CHAPTER – 1(RATIO, PROPORTION, INDICES AND LOGARITHM)

RATIO (1A)

If a and b the similar Quantity and same unit, then ratio is $\frac{a}{b}$ or a: b or a to b

Where a, b called terms of ratio (which are integers)

a = Numerator (Antecedent) and b = Denominator (Consequent)

Q.1. Arun Earns ₹ 80 in 7 hours and Varun ₹ 90 in 12 hours. The ratio of their earning is
[Ans. 32:21]

Q.2. what is ratio b/w diameter and circumference of a circle

[Ans. 7:22]

Q.3the salaries of A,B,C are in Ratio 2:3:5. If the incrementof 15%, 10%, and 20% are allowed respectively in their salaries then what will be the new ratio- **[Ans23:33:60]**

NOTES

1. finding Actual value of Quantity from a given ratio and information (k-method)

Eg. If ratio is a:bThen first value = ak and 2nd value = bk

K is derived from given information.

eg. There are some fruits and nuts in the ratio 3:14. If 5 fruits are eaten, then the ratio will becomes 1:8, find the number of fruits are- **[ans 12]**

eg. Two number are in ratio 3:4, and difference b/w their square is 112. Then larger No. is- **[16]**

eg.A bag contains ₹ 216, in the form of one rupee, 50 paise and 25 paise coins in the ratio of 2:3:4. The number of 50p coins is. **[ans144]**

ans: let 2k, 3k, 4k where $k = \frac{216}{2+1.5+1} = 48 \Rightarrow 3k = 144$

2. when only ratio given b/w a and b, and value of an expression in terms of a,b is to be find

Eg.a:b = 3:2 , then value of$\left(\frac{2a+b}{a+2b}\right)$ is given by

Put$a \rightarrow 3$ and $b \rightarrow 2$ **[8:7]**

Eg If $\frac{2a+b}{a+2b} = \frac{8}{7}$ then a: b is (by options, is the best method to find Ans.)

(a) 2:3 **(b) 3:2** (c) 1:2 (d) 2:1

Eg. If $(6a^2 - ab) : (2ab - b^2) = 6:1$, then a:b

(a) 2:3 (b) 3:4 (c) 5:3 (d) 2:1

3. Forms of a Ratio.

→ $\frac{a}{b} = \frac{4}{5}$ or a:b = 4:5 or $\frac{a}{4} = \frac{b}{5}$

→ a:b:c = 2:1:3 → $\frac{a}{2} = \frac{b}{1} = \frac{c}{3}$

Eg. 2x = 3y → $\frac{2x}{6} = \frac{3y}{6}$ → $\frac{x}{3} = \frac{y}{2}$ →x:y = 3:2

Eg. If $x = \frac{y}{2} = \frac{z}{5}$ → x: y: z = 1:2:5

Eg. If 2x = 3y = 6z, then $\frac{x+y+z}{z}$ is …… [6]

E.g. ₹ 1,980 are divided among A, BandC, so that half Of A's part, one-third of B's part and one-sixth of C's part are equal. Then B's part is **[₹ 540]**

Hint: $\frac{A}{2} = \frac{B}{3} = \frac{C}{6} \Rightarrow B = \frac{1980}{11} \times 3 = 540$

4. Simplifying terms of a ratio

Eg. $\frac{2}{3} : \frac{1}{2} : \frac{4}{5}$

Multiply all with LCM of Drs. (i.e.30)

$\frac{2}{3} \times 30 : \frac{1}{2} \times 30 : \frac{4}{5} \times 30$ → 20:15:24

Eg. a:b:c = 2:3:4 , then the value of $\frac{a}{b}:\frac{b}{c}:\frac{c}{a}$ is **[8:9:24]**

5. Dividing a sum(s) into a given ratio a:b:c

$1^{st} = \frac{s}{a+b+c} \times a$ $2^{nd} = \frac{s}{a+b+c} \times b$ and $3^{rd} = \frac{s}{a+b+c} \times c$

Eg. If ₹ 782 be divided into three parts, in ratio $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ then the First Part is **[₹ 204]**

Eg.1230 oranges were loaded in three trucks. When unloaded, it was found that 5, 10 and 15 oranges were rotten in the trucks respectively, remaining oranges were in ratio 3:4:5. How many oranges were loaded initially in third truck.

Ans 3^{rd} truck $= \frac{1200}{3+4+5} \times 5 = 500 + 15 = 515$

Eg. Two whole number's whose sum is 72 cannot be in the ratio.
(a) 5:7 (b) 3:5 **(c) 3:4** (d) 4:5

6. Mathematical terms in ratio

→inverse ratio a:b → b:a.

→ratio compounded of a:b, c:d, …… is product of all ratio's → ac…**:** bd….

→duplicate ratio of a:b is $a^2 : b^2$

→sub-duplicate ratio of a:b is $\sqrt{a} : \sqrt{b}$

→triplicate of a:b = $a^3 : b^3$

→sub- triplicate of a:b = $\sqrt[3]{a} : \sqrt[3]{b}$

→if a:b is the duplicate ratio of a-x: b-x **then $x^2 = ab$**

→if a:b is the sub- duplicate ratio of a-x: b-x **then $x = \frac{ab}{a+b}$**

eg. (3x + 3): (9x + 7) is the duplicate ratio of 3:5. Then the value of x is [ans.8]

eg. The ratio compounded of (a+b): (a-b) and $(a^2 - b^2) : (a + b)^2$ is [ans 1:1]

7. Continued ratio's Two ratio in which one term is common.

Eg. $\frac{a}{b} = \frac{4}{3}$, $\frac{b}{c} = \frac{5}{2}$ then find a:c and a:b:c **ans:** 10:3 and 20:15:6

- If two continued ratio are equal i.e.
  $\frac{a}{b} = \frac{b}{c}$ **then** $b^2 = ac$

E.g. A:B = 2:3, B:C = 4:5 and C:D= 6:7 then A : B : C : D is 16:24:30:35

Eg. The sum of square of three numbers is 532 and the ratio of the 1st to the second as also of the second to the third is 3:2. The second no is …… **[12]**

Eg. If A's money to B's money as 4:5 and B's money to C's money as 2:3. and A has ₹ 800, then C has, **[₹ 1200]**

8. Increase or Decrease in a given ratio

New value = Old value × Inverse ratio.

eg. **A** reduced his weight in ratio 5:3. New weight = old weight × $\left(\frac{3}{5}\right)$

Eg. In what ratio overall wages of workers will increase or decrease if no of workers reduced in ratio 15:11 and wages increase in ratio 22:25. **[Decrease in ratio 6:5]**

Ans new =old x $\frac{11}{15}$x $\frac{25}{22}$ =old x$\frac{5}{6}$

9. Allegation rule (mixing/ combining rule) A(value C_1) and B (value C_2) are mixed if value of mixture is C **then A:B =** $|C - C_1|:|C - C_2|$

eg. In what proportion should one variety of oil at ₹ 9.5 per kg be mixed with another at ` 10 per Kg to get a mixture worth ₹ 9.6 per Kg **ans.**10-9.6 : 9.6-9.5= 4:1

eg. A dealer mixes tea costing of ₹ 6.92 per kg with tea costing ₹ 7.77 per kg and sells the mixture at `8.8 per kg and earns a profit of 17.5% on his sale price. Find the ratio in which tea are mixed.

Ans: C_1 = 6.92 and C_2 = 7.77 and cost of mixture C = 8.8 – 17.5% = 7.26 hence ratio is given by $|C - C_1|:|C - C_2|$= **3:2**

PROPORTION (1 B)

Proportion means tworatios are equal. if$\frac{a}{b} = \frac{c}{d}$then **ad = bc (cross product rule)**

Where a = 1^{st} proportional, b = 2^{nd} proportional, c = 3^{rd} proportional, d = 4^{th} proportional

a, d called extremes and b, c called means and a, b, c, d called in proportion.

Continued proportion: $if\ \frac{a}{b} = \frac{b}{c}$ **then $b^2 = ac$**

Where a = 1^{st}proportion, b = mean proportion, c = 3^{rd} proportion

Properties of proportion

InvertendoIf $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{b}{a} = \frac{d}{c}$AlternendoIf $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{d}{b} = \frac{c}{a}$

ComponendoIf $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a+b}{b} = \frac{c+d}{d}$DividendoIf $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a-b}{b} = \frac{c-d}{d}$Componendo and dividend If $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

AddendoIf$\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$SubtratendoIf $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$

Eg. The Fraction which bears the same ratio to 3/7 that 1/5 does to 7/15 is, **[9/49]**

Eg. The fourth proportioned to (a+b), $(a+b)^2$,(a-b) is **($a^2 - b^2$)**

Eg.The third proportional to 12 and 24 is **[48]**

Eg. What must be subtracted from each of the numbers 21,38,55,106 so that they becomes in proportional? **[4]**

Eg. The Mean Proportional b/w 1.25 and 1.8 is **[1.5]**

Eg.$\frac{x}{y} = \frac{z}{w} = 10$, then $\frac{x+z}{y+w}$is **[10]**

Eg. If $\frac{x}{y+z} = \frac{y}{x+z} = \frac{z}{x+z} = K$ then K is [½]Eg.

What are two numbers, their sums is 13 and Mean proportion b/w them is 6.A.(3,10) **B.(4,9)** C.(8,5) D. (4,10)

INDICIES (1 C)

Laws of Indices

1. $x^m \times x^n = x^{m+n}$ or $x^{m+n} = x^m \times x^n$ 2. $\frac{x^m}{x^n} = x^{m-n}$ or $x^{m-n} = \frac{x^m}{x^n}$ and $a^0 = 1$ or $a^x = 1 \rightarrow x = 0$

3. $(a \times b)^x = a^x \times b^x$ and 4. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

When LHS = RHS 5. $a^x = b^x \rightarrow a = b$ and 6. $a^x = a^y \rightarrow x = y$

7. $(x^m)^n = x^{mn}$ eg. $(2^3)^2 = 2^6$ But $2^{3^2} = 2^9$

Eg. $16^x = (2^4)^x = 2^{4x}$ Eg. $\left\{(x^n)^{\frac{n^2-1}{n}}\right\}^{\frac{1}{n-1}} = x^{n \times \frac{(n-1)(n+1)}{n} \times \frac{1}{n-1}} = x^{n+1}$

8. $\left(\frac{a}{b}\right)^x = \left(\frac{b}{a}\right)^{-x}$

Eg. $\left(\frac{4}{3}\right)^{2x} = \left(\frac{3}{4}\right)^{x-3}$ Then the value of x is Eg. $17^{3-6x} = 1$ then x is **…**

Eg. Solve for x if $4^{\sqrt{x}\sqrt{x}} = 256$ Eg. if $x^{m^n} = (x^m)^n$ and n=3, then m

Eg If $(25)^{150} = (25x)^{50}$, then the value of x is….. Eg. If $4^x = 5^y = 20^z$, then Z is

Ans: **1,** $\mathbf{\frac{1}{2}}$, **4,** $\mathbf{5^4}$, $\mathbf{\sqrt{3}}$ **and** $\mathbf{\frac{xy}{x+y}}$

Logarithm (1D)

Properties

1. $log_a x = n \rightarrow x = a^n$

2. $\log_a 1 = 0 \rightarrow \log_5 1 = 0, \log_2 1 = 0$

3. $\log_a a = 1 \rightarrow \log_2 2 = 1, \log_3 3 = 1$

4. $\log_a x^N = N.\log_a x \rightarrow \log_a x^2 = 2 . \log_a x$

5. $\log_{b^y} a^x = \frac{x}{y}\log_b a \rightarrow \log_{a^y} a^x = \frac{x}{y} \rightarrow \log_{b^x} a^x = \log_b a$

6. $\log_x(a \times b \times \ldots\ldots) = \log_x a + \log_x b + \ldots\ldots$
7. $\log_x\left(\frac{a}{b}\right) = \log_x a - \log_x b$

8. <u>Base change</u>

$\rightarrow \frac{1}{\log_b a} = \log_a b \rightarrow \log_b a = \frac{\log a}{\log b}$

9. Power rule $\boldsymbol{a^{\log_a x} = x}$ 10. $a^{\log_x b} = b^{\log_x a} \rightarrow 2^{\log 5} - 5^{\log 2} = 0$

Note: If log x= 5 → x is (5+1) ie 6 digits number also

If log x = 5.68→ x is again 6 digits number

Eg. Find no of digits in 4^{64} Ans x is 39 digits number

eg$\log_5 \log_2 x = 2$,then x $(\mathbf{2^{25}})$

eg.if$\log_y x = 50$ *and* $\log_3 y = 100$ then x is $(\mathbf{3^{5000}})$

eg. $\log_{9^2} 27^5$=**.....(15/4)eg.**$\frac{\log_9 11}{\log_5 13} \div \frac{\log_3 11}{\log_{\sqrt{5}} 13}$=......**(1)**

eg. $\log_2 x + \log_2 \sqrt{x+2}$ **=**2, then x is **(A) 2** (B) 4 (C) 3 (D) none

eg. $\log_3 x + \log_9 x + \log_{27} x = \frac{11}{12}$, then x is **(√3)**

eg. The value of$3^{\log_9 4}$**....... (2)**

eg. $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$,then**(A) a=b**(B) a=2 (C) 2a=b (D) none

eg. If$\log\left(x + \frac{1}{x}\right) + \log 2 = \log 5$, then x is (A) 0 **(B) 2 or 1/2** (C) 3 (D) none

eg.$\frac{1}{\log_x 10} + 2 = \frac{2}{\log_5 10}$, Then x is **......(x=0.25)**

CHAPTER – 2(EQUATIONS)

Linear Equations

One Variable: ax + b = 0 , x is Solutions / root

two variables

$a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$:solutions (x, y) is point of Intersection of these two lines.

eg. Solutions of $\frac{2x-1}{3} - \frac{6x-2}{5} = \frac{1}{3}$[x = $-\frac{1}{2}$] eg. Point of Intersection of lines 2x + 3y = 7 and 6x + 5y = 11 lie in which quadrant

a. I **b. II** c. III d. IV

eg. The length of the rectangle in 4 cm more than the breadth and the perimeter is 11 cm more than the breadth. Find length and breadth.

a. (5cm, 1cm) b. (4cm, 2cm) c. (5cm, 2cm) d. none

eg. Divide 300 into two parts, so that half of one part may be less than the other by 48.

a. (168,132) b. (150,150) c. (130,170) d. none

egThe denominator of a fraction exceeds the numerator by 5 and if 3 be added to both fraction becomes ¾ .Find the fraction.

a. 4/6 **b. 12/17**c. 13/18d. none of these

Quadratic Equations

$ax^2 + bx + c = 0$ ($a \neq 0$) is called quadratic equation.

Solutions/ roots (x) can be obtained in two ways

By options i.e. x for which LHS=RHS

by formula $x = \frac{-b \pm \sqrt{D}}{2a}$,where $D=b^2-4ac$.

Important Notes **1. Nature of roots** If $D < 0$, *then roots are* imaginary andunequal, $if D = 0$,thenreal and equal roots

$if D > 0$, then real and unequal roots → If D is perfect square, roots are rational.

→ If D is not a perfect square; roots are irrational and conjugate to each other.

Eg.ifα = $\sqrt{3}$ - 2 = $\beta = -\sqrt{3}$– 2

Eg. If x = m is roots of $x^2 + 2x - m = 0$, then m is … (put x = m, LHS=RHS) ans is**(0, -1)**

Eg. If the roots of the equation ,$3x^2 + 4x + a = 0$ are equal then a = … **(4/3)**

Eg. The roots of the equations ,$x^{2+}(2p - 1)x + p^2 = 0$, are real if

a. $p \geq 1$ b. $p \leq 4$ c. $p \geq \frac{1}{4}$ **d.** $\mathbf{p \leq \frac{1}{4}}$

Note: 2. **The value of a series upto∞**

Eg.the value of $\sqrt{7 + \sqrt{7 + \sqrt{7 +}}}$……….∞$\mathbf{[\frac{1+\sqrt{29}}{2}]}$

Eg.the value of $4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cdots\ldots\ldots\infty}}}$is[$\mathbf{2 + \sqrt{5}}$]

Eg. If 5=$\sqrt{x + \sqrt{x + \sqrt{x + \cdots \ldots \ldots \infty}}}$ then x is **[20]**

Note 3 **Equations from roots**

$x^2 - (\propto + \beta)x + \propto \beta = 0$or x^2 – (sum of roots)x + product of roots = 0

Eg. Find quadratic equations if one root is ($\sqrt{2}$ - 1) **[$\mathbf{x^2 + 2x - 1 = 0}$]**

Eg. Find quadratic equations if arithmetic Mean and Geometric mean of roots are A and G. **[$\mathbf{x^2 - 2Ax + G^2 = 0}$]**

Note 4 **if ∝, β be the roots of $\mathbf{ax^2 + bx + c = 0}$**

Then $\propto + \beta$ = -$\frac{b}{a}$and$\propto . \beta = \frac{c}{a}$from this, we can find the value of

$\propto^2 + \beta^2$,$\propto^3 + \beta^3$, $\propto^2 - \beta^2$, $\propto^3$ - β^3, by using These formula's

- $(\propto+\beta)^2 = \propto^2 + \beta^2 + 2 \propto \beta$
- $(\propto+\beta)^3 = \propto^3 + \beta^3 + 3 \propto \beta(\propto+\beta)$
- $\propto^2 - \beta^2 = (\propto-\beta)(\propto+\beta)$
- $\propto^3 - \beta^3 = (\propto-\beta)(\propto^2+\beta^2 + \propto \beta)$
- $(\propto-\beta) = \sqrt{(\propto + \beta)^2 - 4\alpha\beta}$

Eg. If $\propto, \beta$be the roots of the equations: $2x^2 - 4x - 3 = 0$, then the value of $(\alpha^2 + \beta^2)$**(7)**

Eg. What is the value of $(\frac{\propto}{\beta} - \frac{\beta}{\propto})$ in above example **$[-\frac{4\sqrt{10}}{3}]$**

***Notes 5* if roots are equal in magnitude but opposite in sign, then b = 0**

***Note 6*if roots are reciprocal to each other then c = a**

Eg. If one root of $mx^2 - 2x + 2m + 3 = 0$ be the reciprocal of other, then m**[m = -3]**

***Note 7*Equations and one root in given then other root is given by**

$$\propto \beta = \frac{c}{a}$$

Eg. Roots of $(a-b)x^2 + (b-c)x + c-a = 0$, are

a. (1,a) b. (1,b - c) c. $(1, \frac{b-c}{a-b})$ d. $(1, \frac{c-a}{a-b})$

***Note 8*Equations and difference b/w roots are given,** then for requirements, Apply this $formula$ $(\propto^2 - \beta^2) = (\propto+\beta)^2 - 4 \propto \beta$

Eg. If roots of quadratic equations $x^2 + kx + 12 = 0$, differ by unity. Then k **[$\pm$ 7]**

Eg. If one root of the equation $x^2 - 8x + m = 0$ exceeds other by 2 then m is **[m = 15]**

Note 9 **when quadratic equations ($ax^2 + bx + c = 0$) and ratio b/w roots (p:q) is given** then for requirements, Apply this result. $\frac{b^2}{ac} = \frac{(p+q)^2}{pq}$

Eg. The conditions that one root of $ax^2 + bx + c = 0$ is thrice the other is :

a. $3b^2 = 16c\ a$ b. $b^2 = 9\ c\ a$ c. $3b^2 = -16\ a\ c$ d. None

Eg. The roots of the equations, $lx^2 + mx + n = 0$ are in ratio 3:4, and

$12m^2 = kxl$, where k = **………. [k = 49]**

Note 10 **by options**

Eg. When $\sqrt{2z+1} + \sqrt{3z+4} = 7$, then z is

a. 1 b. 2 c. 3 **d. 4**

Eg. $7^{1+x} + 7^{1-x} = 50$ then x are

a. (1, 0) **b. (1,-1)** c. (-1, 5) d. none

Eg. Total cost of a piece of iron rod is ₹60.if length reduced by 2 meter and each meter cost increase by ₹1, total cost remains unchanged. What is original length?

a. 12m b. 10m c. 15m d. 8m

Cubic equations

$ax^3+bx^2+cx+d=0$, where ($a\neq0$)Solutions/roots is given by two ways

→ first, if making of factors is easy, and then factor method

→2nd, if making of factor is difficult, then, by options

Eg. Rational root of the equation, $2x3-x^2-4x+2=0$ is

(A) **1/2** (B) 2 (C) -2 (D) 3

Eg.If $x^3+x^2-x-1=0$,then the value of (2x+1) are

A.(3,-1,-1) B.(2,-1,-1) C.(3,1,1) D . none

Eg. Roots of equation ,$x^3-7x+6=0$ are A.(1,2,3)**B.(1,2,-3)** C.(3,1,1) D . none

Co-ordinate geometry

Note-1 distance formula i.e. distance b/w two points $A(x_1, y_1)$ and $B(x_2, y_2)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

→If one point is origin (0, 0) and other is (x, y) then distance $= \sqrt{x^2 + y^2}$

→Area of a triangle with vertices $A(x_1,y_1)$ $B(x_2,y_2)$ and $C(x_3,y_3)$ is positive value of A

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

→ Centroid (x, y) of a triangle with vertices $A(x_1,y_1)$ $B(x_2,y_2)$ and $C(x_3,y_3)$ is

$$x = \frac{x_1+x_2+x_3}{3} \text{ and } y = \frac{y_1+y_2+y_3}{3}$$

Note-2 length of perpendicular from a point to a line i.e. shortest distance from a point(x_1,y_1) to a line ax +by + c =0 is positive value of L , $L = \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$

Note-3 slope of a line

→When two points (x_1,y_1) and (x_2,y_2) of the line is given**:** slope, $m = \frac{y_2-y_1}{x_2-x_1}$

→ when equation of the line is given

y = mx+c, slope = m   ax +by +c =0, slope, $m = -\frac{a}{b}$

Use of slope: for two parallel lines, slopes are equal i.e. $m_1 = m_2$

For two perpendicular lines, $m_1.m_2 = -1$

Three points A, B, C are collinear if slope of AB= slope of BC

Note-4 three con-current lines: solve any two lines and put the solution(x, y) into third line if

LHS=RHS ⇒ three lines are con-current.

Note-5 Equation of a line

→ Point slope form: $y-y_1= m(x-x_1)$ and intercepts form: $\frac{x}{a} + \frac{y}{b} = 1$,

Note-6 Two variables and two information are given, and then find the value of one variable when other is given.

