INDEX – ERRORLESS MATHEMATICS

Note from CA Pranav Chandak:

- Hello My Dear students, this book is more than sufficient to get good marks in Mathematics. All you need to do is solve this book atleast twice before the exams.
- Exam Day Chart Book is also available on my website www.pranavchandak.com to help you in securing good marks in CA Foundation Mathematics.
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STUDY PLANNER REVISE किया ? ? ?

SN		NAME OF THE CHAPTER	Fill the Date of Completion		
			Self Study (After Class)	1st Revision	2nd Revision
Chapter 1	1A & 1B	**Ratio and Proportion**			
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Chapter 2		**Equations**			
Chapter 3		**Inequalities**			
Chapter 4		**Time Value of Money**			
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	7C	**Functions**			
Chapter 8	8A	**Differential Calculus**			
	8B	**Integral Calculus**			

CHAPTER 1A. RATIO

INTRODUCTION

- A Ratio is a comparison of two or more quantities of the same kind (units) by division.
- If a & b are two quantities of same kind (same units), then a/b is called ratio of a to b.
- It is written as a: b

PC Note: Quantities to be compared must be in same units or capable of being converted in same units.

CQ1: Ratio b/w 150 gm & 2 kg = Ratio b/w 150 gm & 2000 gm = $\frac{150}{2000} = \frac{3}{40}$ = **3:40.**

CQ2: Ratio b/w 25 mins & 45 sec. = Ratio b/w (25×60) sec & 45 sec. = $\frac{1500}{45} = \frac{100}{3}$ = **100:3**.

CONCEPT 1: ANTECEDENT & CONSEQUENT

- 'a' & 'b' are called terms of the ratio.
- 'a' is called first term or antecedent & 'b' is called second term or consequent.

CQ3. Find the ratio b/w 3 kg & 5 kg.

Solution: '3' is the antecedent & '5' is the consequent. Thus, Ratio = $\frac{\text{Antecedent}}{\text{Consequent}} = \frac{3}{5}$.

CQ4. Ratio of two quantities is 3:4. If antecedent is 15, consequent is ____. **[Ans: 20]**

(a) 16 (b) 60 (c) 22 (d) 20

POINTS TO BE NOTED:

- The order of the terms in a ratio is important. [**Ex:** 3:4 is not same as 4:3].
- Ratio must be expressed in lowest form (simplest form). [**Ex:** 12:16 = $\frac{12}{16} = \frac{3\times4}{4\times4} = \frac{3}{4}$ = 3:4]
- If both terms of a ratio are multiplied or divided by any same number (non-zero), ratio remains same.

  CQ5: 3:4 is a ratio. Now if we multiply both 3 & 4 by any non-zero number (say Ex. by 4), we will get a new ratio 12:16, which is same as 3:4.

- If original quantity increases or decreases in the ratio a:b, then

  New Quantity= Original Quantity × $\frac{b}{a}$ [$\frac{b}{a}$ is called Factor Multiplying Ratio]

  CQ6. Mr. PC weighs 56.7 kg. If he reduces his weight in the ratio 7: 6, find his new weight.

  Solution: Original weight of Mr. PC = 56.7 kg; He reduces his weight in the ratio 7:6.

  His new weight = (56.7 × 6)/7 = **48.6 kg.**

CONCEPT 2: INVERSE RATIO **[Ulta Ratio]**

- Inverse Ratio of a: b = b: a and vice- versa.
- Product of the ratio = 1.

CQ7. Ratio of two quantities is 5:7. If Consequent of its Inverse Ratio is 5, Antecedent is:

(a) 5 (b) $\sqrt{5}$ (c) 7 (d) None

CONCEPT 3: DUPLICATE RATIO **[Multiplication of the Ratio with itself]**

- A ratio multiplied by itself is called its duplicate ratio.
- Duplicate ratio of **a: b** = $\frac{a}{b} \times \frac{a}{b} = a^2: b^2$

Ex: (i) Duplicate ratio of 2:3 = **4: 9**;

CONCEPT 4: SUB-DUPLICATE RATIO **[Ulta of Duplicate Ratio]**

- Sub-duplicate ratio of **a: b** = $\sqrt{a}: \sqrt{b}$
- Sub-duplicate ratio of $a^2: b^2$ = **a: b**

Ex: (i) Sub-duplicate ratio of 9:25 = $\sqrt{9}: \sqrt{25}$ = **3: 5**

CONCEPT 5: TRIPLICATE RATIO **[Ratio of Cubes of Terms]**

- Triplicate ratio of **a: b** = $a^3: b^3$

Ex: (i) Triplicate ratio of 2: 3 = **8: 27**

CONCEPT 6: SUB-TRIPLICATE RATIO **[Ulta of Triplicate Ratio]**

- Sub-triplicate ratio of **a: b** = $\sqrt[3]{a}: \sqrt[3]{b}$
- Sub-triplicate ratio of $a^3: b^3$ = **a: b**

Ex: (i) Sub-triplicate ratio of 8:125 = $\sqrt[3]{8} : \sqrt[3]{125}$ = **2: 5.**

CONCEPT 7: COMPOUND RATIO **[Multiplication of Two Ratios]**

Compound ratio of two ratios **a: b & c: d** = $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ = **ac: bd.**

Ex: (i) Compound ratio of 3:4 & 5:7 = **15: 28.**

Ex: (ii) Compound ratio of 2:3, 5:7 & 4:9 = **40: 189.**

CONTINUED RATIO

- Continued Ratio is the relation between three or more quantities of the same kind.
- The continued ratio of three similar quantities a, b, c is written as a: b: c.

CQ8. A: B = 2: 3; B: C = 4: 5; & C: D = 6: 7, then A: B: C: D is____.

(a) 16: 22: 30: 35 (b) 16: 24: 15: 35 (c) 16: 24: 30: 35 (d) 18: 24: 30: 35

CQ9. If A: B = 2: 3, B: C = 4: 5, C: D = 6: 7 the A: D is_____.

(a) 35:16 (b) 16:35 (c) 2:7 (d) None of these.

Space for PC Class Note:

1B. PROPORTION

INTRODUCTION

- Equality of two ratios is called a proportion.
- Four quantities a, b, c, d are said to be in proportion if a: b = c: d (a: b :: c: d).
- The quantities a, b, c, d are called terms of the proportion;
- 1st & 4th terms are called Extremes; 2nd & 3rd terms are called Means (middle terms).
- **Product of Extremes = Product of Means**

  ☞ If $\frac{a}{b} = \frac{c}{d}$ **then ad = bc.** [Cross Product Rule]

  Ex: If $\frac{3}{5} = \frac{6}{10}$ then LHS = 3 × 10 = 30 & RHS = 6 × 5 = 30

PC NOTE: In a ratio a:b, both quantities must be in same unit but in proportion a: b = c: d, all 4 quantities need not be of the same type. First two quantities should be in same unit & last two quantities should be in same unit.

Ex: Rs. 6: Rs. 8 = 12 toffees: 16 toffees are in a proportion since 1st two quantities are in same unit & last two are in same unit.

CONCEPT 1: CONTINUOUS PROPORTION [Same apply for more than 3 quantities]

- Three quantities a, b, c (same units) are in continuous proportion if a: b = b: c.

  'a' → 1st proportional; 'b' → Mean proportional betn a & c; 'c' → 3rd proportional.

- If $\frac{a}{b} = \frac{b}{c}$, then $b^2 = ac$; OR $b = \sqrt{ac}$.

CQ1. Find the value of x if $\frac{10}{3} : x :: \frac{5}{2} : \frac{5}{4}$. [Ans: $\frac{5}{3}$]

CQ2. Find the fourth proportional to $\frac{2}{3}, \frac{3}{7}, 4$. [Ans: $\frac{18}{7}$]

CQ3. Find the third proportion to 2.4 kg, 9.6 kg. [Ans: 38.4 Kgs]

CQ4. Find the mean proportion betn 1.25 & 1.8. [Ans: 1.5]

CONCEPT 2: PROPERTIES OF PROPORTION → If a:b = c:d then

1	**Invertendo**	b: a = d: c	**Ex:** If $\frac{3}{5} = \frac{6}{10}$ then $\frac{5}{3} = \frac{10}{6}$
2	**Alternendo**	a: c = b: d	**Ex:** If $\frac{3}{6} = \frac{5}{10}$ then $\frac{3}{6} = \frac{5}{10}$
3	**Componendo**	$\frac{a+b}{b} = \frac{c+d}{d}$	**Ex:** If $\frac{3}{5} = \frac{6}{10}$ then $\frac{3+5}{5} = \frac{6+10}{10}$ [Check $\frac{8}{5} = \frac{16}{10}$; 8 × 10 = 5 × 16]
4	**Dividendo**	$\frac{a-b}{b} = \frac{c-d}{d}$	**Ex:** If $\frac{5}{3} = \frac{10}{6}$ then $\frac{5-3}{5} = \frac{10-6}{10}$[Check $\frac{2}{5} = \frac{4}{10}$; 2 × 10 = 5 × 4]
5	**Componendo & Dividendo**	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	**Ex:** If $\frac{5}{3} = \frac{10}{6}$ then $\frac{5+3}{5-3} = \frac{10+6}{10-6}$[Check $\frac{8}{2} = \frac{16}{4}$; 8 × 4 = 2 × 16]
6	**Addendo** If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ = ..., each of ratios **(Addendo)** = (a + c + e + ...): (b + d + f + ...) **Ex:** If $\frac{3}{5} = \frac{6}{10} = \frac{12}{20} = \frac{24}{40}$ = ..., then it comes out as $\frac{3+6+12+24......}{5+10+20+40.......}$		
7	**Subtrahendo**	$\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$	**Ex:** If $\frac{3}{5} = \frac{6}{10} = \frac{12}{20} = \frac{24}{40}$ then, $\frac{3}{5} - \frac{12}{20} = \frac{6}{10} - \frac{24}{40}$ → 0

(Only Addendo and Subtrahendo are equal to the Original Ratio)

CQ5. If a: b = c: d = 2.5:1.5, what are the values of (i) ad: bc & (ii) a+c: b+d?

Ans: (i) $\frac{a}{b} = \frac{c}{d} = \frac{5}{3}$; so, ad = bc, thus ad: bc = ad: ad [Substituting ad = bc], Thus ad: bc = 1:1.

(ii) $\frac{a}{b} = \frac{c}{d} = \frac{2.5}{1.5}$; Using the above given principle, we can say that $\frac{a+c}{b+d} = \frac{5}{3}$.

CONCEPT 3: INVERSE PROPORTION

- If 'a' & 'b' are related to each other such that an increase in 'b' results in proportionate decrease in 'a', then 'a' & b are said to be inversely related or in inverse proportion.
- This is expressed as **a** $\propto \frac{1}{b}$. [a is inversely proportional to b]
- When **a** $\propto \frac{1}{b}$, we can write **a** = $\frac{k}{b}$, where k is the constant of probability.

PC Note: Inverse Proportion of **a: b** is **b: a**.

[**Ex:** 5/4 is in inverse proportion of 4/5].

POINTS TO BE NOTED	
▪ If a ∝ b and b ∝ c, then **a ∝ c.**	▪ If a ∝ b, then **ax ∝ bx.**
▪ If a ∝ bc, then **b** $\propto \frac{a}{c}$ and **c** $\propto \frac{a}{b}$.	
CQ6. X varies inversely as y^2. Given that y = 2 for x = 1. Value of x for y = 6 will be ________. (a) 3 (b) 9 (c) 1/9 (d) – 1/9	

Space for PC Class Note:

RATIO & PROPORTION - QUESTION BANK

SN	CHAPTER 1A. RATIO	Ans
Q1	Ratio exists only between quantities of ______ kind. (a) same (b) bigger (c) smaller (d) None	A
Q2	A ratio is a _____. (a) unit (b) term (c) number (d) function	C
Q3	The order of the terms in a ratio is important. (a) True (b) False (c) Partly True (d) None	A
Q4	A ratio is expressed in ______ form. (a) simplest (b) complicated (c) moderate (d) functional	A
Q5	Ratio has no unit. (a) True (b) Partly True (c) False (d) None	A
Q6	If a: b = c: d then _________. (a) ab = cd (b) ac = bd (c) ad = bc (d) ab = ad	C
Q7	$4^{2.5}:2^3$ is same as _________. (a) 4:1 (b) 2:1 (c) 16:1 (d) 80:1	A
Q8	The ratio 3/2: 1/3: 1/8 is same as _________. (a) 36: 3: 8 (b) 3: 8: 36 (c) 36: 8: 3 (d) 3: 36: 8	C
Q9	If A: B = 2: 3, B: C=4: 5, C: D= 6: 7. the A: D is _________. (a) 35:16 (b) 16:35 (c) 2:7 (d) None	B
Q10	If A: B = 2: 3; B: C = 4: 5 and C: D = 6: 7, then A: B: C: D is _________. (a) 16:22:30:35 (b) 16:24:15:35 (c) 16:24:30:35 (d) 18:24:30:35	C
Q11	The inverse ratio of 11:15 is _________. (a) 15:11 (b) √11: √15 (c) 125:225 (d) None	A
Q12	In the ratio 11/3: 13/4, antecedent is _________. (a) 13/4 (b) 11/3 (c) Both (a) & (b) (d) None	B
Q13	The Duplicate Ratio of 3: 4 is _________. (a) √3: 2 (b) 4:3 (c) 9: 16 (d) None	C
Q14	The Sub Duplicate Ratio of 25: 36 is _________. (a) 6:5 (b) 36:25 (c) 50:72 (d) 5:6	D
Q15	If p: q is the Sub Duplicate Ratio of $p - x^2: q - x^2$ then x^2 is _________. (a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p\,q}$ (d) $\frac{pq}{p+q}$	D
Q16	If 2s: 3t is the Duplicate Ratio of 2s - p: 3t - p then _________. (a) $p^2 = 6st$ (b) p = 6st (c) 2p = 3st (d) None	A
Q17	The Triplicate Ratio of 3: 2 is _________.	A

	(a) 27:8 (b) 6:9 (c) 3:2 (d) 8:27	
Q18	The Triplicate Ratio of 4: 5 is _________. (a) 125:64 (b) 16:25 (c) 64:125 (d) 120:46	**C**
Q19	The Sub Triplicate Ratio of 8: 27 is _________. (a) 27:8 (b) 24:81 (c) 2:3 (d) None	**C**
Q20	If (4x+3): (9x+10) is the Triplicate Ratio of 3: 4, then the value of x is _________. (a) 9 (b) 7 (c) 6 (d) 5	**C**
Q21	Ratio compounded of Duplicate Ratio of √5: √6 & Triplicate Ratio of 3: 5 is _________. (a) 4:75 (b) 2:15 (c) 9:50 (d) 3:10	**C**
Q22	The ratio compounded of Duplicate Ratio of 4: 5, Triplicate of 1: 3, Sub Duplicate Ratio of 81: 256 and Sub Triplicate Ratio of 125: 512 (a) 4:512 (b) 3:32 (c) 1:12 (d) 1:120	**D**
Q23	If $5x^2 - 13xy + 6y^2 = 0$, then x: y is ______. (a) (2:1) only (b) (3:5) or (2:1) (c) (5:3) or (1:2) (d) (3:5)	**B**
Q24	If 2A=3B and 4B=5C, then A:C is ________. (a) 4:3 (b) 15:8 (c) 8:15 (d) 3:4	**B**
Q25	P, Q, and R are three cities. Ratio of average temperature between P & Q is 11:12 and that between P and R is 9:8. Ratio between average temperature of Q and R is _______. (a) 22:27 (b) 27:22 (c) 32:33 (d) None	**B**
Q26	A man divides his property so that his son's share to his wife's share and wife's share to his daughter's share are both in the ratio 3:1. If the daughter gets Rs.10,000 less than son, then total worth of his property is_____. (a) Rs. 16,250 (b) Rs. 8,250 (c) Rs. 15,250 (d) Rs.21,250	**A**
Q27	If 40% of a number is equal to 2/3rd of another number, what is the ratio of first number to second number? (a) 2:5 (b) 3:7 (c) 5:3 (d) 7:3	**C**
Q28	Two numbers are respectively 30% & 40% more than a third number. Ratio of the two numbers is_____. (a) 3:4 (b) 14:14 (c) 13:14 (d) 4:3	**C**
Q29	A recipe for 4 servings requires salt and pepper to be added in the ratio of 2:3. If the recipe is adjusted from 4 to 8 servings, what is the ratio of the salt and pepper that must now be added? (a) 4:3 (b) 2:6 (c) 2:3 (d) 3:2	**C**
Q30	The ages of two persons are in the ratio 5:7. 18 years ago their ages were in the ratio of 8:13 their present ages (in years) are _____. (a) 50,70 (b) 70,50 (c) 40,56 (d) None	**A**
Q31	A bag contains Rs.187 in the form of 1 rupee, 50 paise and 10 paise coins in the ratio 3:4:5. Find the number of each type of coins. (a) 102,136,170 (b) 136,102,170 (c) 170,102,136 (d) None	**A**
Q32	Two numbers are in the ratio 2: 3. If 4 be subtracted from each, they are in the ratio 3:5.	**A**

	The numbers are ______. (a) (16,24) (b) (4,6) (c) (2,3) (d) None	
Q33	What quantity must be added to the terms of the ratio p+q : p-q to make it equal to $(p + q)^2 : (p - q)^2$? (a) (q+p) / 2p (b) (q-p) /2p (c) $(q^2 - p^2)$ / 2p (d) None	**C**
Q34	The ratio between the speeds of two trains is 7:8. If 2nd train runs 400 kms in 5 hours, speed of 1st train is ______. (a) 10 km/hr (b) 50 km/hr (c) 71 km/hr (d) 70 km/hr	**D**
Q35	The angles of a triangle are in ratio 2:7:11. The angles are ______. (a) (20°,70°,90°) (b) (30°,70°,80°) (c) (18°,63°,99°) (d) None	**C**
Q36	If A, B and C started a business by investing Rs.1,26,000, Rs.84,000 and Rs.2,10,000. If at the end of the year profit is Rs. 2,42,000 then the share of each is _____. (a) Rs.72,600, Rs.48,400, Rs.1,21,000 (b) Rs.48,400, Rs.1,21,000, Rs.72,600 (c) Rs.72,000, Rs.49,000, Rs.1,21,000 (d) Rs.48,000, Rs.1,21,400, Rs.72,600	**A**
Q37	The ratio of the number of boys to number of girls in a school of 1,200 Students is 7:5. If 20 boys are newly admitted h the school, find how many new girls may be admitted so that the above ratio may change to 4: 3. (a) 40 (b) 140 (c) 60 (d) 58.	**A**
Q38	Ratio of the number of boys to the number of girls in a school of 720 students is 3:5. If 18 new girls are admitted in the school, find how many new boys shall be admitted so that the ratio of the number of boys to the number of girls may change to 2:3. (a) 40 (b) 48 (c) 42 (d) 58.	**C**
Q39	If a packet containing 12 glasses is dropped, ratio of broken glasses to unbroken glasses cannot be ______. (a) 3:1 (b) 6:1 (c) 4:2 (d) 5:7	**B**
Q40	The ages of A and B are in the ratio 3:1. Fifteen years hence, the ratio will be 2:1. Their present ages are _______. (a) 30 years, 10 years (b) 45 years, 15 years (c) 21 years, 7 years (d) 60 years, 20 years	**B**
Q41	The population of a bacteria culture doubles in number every 12 minutes. The rata of the number of bacteria at the end of 1 hour to the number of bacteria at the beginning of that hour is ________. (a) 64:1 (b) 60:1 (c) 32:1 (d) 16:1	**C**
Q42	Rs.1,360 have been divided among A, B, C such that A gets (2/3) of what B gets and B gets (1/4) of what C gets. Then B's share is ______. (a) Rs.120 (b) Rs.160 (c) Rs.240 (d) Rs. 320	**C**
Q43	A sum of Rs. 53 is to be divided among A, B, C such that A gets Rs. 7 more than what B gets and B gets Rs. 8 more than what C gets. The ratio of three shares is ______. (a) 18:25:10 (b) 18:10:25 (c) 25:18:10 (d) None	**C**
Q44	A & B together have Rs. 1,210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much does B have?	**B**

	(a) Rs. 460 (b) Rs.484 (c) Rs.550 (d) Rs.664	
Q45	Rs. 1,300 is divided amongst p, q, r & s such that $\frac{p\ share}{q\ share} = \frac{q\ share}{r\ share} = \frac{r\ share}{s\ share} = \frac{2}{3}$. Then, P's share is______. (a) Rs. 140 (b) Rs. 160 (c) Rs.240 (d) Rs. 320	**B**
Q46	Salaries of A, B, C are in the ratio 2:3:5. If increments of 15%, 10% & 20% are given to them respectively, what will be new ratio of their salaries? (a) 3:3:10 (b) 10:11:20 (c) 23:33:60 (d) None	**C**
Q47	The ratio of the number of boys and girls in a school is 3:2. If 20% of the boys and 25% of the girls are scholarship holders, what percentage of the students does not get the scholarship? (a) 56 (b) 70 (c) 78 (d) 80	**C**
Q48	The profits of a Firm are to be distributed in a suitable ratio. Suitable Ratio is the ratio whose terms differ by 40 and the measure of which is 2/7. (a) 280:2 (b) 16:56 (c) 80:7 (d) 40:14	**B**
Q49	If (a + b): (b + c): (c + a) = 6: 7: 8 and (a + b + c) = 14, then the value of c = _________. (a) 6 (b) 7 (c) 8 (d) 14	**A**
Q50	If a: b = 3:4, the value of (2a + 3b): (3a + 4b) = _________. (a) 18:25 (b) 8:25 (c) 17:24 (d) None	**A**
Q51	If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ = _________. (a) 7 (b) 2 (c) 1/3 (d) 1/5	**B**
Q52	If x: y = 2:3 then (5x+2y): (3x-y) = _________. (a) 19:3 (b) 16:3 (c) 7:2 (d) 7:3	**B**
Q53	If P: Q = 2:3 & X: Y = 4: 5, then 5PX + 3QY: 10PX + 4QY = _________. (a) 71:82 (b) 27:28 (c) 17:28 (d) None	**C**
Q54	If $\frac{5x-3y}{5y-3x} = \frac{3}{4}$ then x: y is _________. (a) 2:9 (b) 7:2 (c) 7:9 (d) 27:29	**D**
Q55	$If \frac{a}{2} = \frac{b}{5} = \frac{c}{6}, Then \frac{a+b+c}{a+b-c}$ = _________. (a) 13 (b) $\frac{13}{9}$ (c) $\frac{13}{3}$ (d) None	**A**
Q56	If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ then $\frac{pa+qc+re}{pb+qd+rf}$ = _________. (a) k (b) (p + q + r) k (c) $\frac{1}{k}$ (d) None	**A**
Q57	If the value of: $\frac{x+a}{xa} + \frac{x+b}{xb}$, when $x = \frac{2ab}{a+b}$; $a \neq b$ (a) 3 (b) 4 (c) 1 (d) 2	**D**
Q58	Two whole numbers whose sum is 72 cannot be in the ratio _________. (a) 5:7 (b) 3:5 (c) 3:4 (d) 4:5	**C**
Q59	Ratio of two numbers is 7:10 and their difference is 105. The numbers are _________. (a) (200,305) (b) (185,290) (c) (245,350) (d) (350,240)	**C**

Q60	Ratio of numbers is 1:2:3 & sum of their squares is 504, then the numbers are _________. (a) 6,12,18 (b) 3,6,9 (c) 4,8,12 (d) 5,10,15	A
Q61	Three numbers which are in the ratio of 3:4:5 such that sum of their cubes is 1728. (a) 6,8,10 (b) 10,8,6 (c) 12,8,20 (d) None	A
Q62	A person has assets worth Rs. 1,48,200. He wishes to divide it amongst his wife, son & daughter in ratio 3:2:1 respectively. From these assets, share of his son will be _________. (a) Rs. 74,100 (b) Rs. 37,050 (c) Rs. 49,400 (d) Rs. 24,700	C
Q63	Daily earnings of two persons are in the ratio 4:5 and their daily expenses are in the ratio 7:9. If each saves Rs. 50 per day, their daily incomes are ___. (a) (40,50) (b) (50,40) (c) (400,500) (d) None	C
Q64	A person on a tour has Rs. 9600 for his expense. But the tour was extended for another 16 days, so he has to cut down his daily expenses by Rs. 20. The original duration of the tour had been? (a) 48 days (b) 64 days (c) 80 days (d) 96 days	C
Q65	A earns Rs. 150 in 12 hours; B ear ns Rs. 160 in 8 hours. Ratio of their earning is_______. (a) 5:8 (b) 15: 16 (c) 45:32 (d) None	A
Q66	Arun earns Rs. 80 in 7 hours &Varun earns Rs. 90 in 12 hours. Ratio of their earnings is __. (a) 32:21 (b) 23:12 (c) 8:9 (d) None	A
Q67	A bag contains 23 number of coins in the form of 1 rupee, 2 rupee and 5 rupee coins. The total sum of the coins is Rs. 43. The ratio between 1 rupee and 2 rupees coins is 3:2. Then the number of 1 rupee coins (a) 12 (b) 8 (c) 10 (d) 16	A
Q68	Find in what ratio will the total wages of the workers of a factory be increased or decreased if there be a reduction in the number of workers in the ratio of 15:11 and an increment in their wages in the ratio of 22:25. (a) Decrease in the ratio 6:5. (b) Increase in the ratio 6:5 (c) Decrease in the ratio 3:5 (d) Increase in the ratio 3:5	A
Q69	Ratio in which the total wages of the workers of a factory get increased (or decreased), if there be a reduction of workers in the ratio 7:5 & an increase in their wages in the ratio 2:3 is ______. (a) 14:15 (b) 15:14 (c) 4:1 (d) 1:4	A
Q70	$15(2p^2 - q^2) = 7pq$ where p, q are positive then p: q (a) 5: 6 (b) 5: 7 (c) 3: 5 (d) 3:7	A
Q71	If $p^x = q$, $q^y = r$, $r^z = p^6$ then the value of x.y.z is (a) 0 (b) 1 (c) 3 (d) 6	D
Q72	First, second & third month salaries of a person are in the ratio 2:4:5. The difference between the product of the salaries of first 2 months & last 2 months is 4,80,00,000. Find the salary of the second month (a) Rs. 4,000 (b) Rs. 6,000 (c) Rs. 12,000 (d) Rs. 8,000	D
Q73	The number which when subtracted from each of the terms of the ratio 19:31 reducing it to 1:4 is_____.	A

	(a) 15 (b) 5 (c) 1 (d) None	
Q74	If 2x + 3 : 5x - 38 is the duplicate ratio of $\sqrt{5} : \sqrt{6}$, then value of x is _____. (a) 12 (b) 14 (c) 16 (d) 18	C
Q75	The ratio compounded of (a+b): (a-b) and $a^2 - b^2$: $(a + b)^2$ is ______. (a) (a+b):1 (b) (a-b):1 (c) 1:1 (d) None	C
Q76	The ratio of two numbers is 7:10 and their difference is 105. The numbers are ____. (a) (200,305) (b) (185,290) (c) (245,350) (d) 350, 245	C
Q77	In a school, 10% of the boys are same in number as $1/4^{th}$ of the girls. What is the ratio of boys to girls? (a) 3:2 (b) 5:2 (c) 2:1 (d) 4:3	B
Q78	The sides of a triangle are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and its perimeter is 104 cm. The length of the longest side is ______. (a) 52 cm (b) 48 cm (c) 32 cm (d) 26 cm	B
Q79	The ratio of the prices of two types of cars was 16:23. Two years later when the price of the first has increased by 10% and that of the second by Rs. 477, the ratio of the prices becomes 11:20. Find the original prices of the two types of cars. (a) 848 & 1219 (b) 748 & 1319 (c) 948 & 1119 (d) None	A
Q80	Rs. 4,850 have been divided among A, B, C such that if their shares be diminished by Rs. 15, Rs. 10 & Rs. 25 respectively, remainders are in the ratio 3:4:5. Then B's share is ______. (a) Rs.1,595 (b) Rs.1,610 (c) Rs.1,626.66 (d) Rs.1,600	B
Q81	A man spends Rs. 660 on tables and chairs, the price of each table being Rs. 150, the price of each chair being Rs. 20. If he buys the maximum number of tables, what is the ratio of chairs to tables purchased? (a) 4:3 (b) 3:4 (c) 2:5 (d) 2:3	B
Q82	If $\frac{x}{2y} = \frac{3}{2}$, then the value of $\frac{2x+y}{x-2y}$ is ______. (a) 5 (b) 7 (c) 2 (d) 7.1	B
Q83	If $\frac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}} = \frac{7}{3}$ then x equals to _______. (a) 10 (b) 20 (c) 30 (d) 40	B
Q84	If $\frac{a^3+3a}{3a^2+1} = \frac{91}{37}$ then 'a' equals to _______. (a) 8 (b) 7 (c) 6 (d) None	B

SN	CHAPTER 1B. PROPORTION	Ans
Q85	The mean proportional between $12x^2$ and $27y^2$ is _______. (a) 18xy (b) 81xy (c) 8xy (d) None	A
Q86	If 4, x and 9 are in proportional then 'x'= ______. (a) 36 (b) 6.5 (c) 6 (d) 24	C
Q87	The fourth proportional to 4,6,8 is _______. (a) 12 (b) 32 (c) 48 (d) None	A
Q88	The third proportional to 12, 18 is _______. (a) 24 (b) 27 (c) 36 (d) None	B
Q89	If 50 is the third proportional to 8 and X, then the value of X is _______. (a) 20 (b) 2 (c) 10 (d) 1	A
Q90	Mean proportion between 24 and 54 is ________. (a) 33 (b) 34 (c) 35 (d) 36	D
Q91	If 'b' is the mean proportional between a & c, then ___. (a) b × b = ac (b) b = (a + c) / 2 (c) b = a + c (d) b = (a - c) / 2	A
Q92	If a: b = 4:1 then a+b/a = (a) 1 (b) 5/4 (c) 4/5 (d) None	B
Q93	If a:b = c:d = 2.5 : 1.5, what are the values of ad : be and a+c : b+d? (a) 1: 2 and 5: 3 (b) 1: 3 and 4: 3 (c) 1: 1 and 5: 3 (d) 2: 1 and 3: 5	C
Q94	What must be added to each number 10, 18, 22, 38 to make them proportional? (a) 5 (b) 2 (c) 3 (d) 9	B
Q95	The numbers 2.4, 3.2, 1.5, 2 are in proportion & their product of means is 4.8, find the product of extremes. (a) 4.8 (b) 2.4 (c) 8.4 (d) None	A
Q96	The third proportional to $(x^2 - y^2)$ and $(x - y)$ is ______. (a) (x + y) (b) (x - y) (c) $\frac{x+y}{x-y}$ (d) $\frac{x-y}{x+y}$	D
Q97	The fourth proportional to 2a, a^3, c is _______. (a) ac/2 (b) ac (c) 2/ac (d) $a^2c/2$	D
Q98	The fourth proportional to (a + b), $(a + b)^2$, (a - b) is ______. (a) (a+b) (b) $(a^2 - b^2)$ (c) (a-b) (d) $(a + b)^2$	B
Q99	The numbers 14,16,35,42 are not in proportion. The fourth term for which they will be in proportion is _______. (a) 45 (b) 40 (c) 32 (d) None	B
Q100	What least number must be added to each one 6, 14, 18, 38 to make them in	B

	proportion? (a) 1 (b) 2 (c) 3 (d) 4	
Q101	Ratio of 3^{rd} proportional to 12 and 30 & Mean proportional between 9 and 25 is ___. (a) 2:1 (b) 5:1 (c) 7:15 (d) 9:14	B
Q102	Ratio of 3^{rd} proportional to 4 & 6 and mean proportional between 9 & 25 is _____. (a) 5:3 (b) 3:5 (c) 8: 5 (d) 5: 8	B
Q103	If b is mean proportion between a and c, then the mean proportion bet^n (a^2+b^2) & (b^2+c^2) is_______. (a) b (a + c) (b) a (b + c) (c) c (a + b) (d) abc	A
Q104	The number which has the same ratio to 26 that 6 has to 13 is ______. (a) 11 (b) 10 (c) 21 (d) 12	D
Q105	If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is _______. (a) 6/5 (b) 5/6 (c) 15/2 (d) None	C
Q106	Find two numbers such that their AM is 18 and third proportional to them is 144. (a) 9, 36 (b) 29, 56 (c) 18, 72 (d) None	D
Q107	A Dealer mixes Tea costing Rs. 6.92 per kg with Tea costing Rs.7.77 per kg and sells the mixture at Rs. 8.80 per kg and earns a profit 17.5% on his Sale Price. In what proportion does he mix them? (a) 3:2 (b) 4:1 (c) 3:4 (d) 5:3	A
Q108	60 kg of alloy A is mixed with 100 kg of alloy B. If alloy A has lead & tin in ratio 3:2 & alloy B has tin & copper in the ratio 1:4, then amount of tin in new alloy is ____. (a) 36 kg (b) 44 kg (c) 53 kg (d) 80 kg	B
Q109	70 kgs of Alloy I is mixed with 20 kg of Alloy II. If alloy I has Copper and Zinc in the ratio 3:4 and alloy II has Zinc & tin in the ratio 2:3 then the amount of Zinc in the new alloy is ________. (a) 48 kg (b) 52 kg (c) 42 kg (d) None	A
Q110	15 litres of mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed with it, % of alcohol in the new mixture would be ______. (a) 15% (b) $16\frac{2}{3}$% (c) 17% (d) $18\frac{1}{2}$%	B
Q111	Three containers have their volumes in the ratio 3:4:5. They are full of mixtures of milk & water. The mixtures contain milk and water in the ratio of (4:1), (3:1) and (5:2) respectively. The contents of all these three containers are poured into a fourth container. The ratio of milk & water in the fourth container is ________. (a) 4:1 (b) 151: 48 (c) 157:53 (d) 5:2	C
Q112	What is the value of $\frac{P+Q}{P-Q}$ if $\frac{P}{Q} = 7$ (a) 4/3 (b) 2/3 (c) 2/6 (d) 7/8	A

Q113	If a: b = 4:1 then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is ________. (a) 1 (b) 5/2 (c) 4/5 (d) None	B
Q114	An alloy is to contain copper and zinc in the ratio 9:4. The zinc required to be melted with 24 kg of copper is _________. (a) 10.67 kg (b) 10.33 kg (c) $9\frac{2}{3}$ kg (d) 9 kg	A
Q115	If 1 cup of milk is added to a 3 cup mixture that is 2/5 flour &3/5 milk, what % of the 4 cup mixture is milk? (a) 80% (b) 75% (c) 70% (d) 65%	C
Q116	Gold is 19 times as heavy as Water and Copper is 9 times as heavy as Water. In what ratio should these be mixed to get an alloy 15 times as heavy as water? (a) 1:1 (b) 2:3 (c) 1:2 (d) 3:2	D
Q117	20 litres of a mixture contains milk & water in the ratio 5:3. If 4 litres of this mixture be replaced by 4 litres of milk, ratio of milk to water in new mixture will be _____. (a) 2:1 (b) 7:3 (c) 8:3 (d) 4:3	B
Q118	If one type of rice of cost Rs. 13.84 is mixed with another type of rice of cost Rs. 15.54. the mixture is sold at Rs. 17.6 with a profit of 14.6% on selling price then in which proportion the two types of rice mixed? (a) 3: 7 (b) 5: 7 (c) 7: 9 (d) None	A
Q119	What must be added to each of the numbers 6, 15, 20 and 43 to make them proportional? (a) 5 (b) 4 (c) 3 (d) 2	C
Q120	A fraction bears the same ratio to $\frac{1}{27}$ as $\frac{3}{7}$ does to $\frac{5}{9}$. The fraction is ________. (a) $\frac{7}{45}$ (b) $\frac{1}{35}$ (c) $\frac{45}{7}$ (d) $\frac{5}{21}$	B
Q121	If a: b = c: d then _____. (a) ab = cd (b) ac = bd (c) ad = bc (d) ab = ad	C
Q122	If $\frac{1}{x} : \frac{1}{6} = \frac{25}{6} : \frac{1}{x}$ then x = ______. (a) 5:6 (b) 6:5 (c) 5:1 (d) 1:5	B
Q123	Find the value of x if 10/3: x :: 5/2: 5/4. (a) 5/3 (b) 3/5 (c) 2/5 (d) 1/5	A
Q124	If a: b = 3:4, the value of (2a + 3b): (3a + 4b) is _______. (a) 18:25 (b) 8:25 (c) 17:24 (d) None	A
Q125	If a: b=1:2, then a+b: a-b = _____. (a) -3 (b) 1/2 (c) 2 (d) -1/3	A
Q126	If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ = k then $\frac{pa+qc+re}{pb+qd+rf}$ = ________.	A

	(a) k (b) (p + q + r) k (c) $\frac{1}{k}$ (d) None	
Q127	If A = $\frac{B}{2}$ = $\frac{C}{5}$ then A: B: C is ______. (a) 3:5:2 (b) 2:5:3 (c) 2:3:5 (d) 1:2:5	**D**
Q128	If p: q = 2: 3 & x: y = 4: 5, then 5px + 3qy: 10px + 4qy is ___. (a) 71:82 (b) 27:28 (c) 17:28 (d) None	**C**
Q129	If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$ then $\frac{a+b+c}{c}$ is equal to ________. (a) 7 (b) 2 (c) 1/3 (d) 1/5	**B**
Q130	If $\frac{a}{2}=\frac{b}{5}=\frac{c}{6}$, Then $\frac{a+b+c}{a+b-c}$ = ________. (a) 13 (b) $\frac{13}{19}$ (c) $\frac{13}{3}$ (d) None	**A**
Q131	If x: y = 3: 4, the value of $x^2y + xy^2 : x^3 + y^3$ is _________. (a) 13:12 (b) 12:13 (c) 21:31 (d) None	**B**
Q132	If (a + b): (b + c): (c + a) = 6: 7: 8 and (a + b + c) = 14, then c = _____. (a) 8 (b) 7 (c) 6 (d) None	**C**
Q133	If $24(3x^2 - y^2) = 37xy$, then x: y is ______. (a) $\frac{8}{9}$ & $\frac{-3}{8}$ (b) $\frac{3}{5}$ & $\frac{3}{7}$ (c) $\frac{3}{7}$ & $\frac{-2}{5}$ (d) $\frac{2}{5}$ & $\frac{-3}{5}$	**A**
Q134	Electricity Bill of a certain establishment is partly fixed & partly variable as the number of units of electricity consumed. When in a certain month, 540 units are consumed & bill is Rs. 1,800. In another month, 620 units are consumed & bill is Rs. 2,040. In yet another month 500 units are consumed. Find the bill for that month. (a) Rs. 1,560 (b) Rs. 1,680 (c) Rs. 1,840 (d) Rs. 1,950	**B**
Q135	24 carat gold is pure gold; 18 carat gold is (3/4) gold and 20 carat gold is (5/6) gold, ratio of pure gold in 18 carat gold to the pure gold in 20 carat gold is _____. (a) 5:8 (b) 9:10 (c) 15:24 (d) 8:5	**B**
Q136	85 kg of a mixture contains milk and water in the ratio 27:7. How much more water is to be added to get a new mixture containing milk and water in the ratio 3:1? (a) 5 kg (b) 6.5 kg (c) 7.25 kg (d) 8 kg	**A**

1C. INDICES

INTRODUCTION

- **Continued Product:** When two or more numbers are multiplied, it is called continued Product. Each number is called a 'factor'.

  Ex: a × b × c × d. [Here a, b, c, d are factors]
- If the factor gets repeated in a continued product, it is called a 'power'

  Ex: 2 × 2 × 2 = 2^3.
- 'Factor' which multiplies is called the "base" & number of times it is multiplied is called the "power" or the "index". [Thus 'base' is '2' & 'power' is '3'].

LAWS OF INDICES

1. $a^m \times a^n = a^{m+n}$	**Ex:** $3^2 \times 3^1 = 3^{2+1} = 3^3$
2. $a^m \div a^n = a^{m-n}$	**Ex:** $3^2/3^1 = 3^{2-1} = 3^1$
3. $(a^m)^n = a^{mn}$	**Ex:** $(3^2)^2 = 3^{2\times2} = 3^4$
4. $(ab)^m = a^m.b^m$	**Ex:** $(3.2)^2 = 3^2.2^2$
5. $(a/b)^m = a^m/b^m$	**Ex:** $(4/2)^2 = 4^2/2^2$
6. $a^{-m} = \frac{1}{a^m}$ & $\frac{1}{a^{-m}} = a^m$	**Ex:** $x^{-1/4} = 1/x^{1/4}$
7. $x^a = x^b$, then a = b	**Ex:** $3^x = 9$; $3^x = 3^2$; x = 2
8. $x^a = y^a$, then x = y	**Ex:** $a^3 = 27$; $a^3 = 3^3$; a = 3
9. $a^0 = 1$	**Ex:** $5^0 = 1$

SOME IMPORTANT RESULTS

1) $a^{1/n} = \sqrt[n]{a}$

2) $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$

3) $\sqrt{a\sqrt{a\sqrt{a\sqrt{a \ldots \infty}}}} = a$

4) $\sqrt{a\sqrt{a\sqrt{a\sqrt{a \ldots n\ times}}}} = a^{\frac{(2^n - 1)}{2^n}}$

CQ1. Find the value of p from $(\sqrt{4})^{-6} \times (\sqrt{2})^{-4} = 2^p$

(a) 16 (b) 8 (c) -8 (d) 4

CQ2. If $5^{(x+3)} = (25)^{(3x-4)}$, then the value of x is ________.

(a) $\frac{5}{11}$ (b) $\frac{11}{5}$ (c) $\frac{11}{3}$ (d) $\frac{13}{5}$

CQ3. $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

(a) 1 (b) 0 (c) -1 (d) None of these

CQ4. The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

(a) 1 (b) 0 (c) 2 (d) None of these

BASIC FORMULAE

$(a + b)^2 = a^2 + 2ab + b^2$	$a^3 - b^3 = (a - b)(a^2 - ab + b^2)$
$(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)^3 = a^3 + 3ab(a + b) + b^3$
$a^2 - b^2 = (a + b)(a - b)$	$(a - b)^3 = a^3 - 3ab(a - b) - b^3$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

USEFUL RESULTS

1) If $(a + b + c) = 0$, then $a^3 + b^3 + c^3 = 3abc$
2) If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then $(a + b + c)^3 = 27abc$
3) If $a^x = k$, then $a = k^{1/x}$
4) If $a^x = b^y$, then $a = b^{y/x}$
5) If $a^x = b^x$, then $x = y$ $(a \neq 1)$
6) If $a^x = b^x$, then $a = b$ $(x \neq 0, a, b > 0)$
7) If $a^x b^y = a^m b^n$, then $x = m$ & $y = n$ $(a \neq b)$
8) If $x = a^{1/3} - a^{1/3}$, then $(x^3 + 3x) = (a - a^{-1})$
9) If $x = a^{1/3} + a^{1/3}$, then $(x^3 + 3x) = (a + a^{-1})$

Space for PC Class Note:

INDICES - QUESTION BANK

SN	CHAPTER 1C. INDICES	Ans
Q137	$4x^{-1/4}$ is expressed as ______. (a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) None	C
Q138	The value of $2 \times (32)^{1/5}$ is _______. (a) 2 (b) 10 (c) 4 (d) None	C
Q139	The value of $2 \times (256)^{-1/8}$ is ______. (a) 1 (b) 2 (c) 1/2 (d) None	A
Q140	$2^{1/2} \times 4^{3/4}$ = _____. (a) A fraction (b) An Integer (c) 1 (d) None	B
Q141	Simplify $\left(8a^{\frac{3}{2}} \div 27x^{\frac{1}{2}}\right)^{\frac{2}{3}}$ (a) $\frac{4a}{9x}$ (b) $\frac{4a}{9x^{1/3}}$ (c) 4a (d) 1/3	B
Q142	The Value of $\frac{1}{2}\times(216)^{1/3}$ is ________. (a) 2 (b) 3 (c) 2% (d) None	B
Q143	$(64/512)^{1/3}$ = _________. (a) 1/2 (b) 1/4 (c)1/6 (d) None	A
Q144	If $2^x = \sqrt[3]{32}$ then x = _________. (a) 5 (b) 3 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$	D
Q145	The value of $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}}$ is _______. (a) 102 (b) 105 (c) 107 (d) 109	A
Q146	The value of $\sqrt[3]{x^{12}} \times \sqrt[3]{x^6}$ is ________. (a) x^7 (b) x^6 (c) 1 (d) None	B
Q147	The value of $[(10)^{150} \div (10)^{146}]$ is ________. (a) 1000 (b) 10000 (c) 100000 (d) 10^6	B
Q148	The expression $\left(\frac{1}{216}\right)^{-\frac{2}{3}} \div \left(\frac{1}{27}\right)^{-4/3}$ in the simplified form is ________. (a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{8}$	C
Q149	The value of $5^{1/4} \times (125)^{0.25}$ is ________. (a) $\sqrt{5}$ (b) 5 (c) $\sqrt[3]{5}$ (d) 25	B
Q150	$(P^3Q^4Z^6/P^4R^{100})^0$ = _______. (a) 0 (b) 2/3 (c) 1 (d) None	C
Q151	Which one is true?	A

	(a) $x^{2/3} = \sqrt[3]{x^2}$ (b) $x^{2/3} = \sqrt{x^3}$ (c) $x^{2/3} > \sqrt[x]{x^2}$ (d) $x^{2/3} < \sqrt[x]{x^2}$	
Q152	If $10^x/10^y = 100$, then x = ______. (a) y+2 (b) y-2 (c) 2-y (d) 2y	**A**
Q153	$\sqrt{a^{3/4}b^{2/3}c^4} \div \sqrt[3]{a^6b^{-3}c^6}$ (a) $a^{-13/8}b^{4/3}$ (b) $a^{-1/8}b^{1/3}$ (c) $a^{-8}b^3$ (d) 1	**A**
Q154	Find the value of $(2^{7+2a})/(3^{3a+11})$ for a = -4. (a) 2/3 (b) 3/2 (c) 1 (d) -2/3	**B**
Q155	The value of $\left(\frac{x^4}{y^{-8}}\right)^{1/4}$ when x = 2, y = 3 is ______. (a) $\frac{2}{9}$ (b) 18 (c) $2\sqrt{3}$ (d) None	**B**
Q156	If $16 \times 8^{n+2} = 2^m$, then m = ______. (a) n + 8 (b) 2n + 10 (c) 3n + 2 (d) 3n + 10	**D**
Q157	If $3^x - 3^{x-1} = 162$ then the value of x is _______. (a) 5 (b) 4 (c) 6 (d) None	**A**
Q158	If $\frac{9^n \times 3^5 \times 27^3}{3 \times (81)^4} = 27$ then n equals to ______. (a) 0 (b) 2 (c) 3 (d) 4	**C**
Q159	The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is ______. (a) 9/4 (b) 4/9 (c) 2/3 (d) None	**A**
Q160	$x^{a-b} \times x^{b-c} \times x^{c-a}$ = _______. (a) x (b) 1 (c) 0 (d) None	**B**
Q161	If the index of any power function is zero, then the value of that function is (a) 0 (b) 1 (c) -1 (d) ∞	**B**
Q162	If 49 × 49 × 49 × 49 = 7^n, then n equals (a) 4 (b) 7 (c) 8 (d) 16	**C**
Q163	If $x^{-3}y^{-4} \times 8^{-1}x^4y^3$ simplifies to (a) 2xy (b) $\frac{xy}{2}$ (c) $2\frac{x}{y}$ (d) None	**D**
Q164	If $5^{(x+3)} = 25^{(3x-4)}$, then the value of x is (a) $\frac{5}{11}$ (b) $\frac{11}{5}$ (c) $\frac{11}{3}$ (d) $\frac{13}{5}$	**B**
Q165	If $\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$ = x; then x equals to _______. (a) 1 (b) 3 (c) 9 (d) 3^n	**C**
Q166	If $\frac{3}{4} = \frac{6}{x} = \frac{9}{y}$, then x + y = ______. (a) 4 (b) 8 (c) 12 (d) 20	**D**
Q167	If $4(2^n) = 256$; n = _____.	**C**

	(a) 4 (b) 5 (c) 6 (d) None	
Q168	If $2^x - 2^{x-1} = 4$, then the value of x^x= ______. (a) 26 (b) 27 (c) 28 (d) 29	B
Q169	Solve for x if $\sqrt{x}^{\sqrt{x}} = 256$ (a) 2 (b) 16 (c) 4 (d) $\sqrt{2}$	B
Q170	$3^{3x-4}.2^{x+5} = 3^5.2^8$. Find the value of x. (a) 1 (b) 3 (c) 1/3 (d) 0	B
Q171	Solve for 'z' if $z^{-1} = 3^{-1} - 4^{-1}$ (a) 5^{-1} (b) 1 (c) 1 / 12 (d) 12	D
Q172	On simplification $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^x}$ = _____. (a) -1 (b) 0 (c) 1 (d) 10	C
Q173	What minimum integer value of x, expression (3^x / 243) will be greater than 1? (a) 3 (b) 4 (c) 5 (d) 6	D
Q174	Solve for "x" if $\frac{25^{x+2}}{\sqrt{5}} = \left(\frac{1}{5}\right)^{x-7.5}$ (a) 4/3 (b) − 4/3 (c) 3/4 (d) -3/4	A
Q175	Solve for 'b' if $12^{2b+4} = 3^{3b} \times 4^{b+8}$ (a) -1 (b) 2 (c) 4 (d) -2	C
Q176	Solve for x if $x^{a^3}.x^{b^3}.x^{3ab(a+b)} = (2^5)^{25}$ and a + b = 5. (a) 2 (b) 3 (c) 1 (d) 0	A
Q177	If $\frac{9^y.3^2(3^{-y})^{-1}-27^y}{3^{3x}.2^3} = \frac{1}{27}$then x - y = _________. (a) -1 (b) 1 (c) 0 (d) None	B
Q178	$\frac{2^{m+1}.3^{2m-n}.5^{m+n}.6^n}{6^m.10^{n+2}.15^m}$ = _________. (a) $\frac{1}{45}$ (b) $\frac{1}{50}$ (c)$\frac{1}{9}$ (d) None	B
Q179	$\left((x^m)^{1-\frac{1}{m}}\right)^{\frac{1}{m-1}}$ = _________. (a) x (b) 1 (c) 0 (d) None	A
Q180	If $3^a = 729$ and $2^b = 1024$, then find the value of $\frac{4a+6b}{6b-3a}$ (a) 1 (b) 0 (c) 2 (d) 3	C
Q181	Simplification of $\frac{2^{n+3}-10 \times 2^{n+1}}{2^{n+1} \times 6}$ gives (a) -1 (b) 1 (c) 0 (d) None	A
Q182	The expression $\frac{3^{2n+1}+3^{2n-1}}{3^{2n+3}-3^{2n+2}}$ simplifies to (a) $\frac{5}{27}$ (b) 1 (c) $8^{3/7}$ (d) None	A

Q183	If $a^x = b$; $b^y = c$; $c^z = a$ then xyz is ______. (a) 1 (b) 2 (c) 3 (d) None	A
Q184	The value of $\frac{(6^4)^2(8^5)^2(2^2)^3(3^2)^2}{(6^2)^3(8^3)^4(3^3)^2}$ is ______. (a) 1/4 (b) 4 (c) 2 (d) None	B
Q185	If $9^{2x} = \frac{27}{3^{x+2}}$, then the value of x is ______. (a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) 0 (d) None	B
Q186	If x, y, z are all positive, find the value of xyz if $z^x = x$, $z^y = y$, $y^y = x$ (a) 4 (b) $8\sqrt{2}$ (c) 1 (d) 2	B
Q187	If $a^m.a^n = a^{mn}$, then m(n - 2) + n(m - 2) is ________. (a) 1 (b) -1 (c) 0 (d) None	C
Q188	$[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-\frac{1}{2}}$ = ________. (a) x (b) 1/x (c) 1 (d) None	A
Q189	If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ then (b-c) x + (c-a) y + (a-b) z is ________. (a) 1 (b) 0 (c) 5 (d) None	B
Q190	If x + y = a and xy = b then the value of $1/x^3 + 1/y^3$ is ________. (a) $(a^3 - 3ab) / b^3$ (b) $(a^3 - 3a) / b^3$ (c) $(a^3 - 3) / b$ (d) $(a^3 - 3) / b^2$	A
Q191	If $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1, then the value of p+q+r is ________. (a) 1 (b) 0 (c) 1/2 (d) None	B
Q192	If $a^p = b^q = c^r$ and $b^2 = ac$ the value of q(p+r)/pr given by (a) 1 (b) -1 (c) 2 (d) None	C
Q193	If $2^x = 3^y = 6^{-z}$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ = __________. (a) 1 (b) 0 (c) 2 (d) None	B
Q194	If $(5.678)^x = (0.5678)^y = 10^z$ then (a) $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=1$ (b) $\frac{1}{x}-\frac{1}{y}-\frac{1}{z}=0$ (c) $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=-1$ (d) None	B
Q195	If $2^a = 3^b = (12)^c$ then $\frac{1}{c}-\frac{1}{b}-\frac{2}{a}$ = _________. (a) 1 (b) 0 (c) 2 (d) None	B
Q196	If $2^a = 4^b = 8^c$ and abc = 288 then the value of $\frac{1}{2a}+\frac{1}{4b}+\frac{1}{8c}$ is given by (a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{11}{96}$ (d) $-\frac{11}{96}$	C
Q197	If $a^p = b^q = c^r = d^s$ and ab = cd then the value of $\frac{1}{p}+\frac{1}{q}-\frac{1}{r}-\frac{1}{s}$ = __________. (a) $\frac{1}{a}$ (b) $\frac{1}{b}$ (c) 0 (d) 1	C
Q198	If $3^a = 5^b = (75)^c$; then ab — c (2a + b) = ________. (a) 1 (b) 0 (c) 3 (d) 5	B

Q199	Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ tick the correct of these when $x = p^{1/3} - p^{-1/3}$ (a) $x^3 + 3x = p+1/p$ (b) $x^3 + 3x = p - 1/p$ (c) $x^3 + 3x = p+1$ (d) None	B
Q200	If $x = 3^{1/3} + 3^{-1/3}$, then $3x^3 - 9x$ is ________. (a) 15 (b) 10 (c) 12 (d) None	B
Q201	If $x = 5^{1/3} + 5^{-1/3}$ then the value of $5x^3 + 15x$ is ________. (a) 25 (b) 24 (c) 27 (d) 28	B
Q202	On simplification $\left[\frac{\frac{a}{x^{a-b}}}{\frac{a}{x^{a+b}}} \div \frac{\frac{b}{x^{b-a}}}{\frac{b}{x^{b+a}}}\right]^{a+b}$ = ________. (a) 1 (b) -1 (c) 0 (d) None	D
Q203	If $a^b = b^a$ then the value of $\left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\frac{a}{b}-1}$ = ________. [Hint: Put a = 4 & b = 2] (a) a (b) b (c) 0 (d) None	C
Q204	If $x = \sqrt{2 - \sqrt{2 - \sqrt{2} \ldots \infty}}$; X = ________. (a) -2 (b) 1 (c) 2 (d) 0	B
Q205	If p + q + r = 0, $x^{p^2q^{-1}r^{-1}} x^{p^{-1}q^2r^{-1}} x^{p^{-1}q^{-1}r^2}$ = ____. [Hint: a + b+ c = 0; $a^3+b^3+c^3 = 3abc$] (a) x (b) x^2 (c) x^3 (d) x^4	C
Q206	$\frac{1}{1+x^{(b-a)}+x^{(c-a)}} + \frac{1}{1+x^{(a-b)}+x^{(c-b)}} + \frac{1}{1+x^{(b-c)}+x^{(a-c)}}$ = ____ (a) $x^{(a-b-c)}$ (b) 1 (c) 0 (d) None	B
Q207	$\left(\frac{b+c}{x^{c-a}}\right)^{\frac{1}{a-b}} \times \left(\frac{c+a}{x^{a-b}}\right)^{\frac{1}{b-c}} \times \left(\frac{a+b}{x^{b-c}}\right)^{\frac{1}{c-a}}$ = ______. (a) 1 (b) 3 (c) -1 (d) 0	A
Q208	Product of $x^{2^{n-1}} + y^{2^{n-1}}$ and $x^{2^{n-1}} - y^{2^{n-1}}$ = [Hint: Use $(a - b)(a + b) = a^2 - b^2$] (a) $x^{2^n} - y^{2^n}$ (b) $x^2 - y^2$ (c) $x^a - y^b$ (d) None	A
Q209	If $a^m = b^h \times a^n = b^k \times a^p$, find the relationship of 'a' among h, k, m, n and p only. [Hint: Put a =4, b=2, m=5, n=2, k=4, h=6, p= 3] (a) $m = \sqrt[4]{hnpk}$ (b) h(m-p) = k(m-n) (c) $m = \frac{hn}{kp}$ (d) m(h-k) = p(n-p)	B
Q210	$\left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b} \times \left(\frac{x^a}{x^b}\right)^{a+b-c}$ = __________. (a) 1 (b) 0 (c) -1 (d) None	A
Q211	$\left(\frac{x^a}{x^{-b}}\right)^{(a^2-ab+b^2)} \times \left(\frac{x^b}{x^{-c}}\right)^{(b^2-bc+c^2)} \times \left(\frac{x^c}{x^{-a}}\right)^{(c^2-ca+a^2)}$ equals to ________. (a) 1 (b) $x^{-2(a^2+b^2+c^2)}$ (c) $x^{2(a^3+b^3+c^3)}$ (d) $x^{-2(a^3+b^3+c^3)}$	C
Q212	If $x^by = 2x - 3y^2$, then find $(1/2)^b \times \frac{1}{\sqrt{3}}$ [Hint: Put x=2 & y =1] (a) 1 (b) 2 (c) 0 (d) -1	C

Q213	$\sqrt[(a+b)]{\frac{x^{a^2}}{x^{b^2}}} \times \sqrt[(b+c)]{\frac{x^{b^2}}{x^{c^2}}} \times \sqrt[(c+a)]{\frac{x^{c^2}}{x^{a^2}}}$ = ________. (a) 1 (b) 0 (c) -1 (d) None	A
Q214	$.\left(\frac{x^b}{x^c}\right)^{1/bc} \times \left(\frac{x^c}{x^a}\right)^{1/ca} \times \left(\frac{x^a}{x^b}\right)^{1/ab}$ equals to ______. (a) -1 (b) 0 (c) 1 (d) None	C
Q215	The value of $\frac{(x^{a+b})^2.(x^{b+c})^2.(x^{c+a})^2}{(x^a x^b x^c)^4}$ is ________. (a) -1 (b) 1 (c) 0 (d) x	B
Q216	If x = 5 + 2√6, then $\frac{(x-1)}{\sqrt{x}}$ is equal to ________. (a) √2 (b) 2√2 (c) √3 (d) 2√3	B
Q217	$\{(x+y)^{2/3}(x-y)^{3/2}/\sqrt{x+y} \times \sqrt{(x-y)^3}\}^6$ equals ________. (a) 1 (b) $(x+y)^2$ (c) (x-y) (d) (x+y)	D
Q218	If a = xy^{m-1}; b = xy^{n-1}; c = xy^{p-1}, then $a^{n-p} \times b^{p-m} \times c^{m-n}$ =________. (a) 1 (b) -1 (c) 0 (d) None	A
Q219	$1/(1+a^{m-n}+a^{m-p}) + 1/(1+a^{n-m}+a^{n-p}) + 1/(1+a^{p-m}+a^{p-n})$ is equal to _____. (a) 0 (b) a (c) 1 (d) 1/a	C
Q220	The value of $(\frac{x^a}{x^b})^{a+b} \times (\frac{x^b}{x^c})^{b+c} \times (\frac{x^c}{x^a})^{c+a}$ = ________. (a) 1 (b)0 (c) 2 (d) None	A
Q221	$(\frac{x^a}{x^b})^{(a^2+ab+b^2)} \times (\frac{x^b}{x^c})^{(b^2+bc+c^2)} \times (\frac{x^c}{x^a})^{(c^2+ca+a^2)}$ = ________. (a) 1 (b) 0 (c) -1 (d) None	A
Q222	If a = x^{q+r}. y^b, b = x^{r+b}.y^q, c = x^{p+q}.y^r, then $a^{q-r} \times b^{r-q} \times c^{b-q}$ = ________. (a) 0 (b) 1 (c) -1 (d) 2	B
Q223	If xy^{p-1} = a, zy^{q-1} = b, and xy^{r-1} = c then a^{q-r} b^{r-p} c^{p-q} = ________. (a) 1 (b) 0 (c) p+q+r-1 (d) None	A
Q224	$\left[\frac{x^{ab}}{x^{a^2+b^2}}\right]^{a+b} \times \left[\frac{x^{bc}}{x^{b^2+c^2}}\right]^{b+c} \times \left[\frac{x^{ca}}{x^{c^2+a^2}}\right]^{c+a}$ = ________. (a) x^{-2a^3} (b) x^{2a^3} (c)$x^{-2(a^3+b^3+c^3)}$ (d) $x^{2(a^3+b^3+c^3)}$	C
Q225	If abc=1, $\left(\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}\right)$ = _____. (a) 0 (b) 1 (c) $\frac{1}{ab}$ (d) ab	B
Q226	If abc = 2 then the value of $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{b}{2}+c^{-1}} + \frac{1}{1+a^{-1}+c}$ = ________. (a)1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$	A
Q227	If xy^{p-1} = a, xy^{q-1} = b, and xy^{r-1} = c; then $a^{q-r}b^{r-p}c^{p-q}$ = ________. (a) 1 (b) 0 (c) p+q+r-1 (d) None	A

1D. LOGARITHMS

TRANSFORMATION FORMULA

- **If $a^x = b$** [Exponential Form]
- **then $\log_a b = x$** [Logarithmic Form]

PC Note: These are not two different formulae. They are just transformation of each other & should be used to change one form into other form. Following are some examples for better understanding.

CQ	Exponential Form	Logarithmic Form	Read as
1	$2^4 = 16$	$\text{Log}_2 16 = 4$	Log of 16 to the base 2 = 4
2	$10^3 = 1000$	$\text{Log}_{10} 1000 = 3$	Log of 1000 to the base 10 = 3
3	$3^{-4} = \frac{1}{81}$	$\text{Log}_3 \frac{1}{81} = -4$	Log of $\frac{1}{81}$ to the base 3 = - 4
4	$100^{1/2} = 10$	$\text{Log}_{100} 10 = 1/2$	Log of 10 to the base 100 is ½

Mentos Zindagi:

- Log apne side me positive logo ko hi rakhte hai [a & b should be +ve].
- Log 'x' ko apne se dur rakhte hai [Therefore 'x' should be on other side of Log]
- If **NO BASE** is given in the question, it is always **taken as 10** **[In this chapter]**

Some Conditions w.r.t. a, b & x

- a & b > 0; a ≠ 1
- Base of Log > 1 [If Base = 1, then Value of b will always be 1 (1^x).]
- Number (b) > 0 [Log 0 → Does not Exist.]

FUNDAMENTAL LAWS OF LOGARITHMS

1.	**Log 10 = 1** [Because since base is not given, it is taken as 10]
2.	**Log 1 = 0** [Log of 1 to any Base = 0; (Since $a^0 = 1$, $\log_a 1 = 0$)]
3.	**Log M + Log N = Log (M × N)** **[PC Note: Log M + Log N ≠ Log (M + N)]**
	CQ5: Log 6 + Log 5 = Log 30 **CQ6:** Log X + Log X^2 = Log $X.X^2$ = Log X^3
4.	**Log M - Log N = Log (M/N)** **[PC Note: Log (M - N) ≠ Log M - Log N]**

	CQ7: Log 32/4 = Log 32 - Log 4
5.	**Log (M^N) = N. Log M** **[PC Note: $(\text{LogM})^N \neq$ N. Log M]**
	CQ8: Log 25 = Log 5^2 = 2. Log 5
6.	**$\text{Log}_{N^b} M^a = (a \times \frac{1}{b}) \times \text{Log}_N M$**
	i. Jo Number ka Log nikalna hai uska power "jaisa ka waise" bahar aayega. **ii.** Base ka power "reciprocal" me bahar aayega.
7.	**$\text{Log}_M M = 1$** [Log of any number to same base = 1 (Since $a^1 = a$, $\log_a a = 1$)]
8.	**Log 1 = 0** [Log of 1 to any Base = 0; (Since $a^0 = 1$, $\log_a 1 = 0$)]
9.	**$\text{Log}_N M = \frac{Log\ M}{Log\ N}$** [Base Changing Rule.]
	CQ9: $\text{Log}_4 8 = \frac{Log_2\ 8}{Log_2\ 4} = \frac{3\ Log_2\ 2}{2\ Log_2\ 2} = \frac{3}{2}$
10.	**$\text{Log}_C A = \text{Log}_B A \times \text{Log}_C B$**
	LHS → $\text{Log}_C A = \frac{Log\ A}{Log\ C}$ RHS → $\text{Log}_B A \times \text{Log}_C B = \frac{Log\ A}{\cancel{Log\ B}} \times \frac{\cancel{Log\ B}}{Log\ C} = \frac{Log\ A}{Log\ C}$
11.	**$\text{Log}_N M = \frac{1}{Log_M\ N}$**
	CQ10: $\text{Log}_5 10 = \frac{1}{\text{Log}_{10}5} = \frac{1}{\text{Log}_{10}\frac{10}{2}} = \frac{1}{\text{Log}_{10} 10 - \text{Log}_{10} 2} = \frac{1}{1 - 0.3010} = \frac{1}{0.6990} = 1.43$
12.	**$a^{\log_a x} = x$** **$a^{\log_a x} = x^{\log_a a} = x^1 = x$** [Inverse logarithm Property]
13.	**Log 10 = 1** [Because if Nothing is given, base is taken as 10.]

POINTS TO BE NOTED
▪ If NO BASE is given in the question, it is always taken as 10 in numerical calculations. ▪ The Domain of Logarithmic function is (0, ∞) i.e 0 < x < ∞.

LOGARITHMS – QUESTION BANK

SN	CHAPTER 1D. LOGARITHMS	Ans
Q228	Log 0.0001 to the base 0.1 = _______. (a) -4 (b) 4 (c) ¼ (d) None	B
Q229	$\text{Log}_{\sqrt{2}}$ 64 = _______. (a) 12 (b) 6 (c) 1 (d) None	A
Q230	Log (1/81) to the base 9 = _______. (a) 2 (b) 1/2 (c) -2 (d) None	C
Q231	Log (1/81) to the base 3 = _______. (a) 4 (b) 1/4 (c) -4 (d) None	C
Q232	$\text{Log}_{3\sqrt{2}}$ 324 = _______. (a) 2 (b) 3 (c) 4 (d) 1	C
Q233	Value of ($\text{Log}_8 128$) x $\text{Log}_6 \frac{1}{216}$ is ______. (a) -7 (b) 7 (c) 1/7 (d) -2/7	A
Q234	Value of ($\text{Log}_{1/81}$ 729) x Log_2 256 = _______. (a) 12 (b) -12 (c) 1/12 (d) -1/12	B
Q235	Find the base if Logarithm of 32 is 10/3. (a) 5/3 (b) 20/9 (c) $\sqrt{8}$ (d) 4	C
Q236	If 2Log x = 4Log3, then x = (a) 3 (b) 9 (c) 81 (d) 27	B
Q237	$\frac{3+\log_{10}343}{2+\frac{1}{2}\text{Log}\left(\frac{49}{4}\right)+\frac{1}{3}\text{Log}\left(\frac{1}{125}\right)}$ = _________. (a) 3 (b) 3/2 (c) 2 (d) 1	A
Q238	Value of Log_8 25 = _____. [Log 2 = 0.3010 and Log 5 = 0.6989] (a) 1 (b) 2 (c) 1.5482 (d) None	C
Q239	Log (Logx^2) - Log (Logx) = _______. (a) 2 (b) Log 2 (c) Log x (d) Log $\sqrt{x}$	B
Q240	$\text{Log}\left(\sqrt[3]{a^2} \times \sqrt[2]{b^3}\right)$ = _______. (a) $\frac{3}{2}$ Log a + $\frac{2}{3}$ Log b (b) 6Log ab (c) $\frac{2}{3}$ Log a + $\frac{3}{2}$ Log b (d) None	C
Q241	Value of $\log_3 2 \log_4 3 \log_5 4 \ldots\ldots \log_{15} 14 \log_{16} 15$ is _______. (a) 1/3 (b) ½ (c) 1/5 (d) 1/4	D
Q242	Log_3 5 x $\log_5$ 4 x $\log_2$ 3 = _______. (a) 2 (b) 5 (c) -2 (d) None	A

Q243	Value of 16 Log $\frac{64}{60}$ + 12 Log $\frac{50}{48}$ + 7 Log $\frac{81}{80}$+ Log 2 is ________. (a) 0 (b) 1 (c) 2 (d) -1	B
Q244	7Log $\left(\frac{16}{15}\right)$ + 5 Log$\left(\frac{25}{24}\right)$ + 3Log $\left(\frac{81}{80}\right)$ = ________. (a) 0 (b) 1 (c) Log 2 (d) Log 3	C
Q245	$\log_3 \sqrt[4]{729 \sqrt[3]{9^{-1}.27^{\frac{4}{3}}}}$ = ________. (a) -5/3 (b) 5/3 (c) 3/5 (d) -3/5	B
Q246	If $x^{2a-3}y^{2a} = x^{6-a}y^{5a}$ then the value of a.Log (x/y) is ________. (a) 3 Log x (b) Log x (c) 6 Log x (d) 5 Log x	A
Q247	Log$[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-1/2}$ can be written as _______. (a) Log x^2 (b) Log x (c) Log 1/x (d) None	B
Q248	Log (a - 9) + Log a = 1, the value of 'a' is _______. (a) 0 (b) 10 (c) -1 (d) None	B
Q249	If $\frac{1}{\log_x 10} + 2 = \frac{2}{\log_5 10}$, then the value of x is _______. (a) 5 (b) 0.25 (c) 0.5 (d) 25	B
Q250	Find the value of x if Log$\left(x+\frac{1}{x}\right)$ + Log 2 = Log 5 (a) 0 (b) 3 or $\frac{1}{3}$ (c) $\frac{1}{2}$ or 2 (d) 1	C
Q251	If 3 + Log_{10}x = 2Log_{10}y; then value of x in terms of y will be ______. (a) (2/3) y (b) $Y^2/10$ (c) 10y (d) $Y^2/1000$	D
Q252	If $\text{Log}_{10}{}^{y}$ = 1 + 2 Log_{10} x - Log_{10} z; then value of $\frac{yz}{x^2}$ is ________. (a) 10 (b) $\frac{1}{10}$ (c) 100 (d) $\frac{1}{100}$	A
Q253	If $\frac{\text{Log}\, x}{2} = \frac{\text{Log}\, y}{3} = \frac{\text{Log}\, z}{5}$, then yz in terms of x is _______. (a) x (b) x^2 (c) x^3 (d) x^4	D
Q254	If $\frac{log_8 17}{log_9 23} - \frac{log_{2\sqrt{2}} 17}{log_3 23}$ = _________. (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 0	D
Q255	If Log_e M + Log_e N = Log_e (M + N), then find M as a function of N. (a) 1/N (b) N^2 (c) N^2 x (N - 1) (d) N/ (N -1)	D
Q256	On solving Log t + Log (t - 3) = 1 we get the value of t as (base 10) (a) 5 (b) 2 (c) 3 (d) 0	A
Q257	On solving the equation $\log_3[\log_2(\log_3 t)]$ = 1 we get value of t as ____. (a) 8 (b) 18 (c) 81 (d) 6,561	D
Q258	On solving $\text{Log}_{1/2}[\log_t(\log_4 32)]$ = 2 we get the value of t as ______.	C

	(a) 5/2 (b) 25/4 (c) 625/16 (d) None	
Q259	If Log $\left(\frac{a+b}{2}\right) = \frac{1}{2}$ (Log a + Log b), then (a) a = b/2 (b) a = b (c) a = b^2 (d) a^2 = b	**B**
Q260	If Log (x + y) = Log $\left(\frac{3x-3y}{2}\right)$, Log x - Log y = ________. (a) Log 2 (b) Log 3 (c) Log 5 (d) Log 6	**C**
Q261	If $\text{Log}_2[\text{Log}_3(\text{Log}_2\ x)] = 1$, then x = ________. (a) 128 (b) 256 (c) 512 (d) None	**C**
Q262	Value of $\text{Log}_2[\text{Log}_2\{\text{Log}_3(\text{Log}_3(27^3))\}]$ is ________. (a) 1 (b) 0 (c) 2 (d) 3	**B**
Q263	If $\text{Log}_e\ 2.\ \text{Log}_b\ 625 = \text{Log}_{10}16.\ \text{Log}_e10$, then b = ________. (a) 4 (b) 5 (c) 1 (d) e	**B**
Q264	Given that $\text{Log}_{10}2$ = x and $\text{Log}_{10}\ 3$ = y, the value of $\text{Log}_{10}\ 60$ is expressed as _______. (a) x - y + 1 (b) x + y + 1 (c) x - y - 1 (d) None	**B**
Q265	Sum of the series $\text{Log}_a\ b + \text{Log}_{a^2}\ b^2 + \text{Log}_{a^3}\ b^3 + \ldots \text{Log}_{a^n}b^n$ is given by ________. (a) $\text{Log}_a b^n$ (b) $\text{Log}_{a^n}b^n$ (c) n $\text{Log}_{a^n}b^n$ (d) None	**A**
Q266	Value of the following expression $a^{\log_a b . \log_b c . \log_c d . \log_d t}$ is given by ________. (a) t (b) abcdt (c) (a+b+c+d+t) (d) None	**A**
Q267	$\frac{1}{1+\log_a(bc)} + \frac{1}{1+\log_b(ca)} + \frac{1}{1+\log_c(ab)}$ is _____. (a) 0 (b) 1 (c) 3 (d) -1	**B**
Q268	Value of Log $\frac{a^n}{b^n}$ + Log $\frac{b^n}{c^n}$ + Log $\frac{c^n}{a^n}$ is _______. (a) 0 (b) 1 (c) -1 (d) None	**A**
Q269	If Log a = $\frac{1}{2}$ Log b = $\frac{1}{5}$Log c the value of $a^4b^3c^{-2}$ is _________. (a) 0 (b) 1 (c) -1 (d) None	**B**
Q270	If Log 2 + $\frac{1}{2}$Log a + $\frac{1}{2}$ Log b = Log (a + b), then ________. (a) a = b (b) a = - b (c) a = 2, b = 0 (d) a = 10, b = 1	**A**
Q271	If $a^3 + b^3 = 0$, then the value of Log (a + b) - $\frac{1}{2}$(Log a + Log b + Log 3) = ________. (a) 0 (b) 1 (c) -1 (d) 3	**A**
Q272	Log (x - y) - Log 5 - $\frac{1}{2}$ Log x - $\frac{1}{2}$ Log y = 0, then $\frac{x}{y} + \frac{y}{x}$ = _______. (a) 25 (b) 26 (c) 27 (d) 28	**C**
Q273	Given that Log 2 = 0.3010, Log 3 = 0.4771, The value of $\text{Log}_8\ 81$ is _______. (a) $\frac{9542}{4515}$ (b) $\frac{9442}{4515}$ (c) $\frac{4515}{9442}$ (d) None	**A**
Q274	Value of $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$ is _________. **[Q80. Pg 3.18 of SC]**	**C**

	(a) Log 2 (b) 1 (c) 0 (d) None	
Q275	If x = $\log_{2a}$ a; y = $\log_{3a}$ 2a; z = $\log_{4a}$ 3a; xyz + 1 = **[Q109 Pg 3.20 of SC]** (a) 2xy (b) 2yz (c) 2zx (d) None	B
Q276	If $\text{Log}_a b = \text{Log}_b c = \text{Log}_c a$, then ________. (a) a> b > c (b) a < b < c (c) a = b = c (d) a < b < c	C
Q277	If $\text{Log}_a(ab)$ = x, then $\text{Log}_b(ab)$ is _________. (a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$ (c) $\frac{x}{x-1}$ (d) $\frac{x}{1-x}$	C
Q278	Value of $\frac{\text{Log}\, a\, (log_b a)}{\text{Log}\, a\, (log_a b)}$ is ______. (a) -1 (b) 1 (c) Log_a b (d) Log_a (ab)	A
Q279	If a = b^2 = c^3 = d^4 then the value of $\text{Log}_a(abcd)$ (a) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ (b) $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}$ (c) 1 + 2 + 3 + 4 (d) None	A
Q280	Find value of LM + MN + NL - LMN, if L = 1 + Log_abc; M = 1 Log_bca; N = 1 + Log_cab. **[Q114 Pg 3.20 of SC]** (a) 0 (b) 1 (c) -1 (d) 3	A
Q281	If $a^2 + b^2$ = 7ab, then the value of $\text{Log}(\frac{a+b}{3}) - \frac{\text{Log}\, a}{2} - \frac{\text{Log}\, b}{2}$ is _______. (a) 0 (b) 1 (c) -1 (d) 7	A
Q282	If $x^2 + y^2$ =11xy, then 2 Log (x - y) = ______. (a) Log 3 + Log x + Logy (b) 3Log3 + Log x + Log y (c) 2.Log 3 + Log x + Log y (d) None	C
Q283	If $a^3 + b^3$ = 0; then Log (a + b) - $\frac{1}{2}$ (Log a + Log b + Log 3) = ________. [Hint: $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$] (a) 0 (b) 1 (c) -1 (d) None	A
Q284	If $\frac{\text{Log}\, x}{l+m-2n} = \frac{\text{Log}\, y}{m+n-2l} = \frac{\text{Log}\, z}{n+l-2m}$, then $x^2y^2z^2$ = _______. (a) 2 (b) -1 (c) 4 (d) 1	D
Q285	If Log_abc = x, Log_bca = y, Log_cab = z, $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$ = ________. (a) 0 (b) 3 (c) x+y+z (d) 1	D
Q286	If $\frac{\text{Log}\, x}{q-r} = \frac{\text{Log}\, y}{r-p} = \frac{log}{p-q}$, $x^{q+r}y^{r+p}z^{p+q}$= ________. (a) $x^py^qz^x$ (b) 1 (c) 0 (d) xyz	A
Q287	If $\text{Log}_2(3^{2x-2} + 7) = 2 + \text{Log}_2(3^{x-1} + 1)$ then x = ________. (a) 0 (b) 1 (c) 2 (d) 1 or 2	D
Q288	Value of $\text{Log}_5\left(1+\frac{1}{5}\right) + \text{Log}_5\left(1+\frac{1}{6}\right) + \text{Log}_5\left(1+\frac{1}{7}\right) + \text{Log}_5\left(1+\frac{1}{624}\right)$ is ________. (a) 5 (b) 4 (c) 3 (d) 2	C

Q289	$\text{Log}\left\{log_{ab}a + \frac{1}{log_b ab}\right\}$ = ________. (a) Log ab (b) 1 (c) 0 (d) None	C
Q290	Log (1 x 2 x 3) = _______. (a) Log2 (b) Log 3 (c) 1 (d) Log1+Log2+Log3	D
Q291	Log (3 + 7) = _____. (a) 1 (b) 3 (c) 0 (d) ∞	A
Q292	Log ($1^2 + 2^2 + 3^2$) = _________. (a) Log 2 - Log 7 (b) Log 2 + Log 7 (c) 1 (d) None	B
Q293	Log (3 - 2) = _______. (a) 4 (b) 3 (c) 0 (d) ∞	C
Q294	$\text{Log}_2 8$ = ________. (a) 2 (b) 8 (c) 3 (d) None	C
Q295	$log_{2\sqrt{3}}1728$ = _______. (a) $2\sqrt{3}$ (b) 2 (c) 6 (d) None	C
Q296	If $\text{Log}_a\sqrt{2}$ = 1/6, find the value of 'a' (a) 8 (b) 4 (c) 3 (d) 1	A
Q297	Logarithm of 21952 to the base of $2\sqrt{7}$ & 19683 to the base of $3\sqrt{3}$ are. (a) Equal (b) Not equal (c) Different (d) None	A
Q298	Given Log 2 = 0.03010 and Log3 = 0.4771 the value of Log 6 is _________. (a) 0.9030 (b) 0.9542 (c) 0.7781 (d) None	C
Q299	$\frac{1}{2}\text{Log}_{10} 25 - 2\,\text{Log}_{10}3 + \text{Log}_{10}18$ = ________. (a) 0 (b) 1 (c) $\text{Log}_{10} 3$ (d) None	B
Q300	Log $\frac{75}{16}$ - 2 Log $\frac{5}{9}$ + Log $\frac{32}{243}$ reduces to ________. (a) 2 Log 2 (b) 5 Log 2 (c) Log 2 (d) 4 Log 2	C
Q301	$\text{Log}_b(a).\ \text{Log}_c(b)\ \text{Log}_a(c)$ = ________. (a) 0 (b) 1 (c) -1 (d) None	B
Q302	$\text{Log}_{10}(x^2 - 6x + 10) = 0$; then x = ________. (a) 2 (b) 3 (c) 4 (d) None	B
Q303	$\text{Log}_5 3.\ \text{Log}_7 5.\ \text{Log}_9 7.\ \text{Log}_{11} 9.\ \text{Log}_{21} 11$ = _______. (a) $\text{Log}_{21} 3$ (b) $\text{Log}_3 21$ (c) 1 (d) None	A
Q304	Value of Log (1+2+3+ .... +n) = _________. (a) Log 1 + Log 2 + ... + Log n (b) Log n + Log (n+1) - Log 2 (c) 0 (d) 1	B
Q305	The equivalent form of the equation Log (x-2) + Log (x+3) = 0 is _________.	C

	(a) $x^2 + x - 5 = 0$ (b) $x^2 - x - 5 = 0$ (c) $x^2 + x - 7 = 0$ (d) None	
Q306	$\frac{1}{log_{ab}(abc)} + \frac{1}{log_{bc}(abc)} + \frac{1}{log_{ca}(abc)}$ is ________. (a) 0 (b) 1 (c) 2 (d) -1	**C**
Q307	Value of $Log\frac{a^2}{bc}$ + Log $\frac{b^2}{ca}$ + Log $\frac{c^2}{ab}$ is _________. (a) 0 (b) 1 (c) -1 (d) None	**A**
Q308	If $\frac{Log\,a}{y-z} = \frac{Log\,b}{z-x} = \frac{Log\,c}{x-y}$; value of abc is ________. (a) 0 (b) 1 (c) -1 (d) None	**B**
Q309	If $\frac{1}{2}$ Log a = $\frac{1}{3}$ Log b = $\frac{1}{5}$ Log c; value of a^4 - bc is ________. (a) 0 (b) 1 (c) -1 (d) None	**A**
Q310	If $\frac{1}{log_a t} + \frac{1}{log_b t} + \frac{1}{log_c t} = \frac{1}{log_z t}$ then the value of z is ________. (a) abc (b) a+b+c (c) a(b+c) (d) (a+b) c	**A**
Q311	Value of $\frac{1}{log_{xy}(xyz)} + \frac{1}{log_{yz}(xyz)} + \frac{1}{log_{zx}(xyz)}$ is _________. (a) Log xyz (b) 1 (c) 2 (d) None	**C**
Q312	If $a^2 + b^2 = c^2$, $\frac{1}{log_{b+c}a} + \frac{1}{log_{c-b}a}$ is _________. (a) 2 (b) 1 (c) Log abc (d) 0	**A**
Q313	Log 6 +log 5 is expressed as ________. (a) Log11 (b) Log 30 (c) Log 5/6 (d) None	**B**
Q314	Log 32/4 is equal to ________. (a) log 32 / log 4 (b) log 32 - log4 (c) 8 (d) None	**B**
Q315	Given log 2 = 0.3010 and log 3 = 0.4771 then the value of log 24 ________. (a) 1.3081 (b) 1.1038 (c) 1.3801 (d) 1.8301	**C**
Q316	If log_2 x + log_4 x + log_{16} x = $\frac{21}{4}$,then x equals to ________. (a) 8 (b) 4 (c) 2 (d) 16	**A**
Q317	The simplified value of log_2. log_2 $log_2 16$ is ________. (a) 0 (b) 2 (c) 1 (d) None	**C**
Q318	Find the value of $[log_y{}^x . log_z y . log_x z]^3$ = ________. (a) 0 (b) − 1 (c) 1 (d) 3	**C**
Q319	If $\frac{1}{log_a t} + \frac{1}{log_b t} + \frac{1}{log_c t} = \frac{1}{log_z t}$ then the value of z is ________. (a) abc (b) a+b+c (c) a(b+c) (d) (a+b) c	**A**
Q320	If log x = a+b; log y = a-b then log $\frac{10\,x}{y^2}$ = ________. (a) 1-a+3b (b) a-1+3b (c) a+3b+1 (d) 1-b+3a	**A**

Q321	X = 1+ $\log_p qr$, y = 1+ $\log_q rp$, z = 1+$\log_r pq$ then find $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ = ________. (a) 0 (b) 1 (c) 2 (d) -1	**B**
Q322	If x = $Log_a bc$ y = $Log_b ca$ z = $Log_c ab$ then value of xyz - x - y - z is______. (a) 0 (b) 1 (c) -1 (d) 2	**D**
Q323	If x = Log_{2a} a, y = Log_{3a} 2a, z = Log_{4a} 3a then xyz + 1 = _______. (a) 2 xy (b) 2yz (c) 2zx (d) None	**B**
Q324	If $\frac{1}{log_a x}+\frac{1}{log_c x}=\frac{2}{log_b x}$, then a, b, c are in _______. (a) G.P (b) A.P (c) H.P (d) None	**A**
Q325	3.Log x + 3.Log x^3 + 3.Log x^5 + ... + 3.Log x^{2n-1} = _______. (a) $3n^2$ Log x (b) n(n+1) Log a (c) 3n(n+1)Log a (d) None	**A**
Q326	If x = 1983! ; then value of $\frac{1}{log_2 x}+\frac{1}{log_3 x}+\frac{1}{log_4 x}+ \dots +\frac{1}{log_{1983} x}$ is _______. (a) 0 (b) 1 (c) 2 (d) 3	**B**
Q327	Find the number of digits in 2^{64}. [Given that Log 2 = 0.3010] (a) 19 (b) 20 (c) 21 (d) 16.	**B**
Q328	If $Log_4 (x^2 + x) - Log_4(x + 1) = 2$, then the value of x is ________. (a) 2 (b) 4 (c) 8 (d) 16	**D**
Q329	Log_{10} 10 + Log_{10} 100 + Log_{10} 1000 + Log_{10} 10000 + Log_{10} 100000 is __________. (a) 15 (b) Log_{10} 11111 (c) Log_{10} 1111 (d) 14 Log_{10} 100	**A**
Q330	$\frac{1}{log_{a/b}(x)}+\frac{1}{log_{b/c}(x)}+\frac{1}{log_{c/a}(x)}$ is _______. (a) 0 (b) 1 (c) 3 (d) -1	**A**
Q331	$Log_b (a^{1/2}) Log_c (b^3) Log_a(c^{3/2})$ = ________. (a) 0 (b) 1 (c) 4/9 (d) 9/4	**D**
Q332	If $\text{Log}\frac{m}{n}+\text{Log}\frac{n}{m}$ = Log (m + n), then ________. (a) m + n = 1 (b) $\frac{m}{n}$ (c) m - n = 1 (d) $m^2. n^2 = 1$	**A**
Q333	If Log_{10} 2986 = 3.4751, then Log_{10} 0.02986 = ________. (a) 1.5249 (b) $\bar{2}.4751$ (c) 1.2986 (d) $-$ 1.5249	**B & D**
Q334	2Log(a + b) + Log(a - b) - Log($a^2 - b^2$) = Log x, then x = _______. (a) (a + b) (b) a - b (c) $a^2 - b^2$ (d) None	**A**
Q335	If $a^2 + b^2 = 0$, and a + b ≠ 0 then the value of Log (a + b) is ______. (a) Log a + Log b + Log 2 (b) $\frac{1}{2}$(Log a + Log b + Log 2) (c) Log a + Log b (d) None	**B**
Q336	If $Log_{x+2}(x^3 - 3x^2 - 6x + 8) = 3$, then x = _________. (a) 2 (b) -2 (c) 1/2 (d) None	**B**

Q337	If Log $\frac{x+y}{5} = \frac{1}{2}$(Log x + Log y), then $\frac{x}{y} + \frac{y}{x}$ = ________. (a) 20 (b) 23 (c) 22 (d) 21	B
Q338	If Log $\frac{a+b}{3} = \frac{1}{2}$ (Log a + Log b) then the value $\frac{a}{b} + \frac{b}{a}$ is ________. (a) 2 (b) 5 (c) 7 (d) 3	C
Q339	If Log $\frac{x+y}{7} = \frac{1}{2}$ (Log x + Log y), then ________. (a) $\frac{x}{y} + \frac{y}{x}$= 48 (b) $\frac{x}{y} + \frac{y}{x}$ = 49 (c) $\frac{x}{y} + \frac{y}{x}$ = 47 (d) None	C
Q340	If Log (2a - 3b) = Log a - Log b, then a = _________. (a) $3b^2/(2b - 1)$ (b) 3b/(2b - 1) (c) $b^2/(2b + 1)$ (d) $3b^2(2b + 1)$	A
Q341	If $\frac{\text{Log } 3}{x-y} = \frac{\text{Log } 5}{y-z} = \frac{\text{Log } 7}{z-x}$, then $3^{(x+y)} . 5^{(y+z)} . 7^{(z+x)}$ = _______. (a) 2 (b) 10 (c) 1 (d) 0	C
Q342	If Log_{30} 3 = a, Log_{30} 5 = b, then Log_{30} 8 = ____. [Hint: Find (a + b)] (a) 3(1 - a - b) (b) (a - b + 1) (c) (a + b) (d) 1(a - b + 1)	A
Q343	If x = $\text{Log}_a bc$, y = $\text{Log}_b ca$, z = $\text{Log}_c ab$, then ________. (a) xyz = x + y + z + 2 (b) xyz = x + y + z + 1 (c) x + y + z = 1 (d) xyz = 1	A
Q344	If a = Log_{24} 12, b = Log_{36} 24, and c = Log_{48} 36, then 1 + abc = ________. (a) 1 (b) 2 (c) 2bc (d) abc	C
Q345	If x = Log_{2a} a, y = Log_{3a} 2a, z = Log_{4a} 3a then value of yz (2 - x) is ______. (a) 1 (b) -1 (c) 2 (d) -2	A
Q346	$(bc)^{\log \frac{b}{c}} . (ca)^{\log \frac{c}{a}} . (ab)^{\text{Log} \frac{a}{b}}$ = ________. [Hint: Equate it as x & then take log] (a) 0 (b) 1 (c) -1 (d) None	B
Q347	$X^{18} = y^{21} = z^{28}$, then 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in ________. (a) AP (b) GP (c) HP (d) None	A

CHAPTER 2. EQUATION

INTRODUCTION

- **Meaning of Equation:** Equation is defined to be a mathematical statement of equality.
  (Two algebraic expressions are connected by sign of equality (=), they form an equation).
- **Conditional Equation:** If the equality is true for some variables, it is conditional equation.
- **Identity:** If the given equality is true for all variables, it is called an identity.
  [When LHS = RHS for all the values of variables]

  Ex: $\frac{x+2}{3} + \frac{x+3}{2} = 3$ is true only for x =1. So it is a conditional equation.

