 4.5

(c) Both are equal (d) Cannot be determined

II. $2\frac{1}{5} : 3\frac{1}{5}$ or 3.6 : 4.0

(a) $2\frac{1}{5} : 3\frac{1}{5}$ (b) 3.6 : 4.0 (c) Both are equal (d) Cannot be determined

Sol. I. (a) We know,

$3\frac{1}{2} : 4\frac{1}{2}$ can be simplified to 3.5 : 4.5

Now, on comparing 3.5 : 4.5 and 2.5 : 4.5, i.e., $\frac{3.5}{4.5}$ and $\frac{2.5}{4.5}$ we see that the denominators are equal thus we can compare the numerators.

Clearly, 3.5 is greater than 2.5

Thus, 3.5 : 4.5 is greater than 2.5 : 4.5.

Therefore, the ratio $3\frac{1}{2} : 4\frac{1}{2}$ is greater than 2.5 : 4.5.

Hence, the correct option is (a).

II. (b) We know,

$2\frac{1}{5} : 3\frac{1}{5}$ can be simplified to $\frac{11}{5} : \frac{16}{5} = 11 : 16 = \frac{11}{16}$

Also, $3.6 : 4.0 = \frac{36}{40} = \frac{9}{10}$

Making the denominators equal to 80 i.e.,

$\frac{11}{16} \times \frac{5}{5} = \frac{55}{80}$ and $\frac{9}{10} \times \frac{8}{8} = \frac{72}{80}$

On comparing both the ratios, we see

$55 < 72$

$\frac{55}{80} < \frac{72}{80}$

i.e., $2\frac{1}{5} : 3\frac{1}{5} < 3.6 : 4.0$

Therefore, the ratio 3.6 : 4.0 is greater than $2\frac{1}{5} : 3\frac{1}{5}$

Hence, the correct option is (b).

- If a quantity increases or decreases in the ratio a: b. The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.
  E.g.: Let's say you have a quantity of 100 apples and it increases in the ratio of 2 : 3
- Then, New quantity = Original quantity × Factor multiplying ratio
- New quantity $= 100 \times \frac{3}{2} = 150$ apples

Example 4. Earlier shoe company produced 2000 shoes in a day. They increased their production 5 : 6. What will be the new production?

(a) 2500 (b) 2400 (c) 2000 (d) 3000

Sol. (b) Given: Original production of the shoe company in a day = 2000

Since, the increased production ratio is given as 5 : 6, thus the new production is 6/5 times the original production.

Thus, New production $= 2000 \times \left(\frac{6}{5}\right) = 2400$

Therefore, the new production of the shoe company is 2400 shoes in a day.

Hence, the correct option is (b).

Example 5. If Rajni eats 12 chapati in a day. If she reduces her chapati by 6 : 5. How many chapati does she eat now?

(a) 12 (b) 8 (c) 10 (d) 15

Sol. (c) Given: Original number of chapatis Rajni eats = 12

Reduced chapati intake ratio = 6 : 5

Thus, New quantity $= 12 \times \left(\frac{5}{6}\right) = \frac{60}{6} = 10$

Therefore, Rajni now eats 10 chapatis.

Hence, the correct option is (c).

Example 6. Simplify the ratio $\frac{1}{2} : \frac{2}{3} : \frac{4}{5}$

(a) 15 : 20 : 24 (b) 10 : 25 : 24

(c) 2 : 3 : 5 (d) 1 : 2 : 4

Sol. (a) We know that,

L.C.M. (2, 3, 5)

2	2	–3	–5
3	1	–3	–5
5	1	–1	–5
	1	–1	–1

L.C.M. = 2 × 3 × 5 = 30

Thus, making the denominator of each ratio equal to 30, we get

$\frac{1}{2} \times \frac{15}{15} = \frac{15}{30}$

$\frac{2}{3} \times \frac{10}{10} = \frac{20}{30}$

$\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

Therefore, the simplified form of given ratios is 15 : 20 : 24.

Hence, the correct option is (a).

Example 7. Simplify the ratio $\frac{1}{3} : \frac{1}{8} : \frac{1}{6}$ **(ICAI)**

(a) 8 : 3 : 4 (b) 8 : 5 : 1 (c) 7 : 2 : 6 (d) 1 : 3 : 5

Sol. (a) We know that,

L.C.M (3, 8, 6)

2	3	–8	–6
2	3	–4	–3
2	3	–2	–3
3	3	–1	–3
	1	–1	–1

L.C.M (3,8,6) = 2 × 2 × 2 × 3 = 24

Thus, making the denominator of each ratio equal to 24, we get

$\frac{1}{3} \times \frac{8}{8} = \frac{8}{24}$

$\frac{1}{8} \times \frac{3}{3} = \frac{3}{24}$

$\frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$

Therefore, the simplified form of given ratios is 8 : 3 : 4

Hence, the correct option is (a).

Example 8. Anand earns ₹80 in 7 hours and Promod ₹90 in 12 hours. The ratio of their earnings is **(ICAI)**

(a) 32 : 21 (b) 23 : 12 (c) 8 : 9 (d) None of these

Sol. (a) Given that,

Earning of Anand in 7 hours = ₹80

Earning of Pramod in 12 hours = ₹90

Thus, earning of Anand in 1 hour $= \frac{80}{7}$

Earning of Pramod in 1 hour $= \frac{90}{12}$

Therefore, Ratio of their earnings $= \frac{80}{7} \div \frac{90}{12} = \frac{80 \times 12}{7 \times 90} = \frac{32}{21}$

Hence, the correct answer is option (a) i.e., 32 : 21

Example 9. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 5 hours, the speed of the first train is **(ICAI)**

(a) 10 km/hr (b) 50 km/hr (c) 70 km/hr (d) None of these

Sol. (c) Let the speed of two trains be 7x km/hr and 8x km/hr.

It is given that the second train runs 400 km in 5 hours, thus we have

Speed = Distance/Time

$8x = \frac{400}{5}$

$8x = 80$

$x = 10$

Therefore, the speed of the first train is $7 \times 10 = 70$km/hr

Hence, the correct answer is option (c) i.e., 70 km/hr.

Example 10. 40 feet rope is cut into 2. One piece is 18 feet longer than the other. What is the length of the shorter piece?

(a) 11 (b) 12 (c) 18 (d) 22

Sol. (a) Let's assume the length of the shorter piece of the rope is 'x' feet.

According to the problem,

Length of the longer piece = (x + 18) feet

Since, the total length of the rope is 40 feet, thus

$x + (x + 18) = 40$

$\Rightarrow 2x + 18 = 40$

Subtracting 18 from both sides, we get

$\Rightarrow 2x = 22$

Dividing both sides by 2, we get

$x = 11$

Therefore, the length of shorter piece is 11 feet.

Hence, the correct option is (a).

Example 11. The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are **(ICAI)**

(a) (200, 305) (b) (185, 290) (c) (245, 350) (d) None of these

Sol. (c) Given that, Ratio of numbers = 7 : 10
Difference between the numbers = 105
Then, the numbers are 7x and 10x respectively.
As per question,
Difference between the numbers = 105
$10x - 7x = 105$
$\Rightarrow 3x = 105$
$\Rightarrow x = 35$
Therefore, the numbers are:
$7x = 7 \times 35 = 245$ and $10x = 10 \times 35 = 350$
Hence, the correct option is (c) i.e., (245, 350).

Example 12. Division of ₹324 between X and Y is in the ratio 11 : 7. X & Y would get rupees
(a) (204, 120) (b) (200, 124) (c) (180, 144) (d) None of these **(ICAI)**

Sol. (d) We need to divide ₹324 between X and Y in the ratio 11 : 7.
Let X's shares be 11a
Let Y's shares be 7a
Since, sum of their shared amount = ₹324
$11a + 7a = 324$
$\Rightarrow 18a = 324$
$\Rightarrow a = \frac{324}{18}$
$\Rightarrow a = 18$
Therefore, X' shares = $11 \times 18 = 198$
And Y's shares = $7 \times 18 = 126$
Hence, the correct option is (d).

Example 13. If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be
(a) 5 : 8 (b) 8 : 5 (c) 5 : 3 (d) 3 : 5

Sol. (b) Given, Salary of P is 25% lower than that of Q i.e.,

$$P = Q - 25\% \text{ of } Q = Q - \frac{25}{100}Q = \frac{75}{100}Q = 0.75Q$$

$$\Rightarrow \frac{P}{Q} = 0.75 \quad ...(i)$$

Also, salary of R is 20% higher than that of Q i.e,

$$R = Q + 20\% \text{ of } Q = Q + \frac{20}{100}Q = \frac{120}{100}Q = 1.2Q$$

$$\Rightarrow \frac{R}{Q} = 1.2 \quad ...(ii)$$

Thus, $\frac{R}{P} = \frac{R}{Q} \times \frac{Q}{P} = \frac{1.2}{0.75}$

$\Rightarrow \frac{R}{P} = 1.6 = \frac{16}{10} = \frac{8}{5}$

Therefore, the ratio of the salary of R and P is 8 : 5.

Hence, the correct option is (b).

Example 14. The ratio of the number of boys to the number of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, then find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2 : 3.

(a) 40 (b) 42 (c) 45 (d) None of these

Sol. (b) Given: Total students = 720

Ratio of number of boys to girls = 3 : 5

Let the number of boys and number of girls be 3a and 5a.

Thus, 3a + 5a = 720

$\Rightarrow$ 8a = 720

$\Rightarrow$ a = 90

So, number of boys = 3(90) = 270 and number of girls = 5(90) = 450

Now, 18 new girls are admitted in the school

Thus, new girls = 450 + 18 = 468

Let 'x' be the number of boys admitted in the school.

According to the question,

$\frac{270 + x}{468} = \frac{2}{3}$

$\Rightarrow$ 810 + 3x = 936

$\Rightarrow$ 3x = 126

$\Rightarrow$ x = 42

Therefore, the new number of boys admitted in the school is 42.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART B)

1. In a school, where the ratio of boys to girls is 7 : 5, if the total number of students is 2400, how many girls are enrolled in the school?

   (a) 500 (b) 1000 (c) 2000 (d) None of these

2. A bag contains 25 paise, 10 paise, 5 paise are in the ratio 3 : 2 : 1. The total value of coins is Rs. 40, then the number of 5 paise coin is:

   (a) 40 (b) 80 (c) 240 (d) 480

3. If the division of ₹252 between A and B is in the ratio 5 : 4, how much will A receive?

   (a) ₹140 (b) ₹120 (c) ₹112 (d) ₹100

4. The ratio of boys and girls in a college is 7 : 5. If the number of boys increases by 40% and the number of girls increases by 20% , what will be the new ratio of boys and girls?

   (a) 4 : 2 (b) 30 : 49 (c) 49 : 30 (d) 49 : 47

5. The ratio of two numbers is 2 : 9, and their difference is 91. What are the numbers?
(a) (13, 117) (b) (26, 91) (c) (26, 117) (d) None of these

Answer Key
1. (b) **2.** (a) **3.** (a) **4.** (c) **5.** (c)

INVERSE RATIO

One ratio is the inverse of the other if their product is 1.

E.g.: $b : a$ is the inverse ratio of $a : b$ since $\frac{a}{b} \times \frac{b}{a} = 1$

Example 15. The inverse ratio of 11 : 15 is

(a) 15 : 11 (b) $\sqrt{11} : \sqrt{15}$ (c) 121 : 225 (d) None of these

Sol. (a) As per the definition,
The inverse ratio of 11 : 15 is 15 : 11
Hence, the correct option is (a).

Example 16. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 25, the antecedent is

(a) 7 (b) $\sqrt{35}$ (c) 35 (d) None of these

Sol. (c) Given: Ratio of the quantities = 5 : 7
Thus, the inverse ratio of the given ratio = 7 : 5
Also, the consequent of its inverse ratio = 25
Let x be the antecedent of the ratio whose consequent is 25, thus

$$\frac{7}{5} = \frac{x}{25}$$

$\Rightarrow x = 7 \times 5$

$\Rightarrow x = 35$

Therefore, the required antecedent is 35.
Hence, the correct option is (c).

Greater Inequality Ratio	Less Inequality Ratio
A ratio $a : b$ will be of greater Inequality if $a > b$	A ratio $a : b$ will be of less inequality if $a < b$

The ratio 2 : 1 represents a greater inequality because the first term (2) is greater than the second term (1).

The ratio 3 : 4 represents less inequality because the first term (3) is smaller than the second term (4).

Example 17. The ratio of 8 : 15 is

(a) Greater inequality ratio (b) Less inequality ratio
(c) Equal to 1 (d) None of these

Sol. (b) We know that,

A ratio a: b will be of less inequality if a < b

In ratio 8 : 15,

8 < 15 i.e, a < b

Therefore, the ratio of 8 : 15 is less inequality ratio.

Hence, the correct option is (b).

