THEORETICAL DISTRIBUTIONS

Theoritical distribution

- Discrete probability distributions (pmf)
  1. Bernoulli's Distribution
  2. Binomial Distribution
  3. Possion Distribution
  4. Uniform Distribution
- Continuous probability distribution (pdf)
  1. Normal distributions.

Bernoulli's Distribution

Bernoulli trail:-

1. success — p ; failure — q

$$p+q=1$$

2. All the trails are must be independent.
3. For each and every taril
   $p \rightarrow$ remains constant

Ex: Cricket Matchs.

Bernoulli's Distribution defination: A random variable which takes only two values i.e 0 and 1 and it is said to be follows bernoulli's distributions then it's pmf is given by

$$P(X=x) = \begin{cases} p^x \cdot q^{1-x} & ; x=0,1 \\ 0 & ; \text{otherwise} \end{cases}$$

X:	0	1
$P(x)$:	q	p

$q \rightarrow$ probability of failure

$p \rightarrow$ probability of success

$$p+q=1$$

$$q=1-p \text{ or } p=1-q$$

⇒ The parameter of Bernoulli's distribution is 'P'

⇒ Mean of bernoulli's distribution is 'P'

⇒ variance of bernoulli's distribution 'pq'

X	0	1
P(X)	q	P

$E(X) = (0 \times q) + (P \times 1) = P$

$E(X^2) = (0^2 \times q) + (1^2 \times P) = P$

$var(X) = E(X^2) - (E(X))^2$

$= P - (P)^2 = pq$

⇒ standard deviation of bernoulli's distribution $\sqrt{pq}$.

Binomial Distribution

Definition:- A discrete random variable 'x' said to be follow binomial distribution with parameters 'n' and 'p' then its probability mass function is given by

$$P(X=x) = nC_x\, p^x q^{n-x} ; x = 0, 1, 2, \ldots n.$$

$$= 0 \quad ; \text{otherwise}.$$

where $n \rightarrow$ No. of trails

$p \rightarrow$ probability of success.

$q \rightarrow$ probability of failure

and $p+q=1$

⇒ If x is a random variable and it follows binomial distribution, then it is denoted by $x \sim B(n,P)$ and 'x' is called as binomial variable.

⇒ The parameters of B.D are n and p so that B.D is called bi-parametic distribution

⇒ The means of $B.D = np$

⇒ The variance of $B.D = npq$

⇒ The standard deviation of $B.D = \sqrt{npq}$

$\Rightarrow$ In B.D, $\boxed{\text{Mean} > \text{variance}}$ ***

$np \quad npq$

$np(1-p)$

$np-p^2$

$\Rightarrow$ variance :- The variance of BD $= npq$

$= np(1-p)$

$= np - np^2$

$= n(p-p^2)$

$= nq(1-q)$

$= n(q-q^2)$

$= nq - nq^2$

***

The maximum value of variance in binomial distribution $= \frac{n}{4}$.

Moments :-

Movements

Moments about arbitrary point A

(Raw materials)

$M_r' = \frac{1}{n} \Sigma (x_i - A)^r$

$M_1' = \frac{1}{n} \Sigma (x_i - A)$

If $A = 0$ then $M_1' = \frac{1}{n} \Sigma x_i$

$\Rightarrow E(X) = \text{mean}$

$M_2' = E(X^2)$

⋮

Moments about mean

(Central moments)

