Exercise 13(A)

Multiple-choice Questions

1. The two major divisions of probability are
   (a) subjective and statistical probability
   (b) clerical and normative probability
   (c) subjective and objective probability
   (d) estimation and elementary probability

2. An experiment whose results depend on chance only is known as
   (a) simple experiment
   (b) random experiment
   (c) composite experiment
   (d) compound experiment

3. The set of all possible outcomes of a random experiment is known as
   (a) sample space
   (b) deterministic experiment
   (c) random experiment
   (d) None

4. An event which has only one outcome is
   (a) elementary event (b) compound event
   (c) sure event (d) null event

5. An event that can be decomposed into two or more events is known as

(a) elementary event (b) compound event
(c) sure event (d) null event

6. If A is an event, then non occurrence of it is known as
(a) elementary of event A
(b) complement of event A
(c) intersection of event A
(d) None

7. Tossing of a coin is an example for
(a) mutually inclusive events
(b) mutually exclusive events
(c) Both (a) and (b)
(d) None of the above

8. If A and B are two events and if $(A \cap B) = j$, then the two events are
(a) mutually inclusive events
(b) equally likely events
(c) mutually exclusive events
(d) None of the above

9. The probability of occurrence of an event lies between
(a) 1 and –1 (b) 0 and –1
(c) 0 and 1 (d) None of these

10. If $P(A) = 0$, then the event A is known as
(a) mutually exclusive event
(b) sure event
(c) impossible event
(d) possible event

11. If a coin is tossed thrice, then the events– 'occurrence of one head' and 'occurrence of no head' are
(a) independent (b) exhaustive
(c) equally likely (d) not equally likely

12. If A denotes that a student is a topper and B denotes that he sings well, then
(a) $P(A \cap B) = 1$
(b) $P(A \cup B) = 1$
(c) $P(A \cap B) = 0$
(d) $P(A) \doteq P(B)$

13. If A and B are mutually exclusive events, then
(a) $P(B) = P(A - B)$
(b) $P(A) = P(A - B)$
(c) $P(A) = P(A \cap B)$
(d) $P(B) = P(A \cap B)$

14. The limitations of the classical definition of probability is
(a) applicable when the total number of elementary events is finite
(b) applicable if the elementary events are equally likely
(c) applicable if the elementary events are mutually independent
(d) Both (a) and (b)

15. According to the statistical definition of probability, the probability of an event A is the
(a) limiting value of the ratio of the number of times the event A occurs to the number of times the experiment is repeated
(b) the ratio of the frequency of the occurrences of A to the total frequency
(c) the ratio of the frequency of the occurrences of A to the non-occurrence of A
(d) the ratio of the favourable elementary events to A to the total number of elementary events

16. Which of the following pairs of events are mutually exclusive?
(a) A: The student reads in a school B: He studies Philosopy
(b) A: Raju was in India B: He is a fine Engineer
(c) A: Ruma is 16 years old B: She is a good singer
(d) A: Peter is under 15 years of age B: Peter is a voter of Kolkata

17. If $P(A) = P(B)$, then
    (a) A and B are the same events
    (b) A and B must be same events
    (c) A and B may be different events
    (d) A and B are mutually exclusive events

18. If a coin is tossed twice, then the events 'occurrence of one head', 'occurrence of two heads' and 'occurrence of no head' are
    (a) independent (b) equally likely
    (c) not equally likely (d) Both (a) and (b)

19. Let a sample space be $S = \{x_1, x_2, x_3\}$; which of the following defines probability space on S?
    (a) $P(X_1) = 1/4, P(X_2) = 1/3, P(X_3) = 1/3$
    (b) $P(X_1) = 2/3, P(X_2) = 1/3, P(X_3) = 2/3$
    (c) $P(X_1) = 0, P(X_2) = 1/3, P(X_3) = 2/3$
    (d) None of these

20. If two events A and B are mutually exclusive and collectively exhaustive then
    (a) $P(AB) = 0$ and $P(A) + P(B) = 1$
    (b) $P(AB) = 1$ and $P(A) + P(B) = 1$
    (c) $P(AB) = 1$ and $P(A) + P(B) = 0$
    (d) $P(AB) > 0$ and $P(A) + P(B) = 1$

21. Two events A and B are exhaustive, if
    (a) $P(A \cap B) = 1$
    (b) $P(A \cup B) = 1$
    (c) $P(A \cup B) = P(A \cap B)$
    (d) None

22. The probability of occurrence of two events A and B simultaneously is known as
    (a) Conditional probability
    (b) Marginal probability
    (c) Total probability
    (d) Compound probability

23. If A and B are two events and the probability of happening of B when it is known that A has already happened is known as
    (a) the conditional probability of B given A
    (b) the conditional probability of A given B
    (c) Both (a) and (b)
    (d) None of the above

24. If two events are independent, then which of the following is true?
    (a) A and B' are also independent
    (b) A' and B are also independent
    (c) A' and B' are also independent
    (d) All of the above

25. Which of the following statements is true?
    (a) $P(A \cap B) = P(A) \times P(B)$
    (b) $P(A \cap B) = P(A) \times P(B/A)$
    (c) $P(A \cap B) = P(B) \times P(A/B)$
    (d) All of the above