  Identity: $\frac{x+2}{3} + \frac{x+3}{2} = \frac{5x+13}{6}$ is an identity since it satisfy all the values of 'variable x'.

  Variable: It is a quantity whose value varies (changes). Generally denoted by x, y, z.

  Constant: It is a quantity whose value does not change. Generally denoted by a, b, c.

  Solution/Root: Value of variable which satisfies equation. [LHS=RHS when substituted].

SOME IMPORTANT POINTS TO BE KEPT IN MIND WHILE SOLVING THE QUESTIONS:

- Addition/subtraction of same quantity to both sides of an equation does not change equ^n.
- Multiplication/Division of same non- zero number to both sides of an equation does not change the equation.

TRANSPOSITION RULE: Any term of equation taken to the other side by changing its sign.

Transposition is done to take unknown quantities to one side & known quantities to other side

- A term may be transferred from one side to another side by changing its sign.
  [+ve to —ve **or** —ve to +ve]
- A Multiplier may be removed from one side by making it divisor on other side of equation.
- A Divisor may be removed from one side by making it multiplier on other side of equation.

CONCEPT 1: LINEAR EQUATION IN ONE VARIABLE [Highest Degree = 1]

- An equation in which highest power of the variable is **1** is called a Linear (simple) equation.
- A simple equation has only one root.
- It is in the form $ax + b = 0$; (Where a, b are numbers)

CONCEPT 2: SIMULTANEOUS LINEAR EQUATION IN TWO VARIABLES [Highest Degree = 1]

- General form → **ax + by + c = 0**; [a, b ≠ 0 & a, b, c → Constant].

Methods of solving simultaneous linear equation in two variables:

Substitution Method: Any one variable is written in terms of another variable in any one equation & then obtained value is substituted in other equation.

CQ8: Solve: 6x + 5y - 16 = 0 and 3x - y - 1 = 0 we get values of x, y as

Solution: 6x + 5y - 16 = 0 --------(i) and 3x - y - 1 = 0 --------------(ii)

Now from (2), we get y = 3x — 1 ----------(iii);

Substitute the value of y in (i), 6x + 5(3x — 1) — 16 = 0.

6x + 15x — 5 — 16 = 0; 21x — 21 = 0; 21x = 21; **x = 1**

Now Put x = 1 in (iii); we get y = 3(1) — 1 = 3 — 1 = 2. **Thus (x, y) = (1, 2)**

PC Note:

- **Sign of variable with same co-efficient is opposite → Add the equations.**
- **Sign of variable with same co-efficient is same → Subtract the equations.**

TEST OF CONSISTENCY FOR A SYSTEM OF EQUATIONS [$a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$]

- Consistent System → System having **at least one Solution.**
- Inconsistent System → System having **NO Solution.**

No. of Solutions	Condition	System of Equations	Lines intersect at
Unique Solution	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Consistent	One Point
No solution	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Inconsistent	Parallel
Infinite solutions	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Consistent	Coincident

SIMULTANEOUS LINEAR EQUATION WITH THREE VARIABLES → Solve by Option Method.

CONCEPT 3: QUADRATIC EQUATION [Highest degree = 2]

- **General form** $ax^2 + bx + c = 0$; where a ≠ 0 & a, b, c → Constant.
- A quadratic equation has got **two roots**.
- **Pure QE:** If b = 0; → **Affected QE:** When b ≠ 0 **[Not for Exam]**

CONSTRUCTION OF A QUADRATIC EQUATION

1. We have $ax^2 + bx + c = 0$
2. Dividing it by 'a', we will get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
3. Take '—' common from b, $x^2 - (-\frac{b}{a})x + \frac{c}{a} = 0$
4. **x^2 —(sum of roots)x + Product of roots = 0**

ROOTS OF A QUADRATIC EQUATION

(1) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ (2) $\frac{-b-\sqrt{b^2-4ac}}{2a}$

Adding (1) & (2), we get $(-\frac{b}{a})$ & Multiplying (1) & (2), we get $\frac{c}{a}$.

$b^2 - 4ac$ → Discriminant

PC Note: Sum of roots = $(-\frac{b}{a})$ & Product of roots = $\frac{c}{a}$

NATURE OF THE ROOTS

Value of $b^2 - 4ac$	Nature of Roots	Example	Roots
Zero	Real, Equal & rational	$x^2 - 6x + 9 = 0$	3, 3
Perfect Square	Real, unequal & rational	$x^2 - 6x - 16 = 0$	8, -2
Not a Perfect Square	Real, unequal & irrational	$x^2 - 6x + 7 = 0$	$(3 + \sqrt{2}), (3 - \sqrt{2})$
Negative	Imaginary (Complex No.)	$x^2 - 6x + 7 = 0$	No Solution

POINTS TO BE NOTED

- **Irrational** roots occur in **conjugate pairs**. One root is **$(a + \sqrt{b})$**, other root will be **$(a - \sqrt{b})$**.
- Roots are equal in magnitude (value) but opposite in sign, Sum of roots = 0 & so $\frac{b}{a} = 0$ & **b=0.**
- If one root is reciprocal to other root, then their product is 1 & thus $\frac{c}{a} = 1$ i.e. **c = a.**

CQ10: Examine the nature of the roots of $x^2 - 8x^2 + 16 = 0$ [Real & Equal]

CQ11: Examine the nature of the roots of $3x^2 - 8x + 4 = 0$ [Real, rational & unequal]

CQ12: Examine the nature of the roots of $5x^2 - 4x + 2 = 0$ [Imaginary]

CQ13: Examine the nature of the roots of $2x^2 - 6x - 3 = 0$ [Real, irrational & unequal]

SOME USEFUL RESULTS REQUIRED TO SOLVE QUESTIONS OF ROOTS OF QUADRATIC EQ^n

$(a + b)^2 = a^2 + b^2 + 2ab \rightarrow a^2 + b^2 = (a + b)^2 - 2ab$	$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$	$\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2+b^2}{(ab)^2}$	$\frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$
$a - b = \sqrt{(a+b)^2 - 4ab}$			
$a^2 - b^2 = (a + b)(a - b)$			
$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$	$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$		

ROOTS OF EQUATION

- If $p + \sqrt{q}$ is the root, then $p - \sqrt{q}$ is also a root.
- If p + iq is a root, then p - iq is also a root. (Where $i^2 = -1$)
- **Sum of the roots = $\alpha + \beta$ = -b/a,** **Product of the roots = $\alpha\beta$ = c/a.**
- An equation with roots α & β is given by $(x-\alpha)(x-\beta) = 0$, $\mathbf{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$
- If one root is reciprocal of the other roots $(\alpha, 1/\alpha)$ their product is also 1. Also a = c
- If roots are equal in magnitude but opposite in sign $(\alpha, -\alpha)$ then b will be 0
- If a + b + c = 0, then one of the roots = 1, and the other root = c/a [**Ex.** $x^2 + 5x - 6$]
- If a - b + c, then one root is -1 and other is -c/a [**Ex.** $x^2 + 6x + 5 = 0$]
- If α & β are the roots of $ax^2 + bx + c = 0$, then $1/\alpha$, $1/\beta$ will be roots of $cx^2 + bx + a = 0$

RELATIONSHIP BETWEEN SIGN OF A, B, C AND THE ROOTS

Sign of a, b, c	a & c same and b opposite	a, b, c same sign	a & c opposite sign
Sign of Roots	Both are Positive	Both are Negative	Opposite Sign

USEFUL FACTORS TO GET SUM & PRODUCT OF ROOTS I.E. $(\alpha+\beta)$ & $\alpha\beta$

1) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - \alpha\beta$
2) $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$
3) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$
4) $\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$
5) $(\alpha-\beta) = \sqrt{(\alpha+\beta)^2-4\alpha\beta}$
6) $\alpha^2-\beta^2 = (\alpha+\beta)(\alpha-\beta)$
7) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{(\alpha+\beta)}{\alpha\beta}$
8) $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$
9) $\frac{1}{\beta}-\frac{1}{\alpha} = \frac{(\alpha-\beta)}{\alpha\beta}$

CUBIC EQUATION [Highest degree = 3]

- **Format of Cubic equation → $ax^3 + bx^2 + cx + d$ [Where a, b, c, d are number & $a \neq 0$]**

  PC Note: Solve by Option Method to save time & efforts in Exams.

RELATION BETWEEN ROOTS AND CO-EFFICIENT

1) $\alpha+\beta+\gamma = \frac{-b}{a}$
2) $\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a}$
3) $\alpha\beta\gamma = \frac{-d}{a}$
4) $\alpha^2+\beta^2+\gamma^2 = \frac{b^2-2ac}{a^2}$
5) $\alpha^3+\beta^3+\gamma^3 = \frac{3abc-b^3-3a^2d}{a^3}$

RELATIONSHIP BETWEEN SIGN OF a, b, c, d AND OF THE ROOTS

Sign of a, b, c, d	a, b, c, d are Positive	a, c same & b, d opposite sign	a, c same & b, d = 0
Sign of Roots	All roots are Negative	All roots are Positive	No real roots except 0

EQUATIONS – QUESTION BANK

SN	CHAPTER 2. EQUATION	Ans
Q1	If 2x+y = y+14, then x is ______. (a) $\frac{7}{2}$ (b) 7 (c) 14 (d) 21	B
Q2	The root of the equation $\frac{x+4}{4}+\frac{x-5}{3}=11$ is ______. (a) 20 (b) 10 (c) 2 (d) None	A
Q3	The root of the equation $\frac{x+24}{5}=4+\frac{x}{4}$ is _______. (a) 6 (b) 10 (c) 16 (d) None	C
Q4	Pick up the correct value of x for which $\frac{x}{30}=\frac{2}{45}$ (a) x = 7 (b) x = 5 (c) x = 4/3 (d) None	C
Q5	For the values of p = - 5 and q = 6, the value of the expression 6p + 5q is _____. (a) 2 (b) -1 (c) 0 (d) 1	C
Q6	If $\frac{1}{2}x+\frac{1}{4}x+\frac{1}{8}x$ = 14, then x is ______. (a) 4 (b) 8 (c) 12 (d) 16	D
Q7	If 2x+5 = -25 and -3y -6 = 48, then xy is ______. (a) -270 (b) -90 (c) 90 (d) 270	D
Q8	The equation $\frac{12x+1}{4}=\frac{15x-1}{5}+\frac{2x-5}{3x-1}$is true for _______. (a) x = 1 (b) x = 2 (c) x = 5 (d) x = 7	D
Q9	If $\frac{3}{4}=\frac{6}{x}=\frac{9}{y}$, then x + y (a) 4 (b) 8 (c) 12 (d) 20	D
Q10	*If* $\frac{a-b}{b}=\frac{2}{3}$, *what is the value of* $\frac{a}{b}$? (a) 1/2 (b) 3/5 (c) 3/2 (d) 5/3	D
Q11	Solve the equation 17^{3-6x} = 1 for x = _______. (a) -3 (b) 3/2 (c) ½ (d) -1/2	C
Q12	Pick up the correct value of x for which$\frac{x}{0.5}-\frac{1}{0.05}+\frac{x}{0.005}-\frac{1}{0.0005}=0$ (a) x = 0 (b) x =1 (c) x = 10 (d) None	C
Q13	Which one is a linear equation? (a) ax + b < 0 (b) ax + b >0 (c) Both (a) or (b) (d) Not (a) & (b)	D
Q14	Solve $2^{x-2}+2^{3-x}=3$ (a) x = 5 or x = 4 (b) x = 3 or x = 5 (c) x = 2 or x = 3 (d) x = 1 or x = 2	C
Q15	Solving $9^x=3^y$ & $5^{x+y+1}=25^{xy}$ we get the following roots (a) (1, 2), $\left(\frac{-1}{4},\frac{-1}{2}\right)$ (b) 0, 1, 3 (c) 0, 3 (d) 1, 3.	A
Q16	If $2^{2x+3}-3^2.2^x+1=0$ then values of x are _________. (a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, -3	D

Q17	If 6 = 2x + 4y, what is the value of x + 2y is ______. (a) 2 (b) 3 (c) 6 (d) 8	**B**
Q18	Solve for y in the equation $\frac{y+11}{6}-\frac{y+1}{9}=\frac{y+7}{4}$ and the value of y is ______. (a) -1 (b) 7 (c) 1 (d) $-\frac{1}{7}$	**D**
Q19	The solution of the equation (p + 2) (p - 3) + (p + 3) (p - 4) = p (2p - 5) is _______. (a) 6 (b) 7 (c) 5 (d) None	**A**
Q20	The satisfying values of x for the equation $\frac{1}{x+p+q}=\frac{1}{x}+\frac{1}{p}+\frac{1}{q}$ are ________. (a) (p, q) (b) (-p, -q) (c) (p, -q) (d) (-p, q)	**B**
Q21	If $\frac{a}{2}+\frac{b}{2}$ = 3, what is the value of 2a+2b? (a) 6 (b) 8 (c) 12 (d) 16	**C**
Q22	If a+b=5 and $\frac{c}{2}$ = 3, what is the value of 2a+2b+2c? (a) 14 (b) 16 (c) 22 (d) 20	**C**
Q23	If a-b=p and a+b=k, then a^2-b^2 (a) pk (b) $p^2 - k^2$ (c) p + k (d) $\frac{p^2}{k^2}$	**A**
Q24	If b(x+2y) = 60 and by = 15, what is the value of bx? (a) 20 (b) 25 (c) 30 (d) 45	**C**
Q25	If xy + z = y, what is x in terms of y and z? (a) $\frac{y+z}{y}$ (b) $\frac{y-z}{y}$ (c) 1 - z (d) $\frac{z-y}{y}$	**B**
Q26	If $\frac{1}{p+q}$ = r and p ≠ -q, what is p in terms of r and q? (a) $\frac{rq-1}{q}$ (b) $\frac{1+rq}{q}$ (c) $\frac{r}{1+rq}$ (d) $\frac{1-rq}{r}$	**D**
Q27	If $\frac{xy}{x+y}$ = 1 and x ≠ y, what is x in terms of y? (a) $\frac{y+1}{y-1}$ (b) $\frac{y+1}{y}$ (c) $\frac{y}{y-1}$ (d) $\frac{y}{y+1}$	**C**
Q28	The solution of the set of equations 3x + 4y = 7 & 4x — y = 3 is ________. (a) (1, -1) (b) (1, 1) (c) (2, 1) (d) (1, -2)	**B**
Q29	Solve for x and y: x - 3y = 20, y - 2x = 0. The values of x and y are given as ________. (a) x = 4 y = 12 (b) x = 12 y = 4 (c) x = 5 y = 4 (d) None	**D**
Q30	The simultaneous equations 7x - 3y = 31 and 9x - 5y = 41 have solutions given by (a) (-4-1) (b) (-14) (c) (4-1) (d) (3 7)	**C**
Q31	$\frac{x}{p}+\frac{y}{q}$= 2; x + y = (p + q) are satisfied by the values given by the pair ________. (a) (x = p y = q) (b) (x = q y = p) (c) (x = 1 y = 1) (d) None	**A**
Q32	The values of x and y satisfying the equations $\frac{x}{2}+\frac{y}{3}$= 2; x + 2y = 8 are ________. (a) (3, 2) (b) (-2, -3) (c) (2, 3) (d) None	**C**
Q33	Which of the following sets (x, y) will satisfy the equation $23^{xy} = 23^{yx}$& $144^x = 12^y$ (a) (1,1) (b) (0,1) (c) (1,2) (d) (2,1)	**C**

Q34	If $\frac{1}{x}+\frac{1}{y}=\frac{1}{4}$ and $\frac{1}{x}-\frac{1}{y}=\frac{3}{4}$, then x is ______. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2	D
Q35	$\frac{3}{x+y}+\frac{2}{x-y}=3$; $\frac{2}{x+y}+\frac{3}{x-y}=3\frac{2}{3}$. Find the values of x & y which satisfy the equations (a) (1, 2) (b) (-1, -2) (c) $\left(1,\frac{1}{2}\right)$ (d) (2, 1)	D
Q36	When the system is inconsistent, there is _______ solution. (a) No (b) Finite (c) Infinite (d) Identical	A
Q37	$2^x . 4^y = 32$ & $3^x \div 9^y = 3$. Find the solution set. (a) x = 3, y = 1 (b) x = y = 2 (c) x = y = 1 (d) x = y = 3	A
Q38	Solve for x and y: $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10}$ and 3xy = 10(y - x). The value of x and y is ________. (a) (5, 2) (b) (-2, -5) (c) (2, -5) (d) (2, 5)	D
Q39	The pair satisfying the equations x + 5y = 36, $\frac{x+y}{x-y}=\frac{5}{3}$ is given by (a) (16, 4) (b) (4, 16) (c) (4, 8) (d) None	A
Q40	Solve for x, y and z: 2x — y + z = 3; x + 3y - 2z = 11; 3x - 2y + 4z = 1. (a) x= -5, y=4, z= -2 (b) x = 3, y=2, z = -1 (c) x=3, y= -3, z=6 (d) x= -8, y= -5, z= -1	B
Q41	Solve for x, y and z: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=5$; $\frac{2}{x}-\frac{3}{y}-\frac{4}{z}=-11$, $\frac{3}{x}+\frac{2}{y}-\frac{1}{z}=-6$ (a) $x=\frac{1}{2}, y=-\frac{1}{3}, z=\frac{1}{6}$ (b) $x=\frac{1}{2}, y=-\frac{3}{5}, z=\frac{2}{5}$ (c) $x=\frac{4}{5}, y=-\frac{2}{5}, z=\frac{1}{6}$ (d) $x=-\frac{1}{2}, y=\frac{1}{3}, z=-\frac{1}{6}$	A
Q42	Solve for x, y and z: $\frac{xy}{x+y}=70, \frac{xz}{x+z}=84, \frac{yz}{y+z}=140$ (a) x = 105, y = 210, z = 420 (b) x=60, y=80, z=140 (c) x = 100, y = 200, z = 300 (d) x=120, y=150, z=450	A
Q43	Solving 9x + 3y - 4z = 3x + y - z = 0 and 2x - 5y - 4z = -20 following roots as obtained (a) 2, 3, 4 (b) 1, 3, 4 (c) 1, 2, 3 (d) None	C
Q44	$\frac{x}{4}=\frac{y}{3}=\frac{z}{2}$ 7x + 8y + 5z = 62. Solve (a) (4, 3, 2) (b) (2, 3, 4) (c) (3, 4, 2) (d) (4, 2, 3)	A
Q45	$\frac{xy}{x+y}=20$, $\frac{yz}{y+z}=40$, $\frac{zx}{z+x}=24$. Solve (a) (120, 60, 30) (b) (60, 30, 120) (c) (30, 120, 60) (d) (30, 60,120)	D
Q46	2x + 3y + 4z = 0, x + 27y - 5z = 0, 10x + 16y - 6z = 0 Solve. (a) (0, 0, 0) (b) (1, -1, 1) (c) (3, 2, -1) (d) (1, 0, 2)	A
Q47	$\frac{xy}{y-x}=110$, $\frac{yz}{z-y}=132$, $\frac{zx}{z+x}=\frac{60}{11}$. Solve (a) (12, 11, 10) (b) (10, 11, 12) (c) (11, 10, 12) (d) (12, 10, 11)	B
Q48	Find values of x, y and z -3x - 4y + 70z — 0, 2x + 3y - 10z = 0, x + 2y + 3z = 13 (a) (1, 3, 7) (b) (1, 7, 3) (c) (2, 4, 3) (d) (-10, 10, 1)	D
Q49	If α & β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta}-\frac{\beta^2}{\alpha}$ is ________.	D

	(a) $2\sqrt{5}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) $-2\sqrt{5}$	
Q50	If one roots of $5x^2 + 13x + p = 0$ be reciprocal of the other then the value of p is (a) -5 (b) 5 (c) 1/5 (d) -1/5	**B**
Q51	If one root of equation $x^2 + 7x + p = 0$ be reciprocal of the other then value of p is ________. (a) 1 (b) -1 (c) 7 (d) -7	**A**
Q52	If one root of the equation is $2 - \sqrt{3}$, form the equation. (a) $x^2 - 2x + 2 = 0$ (b) $x^2 - 3x + 1 = 0$ (c) $x^2 - 5x + 5 = 0$ (d) $x^2 - 4x + 1 = 0$	**D**
Q53	Root of the equation $x^2 - 8x + m = 0$ exceeds the other by 4 then the value m is ____. (a) m = 10 (b) m = 11 (c) m = 9 (d) m = 12	**D**
Q54	If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal then value of m is ______. (a) -3 (b) -1 (c) 1 (d) -2	**D**
Q55	Equation $\left(\frac{1-m}{2}\right)x^2 - \left(\frac{1+m}{2}\right)x + m = 0$ has got two values of x to satisfy equation given as _____. (a) $\left(1, \frac{2m}{1-m}\right)$ (b) $\left(1, \frac{m}{1-m}\right)$ (c) $\left(1, \frac{21}{1-m}\right)$ (d) $\left(1, \frac{1}{1-m}\right)$	**A**
Q56	The values of $4+\frac{1}{4+\frac{1}{4+\frac{1}{4+\frac{1}{4+\cdots\infty}}}}$ (a) $1 \pm\sqrt{2}$ (b) $2 \pm\sqrt{5}$ (c) $2 \pm\sqrt{3}$ (d) None	**B**
Q57	The condition that one of the roots of $ax^2 + bx + c = 0$ is twice the other is _______. (a) $b^2 = 4ca$ (b) $2b^2 = 9(c + a)$ (c) $2b^2 = 9ca$ (d) $2b^2 = 9(c - a)$	**A**
Q58	The roots of the equation $x^2 + kx + 12$ will differ by unity only if (a) $k = \pm\sqrt{12}$ (b) $k = \pm\sqrt{48}$ (c) $k = \pm\sqrt{47}$ (d) $k = \pm 7$	**D**
Q59	If the roots of $ax^2 + bx + c = 0$ are in the ratio $\frac{p}{q}$ then the value of $\frac{b^2}{(ca)}$ is _______. (a) $\frac{(p+q)^2}{(pq)}$ (b) $\frac{(p+q)}{(pq)}$ (c) $\frac{(p-q)^2}{(pq)}$ (d) $\frac{(p-q)}{(pq)}$	**B**
Q60	If $\frac{x-a^2-b^2}{c^2} + \frac{c^2}{x-a^2-b^2} = 2$ the value of _______. (a) $a^2 + b^2 + c^2$ (b) $-a^2 - b^2 - c^2$ (c) $\frac{1}{a^2+b^2+c^2}$ (d) 1	**C**
Q61	Solving equation $x^2 - (a + b)x + ab = 0$ we find value(s) of x is _________ (a) a, b (b) a (c) b (d) None	**A**
Q62	α & β are roots of equation $x^2 - 5x + 6 = 0$ the eq^n with roots (αβ+α+β) & (αβ - α-β) is __. (a) $x^2 - 12x + 11 = 0$ (b) $2x^2 - 6x + 12 = 0$ (c) $x^2 - 12x + 12 = 0$ (d) None	**A**
Q63	If α&β are the roots of equation $x^2 - 5x + 6 = 0$, then equation with roots $(\alpha^2+\beta)$ & $(\alpha+\beta^2)$ is _______. (a) $x^2 - 9x + 99 = 0$ (b) $x^2 - 18x - 90 = 0$ (c) $x^2 - 18x + 77 = 0$ (d) None	**A**
Q64	Solving equation $z^{10} - 33z^5 + 32 = 0$ the following values of z are obtained (a) 1, 2 (b) 2, 3 (c) 2, 4 (d) 1, 2, 3	**A**
Q65	Solve $4^x - 3.2^{x+2} + 2^5 = 0$ (a) x=3 or x=2 (b) x=4 or x=5 (c) x=5 or x=2 (d) x=3 or x=4	**A**

Q66	Solving $4^x.2^y = 128$ and $3^{3x+2y} = 9^{xy}$ we get the following roots (a) $\frac{7}{4},\frac{7}{2}$ (b) 2, 3 (c) Both (a) and (b) (d) 13	C
Q67	$4^x - 3.2^{x+2} + 2^5 = 0$; x = ________. (a) 4, 8 (b) -2, -3 (c) 2, 6 (d) 2,3	D
Q68	If $\frac{x}{b}+\frac{b}{x}=\frac{a}{b}+\frac{b}{a}$ the roots of the equation are ________. (a) $a,\frac{b^2}{a}$ (b) $a^2,\frac{b}{a^2}$ (c) a, b^2 (d) None	A
Q69	If the roots of the equation p (q - r) x^2 + q (r - p) x + r (p - q) = 0 are equal, then $\frac{2}{q}$ = (a) $\frac{1}{r}+\frac{1}{p}$ (b) $\frac{1}{rp}$ (c) RP (d) $\frac{1}{r}-\frac{1}{p}$	D
Q70	Solving equation $\left(x-\frac{1}{x}\right)^2-6\left(x+\frac{1}{x}\right)+12=0$ we get roots as follows (one of them) (a) 0 (b) 1 (c) -1 (d) None	B
Q71	If $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$ then the values of x are ________. (a) 0, (a +b), (a - b) (b) 0, (a+b), $\frac{a^2+b^2}{a+b}$ (c) 0, (a - b), $\frac{a^2+b^2}{a+b}$ (d) None	B
Q72	The roots of the equation $x^2 + (2p - 1)x + p^2 = 0$ are real if ________. (a) $P \geq 1$ (b) $P \leq 4$ (c) $P \geq 1/4$ (d) $P \leq ¼$	D
Q73	The condition that one of the roots of $ax^2 + bx + c = 0$ is thrice the other is ________. (a) $3b^2 = 16ca$ (b) $b^2= 9ca$ (c) $3b^2 = -16ca$ (d) $b^2 = -9ca$	A
Q74	If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$; the equation having roots as $\frac{p}{q}$and $\frac{q}{p}$ is ________. (a) $x^2 - 19x + 3 = 0$ (b) $3x^2 - 19x - 3 = 0$ (c) $3x^2 - 19x + 3 = 0$ (d)$3x^2+19x+3 =0$	B
Q75	If L+M+N=0 and Ltd N are rational, the roots of the equation $(M+N+L) x^2 + (N+L-M) X + (L+M-N) = 0$ (a) Real & irrational (b) Real & rational (c) Imaginary & equal (d) Real & equal.	B
Q76	Solving equation $x^2 - 24x +135 = 0$ we find value(s) of x is ________. (a) 9, 6 (b) 9, 15 (c) 15, 6 (d) None	B
Q77	Solving equation $z + \sqrt{z} = \frac{6}{25}$ the value of z works out to ________. (a) 1/5 (b) 2/5 (c) 1/25 (d) 2/25	C
Q78	Solution of the quadratic equation $(a + b - 2c) x^2 + (2a - b - c) x + (c + a -2b) = 0$ is ________. (a) x = 1 (b) x = -1 (c) x = 2 (d) x = -2	B
Q79	Solving $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}-\frac{5}{2}=0$ &x + y - 5 = 0, we get the roots as under ________. (a) 1, 4 (b) 1, 2 (c) 1, 3 (d) 1, 5	A
Q80	Solving $x^2 + xy - 21= 0$ and $xy - 2y^2 + 20 = 0$ we get the roots as under ________. (a) ± 1, ± 2 (b) ± 2, ± 3 (c) ± 3, ± 4 (d) None	C
Q81	When $\sqrt{2z+1}+\sqrt{3z+4}$ = 7 the value of z is given by ________. (a) 1 (b) 2 (c) 3 (d) 4	D

Q82	Solving $x^2 + xy + y^2 = 37$ and $3xy + 2y^2 = 68$ we get the following roots (a) ± 3, ± 4 (b) ± 4, ± 5 (c) ± 2, ± 3 (d) None	A
Q83	Solving x + 2y + 2z = 0, 3x - 4y + z = 0 and $x^2 + 3y^2 + z^2 = 11$ following roots are obtained (a) 2, 1, -2 & -2, -1, 2 (b) 2, 1, 2 & -2, -1, -2 (c) Only 2, 1, -2 (d) Only -2, -1, 2	A
Q84	Solving equation $6\left[\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}}\right]$ = 13 following roots are obtained (a) $\frac{4}{13}, \frac{9}{13}$ (b) $\frac{-4}{13}, \frac{-9}{13}$ (c) $\frac{4}{13}, \frac{5}{13}$ (d) $\frac{6}{13}, \frac{7}{13}$	A
Q85	Solving $\frac{x+\sqrt{12p-x}}{x-\sqrt{12p-x}} = \frac{\sqrt{p}+1}{\sqrt{p}-1}$, following roots are obtained (a) 3p (b) Both 3p and -4p (c) Only -4p (d) -3p 4p	B
Q86	Solving $\sqrt{y^2 + 4y - 21} + \sqrt{y^2 - y - 6} = \sqrt{6y^2 + 5y - 39}$ following roots are obtained (a) 2, 3, $\frac{5}{3}$ (b) 2, 3, $-\frac{5}{3}$ (c) -2, -3, $\frac{5}{3}$ (d) -2, - 3, $-\frac{5}{3}$	B
Q87	Solving equation $\left(x - \frac{1}{x}\right)^2 - 10\left(x - \frac{1}{x}\right)$ + 24 = 0 we get roots as follows (a) 0 (b) 1 (c) -1 (d) None	D
Q88	Solving $x^3 - 6x^2 + 11x - 6 = 0$ we get the following roots as ________. (a) -1, -2, 3 (b) 1, 2, -3 (c) 1, 2, 3 (d) -1, -2, -3	C
Q89	Solving $x^3 + 9x^2 - x - 9 = 0$ we get the following roots as ________. (a) ±1, -9 (b) ±1, ± 9 (c) ± 1, 9 (d) None	A
Q90	Solve $x^3 - 7x + 6 = 0$ (a) x = -4, -2, -3 (b)x = 1, 2, -3 (c) x = 5, 6, -1 (d) x = 7, 2, -5	B
Q91	Solve for real x: $x^3 + x + 2 = 0$ (a) x = -4 (b) x = 4 (c) x = -1 (d) x = -3	C
Q92	The solution of the equation $x^3 - 5x^2 + 6x = 0$ is ________. (a) 2, 3 (b) 0, -2, -3 (c) 0, 2, 3 (d) None	C
Q93	The equation $y^3 - 7y + 6 = 0$ is satisfied by ________. (a) 1, 2, -3 (b) 1, 2, 3 (c) -1, -2, 3 (d) 1, -2, 3	A
Q94	The equation $x^3 - x^2 - 12x = 0$ is satisfied by ________. (a) 1, 4, -3 (b) 0, 4, -3 (c) 0, -4, 3 (d) None	B
Q95	Solve $x^3 - 6x^2 + 5x + 12 = 0$ (a) 1, 3, 4 (b) -1, 3, 4 (c) 1, 6, 2 (d) 1, -6, -2	B
Q96	Solve $x^3 - 5x^2 - 2x + 24 = 0$ given that two of its roots being in the ratio of 3:4. (a) -2, 4, 3 (b) -1, 4, 3 (c) 2, 4, 3 (d) -2, -4, -3	A
Q97	The cubic equation $x^3 + 2x^2 - x - 2 = 0$ has 3 roots namely ________. (a) (1, -1, 2) (b) (-1, 1, -2) (c) (-1, 2, -2) (d) (1, 2, 2)	B
Q98	(x-1), $(x^2 + 3x + 2)$ are the factors of the left - hand side of the equation, then (a) $x^3 + 2x^2 - x - 2 = 0$ (b) $x^3 + x^2 - 20x = 0$ (c) $x^3 - 3x^2 - 4x + 12 = 0$ (d) $x^3 - 6x^2 + 11x - 6 = 0$	A

Q99	The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of LHS of $3x^3 + 5x^2 - 3x - 5 = 0$ are ______. (a) (x—1), (x—2), (x-5/3) (b) (x—1), (x+1) (3x-5) (c) (x+1), (x—1) (3x+5) (d) (x-1), (x+1), (x—2)	C
Q100	The roots of $x^3 = x^2 - x - 1$ are _______. (a) (-1, -1, 1) (b) (1, 1, -1) (c) (-1, -1, -1) (d) (1, 1, 1)	A
Q101	The satisfying value of $x^3 + x^2 - 20x = 0$ are _______. (a) (1, 4, -5) (b) (2, 4, -5) (c) (0, -4, 5) (d) (0, 4, -5)	D
Q102	The roots of the cubic equation $x^3 + 7x^2 - 21x - 27 = 0$ are ______. (a) (-3, -9, -1) (b) (3, -9 -1) (c) (3, 9, 1) (d) (-3, 9, 1)	B
Q103	Solve $x^3 + 3x^2 - x - 3 = 0$ give that the roots are in arithmetical progression (a) -1, 1, 3 (b) 1, 2, 3 (c) -3, -1, 1 (d) -3, -2, -1	C
Q104	Solve $x^3 - 7x^2 + 14x - 8 = 0$ given that the roots are in geometrical progression. (a) ½, 1 2 (b) 1, 2, 4 (c) 1/2, -2, 2 (d) -1, 2, -4	B
Q105	The rational root of the equation $2x^3 - x^2 — 4x + 2 = 0$ is _______. (a) 1/2 (b) -½ (c) 2 (d) -2	A
Q106	If the sum of a number and the original number increased by 5 is greater than 11, which could be a possible value of the number? (a) -5 (b) -1 (c) 1 (d) 4	D
Q107	The sum of two numbers is 52 and their difference is 2. The numbers are _______. (a) 17 and 15 (b) 12 and 10 (c) 27 and 25 (d) None	C
Q108	The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age. (a) 60 years (b) 52 years (c) 51 years (d) 50 years	D
Q109	The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man? (a) 65 years (b) 25 years (c) 35 years (d) 45 years	D
Q110	Average age of a group of eight is same as it was 3 years ago, when a young member is substituted for an old member, incoming member is younger to outgoing nests by _______. (a) 11 years (b) 24 years (c) 28 years (d) 16 years	B
Q111	A school has 20 teachers, one of them retires at the age of 60 years and a new teacher replaces him, this change reduces the average age of the staff by 2 years, the age of new teacher is _____. (a) 28 years (b) 25 years (c) 20 years (d) 18 years	C
Q112	If thrice of A's age 6 years ago be subtracted from twice his present age the result would be equal to his present age. Find A's present age. (a) 6 years (b) 9 years (c) 12 years (d) 10 years	B
Q113	Y is older than X by 7 years. 15 years back, the ratio of their ages was 3:4. Their present ages are _______. (a) (X = 36 Y = 43) (b) (X = 50 Y = 43) (c) (X = 43 Y = 50) (d) (X = 40 Y = 47)	A

Q114	If the sum of a number and the original number increased by 5 is greater than 11, which could be a possible value of the number? (a) -5 (b) -1 (c) 1 (d) 4	**D**
Q115	If the difference of the squares of two numbers is 45, the square of the smaller number is 4 times the larger number, then the numbers are _______. (a) 9, 6 or 9, -6 (b) 5, 6, or 5, 4 (c) 9, 5 or 9, -5 (d) 6, 7 or -7, 6	**A**
Q116	A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed, find the number. (a) 54 (b) 53 (c) 45 (d) 55	**C**
Q117	The sum of the digits of a 2 digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is _______. (a) 37 (b) 73 (c) 64 (d) None	**B**
Q118	Sum of numerator and denominator of a fraction is 8. If 3 is added to both the numerator and denominator then the fraction becomes 3/4. Then the fraction is _______. (a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5	**C**
Q119	The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction. (a) $\frac{15}{17}$ (b) $\frac{13}{17}$ (c) $\frac{12}{17}$ (d) $\frac{11}{17}$	**C**
Q120	Difference between a number and its positive square root is 12; find the numbers. (a) 36 (b) 25 (c) 16 (d) 9	**C**
Q121	The ratio between a two digit number and the sum of digits of that number is 4:1. If the digit in the unit place is 3 more than the digit in the tenth place, what is that number? (a) 24 (b) 63 (c) 36 (d) Data insufficient	**C**
Q122	The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are _______. (a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) None	**C**
Q123	The sum of two numbers is 45 and the meal proportional between them is 18. The numbers are _______. (a) (15, 30) (b) (32, 13) (c) (36, 9) (d) (25, 20)	**C**
Q124	There are two consecutive numbers such that the difference of their reciprocals is 1/240. The numbers are _______. (a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)	**A**
Q125	The difference of two positive integers is 3 and the sum of their squares is 89. The integers are ________. (a) (7, 4) (b) (5, 8) (c) (3, 6) (d) (2, 5)	**B**
Q126	A number consists of three digits of which the middle one is zero and the sum of the other digits is 9. The number formed by interchanging the first and third digit is more than the original number by 297. Find the number. (a) 801 (b) 603 (c) 702 (d) 306	**D**
Q127	A number consists of two digits. The digit in the ten's Place is twice the digit in the unit's place. If 18 be subtracted from the number the digits are reversed. Find the number.	**D**

	(a) 96 (b) 62 (c) 38 (d) 42	
Q128	The sum of the digits in a three digit number is 12. If the digits are reversed the number is increased by 495 but reversing only of the tens and unit digits increases the number by 36. The number is ________. (a) 327 (b) 372 (c) 237 (d) 273	**C**
Q129	Two numbers are such that thrice the smaller number exceeds twice the greater one by 18 and 1/3 of the smaller and 1/5 of the greater number are together 21. Numbers are _______. (a) (45, 36) (b) (50, 38) (c) (54, 45) (d) (55, 41)	**B**
Q130	On two numbers 1/5th of the greater is equal to 1/3rd of the smaller and their sum is 16. The numbers are _______. (a) (6, 10) (b) (9, 7) (c) (12, 4) (d) (11, 5)	**A**
Q131	A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is ________. (a) 63 (b) 35 (c) 36 (d) 60	**C**
Q132	Find the fraction which is equal to ½ when both its numerator and denominator are increased by 2. It is equal to 3/4 when both are increased by 12. (a) 3/8 (b) 5/8 (c) 3/8 (d) 2/3	**A**
Q133	If a number of which the half is greater than 1/5th of number by 15 then number is ________. (a) 50 (b) 40 (c) 80 (d) None	**C**
Q134	The fourth part of a number exceeds the sixth part by 4. The number is _______. (a) 84 (b) 44 (c) 48 (d) None	**C**
Q135	Rs. 14 is divided between A and B such that half of the share of A is equal to two thirds of the share of 8, the share of A is ________. (a) Rs.6 (b) Rs.10 (c) Rs.9 (d) Rs.8	**D**
Q136	The number of kilograms of corn needed to feed 5,000 chickens is 30 less than twice the number of kilograms needed to feed 2,800 chickens. How many kilograms of corn are needed to feed 2800 chickens? (a) 70 (b) 110 (c) 140 (d) 190	**C**
Q137	Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are ________. (a) (24, 26) (b) (28, 22) (c) (27, 23) (d) (20, 30)	**D**
Q138	A piece of string is 40 cms long. It is cut into three pieces. The longest piece is 3 times as long as the middle-sized and the shortest pieces are 23 cms shorter than the longest piece. The length of the shortest piece (in cm) is ________. (a) 27 cm (b) 5 cm (c) 4 cm (d) 9 cm	**C**
Q139	A piece of iron rod costs Rs.60. If the rod was 2 metre shorter and each metre costs Rs.1.00 more, the cost would remain unchanged. What is the length of the rod? (a) 12m (b) 22m (c) 20m (d) 32m	**A**
Q140	A train travels first 300 kms at an average rate of 30 Km per tar and further travels the same distance at an average rate of 60 Km per hour then the average speed over the whole distance is _______.	**B**

	(a) 35km per hour (b) 40 Km per hour (c) 42 Km per hour (d) 45 Km per hour	
Q141	On a certain map, 3/8 of an inch represents 120 miles. How many miles does 13/4 inches represent? (a) 300 (b) 360 (c) 400 (d) 560	D
Q142	If four pens cost Rs.1.96, what is the greatest number of pens that can be purchased for Rs. 29.40? (a) 11 (b) 14 (c) 15 (d) 16	C
Q143	A freight train and a passenger train start towards each other at the same time from two towns that are 500 miles apart. After 3 hours the trains are still 80 miles apart. If the average rate of speed of the passenger train is 20 miles per hour faster than the average rate of speed of the freight trains, what is the average rate of speed, in miles per hour, of the freight train? (a) 40 (b) 45 (c) 50 (d) 60	D
Q144	A motor boat traveling at 18 miles per hour traveled the length of a lake in one quarter of an hour less time than it took when traveling at 12 miles per hour. What was the length in miles of the lake? (a) 6 (b) 9 (c) 12 (d) 15	B
Q145	If a car is traveling at a constant rate of 45 miles per hour, how many miles does it travel from 10:40 a.m. to 1:00 p.m. of the same day? (a) 165 (b) 150 (c) 120 (d) 105	D
Q146	The total cost curve of the number of copies photograph is linear. The total cost of 5 and 18 copies of photographs are Rs.80 and Rs.106 respectively. Then the cost for 10 copies of the photograph is ______. (a) Rs.140 (b) Rs.90 (c) Rs.150 (d) Rs.130	B
Q147	A factory produces 300 units and 900 units at a total cost of Rs.6800/- and Rs.10400/- respectively. The linear equation of the total cost line is ______. (a) y = 6x + 1,000 (b) y = 5x + 5,000 (c) y = 6x + 5,000 (d) None	C
Q148	If in Question No. 147, the selling price is Rs.8 per unit the break even point will arise at the level of ________. (a) 1,500 units (b) 2,000 units (c) 2,500 units (d) 3,000 units	C
Q149	If instead in terms of Question No. 147 if a profit of 2000/- is to be earned sale and production levels have to be elevated to (a) 3,000 units (b) 3,500 units (c) 4,000 units (d) 3,700 units	B
Q150	If instead in terms of Question No. 147, if a loss of 3,000/- Is budgeted the factory may maintain production level at (a) 1,000 units (b) 1,500 units (c) 1,800 units (d) 2,000 units	A
Q151	A factory produces 200 bulbs for a total cost of Rs.800/- and 400 bulbs for Rs.1200/-. The equation of the total cost line is _______. (a) 2x - y + 100 = 0 (b) 2x - y + 400 = 0 (c) 1x - y + 400 = 0 (d) None	B
Q152	If in terms of Question No. 151, the factory intends to produce 1000 butts the total cost would be _______. (a) Rs.2,400 (b) Rs.2,200 (c) Rs.2,300 (d) Rs.2,100	A

Q153	A manufacturer produces 60 T.V. sets at a cost of Rs.2,20,000 and 125 T.V. sets at a cost of Rs.2,87,500. Assuming the cost one to be linear, find the equation of the line and then use it to estimate the cost of 95 sets. (a) Rs.3,52,500 (b) Rs.1,32,500 (c) Rs.2,42,500 (d) Rs.3,62,500	C
Q154	If an investment of Rs.1,000 and 100 yield an income of Rs.90 and Rs.20 respectively. For earning Rs.50, investment required be ______. (a) Less than Rs.500 (b) Rs. Over 500 (c) Rs.485 (d) Rs.486	D
Q155	The equation in terms of Question No. 154 is _______. (a) 7x - 9y + 1100 = 0 (b) 7x - 90y + 1000 = 0 (c) 7x - 90y + 1100 = 0 (d) 7x - 90y - 1100 = 0	C
Q156	If an investment of Rs.60,000 and Rs.70,000 respectively yields an income of Rs.5,750 Rs.6,500 an investment of Rs.90,000 would yield income of ________. (a) Rs.7,500 (b) Rs.8,000 (c) Rs.7,750 (d) Rs.7,800	B
Q157	In terms of Question No. 156 an Investment Rs.50,000 would yield income of _____. (a) Exactly Rs.5,000 (b) Little over Rs.5,000 (c) Little less than Rs.5,000 (d) At least Rs.6,000	A
Q158	The equation in terms of Question No. 157 is _______. (a) 3x + 40y + 25,000 = 0 (b) 3x - 40y + 50,000 = 0 (c) 3x - 40y + 25,000 = 0 (d) 3x - 40y - 50,000 = 0	B
Q159	One machine can seal 360 packages per hour & an older machine can seal 140 packages per hour. How many minutes will the two machines working together take to seal a total of 700 packages? (a) 48 (b) 72 (c) 84 (d) 90	C
Q160	If x people working together at the same rate can complete a job in H hours, what part of the same job can one person working along complete in k hours? (a) $\frac{k}{xH}$ (b) $\frac{H}{xk}$ (c) $\frac{k}{x+H}$ (d) $\frac{kH}{x}$	A
Q161	The demand and supply equations for a certain commodity are 4q + 7p = 17 and $p = \frac{q}{3} + \frac{7}{4}$ respectively where p is the market price and q is the quantity then the equilibrium price & quantity are (a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None	A
Q162	For a certain commodity, the demand equation giving demand 'd' in kg. for a price 'p' in rupees per kg. is d=100 (10-p). The supply equation giving the supply 's' in kg. for a price 'p' in rupees per kg is s = 75 (p-3). The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold. (a) 7,500, 600 (b) 6,300, 300 (c) 7,300, 300 (d) 7,600, 300	C
Q163	The wages of 8 men and 6 boys amount to Rs.33. if 4 men earn Rs.4.50 more than 5 boys determine the wages of each man and boy. (a) (Rs.1.50, Rs.3) (b) (Rs.3, Rs.1.50) (c) (Rs.2.50, Rs.2) (d) (Rs.2, Rs.2.50)	B
Q164	One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer then the actual answer is _____.	C

	(a) 320	(b) 400	(c) 480	(d) None	
Q165	If x+4x-3x+8=0, then x ______. (a) -4	(b) -2	(c) 0	(d) 6	A
Q166	If 2x+5=-25 and -3y -6=48, then xy______. (a) -270	(b) -90	(c) 90	(d) 270	D
Q167	If $\frac{1}{x}+\frac{1}{y}=\frac{1}{4}$ and $\frac{1}{x}-\frac{1}{y}=\frac{3}{4}$, then x______. (a) $\frac{1}{4}$	(b) $\frac{1}{2}$	(c) 1	(d) 2	D
Q168	A linear equation has ______. (a) 1 root	(b) 2 roots	(c) 3 roots	(d) No roots	A
Q169	If 4x + 5y = 83 and $\frac{3x}{2y}=\frac{21}{22}$, then y-x = ______. (a) 3	(b) 4	(c) 7	(d) 1	B
Q170	The solution of simultaneous linear equations 2X +3Y = 17, 3X -2Y = 6 is ______. (a) X =4, Y =4	(b) X =3, Y =4	(c) X =4, Y =3	(d) X =3, Y =3	C
Q171	The value of $\frac{16x^{-1}}{4x^{2/3}}$ is ______. (a) $4x^{-3/5}$	(b) $4x^{5/3}$	(c) $4x^{-5/3}$	(d) None	C
Q172	Solving 6x + 5y - 16= 0 and 3x-y-1 = 0 we get values of x, y as ______. (a) 1,1	(b) 1,2	(c) -1,2	(d) 0,2	B
Q173	$\frac{x}{p}+\frac{y}{q}$ = 2, x+y = p+q are satisfied by the values given by the pair ______. (a) (x = p y = q)	(b) (x-q y-p)	(c) (x = 1 y = 1)	(d) None	A
Q174	1.5x + 2.4y = 1.8 2.5(x + 1) = 7y have solutions ______. (a) (0.5, 0.4)	(b) (0.4, 0.5)	(c) $\frac{1}{2},\frac{2}{5}$	(d) (2, 5)	B
Q175	Value of k for which roots are equal of given equation $4x^2$ - 12x + k = 0 is ______. (a) 144	(b) 9	(c) 5	(d) None	B
Q176	Solve $x^2 - 5x + 6 = 0$ (a) 5 and 3	(b) 3 and 2	(c) 4 and 3	(d) 5 and 2	B
Q177	If $\frac{x-bc}{b+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}=a+b+c$ the value of x is ______. (a) $a^2 + b^2 + c^2$	(b) a (a + b + c)	(c) (a + bXb + c)	(d) ab + be + ca	D
Q178	If $\frac{x+2}{x-2}-\frac{x-2}{x+2}=\frac{x-3}{x+3}-\frac{x+3}{x-3}$ then the values of x are ______. (a) 0, ±√6	(b) 0, ±√3	(c) 0, ±2√3	(d) None	A
Q179	The values of x in the equation $7(x + 29)^2 + 59^2 = 35xp + 117p^2$ are ______. (a) (4p, -3p)	(b) (4p, 3p)	(c) (—4p, 3p)	(d) (-4p, -3p)	A
Q180	The solution of the equation $\frac{6x}{x+1}+\frac{6(x+1)}{x}=13$ are ______. (a) (2, 3)	(b) (3, -2)	(c) (-2, -3)	(d) (2, -3)	D
Q181	The solution of the equation $3x^2$ -17x + 24 = 0 are ______. (a) (2, 3)	(b) $\left(2,3\frac{2}{3}\right)$	(c) $\left(3,2\frac{2}{3}\right)$	(d) $\left(3,\frac{2}{3}\right)$	C

Q182	The equation $\frac{3(3x^2+15)}{6} + 2x^2 + 9 = \frac{2x^2+96}{7} + 6$ has got the solution as _______. (a) (1, 1) (b) (1/2, -1) (c) (1, -1) (d) (2, -1)	C
Q183	Number of roots of equation [(x + 2) × (x - 5)] / [(x - 3) × (x + 6)] = (x - 2)/(x + 4) is ___. (a) 1 (b) 2 (c) 3 (d) No root	A
Q184	If $2^{2x+3} - 3^2.2^x + 1 = 0$ then values of x are _______. (a) (0, 1) (b) (1, 2) (c) (0, 3) (d) (0, -3)	D
Q185	Solve $(x - \frac{1}{x})^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$. (a) $x = \frac{-9\pm\sqrt{65}}{4}$ or $x = 2\frac{1}{2}$ (b) $x = \frac{-9\pm\sqrt{55}}{4}$ or $x = 3\frac{1}{2}$ (c) $x = \frac{-9\pm\sqrt{45}}{4}$ or $x = 4\frac{1}{2}$ (d) $x = \frac{-9\pm\sqrt{35}}{4}$ or $x = 2\frac{1}{2}$	A
Q186	Solve $2^{x-2} + 2^{3-x} = 3$ (a) x = 5 or x = 4 (b) x = 3 or x = 5 (c) x = 2 or x = 3 (d) x = 1 or x = 2	C
Q187	The solution of the equation $x - \sqrt{25 - x2} = 1$ is _______. (a) x = -3 (b) x = ±5 (c) x = 1 (d) x = 4	D
Q188	Determine the value of x for the equation $x^2 - 8x + 16 = 0$ (a) 4, -4 (b) -4, -4 (c) 2, 6 (d) 6, 2	A
Q189	Solving equation $\frac{6x+2}{4} + \frac{2x^2-1}{2x^2+2} = \frac{10x-1}{4x}$ we get roots as _______. (a) ±1 (b) +1 (c) -1 (d) 0	B
Q190	Solve for x, $4^x - 3.2^{x+2} + 2^5 = 0$ (a) 4, 8 (b) -2, -3 (c) 2, 6 (d) 2, 3.	D
Q191	Solving $9^x = 3^y$ and$5^{x+y+1} = 25^{xy}$ we get the following roots as _______. (a) (1, 2), $\left(\frac{-1}{4}, \frac{-1}{2}\right)$ (b) 0, 1, 3 (c) 0, 3 (d) 1, 3	A
Q192	Solving $z^2 - 6z + 9 = 4\sqrt{z^2 - 6z + 6}$ following roots are obtained (a) 3 + 2√3, 3 - 2√3 (b) 51 (c) All the above (d) None	C
Q193	Solving equation $2(x - \frac{1}{x})^2 - 5\left(x + \frac{1}{X} + 2\right) + 18 = 0$ we get roots as under _______. (a) 0 (b) 1 (c) -1 (d) - 2 ± √3	D
Q194	Solving $x^2+y^2-25 = 0$ and $x-y-1 = 0$ we get the roots as under _______. (a) ± 3, ± 4 (b) ± 2, ± 3 (c) 0, 3, 4 (d) 0, -3, -4	A
Q195	$\frac{1}{x^2} + \frac{1}{y^2} - 13 = 0$ and $\frac{1}{x} + \frac{1}{y} - 5 = 0$ we get the roots as under _______. (a) $\frac{1}{8}, \frac{1}{5}$ (b) $\frac{1}{2}, \frac{1}{3}$ (c) $\frac{1}{13}, \frac{1}{5}$ (d) $\frac{1}{4}, \frac{1}{5}$	B
Q196	Examine the nature of the roots of $x^2 - 8x^2 + 16 = 0$ (a) Roots are real and equal (b) Roots are real, rational and unequal (c) Roots are imaginary and unequal (d) Roots are real, irrational and unequal	A
Q197	Examine the nature of the roots of $3x^2 - 8x + 4 = 0$ (a) Roots are real and unequal (b) Roots are imaginary and unequal (c) Roots are real, rational and unequal (d) Roots are real, irrational and unequal	C

Q198	Examine the nature of the roots of $5x^2-4x+2=0$ (a) Roots are imaginary and unequal (b) Roots are real and unequal (c) Roots are real, rational and unequal (d) Roots are real, irrational and unequal	A
Q199	Examine the nature of the roots of $2x^2-6x-3=0$ (a) Roots are real and unequal (b) Roots are imaginary and unequal (c) Roots are real, rational and unequal (d) Roots are real, irrational and unequal	D
Q200	The equation $ax^2+bx+c=0$ does not have any solution if ______. (a) $b^2-4ac=0$ (b) $b^2-4ac<0$ (c) $b^2-4ac>0$ (d) $b^2+4ac=0$	B
Q201	The equation $ax^2+bx+c=0$ does not have any solution if ______. (a) $b^2-4ac=0$ (b) $b^2-4ac<0$ (c) $b^2-4ac>0$ (d) $b^2+4ac=0$	B
Q202	In the equation $a x^2+bx+c=0$, the roots are determined from ______. (a) $b^2<4ac$ (b) b^2-4ac (c) $b^2>4ac$ (d) $b^2=4ac$	B
Q203	The roots of $a x^2+bx+c=0$, are real and unequal if ______. (a) $b^2<4ac$ (b) b^2-4ac (c) $b^2>4ac$ (d) $b^2=4ac$	C
Q204	If $b^2-4ac=0$ the roots are ______. (a) Real & Unequal (b) Real & Equal (c) Irrational & Unequal (d) Rational & Unequal	B
Q205	If α & β be the roots of $x^2+7x+12=0$, find equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ (a) $x^2-40x+49-0$ (b) $x^2-35x+39=0$ (c) $x^2-50x+49=0$ (d) $x^2-40x-49=0$	B
Q206	If α, β be the roots of $2x^2-4x-1=0$, find the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$. (a) -42 (b) -22 (c) -32 (d) -52	C
Q207	If α β are roots of equation $x^2-5x+6=0$ the equation with roots $(\alpha^2+\beta)$ and $(\alpha+\beta^2]$ is _____. (a) $x^2-9x+99=0$ (b) $x^2-18x+90=0$ (c) $x^2-18x+77=0$ (d) None	A
Q208	If α β be the roots of the equation $2x^2-4x-3=0$ the value of $\alpha^2+\beta^2$ is ______. (a) 5 (b) 7 (c) 3 (d) -4	A
Q209	If p and q are the roots of $x^2+x+1=0$ then the values of p^3+q^3 becomes ______. (a) 2 (b) -2 (c) 4 (d) -4	D
Q210	The roots of the equation $(q-r)\times x^2+(r-p)\times x+(p-q)=0$ are ______. (a) $(r-p)/(q-r)$, 1 (b) $(p-q)/(q-r)$, 1 (c) $(q-r)/(p-q)$, 1 (d) $(r-p)/(p-q)$, 1	D
Q211	Roots of equation $ax^2-bx+c=0$ are two consecutive integers then b^2-4ac is ___. (a) 3 (b) -2 (c) -1 (d) 1	A
Q212	If α, β be the roots of a quadratic equation if $\alpha+\beta=-2$, $\alpha\beta=-3$ Find quadratic equation (a) $x^2+2x-7=0$ (b) $x^2+2x-3=o$ (c) $x^2-2x-3=0$ (d) $x^2-2x+7=0$	B
Q213	If α, β are the roots of the quadratic equation $2x^2-4x=1$, then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$	C

	(a) -11 (b) 22 (c) -22 (d) 11	
Q214	The sum of the digits of a two digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is ______. (a) 37 (b) 73 (c) 75 (d) None	B
Q215	The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are ______. (a) (16, 20) (b) (60, 20) (c) (60, 30) (d) (80, 40)	D
Q216	Divide 25 into two parts so that sum of their reciprocals is $\frac{1}{6}$. (a) 12 and 13 (b) 9 and 16 (c) 11 and 14 (d) 10 and 15	D
Q217	Divide 56 into two parts such that three times the first part exceeds one-third of the second by 48. The parts are ______. (a) (20,36) (b) (25, 31) (c) (24, 32) (d) None	A
Q218	The hypotenuse of a right-angled triangle is 20 cm. The difference between its other two sides is 4cm. The sides are ______. (a) (11cm, 15cm) (b) (12cm, 16cm) (c) (20cm, 24cm) (d) None	B
Q219	Two squares have sides p cm and (p + 5) cms. The sum of their squares is 625 sq.cm. The sides of the squares are ______. (a) (10cm, 30cm) (b) (12cm, 25cm) (c) (15cm, 20cm) (d) None	C
Q220	Particular company produces some articles on a day. The cost of production per article is Rs. 2 more than thrice the number of articles and the total cost of production is Rs. 800 on a day then the number of articles is ______. (a) 16 (b) 14 (c) 18 (d) 15	A
Q221	The satisfying value of $x^3 + x^2 - 20x = 0$ are ______. (a) (1, 4, -5) (b) (2, 4, -5) (c) (0, -4, 5) (d) (0, 4, -5)	B
Q222	If $4x^3 + 8x^2 - x - 2 = 0$ then value of (2x + 3) is given by ______. (a) 4, -1, 2 (b) -4, 2, 1 (c) 2, -4, -1 (d) None	A
Q223	$ax^3 = c$ is a ______. (a) quadratic eq^n (b) cubic equation (c) linear equation (d) None	B
Q224	Roots of the cubic equation $x^3 - 7x + 6 = 0$ are __________ (a) 1, 2, 3 (b) 1, -2, 3 (c) 1, 2, -3 (d) 1, -2, -3	C
Q225	8 is the solution of the equation (a) $\frac{x+4}{4}+\frac{x-5}{3}=11$ (b) $\frac{x+4}{2}+\frac{x+10}{9}=8$ (c) $\frac{x+24}{5}=4+\frac{x}{4}$ (d) $\frac{x-15}{10}+\frac{x+5}{5}=4$	B
Q226	Solution for the pair of equations $\frac{1}{16x}+\frac{1}{15y}=\frac{9}{20}, \frac{1}{20x}-\frac{1}{27y}=\frac{4}{45}$ is given by ______. (a) $\left(\frac{1}{4},\frac{1}{3}\right)$ (b) $\left(\frac{1}{3},\frac{1}{4}\right)$ (c) (3, 4) (d) (4, 3)	A
Q227	If 5x+y=19 and x-3y=7, then x+y (a) -4 (b) -1 (c) 3 (d) 4	C
Q228	Two variables x and y are related by 7x + 7y + 13 = 0 and x = 7, then y is ______. (a) 8.80 (b) 8.86 (c) -8.80 (d) -8.86	D

Q229	$\frac{4x}{3} - 1 = \frac{14}{15}x + \frac{19}{5}$. Find x = _______. (a) 12 (b) 15 (c) 20 (d) 8	A
Q230	1.5x + 3.6y = 2.1; 2.5(x + 1) = 6y (a) (0.2, 0.5) (b) (0.5, 0.2) (c) (2, 5) (d) (-2, -5)	A
Q231	Solving equation $3x^2 - 14x + 8 = 0$ we get roots as _______. (a) ±4 (b) ±2 (c) 4, 2/3 (d) None	C
Q232	If αβ are the roots of equation $x^2 - 5x + 6 = 0$ equation with roots (α+ β) and (α - β) is _______. (a) $x^2 - 6x + 5 = 0$ (b) $2x^2 - 6x + 5 = 0$ (c) $2x^2 - 5x + 6 = 0$ (d) $x^2 - 5x + 6 = 0$	A
Q233	If α and β are the roots of the equation $ax^2 + bx + c = 0$, then $(\alpha+ \beta)^2$ is ________. (a) $-b^2/a^2$ (b) c^2/a^2 (c) b^2/a^2 (d) bc / a	C
Q234	A quadratic polynomial $f(x) = ax^2 + bx + c$ for all x ∈ R can be factorized into rational factors over R if & only if ________. (a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac = 0$ (c) $b^2 - 4ac < 0$ (d) $b^2 - 4ac > 0$, perfect square or $b^2 - 4ac = 0$	D
Q235	Solving $(b - c) x^2 + (c - a) x + (a - b) = 0$, roots obtained are _______. (a) $\frac{a-b}{b-c}, 1$ (b) (a - b) (a - c), 1 (c) $\frac{b-c}{a-b}, 1$ (d) None	A
Q236	Solving equation $3x^2 - 14x + 16 = 0$ we get roots as ________. (a) ±1 (b) $(2, \frac{8}{3})$ (c) 0 (d) None	B
Q237	Value of $\sqrt{6\sqrt{6\sqrt{6\sqrt{6\sqrt{6 \ldots \infty}}}}}$ = _______. (a) 3 (b) 6 (c) $\sqrt{42}$ (d) $3\sqrt{2}$	B
Q238	12 years after a man will be 4 times as he was 12 years ago, his present age is __ (a) 25 years (b) 20 years (c) 28 years (d) 30 years	B
Q239	10 years ago, age of the father was 4 times age of his son.10 years hence, age of the father will be twice that of his son. Present ages of the father and the son are _______. (a) (50, 20) (b) (60, 20) (c) (55, 25) (d) None	A
Q240	Ten years ago a father was 12 times as old as his son and 10 years hence he will be twice as old as his son. Then their present ages are _____. (a) 12 yrs, 24 yrs (b) 12 yrs, 34 yrs (c) 24 yrs, 42 yrs (d) 12 yrs,42 yrs	B
Q241	Sum of 2 natural numbers is 8 & sum of their reciprocal is 8/15. Numbers are ______. (a) 3 and 5 (b) 6 and 2 (c) 7 and 1 (d) 4 and 4	A
Q242	The sum of two numbers is 38 and their difference is 2. Find them. (a) 20, 18 (b) 10, 12 (c) 17, 15 (d) None	A
Q243	Two numbers are in the ratio 2:3 and the difference of their squares is 320. The numbers are _______. (a) 12,18 (b) 16,24 (c) 14,21 (d) None	B
Q244	The sum of the two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are _______. (a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)	C

Q245	Five times of a positive whole number is 3 less than twice the square of the number. The number is ______. (a) 3 (b) 4 (c) -3 (d) 2	A
Q246	If numerator of a fraction is increased by 2 & denominator by 1 it becomes 1. Again, if numerator is deceased by 4 & denominator by 2 it becomes 1/2. Fraction = __. (a) 3/8 (b) 5/8 (c) 7/8 (d) 1/8	C
Q247	A number consist of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number the digits are reversed. The number is ______. (a) 39 (b) 92 (c) 93 (d) 94	C
Q248	Denominator of a fraction exceeds numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is ________. (a) 5/7 (b) 1/3 (c) 7/9 (d) 3/5	D
Q249	A freight train left a station at 12 noon, going north at a rate of 50 miles per hour. At 1:00 pm, a passenger train left the same station, going south at a rate of 60 miles per hour. At what time were the trains 380 miles apart? (a) 3:00 pm (b) 4:00 pm (c) 4:30 pm (d) 5:00 pm	B
Q250	Julie can type a manuscript in 4 hours. Pat takes 6 hours to type the same manuscript. If Julie and Pat begin working together at 12 noon, at what time will they complete the typing of the manuscript? (a) 2:24 pm (b) 2:30 pm (c) 2:40 pm (d) 3:00 pm	A
Q251	A firm produces 50 units of a product for Rs.320 and 80 units for Rs.380. considering cost curve to be a straight-line the cost of producing 110 units to be estimated as _______. (a) Rs.400 (b) Rs.420 (c) Rs.440 (d) None	C

CHAPTER 3. INEQUALITIES

INTRODUCTION

- Inequalities are statements where two quantities are unequal but a relationship exists between them.
- A quantity may be greater than, less than, ⩾, ⩽ to the other quantity.

  Ex: $x < 10$, $y > 2$, $x + y \geqslant 25$, $x - y \leqslant 12$ etc.

LINEAR INEQUALITIES IN ONE VARIABLE AND THE SOLUTION SPACE

- **Linear Inequality:** Any linear function that involves an inequality sign is linear inequality.
- **Solution Space:** Values of the variables that satisfy an inequality are called the solution space

  POINTS TO REMEMBER

  ✓ If both sides are multiplied/divided by any negative number, inequality sign CHANGES.

  Ex: $- 6X < -18$; If we divide both sides by -6, $X > 3$. The inequality sign will change.

  Ex: If $a > b$ & $c < 0$, then $ac < bc$ & $a/c < b/c$.

  ✓ If both sides are multiplied/divided by positive number, inequality sign 'NO CHANGE'

  Ex: $5X < 20$; If we divide both sides by 5, $X < 4$. The inequality sign won't change.

  Ex: If $a > b$ & $c > 0$, then $ac > bc$ & $a/c > b/c$.

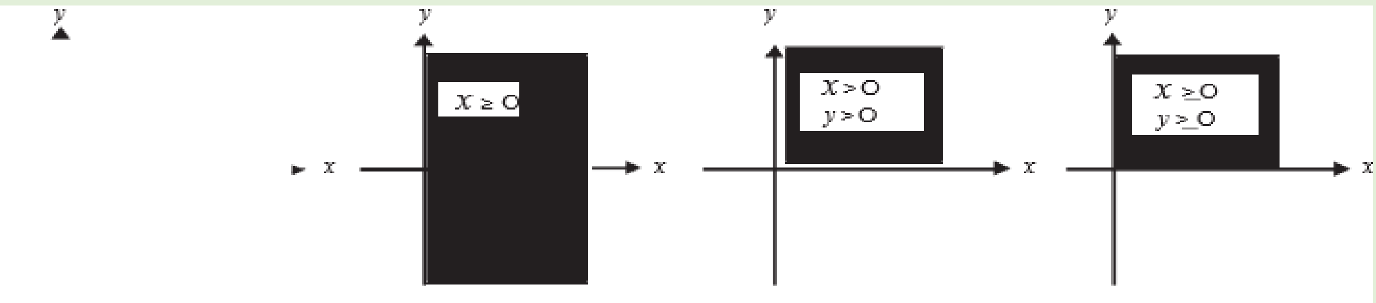
  ✓ NO CHANGE if any number is added or subtracted to both sides of inequality.

  Ex: If $a > b$, then $a + c > b + c$ & $a - c > b - c$.

  ✓ If $a > b$ & $c > d$, then $a + c > b + d$

  ✓ If $a < b$ & $c < d$, then $a + c < b + d$.

SOME FREQUENTLY USED GRAPHICAL REPRESENTATIONS



LINEAR INEQUALITIES IN TWO VARIABLES

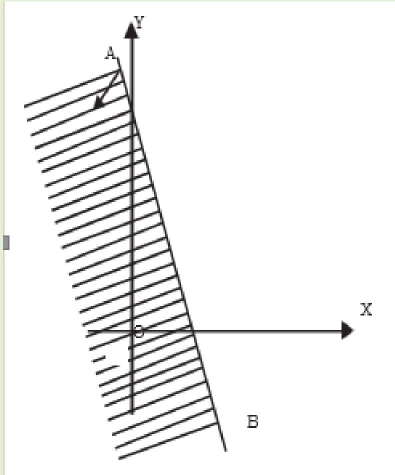
Let us now consider a linear inequality in two variables given by $3x + y < 6$

Inequality mentioned above is true for certain pairs of numbers (x, y) that satisfy $3x + y < 6$.

Steps to solve linear inequalities in two variables:

1. Replace the inequality by an equality & then you will get $3x + y = 6$.
2. Now substitute two convenient values for x & y so that we get two points.

   Let $x = 0$ so that $y = 6$. Let $y = 0$, so that $x = 2$. You will get two points (0,6) & (2, 0).
3. Plot these points on co-ordinate plane & join them to get a line of the linear equation.



PC NOTE

➢ If Plotted line is intersecting (touching) x & y axis, then for

- 'Less than' inequality → Solution = **Part Below** the line.
- 'Greater than' inequality → Solution = **Part Above** the line.

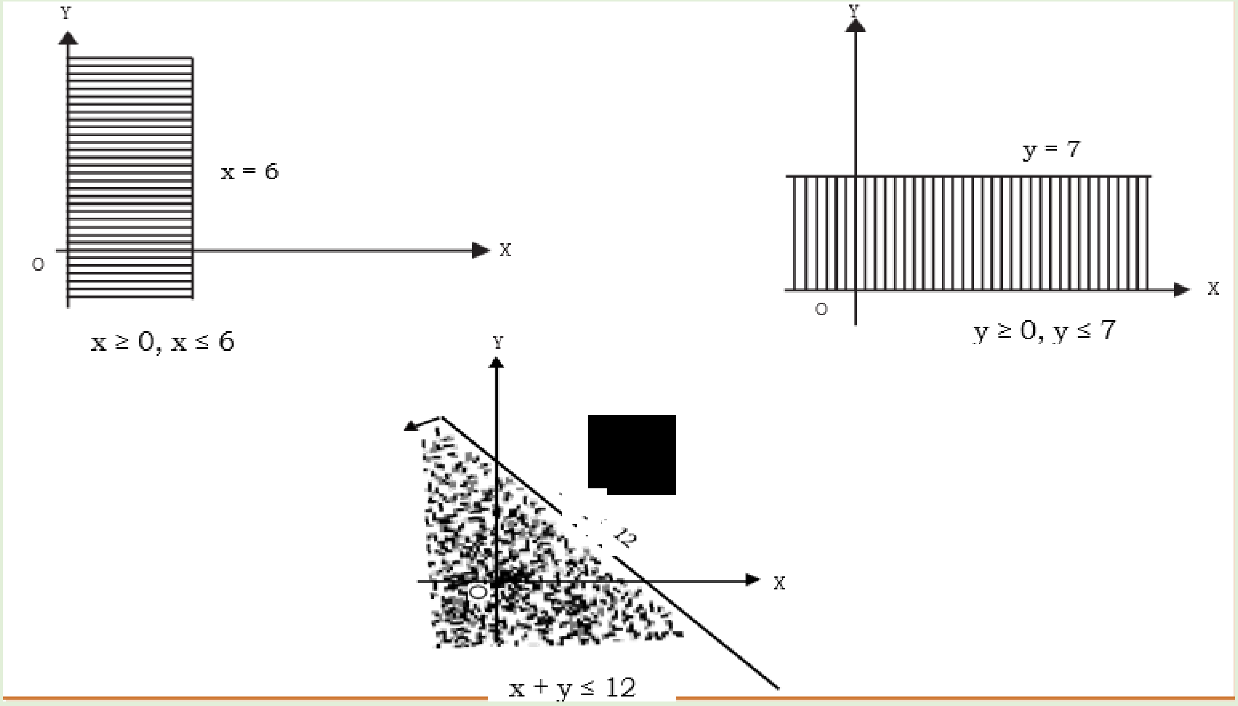
  Since in our example, we had 3x + y < 6, i.e 'Less than' inequality, the solution will be the part below the line as shown in the figure on the left side.

➢ If Plotted line is NOT intersecting (touching) both x & y axis, then we take any point on either side of the line.

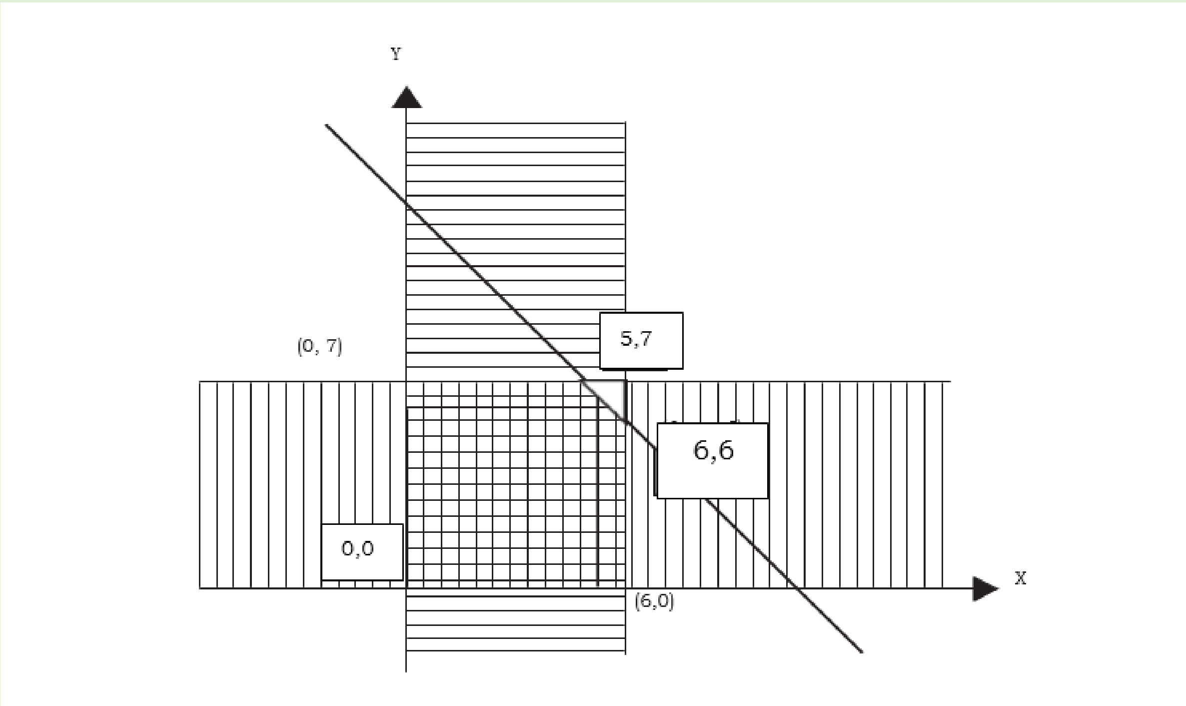
- If that point satisfies the inequality, the part in which the point lies will be our solution.
- If that point does not satisfies the inequality, the part on the other side of the point will be our solution.

CQ1: $x \le 6$, $y \le 7$, $x + y \le 12$, $x \ge 0$, $y \ge 0$. Find the solution space.

Ans:



By superimposing the above three graphs, we determine the common region in the xy plane where all the five inequalities are simultaneously satisfied.



OPTIMAL SOLUTION

- ✓ The objective function attains a maximum or a minimum value at one of the corner points of the feasible solution known as extreme points of the solution set.
- ✓ Once these extreme points (the points of intersection of lines bounding the region) are known, a compact matrix representation of these points is possible. We shall denote the matrix of the extreme points by E.
- ✓ The coefficients of the objective function may also be represented by a column vector. We shall represent this column vector by C.
- ✓ The elements in the product matrix EC shows different values, which the objective function attains at the various extreme points.
- ✓ The largest & the smallest elements in matrix EC are respectively the maximum and the minimum values of the objective function.
- ✓ The row in matrix EC in which this happens is noted and the element in that row indicates the appropriate pairing and is known as the optimal solution.

In the above example;

$$E = \begin{bmatrix} 0 & 0 \\ 0 & 7 \\ 5 & 7 \\ 6 & 0 \\ 6 & 6 \end{bmatrix} \ (\text{columns } X, Y), \quad C = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{matrix} x \\ y \end{matrix}$$

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 7 \\ 5 & 7 \\ 6 & 0 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0\times1+0\times2 \\ 0\times1+7\times2 \\ 5\times1+7\times2 \\ 6\times1+0\times2 \\ 6\times1+6\times2 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \\ 19 \\ 6 \\ 18 \end{bmatrix}$$

The given objective function viz. $Z = x + 2y$ is maximum at the points (5, 7) present in the third row of the matrix E.