Example 18. The ratio of 20 : 15 is

(a) Greater inequality ratio (b) Less inequality ratio

(c) Equal to 1 (d) None of these

Sol. (a) We know that,

A ratio a : b will be of greater inequality if a > b.

In ratio 20:15,

20 > 15 i.e, a > b

Therefore, the ratio of 20 : 15 is a greater inequality ratio.

Hence, the correct option is (a).

COMPOUNDED RATIO

A ratio resultant of compounding two or more ratios.

E.g.: If there are ratios 5 : 6, 7 : 8 and 9 : 10 then the compounded ratio will be:

(5 × 7 × 9) : (6 × 8 × 10)

$$= \frac{5 \times 7 \times 9}{6 \times 8 \times 10} = \frac{7 \times 3}{2 \times 8 \times 2} = \frac{21}{32}$$

Example 19. The ratio compounded of 5 : 6,9 : 2,4 : 3 and 1 : 5 is

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None of these

Sol. (a) We know that,

The compounded ratio of 5 : 6, 9 : 2, 4 : 3 and 1 : 5 is:

$$= \frac{5 \times 9 \times 4 \times 1}{6 \times 2 \times 3 \times 5} = \frac{180}{180} = 1 : 1$$

Hence, the correct option is (a).

Example 20. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None of these

Sol. (a) We know that,

The compounded ratio of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is:

$$= \frac{2 \times 9 \times 5 \times 8}{3 \times 4 \times 6 \times 10} = \frac{720}{720} = 1 : 1$$

Hence, the correct option is (a).

Example 21. If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is

(a) 24 : 35 (b) 8 : 15 (c) 16 : 35 (d) 7 : 15

Sol. (c) Given, $a : b = 2 : 3$, $b : c = 4 : 5$ and $c : d = 6 : 7$

i.e., $\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{4}{5}$ and $\frac{c}{d} = \frac{6}{7}$

Thus, $\frac{a}{d} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35}$

Therefore, $a : d = 16 : 35$

Hence, the correct option is (c).

Example **22.** P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22 : 27 (b) 27 : 22 (c) 32 : 33 (d) None of these

Sol. (b) Given: Ratio of average temperature between P and Q = 11 : 12

Ratio of average temperature between P and R = 9 : 8

i.e., $\frac{P}{Q} = \frac{11}{12}$ and $\frac{P}{R} = \frac{9}{8}$

Thus, $\frac{Q}{R} = \frac{P}{R} \div \frac{P}{Q}$

$= \frac{9}{8} \div \frac{11}{12} = \frac{9}{8} \times \frac{12}{11} = \frac{27}{22}$

Therefore, the ratio between the average temperature of Q and R is 27 : 22.

Hence, the correct option is (b).

- When the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are considered to be **Commensurable**.
- On the other hand, if the ratio cannot be expressed as a ratio of two integers, the quantities are considered to be **Incommensurable**.

DUPLICATE RATIO

A ratio compound to itself is called a Duplicate ratio. For ratio $a : b$, its duplicate ratio will be $a^2 : b^2$

E.g.: For ratio 2 : 3, its duplicate ratio will be $2^2 : 3^2 = 4 : 9$

TRIPLICATE RATIO

For ratio $a : b$, its triplicate ratio will be $a^3 : b^3$

E.g.: For ratio 2 : 3, its triplicate ratio will be $2^3 : 3^3 = 8 : 27$

Example 23. The duplicate ratio of 5 : 9 is

(a) 81 : 25 (b) 9 : 5 (c) 25 : 81 (d) None of these

Sol. (c) We know that, $a^2 : b^2$ is the duplicate ratio of $a : b$.

Thus, the duplicate ratio of 5 :9 is

$5^2 : 9^2 = 25 : 81$

Hence, the correct option is (c).

Example 24. The triplicate ratio of 5 : 7 is

(a) 25 : 49 (b) 5 : 7 (c) 125 : 343 (d) None of these

Sol. (c) We know that, $a^3 : b^3$ is the triplicate ratio of $a : b$

Thus, the triplicate ratio of 5 : 7

$= 5^3 : 7^3 = 125 : 343$

Hence, the correct option is (c).

SUB DUPLICATE RATIO

For $a : b$, the sub duplicate ratio will be $\sqrt{a} : \sqrt{b}$

E.g.: For ratio 4 : 9, its sub duplicate ratio will be $\sqrt{4} : \sqrt{9} = 2 : 3$

SUB TRIPLICATE RATIO

For $a : b$, the sub triplicate ratio will be $\sqrt[3]{a} : \sqrt[3]{b}$

E.g.: For ratio 8 : 27, its sub triplicate ratio will be $\sqrt[3]{8} : \sqrt[3]{27} = 2 : 3$

Example 25. The sub-duplicate ratio of 121 : 25 is

(a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 11 : 5

Sol. (d) We know that, $\sqrt{a} : \sqrt{b}$ is the sub-duplicate ratio of $a : b$.

Therefore, the sub – duplicate ratio of 121 : 25 is

$\sqrt{121} : \sqrt{25} = 11 : 5$

Hence, the correct option is (d).

Example 26. The sub - triplicate ratio of 64 : 27 is

(a) 16 : 9 (b) 4 : 3 (c) 27 : 64 (d) None of these

Sol. (b) We know that, $\sqrt[3]{a} : \sqrt[3]{b}$ is the sub triplicate ratio of $a : b$.

Therefore, the sub – triplicate ratio of 64 : 27 is

$\sqrt[3]{a} : \sqrt[3]{b} = \sqrt[3]{64} : \sqrt[3]{27} = 4 : 3$

Hence, the correct option is (b).

Example 27. The ratio compounded of triplicate ratio of 2 : 3 and the duplicate ratio of 3 : 4 is

(a) 1 : 4 (b) 2 : 3 (c) 1 : 6 (d) None of these

Sol. (c) We know that,

$a^2 : b^2$ is the duplicate ratio of $a : b$.

$a^3 : b^3$ is the triplicate ratio of $a : b$.

Thus, the triplicate ratio of 2 : 3 is $2^3 : 3^3 = 8 : 27$

The duplicate ratio of 3 : 4 is $3^2 : 4^2 = 9 : 16$

Therefore, the compounded ratio of 8 : 27 and 9 : 16 is

$$= \frac{8 \times 9}{27 \times 16} = \frac{1}{6}$$

Hence, the correct option is (c).

Example 28. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 3 : 2, sub duplicate ratio 25 : 81 and sub – triplicate ratio of 1000 : 27 is

(a) 1 : 4 (b) 3 : 25 (c) 4 : 1 (d) None of these

Sol. (c) As per the definitions,

The duplicate ratio of 4 : 5 is $4^2 : 5^2 = 16:25$

The triplicate ratio of 3 : 2 is $3^3 : 2^3 = 27:8$

The sub duplicate ratio 25 : 81 is $\sqrt{25} : \sqrt{81} = 5 : 9$

The sub – triplicate ratio of 1000 : 27 is $\sqrt[3]{1000} : \sqrt[3]{27} = 10 : 3$

Therefore, the compounded ratio of 16 : 25, 27 : 8, 5 : 9 and 10 : 3 is

$$= \frac{16 \times 27 \times 5 \times 10}{25 \times 8 \times 9 \times 3} = \frac{2 \times 2}{1} = \frac{4}{1}$$

Therefore, the required compounded ratio is 4 : 1

Hence, the correct option is (c).

Example 29. If A : B = 2 : 5, then (10A + 3B): (5A + 2B) is equal to:

(a) 7 : 4 (b) 7 : 3 (c) 6 : 5 (d) 7 : 9

Sol. (a) Given: A : B = 2 : 5

Let A = 2x and B = 5x

Then, (10A + 3B) : (5A+2B)

$$= \frac{10A + 3B}{5A + 2B} = \frac{10(2x) + 3(5x)}{5(2x) + 2(5x)} = \frac{20x + 15x}{10x + 10x} = \frac{35x}{20x} = \frac{7}{4}$$

Hence, the correct option is (a).

Example 30. If 2s : 3t is the duplicate ratio of 2s – p : 3t – p, then **(ICAI)**

(a) $p^2 = 6st$ (b) $p = 6st$ (c) $2p = 3st$ (d) None of these

Sol. (a) We know that,

$a^2 : b^2$ is the duplicate ratio of a : b.

Since, 2s : 3t is the duplicate ratio of 2s – p : 3t – p

So, the duplicate ratio of $2s - p : 3t - p = (2s - p)^2 : (3t - p)^2$

According to given condition,

$\Rightarrow (2s - p)^2 : (3t - p)^2 = 2s : 3t$

$\Rightarrow 4s^2 + p^2 - 4sp : 9t^2 + p^2 - 6tp = 2s : 3t$

On cross multiplying,

$\Rightarrow 12s^2t + 3p^2t - 12spt = 18st^2 + 2sp^2 - 12stp$

$\Rightarrow 12s^2t + 3p^2t = 18st^2 + 2sp^2$

$\Rightarrow p^2(3t - 2s) = 6st(3t - 2s)$

$\Rightarrow p^2 = 6st$

Hence, the correct option is (a).

Example 31. If $p : q$ is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is **[ICAI, (2006, 2019)]**

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) None of these

Sol. (c) **Detailed method:**

Given: $p : q$ is the sub-duplicate ratio of $p - x^2 : q - x^2$

Thus, $\frac{p}{q} = \frac{\sqrt{p - x^2}}{\sqrt{q - x^2}}$

On squaring both the sides, we get

$\frac{p^2}{q^2} = \frac{p - x^2}{q - x^2}$

On cross-multiplication, we get

$p^2q - x^2p^2 = pq^2 - q^2x^2$

$\Rightarrow p^2q - pq^2 = x^2(p^2 - q^2)$

$\Rightarrow pq(p - q) = x^2(p - q)(p + q)$

$\Rightarrow pq = x^2(p + q)$

$\Rightarrow x^2 = \frac{pq}{p+q}$

Alternate solution:

Go by choices: For option (c)

When $x^2 = \frac{pq}{p+q}$, then

$p - x^2 = p - \frac{pq}{p+q} = \frac{p^2}{p+q}$

And $q - x^2 = q - \frac{pq}{p+q} = \frac{q^2}{p+q}$

Thus, the sub-duplicate ratio of $p - x^2 : q - x^2$, is

$$\sqrt{\frac{p^2}{p+q}} : \sqrt{\frac{q^2}{p+q}} = \frac{\sqrt{\frac{p^2}{p+q}}}{\sqrt{\frac{q^2}{p+q}}} = \frac{\sqrt{p^2}}{\sqrt{q^2}} = \frac{p}{q}$$

$= p : q$, which is true

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART C)

1. Three friends A, B, and C shared a total amount of ₹3,000 in the ratio of 5 : 3 : 2. How much did B receive?
   (a) ₹700 (b) ₹750 (c) ₹900 (d) ₹300
2. If $x : y = 5 : 2$ then the value of $(8x + 9y) : (8x + 2y)$
   (a) 22 : 29 (b) 26 : 61 (c) 29 : 22 (d) 61 : 26
3. Find the compounded ratio of 3 : 5 , 2 : 3, 5 :1 and 4 : 3.
   (a) 4 : 1 (b) 4 : 5 (c) 3 : 8 (d) 8 : 3
4. The compounded ratio of inverse ratio of 15 : 28, sub-duplicate ratio of 36 : 49 and the triplicate ratio of 5 : 4 is
   (a) 25 : 7 (b) 15 : 4 (c) 25 : 8 (d) None of these
5. Find the compounded ratio of 7 : 4, duplicate ratio of 5 : 8, triplicate ratio of 2 : 7, sub-duplicate ratio of 16 : 25 and sub-triplicate ratio of 125 : 343.
   (a) 1 : 40 (b) 4 : 45 (c) 1 : 56 (d) 7 : 58

Answer Key

1. (c) **2.** (c) **3.** (d) **4.** (c) **5.** (c)

CONTINUED RATIO

It is the comparison between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a,b,c is written as a : b : c.

E.g.: Consider three similar quantities: Lengths of three sides of a triangle.

Let's say the lengths of the sides are a = 2 units, b = 4 units and c = 6 units.

The continued ratio of these side lengths is written as a : b : c, which in this case is 2 : 4 : 6.

This indicates the side lengths in comparison to each other. The ratio can be simplified to 1 : 2 : 3 by dividing all the terms by their greatest common divisor, which in this case is 2.

Example 32. The angles of a triangle are in ratio 2 : 7 : 11. The angles are **(ICAI)**
   (a) (20°, 70°, 90°) (b) (30°, 70°, 80°)
   (c) (18°, 63°, 99°) (d) None of these

Sol. (c) Given: Ratio of angles of triangle = 2 : 7 : 11

Let the angles of the triangle be 2x, 7x, 11x.

We know that, the sum of interior angles of a triangle is 180°.