Eg for output 5 units and 8 units total costs is ₹ 80 and ₹ 116. What is a total cost if output is 10 units? Ans: TC= 116+ $\frac{36}{3} \times 2 = 140$

Eg investments of ₹ 1000 and ₹ 100, incomes are ₹ 90 and ₹ 20. What is a investment for income of ₹ 50.Ans: investment = 100+ $\frac{900}{70} \times 30 = 485.71$

Q. the distance b/w the points(x,-1) and (3, 2) is 5. Then the value of x **[7, -1]**

Q. the points (2, 2) (6, 3) and (4, 11) are the vertices of a triangle which is- **(right angled)**

Q. Area of the triangle with vertices (-3,-5) (5, 2) and (-9,-3) is-- **(29 s. unit)**

Q. the four points (0,3) (0,-1) (-2,3) and (6,7) are vertices of a……….. **(Rectangle)**

Q. if line joining (-1,1) and (2,-2) is perpendicular to the line joining (1,2) and (2,k) , then k.. **(3)**

Q. the centroid of triangle A(-4,6) , B(2,-2) and C is (0,3), then point C is….. **(2, 5)**

Q. the points (k, 1) (5, 5) and (10, 7) are collinear, then the value of k is ….. (k **=5)**

Q. the equation of a line joining the point (3, 5) to the point of intersection of the lines 4x+y-1=0 and 7x-3y-35=0 is….. **(ans: 12x-y-31=0)**

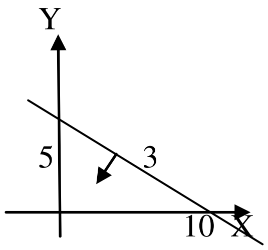
Q. the equation of the line through (5,-3) and perpendicular to 2x-3y+14=0 is-**(3x+2y-9=0)**

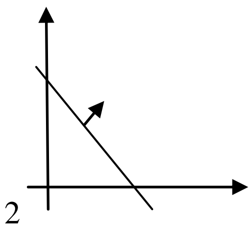
Q. the value of **a** in order that the three lines 3x-y-2=0, 5x+ay-3=0 and 2x+y-3=0, may meet at a point is…. **(a= -2)**

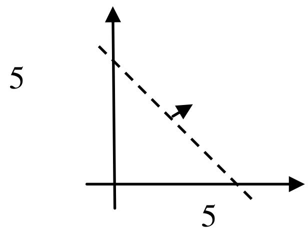
CHAPTER- 3(INEQUALITIES)

Graphs:

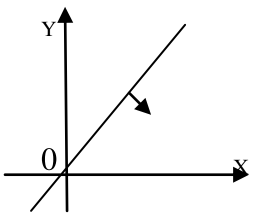
$x + 2y \leq 10$, $3x + 2y \geq 6$, $x + y > 5$

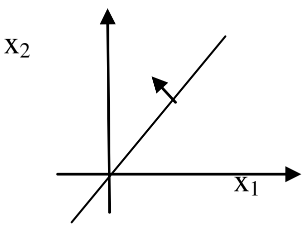




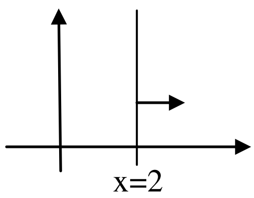


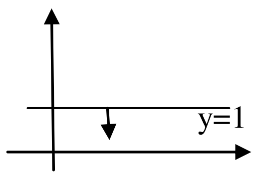
$x \geq 2y$ $2x_1 - x_2 \leq 0 \Rightarrow 2x_1 \leq x_2$

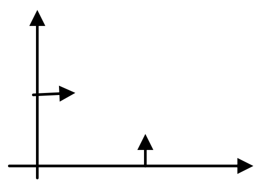




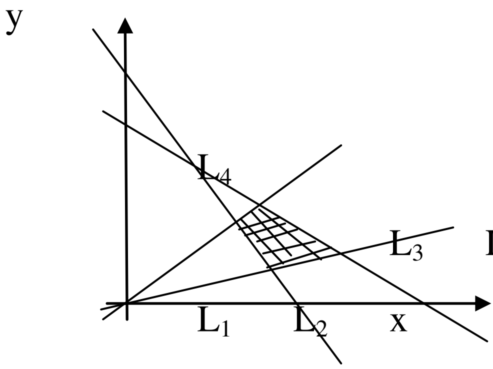
$x \geq 2$ $y \leq 1$ $x, y \geq 0$







Eg. What are the inequalities of this graphical solution?



$L_1: 5x + 3y = 30$

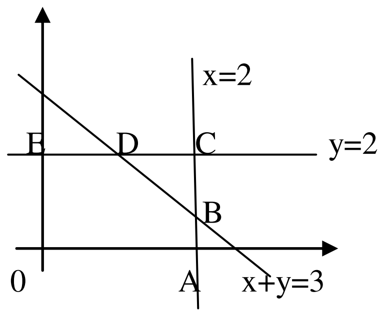
$L_2: x + y = 9$

$L_3: y = x/3$

$L_4: y = x/2$

(a) 5x+3y≤30	**(b)5x+3y≥30**	(c)5x+3y≥30	(d) none of thesex+y≤9
x+y≤9	x+y≥9		
y≤x/3	**y≥x/3**	y≤x/3	
y≤x/2	**y≤x/2**	y≥x/2	

Egsolution of inequalities,x≤2, y≤2 andx+y=3 is given by



(A) Area OABDEO

(B) Area BCDB

(C) Line B to D

(D) point C

Eg. Which point is not in the solution of inequalities, x+2y≤10, 3x+y≤15 and x, y≥0?

(A) (1,4) **(B) (7,1)** (C) (5,0) (D) (0,5)

Eg. Solution space of the inequalities 2x+y≤10, and x-y≤5 (i) includes the origin (ii) includes the point (4, 3), which one is correct?

(A) Only (i) (B) only (ii) (C) both (i) and (ii) (D) none of these

Eg.The solution of the inequality, $\frac{5-2x}{3} \le \frac{x}{6} - 5$

(A) x≥8 (B) x≤8 (C) x=8 (D) none of these

Eg. Solve the inequality,$\frac{x-3}{4} + 2 \ge \frac{3x-1}{2} + \frac{4x+5}{3}$

(A) x≥$\frac{1}{31}$**(B) x≤$\frac{1}{31}$** (C) x≥5 (D) none of these

Eg.Solution of inequalities 3x -2 < 7 and 2x+1 > 3 is

(A)x> 3 (B) x < 1 **(C)1< x < 3** (D) none of these

Eg. Solution of inequalities 3x -2 > 7 and 2x+1 > 3 is

(A) x> 3 (B) x < 1 (C)1< x < 3 (D) none of these

Eg. Solution of inequality 2x +5 < 4x – 1 is,

Ans: 2x – 4x < -1-5 ⇒ -2x < -6 ⇒ 2x > 6, x > 3

Formulation

→ At least / not less than / minimum implies $\geq$ and → At most / not more than / maximum implies $\leq$

No of skilled workers = x and No of unskilled workers = y

Eg. Total no. of workers not more than 10

Ans. $x + y \leq 10$, $x, y \geq 0$

Eg. Skilled workers, produce 5 units daily and unskilled 3 units daily, Daily requirements is at least 50 units.

Ans. $5x + 3y \geq 50$, $x, y \geq 0$.

Eg. No. of skilled workers at least twice of unskilled workers

Ans. $x \geq 2y$ and $x, y \geq 0$

Eg. No. of skilled not more than 5 to each unskilled worker

Ans. Unskilled = 1, Skilled $\leq 5 \Rightarrow x \leq 5y$ and $x, y \geq 0$

Eg. Labor union, forbids less than 2 skilled for each unskilled,

Ans. Un = 1, $S \geq 2 \Rightarrow x \geq 2y$ and $x, y \geq 0$

Eg. No. of skilled workers not less than 10 and no. of unskilled workers at least twice of skilled workers:

A. $x \geq 10$	B. $x \geq 10$	C. $x \leq 10$	**D. $x \geq 10$**
$y \geq 20$	$y \leq 20$	$y \geq 20$	**$y \geq 2x$**
$x, y \geq 0$	$x, y \geq 0$	$x, y \geq 0$	**$x, y \geq 0$**

Eg. A diet for a sick person must at least 4000 units of vitamins, 50 units of minerals and 400 calories. Two foods F1 and F2 are available. One unit of food F1 contains 200 units of vitamins, 1 unit of mineral and 40 calories. Also one unit of food F2 contains 100 units of vitamins, 2 units of minerals and 40 calories. If x and y units of food F1 and F2 are taken, then the linear inequalities are

(a) $200x + 100y \leq 4,000$, $x + 2y \leq 50$, $40x + 40y \leq 400$, $x \geq 0$, $y \geq 0$

(b) $200x + 100y \geq 4,000$, $x + 2y \geq 50$, $40x + 40y \geq 400$, $x \geq 0$, $y \geq 0$

(c) $200x + 100y \leq 4,000$, $x + 2y \leq 50$, $40x + 40y \leq 400$, $x \geq 0$, $y \geq 0$

(d) $200x + 100y \leq 4,000$, $x + 2y \leq 50$, $40x + 40y \leq 400$

CHAPTER-4 (SIMPLE INTEREST, COMPOUND INTEREST AND ANNUITY)

Simple interest

→ $\boxed{SI = \frac{p\ r\ t}{100},\ t\ in\ yrs.}$, $\boxed{A = P + SI}$, $\boxed{rate = \frac{SI\ p.a.}{p} \times 100}$,

$\boxed{P = \frac{A \times 100}{100 + rt}}$

→Double: **r.t = 100**, Triple: **r.t = 200,**5 times : **r.t = 400** etc.

Eg. A sum of money trebles itself in 15 years 6 months. In how many years would it double itself? Ans: 7y and 9 months

Eg. A sum of ₹ 90500 is deposited at SI rate 7.5% but wrongly calculated at rate 5.7%, if difference in interest is ₹ 9774. Then number of yrs for which it was lent.

Ans: In 1 yrs difference is 7.5-5.7= 1.8% i.e. 1.8% of 90500= ₹ 1629, total yrs = $\frac{9774}{1629} = 6\ yrs.$

Eg. In 8 yrs money becomes triple, what is rate of SI.r.t = 200 ⇒ r = $\frac{200}{8}$= **25%**

Eg. 5 times in 10yrs, 3 times in how many yrs. @ SI 4I → 10yrs2 I →10/4× 2 = **5 yrs.**

Eg. Amount in 3 years is ₹5000 and in 5 yrs is ₹ 6,000. Find principal and SI rate.

Ans.SI in 2 yrs = 1000⇒SI p.a. = 500, P = A- SI = 5000 - 3×500 = 3,500

r = $\frac{SI\ p.a.}{p}$×100 =$\frac{500}{3500}$×100 = 14.28% Eg. In how much time would the simple interest on a certain sum be 0.125 times of the principal at 10% p.a.

Ans: SI=$\frac{prt}{100}$, $0.125p = \frac{p \times 10 \times t}{100} \rightarrow t = 1.25y$

E.g. A person invested in all ₹ 1,600 at 4%, 6% & 8% p.a. SI. At the end of year, he got the same interest in all the three cases. What is largest part? **[₹1,200]**

→ Changing rate

Eg. P = 5000 for 5 yrs at rate, 10% pa SI for 1st 3 years and 5% pa SI for remaining years Find SI

Ans. SI = $\frac{p\ r\ t}{100} = \frac{5000 \times 40}{100}$= 2000(rt = 10×3+5×2 = 40)

Compound interest

→ $\boxed{A = P(1+r)^t}$ $, when\ rate\ is\ Annually$

Eg. Ansul father wishes to have ₹75000 in a bank account when his first college expenses begin. How much amount his father should deposit now at 6.5% compounded annually, if Ansul is to start college in 8 yr from now?

Ans: $A = p(1+r)^t, 75000 = p(1.065)^8, p = \frac{75000}{1.065^8}$ =₹ 45317.

Eg. What is the present value of ₹1 to be received after two yr compounded annually at 10% p.a. interest rate.

Ans: $A = p(1+r)^t, 1 = p(1.1)^2, p = \frac{1}{1.1^2}$ =₹ 0.83.

→ $\boxed{A = P(1+i)^n}$ $, when\ rate\ is\ quarterly(m=4), half\ yrly(m=2), monthly(m=12)$

Where $i = \frac{r}{m} and\ \ n = m.t \quad (t\ in\ yrs)$

Eg. In what time will ₹ 390625 amount to ₹ 456976 at 8% p.a., when the interest is compounded semi- annually?

Ans: $A = P(1+i)^n$, $456976 = 390625(1.04)^{2t} \Rightarrow$

$$(1.04)^{2t} = 1.16986 \Rightarrow 2t = 4, t = 2$$

Eg. What is compound interest on ₹ 4000 for $1\frac{1}{2}\ yr$ at 10% p. a. compounded half yearly?

Ans: m =2, r = 10%, t = 1.5 yr$\Rightarrow i = \frac{10\%}{2} = 5\%\ and\ n = 2(1.5) = 3 CI = P[(1+i)^n - 1] = 4000[1.05^3 - 1] = 630.5$

→ CI of n^{th} yr. = $CI\ of\ (n-1)^{th} yr \times (1 + rate)$, also $A_n = (1 + rate). A_{n-1}$

Eg.if rate is 5% p.a. compound interest and CI of 2^{nd} yr. is ₹ 210, then CI of 3^{rd} yr. is ₹ 220.5 (210 x 1.05) and CI of 1^{st} yr. is ₹ 200 ($\frac{210}{1.05} = 200$).

In 1^{st} yr. p x 5% = 200, p = 200/ 5% = ₹ 4,000

Eg. The compound interest for a certain sum at the rate 5% per annum for 2^{nd} year is ₹ 26.25. The SI for the same money at the rate 5% p.a. for 2 years will be

Ans: $CI\ of\ 1st\ yr = \frac{26.25}{1.05}$ =₹ 25 ⇒ SI for 2 yr = 25+25= ₹ 50

→ **Effective rate (r_e)** $\boxed{r_e = [(1+i)^m - 1] \times 100}$

Eg. the effective rate equivalent to nominal rate of 6% compounded monthly is

$\boxed{r_e = [(1+i)^m - 1] \times 100}$ where m =12, i=6%/12= 0.5%=0.005

$$r_e = [(1.005)^{12} - 1] \times 100 = 6.17\%$$

→ **Difference b/w SI and CI**

For 2 yrs. difference = $p \times \frac{r}{100} \times \frac{r}{100}$

For 3 yrs. Difference = CI – SI, where SI = = $\frac{prt}{100}$ $and\ CI = p[(1+r)^t - 1]$

Eg.if the difference of SI and CI is ₹ 72 at 12% for 2 yr, calculate the sum.

Ans; 72 = $p \times \frac{12}{100} \times \frac{12}{100}$, p = ₹ 5000.

Eg.the difference between the simple and compound interest on a certain sum for 3 yr at 5% p.a. is ₹ 228.75. the sum of money is ------**ans 30000**

→ **Population growth** $\boxed{P_1 = P_0(1+r)^t}$

→ **Depreciation (reducing balance Method)**

$\boxed{S = C(1-r)^n}$ where C = original cost,S= scrap value,n = effective life, r = rate of depr.

→**At CI rate:x_1 times in $t_1$yrs andx_2 times int_2 yrs**⇒$x_1^{t_2} = x_2^{t_1}$

Eg. 4 times in 6 yrs then In how many years it become 8 times at CI rate.

Ans : $4^t = 8^6 \rightarrow 2^{2t} = 2^{18} \rightarrow 2t = 18 \Rightarrow t = 9$ yrs

Eg.A machine worth ₹ 490740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 200000

Ans: apply$S = C(1-r)^n$, 200000 = 490740(1-0.15)^n⇒$(0.85)^n$ = 0.4075, n = 5.52

n = 5 year and 6 months

ANNUITY (INSTALLMENT)

→**ordinary annuity** (payments are at end of each period)

A/FV = $R\left[\frac{(1+i)^n-1}{i}\right]$ and loan/PV = $R\left[\frac{1-(1+i)^{-n}}{i}\right]$

→**annuity due**(payments are in the beginning of each period)

A/FV = $R\left[\frac{(1+i)^n-1}{i}\right](1+i)$ and PV =R+ PV of remaining (n-1) installments

→ **Perpetual Annuity/annuity forever i.e. (n →∞)**

PV = $\frac{R}{i}$

Eg. How much amount is required to be invested every year, so as to accumulate ₹ 300000 at the end of 10 year, if interest is compounded annually at 10%?

Ans: A = $R\left[\frac{(1+i)^n-1}{i}\right] \Rightarrow 300000 = R\left[\frac{(1.1)^{10}-1}{0.1}\right] \Rightarrow 300000 = R(15.9374),\ R = 18823.62$

Eg. The present value of annuity of ₹ 5000 per annum for 12 years at 4% per annum compound interest annually is

Ans: PV = $R\left[\frac{1-(1+i)^{-n}}{i}\right] = 5000\left[\frac{1-(1.04)^{-12}}{0.04}\right] = 5000(9.3850) = 46925.37$

Eg. Invest ₹ 10000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of this annuity.

FV = $R\left[\frac{(1+i)^n-1}{i}\right](1+i) = 10000\left[\frac{(1.08)^{10}-1}{0.08}\right](1.08) = 156454.87$

Eg. Suppose your mom decides to gift you ₹10000 every year starting from today for the next 16 years. You deposit this amount in a bank as and when you receive and get 8.5% per annum interest rate compounded annually. What is present value of this annuity?

Ans: PV = 10000 + PV of remaining 15 installments.

$$= 10000 + 10000\left[\frac{1-(1.085)^{-15}}{0.085}\right] = 10000 + 83042.37 = 93042.37$$

Eg. What should be deposit now at rate 10% p.a. to donate ` 600 every year?

Ans: PV = $\frac{R}{i} = \frac{600}{0.1} =$ ₹ 6000.

CHAPTER-5(BASIC CONCEPT OF PERMUTATIONS AND COMBINATIONS)

Factorial n = $\lfloor n$ or n!

→ n$\lfloor$ = n(n-1) (n-2) ×…….× 2 × 1 .

→ n! = n (n-1)! = n (n-1) (n-2)!

→ $0! = 1, 1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120\ etc$

Eg. LCM of 5! , 7! , 8! is highest one i.e. 8!

Eg. HCF of 5! , 7! , 8! Is smallest one i.e. 5!

Eg.$\frac{1}{5!} + \frac{1}{6!} = \frac{x}{7!}$⇒ x = 7^2 =49

Rules of counting

→ Multiple rule: A (l_1 ways) and B(l_2 ways) ⇒ total ways= ($l_1 \times l_2$) ways

→Addition rule: A (l_1 ways) or B (l_2 ways)⇒ total ways = ($l_1 + l_2$) ways

Eg. In how many ways you can go Delhi to Kolkata and return by different train, if 10 trains are runs b/w Delhi and Kolkata.

Ans: D to k and return→ 10 ways × 9 ways = 90 ways

Eg. In how many ways you can purchase a bike if there are two company Honda (5 ways) and yamha(10 ways).

Ans: Honda or Yamha = 5 + 10 = 15 ways.

Permutations(different arrangements)

→ n (different objects), no. of permutations

Taken r at a time

all at a time

$^nP_r = \frac{n!}{(n-1)!}$ =n(n-1)(n-2)………r times i.e. r factors

^nP_n = n!

Eg. 5 persons at 3 chairs

Ans. $5p_3$ = 5.4.3 = 60

Eg. Arrange the word FAILURE

Ans. $^7P_7$ = 7! = 5040

Eg. Total ways of arrangements of 3 letters A, B, C at 5 places is $5p_3$ = 5.4.3 = 60

Eg. Total ways of arrangements of the word TRIANGLE is $^8P_8 = 8! = 40320$

→ total number of re-arrangements = total ways – 1 = 40319

Permutations when not all different

→ **r at a time:** No. of ways $= {}^nP_r \div r_1!\ r_2!$,where r_1 and r_2, are alike of 1st type and 2nd type. Eg. Arrange 4 vowels A, A, A, I at 5 places. Ans. $^5P_4 \div 3! = 20$.→ **All at a time:** No. of ways $= \frac{n!}{n_1!.n_2!}$, out of n objects, n_1, n_2 are alike of 1st and 2nd type and rest are different. **Eg** Arrange the word COLLEGE, Ans. $\frac{7!}{2!.2!}$.

Eg. In how many ways 5 copies of math's book, 3 copies of accounts book and 2 copies of economics book can be arranged?

Ans: total ways $= \frac{10!}{5!.3!.2!}$.

Restricted Permutations

^np_r**(r at a time)**

1. when a particular never included. $^{n-1}p_r$

2. when a particular always include $r \times {}^{n-1}p_{r-1.}$

n!(**All at a time**)

1. When a particular objects always together
2. Never together
3. No two of them are together
4. When position of objects are specified.

Eg. Let the word TRIANGLE

Total ways of arrangements = 8! = 40320 i.e. without restriction

1. When vowels always together

I, A, E → 1 total ways = $6! \times 3!$
Rest →5

Total = 6

2. When vowels Never together = total – always together = $8! - 6! \times 3!$

3. When No. two vowels are together

1st arrange, 5 consonants and then 3 vowels b/w consonants (i.e. 6 places)5! $\times {}^6P_3$

4. When vowels occupying odd places.

Total places =8, in which 4 odd and 4 even places. First arrange vowels (3) at 4 odd places and then 5 consonants at remaining 5 places.${}^4P_3 \times {}^5P_5$ =2880

Note: together but in a Particular way (i.e. always left / right / before / after / a part of the word remain same / unaltered)

Eg. Arrange 10 persons so that A always to the left of B. total no. of ways is-

A, B→ 1 ans: total no. of ways = 9! × 1

rest→ 8

total =9

Eg. Arrange the word TRIANGLE so that ANGLE remains present in all arrangements.

Ans: ANGLE-1, T, R, I-3, total = 4, total ways = 4! × 1 = 24

Circular Permutations

1. Total ways of circular permutations of n different objects= (n-1)! (let n = 5, ans = 24)
2. Clockwise + Anti-clockwise = total cp
3. No of garland / Necklace, from n flowers / beads = $\frac{1}{2}(n-1)!$
4. **Restricted cps:-**

   Eg. 4 boys and 3 girls

   a. Total ways of cps =(7-1)! = 6! = 720

   b. 3 girls always together :total ways= (5-1)! ×3! =144

   c. 3 girls never together= total – together = 720 – 144 = 576

   d. No two girls together

   1st 4 boys in circle in 3! ways and then 3 girls at 4 places b/w boys in ${}^4p_3$ways total ways = $3! \times {}^4P_3$= 144.