Thus, optimal solution is $x = 5$, $y = 7$, & the maximum value of the objective function is 19.

Steps to be followed under graphical solution to a linear programming problem.

- Determine the region that satisfies the set of given inequalities.
- Ensure that the region is bounded*.
- If the region is not bounded, either there are additional hidden conditions which can be used to bound the region or there is no solution to the problem.
- Construct the matrix E of the extreme points, & the column vector C of the objective function.
- Find the matrix product EC.
- For maximization, determine the row in EC where the largest element appears; while for minimization, determine the row in EC where the smallest element appears.

The objective function is optimized corresponding to the same row elements of the extreme point matrix E.

PC Note: If the slope of the objective function be same as that of one side of feasible region, there are multiple solutions to the problem. However, the optimized value of the objective function remains the same.

CQ2: A manufacturer produces two products A and B, and has his machines in operation for 24 hours a day. Production of A requires 2 hours of processing in machine M1 & 6 hours in machine M2. Production of B requires 6 hours of processing in machine M1 & 2 hours in machine M2. The manufacturer earns a profit of Rs. 5 on each unit of A and Rs. 2 on each unit of B. How many units of each product should be produced in a day in order to achieve maximum profit?

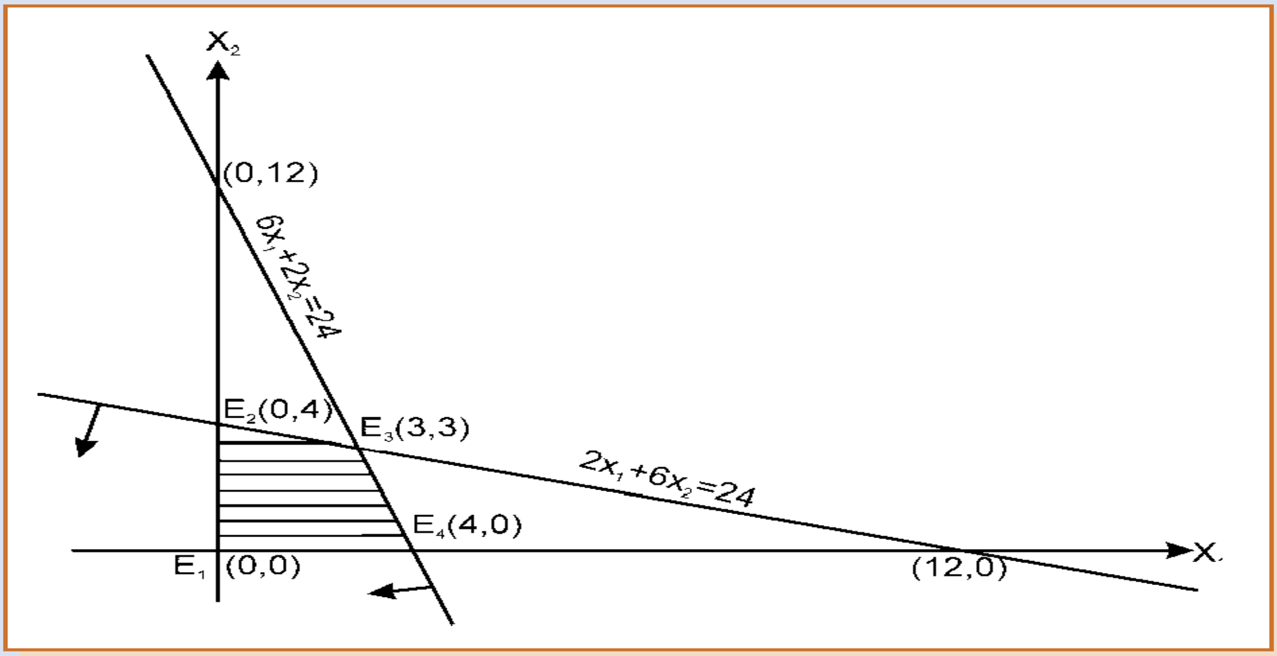
Solution: Let x_1 be the number of units of type A product to be produced, and x_2 is that of type B product to be produced.

Formulation of L.P.P: Maximize $Z = 5x_1 + 2x_2$ subject to the constraints,

$2x_1 + 6x_2 < 24$; $6x_1 + 2x_2 < 24$; $x_1 \geq 0$, $x_2 \geq 0$

For the line $2x_1 + 6x_2 = 24$; Let $x_1 = 0$, so that $x_2 = 4$;   Let $x_2 = 0$, so that $x_1 = 12$.

For the line $6x_1 + 2x_2 = 24$; Let $x_1 = 0$, so that $x_2 = 12$;   Let $x_2 = 0$, so that $x_1 = 4$.



The shaded portion in the diagram is the feasible region and the matrix of the extreme points E_1, E_2, E_3 and E_4 is

$$E = \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 3 & 3 \\ 4 & 0 \end{bmatrix} \begin{matrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{matrix} \quad (\text{columns } X_1, Y_2)$$

The column vector for the objective function is $C = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$

The column vector the values of the objective function is given by

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 3 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0\times5+0\times2 \\ 0\times5+4\times2 \\ 3\times5+3\times2 \\ 4\times5+0\times2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 21 \\ 20 \end{bmatrix} \begin{matrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{matrix}$$

Since 21 is the largest element in matrix EC, therefore the maximum value is reached at the extreme point E_3 whose coordinates are (3,3).

Thus, to achieve maximum profit the manufacturer should produce 3 units each of both the products A and B.

CQ3: Graph $5x_1 + 4x_2 \geq 9$, $x_1 + x_2 \geq 3$, $x_1 \geq 0$, $x_2 \geq 0$; & mark the common region.

Solution: We draw the straight lines $5x_1 + 4x_2 = 9$ and $x_1 + x_2 = 3$.

Table for $5x_1 + 4x_2 = 9$

X_1	0	9/5
X_2	9/4	0

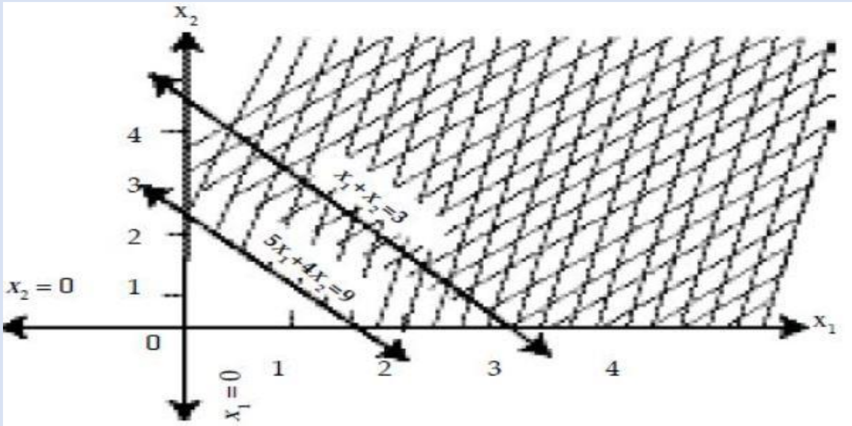
Table for $x_1 + x_2 = 3$

X_1	0	3
X_2	3	0

Now, if we take the point (4, 4), we find $5x_1 + 4x_2 \geq 9$; i.e., $5.4 + 4.4 \geq 9$; or, $36 \geq 9$ (True)

$x_1 + x_2 \geq 3$; i.e., $4 + 4 \geq 3$; $8 \geq 3$ (True);

Hence (4, 4) is in the region which satisfies the inequalities.



HOW TO FORM INEQUATION FROM WORD PROBLEMS

CQ4: A fertilizer company produces two types of fertilizers called Grade I & Grade II. Each of these types is processed through two critical chemical plant units. Plant A has maximum 120 hrs & Plant B has maximum of 180 hrs available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in Plant A and 4 hours in plant B. Manufacturing one bag of Grade II fertilizer requires 3 hrs in Plant A and 10 hours in Plant B.

Answer: Firstly, we need to identify the **key factor (factor having restrictions or conditions).**

Here we have limited Machine Hours & thus Machine hours becomes our Key Factor.

Always arrange 'Key Factor' in columns & other given data in rows.

Particulars	Machine A	Machine B
Chemical Grade I	6 hrs	4 hrs
Chemical Grade II	3 hrs	10 hrs
Maximum Available Time	**120 Hours**	**180 Hours**

Now let's assume that we will produce x units of Chemical Grade I & y units of Chemical Grade II.

Thus, $6x + 3y \leq 120$ & $4x + 10y \leq 180$.

CQ5: Two machines produce two grades of plywood, Grade A & Grade B. In one hour of operation, machine 1 produces 2 units of Grade A & 1 unit of Grade B, while machine II produces 3 units of grade A & 4 units of grade B. Machines are required to meet a production schedule of at least 14 units of grade A & 12 units of grade B.

Answer: Let Machine I operate for x hours & Machine II operate for y hours.

Particulars	Grade A	Grade B
Machine I	2 units	1 unit
Machine II	3 units	4 units
Minimum Quantity required	**14 units**	**12 units**

Thus, $2x + 3y \geq 14$, $x + 4y \geq 12$, $x \geq 0$, $y \geq 0$

INEQUALITIES – QUESTION BANK

SN	CHAPTER 8. INEQUILITIES				Ans
Q1	Solution set of an _____can be represented on a number line.				A
	(a) inequation	(b) equation	(c) Either (a) or (b)	(d) Not (a) & (b)	
Q2	When an inequation is multiplied or divide by sar negative number, inequation ——direction.				A
	(a) Changes	(b) Doesn't Change	(c) Either (a) or (b)	(d) Not (a) & (b)	
Q3	The inequalities x ⩾ 0, y ⩾0 indicates ______.				A
	(a) First quadrant		(b) Second quadrant		
	(c) Third quadrant		(d) Fourth quadrant		
Q4	The inequalities x < 0, y > 0 represents ______.				B
	(a) First quadrant		(b) Second quadrant		
	(c) Third quadrant		(d) Fourth quadrant		
Q5	5X < 20 implies ______.				A
	(a) X < 4		(b) X > 4		
	(c) X less than equal to 4		(d) X greater than equal to 4		
Q6	4x > -16 implies ______.				C
	(a) X ⩾ -4	(b) X <-4	(c) X >-4	(d) X ⩽-4	
Q7	-6X < -18 implies _____.				B
	(a) X < 3	(b) X > 3	(c) X = 0	(d) X = 3	
Q8	X > - 3 Implies ______.				C
	(a) -2x <6	(b) 2x >-6	(c) Both (a) or (b)	(d) Not (a) & (b)	
Q9	X < -3 implies ______.				A
	(a) -2X >6	(b) 2X > -6	(c) Both (a) or (b)	(d) Not (a) & (b)	
Q10	Which of the following values will not satisfy 4x + 3 < 2x + 5?				B
	(a) 0	(b) 1	(c) -1	(d) -2	
Q11	Solve for real 'x' if 5x - 2 ≥2x + 1 & 2x + 3 < 18 - 3x.				C
	(a) 1 < x < 3	(b) - 1 > x > - 3	(c) 1 ≤ x < 3	(d) x = 3	
Q12	If m < n & a < b, then ______.				D
	(a) m - a < n - b	(b) ma <nb	(c) m/a < n/b	(d) m + a < n + b	
Q13	If a < b & c < 0 then ______.				B
	(a) a/c< b/c	(b) a/c > b/c	(c) a/c = b/c	(d) a/c = 0	
Q14	If p - q = - 3 then ______.				A
	(a) p< q	(b) p > q	(c) p = q	(d) p = 0	
Q15	If x ≤ 0, then 2/x + 8/x is ______.				D
	(a) 2 ≤ x ≤ 3	(b) ≥ 0	(c) ≥ 4	(d) ≤ -1	
Q16	What is the smallest integer value of x in 4 - 3x < 11 = ____.				B
	(a) -3	(b) -2	(c) -1	(d) 0	

Q17	What is the largest integer value of p that satisfies the inequality 4 + 3p < p + 1? (a) -2 (b) -1 (c) 0 (d) 1	**A**
Q18	A dealer has only Rs. 5760 to invest in fans (x) & sewing machines (y). Cost per unit of fans and sewing machine is Rs. 360 & Rs.240 respectively. This can be shown by (a) $360x + 240y \geq 5760$ (b) $360x + 240y \leq 5760$ (c) $360x + 240y = 5760$ (d) None of these	**D**
Q19	An employer recruits experienced (x) & fresh workmen (y), But he cannot employ more than 9 people. (a) $x + y \neq 9$ (b) $x + y \leq 9$ (c) $x + y \geq 9$ (d) None	**B**
Q20	Experienced person (x) does 5 units of work while a fresh one (y) does 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as ______. (a) $5x + 3y \leq 30$ (b) $5x + 3y > 30$ (c) $5x + 3y \geq 30$ (d) None	**C**
Q21	Rules demand that employer should employ not more than 5 experienced hands to 1 fresh one. Express as ______. (a) $y \geq x/5$ (b) $5y \geq x$ (c) Both (a) and (b) (d) $5y \leq x$	**C**
Q22	Union forbids him to employ less than 2 experienced persons to each fresh person. This can be expressed as ______. (a) $x \leq y/2$ (b) $y \leq x / 2$ (c) $x \geq 2y$ (d) Both (b) & (c)	**D**
Q23	A scooter company manufactures scooters of two models A &B. Model A requires 15 man-hours for assembly, 5 man-hours for painting &1 man- hour for testing. Model B requires 6 man-hours for assembly, 4-man hours for painting & 2 man hours for testing. There are 300 man-hours available in the assembly shop, 120 man-hours in painting shop and 50 man-hours available in testing division. (a) $15x + 6y \leq 300$, $x + 2y \leq 50$, $5x + 4y \leq 120$ (b) $15x + 6y \leq 300$, $x + 2y \geq 50$, $5x + 4y \geq 120$ (c) $15x + 6y \geq 300$, $x + 2y \leq 50,5x + 4y \geq 120$ (d) $15x + 6y \leq 300$, $x + 2y \leq 50$, $5x + 4y \leq 120$	**A**
Q24	A company produces two types of leather belts, say A and B. Belt A is of superior quality and belt B is of lower quality. Each belt of type A requires twice as much as time required by a belt of type B. If all belts were of type B, the company could produce1000 belt per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires fancy buckles and only 400 fancy buckles are available per day. For belt of type B only 700 buckles are available per day. Assuming that the company produces x unit of belt A and y units of belt B: (a) $x + 2y \geq 1000$, $x + y \geq 800$, $x \geq 400$; $y \leq 700$ (b) $x + 2y \leq 1000$, $x + y \leq 800$, $x \leq 400$; $y \leq 700$ (c) $x + 2y \geq 1000$, $x + y \leq 800$, $x \geq 400$; $y \geq 700$ (d) $x + 2y \leq 1000$, $x + y \geq 800$, $x \leq 400$; $y \geq 700$	**B**
Q25	A firm makes two types of products: Type A and Type B. The profit on product A is Rs.20 each and that on product B is Rs. 30 each. Both types are processed on three machines MI, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine is as follows:	**C**

Machine	A	B	Total Time available
M1	3	3	36
M2	5	2	50
M3	2	6	60

Constraints can be formulated taking x_1 = No. of units A & x_2=No. of unit of B as __.

(a) $x_1 + x_2 \leq 12$; $5x_1 + 2x_2 \leq 50$; $2x_1 + 6x_2 \leq 60$;

(b) $3x_1 + 3x_2 \geq 36$; $5x_1 + 2x_2 \leq 50$; $2x_1 + 6x_2 \geq 60$;

(c) $3x_1 + 3x_2 \leq 36$; $5x_1 + 2x_2 \leq 50$; $2x_1 + 6x_2 \leq 60$;

(d) None of these

Q26 Vitamins A and B are found in food F_1 and F_2. One unit of F_1 contains 20 units of vitamin A &30 units of vitamin B. One unit of food F_2 contains 60 units of vitamin A &40 units of vitamin B. Cost per unit of F_1&F_2 are Rs. 3 &Rs. 4 respectively. The minimum daily requirement of vitamin A &B is 80 &100 units respectively.

Problem is to determine mixture of F_1 & F_2, which meets the requirement at minimum cost.

(a) $20x_1 + 60x_2 \leq 80$, $30x_1 + 40x_2 \leq 100$, $x_1 \leq 0$; $x_2 \leq 0$

(b) $20x_1 + 60x_2 \geq 80$, $30x_1 + 40x_2 \leq 100$, $x_1 \geq 0$; $x_2 \leq 0$

(c) $20x_1 + 60x_2 \geq 80$, $30x_1 + 40x_2 \geq 100$, $x_1 \geq 0$; $x_2 \geq 0$

(d) $20x_1 + 60x_2 \leq 80$, $30x_1 + 40x_2 \geq 100$, $x_1 \leq 0$; $x_2 \geq 0$

C

Q27 A firm produces two types of gadgets A &B, which are first processed in the foundary, and then sent to another machine for finishing. The number of man-hours for the firm available per week are as follows:

Particulars	Foundry	Machine-shop
A	10	5
B	6	4
Capacity per week (man hours)	**100**	**600**

Let the firm manufacture x units of A and y units of 8. The constraints are:

(a) $10x + 6y \leq 1000$, $5x + 4y \geq 600$, $x \geq 0$; $y \leq 0$

(b) $10x + 6y \leq 1000$, $5x + 4y \leq 600$, $x \geq 0$; $y \geq 0$

(c) $10x + 6y \geq 1000$, $5x + 4y \leq 600$, $x \leq 0$; $y \geq 0$

(d) $10x + 6y \geq 1000$, $5x + 4y \geq 600$, $x \leq 0$; $y \leq 0$

B

Q28 A firm plans purchase hens (x) for its canteen. There cannot be more than 20 hens.

(a) $x \leqslant 20$ (b) $x = 20$ (c) $x > 20$ (d) None

A

Q29 In a class of boys (x) & girls (y), maximum seating capacity is 360. This can be shown by ______.

(a) $x + y \leq 360$ (b) $x + y \geq 360$ (c) $x + y \neq 360$ (d) None

A

Q30 Mr. A plans to invest upto Rs. 30,000 in two stocks X and Y. Stock X (x) is priced at Rs.175 & Stock Y (y) at Rs. 95 per share. This can be shown by ______.

(a) $175x + 95y \leq 30{,}000$ (b) $175x + 95y \geq 30{,}000$

(c) $175x + 95y = 30{,}000$ (d) None

A

Q31 A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C &12 units of vitamin D. The vitamin content per Kg. of each food is shown below: **D**

Particulars	A	B	C	D
Food I	2	1	1	2
Food II	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II, expressed as -

(a) $2x+y\le9$; $x+y\le7$; $x+2y\le10$; $2x+3y\le12$; $x>0$, $y>0$

(b) $2x+y\ge30$; $x+y\le7$; $y+2y\ge10$; $x+3y\ge12$

(c) $2x+y\ge9$; $x+y\ge7$; $y+y\le10$; $x+3y\ge12$

(d) $2x+y\ge9$; $x+y\ge7$; $y+2y\ge10$; $2x+3y\ge12$; $x\ge0$, $y\ge0$.

Q32 A man makes two types of furniture: chairs and tables. Profits are 20 per chair and t 30 per table. Both the products are processed on two machines M1 and M2. The time required for each product in hours and total time available in hours per week on each machine are as follows: **C**

Machine	Chair	Table	Available Time
M_1	3	3	36
M_2	5	2	50

(a) $x + y \le 12$, $5x + 2y \ge 50$, $x \le 0$; $y \ge 0$
(b) $x + y \ge 12$, $5x + 2y \le 50$, $x \ge 0$; $y \le 0$
(c) $x + y \le 12$, $5x + 2y \le 50$, $x \ge 0$; $y \ge 0$
(d) $x + y \ge 12$, $5x + 2y \ge 50$, $x \le 0$; $y \le 0$

Q33 Solve for real 'x' if $(x - 4) / (2x - 3) \le 0$ **B**

(a) $x = 1/8$ or $2/3$ (b) $1.5 < x \le 4$ (c) $x = 4$ or $3/2$ (d) $x\ge4$

Q34 If $xy > 1$ and $z < 0$, which of the following statements must be true? **C**

I. $x > z$ II. $xyz < -1$ III. $xy/z < 1/z$

(a) I only (b) II only (c) III only (d) II and III

Q35 If $A = x - 2^{-1}$, $B = x + 2^{-1}$ and $A^2 - B^2 > 0$, then______. **B**

(a) $x > 0$ (b) $x < 0$ (c) $x = 0$ (d) $x = A + B$

Q36 **When $x > 0$, value of $|x|$ is ______.** **C**

(a) 0 (b) $-x$ (c) x (d) 1

Q37 Common region represented by in equalities $2x + y \ge 8$, $x + y \ge 12$, $3x + 2y \le 34$ is ___. **C**

(a) Unbounded (b) In feasible
(c) Feasible and bounded (d) Feasible and unbounded

Q38 The union forbids the employer to employ less than 2 experienced persons (x) to each fresh person (y). This situation can be expressed as ______. **B**

(a) $x\le y/2$ (b) $y\le x/2$ (c) $y \ge x / 2$ (d) None

Q39 A fertilizer company produces two types of fertilizers called Grade I and Grade II. Each of these types is processed through two critical chemical plant units. Plant A has maximum 120 hrs available in a week and Plant B has maximum of 180 hrs available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in Plant A and 4 hours in plant B. **C**

Manufacturing one bag of Grade II fertilizer requires 3 hrs in Plant A & 10 hours in Plant B.

(a) $6x + 3y \leq 120$, $4x + 10y = 180$ (b) $6x + 3y = 120$, $4x + 10y > 180$

(c) $6x + 3y \leq 120$, $4x + 10y \leq 180$ (d) $6x + 3y < 120$, $4x + 10y < 180$

Q40 A man makes two types of furniture: chairs and tables. Profits are Rs. 20 per chair and Rs. 30 per table. Both the products are processed on two machines M1 & M2. Time required for each product in hours and total time available in hours per week on each machine are as follows: **C**

Machine	Chair	Table	Available Time
M_1	3	3	36
M_1	5	2	50

Constraints can be formulated by taking x = no. of chairs, y= no. of tables produced as:

(a) $x + y \leq 12$, $5x + 2y \geq 50$, $x \leq 0$; $y \geq 0$ (b) $x + y \geq 12$, $5x + 2y \leq 50$, $x \geq 0$; $y \leq 0$

(c) $x + y \leq 12$, $5x + 2y \leq 50$, $x \geq 0$; $y \geq 0$ (d) $x + y \geq 12$, $5x + 2y \geq 50$, $x \leq 0$; $y \leq 0$

Q41 Suppose a man needs a minimum of 50 units of carbohydrate, 40 units of proteins per month for good health. He is taking food at two places, viz., A and B, food at A contains 4 and 5 units of carbohydrates and proteins respectively. **C**

Express this in the form of linear inequalities assuming the man is in good health. Let x_1 and x_2 represent carbohydrates and proteins respectively. Then mathematical inequalities are:

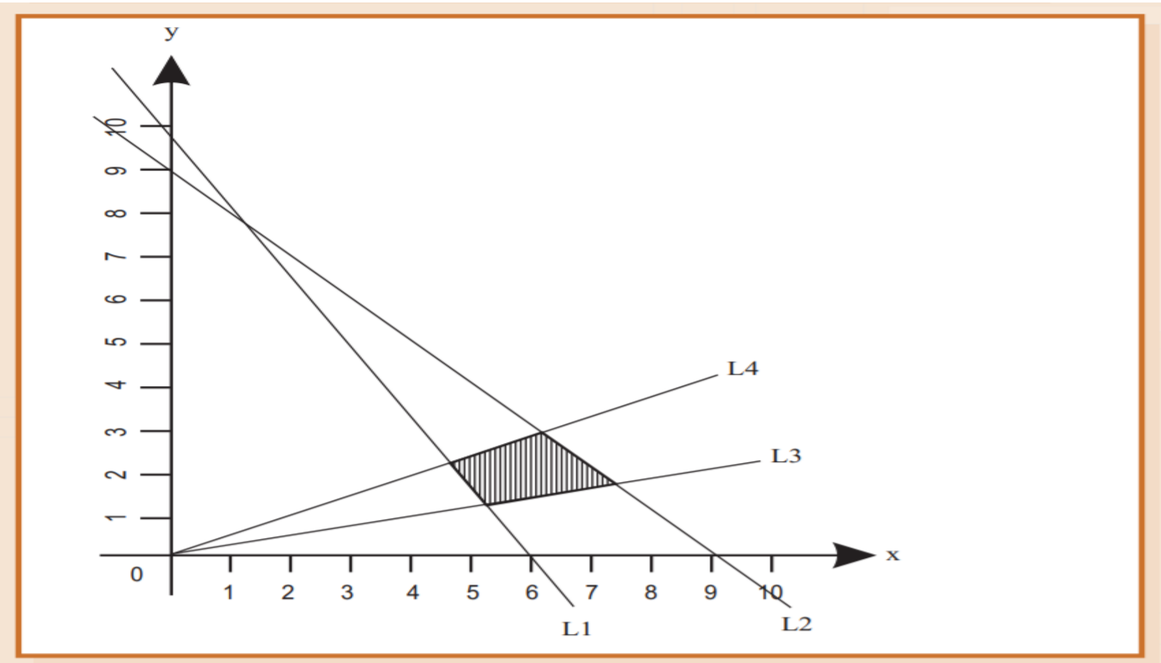
(a) $4x_1 + x_2 \geq 50,\ 5x_1 + 3x_2 \leq 40,\ x_1 \geq 0; x_2 \leq 0$

(b) $4x_1 + x_2 \leq 50,\ 5x_1 + 3x_2 \geq 40,\ x_1 \leq 0;\ x_2 \geq 0$

(c) $4x_1 + x_2 \geq 50,\ 5x_1 + 3x_2 \geq 40,\ x_1 \geq 0; x_2 \geq 0$

(d) $4x_1 + x_2 \leq 50,\ 5x_1 + 3x_2 \leq 40,\ x_1 \leq 0; x_2 \leq 0$

Q42 **L1: 5x + 3y = 30; L2: x + y = 9; L3: y = x/3; L4: y=x/2.** Common region (shaded part) refers to ______. **[ICAI SM Q1(viii)]** **B**

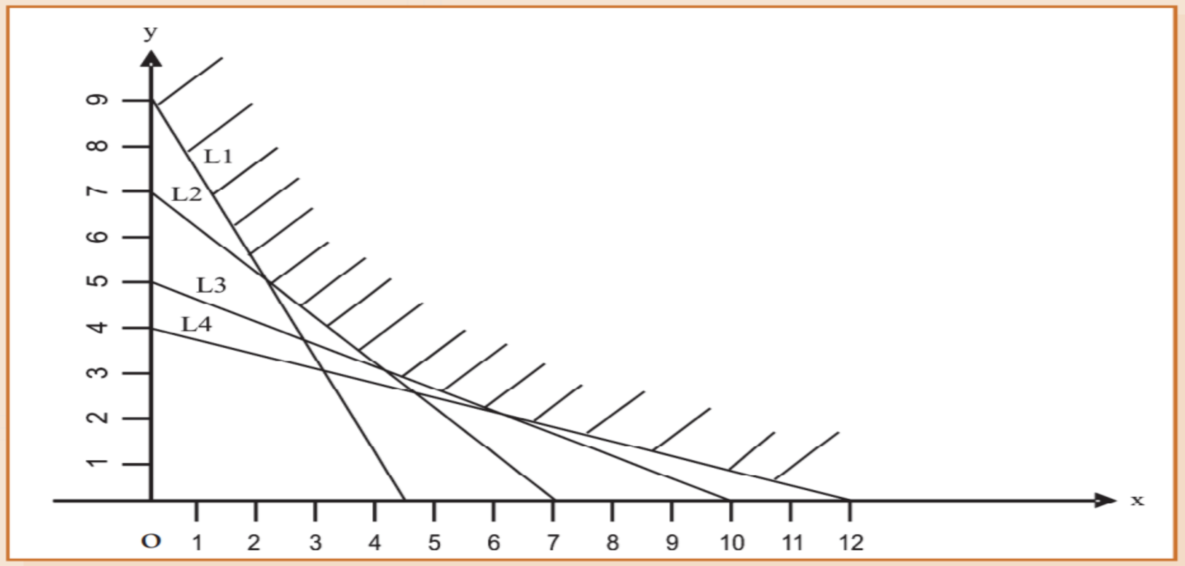


(a) 5x+3y≤30; x+y≤9; y≤1/5x; y≤x/2 (b) 5x+3y≥30; x+y≤9; y≥x/3; y≤x/2; x≥0, y≥0

(c) 5x+3y≥30; x+y≥9; y≥x/3; y≥x/2; x≥0, y≥0 (d) 5x+3y>30; x+y<9; y≥9; y≤x/2; x≥0, y≥0

Q43 Common region (Shaded part) expressed by the set of inequalities is **[ICAI SM Q3]** — **C**

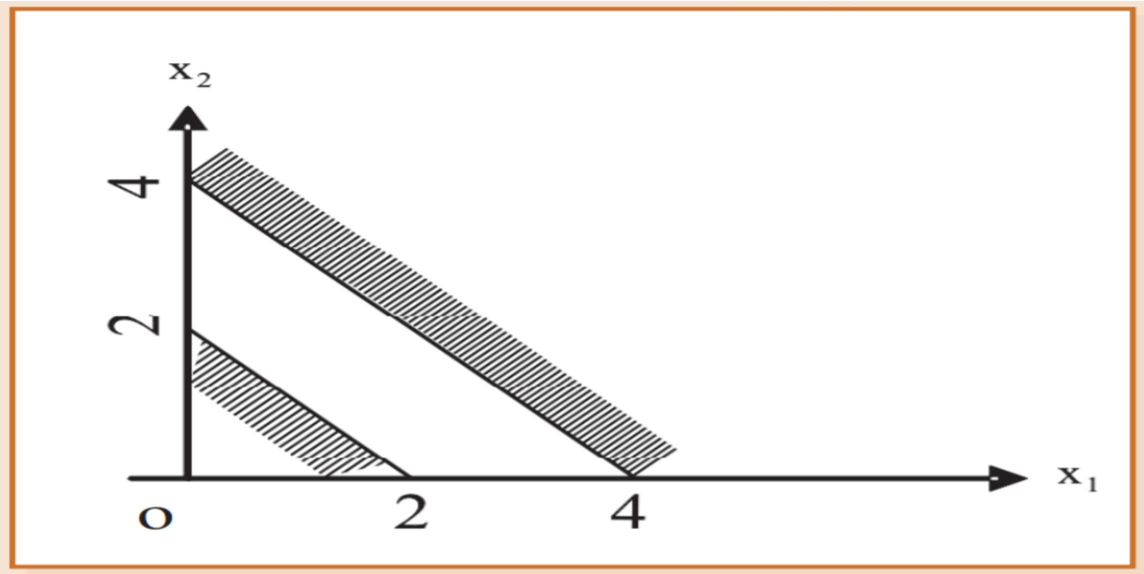
L1: 2x + y = 9; L2: x + y = 7; L3: y = x+2y = 10; L4: x+3y=12



(a) $2x+y \leq 9$; $x+y \geq 7$; $x+2y \geq 10$; $2x+3y \geq 12$ (b) $2x+y \geq 9$; $x+y \leq 7$; $x+2y \geq 10$; $x+3y \geq 12$

(c) $2x+y \geq 9$; $x+y \geq 7$; $y+2y \geq 10$; $x+3y \geq 12$; (d) None of these

Q44 Region indicated by shading in the graph is expressed by inequalities. **[ICAI SM Q5]** — **A**



(a) $x_1 + x_2 \leq 2$; $2x_1 + 2x_2 \geq 8$; $x_1 \geq 0$, $x_2 \geq 0$ (b) $x_1 + x_2 \leq 2$; $2x_1 + x_2 \leq 4$

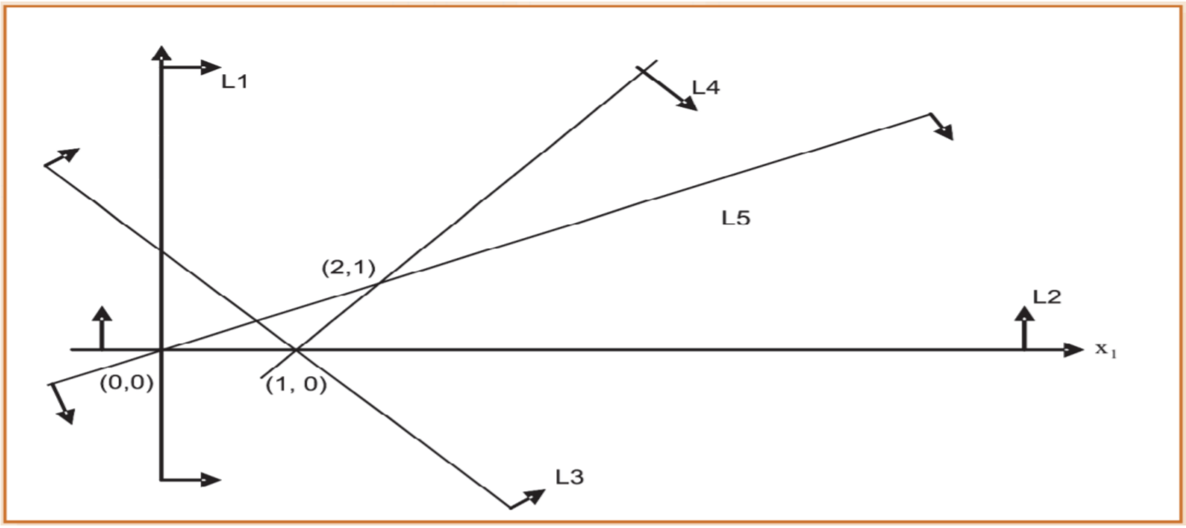
(c) $x_1 + x_2 \geq 2$; $2x_1 + 2x_2 \geq 8$ (d) $x_1 + x_2 \leq 2$; $2x_1 + 2x_2 > 8$

Q45 Two machines (I and II) produce two grades of plywood, Grade A and Grade B. In one hour of operation, machine I produces 2 units of Grade A and one unit of Grade B, while machine II, in one hour of operation produces 3 units of grade A and four units of grade B. The machines are required to meet a production schedule of atleast 14 units of grade A and 12 units of grade B. — **C**

(a) $2x + 3y \geqslant 14$, $x + 4y \geqslant 12$, $x > 0$, $y \geqslant 0$ (b) $2x + 3y \leqslant 14$, $x + 4y = 12$, $x > 0$, $y > 0$

(c) $2x + 3y \geqslant 14$, $x + 4y \geqslant 12$, $x \geqslant 0$, $y \geqslant 0$ (d) $2x + 3y = 14$, $x + 4y = 12$, $x \geqslant 0$, $y \geqslant 0$

Q46 Common region indicated on the graph is expressed by the set of five inequalities. **B**



(a) L1: $x_1 \geq 0$; L2: $x_2 \geq 0$; L3: $x_1 + x_2 \leq 1$; L4: $x_1 - x_2 \geq 1$; L5: $-x_1 + 2x_2 \leq 0$

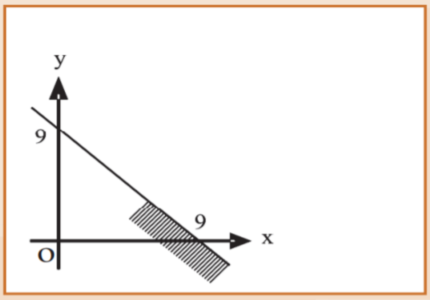
(b) L1:$x_1 \geq 0$; L2: $x_2 \geq 0$; L3: $x_1 + x_2 \geq 1$; L4:$x_1 - x_2 \geq 1$; L5: $-x_1 + 2x_2 \leq 0$

(c) L1:$x_1 \leq 0$; L2: $x_2 \leq 0$; L3: $x_1 + x_2 \geq 1$; L4:$x_1 - x_2 \geq 1$; L5: $-x_1 + 2x_2 \leq 0$

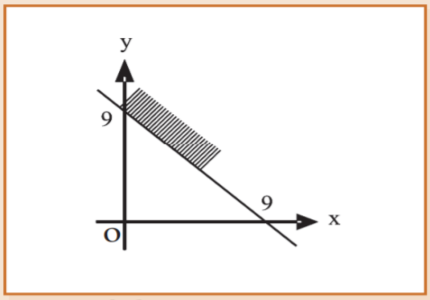
(d) None of these **[ICAI SM Q7]**

Q47 Graph to express the inequality $x + y \leqslant 9$ is **[ICAI SM Q1(v)]** **A**

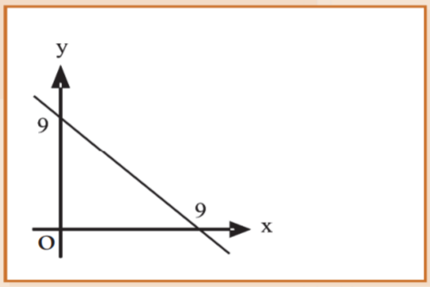
(a)



(b)



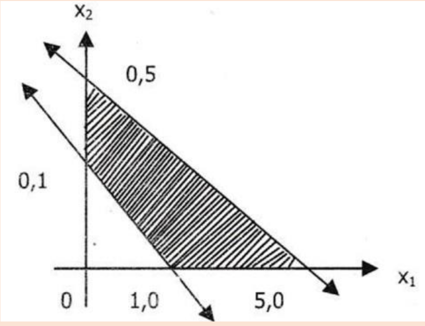
(c)



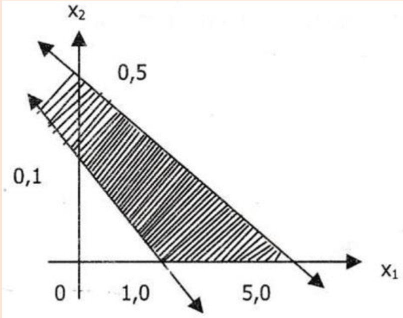
(d) none of these

Q48 The inequalities $X_1 + 2x_2 \leqslant 5$, $x_1 + x_2 \geqslant 1$, $x_1 \geqslant 0$, $x_2 \geqslant 0$ represents the region **A**

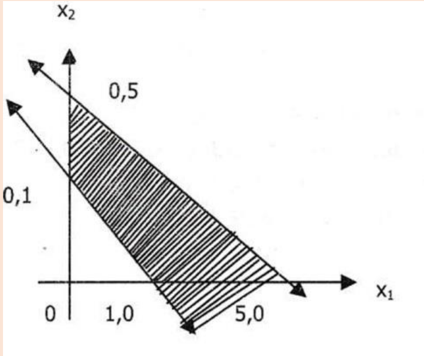
(a)



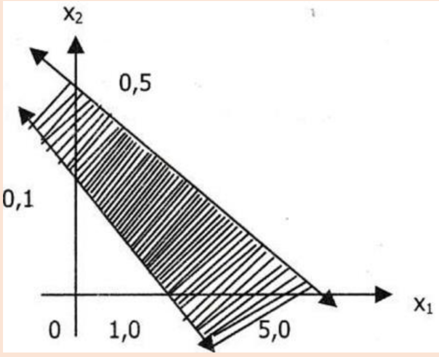
(b)



(c)

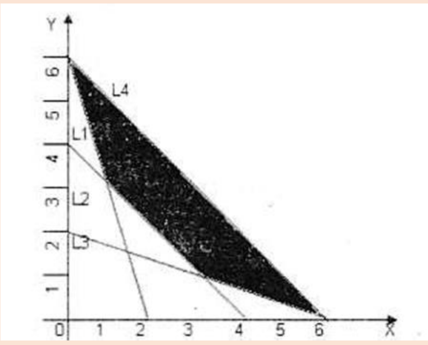


(d)

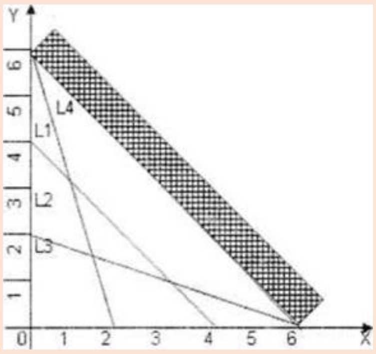


Q49 Common region satisfied by the inequalities L1:3x+y≥6, L2:x+y≥4, L3:3x+3y≥6, and L4:x+y≤6, — **A**

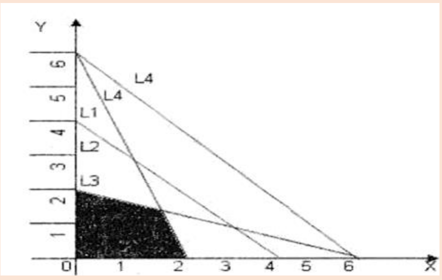
(a)



(b)

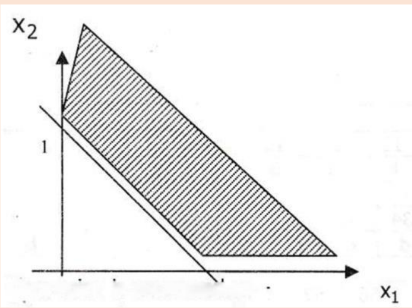


(c)



(d) None

Q50



The region is express as -

(a) $x_1-x_2 \geq 1$

(b) $x_1+x_2 \leq 1$

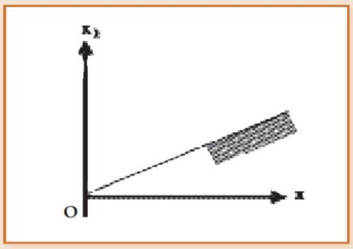
(c) $x_1+x_2 \geq 1$

(d) None of these

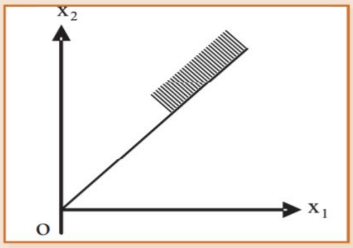
C

Q51 The inequality -x+2y ≤ 0 is indicated on the graph as: **[ICAI SM Q6(iii)]** — Answer: **A**

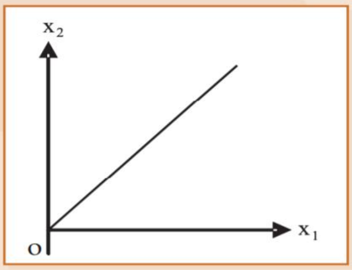
(a)



(b)

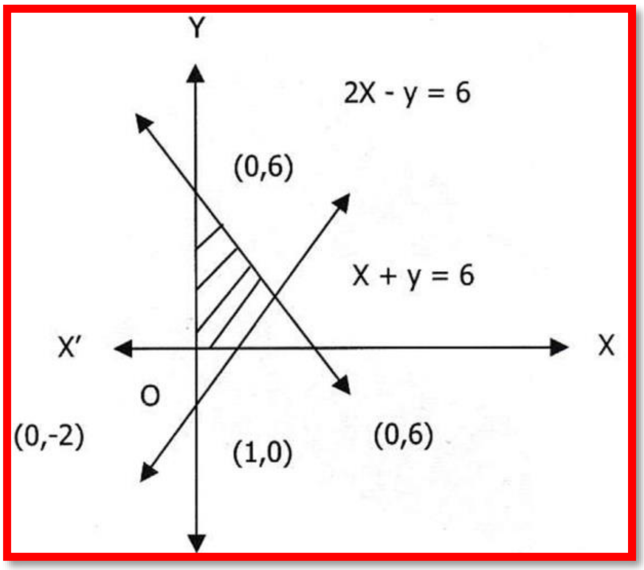


(c)



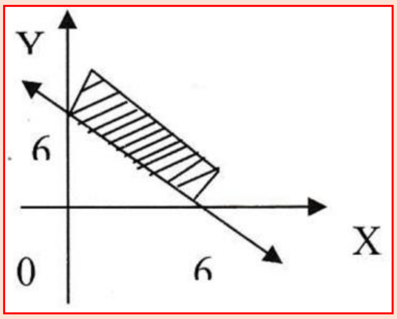
(d) none of these

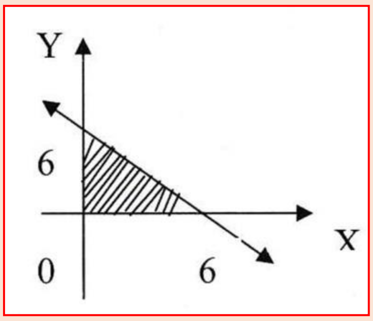
Q52 By lines x + y = 6, 2x - y = 2, the common region shown is the diagram refers to — Answer: **B**

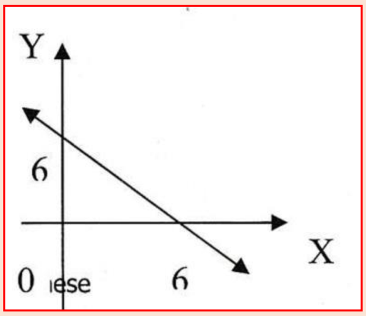


(a) x + y ≥ 6, 2x - y ≤ 2, x ≥ 0, y ≥ 0
(b) x + y≤6, 2x-y≤2,x≥0,y≥0
(c) x + y ≤ 6, 2x - y ≥ 2, x ≥0, y ≥ 0
(d) None of these

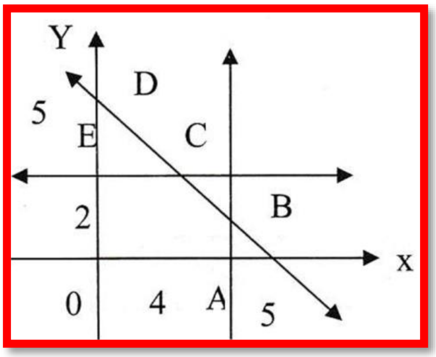
Q53 Which of the following graph represents the inequality x + y ≤ 6 is ______. — Answer: **B**







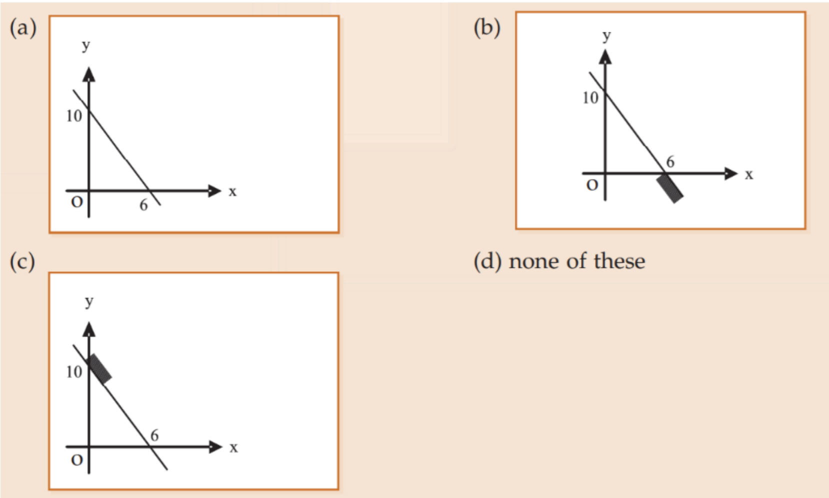
Q54 Given conditions x+y ⩾ 5, x+y ⩽ 5, 0 ⩽ x ⩽ 4 and 0 ⩽ x ⩾ 2 **C**



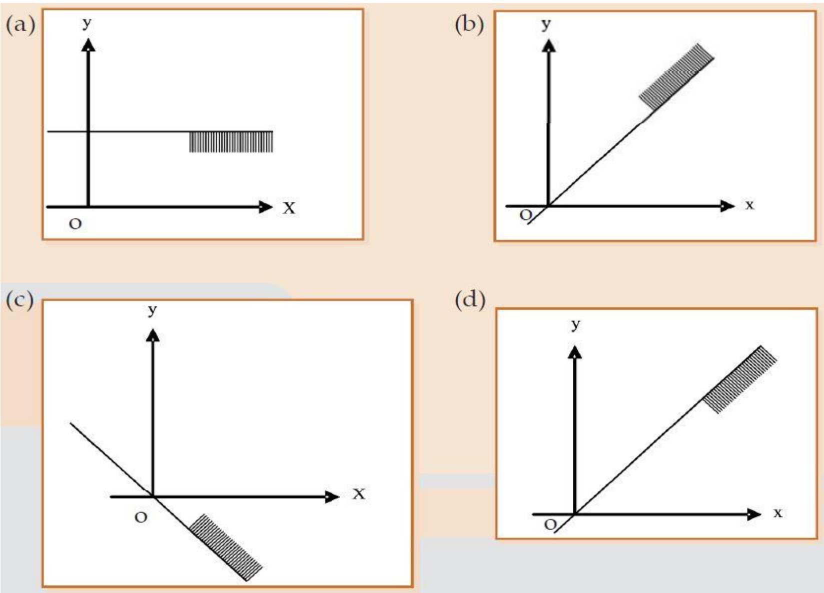
then the common region under these conditions is

(a) ECDE (b) EOABCE (c) Line segment CD (d) Line segment BC

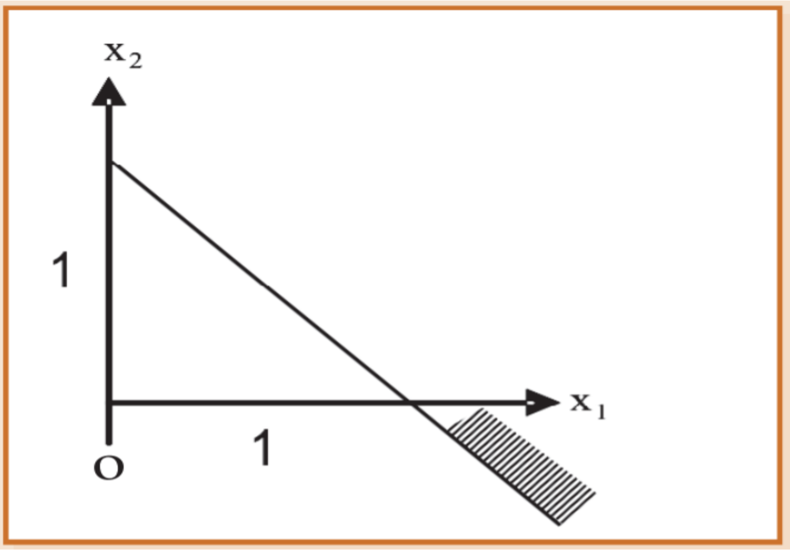
Q55 Graph to express the inequality 5x + 3y ⩾ 30 is ______. **C**



Q56 The graph to express the inequality y ⩽ $\frac{x}{2}$ is indicated by______. **D**



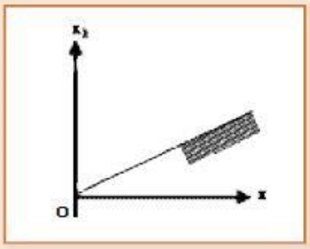
Q57 The region is expressed as ______. **[ICAI SM Q6(ii)]** — **Answer: C**



(a) $x_1 - x_2 \geq 1$ (b) $x_1 + x_2 \leq 1$ (c) $x_1 + x_2 \geq 1$ (d) None

Q58 The inequality $-x_1 + 2x_2 \leq 0$ is indicated on the graph as______. — **Answer: A**

(a)



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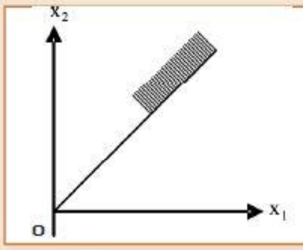
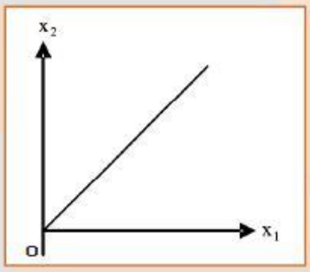


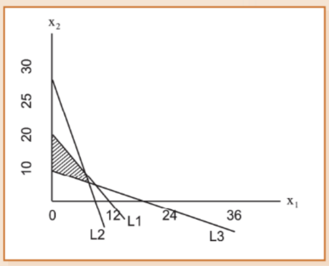
(c)



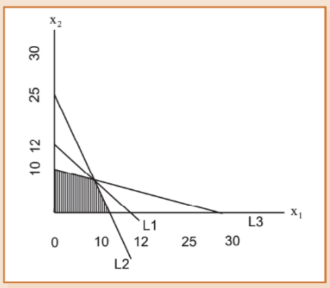
(d) none of these

Q59 The set of inequalities L_1: $x + y \leqslant 12$; L_2: $5x+2y \leqslant 50$; L_3: $x + 3y \leqslant 30$; $x \geqslant 0$ & $y \geqslant 0$ is — **Answer: B**

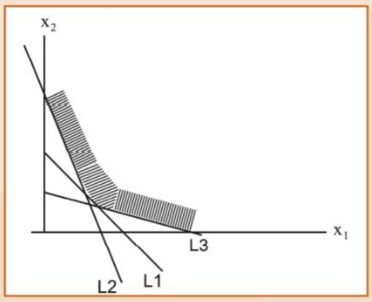
(a)



(b)



(c)



CHAPTER 4. TIME VALUE OF MONEY

Meaning of Some Important Terms

- **Interest:** Interest is the price paid by a borrower for use of a lender's money.
- **Principal:** Principal is the initial amount lent/borrowed.
- **Rate of Interest:** The rate at which the interest is charged for a defined period of time for use of principal (generally on yearly basis) is known as rate of interest. It is usually expressed as percentages.
- **Time:** It is no. of years for which the principle is borrowed or loaned.
- **Accumulated amount (Balance):** It is the final value of an investment. [Principal + Interest].

CONCEPT 1: SIMPLE INTEREST

- Simple interest is the interest computed on the principal for the entire period of borrowing.
- Interest is calculated on the original principal and not on interest previously earned.

NO Interest is paid on Interest Earned.

Simple Interest (SI) = Principal (P) × Rate of Interest (R) × Time in years (T).

Accumulated Amount (A) = P + SI = P + PRT = P (1 + RT).

PC NOTE: Sometimes, we are given two different accumulated amounts for two time period & we have to find out interest, principal & Rate of Interest. Let two accumulated amounts be A_1 & A_2 & time period be T_1 & T_2

Interest per year = $\frac{A_2 - A_1}{T_2 - T_1}$; Rate of Interest = $\frac{A_1 - A_2}{A_1 T_2 - A_2 T_1} \times 100$

How to find Time or Rate to multiply a sum at S.I.

Particular	Sum is 1.5 times	Sum is Doubled	Sum is Trebled	Sum is 4 times
Time Req. (Yrs)	T = $\frac{0.5}{R}$ yrs	T = $\frac{1}{R}$ yrs	T = $\frac{2}{R}$ yrs	T = $\frac{3}{R}$ yrs
Rate Req.	R = $\frac{0.5}{T}$	R = $\frac{1}{T}$	R = $\frac{2}{T}$	R = $\frac{3}{T}$

CQ1: A sum of money amount to Rs. 6,200 in 2 years and Rs. 7,400 in 3 years. The principal & rate of interest are:

(a) Rs. 3,800, 31.57% (b) Rs. 3,000, 20% (c) Rs. 3,500, 15% (d) None

CQ2: Calculate the simple interest on Rs. 50,000 at 12% simple interest for 5 years?

CQ3: Sania Mirza deposited Rs. 50,000 in a bank for 20 years with interest rate of 5.5% p.a. How much interest would she earn? Find the final value of her investment.

CQ4: Find rate of interest if amount owed after 6 months is Rs. 1050 & borrowed amount is Rs. 1000.

CQ5: Katrina gave Rs. 70,000 as loan to Salman Khan @ 6.5% p.a. SI. She received Rs. 85,925 after the end of term. Find out the period for which loan was given by Katrina to Salman Khan.

CQ6: Sharmaji deposited a particular amount in a bank for 7.5 years @ 6% p.a. SI. He received Rs. 1,01,500 at the end of the term. Compute initial deposit of Sharmaji.

CQ7: Rs. 46,875 was lent out at SI & at the end of 1 year & 8 months, total amount was Rs. 50,000. Find R.

CQ8: What amount will produce Rs. 28,600 as an interest in 3 years and 3 months at 2.5% p.a. simple interest?

CQ9: In what time will Rs. 85,000 amount to Rs. 1,57,675 at 4.5 % p.a.?

CQ10: A sum doubles itself in 10 years. Find interest rate.

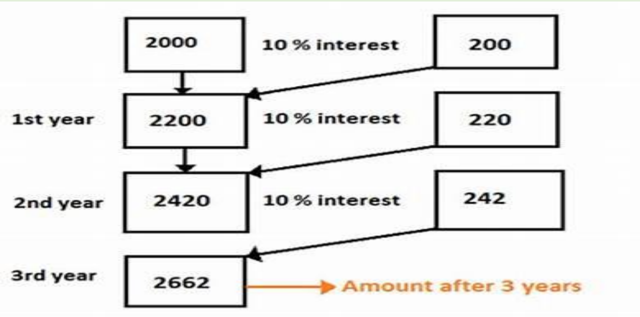
(a) 10 % (b) 12 % (c) 15 % (d) 20 %

CONCEPT 2: COMPOUND INTEREST

- If the interest of a period is added to the principal & interest for next period is calculated on revised principal [Original Principal + Interest], it is called compound interest.
- In CI, principal does not remain same, i.e Principal goes on changing every year.

Interest is charged on Interest Earned.

Amount (A) = $P(1 + R)^T$ **Interest (I)** = A – P



CQ11: PC deposited Rs. 1 crore in a nationalized bank for 3 years. If the rate of interest is 7% p.a. Calculate the interest after 3 years if interest is compounded annually. Also calculate the amount at the end of third year.

Conversion Period

The fixed period at the end of which the interest is calculated & added to the principal is called conversion period.

Ex: When the interest is calculated & added to the principal every 6 months, conversion period is six months. In this case number of conversion periods per year (denoted by K) would be two.

Conversion period & frequency	Number of Conversion Period in a Year (K)	Formula to be used
12 Months (Annually)	1	$A = P(1+R)^{T}$
6 Months (Semi annually)	2	$A = P(1+\frac{R}{2})^{2T}$
3 Months (Quarterly)	3	$A = P(1+\frac{R}{4})^{4T}$
1 Month (Monthly)	12	$A = P(1+\frac{R}{12})^{12T}$
1 Day (Daily)	365	$A = P(1+\frac{R}{365})^{365T}$

Formula to be used: **Amount (A)** $= P(1+\frac{R}{K})^{KT}$ where 'K' is no. of conversion per year.

PC Note:

- If rate of interest is same, CI increases with increase in frequency of compounding.
- If nothing is mentioned in the problem, the interest is taken as 1 yr.
- SI & CI. Are equal for the first conversion period on same sum and same rate
- Amount for CI (P, A_1, A_2,........) form a GP, where $r = (1 + i)$. Also true for intervals.
- CI for each period also forms a GP, where $r = (1 + i)$ [$CI_{2nd} - CI_{1st}$ = SI on CI_{1st})
- CI formula can be used in case of uniform periodical increase at fixed rate like **population growth.** In case of uniform decrease like **depreciation** (W.D.V basis) **i is replaced by -i.**

Years required for a Sum to Double at CI with annual compounding

Thumb Rule	R = 10%	R = 11%	R = 12%	R = 13%	R = 14%
$T = 0.35 + \frac{0.69}{R}$	7.25 Yrs.	6.62 Yrs	6.1 Yrs	5.65 Yrs	5.27 Yrs

CQ12: Rs. 10,000 is invested at annual rate of interest of 10%. What is the amount after 2 years if compounded?

(a) Annually (b) Semi-annually (c) Quarterly (d) Monthly

Points to Remember

- Different Interest Rate for different year (R_1, R_2, R_3) → $A_n = P(1+R_1)(1+R_2)(1+R_3)\ ......(1+R_n)$.

  [Use Calculator as: $A_n = (1 + R_1\% + R_2\% + R_3\% + ... + R_n\%) \times P$
- Time required for a sum to double itself @ 'R' rate of interest (CI) = [$0.35 + \frac{0.69}{R}$] years.
- CI for 1st year = SI for 1st year. But then 2nd year onwards, CI & SI will be different.

  For Annual Compounding only
- CI for 2 years – SI for 2 years = PR^2
- CI for 3 years – SI for 3 years = $PR^2(R+3)$
- $R = \frac{2(CI_2 - SI_2)}{SI_2}$

CONCEPT 3: NOMINAL RATE & EFFECTIVE RATE OF INTEREST

1. **Nominal Rate:** Annual Compound Interest Rate is called N.R. [Compounded annually]
   - It is the stated interest rate. It is the simplest type of interest rate.
   - This rate works according to the simple interest & does not take into account the compounding periods.

CQ13: If a bank pays 5% compounded annually on a savings account, then 5% is the nominal interest rate

2. **Effective Rate of Interest (E):** If the amount is compounded more than once a year, the actual rate of interest (we got) is called effective rate of interest. If we compound the interest more than once a year, effective interest rate for the year will be more than actual interest rate per annum.

   It is the actual equivalent annual rate of interest at which an investment grows in value when interest is credited more often than once a year.

$E = (1 + \frac{R}{K})^K - 1$ [E = Effective interest rate; R = Interest rate per annum; K = No. of conversion period]

PC Note: Effective rate of Interest is relevant when the amount is compounded more than one a year. Effective Interest Rate has nothing to do with Principal. It is related to interest rate & frequency of compounding.

CQ14: Rs. 5,000 is invested in Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? What is effective rate of interest? [Interest = Rs, 306.82; E = 6.13%].

CQ15: Which is better investment? (i) 3% p.a compounded monthly or (ii) 3.2% p.a SI. [$(1+0.0025)^{12}$ =1.0304]

Solution: K = 12 times; E = $(1 + \frac{R}{K})^n - 1$; E = $(1 + \frac{3}{12})^{12} - 1$; = 1.0304 -1 = 0.0304. Thus, **E = 3.04%**

Answer: Effective rate of interest < 3.2% & thus SI @ 3.2% per year is the better investment.

JUST FOR KNOWLEDGE

Real Rate of Return: It is so named because it states the '**real rate**' that lender or investor receives after taking the effect of inflation. [Interest rate that exceeds the inflation rate]

Real Rate of Return = Nominal Rate of Return – Inflation.

How Banks attract customers?

While charging interest, they advertise the nominal rate, which is lower and does not reflect how much interest the consumer would owe on the balance after a full year of compounding. While paying interest on saving deposit accounts, they generally advertise the effective rate because it looks higher than the nominal rate.

PC NOTE: More the compounding period in a year, more expensive the loan becomes. So choose a loan in which the interest is compounded annually.

CONCEPT 4: ANNUITY

- Annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.
- When we pay (or receive) a fixed amount of money periodically over a specified time period we create an annuity.

  Ex: Payment of life insurance premium, EMI of a loan, receipt of pension.

Features of Annuity:

- Amount paid (or received) must be constant over the period of annuity &
- Time interval between two consecutive payments (or receipts) must be the same.

Types of Annuity Based on Mode of Payment.

- **Annuity regular:** Payment is made @ **end** of each period [Preferred when nothing is said in question]
- **Annuity Due/Annuity Immediate:** Payment is made @ **beginning** of each period.

Perpetuity: Annuity where the receipt (or payment) takes place forever. Since the payment is forever we cannot compute a future value of perpetuity. However we can compute the present value of the perpetuity. **P =** $\frac{A}{i}$

SOME TERMS RELATED TO ANNUITY

TERMS	MEANING OF TERMS
Periodic Payment	Size of each Payment of Annuity.
Annual Rent	Sum of all payments made in one year of an annuity
Payment Period	Time between two successive payments of an annuity.
Terms	Total time from first payment period to the last period
Amount	Total worth of all the payments at conclusion of an annity.
Present Value	Sum of the present values of all the payments of an annuity.
Sinking Fund	Money accumulated at CI by regular & equal payments for replacement of a wasting asset or liquidation of a loan

CONCEPT 5: FUTURE VALUE OF ANNUITY

- Future value is the cash value of an investment (done today) in the future.
- It is tomorrow's value of today's money compounded at the given rate of interest.

CQ16: Suppose you invest Rs. 1,000 in FD @ 7% p.a. At the end of 1st year, you will have Rs. 1,070. Rs. 1,070 is the future value of Rs. 1,000 invested for one year at 7%.

We can say that Rs. 1000 today is worth Rs. 1070 in one year's time if the interest rate is 7%.

Thus Rs. 1,144.90 is the future value of Rs. 1,000 invested for two years at 7%.

EXPLANATORY TABLE OF Rs. 1 invested for 4 years @ 6%

End of year	Amount Deposit (Rs.)	Future value at the end of 4th year (Rs)
0	—	—
1	Rs. 1	$1 (1 + 0.06)^3 = 1.191$
2	Rs. 1	$1 (1 + 0.06)^2 = 1.124$
3	Rs. 1	$1 (1 + 0.06)^1 = 1.060$
4	Rs. 1	$1 (1 + 0.06)^0 = 1$
Future Value		**4.375**

A. FUTURE VALUE OF ANNUITY REGULAR [If nothing is given, we consider it "regular"]

FV = **P**$\left[\frac{(1+R)^n - 1}{R}\right]$ where, P = Amount deposited, R = Rate of Interest, N = No. of years (conversion).

CQ17: Find FV of an annuity of Rs. 500 made annually for 7 years @ 14%. [(1.14)7 = 2.5023]

B. FUTURE VALUE OF ANNUITY DUE [FV of annuity regular × **(1+R).]**

CQ18. Find FV of an annuity of Rs. 500 made annually for 7 years at interest rate of 14% compounded annually. Given that (1.14)7 = 2.5023. [**Ans:** Rs. 5365.35]

CQ19: Z invests Rs. 10,000 every year starting from today for next 10 yrs. Interest rate is 8% p.a compounded annually. Find FV of annuity. [(1 + 0.08)10 = 2.15892500] [**Ans:** Rs. 1,56,454.875]

CONCEPT 6: PRESENT VALUE OF ANNUITY

- Present value is today's value of tomorrow's money discounted at the interest rate.
- PV of an annuity = Sum of PV of all the periodic payments discounted @ given rate.

PC Note: FV & PV are related to each other in fact they are the reciprocal of each other.

CQ20: You invested Rs. 1000 at 7% & get Rs. 1,070 at the end of the year. If Rs. 1,070 is FV of today's Rs. 1000; then Rs. 1,000 is the PV of tomorrow's Rs. 1,070. If we invest Rs. 1,000 for two years at 7% p.a, we will get Rs. 1,144.90 after 2 years. It means Rs. 1,144.90 is the FV of today's Rs. 1,000 at 7% & Rs. 1,000 is PV of Rs. 1,144.90.

CQ21: PV of Rs. 1 to be received after 2 yrs compounded annually at 10% interest rate is?

[**Ans:** 0.83]

CQ22: Find PV of Rs. 10,000 to be required after 5 years if interest rate = 9%. [(1.09)5=1.5386]

[**Ans:** 6499.42]

A. PRESENT VALUE OF ANNUITY REGULAR

PV of an annuity (A) = Sum of PV of all the periodic payments discounted @ given rate.

$$PV = \frac{A}{(1+R)^1} + \frac{A}{(1+R)^2} + \frac{A}{(1+R)^3} + \frac{A}{(1+R)^4} + \ldots\ldots \frac{A}{(1+R)^N}$$

PV = A $\left[\frac{(1+R)^n - 1}{R(1+R)^n}\right]$ where, A = Installment Amount, R = Rate of Interest, n = No. of years (conversion).

CQ23: S borrows Rs. 5,00,000 to buy a house. If he pays equal installments for 20 years and 10% interest on outstanding balance what will be the equal annual installment? [**Ans:** 58,730]

CQ24: Rs. 5,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually? [**Ans:** 26,080]

B. PRESENT VALUE OF ANNUITY DUE

- Compute PV of annuity as if it were a annuity regular for one period short.
- Add initial cash payment/receipt to the step 1 value.

CQ25: Your mom decides to gift you Rs. 10,000 every year starting from today for the next 5 years. You deposit this amount in a bank as and when you receive and get 10% p.a compounded annually. Find PV of this annuity?

Sol^n: It is an annuity immediate. For calculating value of the annuity immediate following steps will be followed:

Step 1: Present value of the annuity as if it were a regular annuity for one year less i.e. for four years. = Rs. 10,000 × P (4, 0.10); = Rs. 10,000 × 3.16987; = Rs. 31,698.70.

Step 2: Add initial cash deposit to the step 1 value: Rs. (31,698.70+10,000) = **Rs. 41,698.70.**

CONCEPT 7: SINKING FUND

It is the fund credited for a specified purpose by way of sequence of periodic payments.

Size of Sinking Fund Deposit (A) = **P**× $\left[\frac{(1+R)^N - 1}{R}\right]$

Where, A = Total amount to be saved (FV) P = Periodic Payment

CQ26: How much amount is required to be invested every year so as to accumulate Rs. 3,00,000 at the end of 10 years if interest is compounded annually at 10%?

Answer: A = 3,00,000; N = 10; R = 0.1. we know that **A** = **P**× $\left[\frac{(1+R)^N - 1}{R}\right]$;

Thus, 3,00,000 = P × $\left[\frac{(1+0.1)^{10} - 1}{0.1}\right]$; 3,00,000 = P × 15.9374246; Therefore P = **Rs. 18,823.6.**

SOME OTHER IMPORTANT APPLICATIONS

1. **LEASING:** Leasing is a financial arrangement under which owner of the asset (lessor) allows the user of the asset (lessee) to use asset for a defined period of time for a consideration (lease rental) payable over a given period of time. It is like taking an asset on rent.

   - If Cost of asset > PV of lease rental → Lease
   - If Cost of asset < PV of lease rental → Buy

CQ27: ABC Ltd. wants to lease out an asset costing Rs. 10 lacs for 5 years. It has fixed a rental of Rs. 3.1 lacs p.a payable annually starting from the end of first year. Suppose rate of interest is 12% p.a compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

Answer: Here we have to compute PV of the annuity of Rs. 3,10,000 for 5 years @ 12%p.a.

PV Factor for 5 years @ 12% = 3.604776. Thus, PV of Lease annuity = 3,10,000 × 3.604776 = Rs. 11,17,480.

Since PV of Lease annuity > initial cost of the asset, Leasing is favourable to the lessor.

CQ28: A company is considering proposal of purchasing a machine either by making full payment of Rs. 4,000 or by leasing it for 4 years at lease rent of Rs. 1,250. Which option is preferable if rate is 14% p.a.? [Lease]

2. **CAPITAL EXPENDITURE (INVESTMENT DECISION):** Purchasing an asset (Cash outflows) today in anticipation of Future economic benefits (cash inflow).

   - If PV of cash inflow > PV of cash outflow → Invest
   - If PV of cash inflow < PV of cash outflow → Do NOT invest.

CQ29: A machine with useful life of 7 years costs Rs. 10,000 while another machine with useful life of 5 years costs Rs. 8,000. The first machine saves labour expenses of Rs. 1,900 annually & second one saves labour expenses of Rs. 2,200 annually. Determine preferred course of action. Assume cost of borrowing as 10% p.a.

Answer: (i) PV of annual cost savings for 1st machine = Rs. 1,900 × 4.86842 = Rs. 9,250.

Cost of 1st machine = Rs. 10,000 & it saves Rs. 9,250. Thus, it costs Rs. 750 more than labour cost it saves.

(ii) PV of annual cost savings of 2nd machine = Rs. 2,200 × 3.79079 = Rs. 8,339.74.

Cost of 2nd machine = Rs. 8,000 & it saves Rs. 8339.74. Thus, effective savings in labour cost = Rs. 339.74. **Hence, the second machine is preferable.**

3. **VALUATION OF BOND:** A bond is a debt security in which issuer owes holder a debt and is obliged to repay the principal and interest. They are generally issued for a fixed term.

Value of Bond = PV of Interest Paid + PV of Maturity Amount.

CQ30: An investor intends purchasing a 3 year Rs. 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a return of 14%?

Answer: Interest on bond for every year = Rs. 100. Maturity Amount = Rs. 1,000.

PV of Bond = $\frac{100}{(1.14)^1} + \frac{100}{(1.14)^2} + \frac{100}{(1.14)^3} + \frac{1000}{(1.14)^3}$ = 87.719 + 76.947 + 67.497 + 674.972 = 907.125.

Thus, the bond should be purchased @ Rs. 907.125 or less than it.

CONCEPT 8: PERPETUITY

Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date & continue indefinitely or perpetually.

Ex: Fixed coupon payments on permanently invested (irredeemable) sums of money.

A. PV of "Multi period perpetuity":

$PVA_{\infty} = \frac{P}{R}$ where, P = Payment/Receipt each period; R = Rate of Interest per each period

CQ31: If I want to retire & receive Rs. 30,000 every month & I want my family to receive the same monthly payment after my death. I can earn an interest of 8% p.a. How much will I need to set aside to achieve my perpetuity goal? How much should I invest to get the amount from today itself? [**Ans:** Rs. 45,00,000]

B. PV of "Growing Perpetuity": Perpetuity which grows at constant rate.

$PVA = \frac{P}{R-g}$ where, g = Growth rate

CQ32: I want to receive Rs. 10,000 forever. Interest rate is 8% & the rate at which perpetuity grows is 3%. Advise me the amount to be invested. [Ans: Rs.2,00,000

Answer: PVA = $\frac{P}{R-g} = \frac{10,000}{(8-3)\%} = \frac{10,000}{5\%}$ = Rs. 2,00,000.

CONCEPT 9: NET PRESENT VALUE (NPV)

NPV = PV of Cash Inflow – PV of Cash Outflow.

RULES TO MAKE DECISION: **If NPV > 0 → Accept;** **If NPV < 0 → Reject**

CQ33: Compute NPV for a project with a net investment of Rs. 1,00,000 & net cash inflows for year 1, 2, 3 is Rs. 55,000, Rs. 80,000 & Rs. 15,000 resp. Cost of capital is 10%? [PVIF @ 10% for 3 years: 0.909, 0.826 & 0.751]

Solution: Since NPV of the project is positive, the company should accept the project.

Year	Net Cash Flows	PVIF @ 10%	Discounted Cash Flows
0	(1,00,000)	1.000	(1,00,000)
1	55,000	0.909	49,995
2	80,000	0.826	66,080
3	15,000	0.751	11,265
Net Present Value			**27,340**

COMPOUND ANNUAL GROWTH RATE (CAGR)

- Compounded Annual Growth Rate (CAGR) is a mean annual growth rate of an investment over a specific period of time (generally longer than one year).
- The CAGR calculate is a useful tool when determining an annual growth rate on an investment whose value has fluctuated widely from one period to the next.
- CAGR is often used to describe the growth over a period of time of some element of the business like revenue, units delivered, registered users, etc.

CAGR $(t_o, t_n) = \left[\frac{V(t_n)}{V(t_o)}\right]^{\frac{1}{t_n - t_o}} - 1$ where, t_o = Starting period & t_n = Ending period

CQ34: Revenues of a company for 4 years, Calculate Compound annual Growth Rate.

Year	2013	2014	2015	2016
Revenues	100	120	160	210

Answer: $t_n - t_o$ = 2016 – 2013 = 3.