$\Rightarrow 2x + 7x + 11x = 180°$

$\Rightarrow 20x = 180°$

$\Rightarrow x = \frac{180°}{20}$

$\Rightarrow x = 9$

Thus, the angles of the triangle is $2 \times 9°, 7 \times 9°, 11 \times 9°$ i.e., (18°, 63°, 99°)

Hence, option (c) is correct i.e., (18°, 63°, 99°).

Example 33. Find three numbers which are in the ratio 1 : 2 : 3, such that the sum of their squares is equal to 504. **(Dec 2013)**

(a) 6, 12, 18 (b) 3, 6, 9 (c) 4, 8, 12 (d) 5, 10, 15

Sol. (a) Given: Ratio of three numbers = 1 : 2 : 3

Let the three numbers be x, 2x and 3x

According to the question,

Sum of their squares = 504

$\Rightarrow x^2 + (2x)^2 + (3x)^2 = 504$

$\Rightarrow x^2 + 4x^2 + 9x^2 = 504$

$\Rightarrow 14x^2 = 504$

$\Rightarrow x^2 = 36$

$\Rightarrow x = 6$

Therefore, the three numbers are 6, 6 × 2, 6 × 3 i.e., 6, 12, 18.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART D)

1. If $p : q = 2 : 3$ and $x : y = 4 : 5$, then the value of $5px + 3qy : 10px + 4qy$ is:

(a) $\frac{2}{9}$ (b) $\frac{19}{28}$ (c) $\frac{4}{19}$ (d) None of above

Answer Key

1. (b)

PROPORTION

WHAT IS PROPORTION?

- An equality of two ratios is called a proportion. If we take four quantities a, b, c, d, they will be said to be in proportion if $a/b = c/d$ i.e., $ad = bc$
- a, b, c and d are called its **terms** of the proportion.
- First and fourth terms are called **extremes.**
- Second and third terms are called **means (or middle terms).**
- Also, it can be written as **a : b :: c : d.**

E.g.: Consider four quantities: a = 2, b = 4, c = 3, and d = 6.

In this case, the terms of the proportion are 2, 4, 3 and 6.

The first and fourth terms, a = 2 and d = 6, are called the extremes.

The second and third terms, b = 4 and c = 3, are called the means or middle terms.

We can express this proportion as a : b :: c : d, which indicates the equality of the ratios a/b and c/d.

The ratio a/b is 2/4, which can be simplified to 1/2.

The ratio c/d is 3/6, which can also be simplified to 1/2.

Thus, we can say a = 2, b = 4, c = 3, and d = 6 are in proportion.

CROSS PRODUCT RULE

Product of extremes = Product of means

Let's consider the following proportion: a, b, c and d

i.e., $\frac{a}{b} = \frac{c}{d}$

In this proportion, a and d are the extremes, while b and c are the means.

The cross product rule states that the product of the extremes (a × d) is equal to the product of the means (b × c)

Example 37. 8, 12, * , 15 are in proportion. Then * is

(a) 20 (b) 10 (c) 15 (d) None of these

Sol. (b) We know that,

If a, b, c and d are in proportion, then a/ b = c/d.

Let * be represented by x.

Since, 8, 12, * , 15 are in proportion then

$\frac{8}{12} = \frac{x}{15}$

$\Rightarrow x = \frac{8}{12} \times 15$

$\Rightarrow x = 2 \times 5$

$\Rightarrow x = 10$

Therefore, * is 10.

Hence, the correct option is (b).

Example 38. The fourth proportional to 2, 4, 8 is

(a) 16 (b) 32 (c) 48 (d) None of these

Sol. (a) Let the fourth proportion be x.

We know that,

If a, b, c and d are in proportion, then $\frac{a}{b} = \frac{c}{d}$.

Since, 2, 4, 8 and x are in proportion then

$\frac{2}{4} = \frac{8}{x}$

$\Rightarrow x = \frac{8 \times 4}{2}$

$\Rightarrow x = 16$

Therefore, the fourth proportional to 2, 4, 8 is 16.

Hence, the correct option is (a).

Example 39. The fourth proportional to 2a, a^2, c is **(ICAI)**

(a) $\frac{ac}{2}$ (b) ac (c) $\frac{2}{ac}$ (d) None of these

Sol. (a) Let the fourth proportion be x.

We know that,

If a, b, c and d are in proportion, then $\frac{a}{b} = \frac{c}{d}$..

Since, 2a, a^2, c and x are in proportion, then

$$\frac{2a}{a^2} = \frac{c}{x}$$

$$\Rightarrow x = \frac{c \times a^2}{2a}$$

$$\Rightarrow x = \frac{c \times a}{2}$$

$$\Rightarrow x = \frac{ac}{2}$$

Hence, the correct option is (a).

Example 40. If four numbers $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{x}$ are proportional then x is **(ICAI)**

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{15}{2}$ (d) None of these

Sol. (c) We know that,

If a, b, c and d are in proportion, then a/b = c/d.

Since, $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{x}$ are proportional then

Product of extremes = Product of means

$$\frac{1}{2} \times \frac{1}{x} = \frac{1}{3} \times \frac{1}{5}$$

$$\Rightarrow \frac{1}{2x} = \frac{1}{15}$$

$$\Rightarrow 2x = 15$$

$$\Rightarrow x = \frac{15}{2}$$

Hence, the correct option is (c).

Example 41. The number which has the same ratio to 22 that 5 has to 11 is

(a) 11 (b) 10 (c) $\frac{15}{2}$ (d) None of these

Sol. (b) Let the number which has the same ratio to 22 that 5 has to 11 be x, then

x : 22 :: 5 : 11

$\Rightarrow \frac{x}{22} = \frac{5}{11}$

$\Rightarrow x = \frac{22 \times 5}{11}$

$\Rightarrow x = 2 \times 5$

$\Rightarrow x = 10$

Therefore, the required number is 10.

Hence, the correct option is (b).

CONTINUOUS PROPORTION

- If 3 quantities: a, b, c are of same kind (in same units), they will be in continuous proportion if **a : b = b : c**

  i.e., $\frac{a}{b} = \frac{b}{c}$ or $\mathbf{b^2 = ac}$
- The middle term b is called the **mean proportional** between a and c.
- a is **first proportional** and c is the **third proportional.**

  E.g.: Consider three quantities: a = 2, b = 4 and c = 8

  Here, $\frac{a}{b} = \frac{2}{4} = \frac{1}{2}$

  $\frac{b}{c} = \frac{4}{8} = \frac{1}{2}$

  Since, $\frac{a}{b} = \frac{b}{c}$ thus a, b, c are in continuous proportion.

  We can say, 2 and 8 are first and third proportional respectively and 4 is the mean proportional.

Example 42. The mean proportional between 16, 25 is

(a) 18 (b) 19 (c) 20 (d) None of these

Sol. (c) We know that,

If b is the mean proportional of a and c then $b^2 = ac$.

Let b be the mean proportional between 16 and 25 then

$b^2 = 16 \times 25$

$\Rightarrow b = \sqrt{16 \times 25}$

$\Rightarrow b = 4 \times 5$

$\Rightarrow b = 20$

Therefore, the mean proportional between 16, 25 is 20.

Hence, the correct option is (c).

Example 43. The third proportional to 2, 6 is

(a) 12 (b) 18 (c) 24 (d) None of these

Sol. (b) Let the third proportional to 2, 6 is x, then

$2 : 6 :: 6 : x$

$\Rightarrow \frac{2}{6} = \frac{6}{x}$

$\Rightarrow x = \frac{6 \times 6}{2}$

$\Rightarrow x = \frac{36}{2}$

$\Rightarrow x = 18$

Therefore, the third proportional to 2, 6 is 18.

Hence, the correct option is (b).

Example 44. The mean proportional between $12x^2$ and $27y^2$ is **(ICAI)**

(a) 18xy (b) 81xy (c) 8xy (d) None of these

Sol. (a) We know that,

If b is the mean proportional of a and c then $b^2 = ac$.

Let b be the mean proportional between $12x^2$ and $27y^2$ then

$b^2 = 12x^2 \times 27y^2$

$\Rightarrow b = \sqrt{12x^2 \times 27y^2}$

$\Rightarrow b = 3 \times 3 \times 2 \times x \times y$

$\Rightarrow b = 18xy$

Therefore, the mean proportional between $12x^2$ and $27y^2$ is 18xy.

Hence, the correct option is (a).

Example 45. Find 2 numbers such that the mean proportional between them is 18 and 3rd proportional between them is 144. **(Dec 2012)**

(a) 9; 36 (b) 8; 32 (c) 7; 28 (d) 6; 24

Sol. (a) We know that,

If b is the mean proportional of a and c then $b^2 = ac$

Let the two numbers be x and y

Then, $xy = (18)^2 = 324$ ...(i)

Also, 3rd proportional between them = 144

Then, x, y, 144 are in continued proportion

i.e., $\frac{x}{y} = \frac{y}{144}$

$\Rightarrow y^2 = 144x$

$\Rightarrow y^2 = 144\left(\frac{324}{y}\right)$

$\Rightarrow y^3 = 144 \times 324$

$\Rightarrow y^3 = 12 \times 12 \times 12 \times 3 \times 3 \times 3$

Thus, $y = 12 \times 3 = 36$

So, $x = \frac{324}{36} = 9$

Therefore, the numbers are 9 and 36.

TRICK

Go by choices:

For option (a):

Mean proportion between 9 and 36 $= \sqrt{9 \times 36} = 3 \times 6 = 18$

If 3rd proportion between them is 144, then

$\frac{9}{36} = \frac{36}{144}$

i.e., $\frac{9}{36} = \frac{9}{36}$, which is true.

For option (b):

Mean proportion between 8 and 32 $= \sqrt{8 \times 32} = 8 \times 2 = 16$ which is not true.

Both conditions given in the questions are satisfied only by option (a) i.e., 9 and 36.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART E)

1. If $A = \frac{B}{2} = \frac{C}{5}$, then A : B : C is

   (a) 1 : 1 : 1 (b) 5 : 2 : 1 (c) 1 : 2 : 5 (d) None of these

2. If A : B = 5 : 6 and B : C = 7 : 8, what is the value of A : B : C?

   (a) 48 : 42 : 35 (b) 5 : 42 : 8 (c) 60 : 42 : 58 (d) 35 : 42 : 48

3. The third proportional to 7, 14 is

   (a) 21 (b) 28 (c) 35 (d) None of these

4. The mean proportional of 16 and 9 is

   (a) 12 (b) 14 (c) 15 (d) None of these

5. Find the fourth proportional to 4, 8 and 5

   (a) 8 (b) 10 (c) 15 (d) None of these

6. If a : b : c = 2 : 1 : 3 and $2a - 3b + c = 8$ then a + b + c is

   (a) 8 (b) 6 (c) 12 (d) 15

7. Find 2 numbers such that the mean proportional between them is 14 and 3rd proportional between them is 112.

   (a) 7, 28 (b) 6, 36 (c) 8, 54 (d) None of these

Answer Key

1. (c) 2. (d) 3. (b) 4. (a) 5. (b) 6. (c) 7. (a)

INVERSE PROPORTION

If a ratio is equal to the reciprocal of the other, then either of them is in inverse (or reciprocal) proportion of the other.

Note: In a ratio a : b, both quantities must be of the same kind while in a proportion i.e., a : b = c : d, all the four quantities need not be of the same type.

- **Properties of Proportion:**
  1. If a : b = c: d, then ad = bc **(Product of extremes = Product of means)**
     E.g.: If 2 : 3 = 4 : 6, then (2)(6) = (3)(4), which simplifies to 12 = 12.
  2. If a: b = c: d, then b : a = d : c **(Invertendo)**
     E.g.: If 5 : 10 = 2 : 4, then 10 : 5 = 4 : 2.
  3. If a : b = c : d, then a : c = b : d **(Alternendo)**
     E.g.: If 3 : 9 = 2 : 6, then 3 : 2 = 9 : 6.
  4. If a : b = c : d, then (a + b) : b = (c + d) : d **(Componendo)**
     E.g.: If 4 : 6 = 2 : 3, then (4 + 6) : 6 = (2 + 3) :3, which simplifies to 10 :6 = 5 : 3.
  5. If a : b = c : d, then (a – b) : b = (c – d) : d **(Dividendo)**
     E.g.: If 9 : 4 = 18 : 8, then (9 – 4) : 4 = (18 – 8) : 8, which simplifies to 5 : 4 = 10 : 8
  6. If a : b = c : d, then (a + b) : (a – b) = (c + d) : (c – d) **(Componendo and Dividendo)**
     E.g.: If 5 : 2 = 10 : 4, then (5 + 2) : (5 – 2) = (10 + 4) : (10 – 4), which simplifies to 7 : 3 = 14 : 6.
  7. If a : b = c : d = e : f = ..., then each of these ratios is equal to **(Addendo)**
     (a + c + e +...) : (b + d + f + ----)
     E.g.: If 2 : 4 = 3 : 6 = 1 : 2, then each of these ratios is equal to (2 + 3 + 1) : (4 + 6 + 2), which simplifies to 6 : 12.
  8. If a : b = c : d = e : f = ... , then each of these ratios is equal to (a – c – e – ...) : (b – d – f – .....) (Subtrahendo)
     E.g.: If 5 : 10 =3 : 6 = 1 : 2, then each of these ratios is equal to (5 – 3 – 1) : (10 – 6 – 2), which simplifies to 1 : 2.

