# Sequence And Series

* Formula E list :-

## Arithmetic Progression A.P.

**1. Common difference, d**

L → R "+d"

L ← R "−d"

(i) $d = T_2 - T_1 = T_3 - T_2 = \dots$

(ii) $d = T_n - T_{n-1}$ for $n \geq 2$

**2. $n^{th}$ term, $T_n$**

(i) $T_n = a + (n-1)d$ (from left to right)

(ii) $T_n = l - (n-1)d$ (from right to left)

**3. Assumption of terms in A.P. :-**

A] General A.P :-

$a, a+d, a+2d, \dots, l-2d, l-d, l$

B] Particular A.P :-

(sums of terms given)

3 terms :- $a-d, a, a+d$

4 terms :- $a-3d, a-d, a, a+d, a+3d$

5 terms :- $a-2d, a-d, a, a+d, a+2d$

## Geometric Progression G.P.

**2. Common Ratio, $r$ ($r$ =**

L → R "×"

L ← R "÷"

(i) $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \dots$

(ii) $r = \frac{T_n}{T_{n-1}}$ for $n \geq 2$

**2. $n^{th}$ Term, $T_n$**

(i) $T_n = a \cdot (r)^{n-1}$ (from left to right)

(ii) $T_n = \frac{l}{r^{n-1}}$ (from right to left)

$T_n = S_n - S_{n-1}$, for $n \geq 2$ (common for

**3. Assumption of terms in G.P.**

A] General G.P :-

$a, ar, ar^2, \dots, \frac{l}{r^2}, \frac{l}{r}, l$

B] Particular G.P.

(Product of terms given)

3 terms :- $\frac{a}{r}, a, ar$

4 terms :- $\frac{a}{r^3}, \frac{a}{r}, a, ar, ar^3$

5 terms :- $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

| Arithmetic Progression A.P. | Geomatric progression G.P. |
|---|---|
| $a-3d$, $a-2d$, $a-d$, $a$, $a+d$, $a+2d$, $a+3d$ | $\frac{a}{r^3}$, $\frac{a}{r^2}$, $\frac{a}{r}$, $a$, $ar$, $ar^2$, $ar^3$ |

| | A.P. | | G.P. |
|---|---|---|---|
| 3 terms | ✓ ✓ ✓ | 3 terms | ✓ ✓ ✓ |
| 4 terms | ✓ – ✓ – ✓ – ✓ | 4 term | ✓ – ✓ – ✓ – ✓ |
| 5 terms | ✓✓ ✓ ✓✓ | 5 terms | ✓✓ ✓ ✓✓ |

**Remark** :- (for particular A.P)

1. (i) sum of 3 terms $= 3a$
   (ii) sum of 4 terms $= 4a$
   (iii) sum of 5 terms $= 5a$

2. (i) Sum of Squares 3 terms
   $= 3a^2 + 2d^2$
   (ii) Sum of Squares 4 terms
   $= 4a^2 + 20d^2$
   (iii) sum of Squares 5 terms
   $= 5a^2 + 10d^2$

## Arithmatic Progression (A.P.)

### 4. Sum of terms :

**A] For finite A.P. :-**

(i) $S_n = \frac{n}{2}\{2a+(n-1)d\}$

(ii) $S_n = \frac{n}{2}\{a+l\}$

when 'n' is involved

(iii) $S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

$a \rightarrow$ F.T.

$b \rightarrow$ S.T.

$c \rightarrow l \rightarrow$ L.T.

(iv) $S_n = \frac{l+a}{2} + \frac{l^2-a^2}{2(b-a)}$

when 'n' is not involved

**B] Infinite A.P. :-**

Not In Syllabus

## Geomatric Progression (G.P.)

### 4. Sum of terms :

**A] For finite G.P. :-**

(i) $S_n = \frac{a(r^n-1)}{r-1}$ ; $r > 1$

(ii) $S_n = \frac{a(1-r^n)}{1-r}$ ; $r < 1$

(iii) $S_n = n \times a$ ; $r = b$

when n is involved

(iv) $S_n = \frac{lr-a}{r-1}$ ; $r > 1$

(v) $S_n = \frac{a-lr}{1-r}$ ; $r < 1$

when n is not involved

**B] Infinite G.P :-**

(i) $S_\infty \doteq \frac{a}{1-r}$ , $|r| < 1$

nearly approaching "0"

(ii) $S_\infty = \infty$ , otherwise

not defined

5. Insertion of A.M. / A.M's

A] Inserting only one A.M. between a and b

$$A.M. = \frac{a+b}{2}$$

B] Inserting more than one A.M's between a and b.