CAGR$_{(0,3)}$ **of Revenues** = $\left[\frac{210}{100}\right]^{\frac{1}{3}}$ – 1 = 1.2774 – 1 = 0.2774 = **27.74%**

Space for PC Class Note:

TIME VALUE OF MONEY – QUESTION BANK

SN	CHAPTER 4. TIME VALUE OF MONEY	Ans
	EXERCISE 4.1 – SIMPLE INTEREST	
Q1	The amount charged for a defined length of time for use of the principal, generally on a yearly basis is known as ______. (a) Balance (b) Rate of interest (c) Principal (d) Interest	D
Q2	The principal remains constant for the whole loan period in ______ interest (a) Simple (b) Compound (c) Effective (d) Annuity	A
Q3	In the formula A = P + I, A is known as ______. (a) Simple interest (b) Compound interest (c) Balance (d) Principal	C
Q4	Interest computed on the principal for entire period of borrowing is called ______. (a) Simple Interest (b) Compound Interest (c) Balance (d) All	A
Q5	Simple Interest on Rs. 3,500 for 3 years at 12% p.a. is ______. (a) Rs.1,200 (b) Rs.1,260 (c) Rs.2,260 (d) None	B
Q6	P = 5000 R = 15 T = 4 ½ using I = PRT/ 100. I will be ______. (a) Rs. 3,375 (b) Rs. 3,300 (c) Rs. 3,735 (d) None	A
Q7	Find simple interest on Rs.1,025 at 7.5% p.a. for 4.5 years. (a) Rs. 405.59 (b) Rs. 375.45 (c) Rs. 345.94 (d) Rs.354.94	C
Q8	In what time will Rs. 85,000 amount to Rs. 1,57,675 at 4.5% p.a? (a) 20 years (b) 15 years (c) 22 years (d) 19 years	D
Q9	P = Rs. 12,000; A= Rs. 16,500; T = 2.5 years. Interest rate will be ______. (a) 15% (b) 12% (c) 10% (d) None	A
Q10	A person borrowed Rs. 4,000 & after 6 months, amount paid was Rs. 4,050. Find the rate of interest? (a) 5% (b) 25% (c) 2.5% (d) 20%	C
Q11	A Sum of Rs.46,875 was lent out at simple interest and at the end of 1 yr and 8 months, the total amount was Rs.50,000. Find the rate of interest? (a) 4% (b) 5% (c) 4.5% (d) 6%	A
Q12	A sum doubles itself in 10 years. Find interest rate? (a) 10% (b) 12% (c) 15% (d) 20%	A
Q13	Capital required to earn a monthly interest of Rs.800 p.m. at 5 % at SI is ___. (a) Rs.1,87,000 (b) Rs.40,000 (c) Rs.1,28,000 (d) Rs.1,92,000	D
Q14	A sum of money amounts to Rs. 795 in 4 years and Rs. 850 in 5 years. Sum is ______. (a) Rs. 520 (b) Rs. 630 (c) Rs. 575 (d) Rs. 685	C
Q15	A sum of money amount to Rs.6,200 in 2 years and Rs.7,400 in 3 years. The principal	A

	and rate of interest are ______. (a) Rs.3,800, 31.57% (b) Rs.3,000, 20% (c) Rs.3,500, 15% (d) None	
Q16	Mr. Kapil deposited some amount in a bank for 7.5 years at 6% SI. Mr. Kapil received Rs. 1,01,500 at the end of the term. Compute initial deposit of Kapil. (a) Rs. 1,00,000 (b) Rs.70,000 (c) Rs.75,000 (d) Rs.86,500	**B**
Q17	Rahul invested Rs. 70,000 in a bank at the rate of 6.5% p.a. simple interest rate. He received Rs. 85,925 after the end of the term. Find out the period for which sum was invested by Rahul. (a) 2.5 years (b) 3.5 years (c) 4 years (d) 3 years	**B**
Q18	Simple interest on Rs. 1,500 for 6 years at 5 % p.a. is ______. (a) Rs.400 (b) Rs. 300 (c) Rs.450 (d) Rs.500	**C**
Q19	What will be the final value of investment for the principal value of Rs. 80,000 for 4 years @ 10% p.a. rate of interest? (a) Rs.83,200 (b) Rs. 1,12,000 (c) Rs.82,300 (d) None	**B**
Q20	A = Rs. 5,200; R = 5% p.a; T = 6 years. Principal will be ______. (a) Rs.2,000 (b) Rs.4,000 (c) Rs.3,000 (d) None	**B**
Q21	Sachin deposited Rs.1,00,000 in his bank for 2 years at simple interest of 6%. How much interest would he earn? How much would be the final value of deposit? (a) Rs.6,000, Rs.1,06,000 (b) Rs.15,000, Rs.1,15,000 (c) Rs.11,600, Rs.1,11,600 (d) Rs.12,000, Rs.1,12,000	**D**
Q22	P = Rs.10,000 1= Rs.2,500 R = 12.5% Simple Interest. The number of years T will __. (a) $1\,{}^{1}/_{2}$ years (b) 2 years (c) 3 years (d) None	**B**
Q23	The sum required to earn a monthly interest of Rs. 1,200 at 18% p.a. SI is ______. (a) Rs. 50,000 (b) Rs. 60,000 (c) Rs. 80,000 (d) None	**C**
Q24	Rs. 3,52,000 will produce Rs. 28,600 interest in - years at 2.5% p.a. simple interest. (a) 2 years 2 months (b) 3 years 3 months (c) 4 years 4 months (d) 5 years 5 months	**B**
Q25	Sum of money doubles itself in 10 years. No. of years it would trebles itself is__. (a) 25 years (b) 15 years (c) 20 years (d) None	**C**
Q26	A sum of money that will give Rs. 1, as interest per day at 10% p.a. SI is ______. (a) Rs. 3,800 (b) Rs. 3,000 (c) Rs. 3,650 (d) Rs. 3,500	**C**
Q27	Rs. 80,000 is invested to earn a monthly interest of Rs. 1,200 @ —— p.a. SI. (a) 12% (b) 14% (c) 16% (d) 18%	**D**
Q28	What sum of money produce Rs. 28,600 interest of 3 yrs & 3 mths at 2.5% p.a. SI? (a) Rs. 3,52,000 (b) Rs. 3,65,000 (c) Rs. 3,25,000 (d) Rs.3,56,000	**A**
Q29	Interest on a certain sum of money 2.5 years at 3.25 % p.a. is 390. The sum is __. (a) Rs. 4,800 (b) Rs. 2,100 (c) Rs. 4,700 (d) Rs. 4,900	**A**

Q30	If Rs. 1,600 amounts to Rs. 2,100 is 5 years at a certain rate of simple interest. If the rate of interest is increased by 1 % it would amount to how much? (a) Rs. 2,080 (b) Rs. 2,050 (c) Rs. 2,250 (d) Rs. 2,180	**D**
Q31	A sum was put at simple interest, at a certain rate for 3 years. Had it been put at 1 % higher rate it would have fetched Rs. 63 more. The sum is ______. (a) Rs. 2,400 (b) Rs. 2,200 (c) Rs. 2,100 (d) Rs. 2,480	**C**
Q32	Two equal amounts of money are deposited in two different banks each at 12% p.a. for 8 years and 3.5 years respectively. If the difference between their Interests is Rs. 540, find each sum. (a) Rs. 1,200 (b) Rs. 1,000 (c) Rs. 1,400 (d) Rs. 1,350	**B**
Q33	A certain principal amounts to Rs. 2,800 in 2 years & to Rs. 3,220 in 5 years. The rate of interest p.a. SI is ______. (a) 6.33 % (b) 5.55 % (c) 2.25% (d) 6.6 %	**B**
Q34	Sum of money doubles itself in 25 years. No. of years it would trebles itself is__. (a) 50 years. (b) 37.5 years. (c) 75 years. (d) None	**A**
Q35	Number of years a sum takes to become 4 times @ 12% SI is ______. (a) 24 years. (b) 26 years. (c) 25 years. (d) None	**C**
Q36	If the interest on Rs. 2,400 be more than the interest on Rs. 2,000 by Rs. 64 in 4 years, rate of interest is ______. (a) 5% (b) 4% (c) 3.5 (d) 6 %	**B**
	EXERCISE 4.2 – COMPUND INTEREST	
Q37	Compute the compound interest on Rs. 4,000 for 1 ½ years at 10% p.a. compounded half-yearly. (a) Rs. 360.50 (b) Rs. 600 (c) Rs. 630.50 (d) Rs. 625	**C**
Q38	Determine CI on Rs. 1,000 at 6% compounded semi-annually for 6 yrs. Given that $(1+3\%)^{12} = 1.42576$. (a) Rs. 425.76 (b) Rs. 445.26 (c) Rs. 520.40 (d) Rs. 260.20	**A**
Q39	On what sum will the compound Interest at 5% p.a. for 2 yrs compounded annually be Rs. 1,640? (a) Rs. 16,000 (b) Rs. 17,000 (c) Rs. 18,000 (d) Rs. 19,000	**A**
Q40	On what sum will the compound Interest at 7% p.a. for 3 yrs compounded annually be Rs. 4725.90? (a) Rs. 22,000 (b) Rs. 26,000 (c) Rs. 24,000 (d) Rs. 21,000	**D**
Q41	The C.I. on Rs. 4,000 for 6 months at 12% p.a. payable quarterly is ______. (a) Rs. 243.60 (b) Rs. 240 (c) Rs. 243 (d) None	**A**
Q42	Rs. 4,000 is invested @ 10% p.a. The amount after two years if compounding is done monthly, is ______. (a) Rs. 4,881.16 (b) Rs. 4,818.16 (c) Rs. 4,888.16 (d) None	**A**

Q43	If A = Rs. 1000, n = 2 years, R = 6% p.a. compound interest payable half-yearly, then principal (P) is ______. (a) Rs. 890 (b) Rs. 880 (c) Rs. 800 (d) None	A
Q44	Find the rate, if Rs. 2,00,000 amount to Rs. 2,31,525 in 1½ year interest being compounded half-yearly. (a) 15% (b) 11% (c) 8% (d) 10%	D
Q45	A sum of money yields CI of Rs. 200 & Rs. 220 at the end of 1^{st} & 2^{nd} year respectively. The rate % is ______. (a) 20 (b) 15 (c) 10 (d) 5	C
Q46	CI on half-yearly rates on Rs. 10,000, the rate for 1^{st} & 2^{nd} years being 6% & for 3^{rd} year 9% p.a. (a) Rs. 2,290 (b) Rs. 2,287 (c) Rs. 2,285 (d) Rs. 2,283	A
Q47	A sum of money put at CI amount to Rs. 2,205 in 2 years and to Rs. 2,315.25 in 3 years. Find interest % p.a. (a) 10% (b) 5 % (c) 8 % (d) 6 %	B
Q48	Find the least no. of complete years in which the sum of money put @ 20 % CI will be more than double. (a) 1 year (b) 2 years (c) 3 years (d) 4 years	D
Q49	In how many years will a sum of money double at 5% p.a. compound interest? (a) 15 years 3 months (b) 14 years 2 months (c) 14 years 3 months (d) 15 years 2 months	B
Q50	If A = Rs. 10,000 n = 18 yrs R = 4% p.a C.I, P will be ______. (a) Rs.4,000 (b) Rs.4,900 (c) Rs.4,500 (d) None	D
Q51	The difference between the simple interest and compound interest on a certain sum of money invested for 2 years 5% p.a. is Rs. 30. Then the sum is ______. (a) 10,000 (b) 12,000 (c) 13,000 (d) None	B
Q52	If the sum of money when compounded annually becomes Rs. 1,140 in 2 years and Rs. 1,710 in 3 years, the Rate of Interest is ______. (a) 30% (b) 40% (c) 50% (d) 60%	C
Q53	For a 10-year deposit, what interest rate payable annually is equivalent to 5% interest payable quarterly? (a) 5.1% (b) 4.9% (c) 6.0% (d) None	A
Q54	What annual rate of interest compounded annually doubles an investment in 7 years? [Given that $2^{1/7}$ =1.104090] (a) 10.41% (b) 11.50% (c) 9.65% (d) 10.26%	A
Q55	Rs.16,000 invested at 10% p.a. compounded semiannually amounts to Rs.18,522. Find the time period of investment. (a) 1 year (b) 1 $^1/_2$ years (c) 2 years (d) 1 $^3/_4$ years	B

Q56	In what time will compound interest on Rs. 320 at 12.5% p.a. compounded annually be Rs. 85? (a) 4.5 Years (b) 2.5 Years (c) 2 Years (d) 5 Years	C
Q57	In what time will a sum of Rs. 800 at 5% p.a. compound interest amount to Rs. 882? (a) 1 years (b) 5 years (c) 4 years (d) 2 years	D
Q58	Saina deposited Rs.1,00,000 in a nationalized bank for three years. If the rate of interest is 7% p.a. Calculate the interest that bank has to pay Saina after 3 yrs if interest is compounded annually. Also calculate amount at the end of third year. (a) Rs.1,23,000 (b) Rs.1,22,504.30 (c) Rs.1,20,550.20 (d) Rs.1,35,256	B
Q59	In what time will Rs. 8,000 amounts to Rs. 8820 at 5% p.a. interest compounded half-yearly? (a) 3 years (b) 2 years 5 months (c) 2.5 years (d) 2 Years	D
Q60	At what rate CI does a sum of money becomes four fold in 2 years? (a) 150 % (b) 100 % (c) 200 % (d) 400 %	B
Q61	What interest rate compounded annually which doubles an investment in 2 years? (a) 46.04125 % (b) 14.142135 % (c) 41.42135 % (d) None	C
Q62	The time by which a sum of money would treble itself at 8 % p.a CI is ______. (a) 14.28 years (b) 14 years (c) 12 years (d) 15 years	A
Q63	In how many years a sum of money treble at 5% p.a. CI payable on half-yearly? (a) 18 years 7 months (b) 19 years 6 months (c) 20 years 8 months (d) 22 years 3 months	D
Q64	In how many years a sum will double at 10% p.a. compound interest? (a) 8 years 3 months (b) 7 years 3 months (c) 7 years 6 months (d) 8 years 2 months	B
Q65	Difference b/w SI & CI on a sum in 2 years at 15 % p.a. is Rs. 144. The sum is ___. (a) Rs. 6,000 (b) Rs. 6,200 (c) Rs. 6,300 (d) Rs. 6,400	D
Q66	CI on a certain sum for 2 years is Rs. 41 & SI is Rs. 40. Find interest rate. (a) 4% (b) 5% (c) 6% (d) 8%	B
Q67	CI on a certain sum for 2 years is Rs. 41.60 & SI is Rs. 40. Find the sum. (a) Rs. 500 (b) Rs. 400 (c) Rs. 250 (d) Rs. 300	C
Q68	Difference between the S.I. & the C.I. on Rs.2,400 for 2 years at 5% p.a is ______. (a) Rs.5 (b) Rs.10 (c) Rs.16 (d) None	D
Q69	Difference b/w CI & SI on a sum for 2 years at 6% p.a. is Rs. 13.50. Find the sum? (a) Rs.3,750 (b) Rs.2,750 (c) Rs.4,750 (d) None	A
Q70	Difference b/w CI & SI on a sum for 2 years at 4% p.a. is Rs. 1. The sum is ______. (a) Rs. 625 (b) Rs. 630 (c) Rs. 640 (d) Rs. 635	A

Q71	Difference b/w SI & CI on certain sum for 3 years at 5% pa is Rs. 76.25. Find sum. (a) Rs. 5,000 (b) Rs. 8,000 (c) Rs. 9,000 (d) Rs. 10,000	D
Q72	Difference b/w SI and CI on Rs. 1,200 for 4 years at 10% p.a. is ______. (a) Rs. 77 (b) Rs. 480 (c) Rs. 80 (d) Rs. 557	A
Q73	CI on a certain sum for 2 years at 10 % p.a. is Rs. 420. Find SI at the same rate & for the same time. (a) Rs. 400 (b) Rs. 350 (c) Rs. 380 (d) Rs. 375	A
Q74	Difference b/w CI & SI at 5% pa for 4 years on 20,000 is ______. (a) Rs. 250 (b) Rs. 277 (c) Rs. 300 (d) Rs. 310.	D
Q75	At what rate will a sum double itself in 7 years if interest is compounded annually. (a) 7.0% (b) 8.0% (c) 10.38% (d) 9%	C
Q76	The principal goes on changing every year in ______. (a) simple interest (b) compound interest (C) effective interest (d) All of the above	B
Q77	P = Rs. 1,000; R = 5% p.a; n = 4. Amount and Cl are ______. (a) Rs.1,215, Rs.215 (b) Rs.1,125, Rs.125 (c) Rs.2,115, Rs.115 (d) None	A
Q78	Rs. 10,000 is invested at annual rate of interest of 10%. The amount after two years at annual compounding is ______. (a) Rs. 21,100 (b) Rs. 12,100 (c) Rs. 12,110 (d) None	B
Q79	Rs.100 will become after 20 years at 5% p.a. Calculated CI annually is ______. (a) Rs. 263.32 (b) Rs. 270.50 (c) Rs. 265.32 (d) None	C
Q80	Rs.7,500 is invested at 5% CI for 2 years. The interest for the second year is __. (a) Rs.375 (b) Rs.350 (c) Rs.450 (d) Rs.393.75	D
Q81	The C.I on Rs.16,000 for $1^1/_2$ years at 10% p.a. payable half-yearly is ______. (a) Rs.2,222 (b) Rs.2,522 (c) Rs.2,500 (d) None	B
Q82	Rs.2,000 is invested at annual rate of interest of 10% p.a. The amount after two years if compounding is done half yearly, is ______. (a) Rs.2431 (b) Rs.243.10 (c) Rs.2341 (d) None	A
Q83	C.I on Rs.40,000 at 10% p.a. for 1 years when interest is payable quarterly is ___. (a) Rs.4,000 (b) Rs.4,100 (c) Rs.4,152.51 (d) None	C
Q84	Rs. 3,000 is invested at annual rate of interest of 10% p.a. The amount after two years if compounding is done quarterly, is ______. (a) Rs.3,556.20 (b) Rs.3,565 (c) Rs.3,655.20 (d) None	C
Q85	C.I on Rs.1,000 for 10 years at 4% p.a. the interest being paid quarterly is ______. (a) Rs.786 (b) Rs.586 (c) Rs.489 (d) Rs.186	C

Q86	Rs. 2,000 is invested at 10% p.a. What is the amount after 2 yrs if compounding is done (a) Annually (b) Semi-Annually (c) Quarterly (d) Monthly. (a) 2,430; 2,531; 2,638; 2,700 (b) 2,420; 2,431; 2,437; 2,441 (c) 2,130; 2,483; 2,643; 2,550 (d) 2,420; 2,431; 2,468; 2,712	B
Q87	A sum of money at CI amounts to thrice itself in 3 years. In how many years will it be 9 times itself? (a) 18 (b) 12 (c) 9 (d) 6	D
Q88	A sum of money triples itself in 20 years. The number of years it would double itself. (C.I) ______. (a) 13.2 years (b) 15.2 years (c) 10 years (d) 12.6 years	D
Q89	The population of a town increases every year by 2% of the Population at the beginning of that year. The number of years by which the total increase of population be 40% is ______. (a) 7 years (b) 10 years (c) 17 years (approx) (d) None	C
Q90	The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is ______. (a) 35 yrs (b) 33 yrs (c) 25 yrs (d) None	A
	EFFECTIVE RATE OF INTEREST	
Q91	Effective rate of interest corresponding to a nominal rate 3% p.a. payable half yearly in ______. (a) 3.2% p.a. (b) 3.25% p.a (c) 3.0225 % p.a (d) None	C
Q92	Effective rate of interest for 3% p.a. compounded monthly is______. [Given that $(1+0.0025)^{12} = 1.0304$] (a) 3% (b) 3.02% (c) 3.04% (d) 3.01%	C
Q93	Effective rate of interest corresponding to a nominal rate of 7% p.a. compounded quarterly is ______. (a) 7% (b) 7.5% (c) 7.19% (d) None	C
Q94	Find the effective rate of interest if I = Rs.1,800, P = 18,000, t = 1 year (a) 10% (b) 9% (c) 18% (d) None	A
Q95	Find the compound interest and effective rate of interest if an amount of Rs. 20,000 is deposited in a bank for 1 year at the rate of 8% p.a. compounded semi-annually. (a) Rs. 1426, 7.56% (b) Rs. 1632, 8.16% (c) Rs. 1326, 7.35% (d) Rs. 1744, 8.55%	B
Q96	Ram is confused whether to invest at 9% p.a. compounded monthly or 9.25% p.a. SI. [$(1 + 0.0075)^{12} = 1.09380690$. He decided to find effective rate of interest. (a) 9% (b) 9.25% (c) 9.38% (d) None	C

Q97	In how many years will a sum of Rs. 800 amounts to Rs. 926.10 at 10% interest compounded half yearly? (a) 3 years (b) 2 years (c) 3/2 years (d) 4 years	C
Q98	Find the sum which invested at 4% p.a. compounded twice a year becomes Rs. 78,030 @ end of 1st year. (a) Rs. 73,000 (b) Rs. 75,000 (c) Rs. 74,225 (d) Rs. 76,000	B
EXERCISE 4.3: PRESENT VALUE & FUTURE VALUE OF ANNUITY		
Q99	Present value of Rs. 1 to be received after 2 yrs compounded annually at 10% is __. (a) Rs. 0.9090 (b) Rs. 0.8264 (c) Rs. 0.7513 (d) Rs. 0.6830	B
Q100	Present value of annuity of Rs. 5,000 p.a. for 12 yrs at 4% p.a. C.I. annually is ______. (a) Rs. 46,000 (b) Rs. 46,925 (c) Rs. 15,000 (d) None	B
Q101	The present value of an annuity of Rs. 3,000 for 15 years at 4.5% p.a CI. is ______. (a) Rs. 23,809.41 (b) Rs. 32,219.41 (c) Rs. 32,912.41 (d) None	B
Q102	The present value of an annuity of Rs. 80 p.a for 20 years at 5% p.a is ______. (a) Rs. 997 (appx) (b) Rs. 900 (c) Rs. 1,000 (d) None	A
Q103	A person invested money in bank paying 6% Compounded semi annually. If the person expects to receive Rs. 8000 in 6 years, what is present value of investment? (a) Rs. 5,000 (b) Rs. 4,611.03 (c) Rs. 5,611.03 (d) None	C
Q104	Find PV of ordinary annuity of 8 Quarterly payments of Rs. 500, interest = 8% p.a. compound quarterly. (a) Rs. 4,292.50 (b) Rs. 4,725.00 (c) Rs. 3,662.50 (d) Rs.3,266.50	C
Q105	Company borrows Rs. 10,000 on condition to repay it with CI at 5% p.a. by annual installments of Rs.1,000 each. Number of years by which debt will be clear is ____. (a) 14.2 years (b) 10 years (c) 12 years (d) None	A
Q106	A loan of Rs. 10,000 is to be paid back in 30 equal installments. The amount of each installation to cover the principal and at 4% p.a. CI is ______. (a) Rs. 587.87 (b) Rs. 587 (c) Rs. 578.87 (d) None	C
Q107	Raja aged 40 wished his wife Rani to have Rs. 40 Lacs at his death. If his expectation of life is another 30 years & he starts making equal annual investments commencing now at 3% compound interest p.a. how much should he invest annually? (a) Rs. 82,077 (b) Rs. 83,450 (c) Rs. 84,419 (d) Rs. 84,080	D
Q108	How much amount is required to be invested every year so as to accumulate Rs. 3,00,000 at the end of 10 years if interest is compounded annually at 10%? (a) Rs. 18,222 (b) Rs. 18,823 (c) Rs. 18,725 (d) Rs. 18,955	B

Q109	P Ltd has to make payment of Rs. 20 Lacs in 60 days. The company has decided to invest in CDs of a leading Nationalized Bank at 8% p.a. What money is required to be invested now? (a) Rs. 15,20,912 (b) Rs. 20,26,300 (c) Rs. 19,74,040 (d) Rs. 20,63,000	**C**
Q110	The present value of Rs.10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on yearly basis is ______. (a) Rs.9,070 (b) Rs.9,059 (c) Rs.9,061 (d) Rs.9,060	**A**
Q111	Find the present value of Rs.10,000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5 = 1.5386$. (a) Rs.6,499.42 (b) Rs.7,459.33 (c) Rs.6,544.50 (d) Rs.6,994.62	**A**
Q112	A = Rs.1,200 N = 12 yrs I =0.08 V = ______. using the formula $v= A/I \{1-(1+i)^{-n}\}$ (a) Rs.3,039 (b) Rs.3,990 (c) Rs.9,930 (d) None	**D**
Q113	The present value of an annuity of Rs.3,000 for 15 years at 4.5% p.a Cl is ______. (a) Rs. 23,809.41 (b) Rs. 32,809.41 (c) Rs. 32,908.41 (d) None	**B**
Q114	Suppose your mom decides to gift you Rs.10,000 every year starting form today for next 5 years. You deposit this amount in a bank as and when you receive and get 10% p.a. interest rate compounded annually. What is present value of this annuity? (a) Rs.40,702.70 (b) Rs.42,533.21 (c) Rs.41,698.70 (d) Rs.43,883.33	**C**
Q115	The amount received on an annuity of Rs. 150 for 12 years at 3.5% p.a CI is ______. (a) Rs. 2,190.28 (b) Rs. 1,290.28 (c) Rs. 2,180.28 (d) None	**A**
Q116	Amount of an annuity after 25 years at 5 % C.I. is Rs. 50,000, the annuity will be (a) Rs. 1,406.90 (b) Rs. 1,046.90 (c) Rs. 1,146.90 (d) None	**B**
Q117	Given annuity of Rs. 100 amounts to Rs. 3,137.12 at 4.5% p.a. C.I. No. of years = ____. (a) 25 years (appr) (b) 20 years (appr) (c) 22 years (d) None	**B**
Q118	You invest Rs. 3,000 in a 2-year investment that pays you 12% pa. Calculate FV. (a) Rs. 3,763.20 (b) Rs. 3,360.00 (c) Rs. 3,565.60 (d) Rs. 3,663.55	**A**
Q119	Z invests Rs. 10,000 every year starting from today for next 10 years. Suppose interest rate is 8% p.a. compounded annually. Calculate FV. $[(1+.08)^{10} = 2.15892500]$ (a) Rs. 1,50,580 (b) Rs. 1,56,454 (c) Rs. 1,58,652 (d) Rs. 1,56,902	**B**
Q120	A person invests Rs. 500 at the end of each year with a bank which pays interest at 10% p.a. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is ______. (a) Rs. 11,761.35 (b) Rs. 10,000 (c) Rs. 12,000 (d) None	**A**

Q121	Present value of Rs. 10,000 due in 2 years at 5% p.a. compound interest is_____. (a) Rs. 9,070 (b) Rs. 9,059 (c) Rs. 9,061 (d) Rs. 9,060	**A**
Q122	Find PV of Rs. 500 due after 10 years (R= 10%) is compounded half yearly ___. (a) Rs. 188.40 (b) Rs. 193.94 (c) Rs. 138.94 (d) Rs. 50.00	
Q123	Alibaba borrows Rs. 6 Laths Housing Loan at 6% repayable in 20 annual Installments commencing at the end of the first year. How much annual payment is necessary? (a) Rs. 52,420 (b) Rs. 52,419 (c) Rs. 52,310 (d) Rs. 52,320	**C**
Q124	Johnson left Rs. 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom Dick and Harry aged 9, 12 and 15 years should each received equally after attaining the age 25 years. The rate of interest being 3.5% how much each son will receive after getting 25 years old? (a) Rs. 50,000 (b) Rs. 51,994 (c) Rs. 52,000 (d) None	**D**
Q125	Find the amount received on annuity if payment of Rs. 7,000 is made annually for 7 years at 6% p.a. (a) Rs. 48,756 (b) Rs. 50,857 (c) Rs. 50,363 (d) Rs. 58,756	**D**
Q126	Rs. 200 is invested at the end of each month in an account paying interest 6%p.a compounded monthly. FV of this annuity after 10^{th} payment? [$(1.005)^{10}$ = 1.0511] (a) Rs. 210.22 (b) Rs. 2,050 (c) Rs. 2,025 (d) Rs. 2,044	**D**
	EXERCISE 4.4: SINKING FUND	
Q127	A sinking fund is created for reducing debentures worth Rs. 5 Lacs at the end of 25 years. Now much provision needs to be made out of profits each year if sinking fund investments can earn interest at 4% p.a? (a) Rs. 12,006 (b) Rs. 12,040 (c) Rs. 12,039 (d) Rs. 12,035	**A**
Q128	A machine costs Rs. 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of 25,000. What amount should set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a? (a) Rs. 16,500 (b) Rs. 16,000 (c) Rs. 16,050 (d) Rs. 16,005	**C**
Q129	A person bought a house paying Rs. 20,000 cash &Rs. 4000 at the end of each year for 25 yrs @ 5% p.a C.I. The cash price is ______. (a) Rs. 75,000 (b) Rs. 76,000 (c) Rs. 76,392 (d) None	**C**
Q130	A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being Rs. 23,240 and Rs. 9,000 respectively for how many years the machine was put to use? (a) 7 years (b) 8 years (c) 9 years (d) 10 years	**C**
Q131	A machine is depreciated the rate of 20% on reducing balance. Original cost of the machine was Rs. 1,00,000 and its ultimate scrap value was Rs. 30,000. The effective life of the machine is ______.	**B**

	(a) 4.5 years (b) 5.4 years (c) 5 years (d) None	
Q132	A machine the useful life of which is estimated to be 10 years cost Rs. 10,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is ______. (a) Rs. 3,483 (b) Rs. 4,383 (c) Rs. 3,400 (d) None	**A**
Q133	Appu receiving a pension of Rs. 14,400 per year paid in half yearly installment for the rest of his life. His life expectation is 13 yrs. Interest@ 4% p.a is payable half yearly. What is equivalent lump sum pension? (a) Rs. 1,45,000 (b) Rs. 1,44,900 (c) Rs. 1,44,800 (d) Rs. 1,44,700	**C**
Q134	A man purchased a house valued at Rs. 3 lacs. He paid Rs. 2 lace on purchase &agreed to pay the balance with interest at 12% p.a. compounded half yearly in 20 equal half yearly installments. If 1^{st}installment is paid after 6 months from purchase then the amount at each installment is ______. (a) Rs. 8,719.66 (b) Rs. 8,769.21 (c) Rs. 7,893.13 (d) None	**A**
Q135	A machine can be purchased for Rs. 50,000. Machine will contribute Rs. 12,000 p.a. for next 5 years. Assume borrowing cost is 10% p.a. compounded annually. Decide whether machine should be purchased or not? (a)Yes, Rs. 55,378.65 (b)No, Rs. 48,800.00 (c) No, Rs. 45,489.48 (d)Yes, Rs. 52,366.71	**C**
Q136	Money market instrument with face value of Rs. 100 & discount yield of 6% will mature in 45 days. Compute current price of instrument & effective annual return. (a) Rs. 99.05, 6.00% (b) Rs. 99.00, 5.29% (c) Rs. 99.26, 6.21% (d) Rs. 99.75, 6.08%	**C**
Q137	An investor intends purchasing a 3-year Rs. 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%? (a) Rs. 1,026.29 (b) Rs. 995.22 (c) Rs. 826.36 (d) Rs. 907.125	**C**
Q138	A person desires to create a fund to be invested at 10% CI p.a. to provide for a prize of Rs. 300 every year. Using V - A /I find V and V will be ______. (a) Rs. 2,000 (b) Rs. 2,500 (c) Rs. 3,000 (d) None	**C**
	PRACTICE QUESTION BANK	
Q139	A sum of money kept in a bank amounts to Rs.1,000 in 4 years and Rs.1,400 in 12 years. The sum and interest carried every year are ______. (a) 600,133 $\frac{1}{3}$ (b) 800,50 (c) 750,150 (d) 850,75	**B**
Q140	A sum of money amounts to Rs. 7,803 for one year at the rate of 4% compounded semiannually, sum invested is ______. (a) 7,000 (b) 7,500 (c) 7,750 (d) 8,000	**B**
Q141	Mr. Paul borrows Rs. 25,000 on condition to repaid it with C.I. at 7% p.a. in annual installments of Rs. 3,000 each. The number of years for debt to paid off is ______.	**D**

	(a) 10 years (b) 12 years (c) 11 years (d) 13 years	
Q142	A 6-year bond of Rs. 1,000 has an annual rate of interest of 14%. Interest is paid half-yearly. If required rate of return is 16%, what is the value of the bond? (a) Rs. 925 (b) Rs. 952 (c) Rs. 950 (d) Rs. 945	**D**
Q143	A sum of money will be doubled itself in 8 years at S.I. In how many years the sum will be tripled itself? (a) 20 years (b) 12 years (c) 16 years (d) None	**C**
Q144	A sum of 44,000 is divided into 3 parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal at the rate of simple interest are 6% p.a., 8% p.a., & 6% p.a. respectively. Then the smallest part of sum will be ______. (a) Rs. 4,000 (b) Rs. 8,000 (c) Rs. 10,000 (d) Rs. 12,000	**B**
Q145	A certain sum of money was invested at S.I for 3 years. If it has invested at rate 7% higher, then the interest have been 882/- more, then the sum is ______. (a) Rs. 12,600 (b) Rs. 6,800 (c) Rs. 4,200 (d) Rs. 2,800	**C**
Q146	A machine worth Rs. 4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%? (a) 14 years 6 months (b) 14 years 2 months (c) 14 years 5 months (d) None	**B**
Q147	A machine for which the useful life is estimated to be 5 years cost Rs. 5,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is ______. (a) Rs.2,952.45 (b) Rs.2,500.00 (c) Rs.3,000.00 (d) Rs.2,559.50	**A**
Q148	A machine worth Rs. 4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce to Rs.2,00,000? (a) 4 years 6 months (b) 5 years 7 months (approx.) (c) 4 years 5 months (d) None	**B**
Q149	ABC Ltd wants to lease out an asset costing Rs.3,60,000 for a 5 year period. It has fixed rental of Rs.1,05,000 p.a. payable annually starting from the end of first year. Suppose rate of interest is 14% p.a. compounded annually on which money can be invested by the company. Is this agreement favourable to the company? (a) Favourable, Rs.3,20,022.22 (b) Unfavourable, Rs.2,89,725.22 (c) Unfavourable, Rs.2,99,376.78 (d) Favourable, Rs.3,60,473,40	**D**
Q150	A machine with useful life of 7 years cost Rs.10,000 while another machine with useful life of 5 yrs costs Rs.8,000. The 1^{st} machine saves labour expenses of Rs.1,900 annually and the second one saves labourexpenses of Rs.2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded p.a. [Decision, PV of cost savings] (a) No, Rs.750.36 (b) Yes, Rs.8,339.74 (c) No, Rs.9,250.22 (d) Yes, Rs.5,366.63	**B**

CHAPTER 5A. PERMUTATION

INTRODUCTION OF PERMUTATION & COMBINATION

PERMUTATION:

- Permutation means arrangement of the things (objects) under consideration.
- In permutation, order of the things is important.
- In Permutation (a, b) & b, a) are two different arrangements.

COMBINATION:

- Combination means selection of the things under consideration.
- In combination, order of the things is not important.
- In combination (a, b) & b, a) are same selection.

FUNDAMENTAL PRINCIPLES OF COUNTING

A. Multiplication Rule [AND] **[When two tasks are dependent on each other]**

If certain thing may done in 'm' different ways & after finishing it, a second thing can be done in 'n' different ways, total no. of ways of doing both things one after the another = (m × n) ways.

PC Note: Used when the statements are connected by **"AND".**

CQ1: If one can go to school by 5 different buses and then come back by 4 different buses then total number of ways of going to and coming back from school [**Ans:** 5 × 4 = 20.]

CQ2: There are 4 routes for going from Dumdum to Sealdah & 5 routes for going from Sealdah to Chandni. In how many different ways can you go from Dumdum to Chandni Via Sealdah?

(a) 9 (b) 1 (c) 20 (d) None

B. Addition Rule [OR] **[When two tasks are independent]**

It there are two alternative jobs which can be done in 'm' ways & in 'n' ways respectively then either of two jobs can be done in **(m + n) ways.**

PC Note: Used when the statements are connected by **"OR"**

CQ3: If one wants to go school by bus where there are 5 buses or by auto where there are 4 autos, then total number of ways of going school [**Ans:** 5 + 4 = 9.]

CQ4: A certain Job requires drawing or printing. There are 3 painter & 4 printing machines. The number of ways the job can be completed is:

(a) 12 (b) 1 (c) 10 (d) 2

THE FACTORIAL

- Continuous Product of all integers from 1 to 'n' BOTH Inclusive.
- The factorial 'n' is denoted as n! or n.
- n! = 1.2.3.4.5.6……(n-2) (n-1)n.
- 0! = 1.

PC Note: While solving the question, all the factorials in the question shall be reduced upto the lowest factorial given in the question.

0!	1!	2!	3!	4!	5!	6!	7!	8!	9!	10!
		2 × 1	3 × 2!	4 × 3!	5 × 4!	6 × 5!	7 × 6!	8 × 7!	9 × 8!	10 × 9!
1	1	2	6	24	120	720	5040	40320	362880	3828800

CQ5: Find 4! & 6!

Ans: (i) 4! = 1 × 2 × 3 × 4 = 24; (ii) 6! = 1 × 2 × 3 × 4 × 5 × 6 = 720

CQ6: Find (i) $\frac{9!}{6!} = \frac{9.8.7.6!}{6!} = 9.8.7 = 504$; (ii) $\frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$; (iii) $\frac{11!}{7!} = 11.10.9.8 = 7920$.

CQ7: Find n if (n+1)! = 30(n-1)! **[Answer: n = 5]**

CQ8: Find x if $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$

Ans: Reducing all factorials to the lowest factorial in the question, we have $\frac{1}{9!} + \frac{1}{10.9!} = \frac{x}{11.10.9!}$;

Cancelling $\frac{1}{9!}$ from both sides, we have $1 + \frac{1}{10} = \frac{x}{11.10}$; $\frac{11}{10} = \frac{x}{11.10}$;

Cancelling 10 from both sides, we have $11 = \frac{x}{11}$; **x = 11.11 = 121.**

PERMUTATIONS

- **Definition:** The number of ways of arranging all or some of the given things out of given things is called permutations.
- The order in which person (objects) are arranged is important.
- No. of Permutations of 'r' different object out of 'n' different object = ${}^nP_r = \frac{n!}{(n-r)!}$ [0 ≤ r ≤ n]

CQ9: Calculate ${}^5P_3$; ${}^{10}P_2$; ${}^{11}P_5$.

Solution: ${}^5P_3$ means out of 5 people (objects), we have to select any 3 people (objects).

$^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5.4.3.2!}{2!} = 5\times4\times3 = 60$; $^{10}P_2 = \frac{10!}{(10-2)!} = \frac{10!}{8!} = \frac{10.9.8!}{8!} = 10\times9 = 90$,

$^{11}P_5 = \frac{11!}{(11-5)!} = \frac{11.10.9.8.7.6!}{6!} = 11\times10\times9\times8\times7 = 55440$.

CQ10: If $^nP4 = 5040$, then the value of 'n' is ______.

(a) 8 (b) 9 (c) 10 (d) 6

CQ11: If $^nP_3 : {}^nP_2 = 3:1$, then n is equal to ______.

(a) 7 (b) 4 (c) 5 (d) None of these

CQ12: If $^{x+y}P_2 = 90$ and $^{x-y}P_2 = 30$ then ______.

(a) x = 4y (b) x = 2 (c) x = y `(d) 4x = y

CQ13: If $^{56}P_{r+6} : {}^{51}P_{r+3} = 30800 : 1$, find 'r'.

(a) 31 (b) 41 (c) 51 (d) 21

CQ14: How many 3 letter words can be formed using the letters of the words (a) SQUARE & (b) HEXAGON?

Ans: Since the word 'SQUARE' consists of 6 different letters, the number of permutations of choosing 3 letters out of six equals $^6P_3 = 6\times5\times4 = 120$.

Since 'HEXAGON' contains 7 different letters, number of permutations is $^7P_3 = 7\times6\times5 = 210$.

CQ15: There are 5 guests in a party & only 3 chairs are there. In how many ways can the guests be seated?

Ans: There are 3 chairs & 5 guests. It is obvious that 2 guest will not occupy same chair.

1st Chair → can be occupied by any 1 of the 5 guests = 5 ways &

2nd Chair → can be occupied by any 1 of the remaining 4 guests = 4 ways &

3rd chair → can be occupied by any 1 of the remaining 3 guests = 3 ways.

Total number of ways = 5 × 4 × 3 = 60 ways.

Chair 1	Chair 2	Chair 3
5 Guests (ways)	4 Guests (ways)	3 Guests (ways)

CQ16: In how many different ways can 3 students be associated with 4 CAs, assuming that each chartered accountant can take at most one student?

Ans: 1st student can be associated with any of the 4 CAs = 4 ways;

2nd student can be associated with any of the remaining 3 CAs = 3 ways;

3rd student can be associated with any of the remaining 2 CAs = 2 ways; [**Ans** = 4×3×2 = 24]

Alternate Method: Number of permutations of choosing 3 persons out of 4.

Hence, answer is $^4P_3 = 4\times3\times2 = 24$.

CQ17: When Dr. Ram arrives in his dispensary, he finds 12 patients waiting to see him. If he can see only one patient at a time, find the number of ways, he can schedule his patients (a) if they all want their turn, and (b) if 3 leave in disgust before Dr. Ram gets around to seeing them.

Ans: (a) There are 12 patients and all 12 wait to the see doctor. Therefore number of ways = 12! Ways.

(b) There are 12—3 = 9 patients. They can be seen $^{12}P_9$ = 79,833,600 ways.

CQ18: How many 4 digit numbers can be formed from 1, 2, 3, 4, 5. [Repetition not allowed]

Ans: 5 × 4 × 3 × 2 = 120 ways.

Ten thousand place	Thousand place	Tens Place	Unit Place
Can be filled in 5 ways	Can be filled in 4 ways	Can be filled in 3 ways	Can be filled in 2 ways

CQ19: How many 4 digits numbers can be formed by using 1, 2, 3,4,5,6,7,8,9, no digit being repeated in any number?

Ans: We have 9 digits &we have to find the number of permutations of these taken 4 at a time, which is $^9P_4$. =3024 ways.

CONCEPT 1: PERMUTATION OF 'n' THINGS TAKEN ALL AT A TIME

Number of permutations of n different things taken all n things at a time = **n!**

Here r = n, Thus, $^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$.

CQ20: In how many different ways can five persons stand in a line for a group photograph?

Ans: Here we know that the order is important. Hence, this is the case of number of permutations of five things taken all at a time. Thus 5! = 120 ways.

TABULAR SUMMARY OF DIGITS

Available	Taken at a time	All Possible Arrangements	No. of ways	Formula
1, 2, 3	3 digits	123, 132, 213, 231, 312, 321.	6 ways	$^3p_3$
1, 2, 3	2 digits	25, 27, 52, 72, 75	6 ways	$^3p_2$
1, 2, 3	1 digit	2, 5, 7	3 ways	$^3p_1$

CONCEPT 2: PERMUTATION OF DIFFERENT THINGS WITH RESTRICTIONS

- No. of permutations of n distinct objects taken 'r' at a time when a particular object is not included in any arrangement = $^{(n-1)}P_r$

  Explanation: If there are 'n' person & we have to select 'r' out of them. But one person is not taken. Thus, we have only (n-1) person (objects) to select 'r' person (objects).

- No. of permutations of n distinct objects taken r at a time when a particular object is always included in any arrangement = $^{(n-1)}P_{(r-1)}$. **[Person to be included is fix]**

  Explanation: If there are 'n' person & we have to select 'r' out of them. But one person is always taken. Thus, we have to arrange only (r-1) persons & we have only (n-1) person (objects) left since 1 person is already taken.

CQ21: How many 4 digits numbers can be formed by using 1,2,3,4,5,6,7,8,9 such a that the numbers will (i) begin with a specified digit

(ii) begin with a specified digit and end with a specified digit?

Ans: (i) No. begin with a specified digit, then to arrange 8 digits out of 3. Thus $^8P_3$ **= 336.**

(ii) Numbers begin with a specified digit & end with another specified digit. Then we have to find the number of permutations of 7 things taken 2 at a time, which is $^7P_2$ **= 42 ways.**

- No. of permutations of 'n' distinct objects taken 'r' at a time when a particular object is always to be included in any arrangement = $r.^{(n-1)}P_{(r-1)}$. **[Person to be included is not fix].**

  Explanation: If there are 'n' person & we have to select 'r' out of them. But one person is always to be included which is not fix. So, we can fix any of the 'r' person. Thus, fixing a person can be done in 'r' ways. Now we are left with only (r-1) persons & we have only (n-1) person (objects) left since 1 person is already taken. Thus, we have to arrange (r - 1) persons out of (n — 1) persons.

- No. of Permutations when 2 things are always together out of 'n' things = **(n-1)! × 2!**

  Explanation: Suppose we have to arrange n things out of n things, A1 & A2 should always come together. Thus, we have total (n-2) thing out of which we have to arrange (n-2) things. This can be done in (n-2)! ways. The 2 thing can be arranged in 2! ways. [A1 & A2 or A2 & A1]

- **TWO THINGS ARE NEVER TOGETHER = TOTAL NUMBER OF WAYS — "ALWAYS TOGETHER" WAYS**

  $= n! - (n-1)! \times 2! \Rightarrow$ **(n-1)! (n-2)**

  Explanation: We will subtract the number of ways when things are always together from total number of ways. This will give us the number of ways when 2 things are never together.

CQ22: In how many ways 10 examination papers can be arranged so that best & worst paper never come together?

Ans: (i) Total number of permutations of 10 papers without any restriction is $^{10}P_{10} = 10!$

(ii) Let us regard the worst and the best papers together as one paper.

Now we have (10 - 1) papers which can be arranged in $^9P_9$ ways = 9! Ways.

Now these 2 papers (i.e best and worst papers) can be arranged internally in **2! Ways.**

The number of ways the two papers are always together is (9! × 2!)

(iii) No. of ways that the best and worst paper never come together = Total number of permutations without restrictions - number of ways two papers are always together

= **10! - (9! × 2!) = 9! [10- 2] = 9! × 8.**

CQ23: There are 6 books on Economics, 3 on Mathematics and 2 on Accountancy. In how many ways can these be placed on a shelf if the books on same subject are to be together?

Ans: Consider the books on each subject as one unit. Now there are 3 units. They can be arranged in 3! Ways.

6 Economics books can be arranged among themselves internally in 6! ways.

3 Mathematics books can be arranged internally in 3! ways.

2 books on Accountancy can be arranged internally in 2! ways.

Total number of arrangements = 3! × 6! × 3! × 2! = **51,840.**

CQ24: How many different arrangements can be made by using all the letters of word MONDAY?

Ans: MONDAY has different letters. So, 6 different letters arranged in $^6P_6$ = 6! = **720 ways**.

CQ25: In Q24, how many of these arrangement being with A?

Ans: Suppose all words begin with A. Remaining 5 places filled with remaining 5 letters $\mathbf{^5P_5}$

CQ26: In Q24, how many of this arrangement begin with A & end with D?

Ans: Suppose all words begin with A & end with D. Remaining 4 Places can be filled in

$^4P_4$= 4! Ways = **24 Ways.**

CQ27: In Q24, how many arrangements are there in which vowels A & O occur together?

Ans: The vowels are A & O. Let us take them as one letter, then remaining 5 letters can be arranged in $\mathbf{^5P_5}$**= 5! = 120 ways**. These two vowels can be arranged amongst themselves internally in 2! = 2 ways. So total numbers of ways = 2*120 = **240 ways.**

CQ28: In Q24, how many words can be formed such that consonants occur together?

[**Ans:** 144 ways.]

CQ29: In Q24, how many words can be there such that the vowels A, O occupy even places?

[**Ans:** 144 ways]

CONCEPT 3: PERMUTATION OF 'r' out of 'n' things WHEN REPETITION IS ALLOWED

If repetition is allowed, Number of ways or arranging 'r' things out of 'n' things = n^r.

CQ30: How many telephones connections may be allotted with 8 digits from the no. 0 to 9?

(a) 10^8 (b) 10! (c) $^{10}C_8$ (d) $^{10}P_8$

CONCEPT 4: PERMUTATION OF SIMILAR THINGS TAKEN ALL AT A TIME

The number of ways in which 'n' things can be arranged taking all at a time, when 'p' things are similar of one type, 'q' things are similar of 2nd type, 'r' things are similar of 3rd type & remaining things are different = $\frac{n!}{p! \times q! \times r!}$.

CQ31: In how many ways can 17 billiard balls be arranged, if 7 of them are black, 6 red & 4 white? **[Ans:** $\frac{17!}{7! \times 6! \times 4!}$**]**

CQ32: How many permutations can be made out of the letters of the word?

(i) MATHEMATIC (ii) COMMERCE; (iii) EXAMINATION?

Ans: (i) The word MATHEMATICS Contains 11 words in which, A appears 2 times: T appears 2 times: M appears 2 times and the remaining letters H, E, C, and S appear only once. Therefore required number of permutations = 11!/2!2!2!

(ii) Here M appears 2 times. E appears two times and O appears 2 times out of 8 words of COMMERCE.

Therefore required number of permutations= 8!/2!2!2! =5040.

(iii) The word EXAMINATION has 11 words, out which A appears 2 times, I appear 2 times, N appears 2 times.

Therefore required number of permutations=11! /2!*2!*2! = 4989600.

CQ33: The number of arrangements that can be made with the word 'assassination' is

(a) $13! \div [3! \times 4! \times (2!)^2]$ (b) $13! \div [3! \times 4! \times 2!]$ (c) 13! (d) None

CQ34: (i) How many different words can be formed with the letters of the word BHARAT?

(ii) How many of these begin with B and End T?

(iii) In how many of these B and H are never together?

Ans: (i) **6! /2! =360.**

(ii) **4! /2! =12.**

(iii) **360 – 120 = 240.**

CONCEPT 5: CIRCULAR PERMUTATIONS

- Arrangement of things along a circle is known as circular permutations.
- abcd, dabc, cdab, bcda are different in a line but they are same in circular permutation as there is no beginning nor ending in the circle.
- Number of circular permutation of 'n' different things taken 'r' at a time = $\frac{nPr}{r}$

A. Clockwise & anti-clockwise are different arrangements: No. of circular permutations of n different things chosen at a time is **(n−1)!**

Explanation: In line permutation, no. of ways or arranging n things = n!. Then why do we have (n-1)! in circular permutation. The reason is simple:

Let us assume that we have 6 people and 6 chairs.

The number of ways in which 1st person can sit = 1 way only because for the 1st person, all the arrangements will be same irrespective of the chair he sit at.

2nd person can sit in 5 ways. Because for him, the arrangements won't be same for all chairs because 1 chair is already occupied. 3rd person can sit in 4 ways;

4th person can sit in 3 ways; 5th person in 2 ways & 6th persons in 1 way.

Thus answer = 5.4.3.2.1 = 5! which is equal to (6-1)!

[PC Note: Mostly used in case of "Sitting arrangement of Person" examples]

CQ35: How many ways can 4 persons sit at a round table? **[Ans:** 3! ways**]**

B. Clockwise & anti-clockwise are same arrangements: No. of circular permutations of n different things chosen at a time is $\frac{(n-1)!}{2}$.

[PC Note: Mostly used in case of "Necklace & garlands" examples]

SUM OF ALL Nos FORMED OUT OF 'n' DIGITS

(n-1)! × Sum of digits × (1111....n times)

CQ36: Compute the sum of 4 digits numbers which can be formed with the four digits 1, 3, 5, 7, if each digit is used only once in each arrangement.

Ans: (n-1)! × Sum of digits × (1111....n times) = (4-1)! × (1 + 3 + 5 + 7) × 1111 = 6.16.1111 = 106656.

PC Note: If the digits include 'ZERO', Answer = (i) − (ii)

(i) Solve as per above given formula **including '0'**; (ii) Solve by **ignoring '0'**

CQ37: Find the sum of all numbers greater than 10,000 formed by using the digits 0, 2, 4, 6, 8.

Ans: (i) 53,33,280 - (ii) 1,33,320. Thus, the required Sum 51,99,960.

SOME EXAMPLES

CQ38: How many 4 digits numbers can be formed out of the digits 1,2,3,5,7,8,9, if no digit is repeated in any number? How many of these will be greater than 3000?

Ans: 7 different digits and 4 digit number is to be formed using any 4 of these digits.

This is same as the permutations of 7 different things taken 4 at a time.

Hence, the number of four-digit numbers that can be formed = $^7P_4$ = 7 × 6 × 5 × 4 × = 840 ways.

Next, there is the restriction that the four-digit numbers so formed must be greater than 3,000. Thus, it will be so if the first digit-that in the thousand's position, is one of the five digits 3, 5, 7, 8, 9. Hence, the first digit can be chosen in 5 different ways; when this is done, the rest of the 3 digits are to be chosen from the rest of the 6 digits without any restriction and this can be done in $^6P_3$ ways. [Ans: 5 × $^6P_3$ = 5 × 6 × 5 × 4 = 5 × 120 = 600.]

CQ39: Find the total number of numbers greater than 2000 that can be formed with the digits 1, 2, 3, 4, 5 no digit being repeated in any number.

Ans: 5 Digit Numbers: All the 5 digits numbers that can be formed with the given 5 digits are greater than 2000. This can be done in $^5P_5$ ways = **120 ways.**

4 Digit Number: Greater than 2000. Thus, Thousand place can have 2, 3, 4, 5 only.

Thousand Place	Hundred Place	Ten's Place	Unit place
4 ways [cannot have 1]	4 ways [can have 1 also]	3 ways	2 ways

This can be done in 4 × 4 × 3 × 2 = 96 ways. **Total Number of ways = 120 + 96 = 216 ways.**

CQ40: There are 6 students of whom 2 are Indians, 2 Americans, and the remaining 2 are Russians. They have to stand in a row for a photograph so that the two Indians are together, the two Americans are together and so also the two Russians. Find the number of ways in which they can do so. **[Ans:** 48**]**

CQ41: A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways can they be seated if (i) all the sisters sit together, (ii) no two sisters sit together?

Ans: (i) **5! × 3! ways = 720 ways**.

(ii) **$^5P_3$ × 4! = 60 × 24 = 1440 ways.**

CQ42: 6 boys & 5 girls are to be seated for a photograph in a row such that no two girls sit together and no two boys sit together. Find the number of ways in which this can be done.

Ans: 6! × 5!.

PERMUTATION OF DISSIMILAR THINGS (ALL DIFFERENT) UNDER RESTRICTION

Cases	Formula	If n = 8, r = 3
A particular thing is NOT INCLUDED	$^{n-1}P_r$	$^{7}P_3$
A particular thing is ALWAYS INCLUDED	$r(^{n-1}P_{r-1})$	$3(^{7}P_2)$
'm' particular thing ALWAYS TOGETHER	$(n - m + 1)!\ m!$	
2 particular ALWAYS TOGETHER	$(n - 1)!\ 2!$	$7! \times 2!$
3 particular ALWAYS TOGETHER	$(n - 2)!\ 3!$	$6! \times 3!$
4 particular ALWAYS TOGETHER	$(n - 3)!\ 4!$	$5! \times 4!$
'm' particular thing NEVER TOGETHER	$n! - [(n - m + 1)!\ m!]$	
2 particular NEVER TOGETHER	$(n - 2) \times (n - 1)!$	$6 \times 7!$
3 particular NEVER TOGETHER	$(n - 3) \times (n + 2) \times (n - 1)!$	$5 \times 10 \times 6!$
NO TWO are together out of 'r' things & no restriction on remaining 'q' things	$q! \times {}^{(q+1)}P_r$	
Forming numbers including ZERO	$^{n}P_r - {}^{n-1}P_{r-1}$	$^{8}P_3 - {}^{7}P_2$
SUM of ALL no. formed out of 'n' digits	(n − 1)! × (Sum of all digits) × (111111.. n times)	

PERMUTATION OF SIMILAR THINGS (2 ALIKE GROUPS) UNDER RESTRICTION

Cases	Formula	n = 8, p = 2, q = 3
'm' particular ALWAYS TOGETHER	$[n - m + 1)!\ m!] / (p! \times q!)$	
2 particular ALWAYS TOGETHER	$[n - 1)!\ 2!] / (p! \times q!)$	$(7! \times 2!) / ((2! \times 3!)$
3 particular ALWAYS TOGETHER	$[n - 2)!\ 3!] / (p! \times q!)$	$(6! \times 3!) / ((2! \times 3!)$
4 particular ALWAYS TOGETHER	$[n - 3)!\ 4!] / (p! \times q!)$	$(5! \times 4!) / ((2! \times 3!)$
'm' particular NEVER TOGETHER		
2 particular NEVER TOGETHER	$[(n - 2)\ (n - 1)!] / (p! \times q!)$	$(6 \times 7!) / ((2! \times 3!)$
2 particular NEVER TOGETHER	$[(n - 3)\ (n + 2)\ (n - 1)!] / (p! \times q!)$	$(5 \times 10 \times 6!) / ((2! \times 3!)$

PERMUTATION OF DISSIMILAR THINGS IN A CIRCLE UNDER RESTRICTION

Cases	Formula	If n = 8,
'm' particular ALWAYS TOGETHER	$[(n-1)-m+1)!\ m!$	
2 particular ALWAYS TOGETHER	$[(n-1)-1)!\ 2!$	$6! \times 2!$
3 particular ALWAYS TOGETHER	$[(n-1)-2)!\ 3!$	$5! \times 3!$
4 particular ALWAYS TOGETHER	$[(n-1)-3)!\ 4!$	$4! \times 4!$
'm' particular NEVER TOGETHER		
2 particular NEVER TOGETHER	$[(n-1)-2] \times [(n-1)-1]!$	$5 \times 6!$
2 particular NEVER TOGETHER	$(n-4)\ (n+1)\ (n-3)!$	$4 \times 9 \times 5!$
NO TWO are together out of 'r' things & no restriction on remaining 'q' things	$(q-1)! \times {}^{q}P_{r}$	

Space for PC Class Note:

PERMUTATION – QUESTION BANK

SN	CHAPTER 5A. PERMUTATION	Ans
	FACTORIAL & FUNDAMENTAL RULE OF COUNTING & ^nP_r FORMULA	
Q1	Find n if $^nP_3 = 60$ (a) 4 (b) 5 (c) 6 (d) 7	**B**
Q2	Find the value of n if $(n+1)! = 42\ (n-1)!$ (a) 6 (b) -7 (c) 7 (d) -6	**A**
Q3	$^6P_r = 360$ then find r? (a) 4 (b) 5 (c) 6 (d) None	**A**
Q4	If $^nP_4 = (20)\ ^nP_2$ then the value of n is ______. (a) 5 (b) 6 (c) 7 (d) 8	**C**
Q5	If $^7P_n \div {}^7P_{n-3} = 60$ the value of n is ______. (a) 8 (b) 4 (c) 5 (d) 2	**C**
Q6	If $^5P_r = 60$, then the value of 'r' is ______. (a) 3 (b) 2 (c) 4 (d) None	**A**
Q7	If $^{11}P_r = {}^{12}P_{r-1}$, then the value of 'r' is ______. (a) 6 (b) 7 (c) 9 (d) 8	**C**
Q8	There are 10 trains plying between Calcutta & Delhi. The number of ways in which a person can go from Calcutta to Delhi & return by a different train is ______. (a) 99 (b) 90 (c) 80 (d) None	**B**
Q9	$\frac{0! \times 5!}{2!}$ = ______. (a) 60 (b) 0 (c) 120 (d) None	**A**
Q10	In ^nP_r, n is always ______. (a) An integer (b) A fraction (c) A positive integer (d) None	**C**
Q11	In ^nP_r, the restriction is ______. (a) n>r (b) $n \geq r$ (c) $n \leq r$ (d) None	**B**
Q12	$^nP_r \div {}^{n-1}P_{r-1}$ is ______. (a) n (b) n! (c) (n-1)! (d) ^nC_n	**A**
Q13	In $^nP_r = n.(n-1).(n-2) \ldots\ldots\ldots(n-r-1)$, number of factor is ______. (a) n (b) r- 1 (c) n- r (d) r	**D**
Q14	$^{(n-1)}P_r + r.{}^{(n-1)}P_{(r-1)}$ = ______. (a) ^nC_r (b) $\underline{\lvert n}/(\underline{\lvert r}\ \underline{\lvert n-r})$ (c) ^nP_r (d) None	**C**
Q15	0! = __. (a) 0 (b) 1 (c) ∞ (d) -1	**B**

Q16	Compute the value of 8! (a) 120 (b) 3,62,880 (c) 720 (d) 40,320	**D**
Q17	$4P_4$ is equal to ______. (a) 1 (b) 24 (c) 0 (d) None	**B**
Q18	The value of $^{11}P_9$ is equal to ______. (a) $\frac{11!}{9! \times 2!}$ (b) $\frac{11!}{2!}$ (c) $\frac{11! \times 2!}{9!}$ (d) None	**B**
Q19	If $^nP_4$ = 5040, then the value of 'n' is______. (a) 8 (b) 9 (c)10 (d) 6	**C**
Q20	If. $^nP_3$: $^nP_2$ = 3:1, then n is equal to______. (a) 7 (b) 4 (c) 5 (d) None	**C**
Q21	If $^{56}p_{r+6}$:$^{54}p_{r+3}$ = 30800:1, find 'r'. (a) 31 (b) 41 (c) 51 (d) 21	**B**
Q22	If (n +1)! = 20 (n -1)!, then value of n is ______. (a) 6 (b) 5 (c) 4 (d) None	**C**
Q23	$^{m+n}P_2$ = 56, $^{m-n}P_2$ = 30, then (a) m = 6, n = 2 (b) m = 7, n = 1 (c) m = 4, n = 4 (d) None	**B**
Q24	If $^{x+y}P_2$ = 90 & $^{x-y}P_2$ = 30 then______. (a) x = 4y (b) x = 2 (c) x = y (d) 4x = y	**A**
Q25	1.1! + 2. 2! + 3.3! + 4.4! +......(n-1) (n-1)! + n.n! (a) n(n+1) (n+1)! (b) (n+1)! — 1 (c) (n+1)! + 1 (d) (n+1)!	**B**
Q26	Value of $\sum_{r=1}^{10} r.^r P_r$ is ______. (a) $^{11}P_{11}$ (b) $^{11}P_{11}$-1 (c) $^{11}P_{11}$+1 (d) None	**B**
Q27	If $^{n+3}P_6 \div {}^{n+2}P_4$ = 14 the value of n is ______. (a) 8 (b) 4 (c) 5 (d) 2	**B**
Q28	There are 4 routes for going from Dumdum to Sealdah & 5 routes for going from Sealdah to Chandni. In how many different ways can you go from Dumdum to Chandni Via Sealdah? (a) 9 (b) 1 (c) 20 (d) None	**C**
Q29	There are 6 routes for journey from station A to station B. In how many ways you may go from A to B & return if for returning you make a choice of any of the routes? (a) 6 (b) 12 (c) 36 (d) 30	**C**
Q30	In Question No.29, if you decided to take the same route you may do it in —— number of ways. (a) 6 (b) 12 (c) 36 (d) 3	**A**
Q31	In Question No.29, if you decided not take the same route you may do it in number	**D**

	of ways. (a) 6 (b) 12 (c) 36 (d) 30	
Q32	If six times the number permutations of n things taken 3 at a time are equal to seven times the number of permutations of (n-1) things chosen 3 at a time, find 'n'. (a) 18 (b) 9 (c) 36 (d) 21	**D**
Q33	In a group of boys, the number of arrangements of 4 boys is 12 times the number of arrangements of 2 boys. The number of boys in the group is ______. (a) 10 (b) 8 (c) 6 (d) None	**C**
Q34	A dealer provides you Maruti Car & Van in 2 body patterns & 5 different colours. How many choices are open to you? (a) 2 (b) 7 (c) 20 (d) 10	**C**
	BASIC QUESTIONS WITH SIMPLE RESTRICTIONS	
Q35	How many different words can be formed from letters of the word 'TRIANGLE? (a) 8! (b) 7! (c) 6! (d) 2! x 6!	**A**
Q36	Number of words that can be formed by using all the letters of the word 'DELHI'. (a) 120 (b) 24 (c) 125 (d) 130	**A**
Q37	How many arrangements of the word 'PUBLIC' will begin with B? (a) 6! (b) 5! (c) $^6P_5$ (d) 5	**B**
Q38	How many 7 letter words can be formed using letters of the words "SPECIAL"? (a) 5,040 (b) 6 (c) 840 (d) 450	**A**
Q39	How many arrangements can be made by using all the letters of word "Monday"? (a) 120 (b) 720 (c) 41 (d) 51	**B**
Q40	Find how many five letter words can be formed out of the word "LOGARITHMS". (a) $^{10}P_5$ (b) $^{10}C_5$ (c) $^9C_4$ (d) None	**A**
Q41	Three persons enter a railway carriage, where there are 5 vacant seats. The number of ways they can seat themselves is ______. (a) 60 (b) 50 (c) 70 (d) 40	**A**
Q42	Mr. X & Mr. Y enter into a railway compartment having six vacant seats. The number of ways in which they can occupy the seats is ______. (a) 25 (b) 31 (c) 32 (d) 30	**D**
Q43	The number of arrangements of 10 different things taken 4 at a time in which one particular thing always occurs is ______. (a) 2,015 (b) 2,016 (c) 2,014 (d) None	**B**
Q44	The number of permutations of 10 different things taken 4 at a time in which one particular thing never occurs is ______. (a) 3,020 (b) 3,025 (c) 3,024 (d) None	**C**

Q45	The number of arrangements in which the letters of the word MONDAY be arranged so that the words thus formed begin with M & do not end with N is ______. (a) 720 (b) 96 (c) 120 (d) None	**B**
Q46	In how many ways it is possible to write the word 'ZENITH' in a dictionary? (a) $^6P_6$ (b) $^6C_6$ (c) $^6P_0$ (d) None	**A**
Q47	How many telephones connections may be allotted with 8 digits from the numbers 0,1,2,....9? (a) 10^8 (b) 10! (c) $^{10}C_8$ (d) $^{10}P_8$	**A**
Q48	Eleven students are participating in a race. In how many ways the first 5 prizes can be won? (a) 44,550 (b) 55,440 (c) 120 (d) 90	**B**
Q49	Total number of sitting arrangements of 7 persons in a row if 2 persons occupy the end seats is ______. (a) 5! (b) 6! (c) 2! x 5! (d) None	**C**
Q50	Total number of sitting arrangements of 7 persons in a row if one person occupies the middle seat is ______. (a) 5! (b) 6! (c) 2! x 5! (d) None	**B**
Q51	The number of different ways in which 5 girls may be arranged in a row is_____. (a) 102 (b) 120 (c) 100 (d) 210	**B**
Q52	3 persons go into a railway carriage having 8 seats. In how many ways they may occupy the seats? (a) $^8P_3$ (b) $^8C_3$ (c) $^8C_5$ (d) None	**A**
QUESTIONS BASED ON DIGITS		
Q53	Number of 4-digit numbers greater than 5,000 that can be formed out of the digits 3,4,5,6 & 7 (no. digit is repeated). (a) 72 (b) 27 (c) 70 (d) None	**A**
Q54	How many numbers between 1000 & 10000 can be formed with 1, 2, ... 9? (a) 3024 (b) 60 (c) 78 (d) None	**D**
Q55	How many numbers higher than a million can be formed with the digits 0,4,4,5,5,5,3? (a) 420 (b) 360 (c) 7! (d) None	**D**
Q56	How many three-digit numbers are there, with distinct digits, with each digits odd? (a) 120 (b) 60 (c) 30 (d) 15	**B**
Q57	The number of numbers lying between 100 & 1,000 can be formed with the digits 1,2,3,4,5,6,7 is ______. (a) 210 (b) 200 (c) 110 (d) None	**A**

Q58	How many six digits numbers can be formed with the permutation of digits 9,5,3,1,7,0? (a) 600 (b) 720 (c) 120 (d) None	**A**
Q59	In terms of Question No.58, how many numbers will have 0's in ten's palace? (a) 600 (b) 720 (c) 120 (d) None	**C**
Q60	How many 3 digit numbers are there if repetition of digits is not allowed? (a) 648 (b) 9^3 (c) 3^9 (d) ${}^9C_3$	**A**
Q61	The number of four digit numbers that can be formed using the digits 1, 7, 6 & 9 without repetition is ______. (a) 24 (b) 46 (c) 64 (d) 90	**A**
Q62	No. of 4 digit numbers that can be formed out of the figures 0,1,2,3,4 (no digit is repeated) is ______. (a) 120 (b) 20 (c) 96 (d) None	**C**
Q63	The number of numbers lying between 10 & 1,000 can be formed with the digits 2,3,4,0,8,9 is ______. (a) 124 (b) 120 (c) 125 (d) None	**D**
Q64	How many six digit numbers can be formed out of 4,5,6,7,8,9 (no digits being repeated)? (a) 6! - 5! (b) 6! (c) 6! + 5! (d) None	**B**
Q65	The total number of numbers less than 1,000 & divisible by 5 formed with 0, 1, 2, 9 such that each digit does not occur more than once in each number is ______. (a) 150 (b) 152 (c) 154 (d) None	**C**
Q66	How many four digits number can be formed by using 1, 2 ... 7? (Without repetition of digits) (a) ${}^7P_4$ (b) ${}^7P_3$ (c) ${}^7C_4$ (d) None	**A**
Q67	How many four digits numbers can be formed by using 1, 2, ...7? (Which are greater than 3,400) (a) 500 (b) 550 (c) 560 (d) None	**C**
Q68	The number of even numbers greater than 300 that can be formed with the digits 1,2,3,4,5 without repetition is ______. (a) 110 (b) 112 (c) 111 (d) None	**C**
Q69	How many 4 digit numbers greater than 7,000 can be formed out of the digits 3, 5, 7, 8, 9? (a) 24 (b) 48 (c) 72 (d) 50	**C**
	ALWAYS TOGETHER & NEVER TOGETHER	
Q70	In how many number of ways can 'n' books be arranged on a shelf so that two particular books are not together?	**A**

	(a) (n-2) (n-1)! (b) (n-1) n! (c) (n-2) n! (d) (n-2) (n-1)	
Q71	10 examination papers are arranged in such a way that the best & worst papers never come together. The number of arrangements is_____. (a) 9.8! (b) 10! (c) 8.9! (d) None	**C**
Q72	In how many ways 5 Sanskrit, 3 English & 3 Hindi books be arranged keeping the books of the same language together? (a) 5! × 3! × 3! × 3! (b) 5! × 3! × 3! (c) $^5P_3$ (d) None	**A**
Q73	In how many ways can the word 'STRANGE' be arranged so that the vowels never come together? (a) 7! - 6! X 2! (b) 7! - 6! (c) $^7P_6$ (d) None	**A**
Q74	In how many ways can the word 'strange' be arranged so that the vowels are never separated? (a) 6! x 2! (b) 7! (c) 7! ÷ 2! (d) None	**A**
Q75	There are 5 speakers A, B, C, D & E. the number of ways in which A will speak always immediate& before B is ______. (a) 24 (b) 120 (c) 15 (d) None	**A**
Q76	Number of ways of arranging 5 different books on history, 2 different books on English & 4 different books on physics on a shelf so that books on same subject are not separated. (a) 5,760 (b) 34,560 (c) 120 (d) 11!	**B**
Q77	How many arrangements can be made out of the word DRAUGHT, the vowels never being separated? (a) 720 (b) 360 (c) 840 (d) 670	**A**
Q78	In how many ways can the letters of the word PENCIL be arranged so that N is always next to E ______. (a) 60 (b) 40 (c) 720 (d) 120	**D**
Q79	The total number of sitting arrangements of 7 persons in a row if 3 persons sit together in any order is ______. (a) 5! (b) 6! (c) 2! x 5! (d) None	**B**
Q80	The number of arrangements of the letters in the work FAILURE, so that vowels are always coming together is ______. (a) 576 (b) 676 (c) 570 (d) None	**A**
Q81	The number of ways the letters of the word "TRIANGLE" to be arranged so that the word 'ANGLE" will be always present is ______. (a) 20 (b) 60 (c) 24 (d) 32	**C**
Q82	If 5 books of English, 4 books of Tamil & 3 books of Hindi are to be arranged in a single row so that books of same language come together. (a) 1,80,630 (b) 1,60,830 (c) 1,03,680 (d) 1,30,680	**C**

Q83	In how many ways the letters of the word 'FAILURE' can be arranged with the condition that the four vowels are always together? (a) $(4!)^2$ (b) 4! (c) 7! (d) None	**A**
Q84	In how many ways the word 'ARRANGE' be arranged such that 2 'r's come together? (a) 400 (b) 440 (c) 360 (d) None	**C**
Q85	In how many ways the word 'ARRANGE' be arranged such that the 2 'r's & 2 'a's come together? (a) 120 (b) 130 (c) 140 (d) None	**A**
Q86	A family of 4 brothers & three sisters is to be arranged for a photograph in one row. In how many ways can they be seated if all the sisters sit together? (a) 720 (b) 640 (c) 840 (d) 600	**A**
Q87	A family of 4 brothers & three sisters is to be arranged for a photograph in one row. In how many ways can they be seated if no two sisters sit together? (a) 840 (b) 1,440 (c) 2,210 (d) 1,020	**B**
Q88	There are 6 students of whom 2 are Indians, 2 Germans & the remaining 2 are British. They have to stand in a row for a photograph so that the two Indians are together, the two Germans are together & so also the two British. The number of ways such an arrangement can be made is_____. (a) 48 (b) 8 (c) 16 (d) 24	**A**
	FIXED PLACES (EVEN/ODD) + NO TWO GIRLS/BOYS SIT TOGETHER	
Q89	5 Boys & 4 girls are to be seated in row. If girls occupy even places, then no. of such arrangements are ___. (a) 288 (b) 2808 (c) 2008 (d) 2880	**D**
Q90	The number of ways in which the letters of the word MOBILE be arranged so that consonants always occupy the odd places is ______. (a) 36 (b) 63 (c) 30 (d) None	**A**
Q91	In how many ways the words 'failure' can be arranged so that consonants occupy only the odd positions? (a) 4! (b) $(4!)^2$ (c) $7! \div 3!$ (d) None	**B**
Q92	In how many ways can be letters of the word 'VIOLENT' be arranged so that the vowels occupy even places only? (a) 1,440 (b) 240 (c) 480 (d) 144	**D**
Q93	The number of ways the letters of the word 'SIGNAL' can be arranged such that the vowels occupy only odd position is ______. (a) 1,440 (b) 240 (c) 480 (d) 144	**D**
Q94	In how many ways can the word 'STRANGE' be arranged so that the vowels occupy only the odd places?	**C**

	(a) $^5P_5$ (b) $^5P_5 \times {}^4P_4$ (c) $^5P_5 \times {}^4P_2$ (d) None	
Q95	In how many ways the vowels of the word "ALLAHABAD" will occupy the even places? (a) 120 (b) 60 (c) 30 (d) None	**B**
Q96	In how many ways the word 'Article' can be arranged in a row so that the vowels occupy even places? (a) 132 (b) 144 (c) 72 (d) 160	**B**
Q97	Six boys & five girls are to be seated for a photograph in a row such that no two girls sit together & no two boys sit together. Find the number of ways in which this can be done. (a) 64,500 (b) 76,800 (c) 86,400 (d) 92,500	**C**
	PERMUTATION OF SIMILAR THINGS	
Q98	Number of different arrangements of the letters of the word 'CALCUTTA' is __. (a) | 8 (b) | 5 x | 2 x | 2 x | 2 (c) 5,040 (d) 10,080	**C**
Q99	If you have 5 copies of one book, 4 copies of each of two books, 6 copies each of three books & single copy of 8 books you may arrange it how many number of ways? (a) $\frac{39!}{5!\times(4!)^2\times(6!)^3}$ (b) $\frac{39!}{5!\times8!\times(4!)^2\times(6!)^3}$ (c) $\frac{39!}{5!\times8!\times4!\times(6!)^3}$ (d) $\frac{39!}{5!\times8!\times4!\times6!}$	**A**
Q100	How many different permutations are possible from the letters of word CALCULUS? (a) 4600 (b) 5040 (c) 5400 (d) 4680	**B**
Q101	How many different arrangements are possible from letters of "CALCULATOR"? (a) 4,53,600 (b) 50,400 (c) 45,360 (d) None	**A**
Q102	No. of permutation can be made out the letters of word 'COMMERCE" is ______. (a) 5,040 (b) 8! (c) 6! (d) None	**A**
Q103	No. of arrangements that can be made with the word 'assassination' is______. (a) $13! \div [3!\times4!\times(2!)^2]$ (b) 13! (c) $13! \div [3! \times 4! \times 2!]$ (d) None	**A**
Q104	The number of subsets formed from the letters of the word "ALLAHABAD". (a) 128 (b) 16 (c) 32 (d) None	**C**
Q105	The number of permutation of the word "ALLAHABAD" is ______. (a) $9! \div (4! \times 2!)$ (b) $9! \div 4!$ (c) 9! (d) None	**A**
Q106	In how many ways can the letters of the word 'ARRANGE' be arranged? (a) 1200 (b) 1250 (c) 1260 (d) 1300	**C**
Q107	Number of words that can be formed using the letter A thrice, letter B twice & the letter C once is ______. (a) 80 (b) 50 (c) 70 (d) 60	**D**

CIRCULAR PERMUTATION		
Q108	If 50 different jewels can be set to form a necklace then number of ways is ___. (a) $\frac{1}{2}$.50! (b) $\frac{1}{2}$.49! (c) 49! (d) None	B
Q109	Number of circular permutations of n different things chosen at a time is_____. (a) (n -1)! (b) (n + 1)! (c) n! (d) (n - 2)!	A
Q110	In how many ways can 4 persons sit at a round table for a group discussion? (a) 24 (b) 12 (c) 6 (d) 18	C
Q111	Number of ways in which 7 girls form a ring is_____. (a) 700 (b) 710 (c) 720 (d) None	C
Q112	Number of ways in which 8 different beads be strung on a necklace is ______. (a) 2,500 (b) 2,520 (c) 2,250 (d) None	B
Q113	5 persons are sitting in a round table in such way that Tallest Person is always on the right-side of the shortest person. The number of such arrangements is ______. (a) 6 (b) 8 (c) 24 (d) None	A
Q114	In how many ways can 8 persons be seated at a round table, such that 2 particular persons sit together? (a) 840 (b) 1220 (c) 1,440 (d) 1896	C
Q115	In how many ways 4 men & 3 women are arranged at a round table if women always sit together? (a) 6 x 6! (b) 6! (c) 7! (d) None	B
Q116	In how many ways 4 men & 3 women are arranged at a round table if the women never sit together? (a) 6 x 6! (b) 6! (c) 7! (d) None	A
Q117	The Chief Ministers of 17 States meet to discuss the hike in oil price at a round table. In how many ways they seat themselves if the Kerala & Bengal chief ministers choose to sit together? (a) 15! × 2! (b) 17! × 2! (c) 16! × 2! (d) None	A
Q118	In how many ways can 4 Americans & 4 English men be seated at a round table so that no 2 Americans may be together? (a) 4! × 3! (b) ${}^4P_4$ (c) 3 × ${}^4P_4$ (d) ${}^4C_4$	A
Q119	In how many ways can 6 boys & 6 girls be seated around a table so that no 2 boys are adjacent? (a) 4! × 5! (b) 5! × 6! (c) ${}^6P_6$ (d) 5×${}^6P_6$	B
Q120	Six Persons A, B, C, D, E & F to be seated at a circular table. In how many ways can this be done, if A must always have either B or C on his right & B must always have either C or D on his right? (a) 3 (b) 6 (c) 12 (d) 18	D

MISCELLANEOUS QUESTIONS

Q121	The letters of the words CALCUTTA & AMERICA are arranged in all possible ways. The ratio of the number of these arrangements is______. (a) 1:2 (b) 2:1 (c) 1:1 (d) 1.5:1	**B**
Q122	How many arrangements of the letters of the word 'BHARAT will not have 'B' & 'H' together"______. (a) 360 (b) 240 (c) 120 (d) 60	**B**
Q123	How many words of 3 consonants & 2 vowels can be formed from 6 consonants & 4 vowels? (a) ${}^{6}P_{3}\times{}^{4}P_{2}$ (b) ${}^{6}C_{3}+{}^{4}C_{2}$ (c) ${}^{6}P_{3}\times{}^{4}P_{2}$ (d) $\underline{\lvert 3} \times \underline{\lvert 2}$	**A**
Q124	In how many ways the word 'ARRANGE' be arranged such that the 2 'r's do not come together? (a) 1000 (b) 900 (c) 800 (d) None	**B**
Q125	The total number of 9 digit numbers of different digits is_____. (a) ${}^{10}P_{9}$ (b) ${}^{10}P_{9}$ (c) ${}^{9}P_{9}$ (d) None	**D**
Q126	How many numbers between 3,000 & 4,000 can be formed with 1 2……. 6? (a) 3,024 (b) 60 (c) 78 (d) None	**D**
Q127	How may numbers between 1,000 & 10,000 can be formed with the digits 1,2,3,4,5,6 (a) 720 (b) 360 (c) 120 (d) 60	**B**
Q128	Number of 4-digit numbers that can be formed from 1,2,3,5,7,8,9 such that no digit being repeated in any number, which are greater than 3000 are_____. (a) 120 (b) 480 (c) 600 (d) 840	**C**
Q129	Eight guests have to be seated 4 on each side of a long rectangular table. 2 particular guests desire to sit on one particular side of the table & 3 on the other side. The number of ways in which the sitting arrangements can be made is_____. (a) 1732 (b) 1728 (c) 1730 (d) 1278	**B**

ADVANCE QUESTIONS

Q130	How many different words can be formed from letters of the word 'TRIANGLE? (a) 8! (b) 7! (c) 6! (d) 2! x 6!	**A**
Q131	How many different words can be formed beginning with 'E' of the word 'TRIANGLE? (a) 8! (b) 7! (c) 6! (d) 2! x 6!	**B**
Q132	In Question No.131, how many of them will begin with 'T' & end with 'E'? (a) 8! (b) 7! (c) 6! (d) 2! x 6!	**C**
Q133	In Question No.131, how many of them have 'T' & 'E' in the end places? (a) 8! (b) 7! (c) 6! (d) 2! x 6!	**D**

Q134	In Question No.131, how many of them have consonants never together? (a) 8! - 4! x 5! (b) $^6P_3$ x 5! (c) 2! x 5! x 3! (d) $^4P_3$ x 5!	**A**
Q135	In Question No.131, how many of them have arrangements that no two vowels are together? (a) 8! - 4! x 5! (b) $^6P_3$ x 5! (c) 2! x 5! x 3! (d) $^4P_3$ x 5!	**B**
Q136	In Question No.131, how many of them have arrangements that consonants & vowels are always together? (a) 8! - 4! x 5! (b) $^6P_3$ x 5! (c) 2! x 5! x 3! (d) $^4P_3$ x 5!	**C**
Q137	In Question No.131, how many of them have arrangements that vowels occupy odd Places? (a) 8! - 4! x 5! (b) $^6P_3$ x 5! (c) 2! x 5! x 3! (d) $^4P_3$x 5!	**D**
Q138	Number of 2-digit numbers which are divisible by 6 is_____. (a) 16 (b) 15 (c) 17 (d) 14	**B**
Q139	How many different signals are possible if we wish to make signals by arranging 3 red, 2 yellow & 2 green flags in one post. (a) 210 (b) 6,420 (c) 40,320 (d) 96	**A**
Q140	Let S be the collection of eight points in the plane with no three points on the straight line. Find the number of triangles that have points of S as vertices. (a) 52 choices (b) 55 choices (c) 48 choices (d) 56 choices	**D**
Q141	The number of ways in which 8 sweets of different sizes can be given among 8 persons of different ages so that the largest sweet always goes to be younger assuming that each one of them gets a sweet is_____. (a) 8! (b) 5,040 (c) 5,039 (d) None	**B**
Q142	Number of ways in which arrangements of 4 letters can be made from the word "MATHEMATICS". (a) 1,680 (b) 756 (c) 18 (d) 2454	**D**
Q143	Total number of ways in which six '+' & four '-' signs can be arranged in a line such that no two '-' signs occur together is _______. (a) 7!/3! (b) 6! x (7!/3!) (c) 35 (d) None	**C**
Q144	In how many ways 21 red balls & 19 blue balls can be arranged in a row so that no two blue balls are together. (a) 1,540 (b) 1,520 (c) 1,560 (d) None	**A**
Q145	Find the number of divisors of 21,600 excluding 1 & the number itself_____. (a) 72 (b) 142 (c) 35 (d) 70	**D**
Q146	A computer has 5 terminals & each terminal is capable of four distinct positions including the positions of rest what is the total number of signals that can be made? (a) 20 (b) 1020 (c) 1023 (d) None	**C**

Q147	In order to pass PCA examination minimum marks have to be secured in each of 7 subjects. In how many ways can a pupil fail? (a) 128 (b) 64 (c) 127 (d) 63	C
Q148	In how many ways can 9 letters be posted in 4 letter boxes? (a) 4^9 (b) 4^5 (c) ${}^9P_4$ (d) ${}^9C_4$	A
Q149	If all the permutations of the letters of the word "CHALK" are written in a dictionary the rank of this word will be______. (a) 30 (b) 31 (c) 32 (d) None	C
Q150	Number of ways the letters of the word COMPUTER can be rearranged as______. (a) 40,320 (b) 40,319 (c) 40,318 (d) None	B
Q151	No. of words which can be formed with 2 different consonants & 1 vowel out of 7 different consonants & 3 different vowels. vowel to lie between 2 consonants is__. (a) 3×7×6 (b) 2×3×7×6 (c) 2×3×7 (d) None	A
Q152	If the letter of the word ATTEMPT are written down at random, the chance that all Ts are consecutive is______. (a) 1/42 (b) 6/7 (c) 1/7 (d) 1	C
Q153	There are 50 stations on a railway line how many different kinds of single first class tickets may be printed to enable a passenger to travel from one station to other? (a) 2,500 (b) 2,450 (c) 2,400 (d) None	B
Q154	A letter lock has three rings each marked with 10 different letters. In how many ways it is possible to make an unsuccessful attempt to open the lock? (a) 1,000 (b) 999 (c) 5040 (d) None	B
Q155	In how many different ways 3 rings of a lock can not combine when each ring has digits 0, 1, 2 .....9 leading to unsuccessful events? (a) 999 (b) 10^3 (c) 10! (d) 997	A
Q156	In how many different ways can 7 persons stand in a line for a group photograph? (a) 7 × 6! (b) 6! (c) 7 (d) 24	A

CHAPTER 5B. COMBINATION

INTRODUCTION

- **Definition:** The number of ways of SELECTING all or some of the given things out of given things is called combination.
- The order in which things are arranged is NOT important.
- Number of Permutations of 'r' different objects out of 'n' different objects = $^nC_r = \frac{n!}{(n-r)! \times r!}$

PROPERTIES OF ^nC_r

1) $^nC_r = {^nC_{n-r}}$

2) $^nC_x = {^nC_y}$ ⇒ Either x = y or x + y = n

CQ1: Find 'r' if $^{18}C_r = {^{18}C_{r+2}}$

Ans: r cannot be equal to r + 2. Therefore r + (r +2) = 18 ⇒ 2r + 2 = 18 ⇒ 2r = 16 ⇒ **r = 8.**

3) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

CQ2: Find x if $^{12}C_5 + 2\,^{12}C_4 + {^{12}C_3} = {^{14}C_x}$

Ans: $^{12}C_5 + 2.\,^{12}C_4 + {^{12}C_3}$ ⇒ $^{12}C_5 + {^{12}C_4} + {^{12}C_4} + {^{12}C_3}$ ⇒ $^{13}C_5 + {^{13}C_4} = {^{14}C_5}$.

Using $^nC_r = {^nC_{n-r}}$; $^{14}C_5 = {^{14}C_{14-5}} = {^{14}C_9}$

Thus LHS = $^{14}C_5 = {^{14}C_9}$ & RHS = $^{14}C_x$ ⇒ Either x = 5 or x = 9.

4) $^nC_0 + {^nC_1} + {^nC_2} + \ldots + {^nC_{(n-1)}} + {^nC_n} = 2^n$.

CQ3: $^5C_1 + {^5C_2} + {^5C_3} + {^5C_4} + {^5C_5} =$

(a) 30 (b) 31 (c) 32 (d) 25

5) $^nC_0 = 1$.

6) $^nC_n = 1$. Here r = n, $[^nC_n = \frac{n!}{(n-n)! \times n!} = \frac{n!}{0! \times n!} = 1]$.

7) $^nC_r = \frac{nPr}{r}$ ⇒ $^nP_r = r.\,^nC_r$

CQ4: If $^{10}P_r$ = 6,04,800 and $^{10}C_r$ = 120; find the value of r, **[Ans:** r = 7]

8) $^nC_r = \frac{n}{r} \cdot {}^{(n-1)}C_{(r-1)} \Rightarrow {}^{10}C_3 = \frac{10}{3} \cdot {}^9C_2$

CQ5: Find no. of different poker hands (5 cards) in a pack of 52 playing cards.

Ans: In cards, order is not important. Thus, out of 52 cards, 5 cards at a time. **$^{52}C_5$ = 2,598,960.**

CQ6: A committee is to be formed of 3 persons out of 12. Find the number of ways of forming such a committee. **[Ans:** $^{12}C_3$ = 220 ways.**]**

CQ7: A person has 12 friends of whom 8 are relatives. In how many ways can he invite 7 guests such that 5 of them are relatives? **[Ans:** $^8C_5 \times {}^4C_2$ = 336 ways**]**

CQ8: A building contractor needs 3 helpers & 10 men apply. In how many ways can these selections take place? **[Ans:** $^{10}C_3$ ways**]**

CQ9: A committee of 7 members is to be chosen from 6 CAs, 4 Economists & 5 Cost Accountants. In how many ways can this be done if in committee, there must be at least one member from each group and at least 3 CAs?

Ans: The various methods of selecting the persons from the various groups are shown below:

Committee of 7 members				
	C.A.s [Total 6]	**Economists [Total 4]**	**Cost Accountants [Total 5]**	**Ways**
Method 1	3 ⇒ $^6C_3$ ways = 20	1 ⇒ $^4C_1$ ways = 4	3 ⇒ $^5C_3$ ways = 10	800
Method 2	3 ⇒ $^6C_3$ ways = 20	2 ⇒ $^4C_2$ ways = 6	2 ⇒ $^5C_2$ ways = 10	1200
Method 3	3 ⇒ $^6C_3$ ways = 20	3 ⇒ $^4C_3$ ways = 4	1 ⇒ $^5C_1$ ways = 5	400
Method 4	4 ⇒ $^6C_4$ ways = 15	1 ⇒ $^4C_1$ ways = 4	2 ⇒ $^5C_2$ ways = 10	600
Method 5	4 ⇒ $^6C_4$ ways = 15	2 ⇒ $^4C_2$ ways = 6	1 ⇒ $^5C_1$ ways = 5	450
Method 6	5 ⇒ $^6C_5$ ways = 6	1 ⇒ $^4C_1$ ways = 4	1 ⇒ $^5C_1$ ways = 5	120

Therefore, total number of ways = 800 + 1200 + 400 + 600 + 460 + 120 = **3,570**

CQ10: A box contains 7 red, 6 white & 4 blue balls. How many selections of 3 balls can be made so that (a) all three are red (b) none is red (c) one is of each colour?

Ans: (a) **$^7C_3$ = 35 ways.** (b) **$^{10}C_3$ = 120 ways.** (c) **$^4C_1$ = 4 ways.**

Thus, Number of groups of three balls such that one is of each color = 7 × 6 × 4 = **168 ways**.

