FORMULA MARATHON MATHS

1. New Quantity = $\frac{b}{a}$ × Orgnal Quantity (Ratio a:b)
   Factor moltiplying Ratio
   b- antecedant, a- consequent

2. Inverse Ratio → a:b ⟹ b:a 4:3 ⟹ 3:4

3. Compounded Ratio → a:b & c:d = ac:bd 4:5, 3:7 = 12:35
   (b: Consequent, a: Antecedant)

4. Duplicate Ratio = a:b ⟹ $a^2 : b^2$ (a:b × a:b)

5. Triplicate Ratio = a:b ⟹ $a^3 : b^3$

6. Sub-duplicate = a:b ⟹ $\sqrt{a} : \sqrt{b}$

7. Sub-triplicate = a:b ⟹ $\sqrt[3]{a} : \sqrt[3]{b}$

8. Continued Ratio = a, b, c, d ⟹ a:b:c:d - Partnership Ratio
   a:b = 2:5 b:c = 4:7 a:b:c = ?
   a:b:c = 8:20:35 $\frac{a}{b} = \frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$ $\frac{b}{c} = \frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$

9. Continous Proportion = $\frac{a}{b} = \frac{b}{c} \Rightarrow$ $ac = b^2$; $b = \sqrt{ac}$ = GM of a & c

10. Cross Prod. Rule
    Prod. of Means = Prod. of extremes → $\frac{2}{6} = \frac{3}{x}$ = $2x = 18 \Rightarrow x = 9$
    $\frac{2}{6} = \frac{3}{9}$ ✓