26. For any three events A, B and C, the probability that they occur jointly is given by
    (a) $P(A \cap B \cap C) = P(A) \times P(B) \times P(C/A)$
    (b) $P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/A \cap B)$
    (c) $P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/A)$
    (d) $P(A \cap B \cap C) = P(A/B) \times P(B/A) \times P(C/A \cap B)$

27. For two events A and B
    (a) $P(A - B) = P(A) - P(B)$
    (b) $P(A - B) = P(A \cap B) + P(A \cup B)$
    (c) $P(A - B) = P(A) - P(A \cap B)$
    (d) $P(A - B) = P(B) + P(A \cap B)$

28. If one event is unaffected by the outcome of another event, the two events are said to be
    (a) dependent (b) independent
    (c) mutually exclusive (d) All of the above

29. In Venn diagram, if events A and B do not overlap on each other, then events A and B are
    (a) mutually exclusive
    (b) not mutually exclusive
    (c) independent
    (d) dependent

30. For any two events A and B
    (a) $P(A - B) = P(B) - P(A \cap B)$
    (b) $P(A - B) = P(A) - P(A \cap B)$
    (c) $P(A - B) = P(A) - P(B)$
    (d) $P(B - A) = P(B) + P(A \cap B)$

Exercise 13(B)

Multiple-choice Questions

1. If $p : q$ are the odds in favour of an event then the probability the event is

(a) $\frac{p}{q}$ (b) $\frac{p}{p+q}$

(c) $\frac{q}{p+q}$ (d) None of these

2. The probability that a number selected at random from the set of numbers $\{1, 2, 3, \ldots, 25\}$ is a perfect cube is

(a) 1/25 (b) 2/25

(c) 3/25 (d) None of these

3. A single letter is selected at random from the word PROBABILITY; the probability that is a vowel is

(a) $\frac{3}{11}$ (b) $\frac{4}{11}$

(c) $\frac{2}{11}$ (d) $\frac{1}{11}$

4. Two dice are thrown simultaneously. The probability that the sum of the two numbers on the dice is a prime number is

(a) 7/12 (b) 5/12

(c) 1/2 (d) 1

5. A five digit number is formed by the digits 1, 2, 3, 4, 5, 6 and 8. The probability that the number has even digit at both ends is

(a) 2/7 (b) 3/7

(c) 4/7 (d) None of these

6. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is

(a) 0.30 (b) 0.70

(c) 0.25 (d) 0.20

7. A, B, C are three mutually independent with probabilities 0.3, 0.2 and 0.4 respectively. What is $P(A \cap B \cap C)$?

(a) 0.40 (b) 0.240

(c) 0.024 (d) 0.500

8. If two letters are taken at random from the word HOME, what is the probability that none of the letters would be vowels?

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

9. If A, B and C are mutually exclusive independent and exhaustive events then what is the probability that they occur simultaneously?

(a) 1

(b) 0.50

(c) 0

(d) Any value between 0 and 1

10. It is given that a family of 2 children has a girl, what is the probability that the other child is also a girl?

(a) 0.58 (b) 0.75

(c) 1/3 (d) 2/3

11. Two coins are tossed simultaneously. What is the probability that the second coin would show a tail given that the first coin has shown a head?

(a) 0.50 (b) 0.25

(c) 0.75 (d) 0.12

12. Four digits 1, 2, 4 and 6 are selected at random to form a four digit number. What is the probability that the number so formed, would be divisible by 4?

(a) 1/2 (b) 1/5

(c) 1/6 (d) 1/3

13. The following table gives distribution of wages of 100 workers:

Wages (in ₹)	120–140	140–160	160–180	180–200	200–220	220–240	240–260
No. of Workers	9	20	0	10	8	35	18

The probability that his wages are under ₹140 is

(a) 20/100 (b) 9/100

(c) 29/100 (d) None

14. If a non biased coin is tossed twice, the probability of obtaining at least one tail is

(a) 0.25 (b) 0.50

(c) 0.75 (d) 1.0

15. What is the chance of getting the sum at least 7 in a single cast with 2 dice?

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$

(c) $\frac{1}{4}$ (d) $\frac{17}{3}$

16. If two unbiased dice are rolled together, what is the probability of getting no difference of points?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{5}$ (d) $\frac{1}{6}$

17. If an unbiased coin is tossed three times, what is the probability of getting more than one head?

(a) 1/8 (b) 3/8

(c) 1/2 (d) 1/3

18. If two unbiased dice are rolled, what is the probability of getting sum of points neither 6 nor 9?

(a) 0.25 (b) 0.50

(c) 0.75 (d) 0.80

19. A bag contains 10 white and 10 black balls. A ball is drawn from it. The probability that it will be white is

(a) 1/10 (b) 1

(c) 1/2 (d) None

20. Three coins are tossed together. The probability of getting at least two heads is

(a) 1/2 (b) 3/8

(c) 1/8 (d) None

sum of ... finds the probability of both the events occurring

simultaneously.

i.e., $P(A_1 \cup A_2) = P(A_1)$ $+ P(A_2) - P(A_1 \cap A_2)$.

Exercise 14(C)

Multiple-choice Questions

1. A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?

   (a) 1/3 (b) 1/5

   (c) 2/3 (d) None

2. When a card is drawn at random from a pack, find the probability that it may be either king or queen.

   (a) 11/13 (b) 1/13

   (c) 1/26 (d) 2/13

3. If a card is drawn at random from a pack of 52 cards, what is the chance of getting a Spade or an ace?

   (a) $\frac{4}{13}$ (b) $\frac{5}{13}$

   (c) 0.25 (d) 0.20

4. One counter is drawn at random from a bag containing 70 counters marked with the first 70 numerals. Find the chance that it is a multiple of 2 or 4.

