3 Graph of Linear Inequalities

1.0 LINEAR INEQUALITIES IN ONE VARIABLE AND SOLUTION SPACE

1.1 Any linear function that involves an inequality sign (*viz.* $>$ or $<$ or $\geq$ or $\leq$) is a linear inequality.

1.2 It may be of one variable or may be of more than one variables. The simplest examples of linear equation of one variable are $x > 0$ or $x \leq 0$.

1.3 They can be represented on a number line as given below:

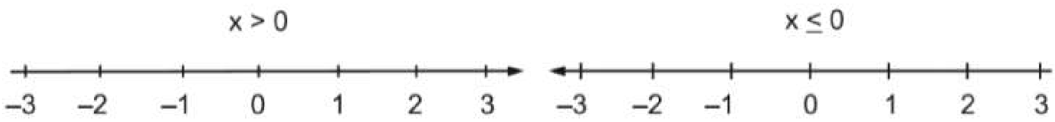


Fig. 3.1

1.4 The values of the variables that satisfy an inequality are called the *solution space*, and is abbreviated as S.S. The solution spaces for (i) $x > 0$, (ii) $x \leq 0$ are shaded in the above diagrams, by using deep lines.

2.0 LINEAR INEQUALITIES IN TWO VARIABLES

2.1 An expression of the types $ax + by \leq c$ or $ax + by \geq c$ or $ax + by < c$ or $ax + by < c$, where a, b and c are real numbers, is a linear inequality in two variables x and y.

2.2 An ordered pair (x_1, y_1) is said to be a solution of $ax + by \leq c$ if $ax_1 + by_1 \leq c$.

2.3 The set of all such solutions is called the solution set of the linear inequality $ax + by \leq c$.

2.4 An equality is sometimes known as inequation.

3.0 GRAPH OF AN INEQUALITY

Practical Steps Involved in Draining a Graph of Inequality

Step 1 → Write down the inequality as an equality.

Step 2 → Form a table of values of the equation $ax + by = c$.

Step 3 → Plot the points of the table obtained in Step 2 in XOY plane and join them.

(a) *If the inequality is* $\leq$ or $\geq$, *the line drawn should be thick.*

(b) *If the inequality is* $<$ *or* $>$, *the line drawn should be dotted.*

(c) This line will divide XOY plane in two regions.

Step 4 → Determine the region in which the given inequation is satisfied. This is done by taking any arbitrary point (x_1, y_1) in one of the regions. Generally, (x_1, y_1) is taken as (0, 0). Substitute its coordinates in the given inequation.

(i) *If the given inequation is satisfied by* (x_1, y_1)*, then the region containing this point is the desired region.*

(ii) *If the point* (x_1, y_1) *does not satisfy the given inequation, then the region not containing this point is the desired region.*

Step 5 → Shade the desired region.

Note: (i) *Thick line is a part of the solution set but dotted line is not.*

(ii) The suitable point *T* (also called **Testing Point**) is generally taken be the origin (0, 0).

(iii) For more than one inequations repeat the above procedure, for each of these.

ILLUSTRATION 1

Solve $y < 3$ graphically.

SOLUTION The graph of equation $y = 3$ is a horizontal line parallel to *x*-axis.

Putting $y = 0$ in the given inequation, we see that $0 < 3$, which is true.

Thus, the solution region is the shaded region below the line $y = 3$ containing the origin. Hence, every point below the line (excluding all points on the line) determines the solution of the given inequation.

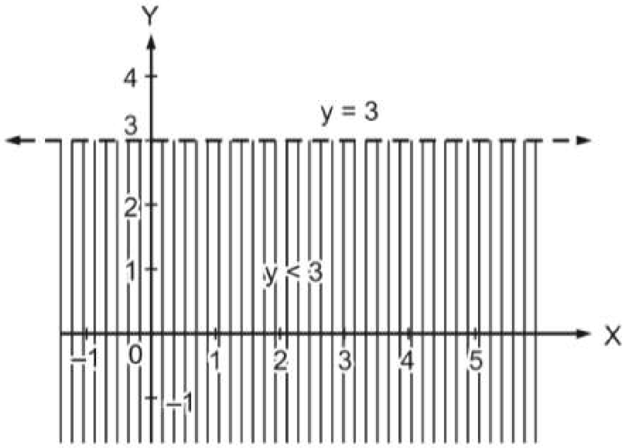


Fig. 3.2

ILLUSTRATION 2

Solve $2x - 3 \geq 0$ graphically.

SOLUTION In two dimensional plane, we note that

$$2x - 3 = 0 \text{ or } x = 3/2$$

represents a line parallel to *y*-axis. Each point of the graph is 3/2 units to the right of *y*-axis. Putting $x = 0$ in the given inequation, we see that $2(0) - 3 \geq 0$ or $-3 \geq 0$, which is false.

Thus, the solution region is the shaded on the right hand side of the line $x = 3/2$ which does not contain the point (0, 0).

Hence, every point on the right hand side of the line (including all points on the line determines the solution of the given inequation.

