

RATIO

Min → 4 Ques

Max → 10 Ques

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Chapter 1

(Same units must be there) ← Ratio & Prop → antecedent
simplest form. a
b → consequent

* Types of Ratio

1. Duplicate Ratio

square of original ratio.

↳ Duplicate ratio of $\frac{a}{b}$ is $\frac{a^2}{b^2}$

Eg: Find Duplicate Ratio of $\frac{3}{5}$ = $\frac{3^2}{5^2} = \frac{9}{25}$

2. Triplicate Ratio

cube of original ratio.

↳ Triplicate ratio of $\frac{a}{b}$ is $\frac{a^3}{b^3}$

Eg: Find Triplicate Ratio of $\frac{1}{2}$ = $\frac{1^3}{2^3} = \frac{1}{8}$

3. Sub-Duplicate Ratio

↳ Sub-duplicate Ratio of $\frac{a}{b}$ is $\frac{\sqrt{a}}{\sqrt{b}}$

4. Sub-Triplicate Ratio

↳ sub triplicate of $\frac{a}{b}$ is $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

5. Inverse Ratio

↳ Inverse Ratio of $\frac{a}{b}$ is $\frac{b}{a}$

6. Compounded Ratio

↳ Multiplication of Ratios.

Eg: Find compounded Ratio of $\frac{3}{5}$ and $\frac{4}{8}$

↳ $\frac{3}{5} \times \frac{4}{8} = \frac{3}{10}$ Ans.

PRACTICE:

Find compounded Ratio of,
Duplicate Ratio of $\frac{1}{2}$, Sub-duplicate
of $\frac{4}{9}$, Triplicate of $\frac{1}{3}$ & Inverse
Ratio of $\frac{3}{4}$

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$$\Rightarrow \frac{1}{4} \times \frac{2}{3} \times \frac{1}{27} \times \frac{4}{3} = \frac{2}{243}$$

Imp. Find compounded Ratio of Duplicate Ratio
of $\frac{1}{2}$, sub-duplicate of $\frac{4}{9}$, Triplicate of $\frac{1}{3}$ &

$$\Rightarrow \frac{1}{4} \times \frac{2}{3} \times \frac{1}{27} \times \frac{4}{3} = \frac{1}{243}$$

separate no.

1. Ratio = $\frac{a}{b}$ $\begin{cases} a > b \Rightarrow \text{Greater Inequality} \\ a < b \Rightarrow \text{less Inequality} \end{cases}$

$\frac{5}{7} = (a < b) \Rightarrow \text{less inequality}$

2. Ratio $\begin{cases} \text{Integers} \Rightarrow \text{Commensurable} = \frac{2}{3} \\ \text{Non-Integers} \Rightarrow \text{Incommensurable} = \frac{13}{7} \end{cases}$

3. Continued Ratio $\Rightarrow a:b:c$

Ratio of 3 or more numbers

$$\Rightarrow 5:6:7$$

Imp.
eg

If Ratio b/w A & B is 2:3, B & C is 4:5

Find Ratio of A:B:C

$$A = 2 \times 4 \Rightarrow 8$$

$$B = 3 \times 4 \Rightarrow 12$$

$$B = 4 \times 3 \Rightarrow 12$$

$$C = 5 \times 3 \Rightarrow 15$$

[Make B same]

$$\text{Ratio} = 8:12:15$$

$$\text{Ratio} = 15:25$$

original no.

$$\leftarrow \frac{15}{25} = \frac{3}{5} \rightarrow \text{Ratio.}$$

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[Here,

Common factor

Comm. factor = 5.]

eliminate

(finish) उत्तर)

* $\text{Ratio} \times \text{Common factor} = \text{original no}$

$$\frac{3}{5} \times \frac{5}{5} \Rightarrow \boxed{\frac{15}{25}}$$

SUMMARY 1. To find Ratio, eliminate the common factor

2. To find original quantity, multiply ratio by common factor.

Imp.

* $\text{Daily earnings} = 4:5$

* $\text{Daily Expenditures} = 7:9$

* $\text{Daily savings} = ₹ 50 \text{ each}$

→ Find Income & Expenditure of A & B

Solⁿ $\text{Ratio} \times \text{Common factor} = \text{original Qty}$
factor.

Let the common factor for income be x
& for Expenditure be y .