   (a) 44/70 (b) 24/70

   (c) 19/70 (d) 28/70

5. From a set of 18 balls marked 1, 2, 3, ... 17, 18, one ball is drawn at random. What is the probability that its number is either a multiple of 3 or of 4?

   (a) 0.5 (b) 0.67

   (c) 0.33 (d) None of these

6. Addition Theorem of Probability states that for any two events A and B,

   (a) $P(A \cup B) = P(A) + P(B)$

   (b) $P(A \cup B) = P(A) + P(B) + P(AB)$

   (c) $P(A \cup B) = P(A) + P(B) - P(AB)$

   (d) $P(A \cup B) = P(A) \cdot P(B)$

7. When two dice are thrown the probability of getting 10 or 11 is

   (a) 7/36 (b) 5/36

(c) 5/18 (d) 7/16

8. A traffic census A and B are independent and $P(A) = 2/3$, $P(B) = 3/5$ then $P(A + B)$ is equal to

(a) 13/15 (b) 6/15

(c) 1/15 (d) None of these

9. There are 10 balls numbered from 1 to 10 in a box. If one of them is selected at random, what is the probability that the number printed on the ball would be an odd number greater than 4 or multiple of 5?

(a) 0.40 (b) 0.50

(c) 0.60 (d) None

10. A bag contains 5 tennis balls and 3 cricket balls. Three balls are randomly selected from the bag. Find the probability at least one of the balls drawn is a cricket ball.

(a) 46/56 (b) 10/56

(c) 23/56 (d) 40/56

11. A bag contains 3 red, 4 green and 3 yellow marbles. Three marbles are randomly drawn from the bag. What is the probability that they are of different colours (one of each colour)?

(a) 3/10 (b) 2/10

(c) 4/10 (d) 6/10

12. Following are the wages of 12 workers in rupees:

50, 62, 40, 70, 45, 56, 32, 43, 50, 42, 60, 32

If one of the workers is selected at random, what is the probability that his wage would be lower than the median wage or the wage selected be an odd number?

(a) 6/12 (b) 7/12

(c) 3/12 (d) 4/12

13. If $P(A) = 1/4$, $P(B) = 1/2$, $P(A \cup B) = 5/8$, then $P(A \cap B)$ is

(a) 3/8 (b) 1/8

(c) 2/8 (d) 5/8

14. A number is selected from the first 1000 natural numbers. What is the probability that the number selected would be a multiple of 7 or 11?

(a) 0.25 (b) 0.22

(c) 32/100 (d) 33/1000

15. Suppose 70% of the tourists who come to India will visit Agra while 60% will visit Goa and 50% will visit both Agra and Goa. What is the probability that the tourist will visit either Goa or Agra?

(a) 0.2 (b) 0.8

(c) 0.95 (d) 0.1

16. Two fair dice are rolled. Find the probability that the sum of the number obtained is 5 or 7.

(a) 0.6 (b) 0.28

(c) –0.28 (d) –0.6

17. A card is drawn at random from a pack. If it is known that the card drawn is red, what is the probability that it is a diamond?

(a) 0.2 (b) 0.3

(c) 0.4 (d) 0.5

18. Two fair dice are rolled. If the sum of the numbers obtained is 6, find the probability that the numbers obtained on both dice are odd.

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$

(c) $\frac{5}{36}$ (d) $\frac{3}{36}$

19. A shooter can hit a target in three out of four trials and another shooter can hit the target in two out of three trials. Find the probability that a target is being hit when both of them try?

(a) 0.685 (b) 0.72

Exercise 13(E)

Multiple-choice Questions

1. Expected value of a random variable
   (a) is always positive
   (b) may be positive or negative
   (c) may be positive or negative or zero
   (d) can never be zero

2. Two fair coins are tossed once. Find the mathematical expectation of numbers of heads obtained.
   (a) 0.5 (b) 0.75
   (c) 0.25 (d) 1

3. Find the variance of the number of points obtained in a throw of a fair dice.
   (a) 2.945 (b) 2.467
   (c) 2.93 (d) 2.92

4. If $E(x)^2 = 90$ and $E(x) = 3$, find the standard deviation of x.
   (a) 3 (b) 81
   (c) 27 (d) 9

5. A random variable x assumes values 1 and 0 with respective probabilities p and $q = 1 - p$. Find the variance.
   (a) p/q (b) pq

(c) $\sqrt{pq}$ (d) $pq/2$

6. A bag contains 4 green and 3 red balls. A man draws 2 balls at random from the bag. If he is to receive ₹15 for every green ball he draws and ₹10 for every red ball, what is his expectation?

(a) 32 (b) 21.5

(c) 25.7 (d) 33.2

7. A person throws a fair coin and gets ₹10 if head appears. Otherwise, he loses ₹5. Find his variance.

(a) 58.25 (b) 56.25

(c) 57.25 (d) 59.25

8. A discrete random variable x assumes all integer values between 1 to n with equal probability. Then the respective value of mean and variance of x is

(a) $\frac{n(n+1)}{2}$ and $\frac{(n-1)(n+1)}{6}$

(b) $\frac{n(n+1)}{2}$ and $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{(n+1)}{2}$ and $\frac{(n^2-1)}{12}$

(d) None

9. A bag has 3 one rupee coins and 4 ten paise coins. A boy picks a coin at random from the bag. What is the expectation of the amount he has picked?