Example 46. If $\frac{x}{y} = \frac{z}{w}$, implies $\frac{y}{x} = \frac{w}{z}$, then the process is called **(ICAI)**

(a) Dividendo (b) Componendo
(c) Alternendo (d) None of these

Sol. (d) We know that,
By the process of Invertendo,
If a : b = c : d, then b : a = d : c
According to the question

If $\frac{x}{y} = \frac{z}{w}$, implies $\frac{y}{x} = \frac{w}{z}$, it is the process of Invertendo.

Hence, the correct option is (d).

Example 47. If $\frac{p}{q} = \frac{r}{s} = \frac{p-r}{q-s}$, the process is called (ICAI)

(a) Subtrahendo (b) Addendo (c) Invertendo (d) None of these

Sol. (a) We know that,

By the process of Subtrahendo,

If $a : b = c : d = a - c : b - d$

According to the question, we have

$$\frac{p}{q} = \frac{r}{s} = \frac{p-r}{q-s}$$

It is the process of Subtrahendo.

Hence, the correct option is (a).

Example 48. $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$, then $\frac{a+b+c}{c}$ is **(ICAI)**

(a) 4 (b) 2 (c) 7 (d) None of these

Sol. (b) Given: $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$

Let $\frac{a}{4} = \frac{b}{5} = \frac{c}{9} = k$, then

$a = 4k$, $b = 5k$ and $c = 9k$

Thus, $\frac{a+b+c}{c} = \frac{4k+5k+9k}{9k} = \frac{18k}{9k}$

$\Rightarrow \frac{18}{9} \Rightarrow \frac{2}{1} = 2$

Hence, the correct option is (b).

Example 49. If $\frac{a}{4} = \frac{b}{5}$ then **(ICAI)**

(a) $\frac{a+4}{a-4} = \frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$

(c) $\frac{a-4}{a+4} = \frac{b+5}{b-5}$ (d) None of these

Sol. (b) We know that,

By the process of Componendo and Dividendo,

If $a : b = c : d$, then $a + b : a - b = c + d : c - d$

Since, $\frac{a}{4} = \frac{b}{5}$ then

$$\frac{a+4}{a-4} = \frac{b+5}{b-5}$$

Hence, the correct option is (b).

Example 50. If $\frac{x}{2} = \frac{y}{3} = \frac{z}{7}$, then the value of x : y is

(a) $\frac{6}{23}$ (b) $\frac{23}{6}$ (c) $\frac{2}{3}$ (d) $\frac{17}{6}$

Sol. (c) Given: $\frac{x}{2} = \frac{y}{3} = \frac{z}{7}$

Since, $x/2 = y/3$

i.e., $\frac{x}{2} = \frac{y}{3}$

$\Rightarrow \frac{x}{y} = \frac{2}{3}$

Hence, the correct option is (c).

Example 51. If $\frac{5x-3y}{5y-3x} = \frac{3}{4}$, then the value of x : y is

(a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) None of these

Sol. (d) Given: $\frac{5x-3y}{5y-3x} = \frac{3}{4}$

$\Rightarrow 4(5x - 3y) = 3(5y - 3x)$

$\Rightarrow 20x - 12y = 15y - 9x$

$\Rightarrow 20x + 9x = 15y + 12y$

$\Rightarrow 29x = 27y$

$\Rightarrow \frac{x}{y} = \frac{27}{29}$

Thus, the value of x : y is 27 : 29

Hence, the correct option is (d).

Example 52. $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$, then $(b - c)x + (c - a)y + (a - b)z$ is equal to

(a) 1 (b) 0 (c) 5 (d) None of these **(ICAI)**

Sol. (b) Given: $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$

Let $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = k$

Thus, $\frac{x}{b+c-a} = k, \frac{y}{c+a-b} = k$ and $\frac{z}{a+b-c} = k$

So, $x = k(b + c - a)$, $y = k(c + a - b)$ and $z = k(a + b - c)$

Now, Putting the values of x, y, z in $(b - c)x + (c - a)y + (a - b)z$, we get

$(b - c)k(b + c - a) + (c - a)k(c + a - b) + (a - b)k(a + b - c)$

$= k[(b - c)(b + c - a) + (c - a)(c + a - b) + (a - b)(a + b - c)]$

$= k[(b - c)(b + c) - a(b - c) + (c - a)(c + a) - b(c - a) + (a - b)(a + b) - c(a - b)]$

$= k(b^2 - c^2 - ab + ac + c^2 - a^2 - bc + ab + a^2 - b^2 - ac + bc) = k(0) = 0$

Therefore, the value of $(b - c)x + (c - a)y + (a - b)z$ is 0.

Hence, the correct option is (b) i.e., 0.

Example 53. The sum of the ages of 3 persons is 150 years. 10 years ago, their ages were in the ratio 7 : 8 : 9. Their present ages are

(a) (45, 50, 55) (b) (40, 60, 50)

(c) (35, 45, 70) (d) None of these

Sol. (a) Given: Ratio of ages of 3 persons 10 years ago = 7 : 8 : 9

Let the ages of three persons 10 years ago were 7x, 8x and 9x.

Thus, their present ages i.e, their ages after 10 years are

$7x + 10$, $8x + 10$ and $9x + 10$

According to the question,

Sum of the ages of 3 persons = 150 years

$\Rightarrow 7x + 10 + 8x + 10 + 9x + 10 = 150$

$\Rightarrow 24x + 30 = 150$

$\Rightarrow 24x = 120$

$\Rightarrow x = 5$

Therefore, the present ages of three persons are:

$7(5) + 10$, $8(5) + 10$ and $9(5) + 10 = 45, 50, 55$

Hence, the correct option is (a).

Example 54. If $a : b = 4 : 1$, then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is : **(ICAI)**

(a) $\frac{5}{2}$ (b) 4 (c) 5 (d) None of these

Sol. (a) Given, $a : b = 4 : 1$

i.e., $\frac{a}{b} = \frac{4}{1}$

Thus, $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$

$= \sqrt{\frac{4}{1}} + \sqrt{\frac{1}{4}} = 2 + \frac{1}{2} = \frac{5}{2}$

Hence, the correct option is (a).

Example 55. A person has assets worth ₹1,48,200. He wishes to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this asset, the share of his son will be:

(a) ₹24,700 (b) ₹49,400 (c) ₹74,100 (d) ₹37,050

Sol. (b) Given: Shares of wife, son and daughter = 3 : 2 : 1

Value of assets = ₹1,48,200

Let the shares of wife, son and daughter be 3x, 2x and x respectively.

According to the question,

$3x + 2x + x = 1, 48,200$

$\Rightarrow 6x = 1,48,200$

$\Rightarrow x = 24,700$

Therefore, the share of the son is 2x.

$= 2(24,700) = 49,400$

Hence, the correct option is (b).

Example 56. The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40 (b) 25, 24 (c) 40, 60 (d) 50, 70

Sol. (d) Given: Ratio of students of two classes = 5 : 7

Let the students of two classes be 5x and 7x respectively.

According to the question,

$$\frac{5x-10}{7x-10} = \frac{4}{6}$$

On cross-multiplication, we get

$6(5x - 10) = 4(7x - 10)$

$\Rightarrow 30x - 60 = 28x - 40$

$\Rightarrow 30x - 28x = 60 - 40$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10$

Therefore, the number of students in each class are 5x and 7x i.e., 50 and 70.

Hence, the correct option is (d).

Example 57. A dealer mixes rice costing ₹13.85 per kg with rice costing ₹15.54 and sell the mixture at ₹17.60 per kg so, he earned a profit of 14.6% on his sale price. the proportion in which he mixes the two qualities of rice is

(a) 3 : 7 (b) 5 : 7 (c) 7 : 9 (d) 9 : 11

Sol. (a) Given,

Cost of type 1 rice = ₹13.85

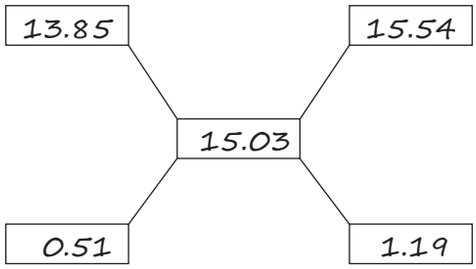
Cost of type 2 rice = ₹15.54

Selling price of mixture = ₹17.60

Profit percent = 14.6%

$$\text{Thus, profit} = 17.60 \times \frac{14.6}{100} = 2.57$$

Cost price = 17.6 – 2.57 = 15.03



Therefore, the required proportion $= \frac{0.51}{1.19} = \frac{51}{199} = \frac{3}{7}$

Hence, the correct option is (a).

Example 58. Two numbers are in the ratio 7 : 8. If 3 is added to each of them, their ratio becomes 8 : 9. The number are: **(2007 Feb)**

(a) 14, 16 (b) 24, 27 (c) 21, 24 (d) 16, 18

Sol. (c) Given: Ratio of numbers = 7 : 8

Let the numbers be 7x and 8x.

According to the question,

If 3 is added to each of them, their ratio becomes 8 : 9 i.e.,

$\frac{7x+3}{8x+3} = \frac{8}{9}$

$\Rightarrow 9(7x + 3) = 8(8x + 3)$

$\Rightarrow 63x + 27 = 64x + 24$

$\Rightarrow 63x - 64x = 24 - 27$

$\Rightarrow -x = -3$

$\Rightarrow x = 3$

Therefore, the numbers are 7(3) and 8(3) i.e., 21 and 24.

Hence, the correct option is (c).

Example 59. Two numbers are in the ratio 2 : 3 and the difference of their squares is 320. The numbers are

(a) 12, 18 (b) 16, 24 (c) 14, 21 (d) None of these

Sol. (b) Given: Ratio of two numbers = 2 : 3

Let the two numbers are 2x and 3x.

According to the question,

The difference between their squares is 320.

i.e., $(3x)^2 - (2x)^2 = 320$

$\Rightarrow 9x^2 - 4x^2 = 320$

$\Rightarrow 5x^2 = 320$

$\Rightarrow x^2 = 64$

$\Rightarrow x = 8$

Therefore, the numbers are 2(8) and 3(8) i.e., 16 and 24.

Hence, the correct option is (b).

Example 60. The monthly income of A and B are in the ratio 4 : 5 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹150 per month, find their monthly incomes.

(a) (₹40, ₹50) (b) (₹50, ₹40) (c) (₹400, ₹500) (d) None of these

Sol. (c) Given: Ratio of monthly incomes of A and B = 4 : 5

Ratio of their expenditures = 5 : 7

Let the monthly income of A be 4x and that of B be 5x

Let the monthly expenses of A be 5y and that of B be 7y

According to the question,

$4x - 5y = 150$ ...(i)

$5x - 7y = 150$ ...(ii)

Multiply eq (i) with 5 and (ii) by 4, thus we get

$20x - 25y = 750$

$20x - 28y = 600$

On solving both equations, we get

$3y = 150$

$y = 50$

Thus, $4x - 250 = 150$

$4x = 400$

$x = 100$

Therefore, the monthly income of A = $4x = 4(100) = ₹400$

Monthly income of B = $5x = 5(100) = ₹500$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART F)

1. A bag contains ₹187 in the form of 1 rupee, 50 paise and 10 paise coins in the ratio 3 : 4 : 5. Find the number of each type of coins: **(May, 2007)**

   (a) 102, 136, 170 (b) 136, 102, 170

   (c) 170, 102, 136 (d) None of these

2. Ratio of earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What is A's earning? **(August, 2007)**

   (a) ₹21,000 (b) ₹26,000 (c) ₹28,000 (d) Data inadequate

3. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹1,500, then B's income is: **(Nov, 2007)**

   (a) ₹6,000 (b) ₹4,500 (c) ₹3,000 (d) ₹7,500

4. In a 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3 : 1. The quantity of water added in the mixture in order to make this ratio 2 : 1 is

   (a) 15 litres (b) 10 litres (c) 8 litres (d) 5 litres **(February, 2008)**

5. In what ratio should tea worth ₹10 per kg be mixed with tea worth ₹14 per kg, so that

the average price of the mixture may be ₹11 per kg? **(June, 2008)**

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 3

6. If A, B and C started a business by investing ₹1,26,000, ₹84,000 and ₹2,10,000. If at the end of the year profit is ₹2,42,000 then the share of each is **(2008, December)**

(a) 72,600; 48,400; 1,21,000 (b) 48,400; 1,21,000; 72,600

(c) 72,000; 49,000; 1,21,000 (d) 48,000; 1,21,400; 72,600

7. In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹1,60,000 and C gets ₹2,50,000. Find the amount received by B? **(2011, June)**

(a) ₹2,00,000 (b) ₹2,50,00 (c) ₹1,00,000 (d) ₹1,50,000

8. If $15(2p^2 - q^2) = 7pq$, where p and q are positive, then p : q will be

(a) 5 : 6 (b) 5 : 7 (c) 3 : 5 (d) 8 : 3

9. The ratio of third proportion of 12, 30 to the mean proportion of 9, 25 is

(a) 2 : 1 (b) 5 : 1 (c) 7 : 15 (d) 3 : 5 **(2015 Dec)**

10. What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers in proportion? **(2015 Dec)**

(a) 2 (b) 4 (c) 8 (d) None of these

11. If x : y = 2 : 3, y : z = 4 : 3 then x : y : z is

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) None of these

12. The numbers 14, 16, 35 are not in proportion. The fourth term for which they will be in proportion is:

(a) 45 (b) 40 (c) 32 (d) None of these

Answer Key

1. (a) **2.** (d) **3.** (c) **4.** (d) **5.** (b) **6.** (a) **7.** (a) **8.** (a) **9.** (b) **10.** (a)
11. (a) **12.** (b)

INDICES

The power, also known as the **index**, tells you how many times you have to multiply the number by itself and number will be called as **base.**

In other words: If n is a positive integer, and 'a' is a real number, i.e. $n \in N$ and $a \in R$ (where N is the set of positive integers and R is the set of real numbers).