(i) Find $d = \frac{b-a}{n+1}$

$a \longrightarrow$ F.T.

$b \longrightarrow$ L.T.

$n \longrightarrow$ No. of F.I.B

(ii) Left to right, keep on adding "d" to fill all F.I.B.

5. Insertion of G.M. / G.M.'s

A] Inserting only one G.M. between a and b

$$G.M. = \sqrt{a \times b}$$

B] Inserting more than one G.M's between a and b

(i) Find $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$a \longrightarrow$ F.T.

$b \longrightarrow$ L.T.

$n \longrightarrow$ No. of F.I.B

(ii) Left to right, keep on multiplying "r" to fill all the F.I.B.

* Summation of series :-

1. Sum of first 'n' natural numbers $= \frac{n(n+1)}{2}$

ie $1+2+3+ \cdots + n = \frac{n(n+1)}{2}$

ie $\sum_{r=10}^{n} r = \frac{n(n+1)}{2}$

2. Sum of squares of first 'n' natural numbers :-

$= \frac{n(n+1)(2n+1)}{6}$

ie $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

ie $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

3. Sum of cubes of first 'n' natural numbers

$= \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2 \cdot (n+1)^2}{4}$

ie $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2 \cdot (n+1)^2}{4}$

ie $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2 \cdot (n+1)^2}{4}$

* Properties / Prove that sums to be remembered without proof :

→ A.P

1. If $m.T_m = n.T_n$ then $T_{m+n} = 0$

ie

If 7 times the $7^{th}$ term of an AP is equal to 11 times the $11^{th}$ term then $18^{th}$ term = 0

$7.T_7 = 11.T_{11}$ then $T_{18} = 0$

2. If $T_m = n$ & $T_n = m$ then (i) $T_r = m+n-r$

$T_{m+n} = 0$

$T_{m-n} = 2n$

ie

If $T_{10} = 15$, $T_{15} = 10$ then (i) $T_{13} = 12$

$T_{25} = 0$

$T_5 = 20$

If $10^{th}$ term of A.P is 15 and $15^{th}$ term is 10.

3. If $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$ then

(i) $T_{mn} = 1$

(ii) $S_{mn} = \frac{1}{2}(mn+1)$

ie $T_7 = \frac{1}{5}$ $T_5 = \frac{1}{7}$

then (i) $T_{35} = 1$

$S_{35} = \frac{1}{2}(35+1)$

4. If $S_m = S_n$, $m \neq n$ then $S_{m+n} = 0$

ie $S_{13} = S_{10}$ $\therefore S_{23} = 0$

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5. If $S_m = n$ & $S_n = m$ then
$S_{m+n} = -(m+n)$

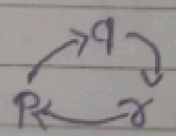
ie
If sum of the 11 terms of AP is 19, sum of 19 term is 11 then
$S_{30} = -(11+19)$
$S_{30} = -30$

6. If $\frac{S_p}{S_q} = \frac{p^2}{q^2}$ then (i) $d = 2a$
(ii) $\frac{T_p}{T_q} = \frac{2p-1}{2q-1}$

7. If ratio of $S_n$ is given and we have to find out ratio of $T_n$ then steps are
CW 84 (i) $n = 2(\text{terms}) - 1$
(ii) Put $n$ in given ratio of $S_n$ to get the ans.