$M_r = \frac{1}{n} \Sigma (x_i - \bar{x})^r$

$M_1 = \frac{1}{n} \Sigma (x_i - \bar{x})^1 = 0$

First central moment, $\mu_1 = 0$

Second central moment,

$\mu_2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2$

$= \text{variance}$

$\mu_3 = 3^{rd}$ central moment

$\mu_4 = 4^{th}$ central movement

Pearson's constant:-

1. skewness, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

2. coefficient of skewness, $\gamma_1 = \sqrt{\beta_1}$

3. kurtosis; $\beta_2 = \frac{\mu_4}{\mu_2^2}$

4. cofficient of kurtosis, $\gamma_2 = \beta_2 - 3$

Moments of Binomial distribution:-

1. First moment about origin $\mu_1' = E(X) = \text{mean} = np$.

2. First central moment, $\mu_1 = 0$

3. Second central moment, $\mu_2 = \text{variance} = npq$

4. Third central moment, $\mu_3 = npq(q-p)$

5. Fourth central moment, $\mu_4 = npq\{3pq(n-2)+1\}$

6. skewness, $\beta_1 = \frac{(q-p)^2}{npq}$

7. cofficient of skewness, $\gamma_1 = \sqrt{\beta_1}$

$= \frac{q-p}{\sqrt{npq}}$

8. kurtosis, $\beta_2 = 3 + \frac{1-6pq}{npq}$

9. cofficient of kurtosis, $\gamma_2 = \beta_2 - 3$

$= \frac{1-6pq}{npq}$

Skewness :- ***

$$\gamma_1 = \frac{q-p}{\sqrt{npq}}$$

$q > p \Rightarrow \gamma_1 > 0$ $\quad$ $q < p \Rightarrow \gamma_1 < 0$

$1-p > p$ $\quad$ $1-p < p$

$1 > 2p$ $\quad$ $1 < 2p$

$\frac{1}{2} > p$ $\quad$ $\frac{1}{2} < p$

$p < \frac{1}{2}$ $\quad$ $p > \frac{1}{2}$.

Binomial distribution is - positively skewed if $p < \frac{1}{2}$

- negatively skewed if $p > \frac{1}{2}$

- Symmetrical if $p = \frac{1}{2}$

Mode of Binomial Distribution:

Let us consider, $m = (n+1)p$

Ex:- $n = 15$ $\quad p = \frac{1}{4}$ $\quad q = \frac{3}{4}$

$(n+1)p = (15+1)\frac{1}{4} = \frac{16}{4} = 4$

Ex: $n = 17$, $p = \frac{1}{5}$, $q = \frac{4}{5}$

$(n+1)p = (17+1)\frac{1}{5} = \frac{18}{5} = 3.6$

Case: I :- If m is not an integer then an integral part of the value 'm' will be the mode of Binomial Distribution.

$\therefore$ In this case Binomial Distribution is uni-modal.

Ex:- If $n = 13$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

$m = (n+1)p = (13+1)\frac{1}{3} = \frac{14}{3} = 4.66$

Mode = Integral part (4.66)

$= 4$.

Case-(ii):- If 'm' is an integer, then the values of 'm' and 'm-1' are the modes of BD.

∴ In this case BD is bi-modal.

Ex: If $n=17,\ p=\frac{1}{6}\cdot q=\frac{5}{6}$

$(n+1)\,p=(17+1)\frac{1}{6}=\frac{18}{6}=3$

Mode = m and m-1

= 3 and 3-1

= 2 and 3.

Additive property:- If x and y are two independent random variables and $X\sim B(n_1,p_1)$, $Y\sim B(n_2,p_2)$ then $X+Y$ follows binomial distribution only if $p_1=p_2=p$.

i.e. $X\sim B(n_1,p)$

$Y\sim B(n_2,p)$

then $X+Y\sim B(n_1+n_2,p)$

Note:- p.m.f of B.D is

$P(X=x)={}^nC_x\, p^x.q^{n-x}\quad ;\ x=0,1,2,\dots n.$

$=q^n+{}^nC_1\,p^1q^{n-1}+{}^nC_2\,p^2q^{n-2}+\dots+p^n.$

$=(q+p)^n=1$

General form of BD is $(q+p)^n$.

1) $B(4,\frac{1}{3})$

$n=4,\ p=\frac{1}{3}$

Mean $=np=\frac{4}{3}$

2) Given $n=4,\ p=\frac{1}{3},\ q=\frac{2}{3}$

variance $=npq$

$=4\times\frac{1}{3}\times\frac{2}{3}=\frac{8}{9}.$

S.D $=\sqrt{\frac{8}{9}}=\frac{\sqrt{8}}{3}$.

