

Equations

Linear Equation

Quadratic Eqⁿ

Cubic Eqⁿ

- One Variable
- Two Variable
- Three Variable

Linear Eqⁿ with one variable

$$\text{If } ax + b = 0 \quad x = \frac{-b}{a}$$

$$\text{If } ax + b = c \quad x = \frac{c-b}{a}$$

$$\text{If } ax + b = cx + d \quad x = \frac{d-b}{a-c}$$

Eg 1: find the value of x in the following cases

Case 1: $2x + 3 = 0 \Rightarrow -x = -3/2$

Case 2: $4x + 5 = 0 \Rightarrow x = -5/4$

Case 3: $3x + 6 = 8 \Rightarrow 3x = 2 \Rightarrow x = 2/3$

Case 4: $13x + 3 = 10x + 27 \Rightarrow (13-10)x = 27-3 \Rightarrow x = \frac{27-3}{13-10}$

Step 1: $13-10 = M+$

Step 2: $27-3 = \therefore MRC = 8$

Case 5: $127x + 1080 = 89x + 1688$

Step 1: $127x - 89x \quad M+$

Step 2: $1688 - 1080 = \therefore MRC = 16$

Case 6: $12.6x + 13.2 = 9.7x + 8.85 - 1.5$

Case 7: $642x - 1491 = 429x - 213 \Rightarrow 213 = 6$

Case 8: $35.5x - 24.7 = 17.5x + 195.8 \Rightarrow 12.25$

Linear Eqⁿ With Two Variable

$$\begin{aligned} 3x + 2y &= 12 \\ 5x + 4y &= 22 \end{aligned}$$

Substitution Method Elimination Method Cross Multiplication Method

- Cross Multiplication Method

~~$3x + 2y = 12$~~
 ~~$5x + 4y = 22$~~

Step 1 $(+3) \times (+4) - (+2) \times (+5)$
 $12 - 10 = 2$

forward

$$\begin{array}{r} 3 \times 4 = M+ \\ 2 \times 5 = M- \end{array} \left. \begin{array}{l}] \\] \end{array} \right\} \begin{array}{l} MRC \\ MRC \end{array} \quad 2$$

Step 2 Value of x

backward

$$\begin{array}{r} 12 \times 4 = M+ \\ 2 \times 22 = M- \end{array} \left. \begin{array}{l}] \\] \end{array} \right\} \begin{array}{l} MRC \\ MRC \end{array} \quad \div -2 = 2$$

Step 3 Value of y

forward

$$\begin{array}{r} 12 \times 4 \\ 3 \end{array}$$

Eg 2:- find the value of x and y in the following cases

Case 1:- $13x + 3y = 28$

$7x + 12y = 67$

Step 1: $13 \times 12 = M+$
 $3 \times 67 = M-$ MRC MRC 135

Step 2 Value of x

$28 \times 12 = M+$

$3 \times 67 = M-$ MRC MRC 135



step 3 value of y

$$13x + 67 = M+$$

$$28x + 7 = M-$$

$$MRC - MRC = -135 = 5$$

Case 2: $4x + 3y = 6$

$$11x + 12y = 9$$

- 2

Case 3: $3x + 2y = 13$

$$7x + 3y = 17$$

step 2: $13 \times 3 = M+$, $17 \times 2 = M-$

step 3: $3 \times 17 = M+$, $13 \times 7 = M-$

$$MRC - MRC = 5 + \frac{1}{2} = 8$$

14

Case 4: $32x - 19y = 6$

$$17x + 3y = 25.5$$

Case 5: $11x - 5y = 17$

$$3x + 2y = 8$$

step 1 = 37

$$x = 2, y = 1$$

Case 6: $2x + 5y + 29 = 0$

$$3x + 2y + 5 = 0$$

$$2x + 5y = -29$$

$$3x + 2y = -5$$

step 2 = -11

$$x = 3, y = -7$$

Case 7: $x + 3y - 16.2 = 0$

$$5y + 3x = 30.2$$

$$x + 3y = 16.2$$

$$3x + 5y = 30.2$$

step 1 = -4

$$x = 2.4$$

Case 8: $3x + 2y = 7$

$$6x + 4y = 14$$

step 1 = 0

$$x =$$

$$y = 4.6$$

a) 0, b) 1, 2

c) Not determine

d) None of these

$$\frac{a_1}{a_2} = \frac{3}{6} = 0.5 \quad \frac{b_1}{b_2} = \frac{2}{4} = 0.5 \quad \frac{c_1}{c_2} = \frac{7}{14} = 0.5$$

Infinite solⁿ.