8. If $T_p = a$, $T_q = b$, $T_r = c$ then
$a(q-r) + b(r-b) + c(p-q) = 0$

$\therefore$ Cyclic expression → Base → Ans. = 0
→ Power → Ans. = 1




9. If $S_p = a$, $S_q = b$, $S_r = c$ then
$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

10. If $S_1$ = Sum of $n$ terms
$S_2$ = Sum of $2n$ terms
$S_3$ = Sum of $3n$ terms
then,
$$S_3 = 3(S_2 - S_1)$$

11. If $S_1, S_2, S_3, \ldots, S_p$ are sums each upto $n$ terms of $p$ A.P.'s whose first terms are $1, 2, 3, \ldots$ & common differences are $1, 3, 5 \ldots$ respectively then,
$$S_1 + S_2 + S_3 + \cdots + S_p = \frac{np(np+1)}{2}$$

12. If sum of first $m$ terms of an A.P. vanishes (i.e. becomes 0) then sum of next $n$ terms $= \frac{-an(m+n)}{m-1}$, $a$ is a first term of series.

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→ G.P.

1. If $T_p = a$, $T_q = b$, $T_r = c$ then,
(i) $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$
(ii) $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$

2. If $T_{p+q} = a$, $T_{p-q} = b$ then,
(i) $T_p = \sqrt{ab} = (ab)^{1/2} = (ab)^{0.5}$
Extra: (ii) $T_q = a^{1-(p/2q)} \cdot b^{1-(p/2q)}$

3. If $T_1 = a$, $T_n = b$, $P$ = Product of GM of $n$ terms
then
$$P^2 = (ab)^n \quad \text{or} \quad P = (ab)^{n/2}$$
ie $\overset{a}{2}, 4, 8, 16, \overset{b}{32}$ $\quad n = 5$
$$P = (2 \times 32)^{5/2} = (64)^{5/2} = 8^5 = 32768$$

4. If $S$ = Sum of $n$ terms
$P$ = Product of $n$ terms
$R$ = Sum of reciprocal of $n$ terms
then
$$P^2 = \left(\frac{S}{R}\right)^n \quad \text{or} \quad P = \left(\frac{S}{R}\right)^{n/2} \quad \text{or} \quad P = \sqrt{S^n \cdot R^{-n}}$$
ie $P$ = GM of $S^n$ and $R^{-n}$
$2, 4, 8 \longrightarrow$ G.P.

$S = 2+4+8 = 14$
$P = 2 \times 4 \times 8 = 64$
$R = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

$P^2 = \left(\frac{S}{R}\right)^n$
$64^2 = \left(\frac{14}{7/8}\right)^3$
$64 = \left(\frac{112}{7}\right)^{3/2}$
$64 = 64$

5. If $S_1$ = Sum of $n$ terms
$S_2$ = Sum of $2n$ terms
$S_3$ = Sum of $3n$ terms
then,
$$S_1(S_3 - S_2) = (S_2 - S_1)^2$$

6. If $S_1, S_2, S_3, \dots S_p$ are the sum of Infinite Geomatric series whose first term are $1, 2, 3, \dots p$ with common ratio
$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} \dots \frac{1}{p+1}$$
then
$$S_1 + S_2 + S_3 + \dots + S_p = \frac{P(P+3)}{2}$$

7. If $x = 1 + a + a^2 + \dots + \infty \quad |a| < 1, \ -1 < a < 1$
$y = 1 + b + b^2 + \dots + \infty \quad |b| < 1, \ -1 < b < 1$
then
$$1 + ab + a^2b^2 + \dots + \infty = \frac{xy}{x+y-1}$$
ie If $x = 1 + 1/3 + 1/3^2 + \dots \infty$
$y = 1 + 1/2 + 1/2^2 + \dots \infty$
then $1 + 1/6 + 1/6^2 + \dots \infty = \dfrac{xy}{x+y-1}$
$$= \frac{(3/2)(2)}{(3/2) + 2 - 1}$$

8. If $a, b, c$ are in G.P. then,
(i) $\log a, \log b, \log c$ are in an A.P.
(ii) $\log a^n, \log b^n, \log c^n$ are in an A.P.

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$\longrightarrow$ A.P. & G.P.

1. If $a, b, c$ are in an AP
$x, y, z$ are in GP
then
(i) $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$
(ii) $(x^b \cdot y^c \cdot z^a) \div (x^c \cdot y^a \cdot z^b) = 1$

2. If $a, b, c$ are in A.P
& $T_a = x$, $T_b = y$, $T_c = z$
then, $x, y, z$ are in G.P.

3. If the $p^{th}$, $q^{th}$, $r^{th}$ & $s^{th}$ term of an AP are in GP
then,
$(p-q), (q-r), (r-s)$ are also in G.P.

4. If $a, b, c$ are in an AP & $a^x = b^y = c^z$
then
(i) $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

(ii) $x, y, z$ are in A.P.