E.g. In how many ways 5 persons can be arranged in circle such that A and B always together. Ans: (4-1)! × 2! = 12

Combinations (Selections)

→Total ways of selections = $^nc_r = \frac{n!}{r!\times(n-r)!} = \frac{n(n-1)(n-2)........r\ times}{r!}$

→ $^nP_r = {}^nC_r \,.\, r!$

→ Restricted Selections

1. When a particular neverSelected, total ways= $^{n-1}C_r$.
2. When a particular always selected ,total ways = $1 \times {}^{n-1}C_{r-1}$

→Selection of One or more (ie at least one)

$^nc_1 + {}^nc_2 + {}^nc_3 + ... + {}^nc_n = 2^n - 1$

(If each has an alternate, then $3^n - 1$)

→ Selection/arrangements from two categories

Eg. In how many ways 4 consonants and 3 vowels can be selected/ arranged from 12 constant and 5 vowels.

C(12)	V(5)	Total ways of selection	total ways of arrangements
4	3	$^{12}c_4 \times {}^5c_3$	$^{12}c_4 \times {}^5c_3 \times 7!$

.

→ **Properties**

∗ $^nc_r = {}^nc_{n-r}$

∗ $^nc_x = {}^nc_y \Rightarrow$ either x = y or x + y = n

∗ $^nc_r + {}^nc_{r-1} = {}^{n+1}c_r$

eg. $^nc_3 = {}^nc_{10} \Rightarrow n = 3 + 10 = 13$

eg. $^{18}c_r = {}^{18}c_{2r-3} \Rightarrow r = 2r-3$ or $r + 2r - 3 = 18 \Rightarrow r = 3$ or $r = 7$

eg. $^{10}c_2 + {}^{10}c_3 = {}^{11}c_x \Rightarrow x = 3$ or 8

eg. $^{10}c_2 + 2.{}^{10}c_3 + {}^{10}c_4 = {}^xc_4 \Rightarrow x = 12$

→ **Division:** Unequal e.g. 10 things into 3 parts 2 things, 3things and 5 things→ ans=$\frac{10!}{2!3!5!}$

Equal: 12 things equally →between 3 persons: ans=$\frac{12!}{4!4!4!}$

and → between 3 groups: ans=$\frac{12!}{4!4!4!3!}$

→ **Use of ^nc_r**

1. No. of handshakes among n guests = $^nc_2$
2. there are 'n' points in a plane No three points are collinear, then
   → No of straight lines = $^nc_2$
   → No of triangles = $^nc_3$
   → No of quadrilaterals = $^nc_4$
3. there are n points in a plane, No three points are collinear except 'p' points.
   → No of straight lines = $^nc_2$ -$^pc_{2\,+\,1}$
   → No of triangles = $^nc_3$ -$^pc_3$
4. no of diagonals in a Polygon, having 'n' sides = $\frac{n\ (n-3)}{2}$
5. Five parallel lines intersecting another set of 3 parallel lines. Total No of parallelograms = $^5c_2 \times {}^3c_{2} = 60$.
6. Maximum number of points of intersection by n circles is $^nP_2$

Eg Six points are on a circle.→ what is no of chords: $^6c_2$=15

Eg there are 54 diagonals in a polygon, what is number of sides:

solve the equation 54= $\frac{n\ (n-3)}{2}$Ans: n =12

Eg. maximum number of points of intersections by 10 circles is : $^{10}P_2 = 90$

Eg How many 4 digits numbers greater than 7000 can be formed out of the digits 3,5,7,8 and 9Ans: 3×4×3×2 =72

EgIn how many ways 5 Sanskrit, 3 English, and 3 Hindi books be arranged keeping the books of the same language together

(a) **5! × 3! × 3! × 3!** (b) 5! ×3! × 3! (c) $^5p_3$ (d) none of these

CHAPTER-6 (PROGRESSIONS A.P./G.P./H.P.)

Progression means numbers in a line separated by commas but in a definite ORDER.

→when difference b/w two consecutive numbers is same, called **A.P. (arithmetic progression)**where **a** is first term and **d** is called common difference.

Eg 2,4,6,8… Where a=2 and d=2

→when ratio b/w two consecutive numbers is same, called **G.P. (geometric progression)**

Eg 2,4,8,16 ….where a=2 and r=2 (common ratio)

→**H.P.(harmonic progression)** is the reciprocal of A.P.

Eg if a,b,c,d are in A.P. ⇒ 1/a,1/b,1/c,1/d are in H.P.

A.M. (arithmetic progressions)

→ **n^{th} term i.e. last term :** $\boxed{\boldsymbol{l, or\ an\ ,\ or\ \ tn\ =\ a + (n-1)\ d}}$

Eg the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots \dots \dots .10\ terms\ , find\ last\ term$

Ans : a=$\sqrt{2}$, d=$\sqrt{2}$ last term ie t_{10}=a+9d=………………$\sqrt{\mathbf{200}}$

Eg the fourth term of an A.P. is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. What is d?

(a) 1 **(b) 2** (c) 3 (d) none of these

→no of terms of an A.P., when last term is given $\boxed{\boldsymbol{n = \frac{l-a}{d} + 1}}$

Eg number of terms in AP: 0.3, 0.31, 0.32……………0.79 **(n =50)**

→when two terms, t_m= x and t_n =y then t_r is calculated in two steps

First find $\boxed{d = \frac{t_m - t_n}{m-n} = \frac{x-y}{m-n}}$ and then $\boxed{t_r\ =\ t_m +\ (r-m)d\ \ or\ t_n +\ (r-n)d}$

EgIf the m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$, then $(mn)^{th}$ term is

(a) 1 (b) 2 (c) 3 (d) none of these

Eg In a AP if the $(p+q)^{th}$ term is m and $(p-q)^{th}$ term is n, then the p^{th} term is

(a) $\boldsymbol{\frac{m+n}{2}}$(b) $\sqrt{mn}$ (c) m^2 (d) none of these

→**if a,b,c are in A.P. then** $\boxed{2b = a + c}$

EgThe numbers 2x, x+10 and 3x+2 are in A.P. then the value of x is **(x =6)**

Egif $\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$,then x, y, z are in (a) A.P. (b) G.P. **(c) H.P.** (d) None of these

→ **Sum of an AP: first** $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$ **or2nd** $\boxed{S_n = \frac{n}{2}(a + l)}$

Egthe sum of all numbers between 500 and 1000 which are divisible by 13 is:

Ans: sum = 507+520+…………..+988=**28405** **2^{nd} formula**

EgA club consists members whose ages are in A.P. , common differences being 3 months. The youngest member of the club is just 7 years old and the sum of the ages of all members is 250 years. The number of members in the club is:

(a) 15 (b) 20 **(c) 25** (d) none of these**1^{st} formula**

→In an AP find S_n from t_n: apply 2^{nd} formula

Egt_n =2n+3 of an AP, find $S_n$ans: put n=1 →a=5 and l=2n+3 , hence $S_n=\frac{n}{2}(5 + 2n + 3)$

→**In an AP, find t_n from S_n**

EgIf$S_n = 5n^2 + 3n$,then t_n is- ans: put n=1, a=8 and $\boxed{d = 2 \times coefficient\ of n^2}$→d=10

Hence t_n= 8+ (n-1)10= 10n-2

Eg if sum $S_n = 2n^2 + n$ of an arithmetic progression, then difference of its 10^{th} and first term is

(a) 15 (b) 20 **(c) 36** (d) none of these

→**Arithmetic means between two numbers x and y**

- Single AM, A = $\frac{x+y}{2}$
- n AMS : x, $A_1, A_2, A_3, \ldots\ldots\ldots A_n$, y are in A.P. where $\boxed{d = \frac{y-x}{n+1}}$

  Hence, A_1=x+d, A_2=x+2d, A_5=x+5d ………
- sum of all n AMS between x and y is $\boxed{n\left(\frac{x+y}{2}\right)}$

Egthe 4 arithmetic means between -2 and 23 are… **3, 8, 13, and 18**

Eg50 AMS are inserted between 2 and 104 . What is 25^{th} AM and what is sum of all AMS.

Ans : (54 and 2650)

G.P. (geometric progression)

n^{th} term : $\boxed{t_n = ar^{n-1}}$

egthe 10^{th} term of G.P. , -6,-1, -1/6, ………. Is

(a) $\frac{1}{6^8}$ (b) $\frac{1}{6^9}$ (c) $-\frac{1}{6^8}$ (d) none of these

Egno of terms in G.P. 16, 8, 4……. $\frac{1}{2^{15}}$ Is

(a) 20 (b) 21 (c) 22 (d) none of these

Egthe second term of a GP is 24 and the fifth term is 81, the series is

(a) 12, 24, 48….. **(b) 16, 24, 36…..** (c) 8, 24, 72….. (d) None of these

Sum of n terms of a G.P.

$\boxed{S_n = \frac{a(r^n-1)}{r-1}, if\ r > 1}$ Or $\boxed{S_n = \frac{a(1-r^n)}{1-r}, if\ r < 1}$ In case r=1, A.P./G.P./H.P. S_n = na

Sum to infinity i.e. n→∞, $\boxed{S_\infty = \frac{a}{1-r}}$

Eg if 256 + 128 +64 +……………..n terms =511. Then n is **(n=9)**

Ans: GP, a=256, r=1/2 and s_n= 511. Use $S_n = \frac{a(1-r^n)}{1-r}$ if r < 1

Eg If $x = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots \infty \ \ and\ y = 1 + \frac{1}{4} + \frac{1}{4^2} + \cdots \infty$ then find xy.**(ans=2)**

Eg find the product of (243), $(243)^{1/6}$, $(243)^{1/36}$, … ∞

Ans: product = $(243) \times (243)^{\frac{1}{6}} \times (243)^{\frac{1}{36}} \times \dots \dots . \infty$

$=(243)^{1+\frac{1}{6}+\frac{1}{36}+\cdots\infty}=729$

Geometric mean

- one GM between x and y, G =$\sqrt{xy}$i.e. $\boxed{if\ a, b, c\ are\ in\ GP\ then\ b^2 = ac}$
- 'n' GMS between x and y is

G_1= xr, G_2= xr^2, …….G_n=xr^n where $r = \left(\frac{y}{x}\right)^{\frac{1}{n+1}}$

EgIf a, b, c form an A.P. and b, c, a are in G.P. then $\frac{1}{c}, \frac{1}{a}, \frac{1}{b}$ are in

(a) AP (b) GP (c) HP (d) none of these

Note-1 Assuming terms in AP and GP

Number of terms	AP	GP
3	a-d, a, a+d	$\frac{a}{r}, \quad a, \quad ar$
4	a-3d, a-d, a+d, a+3d	$\frac{a}{r^3}, \quad \frac{a}{r}, \quad ar, \quad ar^3$

Note-2 sum of special series

- Sum of the first n natural numbers: $\sum n$ = 1+2+3+……..+n =$\frac{n(n+1)}{2}$
- Sum of the square : $\sum n^2$ = $1^2+2^2+3^2+\ldots\ldots+n^2$ = $\frac{n(n+1)(2n+1)}{6}$
- Sum of the cube of first n natural numbers: $\sum n^3$ = $1^3+2^3+3^3+\ldots\ldots\ldots+n^3$ = $\left[\frac{n(n+1)}{2}\right]^2$
- Sum of series: a+aa+aaa+…….. n terms =$\frac{a}{9}\left[\frac{10(10^n-1)}{9} - n\right] or \frac{a}{81}[10^{n+1} - 10 - 9n]$
- Sum of:0.a+0.aa+0.aaa+.... n terms =$\frac{a}{9}\left[n - \frac{1}{9}(1 - 10^{-n})\right] \; or \; \frac{a}{81}[9n - 1 + 10^{-n}]$

Note-3 if a, b, c… are in AP, let a=1, b=2, c=3, ……and a^2, b^2, c^2 are in AP let a=1, b=5, c=7

If a, b, c,…are in, GP let a=1, b=2, c=4,…….and a^2, b^2, c^2 are in GP let a=1, b=2, c=4

EgIf a, b and c are in AP, then the value of $\frac{(a^2+4ac+c^2)}{(ab+bc+ca)}$ is **(ans: 2)**

EgIf a, b, c, d are in GP, then a+b, b+c, c+d are in **(GP)**

EgIf a^2, b^2, c^2 are in AP, then b+c, c+a and a+b are in

(a) AP (b) GP **(c) HP** (d) none of these

Note-4→If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$is AM of a and bthen n=0 and for GM n= -1/2

→ If t_5=40 in an AP, then sum of first 9 terms (1 less than double) **(ans: sum=40×9)**

→ If t_5=k in a GP then products of first 9 terms- **(product=k^9)**

→ If the sum of n terms of two AP^s are in the ratio (3n+1): (n+4), then ratio of the fourth term is- put n = 4×2-1=7 ans is **2:1**

→ express $0.21\dot{7}\dot{5}$ (i.e. 0.217575757575……..) in fraction form.

(a) 2175/10000 **(b) 359/1650** (c) 259/1550 (d) none of these

CHAPTER-7(SETS, FUNCTIONS AND RELATIONS)

Sets: A set is collection of well-defined distinct objects (elements), it denoted by capital letters

E.g.Set of natural numbers = N, set of vowels = V

Representation of sets:

→**Roster form**i.e. listing elements separated by commas within brackets { }

e.g. N =$\{1,2,3,4, \dots\dots\dots\dots\}$, V= $\{a, e, i, o, u\}$

→**Set-builder form** i.e. describing / defining elements within brackets.

e.g. N = $\{x: x\ is\ natural\ numbers\}$, V = $\{ x \mid x\ is\ vowels \}$

Cardinal no of a set A (total no of elements) denoted by n(A)

Egn(N)= ∞ infinite set , n(V) = 5 finite set

Types of sets.

Null or empty set→ has no element, denoted by ϕ or { }

Singleton set → has only one element.

Equal sets → A =B means elements and number of elements of A and B are same and equal.

Eg A = {2, 3, 4} and B = {3 , 2, 4} ⇒ A=B

Equivalent sets → only number of elements are equal, n(A)= n(B)

Eg A = {2, 3, 4} and B = {a, b, c}

e.g. set { $1-(-1)^x$: where x is integers } is (a) null set **(b) finite set** (c) infinite set (d) singleton set

e.g. set { 2^x : where x∈ N} is (a) null set (b) finite set **(c) infinite set** (d) singleton set

Subsets of a setA

Let super set A ={2, 3, 5} then total subsets are {2}, {3}, {5}, {2,3}, {3,5}, {2,5}, {2,3,5}, ϕ.

i.e. total no of subsets = 2^n (n is no of elements in set A) in which one is super set and rest (2^n-1)are called proper subsets (⊂)Eg if n (A) =5, then total no of subsets =2^5=32 and total no of proper subsets =32-1=31

Power set of a set A is set of all subsets of A and denoted by P(A)

Eg if A = {a, b} then P(A) = { {a}, {b}, {a,b}, ϕ} ⇒ no of elements in power set is 2^nie

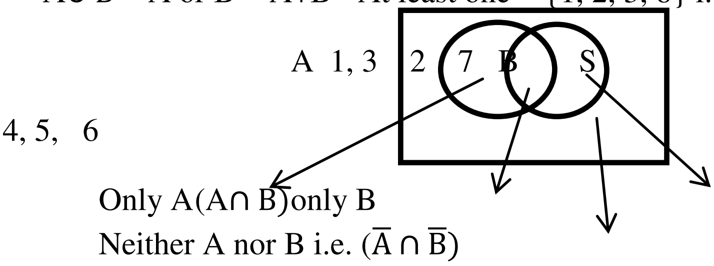
n[P(A)]=2^2=4

Algebra of sets

Let super set S ={1,2,3,4,5,6,7} and there two subsets A ={1,2,3} and B ={2,7}

→ n(S)=8, n(A) =3 and n(B)=2

→ A∪ B = A or B = A+B= At least one = {1, 2, 3, 8} i.e. n(A∪ B)= 4



- n(A∪ B) = n(A) +n(B)- n(A∩ B) = 3+2-1=4
- (A∪ B) = only A + only B+ (A∩ B)=2+1+1=4
- (A∪ B) = A + only B or B+ only A = 3+1 or 2+2=4

→ $A \cap B$ or A and B = {2}

→ Neither A nor B i.e. $(\bar{A} \cap \bar{B})$= total(S) - (A∪ B) ={4, 5, 6}

→ Only A or A but not B or A-B or $(A \cap \bar{B})$ =A- $A \cap B$ = {1, 3}

→ Only B or B but not A or B-A or $(\bar{A} \cap B)$= B- $A \cap B$ = {7}

→ Complement of set A or $\bar{A}$orA^c *or* A' or not A = total(S) - A = {4, 5, 6, 7}

→ Complement of set B or $\bar{B}$orB^c *or* B' or not B = total(S) - B = {1, 3, 4, 5, 6}

Properties:

Let U= universal set, A is a set ⇒ A ⊂ U because every set is subset of universal set.

(i) $A \cup A = A$ $(ii)\ A \cup U = U$ $(iii) A \cup \emptyset = A$ $(iv)\ A \cap A = A$ $(v)\ A \cap U = A$

(vi) $A \cap \emptyset = \emptyset$ $(vii) U' = \emptyset$ $(viii)\ A \cup A' = U$ $(ix)\ A \cap A' = \emptyset$ $(x)(A')' = A$

(xi) $(A \cap B)' = A' \cup B' (xii)(A \cup B)' = A' \cap B' (xiii)\emptyset' = U$

Formulas for three sets

(i) $n(\bar{A} \cap \bar{B} \cap \bar{C}) = total - n(A \cup B \cup C)$

(ii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(iii) Any two but only two

$\rightarrow n(\ A\ and\ B\ but\ not\ C)\ ie\ n(A \cap B \cap \bar{C}) = n(A \cap B) - n(A \cap B \cap C)$

$\rightarrow n(\ A\ and\ C\ but\ not\ B)\ ie\ n(A \cap C \cap \bar{B}) = n(A \cap C) - n(A \cap B \cap C)$

$\rightarrow n(\ B\ and\ C\ but\ not\ A)\ ie\ n(B \cap C \cap \bar{A}) = n(B \cap C) - n(A \cap B \cap C)$

(iv) Any one but only one

→ n (only A) i.e. n($A\cap \bar{B} \cap \bar{C}$) $= n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$)

→ n (only B) i.e. n($B\cap \bar{A} \cap \bar{C}$) $= n(B) - n(B\cap A) - n(B\cap C) + n(A\cap B\cap C)$)

→ n (only C) i.e. n($C\cap \bar{A} \cap \bar{B}$) $= n(C) - n(C\cap A) - n(C\cap B) + n(A\cap B\cap C)$)

Cartesian product of sets:

Let A and B are two non-empty sets, Cartesian product is $A\times B = \{(a,b)|\ a\in A\ and\ b\in B\}$

E.g. if A= {1, 2, 3} and B= {4, 5} then A×B = {1, 2, 3} × {4, 5}

={ (1,4), (1,5), (2,4), (2,5), (3,4), (3,5) }

And n (A×B)=n(A)×n(B)=3×2=6

E.g. If R is the set of positive rational number and E is the set of real numbers, then

(a) R $\subseteq$E **(b) R ⊂ E** (c) E ⊂R (d) none of these

E.g. If A⊂ B, then which one of the following is true

(a) A ∩ B=B **(b) A ∪ B=B** (c) $A \cap B = \emptyset$ (d) none of these

E.g. After qualifying out of 400 professionals, 112 joined industry, 120 started practice and 160 joined as paid assistants. There were 32 who were practice and industry, 40 in both practice and assistantship and 20 in both industry and assistantship. There were 12 who did all the three.

→find how many could not get any of these? **(88)**

→ find how many joined industry and practice but not assistantship. **(20)**

→ find how many started practice only. **(60)**

E.g. there are 20 players, 13 are cricket players and 8 played only cricket but not football, how many played only football? **(ans is 7)**

E.g. Identify the elements of P, if set Q={1, 2, 3} and P×Q= {(4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (6,1), (6,2), (6,3)}.

(a) {3,4,5} **(b) {4,5,6}** (c) {5,6,7} (d) none of these

E.g. If A= {2, 3}, B={4, 5} and C = {5, 6}, then A× (B∪ C)

(a) { (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)}

(b) {(2, 5), (3, 5)}

(c) { (2,4), (2,5), (3,4), (3,5), (4,5), (4,6), (5,5), (5,6)}

(d) None of these.

Relations:

Any subset of the product set X × Y is said to define a relation from X to Y.

Let X ={1, 2, 3} and Y = { 2, 4,} then X × Y= { (1,2), (1,4), (2,2), (2,4), (3,2), (3,4)}

Hence total no of relations from X to Y = 2^{mn}, where m=3, is no of elements in X and n=2 is no of elements in Y. total no of relations = 2^6 =64.

E.g. R_1={(1, 2), (1,4)}, R_2={(1,2), (2,2)}, R_3={(1,4), (2,4), (3,2)}…………..R_{64}

Types of relations

Reflexive(R):A relation R in a set A is said to be reflexive, if (a, a)$\in R, for\ all\ a \in A\ ie$

a R a is true for all a ∈ A i.e. every element of A is related to itself.

E.g. →Let A= {1, 2, 3}, relation R= {(1, 1), (2,2), (3,3)} is reflexive

→ For set of straight lines, relation R='is parallel to' is reflexive, because every line is parallel to itself.

Symmetric(S): A relation R in a set A is said to be symmetric, if$(a, b) \in R \Rightarrow (b, a) \in R$

i.e. if a R b is true then b R a is also true i.e. relation a to b is same as relation b to a.

e.g.→for set A, relation R= {(1, 2), (2, 1)} is symmetric but {(1, 1), (1,2)} is not R, not S

→ For set of straight lines, relation R= 'is perpendicular' is symmetric, if line l_1 is perpendicular to line $l_2 \Rightarrow$ line l_2 is also perpendicular to l_1.

Transitive (T): if (a, b) ∈ R and (b,c) ∈ R then (a,c) ∈ R i.e. if a related to b and b related to c then a is related c , called transitive.

Eg for set A , relation R={ (1,2), (2,3), (1,3)} is transitive.

And relation R= {(1, 1), (2, 2), (1, 2), (2, 1)} is S and T but not R

EgR= 'is parallel to' on set of straight lines is transitive, because l_1 R l_2 and l_2 R $l_3 \Rightarrow l_1$ R l_3.

A relation which is R, S, and T called equivalence (E)

Eg→ R= 'is parallel to' on set of straight lines.

→ R = 'has the same father' on set of children.

Functions: relations from X to Y in which no two different order pairs have same first element is called a function/mapping from X to Y. denoted by f : X → Y

Eg if X={1,2,3} and Y={4,6,8} then relation R= {(1,4), (1,6), (2,4)} is not a function, but R={ (1,4), (2,4), (3,6)} is a function from X to Y.

Where, set of first elements (pre-image i.e. x) are called domain, set of second elements (image i.e. y) are called range and whole set Y is called co-domain.