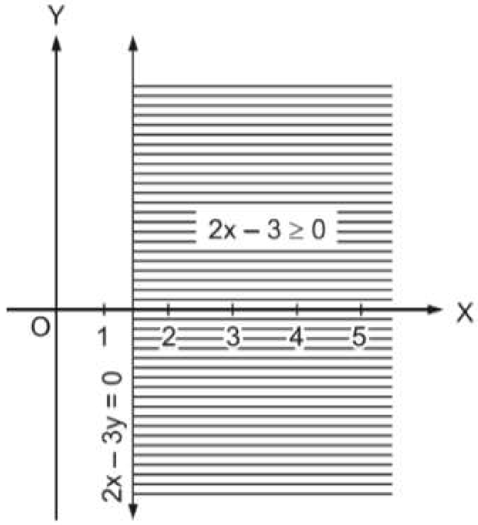


Fig. 3.3

ILLUSTRATION 3

Draw the graph of the inequation : $2x + y \geq 10$.

SOLUTION We shall first draw the graph of the equation

$$2x + y = 10 \text{ or } y = 10 - 2x$$

Table for $y = 10 - 2x$

x	2	4	5
y	6	2	0

Step 1 → We plot the points (2, 6), (4, 2) and (5, 0) on the graph paper and join them to get the line AC.

Step 2 → We notice that the line AC: $2x + y = 10$ divides the region into two half planes, *viz*., I and II.

Step 3 → Let the testing point be (0, 0). Substituting (0, 0) in the inequality $2x + y > 10$, we have $0 + 0 > 10 \Rightarrow 0 > 10$, **which is absurd.** Thus **the desired region will not contain (0, 0)**.

Step 4 → The desired region is region II, including the line *AD*.

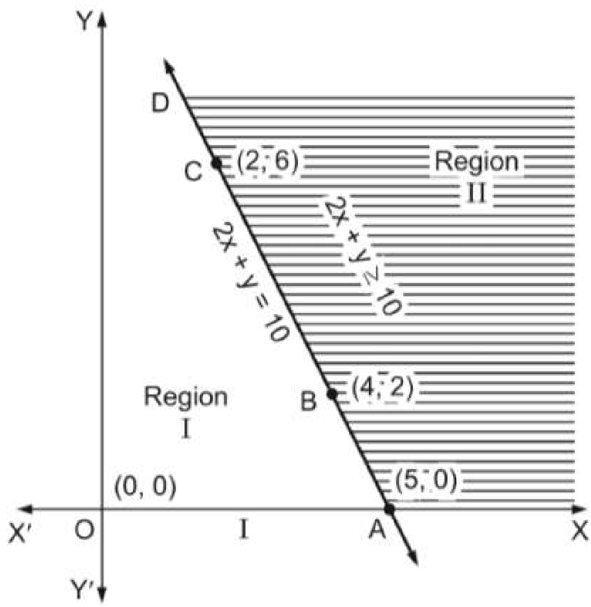


Fig. 3.4

ILLUSTRATION 4

Sketch the graph of the linear inequality: $3x + 4y < 24$.

SOLUTION

Table for $3x + 2y = 24$

x	8	0
y	0	12

Step 1 → We plot the points A (8, 0) and B (0, 12) on the graph and join them.

Step 2 → We have shown: AB as dotted line in the figure as we have strict inequality. Now the line, AB, divides the plane into two regions I and II as shown in the Fig.

Step 3 → Consider the point (0, 0) as the testing point. This lies in *Region I.*

Substituting (0, 0) in the inequality, we get

$3 \times 0 + 4 \times 0 < 24$

$\Rightarrow 0 < 24$

which is true. Thus the desired region will contain (0, 0)

Step 4 → The shaded *Region I* is the graph of the inequation.

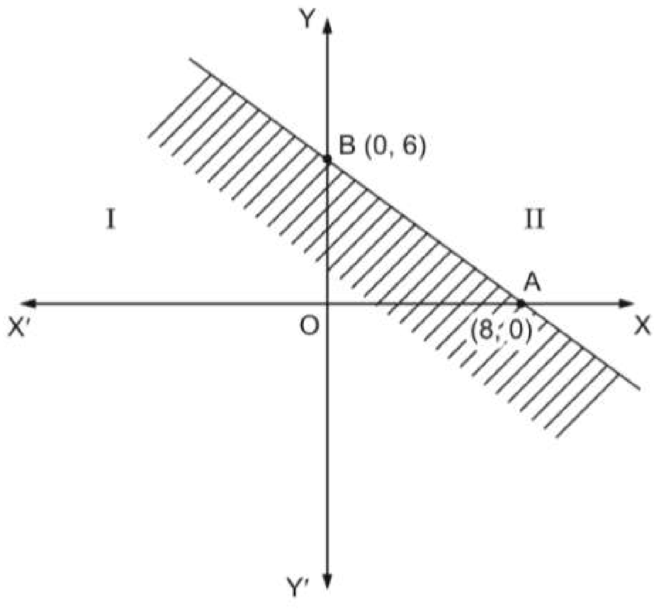


Fig. 3.5

ILLUSTRATION 5

Draw the graph of the inequations

$2x + 3y \leq 6$ and $5x + 3y \leq 15$; $x, y \geq 0$.

SOLUTION Here we shall first draw the graphs of equations

$$2x + 3y = 6 \quad \text{and } 5x + 3y = 15.$$

(i) Graph of $2x + 3y = 6$ or $y = (6 - 2x)/3$

Table for $y = (6 - 2x)/3$

x	0	3
y	2	0

$\therefore$ (0, 2) and (3, 0) are the points on line $2x + 3y = 6$. Joining them by a free hand, we get the required graph as the line *CD*. Obviously the point (0, 0) **satisfies the inequality $2x + 2y \leq 6$.**

(ii) **Graph of $5x + 3y = 15$** or $y = (15 - 5x)/3$

x	3	0
y	0	5

Plot the points (0, 5) and (3, 0) and join them to get *AB* as the graph of the equation $5x + 3y = 15$.