CQ11: Find no. of ways of selecting 4 letters from word 'EXAMINATION'. **[Ans:** 136 ways**]**

CONCEPT 1: SOME STANDARD RESULTS

1. Total no. of ways in which it is possible to form groups by taking all of n things = **$(2^n - 1)$.**

Explanation: We have total 'n' things. Each of 'n' different things may be dealt with in 2 ways

(i) Selected in group; (ii) Not Selected in group

1^{st} thing = 2 ways;
2^{nd} thing = 2 ways;
3^{rd} thing = 2 ways;
.
.
.
.
n^{th} thing = 2 ways;

$2 \times 2 \times 2 \times 2 \times \ldots\ldots n \text{ times} = 2^n$

But this answer of 2^n includes the case when all the things are not selected & thus no group will be formed.

But we have to find the ways of forming the group. Thus, we will have to subtract this case from our answer. & Therefore,

Total number of ways of forming a group by taking all of 'n' different things is $2^n - 1$.

CQ12: An examination paper with 10 questions consists of 6 questions in Algebra & 4 questions in Geometry. At least one question from each section is to be attempted. In how many ways can this be done?

Ans: A student must answer at least one question from each section & he may answer all questions from each section.

Algebra: There are 6 questions and he may answer a question or he may not answer it.

2^6. But this includes the possibility of none of the question from Algebra being attempted.

Thus, we have to subtract 1 from the answer. Thus $(2^6 - 1)$ ways.

Geometry: There are 4 questions and he may answer a question or he may not answer it.

2^4. But this includes the possibility of none of the question from Algebra being attempted.

Thus, we have to subtract 1 from the answer. Thus $(2^4 - 1)$ ways.

Thus, Examination paper can be attempted in **$(2^6 - 1)(2^4 - 1)$** number of ways.

CQ13: A man has 5 friends. In how many ways can he invite one or more of his friends to dinner?

Ans: As he has to select one or more of his 5 friends, he can do so in 25 – 1 = **31 ways**.

Alternate Method: He can invite his friends one by one, in twos, in threes, etc. and hence the number of ways. = ${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 5 + 10 + 10 + 5 + 1 = 31$ ways.

ALIKE GROUPS

2. Combinations of 'n' things taken some or all at a time when 'p' things are same of one kind, 'q' things are same of another kind, 'r' things are same of a third kind & remaining 's' things are different = **$[(p+1)(q+1)(r+1)2^s] - 1$.**
3. The combinations of selecting r_1 things from a set having n_1 objects & r_2 things from a set having n_2 objects where combination of r_1 things, r_2 things are independent = **${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$.**
4. Number of Diagonals of a polygon with 'n' sides = $\frac{n(n-3)}{2}$.
5. No. of Triangles from 'n' points if 'm' points are collinear = **${}^nC_3 - {}^mC_3$** [2nd part gets cancelled if no points are collinear].
6. No. of lines from 'n' points if 'm' points are collinear = **${}^nC_2 - {}^mC_2 + 1$.**
7. No. of parallelogram formed from 'm' parallel lines intersecting another 'n' parallel lines = **${}^mC_2 \times {}^nC_2$.**
8. If there are '(a + b + c)' things which are to be divided in equal groups having 'a' things, 'b' things & 'c' things respectively, [such that a = b = c], it can be done in $\frac{(a+b+c)!}{a!.b!c!(\text{no.of equal groups})!}$.

CQ14: The number of ways in which 12 things can be divided into 3 equal groups = $\frac{9!}{3!.3!.3!.4!}$.

Ans: Each group will have 3 things. Thus, we have 3 equal groups of 4 things each.

Thus no. of equal groups = 34. Thus, answer will be $\frac{9!}{3!.3!.3!.4!}$.

CQ15: If 7 things are to be divided into 3 groups, of 2, 2, 3 things respectively, find the number of ways in which this can be done.

Ans: No. of equal groups = 2 groups [2, 2 ka]. = $\frac{(a+b+c)!}{a!.b!c!(\text{no.of equal groups})!}$ = $\frac{7!}{2!.2!3!2!}$ = 105.

CONCEPT 2: FINDING RANK (POSITION) OF A WORD IN DICTIONARY [Shortcut Trick]

Steps:

1. Write alphabets in alphabetical order in vertical form & give them numbers starting from 0.
2. Now find the number given to 1st alphabet in step 1 we want as per the question. Write that number in the answer followed by factorial of remaining alphabets.
3. Eliminate 1st alphabet & re – number the vertical alphabets starting from '0' except the eliminated alphabets & repeat step 2 until you have only last alphabet left.

The value for last alphabets will be 0!.

CQ16: Find the rank of 'KNIFE' in the dictionary.

Answer:

K	N	I	F	E

Step 1		Step 3	Step 4
E	0	0	0
F	1	1	1
~~I~~	~~2~~	~~2~~	~~2~~
~~K~~	~~3~~	NA	NA
~~N~~	~~4~~	~~3~~	NA

Step 1: Done.

Step 2: 1^{st} alphabet is 'K'. So, we find the number given to 'K' in step 1. The number is 3. Remaining alphabets are N, I, F, E = 4. Thus, the required number is 3.4!

Step 3: Eliminate 'K' from vertical form & re — number alphabets starting from '0' except 'K'.

Now we find the number given to 'N' in vertical form. The number is 3.

Remaining alphabets are I, F, E = 3. Thus, the required number is 3.3!.

Step 4: Eliminate 'N' from vertical form & re—number the alphabets starting from '0' except K & N. Now we find the number given to 'I' in vertical form. The number is 2.

Remaining alphabets are F, E = 2. Thus, the required number is 2.2!.

Step 5: Eliminate 'I' from vertical form & re—number the alphabets starting from '0' except K, N & I. Now we find the number given to 'F' in vertical form. The number is 1.

Remaining alphabets are E = 1. Thus, the required number is 1.1!

Step 6: We have only one alphabet left. Thus, the value for it will be 0!

Rank of KNIFE = Sum of all values = 3.4! + 3.3! + 2.2! + 1.1! + 0! = 3.24 + 3.6 + 2.2 + 1 + 1 = 96^{th} rank.

CQ17: If all permutations of word "CHALK" are written in a dictionary rank of this word will ___.

(a) 30 (b) 31 (c) 32 (d) None

Ans:

C H A L K

Step 1		Step 3	Step 4	Step 5
~~A~~	~~0~~	~~0~~	~~0~~	NA
~~C~~	~~1~~	NA	NA	NA
~~H~~	~~2~~	~~1~~	NA	NA
K	3	2	1	0
L	4	3	2	1

C	H	A	L	K
1.4!	+ 1.3!	+ 0.3!	+ 1.1!	+ 0!

= 24 + 6 + 0 + 1 + 1 = 32^{nd} rank.

COMBINATION OF DISSIMILAR THINGS UNDER RESTRICTION (OUT OF 'n' THINGS)

Cases	Things taken	Formula	Formula
A particular things is NOT ALLOWED	R	${}^{n-1}C_r$	
A particular things is ALWAYS ALLOWED	R	$({}^{n-1}C_{r-1})$	$({}^{n}C_r - {}^{n-1}C_r)$
Selecting 1 or more out of 'n' things	1 or More	$2^n - 1$	
ALIKE GROUPS			
'p' of 1st type, 'q' of 2nd, 'r' of 3rd & 's' different	1 or More	$[(p + 1) (q + 1) (r + 1)2^s] -1$	

DISTRIBUTION OF DISSIMILAR THINGS INTO GROUPS OR PERSONS (OUT OF 'N' THINGS)

No. of things	Relationship	Distributed to	Formula	Formula
p + q = n	p ≠ q	Persons/ Groups	$\frac{n!}{p!\,q!}$	${}^{n}C_p \times {}^{q}C_q$
p + q = n	p = q	Persons	$\frac{n!}{p!\,q!}$	${}^{n}C_p \times {}^{p}C_p$
p + q = n	p = q	Groups	$\frac{n!}{2! \times p!\,q!}$	
p + q + r = n	p = q = r	Persons	$\frac{n!}{p!\,q!\,r!}$	${}^{n}C_p \times {}^{q+1}C_q \times {}^{r}C_r$
p + q + r	p = q = r	Groups	$\frac{n!}{3! \times p!\,q!\,r!}$	No. of equal groups ka fraction

COMBINATION – QUESTION BANK

SN	CHAPTER 5B. COMBINATION	Ans
Q157	^nP_r = 720 and ^nC_r = 120 Find r? (a) 6 (b) 4 (c) 3 (d) 2	C
Q158	Solve for 'n' if $^nC_4$: $^{n+2}C_n$ = 5:18 (a) 5 (b) 7 (c) -8 (d) 7 or 8	B
Q159	If $^{500}C_{92}$= $^{499}C_{407}$ + ^nC_r = 56, then n is ______. (a) 501 (b) 500 (c) 502 (d) 499	D
Q160	If $^{1000}C_{98}$ =$^{999}C_{97}$ + $^xC_{901}$ then the value of x will be______. (a) 999 (b) 998 (c) 997 (d) None	A
Q161	A team of 12 men is to be formed out of n persons. Then number of times 2 men "A" & "B" are together is ______. (a) $^nC_{12}$ (b) $^{n-1}C_{11}$ (c) $^{n-2}C_{10}$ (d) None	C
Q162	Every person shakes hands with each other in a party and the total number of handshakes is 66. The number of guests in the party is ______. (a) 11 (b) 12 (c) 13 (d) 14	B
Q163	Out of 10 different consonants and 4 different vowels how many words can be formed each containing 6 consonant and 3 vowels? (a) $^{10}C_6 \times {}^4C_3$ (b) $^{10}C_6 \times {}^4C_3 \times 9!$ (c) $^{10}C_6 \times {}^4C_3 \times 10!$ (d) None	B
Q164	First, second and third prizes are to be awarded at an engineering fair in which 13 exhibits have been entered. In how many ways can the prizes be awarded? (a) 1,462 (b) 1,716 (c) 1,876 (d) 1,672	B
Q165	You are selecting a cricket team of first 11 players out of 16 including 4 bowlers and 2 wicket-keepers. In how many ways you can do it so that the team contains exactly 3 bowlers and 1 wicket-keeper? (a) 960 (b) 840 (c) 420 (d) 252	A
Q166	In Question No.165, would your answer be different if the team contains at least 3 bowlers and at least 1 wicket-keeper? (a) 2,472 (b) 960 (c) 840 (d) 420	A
Q167	A party of 6 is to be formed from 10 men and 7 women as so as to include 3 men and 3 women. In how many ways the partly can be formed if two particular women refuses to join it? (a) 4,200 (b) 600 (c) 1,200 (d) None	C
Q168	In how many ways can a consonant and a vowel be chosen out of the letters of the word 'LOGARITHM? (a) 18 (b) 15 (c) 3 (d) None	A

Q169	A box contains 7 red, 6 white and 4 blue balls. How many selections of three balls can be made so that (a) all are red (b) none is red (c) one is of each colour? (a) 35 ways, 120 ways, 168 ways (b) 35 ways, 140 ways, 168 ways (c) 30 ways, 120 ways, 168 ways (d) 35 ways, 120 ways, 148 ways	A
Q170	Five bulbs of which three are defective are to be bled in two bulb points in a dark room. Hunter of trials the room shall be lighted ______. (a) 6 (b) 8 (c) 5 (d) 7	D
Q171	A candidate is required to answer 6 out of 12 questions which are divided into two groups containing 6 questions in each group. He is not permitted to attempt not more than four from any group. The number of choices are ______. (a) 750 (b) 850 (c) 800 (d) None	B
	HOMEWORK QUESTIONS	
Q172	If c (n, 8) = c (n, 6), find c (n, 2) (a) 14 (b) 91 (c) 19 (d) 41	B
Q173	If $^nC_{r-1} = {^nC_{r+1}} = 15$ and $^nC_r = 20$, then the value of $^rC_2$ is ______. (a) 3 (b) | 3 (c) | 4 (d) 12	A
Q174	There are 7 men and 3 ladies. Find the number of ways in which a committee of 6 can be formed of them if the committee is to include at least 2 ladies? (a) 120 (b) 160 (c) 140 (d) 150	C
Q175	In how many ways a committee of 5 people can be formed out of 5 males & 6 females such that there are 3 males and 2 females? (a) 150 (b) 200 (c) 1 (d) 461	A
Q176	In Question No.175, how many choices you have to make if there are 2 males? (a) 150 (b) 200 (c) 1 (d) 461	B
Q177	In Question No.175, how many choices you have to make if there is no female? (a) 150 (b) 200 (c) 1 (d) 461	C
Q178	In Question No.175, how many choices you have to make if there is at least one female? (a) 150 (b) 200 (c) 1 (d) 461	D
Q179	In Question No.175, how many choices you have to make if there are not more than 3 males? (a) 200 (b) 1 (c) 461 (d) 401	D
Q180	A person has 8 friends. The number of ways in which he may invite one or more of them to a dinner is ______. (a) 250 (b) 255 (c) 200 (d) None	B
Q181	In how many ways can a consonant and a. vowel be chosen out of the letters of the word 'EQUATION?	B

	(a) 18 (b) 15 (c) 3 (d) None	
Q182	A question paper contains 6 questions, each having an alternative. The number of ways an examinee can answer one or more questions is ______. (a) 720 (b) 728 (c) 729 (d) None	B
Q183	There are 12 points in a plane of which 5 are collinear. The number of triangles is (a) 200 (b) 211 (c) 210 (d) None	C
Q184	A committee is to be formed of 2 teachers and 3 students out of 10 teachers and 20 students. The number of ways in which this can be done is ______. (a) ${}^{10}C_2 \times {}^{20}C_3$ (b) ${}^{9}C_1 \times {}^{20}C_3$ (c) ${}^{10}C_2 \times {}^{19}C_3$ (d) None	A
Q185	In Question No.184, if a particular teacher is included the number of ways in which this can be done is ______. (a) ${}^{10}C_2 \times {}^{20}C_3$ (b) ${}^{9}C_1 \times {}^{20}C_3$ (c) ${}^{10}C_2 \times {}^{19}C_3$ (d) None	B
Q186	In Question No.184, if a particular student is excluded the number of ways in which this can be done is ______. (a) ${}^{10}C_2 \times {}^{20}C_3$ (b) ${}^{9}C_1 \times {}^{20}C_3$ (c) ${}^{10}C_2 \times {}^{19}C_3$ (d) None	C
Q187	A boats crew consists of 8 men 3 of whom can row only on one particular side and 2 only on the other. The number of ways in which the crew can be arranged is (a) ${}^{3}C_3 \times (4!)^2$ (b) ${}^{3}C_1 \times 4!$ (c) ${}^{3}C_1$ (d) None	A
Q188	If 7 distinct things are to be divided in 3 groups, consisting of 2, 2, and 3 things respectively, find the number of ways this can be done. (a) 105 (b) 210 (c) 100 (d) None	A
	PRACTICE QUESTION	
Q189	Number of straight fines obtained by joining 16 points on a plane, no twice of them being on the same fine is ______. (a) 120 (b) 110 (c) 210 (d) None	A
Q190	The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is ______. (a) 6 (b) 18 (c) 12 (d) 9	B
Q191	8 points are marked on the circumference of a circle. Number of chords obtained by joining these in pairs is ______. (a) 25 (b) 27 (c) 28 (d) None	C
Q192	There are 12 points in a plane no.3 of which are collinear except that 6 points which are collinear. The number of different straight lines is ______. (a) 50 (b) 51 (c) 52 (d) None	C
Q193	C_r is equal to _____. (a) $\frac{n!}{(n-r)!}$ (b) $\frac{n!}{r!(n-r)!}$ (c) $\frac{n!r!}{(n-r)!}$ (d) $\frac{n!(n-r)!}{r!}$	B
Q194	The value of ${}^{n}C_0$ _____.	C

	(a) n (b) 0 (c) 1 (d) ∞	
Q195	The value of ${}^nC_n$ is ______. (a) n (b) 1 (c) 0 (d) ∞	B
Q196	${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots\ldots + $ equals ______. (a) $2^n - 1$ (b) 2^n (c) $2^n + 1$ (d) None	A
Q197	Which one is true? (a) ${}^nC_r < {}^nC_{n-r}$ (b) ${}^nC_r > {}^nC_{n-r}$ (c) ${}^nC_r = {}^nC_{n-r}$ (d) ${}^nC_r \neq {}^nC_{n-r}$	C
Q198	${}^nC_r$ has a meaning only when______. (a) 0 < r < n (b) 0 < = r <=n (c) 0 < r <= n (d) 0 < = r <n	B
Q199	The value of ${}^7C_1$ is ______. (a) 1 (b) 7 (c) 6 (d) 8	B
Q200	The value of ${}^8C_3$ is ______. (a) 48 (b) 65 (c) 24 (d) 56	D
Q201	The value of ${}^9C_9$ is ______. (a) 0 (b) 9 (c) 8 (d) 1	D
Q202	The value of ${}^8C_4 + {}^5C_4$ is ______. (a) 75 (b) 24 (c) 30 (d) 27	A
Q203	${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$ is equal to ______. (a) 30 (b) 31 (c) 32 (d) 25	B
Q204	If ${}^{18}C_r = {}^{18}C_{r+2}$, the value of ${}^rC_5$ is ______. (a) 55 (b) 50 (c) 56 (d) None	C
Q205	If ${}^nC_{10} = {}^nC_{14}$, then ${}^{25}C_n$ is ______. (a) 24 (b) 25 (c) 1 (d) None	B
Q206	If ${}^nC_{18} = {}^nC_{12}$, then the value of ${}^{32}C_n$ is ______. (a) 30 (b) $(\lfloor 32 / \lfloor 6)$ (c) $[\lfloor 32 / (\lfloor 26 \times \lfloor 6)]$ (d) 496	B
Q207	Find n if $4 \times {}^nC_2 = {}^{n+2}C_3$ (a) 2,6 (b) 3,8 (c) 5,3 (d) 2,7	D
Q208	If $(n+1)c_{r-1} : n_{c1} : n-1c_{r-1} = 8:3:1$ then find the value of n? (a) 14 (b) 15 (c) 16 (d) 17	B
Q209	Find n if ${}^{n+2}C_n = 45$ (a) 12 (b) 10 (c) 8 (d) 15	C
Q210	If ${}^{18}C_n = {}^{18}C_{n+2}$ then the value of n is ______. (a) 0 (b) -2 (c) 8 (d) None	C
Q211	If ${}^nP_r = 336$ and ${}^nC_r = 56$, then n and r will be ______. (a) (3,2) (b) (8,3) (c) (7,4) (d) None	B

Q212	If $^{10}P_r$ = 6,04,800 and $^{10}C_r$ = 120; find the value of r? (a) 12 (b) 7 (c) 8 (d) 9	B
Q213	Find r if $^{12}C_5 + 2\,^{12}C_4 + ^{12}C_3 = 14C_r$ (a) 5,9 (b) 4,9 (c) 5,8 (d) 4,8	A
Q214	If $^{28}C_{2r}$: $^{24}C_{2r-4}$ = 225: 11, find r? (a) 9 (b) 6 (c) 8 (d) 7	D
Q215	If $^nC_{r-1}$ = 56, ^nC_r = 28 and $^nC_{r+1}$ = 8, then r is equal to _____. (a) 8 (b) 6 (c) 5 (d) None	B
Q216	A committee is to be formed of 3 persons out of 12. Find the number of ways of forming such Committee. (a) 220 (b) 240 (c) 36 (d) 4	A
Q217	Out of 7 gents and 4 ladies a committee of 5 is to be formed. The number of committee such that each committee includes at least one lady is _____. (a) 400 (b) 440 (c) 441 (d) None	C
Q218	5 letters are written and there are five letter-boxes. The number of ways the letters can be dropped into the boxes, one in each. (a) 119 (b) 120 (c) 121 (d) None	B
Q219	A committee of 7 members is to be chosen from 6 Chartered Accountants, 4 Economist and 6 Cost Accountants. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants. (a) 3,450 (b) 3,570 (c) 3,690 (d) 3,200	B
Q220	A committee of 3 ladies and 4 gents is to be formed out of 8 ladies and 7 gents. Mrs.X refuses to serve in a committee in which Mr.Y is a member. The number of such committees is _____. (a) 1,530 (b) 1,500 (c) 1,520 (d) 1,540	D
Q221	Out of 6 members belonging to party "A" and 4 to party "B" in how many ways a committee of 5 can be selected so that members of party "A" are in a majority? (a) 180 (b) 186 (c) 185 (d) 184	B
Q222	A person has 10 friends of which 6 of them are relatives. He wishes to invite 5 persons so that 3 of them are relatives. In how many ways he can invites? (a) 450 (b) 600 (c) 120 (d) 810	C
Q223	In how many ways 4 members can occupy 9 vacant seats in a row? (a) 3204 (b) 3024 (c) 4^9 (d) 9^4	B
Q224	The number of ways in which a person can chose one or more of the four electrical appliances: T.V, Refrigerator, Washing Machine and a cooler is _____. (a) 15 (b) 25 (c) 24 (d) None	A
Q225	A building contractor needs three helpers and ten men apply. In how many ways	A

	can these selections take place? (a) 120 ways (b) 30 ways (c) 150 ways (d) 240 ways	
Q226	A company having 6 departments wishes to simultaneously promote two of its Department's Heads to Asst. Managers. In how many ways these promotions can take place? (a) 15 ways (b) 12 ways (c) 24 ways (d) 30 ways	A
Q227	Total number of Hand shakes in a group of 10 persons to each other are_____. (a) 45 (b) 54 (c) 90 (d) 10	A
Q228	6 seats of articled clerks are vacant in a 'Chartered Accountant firm'. How many different batches of candidates can be chosen out of 10 candidates if one candidate is always selected? (a) 124 (b) 125 (c) 126 (d) None	C
Q229	In your office 4 posts have fallen vacant. In how many ways a selection out of 31 candidates can be made if one candidate is always included? (a) ${}^{30}C_3$ (b) ${}^{30}C_4$ (c) ${}^{31}C_3$ (d) ${}^{31}C_4$	A
Q230	In Q229 would your answer be different if one candidate is always excluded? (a) ${}^{30}C_3$ (b) ${}^{30}C_4$ (c) ${}^{31}C_3$ (d) ${}^{31}C_4$	B
Q231	In your college Union Election you have to choose candidates. Out of 5 candidates 3 are to be elected and you are entitled to vote for any number of candidates but not exceeding the number to be elected. In how ways it can be done? (a) 25 (b) 5 (c) 10 (d) None	A
Q232	Find the number of ways of selecting 4 letters from the word EXAMINATION. (a) 140 ways (b) 136 ways (c) 152 ways (d) 128 ways	B
Q233	Find the number of ways in which a selection of 4 letters can be made from the word "Mathematics" (a) 130 (b) 132 (c) 134 (d) 136	D
Q234	The number of different words that can be formed with 12 consonants and 5 vowels by taking 4 consonants and 3 vowels in each word is _____. (a) ${}^{12}C_4 x {}^{5}C_3$ (b) ${}^{17}C_7$ (c) 4950 x $\underline{\lvert 7!}$ (d) None	C
Q235	How many different numbers can be formed by using any three out of five digits 1, 2, 3, 4, 5, no digit being repeated in any number? (a) 60 (b) 50 (c) 40 (d) 30	A
Q236	How many different numbers can be formed by using any three out of five digits 1, 2, 3, 4, 5, no digit being repeated in any number? How many of these will begin with a specified digit? (a) 8 (b) 10 (c) 12 (d) 18	C
Q237	How many different numbers can be formed by using any three out of five digits 1, 2, 3, 4, 5, no digit being repeated in any number? How many of these will begin	C

	with a specified digit and end with another specified digit? (a) 12 (b) 6 (c) 3 (d) 18	
Q238	How many four digit numbers can be formed out of the digits 1, 2, 3, 5, 7, 8, 9, if no digit is repeated in any number? How many of these will be greater than 3,000? (a) 1,000 (b) 1,200 (c) 600 (d) 400	C
Q239	In how many ways 3 scholarships can be awarded to 5 students when each student is eligible for any of the scholarships? (a) 15 (b) 3^5 (c) 5^3 (d) ${}^5P_3$	C
Q240	You have to make choice of 7 questions out of 10 questions set you can do it in _. (a) ${}^{10}C_7$ (b) ${}^{10}P_7$ (c) $7! \times {}^{10}C_7$ (d) None	A
Q241	You have to make a choice of 4 balls out of one red one blue and ten while balls. The number of ways this can be done to always the red ball is _____. (a) ${}^{11}C_3$ (b) ${}^{10}C_3$ (c) ${}^{10}C_4$ (d) None	A
Q242	In Question No.241, the number of ways in which this can be done to include the red ball but exclude the blue ball always is _____. (a) ${}^{11}C_3$ (b) ${}^{10}C_3$ (c) ${}^{10}C_4$ (d) None	B
Q243	In Question No.241, the number of ways in which this can be done to exclude both the red and blues ball is _____. (a) ${}^{11}C_3$ (b) ${}^{10}C_3$ (c) ${}^{10}C_4$ (d) None	C
Q244	Out of 8 different balls taken three at a time without taking same three together more than once for how many number of times you can select a particular ball? (a) ${}^7C_2$ (b) ${}^8C_3$ (c) ${}^7P_2$ (d) ${}^8P_3$	A
Q245	In Question no.244, for how many number of times you can select any ball? (a) ${}^7C_2$ (b) ${}^8C_3$ (c) ${}^7P_2$ (d) ${}^8P_3$	B
Q246	The number of diagonals in a decagon is _____. (a) 30 (b) 35 (c) 45 (d) None	B
Q247	A regular Polygon has 45 diagonals then the no. of sides are _____. (a) 8 (b) 9 (c) 10 (d) 11	D
Q248	No. of ways in which 15 mangoes can be equally divided among 3 students is __ (a) $\underline{\|15}/\ \underline{\|(5)}^4$ (b) $\underline{\|15}/\ \underline{\|(5)}^3$ (c) $\underline{\|15}/\ \underline{\|(5)}^2$ (d) None	D
Q249	In a school number of students in each section is 36. If 12 new students are added, then the number of sections are increased by 4, and the number of students in each section becomes 30. The original number of sections at first is _____. (a) 6 (b) 10 (c) 14 (d) 18	D
Q250	Raj has 3 books on A/c, 3 books on Economics, 5 on Maths. If these books are to be arranged subjectwise. In how many ways can these can be placed on a shelf. (a) 25,290 (b) 25,920 (c) 4,230 (d) 4,320	B

CHAPTER 6A. ARITHMETIC PROGRESSION

INTRODUCTION

- **SEQUENCE:** A set of numbers arranged in a definite order as per a definite rule or law is called a sequence if we can find out the next unknown term.

  Ex: 1, 2, 3, 4, 5 → Sequence of consecutive natural numbers.

  Ex: -1, -27, -125.... → Sequence of cube of odd numbers in negative. [Next term will be -343]

- **SERIES:** All terms of sequence are added/subtracted, it forms a series. [**Ex:** $t_1 + t_2 + t_3 .... + t_n$]

  Ex: 1 + 3 + 5 + 7 + 9 ....

ARITHMETIC PROGRESSION (AP)

- A sequence in which 'difference between two consecutive terms' is "constant (same)".
- This constant difference is denoted by **'d'** & is called the common difference of the AP.
- First term of AP is denoted by **'a'.**

  Ex: (a) 2, 5, 8, 11, 14, 17 is an AP in which d = 3 is the common diference.

  Ex: (b) 15, 13, 11, 9, 7, 5, 3, 1, —1 is an AP in which —2 is the common difference.

CQ1. If the terms 2x, (x+10) and (3x+2) be in AP, the value of x is ____.

CONCEPT 1: ARITHMETIC MEAN

If a, b, c are in AP, then b - a = c - b; then $b = \frac{a+c}{2}$ which is called Arithmetic Mean.

CQ2. Arithmetic mean bet^n 33 & 77 = $\frac{33+77}{2}$ = 55.

CONCEPT 2: Finding nth term (Tn) of an AP

- In AP, we can find out next term of an AP if we know the first term (a) & 'd'.
- Let T_1 be a, then, $T_2 = T_1 + d = a + d$

  $T_3 = T_2 + d = (a + d) + d = a + 2d$ Substituting the value of T_2 from (i) -------- (ii)

  $T_4 = T_3 + d = (a + 2d) + d = a + 3d$; $T_5 = T_3 + d = (a + 2d) + d = a + 4d$

  T_6 = ...= **a + 5d;** T_7 = ...= **a + 6d**

  $T_n = a + (n-1)\,d$

- We can also use this formula when Sn is known $T_n = S_n - S_{n-1}$.

CONCEPT 3: COMMON DIFFERENCE 'd' OF AP

- Diff^n bet^n two consecutive terms is common difference 'd'.
- **d** = **(T_2 - T_1)** or **(T_3 - T_2)** or **(T_4 - T_3)** ……… or **(T_{n-2} - T_{n-3})** or **(T_n - T_{n-1})** →

  D = T_n - T_{n-1}

CQ3: Find the nth term of the given AP 4,7,10…… [**Ans:** 3n+1]

CONCEPT 4: GENERAL FORM OF T_n

- **General Form of T_n = An + B;** (where A & B are constants which will be given in question)

  PC Note: If you are given **T_n in An + B** format → D = Co-efficient of 'n'. **[d = A & a = (A+B)]**

CQ4: If T_n = 5n + 1, find the AP. [**Ans:** AP is 6, 11, 16, 21..,]

PC NOTE:

- If 2 non-consecutive terms in AP (say T_m & T_n) & their values are given in question & you are asked to find out AP: **D = $\frac{(T_m - T_n)}{m - n}$**

CQ9: If 5th & 12th terms of an AP are 14 & 35 respectively, find AP. [**Ans:** AP is 2, 5, 8, 11.]

CONCEPT 5: INSERTION OF 'n' ARITHMETIC MEANS BETWEEN TWO NUMBERS

- Total number of terms in the required AP will be (n+2).
- Take 1st given number as T_1 & 2nd given number as T_{n+2} & use the above given note.

CQ10: Two AMs between -7 &14 is ____.

Ans: If we insert 2 AMs between -7 & 14, total number of terms will be 4. → -7, AM_1, AM_2, 14.

Take T_1= -7; & T_{2+2} = 14; Thus T_4 = 14. Now we will use the above note.

(4-1) d = 14 — (-7) → 3d = 21 → **d = 7.**

Now, AM_1 which is 2nd term of AP can be calculated using Tn formula;

T_2 = a + d = -7 + 7 = 0 & AM_2 which will be 3rd term of AP; T3 = a + 2d = -7 + 2(7) = 7.

So, the two arithmetic means between -7 & 14 are **0 & 7.**

CQ11: Insert 4 arithmetic means between 4 & 324. [**Ans:** 68, 132, 196, 260]

CONCEPT 6: SUM OF FIRST 'N' TERM OF AP

$S_n = \frac{n}{2} \times (T_1 + T_n)$ (T_n= Last term & T_1= 1st term & n = No. of terms) → Used when T_1 & T_n are given

$S_n = \frac{n}{2} \times [T_1 + a + (n-1)d]$ → By substituting value of T_n = a + (n-1) d in above formula & T_1 = a.

$S_n = \frac{n}{2} \times [2a + (n-1) d]$ → Used when T_1, d & n are given in the question

CQ11: The sum of the series 9, 5, 1.... to 100 terms is ______.

Ans: n = 100, a= 9, d = - 4; Sn = $\frac{n}{2}$ × [2a + (n-1) d]; → Sn = $\frac{100}{2}$ × [(2(9) +(100—1) (-4)] → **Sn = -18900.**

CQ12. Find Sn of the given AP 4, 8, 12, 16......

Ans: Sn = $\frac{n}{2}$ × [2a + (n-1) d] = $\frac{n}{2}$ × [2.4 + (n-1)4] = $\frac{n}{2}$ × [8 + 4n - 4] = $\frac{n}{2}$ × [4n + 4] = $\frac{n}{2}$ ×2[2n+2] = **$2n^2 + 2n$**

CONCEPT 7: GENERAL FORM OF S_n

General Form of $S_n = An^2 + Bn$; (where A & B are constants)

PC Note: If you are given S_n in $An^2 + Bn$ format → d = 2A & (a) = (A+B)

CQ13. The sum of n terms of an AP is $3n^2 + 5n$. Find the series. **[Ans:** AP is 8, 14, 20, 26....,**]**

CONCEPT 8: ASSUMPTIONS OF THE TERMS IN AP

If No. of terms given in question are	Middle Term	Common Difference	Examples of Terms
ODD No. of terms	a	D	**3 terms:** (a-d), a, (a+d); **5 terms:** (a-2d), (a-d), a, (a+d), (a+2d)
EVEN No. of terms	(a — d) & (a + d)	2d	**2 terms:** (a-d) & (a+d); **4 terms:** (a-3d), (a-d), (a+d), (a+3d)

CQ14. 3 numbers are in A.P. whose sum is 69 and the product of first two is 483. Numbers are

(a) 25, 23, 21 (b) 21, 23, 25 (c) 19, 22, 25 (d) None

Ans: Since the number of terms given in the question are 3 (ODD), we assume 3 numbers as:

(a-d), a, (a+d); Thus (a-d) + a + (a+d) = 69. 3a = 69. **a = 23.**

(a-d) × a = 483; (23 — d) = 483/23; (23 — d) = 21 & **d = 2.**

Numbers are (23—2), 23, (23+2) = **21, 23, 25**

PC NOTE: But we will go by OPTION METHOD in such type of questions TO SAVE TIME.

CONCEPT 9: SOME IMPORTANT SERIES

SUM OF	FORMULA	EXAMPLE
1. 1st 'n' **NATURAL** No.	$\sum n = \frac{n(n+1)}{2}$	$1 + 2 + 3 + ...100 = \frac{n(n+1)}{2} = \frac{100(100+1)}{2}$
2. 1st 'n' **ODD** natural No.	$\sum(2n-1) = n^2$	$1 + 3 + 5 + 7 + 9 = 5^2 = 25$
3. 1st 'n' **EVEN** Natural No.	$\sum 2n = n(n+1)$	$2 + 4 + 6 + 8 + 10 = n(n+1) = 5(6) = 30$
4. **SQUARE** of 1st 'n' Natural No.	$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$	$1^2+2^2+...100^2 = \frac{n(n+1)(2n+1)}{6} = \frac{100(100+1)(200+1)}{6}$
5. **CUBES** of 1st 'n' Natural No.	$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$	$1^3+2^3+3^3...100^3 = \left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{100(100+1)}{2}\right]^2$

PROPERTIES OF AP

Particulars	Examples
1. If $S_n = S_m \to S_{(m+n)} = 0$	If $S_7 = S_{11} \to S_{18} = 0$
2. $T_p = \frac{1}{q}$ & $T_q = \frac{1}{p}$; $\to$ $T_{pq} = 1$ & $S_{pq} = \frac{pq+1}{2}$	$T_3 = \frac{1}{2}$ & $T_2 = \frac{1}{3}$; $\to$ $T_6 = 1$ & $S_6 = \frac{6+1}{2} = \frac{7}{2}$
3. If $S_p = q$ & $S_q = p \to S_{(p+q)} = -(p+q)$	If $S_7 = 11$ & $S_{11} = 7$, $\to S_{18} = -(11+7) = -18$
4. If $T_p = q$ & $T_q = p$; then $T_r = (p + q - r)$	5. If $T_p = q$ & $T_q = p$; then $T_{(p+q)} = 0$.
6. If ratio of S_n of 2 APs = $\frac{An^2+Bn}{Cn^2+Dn} = \frac{An+B}{Cn+D}$; Ratio of their $T_m = \frac{A(m-1)+B}{C(m-1)+D}$. **Q.** Sum of 'n' terms of 2 APs are in the ratio of $\frac{(5n+2)}{(11n-7)}$. Ratio of their sixth terms is ____.	
7. We add/subtract/multiply/divide all terms of AP by any no. resulting series is AP.	
8. If we form a series from the reciprocal of all the terms of AP, it becomes HP.	
9. If 3 numbers are given in AP, Put 1st no = 1; 2nd no = 2; & 3rd no. = 3; (If necessary).	
10. If a, b, c are in AP → Put their value as 1, 2, 3 in options & get the answer.	
11. If a^2, b^2, c^2 are in AP → Put value as 1, 5, 7 in options & get answer [1,25,49 → AP]	

ARITHMETIC PROGRESSION – QUESTION BANK

SN	6A. ARTHMETIC PROGRESSION	Ans
Q1	Two APs have the same common difference. If the difference between their 100th terms is 111222333, then the difference between their millionth terms is ______. (a) 123 (b) 112233 (c) 111222333 (d) 112333	C
Q2	n^{th} term of the sequence 2, 4, 6, 8 ..... is ______. (a) 2n (b) 2n-1 (c) 2n + 1 (d) N	A
Q3	Number of terms in the series 1+ 3 +5 +7 +....+ 61 is ______. (a) 30 (b) 28 (c) 31 (d) 29	C
Q4	If 1^{st} term of an AP is 5 & its 100^{th} term is -292, then its 51^{st} term is ______. (a) -142 (b) -149 (c) 155 (d) -145	D
Q5	In a certain arithmetic sequence, if the 24th term is twice the 10^{th} term, then 72^{nd} term is twice the ______. (a) 30^{th} term (b) 40^{th} term (c) 34^{th} term (d) 38^{th} term	C
Q6	If 10th term of an A.P. is twice the 4th term & 23rd term is 'k' times the 8th term, then k = ______. (a) 2.5 (b) 3 (c) 3.5 (d) 4	A
Q7	The two arithmetic means between -6 and 14 is ______. (a) 2/3, 1/3 (b) 2/3, 22/3 (c) -2/3, -22/3 (d) None	B
Q8	The sum of the series $3\frac{1}{2}+7+10\frac{1}{2}+14+....$ to 17 terms is ______. (a) 530 (b) 535 (c) $535\frac{1}{2}$ (d) None	C
Q9	The sum of an A.P. whose first term is - 4 and the last term is 146 is 7171. Find the Value of n. (a) 99 (b) 101 (c) 100 (d) 102	B
Q10	The number of the terms of the series 10+9 $3\frac{2}{3}$ + $9\frac{1}{3}$ + 9.... will amount to 155 is ___. (a) 30 (b) 31 (c) 32 (d) None	D
Q11	a = 14 & sum of first 5 terms & sum of first 10 terms are equal is magnitude but opposite in sign. T_3 is ______. (a) 70/11 (b) 6 (c) 4/11 (d) None	A
Q12	The sum of progression (a+b), a, (a-b) upto n terms is ______. (a) $\frac{n}{2}$[2a + (n-1) b] (b) $\frac{n}{2}$[2a + (3 - n) b] (c) $\frac{n}{2}$[2a + (3 - n)] (d) $\frac{n}{2}$[2a (n -1)]	B
Q13	The maximum sum of the AP series 40, 36, 32, 28 ..... is ______. [Hint: 2 × 10 × 11] (a) 220 (b) 225 (c) 232 (d) 320	A
Q14	How many terms are there in the AP whose 1^{st} & 5^{th} are -14 & 2 respectively & sum	B

	of the term is 40? (a) 2 x d (b) 10 (c) 8 (d) 14	
Q15	P^{th} term of an AP is $\frac{3p-1}{6}$. The sum of the first n terms of the AP is ______. (a) n(3n+1) (b) $\frac{n}{12}$ (3n+1) (c) $\frac{n}{12}$ (3n-1) (d) None	B
Q16	Find the sum of first 25 terms of AP series whose n^{th} term is (n/5) + 2 (a) 105 (b) 115 (c) 125 (d) 135	B
Q17	The sum of n terms of an AP is $2n^2$ +3n. Find the n^{th} term. (a) 4n+1 (b) 4n-1 (c) 2n+1 (d) 2n-1	A
Q18	Sum of all natural numbers from 100 to 300 which are divisible by 4 or 5 is ______. (a) 10200 (b) 15200 (c) 16200 (d) None	A
Q19	The sum of all natural numbers from 100 to 300 which are divisible by 5 is ______. (a) 10200 (b) 30000 (c) 8200 (d) 2200	C
Q20	Sum of all natural numbers from 100 to 300 which are divisible by 4 and 5 is ______. (a) 10200 (b) 30000 (c) 8200 (d) 2200	D
Q21	The sum of natural numbers upto 200 excluding those divisible by 5 is ______. (a) 20100 (b) 4100 (c) 16000 (d) None	C
Q22	Find three numbers in AP whose sum is 6 and the product is -24 (a) -2, 2, 6 (b) -1, 1, 3 (c) 1, 3, 5 (d) 1, 4, 7	A
Q23	The four numbers in AP whose sum is 24 and their product is 945 are ______. (a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None	A
Q24	4 numbers in AP with the sum of 2^{nd} & 3^{rd} being 22 and the product of 1^{st} & 4^{th} being 85 are ______. (a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None	C
Q25	Divide 12.50 in 5 parts in AP such that the first part and the last part are in the ratio 2:3 (a) 2, 2.25, 2.5, 2.75, 3 (b) -2, -2.25, -2.5, -2.75, -3 (c) 4, 4.5, 5, 5.5, 6 (d) -4, - 4.5, -5, -5.5, -6	A
Q26	Find four numbers in AP with the sum of 2^{nd} & 3^{rd} is 22 & product of 1^{st} & 4^{th} is 85. (a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None.	C
Q27	The sum of the series 1 + 2 + 3 + 4 +....... + 100 is ______. (a) $\frac{100(101)}{2}$ (b) $\left[\frac{100(101)}{2}\right]2$ (c) 100 x 101 (d) None	A
Q28	The value of $11^2+12^2+......+20^2$ is ______. (a) 3845 (b) 2485 (c) 2870 (d) 3255	B
Q29	The value of $\frac{1^3+2^3+\cdots+10^3}{1+2+\cdots+10}$ is........ (a) 45 (b) 55 (c) 385 (d) 285	B

Q30	If a, b, c are in AP then the value of $\frac{(a^3+4b^3+c^3)}{b(a^2+c^2)}$ (a) 1 (b) 2 (c) 3 (d) None	C
Q31	If a, b, c are in AP then (b+c), (c+a), (a+b) are in ______. (a) AP (b) GP (c) HP (d) None	A
Q32	If a, b, c are in the p^{th}, q^{th} and r^{th} terms of an AP, value of a(q-r) +b(r-p) +c(p-q) is (a) 0 (b) 1 (c) -1 (d) None	A
Q33	If a^2, b^2, c^2 are in AP then (b+c), (c+a), (a+b) are in ______. (a) AP (b) GP (c) HP (d) None	C
Q34	A person pays Rs.975 by monthly instalment each less then the former by Rs.5. The first instalment is Rs. 100. Time by which the entire amount will be paid is __. (a) 10 months (b) 15 months (c) 14 months (d) None	B
Q35	If n^{th} terms of two A.P's are in the ratio (3n+1):(n+4) the ratio of fourth term is __. (a) 2 (b) 3 (c) 4 (d) None	A
Q36	10^{th} term from the end of the AP 4,9,14,.... 254. (a) 204 (b) -209 (c) 209 (d) 214	C
Q37	Find the sum to *n* terms of (1 - 1/n) + (1 - 2/n) + (1 - 3/n) +...... (a) ½(*n*-1) (b) ½(*n*+1) (c) (*n*-1) (d) (*n*+1)	A
Q38	Sum of *n* terms of $(x + y)^2$, $(x^2 + y^2)$, $(x - y)^2$,is ______. (a) $(x + y)^2 - 2(n-1)xy$ (b) $n(x + y)^2 - n(n-1)xy$ (c) both the above (d) None	B
Q39	Sum of n terms of (1/n) (n-1), (1/n) (n-2), (1/n) (n-3) is ______. (a) 0 (b) (1/2) (n-1) (c) (1/2) (n+1) (d) None	B
Q40	Value of n^2 + 2n [1+2+3+ .....+(n-1)] is ______. (a) n^3 (b) n^2 (c) n (d) None	A
Q41	Which term of series 7+11+15 .... = 403. (a) 50 (b) 100 (c) 101 (d) 51	B
Q42	The sum 1+3+5+7+.... +99 is equal to ______. (a) 2499 (b) 2401 (c) 9801 (d) None	D
Q43	If Sn the sum of first n terms in a series is given by $2n^2$+3n the series is in ______. (a) AP (b) GP (c) HP (d) None	A
Q44	n^{th} term of the series whose sum to n terms is $5n^2$+2n is ______. (a) 3n - 10 (b) 10n - 2 (c) 10n - 3 (d) None	C
Q45	t_1 = n, t_2 = n + 1, t_3 = n + 2 and so on, then t_n= ______. (a) n (b) 2n - 1 (c) 2n + 1 (d) 2n	**B**
Q46	Sum of all natural numbers between 200 and 400 which are divisible by 7 is____.	B

	(a) 7730 (b) 8729 (c) 7729 (d) 8730	
Q47	Sum of all natural numbers between 500 & 1000 which are divisible by 13 is__ (a) 28400 (b) 28405 (c) 28410 (d) None	A
Q48	Number of numbers between 74 and 25556 divisible by 5 is ______. (a) 5090 (b) 5097 (c) 5095 (d) None	B
Q49	Sum $1^2 + 2^2 + 3^2 + 4^2 + ...10^2$ is equal to ______. (a) 385 (b) 386 (c) 384 (d) None	A
Q50	Sum of $1^3 + 2^3 + 3^3 + 4^3 + ... 10^3$ is equal to ______. (a) 4410 (b) 3025 (c) 3470 (d) None	B
Q51	Sum of *n* terms of the series 2 + 6 + 10 + ... is ______. (a) $2n^2$ (b) n^2 (c) $n^2/2$ (d) $4n^2$	A
Q52	Unity is added to sum of any number of terms of the AP 3,5,7,9,... resulting sum is __ (a) 'a' perfect cube (b)'a' perfect square (c) 'a' number (d) None	B
Q53	Find the no. which should be added to the sum of any number of terms of AP so that resultant is also AP (a) -1 (b) 0 (c) 1 (d) None	C
Q54	If a, b, c d are in AP then _____. (a) $a^2 - 3b^2 - 3c^2 - d^2 = 0$ (b) $a^2 + 3b^2 + 3c^2 + d^2 = 0$ (c) $a^2 + 3b^2 + 3c^2 - d^2 = 0$ (d) None	D
Q55	If a, b, c be the sums of p, q, r terms respectively of an AP, the value of $(\frac{a}{p})(q-r) + (\frac{b}{q})(r-p) + (\frac{c}{r})(p-q)$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q56	If a, b, c, d, e are in AP then _____. (a) a - b - d + e = 0 (b) a - 2c + e = 0 (c) b - 2c + d = 0 (d) All	D
Q57	A person saved Rs. 16,500 in 10 years. In each year after the first year he saved Rs. 100 more than he did in the preceding year. The amount of money he saved in the 1st year was _____. (a) Rs. 1000 (b) Rs. 1500 (c) Rs. 1200 (d) none	C
Q58	The sum of *n* terms of a+b, 2a, 3a-b....is ______. (a) n(a-b) +2b (b) n (a+b) (c) both the above (d) None	D
Q59	A sum of Rs. 6240 is paid off in 30 installments such that each installment is Rs. 10 more than the preceding installment. The value of the 1st installment is ______. (a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) None	D
Q60	2, 5, 8, 11, 14, 17... is an A.P in which the common difference is ___.	B

	(a) 2 (b) 3 (c) -2 (d) -3	
Q61	Determine the common difference of progression 16, 13, 10... 25 terms (a) 2 (b) -2 (c) 3 (d) -3	D
Q62	If a, b, c are in A.P., then 2b = ____ (a) a - c (b) a + c (c) $\frac{a+c}{2}$ (d) $\frac{a-c}{2}$	B
Q63	If the terms 2x, (x+10) and (3x+2) be in AP, the value of x is _____. (a) 7 (b) 10 (c) 6 (d) None	C
Q64	The value of x such that 8x+4,6x-2,2x+7 will form an A.P. is _____. (a) 15 (b) 2 (c) $\frac{15}{2}$ (d) None	C
Q65	Find the 7^{th} term of the A.P 8, 5, 2, -1, -4,......... (a) -13 (b) -10 (c) -7 (d) -16	B
Q66	20^{th} term of the progression 1, 4, 7, 10 .... is _____. (a) 58 (b) 52 (c) 0 (d) None	A
Q67	For A.P 2, 5, 8, 11, 14, ..., 12th term is _____. (a) 34 (b) 33 (c) 35 (d) 36	C
Q68	13th term of series 93, 90, 87 .... is _____. (a) 57 (b) -54 (c) 50 (d) 54	A
Q69	n^{th} element of the sequence 1,3,5,7,.... is _____. (a) n (b) 2n -1 (c) 2n +1 (d) None	B
Q70	n^{th} term of the sequence 2, 4, 6, 8 .... is _____. (a) 2n (b) 2n-1 (c) 2n + 1 (d) N	A
Q71	m^{th} term of an A.P is n and n^{th} term is m, the r^{th} term of it is _____. (a) m + r + r (b) n + m - 2r (c) m - 2r (d) m + n - r	D
Q72	If the p^{th} term of an AP is q and q^{th} term is p, the value of the $(p+q)^{th}$ term is _____. (a) 0 (b) 1 (c) -1 (d) None	A
Q73	If the 5^{th} and 12^{th} terms of the A.P are 14 and 35 respectively, find the A.P. (a) -2, 2, 6, 10, 14,... (b) -10, -4, 2, 8, 14,... (c) 6, 8, 10, 12, 14, ... (d) 2, 5, 8, 11, 14,...	D
Q74	Which term of the A. P 11, 8, 5,2 ,... is -10? (a) 10^{th} (b) 8^{th} (c) 12^{th} (d) 14^{th}	B
Q75	Which term of the progression -1, -3, -5,.... is -39? (a) 21^{st} (b) 20^{th} (c) 19^{th} (d) None	B
Q76	Which term of the A.P $\frac{3}{\sqrt{7}}, \frac{4}{\sqrt{7}}, \frac{5}{\sqrt{7}}$, ... is $\frac{17}{\sqrt{7}}$, ? (a) 13 (b) 14 (c) 15 (d) 16	C

Q77	The last term of the series 5,7,9, ... to 21 term is _____. (a) 44 (b) 43 (c) 45 (d) None	C
Q78	The last term of the A.P 0.6,1.2,1.8 to 13 term is (a) 8.7 (b) 7.8 (c) 7.7 (d) None	B
Q79	Determine the first term of an A.P. with common difference 3 & 7th term being 11 (a) -7 (b) 7 (c) 6 (d) 5	A
Q80	If the 10^{th} term of an A.P. is twice the 4^{th} term, and the 23^{rd} term is 'k' times the 8^{th} term, then the value of 'k' is (a) 2.5 (b) 3 (c) 3.5 (d) 4	A
Q81	The sum of__________ between the actual values and the A.M is zero. (a) sums (b) differences (c) product (d) square root	B
Q82	AM between a & c is _____. (a) ac (b) $\frac{(a+c)}{2}$ (c) $\frac{ac}{2}$ (d) $\frac{(a-c)}{2}$	B
Q83	A. M between 2 & 4 is (a) 2 (b) 4 (c) 3 (d) 6	C
Q84	AM between 8 & 20 is (a) 6 (b) 12 (c) 14 (d) 18	C
Q85	AM between 5 and 13 is (a) 9 (b) 10 (c) 8 (d) None	A
Q86	AM between 33 and 77 is (a) 50 (b) 45 (c) 55 (d) None	C
Q87	4 arithmetic means between -2 and 23 are (a) 3,13,8,18 (b) 18,3,8,13 (c) 3,8,13,18 (d) None	C
Q88	If the AM of two numbers is 6 and GM is 6 then find the numbers. (a) 6,6 (b) 10,8 (c) 10,6 (d) 9, 2	A
Q89	Find the numbers whose GM is 5 and AM is 7.5. (a) 12 and 13 (b) 13.09 and 1.91 (c) 14 and 11 (d) 17 and 19	B
Q90	Between the two numbers whose sum is $\frac{13}{6}$, an even number of A.M is inserted. If the sum of arithmetic mean exceeds their number by unity, then number of arithmetic means inserted are (a) 6 (b) 10 (c) 8 (d) 12	D
Q91	Three numbers a,b,c are in A.P, Find a-b+ c (a) a (b) -b (c) b (d) c	C
Q92	In an A.P. if the 3rd term is 18, 7 term is 30 then the sum of first 20 terms is _____. (a) 810 (b) 520 (c) 180 (d) 250	A

Q93	2^{nd} term of A.P. is a_2, its common difference is'd'. Sum of its first 'n' terms = _____. (a) $\frac{n}{2}$ [$2a_{2+}$(n-1) d] (b) $\frac{n}{2}$ [$2a_{1+}$(n-1) d] (c) $\frac{n}{2}$ [$2a_2$ +(n-3) d] (d) $\frac{n}{2}$ [a_{2+}(n-1) d]	C
Q94	The sum of the series 1+2+4+8+ .... to 10 term is _____. (a) 1024 (b) 1023 (c) 1025 (d) None	B
Q95	The sum of series 8, 4, 0 .... to 50 terms is _____. (a) 18900 (b) 9000 (c) -4500 (d) None	C
Q96	The sum of all numbers between 200 and 300 (a) 11,600 (b) 12,490 (c) 12,500 (d) 24,750	D
Q97	The sum 1+2+3+4.......... +70 is equal to _____. (a) 2484 (b) 2485 (c) 2845 (d) None	B
Q98	The sum of series 8, 4, 0 ...... to 50 terms is _____. (a) 18900 (b) 9000 (c) -4500 (d) None	C
Q99	In an A.P. if S_n = $3n^2$ - n & its common difference is '6', then the First term is _____. (a) 2 (b) 3 (c) 4 (d) 6	A
Q100	The sum of $\frac{1}{(x+y)}$ and $\frac{1}{(x-y)}$ is _____. (a) $\frac{2y}{(x^2-y^2)}$ (b) $\frac{2x}{(x^2-y^2)}$ (c) $\frac{2y}{(x^2+y^2)}$ (d) $-\frac{2x}{(x^2-y^2)}$	B
Q101	$\frac{a^2}{a^2-b^2}+\frac{b^2}{b^2-a^2}=$ _____. (a) a - b (b) a + b (c) a^2 - b^2 (d) 1	D
Q102	8^{th} term of the progression 8, 5, 2, -1, -4, ... is ____. (a) -12 (b) -13 (c) 13 (d) 12	B
Q103	Sum of a series in AP is 72 the first term being 17 and the common difference -2. Number of terms is _____. (a) 6 (b) 12 (c) 6 or 12 (d) None	C
Q104	Number of terms of series needed for sum of the series 50 + 45 + 40 +....... becomes zero (a) 22 (b) 21 (c) 20 (d) None	B
Q105	Sum of certain numbers of terms of an AP series -6, -3, 0...... is 225. Number of terms is _____. (a) 16 (b) 15 (c) 14 (d) 13	B
Q106	The number of terms in the A.P. 7, 13, 19,........ 97 is _____. (a) 97 (b) 17 (c) 16 (d) 15	C
Q107	The sum of all natural numbers from 100 to 300 which are divisible by 4 is _____. (a) 10200 (b) 30000 (c) 8200 (d) 2200	A

Q108	Sum of n terms of 2 APs are in the ratio of $\frac{7n-5}{5n+17}$. Then ___ term of the two series are equal (a) 12 (b) 6 (c) 3 (d) None	B
Q109	The sum of the first 100 terms common to the series 17, 21, 25 .... And 16, 21, 26,... is _____. (a) 202200 (b) 100101 (c) 101010 (d) 101100	D
Q110	If the p^{th} term of an AP is q & the q^{th} term is p the value of the rth terms is _____. (a) p-q-r (b) p + q-r (c) p + q + r (d) None	B
Q111	The p^{th} term of an AP is $\frac{1}{q}$ and the q^{th} term is $\frac{1}{p}$. The sum of the pq term is _____. (a) $\frac{1}{2}$ (pq+1) (b) $\frac{1}{2}$ (pq-1) (c) (pq+1) (d) (pq-1)	A
Q112	Sum of p terms of an AP is q and the sum of q terms is p. The sum of p+q terms is (a) -(P+q) (b) (P+q) (c) $(p-q)^2$ (d) P^2-q^2	A
Q113	If S_1, S_2, S_3 be respectively, sum of n, 2n, 3n terms of an AP the value of $S_3 \div (S_2-S_1)$ is _____. (a) 1 (b) 2 (c) 3 (d) None	C
Q114	If S_1, S_2, S_3 be the sums of n terms of three APs the first term of each being unity and the respective common differences 1, 2, 3 then $\frac{(S_1+S_3)}{S_2}$ is _____. (a) 1 (b) 2 (c) -1 (d) None	B
Q115	Sum of 'n' terms of two A.Ps are in the ratio of $\frac{(5n+2)}{(11n-7)}$ the ratio of their sixth terms is ____. (a) 32:59 (b) 1:1 (c) 2:1 (d) 5:11	D
Q116	If m, p, q are consecutive terms in an A.P. then p is _____. (a) $\frac{mq}{2}$ (b) $\frac{(m-q)}{2}$ (c) $2(m^2 + q^2)$ (d) $\frac{(m+q)}{2}$	D
Q117	The five numbers in AP with their sum 25 and sum of their squares 135 are _____. (a) 3, 4, 5, 6, 7 (b) 3, 3.5, 4, 4.5, 5 (c) -3, -4, -5, -6, -7 (d) -2, -3.5, -4, -4.5, -5	A
Q118	Three numbers are in A.P. whose sum is 69 and the product of first two is 483. Numbers are _____. (a) 25, 23, 21 (b) 21,23,25 (c) 19, 22, 25 (d) None	B
Q119	Three numbers are in A.P. of whose sum is 15 and whose product is 105, then numbers are _____. (a) 3,5,7 (b) 2, 5, 8 (c) 0, 5, 10 (d) None	A
Q120	Three number in AP whose sum is 27 and the sum of their squares is 341 are _____. (a) 2, 9, 16 (b) 16, 9, 2 (c) Both (a) and (b) (d) -2, -9, -16	C
Q121	Four numbers in AP whose sum is 20 and the sum of their squares is 120 are _____.	B

	(a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None	
Q122	Divide 69 into 3 parts which are in A.P and are such that product of the 1st two parts is 483. (a) 21,23,25 (b) 23,25,27 (c) 19,21,23 (d) 17,19,21	A
Q123	Sum of 3 numbers in A.P. is 12 and the sum of their cube is 408. Numbers are _____. (a) 3,4,5 (b) 1,4,7 (c) 2,4,6 (d) None	B
Q124	Five numbers in AP with the sum 20 and product of the first and last 15 are _____. (a) 3, 4, 5, 6, 7 (b) 3, 3.5, 4, 4.5, 5 (c) -3, -4, -5, -6, -7 (d) -2, -3.5, -4, -4.5, -5	B
Q125	If sum of first 50 natural numbers is 1275 and the sum of first 50 odd numbers is 2500, then the sum of the first 50 even numbers is _____. (a) 2550 (b) 1275 (c) 1725 (d) 2500	A
Q126	Sum of three integers in AP is 15 and their product is 80. the integers are _____. (a) 2,5,8 (b) 8,5,2 (c) 2,8,5 (d) Both (a) and (b)	D
Q127	Sum of all natural no. from 100 to 300 which are exactly divisible by 4 or 5 is _____. (a) 10200 (b) 15200 (c) 16200 (d) None	C
Q128	In an Ashoka Chakra, central angle made by the smallest sector, two small sectors, three small sectors and so on are _____. (a) In A.P. (b) Equal (c) In G.P. (d) Such that their summation is 360°	A
Q129	A person employed in a company at Rs. 3000 per month and he would get an increase of Rs. 100 per year. Find the total amount which he receives in 25 years and the monthly salary in the last year. (a) 1380000 and 6200 (b) 930000 and 5400 (c) 1480000 and 7200 (d) 1570000 and 4800	B
Q130	A person received the salary for the 1st Year is Rs. 5,00,000 per year and he received an increment of Rs. 15,000 per year then the sum of the salary he taken in 10 years (a) Rs. 56,75,000 (b) Rs. 72,75,000 (c) Rs. 63,75,000 (d) None	A
Q131	The sum of n terms of an AP is $3n^2 + 5n$, which term of AP is 164. (a) 25 (b) 27 (c) 29 (d) 31	B
Q132	Sum of n terms of $(x+y)^2$, (x^2+y^2), $(x-y)^2$... (a) $(x + y)^2 - 2(n - 1)xy$ (b) $n(x + y)^2 - n(n - 1)xy$ (c) $n(x + y)^2 - n(n + 1)xy$ (d) None	B
Q133	Sum of n terms of (n - 1)/n, (n - 2)/n, (n - 3)/n.... is ______. (a) 0 (b) (n-1)/2 (c) (n+1)/2 (d) None	B
Q134	The sum of first n natural number is______.	A

	(a) $(\frac{n}{2})(n+1)$ (b) $(\frac{n}{6})(n+1)(2n+1)$ (c) $[(\frac{n}{2})(n+1)]^2$ (d) None	
Q135	The sum of square of first n natural number is______. (a) $(\frac{n}{2})$ (n+1) (b) $(\frac{n}{2})$ (n+1) (2n+1) (c) $[(\frac{n}{2})(n+1)^2]$ (d) None	B
Q136	The sum of cubes of first n natural number is______. (a) $(\frac{n}{2})$ (n+1) (b)$(\frac{n}{6})$(n+1) (2n+1) (c) $[(\frac{n}{2})(n+1)]^2$ (d) None	C
Q137	The sum of first 'n' odd number is______. (a) $\frac{n(n+1)}{2}$ (b) n^2 (c) $\frac{n}{2}$ (d) $\frac{n(n-1)}{2}$	B
Q138	The sum of n terms of an AP is $2n^2 + 3n$. Find the nth term? (a) 4n+1 (b) 4n-1 (c) 2n+1 (d) 2n-1	A
Q139	The first three terms of sequence when nth term T_n is n^2-2n are______. (a) -1, 0, 3 (b) 1, 0, 2 (c) -1, 0, -3 (d) None	A
Q140	If Sn the sum of first n terms in a series is given by $2n^2+3n$ the series is in______. (a) AP (b) GP (c) HP (d) None	A
Q141	n^{th} term of the series whose sum to n terms is $5n^2+2n$ is ______. (a) 3n - 10 (b) 10n - 2 (c) 10n - 3 (d) None	C
Q142	$t_1 = n$, $t_2 = n + 1$, $t_3 = n + 2$ and so on, then t_n= ______. (a) n (b) 2n - 1 (c) 2n + 1 (d) 2n	B
Q143	A sum of Rs. 6240 is paid off in 30 installments such that each installment is Rs. 10 more than the preceding installment. The value of the 1st installment is ______. (a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) None	D
Q144	If a, b, c, d, e are in AP then (a) a - b - d + e = 0 (b) a - 2c ______.+ e = 0 (c) b - 2c + d = 0 (d) All	D
Q145	The sum of *n* terms of a+b, 2a, 3a-b....is ______. (a) n(a-b) +2b (b) n(a+b) (c) both the above (d) None	D
Q146	Find the sum to *n* terms of (1-1/n) + (1-2/n) + (1-3/n) +...... (a) ½(n-1) (b) ½(n+1) (c) (n-1) (d) (n+1)	A
Q147	Value of n^2 + 2n [1+2+3+ .....+(n-1)] is ______. (a) n^3 (b) n^2 (c) n (d) None	A
Q148	Which term of series 7+11+15 .... = 403. (a) 50 (b) 100 (c) 101 (d) 51	B
Q149	Sum 1+3+5+7+.... +99 is equal to ______. (a) 2499 (b) 2401 (c) 9801 (d) 2500	D
Q150	Sum of all natural numbers between 200 and 400 which are divisible by 7 is____. (a) 7730 (b) 8729 (c) 7729 (d) 8730	B

Q151	Sum $1^2 + 2^2 + 3^2 + 4^2 + ...10^2$ is equal to ______. (a) 385 (b) 386 (c) 384 (d) None	A
Q152	Sum of $1^3 + 2^3 + 3^3 + 4^3 + ... 10^3$ is equal to ______. (a) 4410 (b) 3025 (c) 3470 (d) None	B
Q153	Sum of *n* terms of the series 2 + 6 + 10 + ... is ______. (a) $2n^2$ (b) n^2 (c) $n^2/2$ (d) $4n^2$	A
Q154	If a, b, c d are in AP then _____. (a) $a^2 - 3b^2 - 3c^2 - d^2 = 0$ (b) $a^2 + 3b^2 + 3c^2 + d^2 = 0$ (c) $a^2 + 3b^2 + 3c^2 - d^2 = 0$ (d) $a^2 - 3b^2 + 3c^2 - d^2 = 0$	D
Q155	If a, b, c be the sums of p, q, r terms respectively of an AP, the value of $(\frac{a}{p})(q-r) + (\frac{b}{q})(r-p) + (\frac{c}{r})(p-q)$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q156	If 10^{th} term of AP is twice the 4^{th} term & 23^{rd} term is 'k' times the 8^{th} term, k = __. (a) 2.5 (b) 3 (c) 3.5 (d) 4	A
Q157	Value of $11^2+12^2+......+20^2$ = _____. (a) 3845 (b) 2485 (c) 2870 (d) 3255	B
Q158	Value of $\frac{1^3+2^3+\cdots+10^3}{1+2+\cdots+10}$ = _____. (a) 45 (b) 55 (c) 385 (d) 285	C
Q159	If a, b, c are in AP, then value of $\frac{(a^3+4b^3+c^3)}{b(a^2+c^2)}$ = _____. (a) 1 (b) 2 (c) 3 (d) None	C
Q160	If a, b, c are in AP then (b+c), (c+a), (a+b) are in ______. (a) AP (b) GP (c) HP (d) None	A
Q161	If a, b, c are in p^{th}, q^{th} & r^{th} terms of an AP, value of a(q-r) +b(r-p) +c(p-q) is _____. (a) 0 (b) 1 (c) -1 (d) None	A
Q162	If a^2, b^2, c^2 are in AP then (b+c), (c+a), (a+b) are in ______. (a) AP (b) GP (c) HP (d) None	C

CHAPTER 6B. GEOMETRIC PROGRESSION

INTRODUCTION

- It is a sequence in which 'any term divided by its preceding term' is "same/constant".
- Ratio between two consecutive terms of the series is "constant". Such Ratio is known as Common Ratio & is denoted by **'r'.**
- First term of GP is denoted by **'a'.**
- $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$ --------- $\frac{T_n}{T_{n-1}}$
- Common Ratio of GP **'r' =** $\frac{T_n}{T_{n-1}}$

CONCEPT 1: Finding n^{th} TERM OF GP

- If a = 5 & r = 2
- $T_1 = a$; $T_2 = a.r$;

  $T_3 = T_2.\ r = ar.r = ar^2$; $T_4 = T_3.\ r = r^2r = ar^3$;
- $T_n = a.r^{n-1}$

CQ1: Find the 8^{th} term of series 4, 8, 16 ...... is [**Ans:** 512]

CQ2: 10^{th} term of the G.P. $\frac{1}{2}$, 1, 2, 22, ... is [**Ans:** 256]

CQ3: The last term of the series x^2, x, 1, ... to 31 terms is [**Ans:** $\frac{1}{x^{28}}$]

CQ4: Which term of the G.P. series ¼, -1/2, 1....is -128?

CQ5: The number of terms in 6,18,54, ... upto 1458 is _____.

CQ6: Which term of series 3, $\sqrt{3}$, 1, $\frac{1}{\sqrt{3}}$....is $\frac{1}{243}$?

CONCEPT 2: GEOMETRIC MEAN

- If a, b, c are in GP, b/a = c/b = $b^2 = ac$, b is called GM between a & c.

CQ7: If (k+9), (k-6) & 4 forms three consecutive terms of a G.P, then the value of 'k' is_____.

PC NOTE

If two non-consecutive terms in GP (say T_m & T_n) & their values are given in question & you are asked to find out GP. $r^{m-n} = \frac{T_m}{T_n}$

CQ8: Find GP where T_3 is 36 & T_5 is 324.

Ans: $r^{5-3} = 324/36$; $r^2 = 9$ & thus $r = \pm 3$.

$ar^2 = 36$. $a.9 = 36$. $a = 4$.

GP will be 4, ±12, ±36, ± 108….

CONCEPT 3: INSERTION OF 'n' GEOMETRIC MEANS BETWEEN TWO NUMBERS

- **Total number of terms** in the required GP will be **(n+1)**.
- Take the 1st given number as T_1 & 2nd given number as T_{n+2} $r = \left(\frac{b}{a}\right)^{\frac{1}{n+!}}$

CQ9: Insert 3 geometric means between 1/9 & 9.

Ans: Insert 3 GMs between 1/9 & 9, total number of terms will be 5 → 1/9, GM_1, GM_2, GM_3, 9.

Take T_1= 1/9; &Thus T_5 = 9. [Now we will use the above note.]

$r^{5-1} = 9/1/9$ **$r^4 = 81$** **& thus r = 3.**

GM_1 = 1/9 × 3 = **1/3,** GM_2 = 1/3 × 3 = **1,** GM_3 = 1 × 3 = **3.** **GP will be 1/9, 1/3, 1, 3, 9.**

HQ1: Second terms of a GP is 24 and fifth term is 81. The series is ________.

(a) 16, 36, 24, 54 (b) 24, 36, 53 (c) 16, 24, 36, 54 (d) None

CONCEPT 4: SUM OF FIRST 'N' TERM OF GP

$S_n = a \times \frac{1-r^n}{(1-r)}$ **when r < 1**

$S_n = a \times \frac{r^n-1}{(r-1)}$ **when r > 1**

CONCEPT 5: SUM OF INFINITE GP (S_∞)

- It is denoted by S_∞
- $S_\infty = \frac{a}{1-r}$

CONCEPT 5: ASSUMPTIONS OF THE TERMS IN GP

If No. of terms given in question are	Middle Term	Common Difference	Examples of Terms
ODD No. of terms	A	r	**3 terms:** (a/r), a, (ar); **5 terms:** (a/r^2), (a/r), a, (ar), (ar^2)
EVEN No. of terms	(a/r) & (ar)	r^2	**2 terms:** (a/r) & (ar); **4 terms:** (a/r^3), (a/r), (ar), (ar^3)

PC NOTE: But we will go by **OPTION METHOD** in such type of questions **TO SAVE TIME.**

CQ10: In a GP series, the product of the first three terms 27/8. The middle term is ________.

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) None

CONCEPT 6: PROPERTIES OF GP

Particulars	Examples
1. If we add/subtract all the terms of GP by any number, resulting series is NOT a GP.	
2. If we Multiply/divide all the terms of GP by any number, resulting series is a GP.	
3. Reciprocal of all the terms of a GP will be in GP (New GP).	
4. All the numbers of GP raised to the power k (any number) will also be in GP.	
5. If a, b, c OR a^2, b^2, c^2 are in GP → Put a, b, c value as 1, 2, 4 in options & get the answer.	
6. Log of all terms of a GP, it will become AP.	
7. If there are 'n' terms in a GP, m^{th} term from the end will be $(m-n+1)^{th}$ term from the start. **Ex:** If there are 7 terms in a GP, 2^{nd} term from the end will be $(7-2+1)^{th}$ term from the start.	

PROPERTIES OF A.P. & G.P.

- A sequence is both A.P. & G.P., if it is constant sequence, i.e. all the terms are equal (d = 0, r = 1).
- If A.M. & G.M. of 2 no. is known, the two no. are: **A.M.** $\pm \sqrt{(A.M.)^2 - (G.M.)^2}$
- If A.M. & G.M. of 2 no. is in ratio m : n, then no. are in ratio **(m +** $\sqrt{(m)^2 - (n)^2}$**) : (m -** $\sqrt{(m)^2 - (n)^2}$**)**
- If $T_n = An^3 + Bn^2 + Cn + D$, then $S_n = \sum T_n = A\sum n^3 + B\sum n^2 + C\sum n + nD$

HARMONIC MEAN (H.P.)

A sequence of non-zero number $a_1, a_2, a_3,$......... are in H.P. if $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3},$........... are A.P.

Ex: The sequence 1, 1/3, 1/5, 1/7,......... are in H.P. since 1, 3, 5, 7,........ are in A.P.