Eg F= {(1,2), (3,5), (4,5)}, domain={1,3,4} and range={2, 5}

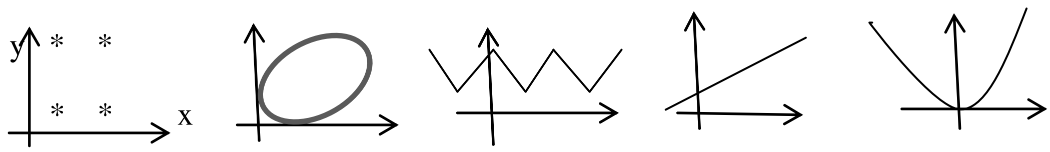
Note: In general domain means values of x (all real numbers) for which y is defined.

Eg domain of y=$\sqrt{x}$ is non-negative real numbers i.e. x≥0 or [0,∞)

Eg If f: A→R is a real valued function defined by f(x) =$\frac{1}{x}$, then A is R-{0}

Eg The range of the function f(x) =$\log_{10}(1+x)$ for the domain of real values of x when 0≤x≤9 is (a) (0,1) (b) {0, 1} **(c) [0,1]** (d) none of these

Eg which one is function?



No No Yes Yes (y = x+1) Yes (y = x^2)

Types of functions/ mapping

(i) One-one function: let f: A→ B, different elements in A have different image in B.

(ii) Many –one function: for a given image in B, have more than one pre-image.

Eg constant function, y =5

(iii) Into function (When range ⊂ co-domain) and onto function (when range = co-domain)

Eg If N= all natural numbers, E= all even natural numbers and f: N→E is defined by f(x) =2x, x ∈ N then 'f' is(a) One-one into **(b) one-one onto** (c) many to one (d) one to many

(iv) Inverse function: It is denoted by f^{-1} and defined as if f: A → B, then $f^{-1}: B \rightarrow$ A

Eg If A = {1, 2, 3, 4} and B = {2, 4, 6, 8},

$f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8\ and\ f: A \rightarrow B, then\ f^{-1}$is

Ans–{(2, 1), (4, 2), (6, 3), (8, 4)}

EgIf f(x) =$\frac{2+x}{2-x}$, then $f^{-1}(x)$ is Ans – first change, x→y and y→x and then find y= inverse function.

Here y=$\frac{2+x}{2-x}$change, we get x=$\frac{2+y}{2-y}$, Cross multiply, 2x-xy =2+y ⇒2x-2 = y+xy⇒2(x-1) =y(x+1),y=$\frac{2(x-1)}{(x+1)}$, Hence $f^{-1}(x)$=$\frac{2(x-1)}{(x+1)}$

Eg If y =$\log_{10} x$, *then inverse of y is –*

(a)10^x (b) x^{10} (c) 10x (d) none of these

(v) **Composite functions:** If f(x) and g(x) are two functions then fof, fog, gof and gog are called composite functions.
Eg f(x)= x+3, g(x)= x^2, then

fof= f{f(x)}=f(x)+3=x+3+3=x+6.

fog=f{g(x)}=g(x)+3=x^2+3.

gof=g{f(x)}=$f^2(x)=(x+3)^2$

gog=g{g(x)}=$g^2(x)=(x^2)^2=x^4$.

E.g. If $f(x) = \frac{1}{1-x}$ *and* $g(x) = \frac{x-1}{x}$, *then* $gof(2)$ *is*

(a) -1 (b) 1 **(c) 2** (d) none of these

E.g. If f: R →R is defined by f(x) =$(x+1)^2$, then find (fof)

E.g. If $f(x) = \frac{x}{\sqrt{1+x^2}}$ *and* $g(x) = \frac{x}{\sqrt{1-x^2}}$, *then find fog.*

(a) x (b) $\frac{1}{x}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$

E.g. If f(x-1) = x^2-4x+8, then f(x+2) is equal to

(a) x^2+8 (b) x^2+7 (c) x^2+4 **(d) x^2+2x+5**(hint: put x-1→ x+2 i.e. x→ x+3)

(vi) Even and odd functions: If a function f(x) is such that f(-x)=f(x), then it is an even function and f(-x)= - f(x), then it is an odd function.

Eg f(x) = x^2+2 is an even function and f(x) =$\left(x - \frac{1}{x}\right)$ is an odd function.

Eg [f(x) + f (-x)] is always even and [f(x) – f (-x)] is always odd.

If f(x) = $\frac{e^x+e^{-x}}{2}$ *is an even* and f(x) = $\frac{e^x-e^{-x}}{2}$ is an odd

(vii) Modulus function:

$$f(x) = |x| \quad \Rightarrow f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

CHAPTER-8 (LIMITS AND CONTINUITY)

- When LHL =RHL = l , ⇒ limit = $\underset{x\to k}{limit} f(x)$ = l
- When LHL ≠RHL, ⇒ limit does not exit.

Important results.

1. $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ sub-result , when a→1 i.e. $\lim_{x\to 1}\frac{x^n-1}{x-1}=n$

2. $\lim_{x\to 0}\frac{\log(1+kx)}{x}=k$ eg. $\lim_{x\to 0}\frac{\log(1+3x)}{x}=3$

3. $\lim_{x\to 0}\frac{e^{kx}-1}{x}=k$ eg. $\lim_{x\to 0}\frac{e^{-2x}-1}{x}=-2$, $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}=2$

4. $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$ eg. $\lim_{x\to 0}\frac{2^x-1}{x}=\log_e 2$

   Eg $\lim_{x\to 0}\frac{a^x+b^x-2}{x}=\log_e(a.b)$, $\lim_{x\to 0}\frac{a^x-b^x}{x}=\log_e\left(\frac{a}{b}\right)$, $\lim_{x\to 0}\frac{6^x-2^x}{4^x-2^x}=\log_2 3$

5. $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{ax}=\lim_{x\to 0}(1+kx)^{a/x}=e^{ka}$

   Eg. $\lim_{x\to 0}(1+3x)^{5/x}=e^{15}$, $\lim_{x\to\infty}\left(1+\frac{9}{x}\right)^{x}=e^9$

   Eg $\lim_{x\to\infty}\left(\frac{x+6}{x+3}\right)^{x}=\frac{e^6}{e^3}=e^3$

Q1. $\lim_{x\to a}\frac{x^5-a^5}{x^2-a^2}$ Q2. $\lim_{x\to 1}\frac{x^6-1}{x^5-1}$ Q3. $\lim_{x\to 1}\left(\frac{\sqrt[5]{x}-1}{\sqrt[3]{x}-1}\right)$

Q4. $\lim_{x\to 3}\frac{x^n-3^n}{x-3}=108, then\ n\ is$

Q5. $\lim_{x\to p}\frac{(x+2)^{5/3}-(p+2)^{5/3}}{x-p}$ Q6. $\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}$ Q7. $\lim_{x\to 0}\frac{4^{x+1}-4}{2x}$

Q8. $\lim_{x\to 0}\frac{8xe^x+6x}{\log(1+2x)}$ Q9. $\lim_{x\to 0}\frac{a^x-1}{b^x-1}$

Ans: (1) $\frac{5}{2}a^3$ (2) 6/5 (3) 3/5 (4) n=4 (5) $\frac{5}{3}(p+2)^{2/3}$ (6) log3. Log2 (7) 4log2 (8) 7 (9) $\log_b a$

Limit of rational functions$\left(\frac{p}{q}\ form\right)$

- When x → a (constant)
  Eg1. $\lim_{x\to 2}\frac{2x+3}{x^2+1}=\frac{7}{5}$Eg2. $\lim_{x\to 2}\frac{2x-4}{x^2+1}=\frac{0}{5}=0$

  Eg3. $\lim_{x\to 1}\frac{2x+3}{x^2-1}=\frac{5}{0}=\ \infty, limit\ does\ not\ exist.$

  Eg4. $\lim_{x\to 1}\frac{2x^2+x-3}{x^2-1}=\frac{0}{0}\ form.$ In this case there are two methods to find limit.

  Factor method: $\lim_{x\to 1}\frac{2x^2+x-3}{x^2-1}=\frac{(x-1)(2x+3)}{(x-1)(x+1)}=\frac{(2x+3)}{(x+1)}, put\ x\to 1\ ans{:}\ \frac{5}{2}$
  L hospital method: $\frac{derivative\ \ of\ Nr.}{derivative\ \ of\ Dr.}=\frac{4x+1}{2x}\ , then\ put\ x\to 1\ ans{:}\ \frac{5}{2}$

  Q1. $\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}$, Q2. $\lim_{x\to 0}\frac{e^x-(1+x)}{x^2}$, Q3. $\lim_{x\to 1}\frac{\log_e x}{x-1}$
  Q4. $\lim_{x\to\sqrt{2}}\frac{x^{5/2}-2^{5/4}}{x^{1/2}-2^{1/4}}$ Q5. $\lim_{x\to 1}\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}$

  Ans: (1) -5 (2) ½(3) 1(4) 10 (5) $\frac{n(n+1)}{2}$

- When x → ∞
  If highest power of Nr. is less than highest power of Dr.ans is: 0
  If highest power of Nr. is more than highest power of Dr. ans is: ∞
  If highest power of Nr. is equal to highest power of Dr. ans is: ratio of coefficient of highest power.
  Eg1. $\lim_{x\to\infty}\frac{2x^2-5x^2+2x+2}{3x^3+2x^2-6x+3}=0$ Eg2. $\lim_{x\to\infty}\frac{x^3-5x^2+2x+2}{x^2+2x^2-6x+3}=\infty$

  Eg3. $\lim_{x\to\infty}\frac{5x^3-5x^2+2x+2}{2x^3+2x^2-6x+3}=\frac{5}{2}$

  Q1. $\lim_{n\to\infty}(2n-1)2^n(3n+1)^{-1}2^{1-n}$ Q2. $\lim_{x\to\infty}\frac{e^x-1}{2e^x+3}$ Q3. $\lim_{x\to\infty}\frac{\sqrt{2x^2+3}}{4x+1}$

  Q4. $\lim_{n\to\infty}\left[\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\cdots+\frac{1}{5^n}\right]$ Q5. $\lim_{x\to\infty}\frac{(1^3+2^3+3^3+\cdots+x^3)}{x^4}$

  Q6. $\lim_{n\to\infty}(2n-1)(2+n)n^2(2n+1)^{-2}(2n+2)^{-2}$

  Ans : (1) 4/3 (2) ½ (3) $\frac{1}{2\sqrt{2}}$ (4) ¼ (5) ¼ (6) 1/8

Limit of irrational functions, $\frac{0}{0}$ form.

First rationalize and then cancel common factors in Nr. and Dr. and then put the limit. Or apply L hospital rule.

Eg1. $\lim_{x\to 4} \frac{3-\sqrt{5+x}}{x-4} =$

$$\lim_{x\to 4} \frac{3-\sqrt{5+x}}{x-4} \times \frac{3+\sqrt{5+x}}{3+\sqrt{5+x}} = \lim_{x\to 4} \frac{9-5-x}{(x-4)\left(3+\sqrt{5+x}\right)} = \frac{-1}{3+\sqrt{5+x}} \quad put\ x \to 4$$

$ans: \frac{-1}{6}$

Q1. $\lim_{x\to 1} \frac{x^2-1}{\sqrt{3x+1}-\sqrt{5x-1}}$ Q2. $\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{a+3x}-2\sqrt{x}}$

Q3. $f(x) = \frac{(x+1)}{3x+\sqrt{6x^2+3}}$, then $\lim_{x\to -1} f(x) is$

Ans: (1)-4 (2) $\frac{2}{\sqrt{3}}$ (3) 1

One sided limit.

When f(x) is given into two parts, for $x < k$ one function but for $x > k$ other function, then limit is calculated in two steps first LHL and 2nd RHL

LHL= $\underset{x\to k^-}{limit} f(x)\ where\ x < k$ and RHL= $\underset{x\to k^+}{limit} f(x)\ where\ x > k$.

When LHL=RHL, then limit exist but LHL≠RHL, limit does not exist.

In case of modulus function limit x→0, LHL and RHL is calculated, for LHL put $|x| = -x$ and for RHL put $|x| = x$.

Eg1. $f(x) = \begin{cases} \frac{1}{2} + x, & when\ x < \frac{1}{2} \\ \frac{3}{2} - x, & when\ x > \frac{1}{2} \end{cases}$ Find $\lim_{x\to \frac{1}{2}} f(x)$ and f (1)

LHL= $\lim_{x\to \frac{1}{2}} \frac{1}{2} + x = \frac{1}{2} + \frac{1}{2} = 1$ and RHL= $\lim_{x\to \frac{1}{2}} \frac{3}{2} - x = \frac{3}{2} - \frac{1}{2} = 1$ As LHL=RHL =1 ⇒ limit = 1

f (1) is given by function, $\frac{3}{2} - x = \frac{3}{2} - 1 = \frac{1}{2}$

Eg2. $\lim_{x\to 0} \frac{3x+|x|}{7x-5|x|}$

LHL= $\lim_{x\to 0} \frac{3x+(-x)}{7x-5(-x)} = \lim_{x\to 0} \frac{2x}{12x} = \frac{1}{6}$ and RHL = $\lim_{x\to 0} \frac{3x+(x)}{7x-5(x)} = \frac{4x}{2x} = 2$

As LHL ≠RHL, limit does not exist.

Continuity and discontinuity

f(x) is continuous at x= a when limit x→a and f (a) are equal i.e. limiting value = functional value.

Sometime, LHL = RHL= VALUE.

Note: 1. Dr. Zero is undefined, means for x=a, Dr. is zero, value does not exist⇒ function is discontinuous at x= a.

Note: 2. Values of x for which f(x) is defined called its domain, and every rational, inverse, exponential, logarithm, modulus, constant, identity etc are continuous in its domain.

Note: 3. if f and g are continuous at x= a, then f+g, f-g, f.g, f/g ($g \neq 0$) is also continuous at x=a.

.Q1. Let $f(x) = \begin{cases} \frac{x^2-16}{x-4} & for\ x \neq 4 \\ 10 & for\ x = 4 \end{cases}$

Ans: f(x) is not continuous, because limit (8) is not equal to functional value (10)

Q2. f(x) = 2x-1, $x \leq 1$ f(x) is continuous at **(A) x = 1 only** (B) x= 1, 2

= x^2, 1< x <2 (C) x =2 only (D) none of these

=3x-4, 2 ≤x <4

Q3. $f(x) = 2x - |x|$ is(A) discontinuous at x = 0 (B) undefined at x=0 **(C) Continuous at x =0** (D) none of these

Q4. If *f(x)* =3, when x<2 and f (x) =kx^2, when x≥2 is continuous at x=2, then k is

Put LHL=RHL ⇒ 3=4k**, k= ¾**

Q5. $f(x) = \frac{x^2-3x+2}{x-1}$, $x \neq 1$ becomes continuous at x=1. Then the value of f (1) is:

Value =limit, **f (1) = -1.**

Q6. If f(x) =-2x+a, $x \leq 2$ and f(x) = x-1, x>2 is continuous at x =2 then a= **(a=5)**

Q8.A function f(x) = x+1, when x≤ 1 and f(x) =3-px, when x>1. The value of p for which f(x) is continuous at x=1 is: **(p = 1)**

Q9.If $f(x) = \frac{x^3+x^2-16x+20}{(x-2)^2}$, $x \neq 2$ and f(x) =k, x=2 is continuous for all x then k= **(k=7)**

Q10.the points of discontinuity of the function $f(x) = \frac{x^2+2x+5}{x^2-3x+2}$, are Put Dr. =0, x = 1,2

CHAPTER-9(BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS)

Differential, derivative, differential coefficient, slope, gradient, rate of change, all are same.

For y derivative is denoted by $\frac{dy}{dx}$ and for f(x) derivative is denoted by $f'(x)$.

Result 1. $y = x^n$, $\frac{dy}{dx} = n.x^{n-1}$

eg . $y = k, \frac{dy}{dx} = 0$ eg. $y = x, \frac{dy}{dx} = 1$ eg. $y = x^2, \frac{dy}{dx} = 2x$ eg. $y = x^3, \frac{dy}{dx} = 3x^2$

eg. $y = \sqrt{x}, \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ **eg.** $y = \frac{1}{x} = x^{-1}, \frac{dy}{dx} = -1\, x^{-1-1} = \frac{-1}{x^2}$ **eg.** $y = \frac{1}{\sqrt{x}} = x^{-1/2}, \frac{dy}{dx} = \frac{-1}{2x\sqrt{x}}$

eg. Derivative of $5x^2$ is $5(2x) = 10x$

eg. Derivative of $8x^3$ is $8(3x^2)=24x^2$

eg. Derivative of $8+2x+5x^2-3x^3$ is $0+2+10x-9x^2$.

Q1. The gradient of the curve $y = 2x^3-5x^2-3x$ at x=2 is ------- **(-7)**

Q2. Let $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then $f'(2)$ will be ------- **(3/4)**

Q3. If xy = 1, then $y^2 + \frac{dy}{dx}$ is **(0)**

Q4. If $f(x) = x^k$, and $f'(1)= 10$ then k = ------ **(k=10)**

Q5. If $f(x) = x^2-6x+8$, then $f'(5) - f'(8)$ is equal to : **($3f'(2)$)**

Q6. If $f(x) = {}^xC_3$, then $f'(1)$ is equal to **(- 1/6)**

Q7. The gradient of the curve y +px +3y = 0 is 2, then p is **(p= -8)**

Result 2. $y = \log_e x, \frac{dy}{dx} = \frac{1}{x}$ When base is not e, $y = \log_a x = \frac{\log_e x}{\log_e a} = \log_a e \,.\log_e x, \frac{dy}{dx} = \frac{1}{x}\log_a e$ Eg. if $y= \log_{10} x, \frac{dy}{dx} = \frac{1}{x}\log_{10} e$

Result 3. $y = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty, \frac{dy}{dx} = e^x$

Result 4. $y = a^x, \frac{dy}{dx} = a^x.\log_e a$ eg. $y = 2^x, \frac{dy}{dx} = 2^x.\log_e 2$

Q1. if $y = 2^{\log_2 x}, \frac{dy}{dx}$ is **(1)**

Q2. if $f(x) = e^{3logx}$, then $f'(x)$ is **($3x^2$)**

Q3. $if\ y = e^{x\log 3}, then\ \frac{dy}{dx}\ is$ **($3^x.\log_e 3$)**

Q4. $if\ y = 2^{x+3} + \frac{4}{\log_x 3}\ , then\ \frac{dy}{dx}\ is$ **($8.2^x \log 2 + \frac{4}{x}\log_3 e$)**

Q5. If $y = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots\ \infty, then\ \frac{dy}{dx} - y =$ **(0)**

Rule of derivative

1. Product rule. y = u. v, $\frac{dy}{dx} = \frac{du}{dx}\ .v + u.\frac{dv}{dx}$

Eg $y = x.\log x,\ \frac{dy}{dx} = \frac{dx}{dx}.\log x + x.\frac{d\ \log x}{dx} = 1.logx + x.\frac{1}{x} = logx + 1.$

Eg. $f(x) = x.e^x, then\ f^{'}(x) = \frac{dx}{dx}.e^x + x.\frac{d\ e^x}{dx} = 1.e^x + x.e^x = e^x(x+1).$

2. Quotient rule. y = $\frac{u}{v}$, $\frac{dy}{dx} = \frac{\frac{du}{dx}.\ v - u\ .\ \frac{dv}{dx}}{v^2}$

Eg $y = \frac{x}{logx}\ ,\frac{dy}{dx} = \frac{\frac{dx}{dx}.\ logx - x\ .\frac{d\ logx}{dx}}{(logx\ )^2} = \frac{1.logx - x.\frac{1}{x}}{(logx\ )^2} = \frac{logx - 1}{(logx\ )^2}$

Eg. If $f(x) = \frac{x^2}{e^x}, then\ f^{'}(1)\ is$

Ans; $f^{'}(x) = \frac{\frac{dx^2}{dx}.e^x - x^2\ .\frac{d\ e^x}{dx}}{(e^x)^2} = \frac{2x.e^x - x^2.e^x}{e^{2x}}, put\ x = 1, f(1) = \frac{2e-e}{e^2} = \frac{1}{e}$

3. Parametric rule. If x = f (t) and also y = f (t), then $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{dy}{dt} \times \frac{dt}{dx}$

Eg find $\frac{dy}{dx}, if\ x = at^3, y = \frac{a}{t^3}.$

Ans: x = at^3,then $\frac{dx}{dt} = 3at^2$ and y = $\frac{a}{t^3} = at^{-3}\ , then\ \frac{dy}{dt} = \frac{-3a}{t^4}$.

$Now, \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{-3a}{t^4}}{3at^2} = \frac{-1}{t^6}.$

Eg. $if\ x = t\ e^t, y = 1 + logt, find\ \frac{dy}{dx}.$

Ans: $x = t.e^t,\ \frac{dx}{dt} = t.e^t + e^t.1 = e^t(t+1)$

y=1+logt, $\frac{dy}{dt} = \frac{1}{t}$

Hence $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1}{t} \times \frac{1}{e^t(t+1)} = \frac{1}{t(t+1)e^t}$

4. Chain rule. Is used when it is f(x) in place of x.

→ $y = [f(x)]^n, \frac{dy}{dx} = n[f(x)]^{n-1} . f'(x)$

→ $y = log f(x), \frac{dy}{dx} = \frac{1}{f(x)}. f'(x)$

→ $y = e^{f(x)}, \frac{dy}{dx} = e^{f(x)}. f'(x)$

→ $y = a^{f(x)}, \frac{dy}{dx} = a^{f(x)}. \log_e a \, . f'(x)$

Eg.$y = x^6, \frac{dy}{dx} = 6x^5. Now\ y = (3x + 2)^6, \frac{dy}{dx} = 6(3x + 2)^5 \times derivative\ of\ (3x + 2) = 18(3x + 2)^5$

Eg.$y = \sqrt{x}, \frac{dy}{dx} = \frac{1}{2\sqrt{x}}. Now\ y = \sqrt{2x + 3}, \frac{dy}{dx} = \frac{1}{2\sqrt{2x+3}} \times derivative\ of\ (2x + 3) = \frac{1}{\sqrt{2x+3}}$

Eg.$y = \sqrt{3x^2 + 2}, \frac{dy}{dx} = \frac{1}{2\sqrt{3x^2+2}} \times derivative\ of (3x^2 + 2) = \frac{3x}{\sqrt{3x^2+2}}$.