11. Invertendo → If $a:b = c:d$ then $b:a = d:c$

$1:2 = 4:8$ then $2:1 = 8:4$

12. Alternendo → $a:b = c:d$ then $a:c = b:d$

$5:7 = 10:14$ then $5:10 = 7:14$ $(1:2 = 1:2)$

13. Componendo → If $a:b = c:d$; then $a+b:b = c+d:d$

14. Dividendo → If $a:b = c:d$; then $a-b:b = c-d:d$

15. 13 & 14 → If $a+b = c:d$ then $a+b:a-b = c+d:c-d$

$a-b:a+b = c-d:c+d$

16. Addendo → If $2:3 = 4:6 = 6:9 = 10:15 = K(0.66666)$

$= 2+4+6+10 : 3+6:9+15$

$= 22:33 = 0.6666(K)$

17. Subtrahendo → $\frac{2-4-6-10}{3-6-9-15} = \frac{-18}{-27} = 0.6666(K)$

18. $a^0 = 1$

19. $\sqrt[n]{a} = a^{1/n}$

20. $a^m \times a^n = a^{m+n}$

21. $a^m \div a^n = a^{m-n}$

22. $(a^m)^n = a^{mn}$

$(2^3)^4 = 2^{3\times4} = 2^{12} = 8^4 = 4096$

23. $(a \times b)^n = a^n \times b^n$

24. Base x $\overset{2\ 3\ 4\ 5}{=\ =\ =\ =}$

Sq Cu

$2x = 4 = 8 = 16$

25: Reciprocal / Negative Power

$60000 = AI \times 10.73$

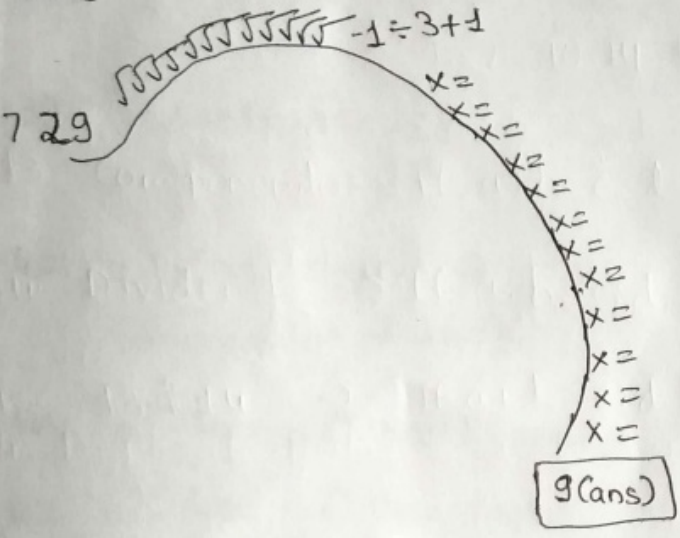
$10.73 \div = 60000 = 5591.7986$

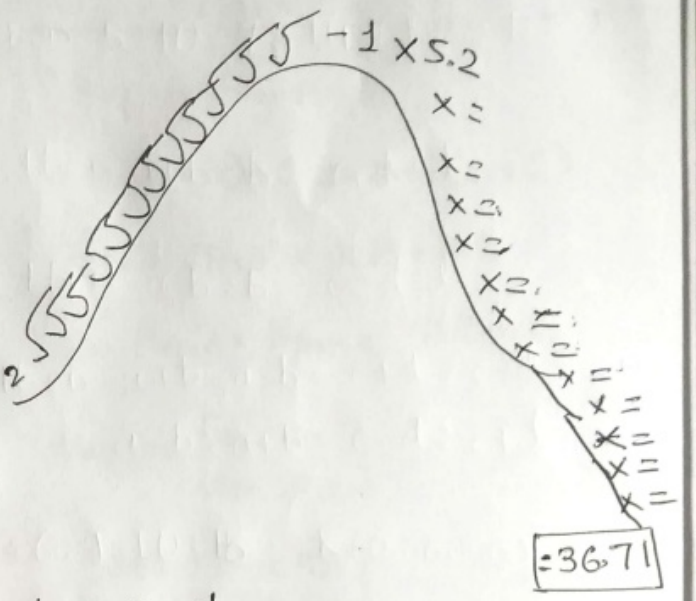
$\frac{1}{3^4} = \frac{1}{81}$

26. Calculator Trick for any root

Base ($\sqrt{}$ 12 times $-1 \div n + 1 (x =)$ 12 times

Power $\sqrt[3]{729}$ = ($\sqrt{}$ 12 times $-1 \times n + 1 (x =)$ 12) — Power trick





27. $a^{4} = 81$ $\log_a 81 = 4$

$n > 0, a > 0, a \neq 1$

28. $\log_a a = 1$

29. $\log_a 1 = 0$ $\log_2 1 = 0$

$2^0 = 1$

30. $\log_a mn = \log_a m + \log_a n$

31. $\log_a \frac{m}{n} = \log_a m - \log_a n$

32. $\log_a m^n = n \log_a m$

$\log_3 3^8 = 8 \log_3 3 = 8$

33. $\log_b m = \frac{\log m}{\log b} = \frac{\log_a m}{\log_a b}$

34. $\log_b a = \log_a b = \frac{\log a}{\log b} \times \frac{\log b}{\log a} = 1$

35. Common ratio Log Base = 10

-Natural Log Base = e

Calculus & stats = 2,71828

36. $ax^2 + bx + c = 0$

$a = x^2$ coefficient

$b = x$ coefficient

$c =$ constant

37. $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

38. Sum of roots $\alpha + \beta = -\frac{b}{a}$

39. Prod. of roots $\alpha\beta = \frac{c}{a}$

40. $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

Construction of Qd. Eq.