- Standard form of a H.P. is: $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d},$.........
- n^{th} term of a H.P. is $t_n = \frac{1}{a+(n-1)d}$
- If 3 terms are in H.P. $b = \frac{2ac}{a+c}$, b is the H.M. between 'a' & 'c'
- For any two distinct positive numbers, **A.M. > G.M. > H.M.** & **$(G.M.)^2$ = A.M. × H.M.**
- If a, b, c are in G.P. then a + b, 2b, c + d are in H.P. (**Ex:** 1, 2, 4 = 3, 4, 6)

Space for PC Class Note:

GEOMETRIC PROGRESSION - QUESTION BANK

SN	6B. GEOMETRIC PROGRESSION	Ans
Q163	6^{th} term of series ab, a^2b^3, a^3b^5 = ______. (a) a^6b^{11} (b) $a^{11}b^{30}$ (c) $a^{15}b^{36}$ (d) Cannot say	A
Q164	If the fifth term of a G.P. is 3^4 & second term is $3(2)^3$ then the first term is __. (a) 2^4 (b) 8 (c) 32 (d) 3.2^3	A
Q165	If n, p, q are in G.P, then the *expression for p* in terms of n & q is ______. (a) $\frac{n}{q}$ (b) $(nq)^{1/2}$ (c) q^{2n} (d) Nq	B
Q166	n^{th} root of the product of n observations is ______. (a) G.M (b) H.M (c) Median (d) A.M	A
Q167	If an observation in the data set in zero, then its geometric mean is ______. (a) Positive (b) Negative (c)Zero (d) Indeterminant	C
Q168	The AM of two positive numbers is 40 and their GM is 24. The numbers are ______. (a) (72,8) (b) (70,10) (c) (60,20) (d) None	A
Q169	AM is never __________ than GM. (a) more (b) less (c) maximum (d) minimum	B
Q170	If A be the AM of two positive unequal quantities x and y and G be their GM, then (a) A<G (b) A>G (c) A≥G (d) A≤G	B
Q171	1^{st} term is 1 & 6^{th} term is 32, find 'r'. (a) 3 (b) 32/5 (c) 2 (d) 160	C
Q172	Four geometric means between 4 and 972 are ______. (a) 12, 48, 192, 768 (b) 16, 64, 256, 512 (c) 12, 36, 108, 324 (d) None	C
Q173	The sum of the series $\frac{1}{\sqrt{3}}$ +1+ $\frac{3}{\sqrt{3}}$ +... to 18 terms is ______. (a) $9841\left(1+\frac{1}{\sqrt{3}}\right)$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None	A
Q174	The sum of the series 1+2+4+8+... to n term (a) 2^n -1 (b) 2n -1 (c) $1/2^n$ -1 (d) None	A
Q175	The sum of n terms of a GP is $1\frac{127}{128}$, its first term is 1 and the common ratio is $\frac{1}{2}$. The value of n is ______. (a) 7 (b) 8 (c) 6 (d) None	B
Q176	If r = 3 & the last term is 486. If the sum of these terms be 728, then the value of first term is ______. (a) 4 (b) 2 (c) 9 (d) 1	B
Q177	The sum of the first 20 terms of a GP is 244 terms the sum of its first 10 terms. The common ratio is ______.	A

	(a) $\pm\sqrt{3}$ (b) ± 3 (c) $\sqrt{3}$ (d) None	
Q178	Find the sum to Infinity of the Following series : 1 -1+1-1+1-1... ∞ (a) 1 (b) 1/2 (c) 0 (d) None	B
Q179	Sum upto ∞ of the series $8 + 4\sqrt{2} + 4.....$ is ______. (a) $8(2 + \sqrt{2})$ (b) $8(2 - \sqrt{2})$ (c) $4(2 + \sqrt{2})$ (d) $4(2 - \sqrt{2})$	A
Q180	The sum of the first two terms of a GP is $\frac{5}{3}$ and the sum to infmity of the series is 3. The common ratio is ______. (a) 1/3 (b) 2/3 (c) — 2/3 (d) Both (b) and (c)	D
Q181	The infinite GP series with first term $\frac{1}{4}$ and sum $\frac{1}{3}$ is ______. (a) $\frac{1}{4}, \frac{1}{16}, \frac{1}{64},....$ (b) $\frac{1}{4}, \frac{1}{16}, \frac{1}{64},....$ (c) $\frac{1}{4}, \frac{1}{18}, \frac{1}{16}....$ (d) None	D
Q182	The sum of 3 numbers of a GP is 39 and their product is 729. The numbers are ___. (a) 3, 27, 9 (b) 9, 3, 27 (c) 3, 9, 27 (d) None	C
Q183	If the sum of three numbers in GP is 21 and the sum of their squares is 189 the numbers are ______. (a) 3, 6, 12 (b) 12, 6, 3 (c) Both (d) None	C
Q184	If continued product of three numbers in GP is 27 & sum of their products in pairs is 39. The numbers are ______. (a) 1, 3, 9 (b) 9, 3, 1 (c) Both (a) and (b) (d) None	C
Q185	If a, b, c are in GP, then the value of $a(b^2 + c^2) - c(a^2 + b^2)$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q186	If a, b, c, d are in GP, (a+b), (b+c), (c+d) are in ______. (a) AP (b) GP (c) HP (d) None	B
Q187	If a, (b-a), (c-a) are in GP and $a = \frac{b}{3} = \frac{c}{5}$ then a, b, c are in ______. (a) AP (b) GP (c) HP (d) None	A
Q188	If a, b, c are in AP and x, y, z in GP, then the value of $x^{b-c}. y^{c-a}. z^{a-b}$ is ______. (a) 0 (b) 1 (c) -1 (d) None	B
Q189	If a, b, c are the p^{th}, q^{th} and r^{th} terms of a GP, the value of $a^{q-r}\, b^{r-p}.c^{p-q}$ is ______. (a) 0 (b) 1 (c) -1 (d) None	B
Q190	If a, b, c are in AP & a, x, b are in GP & b, y, c are in GP then x^2, b^2, y^2 are in ____. (a) AP (b) GP (c) HP (d) None	A
Q191	Three numbers are in AP & their sum is 15. If 8, 6, 4 be added to them respectively, they will be GP. They are ______. (a) 2, 6, 7 (b) 4, 6, 5 (c) 3, 5, 7 (d) None	C
Q192	The least value of n for which the sum of n terms of the series $1+3+3^2+....$ is greater than 7000 is ______.	A

	(a) 9 (b) 10 (c) 8 (d) 7	
Q193	6^{th} term from the end of the geometric progression 8, 4, 2, 1, ½, ¼, …… 1/1024 is (a) 1/4 (b) 1/16 (c) 1/32 (d) 1/64	C
Q194	The numbers x, 8, y are in GP and the numbers x, y,-8 are in AP. The value of x and y are ______. (a) (-8, -8) (b) (16,4) (c) (%8) (d) None	B
Q195	The sum of four numbers in GP is 60 and the AM of 1^{st} and the last term is 18. The numbers are ______. (a) 4, 8, 16, 32 (b) 4, 16, 8, 32 (c) 16, 8, 4, 20 (d) None	A
Q196	The sum of the series 1-1+1-1+1-1+ ….. to 100 terms is equal to ______. (a) 1 (b) -1 (c) 0 (d) 50	C
Q197	Find the sum to n terms of the series 3+33+333+… (a) $\frac{1}{27}(10^{n+1} - 9n - 10)$ (b) $\frac{1}{27}(10^{n-1} - 9n - 10)$ (c) $\frac{1}{27}(10^{n-1} + 9n + 10)$ (d) $\frac{1}{27}(10^{n+1} + 9n + 10)$	C
Q198	The sum upto infinity of the series $\frac{2}{3} + \frac{5}{9} + \frac{2}{27} + \frac{5}{81}$ +….. is ______. (a) 11/8 (b) 8/11 (c) 3/11 (d) None	A
Q199	If $x = a+\frac{a}{r} - \frac{a}{r^2} +....\alpha$, $y = b- \frac{b}{r} + \frac{b}{r^2} -....\alpha$, $z = c + \frac{c}{r} + \frac{c}{r^3} +.....\alpha$ then the value of $\frac{xy}{z} - \frac{ab}{c}$ is (a) 0 (b) 1 (c) -1 (d) None	A
Q200	Given x, y, z are in GP and xp = yq =zr, then $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in ______. (a) AP (b) GP (c) Both AP and GP (d) None	B
Q201	If a, b, x, y, z are positive numbers such that a, x, b are in AP and a, y, b are in GP and $z = \frac{(2ab)}{(a+b)}$ then ______. (a) x, y, z are in GP (b) x ≥ y ≥ z (c) Both (d) None	C
Q202	A radioactive sample decays & remaining sample at infinite time is given by b = 1 $-(\frac{1}{2} + \frac{1}{4}$ +… to ∞), then b is ______. (a) 0 (b) 1 (c) $1/\sqrt{2}$ (d) ½	A
Q203	The value of $A^{\frac{1}{2}}. A^{\frac{1}{4}} . A^{\frac{1}{8}}$ to infinity is ______. (a) Zero (b) Infinity (c) 1/2 (d) A	B
Q204	The sum upto infinity of the series $\frac{4}{7} - \frac{5}{7^2} + \frac{4}{7^3} - \frac{5}{7^4} +$ …. is ______. (a) $\frac{23}{48}$ (b) $\frac{25}{48}$ (c) $\frac{1}{2}$ (d) None	A
Q205	Sum upto ∞ of the series $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} + \frac{1}{3^6}$ +…… is ______. (a) 19/24 (b) 24/19 (c) 5/24 (d) None	A

Q206	If $1 + a + a^2 + ... \infty = x$; $1 + b + b^2 + ...\infty = y$ and $1 + ab + a^2b^2 + ...\infty$ is given by - (a) $\frac{xy}{x+y-1}$ (b) $\frac{xy}{x-y+1}$ (c) $\frac{xy}{x+y+1}$ (d)None	A
Q207	If $S_1, S_2,..S_n$ are the sum of Infinite GPs whose first terms are 1, 2, 3...n & whose common ratios are $\frac{1}{2}, \frac{1}{3}, ..., \frac{1}{n+1}$ then the value of $S_1+S_2+S_3+...S_n$, is _______. (a) $\frac{n(n+3)}{2}$ (b) $\frac{n(n+2)}{2}$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n^2}{n+1}$	A
Q208	The least vale of 'n' satisfying $1 + 2 + 2^2 + ...+ 2^{n-1} > 300$ is _______. (a) 8 (b) 9 (c) 10 (d) 6	B
Q209	Find the sum of n terms of the series 0.7+0.77+0.777+... to n terms. (a) $\frac{7}{81}\{9n + 1 + 10^{-n}\}$ (b) $\frac{7}{81}\{9n - 1 + 10^{-n}\}$ (c) $\frac{7}{81}\{9n + 1 + 10^{n}\}$ (d) $\frac{7}{81}\{9n - 1 - 10^{n}\}$	B
Q210	Three real numbers are such that their integer parts are in A.P. with common differences=3 and their decimal parts are in G.P. with common ratio = 2, and sum of three numbers is 25.4. Find the middle number (a) 6.4 (b) 11.2 (c) 5.2 (d) 8.4	D
Q211	If geometrical progressions 5, 10, 20,... & 1280, 640, 320 ... have their p^{th} terms equal, then value of 'p' is _______. (a) 10 (b) 75 (c) 5 (d) 40	C
Q212	In a GP if the $(p+q)^{th}$ terms is m and the (p-q)th term is n then the pth term is ____. (a) $(mn)^{1/2}$ (b) mn (c) m + n (d) m – n	A
Q213	The Lt $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... + \frac{1}{3^{n-1}} = n\rightarrow\infty$ (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{5}$ (d) None	B
Q214	The sum of n terms of $(x+y)^2, (x^2+y^2), (x-y)^2$... (a) $(x + y)^2 - 2(n-1)xy$ (b) $n(x + y)^2 - n(n - 1)xy$ (c) Both the above (d) None	B
Q215	The sum of n terms of (n – 1)/n, (n – 2)/n, (n – 3)/n.... is _______. (a) 0 (b) (n–1)/2 (c) (n + 1)/2 (d) None	B
Q216	The sum of n terms of the series 1.2 + 2.3 + 3.4 +.... is _______. (a) $\left(\frac{n}{3}\right)(n+1)(n+2)$ (b) $\left(\frac{n}{2}\right)(n+1)(n+2)$ (c) $\left(\frac{n}{3}\right)(n+1)(n-2)$ (d) None	A
Q217	The sum of n terms of the series 1.4 + 3.7 + 5.10 +... is _______. (a) $n(4n^2 + 5n - 1)/2$ (b) $n(5n^2 + 4n - 1)/2$ (c) $n(4n^2 + 5n + 1)/2$ (d) None	A
Q218	If a, b, c are in G.P. the b^2 =________. (a) ac (b) -ac (c) a+ b (d) a – c	A
Q219	If a, ar, ar^2, ar^3, ... be in G.P. Find the common ratio. (a) a (b) ar (c) r (d) $\frac{1}{r}$	C

Q220	Suppose x, y, z form a geometric sequence with common ratio r ($0 < r < 1$), if x, 2y, 3z form an arithmetic sequence, then value of Y is______. (a) $\frac{1}{3}$ (b) 1 (c) $\frac{1}{4}$ (d) Dependent of x, y, z	A
Q221	The common ratio of the G.P 2,-6, 18, -54 is______. (a) 3 (b) -3 (c) 4 (d) -4	B
Q222	In 5, 15, 45, 135,... the common ratio is______. (a) 3 (b) 5 (c) 10 (d) 30	A
Q223	The sum of first eight terms of GP is five times the sum of the first four terms. The common ratio is______. (a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) Both (d) None	C
Q224	The number of terms in 6,18,54,....... 1458 is______. (a) 5 (b) 7 (c) 8 (d) 6	D
Q225	Third term of geometric progression is 4. Then product of the first 6 terms is______. (a) 4^6 (b) $4^{7.5}$ (c) 4^5 (d) 4^{15}	B
Q226	If the $(p+q)^{th}$ term of a G.P. is X and the $(p-q)^{th}$ term is Y, then p^{th} term is______. (a) XY (b) $\frac{(X+Y)}{2}$ (c) $\sqrt{XY}$ (d) $\sqrt{\frac{X^2+Y^2}{2}}$	C
Q227	Which term of the progression 1,2,4, 8......... is 64 (a) 7 (b) 5 (c) 6 (d) 9	A
Q228	Which term of series 3, $\sqrt{3}$, 1, $\frac{1}{\sqrt{3}}$ ..... is $\frac{1}{243}$? (a) 13 (b) 14 (c) 15 (d) 12	A
Q229	Which term of the progression is 1, 2, 4, 8,... is 256? (a) 10 (b) 9 (c) 12 (d) 13	B
Q230	The 4^{th} term of the series 0.04,0.2,1,... is______. (a) 0.5 (b) $\frac{1}{2}$ (c) 5 (d) None	C
Q231	The sixth term of a G.P with common ratio as 2 and first term being 5 is______. (a) 160 (b) 32 (c) 800 (d) 64	A
Q232	The 7^{th} term of the series 6, 12, 24, ... is______. (a) 384 (b) 834 (c) 438 (d) None	A
Q233	t_8 of the series 6, 12, 24, is______. (a) 786 (b) 768 (c) 867 (d) None	B
Q234	t_{12} of the series -128, 64,-32,.... is______. (a) $-\frac{1}{16}$ (b) 16 (c) $\frac{1}{16}$ (d) None	C

Q235	In a GP series, the product of the first three $\frac{27}{8}$. The middle term is______. (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) None	A
Q236	In a GP, the 6^{th} term is 729 and the common ratio is 3, then the 1^{st} term is______. (a) 2 (b) 3 (c) 4 (d) 7	B
Q237	In a GP series the product of first three term is $\frac{729}{8}$. The middle term is______. (a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $\frac{2}{9}$ (d) None	B
Q238	The last term of the series 1,2,4...... to 10 terms is______. (a) 512 (b) 256 (c) 1024 (d) None	A
Q239	The last term of the series 1-3,9,-27,upto 7 terms is______. (a) 297 (b) 729 (c) 927 (d) None	B
Q240	The last term of the series x^2, x,1,... to 31 terms is______. (a) x^{28} (b) $\frac{1}{x}$ (c) $\frac{1}{x^{28}}$ (d) None	C
Q241	The nth element of the sequence -1, 2 -4, 8.... is______. (a) $(-1)^n\ 2^{n-1}$ (b) 2^{n-1} (c) 2^n (d) None	A
Q242	The second terms of a GP is 24 and fifth term is 81. The series is______. (a) 16, 36, 24, 54 (b) 24, 36, 53 (c) 16, 24, 36, 54 (d) None	C
Q243	The sum of the series 1-1+1-1+1-1+........ to 101 terms is equal to______. (a) 1 (b) -1 (c) 0 (d) 100	A
Q244	Product of 3 numbers in GP is 729 and Sum of squares is 819. the numbers are______. (a) 9,3,27 (b) 27, 3, 9 (c) 3,9,27 (d) None	C
Q245	Sum of three numbers in GP is 35 and their product is 1000 the numbers are______. (a) 20 10 5 (b) 5 10 20 (c) Both (d) None	C
Q246	The numbers in GP with their sum 130 and their product 27000 are______. (a) 10 30 90.... (b) 90 30 10.... (c) Both (d) None	C
Q247	Three numbers in GP with their sum $\frac{13}{3}$ and sum of their squares $\frac{91}{9}$ are______. (a) $\frac{1}{3}$,1,3 (b) 3,1,$\frac{1}{3}$ (c) Both (d) None	C
Q248	Find five numbers in GP such that their product is 32 and product of last two is 108. (a) $\frac{2}{9}$, $\frac{2}{3}$,2,6,18 (b) 18,6,2, $\frac{2}{3}$, $\frac{2}{9}$ (c) Both (d) None	A
Q249	Find three numbers in G.P whose sum is 52 and Sum of their product in pairs is 624. (a) 4, 12, 36 (b) 10, 16, 26 (c) 5, 17, 30 (d) None	A
Q250	Numbers a,X,c are in AP if X=25 & a,Y,c are in GP if Y=7, then value of (a, c) are___.	C

	(a) 5,7 (b) 25,7 (c) 1,49 (d) 39, 11	
Q251	The G.M between 2 and 8 is______. (a) 4 (b) 10 (c) 6 (d) 8	A
Q252	The geometric mean between 6 and 96 is______. (a) 24 (b) 4 (c) 2 (d) 16	A
Q253	Let S be the sum, P be the product and R be the sum of reciprocals of n terms of a G.P. then P^2R^n: (a) S^{2n} (b) S^{-n} (c) S^n (d) S^{-2n}	C
Q254	The A.M and G.M of two positive numbers is 10. The numbers are______. (a) (10,10) (b) (15,5) (c) (5,15) (d) (20,0)	A
Q255	A.M. and G.M. of 2 observations are 5 & 4 respectively, then 2 observations are___. (a) 8,2 (b) 7, 3 (c) 6, 4 (d) 5, 5	A
Q256	If x, y, z are in GP., then______. (a) $x(y^2+z^2) = z(x^2+y^2)$ (b) $y(z^2+x^2) = x(z^2+y^2)$ (c) $z(x^2+y^2) = y(z^2+x^2)$ (d) None	A
Q257	If a, b, c are in the p^{th}, q^{th} and r^{th} terms of an AP value of a(q-r)+b(r-p)+c(p-q) is_____. (a) 0 (b) 1 (c) -1 (d) None	A
Q258	If a, b, c be the sums of p, q, r terms respectively of an AP, the value of $\left(\frac{a}{p}\right)(q-r)+\left(\frac{b}{q}\right)(r-p)+\left(\frac{c}{r}\right)(p-q)$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q259	If a, b, c are in AP then the value of $\frac{(a^2+4ac+c^2)}{(ab+be+ca)}$ is______. (a) 1 (b) 2 (c) 3 (d) None	B
Q260	If a, b, c are in AP then (b + c), (c + a), (a + b) are in______. (a) AP (b) GP (c) HP (d) None	A
Q261	If a, b, c are in AP then $\left(\frac{a}{bc}\right)(b+c), \left(\frac{b}{ca}\right)(c+a), \left(\frac{c}{ab}\right)(a+b)$ are in______. (a) AP (b) GP (c) HP (d) None	A
Q262	If a, b, c are in AP then the value of $\frac{(a^3+4b^3+c^3)}{b(a^2+c^2)}$ is______. (a) 1 (b) 2 (c) 3 (d) None	C
Q263	If $(b+c)^{-1},(c+a)^{-1},(a+b)^{-1}$ are in AP the a^2,b^2,c^2 are in______. (a) AP (b) GP (c) HP (d) None	A
Q264	If a^2,b^2,c^2 are in AP then (b + c),(c + a),(a + b) are in______. (a) AP (b) GP (c) HP (d) None	C

Q265	If a, b, c are in AP a x b are in GP and b y c are in GP then x^2,b^2,y^2 are in______. (a) AP (b) GP (c) HP (d) None	A
Q266	If a, b, c are in GP a^2+b^2,ab + bc,b^2+ c^2 are in______. (a) AP (b) GP (c) HP (d) None	B
Q267	If a, b, c are in GP then value of $(a - b + c)(a + b + c)^2-(a + b + c)(a^2 + b^2 + c^2)$ is______. (a) 0 (b) 1 (c) -1 (d) None	A
Q268	If a, b, c are in GP then value of $a^2b^2c^2(a^{-3} + b^{-3} + c^{-3})- (a^3 + b^3 + c^3)$ is given by______. (a) 0 (b) 1 (c) -1 (d) None	A
Q269	If a, b, c d are in AP then______. (a) $a^2 - 3b^2 + 3c^2 - d^2 = 0$ (b) $a^2 + 3b^2 + 3c^2 + d^2 = 0$ (c) $a^2 + 3b^2 + 3c^2 - d^2 =0$ (d) None	A
Q270	If a, b, c, d, e are in AP then______. (a) a-b-d + e = 0 (b) a-2c + e = 0 (c) b - 2c + d = 0 (d) All	D
Q271	If a, b, c, d are in GP. Then the value of $b(ab - cd)- (c + a)(b^2 - c^2)$ is______. (a) 0 (b) 1 (c) -1 (d) None	A
Q272	If a, b, c, d are in GP then $(a-b)^2$,$(b-c)^2$ $(c-d)^2$ are in______. (a) AP (b) GP (c) HP (d) None	B
Q273	If a, b, c, d are in GP then value of $(b - c)^2 + (c - a)^2 + (d - b)^2 - (a - d)^2$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q274	If (a - b), (b - c), (c - a) are in GP then value of $(a + b + c +)^2 - 3(ab + bc + ca)$ is_____. (a) 0 (b) 1 (c) -1 (d) None	A
Q275	Numbers x, 8, y are in GP and numbers x, y,-8 are in AP. Value of x and y are______. (a) (-8-8) (b) (16,4) (c) (8,8) (d) None	B
Q276	The sum of 3 numbers in AP is 15. If 1,4 and 19 be added to them respectively, the results are is GP. The numbers are______. (a) 26, 5, -16 (b) 2, 5, 8 (c) 5, 8, 2 (d) Both (a) and (b)	A
Q277	The sum of three numbers in GP is 70. If the two extremes be multiplied each by 4 and the mean by 5, the products are in AP. The numbers are______. (a) 12, 18, 40 (b) 10, 20, 40 (c) 40, 20, 10 (d) Both (b) and (c)	B
Q278	A person borrows Rs. 8000 at 2.76% simple interest per annum. The principal and the interest are to be paid in 10 monthly installments. If each installment is double the preceding one, find the value of the first and the last installment. (a) 12 and 6048 (b) 6 and 3036 (c) 4 and 2024 (d) 8 and 4096	D
Q279	A sum of Rs. 6240 is paid off in 30 installments such that each instalment is Rs. 10 more than the preceding instalment. The value of the 1st instalment is______.	D

	(a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) None	
Q280	10% CL p.a sum of money accumulate to Rs. 8650 in 5 yr. Sum invested initially is____. (a) Rs. 5976.37 (b) Rs. 5970 (c) Rs. 5975 (d) None	D
Q281	The population of a country was 55 crores in 2005 and is growing at 2% p.a. C.I. the population in the year 2015 is estimated as______. (a) 5705 (b) 6005 (c) 6700 (d) None	D
Q282	If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so or, then your total savings in two weeks will be______. (a) Rs. 163 (b) Rs. 18 (c) Rs. 163.83 (d) None	C
Q283	In the series 2 + 8 + 32 +......... common ratio is______. (a) 24 (b) 6 (c) 4 (d) 10	C
Q284	The sum of 1 + 2 + 4 + 8 +........ to 8 terms is______. (a) 255 (b) 252 (c) 254 (d) 256	A
Q285	The sum of the series -2,6-18,.... to 7 terms is______. (a) -1094 (b) 1094 (c) -1049 (d) None	A
Q286	Find the sum of progression 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$,.......... 10 terms (a) 1.9 (b) 1.989 (c) 1.998 (d) 1.89	C
Q287	The sum of 1.03+$(1.03)^2$ +$(1.03)^3$+... to n terms is______. (a) 103{$(1.03)^n$-1} (b) $\frac{103}{3}\{(1.03)^n - 1\}$ (c) $(1.03)^{n-1}$ (d) None	B
Q288	Sum of the series 1+3+9+27....... is 364. The number of terms is______. (a) 5 (b) 6 (c) 11 (d) None	B
Q289	How many terms of the GP 1 4 16 .... Are to be taken to have their sum 341? (a) 8 (b) 5 (c) 3 (d) None	B
Q290	Sum of all natural numbers from 100 to 300 which are exactly divisible by 4 & 5 is__. (a) 2200 (b) 2000 (c) 2220 (d) None	A
Q291	The GP series whose 3rd and 6th terms are 1, $-\frac{1}{8}$ respectively is______. (a) 4, -2, 1... (b) 4, 2, 1... (c) 4, -1, $\frac{1}{4}$ (d) None	A
Q292	Sum of n terms of a GP with last term 128 & common ratio 2 is 255 value of n is______. (a) 8 (b) 5 (c) 3 (d) None	A
Q293	The nth term of the series 16,8,4,...... is $\frac{1}{2^{17}}$. The value of n is______. (a) 20 (b) 21 (c) 22 (d) None	C
Q294	The sum of n terms of the series 1.03+1.03^2+1.03^3 +... is______.	A

	(a) $\left(\frac{103}{3}\right)(1.03^n - 1)$ (b) $\left(\frac{103}{3}\right)(1.03^n + 1)$ (c) $\left(\frac{103}{3}\right)(1.03^{n+1} - 1)$ (d) None	
Q295	Sum of n terms of the series 4+44+444 +.... is_______. (a) $\frac{4}{9}\left(\frac{10}{9}(10^n - 1) - n\right)$ (b) $\frac{10}{9}(10^n - 1) - n$ (c) $\frac{4}{9}(10^n - 1) - n$ (d) None	B
Q296	-5 + 25 -125 + 625,... can be written as_______. (a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty} 5^k$ (c) $\sum_{k=1}^{\infty} -5^k$ (d) None	A
Q297	The sum of the series 1, $\frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}$ ....to ∞ is_______. (a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{3}$ (d) None	A
Q298	The sum of the infinite GP 14 - 2 + $\frac{2}{7} - \frac{2}{49}$+... is_______. (a) $4\frac{1}{12}$ (b) $12^1/_4$ (c) 12 (d) None	D
Q299	The sum of the infinite GP 0.171-0.114+0.076 is_______. (a) 0.1226 (b) 0.1020 (c) 0.1026 (d) None	B
Q300	If S = 1 + $(1.04)^{-1}$ + $\frac{1}{(1.04)^2}$ + $(1.04)^{-3}$ +.... to infinity, then the value of 'S' is_______. (a) 25 (b) 26 (c) 2.74 (d) 27.4	C
Q301	The sum upto infinity of the series 0.4+0.8+0.16+.... is_______. (a) 5 (b) 10 (c) 8 (d) None	A
Q302	The sum upto infinity of the series $(1+ 2^{-2}) + (2^{-1} + 2^{-4}) + (2^{-2} + 2^{-6})$ + ... is_______. (a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) None	A
Q303	The sum of an infinite GP is 15 and the sum of their squares is 45. Series is_______. (a) 5,10,20.... (b) $5 + \frac{5}{3} + \frac{5}{9}$+.... (c) $5 + \frac{10}{3} + \frac{20}{9} + \cdots$ (d) None	A
Q304	If the first term of a GP exceeds the second term by 2 and the sum to infinity is 50 the series is ______. (a) 10 8 $\frac{32}{5}$ (b) 108 $\frac{5}{2}$ (c) 10 $\frac{10}{3}$ $\frac{10}{9}$.... (d) None	A
Q305	1st term is 1 & 6th term is 32, find 'r'. (a) 3 (b) 32/5 (c) 2 (d) 160	C
Q306	If r = 3 & last term is 486. If sum of these terms be 728, then first term is _______. (a) 6 (b) 2 (c) 18 (d) 162	B
Q307	If sum of three numbers in GP is 21 & sum of their squares is 189, numbers are ____. (a) 3, 6, 12 (b) 12, 6, 3 (c) Both (d) None	C
Q308	If a, b, c are in AP & a, x, b are in GP and b, y, c are in GP then x^2, b^2, y^2 are in (a) AP (b) GP (c) HP (d) None	A

Q309	6^{th} term from the end of GP 8, 4, 2, 1, ½, ¼, ...1/1024 is ___. (a) 1/4 (b) 1/16 (c) 1/32 (d) 1/64	C
Q310	Given x, y, z are in GP and xp = yq =zr, then $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in ______. (a) AP (b) GP (c) Both AP and GP (d) None	B
Q311	Sum upto infinity of the series $\frac{4}{7}-\frac{5}{7^2}+\frac{4}{7^3}-\frac{5}{7^4}+$ .... is ______. (a) $\frac{23}{48}$ (b) $\frac{25}{48}$ (c) $\frac{1}{2}$ (d) None	A
Q312	The geometric mean between 6 & 96 is ______. (a) 24 (b) 4 (c) 2 (d) 16	A
Q313	If the A.M. and G.M. of two observations are 5 and 4 respectively, then the two observations are ______. (a) 8, 2 (b) 7, 3 (c) 6.4 (d) 5, 5	A
Q314	The AM & GM of two positive numbers is 10. The numbers are ______. (a) (10, 10) (b) (15, 5) (c) (5, 15) (d) (20, 0)	A

SPECIAL SERIES ON AP & GP

SN	6C. SPECIAL SERIES ON AP & GP	Ans
Q315	Find the sum to infinity of the series $\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+....$ (a) $\left(\frac{n}{4}\right)(3n+4)^{-1}$ (b) $\left(\frac{n}{4}\right)(3n-4)^{-1}$ (c) $\left(\frac{n}{2}\right)(3n+4)^{-1}$ (d) None	A
Q316	The sum of n terms of the series 4 + 6 + 9 + 13... is ______. (a) $\left(\frac{n}{6}\right)(n^2+3n+20)$ (b) $\left(\frac{n}{6}\right)(n+1)(n+2)$ (c) $\left(\frac{n}{2}\right)(n+1)(n+2)$ (d) None	A
Q317	The sum of n terms of 1, (1+2), (1+2+3)..... is ______. (a) $\left(\frac{n}{6}\right)(n+1)(n+2)$ (b) $\left(\frac{n}{3}\right)(n+1)(n+2)$ (c) $n(n+1)(n+2)$ (d) None	A
Q318	The sum of n terms of the series $\frac{1}{(4.9)}+\frac{1}{(9.14)}+\frac{1}{(14.19)}+\frac{1}{(19.24)}+...$ is ______. (a) $\left(\frac{n}{4}\right)(5n+4)^{-1}$ (b) $\left(\frac{n}{3}\right)(5n+4)$ (c) $\left(\frac{n}{2}\right)(5n-4)^{-1}$ (d) None	A
Q319	The sum of n terms of the series $1^2+(1^2+2^2)+(1^2+2^2+3^2)+...$ is ______. (a) $\left(\frac{n}{12}\right)(n+1)^2(n+2)$ (b) $\left(\frac{n}{12}\right)(n-1)^2(n+2)$ (c) $\left(\frac{n}{12}\right)(n^2-1)(n+2)$ (d) None	A
Q320	The sum of n terms of the series $1+(1+\frac{1}{3})+(1+\frac{1}{3}+\frac{1}{3^2})+...$ is ______. (a) $\left(\frac{3}{2}\right)(1-3^{-n})$ (b) $\left(\frac{3}{2}\right)[n-(1/2)(1-3^{-n})]$ (c) Both (d) None	B
Q321	The sum of n terms of the series $\frac{1^2}{1}+\frac{(1^1+2^2)}{(1+2)}+\frac{(1^2+2^2+3^2)}{(1+2+3)}+....$ is ______. (a) n (n + 2)/3 (b) n (n + 1)/3 (c) n (n + 3)/3 (d) None	A
Q322	The sum of n terms of the series $\frac{1^3}{1}+\frac{(1^3+2^3)}{2}+\frac{(1^3+2^3+3^3)}{3}+....$ is ______. (a) $\left(\frac{n}{48}\right)(n+1)(n+2)(3n+5)$ (b) $\left(\frac{n}{3}\right)(n+1)(n+2)(3n+5)$ (c) $\left(\frac{n}{2}\right)(n+1)(n+2)(5n+3)$ (d) None	A
Q323	Three numbers whose sum is 15 are in AP. If they are added by 1, 4, 19, they are in GP. The numbers are ______. (a) 2, 5, 8 (b) 26, 5, -16 (c) Both (d) None	C
Q324	Three numbers in GP with their sum $\frac{13}{3}$ and sum of their squares $\frac{91}{9}$ are ______. (a) $\frac{1}{3}$, 1, 3 (b) 3, 1, $\frac{1}{3}$ (c) Both (d) None	C
Q325	The number of terms to be taken so that 1+2+4+8+ will be 8191 is ______. (a) 10 (b) 13 (c) 12 (d) None	B

Q326	If you save 1 paise today, 2 paise next day, 4 paise succeeding day & so on, then total savings in two weeks = _______. (a) Rs. 163 (b) Rs. 18 (c) Rs. 163.83 (d) None	C
Q327	The sum of the series 1, $\frac{1}{3}$, $\frac{1}{3^2}$, $\frac{1}{3^3}$,..., to ∞ is _______. (a) 4/3 (b) 3/2 (c) 1/3 (d) None	B
Q328	The sum of the infinite GP 14 -2 + $\frac{2}{7}$ - $\frac{2}{49}$ +....is _______. (a) 9/2 (b) 49/4 (c) 42/4 (d) None	B
Q329	Sum of n terms of a GP with last term 128 & common ratio 2 is 255 value of n is __. (a) 8 (b) 5 (c) 3 (d) None	A
Q330	If a, b, c are in GP, (a^2+b^2), $(ab+bc)$, (b^2+c^2) are in _______. (a) AP (b) GP (c) HP (d) None	B
Q331	The sum upto infinity of the series $(1+2^{-2}) + (2^{-1}+2^{-4}) + (2^{-2}+2^{-6})$ +.... is _______. (a) 7/3 (b) 3/7 (c) 4/7 (d) None	A
Q332	The sum of n terms of the series $1.03+1.03^2+1.03^3$+... is _______. (a) $(\frac{103}{3})(1.03^n-1)$ (b) $(\frac{103}{3})(1.03^n+1)$ (c) $(\frac{103}{3})(1.03^{n+1}-1)$ (d) None	A
Q333	The sum of n terms of the series 1.2.3 + 2.3.4 + 3.4.5 +.... is _______. (a) n (n+1) (n+2) (n+3)/4 (b) n (n+1) (n+2) (n+3)/3 (c) n (n+1) (n+2) (n+3)/2 (d) None	A
Q334	Evaluate $(a+b)+(a^2+2b)$+...... to 4 terms if a=3, b=-7 (a) 190 (b) 50 (c) 110 (d) 170	B
Q335	The average of 15 numbers is 18. The average of first 8 is 19 and that last 8 is 17, then the 8th number is_______. (a) 15 (b) 16 (c) 18 (d) 20	C
Q336	$t_1 = n$, $t_2 = n + 1$, $t_3 = n + 2$ and so on, then t_n =_______. (a) n (b) 2n – 1 (c) 2n + 1 (d) 2n	B
Q337	In the sequence whose $t_n = \frac{3n-2}{4}$;n ∉ N the first term of the sequence is_______. (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) ½ (d) 1	A
Q338	The weighted mean of first n natural numbers whose weights are equal to the squares of corresponding numbers is_______. (a) $\frac{(n+1)}{2}$ (b) $\frac{[3n(n+1)]}{[2(2n+1)]}$ (c) $\frac{[(n+1)(2n+1)]}{6}$ (d) $\frac{n(n+1)}{2}$	B
Q339	If nth term of a sequence be $2^{3n}(-5)^n$, then the common ratio of sequence is_______. (a) -40 (b) 40 (c) 80 (d) -80	A
Q340	The mean of the cubes of the first n natural numbers is_______. (a) $\frac{n^2(n+1)^2}{4}$ (b) $\frac{n(n+1)^2}{4}$ (c) $\frac{[n\times(n+1)\times(n+2)]}{8}$ (d) $n^2 + n + 1$	B

Q341	The mean of the squares of the first n natural number is______. (a) $n^2 + 1$ (b) $(n^4 + 1)$ (c) $\frac{[(n+1)(2n+1)]}{6}$ (d) $\frac{n(n-1)}{2}$	C
Q342	The sum of n terms of the series 1+3+5+... is______. (a) n^2 (b) $2n^2$ (c) $\frac{n^2}{2}$ (d) None	A
Q343	The value of $11^2+12^2+........+20^2$ is______. (a) 3845 (b) 2485 (c) 2870 (d) 3255	B
Q344	If $1^2 + 2^2 + ...+ 10^2 = 385$, then $2^2 + 4^2 + 6^2 + ...+ 20^2$ is______. (a) 770 (b) 1150 (c) 1540 (d) 385 × 385	C
Q345	Find the sum of n terms of $\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\cdots$______. (a) $\frac{1}{2}(n-1)$ (b) $\frac{1}{2}(n+1)$ (c) $(n-1)$ (d) $(n+1)$	A
Q346	The sum to n terms of the series 11, 23, 59, 167......... is______. (a) $3^{n+1} + 5n - 3$ (b) $3^{n+1} + 5n + 3$ (c) $3^n + 5n - 3$ (d) None	A
Q347	Find the sum to n terms of 6+27+128+629+...______. (a) $\left\{\frac{5(5^n-1)}{4}\right\}+\left\{\frac{n(n+1)}{2}\right\}$ (b) $\left\{\frac{5(5^n-1)}{4}\right\}-\left\{\frac{n(n+1)}{2}\right\}$ (c) $\left\{\frac{5(5^n-1)}{4}\right\}-\left\{\frac{n(n+1)}{2}\right\}$ (d) $\left\{\frac{5(5^n+1)}{4}\right\}+\left\{\frac{n(n+1)}{4}\right\}$	A
Q348	1+3-5+7+9-11+13......... 3n terms (a) $2n^2 + 3$ (b) $5n^2 + 3$ (c) $3n^2 - 4n$ (d) $3n^2$	C
Q349	The sum of n terms of $(x + y)^2, (x^2 + y^2), (x - y)^2$, is______. (a) $(x + y)^2 - 2(n-1)xy$ (b) $n(x + y)^2 - n(n - 1)xy$ (c) Both the above (d) None	B
Q350	Find the sum to infinity of the series $\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+$ .... (a) $\left(\frac{n}{4}\right)(3n+4)^{-1}$ (b) $\left(\frac{n}{4}\right)(3n-4)^{-1}$ (c) $\left(\frac{n}{2}\right)(3n+4)^{-1}$ (d) None	A
Q351	The sum of n terms of the series $1.3^2+4.4^2+7.5^2+10.6^2+...$ is______. (a) $\left(\frac{n}{12}\right)(n+1)(9n^2+49n+44)-8n$ (b) $\left(\frac{n}{12}\right)(n+1)(9n^2+49n+44)+8n$ (c) $\left(\frac{n}{6}\right)(2n+1)(9n^2+49n+44)-8n$ (d) None	A
Q352	The sum of n terms of the series 1.2+2.3+3.4+.... is______. (a) $\left(\frac{n}{3}\right)(n+1)(n+2)$ (b) $\left(\frac{n}{2}\right)(n+1)(n+2)$ (c) $\left(\frac{n}{3}\right)(n+1)(n-2)$ (d) None	A
Q353	The sum of n terms of the series 1.4+3.7+5.10+... is______. (a) $\left(\frac{n}{2}\right)(4n^2+5n-1)$ (b) $\left(\frac{n}{2}\right)(5n^2+4n-1)$ (c) $\left(\frac{n}{2}\right)(4n^2+5n+1)$ (d) None	A

Q354	The sum of n terms of the series $\frac{1}{(4.7)}+\frac{1}{(7.10)}+\frac{1}{(10.13)}+\cdots$. is______. (a) $\left(\frac{1}{3}\right)[(3n+1)^{-1}-(3n+4)^{-1}]$ (b) $\left(\frac{1}{3}\right)(3n-1)^{-1}-(3n+4)^{-1}]$ (c) $\left(\frac{1}{3}\right)[(3n+1)^{-1}-(3n-4)^{-1}]$ (d) None	A
Q355	The sum of n terms of $\left(\frac{1}{n}\right)(n-1),\left(\frac{1}{n}\right)(n-2),\left(\frac{1}{n}\right)(n-3),\ldots$ is______. (a) 0 (b) $(\frac{1}{2})$(n-1) (c) $(\frac{1}{2})$ (n+1) (d) None	B
Q356	The value of $\frac{1^3+2^3+\cdots+10^3}{1+2+\cdots+10}$ is______. (a) 45 (b) 55 (c) 385 (d) 285	B
Q357	The value of $1^3+2^3+3^3+ m^3$ is equal to______. (a) $[\frac{m(m+1)}{2}]^3$ (b) $\frac{m(m+1)(2m+1)}{6}$ (c) $[\frac{m(m+1)}{2}]^2$ (d) None	C
Q358	The sum of m terms of the series is 1+11+111+........ is equal to______. (a) $\frac{1}{81}[10^{m+1}-9m-10]$ (b) $\frac{1}{27}[10^{m+1}-9m-10]$ (c) $[10^{m+1}-9m-10]$ (d) None	A
Q359	1+11+111+............ n terms (a) [10n+1 - 9n - 10] (b) [10n+1 - 9n - 10] (c) [10n+1 - 9n - 10] (d) None	C
Q360	Sum of first n terms of an A.P is 6n2+6n. Then find 4th term of series. (a) 120 (b) 72 (c) 48 (d) 24	C
Q361	In an A.P. If $S_n=n^2p$ and $S_m = m^2p$, (m≠n) the S_p =______. (a) P^3 (b) P^2 (c) $2p^3$ (d) P^4	B
Q362	If the numbers x,y,z are in G.P then the numbers x^2+y^2, xy+yz, y^2+z^2 are in______. (a) A.P (b) G.P (c) H.P (d) None	B
Q363	2.353535 =______. (a) $\frac{233}{99}$ (b) $\frac{234}{99}$ (c) $\frac{232}{99}$ (d) $\frac{235}{99}$	A
Q364	Sum of n terms of the series 1.2 + 2.3 + 3.4 +.... is ______. (a) $\left(\frac{n}{3}\right)$(n+1) (n+2) (b) $\left(\frac{n}{2}\right)$(n+1) (n+2) (c) $\left(\frac{n}{3}\right)$(n+1) (n-2) (d) None	A
Q365	Sum of n terms of the series 1.4+3.7+5.10+... is ______. (a) n $(4n^2 + 5n - 1)/2$ (b) n $(5n^2 + 4n - 1)/2$ (c) n $(4n^2 + 5n + 1)/2$ (d) None	A
Q366	The sum of n terms of the series 1+5+12+22+ ...is ______. (a) $n^2(n+1)/2$ (b) $n^2(n+1)$ (c) $n^2(n+2)/2$ (d) None	A
Q367	The sum of n terms of the series 4 + 14 + 30 + 52 + 80 +......is ______. (a) $n(n+1)^2$ (b) $n(n-1)^2$ (c) $n(n^2-1)$ (d) None	A

Q368	The sum of n terms of the series 1+(1+3) + (1+3+5) +... is ______. (a) $\left(\frac{n}{6}\right)$(n + 1) (2n + 1) (b) $\left(\frac{n}{6}\right)$(n + 1) (n + 2) (c) $\left(\frac{n}{3}\right)$(n + 1) (2n + 1) (d) None	A
Q369	The sum of n terms of the series $2.3^2+5.4^2+8.5^2+....$ is ______. (a) $\left(\frac{n}{12}\right)(9n^3 + 62n^2 + 123n + 22)$ (b) $\left(\frac{n}{12}\right)(9n^3 - 62n^2 + 123n + 22)$ (c) $\left(\frac{n}{6}\right)(9n^3 + 62n^2 + 123n + 22)$ (d) None	A
Q370	The sum of n terms of $1^2, 3^2, 5^2, 7^2,...$ is ______. (a) $n(4n^2 - 1)/3$ (b) (n) $(4n^2 - 1)$ (c) $n(4n^2 + 1)/3$ (d) None	A
Q371	The sum of n terms of the series $2^2+5^2+8^2+...$ is ______. (a) $n(6n^2 + 3n - 1)$ (b) $n(6n^2 - 3n - 1)/2$ (c) $n(6n^2 + 3n + 1)$ (d) None	A
Q372	The sum of n terms of the series 2.4.6+4.6.8+6.8.10+.... is ______. (a) $2n(n^3+6n^2+11n+6)$ (b) $2n(n^3-6n^2+11n-6)$ (c) $n(n^3+6n^2+11n+6)$ (d) $n(n^3 - 6n^2+11n - 6)$	A
Q373	The sum of n terms of the series $\frac{1}{(3.8)} + \frac{1}{(8.13)} + \frac{1}{(13.18)} +....$ is ______. (a) $\left(\frac{n}{3}\right)(5n + 3)^{-1}$ (b) $(n)(5n + 3)^{-1}$ (c) $\left(\frac{n}{2}\right)(5n + 3)^{-1}$ (d) None	A
Q374	The sum of n terms of the series $\frac{1}{1} + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} +....$is ______. (a) $2n(n + 1)^{-1}$ (b) $n(n + 1)^{-1}$ (c) $2n(n - 1)^{-1}$ (d) None	A

CHAPTER 7A. SET

INTRODUCTION

- **Sets:** A set is a well-defined collection of objects. [If we can clearly say whether a given object belongs to it or not].

  Ex: The collection of all English Alphabets is a set [Say Set A].
- **Element:** Each object in a set is called an element of the set.

  Ex: A = {a, b, c, d, e,........, x, y, z}
- A set is denoted by 'capital letters' & their elements are denoted by 'small letters'.

Example: A = {a, e, i, o, u},

'a' is an element and we write **a∈A** & read as 'a' belongs to 'A'. But 3 is not an element of B = {2, 4, 6, 8, 10} & we write b ∉B & read as '3' does not belong to 'B'.

METHODS OF WRITING A SET

- **Roster or Braces Method:** All elements of the set are listed and put it within braces { }.

  Ex: A = {a, b, c, d, e,........, x, y, z}.
- **Set Builder Method:** In this method, Rules or properties to write down a set is given.

  Ex: A = {x: x is a set of all English Alphabets}.

CQ1: Represent the following sets in set notations:-

(i) Set of all alphabets in English language.

(ii) Set of all odd integers less than 25.

(iii) Set of all odd integers.

(iv) Set of positive integers 'x' satisfying' the equation $x^2 + 5x + 7 = 0$.

Ans:

(a) A = {x: x is an alphabet in English}; {x:x is an odd integer > 25}; {2 4 6 8...}; {x: $x^2 + 5x + 7 = 0$}

(b) A= {x: x is an alphabet in English}; {x:x is an odd integer < 25}; {1 3 5 7....}; {x: $x^2 + 5x + 7 = 0$}

(c) A= {x: x is an alphabet in English}; {x:x is an odd integer ≤ 25}; {1 3 5 7....}; (x:$x^2 + 5x + 7 = 0$)

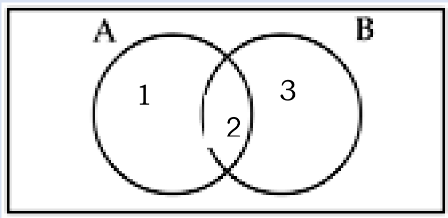
(d) None

- **Repetition** of elements in a set is **MEANINGLESS.**
- **Order** of the elements in a set is **NOT RELEVANT.**

CONCEPT 1: TYPES OF SETS	
Universal Set	A set containing all the possible elements of a particular situation. **Ex:** A **=** {x: x is the set of All English Alphabets}
Null Set	Set having NO element is called Null set (Empty set/void set). [{ } or ∅] **Ex:** A = {x: x is odd no. divisible by 2} = { } or ∅;
Singleton Set	A set having only one element is called singleton set. **Ex:** A **=** {1}
Equal Set	If every element of A is in B & every element of B is in A, A & B are equal sets. **Ex:** If A = {2, 4, 6} and B = {6, 2, 4} then A = B. **[Order of element is NOT relevant]**
Equivalent Set:	If number of elements in Set A & Set B are SAME, they are equivalent sets. **Ex:** A = {a, b, c} & B = {1, 2, 3}; n(A) = 3 & n(B) = 3, A & B are equivalent sets.
Subset	If all the elements of set A are present in Set B, A is a subset of B. [**A** ⊆ **B**]. **Ex:** A = {1, 2} & B = {1, 2 ,3} then A is subset of B. [**B is said to be a superset of A**] ❖ **PC Note:** In subset, there exist an equal set & null set also. **Ex:** {1,2,3} **Number of Subsets of a set = 2^n** [where 'n' = Number of elements]
Proper Subset	Set A is a proper subset of B if Set A is a subset of Set B but not equal set. A ⊆ B & A ≠ B. **Ex:** A = {1, 2, 3}; Proper Subset of A includes {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3} & { }. ❖ **PC Note:** Proper Subset **does not include Equal set** of the given set. A Null set does not have a Proper subset. **Number of Subsets of a set = 2^n - 1** [where 'n' = Number of elements] **Ex:** Set containing 3 elements has $(2^3 - 1)$ = 7 Proper subsets
Power Set:	The set of all subsets of a set is called Power set. **Ex:** A = {1, 2, 3}; Subset of A include {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3} & { }. **Power set of A = {**{1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, { }**}.**
Disjoint Sets	If Set A & Set B has NO Common element, they are disjoint Sets. **[A∩B = ∅]** **Ex:** A = {a, b, c} & B = {1, 2, 3}; A & B are disjoint sets (no common element.)

CONCEPT 2: OPERATIONS ON SETS

Let A = {1, 2, 3, 6, 8, 9} & B = {2, 4, 6, 8, 10}

Union Of Sets (A∪B)	It contains all elements which are **EITHER** in Set A **OR** in Set B. **Ex:** (A∪B) = {1, 2, 3, 4, 6, 8, 9, 10}.
Intersection Of Sets (A∩B)	It contains all the elements which are in Set A ***AND*** Set B. **Ex:** (A∩B) = {2, 6 8}.
Difference Of Sets (A-B)	Set of elements which are in Set A **but not** in Set B **(B − A):** Set of elements which are in Set B but not in Set A. **Ex:** If A = {1, 2, 3, 5, 7} & B = {1, 3, 6, 7, 15}, **A − B =** {2, 5) & **B − A =** {6, 15}. **CQ2:** U = {1, 2, 3, 4, 5}; A = {1, 2, 5}; A' = {3, 4}.  **1** → A − B **2** → A ∩ B **3** → B − A 1 + 2 + 3 → A∪B
Complimentary Set (A')	Set of elements which are in Universal set **but not** in Set A are called complementary set of A **CQ3:** U = {1, 2, 3, 4, 5, 6, 7, 8, 9}; P = {2, 4, 6, 8}; Q = {1, 2, 3, 4, 5). **Ans:** (i) (P∪Q) = {1, 2, 3, 4, 5, 6, 8}; (ii) (P∪Q)' = {7, 9}; (iii) (P∩Q) = {2, 4}; (iv) (P∩Q)' = {1, 3, 5, 6, 7, 8, 9}; (v) P' = {1, 3, 5, 7, 9}; (vi) Q' = {0, 6, 7, 8, 9}; (vii) P − Q = {6, 8}; (viii) Q − P = {1, 3, 5}. **CQ4:** If U = {x: x is a positive integer < 25}, A = {2,6,8,14,22}, B = {4,8,10,14} then ________. (a) (A ∪ B)'=A' ∪ B' (b) (A ∩ B)' = A'∪ B' (c) (A' ∩ B)'= ϕ (d) None

CONCEPT 3: ALGEBRA OF SETS

A∪B = B∪A	(A∪B)∪C = A∪(B∪C)	A∩(B∪C) = (A∩B) ∪(A∩C)	(A∪B)' = A' ∩ B'	A ∩ A' = ∅
A∩B = B∩A	(A∩B)∩C = A∩(B∩C)	A∪(B∩C) = (A ∪ B) ∩(A ∪ C)	(A∩B)' = A' ∪ B'	A ∪ A' = U
A ∩ A = A	A ∪ A = A	A ∪ ∅ = A	A ∩ U = A	

CQ5: If A = {a, b, c, d, e, f} & B = {a, e, i, o, u} & C = {m, n, o, p, q, r, s, t, u} then

(i) A ∪ B = ______.

(a) {a, b, c, d, e, f, i, o, u} (b) {a, b, c, s, t, u}

(c) {d, e, f, p, q, r} (d) None

(ii) A ∪ C = ______.

(a) {a, b, c, d, e, f, m, n, o, p, q, r, s, t, u} (b) {a, b, c, s, t, u}

(c) {d, e, f, p, q, r} (d) None

(iii) B ∪ C = ______.

(a) {a, e, i, o, u, m, n, p, q, r, s, t} (b) {a, e, i, r, s, t}

(c) {i, o, u, p, q, r} (d) None

(iv) A - B = ______.

(a) {b, c, d, f} (b) {a, e, i, o} (c) {m, n, p, q} (d) None

(v) A ∩ B = ______.

(a) {a, e} (b) {i, o} (c) {o, u} (d) None

(vi) B ∩ C = ______.

(a) {a, e} (b) {i, o} (c) {o, u} (d) None

(vii) A ∪ (B - C) = ______.

(a) {a, b, c, d, e, i, f} (b) {a, b, c, d, e, f, o}

(c) {a, b, c, d, e, f} (d) None

(viii) A ∪ B ∪ C = ______.

(a) {a, b, c, d, e, f, i, o, u, m, n, p, q, r, s, t} (b) {a, b, c, r, s, t}

(c) {d, e, f, p, q} (d) None

(ix) A ∩ B ∩ C = ______.

(a) ϕ (b) {ae} (c) {m n} (d) None

CONCEPT 4: APPLICATIONS OF SET THEORY

▪ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$	▪ $n(B) = n(B-A) + n(A \cap B)$.
▪ $n(A) = n(A-B) + n(A \cap B)$.	▪ $n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$.

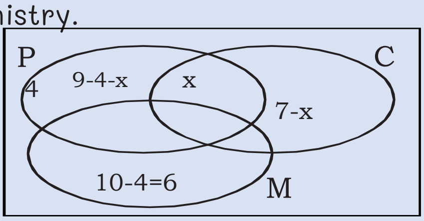
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.
- $n(A \Delta B)$ = No. of elements which belongs to **exactly one** of A or B = $n(A) + n(B) - 2n(A \cap B)$.
- No. of elements in **exactly two** of the sets A, B, C = $n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$.
- No. of elements in **exactly one** of the sets
  A, B, C = $n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$.

SOLVED EXAMPLES

CQ6: Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics? How many teach only Physics?

Ans: Let x be the no. of teachers who teach both Physics & Chemistry.

Thus, $9 - 4 - x + 6 + 7 - x + 4 + x = 20$; $22 - x = 20$; $x = 2$.



Hence, 2 teachers teach both Physics and Chemistry &

3 **(9 − 4 − 2)** teachers teach only Physics.

CQ7: 74% of the Indians like grapes, whereas 68% like bananas. What % of Indians like both grapes & bananas?

Ans: Let P & Q denote the sets of Indians who like grapes and bananas respectively.

Thus, $n(P) = 74$, $n(Q) = 68$ & $n(P \cup Q) = 100$.

We know that $n(P \cap Q) = n(P) + n(Q) - n(P \cup Q) = 74 + 68 - 100 = 42$

Hence, 42% of the Indians like both grapes and bananas.

CQ8: In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like

(i) Maths only.   (ii) Science only.   (iii) Maths or Science.   (iv) Not Maths & Science.

Ans: Let M = students who like Maths & S = students who like Science;-

$n(M) = 40$, $n(S) = 36$ & $n(M \cap S) = 24$.

(i) $n(M) - n(M \cap S) = 40 - 24 = 16$.   (ii) $n(S) - n(M \cap S) = 36 - 24 = 12$.

(iii) $n(M \cup S) = n(M) + n(S) - n(M \cap S) = 40 + 36 - 24 = 52$. (iv) $n(M \cup S)^C = 60 - n(M \cup S) = 60 - 52 = 8$.

CONCEPT 5: PRODUCT SET

ORDERED PAIR:- Two elements 'a' & 'b', listed in a specific order, form an ordered pair. It is denoted by (a, b).

Here 'a' is called '1st element' or 1st co-ordinate & 'b' is called 2nd element or 2nd co-ordinate.

Note: (a, b) ≠ {a, b}.

If a ≠ b, then (a, b) ≠ {b, a}. Thus if (a, b) = (c, d), it means that a = c & b = d.

- In set theory, repetition of elements is meaningless & thus if we have set A = {5, 5}, it means we have only one element in the set.
- But for ordered pairs, (5, 5) means 5 belongs in both the sets under consideration.

CARTESIAN PRODUCT OF SETS:- Set of all ordered pairs (a, b) such that a ∈ A & b ∈ B, is called Cartesian product of A & B. It is denoted by A × B. Thus, **A × B = {(a, b): a ∈ A & b ∈ B}.**

Cardinal Number:

- Number of elements in a set is known as its cardinal number.
- Cardinal number of set A is denoted as n(A).

Number of Elements of n(A × B) = n(A) × n(B).

CQ9: If P = {1, 3, 6} & Q {3, 5}. Find P × Q & Q × P.

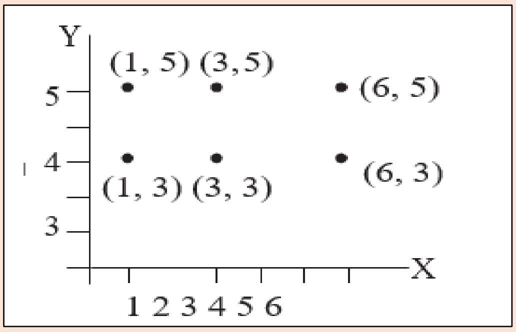
Ans:

P × Q = {(1, 3), (1, 5), (3, 3), (3, 5), (6, 3), (6, 5)};

Q × P = {(3, 1), (3, 3), (3, 6), (5, 1), (5, 3), (5, 6)}

It is noted that ordered pairs (3, 5) & (5, 3) are not equal.

So, P×Q ≠ Q×P; but n (P×Q) = n(Q×P).



CQ10: If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)}, find A and B. **[Ans:** A = {3, 5} & B = {2, 4}]

CQ11: A = {1, 2, 3}, B = {4, 5}. Find A×B & n(A×B). **[Ans:** 6]

CQ12: If the set P has 3 elements, Q has 4, & R has 2, then the set P x Q x R contains ________.

(a) 9 elements (b) 20 elements (c) 24 elements (d) None.

CQ13: If A = (1,2,3,5,7) and B = (1,3,6,10,15) then cardinal number of A - B is ________.

(a) 3 (b) -4 (c) 6 (d) None of these

SETS – QUESTION BANK

SN	CHAPTER 7A. SETS	Ans
Q1	If A = {a, b, c}, then n(p(A)) is ______. (a) 3 (b) 8 (c) 6 (d) 1	B
Q2	The set {2^x: x is any positive rational number} is ______. (a) Infinite set (b) Null set (c) Finite set (d) None	A
Q3	{$\frac{n(n+1)}{2}$: n is a positive integer} is ______. (a) A finite set (b) An infinite set (c) Is an empty set (d) None	B
Q4	State whether the following sets are finite, infinite or empty (i) X= {1, 2, 3,.... 500} (ii) Y = {y: y = a^2; a is an integer) (iii) A = {x: x is a positive integer multiple of 2} (iv) B = {x: x is an integer which is a perfect root of 26 < x < 3.5} (a) Finite, Infinite, Infinite, Empty (b)Infinite, Infinite, Finite, Empty (c) Infinite, Finite, Infinite, Empty (d) None	A
Q5	If E = {1, 2, 3, 4, 5, 6, 7, 8, 9}, the subset of E satisfying 5 + x >10 is ______. (a) {5,6,7,8,9} (b) {6,7,8,9} (c) {7,8,9} (d) None	B
Q6	If A = {1 2 ....9}; B = {2 4 6 8}; C = {1 3 5 7 9}; D = {3 4 5}; E = {3 5}	
	(i) What is the set S if it is also given that S ⊂ D and S ⊄ A (a) {3 5} (b) {2 4} (c) {7 9} (d) None	A
	(ii) What is set S if it is also given that S⊂B and S ⊄C (a) {3 5} (b) {2 4} (c) {5 6 7 8 9} (d) {57 9}	B
Q7	Following set notations represent: A ⊂ B; x ∉ A; A ⊃ B; {0}; A⊄B (a) A is a proper subset of B; x is not an element of A; A contains B; singleton with an only element zero; A is not contained in B. (b) A is a proper subset of B; x is an element of A; A contains B; singleton with an only element zero; A is contained in B. (c) A is a proper subset of B; x is not element of A; A does not contains B; contains elements other than zero; A is not contained in B. (d) None	A
Q8	If P = {1,2,3,4}; Q = {2,4,6} then P ∪ Q = ______. (a) {1,2,3,6} (b) {1,4,6} (c) {1,2,3,4,6} (d) None	C
Q9	A = {2, 3, 5, 7} & B = {4, 6, 8, 10} then A ∩ B can be written as ______. (a) {} (b) {ϕ} (c) (A ∪B)' (d) None	A
Q10	If A △ B = (A-B) ∪ (B-A) and A = {1,2,3,4}, B = {3,5,7} then A △ B is ______. (a) {1,2,4,5,7} (b) {3} (c) {1,2,3,4,5,7} (d) None	A

Q11	Identify the elements of P if set Q = {1, 2, 3} and PxQ = {(4,1); (4,2); (4,3); (5,1); (5,2); (5,3); (6,1); (6,2); (6,3)} (a) {3 ,4,5} (b) {4 ,5 ,6} (c) {5 ,6 ,7} (d) None	**B**
Q12	If A = {2 ,3}; B = {4, 5}; C= {5, 6} then	
	(i) A x (B∪ C) (a) {(2 ,4) ;(2 ,5) ;(2 ,6) ;(3 ,4); (3 ,5); (3 ,6)} (b) {(2 ,5); (3 ,5)} (c) {(2 ,4); (2 ,5) ;(3 ,4) ;(3 ,5) ;(4 ,5); (4 ,6); (5 ,5); (5 ,6)} (d) None	**A**
	(ii) The set A x (B ∩ C) is _______. (a) {(2, 4); (2 ,5); (2, 6) ;(3 ,4); (3, 5); (3, 6)} (b) {(2 ,5); (3, 5)} (c) {(2 ,4); (2,5) ;(3 ,4); (3 ,5) ;(4,5); (4 ,6); (5, 5); (5, 6)} (d) None	**B**
	(iii) The set (A x B) ∪ (B x C) is _______. (a) {(2 ,4); (2, 5); (2 ,6) ;(3 ,4) ;(3 ,5) ;(3, 6)} (b) {(2, 5); (3, S)} (c) {(2, 4); (2, 5); (3, 4); (3, 5) ;(4,5); (4, 6); (5, 5); (5, 6)} (d) None	**C**
Q13	S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {0, 2, 4, 6, 8}, and Q = {1, 2, 3, 4, 5}, then Q' is ___. (a) {0, 6, 7, 8, 9} (b) {1, 2, 4, 5, 6} (c) {1, 3, 5, 7, 9} (d) {0, 2, 4, 6, 8}	**A**
Q14	If A = {(1 2 3 4} B = {2 3 7 9} and C = {1 4 7 9} then _______. (a) A∩B ≠ϕ, B∩C≠ϕ, A∩B∩C=ϕ (b) A∩B = ϕB∩C = ϕ A∩B∩C = ϕ (c) A∩B ≠ϕA∩C≠ A∩B∩C=ϕ (d) None	**A**
Q15	N is the set of natural numbers and I is the set of positive integers, then _______. (a) N = I (b) N⊂ I (c) N⊃ I (d) None	**A**
Q16	R is the set of positive rational number and E is the set of real numbers then _____. (a) R ⊂ E (b) R ⊆ E (c) E ⊇ R (d) None	**B**
Q17	E is a set of positive even no. & O is a set of positive odd no., then E ∪ O is _______. (a) Set of whole numbers (b) N (c) A set of rational number (d) None	**B**
Q18	In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is _______. (a) 6 (b) 7 (c) 1 (d) None	**D**
Q19	If A has 32 elements, B has 42 elements & A ∪ B has 62 elements, the number of elements in A ∩ B = _______. (a) 12 (b) 74 (c) 10 (d) None	**A**
Q20	In a group of 40 children 16 like wicket but not movie and 26 like cricket. The number of children like movie but not cricket are _______. (a) 12 (b) 24 (c) 2 (d) None	**D**
Q21	Sample of income group of 1,172 families was surveyed and noticed for income groups <Rs.6000/- Rs.6000/- to Rs.10999/-, Rs.11000/- to Rs.15999/-, Rs.16000 and	

	above, no TV set is available to 70, 50, 20, 50 families, one set is available to 152, 308, 114, 46 families, and two or more sets are available to 10, 174, 84, 94 families. A = {x|x is a family owning two or more sets} B = {x|x is a family with one set} C = {x|x is a family with income less than Rs.6000/-} D = {x|x is a family with income Rs.6000/- to Rs.10999/-} E = {x|x is a family with income Rs.11000/- to Rs.15999/-}	
	(i) Find the number of families in each of the following sets (i) C ∩ B (ii) A ∪ E (a) 152, 580 (b) 152 20 (c) 152 50 (d) None	A
	(ii) Find the number of families in each of the following sets (1) (A ∪ B') ∩ E & (2) (C∪D∪E)∩(A∪ B)' (a) 20, 50 (b) 152, 20 (c) 152, 50 (d) None	A
	(iii) Express the following sets in set notation (1) {x|x is a family with one set and income of less than Rs.11000/-) (2) {x|x is a family with no set and income over Rs.16000/-) (a) (C ∪D) ∩B (b) (A ∪B)' ∩ (C'∪D'∪E') (c) both (d) None	C
	(iv) Express the following sets in set notation (i) {x|x is a family with two or more sets or income of Rs. 11000/-1 o Rs. 15999/-} (ii) {x|x is a family with no set} (a) (A U E) (b) (A U B)' (c) Both (d) None	C
Q22	Out of 60 students 25 failed in paper (1), 24 in paper (2), 32 paper in (3), 9 in paper (1) alone, 6 in paper(2) alone, 5 in papers (2) and (3), and 3 in papers (1) and (2).	
	(i) Find how may failed in all the three papers. (a) 10 (b) 60 (c) 50 (d) None	A
	(ii) How many passed in all the three papers? (a) 10 (b) 60 (c) 50 (d) None	A
Q23	At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. The number of women doctors attending the conference is ______. (a) 2 (b) 4 (c) 1 (d) None	C
Q24	On a survey of 100 boys it was found that 50 used white shirt 40 red and 30 blue. 20 were habituated in using both white and red shirts 15 both red and blue shirts and 10 blue and white shirts. Find the number of boys using all the colours. (a) 20 (b) 25 (c) 30 (d) None	B
Q25	Out of total 150 students 45 passed in Accounts, 50 in Maths, 30 in Costing, 30 in both Accounts and Maths, 32 in both Maths and Costing, 35 in both Accounts and	B

	Costing. 25 students passed in all the three subjects. Find the number who passed at least in any one of the subjects. (a) 63 (b) 53 (c) 73 (d) None	
Q26	If four members a, b, c, d of a decision making body are in a meeting to pass a resolution where rule of majority prevails. Given that a, b, c, d owns 50%, 20%, 15%, 15% shares each.	
	(i) List the winning conditions. (a) {a, b}; {a, c}; {a, d}; {a, b, c}; {a, b, d}; {a, b, c, d} (b) {b, c, d}; {a} (c) {b, c}; {b, d}; {c, d}; {b}; {c}; {d} (d) None	**A**
	(ii) List the blocking conditions. (a) {a b} {a c} {a d} {a b c} {a b d} {a b c d} (b) {b c d}, {a} (c) {b c} {b d} {c d} {b} {c} {d} ϕ (d) None	**B**
	(iii) List the losing conditions. (a) {a b} {a c} {a d} {a b c} {a b d} {a b c d} (b) {b c d}, {a} (c) {b c} {b d} {c d} {b} {c} {d} ϕ (d) None	**C**
Q27	Out of 1000 students 658 failed in the aggregate 16 in the aggregate and in group-I 434 in aggregate and in group-II 372 in group-I 590 in group-II and 126 in both the groups.	
	(i) Find out how many failed in all the three (a) 106 (b) 224 (c) 206 (d) 464	**A**
	(ii) How many failed in the aggregate but not in group-II? (a) 106 (b) 224 (c) 206 (d) 464	**B**
	(iii) How many failed in group-I but not in the aggregate. (a) 106 (b) 224 (c) 206 (d) 464	**C**
	(iv) How many failed in group-II but not in group-I? (a) 106 (b) 224 (c) 206 (d) 464	**D**
	(v) How many failed in the aggregate or group-II but not in group-I? (a) 206 (b) 464 (c) 628 (d) 164	**C**
	(vi) How many failed in the aggregate but not in group-I and group-II? (a) 206 (b) 464 (c) 628 (d) 164	**D**
Q28	If A = {2, 5, 6, 8}, then n (A) is _______. (a) 2 (b) 4 (c) 5 (d) 1	**B**
Q29	If A= (a, b, c, d) list the element of power set P(A) (a) {ϕ}, {a}, {b}, {c}, {d}, {a b}, {a c), {a d}, {b c}, {b d}, {c d} (b) {a b c} {a b d} {a c d) {b c d} (c) {a b c d}	**D**

	(d) All the above elements are in P(A)	
Q30	The set of cubes of the natural number is ______. (a) A finite set (b) An infinite set (c) A null set (d) None	**B**
Q31	The set of squares of positive integers is ______. (a) A finite set (b) Null set (c) Infinite set (d) None	**C**
Q32	Equal sets are ______. (a) Equivalent (b) Null (c) Disjoint (d) None	**A**
Q33	If cardinal number of two finite sets is same, then the sets are ______. (a) Equivalent (b) Equal (c) Null (d) Singleton	**A**
Q34	The range set of a constant function is a ______. (a) Disjoint set (b) Singleton set (c) Void set (d) Infinite set	**B**
Q35	Let A = {a, b}. Set of subsets of A is called power set of A denoted by P(A). Now n(P(A)) is ______. (a) 2 (b) 4 (c) 3 (d) None	**B**
Q36	The number of subsets of the set {2, 3, 5} is ______. (a) 3 (b) 8 (c) 6 (d) None	**B**
Q37	$A \cup A$ is equal to ______. (a) A (b) E (c) φ (d) None	**A**
Q38	$A \cup A'$ is equal to ______. (a) A (b) E (c) ϕ (d) None	**B**
Q39	$A \cup E$ is equal to ______. (a) A (b) E (c) ϕ (d) None	**B**
Q40	$A \cap A$ is equal to ______. (a) ϕ (b) A (c) E (d) None	**A**
Q41	$A \cap \varphi$ is equal to ______. (a) A (b) E (c) φ (d) None	**C**
Q42	$A \cap A'$ is equal to ______. (a) E (b) φ (c) A (d) None	**B**
Q43	If A = {1,2,3,4} and B = {2,4} then $A \cap B$ can be written as ______. (a) ϕ (b) {1,3} (c) {2,4} (d) {0}	**C**
Q44	If A = {1, 2, 3, 4}, B = {5, 6, 7} then cardinal number of the set A x B is ______. (a) 7 (b) 1 (c) 12 (d) None	**C**
Q45	If A = {1, 2, 3}, B = {4, 5}, then A x B is ______. (a) {(1,4), (1,5), (2,4), (2,5), (3, 4), (3, 5)} (b) {(1,2), (2,3), (3, 4), (4, 5), (5,1), (5,2)}	**A**

	(c) {(4,1), (4,2), (5, 1), (5, 2), (3,1), (3,2)} (d) {(1,2), (2,3), (3,4), (4,5)}	
Q46	S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {0, 2, 4, 6, 8}, and Q = {1, 2, 3, 4, 5}, then P' is ___. (a) {0, 6, 7, 8, 9} (b) {1, 2, 4, 5, 6} (c) {1, 3, 5, 7, 9} (d) {0, 2, 4, 6, 8}	**C**
Q47	S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {0, 2, 4, 6, 8}, and Q = {1, 2, 3, 4, 5}, then P'∩Q' is__. (a) {7,6} (b) {2,4} (c) {5, 9} (d) {7, 9}	**D**
Q48	S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {0, 2, 4, 6, 8}, and Q = {1, 2, 3, 4, 5}, then P'∪Q' is__. (a) {0, 1, 3, 5, 6, 7, 8, 9} (b) {1, 2, 4, 5, 6, 7, 8, 9} (c) {0, 1, 2, 3, 5, 7, 9} (d) {0, 2, 4, 6, 8}	**A**
Q49	If A = {3, 4, 5, 6}; B = {3, 7, 9, 5} &C = {6, 8, 10, 12, 7} & U = {3, 4, ...11, 12, 13} then	
	(i) A' is _______. (a) {7 8 12 13} (b) {4 6 8 10 ....13} (c) {3 4 5 7 9 11 13} (d) None	**A**
	(ii) The set B' is _______. (a) {7 8 12 13} (b) {4 6 8 10 ....13} (c) {3 4 5 9 11 13} (d) None	**B**
	(iii) The set C' is _______. (a) {7 8 12 13} (b) {4 6 8 10 ....13} (c) {3 4 5 9 11 13} (d) None	**C**
	(iv) The set (A')' is _______. (a) {3 4 5 6} (b) {3 7 9 5} (c) {8 10 11 12 13} (d) None	**A**
	(v) The set (B')' is _______. (a) {3 4 5 6} (b) {3 7 9 5} (c) {8 10 11 12 13} (d) None	**B**
	(vi) The set (A ∪ B)' is _______. (a) {3 4 5 6} (b) {3 7 9 5} (c) {8 10 11 12 13} (d) None	**C**
	(vii) The set (A ∩ B)' is _______. (a) {8 10 11 12 13} (b) {4 6 7 ... 13} (c) {3 4 5 7 8 ... 13} (d) None	**B**
	(viii) The set A' ∪ C' is _______. (a) {8 10 11 12 13} (b) {4 6 7 ...13} (c) {3 4 5 7 8 ....13} (d) None	**C**
Q50	A has 70 elements, B has 32 elements and A ∩ B has 22 elements then A∪ B is ____. (a) 60 (b) 124 (c) 80 (d) None	**C**
Q51	If n(P) = 3 and n(Q) = 4, then n(PxQ) is _______. (a) 3 (b) 4 (c) 12 (d) 1	**C**
Q52	When 5x < 24 & x belongs to set of natural numbers then the solution set is _______. (a) {1, 2, 3, 4, 5} (b) {1, 2, 3, 4} (c) {1, 2, 3} (d) {0, 1, 2, 3, 4}	**B**
Q53	If V= {x: x+2 = 0} R = {x: $x^2+2x = 0$} and S = {x: $x^2+x-2=0$} then V, R, S are equal for which value of x?	**C**

	(a) 0 (b) -1 (c) -2 (d) None	
Q54	For $3x + 1 \leq 16$ & x belongs to set of natural number, the solution set is ______. (a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5} (c) {1, 2, 3} (d) {1, 2, 3, 4, 5, 6}	B
Q55	If A = {1,2,3,5,7}, and B = $\{x^2 : X \in A\}$ (a) n(b) = n(A) (b) n(B) > n(A) (c) n(A) ≠ n(B) (d) n(A) < n(B)	A
Q56	If A = {1, 2} and B = {2,3} then A x B is equal to ______. (a) {(1,2), (1,3), (2,2), (2,3)} (b) {(2,1), (2,2), (3,1), (3,2)} (c) {(1,1), (1,2), (2,2), (2,1)} (d) {(3,1), (2,1), (3,3), (2,3)}	A
Q57	A survey shows that 68% of women like apples, 74% of women like orange. What percentage like both? (a) 12% (b) 6% (c) 21% (d) 42%	D
Q58	A survey shows that 74% of the Indians like grapes, whereas 68% like bananas. What % of Indians like both grapes and bananas? (a) 32% (b) 26% (c) 42% (d) 50%	C
Q59	In a class 30 students, 20 students like maths, 18 like science and 12 like both the subject. Find the number of student who likes no subject. (a) 4 (b)5 (c) 8 (d) None	A
Q60	Complaints about works canteen had been about Mess(M) Food(F) and Services(S). Total complaints 173 were received as follows - n(M) = 110; n(F) = 55; n(S) = 67; $n(M \cap F \cap S') = 20$; $n(M \cap S \cap F') = 1$; $n(F \cap S \cap M') = 16$.	
	(i) Determine the complaints about all the three. (a) 6 (b) 43 (c) 35 (d) None	A
	(ii) Determine the complaints about two or more than two. (a) 6 (b) 53 (c) 35 (d) None	B
Q61	After qualifying out of 400 professionals 112 joined industry 120 started practice and 160 joined as paid assistants. There were 32 who were in both practice and service 40 in both practice and assistantship and 20 in both industry and assistantship. There were 12 who did all the three.	
	(i) Find how many could not get any of these (a) 88 (b) 244 (c) 122 (d) None	A
	(ii) Find how many of them did only one of these (a) 88 (b) 244 (c) 122 (d) None	A
Q62	The number of subsets of the sets {6, 8,11} is ______. (a) 9 (b) 6 (c) 8 (d) None	C
Q63	If A = {1, 3, 5}, B = {0, 2} then $A \cup B$ is ______. (a) {0, 1, 2, 3, 5) (b) 0 (c) {1,3,5,7,9,13} (d) None	A
Q64	If A = {3,5,7} B = {0,2,4,6} then $A \cup B$ is ______.	B

	(a) φ (b) {0,2,3,4,5,6, 7} (c) {0} (d) None	
Q65	If A = {1, 3, 5, 7, 9}, D = {2, 4, 6, 8, 10} then A ∪ B is ______. (a) {0} (b) {1,2,3,4,5,6,7,8,9,10} (c) φ (d) None	B
Q66	If P = {1, 2, 3, 5, 7} & Q = {1, 3, 6, 10, 15}	
	(i) The cardinal number of P ∩ Q is ______. (a) 3 (b) 2 (c) 0 (d) None	B
	(ii) The cardinal number of P ∪ Q is ______. (a) 10 (b) 9 (c) 8 (d) None	B
Q67	If P = {3, 4, 5, 6} then cardinal number of P is ______. (a) 3 (b) 5 (c) 4 (d) 6	C
Q68	The null set is represented by ______. (a) {Φ} (b) {0} (c) Φ (d) None	C
Q69	If A = {1, 2, 3}, then P (A) is ______. (a) 3 (b) {{1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, Φ} (c) {1,2,3} (d) {{1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}}	B
Q70	The number of subsets of a set containing n elements is ______. (a) 2^n (b) 2^{-n} (c) n (d) None	A
Q71	A set containing 4 elements have ______. (a) 15 subsets (b) 16 subsets (c) 14 subsets (d) 13 subsets	B
Q72	What is the relationship between the following sets? A = {x:x is a letter in the word flower} B = {x:x is a letter in the world flow} C = {x:x is a letter in the world wolf} D = {x:x is a leter in the word follow} (a) B = C = D and all these are subsets of the set A (b) B = C ≠ D (c) B ≠ C ≠ D (d) None	A
Q73	If P = {1,2,3,4}: Q = {2,4,6} then P∪Q (a) {1,2,3,6} (b) {1,4,6} (c) {1,2,3,4,6} (d) None	C
Q74	If P is a set of natural number then P ∩ P' is ______. (a) ϕ (b) Sample Space. (c) 0 (d) (P ∪ P')'	A
Q75	(A ∪ B)' is equal to ______. (a) (A∩B)' (b) A∪B' (c) A'∩ B' (d) None	C
Q76	(A ∩ B)' is equal to ______. (a) (A'∪B') (b) A∪B' (c) A'∩ B' (d) None	A
Q77	If A = {a b c d e f} B = {a e l o u} and C = {m n o p q r s t u} then	
	(i) A ∪ B is ______.	A

	(a) {a b c d e f i o u} (b) {a b c s t u} (c) {d e f p q r} (d) None	
	(ii) A ∪ C is _______. (a) {a b c d e f m n o p q r s t u} (b) {a b c s t u} (c) {d e f p q r} (d) None	A
	(iii) B ∪ C is _______. (a) {a e I o u m n p q r s t} (b) {a e i r s t} (c) {i o u p q r} (d) None	A
	(iv) A - B is _______. (a) {b c d f} (b) {a e i o} (c) {m n p q} (d) None	A
	(v) A ∩ B is _______. (a) {a e} (b) {i o} (c) {o u} (d) None	A
	(vi) B ∩ C is _______. (a) {a e} (b) {i o} (c) {o u} (d) None	C
	(vii) A ∪ (B - C) is _______. (a) {a b c d e f i} (b) {a b c d e f o} (c) {a b c d e f u} (d) None	A
	(viii) A ∪ B ∪ C is _______. (a) {a b c d e f i o u m n p q r s t} (b) {a b c r s t} (c) {d e f n p q} (d) None	A
	(ix) A ∩ B ∩ C is _______. (a) φ (b) {a e} (c) {m n} (d) None	A
Q78	If the set P has 3 elements, Q four and R two then the set P×Q×R contains (a) 9 elements (b) 20 elements (c) 24 elements (d) None	C
Q79	If the set P has 6, Q has 5 and R has 2 elements then the set P×Q×R contains (a) 13 (b) 9 (c) 60 (d) None	C
Q80	If A x B = {(3, 2), (3, 4), (5, 2), (5, 4)}, then find A and B. (a) A = {3, 5} and B = {2, 4} (b) A = {3, 4} and B = {2, 5} (c) A = {3, 2} and B = {5, 4} (d) A = {5, 4} and B = {2, 3}	A
Q81	If A = (1,2,3,5,7) and B= (1,3,6,10,15) then cardinal number of A - B is _______. (a) 3 (b) -4 (c) 6 (d) None	A
Q82	If V = {0 1 2 ....9} X = {0 2 4 6 8} Y = {3 5 7} and Z = {3 7} then	
	(i) Y ∪ Z, (V ∪ Y) ∩ X, (X ∪ Z) ∪ V are respectively (a) {3 5 7} {0 2 4 6 8} {0 1 2 ....9} (b) {2 4 6} {0 2 4 6 8} {0 1 2 ...9} (c) {2 4 6} {0 1 2 ....9} {0 2 4 6 8} (d) None	A
	(ii) (X ∪ Y) ∩ Z and (Φ ∪ V) ∩ Φ are respectively	B

	(a) {0 2 4 6 8} Φ (b) {3 7} Φ (c) {3 5 7} Φ (d) None	
Q83	$\{1-(-1)^x\}$ for all integral x is the set is ______. (a) {0} (b) {2} (c) {0,2} (d) None	**C**
Q84	The set {x|0<x<5} represents the set when x may take integral values only (a) {0,1,2,3,4,5} (b) {1,2,3,4} (c) {1,2,3,4,5} (d) None	**B**
Q85	If the universal set is X = {x: x ∈ N l≤x≤12} and A {1 9 10} B ={3 4 6 11 12} and C = {2 5 6} are subsets of X	
	(i) The set A ∪(B ∩ C) is ______. (a) {3 4 6 12} (b) {16 9 10} (c) {2 5 6 11} (d) None	**B**
	(ii) The set (A∪B) ∩ (A∪C) is ______. (a) {3 4 6 12} (b) {16 9 10} (c) {2 5 6 11} (d) None	**B**
Q86	Universal set E = {x|x is a positive integer < 25}, A = {2,6,8,14,22}, B = {4,8,10,14} then (a) (A ∪ B)'=A' ∪ B' (b) (A ∩ B)'=A' ∪ B' (c) (A' ∩ B)'= Φ (d) None	**B**
Q87	Represent the following sets in set notation set of all alphabets in English language set of all odd integers less than 25 set of all odd integers set of positive integers x satisfying the equation $x^2 + 5x + 7 = 0$ - (a) A = {x:x is an alphabet in English} | = {x:x is an odd integer > 25} = {2 4 6 8....} | = $\{x:x^2+5x+7=0\}$ (b) A = {x:x is an alphabet in English} | = {x:x is an odd integer < 25} = {1 3 5 7....} | = $\{x:x^2+5x+7=0\}$ (c) A = {x:x is an alphabet in English} | = {x:x is an odd integer ≤ 25} = {1 3 5 7....} | = $\{x:x^2+5x+7=0\}$ (d) None	**B**
Q88	Re-write the following sets in a set builder form A={a e I o u} B={1 2 3 4 ......} C is a set of integers between - 15 and 15. (a) A = {x:x is a constant} B = {x:x is an irrational number} C= {X:-15<X<15∧x is a fraction} (b) A = {x:x is a vowel} B = {x:x is a natural number} C= {X:-15≥X≥15∧X is a whole number} (c) A = {x:x is a vowel} B = {x:x is a natural number} C= {X:-15<X<15∧X is a whole number} (d) None	**C**
Q89	Comment on the correctness or otherwise of the following statements (i) {a b c} = {c b a} (ii) {a c a d c d} ⊃ {a c d} (iii) {b} ∉ {{b}} (iv) {b}⊂ {{b}}and Φ⊂{{b}} (a) Only (iv) is incorrect (b) (i) (ii) are incorrect (c) (ii) (iii) are incorrect (d) All are incorrect	**A**

Q90	If A = {a b c} B = {a b} c = {a b d} D = {c d} m E ={d} state which of the following statements are correct (i) B⊂ A (ii) D ≠C (iii) C⊃E (iv) D = E (v) D⊂B (vi) D = A (vii) B ⊄ C (viii) E⊂ A (ix) E⊄B (x) a ∈ A (xi) a⊂A (xii) {a}∈ A (xiii) {a}⊂ A (a) (i) (ii)(iii) (ix) (x) (xiii) only are correct (b) (ii) (iii) (iv) (x) (xii) (xiii) only are correct (c) (i) (ii) (iv) (ix) (xiii) only are correct (d) None	**A**
Q91	Let A = {0} B = {0 1} C = Φ D = {Φ} E = {x|x is a human being 300 years old} F = {x|x ∈A and x ∈ B} state which of the following statements are true (i) A⊂ B (ii) B = F (iii) C⊂ D (iv) C = E (v) A = F (vi) F = 1 (vii) E = C =D (a) (iii) (iv) and (v) only are true (b) (i) (ii) (iii) and (iv) only are true (c) (i) (ii) (iii) and (vi) only are true (d) None	**B**
Q92	If A = {0 1} state which of the following statements are true (i) {1} ⊂ A (ii) {1} ∈ A (iii) Φ ∈ A (iv) 0 ∈ A (v) 1 ⊂ A (vi) {0} ⊂ A (vii) Φ ⊂ A (a) (i) (iv) and (vii) only are true (b) (i), (iv) and (vi) only are true (c) (ii), (iii) and (vi) only are true (d) None	**A**
Q93	Out 2000 staff 48% preferred coffee 54% tea and 64% cocoa. Of the total 28% used coffee and tea 32% tea and cocoa and 30% coffee and cocoa. Only 6% did none of these.	
	(i) Find the number having all the three. (a) 360 (b) 280 (c) 160 (d) None	**A**
	(ii) Find the number having tea and cocoa but not coffee. (a) 360 (b) 280 (c) 160 (d) None	**B**
	(iii) Find the number having only coffee. (a) 360 (b) 280 (c) 160 (d) None	**C**
Q94	Out of 1000 students 658 failed in the aggregate 16 in the aggregate and in group-I 434 in aggregate and in group-II 372 in group-I 590 in group-II and 126 in both the groups.	
	(i) Find out how many failed in all the three (a) 106 (b) 224 (c) 206 (d) 464	**A**

	(ii) How many failed in the aggregate but not in group-II? (a) 106 (b) 224 (c) 206 (d) 464	B
	(iii) How many failed in group-I but not in the aggregate. (a) 106 (b) 224 (c) 206 (d) 464	C
	(iv) How many failed in group-II but not in group-I? (a) 106 (b) 224 (c) 206 (d) 464	D
	(v) How many failed in the aggregate or group-II but not in group-I? (a) 206 (b) 464 (c) 628 (d) 164	C
	(vi) How many failed in the aggregate but not in group-I and group-II? (a) 206 (b) 464 (c) 628 (d) 164	D
Q95	Asked if you will cast your vote for a party the following feed back is obtained	

	Yes	**No**	**Don't know**
Adult Male	10	20	5
Adult Female	20	15	5
Youth over 18 years	10	5	10

If A = set of Adult Males, C =Common set of Women and Youth Y= set of positive opinion, N = set of negative opinion then

	(i) n(A') is _______. (a) 25 (b) 40 (c) 20 (d) None	A
	(ii) The set n (A ∩ C) is _______. (a) 25 (b) 40 (c) 20 (d) None	B
	(iii) The set n (Y ∪ N)' is _______. (a) 25 (b) 40 (c) 20 (d) None	C
	(iv) The set n (A ∩ (Y ∩ N)' is _______. (a) 25 (b) 40 (c) 20 (d) None	C
Q96	A survey of 1000 customers revealed the following in respect of their buying habits of different grades:	

A grade only	180
A and C grades	80
C grades	480
A grade but not B grade	230
A grade	280
C and B grades	80
None	240

	(i) How many buy B grade? (a) 280 (b) 180 (c) 50 (d) none	B
	(ii) How many buy C grade if any only if they do not buy B grade? (a) 280 (b) 400 (c) 50 (d) none	B
	(iii) How many buy C and B grades but not the A grade? (a) 280 (b) 400 (c) 50 (d) none	C
Q97	A marketing research team interviews 100 people about their drinking habits of tea coffee or milk or A B C respectively. Following data is obtained but the Manager is not sure whether these are consistent.	A

Category	**No.**
ABC	3
AB	7
BC	13
AC	18
A	42
B	17
C	27

(a) Inconsistent since 42+17+27-7-13-18+3 ≠ 50 (b) Consistent
(c) Cannot determine due to data insufficiency (d) None

Q98 In a market survey you have obtained the following data which you like to examine regarding its correctness — **A**

% did not use the Brand		Percentage answering "Yes"	
April	59	May & June	33
May	62	April & June	31
June	62	April, May, June	22
April & May	35		

(a) Inconsistent since 59+62+62-35-33- 31+22≠ 100 (b) Consistent
(c) Cannot determine due to data insufficiency (d) None

Q99 In his report an Inspector of an assembly line showed in respect of 100 units the following which you are require to examine. — **A**

Defect	**No. of pieces**
Strength (S)	35
Flexibility (F)	40
Radius (R)	18

S and F	7
S and R	11
F and R	12
S F R	3

Is the report consistent and be accepted?
(a) No. of pieces with radius defect alone was -2 which was impossible. The report is inconsistent.
(b) Report may be accepted
(c) Cannot be determined due to data insufficiency
(d) None

Q100	A = {2,3}, B = {4,5}, C = {5,6} then Ax(B∩C) = ______. (a) {(5,2), (5,3)} (b) {(2,5), (3,5)} (c) {(2,4), (5,3)} (d) {(3,5), (2,6)}	**B**
Q101	Number of subsets of the set {1,2,3,4} is ______. (a) 13 (b) 12 (c) 16 (d) 15	**C**
Q102	Number of subsets of the set A = {1,2,3,4,5,6,7,8} is ______. (a) 36 (b) 128 (c) 256 (d) None	**C**
Q103	If A = {1, 3, 5, 7, ...}, B = {2, 4, 6, 8, ...} then A ∩ B is equal to ______. (a) Set of all integers (b) Set of all positive integers (c) ϕ (d) None of these	**B**
Q104	If A = {1, 2, 3, 4, 5} & B = {6, 7, 8} then cardinal number of AxB is ______. (a) 15 (b) 5 (c) 3 (d) 8	**A**
Q105	{x | 0 < x < 6, x take integral values} represents the set ______. (a) {0, 1, 2, 3, 4, 5} (b) {1,2,3,4,5} (c) {1,2,3,4,5,6} (d) {1,2,3,4}	**B**
Q106	If U = {1, 2, ...9} be the universal set A = {1, 2, 3 ,4} & B = {2 ,4, 6, 8}	
	(i) Then the set A U B is ______. (a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6 ,7 ,8,9} (d) {1 ,3,5,6 ,7 ,9}	**A**
	(ii) Set A ∩ B is ______. (a) {1 ,2 ,3 ,4 ,6 ,8} (b) {2, 4} (c) {5 ,6, 7 ,8, 9} (d) {1,3,5,6, 7 ,9}	**B**
	(iii) The set A' is ______. (a) {1,2 ,3 ,4 ,6 ,8} (b) {2 ,4} (c) {5 ,6 ,7, 8, 9} (d) {1 ,3 ,5 ,6 ,7, 9}	**C**
	(iv) The set (A ∪ B)' is ______. (a) {1, 2 ,3 ,4 ,6 ,8} (b) {2 ,4} (c) {5, 7, 9} (d) {6, 8, 9}	**C**
	(v) The set (A ∩ B)' is ______. (a) {1, 2 ,3 ,4 ,6 ,8} (b) {2 ,4} (c) {5, 6 ,7 ,8 ,9} (d) {1 ,3, 5, 6, 7, 8, 9}	**D**
Q107	Let P = (1, 2, x); Q = (a, x, y); R = (x, y, z) then ______.	

