Mathematical Progressions — 2009 to 2014 past papers — answers

Answer # 1 (Spring 2010 Question # 1a)

We know that:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Here r = 3 and $S_n = 364a$

$$\Rightarrow 364a = \frac{a(3^n - 1)}{3 - 1}$$
$$\Rightarrow 364 = \frac{(3^n - 1)}{2}$$

$$\Rightarrow$$
 729 = 3^n

$$\Rightarrow$$
 3⁶ = 3ⁿ

Since bases are same, exponents will be equal:

$$\Rightarrow n = 6$$

Answer # 2 (Autumn 2011 Question # 2a)

Since it is a Geometric Progression, we would use the relation:

$$S = \frac{a(r^n - 1)}{r - 1}$$

Here:

s = Summation of payments

$$a = Re. 1$$

$$n = 7$$

$$r=\frac{3}{1}=3$$

$$S = \frac{1(3^7 - 1)}{3 - 1} = \frac{(2187 - 1)}{2} = 1,093$$

Since the amount obtained through alternative is less than Rs. 1,200, hence Bashir has taken a wise decision by accepting the pocket money of Rs. 1,200.

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Answer # 3 (Spring 2012 Question # 1b)

1st number after 170 divisible by 8 = 176Last number before 1,000 divisible by 8 = 992Thus, the A.P. =176, 184,..... 992 $S_n = 176 + 184 + \dots +992$

Where,

first term: a=176; common difference: d=184-176=8 and last term: L=992We know that a=L-(n-1)d

$$\therefore n = \left(\frac{L-a}{d}\right) + 1$$

$$\Rightarrow n = \left(\frac{992 - 176}{8}\right) + 1 = \frac{816}{8} + 1 = 102 + 1 = 103$$

Since
$$S_n = \frac{n}{2}(a+l)$$

$$\therefore S_n = \frac{103}{2} (176 + 992) = \frac{103}{2} (1168) = 103(584) = 60,152$$

Answer # 4 (Autumn 2012 Question # 1a)

The given scenario is an example of arithmetic series.

The sum of an arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

In the given situation, $S_n = 500,000$; a = 10,000 and d = 500

$$500,000 = \frac{n}{2} [2(10,000) + (n-1)(500)]$$

$$1,000,000 = n(20,000 + 500n - 500)$$

$$1,000,000 = 500$$
n² + 19,500n

$$500n^2 + 19,500n - 1,000,000 = 0$$

Dividing both sides by 500 we get:

$$n^2 + 39n - 2,000 = 0$$

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According to Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
where $x = n$, $a = 1$, $b = 39$ and $c = -2,000$

$$n = \frac{-39 \pm \sqrt{(39)^2 - 4(1)(-2,000)}}{2(1)} = \frac{-39 \pm \sqrt{1521 + 8,000}}{2} = \frac{-39 \pm \sqrt{9,521}}{2}$$

$$n = \frac{-39 \pm 97.58}{2} = 29.29 \text{ or } -68.29$$

Since n cannot be negative, hence Sadiq's aggregate savings would **exceed** Rs.500,000 in 30 months.