41. $b^2 - 4ac = 0$ Real & Equal

42. $b^2 - 4ac < 0$ Imaginary

43. $b^2 - 4ac > 0$ Real & Unequal

44. $b^2 - 4ac > 0$ & per. sq. — Real, Unequal & Rational

45. $b^2 - 4ac > 0$ & not per. sq. — Real, Unequal & Irrational

46. $m+\sqrt{n}$ other will be $m-\sqrt{n}$
Conjugate pair

47. $ax+b=0$ Simple equation

$3x - 5 = 13$

$3x = 5 + 13$

$3x = 18$

$x = 6$

48. $a_1x + b_1y + c_1 = 0$ - Simultan
$a_2x + b_2y + c_2 = 0$

49. $ax^3 + bx^2 + cx + d = 0$

Trial & Error

MATRICES

50. $A + B = B + A$

51. $(A+B)+C = A+(B+C)$

52. $k(A+B) = kA + kB$

53. $A_{m\times n} \times B_{n\times p} = AB_{m\times p}$

54. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$\det A \begin{bmatrix} a_{22} & a_{21} \\ a_{12} & a_{11} \end{bmatrix}$

55. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$

$+ a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

56. $C_{ij} = (-1)^{i+j} M_{ij}$

57. $A^{-1} = \frac{1}{\det A} \times \text{adj } A$

Inverse of A; $\det A \neq 0$

Cramer's Rule

58. $2x+3y-4z=10$

$7x+y+2z=19$

$-2x-5y+3z=8$

$\Delta, \Delta x, \Delta y, \Delta z$

$$\Delta = \begin{vmatrix} 2 & 3 & -4 \\ 7 & 1 & 2 \\ -2 & -5 & 3 \end{vmatrix}$$

$$\Delta x = \begin{vmatrix} 10 & 3 & -4 \\ 19 & 1 & 2 \\ 8 & -5 & 3 \end{vmatrix}$$

$$\Delta y = \begin{vmatrix} 2 & 10 & -4 \\ 7 & 19 & 2 \\ -2 & 8 & 3 \end{vmatrix}$$

$$\Delta z \begin{vmatrix} 2 & 3 & 10 \\ 7 & 1 & 19 \\ -2 & -5 & 8 \end{vmatrix}$$

$x = \Delta x / \Delta$

$y = \Delta y / \Delta$

$z = \Delta z / \Delta$

Time Value of money

59. $SI = \frac{PRT}{100}$

60. $A = P + SI$

$= P + \frac{PRT}{100}$

$= P\left(1 + \frac{RT}{100}\right)$

61. $A = P(1+i)^n$

$i = \frac{r\%}{\text{nocppy}}$

$n = t \times \text{nocppy}$

62. $P = 1000, i = 10\%, n = 3$

$1000 + 10\% + 10\% + 10\%$

$A = 1331$

$CI = 1331 - 1000 = 331$

63. Amt. as per CI = ?

$P = 20000 \quad t = 3$ years

$r = 8\%$ for first 2yrs

$r = 12\%$ for last yr.