Obviously, the point (0, 0) satisfies the inequality $5x + 3x \leq 15$.

The region satisfying both the inequalities simultaneously, *viz.*, $2x + 3y \leq 6$ and $5x + 3y \leq 15$, is the cross-hatched portion in Fig given below:

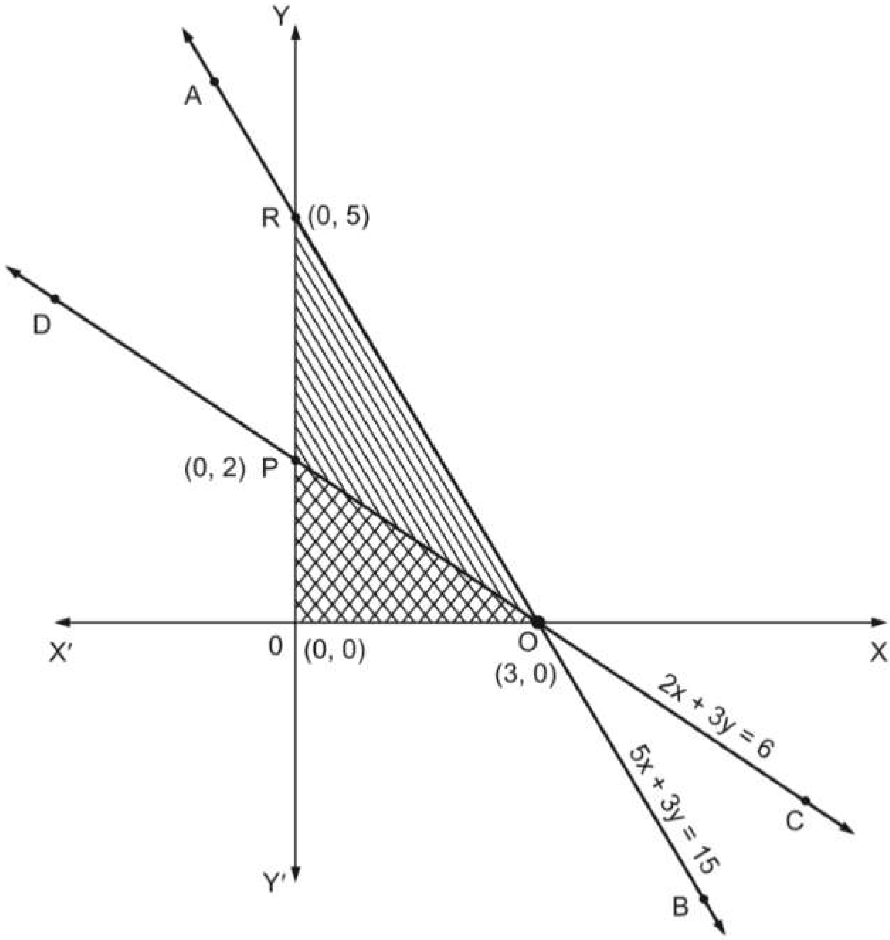


Fig. 3.6

ILLUSTRATION 6

Draw the graph of the following linear inequalities:

$x + 4y \leq 12,\ 2x + 5y \leq 20,\ y \geq 0,\ 1 \leq x \leq 8.$

Indicate the common region also.

SOLUTION Let us first draw the straight lines

$$x + 4y = 12 \text{ and } 2x + 5y = 20$$

Table for $x = 4y = 12$

x	0	12
y	3	0

Table for $2x + 5y = 20$

x	0	10
y	4	0

Table region for the inequality is shown by pointing arrows in the graph.

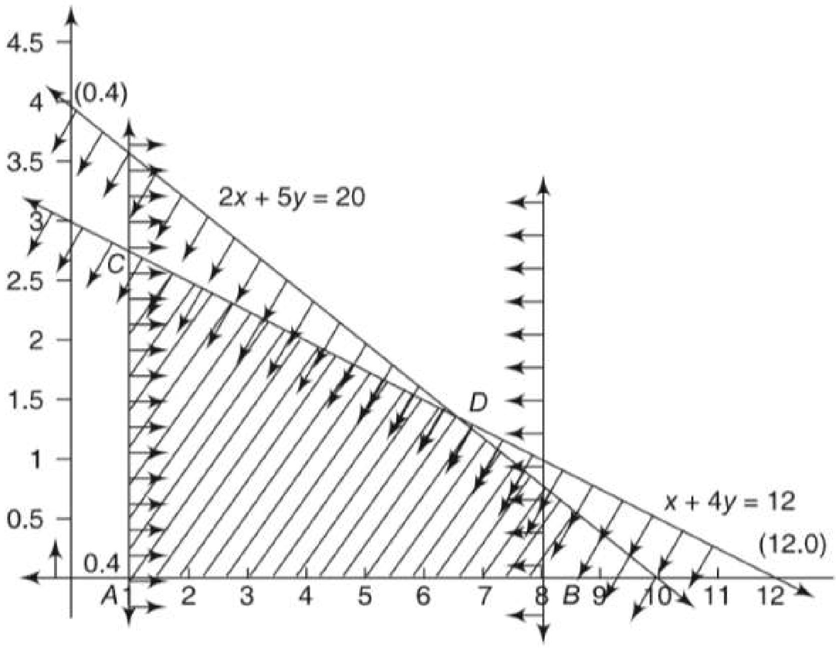


Fig. 3.7

$y = 0$ indicates x-axis and $y \geq 0$ indicates region above x-axis as shown in the graph by pointing arrows.