Income
A = $\frac{4}{5} = \frac{4 \times x}{5 \times x} = \frac{4x}{5x}$

Expenditure
A = $\frac{7}{9} = \frac{7 \times y}{9 \times y} = \frac{7y}{9y}$

$$\text{Income} = \text{Expenditure} + \text{saving}$$

$$\frac{4x}{5x} = \frac{7y}{9y} + 50$$

$$36y + 200 = 35y + 250$$

$$\boxed{y = 50}$$

$$4x = 7y + 50$$

$$4x = 7(50) + 50$$

$$x = \frac{400}{4} \Rightarrow \boxed{100 = x}$$

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Ans.

Income

$$A = 4x = 4(100) = 400$$

$$B = 5x = 5(100) = 500$$

Expenditure

$$A = 7y = 7(50) = 350$$

$$B = 9y = 9(50) = 450$$

Imp

Q

Weight of the person increases in 8:9
If old weight was 32 Kg, what is
the new weight.

Solⁿ

Weight \uparrow (यदि 8 भा, अतः 9 हो गया)

$$\text{Old} = 8 = 32$$

$$\text{New} = 9 = x$$

$$x = \frac{32 \times 9}{8} \Rightarrow \Rightarrow \boxed{36 \text{ Kg}}$$

PROPORTION

Equality of Ratios.

$$a:b = c:d$$

$$\frac{a}{b} = \frac{c}{d}$$

1.

$$a:b :: c:d$$

2.

a & d = Extremes

b & c = Means

$$a:b :: c:d$$

[means]
[Extremes]

3.

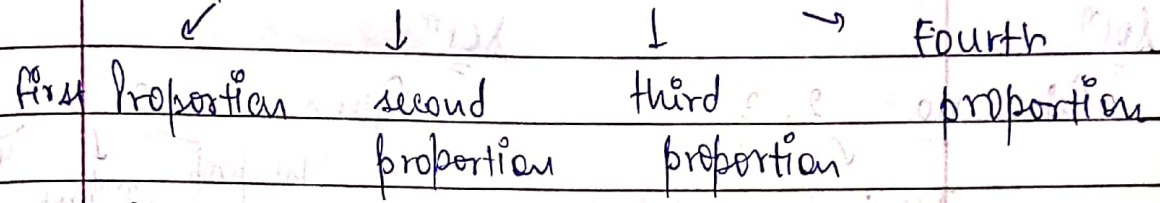
$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Product of Extremes = Product of Means

4.

$$a : b :: c : d$$



Q

Find third proportion of 8, 6 & 12
I II IV

$$\frac{8}{6} = \frac{12}{x}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$x = 16$$

Q

Find 4th proportion of 5, 10 & 12
I II III

$$\frac{5}{10} = \frac{12}{n}$$

$$n = 12 \times 2 = 24$$

NEW CONCEPT

Proportion from three numbers

$$a : b :: b : c$$

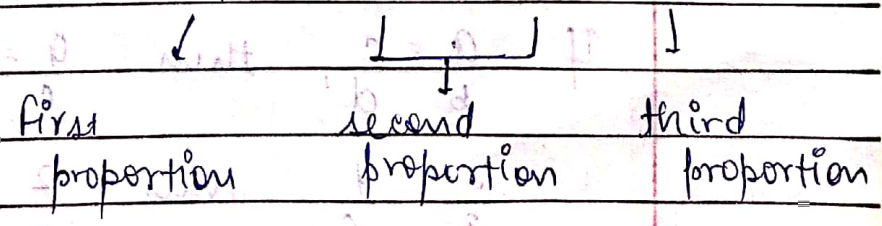
$$\frac{a}{b} = \frac{b}{c}$$

$$b^2 = ac$$

$$b = \sqrt{ac}$$

mean proportion

$$a : b :: b : c$$



[a : b :: b : c]

Q Find third proportion from 2 & 3.
Solⁿ
Given no. = 2, 3

Q Find third prop from 2, 3 & 5.
Solⁿ
Given no. = 2, 3, 5

$\frac{a}{b} = \frac{b}{c}$
 $\frac{2}{3} = \frac{3}{c}$

$\frac{a}{b} = \frac{c}{d}$
 $\frac{2}{3} = \frac{c}{5}$

$c = \frac{9}{2}$

$c = \frac{10}{3}$

Q Find Mean proportion of 25 & 16.
 $b = \sqrt{ac}$
 $= \sqrt{400} = 20$

* PROPERTIES OF PROPORTION

1 Invertendo

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$

$\frac{2}{3} = \frac{4}{6}$ then $\frac{3}{2} = \frac{6}{4}$

2 Alternendo

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

$\frac{2}{3} = \frac{4}{6}$ then $\frac{2}{4} = \frac{3}{6}$

3.