8) Expected frequenceny function :-

In an expirement with (n) number of trails is repeated N times then the expected No. of success can be determined by

Expected frequency function $= N \cdot p(x)$.

Poision Distribution

$n \to \infty$, $p \to 0$, $q \to 1$ (Countable infinite).

Ex:-

P	q
0.4	0.6
0.3	0.7
0.2	0.8

$\boxed{np = \text{finite}}$ → This is called as $\boxed{\lambda \text{ lamda}}$

(largenvalue, small p value).

⇒ Small interval of time happening of probability towards 0.

⇒ poision distribution deals with rare events

⇒ Also called as distribution of rare events.

In binomial distribution, $n \to \infty$

$p \to 0$, $q \to 1$ and np = finite (λ or m), then binomial distribution is approximation to posion distribution.

poision distribution also called limiting case of Binomial distribution.

poision distribution also called A distribution of rare events (since $p \to 0$)

Definition: A discrete random variable 'x' said to be follow poision distribution with parameter (λ or m) then it's p.m.f is given by

$$\boxed{P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}} \; ; x = 0, 1, 2, \ldots \infty$$

$= 0$, otherwise
(Decimal values)

⇒ If a variable 'x' is follows poision distribution then it is denoted $\boxed{X \sim P(\lambda)}$ and 'x' is called poision variante

⇒ The parameter of posion distribution is λ so, that it is called as uniparametric distribution

$\boxed{\lambda \text{ is always positive only}}$

⇒ Mean of P.D $= \lambda$

⇒ Variance of P.D $= \lambda$

*** ⇒ $\boxed{\text{Mean} = \text{Variance} = \lambda}$

⇒ Moments:-

1^{st} raw moment, $\mu_1' = \lambda$

1^{st} central moment, $\mu_1 = 0$

2^{nd} central moment, $\mu_2 = \lambda$

3^{rd} central Moment, $\mu_3 = \lambda$

4^{th} central Moment, $\mu_4 = 3\lambda^2 + \lambda$

$\boxed{\text{Skewness}, \beta_1 = \frac{1}{\lambda}}$

Coefficient of Skewness, $\gamma_1 = \sqrt{\beta_1} = \frac{1}{\sqrt{\lambda}}$

Kurtosis, $\beta_2 = \frac{3\lambda^2 + \lambda}{\lambda^2} = 3 + \frac{1}{\lambda}$

Coefficient of kurtosis, $\gamma_2 = \beta_2 - 3 = \frac{1}{\lambda}$

Note:- poison distribution is always positively skewed distribution since $\boxed{\lambda > 0}$.

poison distribution is always leptokurtic distribution since $\boxed{\beta_2 > 3}$.

Mode:- case(i):- If λ is not an integer is not an integer then an integral part of value of λ will be the mode of poisson distribution.

Ex:- $\lambda = 5.35$ then mode $= 5$.

case(ii):- If λ is an integer then λ and $\lambda - 1$ are the modes of P.D.

∴ In this ~~case~~ P.D. is $\boxed{\text{bi-Modal.}}$

Ex: $\lambda = 4$ then mode $= 3$ and 4.

Addictive property:- If $\boxed{X \sim P(\lambda_1)}$ and $\boxed{Y \sim P(\lambda_2)}$ then $\boxed{X+Y \sim P(\lambda_1+\lambda_2)}$

Also $\boxed{X- \sim P(\lambda_1-\lambda_2)}$

Note: The p.m.f of possion distribution $\boxed{P(X=x) = \frac{e^{-\lambda}\cdot \lambda^{x}}{x!}}$

$\boxed{\therefore e = 2.7183}$

Ex:- $P(X=0) = \frac{e^{-\lambda}\cdot \lambda^{0}}{0!} = e^{-\lambda}.$

$P(X=1) = \frac{e^{-\lambda}\cdot \lambda^{1}}{1!} = \lambda \cdot e^{-\lambda}$

$P(X=2) = \frac{e^{-\lambda}\cdot \lambda^{2}}{2!} = e^{-\lambda}\cdot \frac{\lambda^2}{2}$

$P(X=3) = \frac{e^{-\lambda}\cdot \lambda^{3}}{3!} = e^{-\lambda}\cdot \frac{\lambda^3}{6}$

$\vdots$

$e^{-2} = \frac{1}{e^2} = \frac{1}{(2.7183)^2} = 0.13534$ [previous Question].