(a) 49.57 (b) 48.57

(c) 59.57 (d) 63.57

10. There are 6 tickets which are numbered 1, 2, 3, 4, 5, 6. Two tickets are drawn randomly. Find the expectation of the sum of the 2 numbers drawn.

(a) 21/6 (b) 42

(c) 7 (d) 21

11. If x and y are two independent random variables and if $E(x) = 3$ and $E(y) = 6$, then $E(xy)$ is

(a) 3 (b) 6

(c) 18 (d) 24

12. Sheehan draws 2 balls from a bag containing 3 white and 5 red balls. She gets ₹500 if she draws a white ball and ₹200 if she draws a red ball. What is the expectation if she is asked to pay ₹400 for participating in the game?

(a) 205 (b) 215

(c) 225 (d) 235

13. For the following probability distribution. Find the variance of x.

X:	−1	0	1	2
P(x):	1/5	1/10	2/10	3/5

(a) 1.29 (b) 1.36

(c) 1.73 (d) 1.42

14. The probability that there is atleast one error in an account statement prepared by A is 0.3, B is 0.2 and C is 0.4. A, B and C prepared 10, 15 and 20 statements respectively. Find the expected number of correct statements in all.

(a) 20.5 (b) 31

(c) 18.6 (d) 35

15. If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50, 0.20, then its expected value is

(a) 1.50 (b) 3

(c) 0.90 (d) 1

16. A random variable x has the following probability distribution:

X:	4	5	6	8
Probability:	0.1	0.3	0.4	0.2

Find the expectation of the random variable x.

(a) 4.58 (b) 5.01

(c) 5.9 (d) 6.4

17. The probability distribution of a random variable x is given below

X:	1	2	4	5	6
P(x):	0.15	0.25	0.20	0.30	0.10

What is the standard deviation of x?

(a) 1.49 (b) 1.69

(c) 0.29 (d) 0.80

18. In a business venture, a man can make a profit of ₹2000 with a probability of 0.4 or a have a loss of ₹1000 with a probability of 0.6. What is his expected profit?

(a) Profit of ₹100 (b) Loss of ₹100

(c) Loss of ₹200 (d) Profit of ₹200

19. The monthly demand for radio is known to have the following probability distribution:

Demand:	1	2	3	4	5	6
Probability:	0.10	0.15	0.20	0.25	0.20	0.10

What is the variance of the distribution of demand for radios?

(a) 2.41 (b) 2.59

(c) –3.39 (d) 2.14

20. A person tosses a coin once and is to receive ₹4 for head and is to loss ₹2 for tail. Find the expectation and variance of his gain?

(a) ₹1.00, ₹8.00 (b) ₹1.00, ₹9.00

(c) ₹2.00, ₹9.00 (d) ₹2.00, ₹800

Difficult Problems

Answer Key

Exercise 13(A)

1. (c)	2. (b)	3. (a)	4. (c)	5. (b)	6. (b)	7. (b)	8. (c)	9. (c)
10. (c)	11. (d)	12. (c)	13. (b)	14. (d)	15. (a)	16. (d)	17. (c)	18. (c)
19. (c)	20. (a)	21. (b)	22. (d)	23. (a)	24. (d)	25. (d)	26. (b)	27. (c)
28. (b)	29. (a)	30. (b)						

Exercise 13(B)

1. (b)	2. (b)	3. (b)	4. (b)	5. (a)	6. (b)	7. (b)	8. (a)	9. (c)
10. (c)	11. (a)	12. (d)	13. (b)	14. (c)	15. (b)	16. (d)	17. (c)	18. (a)
19. (c)	20. (a)							

Exercise 13(C)

1. (a) 2. (d) 3. (a) 4. (a) 5. (a) 6. (c) 7. (b) 8. (a) 9. (a)
10. (a) 11. (a) 12. (b) 13. (b) 14. (b) 15. (b) 16. (b) 17. (d) 18. (b)
19. (d) 20. (d)

E

(d) 3. (b) 4. (a) 7. (a) 18. (a)
19. (b) 20. (a)

Exercise 13(E)

1. (c) 2. (d) 3. (d) 4. (d) 5. (b) 6. (c) 7. (b) 8. (c) 9. (b)
10. (c) 11. (c) 12. (c) 13. (b) 14. (b) 15. (c) 16. (c) 17. (b) 18. (d)
19. (a) 20. (b)

Exercise 13(A)

1. The two major divisions of probability are subjective and objective probability.
2. An experiment whose results depend on chance only is known as random experiment.
3. The set of all possible outcomes of a random experiment is known as sample space.
4. An event which has only one outcome is sure event.
5. An event that can be decomposed into two or more events is known as compound event.
6. If A is an event, then non occurrence of it is known as complement of event A.
7. Tossing of a coin is an example for mutually exclusive events.
8. If A and B are two events and if $(AB) =$ (AB)= then the two events are mutually exclusive events.
9. The probability of occurrence of an event lies between 0 and 1.
10. If $P(A) = 0$, then the event A is known as impossible event.
11. If a coin is tossed thrice, then the events—'occurrence of one head' and occurrence of no head' are not equally likely.
12. If A denotes that a student is a topper and B denotes that he sings well, then $P(A \cap B) = 0$.
13. If A and B are mutually exclusive events, then $P(A) = P(A - B)$.
14. The limitations of the classical definition of probability is applicable when the total number of elementary events is finite and

it is applicable if the elementary events are equally likely.