Eg.$y = \log(x + \sqrt{x}), \frac{dy}{dx} = \frac{1}{x+\sqrt{x}} \times derivative\ of\ (x + \sqrt{x}) = \frac{1}{x+\sqrt{x}} \times (1 + \frac{1}{2\sqrt{x}})$

Eg.$if\ f(x) = e^{ax^2+bx+c}, the\ f'(x)\ is$

Ans :$f'(x) = e^{ax^2+bx+c} \times derivative\ of\ (ax^2 + bx + c) = e^{ax^2+bx+c} \times (2ax + b)$

Eg.$if y = e^{\sqrt{2x}}, \frac{dy}{dx} = e^{\sqrt{2x}} \times derivative\ of\ \sqrt{2x} = e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}} \times 2 = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

Eg.$if\ y = 3^{(x^2+2x)}, \frac{dy}{dx} = 3^{(x^2+2x)}. log3 \times derivative\ of\ (x^2 + 2x)$

$=(2x + 2). 3^{(x^2+2x)}. log3$

Eg. Derivative of y →$\frac{dy}{dx}$, y^2 → 2y.$\frac{dy}{dx}$, logy → $\frac{1}{y}.\frac{dy}{dx}$

Derivative of implicit function.

A function in the form of f(x, y)= 0, called implicit function.

Eg. Find $\frac{dy}{dx}$ of x^2+ 3xy +y^2 =5

Ans: d. w. r. to x of x^2+3xy+ y^2 =5, we get

2x+ 3y+ 3x$\frac{dy}{dx}$ + 2y.$\frac{dy}{dx}$ =0 ⇒$\frac{dy}{dx} = \frac{-(2x+3y)}{(3x+2y)}$

Integration:

Integration is reciprocal process of differential.Hence, y or f(x) =$\int (slope)\ dx$.

Formulae:

1. $\boldsymbol{\int x^n\ dx = \frac{x^{n+1}}{n+1} + c,\ \ n \neq -1}$

Eg. $\int x\ dx = \frac{x^2}{2} + c$, $\int x^2\ dx = \frac{x^3}{3} + c$, $\int x^3\ dx = \frac{x^4}{4} + c$, $\int 2x\ dx = 2.\frac{x^2}{2} = x^2 + c$

Eg. $\int k\ dx = kx + c, \int 1\ dx = x + c, \int 2\ dx = 2x + c, \int 6x^2\ dx = 6.\frac{x^3}{3} = 2x^3 + c$

Eg. $\boldsymbol{\int \sqrt{x}\ dx = \frac{2}{3}x^{3/2} + c,\quad \int \frac{1}{\sqrt{x}}\ dx = 2\sqrt{x} + c,\quad \int \frac{1}{x^2}\ dx = -\frac{1}{x} + c}$

Eg. $\int (5x^3 + 3x^2 + 2x + 3)\ dx = 5.\frac{x^4}{4} + 3.\frac{x^3}{3} + 2.\frac{x^2}{2} + 3.x$

Eg. The equation of a curve passing through the point (1, 0) and if slope, $f'(x) = x - 1$is given by-

Ans: $y = \int slope\ dx = \int (x-1)dx = \frac{x^2}{2} - x + k$, put x=1 and y=0 then find k

0=$\frac{1^2}{2} - 1 + k\ , k = \frac{1}{2}$, hence $y = \frac{x^2}{2} - x + \frac{1}{2}$

2. $\boldsymbol{\int \frac{1}{x} = \log|x| + c}$

3. $\boldsymbol{\int a^x\ dx = \frac{a^x}{loga} + c,}$ $\quad$ $eg. \int 3^x\ dx = \frac{3^x}{log\,3} + c$

4. $\boldsymbol{\int e^x\ dx = e^x + c}$

Eg. $\int 2^x.5^{2x}\ dx = \int (2.5^2)^x\ dx = \int 50^x\ dx = \frac{50^x}{log\,50} = \frac{2^x.5^{2x}}{log\,50} + c$

Eg. $\int e^{2logx}\ dx = \int e^{log\,x^2}\ dx = \int x^2\ dx = \frac{x^3}{3} + c$

Eg. $\int (e^{3a.logx} + e^{3x.loga})\ dx = \int (x^{3a} + a^{3x})dx = \frac{x^{3a+1}}{3a+1} + \frac{a^{3x}}{3.loga} + c$

Eg. $\int 2^{\log_2 x}\ dx = \int x\ dx = \frac{x^2}{2} + c$

Types of integration:

Type1. $if\ \int f(x)\ dx = F(x) + c,\ \ then\ \int f(ax+b)\ dx = \frac{F(ax+b)}{a} + c$

Eg. $\int x^6\ dx = \frac{x^7}{7},\quad \int (2x+3)^6\ dx = \frac{(2x+3)^7}{7\times 2} = \frac{(2x+3)^7}{14} + c$

Eg. $\int \sqrt{x}\ dx = \frac{2}{3}.x^{3/2},\ \int \sqrt{3x+2}\ dx = \frac{2}{3}.\frac{(3x+2)^{3/2}}{3} = \frac{2}{9}(3x+2)^{3/2} + c$

Eg. $\int \frac{1}{\sqrt{x}}\ dx = 2\sqrt{x},\ \int \frac{1}{\sqrt{4x+5}}\ dx = \frac{2\sqrt{4x+5}}{4} = \frac{\sqrt{4x+5}}{2} + c$

Eg. $\int e^x\ dx = e^x,\ \int e^{2x+3}\ dx = \frac{e^{2x+3}}{2} + c$

Eg. $\int \frac{1}{x}\ dx = logx,\quad \int \frac{1}{2x+3} dx = \frac{\log(2x+3)}{2} + c$

Type2. $if\ \int \frac{p(x)}{q(x)}\ dx, where\ degree\ of\ p(x)\ \geq degree\ of\ q(x).$ Then first divide and then write the Q. in form $\left(quotient + \frac{remainder}{divisor}\right)$ and then integrate.

Eg. Evaluate: (i) $\int \frac{x^2}{(2x+1)}\ dx$ $\quad (ii) \int \frac{2x}{1+x} dx.$

Ans: (i) $\int (\frac{x}{2} - \frac{1}{4} + \frac{\frac{1}{4}}{2x+1})\ dx = \frac{x^2}{4} - \frac{1}{4}x + \frac{1}{4}.\frac{\log(2x+1)}{2} + c$ (ii) $\int \left(2 - \frac{2}{x+1}\right) dx = 2x - 2\log x+1+c$

Type3. $\int \frac{f'(x)}{f(x)}\ dx = logf(x) + c$

Eg. $\int \frac{(e^x - e^{-x})}{(e^x + e^{-x})}\ dx = log(e^x + e^{-x}) + c$

Eg. $\int \frac{2x}{1+x^2}\ dx = log(1+x^2) + c$

Eg. $\int \frac{dx}{x.logx} = \log(logx) + c$

Eg. $\int \frac{2x+1}{x^2+x+1}\ dx = \log(x^2+x+1) + c$

Eg. $\int \frac{x^2}{1+x^3}\ dx = \frac{1}{3}\int \frac{3x^2}{1+x^3}\ dx = \frac{1}{3}\log(1+x^3) + c$

Type4. Method of substitution (change of variable)

When $\int f[\emptyset(x)].\emptyset'(x)dx, then\ put\ \emptyset(x) = t\ and\ change\ the\ Q.in\ terms\ of\ t.$

Eg. $(i) \int \frac{2x}{(2x+1)^2}\ dx, put\ 2x+1 = t\ so\ that\ 2dx = dt\ or\ dx = \frac{dt}{2}$

$= \int \frac{(t-1)}{t^2}.\frac{dt}{2} = \frac{1}{2}\int \frac{(t-1)}{t^2}\ dt = \frac{1}{2}\int \left(\frac{1}{t} - \frac{1}{t^2}\right) dt = \frac{1}{2}\left(logt + \frac{1}{t}\right) = \frac{1}{2}\log(2x+1) + \frac{1}{2(2x+1)} + c$

(ii) $\int \frac{(x-1)}{\sqrt{x+4}}\ dx, put\ x+4 = t^2 \rightarrow dx = 2t\ dt$

$= \int \frac{(t^2-4-1)}{t}.2t\ dt = 2\int (t^2-5)dt = \frac{2t^3}{3} - 10t = \frac{2}{3}(x+4)^{\frac{3}{2}} - 10\sqrt{x+4} + c$

(i) $\int \frac{x^3}{(x^2+1)^3} dx = \int \frac{x^2(x\,dx)}{(x^2+1)^3}$, $put\ x^2 + 1 = t \to 2x\,dx = dt\ or\ x\,dx = \frac{dt}{2}$

$$= \frac{1}{2}\int \frac{(t-1)dt}{t^3} = \frac{1}{2}\int \left(\frac{1}{t^2} - \frac{1}{t^3}\right) dt$$

Integration by parts: $\int (I.II)\,dx = I\int II\,dx - \int \left[\frac{dI}{dx} \times \int II\,dx\right] dx$

Order of first or second→ log then algebraic and then exponential (L A E)

Eg. $\int x.logx\,dx = \int x(II).logx(I)\,dx$

$$= logx.\int x\,dx - \int \left[\frac{d\,logx}{dx} \times \int x\,dx\right] dx = logx.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}\,dx = \frac{x^2}{2}logx - \frac{x^2}{4} + c$$

Eg. Evaluate $\int 2x^3 . e^{x^2}\,dx, put\ x^2 = t\ so\ that\ 2x\,dx = dt\ = \int x^2 . e^{x^2} . 2x\,dx =$
t(I).et(II) dt=tetdt−d tdt×etdtdt=tet−1.etdt=tet−et=ett−1=ex2x2−1+c

Eg. $\int logx\,dx = \int 1(II).logx(I)\,dx = logx\int 1\,dx - \int \left[\frac{d\,logx}{dx} \times \int 1\,dx\right] dx = x.logx -$
$\int \frac{1}{x} \times x dx = x\,logx - x = x(logx - 1) + c$

Note: **I=** $\int \boldsymbol{e^x\,[f(x) + f'(x)]\,dx = e^x.f'(x) + c}$

Eg. $\int e^x(\frac{x-1}{x^2})\,dx = \int e^x(\frac{1}{x} - \frac{1}{x^2})\,dx = e^x(\frac{1}{x}) + c$

Eg. $\int e^x \frac{(xlogx + 1)}{x} dx = \int e^x\left(logx + \frac{1}{x}\right) dx = e^x.logx + c$

Eg. $\int \frac{xe^x}{(1+x)^2}\,dx = \int e^x \frac{(x+1-1)}{(1+x)^2} dx = \int e^x\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) dx = e^x\left(\frac{1}{1+x}\right) + c$

Eg. $\int \frac{(2-x)e^x}{(1-x)^2}\,dx = \int e^x \frac{(1-x+1)}{(1-x)^2}\,dx = \int e^x\left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2}\right] dx = e^x\left(\frac{1}{1-x}\right) + c$

Definite integration $\int_a^b \boldsymbol{f(x)dx} = [\emptyset(\boldsymbol{x})]_a^b = \emptyset(\boldsymbol{b}) - \emptyset(\boldsymbol{a})$

Eg. $\int_1^2 (2x + 1)dx = [x^2 + x]_1^2 = (2^2 + 2) - (1^2 + 1) = 4$

Eg. $\int_0^2 \sqrt{6x + 4}\,dx = \frac{2}{3}.(6x + 4)^{3/2}.\frac{1}{6} = \frac{1}{9}\left[(6x + 4)^{3/2}\right]_0^2 = \frac{1}{9}\left(16^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) = \frac{56}{9}$

Eg. $\int_2^4 \frac{2x}{1+x^2}\,dx = [\log(1 + x^2)]_2^4 = log17 - log5$

Eg. $\int_0^1 xe^x\,dx = [e^x(x - 1)]_0^1 = 0 - (-1) = 1$

CHAPTER-10(STATISTICAL DESCRIPTION OF DATA)

*** History:** The words statistics is derived from –

1st: Latin word → Status
2nd: Italian word → Statista
3rd: German word → Statistik
4th: French word → Statistique.

*** Definition / Meaning:-**

The word statistics is used in Two senses:-

1. In Plural sense: - it is<u>Statistical or numerical data.</u>
2. In Singular Sense: - it is<u>Statistical or Scientific Method.</u>

1. Statistical data: - Group Study / Aggregate Study.

Variables (x) over a group, Observed and recorded, called statistical data

Example: - Which one is Statistical Data.

A. Age of A 20 Year
B. Height of A = 160 cm
C. Weight of A = 50 kg
D. Weight A (50 Kg), B (60 kg), C (65 kg).

Variable

- Qualitative /Attribute (ordinal)
- Quantitative (Cardinal)
  - Discrete Variables → Only Integral Values
  - Continuous Variables → Can be any Values within a range

Qualitative /Attribute (ordinal):

- ➢ Intelligency
- ➢ Honesty
- ➢ Gender
- ➢ Marital Status
- ➢ Blindness
- ➢ Nationality
- ➢ Drinking habitect.

- Discrete variables :
  - ➢ No. of children in a family.
  - ➢ No. of Miss - print in a page.
  - ➢ No. of road accident in a day.
  - ➢ No. of Shares distributed by a company.
  - ➢ <u>Annual Income</u> ,<u>Marks Obtained</u>Etc.

- Continuous Variables :
  - ➢ Height, weight, Age, Profit, % marks etc.

2. Statistical Method:- Deals with four Things

- ➢ Collection of data
- ➢ Presentation of Data.
- ➢ Analysis of data and
- ➢ Interpretation of the result.

***Collection of Data**

Primary Data
(Which is collected for first time?)
↓
Methods
↓

→ Interview Method
- o Direct (Personal) Interview
- o Indirect Interview
- o Telephonic Interview

→ MailedQuestionnaire
→ Observation Method

Secondary Data
(Already collected data, used by others)
↓
Sources

- ➢ **Personal :-** Interview is Suitable In case of Natural calamity.
- ➢ **Indirect :-** In case of rail / road / air accident
- ➢ **Telephone:-**
  - Fast and wide Area is covered.
  - Less Response.
- ➢ **Mailed :-**
  - Widest Area is covered.
  - Least response
    i.e. Amount of Non-response is maximum.
- ➢ **Observation Method:-** Data on Height , weight, age , Marks etc. are 1^{st} Observed / Measured and then recorded.

<u>Sources of Secondary data</u>

- ➢ International Source: - Word bank, WHO, IMF etc.
- ➢ Government Source: - NSSO, CSO etc.
- ➢ Private Source
- ➢ Unpublished Source.

* **Use of Stats:-**
  - Business / Commerce
  - Economics
  - Industry

* **Scrutiny of data: -** Checking consistency and reliability of data.

Internal Check → Is possible for related series of data

eg. Sales – Cost = Profit

1,000 – 500 = 400.

External Check → For Confirmation

Classification of data: -

1. Qualitative Data
2. Quantitative Data
3. Geographical/special/ data varying over space i.e. on the basis of area.
4. Time Series Data/ temporal data/ chronological data i.e. on the basis of time.

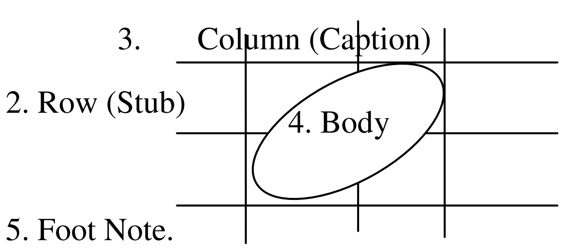
<u>Presentation:-</u>

- Textual Presentation
- Tabular Presentation
- Diagrammatic Presentation
- Frequency Distribution
- Graphical Presentation

→ **Textual Presentation:** This is simplest mode of presentation in which data is presented in the form of text and paragraph by paragraph.

→**Tabular Presentation:** Textual data is re-presentation into a table.

1. Box head (Title and Head Note)



- **Stub: -** Left part of the table, describing row.
- **Caption: -** Upper Part of the table, describing columns.
- **Body: -** Common Part of rows and columns. It contains numerical data of Information
- **Head Note: -** Explain unit of numerical data.
- **Foot Note: -** Explain any Short-Form used in rows/columns and Source if any

Note: → Tabulation the best mode of presentation i.e. Accurate mode of Presentation
→ In a Table any two row and any two columns can be compared.

E.g. In the year 2005, there are 1000 workers in a factory out of which 25% are female and rest are male workers. 700 workers are experience and 30% of experience workers are female.

Find number of male-experienced workers **[260]**

→ Diagrammatic Presentation: -

- Charts, Bars, Pictures etc.
- Rough Sketch. (Less accurate)
- Attractive Mode of Presentation.
- Hidden trend if any is reflected under this mode of presentation.

→ Charts: -

- Simple Line Chart: - For one Variable.
- Multiple Line Chart / Comparative Line Chart : For two Variables having same unit.
- Multiple axis Line Chart: - For Two variables, having different unit.
- Ratio Chart: - Line Chart between Logarithm of variable and time when big fluctuations and for small fluctuations, diagram is false base line.

Bar Diagrams

Horizontal Vertical

Qualitative data/ data varying over space Quantitative Data/ time series data

→Pie chart or divided/group bar diagram

A total Figure, which is sum of two or more components, is presented into a pie chart or divided bar diagram. In which we can compare any two components and measure one component as a percentage of total figure.

Eg. Family budget of monthly expense

Eg. Total GDP from different sectors

.

Frequency Distribution

Individual Series: (n → by counting), where n = number of observations

X: $x_1, x_2, x_3, x_4, x_5, \ldots, x_n$

1. Ungrouped Frequency Distribution
(discrete series)
Mutually inclusive/mutually exclusive/

2. Grouped Frequency Distribution

Non-overlapping/
For discrete variabl

overlapping/
continuous variable

Conversion of inclusive type(includes both limits) into exclusive type / continuous series

Class limits	f		Class boundaries	f
10-19	2		9.5-19.5	2
20-29	5	→	19.5-29.5	5
30-39	3		29.5-39.5	3

Note: → Most extremes values of a class is class boundaries

→ Class length (i) = UCB-LCB

→ Class mid-value / class mark (m) = $\frac{LCL+UCL}{2}$ or $\frac{UCB+LCB}{2}$

→ Frequency density = $\frac{class\ \ frequency}{class\ \ leng\,ht} = \frac{f}{i}$

→ Relative frequency = $\frac{f}{\sum f} = \frac{frequency}{total\ \ frequency}$

→ Percentage frequency = $\frac{f}{\sum f} \times 100 = \frac{frequency}{total\ \ frequency} \times 100$

→ Number of classes (k) = $\frac{range}{class\ \ len\,ght} = \frac{R}{i} = \frac{L-S}{i}$

E.g. From the following data find the number of class-intervals / number of classes, if class length is given as 5 and data is 73, 72, 65, 41, 54, 80, 50, 46, 49, 53.

Ans: Number of classes (k) = $\frac{range}{class\ length} = \frac{R}{i} = \frac{80-41}{5} = 7.8 = 8$

Graphical Presentation

1. Histogram
2. Frequency Polygon
3. Frequency Curve
   (1–3: Graph between x& f)
4. O-Give → graph between x and cf (cumulative frequency)

Histogram/polygon/curve

- Histogram is prepared from Continuous Series.
- For histogram / Mode, class Length is equal.
- Histogram appears Like bar diagram , but histogram is area diagram (rectangle)
- Mode is calculated graphically from histogram
- Mid-points of histogram are joined by straight lines, called frequency polygon.
- Mid-points of histogram are joined by free hand i.e. smooth curve, called frequency curve

Frequency Curve:-

- Total areas assume to be unity.
- Frequency curve is limiting form of histogram or frequency polygon.

Bell Shape: Most common frequency curve is bell shape. Data on height, weight, marks, profit ect have bell shape frequency curve.

U-shape, J-Shape, Mixed Shape

O-give:

x:	5-10	10-15	15-25	25-30	n = $\sum f = 50$
f:	5	10	20	15	

Cumulative frequency table

Less than cf table (From upper limit) More than cf table (From lower Limit)

Less than x	cf		More than x	cf
Less than 10	5		More than 5	50
Less than 15	15		More than 10	45
Less than 25	35		More than 15	35
Less than 30	50		More than 25	15

- Largest cf = n = 50
- Cf Less than x_1 + Cf more than x_1 = n
- Number between x_1 (15)and x_2 (25) = difference between their cf = 35-15=20
- Less than 15 = total – more than 15 = 50 – 35 = 15
- More than 10 = total – less than 10 = 50 – 5 = 45

Median, Quartiles, deciles, percentiles (Partition values) are graphically calculated from o-give, specially less than o-give (J shape)

Note:

→**One dimensional (1-D):** having only length **e.g.** All type of bar diagrams and all type of line charts

→ **Two dimensional (2-D):** having length and width i.e. area diagram **e.g**. rectangles, squares, circles, pie charts, histograms etc.

→ **Three dimensional (3-D):** having length, width, and depth i.e. volume e.g. cube, cuboid, cylinder etc.

CHAPTER-11(MEASURES OF CENTRAL TENDENCY AND DISPERSION)

Arithmetic Mean

- Individual Series : $\bar{x} = \frac{\sum x}{n}$
- Discrete Series : $\bar{x} = \frac{\sum fx}{n}, n = \sum f$
- Continuous Series : $\bar{x} = \frac{\sum fm}{n}, n = \sum f$

E.g. AM of X: 5, 6, 4, 6, and 8. $\sum X = 5 + 6 + 4 + 6 + 8 = 29$, n = 5 hence
$\bar{x} = \frac{\sum x}{n} = \frac{29}{5} = 5.8$

E.g.AM of x = 1, 2, 3, 4, 5
f = 1, 2, 3, 4, 5
fx = 1, 4, 9, 16, 25 $\bar{x} = \frac{\sum fx}{n} = \frac{55}{15} = \frac{11}{3}$

Example: AM of following data

x:	10-20	20-30	30-40
f:	5	2	3
m:	15	25	35
fm:	75	50	105

$\sum fm$= 75 + 50 + 105 = 230 and n = $\sum f$ = 10 $\bar{x} = \frac{230}{10} = 23$.

Note: In grouped frequency distribution, it is assumed that each value of a class is equal to **Mid-value / class mark.**

Properties of AM:

1. If X: 10, 10, 10 ………..then, average = 10 ⇒ mean, median, mode, etc. =10

2. Sum of all observations is equal to product of no. of observation with AM mean.
$\bar{x} = \frac{\sum x}{n} \Rightarrow \sum x = n.\bar{x}$

$Example : n = 5,\ \bar{x} = 80$

Sum of all values $\sum x = \bar{x}n = 80 \times 5 = 400.$

E.g. If an average mark of 10 Students is 75. Two more students with marks 60 and 65 joined this group, what is new average.