x	0	1	2	3
y	3.5	2	0.5	-1

Given $a_1x + b_1y = c_1$
 $a_2x + b_2y = c_2$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$a_1b_2 - b_1a_2 \neq 0$$

Unique solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$a_1b_2 - b_1a_2 = 0$$

$$\frac{a_1 - b_1 - c_1}{a_2 - b_2 - c_2}$$

$$\frac{a_1 - b_1 \neq c_1}{a_2 - b_2 - c_2}$$

Infinite Solⁿ No Solution

Case 9: $3x + 2y = 7$
 $6x + 4y = 21$

Step 1 = 1

$$2 = \frac{a_1}{a_2} = \frac{3}{6} = 0.5 \quad \frac{b_1}{b_2} = \frac{2}{4} = 0.5 \quad \frac{c_1}{c_2} = \frac{7}{21} = 0.333$$

No Solution

Linear Eqⁿ with Three Variable.

Such Equation can be solved through Matrices and determinant, However since exam is objective, it is advisable to put options to get answer.

Eq: $x + y + z = 9$

$2x - y + z = 5$

$3x + 2y - z = 8$

a) 4, 3, 2 b) 4, 4, 1 c) 5, 1, 3 d) 2, 3, 4

Let Quadratic eqⁿ be
 $ax^2 + bx + c = 0$

Then the roots of this eqⁿ i.e. α & β are as follow

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Eg:- $x^2 - 5x + 6 = 0$

$a = 1, b = -5, c = 6$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$\frac{-5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2}$$

3 or 2

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha \beta = \frac{c}{a}$$

$$\frac{(-b \pm \sqrt{b^2 - 4ac})}{2a} \cdot \frac{(-b \mp \sqrt{b^2 - 4ac})}{2a} = \frac{c}{a}$$

Eg:- find $\alpha + \beta$ and $\alpha \beta$ from the following.

Case 1:- $x^2 - 5x + 6 = 0$ $a = 1, b = -5, c = 6$

$$\alpha + \beta = 5 = \alpha \beta = 6 - 6$$

Case 2:- $2x^2 + 5x + 6 = 0$ $a = 2, b = 5, c = 6$

$$\alpha + \beta = \frac{-5}{2}, \alpha \beta = 3$$

Case 3:- $3x^2 - 7x + 11 = 0$ $a = 3, b = -7, c = 11$

$$\alpha + \beta = \frac{7}{3}, \alpha \beta = \frac{11}{3}$$

Case 4:- $\sqrt{2}x^2 + 3x - \sqrt{5}x + 13 = 0$

$$\alpha + \beta = \frac{\sqrt{5} - 3}{\sqrt{2}}, \alpha \beta = \frac{13}{\sqrt{2}}$$

$a = \sqrt{2}$
 $b = 3 - \sqrt{5}$
 $c = 13$

Eg:- find α and β from the following :-
 Case 1:- $x^2 - 5x + 4 = 0$. $a=1, b=-5, c=4$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
 $\alpha, \beta = 1 \text{ and } 4$

Case 2:- $x^2 - 5x + 3 = 0$. $a=1, b=-5, c=3$

Case 3:- $4x^2 - 20x + 25 = 0$ $\alpha, \beta = \frac{5}{2}$

Case 4:- $x^2 - 5x + 7 = 0$. $\alpha, \beta = \frac{5 \pm \sqrt{3}}{2}$

- ★
- If a, b and c are Integer
 - If $b^2 - 4ac < 0$, imaginary root -ve $\sqrt{-3}$
 - If $b^2 - 4ac \geq 0$; Real Root +ve value
 - If $b^2 - 4ac > 0$ and is not a perfect square, Real and Irrational roots.
 - If $b^2 - 4ac > 0$ and is a perfect square is Real, rational and unequal.
 - If $b^2 - 4ac = 0$; real and equal root, rational
 - If $b^2 - 4ac \neq 0$, unequal roots.

If α & β are roots of quadratic Eqⁿ, then such Eqⁿ can be expressed as.

$$[(x - \alpha)(x - \beta) = 0]$$

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$\rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Eg:- find the quadratic eqⁿ from the following:-

Case 1:- $\alpha = 2, \beta = 5$

$$x^2 - 7x + 10 = 0$$

Case 2:- $\alpha = -1, \beta = 4$

$$x^2 - 3x - 4 = 0$$

Case 3: $\alpha = -3, \beta = -5$
 $x^2 + 8x + 15 = 0.$

Case #:- Solve for x

Case 1:- $x^2 = 4,$
 $\pm 2.$

$x^2 - 4 = 0$
 $x^2 - 2^2 = 0$
 $(x-2)(x+2) = 0$
 $x = \pm 2$

Case 2:- $x = \sqrt{4},$
 $x = 2.$

[The value of $\sqrt{\quad}$ is always positive]