15. According to the statistical definition of probability, the probability of an event A is the limiting value of the ratio of the number. Of times the event A occurs to the number of times the experiment is repeated.

16. A: Peter is under 15 years of age B: Peter is a voter of Kolkata is a pair of event which are mutually exclusive.

17. If $P(A) = P(B)$, then A and B may be different events.

18. If a coin is tossed twice, then the events 'occurrence of one head' , 'occurrence of 2 heads' and 'occurrence of no head' are not equally likely.

19. $P(X_1) = 0$, $P(X_2) = 1/3$, $P(X_3) = 2/3$ defines probability space on S.

20. If two events A and B are mutually exclusive and collectively exhaustive then $P(AB) = 0$ and $P(A) + P(B) = 1$.

21. Two events A and B are exhaustive, if $P(A \cup B)$.

22. The probability of occurrence of two events A and B simultaneously is known as compound probability.

23. If A and B are two events and the probability of happening of B when it is known that A has already happened is known as the conditional probability of B given A.

24. A and B' are also independent A' and B are also independent A' and B' are also independent all of the above are true.

25. $P(A \cap B) = P(A) \times P(B)$,
$P(A \cap B) = P(A) \times P(B/A)$,
$P(A \cap B) = P(B) \times P(A/B)$
all of the above are true.

26. For any three events A, B and C, the probability that they occur jointly is given by
$P(A \cap B \cap C)$
$= P(A) \times P(B/A) \times P(C/A \cap B)$

27. For two events A and B
$P(A - B) = P(A) - P(A \cap B)$

28. If one event is unaffected by the outcome of another event, the two events are said to be independent.

29. In Venn diagram, if events A and B do not overlap on each other, then events A and B are mutually exclusive.

30. For two events A and B
$P(A - B) = P(A) - P(A \cap B)$

Exercise 13(B)

1. Occurrence of one head can happen in three ways: HTT, THT, and TTH.

   Probability of each of the three outcomes is 1/8. Hence, probability of getting one head will be $3(1/8) = 3/8$.

   No head can occur in only one way: TTT.

   The probability is 1/8. Since the probability ties are different, the events are not equally likely.

2. 1 and 8 are perfect cubes between 1 and 25; thus, 2/25

3. Total number of letters = 11

   Number of vowels $(O, A, I, I) = 4$

   Required probability = 4/11

4. $E = \{(1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6), (6, 5)\}$

   (getting numbers whose sums are 2, 3, 5, 7, 11)

   Hence, $n(E) = 15. n(s) = 6 \times 6$

   $P(E) = 15/6 \times 6 = 5/12$

5. We can arrange 5 digits out of 7 digit in ${}^7p_5 = 2520$ ways. There are 4 even numbers (2, 4, 6, 8). Two can be arranged at the end in ${}^4p_2 = 12$ ways. Remaining three places of five digit number can be filled with remaining 5 digits in ${}^5p_3 = 60$ ways.
Required probability
$$= \frac{12 \times 60}{2520} = \frac{2}{7}$$

6. $15 \rightarrow$ ₹1
$25 \rightarrow$ ₹2
$10 \rightarrow$ ₹5
(₹1 coin not selected)
$$P(A) = \frac{n(a)}{n(s)} = \frac{35}{50} = \frac{7}{10} = 0.7$$

7. $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$
$= 0.3 \times 0.2 \times 0.4 = 0.024$

8. $$P(A) = \frac{n(A)}{n(s)} = \frac{{}^2c_2}{{}^4c_2} = \frac{1}{6}$$

9. A, B, C are mutually exclusive
$P(A \cap B \cap C) = P() = 0$

10. In a two child family, there are four possible combination of children then
$S = \{BB, BG, GB, GG\}$
We know that one child is a girl, then
$A = \{BG, GB, GG\}$
$P(A) = n(A)/n(s) = 3/4$
Let B be a event that the family has other child is also girl then
$B = \{GG\}$
$P(B) = n(B)/n(s) = 1/4$
$A \cap B = \{GG\}$
$P(A \cap B) = n(A \cap B)/n(s) = 1/4$
Required probability $= P(B/A)$
$= P(A \cap B)/P(A) = (1/4)/(3/4) = 1/3$

11. $P(B/A) = (1/4)/(2/4) = 1/2 = 0.50$

12. Total number of four digit number can be formed $= 4! = 24$
A four digit number would be divisible by 4 if the number formed by the last two digits is divisible by 4.
This could happen when the four digit number ends with 12, 16, 24, 64.
The number of four digit number that are divisible by 4 is $2! \times 4 = 8$
Required probability $= 8/24 = 1/3$

13. $P(\text{wage} < 140) = 9/100$

14. $S = \{HH, HT, TH, TT\}$
Let A be the event of getting atleast one tail
$A = \{HT, TH, TT\}$
$P(A) = n(A)/n(s) = 3/4 = 0.75$