Ans: new average = $\frac{10\times75+60+65}{12} = 72.92$

E.g. AM of 10 observations calculated as 40, later on it found that two values 35, 50 wrongly taken as 25 and 45. What is correct AM

$$correct\ mean = \frac{10 \times 40 - 25 - 45 + 35 + 50}{10} = 41.5$$

3. Desirable mathematical property/combined mean (AM) / Pooled AM.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Example: $n_1 = 6$ $n_2 = 4$ $\bar{x}_1 = 12$ $\bar{x}_2 = 20$ Combined mean= $\frac{72+80}{10} = \frac{152}{10} = 15.2$

4. Sum of Deviations from AM is always zero

$$\Sigma(x - \bar{x}) = 0$$

5. Sum of Squares of deviations from AM is always "Least / Minimum"

$$\Sigma(x - \bar{x})^2 = Least$$

6. Change / Shift of origin and scale

Arithmetic mean (Median, Mode) are affected by both –

- ✓ Change of origin (adding/subtracting a number in all values)
- ✓ Change of scale (multiply/ divide all values with a number)

If y = ax + b where scale = a and origin = b then,

$\bar{y} = a\bar{x} + b$, median of y = a. Median (x) + b, Mode of (y) = a Mode (x) + b

This property is also called **<u>Linear relation property.</u>**

Example:→ if $y = 10 + 2x$ and $\bar{x}$ = 5 then, $\bar{y}$ = 10+2(5) = 20

→ if x and y are related as 3x + 5y – 60 = 0 and mode of x is 12 then mode of y is …

3(12) + 5 (mode of y) – 60 = 0 ⇒ mode of y = 4.8

→ M_e (x) = 50, then M_e (x +2) = 52, M_e (2x-3) = 2(50)-3 = 97

Geometric Mean:

GM of 'n' observation is equal to n^{th} root of their products.

Example: GM of 4, 9

GM = $\sqrt{36}$ = 6

Example: GM of 2, 4, 8

GM = $\sqrt[3]{64}$ = 4.

Example: GM of -2, 8

GM is not define

If some values are –ve, and some values are +ve, we should not consider GM as an average.

Like: average Profit/Loss

Example: GM of P, P^2 , P^3,……………..P^n

$= (P^1, P^2, P^3, \ldots.. P^n)^{1/n}$

$= (P^{1+2+3+\ldots\ldots n})^{1/n} = \left(P^{\frac{n(n+1)}{2}}\right)^{1/n} = P^{\frac{n+1}{2}}$

E.g. If GM of a, b, c, d…. is k, then GM of 1/a, 1/b, 1/c, 1/d……. is 1/k

E. g. what is GM of 4, 4^2, 4^3, 4^4 …4^{n-1}

Ans: GM = $(4.4^2.4^3.4^4. \ldots..4^{n-1})^{1/n-1}$ = 2^n

Properties of GM

1. X = 10, 10, 10 …
   GM = 10.
2. GM (x.y) = GM (x) × GM(y)

3. GM (x/y) = GM (x) / GM(y)

Example: if GM (x) = 6 and GM (y) = 10, then

→ GM (xy) = 6×10 = 60.

$$\rightarrow GM\left(\frac{3y}{x}\right) = \frac{GM(3y)}{GM(x)} = \frac{3\times 10}{6} = 5$$

4. Combined GM /pooled GM is calculated.

Harmonic Mean

- Individual Series : HM = $\frac{n}{sum\ of reciprocals} = \frac{n}{\sum\frac{1}{x}}$

HM is defined when no observation is zero

Example: HM of 4, 6 is $\frac{n}{\sum\frac{1}{x}} = \frac{2}{\frac{1}{4}+\frac{1}{6}} = 4.8$

Example: HM of 3, 4, 6 is $\frac{3}{\frac{1}{4}+\frac{1}{3}+\frac{1}{6}} = 4.$

Example: HM of 1, ½, 1/3 ……..1/n

HM = $\frac{2}{n+1}$

Properties:

1. X: 10, 10, 10……..HM = 10

2. Combined / Pooled HM (Desirable mathematical property)

Example: $n_1 = 6$ $n_2 = 4$

$H_1 = 25$ $H_2 = 35.$

$$\text{HM} = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}} = \frac{12}{\frac{1}{5} + \frac{1}{5}} = \frac{12}{2} \times 5 = 30.$$

Relation b/w AM, GM, HM

- For equal values,
  AM = GM = HM
- For unequal (district values)
  AM > GM > HM
- In General : AM $\geq$ GM $\geq$ HM

- For only 2 positive values (x, y) $(GM)^2$ = AM x HM

Example: For two positive numbers AM = 10 and GM = 8

→ Then HM is: $8^2/10 = 6.4$

→ Two values are **(a) 4, 16** (b) 8, 12 (c) 10, 10 (d) none of these

Use of AM / GM / HM

→ GM / HM is used to find average rate/ ratio: when rate in percentage then GM is best and when rate is per kg, per hour, per unit etc. Then HM is the best.

→ AM is the best average / most common average generally: Average weight, Average marks, Average Profit, Average Length etc. is calculated by AM.

E.g. A bus travel from A to B @ 40 k/h and return @ speed 60 k/h, what is average speed

Ans: Average Speed = HM of 40 and 60 $HM = \frac{2}{\frac{1}{40} + \frac{1}{60}} = 48k/h$

Partition Values: Median, Quartiles, deciles, Percentiles

First arrange all values in ascending order and apply formula as below

Example: X (`):10, 8, 15, 20, 40, 12, 18, 25, and 32.

Ascending order: 8, 10, 12, 15, 18, 20, 25, 28, 32, 40

Find Me, Q_1, Q_3, D_3, P_{45}.

$$Median = value of \frac{1}{2}(10+1) = \frac{11}{2} = 5.5 \rightarrow Simple average of 2 middle Terms 18,20 = 19$$

$$Q_1 = value of \frac{1}{4}(11) term = 2.75 \rightarrow 10 + 75\%(2) = 10 + 1.5 = 11.5$$

$$Q_3 = value of \frac{3}{4}(11) = 8.25 term \rightarrow 28 + 25\%(4) = 28 + 1 = 29.$$

$$D_3 : \frac{3}{10}(11) = 3.3 \rightarrow 12 + 30\%(3) = 12.9$$

$$P_{45} : \frac{45}{100}(11) = 4.95 \rightarrow 15 + 95\%(3) = 15 + 2.85 = 17.85$$

Note: when 'n' is even then median is simple average of two middle terms

Example: Median of $\frac{x}{2}, \frac{x}{5}, \frac{x}{8}, \frac{x}{3}$ $is\ 15\ then\ the\ value\ of\ x$[x = 36]

Example: There are 11 students in a class, 4 failed in exam and marks of remaining studentsare 20, 18, 10, 12, 15, 26, 17. What is the median mark of the class?

Ans: median is $\frac{1}{2}(11+1)th\ term\ i.e.6th\ term\ in$ F_1, F_2, F_3, F_4, 10, 12, 15, 17, 18, 20, 26 = 12

Median is not dependent an all values i.e. median is not affected by extremes values hence in case of open ended classification median is the best average.

Mode: variable (x) having largest frequency is called mode

E.g. X: 2, 5, 4, 5, 6, 5 Mode = 5.

→ uni-modal i.e. mode is well define.

E.g.

x:	2	4	5	6	7	8
f:	1	1	10	10	1	1

Mode 5, 6

Bi-Model i.e. mode is ill-define.

→ Mode may not uniquely define.

→ Frequency distribution is necessary for mode

E.g.

x:	5	6	8
f:	10	10	10

Mode is not defined.

Note: Empirical relation between Mean, Median and mode

Mo = 3 Me - 2$\bar{x}$ *or* $M_o - \bar{x}$ = 3 (Me - $\bar{x}$)

E.g. mean = 10 and median = 12 then mode = 3(12)-2(10) = 16

E.g. difference b/t mode and mean is 63 then what is difference b/w median and mean

Ans: 63/3 = 21

Note: **Weighted Average**

There are some values and each having different importance (weight) then in this case weighted average is calculated.

If weights are equal then there is no difference between simple mean and weighted mean

Measures of Variation(Dispersion / scatterness)

Range, QD, MD, and SD

Note: → S.D is the best dispersion generally i.e. most common dispersion

→ Variation (SD) and CV = co-efficient of SD

→ Relative measures are used for comparison

Range: R = L – S and Co-efficient of Range = $\frac{L-S}{L+S}\times 100$.

E.g.x (₹) :10, 5, 15, 40, 30, 45, 6, 10, 18Range = 45-5 = `40Co-e*ff.* of Range =
$\frac{45-5}{45+5}\times 100 = \frac{4,000}{50} = 80$ **Co -** efficients **are unit free**

Example:

x:	2, 5, 8
f:	1, 10, 2

Range = 8 - 2 = 6 Co-e*ff.* of Range = $\frac{6}{10}\times 100 = 60.$

Range is independent of f' and range is quickest to calculate.

E.g.	*x:*	10-19	20-29	30-39
	f:	-	-	-

Range = 39.5 – 9.5 = 30.

Note: **Open Ended Classification**

Largest, Smallest i.e. extreme value is absent.

- In open ended classification Q.D is the best dispersion.
- Median is the best average in open ended classification because median is not affected by extreme values.

<u>Quartile Deviation</u>

- Inter Quartile range = $Q_3 - Q_1$(Middle 50% Value)
- Semi – Inter Quartile range = Q.D.
- $QD = \frac{Q_3 - Q_1}{2}$
- $Co - eff.of\ Q.D = \frac{Q_3 - Q_1}{Q_3 - Q_1}\times 100.$
- $Me = Q_1 + QD = Q_3 - QD = \frac{Q_1 + Q_3}{2}$
- $Co - eff.ofQD = \frac{QD}{Me}\times 100.$

E.g. what is QD and it's co-efficient to the following data: 10, 5, 25, 8, 22, 18, and 15

Ans: ascending order: 5, 8, 10, 15, 18, 22, 25 where Q_1 = 2nd term = 8 and Q_3 = 6th term = 22

Hence $QD = \frac{Q_3 - Q_1}{2} = \frac{22-8}{2} = 7$

E.g. mean = 8, median = 5 and QD = 2 then find Q_1 and co-efficient of QD.

Ans: Q_1 = median – QD = 5 – 2 = 3 and co-efficient of QD = $\frac{QD}{M_e} \times 100 = 40$

Mean deviation:

$$MD = \frac{\sum|D|}{n}\ , \quad co - efficient\ of\ MD = \frac{MD}{mean\ or\ M_e\ or\ M_o} \times 100$$

→Where: D = X-mean or M_e or M_o i.e. deviation about mean or median or mode as given.

→ MD is based on absolute/positive deviations

→ Sum of positive deviations about median is always least i.e. $\sum|X - M_e|$is minimum.

E.g. find co-efficient MD of first 9 natural numbers (1, 2, 3, 4, 5, 6, 7, 8, 9)

Ans: n = 9, mean = 5, $\sum|D| = 4+3+2+1+0+1+2+3+4 = 20$

$$MD = \frac{\sum|D|}{n} = \frac{20}{9}\ , \quad co - efficient\ of\ MD = \frac{MD}{mean} \times 100 = \frac{\frac{20}{9}}{5} \times 100 = \mathbf{\frac{400}{9}}$$

E.g. what is the value of mean deviation about mean for the following numbers?

X: 5, 8, 6, 3, 4 **ans: 1.44**

Standard deviation:

σ = $\sqrt{\frac{\sum X^2}{n} - \bar{X}^2}$, direct method	σ = $\sqrt{\frac{\sum(X-\bar{X})^2}{n}}$	σ = $\sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$ and $\bar{X} = A + \frac{\sum d}{n}$, d = X – A

→Co-efficient of variation (CV)/ co-efficient of SD = $\frac{SD}{mean} \times 100$ i.e. SD as a percentage of mean.

→Variance = SD^2 and $SD = \sqrt{variance}$

E.g. The sum of squares of deviation from mean of 10 observations is 250. Mean of data is 10, find the co-efficient of variation

Ans: here n = 10, $\bar{X} = 10$ *and* $\sum(X - \bar{X})^2 = 250$

σ = $\sqrt{\frac{\sum(X-\bar{X})^2}{n}} = 5, hence\ CV = \frac{5}{10} \times 100 = 50$

E.g. Σx^2=3390, n = 30, σ =7 then AM of x -- a) 113 b) 210 **c) 8** d) None

E.g. if SD = 5, then variance = 5^2 =25

E.g. if mean = 20 and co-efficient of variation = 40 then SD is -- **[8]**

Important notes:

1. If all values are equal, then variation is zero. i.e. X → 5, 5, 5, 5……..then

Range = QD = MD = SD = 0

2. If there are only two values **a** and **b,** then

$AM = \frac{a+b}{2}\ and\ SD = \frac{|a-b|}{2} = \frac{difference}{2} = \frac{range}{2}$ i.e. half of range

E.g. If AM is given by $\frac{2+5}{2}$ then variance is ….

Ans: SD = $\frac{5-2}{2} = 1.5\ then\ variance = 1.5^2 = 2.25$

3. AM of first n natural numbers (1, 2, 3, ……,n) = $\frac{n+1}{2}$ and its SD = $\sqrt{\frac{n^2-1}{12}}$

E.g. If X → 1, 2, 3 …7, Then AM = $\frac{n+1}{2} = \frac{7+1}{2} = 4$ and SD = $\sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{7^2-1}{12}} = 2$

4. <u>Combined /pooled measures property, also called desirable mathematical property</u>

AM/GM/HM only in SD i.e. SD can be measured after combining several groups

5. Relation between QD/MD/SD: $QD = \frac{2}{3}SD \text{ and } MD = \frac{4}{5}SD \Rightarrow SD > MD > QD$

6. Variation(R, QD, MD, SD) are independent / not affected by change of origin but dependent/affected by change of scale positively.

E.g. if $SD_x = 5$ then

→SD (X+4) = 5, SD (2X) = 2 x 5 = 10, SD(-2X) = 2 x 5=10, SD (2-3X) = 15

→ Variance of (5 – 2x) = $2^2.V_x$ = 4 x 25 = 100

E.g. AM of X = 50 and SD_x = 3 then

→$AM\left(\frac{X-50}{3}\right) = \frac{\bar{X}-50}{3} = 0 \text{ and } SD\left(\frac{X-50}{3}\right) = \frac{SD_x}{3} = \frac{3}{3} = 1$

E.g.AM and SD of x is 1500 and 80, now all values are increased by 100 and then increased by 25%, what will be the new AM and SD.

→ new AM = 1500 + 100 = 1600 ↑by 25% = 2000

→new SD = 80 + 0 = 80 ↑by 25% = 100

Note:→ If y = a + bx then $\bar{y} = a + b\bar{x}$ i.e. affected by both origin and scale

→ If y = a + bx then $R_y = |b|R_x$, $QD_y = |b|QD_x$, $MD_y = |b|MD_x$,$SD_y = |b|SD_x$

→$V_y = |b|^2.V_x$

E.g. if y = 2x + 5 and AM of x is 10 and SD of x is 5 then co-efficient of variation of y is-

Ans: $\bar{y} = 5 + 2\bar{x} = 25 \text{ and } SD_y = |2|SD_x = 10 \text{ hence } CV_y = \frac{\sigma_y}{\bar{y}} \times 100 = \mathbf{40}$

E.g. if x, y are related as 5x + 2y = 17 and$MD_x = 5$, then mean deviation of y is – **[2]**

E.g. if 2x + 3y + 5 = 0 and $V_x = 6$, then variance of y is .... **[8/3]**

CHAPTER-12(CORRELATION AND REGRESSION ANALYSIS)

Bi-variate data: Two variables measured / recorded at same point of time.

(X, Y): (5, 4) (4, 6) (7, 8)…**Where n = no of pairs of observations.**

Bi- variate frequency table: one variable is taken in row and other in column, frequencies are put into cells

→ Let size of table p x q i.e. 3 x 4 where p = no of rows and q = no of columns

→ total no of cells = p.q = 12

→ the cell frequency are 0, 1, 2, 3…

→ Maximum number of marginal distribution in bi- variate frequency table is 2.

→ Maximum number of conditional distribution in bi- variate frequency table is (p+q) = 7

Analysis of bi-variate data:

Correlation analysis: means establishing relation and amount (extent) of relation between two variables.	Regression analysis: means establishing mathematical relation (regression equation) between two variables and then estimating dependent variable for a given value of independent variable.

Correlation analysis: means relation and amount of relation between two variables.

→ Relation between two variables is called just **correlation** i.e. nature of correlation (positive, negative, zero)

Positive: direct relation, changes in same direction b/w two variables i.e. either both are increasing or both are decreasing. E.g. relation b/w price and supply

Negative: inverse relation, changes in opposite direction b/w two variables i.e. one ↑, other ↓ e.g. price and demand.

Zero: change in one variable not effects other and also not affected by other e.g. shoe size and intelligence.

E.g. relation b/w day temperature and sale of cold- rinks is positive.

E.g. relation b/w age and life insurance premium is positive.

E.g. relation b/w speed and distance travel after applying brakes is negative.

E.g. spurious correlation means no relation b/w two variables.

→ Amount of relation is called **correlation co-efficient (r) – properties**

(i) It is a pure number i.e. unit free.

(ii) It lies from -1 to +1 i.e. $-1 \leq r \leq +1$

E.g. r = -1, perfect disagreement and r = 1, perfect agreement

(iii) It is independent of (not affected by) change of origin and change of scale.

E.g. If $r_{xy} = 0.8$ then correlation co-efficient b/w

→ r (x+2, y-3) = 0.8

→ r (2x, 5y) = 0.8 and r (2x+5, 3y-7) = 0.8

If change of scale with opposite sing, then nature of r will change

→ r (3x, -2y) = - 0.8

→ r (2x+5, 3-2y) = - 0.8 and r (6-2x, 5-6y) = 0.8

E.g. If it is given that, $r_{xy} = 0.6$

→ u = 2x+5 and v = 3y-7 then $r_{uv} = 0.6$

→ u = 5-2x and v = 2 + 6y then $r_{uv} = - 0.6$

→ u = 3-2x and v = 6 -3y then $r_{uv} = 0.6$

→ When 3x + 5u = 10 and 3v -6y = 5, then $r_{uv} = - 0.6$

→ When 2x + 3u -5 = 0 and 3v –y +6 = 0 then $r_{uv} = - 0.6$

→ u = 4-5x then $r_{uy} = - 0.6$

E.g. If correlation coefficient b/w x and y is r, then coefficient of correlation between $\left(\frac{x-2}{5}, \frac{5-3y}{7}\right)$is – **r**

.Methods to find value of r:

1. Scatter diagram method

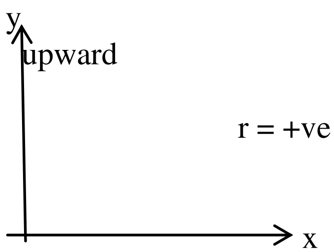
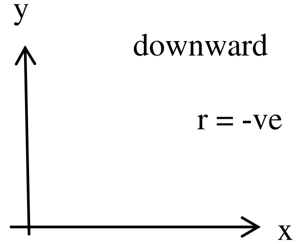
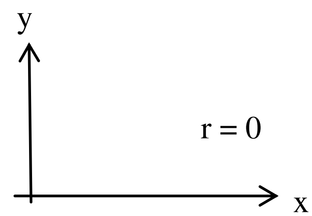
2. Karl Pearson's co-variance method / product moment correlation co-efficient method

3. Spearman's rank correlation co-efficient

4. co-efficient of con-current deviation method

→ **Scatter diagram method**

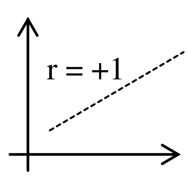
Under this method only nature of correlation can be judge, positive or negative or zero.

 y upward r = +ve x lower left to upper right	 y downward r = -ve x upper left to lower right	 y r = 0 x uniform / even distribution of points

Note: if there are perfect linear relation between x, y (y = a + b.x, b = slope) then r = +1 or -1. it's depends on slope. If b < 0 ⇒ r = -1 and if b > 0 ⇒ r = +1

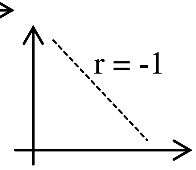
E.g. if x, y are related as y = 3 + 2x

b = 2 > 0 ⇒ r = +1



E.g. if x, y are related as 2x + 3y +8 = 0, then r = -1

Because b = -2/3 < 0



→ Karl Pearson's co-variance method

$$r = \frac{co-variance\ (x,\ y)}{\sigma_x\ .\ \sigma_y}, \qquad restriction: \sigma_x.\sigma_y \geq co-variance(x,y)$$

Correlation co-efficient (r) is directly related with co-variance between x and y.

Co-variance (+ve or –ve or zero) ⇒ r (+ve or –ve or zero)

E.g. if co-variance (x, y) = 40, variance of x = 256 and variance of y = 16, then co-efficient of correlation is

Ans: $r = \frac{co-variance\ (x,\ y)}{\sigma_x\ .\ \sigma_y} = \frac{40}{16\times4} = 0.625$

E.g. if co-variance (x, y) = 40, variance of x = 16 and variance of y is

Ans: $\sigma_x.\sigma_y \geq co-variance(x,y)\ \Rightarrow 4\times\sigma_y \geq 40, \sigma_y \geq 10\ hence\ variance\ of\ y \geq 100.$

→ Spearman's rank correlation co-efficient

This is most suitable to find correlation on basis of quality / attributes i.e. when there is no numerical data, then ranking is done on basis of attributes and then correlation is calculated by using formula: $r = 1 - \frac{6\sum d^2}{n^3-n}, \quad where\ d = difference\ in\ ranks = R_x - R_y\ and\ \sum d = 0$

E.g. if sum of square of difference in ranks for 6 students in marks of math's and accounts is 21, then rank correlation co-efficient is

Ans: $r = 1 - \frac{6\sum d^2}{n^3-n} = 1 - \frac{6\times21}{6^3-6} = 0.4$

E.g. If sum of squares of difference in ranks is 66 and rank correlation co-efficient is 0.6, then no of pairs of observation is

Ans: : $r = 1 - \frac{6\sum d^2}{n^3-n} \Rightarrow 0.6 = 1 - \frac{6\times66}{n^3-n}, n = 10$

→ co-efficient of con-current deviation method

This is quickest method to find correlation co-efficient and this method not much dependent on magnitude of observations.

$r = \pm\sqrt{\pm\left(\frac{2c-m}{m}\right)}$ Where m = n-1 and c = no of con-current deviation.

E.g. co-efficient of concurrent deviation for n pairs of observation is $\frac{1}{\sqrt{3}}$ if no of concurrentdeviation is 6 then find n.