Then, $a^n = a \times a \times a \times a$.... to n factors

Example 61. The values of

(I) 2^4 (II) 3^3 (III) 18^2

(a) 8, 9, 36 (b) 16, 27, 36 (c) 16, 27, 324 (d) None of these

Sol. (c) $2^4 = 2 \times 2 \times 2 \times 2 = 16$

Now, $3^3 = 3 \times 3 \times 3 = 27$

Also, $18^2 = 18 = 324$

Therefore, $2^4 = 16, 3^3 = 27$ and $18^2 = 324$

Hence, the correct option is (c).

- Any base to the power (or index) 0 is 1 i.e., $a^0 = 1$ for $a \neq 0$.

Example 62. The values of

(I) 12° (II) 32° (III) $\left(\frac{1}{18}\right)^\circ$

(a) 12, 32, 18 (b) 12, 32, $\frac{1}{18}$ (c) 1, 1, 1 (d) None of these

Sol. (c) We know that,

$a^\circ = 1$ for $a \neq 0$

Here, $12^\circ = 1$

Also, $32^\circ = 1$

Now, $\frac{1}{18^\circ} = \frac{1}{1} = 1$

Therefore, $12^\circ = 1$, $32^\circ = 1$ and $\frac{1}{18^\circ} = 1$

Hence, the correct option is (c).

Finding the roots : If you have to find n^{th} root of base a i.e. $\sqrt[n]{a}$ it will give value equal to $a^{\frac{1}{n}}$.

Negative Powers: If you have power as negative integer, –n (let) for base a, then $a^{-n} = \frac{1}{a^n}$.

Calculator trick to find n^{th} root:

1. Type the number
2. Press '√' for 12 times
3. Subtract 1 from it and then divide by n
4. Add '1'
5. Press '× =' for 12 times

E.g.: To find $(16)^{\frac{1}{4}}$

1. Type '16'
2. Press '√' for 12 times
3. Subtract 1 from it and then divide by 4
4. Add '1'
5. Press '× =' for 12 times

We will get $2.00035 \approx 2$

Therefore, $(16)^{\frac{1}{4}} = 2$

Example 62. Find out the values of each of the indices:

(I) $\sqrt{4}$ (II) 16^{-4} (III) $(3)^{-3}$ (IV) 9^{-5}

(a) $2, \frac{1}{2}, \frac{1}{9}, \frac{1}{3^{10}}$ (b) $2, \frac{1}{2}, \frac{1}{27}, \frac{1}{3^5}$

(c) $2, \frac{1}{2}, \frac{1}{27}, \frac{1}{3^{10}}$ (d) None of these

Sol. (c) (I) $\sqrt{4} = \sqrt{2 \times 2} = 2$

(II) We have, 16^{-4}

$= (2^4)^{-4} = 2^{4x-4} = 2^{-1} = \frac{1}{2}$

Thus, $16^{-4} = \frac{1}{2}$

(III) $(3)^{-3} = \frac{1}{3^3} = \frac{1}{27}$

Thus, $(3)^{-3} = \frac{1}{27}$

(IV) $9^{-5} = \frac{1}{9^5} = \frac{1}{(3^2)^5} = \frac{1}{3^{10}}$

Thus, $9^{-5} = \frac{1}{3^{10}}$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART G)

1. Find out the values of each: (i) $16^{\frac{1}{4}}$ and (ii) $343^{\frac{1}{3}}$

(a) 2, 7 (b) 4, 343 (c) 3, 27 (d) None of these

2. The value of $16^{-\frac{3}{4}}$ is

(a) 8 (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) None of these

3. Simplify $\left(\frac{1}{5}\right)^{-2}$.

(a) 5 (b) 10 (c) 25 (d) $\frac{1}{25}$

4. The value of $\left(\frac{8}{125}\right)^{\frac{1}{3}}$ is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{25}$ (d) None of these

5. The value of $\left(\frac{2p^2q^3}{3xy}\right)^{\circ}$ where $p, q, x, y \neq 0$ is equal to

(a) 0 (b) 1 (c) 2 (d) None of these

Answer Key									
1. (a)	2. (b)	3. (d)	4. (a)	5. (b)	6. (a)	7. (a)	8. (a)	9. (b)	10. (a)

LAW 1

$a^m \times a^n = a^{m+n}$

- **Case-I:** When m and n are positive integers; then $a^m \times a^n = a^{m+n}$
  E.g.: $6^7 6^2 = 6^{7+2} = 6^9$
- **Case-II:** When $n = -m$, m is positive integer then $a^m \times a^{-m} = a^0 = 1$
  E.g.: $2^3 2^{-3} = 2^{3-3} = 2^0 = 1$
- **Case-III:** When m and n are any integers
  E.g.: $7^{\frac{4}{3}} 7^{-2} = 7^{\frac{4}{3}-2} = 7^{\frac{4-6}{3}} = 7^{-\frac{2}{3}}$

Example 64. Find the value of $5^{\frac{3}{5}} . 5^{-\frac{7}{2}}$.

(a) $5^{\frac{-4}{7}}$ (b) $5^{-\frac{41}{10}}$ (c) $5^{\frac{29}{10}}$ (d) $5^{-\frac{29}{10}}$

Sol. (d) We have, $5^{\frac{3}{5}} . 5^{-\frac{7}{2}}$

Thus, $5^{\frac{3}{5}+\left(-\frac{7}{2}\right)} = 5^{\frac{6-35}{10}} = 5^{-\frac{29}{10}}$

Hence, the correct option is (d).

Example 65. The value of $8^{\frac{1}{3}}$ is **(ICAI)**

(a) $3\sqrt{2}$ (b) 4 (c) 2 (d) None of these

Sol. (c) To find: $8^{\frac{1}{3}}$

$= \sqrt[3]{2 \times 2 \times 2} = 2$

Hence, the correct option is (c).

Example 66. The value of $2 \times \left(32\right)^{\frac{1}{5}}$ is **(ICAI)**

(a) 2 (b) 10 (c) 4 (d) None of these

Sol. (c) To find: $2 \times (32)^{\frac{1}{5}}$

$= 2 \times \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2} = 2 \times 2 = 4$

Therefore, the value of $2 \times (32)^{\frac{1}{5}}$ is 4.
Hence, the correct option is (c).

Example 67. The value of $2(256)^{-\frac{1}{8}}$ is **(ICAI)**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) None of these

Sol. (a) To find: $2(256)^{-\frac{1}{8}}$

$$= \frac{2}{(256)^{\frac{1}{8}}} = \frac{2}{\sqrt[8]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}} = \frac{2}{2} = 1$$

Therefore, the value of $2(256)^{-\frac{1}{8}}$ is 1.
Hence, the correct option is (a).

Example 68. $4x^{-1/4}$ is expressed as

(a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) None of these

Sol. (c) We have, $4x^{-1/4} = \frac{4}{x^{\frac{1}{4}}}$

Therefore, $4x^{-1/4} = \frac{4}{x^{\frac{1}{4}}}$

Hence, the correct option is (c).

Example 69. Simplify $4x^{1/3} \times 2x^{-1}$ if $x = 4$.

(a) $2^{\frac{1}{3}}$ (b) 2 (c) $2^{\frac{5}{3}}$ (d) None of these

Sol. (c) We have, $4x^{1/3} \times 2x^{-1}$
At $x = 4$

$4x^{1/3}2x^{-1} = 4(4)^{\frac{1}{3}} \times 2(4)^{-1}$

$= 4^{1+\frac{1}{3}-1} \times 2 = 4^{\frac{1}{3}} \times 2 = (2^2)^{\frac{1}{3}} \times 2 = 2^{\frac{2}{3}+1} = 2^{\frac{5}{3}}$

Hence, the correct option is (c).

Example 70. $x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to

(a) x (b) 1 (c) 0 (d) None of these

Sol. (b) To find: $x^{a-b} \times x^{b-c} \times x^{c-a}$
We know that,
$a^m \times a^n = a^{m+n}$

Thus, $x^{a-b} \times x^{b-c} \times x^{c-a} = x^{a-b+b-c+c-a} = x^0$

Since, $a^0 = 1$ for $a \neq 0$

Therefore, $x^{a-b} \times x^{b-c} \times x^{c-a} = x^0 = 1$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART H)

1. The value of $4^{-3} \times 4^5$ is

   (a) 4 (b) 8 (c) 16 (d) None of these

2. The value of $9^2 \times 9^3 \times 9^5$ is

   (a) 90 (b) 9^{10} (c) 9^{30} (d) None of these

3. The value of $5^3 \times 7^2 \times 5^4 \times 7^3$ is

   (a) 35^{12} (b) $5^3 \times 7^2$ (c) $5^7 \times 7^5$ (d) None of these

4. The value of $(125)^{\frac{2}{3}} \times (625)^{\frac{3}{4}}$ is :

   (a) 150 (b) 625 (c) 3125 (d) None of these

Answer Key

1. (c) **2.** (b) **3.** (c) **4.** (c)

LAW 2

$\frac{a^m}{a^n} = a^{m-n}$, when m and n are positive integers and m > n.

E.g.: $\frac{2^3}{2^4} = 2^{3-4} = 2^{-1} = \frac{1}{2}$

Example 71. Which of the following option is correct when you simplify the following:

(I) $4/(16)^{1/4}$ (II) $(27)^{2/3}/(9)^{3/2}$

(a) $2, \frac{1}{3}$ (b) $\frac{1}{2}, \frac{1}{3}$ (c) 2, -3 (d) None of these

Sol. (a) (I) $4/(16)^{1/4}$

$$= \frac{4}{16^{\frac{1}{4}}} = \frac{4}{(2 \times 2 \times 2 \times 2)^{\frac{1}{4}}} = \frac{4}{2} = 2$$

(II) $(27)^{2/3}/(9)^{3/2}$

$$= \frac{27^{\frac{2}{3}}}{9^{\frac{3}{2}}} = \frac{(3^3)^{\frac{2}{3}}}{(3^2)^{\frac{3}{2}}} = \frac{3^2}{3^3} = 3^{2-3} = 3^{-1} = \frac{1}{3}$$

Hence, the correct option is (a).

Example 72. The value of $\frac{4}{(32)^{\frac{1}{5}}}$ is **(ICAI)**

(a) 8 (b) 2 (c) 4 (d) None of these

Sol. (b) We have, $\frac{4}{(32)^{\frac{1}{5}}}$

$= \frac{4}{(32)^{\frac{1}{5}}} = \frac{4}{(2\times2\times2\times2\times2)^{\frac{1}{5}}} = \frac{4}{2} = 2$

Therefore, $\frac{4}{(32)^{\frac{1}{5}}} = 2$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART I)

1. The value of $(10)^7 \div (10)^5$ is
   (a) 1 (b) 10 (c) 20 (d) 100
2. The value of $3^5 \times 3^6 \div 3^2$ is
   (a) 3 (b) 3^3 (c) 3^9 (d) 3^{12}
3. The value of $12y^6 \div 6y^{-5}$ is
   (a) 2 (b) 2y (c) $2y^{-1}$ (d) None of these
4. Simplify $\frac{2a^{\frac{1}{2}} \times a^{\frac{2}{3}} \times 6a^{-\frac{7}{3}}}{9a^{a^{-\frac{5}{3}}} \times a^{a^{\frac{3}{2}}}}$ if $a = 4$

   (a) 3 (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) 4

Answer Key
1. (d) **2.** (c) **3.** (b) **4.** (b)

LAW 3

$(a^m)^n = a^{mn}$ when m and n are positive integers

E.g.: $((16)^{1/4})^5 = (16)^{\frac{1}{4}\times5} = 16^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^{4\times\frac{5}{4}} = 2^5$

Example 74. Find the value of $\left[(27)^{\frac{2}{3}} \cdot (9)^{\frac{3}{2}}\right]^{\frac{1}{5}}$

(a) 27 (b) 3 (c) 3^5 (d) None of these

Sol. (b) We have, $\left[(27)^{\frac{2}{3}} \cdot (9)^{\frac{3}{2}}\right]^{\frac{1}{5}}$

$= \left[(3^3)^{\frac{2}{3}} \cdot (3^2)^{\frac{3}{2}}\right]^{\frac{1}{5}} = \left[3^{3\times\frac{2}{3}} \cdot 3^{2\times\frac{3}{2}}\right]^{\frac{1}{5}} = \left[3^2 \cdot 3^3\right]^{\frac{1}{5}} = \left[3^5\right]^{\frac{1}{5}} = 3$

Hence, the correct option is (b).