$20000 + 8\% + 8\% + 12\%$

$= 26127.36$

$CI = 6127.36$

64. $E = [(1+i)^n - 1]$

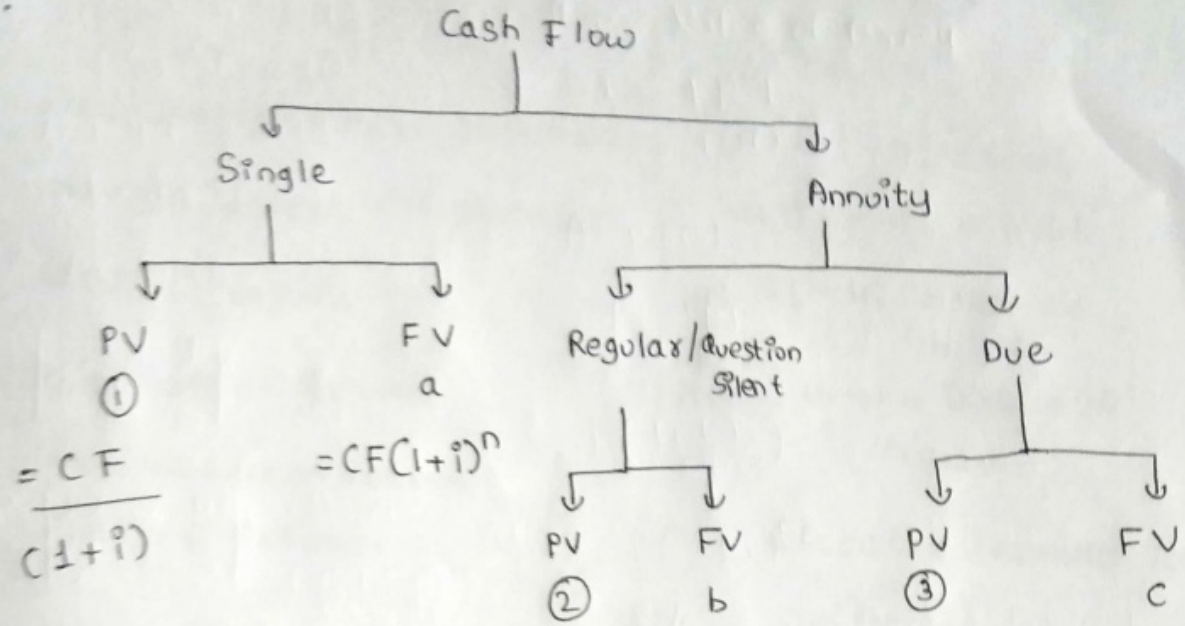
PNB = 8% p.a. Semi Annually

$i = \frac{8}{2} = 4\%$

$= [(1.04)^2 - 1] \times 100$

$= 8.16\%$

65.



FVAR

$= A_i \times FVAF(n,i)$

$= A_i \times \left\{ \frac{[(1+i)^n - 1]}{i} \right\}$

$y_1 = 10\,000$
$y_2 = 10\,000$
$y_3 = 10\,000$ → Balance

$= 10000 \times [3 \blacksquare + \text{Something}]$ ← Fut. Val. Ann. Fac.

66. FVA Due $= A_i \times FVAF(n,i)$

67. PVA Reg = [illegible]

$1+i = = \ldots$ n times GT

68. PVA Due

69. $PVP = \frac{A_i}{i}$

70. Growing Perpetuity

$PVGP = \frac{A_i}{i-g}$

69. 3000, 3000, 3000
10% p.a

$PVP = \frac{3000}{10\%} = 30000$

70. 1000, 1050, 1102.5

$PVGP = \frac{1000}{0.10 - 0.05} = 20000$

71. NPV

= PV Inflows - PV of Outflows

NPV ≥ 0, accept the proposal

NPV < 0, reject

72. Real Rate of Return

= Nominal Rate of Return

- Rate of Return.

73. Com. Anval Growth Rate

Permutation & Combination

74. $n! = n(n-1)(n-2)\ldots 3.2.1$

$7! = 7\times6\times5\times4\times3\times2\times1 = 5040$

$1=1,\ 2=2,\ 3=6,\ 4=24,\ 5=120$

$6=720,\ 7=5040,\ 8!=40320,$

$9!=362880,\ 10!=3628800$

75. $^nP_r = \dfrac{n!}{(n-r)!}$

$n \geq r$

$^{10}P_4 = \dfrac{10!}{(10-4)!}$ or $10\times9\times8\times7$

no. of factors $= r$

76. $^nP_n = n!$

$(n+1)! - n! = n.n!$

$(4+1)! - 4! = 4.4!$

LHS $5! - 4! = 120 - 24$

$= 96$

RHS $= 4.4! = 4\times24 = 96$

Proved

77. Circular Permutation

$= (n-1)!$

78. Circular Permutation (TII)

$\frac{1}{2}(n-1)!$ [No two neighbours are same]

[necklaces, garland]

79. Permutation with Restriction

$^{n-1}P_r$ 1 excluded

80. Particular obj is always inc.

$r.\ ^{n-1}P_{r-1}$

81. $^{n-1}P_r + r.\ ^{n-1}P_{r-1} = {}^nP_r$

82. No. of ways when things are never together

1 → Together

83. $^nC_r = \frac{^nP_r}{r!}$

$^nC_r = \frac{n!}{r!(n-r)!}$

$^{10}C_4 = \frac{^{10}P_4}{4!}$

$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$

$= 210$

84. $^nC_0 = 1$, $^nC_n = 1$

85. $^{10}C_8 = {}^{10}C_2$

Complimentary Combination

$^{14}C_{11} = {}^{14}C_3$

$^nC_r = {}^nC_{n-r}$

$^{14}C_r = {}^{14}C_{14-r}$

$^{14}C_{11} = {}^{14}C_3$

5. 6, 11, 16, ...