15. $P(\Sigma \text{ at least } 7)$
$= P(\Sigma = 7) + P(\Sigma = 8) + P(\Sigma = 9)$
$+ P(\Sigma = 10) + P(\Sigma = 11) + P(\Sigma = 12)$
$= 7/12$

16. When two unbiased dice are rolled.
Exhaustive no. of cases $= 6^2 = 36$
Let A be the event of getting no. of difference of points on the two dice. Favourable cases to event A are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)
No. of favourable cases = 6
Required Probability $= P(A) = 6/36 = 1/6$

17. When an unbiased coin is tossed three times.
Exhaustive no. of cases $= 2^3 = 8$
Let A be the event of getting more than one head. Favourable cases to the event A are HHT, HTH, THH, HHH
Number of favourable cases = 4
Required probability $= P(A) = 4/8 = 1/2$

18. When an unbiased coin is tossed three times.
Exhaustive number of cases $= 2^3 = 8$
Let A be the event of getting more than one head.
Favourable cases to the event A are HHT, HTH, THH, HHH
Number of favourable cases = 4
Required probability $= P(A) = \frac{4}{8} = \frac{1}{2}$.

19. $\begin{pmatrix} 10W \\ 10B \end{pmatrix} P(\text{white}) = P(A) = \dfrac{n(A)}{n(s)}$

$= \dfrac{^{10}c_1}{^{20}c_1} = \dfrac{10}{20} = \dfrac{1}{2}$

20. $P(A) = \dfrac{n(A)}{n(s)} = \dfrac{4}{8} = \dfrac{1}{2}$

Exercise 13(C)

1. $n(s) = 12$

$n(A) = 2$

$n(B) = 2$

$n(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{n(A)}{n(s)} + \dfrac{n(B)}{n(s)} - \dfrac{n(A \cap B)}{n(s)}$

$= \dfrac{2}{12} + \dfrac{2}{12} - \dfrac{0}{12}$

$= \dfrac{4}{12}$

$= \dfrac{1}{3}$

2. $n(s) = 52$

$n(A) = 4$

$n(B) = 4$

$n(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{n(A)}{n(s)} + \dfrac{n(B)}{n(s)} - \dfrac{n(A \cap B)}{n(s)}$

$= \dfrac{4}{52} + \dfrac{4}{52} - \dfrac{0}{52}$

$= \dfrac{8}{52}$

$= \dfrac{2}{13}$

3. $n(s) = 52$

$n(A) = 13$

$n(B) = 4$

$n(A \cap B) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{n(A)}{n(s)} + \dfrac{n(B)}{n(s)} - \dfrac{n(A \cap B)}{n(s)}$

$= \dfrac{13}{52} + \dfrac{4}{52} - \dfrac{1}{52} = \dfrac{4}{13}$

4. $n(s) = 70$

$n(A) = 35$

$n(B) = 17$

$n(A \cap B) = 8$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{n(A)}{n(s)} + \dfrac{n(B)}{n(s)} - \dfrac{n(A \cap B)}{n(s)}$

$= \dfrac{35}{70} + \dfrac{17}{70} - \dfrac{8}{70} = \dfrac{44}{70}$

5. $n(s) = 18$

$n(A) = \{3, 6, 9, 12, 15, 18\} = 6$

$n(B) = \{4, 8, 12, 16\} = 4$

$n(A \cap B) = \{12\} = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{n(A)}{n(s)} + \dfrac{n(B)}{n(s)} - \dfrac{n(A \cap B)}{n(s)}$

$= \dfrac{6}{18} + \dfrac{4}{18} - \dfrac{1}{18} = \dfrac{9}{18} = \dfrac{1}{2} = 0.5$

6. Use the formula,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

7. $n(s) = 36$

$n(A) = \{46, 55, 64\} = 3$

$n(B) = \{56, 65\} = 2$

$n(A \cap B) = \{\} = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 3/36 + 2/36 - 0/36$

$= 5/36$

8. $P(A) = \frac{2}{3}$

$P(B) = \frac{3}{5}$

$P(A + B)$

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \times P(B)$

$= \frac{2}{3} + \frac{3}{5} - \frac{2}{5} \times \frac{3}{5}$ {as independent events}

$= \frac{2}{3} + \frac{3}{5} - \frac{6}{15}$

$= \frac{19-6}{15} = \frac{13}{15}$

9. $n(s) = 10$

$n(A) = \{5, 7, 9\} = 3$

$n(B) = \{5, 10\} = 2$

$n(A \cap B) = \{5\} = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 3/10 + 2/10 - 1/10$

$= 4/10 = 0.40$

10. P(atleast one of the balls drawn is a cricket ball) = 1 - p (none of the ball is cricket ball)

$= 1 - {}^5C_3/{}^8C_3 = 46/56.$

11. The marbles should be one of each colour. Therefore, ${}^3c_1 \times {}^4c_1 \times {}^3c_1$ outcomes are favourable. And so,

P(marbles of different colours)

$= \frac{{}^3c_1 \times {}^4c_1 \times {}^3c_1}{{}^{10}c_3} = 3/10$

12. $n(s) = 12$

$n(A) = p$(lower than mean wage){40, 45, 32, 43, 42, 32} = 6

$n(B) = p$(wage selected is odd no.) {45, 43} = 2

$n(A \cap B) = \{45\} = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 6/12 + 2/12 - 1/12$

$= 7/12 = 0.5833$

13. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$1/4 + 1/2 - 5/8 = 1/8$

14. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{142 + 90 - 12}{1000}$

$= \frac{220}{1000} = 0.22$

15. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.6 - 0.5$

$= 0.8$

16. A sum of 5 can be obtained by the following outcomes:

(1, 4), (4, 1), (2, 3), (3, 2)

i.e., 4 outcomes

A sum of 7 can be obtained by the following outcomes:

(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

i.e., 6 outcomes

A sum of 5 or 7 will have 4 + 6 = 10 favourable outcomes.