Uniform Distribution:

Let x be a discrete random variable and it is said to be follows uniform distribution with p.m.f is

$$P(X=x) = \frac{1}{n} \quad \forall\ x = 1, 2, 3, \cdots n$$

$$\boxed{\begin{aligned} \text{Mean} &= \frac{n+1}{2} \\ \text{variance} &= \frac{n^2-1}{12} \end{aligned}}$$

Ex:-

① An coin tossed one time

X:	H	T
P(x)	$\frac{1}{2}$	$\frac{1}{2}$

② A dice is thrown once

x	1	2	3	4	5	6
P(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

③ A set of number is selected to form a numbers 11, 13, 16, 20, 27, 29, 32

x	11	13	16	20	27	29	32
y	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Normal distribution:- Development by Gaussian

→ It is most importance continoues probability distribution.

→ It is also called as Gaussian distribution.

Definition:-

A continuous random varaibe x is said to follow normal distribution with parameters μ and σ^2 and its p.d.f is given by.

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; -\infty < x < \infty}$$

→ A continuous random variable follows normal distribution then it denoted by $\boxed{X \sim N(\mu, \sigma^2)}$ and x is called as Normal variate.

→ The parameters of normal distribution are $\boxed{\mu \text{ and } \sigma^2}$

∴ Normal distribution is $\boxed{\text{bi-parametric}}$

→ Mean of Normal distribution $= \mu$

→ Variance of Normal distribution $= \sigma^2$

→ Standard deviation of Normal distribution $= \sigma$

→ In Normal distribution Mean = Medial = Model = μ.

→ Mode of Normal distribution 'μ'. so that N.D is called as a distribution of single peak. (Max. freq)

→ In Normal distribution $Q_1 = \mu - 0.6745\,\sigma$

$Q_2 = \mu$

$Q_3 = \mu + 0.6745\,\sigma$

→ IN. N.D 6QD = 5MD = 5

→ $QD = \frac{2}{3} SD \qquad M = \frac{4}{5} \cdot \sigma$

$QD = 0.67\sigma \qquad MD = 0.8\sigma$.

→ In N.D = $\boxed{QD : MD : SD} = \boxed{\frac{2}{3} SD : \frac{4}{5} SD : SD}$

$= \frac{2}{3} : \frac{4}{5} : 1$

$\boxed{= 10 : 12 : 15}$

~~→ The shape of Normal distribution is bell shaped~~

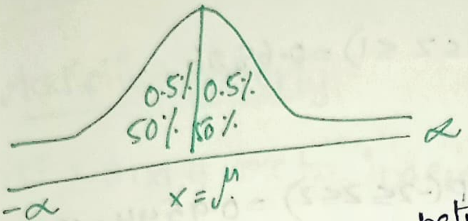
~~→ The Normal distribution probability curve is called probability Normal~~

~~→ The total area in the N.P curve distribution curve is one.~~

→ The shape of ND is <u>bell shaped</u>.

→ The ND curve is also called as Normal probability curve.

→ The total area in Normal probability curve is <u>One</u>.