Example 75. The value of $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ is **(ICAI)**

(a) 2/3 (b) 3/2 (c) 2/9 (d) None of these

Sol. (a) We have, $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$= \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}} = \left(\frac{2}{3}\right)^{3\times\frac{1}{3}} = \left(\frac{2}{3}\right)^{1} = \frac{2}{3}$

Hence, the correct option is (a).

Example 76. $\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$ **(ICAI)**

(a) x^n (b) x^{n+1} (c) x^{n-1} (d) None of these

Sol. (c) We have, $\left[(x^n)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$

$= \left[(x^n)^{\frac{n^2-1}{n}}\right]^{\frac{1}{n+1}} = \left[(x^n)^{\frac{(n-1)(n+1)}{n}}\right]^{\frac{1}{n+1}} = (x^n)^{\frac{(n-1)(n+1)}{n(n+1)}}$

$= (x^n)^{\frac{(n-1)}{n}} = x^{\frac{n(n-1)}{n}} = x^{n-1}$

Hence, the correct option is (c).

Example 77. The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ **(ICAI)**

(a) 1 (b) 0 (c) 2 (d) None of these

Sol. (a) We have,

$\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} = x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} = x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$

Hence, the correct option is (a).

Example 78. If $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, then the simplified form of **(ICAI)**

$$\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$$

(a) 0 (b) 1 (c) x (d) None of these

Sol. (b) We have, $\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$

$= \left[x^{l-m}\right]^{l^2+lm+m^2} \times \left[x^{m-n}\right]^{m^2+mn+n^2} \times \left[x^{n-l}\right]^{l^2+ln+n^2}$

We know that,

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$= \left[x^{(l-m)\left(l^2+lm+m^2\right)}\right] \times \left[x^{(m-n)\left(m^2+mn+n^2\right)}\right] \times \left[x^{(n-l)\left(l^2+ln+n^2\right)}\right]$

$= x^{l^3-m^3} \times x^{m^3-n^3} \times x^{n^3-l^3} = x^{l^3-m^3+m^3-n^3+n^3-l^3} = x^0 = 1$

Hence, the correct option is (b).

Example 79. If $(25)^{150} = (25x)^{50}$, then the value of x will be

(a) 5^3 (b) 5^4 (c) 5^2 (d) 5

Sol. (b) Given: $(25)^{150} = (25x)^{50}$

$\Rightarrow (25)^{150} = (25^{50})(x^{50})$

$\Rightarrow x^{50} = \frac{(25)^{150}}{(25)^{50}}$

$\Rightarrow x^{50} = (25)^{150-50}$

$\Rightarrow x^{50} = (25)^{100}$

$\Rightarrow x^{50} = (5^2)^{2\times 50}$

$\Rightarrow x^{50} = (5^4)^{50}$

On comparing, we get

$x = 5^4$

Hence, the correct option is (b).

Example 80. The value of $\frac{64(b^4a^3)^6}{\left[4(a^3b)^2 \times (ab)^2\right]}$ is

(a) $16a^{10}b^{20}$ (b) $4a^{20}b^{10}$ (c) $8a^{10}b^{20}$ (d) $4a^{10}b^{20}$

Sol. (a) $\frac{64(b^4a^3)^6}{\left[4(a^3b)^2 \times (ab)^2\right]}$

$= \frac{64b^{24}a^{18}}{\left[4a^6b^2 \times a^2b^2\right]} = \frac{64b^{24}a^{18}}{\left[4a^8b^4\right]} = 16b^{24-4}a^{18-8} = 16b^{20}a^{10}$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART J)

1. The value of $(4^2)^5$ is
   (a) 4^7 (b) 4^{10} (c) 4^{20} (d) None of these
2. If $(3^5)^6 = 3^m$ $(3^5)^6 = 3^m$, then the value of m is
   (a) 10 (b) 30 (c) 20 (d) None of these
3. On simplification, $\frac{1}{1+a^{m-n}+a^{m-p}} + \frac{1}{1+a^{n-m}+a^{n-p}} + \frac{1}{1+a^{p-m}+a^{p-n}}$ is equal to **(ICAI)**
   (a) 0 (b) a (c) 1 (d) $\frac{1}{a}$
4. $[\{(2)^{1/2}.(4)^{3/4}.(8)^{5/6}.(16)^{7/8}.(32)^{9/10}\}^4]^{3/25}$ is **(ICAI)**
   (a) A fraction (b) an integer (c) 1 (d) None of these

Answer Key
1. (b) **2.** (b) **3.** (c) **4.** (b)

LAW 4

$(ab)^n = a^n b^n$ when n can take all of the values.
E.g.: 6^3 can be written as $(2 \times 3)^3 = 2^3 \times 3^3$

Example 81. The value of $(8)^3(27)^3$ is
(a) 6 (b) 6^{18} (c) 6^9 (d) None of these

Sol. (c) We have, $(8)^3(27)^3$
$= (23)^3(3^3)^3 = (2^9).(3^9) = (2 \times 3)^9 = 6^9$
Hence, the correct option is (c).

Example 82. Simplify $6a^2b^3c^2 \times 4b^{-3}c^{-2}d$. **(ICAI)**
(a) 0 (b) $24a^2d$ (c) $24a^2bcd$ (d) None of these

Sol. (b) We have, $6a^2b^3c^2 \times 4b^{-3}c^{-2}d$
$= 6 \times 4 \times a^2 \times b^3 \times b^{-3} \times c^2 \times c^2 \times c^{-2} \times d$
$= 24 \times a^2 \times b^{3-3} \times c^{2-2} \times d = 24a^2b^0c^0d = 24a^2d$
Hence, the correct option is (b).

Example 83. The simplified value of $16x^{-3}y^2 \times 8^{-1}x^3y^{-2}$ is **(ICAI)**
(a) 2xy (b) $\frac{xy}{2}$ (c) 2 (d) None of these

Sol. (c) We have, $16x^{-3}\ y^2 \times 8^{-1}x^3\ y^{-2}$
$= \frac{16}{8} \times x^{-3} \times x^3 \times y^2 \times y^{-2} = 2 \times x^{-3+3} \times y^{2-2} = 2 \times x^0 \times y^0 = 2$
Therefore, $16x^{-3}y^2 \times 8^{-1}x^3y^{-2} = 2$
Hence, the correct option is (c).

Example 84. If $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{z}}$ and $xyz = 1$, then the value of $p + q + r$ is **(ICAI)**

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) None of these

Sol. (b) Given: $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}}$ and $xyz = 1$

Let $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}} = k$

Since $x^{\frac{1}{p}} = k$, thus

$\left(x^{\frac{1}{p}}\right)^p = k^p \Rightarrow x = k^p \quad \left[\because (x^m)^n = x^{mn}\right]$

$[\therefore (x^m)^n = x^{mn}]$

Similarly, $y^{\frac{1}{q}} = k$

$\left(y^{\frac{1}{q}}\right)^q = k^q \Rightarrow y = k^q$

Also, $z^{1/r} = k$

$\left(x^{\frac{1}{r}}\right)^r = k^r \Rightarrow z = k^r$

Substituting the values of x, y and z in $xyz = 1$, we get

$k^p \times k^q \times k^r = 1$

$\Rightarrow k^{p+q+r} = 1 \ (\therefore a^m \cdot a^n = a^{m+n})$

$\Rightarrow k^{p+q+r} = k^0 \ (\therefore a^0 = 1)$

On comparing, we get $p + q + r = 0$

Hence, option (b) is correct.

Example 85. On simplification $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^x}$ reduces to

(a) 0 (b) 1 (c) -1 (d) None of above

Sol. (b) $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 2^{y+1} \times 3^{y+1}}{2^{x+1} \times 3^{x+1} \times 2^{y+3} \times 5^{y+3} \times 3^x \times 5^x}$

$\Rightarrow \frac{2^{x+3} \times 2^{y+1}}{2^{x+1} \times 2^{y+3}} \times \frac{3^{2x-y} \times 3^{y+1}}{3^{x+1} \times 3^x} \times \frac{5^{x+y+3}}{5^{y+3} \times 5^x}$

$2^{x+3+y+1-x-1-y-3} \times 3^{2x-y+y+1-x-1-x} \times 5^{x+y+3-y-3-x}$

$\Rightarrow 2^0 \times 3^0 \times 5^0 = 1$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART K)

1. If $3^{x+2} = 27$, what is the value of x?

   (a) 1 (b) 0 (c) 2 (d) None of these

2. If $2^x = 3^y = 6^{-z}$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is **(ICAI)**

   (a) 1 (b) 0 (c) 2 (d) None of these

3. The value of $\{(x+y)^{2/3}(x-y)^{3/2} \Big/ \sqrt{x+y} \times \sqrt{(x-y)^2}\}^6$ is **(ICAI)**

   (a) $(x+y)^2$ (b) $(x-y)$ (c) $x+y$ (d) None of these

4. $\{(3^3)^2 \times (4^2)^3 \times (5^3)^2\} / \{(3^2)^3 \times (4^3)^2 \times (5^2)^3\}$ is **(ICAI)**

   (a) 3/4 (b) 4/5 (c) 4/7 (d) None of these

5. The value of $\left(\frac{6^{-1}7^2}{6^2 7^{-4}}\right)^{7/2} \times \left(\frac{6^{-2} - 7^3}{6^3 7^{-5}}\right)^{-5/2}$ is

   (a) 36 (b) 252 (c) 600 (d) None of these

Answer Key

1. (a) **2.** (b) **3.** (c) **4.** (d)

LOGARITHM

WHAT IS LOGARITHM?

Definition:

- The logarithm is the inverse operation of an exponential equation of the form $n = a^x$, where 'a' represents the base and 'x' represents the exponent. The logarithm can be found by interchanging the positions of 'x' (index) and 'n' in the equation.

  E.g.: For $2^3 = 8$, then $\log_2 8 = 3$

- For all real numbers n and all positive numbers a and x where $a \neq 1$, $x = \log_a n$ if and only if $n = a^x$.

  E.g.: For $n = 1000$ and $a = 10$.

  Here, $n = 1000 = (10)^3$ which is of the form $n = a^x$

- The expression $\log_a n$ represents the base or index to which the base "a" must be raised in order to obtain x.

Example 88. $\log_2 8$ is equal to

(a) 2 (b) 8 (c) 3 (d) None of these

Sol. (c) We have, $\log_2 8$

We know that, $\log_a b = \frac{\log b}{\log a}$

$$\log_2 8 = \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2} = \frac{3\log 2}{\log 2} = 3 \times 1 = 3$$

Therefore, $\log_2 8$ is equal to 3.
Hence, the correct option is (c).

Example 89. $\log_{\sqrt{2}} 64$ is equal to

(a) 12 (b) 6 (c) 1 (d) None of these

Sol. (a) We have, $\log_{\sqrt{2}} 64$

We know that, $\log_a b = \frac{\log b}{\log a}$

Thus, $\log_{\sqrt{2}} 64 = \frac{\log 64}{\log \sqrt{2}}$

$$= \frac{\log 2^6}{\log 2^{\frac{1}{2}}} = \frac{6\log 2}{\frac{1}{2}\log 2} = 6 \times 2 = 12$$

Therefore, $\log_{\sqrt{2}} 64 = 12$
Hence, the correct option is (a).

Example 90. $\log_{2\sqrt{3}} 1728$ is equal to

(a) $2\sqrt{3}$ (b) 2 (c) 6 (d) None of these

Sol. (c) We have, $\log_{2\sqrt{3}} 1728$

$= \log_{2\sqrt{3}}(2^6 \times 3^3) = \log_{2\sqrt{3}}(2^6 \times (\sqrt{3})^6)$ $(\because 3 = \sqrt{3} \times \sqrt{3} \Rightarrow 3^3 = (\sqrt{3})^6)$

$= \log_{2\sqrt{3}}(2\sqrt{3})^6$ $\left[\because (x \times y)^m = x^m \times y^m\right]$

$= 6\log_{2\sqrt{3}}(2\sqrt{3})$ $\left[\because \log_a x^n = n\log_a x\right]$

$= 6 \times 1 = 6$ $\left[\because \log_a a = 1\right]$

Therefore, the value of $\log_{2\sqrt{3}} 1728$ is 6.
Hence, option (c) is correct.