$a = 6 \quad d = +5$

$6 + 5 = = = = 101$

$t_8 = a_0 = \ldots$

86. Special Combination Formula

$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$^{600}C_8 + {}^{600}C_9 = {}^{601}C_9$

87. Combination of one or more

$2^n - 1$

5 - One or more

$^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 32 = 2^5$

$= 2^5 = 1$

88. No. of straight lines $= {}^nC_2$

89. No. of triangles $= {}^nC_3$

90. No. of Collinear = Linear - Collinear

91. No. of parallelogram $= {}^nC_2 \times {}^mC_2$

92. No. of Diagonals $= {}^nC_2 - n$

AP, GP

93. $d = t_2 - t_1 = t_3 - t_2 = t_n - t_{n-1}$

Common diff. of AP

94. $t_n = a + (n-1)d$ (Gen. Term of AP)

95. $a \pm d = = = \ldots$ n times

$t_2 \; t_3 \; t_4 \quad t_n$

Shortcut Trick

96. $S_n = \frac{n}{2}(a + t_n)$

97. $S_n = \frac{n}{2}\{2a + (n-1)d\}$

98. $a + d = = \ldots$ upto n terms GT + a
$t_2\ t_3$

$S_{10} = \frac{10}{2}\{2 \times 20 + 9 \times (-4)\}$

$= 5(40 - 36) = 20$

$20 - 4 = = = \ldots t_{10}$ GT + 20

99. $\frac{n(n+1)}{2}$

100. $S = n^2$

101. $S = n(n+1)(2n+1)$

102. $S = \{n(n+1)\}^2$

103. $r = \frac{t_2}{t_1} = \frac{t_n}{t_{n-1}} = \frac{t_3}{t_2}$

104. $t_n = ar^{n-1}$

$r \times a = = = \ldots =$
$t_2\ t_3 t_4 \ldots\ t_n$

2, 6, 18, 54

$a = 2, r = 3$

$t_8 = ar^7 = 2 \times 3^7 = 4374$

$3 \times 2 = = =$ upto 8 times
$= 4\ 374$

GP

105. $S_n = a\frac{(1 - r^n)}{1 - r}$
$r < 1$

106. $S_n = a\frac{(r^n - 1)}{r - 1}$
$r > 1$

107. $r \times a = = = \ldots = $ GT + a
$t_2\ t_3 t_4\ \ t_n$

20, 10, 5, 5/2

$S_5 = \frac{a(r^n - 1)}{r - 1}$

$= \frac{20(0.5^5 - 1)}{0.5 - 1}$

$20 \times \frac{+0.96875}{+0.5}$

$= 38.75$

$0.5 \times 20 = = = =$ GT + 20
$t^2 t^3 t^4 t^5$

108. $S_\infty = \frac{a}{1 - r}$

109. $A = \{2,5,7\}$

$\{2\}$ $\{5\}$ $\{7\}$

$\{2,5\}$ $\{5,7\}$ $\{2,7\}$

$\{2,5,7\}$ ϕ

(8)

Total Subsets $= 2^n$.