Ans: $r = \sqrt{\left(\frac{2c-m}{m}\right)} \Rightarrow \frac{1}{\sqrt{3}} = \sqrt{\frac{12-m}{m}}, m = 9, n = m + 1 = 10$

Note: if correlation between x and y is, r = 0.6

→ co-efficient of determination = r^2 = 0.36 and
percentage of accounted (explained) variation is 100(r^2) = 36%

→ co-efficient of non-determination = 1-r^2 = 1-0.36 = 0.64 and

Percentage of unaccounted (unexplained) variation is 100 (1-r^2) = 64%

Regression analysis:

→ there are two regression equations

1. X on Y (for estimating x when y is given): $X = a + b_{xy}Y$ where
   $b_{xy} = regression\ coefficient\ x\ on\ y$

2. Y on X (for estimating y when x is given): $Y = a + b_{yx}X$ where
   $b_{yx} = regression\ coefficient\ y\ on\ x$

→ difference between actual value/observed value and estimated value is called error or residual.

Error = actual – estimated, which may be positive or negative or zero.

→ The regression line passing through scatter diagram in such a way that equal no of points bearing on both sides and distance of all points from the line is minimum.

If horizontal distance is minimum ⇒ line is x on y

If vertical distance is minimum ⇒ line is y on x

→ Method of fitting regression line is called least square method.

Formula:

X on Y: $\boldsymbol{X - \overline{X} = b_{xy}(Y - \overline{Y})\ \ where\ b_{xy} = r.\frac{\sigma_x}{\sigma_y}}$

Y on X: $\boldsymbol{Y - \overline{Y} = b_{yx}(X - \overline{X})\ where\ b_{yx} = r.\frac{\sigma_y}{\sigma_x}}$

E.g. estimate profit if sales target is 150 from the data given below.

statistics	sales	profit
mean	120	80
SD	10	4
r	0.8	

Ans: let sales = x and profit = y then we have $\bar{x} = 120, \bar{y} = 80, \sigma_x = 10, \sigma_y = 4\ and\ r = 0.8$

Now estimate y if x = 150 i.e. from y on x: $b_{yx} = r.\frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{4}{10} = 0.32$

$$y - \bar{y} = b_{yx}(x - \bar{x}) \Rightarrow y - 80 = 0.32(x - 120), y = 0.32x + 41.6\ \ put\ x = 150$$

Hence **y = 89.6**

E.g. If regression equation x on y is: 3x + 5y +7=0 then b_{xy} is --- [**ans is -5/3**]

E.g. if regression equation y on x is: y = 5-2x then b_{yx} is – **2**

Properties of regression co-efficients

1. Sign of both regression co-efficients are same either both are positive or both are negative.

2. Sign of correlation co-efficient (r) is same as regression co-efficients

3. Correlation co-efficient (r) is GM of two regression co-efficients i.e. $r = \pm\sqrt{b_{xy}.b_{yx}}$

E.g. if $b_{xy} = -2\ and\ b_{yx} = -\frac{1}{8}\ then\ r\ is\ -\sqrt{-2 \times -\frac{1}{8}} = -\frac{1}{2}$

4. Products of two regression co-efficients cannot exceed unity i.e. $b_{xy}.b_{yx} \leq 1$

E.g. if regression equation y on x is 20x -4y +5 = 0 then $b_{xy}\ is$

(a) 1/2 **(b) 1/5** (c) 1/3 (d) 5

5. Regression co-efficients are not affected by change of origin but affected by change of scale.

E.g. if$b_{yx} = 5$, u = x+ 5 and v = y -2 then b_{vu} is 5

E.g. if$b_{xy} = 1.5$, u = x+ 5 and v = y -2 then b_{uv} is 1.5

Scale: $u = \frac{x}{p} + a \text{ and } v = \frac{y}{q} + b,\ \text{then apply this for } b_{uv} \text{ and } b_{vu}$

$b_{xy} = \frac{p}{q} . b_{uv}$	$b_{yx} = \frac{q}{p} . b_{vu}$

E.g. if $b_{xy} = 2.4\ , u = \frac{x}{2} + 3 \text{ and } v = \frac{y}{-3} - 2 \text{ then } b_{uv} \text{ is}$

Ans: here p = 2 and q = -3, apply $b_{xy} = \frac{p}{q} . b_{uv} \Rightarrow 2.4 = \frac{2}{-3} . b_{uv} \Rightarrow b_{uv} = -3.6$

E.g. if $b_{yx} = 2\ ,\ u = 5x + 3 \text{ and } v = -2y - 2 \text{ then } b_{vu} \text{ is}$

Ans: here p = 1/5 and q = -1/2, apply $b_{yx} = \frac{q}{p} . b_{vu} \Rightarrow 2 = \frac{-1/2}{1/5} . b_{uv} \Rightarrow b_{uv} = -0.8$

E.g. find correlation co-efficient (r), from two regression equations as below

(I) 2 x + 5 y -1 = 0	$r = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$
(II) x + 2 y +5 =0	

E.g. find type of regression equations:

(I) 3x + 5y -8 = 0	Co-efficient of y is greater, hence y on x
(II) 2x + y +1 = 0	Co-efficient of x is greater, hence x on y

E.g. if two regression lines are (I) 2x + 5y -1=0 and (II) 3x – 2y +2 = 0 then y on x is

(a) I (b) II (c) both **(d) none of these**

Properties of regression lines:

1. Two regression lines are intersecting at means $(\bar{x}, \bar{y})$

E.g. if two regression lines are 2x + 3y +1 = 0 and 4x +5y +1=0 then AM of x and y are **(1, -1)**

2.→ when r = 0, then two regression lines are right angled / perpendicular to each other

→ When r = +1 or -1 then two regression lines becomes identical/overlapping/coinciding/equal/same i.e. angle between two lines is 0 degree.

CHAPTER-13(PROBABILITY AND EXPECTED VALUE)

Definition: random experiment (E) i.e. having two or more outcomes called random experiment.

↓

Let **n** outcomes.

↓

Let an event A (result of random experiment),

Then probability of event A, P (A) = $\frac{m}{n}$ where m = favorable cases of A i. e. P (A) = $\frac{favorable\ \ cases}{total\ \ cases}$ and P lies from 0 to 1

E.g. let experiment: tossing a coin, n = 2 S = {H, T}

P (H) = 1/2 and P (T) = 1/2

E.g. Tossing two coins, n = 2^2 = 4, S = {HH, HT, TH, TT}

Let A = both are heads i.e. favorable cases is 1, {HH}

P (A) = 1/4

Let B = at least one tail i.e. favorable cases are 3, {HT, TH, and TT}

P (B) = 3/4

Let C = at least one head i.e. favorable cases are 3, {HT, TH, and HH}

P (C) = 3/4

Types of events:

→**Simple event:** having only one favorable case e.g. event A

→**Compound/composite event:** having two or more favorable cases e.g. B, C

→**Equally likely events:** two different events having equal probability.

→**Mutually exclusive events:** two or more events having no common favorable cases. Intersection = ϕ i.e. they cannot occur simultaneously / together.

E.g.$A \cap B = \emptyset\ hence\ A, B\ are\ mutually\ exclusive\ events. P\ (A \cap B) = 0$

$$B \cap C = \{HT, TH\}, hence\ B, C\ are\ not\ mutually\ exclusive\ events. P\ (B \cap C) = \frac{1}{2}$$

→ **Exhaustive events:** union of two or more events is equal to total outcomes an experiment.

E.g.$A \cup B = \{HH, HT, TH, TT\}$ $hence\ A, B\ are\ exhaustive\ events.$ $P\ (A \cup B) = 1$

$B \cup C = \{HH, HT, TH, TT\}$ $hence\ B, C\ are\ exhaustive\ events.$ $P\ (B \cup C) = 1$

Note: If events (A, B) are both mutually exclusive and exhaustive events then

P (A) + P (B) = 1,$P\ (A \cap B) = 0$, and $P\ (A \cup B) = 1$

E.g. If A, B, C is mutually exclusive and exhaustive events and P (A) = 2 P (B) = 3 P (C), then P (B) is …..

Ans: use two equations P (A) = 2 P (B) = 3 P (C) and P (A) + P (B) + P (C) = 1 and solve

$$P(A) = 2\ P(B) = 3\ P(C) = K \Rightarrow P(A) = K, P(B) = \frac{K}{2}, P(C) = \frac{K}{3}\ \ put\ these\ values\ in$$

Equation P (A) + P (B) + P (C) = 1 ⇒ k + k/2 + k/3 = 1, k =$\frac{6}{11}$ hence $P(B) = \frac{K}{2} = \frac{6/11}{2} = \frac{3}{11}$

Eg. Throwing 2 dice, n = 6^2 =36

S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

→Let event A = difference between numbers is zero i. e. doublets P (A) = 6/36 = 1/6

→Event B = difference between numbers is 2 i.e. (1, 3) (2, 4) (3, 5) (4, 6) (3, 1) (4, 2) (5, 3) (6, 4).

P (B) = 8/36 = 2/9.

→X = sum of numbers, f = number of cases

x	2	3	4	5	6	**7**	8	9	10	11	12
f	1	2	3	4	5	**6**	5	4	3	2	1

E.g. P (sum = 10) = 3/36, P (sum = 6) = 5/36, if probability of sum is maximum ⇒ sum = 7

P (sum = 15) = 0

Note: odds in favor of an event is favorable cases: unfavorable cases

Odds of against of an event is unfavorable cases: favorable cases

Eg. What is an odd in favor of sum of 7?

Ans: favorable cases = 6, total cases = 36 ⇒ unfavorable cases = 30

Odds in favor = 6:30 = 1:5

Eg. If P (A) = 8/11 then odds of against of A is

Ans: favorable cases = 8, total cases = 11 ⇒ unfavorable cases = 3

Odds of against of A = 3:8

Eg. If an odd of against of an event A is 4:7 then P (A) is

Ans: P (A) = $\frac{favorable \ \ cases}{total \ \ cases} = \frac{7}{11}$

Problems relating to playing cards

<u>Total number of cards = 52 (divided into 4 suits)</u>

Heart = 13(red) diamond = 13(red) spade = 13(black) club = 13(black)

Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king

No of jack = 4, no of queen = 4 and no of king = 4 ⇒ total face cards = 12

Eg. One card is selected from well shuffled pack of 52 cards. What is

P (king) = 4/52, P (spade) = 13/52, P (red card) = 26/52, P (king of spade) = 1/52

Eg.Two cards is selected from well shuffled pack of 52 cards.

P (both are king) = ${}^{4}C_{2}/{}^{52}C_{2}$, P (1king and 1 queen) = ${}^{4}C_{1} . {}^{4}C_{1} / {}^{52}C_{2}$

Eg. In a box there are 2 defective and 4 non-defective bulbs.

→One bulb is taken, what is probability that it is defective. Ans: P = 2/6

→Two bulbs are taken, what is probability that both are non-defective. Ans: P = ${}^{4}C_{2} / {}^{6}C_{2}$

Eg.two letters from the word HOME is selected, what is probability that non is vowels.

Ans: Vowels-2, Consonants-2 probability = ${}^{2}C_{2} / {}^{4}C_{2}$ = 1/6

Eg. two cards are drawn one by one from a well shuffled deck of 52 cards. What is the probability that the first card is an ace and second is a king.

Ans: case (i) with replacement: $\frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

case(ii) without replacement: $\frac{4}{52} \times \frac{4}{51}$

Eg. A bag contains 7 white and 3 red balls. Two balls are drawn without replacement. What is the probability that one ball is white and other red.

Ans: P (WR) + P (RW) = $\frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9} = \frac{7}{15}$

Eg. In a pack of playing cards with two joker's probability of getting king of spade is

Ans: total number of cards = 52 + 2 = 54 and number of king of spade = 1

Hence P = 1/54

Eg. Probability of getting 53 Sundays

→ In a leap year: 366 days i.e. 52 weeks and 2 days i.e. 52 weeks means 52 Sundays confirm, For one more Sunday possibilities of 2 days are
{sun-mon, mon-tue, tue-wed, wed-thur, thur-fri, fri-sat, sat-sun} n = 7 and m= 2, prob. = 2/7

→In a non-leap year: 365 days i.e. 52 weeks and 1 day {S, M, T, W, TH, F SAT.} P = 1/7

Random variable and expected value:

E(x)= $\sum PX$ = $X_1P_1 + X_2P_2 + \ldots\ldots X_nP_n$

Random variable(x) can be any real number and E(x) can be negative, positive or zero.

Eg. Two coins are tossed, what is expected value of number of heads.

Ans: outcomes are HH, HT, TH, and TT. Let x = number of heads⇒ x = 2, 1, 1, 0

X	P	PX
0	1/4	0
1	2/4	1/2
2	1/4	1/2
total	$\sum P$ =1	$\sum PX$ =1

Hence expected value of number of heads, E(x) = 1

Note:

→E(x) =$\sum PX$, E(x^2) = $\sum PX^2$, $SD_x = \sqrt{E(X^2) - [E(X)]^2}$

→ E (K) = K eg. E (2) = 2 → E (KX) = K. E(X) eg. E (3x) = 3. E(x)→ E (X + Y) = E (X) + E (Y) →E (XY) = E (X). E (Y)

Eg. A random variable X has the following probability distribution

x	0	1	2	3
P(x)	0.2	k	2k	k

Find (i) k, (ii) P (x < 3), (iii) E (x), (iv) E(x^2), (v) SD_x

Ans: (i) $\sum P(x) = 1,\ \ 0.2 + k + 2k + k = 1,\ \ k = 0.2$

(ii) P (x < 3) = P (0) + P (1) + P (2) = 0.2+k+2k = 0.2+0.2+0.4 = 0.8

	(iii) E (x) = 1.6	(iv) E (x^2) =3.6
(v) $SD_x = \sqrt{E(X^2) - [E(X)]^2} = \sqrt{3.6 - 1.6^2} = 1.02$		

Note: → SD (K) = 0 eg. x = 2, 2, 2, 2….., SD = 0

→ SD (x + a) = SD_x eg. $if\ \sigma_x = 3, then\ SD\ (x + 5) = \sigma_x = 3$

→$SD(a + bx) = \ |b|.\sigma_x \quad eg. if\ \sigma_x = 3, then\ SD(\ 5 - 2x) = 2.\sigma_x = 6$

Also $y = a + bx, then\ \ \sigma_y = |b|.\sigma_x \ \Rightarrow\ V_y = \ b^2.V_x$

Eg. If x, y are related as 2x + 3y +6 =0 and V_x= 6 then V_y

Ans: $y = \ -2 - \frac{2}{3}\ x \ \Rightarrow\ V_y = \ \left(\frac{2}{3}\right)^2.V_x = \ \frac{4}{9} \times 6 = \frac{8}{3}$

Note: → P (A) = $\frac{m}{n}$ is classical / mathematical definition of probability.

→ the limiting value of relative frequency of an event is called its probability, is statistical definition of probability.

Conditional probability $P\left(\frac{A}{B}\right)$, when A, B are dependent events Probability of an event A under a given condition that event B is occurred. $P\left(\frac{A}{B}\right)$ Means probability of A in sample of space B.

Eg.If a family has two children, what is probability that both are girls, given that?

(i) One is girl (ii) 2nd is girl

Ans: (i) let A = both are girls (GG) and given that, B = one is girl {GB, BG, GG}

Hence $P\left(\frac{A}{B}\right) = \frac{1}{3}$, i.e. probability of A in B.

(ii) B = {BG, GG}, $P\left(\frac{A}{B}\right) = \frac{1}{2}$

Eg. Two dice are rolled, what is probability that 3 is appeared given that sum 7 is occur.

Ans: let A = 3 is appear and given that B = {(1, 6) (2, 5) (3, 4) (6, 1) (5, 2) (4, 3)}

Probability of A in B, $P\left(\frac{A}{B}\right) = \frac{2}{6} = \frac{1}{3}$

Eg. A dice is tossed once. Find the probability of getting a number greater than 3, if it is given that an even number has occurred. Ans: 2/3

Formulas:

dependent events: $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Independent events: P (A) = $\frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A).P(B)$

If A, B are independent events then $(\bar{A} \cap B), (A \cap \bar{B})$ and $(\bar{A} \cap \bar{B})$ are also independent.

→ P $(\bar{A} \cap B) = P(\bar{A}).P(B)$

→$P(A \cap \bar{B}) = P(A).P(\bar{B})$

→$P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B})$

Always applicable:

→ P (only B), P $(\bar{A} \cap B) = P(B) - P(A \cap B)$

→ P (only A), $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

→ P (neither A nor B), $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

→$P(A + B)$, P $(A$ or $B)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Eg. P (A) = p, P (B) = q then $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \leq \frac{p}{q}$

Eg. P (A) = 0.6 and P (B) = 0.5, find P (A+B) If A, B are independent

Ans: P (A + B) = P (A) + P (B) - P (A) .P (B) = 0.6 + 0.5 – (0.6). (0.5) = 0.8

Eg.if odds in favor that A will solve a question is 2:3 and odds of against that B will solve the same question is 5:4, what is probability that question will be solved if both try?

Ans: P (A) = 2/5 an P (B) = 4/9, P (Q will be solved) = $P(A \cup B)$Here A, B are independent, hence $P(A \cup B) = P(A) + P(B) - P(A).P(B)$

$P(A \cup B) =$ 2/5+ 4/9 -2/5 .4/9 = 2/3

Eg. If$P(A \cup B) = 0.8$, P (A) = 0.5 then find P (B) when

→ (i) A, B are mutually exclusive and

→ (ii) A, B are independent

Ans: (i) When A, B are mutually exclusive, then $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$⇒ 0.8 = 0.5 + P (B), P (B) = 0.3

(ii) When A, B are independent events, then $P(A \cap B) = P(A).P\,(B)$

$P(A \cup B) = P(A) + P(B) - P(A).P(B)$

0.8 = 0.3 + P (B) – (0.3).P (B) ⇒ 0.5 = P (B). (0.7), P (B) = 5/7

Eg. If P (A) =$\frac{2}{3}, P\,(B) = \frac{3}{5}, P\,(A \cup B) = \frac{5}{6},\ then\ find\ P\left(\frac{B}{A}\right)$ **Ans: 13/20**

Eg. If a class there 60 boys and 30 girls, 15 boys and 20 girls drink tea and other drink coffee. A student is selected at random from the class. What is probability that the student selected is a boy, if he drinks coffee i

Ans: let A = student is a boy and B = coffee drinker then find $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{9}{11}$

CHAPTER-14(THEORETICAL DISTRIBUTION)

Theoretical distributions exist only in theory not in real life, where formula for finding probability is given in different situations. It is of two types

Discrete variable probability distribution

Formula is called **Probability mass function**

1. Binomial (Bernoulli's) distribution (B.D.)

2. Poisson distribution (P.D.)

continuous variable probability distribution

Formula is called **probability density function**

3. Normal distribution (N.D.) → **chi-square distribution, T distribution, F distribution**

1. Binomial distribution:

Conditions under which B.D. is used

→when an experiment is repeated n times or performed on n objects.

→each trial or object are independent.

→in any trial there are only two outcomes, 1st success and 2nd failure where p ,q are probability of success and failure in any trial, and p + q = 1.

→ no of success is variable (x) which is discrete as x = 0, 1, 2, 3, ……………n

In this case for probability of r success out of n trials is given by B.D. which is
$\boldsymbol{P(r) = {}^nC_r p^r . q^{n-r}}$ **where r = 0, 1, 2, …….n**

Eg. What is the probability of making 3 correct guesses in 5 true-false answer type questions?

Ans: B.D. with n = 5, where success = correct guess and $p = \frac{1}{2}$ and $q = 1 - p = \frac{1}{2}$

Probability of 3 success, P(r=3) = ${}^5C_3\left(\frac{1}{2}\right)^3 . \left(\frac{1}{2}\right)^2 = 0.3125$

Eg. If the overall percentage of success in an exam is 60, what is probability that out of a group of 4 students, at least one has passed?

Ans: B.D. with n = 4, p = 0.6 and q = 0.4 now $P(r \geq 1) = 1 - P(r=0) = 1 - {}^4C_0\,(0.6)^0(0.4)^4 = 0.9744$

Properties of B.D.

1. $P(x) \geq 0$ and $\sum P(x) = 1$ $i.e.$ $P(0) + P(1) + \cdots + P(n) = 1$

2. B.D. is bi-parametric, its parameter is n and p and denoted by $x \in B(n, p)$

3. In B.D. Mean = np is always more than variance = npq and SD = $\sqrt{npq}$

Eg. For 48 patients, chance of recovery is 75%, find mean and SD

Ans: n = 48, p = 0.75 hence mean= 48(0.75) =36 and SD = $\sqrt{48(0.75)(0.25)} = 3$

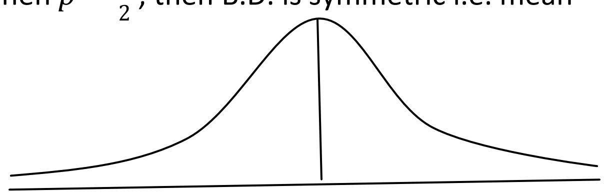
Eg.for B.D. mean = 3 and SD = 1.5, what is parameter of B.D.?

Ans: np = 3 and $\sqrt{npq}$ = 1.5 ⇒ npq = 2.25

Put np = 3 in npq = 2.25 ⇒q = 0.75 and p = 0.25, n = 12

→When $= \frac{1}{2}$, then variance of B.D. is maximum and $V_{max} = \frac{n}{4}$

→when $p = \frac{1}{2}$, then B.D. is symmetric i.e. mean = median = mode



Mean= median = mode

4. B.D is uni or bi -modal

→When (n+1) p is an integer then bi-modal and modes are (n+1) p and (n+1) p-1

→when (n+1) p is not an integer then uni-modal and mode is integral part of (n+1) p.

Eg. The mode of the binomial distribution x Є B (7, 1/3)

Ans: (n+1) p = (7+1). 1/3 = 2.666 hence uni-modal i. e. mode = 2

5. Additive property

X Є B(n_1, p) and Y Є B(n_2, p) ⇒ (X+Y) Є B(n_1+n_2, p) i.e. two B.D. is added when there probability of success is same.

Eg.x Є B(5,0.6) and y Є B(3,0.6) then (x+y) Є B(8, 0.6)

2. Poisson distribution.

A binomial distribution with parameter n and p, approaches Poisson distribution when $n \to \infty$ *and* $p \to 0$ *so that mean*$(m = np)$ *finite and moderate.*

In this case for probability of r success, we apply P.D.

$$\boldsymbol{P(r) = \frac{e^{-m}.m^r}{r!} \quad where,\ e = 2.718}$$

Eg.If 1% of calculators produce by a company is known to be defective. If a random sample of size 100 calculators is selected for inspection, calculate the probability of no defectives.

Ans: here n =100 (large) , p = 0.01(small) and mean, m = np = 1 which is finite and moderate.