PRACTICE QUESTIONS (PART I)

1. The value of $\log_3 27$ is

(a) 9 (b) 3 (c) 1 (d) Cannot be determined

2. Find the value of $\log_3\left(\frac{1}{9}\right)$

(a) 2 (b) -2 (c) 3 (d) None of these

3. The value of $\log_{3\sqrt{2}} 5832$ is

(a) $2\sqrt{3}$ (b) 6 (c) 12 (d) Cannot be determined

Answer Key

1. (a) **2.** (b) **3.** (c)

CHARACTERISTIC

The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of digits to the left of the decimal point in the given number.

Mantissa: The mantissa is the fractional part of the logarithm of a given number.

E.g.: log 27 = 1.431, here 1 is the characteristic and 0.431 is mantissa.

- **Calculator Trick to find out Logarithm:**
  1. Type number and press root for 15 times.
  2. Subtract 1 after step 1.
  3. Divide number by 0.00007027 or multiply by 14230.

Example 91. Find

(I) log 4.221 (II) log 0.2312 (III) log 0.001294

Solution:

(I) First type 4.221 and press root for 15 times. After that subtract 1 from it . Now divide it by 0.00007027 or multiply by 14230, thus we get

log 4.221 = 0.6254

(II) Following the above steps, we get

log 0.2312 = -0.636

(III) Following the above steps, we get

log 0.001294 = -2.88758

FUNDAMENTAL LAW OF LOGARITHMS

LAW 1

Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base, i.e. $\log_a mn = \log_a m + \log_a n$

E.g.: $\log_a(10) = \log_a(2 \times 5) = \log_a 2 + \log_a 5$

Example 92. log 20 can be expressed as

(a) log 2 + log 5 (b) log 5 + log 8

(c) log 5 + log 4 (d) None of these

Sol. (c) We know that, log ab = log a + log b

We have, log 20

= log(4 × 5) = log 4 + log 5

Hence, the correct option is (c).

Example 93. log 6 + log 5 is expressed as

(a) log 11 (b) log 30 (c) log 5/6 (d) None of these

Sol. (b) We know that, log ab = log a + log b

Thus, log 6 + log 5 = log(6 × 5) = log 30

Hence, the correct option is (b).

Example 94. log (1 × 2 × 3) is equal to

(a) log 1 + log 2 + log 3 (b) log 3

(c) log 2 (d) None of these

Sol. (a) We know that, log ab = log a + log b

Thus, log (1 × 2 × 3) = log 1 + log 2 + log 3

Hence, the correct option is (a).

Example 95. If log x + log y = log (x + y), y can be expressed as

(a) x - 1 (b) x (c) $\frac{x}{x-1}$ (d) None of these

Sol. (c) We have, log x + log y = log (x + y) i.e., log xy = log (x + y)

Thus, xy = x + y

$\Rightarrow$ xy − y = x

$\Rightarrow$ y(x − 1) = x

$\Rightarrow y = \frac{x}{x-1}$

Hence, the correct option is (c).

Example 96. Given log 2 = 0.3010 and log 3 = 0.4771, the value of log 6 is

(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) None of these

Sol. (c) We know that,

log ab = log a + log b

Thus, log 6 = log(2 × 3) = log 2 + log 3

= 0.3010 + 0.4771 = 0.7781

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART M)

1. Which of the following statements does not hold?

(a) $\log_{100} 1 = 0$ (b) $\log_5 5 = 1$

(c) log(2 + 3) = log(2 × 3) (d) log (1 × 2 × 3) = log 1 + log 2 + log 3

2. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 60$ is expressed in terms of x and y

(a) x − y + 1 (b) x + y + 1 (c) x − y − 1 (d) None of these

Answer Key

1. (c) **2.** (b)

LAW 2

The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base, i.e., $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

E.g.: $\log_4\left(\frac{27}{5}\right) = \log_4(27) - \log_4(5)$

Example 97. $\log\left(\frac{32}{4}\right)$ is equal to

(a) $\frac{\log 32}{\log 4}$ (b) $\log 32 - \log 4$

(c) 23 (d) None of these

Sol. (b) Using, $\log\left(\frac{a}{b}\right) = \log a - \log b$

Thus, $\log\left(\frac{32}{4}\right) = \log 32 - \log 4$

Hence, the correct option is (b).

Example 98. $\log 20 - \log 5$ is equal to

(a) log 6 (b) log 16 (c) log 4 (d) None of these

Sol. (c) We know that,

$\log\left(\frac{a}{b}\right) = \log a - \log b$

Thus, $\log 20 - \log 5 = \log\left(\frac{20}{5}\right) = \log 4$

Hence, the correct option is (c).

Example 99. The simplified value of $\log_{10} 5 + \log_{10} 8 - \log_{10} 4$ is

(a) $\frac{1}{2}$ (b) 4 (c) 1 (d) None of these

Sol. (c) We know that,

$\log_a mn = \log_a m + \log_a n$

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

Thus, $\log_{10} 5 + \log_{10} 8 - \log_{10} 4 = \log_{10}\left(\frac{5 \times 8}{4}\right)$

$= \log_{10}(10) = 1$

Therefore, $\log_{10} 5 + \log_{10} 8 - \log_{10} 4 = 1$.

Hence, the correct option is (c).

Example 100. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 1.2$ is expressed in terms of x and y as

(a) $x + 2y - 1$ (b) $x + y - 1$ (c) $2x + y - 1$ (d) None of these

Sol. (c) We know that,

$\log_a mn = \log_a m + \log_a n$

Thus, $\log_{10} 1{\cdot}2 = \log_{10}\left(\frac{12}{10}\right) = \log_{10}(12) - \log_{10}(10)$

$= \log_{10}(2 \times 2 \times 3) - \log_{10}(10) = \log_{10}(2^2 \times 3) - 1$

$= \log_{10}(2^2) + \log_{10}(3) - 1 = 2\log_{10}(2) + \log_{10}(3) - 1 = 2x + y - 1$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART N)

1. The value of log 500 – log 5 is

(a) 2 (b) 1 (c) 10 (d) None of the above

2. If $\log_{10}2 = y$ and $\log_{10}3 = x$, then the value of $\log_{10}15$ is **(Dec 2022)**

(a) $x - y + 1$ (b) $x + y + 1$

(c) $x - y - 1$ (d) $y - x + 1$

3. If $\log_3 x = 2$ and $\log_3 y = 3$, what is the value of $\log_3\left(\frac{x}{y}\right)$?

(a) 1 (b) -1 (c) 0 (d) 5

4. Given that $\log x = m + n$ and $\log y = m - n$, the value of $\log \frac{10x}{y^2}$ is expressed in terms of m and n as

(a) $1 - m + 3n$ (b) $m - 1 + 3n$ (c) $m + 3n + 1$ (d) None of these

Answer Key

1. (a) **2.** (a) **3.** (b) **4.** (a)

LAW 3

Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base i.e. $\log_a m^n = n\log_a m$

E.g.: $\log_2 2^4 = 4 \log_2 2$

Example 103. $\log_3 81$ is equal to

(a) 2 (b) 8 (c) 3 (d) None of these

Sol. (d) We have,

$\log_3 81 = \log_3(3^4) = 4 \log_3(3) = 4(1) = 4$

Therefore, $\log_3 81 = 4$

Hence, the correct option is (d).

Example 104. The value of log 0.0001 to the base 0.1 is

(a) −4 (b) 4 (c) $\frac{1}{4}$ (d) None of these

Sol. (b) We have,

log 0.0001 to the base 0.1 which can be written as

$= \log_{0.1}(0.0001) = \log_{0.1}(0.1^4) = 4\log_{0.1}(0.1) = 4(1) = 4$

Therefore, the value of log 0.0001 to the base 0.1 is 4.

Hence, the correct option is (b).

Example 105. $\log\left(\frac{1}{81}\right)$ to the base 9 is equal to

(a) 2 (b) $\frac{1}{2}$ (c) −2 (d) None of these

Sol. (c) $\log\left(\frac{1}{81}\right)$ to the base 9 can be written as

$$= \log_9\left(\frac{1}{81}\right) = \frac{\log\frac{1}{81}}{\log 9} = \frac{\log 9^{-2}}{\log 9} = \frac{-2\log 9}{\log 9} = -2$$

Therefore, $\log\left(\frac{1}{81}\right)$ to the base 9 is equal to −2.

Hence, the correct option is (c).

Example 106. The value of $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$ is equal to

(a) 1 (b) 2 (c) 0 (d) None of these

Sol. (c) We have,

$\log_2[\log_2\{\log_3(\log_3 27^3)\}]$

$= \log_2[\log_2\{\log_3(\log_3 27^3)\}] = \log_2[\log_2\{\log_3(\log_3(3^3)^3)\}]$

$= \log_2[\log_2\{\log_3(\log_3 3^9)\}] = \log_2[\log_2\{\log_3(9\log_3 3)\}] = \log_2[\log_2\{\log_3 9\}]$

$= \log_2[\log_2\{\log_3 3^2\}] = \log_2[\log_2\{2\log_3 3\}] = \log_2[\log_2 2] = \log_2 1 = 0$

Therefore, the value of $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$ is equal to 0.

Hence, the correct option is (c).

Example 107. $\log_{11}\left(1-\frac{1}{3}\right) + \log_{11}\left(1-\frac{1}{4}\right) + \log_{11}\left(1-\frac{1}{5}\right) + \ldots + \log_{11}\left(1-\frac{1}{242}\right)$

(a) 1 (b) 2 (c) −2 (d) None of these

Sol. (c) We have, $\log_{11}\left(1-\frac{1}{3}\right) + \log_{11}\left(1-\frac{1}{4}\right) + \log_{11}\left(1-\frac{1}{5}\right) + \ldots + \log_{11}\left(1-\frac{1}{242}\right)$

$$\Rightarrow \log_{11}\left(\frac{3-1}{3}\right) + \log_{11}\left(\frac{4-1}{4}\right) + \log_{11}\left(\frac{5-1}{5}\right) + \ldots \log_{11}\left(\frac{242-1}{242}\right)$$

$\Rightarrow \log_{11}\left(\frac{2}{3}\right) + \log_{11}\left(\frac{3}{4}\right) + \log_{11}\left(\frac{4}{5}\right) + \cdots + \log\frac{240}{241} + \log\frac{241}{242}$

$\Rightarrow \log_{11}\left(\frac{2}{242}\right) \Rightarrow \log_{11}\left(\frac{1}{121}\right) \Rightarrow \log_{11}\left(\frac{1}{11^2}\right) \Rightarrow \log_{11}\left(11^{-2}\right) \Rightarrow -2\log_{11} 11 \Rightarrow -2(1) \Rightarrow -2$

Hence, the correct option is (c).

