08-10.

Random Variables

Random Epesiment:- An explorement conducted without knowing | guessing the result in advance is called random experiment.

Ex:- 1) Tossing a coin

2) Rolling a dice

3) selecting a card form a pack of 52 cards, ectc, ...

Random variable :- It is also called as stochastic variable and it is a function. deifined on sample space associated with a random expiriement and assining a real value to each and every sample point of sample space.

usually random variable are denoted by x, y, z, ... etc.,

Ex:- 1) A coin tossed 2 times

x : No. of heads

R.V : x :- 0 1 2

2) Two dice are thrown simultaneously

x : No. of sixes

R.V. x : 0 1 2

Type of Random variables :

these are two types of random variables:-

1) Discrete random variable

2) continous random variable

Discrete random variable :

A random variable which assumes finite or countable infinite and always expressed as integers. is called as discrete random variables.

1) Tossing a coin
2) Rolling a dice

Continous random variables:

- A random variable which is defined on an interval says [a,b] and it assumes all possible values between the limits of an interval is called as continous random variable.

$\boxed{x \in (1,10)}$

Probability distribution: It is a systematic arrangement of the values of r.v and their probabilities.

coins = 3

x = No. of heads

$P(x) = \frac{1}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} \cdot \frac{1}{8}$

x = 0 1 2 3

Discrete probability Distribution: The probability distribution of discrete random variable is called discrete probability distribution.

Ex: If a coin tossed 3 times.

x: No. of heads

x:	0	1	2	3
P(x):	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Continous probability distribution: The probability distribution of continous random variables is called continous probability distribution.

Ex:

weight (x)	40-45	45-50	50-55	55-60
No. of Persons	7	12	8	3
P(x)	7/30	12/30	8/30	3/30

A Coin tossed 20 times

X : No. of heads

X : 0 1 2 20

$\frac{1}{2^{20}}$ $\frac{1}{2^{20}}$

∴ If 14 then $P(X=14) = \frac{{}^{20}C_{14}}{2^{20}} \left[\frac{{}^{n}C_{r}}{2^{n}}\right]$.

Probability Function : A function which generate the probabilities to all the values taken by a random values.

Probability Mass Function :- The probability or relative frequency function of discrete random variable is called probability Mass function.

for pmf, (i) $P(x) \geq 0$

(ii) $\Sigma P(x) = 1$

ex: The pmf of binomial distribution
(i)

$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$.

(ii) The pmf of poisson distribution

$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$

Probability density function (Pdf):

The probability or relative frequency function of continous random variable is probability density function.

for pdf = (i) $f(x) \geq 0$ $\Sigma P(x) = 1$

(ii) $\int_a^b f(x)\,dx = 1$

Ex(i) The pdf of Normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Mathematical Expectations:

If x is a random variable then its expectations is denoted by $E(X)$..

Mathematical Expectations of random variable is defined as the sum of products of values of random variables and their corresponding probabilities.

In other words, Mathematical expectation can be defined as "the mean of probability distribution"

For a discrete probability distribution :-

$$\begin{array}{l|l} X : x_1 \quad x_2 \quad x_3 \;\cdots\; x_n & E(PX) = 1 \\ P(X) : P_1 \quad P_2 \quad P_3 \;\cdots\; P_n & P_1 + P_2 + P_n = 1 \end{array}$$

$$E(X) = x_1P_1 + x_2P_2 + x_3P_3 + \cdots + x_nP_n$$

$$\boxed{E(X) = \sum x\, p(x)}$$

$$E(x)^2 = x_1^2P_1 + x_2^2P_2 + x_3^2P_3 + \cdots + x_n^2P_n$$

$$\boxed{E(x^2) = \sum x^2\, p(x)}$$

For continous probability distribution :-

$$E(x) = \int_a^b x\, f(x)\, dx \qquad \boxed{\sum x\, P(x)}$$

$$E(x^2) = \int_a^b x^2 f(x) dx.$$

$$\Rightarrow \boxed{\text{variance}(x) = E(x^2) - [E(x)]^2}$$

For discrete random variable:-

$$\text{variance}(x) = \Sigma x^2 \cdot p(x) - [\Sigma x\, p(x)]^2$$

For continuous random variables:-

$$\text{variance}(x) = \int_a^b x^2 f(x)\,dx - \left[\int_a^b x f(x)\,dx\right]^2$$

Properties of Mathematical Expectations:-

1) E(constant) = constant

2) $E(kx) = kE(x)$

3) $E(x+y) = E(x) + E(y)$

4) $E(x-y) = E(x) - E(y)$

5) For independent of random variables
$E(xy) = E(x) \cdot E(y)$

6) If Expectations of x is positive then the game is favour to player.

7) If Expectations of x is negative then the game is against to player.

8) If Expectations of x is zero then the game is said to be fair game.

Properties of variance:

1) var(constant) = 0

2) $\text{var}(ax+b) = a^2\,\text{var}(x)$

3) $\text{var}(ax+by) = a^2\,\text{var}(x) + b^2\,\text{var}(x) + 2ab\,\text{cov}(x,y)$.

4) $\text{var}(ax-by) = a^2\,\text{var}(x) + b^2\,\text{var}(y) - 2ab\,\text{cov}(x,y)$

5) for independent variables,
$\text{var}(ax \pm by) = a^2\,\text{var}(x) + b^2\,\text{var}(y)$.