Hence apply P.D. $P(r = 0) = \frac{e^{-1}.1^0}{0!} = e^{-1} = 0.368$

Eg.On an average 2 customer per minute arrive at a certain bank counter. What is probability that during a given minute exactly one customer arrive?

Ans: here m = 2 and r = 1, apply P.D. $P(r = 1) = \frac{e^{-2}.2^1}{1!} = \frac{2}{e^2} = 0.27$

Properties of P.D.

1. $P(x) \geq 0\ and\ \sum P(x) = 1\ i.e.\ P(0) + P(1) + \cdots + P(n) = 1$

2. P.D. is uni-parametric, parameter is m and it is denoted by x Є P (m).

3. In P.D. mean = variance i.e. SD = $\sqrt{m}$ and $CV = \frac{\sqrt{m}}{m} \times 100 = \frac{100}{\sqrt{m}}$

4. Like B.D., P.D. is also uni or bi-modal

→when m is an integer, P.D. is bi-modal and mode = m, m-1

→when m is non-integer, P.D. is uni-modal and mode = integral part of m

Eg. If coefficient of variation of P.D. is 50 then mode is

Ans: $\frac{100}{\sqrt{m}} = 50$ ⇒ m = 4 hence bi-modal, mode = 4, 3

Eg.if SD of P.D. is 1.8 then mode is

Ans: $\sqrt{m}$ = 1.8 ⇒ m = 3.24 hence uni-modal and mode is 3

5. Additive property

Two P.D. with different or same mean is always added.

Eg. If x Є P (m_1) and y Є P (m_2) then (x+y) Є P (m_1+m_2)

Eg.Ina P.D., SD_x = 3 and SD_y = 2 then SD of (x+y) is

Ans: mean of x = 9 and mean of y = 4 hence mean of (x+y) is 13 , then SD of (x+y) = $\sqrt{m} = \sqrt{13}$

Q 1) for a Poisson distribution P (x = 3) = 5.P (x = 5), then SD is

Ans: P (x = 3) = 5.P (x = 5)

$$\frac{e^{-m}.m^3}{3!} = 5. \frac{e^{-m}.m^5}{5!} \Rightarrow \text{m} = 2 \text{ hence SD} = \sqrt{2}$$

Q2) number of misprint per page of a thick book follows

Ans: Poisson distribution, thick book means n is large.

Q3) Number of success in one day match between India and Pakistan follows

Ans: binomial distribution, because in one day match series 3 or 5 matches is there.

Q4) In P.D. with SD = 2, what is P (1.5 < x < 2.3)

Ans: mean m = 4, P (1.5 < x < 2.3) means P (x =2),

Hence P (x =2) = $\frac{e^{-m}.m^x}{x!} = \frac{e^{-4}.4^2}{2!} = \frac{8}{e^4} = 0.1466$

Q5) for a binomial experiment, the standard deviation of the distribution of the number of success is 3.6 and the probability of success is 0.4. What is the mean of the distribution of the number of success?

Ans: p = 0.4 ⇒ q = 0.6 and SD = 3.6 ⇒ variance = 12.96, npq = 12.96 put q = 0.6

np = 21.6 ⇒ mean = 21.6

3. Normal distribution

Population/universe

Variable = x follows N.D., whose

Mean (μ) and variance (σ^2) are given

↓

One element is taken from this population, probability that its value is x is given by N.D. and

The N.D. is P (x) = $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $\quad where \quad -\infty < x < \infty$ and P(x) > 0

Eg. X follows normal distribution with mean = 10 and variance = 16 then P (x = 12) is given by

P (x = 12) $=\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{4\sqrt{2\pi}} e^{-\frac{(12-10)^2}{2.4^2}} = \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{8}}$

Eg. What is the coefficient of variation of X, characterized by the following probability density function $f(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-10)^2}{18}}$

Ans: μ = 10 and $2\sigma^2 = 18 \Rightarrow \sigma = 3, hence\ CV = \frac{\sigma}{\mu} \times 100 = 30$

Eg. If $f(x) = \frac{2}{\sqrt{\pi}} e^{-4x^2}, -\infty < x < \infty$ is the p.d.f. of a normal distribution, then the mean and variance are----

Ans: μ = 0 and $2\sigma^2 = \frac{1}{4} \Rightarrow \sigma^2 = \frac{1}{8}$, hence mean = 0 and variance = $\frac{1}{8}$

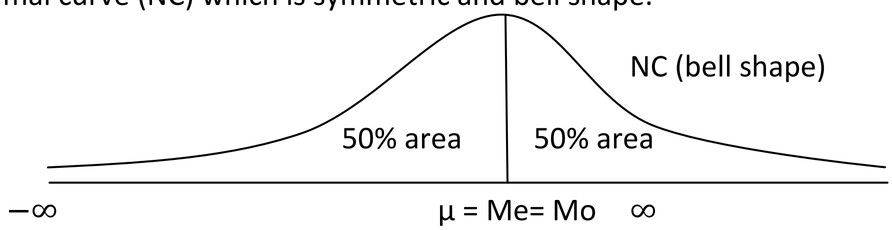
Features of normal distribution:

1. N.D. is bi-parametric, its parameter is mean and variance and it is denoted by x Є N (μ, σ^2)

2. Probability of N.D. is maximum at x = μ and its maximum probability is $\frac{1}{\sigma\sqrt{2\pi}}$ and either side of mean probability decline at same rate.

3. N.D. is uni-modal i.e. it has only one mode and mode = mean

4. N.D. is always symmetric i.e. it's mean = median = mode and the curve of N.D. is called normal curve (NC) which is symmetric and bell shape.



5. There are two points of inflexion on NC, 1st = $\mu - \sigma$ and 2nd $\mu + \sigma$

Eg If x Є N (10, 16) then points of inflexion are

Ans: here μ = 10 and $\sigma^2 = 16 \Rightarrow \sigma = 4$

Hence points of inflexion are $\mu - \sigma = 6$ and $\mu + \sigma = 14$

6. Area under normal curve

→ $\mu \pm \sigma$ covers 68.27% of total area

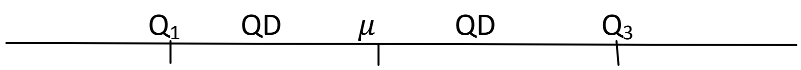
→ $\mu \pm 2\sigma$ *covers* 95.45% *of total area*

→ $\mu \pm 3\sigma$ *covers* 99.73% *of total area*

→ $Area(-\infty \ to \ \mu) = 50\% \ and \ Area \ (\mu \ to \ \infty) = 50\%$

7. In N.D. $QD = 0.675\ SD \ and \ MD = 0.8\ SD \ \Rightarrow SD > MD > QD$

And also $QD = \frac{Q_3 - Q_1}{2}$, $\mu = Q_1 + QD = Q_3 - QD = \frac{Q_3 + Q_1}{2}$



Eg.if the two quartiles of N (μ, σ^2) are 14.6 and 25.4 respectively, what is the standard deviation of the distribution?

Ans: first find $QD = \frac{Q_3 - Q_1}{2} = \frac{25.4 - 14.6}{2} = 5.4$ *and then apply* $QD = 0.675\ SD$

$$5.4 = 0.675\ SD \Rightarrow SD = 8$$

Eg.If $x \in N(10, 16)$ then two quartiles are -----

Ans: here $\mu = 10 \ and \ \sigma^2 = 16 \Rightarrow \sigma = 4 \ and \ QD = 0.675\sigma = 2.7,$ *now apply*

$$Q_1 = \mu - QD = 10 - 2.7 = 7.3 \ and Q_3 = \mu + QD = 10 + 2.7 = 12.7$$

8. Additive property

If $x \in N(\mu_1, \sigma_1^2)$ and $y \in N(\mu_2, \sigma_2^2) \Rightarrow (x+y) \in N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

I.e. for (x+y), mean = $\mu_1+\mu_2$ and variance = $\sigma_1^2+\sigma_2^2$

Eg. If x follows N.D. with mean 10, SD =4 and y follows N.D. with mean 12, SD =3. What is mean and SD of (x+y)

Ans: mean = 10+12 =22 and variance = $4^2+3^2 = 25 \Rightarrow$ SD = 5

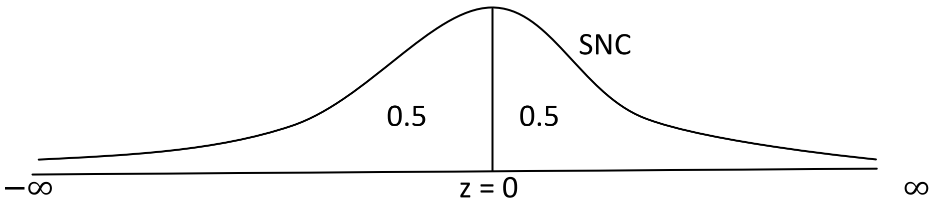
Eg.If $X \in N(3, 36)$ and $Y \in N(5, 64)$ are two independent normal variate with their standard parameters of distribution, then if $(X+Y) \in N(8, A)$ also follows normal distribution. Then the value of A will be ….

Ans: A = 100

9. Standard normal distribution

Normal distribution with mean (μ) = 0 and variance (σ^2) = 1 and variable = z

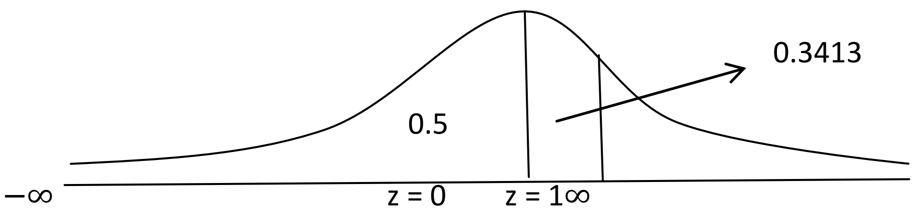
The S.N.D. is P (z) = $\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$ $where \ -\infty < z < \infty$



→ QD = 0.675, MD = 0.8 and points of inflection are -1 and + 1

→ Total area of SNC is one

→ϕ (a) = area under SNC from z = −∞ to z = aeg. ϕ (1) = area under SNC from z = −∞ to z = 1



Hence ϕ (1) = 0.5 + 0.3413 = 0.8413

Note: z = 1, area is 0.3413 (given) means area under SNC from z = 0 to z = 1

CHAPTER-16 (INDEX NUMBER)

Meaning: index number measures percentage change in variable (price or quantity) of one period (called current period) as compare to another period (called the base period).

Index number of base period is always taken as 100 and if index number of current period is 125, means variable increase by 25% and if index number of current period is 80, means variable decrease by 20%.

Eg.→ If variable increase by 30%, means index number is 130.

→If variable decrease by 35%, means index number is 65.

→If variable increase by 1.25 times, means index number is 225.

Use: index number is useful in policies formation, measuring economic activities like inflation/deflation, knowing trend and purchasing power of money.

Symbol: let year 2000 is base year (0) and 2001 is current year (1) then

$$P_{01} = price\ index\ number\ of\ 1\ to\ the\ 0\ base, Q_{01} = quantity\ index\ number\ of\ 1\ on\ 0\ base.$$

Definition: index number is ratio of current value to the base value expressed in percentage,

i.e. index number = $\frac{current\ \ value}{base\ \ value} \times 100$

eg. if price of a commodity is ₹ 5 in current year and it was ₹ 4 in base year.

What is price index number?

Ans: let $p_0 = 4\ and\ p_1 = 5$, $P_{01} = \frac{p_1}{p_0} \times 100 = \frac{5}{4} \times 100 = 125$ ie

price increase by 25%.

Factors to be consider in construction of index number.

(i) Selection of base period.
→ relatively current not too old i.e. base period is temporary solution.
→ it is normal period ie not affected by natural calamity.
→ it is a standard point of reference in comparing various data.
→ base period is fixed or chain. Fixed means one year is taken as base year for all other years. Chain base means just previous year of current year is the base year.

(ii) Selection of average.
There are various data available for construction of index number, out of which data is selected on **sample basis** and then average value is calculated i.e. index number is a **special type of average.**
Theoretically geometric mean is the best (most suitable) average for construction of index number but for simplicity (practically) arithmetic mean is used.

(iii) Selection of weights.
For weighted index number, weights plays important role in construction of index number. For price index quantity is the weight and for quantity index price is weight.

Methods of construction of index numbers

Simple / un weighted index

When P or Q is given

1. Simple aggregative method
2. Simple average of relatives method

weighted index

when P and Q is given

1.weighted aggregative method
2. Weighted average of relative method

Simple index:

1. Simple aggregative: $P_{01} = \frac{\sum p_1}{\sum p_0} \times 100 \quad or \quad Q_{01} = \frac{\sum q_1}{\sum q_0} \times 100$

Eg. $P_{01} = \frac{\sum p_1}{\sum p_0} \times 100 = \frac{24}{16} \times 100 = 150$

commodity	p_0	p_1
A	5	8
B	7	10
C	4	6
total	16	24

2. Simple average of price relatives: $P_{01} = \frac{\sum P}{n}, \ where\ price\ relative(P) = \frac{P_1}{P_0} \times 100.$

P is price index number of individual commodity, also called group index number.

Eg.

commodity	p_0	p_1	$P = \frac{P_1}{P_0} \times 100.$	Where n = 3 and $\sum P =$ *452.86, P01=452.863* =150.95
A	5	8	$\frac{8}{5} \times 100 = 160$	
B	7	10	$\frac{10}{7} \times 100 = 142.86$	
C	4	6	$\frac{6}{4} \times 100 = 150$	

Weighted index: (when price and quantity both are given)

1. Weighted aggregative index

→Laspeyre's index (L): $P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 \quad or \quad Q_{01} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$

Base year value is taken as weight, for price index base year quantity is taken as weight and for quantity index base year price is taken as weight.

→ Paasche's index(P): : $P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 \quad or \quad Q_{01} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$

Current year value is taken as weight.

→ Dorbish and bowley index:

In this method index number is **arithmetic mean** of L and P i.e. index number = $\frac{L+P}{2}$

→ Fisher's ideal index number:

In this method index number is **geometric mean of** L and P i.e. index number = $\sqrt{L.P}$

Bowley index > fisher's index.

→ Marshal – adge worth index:

In this method index number is calculated taking average value of base year and current year is taken as weight.$P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100, where\ q = \frac{q_0+q_1}{2}$

$$P_{01} = \frac{\sum p_1(q_0+q_1)}{\sum p_0(q_0+q_1)} \times 100 = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100.$$

→When price and quantity both are given then value index is also calculated.

Value / expenditure = price x quantity = p.q

Value index / expenditure index, $V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} \times 100$

commodity	Base year		Current year		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	p_0	q_0	p_1	q_1				
A	5	8	8	10	40	50	64	80
B	7	10	9	11	70	77	90	99
C	6	5	7	8	30	48	35	56

Now $\sum p_0 q_0 = 140$, $\sum p_0 q_1 = 175, \sum p_1 q_0 = 189, \sum p_1 q_1 = 235$ **Laspeyre's index:**$P_{01} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100 = \frac{189}{140} \times 100 = 135,$ $Q_{01} = \frac{\sum q_1p_0}{\sum q_0p_0} \times 100 = \frac{175}{140} \times 100 = 125$

Paasche's index:$P_{01} = \frac{\sum p_1q_1}{\sum p_{0q_1}} \times 100 = \frac{235}{175} \times 100 = 134.28$

$$Q_{01} = \frac{\sum q_1p_1}{\sum q_0p_1} \times 100 = \frac{235}{189} \times 100 = 124.34$$

Bowley index:$p_{01} = \frac{L+P}{2} = \frac{135+134.28}{2} = 134.64$

Fisher's index:$p_{01} = \sqrt{L.P} = \sqrt{135 \times 134.28} = 134.639$

Eg. Bowley index = 120 and laspeyre's index =80 then fisher's index number is----- **(113.14)**

Eg.ifBowley index is 155 and fisher's index is 154.92. Then laspeyre's index is

(a) **160** (b) 140 (c) 145 (d) none of these

Marshal's index: $P_{01} = \frac{\sum p_1q_0+\sum p_1q_1}{\sum p_0q_0+\sum p_0q_1} \times 100 = \frac{189+235}{140+175} \times 100 = 134.6$

Note: marshal index number is good approximation of fisher's ideal index number.

Value index:$V_{01} = \frac{\sum p_1q_1}{\sum p_0q_0} \times 100 = \frac{235}{140} \times 100 = 167.857$

Means current year expenditure increase by 67.857% as compare to base year

Note: purpose determines which formula of index number is to be used.

2. Weighted average of price relatives:$P_{01} = \frac{\sum PW}{\sum W}$, $where\ price\ relative\ P = \frac{P_1}{P_0} \times 100\ and\ W = p_0q_0\ (expenditure\ of\ base\ year)$

→This index number is also equal to laspeyre's price index number, $P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$

→This index number is also called **cost of living index number (C.L.I.) / Consumer price index number / general index number / whole sale price index number when price and quantity is taken from whole sale market.**

commodity	Base year		Current year		$W = p_0 q_0$	$P = \frac{p_1}{p_0} \times 100$	PW
	p_0	q_0	p_1	q_1			
A	5	8	8	-	40	160	6400
B	7	10	9	-	70	128.57	9000
C	6	5	7	-	30	116.67	3500

$\sum W = 140,\ \ and \sum PW = 18900$. Hence, $P_{01} = \frac{\sum PW}{\sum W} = \frac{18900}{140} = 135$

Means cost of living increase by 35% i.e. ₹ 135 of current year is equal to ₹100 of base year.

Eg.Find cost of living index number.

commodity	Expenditure (%)	Group index number
food	45	120
rent	-	130
other items	10	105

Ans: expenditure = W and group index number = P

$\sum W = 100 \ \Rightarrow W\ of\ rent\ is\ 45,\ \ and \sum PW = 45 \times 120 + 45 \times 130 + 10 \times 105 = 12300$

Hence, C.L.I. = $\frac{\sum PW}{\sum W} = \frac{12300}{100} = 123$

Use of C.L.I.→ For D. A. (dearness allowance) / salary calculation

Eg. Monthly salary of on employee was ₹ 10, 000 in the year 2000 and it was increase to ₹ 20, 000 in the year 2013 while the consumer price index number is 240 in year 2013 with the base year 2000, what should be his salary in comparison of consumer price index in the year 2013?a) 2, 1000 b) 16,000 **c) 24000** d) none of these

Eg. Net monthly salary of an employee was ₹ 3000. The consumer price Index in 1985 is 250 with 1980 as base year. If he has to be rightly compensated, then additional Dearness Allowance to be paid to the employee is ____

a) `4000 b) ₹4800 c) ₹ 5500 **d) ₹ 4500**

→ **For real value / purchasing power / deflated value calculation**

$= \frac{current\ \ value}{C.\ L.\ I.\ of\ current\ \ year} \times 100$ i.e. purchasing power is reciprocal of index number.

Eg. If current year GDP is 1815crores and inflation during this period is 10% then

Real GDP= $\frac{current\ \ value}{C.\ L.\ I.\ of\ current\ \ year} \times 100 = \frac{1815}{110} \times 100 = 1650$ crores

Eg. If nominal rate of return is 20% and during this period inflation is 10% then real rate of is---

Ans: after one year 100 becomes 120 and index number become 110, hence real value of 120 is $\frac{120}{110} \times 100 = 109.09\ i.e. real\ rate\ of\ return\ is\ 9.09\%$.

Eg. If index number of 2016 with base 2015 is 150 and salary of an employ in the year 2015 was ₹ 25000 and in the year 2016 is ₹ 30000. What is real gain / loss.

Ans: Real value of salary of 2016

$=\frac{30000}{150} \times 100 = 20000\ which\ is\ compare\ from\ base\ year\ salary\ 25000$

Hence real loss of ₹ 5000

Eg. If cost of living index in the year 1975 and 1976 was 110 and 200 respectively, during that period wages of a worker increases from ₹ 330 to ₹ 500. What were real gain / loss?

Ans: In this case both year wages is to be converted into real wages and then compare.

1[st]real value $= \frac{330}{110} \times 100 = 300$

2[nd] real value $= \frac{500}{200} \times 100 = 250,$ hence real loss of ₹50

Eg. Expenditure of a family on 3 items are in the ratio of 2:5:3. The prices of these commodities rise by 30%, 20% and 40% respectively. What is cost of living index number?

Item	Index number(P)	Weight (W)	WP
A	130	2	260
B	120	5	600
C	140	3	420
total		10	1280

Hence, cost of living index number

$$P_{01} = \frac{\sum PW}{\sum W} = \frac{1280}{10} = 128\ i.e. cost\ of\ living\ increase\ by\ 28\%$$

Test of adequacy / consistency / sufficiency.

There are 4 test of adequacy to determine which formula of index number is most adequate. The formula which satisfies maximum number of test is called most adequate.

Unit test

→ This test requires that formula should be independent of the unit of price and quantity.
→ Except the simple aggregative index all other formula satisfy this test.

Time reversal test

→ Formula by which, $P_{01} \times P_{10} = 1, when\ multiplier\ 100\ is\ ignored\ is\ satisfy\ this\ test.$
→ laspeyre's and paasche do not met this test but fisher's formula pass this test and also simple aggregative and marshal formula satisfy this test.

Factor reversal test

→ Formula by which,

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0\, q_0}, when\ multiplier\ 100\ is\ ignored\ is\ satisfy\ this\ test.$$

→ laspeyre's and paasche not pass this test but fisher's formula pass this test.
Note: fisher price or quantity index formula pass only time reversal test.

Circular test

→ Formula by which,

$$P_{01} \times P_{12} \times P_{20} = 1, when\ multiplier\ 100\ is\ ignored\ is\ satisfy\ this\ test.$$

This is an extension time reversal test in which base period is shifting.
→ L, P, F do not pass this test.
→ Simple GM of price relatives and weighted aggregative with fixed weight pass this test and also simple aggregative index pass this test.

Index number series

(i) Fixed base index (FBI): one year is taken as base year for all years
→ shifting technique: means shifting base year in a FBI series to a new base year to find new fixed base index series. Formula is

$$shifted\ index = \frac{old\ \ indices}{old\ \ index\ \ of\ \ new\ \ base\ \ year} \times 100$$

→ splicing technique: combining two or more series of overlapping index numbers to obtain a single index number on a common base is called splicing of index numbers i.e. Splicing means constructing one continuous series from two different indices on the basis of common base.

(ii) Chain base index (CBI): A chain base index number is an index number with previous year as base.

$$chain\ index\ of\ current\ year = \frac{link\ relative\ of c.y. \times chain\ index\ of\ p.y.}{100}$$

And $link\ relative\ of c.y. = \frac{c.y.value}{p.y.value} \times 100$