$x = 1$ indicates line parallel to y-axis at distance 1.

$x = 8$ indicates line parallel to y-axis at distance 8.

$1 \leq x \leq 8$ indicates region between the lines $x = 1$ and $x = 8$ as marked in the graph by pointing arrows.

The common region is the polygon ABCDE in the graph and it is marked by parallel lines.

ILLUSTRATION 7

A company manufactures two types of cloth using three different colours of wool. One yard length of type *A* cloth requires 100 gm of red wool, 125 gm of green wool and 75 gm of yellow wool. One yard length of type *B* cloth requires 125 gm of red wool, 50 gm of green wool and 200 gm of yellow wool. The wool available for manufactures is 25 kg of red as well as green wool and 30 kg of yellow wool.

Express the above in terms of linear inequalities, draw the graph of these inequalities and indicate the common region.

SOLUTION Let us assume that x yard length of cloth of type *A* and y yard length of cloth of type *B* have been manufactured. The given conditions allow us to write:

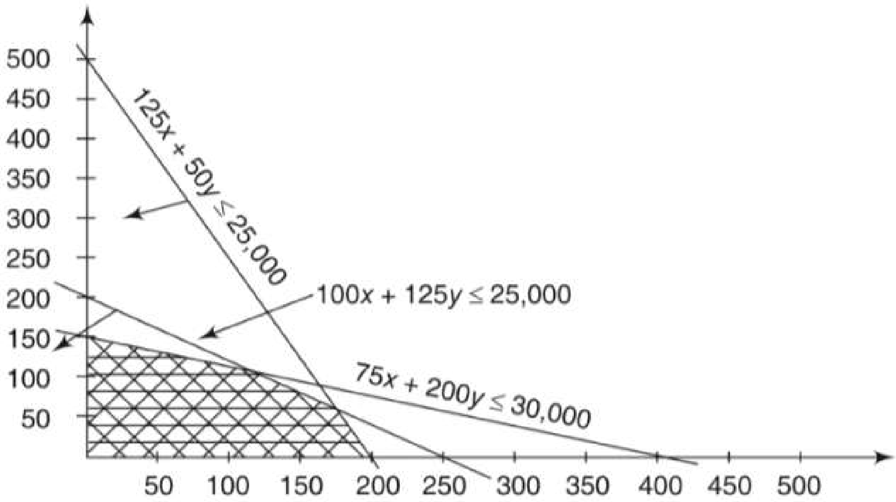
$$100x + 125y \leq 25{,}000 \text{ i.e. } 4x + 5y \leq 1{,}000 \Rightarrow \frac{x}{250} + \frac{y}{200} \leq 1$$

$$1125x + 50y \leq 25{,}000 \qquad 5x + 2y \leq 1{,}000 \Rightarrow \frac{x}{200} + \frac{y}{500} \leq 1$$

$$75x + 125y \leq 30{,}000 \qquad 3x + 8y \leq 1{,}200 \Rightarrow \frac{x}{400} + \frac{y}{150} \leq 1$$

$$x \geq 0,\ y \geq 0 \qquad x \geq 0,\ y \geq 0$$

Graph of linear inequalities is drawn below.



The common region is shown cross-hatched in the figure therein.

ILLUSTRATION 8

A company manufacturing two types of products *A* and *B*. Production is limited 80 units of product *A* and 60 units of product *B*. Production of each of these products require 5 units and 6 units of electronic components respectively. The electronic components are supplied by another manufacturer and the supply is limited to 600 units per day. The company has 160 employees; i.e. the labour supply amounts of 160 man-days. The production of one unit of product *A* requires one man-day of labour and one unit of product *B* requires two man-days of labour.
Express this using linear inequalities. Draw graph of these inequalities and then mark the feasible region.

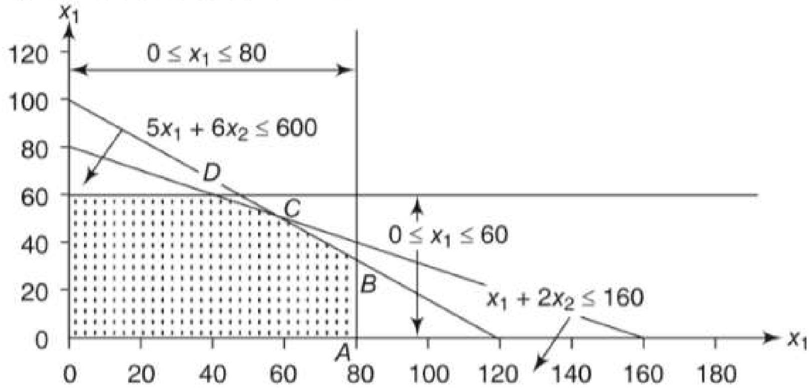
SOLUTION Let x_1 and x_2 be the quantities produced for Products A and B respectively.

X	A	B	Supply
Number of electronic components required (per unit)	5	6	600
Number of Man-days required (per unit)	1	2	160
Production Limit	80	60	

Required inequalities are given by

$$0 \le x_1 \le 80, \quad 0 \le x_2 \le 60$$
$$5x_1 + 6x_2 \le 600$$
$$x_1 + 2x_2 \le 160$$

Graph of the inequalities is drawn below:



The feasible region OABCDE is shaded.