5. If $a, b, c$ are in AP as well as G.P.
then (i) $a, b, c$ are in HP also
(ii) Their reciprocals
ie $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

→ AM., GM & HM. :

1. For any two positive numbers $a$ & $b$ with,
$AM = \frac{a+b}{2}$, $GM = \sqrt{ab}$, $HM = \frac{2ab}{a+b}$

(i) AM > GM > HM , for $a \neq b$
(ii) AM = GM = HM , for $a = b$
(iii) AM ⩾ GM ⩾ HM , If nothing is clear about $a$ & $b$
(iv) $(GM)^2 = (AM) \times (HM)$ → always true

2. $\left(\frac{a^{n+1} + b^{n+1}}{a^n + b^n}\right)$ may be

(i) HM between $a$ & $b$ for $n = -1$ — $\frac{2ab}{a+b}$
(ii) GM between $a$ & $b$ for $n = -1/2$ — $\sqrt{ab}$
(iii) AM between $a$ & $b$ for $n = 0$ — $\frac{a+b}{2}$

A > G > H.

3. If $\frac{\text{AM between two positive no.s } a \,\&\, b}{\text{GM between two positive no.s } a \,\&\, b} = \frac{m}{n}$

then,
$$\frac{a}{b} = \frac{m + \sqrt{m^2 - n^2}}{m - \sqrt{m^2 - n^2}} \; ; \text{ with } a > b$$

4. If A is AM & G is GM respectively of 2 no. then,
(i) The numbers are $A \pm \sqrt{A^2 - G^2}$
(ii) Q.E. is $x^2 - (2A)x + G^2 = 0$

5. If one AM, A and two GM's $G_1$ and $G_2$ are to be, inserted between any two numbers then

$$G_1^3 + G_2^3 = 2A\,G_1 G_2$$

ie $$\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = 2A$$

6. If one GM, G and Two AM'S $A_1$ and $A_2$ are to be, inserted between any two numbers then

$$G^2 = (2A_1 - A_2).(2A_2 - A_1)$$

7. Sum of n AM's inserted betwee a and b is $\frac{n}{2}(a+b)$

ie $$A_1 + A_2 + A_3 + \dots + A_n = \frac{n}{2}(a+b)$$

- The sum of n AM's between the two given numbers is equal to n times the single AM between them.

8. Product of n GM's inserted between a and b is $(ab)^{n/2}$

ie $$G_1 . G_2 . G_3 \dots G_n = (ab)^{n/2}$$

- The product of n GM's between the two given numbers is equal to the $n^{th}$ power of the single GM between them.

Remark:-

1. th, rd, st $\longrightarrow$ $T_n$ formula e.g. $20^{th}, 23^{rd}, 21^{st}$
2. $T_n \longrightarrow n^{th}$ element / $n^{th}$ term
3. To 21 terms
upto 21 terms $\Big\}$ $n = 21$
4. $n = ?$ $\longrightarrow$ Which term?
$\longrightarrow$ How many terms?
$\longrightarrow$ Number of terms?
$\longrightarrow$ Certain Number of terms.
5. Whenever options are given in terms of '$n$' then always use $S_1 = T_1$, $S_2 = T_1 + T_2$.
6. Whenever options contain listing of numbers separated by ( , ) then always use method of checking given conditions.
Since product of 3 terms in G.P. is involved hence use 3 terms from perticular G.P.
7. Cyclic expression in base $\longrightarrow$ ans. 0
Cyclic expression in power $\longrightarrow$ ans. 1
$\longrightarrow$ If cyclic expression breaks then use dummy values.
8. $\sum_{r=10}^{27} 2r^3$ $\longrightarrow$ Summation of $2r^3$, $r$ running from 10 to 27.
9. Sum of Odd numbers $= n^2$
Sum of even numbers $= n(n+1)$ $\Big\}$ $\boxed{\text{HW 2 - 28}}$
10. If 2 APs have same difference (d) then the difference between their $n^{th}$ term is always same. $\boxed{\text{HW2 1}}$