$P(-\alpha < x < \alpha)$ = area between $-\alpha$ to $\alpha = 1$

$P(-\alpha < x < \mu)$ = area between $-\alpha$ to $\mu = 0.5$

$P(\mu < x < \alpha)$ = area between μ to $\alpha = 0.5$

<u>Standard Normal distribution</u>:-

→ In p.d.f of N.P $\boxed{\frac{x-\mu}{\alpha} = z}$ is called $\boxed{S.N.D}$

→ The probability density function S.N.D is

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} \times e^{-1/2(z)^2}}$$

∴ Mean of S.N.D = <u>0</u>

∴ Varaince of S.N.D = <u>1</u>

→ The <u>parameter of S.N.D</u> is $\boxed{0 \; \& \; 1}$

If $x \sim N(\mu, \sigma^2)$ then $z \sim N(0,1)$

$Q_1 = -0.6745$

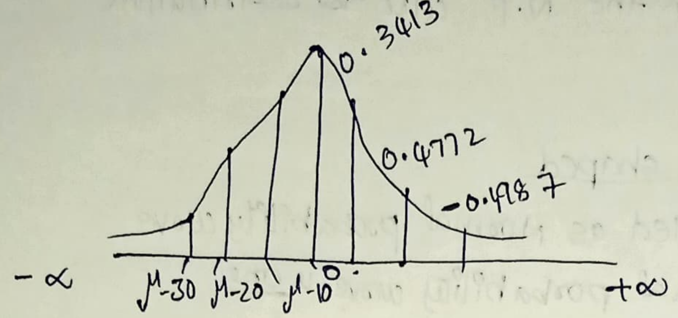
$Q_2 = 0$

$Q_3 = 0.6745$

$Qd = \frac{2}{3} sd = 0.675$

$md = \frac{4}{5} sd = 0.8$

Area property of Normal distribution:-



$\therefore$ Probability of $\mu-\sigma < x < \mu+\sigma) = P(-1 \le z \le 1) = 0.6826$

Area $= 68.2\%$

$\therefore$ Probability of $\mu - 2\sigma \le x \le \mu+2\sigma) = P(-2 \le z \le 2) = 0.9544 = 95.44\%$

$\therefore P(\mu-3\sigma \le z \le \mu+3\sigma) = 0.9973$; Area $= 99.73\%$.

$\therefore P(\mu - 1.96\sigma \le x \le \mu+1.96\sigma) = P(-1.96\sigma \le z \le 1.96) = 0.95$

$\boxed{\text{Area} = 95\%}$

$\therefore P(\mu - 2.58\sigma \le x \le \mu+2.58\sigma) = P(-2.58 \le z \le 2.58) = 0.99$

$\boxed{\text{Area} = 99\%}$

Finding probability by using by using normal probability curve:- (NIL)

Movements of Normal distribution:-

1) 1st raw movement $= \mu_1' = \mu$

2) 1st central movement $= \mu_1 = 0$

3) 2nd " " $=$ variance $= \mu_2 = \sigma^2$

4) 3rd " " $= \mu_3 = 0$

5) 4th " " $= \mu_4 = \sigma$

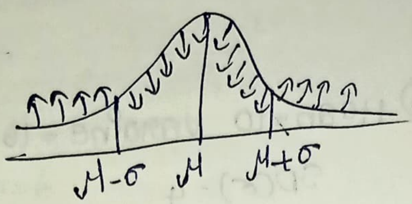
6) Skewness, $\beta_1 = 0$ (Normal distribution is symmetric).

7) Coefficient of Skewness $= \gamma_1 = \sqrt{\beta_1} = 0$.

8) kurtosis = $\beta_2 = 3$

9) Coefficient of kurtosis = $\gamma_2 = \beta_2 - 3 = 0$

10) ⇒ The points of ~~inflation~~ inflexion of Normal distribution are $\boxed{\mu \pm \sigma}$



Mode : μ (single peak).

Additive property:-

→ If x and y are two independent random variables.

$x \sim N(\mu, \sigma^2)$ and $y \sim N(\mu_2, \sigma_2)^2$

then $x+y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$ and

$x-y \sim N(\mu_1-\mu_2), \sigma_1^2+\sigma_2^2)$

variance formula.

$\boxed{V(x \pm y) = V(x) + V(y)}$

Normal	Standard Normal.
Pdf is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	Pdf is $f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-z^2}{2}}$
Parameters : μ and σ^2	parameters : 0 and 1.
$x \sim N(\mu, \sigma^2)$	$Z \sim N(0,1)$
Mean = μ	Mean = 0
Variance = σ^2	variance = 1
SD = σ	SD = 1
QD = 0.675 σ	QD = 0.675
MD = 0.800 σ	MD = 0.80
Point of inflexion = $\mu \pm \sigma$	point of inflexion ± 1