Cubic Equation

$$x^3 + 4x^2 - 7x - 10 = 0$$

$$x = 0$$

$$0 + 0 - 0 - 10 \neq 0$$

$$1 + 4 - 7 - 10 \neq 0$$

$$x = 1$$

$$-1 + 4 + 7 - 10 = 0$$

$$x = -1$$

$$x + 1 = 0$$

$$x + 1$$

$$x^2 + 3x - 10$$

$$x^3 + 4x^2 - 7x - 10$$

$$-x^2 + x^2$$

$$3x^2 - 7x - 10$$

$$3x^2 + 3x$$

$$-10x - 10$$

$$-10x - 10$$

x

$$(x+1)(x^2+3x-10) = 0$$

either $x+1=0$

$$x = -1$$

$$\text{or } x^2 + 3x - 10 = 0$$

$$-3 \pm \sqrt{9 - (4)(1)(-10)}$$

2

$$\frac{-3 \pm 7}{2} = 2, -5$$

$$x = -1, 2, -5$$

Eg a) 1, 2, 5 b) -1, 2, 5 c) -1, 2, -5, d) 1, -2, 5

add or subtract
4

$$x^3 + 4x^2 - 7x - 10 = 0$$

put

$$-1^3 + 4(-1)^2 - 7(-1) - 10 = 0$$

$$1(-1) \times = M+$$

$$1(+1) \times 4 = M+$$

$$1(+1) \times 7 = M-$$

$$\Delta \pm B \pm \sqrt{\frac{-b}{a}} = -4$$

$$\Delta B \pm \sqrt{10}$$

Eg: $x^3 - 7x^2 - 5x + 315 = 0$

- a) 7, -5, 9 b) 7, -5, -9 c) -7, 5, -9 d) -7, 5, 9

$\alpha + \beta + \gamma = 7$

$7 \times 7 = M+$

$7 \times -5 = M-$

$7 \times 53 = M-$

315 M+

MRC

If α, β and γ are roots of Cubic equation, then Such equation can be expressed as

$(x-1)(x-2)(x-3) = 0$

$(x-\alpha)(x-\beta)(x-\gamma) = 0$

If $ax^3 + bx^2 + cx + d = 0$

$x^3 [(\alpha + \beta + \gamma)]x^2 + [\alpha\gamma + \beta\gamma + \alpha\beta]x - \alpha\beta\gamma = 0$

$x^3 + \frac{bx^2}{a} + \frac{cx}{a} + \frac{d}{a} = 0$

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \alpha\gamma = c/a$

$\alpha\beta\gamma = -d/a$

Eg.

Application of Equations

Cost and profit function

Total cost = fixed cost + variable cost

Eg: Rent

If FC is "a" and VC per ^{unit} is "b"

and no. of units produced be x , then

$$TC = y = a + bx$$

Eg: If FC = 50000 and VC pu = Rs 100, then

Total Cost

$$TC = y = 50000 + 100x$$

TC if 1 unit is produced = Rs 50100

$$50000 + 100 \times 20$$

20 unit

$$= 52000$$

$$50000 + 100 \times 200$$

200

$$70000$$

Similarly Total Revenue = FR + VR

If FR is "c" and VR pu is "d"

and no. of units sold be x .

$$TR = c + dx$$

Eg: If FR = 10000 and Variable Revenue 90 per unit

then $TR = y = 10000 + 90x$

TR if 1 unit is sold \therefore Rs 10900

if ~~20000~~ ^{20 x 90} 29000 are sold

$$28000$$

200

$$19000$$

profit = TR - TC

$$\text{If } TR = 10000 + 90x$$

$$TC = 50000 + 100x$$

$$\text{then } TP = 800x - 40000$$

if 500 unit are sold 120000

$$= \text{Rs } 240000$$

Break Even point

Minimum point / No. of units to be sold to avoid losses
i.e. when Total loss = 0

$$\text{If } TP = 800x - 40000$$

$$\text{BEP } TL = 0$$

$$800x - 40000 = 0$$

$$x = 50 \text{ units}$$

$$TP = x = 50$$

$$TP = 0$$

$$x = 49$$

$$TP = -800$$

$$x = 48$$

$$TP = -1600$$

$$x = 51$$

$$TP = +800 \text{ profit}$$

Eg: Compute BEP if $TP = 70x - 25000$

a) 357

b) ~~357.14~~

c) 358

d) None of these

If x is in fraction, then always take next integral value.

beginning step