	(i) P x Q is ______. (a) {(1a), (1x), (1y); (2a), (2x), (2y); (xa), (xx), (xy)} (b) (1x), (1y), (1z); (2x), (2y), (2z); (xx), (xy), (xy)} (c) {(ax), (ay), (az); (xx), (xy), (xz); (yx), (yy), (yz)} (d) {(1x), (1y); (2x). (2y); (xx), (xy)}	A
	(ii) The set P x R is ______. (a) {(1a), (1x), (1y); (2a), (2x), (2y); (xa), (xx), (xy)} (b) {(1x), (1y), (1z); (2x), (2y), (2z); (xx), (xy), (xz)} (c) {(ax), (ay), (az); (xx), (xy), (xz); (yx), (yy), (yz)} (d) {(1x), (1y); (2x), (2y); (xx), (xy)}	B
	(iii) The set Q x R is ______. (a) {(1a), (1x), (1y); (2a), (2x), (2y); (xa), (xx), (xy)} (b) {(1x), (1y), (1z); (2x), (2y), (2z); (xx), (xy), (xy)} (c) {(ax), (ay), (az); (xx), (xy), (xz); (yx), (yy), (yz)} (d) {(1x), (1y); (2x), (2y); (xx), (xy)}	C
	(iv) The set (P x Q) ∩ (P x R) is ______. (a) {(1a), (1x), (1y); (2a), (2x), (2y); (xa), (xx), (xy)} (b) {(1x), (1y), (1z); (2x), (2y), (2z); (xx), (xy), (xy)} (c) {(ax), (ay), (az); (xx), (xy), (xz); (yx), (yy), (yz)} (d) {(1x), (1y); (2x), (2y); (xx), (xy)}	D
	(v) The set (R x Q) ∩ (R x P) is ______. (a) {(ax), (ay), (az), (xx), (xy), (xz), (yx), (yy), (yz)} (b) {(1x), (1y), (2x), (2y)} (c) {(xx), (yx), (zx)} (d) {(1a), (1x), (1y), (2a), (2x), (2y), (xa), (xy), (x1), (x2), (y1), (y2), (yx), (z1), (z2), (zx)}	C
	(vi) The set (P x Q) ∪ (R x P) is ______. (a) {(ax), (ay), (az), (xx), (xy), (xz), (yx), (yy), (yz)} (b) {(1x), (1y), (2x), (2y), (xx), (yx), (zx) (c) {(x), (yx), (zx)} (d){(1a), (1x), (1y), (2a), (2x), (2y), (xa), (xx), (xy), (x1), (x2), (y1), (y2), (yx), (z1), (z2), (zx)}	D
Q108	Out of 2000 staff, 48% preferred coffee, 54% tea and 64% cocoa. Of the total 28% used coffee and tea; 32% tea and cocoa; 30% coffee and cocoa. Only 6% did none of these.	
	(i) Find the number having all the three. (a) 360 (b) 280 (c) 160 (d) None	A
	(ii) Find the number having tea and cocoa but not coffee.	B

	Question	Ans
	(a) 360 (b) 280 (c) 160 (d) None	
	(iii) Find the number having only coffee. (a) 360 (b) 280 (c) 160 (d) None	C
Q109	A town has a total population of 50,000. Out of it, 28,000 read the newspaper X, 23000 read Y while 4000 read both the papers. The number of persons not reading X and Y both is ______. (a) 2000 (b) 3000 (c) 4000 (d) None	B
Q110	Consider the following data	

Worker Term	Skilled direct	Unskilled Direct	Skilled indirect	Unskilled indirect
Short	6	8	10	20
Medium	7	10	16	9
Long	3	2	8	0

Question	Ans
(i) If S, M, L, T, I denote short medium long terms skilled and Indirect workers respectively find the number of workers in set M. (a) 42 (b) 8 (c) 10 (d) 43	A
(ii) Find the number of workers in set L ∩ I. (a) 42 (b) 8 (c) 10 (d) 43	B
(iii) Find the number of workers in set S ∩ T ∩ I. (a) 42 (b) 8 (c) 10 (d) 43	C
(iv) Find the number of workers in set (M U L) ∩ (T U I). (a) 42 (b) 8 (c) 10 (d) 43	D
(v) Find the number of workers in set S' U (S' ∩ I)'. (a) 42 (b) 44 (c) 43 (d) 99	D
(vi) Find the set of pair has more workers as its masters. Pair is (S U M)' or L (a) (S U M)' > L (b) (S U M)' < L (c) (S U M)' = L (d) None	C

CHAPTER 7B. RELATIONS

INTRODUCTION

- Any subset of the product set **A × B** is called a relation from A to B. It is denoted by R.
- R ⊆ **A × B.**

Domain & Range of a relation: If R is a relation from A to B;

- Set of all first elements of the ordered pair that belongs to R is called the domain of R.
- Set of all second elements of the ordered pair that belongs to R is called the range of R.

So, Dom (R) = {a: (a, b) ∈ R} & Range (R) = {b: (a, b) ∈ R}.

CQ1: Let Set A = {1, 2, 3} & Set B = {2, 4, 6}.

Then A × B = {(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)}

We know that every subset of the product set **A × B** is called a relation from A to B.

Now we consider the relation which is the subset of **A × B**. Let R = {(1, 2), (1, 4), (3, 2), (3, 4)}.

Domain of R = 1^{st} Elements = {1, 3} & Range of R = 2^{nd} Element = {2, 4}

TYPES OF RELATION

Identity Relation	If both the elements of ordered pairs are same, it is an identity relation. The relation I = {(a, a): a∈ A} is called the identity relation on A. **Ex:** Let A = {1, 2, 3} then I = {(1, 1), (2, 2), (3, 3)}
Reflexive Relation	R is reflexive relation if (a, a) ∈ R & a **=** a. **PC Note:** R is reflexive if it contains **ALL POSSIBLE ORDERED PAIRS** of the type (x, x). **Ex:** Let A = {1, 2, 3}; **A×A =** {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}. Now let us consider a relation R which is a subset of A×A. (i) If R = {(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)}; It is a reflexive relation because all the possible ordered pair of the form (x, x) are present in the given relation. (ii) If R = {(1,1), (1,3), (2,3), (3,1), (3,3)} is NOT a reflexive relation because (2,2) is missing in R
Symmetric Relation	R is symmetric relation if (a, b) ∈ R; then (b, a) should also ∈ R. **PC NOTE:** For each ordered pair (a, b), the reverse pair (b, a) should also be present in R.

	Ex: R = {(1,1), (1, 3), (1,2), (2,1), (3,1)}; In a symmetric relation becoz all reverse pair are present Reverse pair of (1,1) is (1,1) itself & repetition is meaningless. Thus, it is given only once. Reverse pair of (1,3) is (3,1) which is present & Reverse pair of (1,2) is (2,1) which is also present.
Transitive Relation	R is transitive relation if (a, b) ∈ R & (b, c) ∈ R, then (a, c) should ∈ R. **Ex:** a∥b, b ∥c a ∥c.
Equivalence relation	A relation which is reflexive, symmetric & transitive is called an equivalence relation. **Ex:** "is equal to" is an equivalence relation.
Inverse Relation	R is a relation from A to B, then relation R^{-1} from B to A = {(b, a): (a, b) ∈ R}. **Dom of (R^{-1}) = Range of (R) & Range of (R^{-1}) = Dom of (R).** **Ex:** Let A = {1, 2, 3} & R = {(1, 2), (2, 2), (3, 1), (3, 2)}. R being a subset of A×A, is a relation on A. Dom of (R) = {1, 2, 3} & Range of (R) = {2, 1}. Now, R^{-1} = {(2, 1), (2, 2), (1, 3), (2, 3)}. Dom (R^{-1}) = {2, 1} = Range (R) & Range (R^{-1}) = {1, 2, 3} = Dom (R).
Universal Relation	A relation R from A to B is said to be universal relation if R = A × B **Ex:** Let A = {1, 2} then, R = A × A = {(1, 1), (1, 2), (2, 1), (2, 2)} is universal relation on A
Void Relation	A relation R from A to B is said to be void relation if R = ∅ **Ex:** Let A = {7, 11} and B = {3, 5}. Let R = [(a, b): a ∈ A, b ∈ B, a − b is odd}, then R = ∅

"IS EQUAL TO" Relation	**"IS PARALLEL TO" Relation**
(a) Reflexive: a = a. (b) Symmetric: a = b ⇒ b = a. (c) Transitive: a = b, b = c ⇒ a = c.	(a) Reflexive: a ∥a. (b) Symmetric: a ∥b ⇒ b ∥a. (c) Transitive: a∥b, b∥c ⇒ a∥c

RELATIONS – QUESTION BANK

SN	CHAPTER 7B. RELATION	Ans
Q111	"Is equal to" over the set of all rational numbers is _______. (a) Transitive (b) Symmetric (c) Reflexive (d) Equivalence	D
Q112	"Is smaller than" over the set of eggs in a box is _______. (a) Transitive (T) (b) Symmetric (S) (c) Reflexive (R) (d) Equivalence(E)	A
Q113	"Is greater than" over the set of all natural number if known as _______. (a) Transitive (b) Symmetric (c) Reflexive (d) Equivalence	A
Q114	Relation "is parallel to" on the set of all straight lines in a plane is _______ Relation. (a) an Equivalence (b) an Equal (c) Reflexive (d) Transitive	A
Q115	"Is perpendicular to" over the set of straight lines in a given plane is _______. (a) Symmetric (b) Reflexive (c) Transitive (d) Equivalence	A
Q116	"Is the reciprocal of" over the set of non zero real numbers is ______. (a) Symmetric (b) Reflexive (c) T ransitive (d) None	A
Q117	"Is the square of" over n set of reai numbers is _______. (a) Reflexive (b) Symmetric (c) Transitive (d) None	D
Q118	"Has the same father" as _______over the set of children (a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence	D
Q119	{(x, y) | x, x, yy, y = x} is _______. (a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence	D
Q120	{(x, y) | x + y=2x where x and y are positive integers} is _______. (a) Reflexive (b) Symmetric (c) Transitive (d) Both (a) and (b)	D
Q121	If A = {1, 2, 3} then R = {(1,1), (2,2), (3,3), (1,2)} is _______. (a) reflexive & transitive but not symmetric (b) reflexive & symmetric but not transitive (c) symmetric and transitive but not reflexive (d) identity relation	A
Q122	If a = {1, 2, 3} then a relation {(1,1), (2,2), (3,3)} is _______ Relation (a) an Into (b) an Identity (c) Symmetric (d) Transitive	B
Q123	In inverse relation R _______. (a) domain (R inverse) = range (R inverse) (b) domain (R) = range (R) (c) domain (R inverse) = range (R) (d) domain (R) = range (R inverse)	C
Q124	If a relation R = {(1,1), (2,2), (1,2), (2,1)} is symmetric on A = {1,2,3} then R is ______. (a) Reflexive but not Transitive (b) Transitive but not Reflexive (c) Reflexive and Transitive (d) Neither Reflexive nor Transitive	B

CHAPTER 7C. FUNCTIONS

INTRODUCTION

- Function means any relation from X to Y in which two different ordered pairs should not have same first element.
- If any ordered pair of a relation have same first element, then such relation is not a function.
- If each element 'x' of A is related with a unique element f(x) of B, it is called a function or mapping from A to B and it is written as **f: A→B.**
- The element f(x) is called the image of x, while 'x' is called the pre-image of f (x).
- **Let f: A→B, 'A' is called the domain; while 'B' is called the range.**

Ex: A = {1, 2, 3} & B = {a, b}. Let us consider a function **{(1, a), (2, b)}.**

In this case, no ordered pair have same first element, so it is a function.

Ex: Let N be the set of all natural numbers.

Then, rule f(x) = 2x for all x∈N is a function from N to N, since twice a natural number is unique.

Now, f (1) = 2; f (2) = 4; f(3) = 6 and so on.

Here domain of function = {1, 2 ,3, 4 ,..........} & range of function = { 2, 4, 6,.........}

Ex: Let A = {1, 2, 3, 4} & B = {1, 2, 3}.

A×B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3),(4, 1), (4, 2), (4, 3)}

(i)	R = Subset {(1, 2), (1, 3), (2, 3)} is a relation on A×B. This relation contains all ordered pair in A×B for which A < B. So, it is "less than" relation. This relation is not a function because it includes two different ordered pairs (1,2), (1,3) have same 1^{st} element.
(ii)	Subset {(1, 1), (2, 2), (3, 3)} defines the function y = x as different ordered pairs of this subset have different 1^{st} element.

CQ1. If $f(x) = 2x^2 - 5x + 4$ then 2f(x) = f(2x) for **[Ans: C]**

(a) x = 1 (b) x = -1 (c) x = ±1 (d) None

CQ2. If $f(x) = x^2 - 5$, evaluate f (3), f (-4), f (5) and f(1). **[Ans: C]**

(a) 0, 11, 20, 4 (b) -4, 11, -2, 4 (c) 4, 11, 20, - 4 (d) 4,10, 20, 5

CQ3. Which of these is a function from A → B A = {x, y, z} B = {a, b, c, d} **[Ans: C]**

(a) {(x,a) (x,b) (y,c)} (b) {(x,a) (x,b) (y,c) (z,d)} (c) {(x,a) (y,b) (z,d)} (d) {(a,x) (b,z) (c,y)}

TYPES OF FUNCTIONS	
1	**One - One function (Injective function)**
	▪ If every element in Set A have different images in Set B, such function is one-one function. ▪ If f(a) = f(b) ⇒ a = b. **Ex:** Let A = {1, 2, 3} and B = {2, 4, 6}. Thus the function is f: A→B: **f(x) = 2x.** Then f(1) = 2; f(2) = 4; f(3) = 6. Since every element in A have different images in B, it is one-one function.
2	**Many-one function**
	▪ If two or more elements in A have same image (corresponding value) in B, such function is many-one function. ▪ **Ex:** f(x) = x^2; x ∈ R. f(1) = $(1)^2$ = 1 & f(-1) = $f(-1)^2$ = 1. So, two elements of Set A have the same image in Set B. Hence it is a many – one function.
3	**Onto function (Surjective function)**
	▪ f: A→B is called onto function if for all b∈B, there is at least one a∈A with f(a) = b. **PC Note: If every element in B has at least one pre-image in A, it is onto function.** **Range = Co-domain.** **Ex:** A = {1, 2, 3} & B = {a, b}. Let f = {(1, a), (2, a), (3, a)}. In this case, no ordered pair have same first element, so it is a function. In the given function, 'a' have 3 pre images but 'b' does not have any pre image. Hence it is not onto function. It is into function.
4	**Into function**
	If at least one element in B has no pre-image in A, then the function is into function. **Ex:** A = {1, 2, 3} & B = {a, b}. Let f = {(1, a), (2, a), (3, a)}. In this case, no ordered pair have same first element, so it is a function. In the given function, 'a' have 3 pre images but 'b' does not have any pre image. Hence it is not onto function. It is into function. **Ex:** A = {1, 2, 3} & B = {a, b, c, d}. Let f = {(1, a), (2, b), (3, c)}. In this case, no ordered pair have same first element, so it is a function. Here the element 'd' in B does not have a pre images in A. Thus it is into function. **Ex:** A = {2, 3, 5, 7} & B = {0, 1, 3, 5, 7}. Let us consider f: A → B; f(x) = x – 2. Then f(2) = 0; f(3) = 1 ; f(5) = 3 & f(7) = 5. Here there exists an element 7 in B, having no pre-mage in A. Thus it is into function.

5 Bijective Function

- One-One onto function is called Bijective function. **[One to One]**

6 Constant Function [Ex: $f(x) = 5$]

- All the elements in 'A' have the same image in 'B', then it is a constant function.
- Range of a constant is a singleton set.

Ex: Let f(x) = {(1, 3), (2, 3), (3, 3), (4, 3)}.
It is a constant function since all the elements in A have the same image in B.

7 Identity function

- If every element is A has the same image (i.e A).
- $F(x) = x$ [$\forall$ x $\in$ X, f(x) $\in$ X].
- It is a one-to-one onto function with domain A and range A.

PC Note: If every element in A is mapped to itself, it is an identity function.
Ex: Let x = {1, 2, 3, 4} then f(1) = 1; f(2) = 2; f(3) = 3; f(4) = 4 is an identity function.

8 Equal Function

- Two functions f(x) & g(x) are said to be equal if (i) they have same domain; (ii) f(x) = g(x).

Ex: Let $f(x) = x^2$, $\forall$ x $\in$ R & $g(y) = y^2$, $\forall$ y $\in$ R. Then two function f & g are equal.

9 Inverse Function **[Don't even dare to see the definition]**

- If f(x) = y; then $f^{-1}(y) = x$.
- Only one – one onto functions are invertible (can have inverse function).

STEPS TO FIND INVERSE FUNCTION

1. Substitute f(x) = y.
2. Find the value of x in terms of y.
3. Replace 'x' with $f^{-1}(x)$ & 'y' with x.
4. The resultant will be the answer.

Ex: f(x) = 2x. Find $f^{-1}(x)$.
Ans: Step 1: Let f(x) = y. Thus y = 2x; Step 2: x = y/2; Step 3: $f^{-1}(x) = x/2$.

10 Composite Function **[Function of a function]**

PC TIPS TO FIND COMPOSITE FUNCTION

▪ f[g(x)]: Replace 'x' with g(x) in f(x).	▪ g[f(x)]: Replace 'x' with f(x) in g(x).

Ex: Let f(x) = 2x & $g(x) = 3x^2$. Find f[g(x)] & g[f(x)].
Ans:
(i) f[g(x)] = Replace 'x' with g(x) in f(x); $f[g(x)] = 2(3x^2) = 6x^2$.
(ii) g[f(x)] = Replace 'x' with f(x) in g(x); $g[f(x)] = 3(2x)^2 = 12x^2$.

FUNCTIONS – QUESTION BANK

SN	CHAPTER 7C. FUNCTIONS	Ans
Q125	A = {1,2,3,4,...} & B = {1,4,9,16...} & if f is a mapping from A → B such that f(x) = x^2, then ______. (a) domain (f) = {1,2,3,4,...} & range (f) = {1,4,9,...} (b) range (f)={1,2,3,4,...} & domain (f) = {1,4,9,...} (c) domain (f) ={1,2,9,16,....} & range (f) = {1,2,3,....} (d) range (f)={1,2,9,16,....} & domain (f) = 1,2,3,....}	A
Q126	Domain of {(1,7), (2,6)} is ____. (a) (1,6) (b) (7,6) (c) (1,2) (d) (6,7)	C
Q127	Range of {(3,0),(2,0),(1,0),(0,0)} is ____. (a) (0,0) (b) (0) (c) {0,0,0,0} (d) None	B
Q128	The range of {(1,6),(2,7)} is ______. (a) (6, 7) (b) (1, 7) (c) (1, 2) (d) (6, 2)	A
Q129	Domain & range of {(x,y): Y = x^2 is ______ (a) (Real, Natural No.) (b) (Real, +ve Real) (c) (Real, Real) (d) None	B
Q130	Let the domain of x be the set {0,1}. Which of the following functions is equals to 1. (a) f(x) = x^2, g(x) = x (b) f(x) = x, g(x) = 1 - x (c) f(x) = x^2+x+2, g(x) = $(x+1)^2$ (d) None	A
Q131	Range of function f(x) = $\frac{1}{1-x}$ is ______. (a) Set of rational numbers (b) Set of real numbers (except 0) (c) Set of natural numbers (d) Set of integers.	B
Q132	Range of function f(x) = $\log_{10}(1+x)$ for domain of real values of x when $0 \le x \le 9$ is ___. (a) {0} (b) {0,1,2} (c) {0,1} (d) None	C
Q133	For function h(x) = 10^{1+x} domain of real values of x where $0 \le h(x) \le 9$, range is ______. (a) $10 \le h(x) \le 10^{10}$ (b) $0 \le h(x) \le 10$ (c) $0 < h(x) < 10$ (d) None	A
	Finding Value of Function	
Q134	If f(x) = $\frac{x+1}{x^2-3x-4}$, find f(0), f(1), f(-1). (a) 1, 3, 0 (b) $\frac{1}{4},-\frac{1}{3},0$ (c) $-\frac{1}{4},-\frac{1}{3},\frac{0}{0}$ (d) 0, 1, 0	C
Q135	If f(x) = $x^3 - x^2 + x + 1$ then the value of [f(1) + f(-1)] will be _____. (a) 5 (b) 2 (c) 0 (d) -2	C
Q136	If f(x) = $x^2 + 3x$ then f(2) - f(4) is equal to ______. (a) -15 (b) -18 (c) 18 (d) 12	B
Q137	If f(x) = 2x + 3 then f(2x) - 2f(x) + 3 = ______.	B

	(a) 1	(b) 0	(c) -1	(d) None	
Q138	Given the function $f(x) = x^2 - 5$; f(5) is ______.				**D**
	(a) 0	(b) 5	(c) 10	(d) 20	
Q139	If $f(x) = x^2 - x$ then f(h + 1) is equal to ______.				**B**
	(a) f(h)	(b) f(-h)	(c) f(-h+1)	(d) None	
Q140	If $f : R \to R$, $f(x) = x^2 + 8$, then f(-3) is ______.				**B**
	(a) 1	(b) 17	(c) -1	(d) -17	
Q141	If f(x) = |x| + |x - 2|, then redefine the function. Hence find f (3,5), f (-2), f (1.5).				**A**
	(a) 5, 6, 2	(b) 2, 4, 5	(c) 7, 6, 5	(d) 0, 2, 5	
Q142	If $f(x) = x^3 + \frac{1}{x^3}$ then value of f(x) - f(1/x) is equal to ______.				**A**
	(a) 0	(b) 1	(c) $x^3 + \frac{1}{x^3}$	(d) None	
Q143	If $f(x) = \frac{5}{x}$, then f(0) is ______.				**D**
	(a) + ∞	(b) - ∞	(c) 5	(d) Undefined	
Q144	If $f(x) = \frac{1-x}{1+x}$ then f {f(1/x)} = ______.				**A**
	(a) 1/x	(b) x	(c) -1/x	(d) None	
Q145	If f(x+1) = 2x + 7 then f(0) = ______.				**A**
	(a) 5	(b) 4	(c) 3	(d) 0	
Q146	If $f(x) = x^2 - 1$ and $g(x) = \frac{x+1}{2}$ then $\frac{f(3)}{f(3)+g(3)}$ is ______.				**B**
	(a) 5/4	(b) 4/5	(c) 3/5	(d) 5/3	
Q147	If $f(x) = \frac{q\times(x-p)}{(q-p)} + \frac{p\times(x-q)}{(p-q)}$ then f(p) + f(q) is equal to ______.				**A**
	(a) f(p + q)	(b) f(pq)	(c) f(p - q)	(d) None	
Q148	If f(x) = log x (x > 0) that f(p)+ f(q) + f(r) is ______.				**A**
	(a) f(pqr)	(b) f(p)f(q) f(r)	(c) f(1/pqr)	(d) None	
Q149	If $y = h(x) = \frac{px-q}{qx-p}$; then x = ______.				**C**
	(a) h(1/y)	(b) h(-y)	(c) h(y)	(d) None	
	COMPOSITE FUNCTION				
Q150	If f(x) = x+3, $g(x) = (x)^2$ then gof(x) is ______.				**A**
	(a) $(x+3)^2$	(b) x^2+3	(c) $x^2(x+3)$	(d) $x^2 + (x+3)$	
Q151	Find fog(x) for the functions $f(x) = x^8$, $g(x) = 2x^2 + 1$				**D**
	(a) $x^8(2x^2+1)$	(b) x^8	(c) $2x^2 + 1$	(d) $(2x^2+1)^8$	
Q152	Find fog(x) for the functions $f(x) = x^2$, g(x) = x+1				**D**
	(a) $x^2(x+1)$	(b) x^2	(c) x+1	(d) $(x+1)^2$	
Q153	If f(x) = x+3, $g(x) = x^2$, then gof(x) is ______.				**A**

	(a) $(x+3)^2$ (b) x^2+3 (c) $x^2(x+3)$ (d) None	
Q154	If $f(x) = x+3$, $g(x) = x^2$ then fog(x) is ______. (a) x^2+3 (b) x^2+x+3 (c) $(x+3)^2$ (d) None	A
Q155	If $f(x) = x^2 + 3$, $g(x) = x$ then fog(x) is ______. (a) $x^2 + 3$ (b) $(x)^2 + (x^2+ 3)$ (c) $(x + 3)^2$ (d) $(x)^2(x^2+ 3)$	A
Q156	If $f(x) = x^2 + 3$, $g(x) = x$ then gof(x) is ______. (a) $x^2 + 3$ (b) $(x)^2 +(x^2+ 3)$ (c) $(x + 3)^2$ (d) $(x)^2 + (x^2+ 3)$	A
Q157	Find gof(x) for the functions $f(x) = \sqrt{x}$, $g(x) = 2x^2 +1$ (a) $2x^2 +1$ (b) $2x + 1$ (c) $(2x^2+1)\sqrt{x}$ (d) $\sqrt{x}$	B
Q158	$f(x) = 2x+2$, $g(x) = x^2$, fog(4) = ? (a) 100 (b) 10 (c) 34 (d) 36	C
Q159	If $f(x) = \|x+1\|$ & $g(x) = 3x^2 - 5$, find the value of gof = ______. (a) $3x^2+6x-2$ (b) $2x^2-6x + 3$ (c) $\|3x -5\|$ (d) $x - 5$	A
Q160	If $f(x) = x + 3$, $g(x) = x^2$ then f(x).g(x) is ______. (a) $(x+3)^2$ (b) x^2+3 (c) x^3+3x^2 (d) None	C
Q161	If $f(x) = \frac{1}{1-x}$ and $g(x) = \frac{x-1}{x}$, then gof(x) is ______. (a) x (b) 1/x (c) -x (d) None	A
Q162	If $f(x) = \frac{1}{1-x}$ and $g(x) = \frac{x}{x-1}$, then fog(x) is ______. (a) x (b) 1/x (c) -x (d) None	A
Q163	If $f(x) = x + 2$, $g(x) = 7^x$, then g o f (x) = ______. (a) $7^x.x + 2.7^x$ (b) 7^{x+2} (c) $(7^x) + 2$ (d) None	B
Q164	If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2x}{1+x^2}\right)$ = ______. (a) f(x) (b) 2 f(x) (c) 3 f(x) (d) -f(x)	B
Q165	If $f(x) = x + 2$, $g(x) = 7^x$, then g o f (x) = ______. (a) $7^x.x + 2.7^x$ (b) 7^{x+2} (c) $(7^x) + 2$ (d) None	B
Q166	If $f(x) = ax^2 + b$, find $\frac{f(x+h)-f(x)}{h}$ = ______. (a) 2x+h (b) a(2x+h) (c) a(2x-h) (d) 2x-h	B
Q167	If $f(x) = 2x^2 -5x + 2$ then the value of $\frac{f(4+h)-f(4)}{h}$ = ______. (a) 11-2h (b) 11+2h (c) 2h-11 (d) None	B
Q168	$f(x) = \frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)}$ = ______. (a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $-\frac{x}{y}$ (d) $-\frac{y}{x}$	C
	Inverse Function	
Q169	Inverse $h^{-1}(x)$ when $h(x) = \log_{10} x$ is ______.	B

	(a) $Log_{10}x$ (b) 10^x (c) $Log_{10}(1/x)$ (d) None	
Q170	If $f(x) = \frac{1}{1-x}$, then $f^{-1}(x)$ is _____. (a) 1-x (b) (x-1)/x (c) x/x-1 (d) None	B
Q171	Find $f^{-1}(x)$ when $f(x)=x^2$ is _____. (a) $1/x^2$ (b) $\sqrt{x}$ (c) 1/x (d) None	B
Q172	If f(x) = 100x; then $f^{-1}(x)$ = _____. (a) $\frac{X}{100}$ (b) $\frac{1}{100X}$ (c) $\frac{1}{100}$ (d) None	A
Q173	A function is invertible if and only if f is _____. (a) one -one (b) one-one, into (c) one-one, onto (d) many -one, into	C
	MISCELLANEOUS QUESTIONS	
Q174	If A = {1,2,3} and B = {4,6,7} then the relation = {(2,4) (3,6)} is _____. (a) Function from A to B (b) Function from B to A (c) Both (a) & (b) (d) Not a Function	D
Q175	{(x,y) such that $y = x^2$} is _____. (a) Not a function (b) A function (c) Inverse mapping (d) None	B
Q176	If $f(x) = x^2$, x > 0, then the function is _____. (a) Not one to one (b) One to one (c) Into (d) None	B
Q177	N is the set of all natural numbers and E is the set of all even numbers. If f : N E defined by f(x) = 2x, for all x ε N is: A (a) One - one and onto (b) One - one into (c) Many one onto (d) Can't say	A
Q178	{(x, y) such that x<y} is a ____. (a) Not a function (b) A function (c) One-one mapping (d) None	A
Q179	{(x, y); x= 4} is a ____. (a) Not a function (b) Function (c) One-one mapping (d) None	A
Q180	{(x, y) such that (x+y = 5)} is _____ function. (a) Not a function (b) Composite (c) One-one mapping (d) None	C
Q181	Function $f(x) = 2^x$ is _____. (a) One one mapping (b) One many (c) Many one (d) None	A

Q182	If f(x) = |x| ∀ x∈R, then the function is ____. (a) Not one to one (b) One to one (c) Into (d) Not into	**C**
Q183	Let A = {2,3,5,7} and B = [0,1,3,5,7}. If f is a mapping from A to B such that f(x) = x – 2; f is ___. (a) Into (b) an onto (c) constant (d) identical	**A**
Q184	If A = {0,1,3,5,6} & B = {2,4,8,9}; then function is ____. (a) onto function (b) into function (c) Many one onto (d) None	
Q185	F: R → R is defined by f(x) = 2^x then f is _____ (a) One - one & onto (b) One - one & into (c) Many to one (d) None	**B**
Q186	If A = {x, y, z}, B= {p, q, r, s} which of the relation on A, B are function. (a) {(x,p), (y,r), (z,s)} (b) {(x,s), (y,s), (z,s)} (c) {(n,p), (x,q), (y,r),(z,s)} (d) Both (a) and (b)	**D**
	EVEN & ODD FUNCTION	
Q187	If g(x) = 3 - x^2 then g(x) is _____ function. (a) Odd (b) Periodic (c) Even (d) None	**C**
Q188	A function f(x) is an even function if ___. (a) - f(x) = f(x) (b) f(-x) = f(x) (c) f(-x) = - f(x) (d) None	**B**
Q189	If f(x) = $\frac{5^x+1}{5^x-1}$ then f(x) is _____. (a) Even (b) Odd (c) Composite (d) None	**B**

CHAPTER 8A. DIFFERENTIAL CALCULUS

INTRODUCTION

Ex: Let us consider a function y = f(x) = $3x^2 + 5x + 2$.

The value of f(x) i.e 'y' will depend on value of 'x'. [**Note:** x can take any value]

Thus, we can say that 'y' is a dependent variable & 'x' is an independent variable.

If x = 1, y = $3(1)^2 + 5(1) + 2 = 10$; If x = 2, y = $3(2)^2 + 5(2) + 2 = 24$.

Thus, we can say that if we change the value of x from 1 to 2, value of y changes from 10 to 24.

Now let's jump on to the definition of derivative.

MEANING OF DERIVATIVE [DIFFERENTIATION]

- It is a process of finding "change in dependent variable" w.r.t "change in independent variable".
- It measures the rate at which the changes are taking place.
- Change in 'x' is denoted by Δx & Change in 'y' is denoted by Δy. [Called as 'delta' x]
- It involves a very small change in dependent variable (i.e y) w.r.t a very small change in independent variable (i.e x). Thus, it studies "Instantaneous rate of change of a function".

Differentiation is the process of finding "change in value of y" w.r.t "change in value of x".

- Change in 'x' is so small that it tends to Zero. [Δx → 0] & thus we say that it studies "instantaneous rate of change of a function".
- It is defined as the limiting value of the ratio of change (increment) in the function corresponding to a small change (increment) in independent variable as the later tends to zero.
- The derivative of f(x) is also known as differential coefficient of f(x) with respect to x.
- This is denoted as $\frac{dy}{dx}$ or f '(x) [Derivative of 'y' w.r.t 'x']

PC NOTE: To differentiate a function, we have to differentiate it w.r.t independent variable only.

Note: (i) $\frac{d}{dx}f(x) \neq \frac{d}{dx} \times f(x)$. (ii) $\frac{dy}{dx} \neq$ dy ÷ dx.

(iii) $\frac{dy}{dx}$ represents **slope of tangent** to the curve y = f(x) & is known as "**gradient**" of the curve.

SOME STANDARD RESULTS BASED ON FIRST PRINCIPLE

Function f(x)	Derivative	When to apply the formula
(i) x^n	$n.x^{(n-1)}$	When we have a constant number in power. [n → denotes a constant number (+ve/—ve)].
(ii) e^x	e^x	When we have 'e' in base. [Value of 'e' = 2.71828 is irrelevant]
(iii) a^x	a^x.log a	When we have a number in base. [a → denotes a constant number (**a > 0 & a ≠ 1)**]
(iv) Log x	(1/x)	When we have 'log'.
(v) Constant (C)	**ZERO**	Derivative of a "constant" is "Zero". [Note: **e^n & a^a** are constants].
(vi) C. f(x)	**C. f'(x)**	Take 'C' outside; differentiate f(x) & then multiply **f'(x) by C.**

FORMULAE WITH EXAMPLE

Formula	Function	Derivatives of Function
$\frac{d}{dx} x^n = n.x^{(n-1)}$	x^5	$\frac{dy}{dx} = 5.x^{(5-1)} = 5.x^4$
	$\sqrt{x}$;	$Y = x^{1/2}; \frac{dy}{dx} = (½).\ x^{(\frac{1}{2}-1)} = (½)x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$
	$x\sqrt{x}$	$Y = x^{3/2}; \frac{dy}{dx} = \frac{3}{2} \cdot x^{(\frac{3}{2}-1)} = \frac{3}{2} \cdot x^{\frac{1}{2}} = \frac{3}{2\sqrt{x}}$
	$\frac{1}{x}$	$Y = x^{(-1)}; \frac{dy}{dx} = (-1).\ x^{(-1-1)} = (-1)x^{-2} = -\frac{1}{x^2}$
	$\frac{1}{\sqrt{x}}$	$Y = x^{-1/2}; \frac{dy}{dx} = (-½).\ x^{(-\frac{1}{2}-1)} = (-½)x^{-\frac{3}{2}} = -\frac{1}{2.x\sqrt{x}}$
	$x^{-7/3}$;	$\frac{dy}{dx} = -\frac{7}{3}.\ (x^{-\frac{7}{3}-1}) = -\frac{7}{3}.\ x^{-\frac{10}{3}}.$
	x	$Y = x^1; \frac{dy}{dx} = 1.x^{(1-1)} = 1.x^0 = 1.1 = 1$

Class work:

$\frac{d}{dx}(e^x) = e^x$	e^x	$\frac{dy}{dx} = e^x$
	e^2	$\frac{dy}{dx}$ = Zero since e^2 is a constant.
$\frac{d}{dx}(a^x) = a^x.\log a$	a^x	$\frac{dy}{dx} = a^x. \log a$
	2^x	$\frac{dy}{dx} = 2^x. \log 2$
$\frac{d}{dx}\log x = \frac{1}{x}$	Log x	$\frac{dy}{dx} = \frac{1}{x}$
	2^x	$\frac{dy}{dx} = 2^x. \log 2$
$\frac{d}{dx}C. f(x) = \mathbf{C. f'(x)}$	$12x^5$	$\frac{dy}{dx} = 12. \frac{d}{dx}(x^5) = 12. 5x^4 = 60. x^4$
	ax^3	$a. \frac{d}{dx}(x^3) = a. 3x^2 = 3ax^2$.
	$(-3)x^{-2}$	$(-3). \frac{d}{dx}(x^{-2}) = (-3). (-2). x^{(-2-1)} = 6x^{-3}$.
	$\frac{x^5}{2}$	$(½). \frac{d}{dx}(x^5) = (½).5x^4 = \frac{5}{2}x^4$

BASIC LAWS FOR DIFFERENTIATION

SUM/DIFFERENCE RULE: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Ex: $\frac{d}{dx}[ax^2 + bx + c] = \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c) = a.\frac{d}{dx}(x^2) + b.\frac{d}{dx}(x) + \frac{d}{dx}(c) = a.2x + b.1 + 0 = 2ax + bx + 0$

Ex: $\frac{d}{dx}[3x^2 + 5x - 2] = \frac{d}{dx}(3x^2) + \frac{d}{dx}(5x) - \frac{d}{dx}(2) = 3.\frac{d}{dx}(x^2) + 5.\frac{d}{dx}(x) - \frac{d}{dx}(2) = 3.2x + 5.1 - 0 = 6x + 5$.

Ex: $\frac{d}{dx}[a^x + x^a + a^a] = \frac{d}{dx}(a^x) + \frac{d}{dx}(x^a) + \frac{d}{dx}(a^a) = a^x. \log a + a.x^{(a-1)} + 0$.

Let f(x) = U & g(x) = V; **PRODUCT RULE:** $\frac{d}{dx}[U \times V] = U.\frac{d}{dx}[V] + V.\frac{d}{dx}[U]$

Ex: $\frac{d}{dx}(2^x.x^5) = 2^x.\frac{d}{dx}(x^5) + x^5.\frac{d}{dx}(2^x) = 2^x.(5x^4) + x^5.(2^x.\log 2) = 2^x.x^4[5 + x.\log 2]$

Ex: $\frac{d}{dx}(2^x.\log x) = 2^x.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(2^x) = 2^x.(\frac{1}{x}) + \log x\,(2^x.\log 2) = 2^x[(\frac{1}{x}) + \log x . \log 2]$

Ex: $\frac{d}{dx}(x^2.\log x) = x^2.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x^2) = x^2.(\frac{1}{x}) + \log x.(2x) = x + 2x.\log x = x(1 + 2.\log x)$

QUOTIENT RULE: $\frac{d}{dx}\left[\frac{U}{V}\right] = \frac{V.\frac{d}{dx}[U] - U.\frac{d}{dx}[V]}{V^2}$

Ex: $\frac{d}{dx}\frac{e^x}{\log x} = \frac{\log x.\frac{d}{dx}(e^x) - e^x.\frac{d}{dx}(\log x)}{(\log x)^2} = \frac{\log x.(e^x) - e^x.\frac{1}{x}}{(\log x)^2} = \frac{e^x[\log x - \frac{1}{x}]}{(\log x)^2}$

Ex: $\frac{d}{dx}(\frac{x^2}{e^x}) = \frac{e^x.\frac{d}{dx}(x^2) - x^2.\frac{d}{dx}(e^x)}{(e^x)^2} = \frac{e^x.2x - x^2.(e^x)}{(e^x)^2} = \frac{x.e^x[2-x]}{(e^x)^2} = \frac{x[2-x]}{e^x}$

Ex: $\frac{d}{dx}\frac{3-5x}{3+5x} = \frac{(3+5x)\frac{d}{dx}(3-5x) - (3-5x)\frac{d}{dx}(3+5x)}{(3+5x)^2} = \frac{(3+5x)(-5) - (3-5x)(5)}{(3+5x)^2}$

$= \frac{[-15-25x] - [15-25x]}{(3+5x)^2} = \frac{[-15-25x] - 15 + 25x]}{(3+5x)^2} = \frac{-30}{(3+5x)^2}$

"DERIVATIVE OF ONE FUNCTION" WITH RESPECT TO "ANOTHER FUNCTION".

Let f(x) be one function & g(x) be another function, then **Derivative of f(x) w.r.t g(x) =** $\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$

Ex: Differentiate 'log x' w.r.t (x^2).

Ans: $\frac{f'(x)}{g'(x)} = \frac{\frac{d}{dx}\log x}{\frac{d}{dx}x^2} = \frac{\frac{1}{x}}{2x} = \frac{1}{2x^2}.$

Ex: Differentiate (x^2) w.r.t e^x.

Ans: $\frac{f'(x)}{g'(x)} = \frac{\frac{d}{dx}x^2}{\frac{d}{dx}e^x} = \frac{2x}{e^x}.$

Ex: Differentiate (a^x) w.r.t log x.

Ans: $\frac{f'(x)}{g'(x)} = \frac{\frac{d}{dx}a^x}{\frac{d}{dx}\log x} = \frac{a^x.\log a}{\frac{1}{x}} = x.\ a^x.\log a$

CHAIN RULE

We have studied the following formulae earlier:

f(x)	Derivative
$(x)^n$	$n.x^{(n-1)}$
e^x	e^x
a^x	$a^x.\log a$
Log x	$\frac{1}{x}$.
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$.
x	1

PC NOTE: ⇒ If there is anything other than 'x' in the above formulae; Take it as 'y' & use the same rule (replace x with 'y' in the formula & multiply it with additional dy/dx.

So, the above formulae will look like this:

f(x)	Derivative
$(y)^n$	$n.y^{(n-1)}.\frac{dy}{dx}$
e^y	$e^y.\frac{dy}{dx}$
a^y	$a^y.\log a.\frac{dy}{dx}$
Log y	$\frac{1}{y}.\frac{dy}{dx}$
$\sqrt{y}$	$\frac{1}{2\sqrt{y}}.\frac{dy}{dx}$
y	$1.\frac{dy}{dx} = \mathbf{\frac{dy}{dx}}$

SOLVED EXAMPLES	HOMEWORK QUESTIONS
CQ1: Find $\frac{d}{dx}(3x^3 - 5x^2 + 8)^3$. **Ans:** Referring formula 1, we see that $y = (3x^3 - 5x^2 + 8)$; $\frac{dy}{dx} = 9x^2 - 10x$. Thus $\frac{d}{dx}(3x^3 - 5x^2 + 8)^3 = 3(3x^3 - 5x^2 + 8)^2(9x^2 - 10x)$.	**1)** $\frac{d}{dx}[(\log x)^2]$ **2)** $\frac{d}{dx}[(6x^5 - 7x^3 + 9)^{-1/3}]$
CQ2: $\frac{d}{dx}[e^{ax^2+bx+c}]$ **Ans:** Referring formula 2, we see that $y = (ax^2 + bx + c)$; $\frac{dy}{dx} = (2ax+b)$ Thus $\frac{d}{dx}e^{ax^2+bx+c} = e^{ax^2+bx+c}.(2ax+b)$	**3)** $\frac{d}{dx}[e^{(2\log x)}]$ **4)** $\frac{d}{dx}e^{(x-y)}$ **5)** $\frac{d}{dx}[e^{(xy)}]$
CQ3: $\frac{d}{dx}[a^{\log x}]$ **Ans:** Referring formula 3, we see that $y = (\log x)$; $\frac{dy}{dx} = \frac{1}{x}$ Thus $\frac{d}{dx}[a^{\log x}] = [a^{\log x}].\log a.\frac{1}{x}$	**6)** $\frac{d}{dx}a^{x^2}$ **7)** $\frac{d}{dx}5^{(3x+2)}$
CQ4: $\frac{d}{dx}[\log(1+x^2)]$ **Ans:** Referring formula 4, we see that $y = (1+x^2)$; $\frac{dy}{dx} = 2x$. Thus $\frac{d}{dx}[\log(1+x^2)] = \frac{1}{1+x^2}.2x = \frac{2x}{1+x^2}$	**8)** $\frac{d}{dx}[\log(5x)]$ **9)** $\frac{d}{dx}[\log(x.e^x)]$

CQ5: Find $\frac{d}{dx}\sqrt{x+\sqrt{x}}$. **Ans:** Referring formula 5. We see that y = x + $\sqrt{x}$; & $\frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}}$ Thus $\frac{d(\sqrt{x+\sqrt{x}})}{dx} = \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x}}}.[1+\frac{1}{2\sqrt{x}}]$	**10)** $\frac{d}{dx}[\sqrt{(1+x^2)}]$ **11)** $\frac{d}{dx}\sqrt{(\log x)}$

IMPLICIT FUNCTIONS

- A function in the form f(x, y) = 0.
- In Implicit function, y cannot be directly defined as a function of x.

Ex: $5x^2y^2 + x^2y + xy^2 + x + y = 0$

PC Note: In Implicit function, x & y are related in such a way that neither 'x' nor 'y' cannot be expressed in terms of each other.

STEPS TO DIFFERENTIATE IMPLICIT FUNCTION

1. Differentiate both sides w.r.t 'x'. [If RHS = 0, Its derivative will also be 0]
2. All the terms having $\frac{dy}{dx}$ shall be brought to one side & all other terms (not having $\frac{dy}{dx}$) shall be taken to another side.
3. Take $\frac{dy}{dx}$ common from all the terms having $\frac{dy}{dx}$ & remainder shall be sent to another side (division)

CQ6: If $x^3 - 2x^2y^2 + 5x + y + 5 = 0$, find $\frac{dy}{dx}$.

Ans: Differentiating both sides w.r.t x, we get

$\Rightarrow 3x^2 - 2\times\frac{d}{dx}[x^2 \times y^2] + 5 + \frac{dy}{dx} + 0 = 0$; $\Rightarrow 3x^2 - 2\left[x^2.\frac{d}{dx}(y^2) + y^2.\frac{d}{dx}(x^2)\right] + 5 + \frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 2\left[x^2.2y\frac{dy}{dx} + y^2.2x\right] + 5 + \frac{dy}{dx} = 0$ $\Rightarrow 3x^2 - 4x^2y.\frac{dy}{dx} - 4xy^2 + 5 + \frac{dy}{dx} = 0$

Taking all the terms containing $\frac{dy}{dx}$ to one side & other terms on another side,

$\Rightarrow 3x^2 - 4xy^2 + 5 = 4x^2y.\frac{dy}{dx} - \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx}(4x^2y - 1) = 3x^2 - 4xy^2 + 5$: $\mathbf{\frac{dy}{dx} = \frac{3x^2 - 4xy^2 + 5}{(4x^2y - 1)}}$

PARAMETRIC FUNCTIONS

In parametric function, both 'x' & 'y' are expressed in terms of a third variable (generally t).

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

PC Note: While calculating $\frac{dy}{dx}$, replace 't' with 'x' & use the normal rule & then again replace 'x' with 't'.

$$\frac{dy}{dx} = \frac{\text{" Derivative of y" after applying the above note}}{\text{" Derivative of x" after applying the above note}}$$

Ex: Given x = 2t +5; y = t^2 — 2, find $\frac{dy}{dx}$. **Ans:** x' = 2; y' = 2t; $\frac{dy}{dx} = \frac{\text{Derivative of y}}{\text{Derivative of x}} = \frac{2t}{2}$ = t.	**Ex:** Given x = at^2; y = 2at; find $\frac{dy}{dx}$. **Ans:** x' = 2at; y' = 2a; $\frac{dy}{dx} = \frac{\text{Derivative of y}}{\text{Derivative of x}} = \frac{2a}{2at} = \frac{1}{t}$.
Ex: If u = $(x^3 +1)^5$ and y = $(x^3 +7)$ then $\frac{du}{dy}$ = **Ans:** u' = $5(x^3 + 1)^4 . 3x^2$; y' = $3x^2$; $\frac{du}{dy} = \frac{\text{Derivative of u}}{\text{Derivative of y}} = \frac{5(x^3+1)^4 . 3x^2}{3x^2} = 5(x^3 + 1)^4$.	**Ex:** If x = $3t^2$ -1, y = t^3, then $\frac{dy}{dx}$ = **Ans:** $\frac{dy}{dt} = 3t^2$; $\frac{dx}{dt}$ = 6t; $\frac{dt}{dx} = \frac{1}{6t}$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$; $= 3t^2 \times \frac{1}{6t} = \frac{t}{2}$
HW. x = at^3; y = $\frac{a}{t^3}$; find $\frac{dy}{dx}$. [**Ans:** $\frac{-1}{t^6}$]	
HW. If x = $\frac{1-t^2}{1+t^2}$; y = $\frac{2t}{1+t^2}$ then $\frac{dy}{dx}$ @ t = 1 is [**Ans:** $\frac{dy}{dx} = \frac{t^2-1}{2t}$ = 0]	

LOGARITHMIC DIFFERENTIATION

The process of finding derivative by taking logarithm of both sides & then applying antilog is called logarithmic differentiation.

When to use Logarithmic Differentiation:

1. The given function involves function in its power. [**Ex:** x^x since neither x^n nor a^x formula is applicable in this case].
2. The given function is the product of number of functions. [**Ex:** $x^y + y^x$]
3. If using basic formulae will consume more time. [Depends on judgment of the student].

CQ7: Differentiate x^x w.r.t 'x'.

Ans: Let y = x^x; Taking log of both sides, we get Log y = log x^x

⇒ Log y = x.logx [using $\log m^n = n.\log m$]

Differentiating w.r.t x we get $\frac{1}{y} \times \frac{dy}{dx} = x \times \frac{1}{x} + \log x \times 1$

$\Rightarrow \frac{dy}{dx}$ = y [1 + logx] & $\frac{dy}{dx}$ = x^x [1 + logx].

SOME ADVANCED QUESTIONS

CQ8: Differentiate x^{x^x} w.r.t 'x'.

Ans: Let y = x^{x^x}; Taking log of both sides, we get Log y = log x^{x^x}

Log y = x^x . log x [Using $\log m^n = n.\log m$]

Differentiating w.r.t x we get $\frac{1}{y} \times \frac{dy}{dx} = x^x \times \frac{1}{x}$ + logx $[x^x\,(1 + \log x)]$;

$\Rightarrow \frac{dy}{dx}$ = y $[x^x\,\{\frac{1}{x} + \log x.(1 + logx)\}]$ & thus $\frac{dy}{dx} = x^{x^x} \times x^x\,[\frac{1}{x} + \log x.(1 + logx)]$

CQ9: If $x^m . y^n = (x+y)^{m+n}$, find $\frac{dy}{dx}$

Ans: Taking log of Both Sides, log $(x^m.y^n)$ = log $(x+y)^{m+n}$

$\Rightarrow$ log x^m + log y^n = log $(x+y)^{m+n}$ [Using log mn = log m + log n]

$\Rightarrow$ m. log x + n. log y = $(m + n)$. log (x + y) [using $\log m^n = n\,.\log m$]

Differentiating both sides w.r.t 'x' we get

$\Rightarrow$ m. $\frac{1}{x}$ + n. $\frac{1}{y}.\frac{dy}{dx}$ = $(m + n) \times \frac{1}{(x+y)}\left[1 + \frac{dy}{dx}\right]$; $\Rightarrow \frac{m}{x} + \frac{n}{y}.\frac{dx}{dy} = \frac{m+n}{x+y} + \frac{m+n}{x+y}.\frac{dy}{dx}$

$\Rightarrow \frac{n}{y}.\frac{dx}{dy} - \left(\frac{m+n}{x+y}\right)\times\frac{dy}{dx} = \frac{(m+n)}{x+y} - \frac{m}{x}$; $\Rightarrow \frac{dy}{dx}\left[\frac{n}{y} - \frac{m+n}{x+y}\right] = \frac{(m+n)}{x+y} - \frac{m}{x}$

$\Rightarrow \frac{dy}{dx}\left[\frac{n\,(x+y)-(m+n)y}{(x+y)(y)}\right] = \frac{(m+n)x - m(x+y)}{x\,(x+y)}$; $\Rightarrow \frac{dy}{dx} = \frac{\frac{mx+nx-mx-my}{x}}{\frac{nx+ny-my-ny}{y}} = \frac{\frac{nx-my}{x}}{\frac{nx-my}{y}}$ & thus, $\frac{dy}{dx} = \frac{y}{x}$.

CQ10: If y = $\sqrt{\frac{1-x}{1+x}}$ show that $(1 - x^2)\frac{dy}{dx} + y = 0$

Ans: Taking log of both sides we get, log y = $\frac{1}{2}$ [log (1-x) — log (1 + x)]

Differentiating both sides w.r.t 'x', we have,

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}$ [log (1-x) — log (1 + x)] = $\frac{1}{2}\left(\frac{-1}{1-x} - \frac{1}{1+x}\right) = -\frac{1}{1-x^2}$

By cross — multiplication $(1 - x^2)\frac{dy}{dx}$ = -y; $(1 - x^2)\frac{dy}{dx}$ +y = 0.

CQ11: If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{logx}{(1+logx)^2}$

Ans: Taking log of both sides, we have y. log x = (x —y) log e [Log e = 1]

⇒ y. log x = x — y; ⇒ y. log x + y = x; ⇒ y (log x + 1) = x ⇒ y = $\frac{x}{(\log x+1)}$

⇒ Differentiating w.r.t x we get $\frac{dy}{dx} = \frac{(\log x+1)[1]-(x\,[1/x])}{(\log x+1)^2}$;

⇒ $\frac{dy}{dx} = \frac{(\log x+1-1)}{(\log x+1)^2} = \frac{logx}{(1+logx)^2}$

CQ12: $\frac{d}{dx}[\log(x + \sqrt{x^2 + a^2})]$

Ans: Let y = $(x + \sqrt{x^2 + a^2})$

Thus $\frac{dy}{dx} = [1 + \frac{1}{2\sqrt{x^2+a^2}}. \frac{d}{dx}(x^2 + a^2)] = [1 + \frac{1}{2\sqrt{x^2+a^2}}.2x] = [1 + \frac{x}{\sqrt{x^2+a^2}}] = \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}} = \frac{y}{\sqrt{x^2+a^2}}$

Thus $\frac{dy}{dx} = \frac{y}{\sqrt{x^2+a^2}}$

Now, $\frac{d}{dx}[\log y] = \frac{1}{y} \times \frac{dy}{dx}$; $= \frac{1}{y} \times \frac{y}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

HIGHER ORDER DERIVATIVE

- $\frac{dy}{dx}$ is known as first order derivative of 'y' w.r.t 'x'.
- If we differentiate $\frac{dy}{dx}$ again w.r.t 'x', we will get 2nd order derivative of 'y' w.r.t. 'x', written as $\frac{d^2y}{dx^2}$.

CQ13: If y = $ae^{mx} + be^{-mx}$ prove that $\frac{d^2y}{dx^2} = m^2$y.

Ans: $\frac{dy}{dx} = \frac{d}{dx}(ae^{mx} + be^{-mx}) = ame^{mx} - bme^{-mx}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(ame^{mx} - bme^{-mx})$

$= am^2e^{mx} + bm^2e^{-mx} = m^2(ae^{mx} + be^{-mx}) = m^2y.$

CQ14: Find third order derivative of log $[(3x + 4)^{1/2}]$

Ans: y' $= \frac{1}{2} \cdot \frac{1}{(3x+4)}. 3 = \frac{3}{2(3x+4)}$

y'' $= \frac{3}{2}. \frac{d}{dx}[\frac{1}{(3x+4)}] = \frac{3}{2}. (-1)\frac{3}{(3x+4)^2}] = -\frac{3}{2}.\frac{3}{(3x+4)^2} = -\frac{9}{2}.\frac{1}{(3x+4)^2}$

y''' $= -\frac{9}{2}.\frac{d}{dx}[\frac{1}{(3x+4)^2}] = -\frac{9}{2}.(-2).[\frac{3}{(3x+4)^3}] = \frac{27}{(3x+4)^3}$

CQ15: Find the second differential coefficient of y, = $x^2 \log x$

Ans: $\frac{dy}{dx} = x^2.\frac{1}{x} + \log x.\ 2x = x + 2x.\ \log x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}[x + 2x.\ \log x]$

$= 1 + 2.\frac{d}{dx}[x.\ \log x]$ ⇒ $1 + 2[x.\frac{1}{x} + \log x.\ 1]$ ⇒ $1 + 2[1 + \log x]$ ⇒ $1 + 2 + 2\log x$

$= 3 + 2\log x =$ **$3 + \log x^2$**

CQ16: If $f(x) = x^3 - 2x$; 2nd order derivative of f(x) is ______.

Ans: $\frac{dy}{dx} = 3x^2 - 2$; $\frac{d^2y}{dx^2}$ **= 6x.**

CQ17: If $x = at^2$ and $y = 2at$ then $\frac{d^2y}{dx^2} =$

Ans: Here $x = at^2$ and y = 2at. Differentiating, we get:

⇒ $\frac{dx}{dt} = 2at$ & $\frac{dy}{dt} = 2a$; ⇒ $\frac{dx}{dy} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$ .... (1)

⇒ $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{t}\right) = \frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{t^2} \times \frac{1}{2at}$ $\left[From\ (1), \frac{dx}{dt} = 2at \therefore \frac{dt}{dx} = \frac{1}{2at}\right]$

$\frac{d^2y}{dx^2} = \frac{1}{2at^3}$

APPLICATIONS OF DIFFERENTIAL CALCULUS

❖ **Gradient (slope) of the curve** is given by $\frac{dy}{dx}$.

CQ18: Find the gradient of the curve $y = 3x^2 - 5x + 4$ at the point (1, 2).

Ans: $\frac{dy}{dx} = 6x - 5 = 6(1) - 5 = 1$. Thus, gradient of the curve at (1, 2) is 1.

❖ **To find out Minima & Maxima of the function.**

Steps to find out Minima & Maxima of the function:

1. Find f '(x).
2. Put f '(x) = 0 & obtain the values of 'x' from the equation formed.
3. Find f ''(x).
4. Put the values of 'x' obtained in step 2 in f ''(x).
   - If result > 0, then that value of 'x' is Minima.
   - If result < 0, then that value of 'x' is Maxima.
   - If Result = 0, it means 2nd order derivative test failed.

We will use 1st order derivative test. If it also fails, then such point is neither minima nor maxima.

Such point is called **"Point of Inflexion".**

PC Note: By Putting Minima in f(x), we will get the **minimum value** of the function.
By Putting Maxima in f(x), we will get the **maximum value** of the function.

Q19. Find the minimum & maximum value of $f(x) = x^3 + 2x^2 - 4x + 6$.

Ans: Step 1: $f'(x) = 3x^2 + 4x - 4$.

Step 2: $3x^2 + 4x - 4 = 0 \quad \Rightarrow x = -2, \frac{2}{3}$

Step 3: $f''(x) = 6x + 4$.

Step 4: Putting $x = -2$ in $f''(x) \Rightarrow 6(-2) + 4 = -12 + 4 = -8$ which is less than 0.

Thus $x = -2$ is Minima.

Putting $x = \frac{2}{3}$ in $f''(x) \Rightarrow 6(\frac{2}{3}) + 4 = 4 + 4 = 8$ which is greater than 0. Thus $x = \frac{2}{3}$ is Maxima

$\Rightarrow$ Minimum value of function $= (-2)^3 + 2(-2)^2 - 4(-2) + 6 = -8 + 8 + 8 + 6 = 14$.

$\Rightarrow$ Maximum value of function $= (\frac{2}{3})^3 + 2(\frac{2}{3})^2 - 4(\frac{2}{3}) + 6 = \frac{8}{27} + \frac{8}{9} - \frac{8}{3} + 6 = \mathbf{\frac{122}{27}}$.

- **Total Cost Function C(x):** Total cost consists of two parts (i) Variable Cost (ii) Fixed Cost. Variable cost depends upon the number of units produced (i.e value of x) whereas fixed cost is independent of the level of output x.
  - **Total Cost** $C(x) = VC + FC = V(x) + F(x)$
  - **Average cost** $= \frac{\text{Total Cost}}{\text{No.of units}} = \frac{C(x)}{x}$.
- **Total Revenue Function R(x):** It is the amount received by selling 'x' units @ Rs. 'p' per unit.
  - **Total Revenue** $R(x) = p \times x$.
  - **Average Revenue** $= \frac{\text{Total Revenue}}{\text{No.of units}} = \frac{R(x)}{x}$
- **Profit Function** P(x): Revenue Function − Cost Function = **R(x) - C(x)**.
- **Break − Even Point** (BEP): It is the point at which revenue = cost. **[@ BEP: R(x) = C(x)]**.
- **Marginal Cost (MC):** Cost of producing an additional unit. $\mathbf{MC = \frac{d}{dx}[C(x)]}$.
- **Marginal Revenue (MR):** Revenue from selling an additional unit. $\mathbf{MR = \frac{d}{dx}[R(x)]}$.
- **Marginal Profit (MP):** Profit from selling an additional unit. $\mathbf{MP = \frac{d}{dx}[P(x)]}$

❖ **Marginal Propensity to Consume (MPC):** The consumption function C = F(Y) expresses the relationship between the total consumption and total Income (Y), then the marginal propensity to consume is defined as the rate of Change consumption per unit change in Income i.e. $\frac{dC}{dY}$.

By consumption we mean expenditure incurred in on Consumption.

❖ **Marginal Propensity to save (MPS):** Saving (S) is the difference between income (I) & consumption (c) given by $\frac{dS}{dY}$.

CQ20: Total cost of producing 20 items of a commodity is Rs. 205, while total cost of producing 10 items is Rs. 135. Assuming that the cost function is a linear function, find the cost function and marginal cost function.

Ans: Let cost function be C(x) = ax + b [x being no. of items and a, b being constants] - (i)
Given, C(x) = 205 for x = 20 and C(x) = 135 for x = 10.
Putting these values in (i), 205 = 20a+b - (ii) & 135 = 10a+b - (iii)
(ii) – (iii) gives, 70 = 10a or, a = 7
From (iii), b = 135 – 10a = 135 – 70 = 65
Required cost function is given by C(x) = 7x + 65. **Marginal cost function = $\frac{d}{dx}$C(x) = 7.**

CQ21: A company decided to set up a small production plant for manufacturing electronic clocks. The total cost for initial set up (fixed cost) is Rs. 9 lacs. The additional cost for producing each clock is Rs. 300. Each clock is sold at Rs. 750. During the first month, 1,500 clocks are produced and sold.

(i) What profit or loss company incurs during the first month, when all the 1,500 clocks are sold?

(ii) Determine the break-even point.

Ans: Cost function C(x) for 'x' clocks = 9,00,000 (FC) + 300x (VC).
Revenue function R(x) from 'x' clocks = p × x = 750 × x = 750x.
(i) Profit function P(x) = R(x) – C(x) = 750x – [9,00,000 + 300x] = 450x – 9,00,000.
Thus, when all 1500 clocks are sold = 450 × 1500 – 9,00,000 = - Rs. 2,25,000 = Loss of Rs. 2,25,000
(ii) At BEP, C(x) = R(x);
⇒ 9,00,000 + 300x = 750x; ⇒ 450x = 9,00,000 ⇒ x = 2,000 units.
Hence, 2000 clocks have to be sold to achieve the break-even point.

CQ22: A computer software company wishes to start the production of floppy disks. It was observed that the company had to spend Rs. 2 lakhs for the technical informations. The cost of setting up the machine is Rs. 88,000 and the cost of producing each unit is Rs. 30, while each floppy could be sold at Rs. 45. Find:

(i) Total cost function for producing x floppies; & (ii) Break- Even point.

Ans: (i) Total Cost function C(x) = FC + VC = 2,88,000 + 30x. Revenue function R(x) = p × x = 45x.
(ii) At BEP, C(x) = R(x);
⇒ 2,88,000 + 30x = 45x; ⇒ 15x = 2,88,000 ⇒ x = 19,200 units.
Hence, 19,200 units have to be sold to achieve the break-even point.

CQ23: The total cost function of a firm is C(x) = $\frac{x^3}{3}$ - $5x^2$ + 28x + 10, where C is the total cost and x is output. A tax at Rs. 2 per unit of output is imposed and the producer adds it to his cost. If the market demand function is given by p = 2530 – 5x, where p is price p.u of output, find (i) Profit maximizing output & (ii) Price for maximum profit.

Ans:

After imposition of tax of Rs. 2 per unit, the total new cost is C(x) = $\frac{x^3}{3}$ - $5x^2$ + 28x + 10 + 2x;

Revenue Function R(x) = p × x = (2530 – 5x) × x = 2530x – $5x^2$;

(i) P(x) = R(x) – C(x) = [2530x – $5x^2$]– [$\frac{x^3}{3}$ - $5x^2$ + 28x + 10 + 2x] = - $\frac{x^3}{3}$ + 2500x – 10.

We know that P(x) = profit per unit & P'(x) is change in profit for additional unit.

We want profit maximizing output [i.e output at which profit is maximum] & P'(x) = 0.

P'(x) = $\frac{-3x^2}{3}$ + 2500 = $-x^2$ + 2500.

Putting P'(x) = 0, we get 'x' = ± 50. Since output cannot be negative, we consider x = 50.

P ''(x) = -2x.

Putting the value of 'x' = 50 in P''(x), we get -2.50 = - 100 which is less than '0'.

Thus x = 50 is maxima. Thus, the profit is maximum at x = 50.

(ii) Putting x = 50 in demand function, the corresponding price is p = 2530 – 5 × 50 = Rs. 2280.

Price for maximum profit = Rs. 2280.

CQ24: The cost function of a company is given by: C(x) = 100x- $8x^2 + \frac{x^3}{3}$.

Find the level of output at which: (i) Marginal cost is minimum& (ii) Average cost is minimum.

Ans: Average Cost A(x) = $\frac{C(x)}{x}$ = [100x- $8x^2 + \frac{x^3}{3}$]/x = 100 – 8x + $\frac{x^2}{3}$.

A'(x) = -8 + $\frac{2x}{3}$; & A''(x) = $\frac{2}{3}$;

Marginal Cost M(x) = C'(x) = $\frac{d}{dx}$[100x- $8x^2 + \frac{x^3}{3}$] = 100 – 16x + x^2

M'(x) = -16 + 2x; & M''(x) = 2.

(i) Marginal Cost M(x) is Minimum or Maximum when M'(x) = 0. -16 + 2x = 0 ⇒ **x = 8.**

Putting x = 2 in M''(x), we get '2' which is greater than 0, thus x = 2 is Minima.

Thus, Marginal cost is minimum at x = 8.

(ii) Average Cost A(x) is Minimum or Maximum when A'(x) = 0. -8 + $\frac{2x}{3}$ = 0 ⇒ **x = 12.**

Putting x = 12 in A''(x), we get $\frac{2}{3}$ which is greater than 0, thus x = 12 is Minima.

Thus, Average cost is minimum at x = 12.

Minimum Average Cost = 100 – 8(12) + $\frac{(12)^2}{3}$ = 100 – 96 + 144/3 = 52.

Space for PC Class Note:

DIFFERENTIAL CALCULUS – QUESTION BANK

SN	8A. DIFFERENCIAL CALCULAS	Ans
Q1	D_{XY} represents ______. (a) dy/dx (b) dx/dy (c) f(x) (d) f(y)	A
Q2	If $y = 5x^2$ then $\frac{dy}{dx}$ is ______. (a) 10x (b) 5x (c) 2x (d) None	A
Q3	If $y = x^3$ then $\frac{dy}{dx}$ is ______. (a) $\frac{x^4}{4}$ (b) $-\frac{x^4}{4}$ (c) $3x^2$ (d) $-3x^2$	C
Q4	The derivative of $\frac{x^3}{2}(x > 0)$ is ______. (a) $2\frac{x^2}{3}$ (b) $3\frac{x^2}{2}$ (c) $5^{2x/5}$ (d) $5^{5x/2}$	B
Q5	Find $\frac{dy}{dx}$, when $y = 10x^8$ (a) $80x^7$ (b) $10x^7$ (c) $80x^8$ (d) None	A
Q6	If $f(x) = x^k$ and $f'(1) = 10$ the value of k is ______. (a) 10 (b) -10 (c) 1/10 (d) None	A
Q7	If $y = -3x^{-7/3}$ then $\frac{dy}{dx}$ is ______. (a) $7x^{-10/3}$ (b) $-7x^{-10/3}$ (c) $-\frac{7}{3}x^{-10/3}$ (d) None	A
Q8	If 1st order derivative of $f(x) = 3x^2 + 2$ and $f(0) = 0$ then f(2) is ______. (a) 12 (b) 21 (c) 10 (d) 1	A
Q9	If $y = 2x + x^2$ then $\frac{dy}{dx}$ is ______. (a) 2(x+1) (b) 2(x-1) (c) x+1 (d) x-1	A
Q10	If $y = 4x^3 - 7x^4$ then $\frac{dy}{dx}$ is ______. (a) $2x(-14x^2 + 6x)$ (b) $2x(14x^2 + 6x)$ (c) $2x(14x^2 - 6x)$ (d) None	A
Q11	If $f(x) = x^3 + 5x^2 - 8$ the value of 1st derivative of f(x) when x = 2 is ______. (a) 32 (b) 33 (c) 23 (d) 34	A
Q12	Differentiate $3x^2 + 5x - 2$ with respect to x. (a) 6 (b) 6x + 5 (c) $3x^2 + 5$ (d) 5	B

Q13	$\frac{d}{dx}$ (x - 1) (x - 2) is equal to ______. (a) 2x-3 (b) 3x-2 (c) 1 (d) None	A
Q14	If y = x(x-1) (x-2) then $\frac{dy}{dx}$ is ______. (a) $3x^2-6x + 2$ (b) $- 6x^2 + 2$ (c) $3x^2 + 2$ (d) $3x^3 + 5$	A
Q15	The derivative of $\frac{x^2-1}{x}$ is ______. (a) $1+\frac{1}{x^2}$ (b) $1-\frac{1}{x^2}$ (c) $\frac{1}{x^2}$ (d) None	A
Q16	The differential coefficients of $\frac{x^2-1}{x}$ is ______. (a) $1+\frac{1}{x^2}$ (b) $1-\frac{1}{x^2}$ (c) $\frac{1}{x^2}$ (d) None	B
Q17	If $y=\left[\frac{(1-x)}{x}\right]^2$ then $\frac{dy}{dx}$ is ______. (a) $2(x^{-3} + x^{-2})$ (b) $2(-x^{-3} + x^{-2})$ (c) $2(x^{-3}-x^{-2})$ (d) None	B
Q18	$y = 9x^4 - 7x^3 + 8x^2 - \frac{8}{x} + \frac{10}{x^3}$ then $\frac{dy}{dx}$ is ______. (a) $36x^3-21x^2+16x+8x^{-2}-30x^{-4}$ (b) $36x^3-21x^2+16x- 8x^{-2}+30x^{-4}$ (c) $36x^3+21x^2+16x+8x^{-2}+30x^{-4}$ (d) None	A
Q19	If $y=(3x^2+1)(x^3+2x)$ then $\frac{dy}{dx}$ is ______. (a) $15x^4+21x^2+2$ (b) $15x^3+21x^2+2$ (c) $15x^3+21x+2$ (d) None	A
Q20	Differentiate y w.r.t. x when $y = (x^2 - 2x)(x^2 +1)$ (a) $4x^3 + 6x^2 - 2x+2$ (b) $4x^3 -6x + 2$ (c) $4x^3 - 6x^2 + 2x - 2$ (d) None	C
Q21	If $f(x) = x^2-6x + 8$ then f'(5)-f'(8) is equal to ______. (a) f' (2) (b) 3f' (2) (c) 2 f' (2) (d) None	B
Q22	If $x^2-y^2+3x-5y = 0$ then 3) $\frac{dy}{dx}$ is _____. (a) $(2x + 3) (2Y+5)^{-1}$ (b) $(2x + 3) (2y-5)^{-1}$ (c) $(2x-3) (2y-5)^{-1}$ (d) None	A
Q23	If $x^2 + y^2 - 2x = 0$ then $\frac{dy}{dx}$ is ______. (a) $\frac{(1-x)}{y}$ (b) $\frac{(1+x)}{y}$ (c) $\frac{(x-1)}{y}$ (d) None	A
Q24	If $y=ax^3+bx^2+cx+d$ then $\frac{dy}{dx}$ is equal to ______. (a) $3ax^2+2bx+c$ (b) $\frac{ax^2}{4}+\frac{bx^3}{3}+\frac{\alpha^2}{2}+dx$ (c) 0 (d) None	A

Q25	If $y = (x - x^{-1})^2$ then $\frac{dy}{dx}$ is ______. (a) $2x - 2x^{-3}$ (b) $2x + 2x^{-3}$ (c) $2x + 2x^3$ (d) $2x - 2x^3$	A
Q26	If y = $(x^{1/3}-x^{-1/3})$then $\frac{dy}{dx}$ is ______. (a) $1-x^{-2} + x^{-2/3} - x^{-4/3}$ (b) $1 + x^{-2} + x^{-2/3} - x^{-4/3}$ (c) $1 + x^{-2} + x^{-2/3} + x^{-4/3}$ (d) None	A
Q27	y = $2x^{3/2}(x^{1/2}+2(x^{1/2}-1)$ then dy/dx is ______. (a) $4x+5x(x-6)^{1/2}x^{1/2}$ (b) $4x+5x(x-3)^{1/2}x^{1/2}$ (c) $4x+5x(x-2)^{1/2}x^{1/2}$ (d) None	A
Q28	Find $\frac{dy}{dx}$ of $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$ (a) $-b^2x/a^2y$ (b) $-b^2y/a^2x$ (c) $-b^2/a^2$ (d) None	A
Q29	The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8$ at x = 0 is ______. (a) -12 (b) 12 (c) 0 (d) 1	A
Q30	The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at x = 0 is ______. (a) 3 (b) -3 (c) 1/3 (d) -1	B
Q31	If $x^3-2x^2y^2 + 5x + y - 5 = 0$ then $\frac{dy}{dx}$ at x =1 y =1 is equals to ______. (a) 4/3 (b) -4/3 (c) ¾ (d) None	A
Q32	$If \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1; \frac{dy}{dx}$ can be expressed as ______. (a) $\frac{x}{a}$ (b) $\frac{x}{\sqrt{x^2-a^2}}$ (c) $\frac{1}{\sqrt{\frac{x^2}{a^2}-1}}$ (d) $\frac{x}{y}$	D
Q33	If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + .. + \frac{x^n}{n} + \cdots \infty$ then $\frac{dy}{dx} - y$ is ______. (a) 1 (b) -1 (c) 0 (d) None	C
Q34	The derivative of e^o is ______. (a) 0 (b) 1 (c) e (d) ∞	A
Q35	If f(x) = e^{ax^2+bx+c} then f '(x) is ______. (a) e^{ax^2+bx+c} (b) $e^{ax^2+bx+c}(2ax + b)$ (c) 2ax + b (d) a+b	B
Q36	If y = $e^x + e^{x}$ then $\frac{dy}{dx}$ - $\sqrt{y^2 - 4}$ is equal to ______.	C

	(a) 1 (b) -1 (c) 0 (d) None	
Q37	If y = $e^{\sqrt{2x}}$ $\frac{dy}{dx}$ is calculated as _______. (a) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (b) $e^{\sqrt{2x}}$ (c) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (d) None	**A**
Q38	$\frac{d}{dx}$ e^{2logx} is equal to _______. (a) 2 (b) 2x (c) x^2 (d) 0	**B**
Q39	If $x^y y^x$ = M, M is constant then $\frac{dy}{dx}$ is equal to _______. (a) $\frac{-y}{x}$ (b) $\frac{-y(y+x\log y)}{x(y\log x+x)}$ (c) $\frac{y+x\log y}{y\log x+x}$ (d) None	**B**
Q40	If f(x) = $5x^a$ + $10a^x$ + $3a^a$ then $\frac{dy}{dx}$ equals to _______. (a) $5ax^{a-1}$ + $10xa^{x-1}$ + $3a.a^{a-1}$ (b) $5ax^{a-1}$+$10a^x$loga (c) $5x^a$logx+$10xa^{x-1}$ (d) None	**B**
Q41	The derivative of y = $\sqrt{x+1}$ is _______. (a) 1/$\sqrt{x+1}$ (b) -1/$\sqrt{x+1}$ (c) 1/2$\sqrt{x+1}$ (d) None	**C**
Q42	If $y = \frac{1}{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to _______. (a) $\frac{1}{2x\sqrt{x}}$ (b) $\frac{-1}{x\sqrt{x}}$ (c) $-\frac{1}{2x\sqrt{x}}$ (d) $2x$	**C**
Q43	If $y = x^{-1/2}$ then $\frac{dy}{dx}$ is _______. (a) $(-1/2)x^{-3/2}$ (b) $(1/2)x^{-3/2}$ (c) $(1/2)x^{3/2}$ (d) None	**A**
Q44	The derivation of the function $\sqrt{x+\sqrt{x}}$ is _______. (a) $\frac{1}{2\sqrt{x+\sqrt{x}}}$ (b) $1+\frac{1}{2\sqrt{x}}$ (c) $\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$ (d) None	**C**
Q45	Differentiate $\sqrt{1+x^2}$ w.r.t.x, we get _______. (a) $\frac{2x}{\sqrt{1-x^2}}$ (b) $\frac{x}{\sqrt{1+x^2}}$ (c) $\frac{x^2}{\sqrt{1+x^2}}$ (d) None	**B**
Q46	Let $f(x) = \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 . f'(2)$ will be _______. (a) ¾ (b) ½ (c) 0 (d) None	**A**
Q47	Find the first derivative of y = $\log_e$ x (a) $\frac{1}{x}$ (b) e.log x (c) $\frac{1}{x}$e (d) None	**A**
Q48	If y = log 5x then $\frac{dy}{dx}$ is _______.	**A**

	(a) x^{-1A} (b) x (c) $5x^{-1}$ (d) 5x	
Q49	Differentiate $a^x + x^a + a^a$ with respect to x. (a) $a^x \log a + ax^{3-1}$ (b) $a^x \log a + ax^a$ (c) a^x (d) $a^x + ax^{a-1}$	A
Q50	$\frac{d}{dx}\sqrt{logx}$ (a) $\frac{1}{2x\sqrt{logx}}$ (b) $2x.\sqrt{logx}$ (c) $\frac{1}{\sqrt{logx}}$ (d) $\frac{1}{x}$	A
Q51	If $y = x^{10} + 5log3x + 6e^{2x} + 10$ then $\frac{dy}{dx}$ is ______. (a) $10x^9+15x+ 12e^{2x}$ (b) $10x^9 + \frac{5}{x} + 12e^{2x}$ (c) $10x^9 + \frac{5}{x} + 6e^{2x}$ (d) None	B
Q52	$\frac{d}{dx}\left(log\left(\sqrt{x-1}+\sqrt{x+1}\right)\right)$ (a) $\frac{1}{2\sqrt{x^2-1}}$ (b) $\frac{1}{2\sqrt{x^2+1}}$ (c) $\frac{1}{\sqrt{x-1}+\sqrt{x+1}}$ (d) None	A
Q53	Differentiate $2^x x^5$ with respect to x. (a) $x^5 2^x \log_e 2 + 5.2^x x^4$ (b) $x^5 2^x \log x + 2^x \log x$ (c) $2^x \log x + x^5$ (d) $x^4 \log_e x + 2^x$	A
Q54	Differentiate $2^x.\log x$ with respect to x. (a) $2^x\log x + 22$ (b) $\frac{2}{x}\log x$ + xlogx x (c) $\frac{2^x}{x} + 2^x\log 2\log x$ (d) $\log 2x + \frac{1}{x}$	C
Q55	$\frac{d}{dx}$ (x log x) is equal to ______. (a) (1+ log x) (b) $\frac{1}{log\ x}$ (c) log x (d) $\frac{x}{log\ x}$	A
Q56	The derivative of x^2 log x is ______. (a) 1+2log x (b) x (1 + 2 log x) (c) 2 log x (d) None	B
Q57	Differentiate e^x logx with respect to x. (a) $\frac{e^x}{x}$ (b) $\frac{e^x}{x}logx$ (c) $\frac{e^x}{x}(1+xlogx)$ (d) $e^{\log x}$	C
Q58	If xy = 1 then $y^2 + \frac{dy}{dx}$ is equal to ______. (a) 1 (b) 0 (c) -1 (d) 2	B
Q59	Given $e^{xy} - 4xy = 0$; $\frac{dy}{dx}$ can be proved to be ______. (a) -y/x (b) y/x (c) x/y (d) None	A
Q60	If $x^3-xy^2+y^2+2 = 0$ then $\frac{dy}{dx}$ is ______.	A

	(a) $\frac{(y^2-3x^2)}{2y(3-x)}$ (b) $\frac{(y^2-3x^2)}{2y(x-3)}$ (c) $\frac{(y^2-3x^2)}{2y(3+x)}$ (d) $\frac{(y^2-3x^2)}{(3-x)}$	
Q61	If f(xy) = $x^3 + y^3 - 3axy = 0$ $\frac{dy}{dx}$ can be found out as ______. (a) $\frac{ay-x^2}{y^2+ax}$ (b) $\frac{ay-x^2}{y^2-ax}$ (c) $\frac{ay+x^2}{y^2+ax}$ (d) None	B
Q62	Find $\frac{dy}{dx}$ for $x^2y^2 + 3xy + y = 0$ (a) $\frac{(2xy+y)}{(x+2x)}$ (b) $-\frac{(2xy^2+3y)}{(2x^2y+3x+1)}$ (c) $\frac{x^2y^2-2y}{2xy}$ (d) $-\frac{(2x^2y-3y)}{(x^2y+3x)}$	B
Q63	If $x(1 + y)^{1/2} + y(1 + x)^{1/2} = 0$ then $\frac{dy}{dx}$ is ______. (a) $-(1 + x^2)^{-1}$ (b) $(1 + x^2)^{-1}$ (c) $-(1 + x^2)^{-2}$ (d) $(1 + x^2)^{-2}$	A
Q64	If $ax^2+ 2hxy+by^2+2gx+2fy + c = 0$ $\frac{dy}{dx}$ is ______. (a) $-\frac{(ax+hy+g)}{(hx+by+f)}$ (b) $\frac{(ax+hy+g)}{(hx+by+f)}$ (c) $\frac{(ax-hy+g)}{(hx-by+f)}$ (d) $\frac{h(ax-y+g)}{(x-by+f)}$	A
Q65	If $x^2 + 3xy + y^2 - 4 = 0$ then $\frac{dy}{dx}$ is ______. (a) $-\frac{(2x+3y)}{(3x+2y)}$ (b) $\frac{(2x+3y)}{(3x+2y)}$ (c) $-\frac{(3x+3y)}{(2x+3y)}$ (d) $\frac{(3x+3y)}{(2x+3y)}$	A
Q66	If $x^2e^y + 4\log x = 0$ then $\frac{dy}{dx}$ is ______. (a) $\frac{e^y 2x^2+4+8x}{x^3e^y}$ (b) $\frac{e^y 2x^2-4}{x^3e^y}$ (c) $\frac{-e^y 2x^2-4}{x^3e^y}$ (d) None	C
Q67	$F(x) = log_e\left(\frac{x-1}{x+1}\right)$ and $f'(x) = 1$ then the value of x = ______. (a) 1 (b) 0 (c) $\pm\sqrt{3}$ (d) $\pm\sqrt{2}$	A
Q68	Let p = x^3 log x, so what is the value of $\frac{d^2p}{dx^2}$? (a) 5x + 6x log x (b) $5x^2 + \log x^2$ (c) $5x^2 + 6x \log x$ (d) None	A
Q69	Differentiate $\frac{x^2}{e^x}$ with respect to x. (a) $e^x + \frac{2}{x}$ (b) $\frac{x(2-x)}{e^x}$ (c) $e^x \log x$ (d) e^{2x}	B
Q70	The derivative of $\frac{3-5x}{3+5x}$ is ______. (a) $30(3+ 5x)^{-2}$ (b) $1 / (3+5x)^2$ (c) $-\frac{30}{(3 + 5x)^2}$ (d) None	C
Q71	If $f(x) = \frac{x^2+1}{x^2-1}$ then $f'(x)$ is ______. (a) $-\frac{4x}{(x^2-1)^2}$ (b) $4x(x^2-1)^2$ (c) $\frac{x}{(x^2-1)^2}$ (d) 4 × +1	A