Total possible outcomes = 36

Thus, required probability = 10/36 = 5/18 = 0.28.

17. Let A be the event that card drawn is red and B be the event that it is a diamond.

There are 52 cards in total, out of which 26 are red and 13 are diamonds.

We have to find the probability of getting a diamond when it is known that the card drawn is red. Thus, it is a case of conditional probability and we have to find $P(B/A)$.

Probability of getting a red card = $P(A)$ = 26/52 = 1/2

Probability of getting a red card, which is a diamond = $P(A \cap B)$ = 13/52 = 1/4

Now, $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1/4}{1/2}$

$= \dfrac{2}{4} = \dfrac{1}{2} = 0.5$

18. Let event A be that the sum of numbers obtained is 6 and event B be that the numbers on both dice are odd.

There are 36 total outcomes and a sum of 6 can occur in the following outcomes:

(1, 5), (5, 1), (2, 4), (4, 2) and (3, 3)

i.e., 5 outcomes

We have to find the probability of getting both numbers odd when it is known that the sum is 6. Thus, it is a case of conditional probability and we have to find $P(B/A)$. Probability of getting a sum of 6 $= P(A) = 5/36$

Probability of getting a sum of 6 with both numbers odd $= P(A \cap B) = 3/36 = 1/2$

Now, $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1/2}{5/36}$

$= \dfrac{36}{5 \times 12} = \dfrac{3}{5}$

19. P(shooter 1 hits) $= 3/4 = 0.75$

P(shooter 2 hits) $= 2/3 = 0.67$

These are independent events. Thus,
$P(1 \cap 2) = 0.75 \times 0.67 = 0.5$.

Probability of hitting the target when both shooters try will be given by.

$P(1 \cup 2) = P(1) + P(2) - P(1 \cap 2)$

$= 0.75 + 0.67 - 0.5 = 0.92$

20. If there are two ladies, there would be five men in the committee.

Required probability $= \dfrac{{}^{8}C_5 \times {}^{5}C_2}{{}^{13}C_7}$

$= \dfrac{56 \times 10}{1716} = \dfrac{140}{429}$

Exercise 13(E)

1. Expected value of a random variable may be positive or negative or zero.

2. Let x denotes the number of heads obtained when two fair coins are tossed
Probability distribution of $r\ v\ x$

$X = x$	0	1	2	Total
$P(X = x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	1

$$E(X) = \sum xp$$
$$= 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4}$$
$$= 1$$

3. Let x_i denote the outcomes on the dice and P_i be the probability of each outcome.

x_i	p_i	$p_i x_i$	X_i^2	$P_i x_i^2$
1	1/6	1/6	1	1/6
2	1/6	2/6	4	4/6
3	1/6	3/6	9	9/6
4	1/6	4/6	16	16/6
5	1/6	5/6	25	25/6
6	1/6	6/6	36	36/6
		$E(x) = \mathbf{21/6}$		$E(x^2) = \mathbf{91/6}$

Variance $= E(x^2) - [E(x)]^2 = 91/6 - [21/6]^2$
$= 15.167 - 12.25 = 2.92$

4. Variance $= E(x^2) - [E(x)]^2 = 90 - [3]^2 = 90 - 9 = 81$

$$SD = \sqrt{var} = \sqrt{81} = 9$$

5. $E(x) = 1 \times p + 0 \times q = p$

$E(x^2) = 1^2 \times p + 0^2 \times q = p$

Variance $= E(x^2) - [E(x)]^2 = p - p^2 = p$

$(1 - p) = pq$

6. There are possible cases of the two balls drawn--- *RR*, *GG*, *GR*

$$P(RR)\frac{{}^{3}C_2}{{}^{7}C_2} = \frac{1}{7}E(RR) = (10+10)\times\frac{1}{7} = \frac{20}{7}$$

$$P(GG)\frac{{}^{4}C_2}{{}^{7}C_2} = \frac{2}{7}E(GG) = (15+15)\times\frac{2}{7} = \frac{60}{7}$$

$$P(RG)\frac{{}^{3}C_1 \times {}^{4}C_1}{{}^{7}C_2} = \frac{4}{7}E(RG)$$

$$= (10+15)\times\frac{4}{7} = \frac{100}{7}$$

Total expectation = (20 + 60 + 10)/7 = 180/7 = 25.7

7. $E(x) = E(H) + E(T) = 10 \times 1/2 + -5 \times 1/2 = 5 - 5/2 = 5/2$

$E(x^2) = 10^2 \times 1/2 + (-5)^2 \times 1/2 = 50 + 25/2 = 125/2$

Variance = $E(x^2) - [E(x)]^2 = 125/2 - 25/4 = 225/4 = 56.25$

8. Numbers between 1 and n have equal probability, so the probability will be $1/n$.

$$E(x) = 1\times\frac{1}{n} + 2\times\frac{1}{n} + 3\times\frac{1}{n}\cdots n\times\frac{1}{n}$$