MULTIPLE CHOICE QUESTIONS

1. An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people. x and y can be related by the inequality
   (a) $x + y \neq 9$ (b) $x + y \le 9$ (c) $x + y \ge 9$ (d) none of these

2. On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as
   (a) $5x + 3y \le 30$ (b) $5x + 3y > 30$ (c) $5x + 3y \ge 30$ (d) none of these

3. A firm makes two types of products. Type A and type B. The profit on product A is Rs 20 each and that on product B is Rs 30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

   The constraints can be formulated taking x_1 = number of units of A and x_2 = number of unit of B as

   (a) $x_1 + x_2 < 12$
   $5x_1 + 2x_2 < 50$
   $2x_1 + 6x_2 < 60$
   $x_1 \ge 0, x_2 \ge 0$

   (b) $3x_1 + 3x_2 \ge 36$
   $5x_1 + 2x_2 \le 50$
   $2x_1 + 6x_2 \ge 60$
   $x_1 \ge 0, x_2 \ge 0$

(c) $3x_1 + 3x_2 \leq 36$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \leq 60$
$x_1 \geq 0, x_2 \geq 0$

(d) none of these

4. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg. of each food is shown below:

	A	B	C	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II the situation can be expressed as

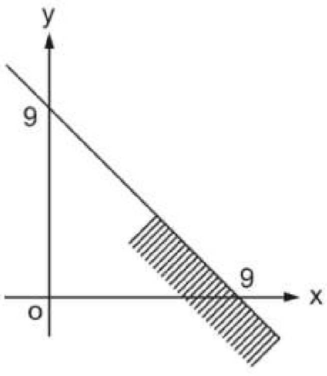
(a) $2x + y \leq 9$
$x + y \leq 7$
$x + 2y \leq 10$
$2x + 3y \leq 12$
$x > 0, y > 0$

(b) $2x + y \geq 30$
$x + y \leq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

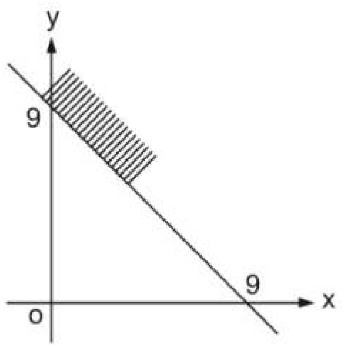
(c) $2x + y \geq 9$
$x + y \leq 7$
$x + y \leq 10$
$x + 3y \geq 12$

(d) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$2x + 3y \geq 12$
$x \geq 0, y \geq 0$

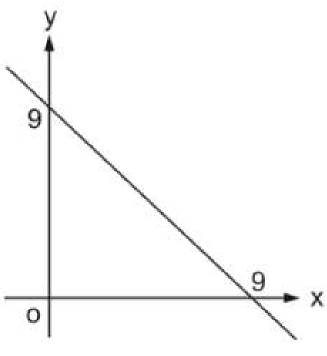
5. The graph to express the inequality $x + y \leq 9$ is



(a)



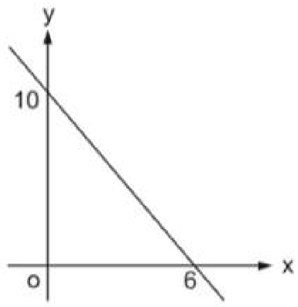
(b)

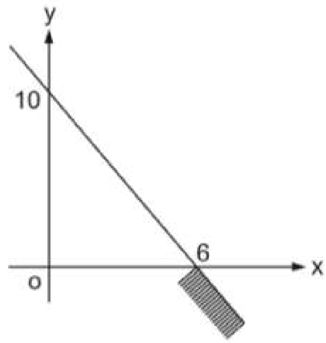


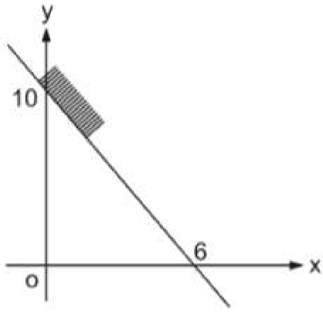
(c)

(d) None of these

6. The graph to express the inequality $5x + 3y \geq 30$ is

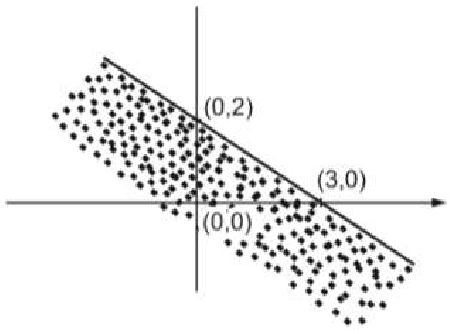
(a) 

(b) 

(c) 

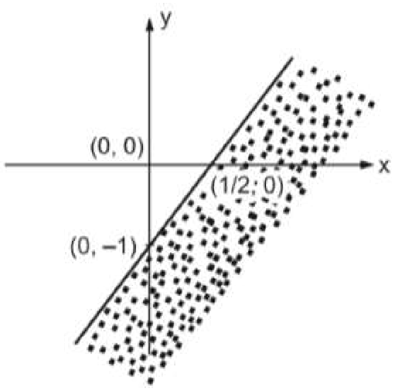
(d) None of these

7. The following region is represented by



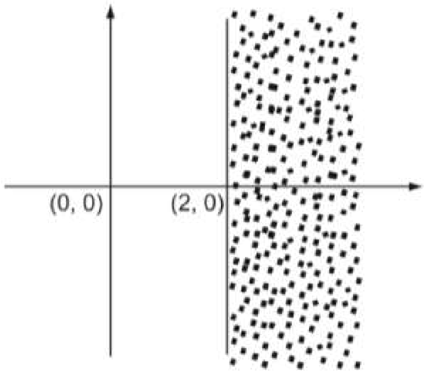
(a) $2x + 3y \leq 6$ (b) $2x + 3y \geq 6$ (c) $2x + 3y = 6$ (d) $2x + 4y \leq 6$