Q72	If $y = \frac{x^2-1}{x^2+1}$ then $\frac{dy}{dx}$ is ______. (a) $4x(x^2+1)^{-2}$ (b) $4x(x^2+1)^2$ (c) $4x(x^2-l)^{-2}$ (d) None	A
Q73	Find value of $\frac{dy}{dx}$ if $y = x^x$ (a) $x^x(1 + \log x)$ (b) $1+\log x$ (c) $y. \log x$ (d) None	D
Q74	If $y = f(x) = \frac{ax+b}{ax-a}$ then $f'(y)$ is ______. (a) -x (b) 2x (c) x (d) None	A
Q75	If $y = \frac{x^{1/2}+2}{x^{1/2}}$ then $\frac{dy}{dx}$ is ______. (a) $-x^{-3/2}$ (b) 3x (c) x (d) None	A
Q76	If $y = \frac{x^{1/2}(5-2x)^{2/3}}{(4-3x)^{3/4}(7-4x)^{4/5}}$ then the value of $\frac{dy/dx}{y}$ is ______. (a) $\frac{1}{2x} - \frac{4}{3(5-2x)} + \frac{9}{4(4-3x)} + \frac{16}{5(7-4x)}$ (b) $\frac{1}{2x} - \frac{3}{4(5-2x)} + \frac{4}{9(4+3x)} + \frac{16}{(7+4x)}$ (c) $\frac{1}{x} - \frac{3}{4(5-2x)} + \frac{4}{9(4-3x)} + \frac{16}{5(7-4x)}$ (d) None	A
Q77	If $y = \frac{(x+a)(x+b)(x+c)(x+d)}{(x-a)(x-b)(x-c)(x-d)}$ ffien value al $\frac{dy/dx}{y}$ is ______. (a) $(x + a)^{-1} + (x + b)^{-1} + (x + c)^{-1} + (x + d)^{-1} - (x - a)^{-1} - (x - b)^{-1} - (x - c)^{-1} - (x - d)^{-1}$ (b) $(x + a)^{-1} - (x + b)^{-1} + (x + c)^{-1} - (x + d)^{-1} - (x - a)^{-1} - (x - b)^{-1} + (x - c)^{-1} - (x - d)^{-1}$ (c) $(x - a)^{-1} + (x - b)^{-1} + (x - c)^{-1} + (x - d)^{-1} - (x + a)^{-1} - (x + b)^{-1} - (x + c)^{-1} - (x + d)^{-1}$ (d) None	A
Q78	If $y = \frac{(x+1)(2x-1)}{(x-3)}$ then $\frac{dy}{dx}$ is ______. (a) $\frac{2(x^2-6x-1)}{(x-3)^2}$ (b) $\frac{2(x^2+6x-1)}{(x-3)^2}$ (c) $\frac{2(x^2+6x+1)}{(x-3)^2}$ (d) None	A
Q79	If $y = \frac{5x^4-6x^2-7x+8}{5x-6}$ then $\frac{dy}{dx}$ is ______. (a) $(75x^4 - 120x^3 - 30x^2 + 72x + 2)(5x - 6)^{-2}$ (b) $\frac{(75x^4-120x^3+30x^2-72x+2)}{5x-6}$ (c) $\frac{(75x^4-120x^3-30x^2+72x-2)}{(5x-61}$ (d) None	A
Q80	Differentiate $\frac{e^x}{logx}$ with respect to x. (a) $\frac{e^x(x \log -1)}{x(\log x)}$ (b) $\frac{e^x(x \log x-1)}{x(logx)^2}$ (c) $e^x \log x$ (d) None	B

Q81	If $y = \frac{e^x+1}{e^x-1}$ then $\frac{dy}{dx}$ is equal to ______. (a) $\frac{-2e^x}{(e^x-1)^2}$ (b) $2e^x(e^x-1)^2$ (c) $2(e^x-1)^2$ (d) None	A
Q82	Given $x = 2t+5; y = t^2 - 2$ $\frac{dy}{dx}$ is calculated as ______. (a) t (b) -1/t (c) 1/t (d) None	A
Q83	If x = $3t^2$ -1, y = t^3 then $\frac{dy}{dx}$ is equal to ______. (a) $\frac{3t^2}{6t}$ (b) $3t^2$ -1 (c) 3t + 1 (d) None	A
Q84	Given x = at^2; y = 2at $\frac{dy}{dx}$ is ______. (a) t (b) -1/t (c) 1/t (d) None	C
Q85	If x = at^2; y = 2at; $\frac{dy}{dx}_{t=2}$ is equal to ______. (a) 1/2 (b) -2 (c) -1/2 (d) None	A
Q86	If $x = \frac{1-t^2}{1+t^2}; y = \frac{2t}{1+t^2}$ then $\frac{dy}{dx}$ at $t = 1$ is ______. (a) ½ (b) 1 (c) 0 (d) None	C
Q87	If $u = (x^3+1)^5$ and $y = (x^3+5x+7)$ then $\frac{du}{dy}$ is ______. (a) $\frac{15x^2(x^3+1)^4}{3x^2+5}$ (b) $\frac{10(x^2+1)^4}{3x^2+5}$ (c) $5x(x^2+1)^4$ (d) None	D
Q88	If y = x^{2x} then $\frac{dy}{dx}$ is______. (a) $2x^{2x}$(1 + logx) (b) 2(1 + logx) (c) x^{2x}(1 + logx) (d) None	A
Q89	If $y = (3x^2-7)^{1/2}$ then $\frac{dy}{dx}$ is ______. (a) $3x(3x^2-7)^{-1/2}$ (b) $6x(3x^2-7)^{-1/2}$ (c) $3x(3x^2+7)^{-1/2}$ (d) None	A
Q90	If $y = (6x^5-7x^3+9)^{-1/3}$ then $\frac{dy}{dx}$ is ______. (a) $\left(-\frac{1}{3}\right)(6x^5-7x^3+9)^{-4/3}(30x^4-21x^2)$(b) $\left(\frac{1}{3}\right)(6x^5-7x^3+9)^{-4/3}(30x^4-21x^2)$ (c) $\left(-\frac{1}{3}\right)(6x^5-7x^3+9)^{4/3}(30x^{4'}-21x^2)$(d) None	A
Q91	If y = $5x^x$, then $\frac{dy}{dx}$ is equal to ______. (a) $5x^x$(1 - logx) (b) $5x^{x-1}$ (c) $5x^x$(1 + logx) (d) None	C
Q92	Let y = $\sqrt{2x}$ + 3^{2x} then $\frac{dy}{dx}$ is equal to ______.	A

	(a) $\frac{1}{\sqrt{2x}} + 2.3^{2x} \log e^3$ (b) $\frac{1}{\sqrt{2x}}$ (c) 2. 3^{2x} log e^3 (d) None	
Q93	Let $f(y) = x^{x^3}$ then $f'(y)$ is _____. (a) $x^3[x^2 + 3x.\text{logx}]$ (b) $x^{x^3}[x^2 + 3x^2.\text{logx}]$ (c) $x^{x^3}[x^2 - 3x.\text{logx}]$ (d) None	B
Q94	If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is ______. (a) $\frac{logx}{(1-\log x)^2}$ (b) $\frac{logx}{(1+\log x)^2}$ (c) $\frac{logx}{(1-\log x)}$ (d) $\frac{logx}{(1+\log x)}$	B
Q95	If $y = (1+x)^{2x}$ then the value of $\frac{1}{y} \times \frac{dy}{dx}$ is ______. (a) $2[x(x+1)^{-1}+\log(x+1)]$ (b) $x(x+1)^{-1}+\log(x+1)$ (c) $2[x(x+1)^{-1}-\log(x+1)]$ (d) None	A
Q96	If $y = x^a + a^x + x^x + a^a$ then the value of $\frac{1}{y} \times \frac{dy}{dx} \times$ is ______. (a) $x^{-2}(1 - \text{logx})$ (b) $x^2(1 - \text{logx})$ (c) $x^2(1 + \text{logx})$ (d) None	A
Q97	If y = x^{x^x} then the value of $\frac{dy}{dx}$ is ______. (a) x^{x^x} $[x^{x-1} + \text{logx}.x^x(1 + \text{logx})]$ (b) x^{x^x} $[x^{x-1} + \text{logx}.(1 + \text{logx})]$ (c) x^{x^x} $[x^{x-1} + \text{logx}.x^x(1 - \text{logx})]$ (d) x^{x^x} $[x^{x-1} - \text{logx}.x^x(1 - \text{logx})]$	A
Q98	If $y = \sqrt{x}^{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to ______. (a) $\frac{y^2}{2-y\log x}$ (b) $\frac{y^2}{x(2-y\log x)}$ (c) y logx (d) $\frac{y(\log x+2)}{4\sqrt{x}}$	B
Q99	If $y = x^{logx}$ then $\frac{dy}{dx}$ is ______. (a) $x^2 - y^2 + 3x - 5y = 0$ (b) $(2x+3)(2y+5)^{-1}$ (c) 2 $x^{\log x - 1}.logx$ (d) None	A
Q100	If $y = x^{x^{x\ldots\infty}}$ then $\frac{dy}{dx}$ is ______. (a) $\frac{y^2}{[x(1-y\log x)]}$ (b) $\frac{y}{[(1-y\log x)]}$ (c) $\frac{y}{[x(1+y\log x)]}$ (d) $\frac{y^2}{[(1+y\log x)]}$	A
Q101	The derivative of log $x.e^x$ is _____. (a) $\frac{e^x}{x} + e^x(logx)$ (b) $e^x\left(\frac{1}{x} - logx\right)$ (c) $e^x(1 + \log x)$ (d) None	A
Q102	If $y = (3x^3 - 5x^2 + 8)^3$ then $\frac{dy}{dx}$ is ______. (a) $3(3x^3 - 5x^2 + 8)^2(9x^2 - 10x)$ (b) $3(3x^3 - 5x^2 + 8)^2(9x^2 + 10x)$ (c) $3(3x^3 - 5x^2 + 8)^2(10x^2 - 9x)$ (d) None	A

Q103	Differentiate $log(x+\sqrt{x^2+a^2})$ with respect to x. (a) $\frac{1}{\sqrt{x}}$ (b) $\frac{1}{\sqrt{x^2-a^2}}$ (c) $\frac{1}{\sqrt{x^2+a^2}}$ (d) $\frac{x}{\sqrt{x^2-a}}$	C
Q104	Differentiate $log(\sqrt{x-a}+\sqrt{x-b})$ with respect to x. (a) $\frac{1}{2(x-a)(x-b)}$ (b) $\frac{1}{2\sqrt{x-a}\sqrt{x-b}}$ (c) $\frac{1}{2(\sqrt{x-ab})}$ (d) $\frac{1}{\sqrt{x-a}+\sqrt{x-b}}$	B
Q105	If $y = log\,[(x-1)^{1/2}-(x+1)^{1/2}]$ then $\frac{dy}{dx}$ is ______. (a) $\left(\frac{1}{2}\right)(x^2-1)^{-1/2}$ (b) $\left(-\frac{1}{2}\right)(x^2-)^{-1/2}$ (c) $\left(\frac{1}{2}\right)(x^2-1)^{1/2}$ (d) None	A
Q106	If $y = log\left[e^x\frac{(x-2)}{(x+3)}\right]^{3/4}$ then $\frac{dy}{dx}$ is ______. (a) $1+\left(\frac{3}{4}\right)(x-2)^{-1}-\left(\frac{3}{4}\right)(x+3)^{-1}$ (b) $1-\left(\frac{3}{4}\right)(x-2)^{-1}+\left(\frac{3}{4}\right)(x+3)^{-1}$ (c) $1+\left(\frac{3}{4}\right)(x-2)^{-1}+\left(\frac{3}{4}\right)(x+3)^{-1}$ (d) None	A
Q107	If f(x) $=x^3$ -2x then 2nd order derivative of f(x) is ______. (a) 6 (b) 6x (c) $3x^2-2$ (d) 3x	B
Q108	If f(x) = x^4then 3rd order derivative of f(x) when x = 3 is ______. (a) 72 (b) 108 (c) 27 (d) 81	A
Q109	If x = at^2 and y = 2at then $\frac{d^2y}{dx^2}$ is ______. (a) $\frac{1}{2at^3}$ (b) $-\frac{1}{2at^3}$ (c) $2at^3$ (d) None	B
Q110	If $x=\frac{1-t}{1+t}$ and $t=\frac{2t}{1+t}$ then $\frac{d^2y}{dx^2}$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q111	y = e^t and x = log t, then $\frac{dy}{dx}$ = ______. (a) $\frac{1}{t}$ (b) $t.e^t$ (c) $-\frac{1}{t^2}$ (d) None	B
Q112	Find the second differential coefficient of y = x^2 log x (a) x + 2x log x (b) 3 + 2 log x (c) 3 log x (d) 2x log x	B
Q113	If y = $ae^{mx}+be^{-mx}$ then $\frac{d^2y}{dx^2}$ is ______. (a) m^2y (b) my (c) $-m^2y$ (d) -my	A
Q114	If y = x^me^{nx} then $\frac{d^2y}{dx^2}$ is ______. (a) $m(m+1)x^{m-2}e^{nx}+2x^{m-1}e^{nx}+n^2x^m$ (b) $m(1-m)x^{m-2}+2mnx^{m-1}e^{nx}+x^me^{nx}$	D

	(c) $m(1 - m)x^{m-2} + 2mnx^{m-1}e^{nx} + e^{nx}$ (d) $m(m - 1)x^{m-2}e^{nx} + 2mnx^{m-1}e^{nx} + n^2x^me^{nx}$	
Q115	Find the fourth derivative of $\log[(3x + 4)^{1/2}]$ (a) $-243(3x + 4)^{-4}$ (b) $243(3x + 4)^{-4}$ (c) $-243(4x + 3)^{-4}$ (d) None	A
Q116	If $y = \sqrt{x^2 + m^2}$ then y y1 (Where y1 = $\frac{dy}{dx}$) is equal to ______. (a) $-x$ (b) x (c) $1/x$ (d) None	B
Q117	If $y = (x + \sqrt{x^2 + m^2})^n$ then $\frac{dy}{dx}$ equals to ______. (a) ny (b) $\frac{ny}{\sqrt{x^2+m^2}}$ (c) $-\frac{ny}{\sqrt{x^2+m^2}}$ (d) None	B
Q118	If $(x + y)^{m+n} - x^my^n = 0$ then $\frac{dy}{dx}$ is ______. (a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $-x/y$ (d) None	A
Q119	If $y = \sqrt{\frac{x}{m}} + \sqrt{\frac{m}{x}}$ then $2xy\frac{dy}{dx} - \frac{x}{m} + \frac{m}{x}$ is equal to ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q120	If $y = (x + \sqrt{x^2 - 1})m$, then the value of $(x^2 - 1)(\frac{dy}{dx})^2 - m^2y^2$ (a) -1 (b) 1 (c) 0 (d) None	C
Q121	If $y = ae^{2x} + bxe^{2x}$ where a & b are constants, value of expression $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$ is (a) 0 (b) 1 (c) -1 (d) None	A
Q122	If $y = (x + 1)^{1/2} - (x - 1)^{1/2}$ value of expression $(x^2 - 1)\frac{d^2y}{dx^2} + s\frac{dy}{dx} - \frac{y}{4}$ is given by (a) 0 (b) 1 (c) -1 (d) None	A
Q123	If $y = log[x + (1 + x^2)^{1/2}]$ the value of the expression $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is ______. (a) 0 (b) 1 (c) -1 (d) None	A
Q124	If $x^2 + y^2 = a^2$, then $\frac{dy}{dx}$ at (-2, 2) is ______. (a) 2 (b) 2 (c) 1 (d) 3	C
Q125	If $f(x) = 2x^3 - 9x^2 + 12x + 5$, then 1st order derivative of f(x) equal to zero implies __. (a) x = 1 and x = 2 (b) x = 2 and x = - 1 (c) x = 1 and x = 1 (d) x = 2 and x = 2	B
Q126	If $y = 2x^2 + 3x + 10$ then $\frac{dy}{dx}$ at (0,0) is ______. (a) 10 (b) 0 (c) 3 (d) None	C

Q127	The slope of the tangent to the curve X $=\frac{(t-1)}{(t+1)}, Y = \frac{(t+1)}{(t-1)}$ at the point $t = 2$ is ______. (a) 9 (b) $\frac{1}{9}$ (c) -9 (d) $-\frac{1}{9}$	C
Q128	Find slope of tangent of curve $Y = \frac{x-1}{x+2}$ at $x = 2$. (a) 3/16 (b) 5/17 (c) 9/11 (d) None	A
Q129	The curve $4y = ux^2 + v$ passes through the point p at (2, 3) and $\frac{dy}{dx}$ = 4 this point 'p'. So the values of u and v are ______. (a) u = 2, v = 2 (b) u = -4, v = -4 (c) u = 4, v = 4 (d) None	C
Q130	The gradient of the curve $y=-2x^3 + 3x + 5$ at x = 2 is ______. (a) -20 (b) 27 (c) -16 (d) -21	D
Q131	The gradient of curve $y = x^3 - x^2$ at (0, 0) (a) 1 (b) 0 (c) -1 (d) None	B
Q132	The gradient of the curve y = xy + 2px + 3qy at the point (3, 2) is $\frac{-2}{3}$. The values of p and q are ______. (a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) (2, 2) (c) $\left(-\frac{1}{2},-\frac{1}{6}\right)$ (d) (0,0)	C
Q133	The slope of the tangent to the curve $y = \sqrt{4x^2}$ at the point where the ordinate and the abscissa are equal is ______. (a) -1 (b) 1 (c) 0 (d) None	A
Q134	The slope of tangent at the point (2 -2) to curve $x^2 + xy + y^2 - 4 = 0$ is given by ______. (a) 0 (b) 1 (c) -1 (d) None	B
Q135	The slope of the tangent to the curve $y = x^2 - x$ at the point where the line y = 2 cuts the curve in the 1st quadrant is ______. (a) 2 (b) 3 (c) -3 (d) None	B
Q136	The curve $y= -e^{-x}$ is ______. (a) Concave upward for x > 0. (b) Concave downward for x > 0. (c) Everywhere concave upward. (d) Everywhere concave downward.	D
Q137	A function f(x) is maximum at x = c if ______. (a) (2nd order derivative of f(x) when x =c) > 0 (b) (2nd order derivative of f(x) when x =c) < 0 (c) (2nd order derivative of f(x) when x =c) = 0	B

	(d) (2nd order derivative of f(x) when x ⩾ f(c))	
Q138	A function f(x) is minimum at x = b if ______. (a) (2nd order derivative of f(x) when x =b) > 0 (b) (2nd order derivative of f(x) when x =b) < 0 (c) (2nd order derivative of f(x) when x = b) = 0 (d) (2nd order derivative of f(x) when x ⩾ f(b))	A
Q139	Find the maximum and minimum value of y = x^3 - $2x^2$ - 4x -1 (a) $ax\ \frac{13}{27}, min\ -9$ (b) Max $\frac{1}{2}, min\ -9$ (c) Max $9, min\ -\frac{13}{27}$ (d) Max $9, min\ -\frac{1}{2}$	A
Q140	Find the maximum and minimum value of y = $2x^3$ -$15x^2$ + 36x + 12 (a) Max 40, Min39 (b) Max 39, Min38 (c) Max 41, Min 40 (d) None	A
Q141	In question above, at which values of x maximum and minimum occur respectively? (a) 2, 3 (b) 3, 2 (c) -2, -3 (d) -3, -2	A
Q142	Find the maximum and minimum value of y = $\frac{x^3}{3+x^2-3x}$ (a) -5 (b) 5 (c) 5 (d) -5	A
Q143	In question above, at which values of x maximum and minimum occur respectively? (a) -3, 1 (b) -3, -1 (c) 3, 1 (d) 3, -1	A
Q144	The point of inflexion of the curve y=x^4 is at ______. (a) x = 0 (b) x = 3 (c) x = 12 (d) No where	D
Q145	At which values of x maximum and minimum occur respectively in respect of y = x^5 - $5x^4$ + $5x^3$ -1? (a) 1 3 (b) 0 3 (c) Both (d) None	C
Q146	At x = 3, y = $(x-2)^6(x-3)^5$ is ______. (a) A maxima (b) A minima (c) A point of inflexion (d) None	C
Q147	y = x^3 - $3x^2$ + 3x + 7 has ______. (a) A maxima (b) A minima (c) Both (d) None	D
Q148	y = x^2 -6x + 13 has ______. (a) A maxima (b) A minima (c) Both (d) None	B

Q149	In question above, the extreme value of y is ______. (a) 4 (b) 3 (c) -4 (d) -3	A
Q150	U= $5t^4 + 4t^3 + 2t^2 + t + 4$ at t=-1 find du/dt (a) -11 (b) 11 (c) -16 (d) 16	A
Q151	If $e^{xy} - 4^{xy} = 4$ then $\frac{dx}{dy}$: ______. (a) $\frac{y}{x}$ (b) $\frac{-y}{x}$ (c) $\frac{x}{y}$ (d) $\frac{-x}{y}$	B
Q152	If $x^p . y^q = (x+y)^{p+q}$ then $\frac{dx}{dy}$: ______. (a) $\frac{y}{x}$ (b) $\frac{-y}{x}$ (c) $\frac{p}{q}$ (d) $\frac{-p}{q}$	B
Q153	If $= 1 + \frac{x}{1^1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \ldots + \frac{x^n}{n!} + \cdots \ldots . \infty$, then $\frac{dx}{dy} y =$ ______. (a) 1 (b) 0 (c) -1 (d) None	B
Q154	$\int_0^2 \lvert 1 - x \rvert dx =$ ______. (a) 23 (b) 21 (c) 0 (d) 1	D

CHAPTER 8B. INTEGRAL CALCULUS

INTRODUCTION

Integration is the reverse (inverse) process of differentiation & is denoted by the symbol $\int$.

f(x) —— Differciation ——▶ f'(x) —— Integration ——▶ f(x)

BASIC FORMULAE

DIFFERENTIATION	INTEGRATION	Examples
1. $\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}\right] = x^n$; $(n \neq -1)$	$\int x^n.dx = \frac{x^{n+1}}{n+1} + C$; $(n \neq -1)$	$\int \mathbf{x^3} = \frac{x^{3+1}}{3+1} + C = \frac{x^4}{4} + C$
2. $\frac{d}{dx}(x) = 1$	$\int 1.dx = x + C$	$\int \sqrt{\mathbf{x}} = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2(x^{\frac{3}{2}})}{3} + C$
3. $\frac{d}{dx}[\text{Log } x] = \frac{1}{x}$	$\int \frac{1}{x}.dx = \text{Log } x + C$	$\int \frac{\mathbf{1}}{\sqrt{\mathbf{x}}} = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{2(x^{\frac{1}{2}})}{1} = 2\sqrt{x}$
4. $\frac{d}{dx}e^x = e^x$	$\int e^x.dx = e^x + C$	$\int \mathbf{x\sqrt{x}} = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{2(x^{\frac{5}{2}})}{5} + C$
5. $\frac{d}{dx}a^x = a^x.\text{ Log } a$	$\int a^x.dx = \frac{a^x}{\log a} + C$	$\int \mathbf{3^x} = \frac{3^x}{\log 3} + c$

CONSTANT OF INTEGRATION (C)

- In integration of every function, we add "+c" (constant of integration) since $\frac{d}{dx}$(Constant) = 0.

Let us understand this concept.

$\frac{d}{dx}(x^2) = 2x$ & $\frac{d}{dx}(x^2 + 5) = 2x$. Because derivative of a constant is always 'Zero.

$\int 2x.dx = x^2$. & $\int (2x + 5).dx = x^2$.

There may be cases when the constant was there in f(x) but it doesn't appear in f'(x) because of its derivative being 'Zero. So we always have to add a constant in integration. Such constant is "Constant of Integration".

ELEMENT OF INTEGRATION

- *'dx'* is called element of integration. It indicates the variable w.r.t which f(x) is to be integrated.
- In differentiation we use to write $\frac{d}{dx}$, & in Integration we write 'dx'.

In $\int x^5.dx$; dx indicates that x^5 is to be integrated w.r.t 'x'

CHAIN RULE [Here we have to DIVIDE by $\frac{dy}{dx}$]

Basic Rules	Chain Rule	Example
$\int x^n.dx = \frac{x^{n+1}}{n+1} + C$;	$\int y^n.dx = \frac{y^{n+1}}{(n+1)} \div \frac{dy}{dx}$	$\int (4x+5)^6.dx = \frac{(4x+5)^{6+1}}{(6+1).4} = \frac{(4x+5)^7}{28} + C$
$\int \frac{1}{x}.dx = \text{Log } x + C$	$\int \frac{1}{y}.dx = \text{Log } y \div \frac{dy}{dx}$	$\int \frac{1}{(2x+5)}.dx = \frac{\text{Log}(2x+5)}{2} + C$
$\int e^x.dx = e^x + C$	$\int e^y.dx = e^y \div \frac{dy}{dx}$	$\int e^{-3x)} = \frac{e^{-3x}}{-3} = -\frac{1}{3.e^{3x}} + C$
$\int a^x.dx = \frac{a^x}{\log a} + C$	$\int a^y.dx = \frac{a^y}{\log a.} \div \frac{dy}{dx}$	$\int 5^{(3x+5)} = \frac{5^{(3x+5)}}{(\log 5).3} + C$

RULES FOR INTEGRATION

Rules	Examples
1. $\int C.f(x) = C.\int f(x)$	$\int [7x^5].dx = 7.\int x^5.dx = 7.\frac{x^{5+1}}{5+1} = 7.\frac{x^6}{6} = \frac{7}{6}.x^6 + C$
2. $\int [f(x) \pm g(x)] = \int f(x) \pm \int g(x)$	$\int [5x^4 + 3x^3 - 2].dx = 5.\int x^4.dx + 3.\int x^2.dx - 2\int 1.dx$ $= 5.\frac{x^5}{5} + 3.\frac{x^3}{3} - 2x = x^5 + x^3 - 2x + C$

SOME SOLVED EXAMPLES

1) $\int (x + \frac{1}{x})^2.dx = \int x^2.dx + 2\int dx + \int \frac{1}{x^2}.dx$

i. $= \frac{x^3}{3} + 2x + \frac{x^{-2+1}}{-2+1}$ $\quad = \frac{x^3}{3} + 2x - \frac{1}{x} + c$

2) $\int \sqrt{x}\,(x^3 + 2x - 3)dx = \int x^{7/2}\,dx + 2\int x^{3/2}\,dx - 2\int x^{1/2}\,dx$

i. $= \frac{x^{7/2+1}}{7/2+1} + \frac{2x^{3/2+1}}{3/2+1} - \frac{3x^{1/2+1}}{1/2+1}$ $\quad = \frac{2x^{9/2}}{9} + \frac{4x^{5/2}}{5} - 2x^{3/2} + c$

3) $\int (e^{2x} + e^{-4x}).dx = \int e^{2x}.dx + \int e^{-4x}.dx$ $\quad = \frac{e^{2x}}{2} + \frac{e^{-4x}}{-4} = \frac{e^{2x}}{2} - \frac{1}{4e^{4x}} + c$

4) $\int \frac{x^2}{x+1}.dx$ = $\int \frac{x^2-1+1}{x+1} dx$ = $\int \frac{(x^2-1)}{x+1} dx + \int \frac{dx}{x+1}$

= $\int (x-1).dx + + \int \frac{dx}{x+1}$ = $\frac{x^2}{2} - x + \log(x+1) + c$

METHOD OF SUBSTITUTION

- Sometimes, integration of a given function becomes simple by substitution of a new variable (say t) in place of the given variable 'x'.
- Element of integration (dx) is also changed to 'dt' after proper adjustments.

PC Note: Generally (not always), term (variable) on complex side is taken as 't'.

SOME SOLVED EXAMPLES

CQ1: $\int \frac{x^3}{(x^2+1)^3}.dx.$

Ans: $t = (x^2 + 1)$ ---------(i)

Now we have to replace 'dx' with 'dt'. SO we find relation between 'dx' & 'dt'.

Differentiating B.S w.r.t 'x', we get $\frac{dt}{dx} = 2x$;

If we observe the question carefully, we have 'x.dx' in the numerator. So we will find its value in terms of dt.

We get **x.dx = $\frac{dt}{2}$ ----------(ii);** ⇒ we have $x^2 = (t - 1)$ from (i)------------(iii)

Thus $\int \frac{x^2.x.dx}{(x^2+1)^3}$ ⇒ $\int \frac{(t-1).dt}{2t^3}$ ---Substituting value of x^2 & x.dx from (ii) & (iii)

$\Rightarrow \frac{1}{2}[\int \frac{(t)}{t^3}.dt - \frac{1}{t^3}.dt$ $\Rightarrow \frac{1}{2}[\int \frac{(1)}{t^2}.dt - \int \frac{1}{t^3}.dt]$

$\Rightarrow \frac{1}{2}[\frac{t^{-2+1}}{-2+1} - \frac{t^{-3+1}}{-3+1}]$ $\Rightarrow \frac{1}{2}[\frac{t^{-1}}{-1} - \frac{t^{-2}}{-2}]$

$\Rightarrow \frac{1}{2}[-\frac{1}{t} + \frac{1}{2t^2}]$ $\Rightarrow \frac{1}{4t^2} - \frac{1}{2t} + C$ $\Rightarrow \mathbf{\frac{1}{4(x^2+1)^2} - \frac{1}{2(x^2+1)} + C}$

CQ2: $\int \frac{x-1}{\sqrt{(x+4)}}.dx$

Ans: $t = \sqrt{(x+4)}$ & thus $t^2 = x + 4$ ---------(i)

Now we have to replace 'dx' with 'dt'. SO we find relation between 'dx' & 'dt'.

Differentiating B.S w.r.t 'x', we get $2t.\frac{dt}{dx} = 1$;

If we observe the question carefully, we have 'dx' in the numerator. So we will find its value in terms of dt.

We get **dx = 2t.dt.** we have $x = (t^2 - 4)$ from (i)

$\Rightarrow \int \frac{((t^2-4)-1).2tdt}{t}$ $\Rightarrow 2\int (t^2 - 5)$

$\Rightarrow 2\,[\int t^2.dt - \int 5.dt]$ $\Rightarrow \int \frac{1}{t}.dt - \int 5.dt]$

$\Rightarrow 2\,[\frac{t^3}{3} - 5t] + C$ $\Rightarrow \frac{2.t^3}{3} - 10t + C$

$\Rightarrow \frac{2.(x+4)^{3/2}}{3} - 10\sqrt{(x+4)} + C$

CQ3: $\int \frac{dx}{x(x^3+1)} = \int \frac{x^2 dx}{x^3(x^3+1)}$

Ans: Let $t = x^3$; $\frac{dt}{dx} = 3x^2$ $\Rightarrow \frac{dt}{3} = x^2.dx$ $= \int \frac{dt}{3.t(t+1)}$ $= \frac{1}{3}\int(\frac{1}{t} - \frac{1}{t+1}).dt$

$= \frac{1}{3}[\log t - \log(t-1)]$ $= \frac{1}{3}\log(\frac{x^3}{x^3-1}) + C$

INTEGRATION BY PARTS

Let f(x) = u & g(x) = v, $\int(u,v) = u\int v - \int\{\frac{du}{dx}.\int v\}$

How to find 'u' & 'v':

Sequence shall be **LAE**:

L	A	E
Logarithmic function	Algebraic functions [Involving x]	Exponential function [Involving x]

Different Cases: [**Note:** Sequence of the functions given in the question is NOT RELEVANT]

Question Consists of	U	V
1. Logarithmic function & Algebraic function	Logarithmic function	Algebraic function
2. Logarithmic function & Exponential function	Logarithmic function	Exponential function
3. Algebraic function & Exponential function	Algebraic function	Exponential function

SOME SOLVED EXAMPLES

(i) $\int xe^x\,dx$

Ans: x → Algebraic Function & e^x → Exponential Function; Thus u = 'x' & v = e^x.

$= x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x\,dx\right\}dx$

$= xe^x - \int 1.e^x.dx = \boldsymbol{xe^x - e^x + c}$

(ii) $\int x\log x\,dx$

Ans: x → Algebraic Function & Log x → Logarithmic Function; Thus u = 'log x' & v = 'x'.

= $\log x \int x\,dx - \int\left\{\frac{d}{dx}(\log x)\int x dx\right\}dx$

= $\log x.\frac{x^2}{2} - \int\left[\frac{1}{x}.\frac{x^2}{2}\right]dx$ = $\frac{x^2}{2}\log x - \frac{1}{2}\int x.dx$

= $\frac{x^2}{2}\log x - \frac{1}{2}.\frac{x^2}{2} + c$ **= $\frac{x^2}{2}\log x - \frac{x^2}{4} + c$**

(iii) $\int x^2e^x$ dx;

Ans: x^2 → Algebraic Function & e^x→ Exponential Function; Thus u = 'x^2' & v = 'e^x'.

= $x^2\int e^x dx - \int\left[\left(\frac{d}{dx}x^2\right).\int e^x.dx\right]$

= $x^2e^x - \int(2x\,.e^x)$ = $x^2e^x - 2\int(xe^x)dx$

We will have to integrate $\int(xe^x)$ again. *Thus u = 'x' & v = 'e^x'.*

= $x^2e^x - 2[\,x.\int e^x.dx - \left[\int\left[\frac{dx}{dx}\right]\int e^x\,dx\right]$ = $x^2e^x - 2[xe^x - \int 1.e^x.dx]$

= $x^2e^x - 2[xe^x - e^x]$ = $x^2e^x - 2xe^x + 2e^x$

= $e^x[x^2 - 2x + 2] + C$

(iv) $\int x^2 e^{ax}$ dx

Ans: x^2 → Algebraic Function & e^{ax}→ Exponential Function; Thus u = 'x^2' & v = 'e^{ax}'.

= $x^2\int e^{ax}dx - \int\left\{\frac{d}{dx}(x^2)\int e^{ax}dx\right\}dx$

= $x^2.\frac{e^{ax}}{a} - \int 2x.\frac{e^{ax}}{a}dx$ = $\frac{x^2}{a}e^{ax} - \frac{2}{a}\int x.e^{ax}dx$

We will have to integrate $\int(xe^{ax})$ again. *Thus u = 'x' & v = 'e^{ax}'.*

= $\frac{x^2}{a}e^{ax} - \frac{2}{a}\{x.\int e^{ax}.dx - \int\left[\frac{d}{dx}(x)\int e^{ax}dx\right]$ = $\frac{x^2}{a}e^{ax} - \frac{2}{a}\left[x.\frac{e^{ax}}{a} - \int 1.\frac{e^{ax}}{a}dx\right]$

= $\frac{x^2}{a}e^{ax} - \frac{2}{a}\left[x.\frac{e^{ax}}{a} - \frac{e^{ax}}{a^2}\right]$ = $\frac{x^2}{a}e^{ax} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a^3} + c$

IMPORTANT STANDARD FORMULAE	
1. $\int\frac{f'(x)}{f(x)}.dx = \log f(x) + c$	**2.** $\int e^x[f(x) + f'(x)].dx = e^x.f(x) + c$
3. $\int\frac{dx}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$	**4.** $\int\frac{dx}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$
5. $\int\frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c$	**6.** $\int\frac{dx}{\sqrt{x^2-a^2}} = \log\left(x + \sqrt{x^2-a^2}\right) + c$
7. $\int\sqrt{x^2+a^2}.dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x + \sqrt{x^2+a^2}\right) + c$	**8.** $\int\sqrt{x^2-a^2}.dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x + \sqrt{x^2-a^2}\right) + c$

Example

(a) $\int \frac{e^x}{e^{2x}-4}$ dx = $\int \frac{dz}{z^2-2^2}$ where z = e^x dz = e^x dx

= $\frac{1}{4}\log\left(\frac{e^x-2}{e^x+2}\right)$ + c

(b) $\int \frac{1}{x+\sqrt{x^2-1}}$ dx = $\int \frac{x-\sqrt{x^2-1}}{(x+\sqrt{x^2-1})(x-\sqrt{x^2-1})}$ dx = $\int (x-\sqrt{x^2-1})$ dx

= $\frac{x^2}{2} - \frac{x}{2}\sqrt{x^2-1} + \frac{1}{2}\log\left(x+\sqrt{x^2-1}\right)$ + c

(c) $\int e^x\,(x^3+3x^2)$ dx = $\int e^x$ {f (x) + f' (x)} dx, where f (x) = x^2

[by (e) above] = $e^x x^3$ + c

INTEGRATION BY PARTIAL FRACTION

- If f(x) & g(x) are polynomials in x, then $\frac{f(X)}{g(x)}$ is called a rational function.
- If degree of f(x) < degree of g(x), it is a proper rational function. [**Ex:** $\frac{8x+1}{5x^3+7}$ i.e $\frac{\text{Degree 1}}{\text{Degree 3}}$.]
- If degree of f(x) > degree of g(x), it is an improper rational function. [**Ex:** $\frac{5x^3+7}{8x+1}$ i.e $\frac{\text{Degree 3}}{\text{Degree 1}}$.]

An improper rational function can be written as a sum of a polynomial & a proper rational function by dividing f(x) by g(x).

If we break any fraction into parts, then the fractions into which the original fraction is broken up are called partial fractions. [**Ex:** $\frac{4}{x-3}$ & $\frac{-3}{x-2}$ are the partial fractions of $\frac{x+1}{x^2-5x+6}$]

STEPS TO BREAK $\frac{f(X)}{g(x)}$ INTO PARTIAL FRACTION

❖ If $\frac{f(X)}{g(x)}$ is not a proper function, then reduce it to a sum of a polynomial & a proper function by dividing the numerator by the denominator as stated above.

❖ Resolve the denominator into simple factors (linear/quadratic) as far as possible.

The factors of the denominator g(x) may consist of the following forms:

Case 1: When denominator has all distict linear factors [say (ax + b), (cx + d)]:

- For every distinct linear factor, there exists a single partial fraction of the form, $\frac{A}{ax+b}$, $\frac{B}{cx+d}$ **where A & B are constants to be determined.**

Ex. $\int \frac{(3x+2)dx}{(x-2)(x-3)}$

Ans: [Degree of numerator must be < degree of denominator; denominator contains non-repeated linear factor]

Let $\frac{(3x+2)}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$

⇒ 3x + 2 = A (x – 3) + B (x – 2) -------------(i)

⇒ We have to find the values of A & B; Thus we will put such value of 'x' which will make co-efficient of either 'A' or 'B' = 0 & we can get the value of other term.

If we put x = 2 in (i); it will make 'B' = 0 & thus we can get 'A'.

⇒ 3.2 + 2 = A (2-3) + B (2-2) ⇒ **A = - 8.**

If we put x = 3 in (i); it will make 'A' = 0 & thus we can get 'B'.

⇒ 3.3 + 2 = A (3-3) + B (3-2) ⇒ **B = 11.**

$\int \frac{(3x+2)dx}{(x-2)(x-3)} = \int \frac{-8}{(x-2)}.dx + \int \frac{11}{(x-3)}.dx$ ⇒ - 8. log(x-2) + 11.log (x-3) + c

Case 2: When denominator has repeated linear factors, (say ax + b, occurs n times):

- To every repetition n times, there corresponds sum of n partial fractions of form, $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \ldots + \frac{A_n}{(ax+b)^n}$ where **A_1, A_2 ……. A_n are constants to be determined.**

Ex: $\int \frac{(3x+2)}{(x-2)^2(x-3)}.dx$

Ans: Let $\frac{(3x+2)dx}{(x-2)^2(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)} =$

3x + 2 = A(x – 2) (x – 3) + B (x – 3) + C $(x-2)^2$

Comparing coefficients of x^2, x and the constant terms of both sides, we find

A + C = 0 --------- (i); - 5A + B – 4C = 3 -----------(ii); 6A – 3B + 4C = 2 ------------(iii)

By (ii) + (iii) ⇒ A – 2B = 5 ---------(iv)

By (i) – (iv) ⇒ 2B + C = - 5 --------(v)

From (iv) ⇒ A = 5 + 2B;

From (v) ⇒ C = -5 – 2B

From (ii) ⇒ -5 (5 + 2B) + B – 4 (-5 - 2B) = 3

⇒ -25 -10B + B + 20 + 8B = 3

⇒ – B – 5 = 3

⇒ **B = - 8**

⇒ **A** = 5 – 16 = **- 11** from (iv)

⇒ **C = - A = 11**

Therefore $\int \frac{(3x+2)dx}{(x-2)^2(x-3)}$ can be written as:

$= \int \frac{-11}{(x-2)}.dx + \frac{-8}{(x-2)^2}.dx + \int \frac{11}{(x-3)}.dx$

$= -11\int \frac{dx}{(x-2)} - 8\frac{dx}{(x-2)^2} + 11\int \frac{dx}{(x-3)}$

$= -11.\log(x-2) + \frac{8}{(x-2)} + 11.\log(x-3)$

$= 11 \log\frac{(x-3)}{(x-2)} + \frac{8}{(x-2)} + c$

Case 3: When denominator has a quadratic factors, [say (ax^2 + bx + c)]:

- To every quadratic factor, there corresponds a partial fraction of the form, $\frac{Ax+B}{ax^2+bx+c}$ **where A & B are constants to be determined.**

Ex: $\int \frac{(3x^2-2x+5)}{(x-1)^2(x^2+5)}.dx$

Ans: Let $\frac{(3x^2-2x+5)}{(x-1)^2(x^2+5)} = \frac{A}{x-1} + \frac{Bx+C}{(X^2+5)}$

Thus $3x^2$- 2x + 5 = A(x^2 + 5) + (Bx + C)(x – 1)

Equating the coefficients of x^2,x and the constant terms from both sides we get,

A + B = 3 --------(i); C – B = - 2 --------(ii); 5A – C = 5 -------- (iii)

From (i) + (ii): A + C = 1 -------- (iv); From (iii) + (iv) 6A = 6 --------(v)

⇒ A = 1 **⇒ B = 3 -1 = 2 &** **⇒ C = 0**

Thus $\int \frac{(3x^2-2x+5)}{(x-1)^2(x^2+5)}$dx $= \int \frac{1}{x-1}.dx + \frac{2x+0}{x^2+5}$.dx

= log(x – 1) + log (x^2 + 5) = log (x -1)(x^2 + 5) + c

SUMMARY TABLE FOR PARTIAL FRACTION

Rational Form	$\frac{px+q}{(x-a)(x-b)}$	$\frac{px+q}{(x-a)^2}$	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$
Partial Form	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{Bx+c}{x^2+bx+c}$

SOME SOLVED EXAMPLES:

CQ4: $\int e^{\sqrt{x}}$ dx

Ans: Let t = $\sqrt{x}$; Differentiating both sides w.r.t $\frac{dt}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2t}$; dx = 2t.dt

⇒ $\int e^{\sqrt{x}}$.dx ⇒ $\int e^t$. 2t.dt.

⇒ 2 $\int (e^t$.t).dt; Apply u.v rule, u = 't' & v = 'e^t'

⇒ 2[t.$\int e^t$ - $\int \frac{dt}{dt}.e^t$]

⇒ 2[t.e^t – e^t] = 2[$\sqrt{x}.e^{\sqrt{x}}$ - $e^{\sqrt{x}}$] + c.

CQ5: Find equation of the curve where slope at (x,y) is 9x and which passes through the origin.

Ans: We are given that slope is 9x & slope means $\frac{dy}{dx}$.

We know that integration of the derivative of a function is that function itself.

Thus $\int \frac{dy}{dx}$ = Y ⇒ $\frac{9x^2}{2}$ + C

Since it passes through the origin, C = 0; Thus the required equation is **$9x^2$ = 2y.**

DEFINITE INTEGRATION

- Let a function be f (x).
- As 'x' changes from 'a' to 'b', value of the integral changes from f(a) to f(b). This is as

  $\int_a^b f(x)$ =f(b) -f(a) 'b' is called the upper limit & 'a' the lower limit of integration.
- No need to add "constant of integration" in definite integration.

How To Solve Definite Integration:

- We shall first find out the integration & then find f(a) & f(b). Answer = f(b) – f(a).

CQ6: $\int_0^2 x^5.dx$

Ans: Firstly, we will integrate the function. $\int x^5.dx = \frac{x^6}{6}$.

Now we will put the upper limit & lower limit respectively.

f(2) = $\frac{x^6}{6} = \frac{2^6}{6} = \frac{64}{6} = \frac{32}{3}$ & f(0) = $\frac{0^6}{6}$ = 0.

f(2) – f(0) = $\frac{32}{3}$ - 0 = $\frac{32}{3}$.

CQ7: $\int_1^2 (x^2 - 5x + 2).dx$

Ans: Firstly, we will integrate the function. $\int (x^2 - 5x + 2).dx = \frac{x^3}{3} - \frac{5x^2}{2} + 2x$

Now we will put the upper limit & lower limit respectively.

f(2) = $\frac{(2)^3}{3} - \frac{5(2)^2}{2} + 2(2) = \frac{8}{3} - 10 + 4 = -\frac{10}{3}$ & f(1) = $\frac{(1)^3}{3} - \frac{5(1)^2}{2} + 2(1) = \frac{1}{3} - \frac{5}{2} + 2 = -\frac{1}{6}$

f(2) – f(1) = $-\frac{10}{3} - [-\frac{1}{6}] = -\frac{19}{6}$

IMPORTANT PROPERTIES OF DEFINITE INTEGRAL

1. $\int_a^b f(x).dx = \int_a^b f(t).dt$	2. $\int_a^b f(x).dx = -\int_b^a f(x).dx$
3. $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$ [a < c < b].	4. $\int_0^a f(x).dx = \int_0^a f(a-x).dx$

5. $\int_{-a}^a f(x).dx = 2\int_0^a f(a).dx$ if f(-x) = f(x) [i.e If even Function]
 = 0 if f(-x) = -f(x) [i.e If odd Function]

6. When f(x) = f(a + x) ⇒ $\int_0^{na} f(x).dx = n.\int_0^a f(x).dx$.

CQ8: $\int_0^2 \frac{x^2\,dx}{x^2+(2-x)^2}$

Ans: Let I = $\int_0^2 \frac{x^2\,dx}{x^2+(2-x)^2}$ & by Property IV; I = $\int_0^2 \frac{(2-x)^2\,dx}{(2-x)^2+x^2}$

I + I = 2I = $\int_0^2 \frac{x^2\,dx}{x^2+(2-x)^2}$ + $\int_0^2 \frac{(2-x)^2\,dx}{(2-x)^2+x^2}$

2I = $\int_0^2 \frac{x^2+(2-x)^2}{x^2+(2-x)^2}$ dx

2I = $\int_0^2 dx$ = $[x]_0^2$ = 2 – 0 = 2

2I = 2 & thus **I = 1.**

CQ9: Evaluate $\int_{-2}^2 \frac{x^4\,dx}{a^{10}-x^{10}}$ (a> 2)

Ans: $\frac{x^4\,dx}{a^{10}-x^{10}} = \frac{x^4\,dx}{(a^5)^2-(x^5)^2}$ -------------(i)

Let t = x^5; Differentiating both sides w.r.t 't', we get $\frac{dt}{dx} = 5x^4$ $\Rightarrow x^4.dx = \frac{dt}{5}$

= $\frac{1}{5}\int \frac{dt}{(a^5)^2-t^2}$ [Substituting the value of $x^4.dx = \frac{dt}{5}$ in (i)]

= $\frac{1}{5}.\frac{1}{2a^5}\log\frac{a^5+x^5}{a^5-x^5}$ [Using the formula $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}$ + c]

Therefore, $\int_{-2}^2 \frac{x^4\,dx}{a^{10}-x^{10}}$ **=** $2\int_0^2 \frac{x^4\,dx}{a^{10}-x^{10}}$ **[Using Property V]**

= 2 × $\frac{1}{10a^5}\log\left[\frac{a^5+x^5}{a^5-x^5}\right]_0^2$ = $\frac{1}{5a^5}\log\frac{a^5+32}{a^5-32}$

INDEFIITE INTEGRAL – QUESTION BANK

SN	8B. INDEFINITE INTEGRALS CALCULUS	Ans
Q155	Integrate $(x + a)^n$ (a) $\frac{(x+a)^{n+1}}{n+1}$ (b) $\frac{(x+a)^n}{n}$ (c) $\frac{(x+a)^{n-1}}{n-l}$ (d) None	A
Q156	Evaluate $\int 5x^2 dx$ and the answer will be ______. (a) $\frac{5}{3}x^3 + k$ (b) $\frac{5x^3}{3}$ (c) $\frac{5}{x^{-3}}$ (d) None	A
Q157	Integration of 3 - 2x - x^4 will become ______. (a) $-x^2 - \frac{x^5}{5}$ (b) $3x - x^2 - \frac{x^5}{5}$ (c) $3x - x^2 - \frac{x^5}{5} + k$ (d) None	C
Q158	Evaluate result of $\int (x^2 - 1)^2 dx$ is ______. (a) $\frac{x^5}{5} - \frac{2}{3}x^3 + x + k$ (b) $\frac{x^5}{5} - \frac{2}{3}x^3 + k$ (c) 2x (d) None	A
Q159	Find $\int \sqrt{x}$dx (a) $\frac{2x^{\frac{3}{2}}}{3} + c$ (b) $\frac{2x}{3} + c$ (c) $-\frac{2x^{\frac{1}{2}}}{5} + c$ (d) $\frac{2}{x^2} + c$	A
Q160	Find $\int \frac{1}{\sqrt{x}}$ dx. (a) $2x + c$ (b) $\frac{\sqrt{x}}{2} + c$ (c) $2\sqrt{x} + c$ (d) $\frac{\sqrt{x}+c}{2}$	C
Q161	Integrate, $x^{-1/2}$ (a) $2x^{1/2}$ (b) $\frac{1}{2}x^{1/2}$ (c) $-\frac{3}{2}x^{-3/2}$ (d) None	A
Q162	Find $\int x\sqrt{x}$ dx. (a) $\frac{2}{5}x^{\frac{5}{2}} + c$ (b) $\frac{3}{5}x^{\frac{3}{2}} + c$ (c) $\frac{2}{3}x^{\frac{1}{2}} + c$ (d) $x^2 + c$	A
Q163	Evaluate $\int (x + \frac{1}{x})^2 dx$ (a) $\frac{x^3}{2} + 2x + c$ (b) $\frac{3x}{2} - \frac{1}{x} + c$ (c) $\frac{x^3}{3} + 2x - \frac{1}{x} + c$ (d) $\frac{x^2}{3} - \frac{2}{x} + c$	C
Q164	Evaluate $\int \sqrt{x}\,(x3 + 2x - 3)$ dx. (a) $\frac{x^{\frac{7}{2}}}{5} + \frac{3x^2}{7} - 8x + c$ (b) $\frac{2x^{\frac{9}{2}}}{9} + \frac{4x^{\frac{5}{2}}}{5} - 2x^{\frac{3}{2}} + c$ (c) $\frac{3x^{\frac{7}{2}}}{7} + \frac{x^{\frac{3}{2}}}{5} - 2x^{\frac{3}{2}} + c$ (d) $\frac{2x^{\frac{5}{2}}}{7} - \frac{x^{\frac{3}{2}}}{9} - 2x^{\frac{5}{2}} + c$	B
Q165	$\int (7x^2 - 3x + 8 - x^{-1/2} + x^{-1} + x^{-2})\, dx$ (a) $\frac{7}{3}x^3 - \frac{3}{2}x^2 + 8x - 2x^{1/2} + \log x - x^{-1}$ (b) $\frac{3}{7}x^3 - \frac{2}{3}x^2 + 8x - \frac{1}{2}x^{1/2} + \log x + x^{-1}$	A

	(c) $\frac{7}{3}x^3 + \frac{3}{2}x^2 + 8x - 2x^{1/2}$ + logx+x −1 (d) None	
Q166	Integrate $\frac{(ax^3+bx^2+cx+d)}{x}$ (a) $\frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx +$ dlog x (b) $3ax^3 + 2bx^2 + cx +$ dlog x (c) $2ax + b - dx^{-2}$ (d) None	**A**
Q167	Integrate $\frac{(4x^6+3x^5+2x^4+x^3+x^2+1)}{x^3}$ (a) $x^4 + x^3 + x^2 + x +$ logx$-\frac{1}{2x^2}$ (b) $x^4 + x^3 + x^2 + x + Iogx + (1/2)x^{-2}$ (c) $x^4 + x^3 + x^2 + x + \log x + 2x^{-2}$ (d) None	**A**
Q168	Integrate $4x^3 + 3x^2 - 2x + 5$ (a) $X^4 + X^3 - X^2 + 5X$ (b) $X^4 - X^3 + X^2 - 5X$ (c) $X^4 + X^3 + X^2 - 5$ (d) None	**A**
Q169	The integral of $px^3 + qx^2 + rk + \frac{w}{x}$ is ________. (a) $px^2 + qx + r + k$ (b) $\frac{px^3}{3} + \frac{qx^2}{2} + rx$ (c) $3px + 2q - \frac{w}{x^2}$ (d)' $\frac{px^4}{4} + \frac{qx^3}{3} + wlogx + rkx$	**D**
Q170	Integrate $(x^4 + 1)/x^2$ (a) $\frac{x^3}{3} - \frac{1}{x}$ (b) $\frac{1}{x} - \frac{x^3}{3}$ (c) $\frac{x^3}{3} + \frac{1}{x}$ (d) None	**A**
Q171	Integrate $(4x + 5)^6$ (a) $\frac{1}{128}(4x + 5)^7$ (b) $\frac{1}{7}(4x + 5)^7$ (c) $\frac{7}{(4x+5)^{-7}}$ (d) None	**A**
Q172	$\int \frac{1}{\sqrt{1+x}}$ is equal to ________. (a) $\frac{2}{(1+x)^{1/2}}$ (b) $(1 + x)^{1/2}$ (c) $2(1 + x)^{1/2}$ (d) None	**C**
Q173	$\int e^{ax}\, dx$ (a) e^x (b) $\frac{e^{ax}}{a}$ (c) $logx$ (d) $\frac{1}{e^{-ax}}$	**B**
Q174	$\int e^{3x+5}$ dx is equal to ________. (a) $\frac{e^{3x+5}}{3} + c$ (b) $\frac{e^{3x}}{5} + c$ (c) $\frac{-e^{3x+5}}{3} + c$ (d) None	**A**
Q175	The value of $\int(6x^5 + 3e^{2x} + 5)$ dx is equal to ________. (a) $x^6 + \frac{3}{2}e^{2x} + 5x + k$ (b) $30x^4 + 6e^{2x}$ (c) $x^6 + \frac{3}{2}e^{2x}$ (d) None	**A**

Q176	Find $\int e^{-3x}$ dx. (a) $-(1/3)e^{-3x}+c$ (b) $e^{-3x}+c$ (c) $(1/3)e^{-x}+c$ (d) $(1/3)e^{x}+c$	A
Q177	Evaluate $\int \frac{e^{3x}+e^{-3x}}{e^x}$ dx. (a) $\frac{e^{3x}}{3}-\frac{1}{2x}+c$ (b) $\frac{e^{2x}}{2}-\frac{1}{4e^{4x}}+c$ (c) $\frac{e^{3x}}{2}+\frac{1}{3e^{2x}}+c$ (d) $-\frac{e^{2x}}{2}-\frac{1}{3e^{2x}}+c$	B
Q178	Find $\int 3^x$ dx. (a) $log_e 3+c$ (b) $\frac{e^x}{3}\ log\ 3+c$ (c) $\frac{3^x}{log_e 3}+c$ (d) 3^x+c	C
Q179	Integrate $\sqrt{x}-\frac{x}{2}+\frac{2}{\sqrt{x}}$ (a) $\frac{2}{3}x\sqrt{x}-\frac{1}{4}x^2+4\sqrt{x}+c$ (b) $\frac{3}{2}\sqrt{x}-\frac{1}{4}x^2+\sqrt{x}+c$ (c) $\frac{2}{3}\sqrt{x}-\frac{1}{2}x^2-\frac{1}{2}\sqrt{x}+c$ (d) None	A
Q180	Integrate $\frac{3}{x}$ + 4x² - 3x + 8 (a) $3Iogx-\frac{4}{3}x^3+\frac{3}{2}x^2-8x+c$ (b) $3Iogx+\frac{4}{3}x^3-\frac{3}{2}x^2+8x+c$ (c) $3Iogx+\frac{4}{3}x^3+\frac{3}{2}x^2+8x+c$ (d) None	B
Q181	Integrate $\left(ax+\frac{b}{x^3}+\frac{c}{x^7}\right)x^2$ (a) $\frac{1}{4}ax^4$ + blog $x-\frac{1}{4}cx^{-4}+k$ (b) $4ax^4+b\ log\ x-4cx^{-4}+k$ (c) $\frac{1}{4}ax^4$ + blog $x+\frac{1}{4}cx^{-4}+k$ (d) None	A
Q182	Integrate $[2^x+\frac{1}{2}e^{-x}+\frac{4}{x}-x^{-1/3}]$ (a) $\frac{2^x}{Iog2}-\frac{1}{2}e^{-x}+4Iogx-\frac{3}{2}x^{2/3}+k$ (b) $\frac{2^x}{Iog2}+\frac{1}{2}e^{-x}+4Iogx+\frac{3}{2}x^{2/3}+k$ (c) $\frac{2^x}{log\ 2}-2e^{-x}+4\|ogx-\frac{2}{3}x^{2/3}+k$ (d) None	A
Q183	$\int\left(x^4+\frac{3}{x}\right)$ dx is equal to ________. (a) $\frac{x^5}{5}+3log\|x\|$ (b) $\frac{1}{5}x^5+3Iog\|x\|+k$ (c) $\frac{1}{5}x^5+k$ (d) None	A
Q184	Evaluate the integral $\int\frac{(1-x)^3}{x}dx$ (a) $Iog\|x\|-3x+\frac{3}{2}x^2+k$ (b) logx−2 $+3x^2+k$ (c) logx $+3x^2+k$ (d) $Iog\|x\|-\frac{x^3}{3}-3x+\frac{3x^2}{2}+k$	D

Q185	Integrate $\frac{x^2}{(x^3+2)^{1/4}}$ (a) (4/9) $(x^3+2)^{\frac{3}{4}}+k$ (b) (9/4) $(x^3+2)^{3/4}+k$ (c) (3/4) $(x^3+2)^{3/4}+k$ (d) None	**A**
Q186	Evaluate $\int \frac{x^2}{x+1}dx.$ (a) $\frac{3x^2}{4}+x-Iog(x+1)+c$ (b) $\frac{x^2}{2}-x+log(2x-1)+c$ (c) $\frac{x^2}{2}-x+Iog(x+1)+c$ (d) None	**C**
Q187	Evaluate $\int \frac{x^3+5x^2-3}{(x+2)}$ dx. (a) $\frac{x^3}{3}+\frac{2x^2}{5}+4x+6Iog(x+3)+c$ (b) $\frac{x^3}{5}+\frac{7x^2}{2}-5x-9Iog(x-8)+c$ (c) $\frac{x^3}{2}-\frac{7x^2}{9}-6x-9Iog(x-4)+c$ (d) $\frac{x^3}{3}+\frac{3x^2}{2}-6x+9Iog(x+2)+c$	**D**
Q188	$\int \frac{8x^2}{(x^3+2)^3}$ dx is equal to _______. (a) $(-4/3)(x^3+2)^2$ (b) $\frac{-4}{3(x^3+2)^2}+k$ (c) $\frac{4}{3}(x^3+2)^2+k$ (d) None	**B**
Q189	Evaluate $\int x(x^2+4)^5\,dx$ (a) $(x^2+4)^6+k$ (b) $\frac{1}{12}(x^2+4)^6+k$ (c) $\frac{(x^2+4)^6}{k}$ (d) None	**B**
Q190	Evaluate $\int \frac{x^3}{(x^2+1)^3}$ dx. (a) $\frac{1}{4}.\frac{1}{(x^2+1)^2}-\frac{1}{2}.\frac{1}{x^2+1}+c$ (b) $\frac{3}{4}.\frac{1}{(x^3+1)^2}-\frac{3}{2}.\frac{1}{x^2-1}+c$ (c) $\frac{5}{4}.\frac{1}{(x^2-1)^2}-\frac{3}{2}.\frac{1}{x^2+1}+c$ (d) $\frac{7}{4}.\frac{1}{(x^2+1)^2}+\frac{1}{2}.\frac{1}{x^2+1}+c$	**A**
Q191	Evaluate $\int \frac{dx}{x(x^3+1)}$ (a) $Iog(x/x+1)+c$ (b) (1/3) log $\frac{x^3}{x^3+1}+c$ (c) (1/3) log $\frac{x}{x^3+1}+c$ (d) $\frac{1}{3}log\frac{x^3}{x^3-1}+c$	**D**
Q192	Integrate $(x^2+2)^{-3}x^3$ (a) $-\frac{2x^2+3}{2(x^2+2)^2}$ (b) $\frac{1}{2}\frac{(2x^2+3)}{(x^2+1)^2}$ (c) $-\frac{1}{4}\frac{(2x^2+1)}{x^2+1}$ (d) $\frac{1}{4}\frac{(2x^2+1)}{x^2+1}$	**A**
Q193	Integrate $x(x^2+3)^{-2}$ (a) $-\frac{1}{2(x^2+3)}$ (b) $\frac{1}{2(x^2+3)}$ (c) $\frac{2}{x^2+3}$ (d) None	**A**
Q194	Evaluate $\int \frac{(2-x)e^x}{(1-x)^2}$ dx and the value is _______.	**A**

	(a) $\frac{e^x}{1-x}+k$ (b) e^x+k (c) $1-x+k$ (d) None	
Q195	Evaluate $\int\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$ and the value is ______. (a) $log_e\|e^x+e^{-x}\|$ (b) $log_e\|e^x+e^{-x}\|+k$ (c) $log_e\|e^x-e^{-x}\|+k$ (d) None	**B**
Q196	Integrate $(x^3+2)^2 3x^2$ (a) $\frac{1}{3}(x^3+2)^3+c$ (b) $3(x^3+2)^3+c$ (c) $3x^2(x^3+2)^3+c$ (d) $9x^2(x^3+2)^3+c$	**A**
Q197	Integrate $(x^3+2)^{1/2}x^2$ (a) $\frac{2}{9}(x^3+2)^{3/2}+c$ (b) $\frac{2}{3}(x^3+2)^{3/2}+c$ (c) $\left(\frac{9}{2}\right)(x^3+2)^{3/2}+c$ (d) None	**A**
Q198	The integral of $\frac{x^3}{x^2+1}$ is equal to ______. (a) $\frac{1+2x^2}{4(x^2+1)^2}+c$ (b) $\frac{1-2x^2}{4(x^2+1)^2}+c$ (c) $\frac{-(1+2x^2)}{4(x^2+1)}+c$ (d) None	**D**
Q199	Integrate $\frac{3x}{(x^2+1)^n}$ (a) $\frac{3}{2}\frac{(x^2+l)^{1-n}}{1-n}$ (b) $\frac{3}{2}\frac{(x^2+l)^{n-1}}{1-n}$ (c) $\frac{2}{3}\frac{(x^2+l)^{1-n}}{1-n}$ (d) None	**A**
Q200	$\int\frac{dx}{e^x+1}$ is equal to ______. (a) $-\log(1+e^{-x})+K$ (b) $(e^x+1)^{-2}+K$ (c) $\frac{1}{1+e^x}+K$ (d) None	**A**
Q201	$\int_0^5\frac{x^2}{x^2+(5-x)^2}dx$ is equal to ______. (a) 5 (b) 5/2 (c) 1 (d) None	**B**
Q202	If $f(x)=\sqrt{1+x^2}$ then $\int f(x)$ dx is ______. (a) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}+k2/3(1+x^2)^{\frac{3}{2}}+k$ (b) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}log(x+\sqrt{x^2+1})$ (c) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}}+k$ (d) None	**B**
Q203	Value of $\int\frac{dx}{16-9x^2}$ (a) $\frac{1}{24}log\left\|\frac{4+3x}{4-3x}\right\|+c$ (b) $\frac{16}{9}log\left\|\frac{4+x}{4-x}\right\|+c$ (c) $\frac{1}{4}log\left\|\frac{3x}{4}\right\|+c$ (d) $log\left\|\frac{4+3x}{4-3x}\right\|+c$	**A**
Q204	The integral of $\int\frac{dx}{x^2-a^2}$ will be ______. (a) $\frac{1}{2a}log\frac{(x-a)}{(x+a)}$ (b) $\frac{1}{2a}log\frac{(x+a)}{(x-a)}$ (c) $\frac{1}{2a}log\frac{x}{(x+a)}$ (d) None	**A**

Q205	$\int \sqrt{x^2+a^2}\,dx$ is equal to ______. (a) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}log\left\|x^2+\sqrt{x^2+a^2}\right\|$ (b) $\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}log\left\|x^2-\sqrt{x^2-a^2}\right\|$ (c) $\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}log\left\|x^2+\sqrt{x^2+a^2}\right\|$ (d) None	A
Q206	Evaluate $\int \frac{(3x+2)dx}{(x-2)(x-3)}$. $(a)\ -Iog(x-2)+11Iog(x-3)+c$ (b) $Iog(x-2)(x-3)+c$ (c) $Iog(3x+2)+c$ (d) $-Iog(x-2)+Iog(x-3)+c$	A
Q207	Evaluate $\int \frac{(3x+2)dx}{(x-2)^2(x-3)}$ (a) $11Iog\frac{(x-3)}{(x-2)}+\frac{8}{(x-2)}+c$ (b) $Iog(x-2)+Iog(x-3)+c$ (c) Iog $\frac{(x-3)}{(x-2)}+log(3x+2)+c$ (d) $Iog(3x+2)+c$	A
Q208	Evaluate $\int \frac{(3x^2-2x-5)}{(x-1)(x^2+5)}$ dx. (a) log($3x^2$ - 2x - 5) + c (b) log(x^2 + 5)(x - 1) + c (c) log(3x - 5) + c (d) log$(x - 1)^2$ + c	B
Q209	$\int \frac{xe^x}{(x+1)^2}dx$ is equal to ______. (a) $\frac{e^x}{x+1}+k$ (b) $\frac{e^x}{x}+k$ (c) e^x+k (d) None	A
Q210	Integrate $\frac{1}{x^2-a^2}$ is ______. $(a)\ Iog\left\|\frac{x-a}{x+a}\right\|+k$ (b) $Iog(x-a)-log(x+a)$ (c) $\frac{1}{2a}log\left\|\frac{x-a}{x+a}\right\|+k$ (d) $\frac{1}{2}log\left\|\frac{x+a}{x-a}\right\|+k$	C
Q211	Evaluate $\int \frac{e^x}{e^{2x}-4}dx$ (a) $\frac{3}{4}Iog\left(\frac{e^x+2}{e^x-2}\right)+c$ (b) $-\frac{5}{4}Iog\left(\frac{e^x-2}{e^x+2}\right)+c$ (c) $\frac{1}{4}log\left(\frac{e^x-2}{e^x+2}\right)+c$ (d) $\frac{7}{4}Iog\left(\frac{e^x+2}{e^x-2}\right)+c$	C
Q212	Evaluate $\int \frac{x+5}{(x+1)(x+2)^2}$ dx (a) $4\ Iog(x+1)-4Iog(x+2)+\frac{3}{x+2}+k4$ (b) $4Iog(x+2)-\frac{3}{x+2}+k$ (c) $4\ log\ (x+1)-4Iog(x+2)$ (d) None	A
Q213	Evaluate $\int \frac{x^2-1}{x^4+x^2+1}dx$	B

	(a) $\frac{1}{4} log \left\|\frac{x^2-x+1}{x^2+x+1}\right\|$ (b) $\frac{1}{2} log \left\|\frac{x^2-x+1}{x^2+x+1}\right\|$ (c) $\frac{1}{3} log \left\|\frac{x^2-x+1}{x^2+x+1}\right\|$ (d) $\frac{1}{3} log \left\|\frac{x^2+x+1}{x^2-x+1}\right\|$	
Q214	Integrate $\frac{1}{x-x^3}$ (a) $\frac{1}{2} log[x^2/(1-x^2)]$ (b) $\frac{1}{2} log[x^2/(1-x)^2]$ (c) $\frac{1}{2} log[x^2/(1+x)^2]$ (d) $\frac{1}{2} log[x^2/(1+x^2)]$	**A**
Q215	The value of $\int \frac{dx}{x(x^2-1)}$ is equal to ________. (a) $\frac{1}{2} log\left(1+\frac{1}{x^2}\right)$ (b) $\frac{1}{2} log\left(1-\frac{1}{x^2}\right)+k$ (c) $Iog\left(1-\frac{1}{x^2}\right)$ (d) None	**B**
Q216	Evaluate the integral of $\int x.e^x$ dx $(a)\ e^x(x^2+1)+c$ (b) $e^x(x+1)+c$ (c) $e^x(2x+1)+c$ (d) $e^x(x-1)+c$	**D**
Q217	The value of $\int(5x.e^x+10)\,dx$ is equal to ________. $(a)\ 5xe^x-5e^x+10x+c$ (b) $5xe^x+5e^x+10x+c$ (c) $xe^x-5e^x+10x+c$ (d) None	**B**
Q218	Integrate logx (a) x(logx -1) (b) x(log x +1) (c) log x -1 (d) logx + 1	**A**
Q219	$\int \frac{log(logx)}{x} dx$ dx is ________. (a) log(logx - 1) + k (b) logx — 1 + k (c) [log(log x - 1)]log x + k (d) None	**C**
Q220	$\int_1^e \frac{e^x(\times log_e x+1)}{x}$ dx = ________. $(a) e^e-1$ (b) e^e (c) $e-1$ (d) none	**B**
Q221	$\int (Iogx)^2\, x\, dx$ is equal to ________. (a) $\frac{x^2}{2}[(Iogx)^2-Iogx+\frac{1}{2}]+c$ (b) (logx)²$-Iogx+\frac{1}{2}+k$ (c) $\frac{x^2}{2}[(Iogx)^2+1/2]+k$ (d) None	**A**
Q222	Integrate x^3logx (a) $x^4/16+k$ (b) $x^4/16(4Iogx-1)+k$ (c) $4Iogx-1+k$ (d) None	**B**
Q223	Evaluate $\int x^3\, e^x dx$ (a) $(x^3-3x^2+6x-6)e^x+c$ (b) $(x^3+3x^2+6x-6)e^x+c$ (c) $(x^3-3x^2-6x-6)e^x+c$ (d) $(x^3+3x^2+6x+6)e^x+c$	**A**

Q224	Evaluate $\int x\, logx$ dx. (a) xlog $x + c$ (b) xlog $x - \frac{x}{3} + c$ (c) $\frac{x^2}{2} Iogx - \frac{x^2}{4} + c$ (d) $\frac{1}{x\, log\, x} + c$	**C**
Q225	Evaluate $\int x^2\, e^{ax}$ dx. (a) $\frac{x^2e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2}{a^3}e^{ax} + c$ (b) $2xe^{ax} + c$ (c) $\frac{x^2}{a} - \frac{2}{a^2}e^{ax} + xe^x - \frac{x}{a} + c$ (d) $e^{ax} + c$	**A**
Q226	$\int (Iogx)^2\, dx$ and the results is ________. (a) $x(Iogx)^2 - 2x\, log\, x + 2x$ (b) $x(Iogx)^2 - 2x$ (c) $2x\, log\, x - 2x$ (d) $x(Iogx)^2 - 2x\, log\, x + 2x + k$	**D**
Q227	$\int Iogx^2\, dx$ is equal to ________. (a) $x(Iogx - 1) + k$ (b) $2x(Iogx - 1) + k$ (c) $2(Iogx - 1) + k$ (d) None	**B**
Q228	Integrate $\frac{l}{xlogxlog(logx)}$ (a) $log\, [Iog(Iogx)]$ (b) log(log x) (c) $Iogx$ (d) $1/x$	**A**
Q229	Integrate $\frac{1}{x(\, log\, x)^2}$ (a) $\frac{-1}{Iogx}$ (b) $\frac{1}{Iogx}$ (c) $Iogx$ (d) None	**A**
Q230	Integrate x^2e^x (a) $e^x(x^2 - 2x + 2)$ (b) $e^x(x^2 + 2x + 2)$ (c) $e^x(x + 2)^2$ (d) None	**A**
Q231	Integrate x^2e^{3x} (a) $\frac{1}{3}(x^2e^{3x}) - \frac{2}{9}(xe^{3x}) + \frac{2}{27}e^{3x}$ (b) $\frac{1}{3}(x^2e^{3x}) + \frac{2}{9}(xe^{3x}) + \frac{2}{27}e^{3x}$ (c) $\frac{1}{3}(x^2e^{3x}) - \frac{1}{9}(xe^{3x}) + \frac{2}{27}e^{3x}$ (d) None	**A**
Q232	Integrate x^n log x (a) $\frac{x^{n+1}}{n+1}\left[Iogx - \frac{1}{n+1}\right]$ (b) $\frac{x^{n-1}}{n-1}\left[Iogx - \frac{1}{n-1}\right]$ (c) $\frac{x^{n+1}}{n+1}\left[Iogx + \frac{1}{n+1}\right]$ (d) None	**D**
Q233	Integrate $\frac{xe^x}{(x+1)^2}$ (a) $\frac{e^x}{x+1}$ (b) $\frac{e^x}{(x+1)^2}$ (c) $\frac{xe^x}{x+1}$ (d) None	**A**
Q234	Integrate xlog x	**A**

	(a) $\frac{1}{4}x^2\ log\ (x^2/e)$ (b) $\frac{1}{2}x^2 Iog(x^2/e)$ (c) $\frac{1}{4}x^2$ log(x/e) (d) None	
Q235	Integrate $\frac{e^x(1+x)}{(x+2)^2}$ (a) $\frac{e^x}{x+2}$ (b) $\frac{-e^x}{2+x}$ (c) $\frac{e^x}{2(2+x)}$ (d) None	A
Q236	Evaluate $\int e^x\,(x^3+3x^2)dx$ (a) e^x+3x+c (b) $e^{3x}+3x+c$ (c) $e^x.x^3+c$ (d) $e^{3x}+3x+x^3+c$	C
Q237	$\int\frac{Iogx}{x}\,dx$ is equal to _______. (a) $\frac{1}{2}Iogx+k$ (b) $\frac{1}{2}(Iogx)^2+k$ (c) $\frac{1}{2}x^2+k$ (d) None	B
Q238	Integrate $e^x\frac{(1+x\ log\ x)}{x}$ (a) $e^x logx$ (b) $-e^x$logx (c) $e^x x^{-1}$ (d) None	A
Q239	$\int\frac{Iog(\ log\ x)}{x}dx$ is equal to _______. (a) $Iog(Iogx)-1+k$ (b) log(log x)+k (c) log x[log(log x) $-$ 1] +k (d) None	C
Q240	The value of the integral $\int\ \frac{1}{x\ log\ x}dx$ is _______. (a) $\frac{1}{(X\ log\ x)^2}+c$ (b) log (x log x) +c (c) log (log x) +c (d) None	C
Q241	Evaluate $\int\frac{Iogx}{(1+\ log\ x)^2}\ dx$ (a) xlog $(x+1)+c$ (b) $Iog(x+1)+c$ (c) $\frac{x}{(\ log\ x+1)}+c$ (d) $Iogx+c$	C
Q242	Evaluate $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ (a) $\frac{e^x}{x}+c$ (b) $\frac{e^x}{x^2}+c$ (c) $\frac{e^x}{x-x^2}+c$ (d) e^x+c	A
Q243	Evaluate $\int e^x\frac{x}{(x+1)^2}dx$ (a) $\frac{e^x}{(x+1)^2}+c$ (b) $\frac{e^x}{x\ 1}+c$ (c) $\frac{e^x}{x+1}+c$ (d) $\frac{e^x}{(x+1)^{\frac{1}{2}}}+c$	C
Q244	$\int(x-1)\frac{e^x}{x^2}$ dx is equal to _______. (a) $\frac{e^x}{x}+k$ (b) $\frac{e^{-x}}{x}+k$ (c) $\frac{-e^x}{x}+k$ (d) None	A
Q245	$\int\frac{e^x(xIog+1)}{x}dx$ is equal to_______. (a) $e^x logx+k$ (b) e^x+k (c) $Iogx+k$ (d) None	A

Q246	Evaluate $\int \frac{1}{x\{6(logx)^2+7logx+2\}}dx$ (a) $Iog\left\|\frac{2\log x-1}{3\log x-2}\right\|+c$ (b) $Iog\left\|\frac{2\log x+1}{3\log x+2}\right\|+c$ $t(c)$ $Iog\left\|\frac{3\log x+1}{2\log x+2}\right\|+c$ (d) $Iog\left\|\frac{3\log x+1}{2\log x+2}\right\|+c$	**B**
Q247	$\int \frac{(x^2+1)}{\sqrt{x^2+2}}$ is equal to ________. (a) $2\sqrt{x^2+2}+k$ (b) $\sqrt{x^2+2}+k$ (c) $(x^2+2)^{3/2}+k$ (d) None	**D**
Q248	$\int (e^x+e^{-x})^2(e^x-e^{-x})dx$ is ________. (a) $\frac{1}{3}(e^x+e^{-x})^3+k$ (b) $\frac{1}{2}(e^x-e^{-x})^2+k$ (c) e^x+k (d) None	**A**
Q249	$\int_0^{1/2} \frac{1}{\sqrt{3-2x}}dx$ is equal to ________. (a) 1 (b) $1-\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}-\sqrt{2}$ (d) 2- $\sqrt{3}$	**C**
Q250	$\int_0^1 xe^{x2}$ dx: (a) 1 (b) e– 1 (c) $\frac{e}{2}-1$ (d) $\frac{1}{2}$ (e—1)	**D**
Q251	The equation of the curve which passes through the point (1, 3) and has the slope 4x-3 at any point (x y) (a) y = $2x^3$ - 3x + 4 (b) y = $2x^2$ - 3x + 4 (c) x = $2y^2$ -3y + 4 (d) None	**B**
Q252	The equation of the curve in the form y = f(x) if the curve passes through the point (1 0) and f(x)=2x-1 is ________. (a) y = x^2 — x (b) x = y^2 — y (c) y = x^2 (d) None	**A**

DEFINITE INTEGRAL – QUESTION BANK

SN	8C. DEFINITE INTEGRAL CALCULUS	Ans
Q253	$\int_0^a [f(x)+f(-x)]dx$ is equal to ______. (a) $\int_0^a 2f(x)dx$ (b) $\int_{-a}^a f(x)dx$ (c) 0 (d) $\int_{-a}^a -f(-x)dx$	B
Q254	Evaluate $\int_2^4 (3x-2)^2 dx$ and the value is ______. (a) 104 (b) 100 (c) 10 (d) None	A
Q255	Evaluate $\int_0^1 (2x^2-x^3)dx$ and the value is ______. (a) 4/3 (b) 5/12 (c) -4/3 (d) None	B
Q256	$\int_0^2 3x^2 dx$ is ______. (a) 7 (b) -8 (c) 8 (d) None	C
Q257	Evaluate $\int_1^4 (2x+5)dx$ and the value is ______. (a) 3 (b) 10 (c) 30 (d) None	C
Q258	The value of $\int_0^1 (2x+5)dx$ Is ______. (a) 54 (b) 6 (c) 19 (d) None	B
Q259	$\int_0^4 \sqrt{3x+4}\,dx$ Is equal to ______. (a) 9/112 (b) 125/9 (c) 11/9 (d) None	B
Q260	$\int_0^1 10x^5 dx$ is equal to ______. (a) $\frac{5}{3}x^6$ (b) $\frac{3}{5}$ (c) $\frac{5}{3}$ (d) None	C
Q261	Evaluate $\int_0^1 (2x^2-x^3)dx$ and the value is ______. (a) $\frac{4}{3}+k$ (b) 5/12 (c) $-4/3$ (d) None	B
Q262	Find the Value of $\int_3^3 x\sqrt{8-x^2}$ dx (a) -1 (b) 1 (c) 0 (d) None	C
Q263	Evaluate $\int_3^3 \lvert x^2+2x-3\rvert\, dx$ (a) 1 (b) -6 (c) 4 (d) 2	C
Q264	Evaluate $\int_1^4 (\lvert x-1\rvert+\lvert x-2\rvert+\lvert x-3\rvert)dx$ (a) 17/2 (b) 15/2 (c) 19/2 (d) 7	C
Q265	Evaluate $\int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx$	B

	(a) $\frac{7}{2}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) 2	
Q266	Evaluate $\int_{-3}^{3}(x^3+x)dx$ (a) 0 (b) 3 (c) -3 (d) 1	A
Q267	Evaluate $\int_{2}^{4}(3x-2)^2dx$ and the value is _______. (a) 104 (b) 100 (c) 10 (d) None	A
Q268	Evaluate $\int_{0}^{1}x\,e^x dx$ and the value is _______. (a) -1 (b) 10 (c) 10/9 (d) None	D
Q269	Evaluate $\int_{1}^{4}(2x+5)dx$ and the value is _______. (a) 3 (b) 10 (c) 30 (d) None	C
Q270	$\int_{1}^{2}\frac{2x}{1+x^2}dx$ is equal to _______. (a) $\frac{log_e 5}{2}$ (b) $log_e 5 - log_e 2 + k$ (c) $log_e 2/5$ (d) None	A
Q271	$\int_{0}^{2}\frac{x+2}{x+1}dx$ is _______. (a) $2+\ log\ 2$ (b) $2+log_e 3$ (c) $log_e 3$ (d) None	B
Q272	Evaluate $\int_{1}^{e^2} d\,x/x(1+logx)^2$ and the value is _______. (a) $\frac{3}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) None	C
Q273	$\int_{0}^{1}\frac{(x+1)(x-4)}{\sqrt{x}}dx$ is equal to _______. (a) $-\frac{48}{5}$ (b) 48/5 (c) 48 (d) None	A
Q274	The value of $\int_{2}^{3}f(5-x)dx - \int_{2}^{3}f(x)dx$ is _______. (a) 1 (b) 0 (c) -1 (d) None	B
Q275	$\int_{1}^{2}x\ log\ xdx$ is equal to _______. (a) 2 log2 (b) $-3/4$ (c) $2\ log2 - 3/4$ (d) None	C
Q276	Evaluate $\int_{1}^{2}\frac{(x^2-1)}{x^2e^{x+1/x}}dx$ and value is _______. (a) $e^2[\sqrt{e}-1]$ (b) $e^2[\sqrt{e}-1]+k$ (c) $e^{2\sqrt{e}}$ (d) None	A
Q277	The value of $\int_{0}^{1}\frac{\sqrt{x}}{\sqrt{x}+\sqrt{1-x}}dx$ is _______. (a) ½ (b) 1 (c) 2 (d) 0	A
Q278	Evaluate $\int_{0}^{7}\frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}dx$ (a) $\frac{7}{2}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) 2	A

Q279	The value of $\int_{2}^{3} \frac{x+3}{x+1} dx$ (a) $1 + 2log\frac{4}{3}$ (b) 1— $2log(4/3)$ (c) $1 + log\frac{3}{4}$ (d) None	A
Q280	$\int_{2}^{e} log\ x\ dx$ is equal to ______. (a) $log2 - 1$ (b) – (2 log2 – 2) (c) 2 log2 — 1 (d) 0	B
Q281	The value of $\int_{0}^{1} x(1-x)^{n} dx$ is equal to ______. (a) 0 (b) 1 (c) $\frac{1}{(n+1)(n+2)}$ (d) $(n+1)(n+2)$	C
Q282	Evaluate $\int_{-3}^{3}(x^3 + x)dx$ (a) 0 (b) 3 (c) −3 (d) 1	A
Q283	Evaluate the value of $\int_{0}^{3}(3x^2 + 5x + 2)\ dx$ (a) 55 (b) 57 (c) 55.5 (d) 56	C
Q284	$\int x e^{x} dx$ with upper limit 1 and lower limit 0 is ______. (a) −1 (b) 0 (c) 1 (d) ∞	C
Q285	$\int_{3}^{4} \frac{1}{25-x^2} dx$ (a) (3/4)log(1/5) (b) (1/5)log(3/4) (c) (1/5)log(4/3) (d) (3/4)log5	B
Q286	Integrate $\int_{3}^{11}(2x+3)^{1/2}dx$ (a) 33 (b) 100/3 (c) 98/3 (d) None	C
Q287	If $\int_{0}^{1}(3x^2 + 2x + k)dx = 0$, find k. (a) 0 (b) −1 (c) −2 (d) 1	C
Q288	If $\int_{a}^{b} x^3\ dx = 0$ and if $\int_{a}^{b} x^2\ dx = \frac{2}{3}$, find a and b, (a) 0 and 1 (b) 1 and −1 (c) −1 and 1 (d) 0 and −1	C
Q289	Evaluate $\int_{1}^{2} \frac{logx}{x^2} dx$ (a) $log(e^2/2)$ (b) (1/2) log(e/2) (c) $Log_2 e$ (d) $log\ 2^e$	B
Q290	Evaluate $\int_{0}^{4} \frac{1}{x+\sqrt{x}} dx$ (a) log6 (b) log3 (c) 2 log3 (d) 2 loge	C