Componendo.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{b} = \frac{c+d}{d}$$

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$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{2+3}{3} = \frac{4+6}{6}, \quad \frac{5}{3} = \frac{10}{6}$$

4.

Dividendo.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a-b}{b} = \frac{c-d}{d}$$

$$\frac{2}{3} = \frac{4}{6}, \text{ then } \frac{-1}{3} = \frac{-2}{6}$$

5.

Componendo & dividendo

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\frac{2}{3} = \frac{4}{6}, \text{ then } \frac{2+3}{2-3} = \frac{4+6}{4-6}$$

$$\frac{5}{-1} = \frac{10}{-2}$$

6.

Addendo

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

$$\text{then, } a+c+e = k$$

$$b+d+f$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = k \rightarrow \boxed{2/3}$$

$$\frac{2+4+6}{3+6+9} = \frac{12}{18} = \boxed{2/3}$$

7. Subtrahendo

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

$$\text{then } a-c-e = k$$

$$b-d-f$$

Q

$$A = \frac{B}{2} = \frac{C}{5}$$

$$A:B:C = ?$$

$$\text{Put } A = \frac{B}{2} = \frac{C}{5} = \boxed{k}$$

* Simplification of Ratio

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Q = $\frac{1}{8}, \frac{1}{4}, \frac{1}{6}$

Take LCM

2	8, 4, 6
2	4, 2, 3
2	2, 1, 3
3	1, 1, 3

⇒ $\boxed{24}$

$\frac{1 \times 3}{8 \times 3} = \frac{1 \times 6}{4 \times 6} = \frac{1 \times 4}{6 \times 4}$

$\boxed{3:6:4}$

* Product of Extremes = Product of Mean
(CROSS PRODUCT RULE)

* $a:b::b:c$ [CONT. PROPORTION]

INDICES

Concepts

- $(a+b)^2 = a^2 + b^2 + 2ab$ → $(a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(a+c)$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$
- $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$
- $a^2 - b^2 = (a-b)(a+b)$
- $a^m \times a^n = a^{m+n}$
 $2^3 \times 2^5 = 2^8$
- $a^m = a^{m-n}$
 $2^5 = 2^2$
- $a^0 = 1$
- $a^{-a} = \frac{1}{a^a}$
- $\sqrt[n]{x} = x^{1/n}$
- $\sqrt{x^3} = x^{3/2}$

Log

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Tool to simplify the calculation

* PROPERTIES

1 log A + log B = log (A.B)

Cg: -> log 10 + log 5 = log (50)

-> log 12 = log (4x3) = log 4 + log 3

2 log (A/B) = log A - log B

Cg: log (12/5) = log (12) - log 5

log (6) = log (24/4) => log 24 - log 4

3. log_a (b^n) => n log_a b

Cg: log_3 8 = log_3 (2^3) = 3 log_3 2

Diagram of log_a b with labels: 'a' is base, 'b' is no. Specifically given & if not given assumed to be 'e'.

log_4 125 = log_4 (5^3) = 3 log_4 5

4. log_a^n b = 1/n log_a b Cg: log_8 11

1/n log_a b = log_a b / n Cg: log_8 11 = log 11 / log 8

log 11 / 3 log 2 => 1/3 log_2 11

5.

$$\log_{a^m} b^n$$

$$\Rightarrow \frac{n \log a b}{m}$$

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$$\log_8 27 = \log_{2^3} (3^3)$$

$$\Rightarrow \frac{3 \log_2 3}{3} \Rightarrow \boxed{\log_2 3}$$

6.

$$\log_a a \Rightarrow 1$$

$$\log_5 5 = 1 \quad \log_8 8 = 1$$

7.

$$\log_a b \Rightarrow \frac{\log b}{\log a}$$

Ceg: $\log_3 10 = \frac{\log_e 10}{\log_e 3}$

BASE
CHANGING
FORMULA

$$\frac{\log 5}{\log 3} \rightarrow \log_3 5$$

base $\rightarrow e$

base 3

8.

$$\log 1 = 0$$

9.

$$\log 10 = 1$$

*

Common logarithm \rightarrow log having base 10

*

Natural logarithm \rightarrow log having base e

$$* e = 2.718 \text{ (approx)}$$

exponential no.