8. The following region is represented by



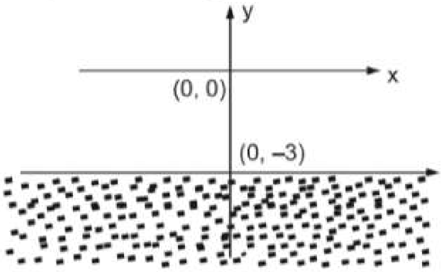
(a) $2x + y \geq 1$ (b) $2x - y = 1$ (c) $2x - y \geq 1$ (d) $2x - y \leq 1$

9. The following region is represented by



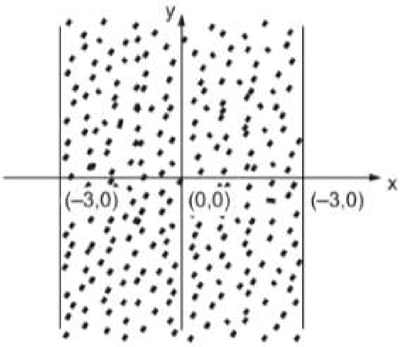
(a) $x \leq 2$ (b) $x = 2$ (c) $y \geq 2$ (d) $x \geq 2$

10. The following region is represented by



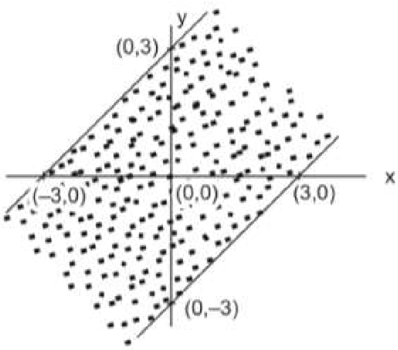
(a) $y \leq -3$ (b) $x \leq -3$ (c) $x \geq -3$ (d) $y \geq -3$

11. The following region is represented by



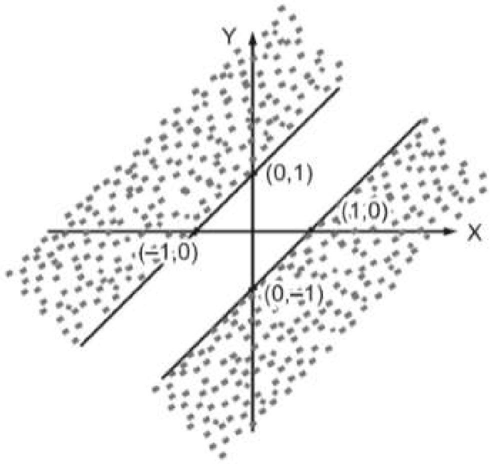
(a) $|x| = 3$ (b) $|y| \leq 3$ (c) $|y| \geq 3$ (d) $|x| \leq 3$

12. The following region is represented by



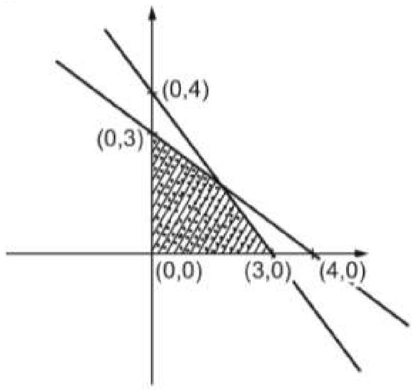
(a) $|y - x| = 3$ (b) $|y - x| \leq 3$ (c) $|y - x| \geq 3$ (d) $|x + y| \leq 3$

13. The following region is represented by



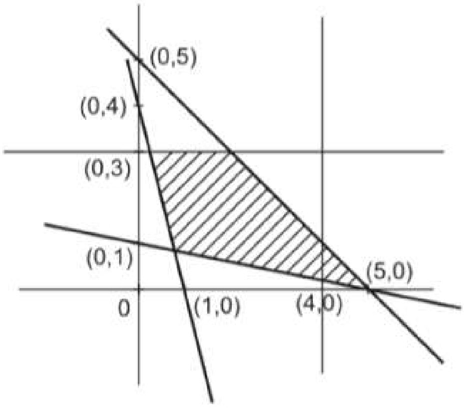
(a) $|x - y| \leq 1$ (b) $|x + y| \geq 1$ (c) $|x - y| \geq 1$ (d) $|x + y| \leq 1$

14. The following shaded region is the solution set of the linear inequations



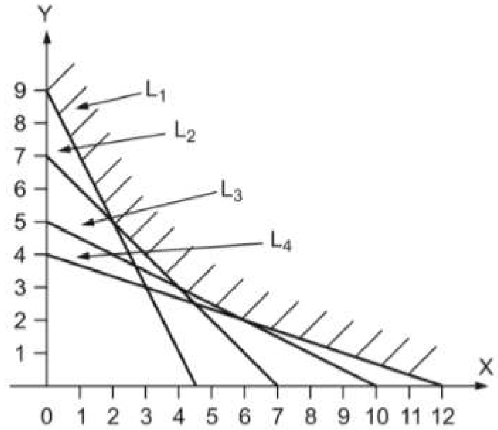
(a) $3x + 4y \geq 12$, $y + 3x \geq 3$, $x \geq 0$, $y \geq 0$
(b) $3x + 4y \leq 12$, $4x + 3y \leq 12$, $x \geq 0$, $y \geq 0$
(c) $3x + 4y \leq 12$, $4x + 3y \geq 12$, $x \geq 0$, $y \geq 0$
(d) $3x + 4y = 12$, $4x + 3y = 12$, $x \geq 0$, $y \geq 0$