$$= \frac{1}{n}(1+2+3\cdots+n) = \frac{1}{n}\times\frac{n(n+1)}{2}$$

sum of first n natural numbers $= \frac{n(n+1)}{2}$

$$\Rightarrow \quad E(x) = mean = \frac{n+1}{2}$$

$$E(x^2) = 1^2\times\frac{1}{n} + 2^2\times\frac{1}{n} + 3^2\times\frac{1}{n}\cdots + n^2\times$$

$$= \frac{1}{n}(1^2 + 2^2 + 3^2\cdots + n^2)$$

$$= \frac{1}{n}\times\frac{n(n+1)(2n+1)}{6}$$

sum of squares of first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow \quad E(x^2) = \frac{n(n+1)(2n+1)}{6}$$

Variance $E(x^2) - [E(x)]^2$

$$= \frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$2(2n^2 + 3n + 1) - 3(n^2 + 2n + 1)$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{2(2n^2+3n+1) - 3(n^2+2n+1)}{12}$$

$$= \frac{n^2 - 1}{12}$$

9. P(1rupee coin) = 3/7, P(10 paise coin) = 4/7

$E(x)$ = (100 paise)(3/7) + (10 paise)(4/7)

= 340/7 paise = 48.57 paise

10. There will be a total of ${}^{6}c_2 = 15$ combinations of 2 numbers possible each of which will be equally likely. Thus, probability of each pair of numbers will be 1/15.

Sum (x)	Probability (p)	px	Sum (x)	Probability (p)	Px
1 + 2 = 3	1/15	3/15	2 + 6 = 8	1/15	8/15
1 + 3 = 4	1/15	4/15	3 + 4 = 7	1/15	7/15
1 + 4 = 5	1/15	5/15	3 + 5 = 8	1/15	8/15

1 + 5 = 6	1/15	6/15	3 + 6 = 9	1/15	9/15
1 + 6 = 7	1/15	7/15	4 + 5 = 9	1/15	9/15
2 + 3 = 5	1/15	5/15	4 + 6 = 10	1/15	10/15
2 + 4 = 6	1/15	6/15	5 + 6 = 11	1/15	11/15
2 + 5 = 7	1/15	7/15			

$E(x) = \sum Px = 105/15 = 7$

11. $E(xy) = E(x) \times E(y) = 18$

12. There are three possible cases of the two balls drawn—*RR*, *WW*, *WR*.

Outcome	Amount	Probability
WW	1000 - 400 = 600	$\frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$
RR	400 - 400 = 0	$\frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$
WR	700 - 400 = 300	$\frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28}$

$$\text{Expected amount} = 600 \times \frac{3}{28} + 0 \times \frac{10}{28} + 300 \times \frac{15}{28}$$
$$= 225$$

13.

x	P	Px	x^2	Px^2
–1	1/5	–1/5	1	1/5
0	1/10	0	0	0
1	2/10	2/10	1	2/10
2	3/5	6/5	4	12/5
		$\sum Px = 6/5$		$\sum Px^2 = 14/5$

$V(x) = E(x^2) - (E(x))^2$
$= 14/5 - (6/5)^2$
$= 34/25 = 1.36$

14. P_1 = 1 - (probability of atleast one error)
$P_1 = 1 - 0.3 = 0.7$
$P_2 = 1 - 0.2 = 0.8$
$P_3 = 1 - 0.4 = 0.6$
$E(x) = \sum Px = (0.7 \times 10 + 0.8 \times 15 + 0.6 \times 20)$
$= 7 + 12 + 12 = 31$

15. Expectations

x	P	Px
0	0.3	0.5
1	0.5	0.5
2	0.2	0.4
		$\sum Px = 0.9$

$E(x) = \sum Px = 0.9$

16. $\sum x(Px) = 4 \times 0.1 + 5 \times 0.3 + 6 \times 0.4 + 8 \times 0.2$
$= 5.9$

17.

x	P	Px	x^2	Px^2
1	0.15	0.15	1	0.15
2	0.25	0.5	4	1
4	0.20	0.8	16	3.2
5	0.30	1.5	25	7.5
6	0.10	0.6	36	3.6
		$\sum Px$ = **3.55**		$\sum Px^2$ = **15.45**

$E(x) = \sum Px = 3.55$
$V(x) = \sum Px^2 - (\sum Px)^2 = 15.45 - (3.55)^2$
$= 2.84$
$\sigma_x = \sqrt{2.84} = 1.69$

18. $2000 \times 0.4 - 1000 \times 0.6$
$= 800 - 600 = 200$
i.e., profit is 200

19.

x	P(x)	xP(x)	x^2	$x^2p(x)$
1	0.1	0.1	1	0.1
2	0.15	0.3	4	0.6
3	0.2	0.6	9	1.8
4	0.25	1	16	4
5	0.2	1	25	5
6	0.1	0.6	36	3.6
		$\sum xP(x)$ = **3.6**		$\sum x^2p(x)$ = **15.1**

$V(x) = 15.1 - (3.6)^2$
$= 15.1 - 12.96$

$= 2.14$

20.

x	P(x)
4	0.5
2	0.0

$E(x) = 1$

$V(x) = 9$