110. Proper Subsets $= 2^n - 1$

111. De Morgan's Law

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

112. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

113. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

3 set operations

114. $f \circ g = f \circ g(x) = f[g(x)]$

$f(x) = x^2$, $g(x) = (2x-1)$

$f \circ g = f(g(x))$

$= (2x-1)^2$

$g \circ f = g(f(x))$

$= 2x^2 - 1$

115. $f(x) = 2x + 1$

$y = 2x + 1 \Rightarrow 2x = y - 1$

$x = \frac{y-1}{2}$

$y = \frac{x-1}{2} - f^{-1}(x)$

Calculus

116. $\frac{d(x^n)}{dx} = n x^{n-1}$

117. $\frac{d}{dx} e^x = e^x$

118. $\frac{d a^x}{dx} = a^x \log_e a$

119. $\frac{d}{dx} k = 0$

120. $\frac{d}{dx} e^{ax} = a e^{ax}$

121. $\frac{d}{dx}(\log x) = \frac{1}{x}$

122. $h(x) = c \cdot f(x) = \frac{d[h(x)]}{dx} = c \cdot \frac{d}{dx}[f(x)]$

123. ~~$h(x) = f(x) \pm g(x)$ $\frac{d}{dx}$~~

$$\frac{d}{dx} u \cdot v = u \frac{dv}{dx} + v \frac{du}{dx}$$

124. $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

125. $y = C(x)$

126. $A(x) = \frac{C(x)}{x}$

127. $A'(x) = 0 =$ A.C min or max.

128. $MC = \frac{dC}{dx}$

129 $M'(x) = 0$

130. $MR(x) = \frac{dR}{dx}$

DIFFERENTIATION

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} K = 0$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \log x = \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \log_e a}$$

$$\frac{d}{dx} a^x = a^x \log_e a$$

$$\frac{d}{dx} \frac{I}{II} = \frac{II \frac{d}{dx} I - I \frac{d}{dx} II}{(II)^2}$$

$$\frac{d}{dx} I.II = I \frac{d}{dx} II + II \frac{d}{dx} I$$

Chain Rule

$$\frac{d}{dx} f(x)^n = n (F(x))^{n-1} \frac{d}{dx} F(x)$$

$$\frac{d}{dx} \sqrt{F(x)} = \frac{1}{2\sqrt{F(x)}} \frac{d}{dx} F(x)$$

$$\frac{d}{dx} a^{f(x)} = a^{f(x)} \log_e a \frac{d}{dx} f(x)$$

$$\frac{d}{dx} e^{F(x)} = e^{F(x)} \times \frac{d}{dx} F(x)$$

$$\frac{d}{dx} \log_e f(x) = \frac{1}{f(x)} \frac{d}{dx} f(x)$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{f(x) \log_e a} \frac{d}{dx} f(x)$$

INTEGRATION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Exception $\int \frac{1}{x} = \int x^{-1} = \log|x| + c$

$$\int k\, dx = \int kx + c$$

$$\int 1\, dx = x + c$$

$$\int e^x dx = e^x + c$$

$$\int f'(x)\, dx = f(x)$$

$$\star 1 \quad \int (ax+b)^n = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + c$$

Exception

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\log|ax+b| + c$$

$$\int (ax+b)^{-1} dx = \uparrow$$

$$\int a^{cx+d} dx = \frac{1}{c}\frac{a^{cx+d}}{\log_e a} + c$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$\int x\sqrt{x}\, dx$

$\int e^{-3x}$ $\quad \frac{2}{5}x^{5/2}$ $\quad = \int x^{3/2} dx$

$= \frac{1}{e^{3x}} + c$

$= -\frac{1}{3} e^{-3x} + c$

$$\int I.II\, dx = I\int II\, dx - \int \left(\frac{d}{dx} I \int II\, dx\right) dx$$

$$\int \frac{xe^x}{(x+1)^2} dx$$

a) $\frac{e^x}{x+1} + c$

$$\frac{d}{dx}\frac{e^x}{x+1}$$

$$= (x+1)\frac{d}{dx}e^x - e^x \frac{d}{dx}(x+1)$$

$$\frac{}{(x+1)^2}$$

$$= \frac{(x+1)e^x - e^x}{(x+1)^2}$$