15. The following shaded region is the solution set of the linear inequations



(a) $x + y \geq 5$, $4x + y \geq 4$, $x + 5y \leq 5$
(b) $x \leq 4$, $y \leq 3$, $x + y \leq 5$, $4x + y \geq 4$
(c) $x + y \leq 5$, $4x + y \geq 4$, $x + 5y \geq 5$, $y \leq 3$
(d) $x + y \leq 5$, $4x + y \leq 4$, $x + 5y \leq 5$

16.



$L_1 : 2x + y = 9,\ L_2 : x + y = 7,\ L_3 : x + 2y = 10,\ L_4 : x + 3y = 12$

The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

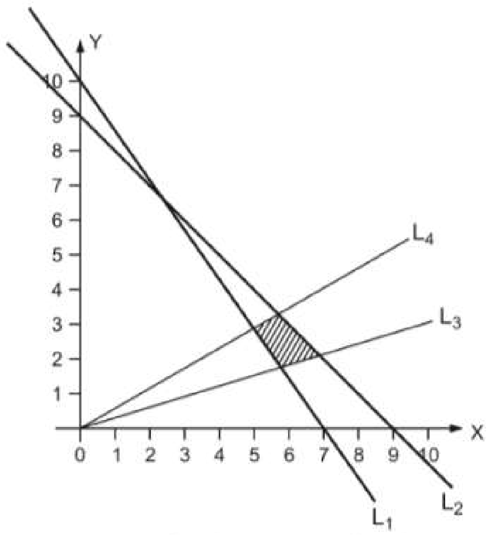
(a) $2x + y \leq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

(b) $2x + y \geq 9$
$x + y \leq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

(c) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$
$x \geq 0,\ y \geq 0$

(d) none of these

17.



$L_1 : 5x + 3y = 30,\ L_2 : x + y = 9,\ L_3 : y = x/3,\ L_4 : y = x/2$

The common region (shaded part) shown in the diagram refers to

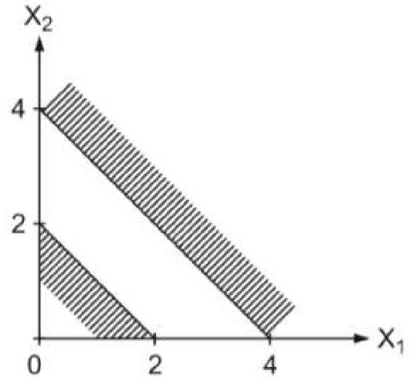
(a) $5x + 3y \leq 30$
$x + y \leq 9$
$y \leq 1/5x$
$y \leq x/2$

(b) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq x/3$
$y \leq x/2$
$x \geq 0,\ y \geq 0$

(c) $5x + 3y \geq 30$
$x + y \geq 9$
$y \leq x/3$
$y \geq x/2$
$x \geq 0, y \geq 0$

(d) $5x + 3y > 30$
$x + y < 9$
$y \geq 9$
$y \leq x/2$
$x \geq 0, y \geq 0$

18. The region indicated by the shading in the graph is expressed by inequalities



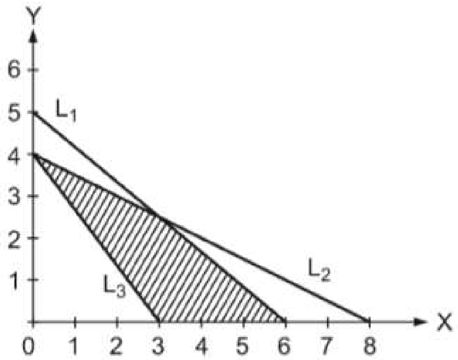
(a) $x_1 + x_2 \leq 2$
$2x_1 + 2x_2 \geq 8$
$x_1 \geq 0, x_2 \geq 0$

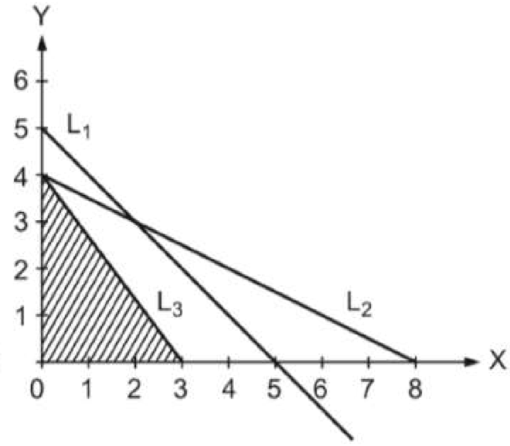
(b) $x_1 + x_2 \leq 2$
$x_1 + x_2 \leq 4$

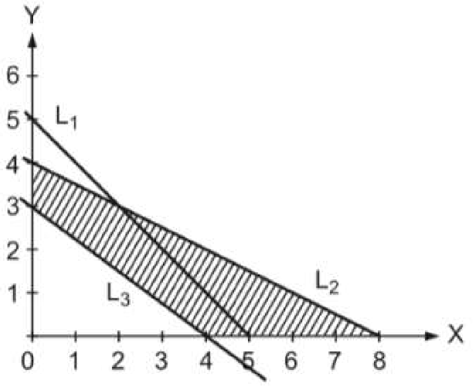
(c) $x_1 + x_2 \geq 2$
$2x_1 + 2x_2 \geq 8$