Example: 108. The value of $\log(1^3 + 2^3 + 3^3 + \ldots + n^3)$ is equal to

(a) $3\log 1 + 3\log 2 + \ldots + 3\log n$

(b) $2\log n + 2\log(n+1) - 2\log 2$

(c) $\log n + \log(n+1) + \log(2n+1) - \log 6$

(d) 1

Sol. (b) We know, Sum of cube of 'n' natural numbers = $\left(\frac{n(n+1)}{2}\right)^2$

Thus, $\log(1^3 + 2^3 + \ldots + n^3)$

$= \log\left(\frac{n(n+1)}{2}\right)^2 = 2\log\left(\frac{(n)(n+1)}{2}\right)$

$= 2(\log n + \log(n+1) - \log 2 = 2\log n + 2\log(n+1) - 2\log 2$

PRACTICE QUESTIONS (PART 0)

1. If $\log_{10000} x = \frac{-1}{4}$, then x is given by:

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$ (c) $\frac{1}{20}$ (d) None of these

2. If $2\log x = 4\log 3$, then x is equal to

(a) 9 (b) 27 (c) 81 (d) None of these

3. The value of $\log_2 16$ is

(a) 4 (b) 8 (c) 16 (d) None of these

4. The value of $\log_{60} 3 + \log_{60} 4 + \log_{60} 5$ is

(a) $\log_{60} 12$ (b) $\log_{60} 30$ (c) 1 (d) 0

5. The value of $\log_5\left(\frac{5}{6}\right) + \log_5\left(\frac{6}{7}\right) + \log_5\left(\frac{7}{8}\right) + \log_5\left(\frac{8}{5}\right)$ is

(a) 1 (b) 0 (c) -1 (d) -2

6. Solve: $\frac{1}{2}\log_{10} 4 - 2\log_{10} 3 + \log_{10} 18$

(a) 2 (b) $\frac{1}{2}$ (c) -2 (d) None of these

7. The simplified value of $2\log_{10}5 + \log_{10}8 - \frac{1}{2}\log_{10}4$ is

(a) $\frac{1}{2}$ (b) 4 (c) 2 (d) None of these

8. $\log[1 - \{1-(1-x^2)^{-1}\}^{-1}]^{-1/2}$ can be written as

(a) $\log x^2$ (b) $\log x$ (c) $\log\frac{1}{x}$ (d) None of these

Answer Key

1. (b) **2.** (a) **3.** (a) **4.** (c) **5.** (b) **6.** (d) **7.** (c) **8.** (b)

CHANGE OF BASE

$$\log_b m = \frac{\log_a m}{\log_a b}$$

E.g.: $\log_3 27 = \frac{\log_{10}27}{\log_{10}3} = \frac{\log_{10}3^3}{\log_{10}3} = \frac{3\log_{10}3}{\log_{10}3} = 3$

Example 108. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$, these x is equal to

(a) 8 (b) 4 (c) 16 (d) None of these

Sol. (a) Given: $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\Rightarrow \log_2 x + \log_{2^2} x + \log_{2^4} x = \frac{21}{4}$

$\Rightarrow \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4} \quad \left(\because \log_{a^n} x = \frac{1}{n}\log_a x\right)$

$\Rightarrow \log_2 x \times \left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4}$

$\Rightarrow \log_2 x \times \frac{7}{4} = \frac{21}{4}$

$\Rightarrow \log_2 x = \frac{21}{4} \times \frac{4}{7} = 3$

$\Rightarrow \log_2 x = 3$

$\Rightarrow x = 2^3 \quad (\because \log a\ b = c \Rightarrow b = a^c)$

$\Rightarrow x = 8$

Thus, the value of x is 8.

Hence, the correct option is (a).

Example 109. The value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to

(a) 3 (b) 0

(c) 1 (d) None of these

Sol. (c) We have, $(\log_b a \times \log_c b \times \log_a c)^3$

We know that, $\log_a b = \frac{\log b}{\log a}$

Thus, $(\log_b a \times \log_c b \times \log_a c)^3 = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right)^3 = (1)^3 = 1$

Therefore, the value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to 1.

Hence, the correct option is (c).

ANTILOGARITHM

- If x is the logarithm of a given number n with a given base then n is called the antilogarithm (antilog) of x to that base. This can be expressed as follows:
- If $\log_a n = x$ then $n = \text{antilog } x$

To find Antilogarithm:

1. Multiply number by 0.000070274 or divide by 14230
2. Add 1 after step 1
3. Press "multiply & =" button for 15 times

Example 110. Find the antilog of

(i) 2.4523 (ii) -1.0451

Sol. (i) First multiply 2.4523 by 0.000070274 or divide by 14230 and then add 1 to it. Now, press "multiply & =" button for 15 times, thus we get antilog (2.4523) = 283.305

(ii) Type 1.0451 and then '+ -' sign in the calculator to get -1.0451 or type '0 - 1.0451'.

Now multiply it by 0.000070274 or divide by 14230 and then add 1 to it. After that, press "multiply & =" button for 15 times, thus we get antilog (-1.0451) = 0.090114

PRACTICE QUESTIONS (PART P)

1. $x^{a-c} \times x^{b-a} \times x^{c-b}$ is equal to

(a) x (b) 1 (c) 0 (d) None of these

2. $\log_2 16$ is equal to

(a) 2 (b) 8 (c) 4 (d) None of these

3. The value of $\left(\frac{8}{27}\right)^{\frac{-1}{3}} \times \left(\frac{32}{243}\right)^{\frac{-1}{5}}$ is

(a) 9/4 (b) 4/9 (c) 80 (d) 480

4. $\left[(x^n)^{\frac{n^2-1}{n}}\right]^{\frac{1}{n+1}}$ is equal to

(a) x^n (b) x^{n+1} (c) x^{n-1} (d) None of these

5. $\log_{\sqrt{2}} 64$ is equal to

(a) 12 (b) 6 (c) 1 (d) None of these

6. If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$ then the value of x is **[July 2021]**

(a) 64 (b) 4 (c) 16 (d) 2

7. The simplified value of $\log \sqrt[4]{729\sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$ is

(a) log 3 (b) log 2 (c) $\log^{1/2}$ (d) None of these

8. $\log_b\left(a^{\frac{1}{2}}\right) \cdot \log_c\left(b^3\right) \cdot \log_a\left(c^{\frac{2}{3}}\right) = ?$

(a) 2 (b) 0 (c) 1 (d) None of these

Answer Key

1. (b) **2.** (c) **3.** (a) **4.** (c) **5.** (a) **6.** (c) **7.** (a) **8.** (c)

PRACTICE QUESTIONS FOR RATIO AND PROPORTION, INDICES AND LOGARITHM

1. Simplify $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) 1/2 (b) 3/2 (c) 2/3 (d) 1/3

2. The value of $\log_4 9 \cdot \log_3 2$ is

(a) 3 (b) 2 (c) 9 (d) 1

3. If $\log\left(\frac{x-y}{2}\right) = \frac{1}{2}(\log\log x + \log\log y)$, then the value of $x^2 + y^2$ is

(a) 2xy (b) 4xy (c) $2x^2y^2$ (d) 6xy

4. If $\log_3 [\log_4 (\log_2 x)] = 0$, then the value of 'x' will be

(a) 4 (b) 8 (c) 16 (d) 32

5. The ratio of the number of ₹5 coins and ₹10 coins is 8 : 15. If the value of ₹5 coins is ₹360, then the number of ₹10 coins will be

(a) 72 (b) 120 (c) 135 (d) 185

6. There are a total of 23 coins of ₹1, ₹2 and ₹5 in a bag. If their value is ₹43 and the ratio of coins of ₹1 and ₹2 is 3 : 2. Then the number of coins of ₹1 is

(a) 12 (b) 5 (c) 10 (d) 14

7. If log 2 = 0.3010 and log 3 = 0.4771, then the value of log 24 is

(a) 1.0791 (b) 1.7323 (c) 1.3801 (d) 1.8301

8. If $3^x = 5^y = 75^z$, then

(a) $x + y - z = 0$ (b) $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$ (c) $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ (d) $\frac{2}{x} + \frac{1}{z} = \frac{1}{y}$

9. The value of $\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}$ is

(a) 0 (b) 1 (c) 5 (d) 60

10. An alloy is to certain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is

(a) $10\frac{2}{3}$ kg (b) $10\frac{1}{3}$ kg (c) $9\frac{2}{3}$ kg (d) 9 kg

11. A box contains ₹56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coins is double the number of 25 paise coins and four times the number of one rupee coins. The numbers of 50 paise coins in the box is

(a) 64 (b) 32 (c) 16 (d) 14

12. If $\log_{10000} x = \frac{-1}{4}$, then x is given by

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$ (d) $\frac{1}{20}$ (d) None of these

13. Ratio of earning of A and B is 4 :7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What is A's earning?

(a) ₹21000 (b) ₹28000

(c) ₹28000 (d) Data inadequate

14. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 3 : 2, sub duplicate ratio 25 : 81 and sub–triplicate ratio of 1000 : 27 is

(a) 1 : 4 (b) 3 : 25 (c) 4 : 1 (d) None of these

15. The third proportional between $(a^2 - b^2)$ and $(a + b)^2$ is

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$ (c) $\frac{(a-b)^2}{a+b}$ (d) $\frac{(a+b)^3}{a-b}$

16. If $2^x - 2^{x-1} = 4$, then x^x is equal to

(a) 7 (b) 3 (c) 27 (d) 9

17. log 144 is equal to

(a) 2 log 4 + 2 log 2 (b) 4 log 2 + 2 log 3

(c) 3 log 2 + 4 log 3 (d) 3 log 2 - 4 log 3

18. The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is **[ICAI]**

(a) 15 (b) 5 (c) 1 (d) None of these

19. $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$ equal to **[Dec. 2022]**

(a) 3 (b) 2 (c) 1 (d) 0

20. If $\log_a(\sqrt{3}) = \frac{1}{6}$, find the value of 'a' **[Dec. 2020]**

(a) 81 (b) 9 (c) 27 (d) 3

21. The value of $\frac{1}{1+a^{y-x}} + \frac{1}{1+a^{x-y}}$ is given by **[ICAI]**

(a) −1 (b) 0 (c) 1 (d) None of these

22. $\log(1 + 2 + 3)$ is equal to **[ICAI]**

(a) log1 + log2 + log3 (b) log (1 × 2 × 3)

(c) Both (d) None of these

23. $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$, then find the value of x **[ICAI]**

(a) 6 (b) 2 (c) 5 (d) 9

24. Find the value of $\frac{3t^{-1}}{t^{-\frac{1}{3}}}$ **[ICAI]**

(a) $t^{\frac{3}{3}}$ (b) $t^{\frac{3}{2}}$ (c) $\frac{3}{t^{\frac{1}{3}}}$ (d) $\frac{3}{t^2}$

25. 4, *, 9, $13\frac{1}{2}$ are in proportion. Then * is **[ICAI]**

(a) 6 (b) 8 (c) 9 (d) None of these

Answer Key

1. (b)	2. (d)	3. (d)	4. (c)	5. (c)	6. (a)	7. (c)	8. (c)	9. (b)	10. (a)
11. (a)	12. (b)	13. (d)	14. (c)	15. (d)	16. (c)	17. (b)	18. (a)	19. (b)	20. (c)
21. (c)	22. (c)	23. (a)	24. (a)	25. (a)					

SUMMARY

RATIO

- If a and b are two quantities in the same units, then the fraction $\frac{a}{b}$ is called the ratio of a to b. It is written as a : b.
- The quantities a and b are called the terms of the ratio, a is called the first term or antecedent and b is called the second term or consequent.

- The ratio compounded of the two ratios $a : b$ and $c : d$ is $ac : bd$.
- A ratio compounded of itself is called its duplicate ratio given by $a^2 : b^2$ and the triplicate ratio of $a : b$ is $a^3 : b^3$.

  Similarly, Sub- duplicate ratio of $a : b$ is $a^{1/2} : b^{1/2}$ and the sub-triplicate ratio of $a : b$ is $a^{1/3} : b^{1/3}$.

  Inverse ratio for any ratio $a : b$ is $b : a$
- Continued Ratio is the relation between three or more Quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as $a : b : c$.

PROPORTION

- The quantities a, b, c, d are called terms of the proportion; a, b, c and d are called its first, second, third and fourth terms respectively and $a : b = c : d$.
- First and fourth terms are called extreme terms, whereas Second and third terms are called means (or middle terms).
- If $a : b = c : d$ are in proportion then $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ i.e.

  Product of extremes = Product of means. This is called the cross product rule.
- Three quantities a, b, c of the same units are said to be in continuous proportion, then the middle term b is called the mean proportional between a and c, a is the first proportional and c is the third proportional.
- Few properties of proportions :
  - If $a : b = b : c$ i.e., $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$
  - Invertendo: $p : q = r : s \Rightarrow q : p = s : r$
  - Alternendo: $a : b = c : d \Rightarrow a : c = b : d$
  - Componendo: $a : b = c : d \Rightarrow a + b : b = c + d : d$
  - Dividendo: $a : b = c : d \Rightarrow a - b : b = c - d : d$
  - Componendo & Dividendo: $a : b = c : d \Rightarrow a + b : a - b = c + d : c - d$
  - Addendo: $a : b = c : d = a + c : b + d$
  - Subtrahendo: $a : b = c : d = a - c : b - d$
  - Generalising: If $a : b = c : d = e : f = \ldots$ each of these ratios $= (a - c - e - \ldots) : (b - d - f - \ldots)$
- **Indices:** (If base is same)
  - $a^m \times a^n = a^{m+n}$
  - $a^m / a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $a^\circ = 1$
  - $a^{-m} = \frac{1}{a^m}$ & $\frac{1}{a^{-m}} = a^m$
  - If $a^x = a^y$, then $x = y$
  - If $x^a = y^a$, then $x = y$
  - $\sqrt[n]{a} = a^{1/n}$

- If $a^x = n$, then $x = \log_a n$
- $\log_a a = 1$

LOGARITHMS

- If base of log is e, then it is a natural logarithm and if base is 10 then it is called common logarithm.

  $\log_a mn = \log_a m + \log_a n$

  $\log_a (m/n) = \log_a m - \log_a n$

  $\log_a m^n = n\log_a m$

  $\log_a 1 = 0$
- $\log_b a \times \log_a b = 1$
- $\log_b a = \dfrac{\log a}{\log b}$
- $\log_b a \times \log_c b = \log_c a$
- $\log_b a = 1/\log_a b$
- $a^{\log_a x} = x$ (Inverse logarithm)

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