(d) $x_1 + x_2 \leq 2$
$2x_1 + 2x_2 > 8$

19. The common region satisfying the set of inequalities $x \geq 0$, $y \geq 0$, L_1: $x + y \leq 5$, L_2: $x + 2y \leq 8$ and L_3: $4x + 3y \geq 12$ is indicated by

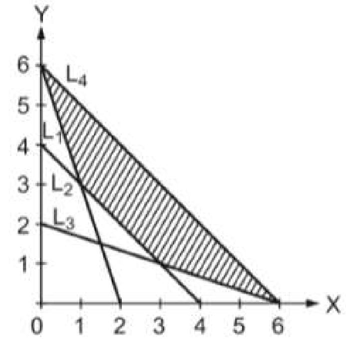
(a)


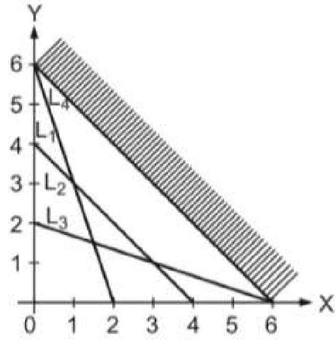
(b)


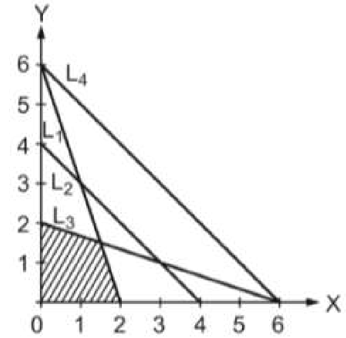
(c)


(d) none of these

20. The common region satisfied by the inequalities L_1: $3x + y \geq 6$, L_2: $x + y \geq 4$, L_3: $x + 3y \geq 6$, and L_4: $x + y \leq 6$ is indicated by

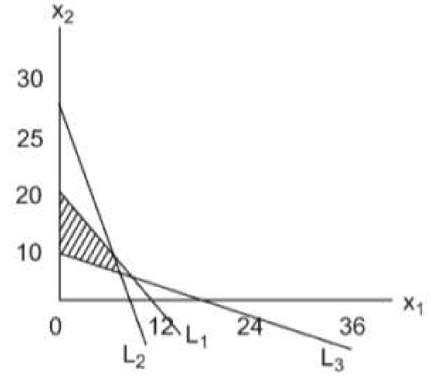
(a) 

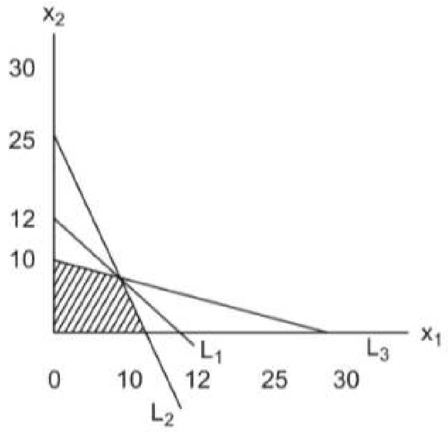
(b) 

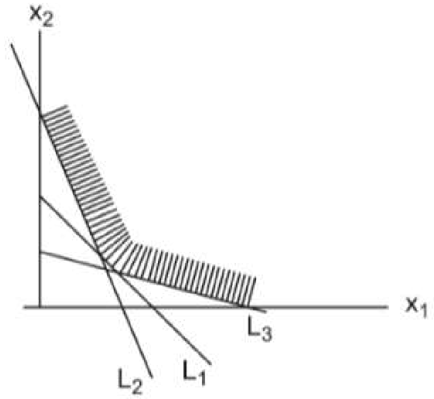
(c) 

(d) none of these

21. The set of inequalities L_1: $x_1 + x_{12} \leq 12$, L_2: $5x_1 + 2x_2 \leq 50$, L_3: $x_1 + 3x_2 \leq 30$, $x_1 \geq 0$, and $x_2 \geq 0$ is represented by

(a) 

(b) 

(c) 

(d) none of these

22. Two machines (I and II) produce two grades of plywood, grade A and grade B. In one hour of operation machine I produces two units of grade A and one unit of grade B, while machine II, in one hour of operation produces three units of grade A and four units of grade B. The machines are required to meet a production schedule of at least fourteen units of grade A and twelve units of grade B. The constraints can be formulated taking x = number of hours required on machine I and y = number of hours required on machine II

(a) $2x + 3y \leq 14$
$x + 4y \leq 12$
$x \geq 0$ and $y \geq 0$

(b) $2x + 3y \geq 14$
$x + 4y \geq 12$
$x \geq 0$ and $y \geq 0$

(c) $2x + 3y \geq 14$
$x + 4y > 12$
$x \geq 0$ and $y \geq 0$

(d) $2x + 3y \geq 14$
$x + 4y \geq 12$
$x \leq 0$ and $y \leq 0$

ANSWERS TO MCQ

1. (b)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (c)
9. (d)	**10.** (a)	**11.** (d)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (c)
17. (b)	**18.** (a)	**19.** (a)	**20.** (a)	**21.** (b)	**22.** (b)		