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Measures of Central Tendency and Dispersion — 8-12 marks

i) **Mean**

- Arithmetic mean (AVERAGE)
- Harmonic mean
- Geometric mean

ii) **Median** (Partition value)

- Median
- Quartile
- Decile
- Percentile

iii) **Mode**

✱ Arithmetic mean

$$A \cdot M = \frac{\text{Sum of all observations}}{\text{Total no. of observations}} = \frac{\Sigma x_i}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Sigma (summation)

✱ Arithmetic mean of Frequency Distribution

$$\boxed{\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i}}$$ Direct method

$$d_i = \frac{A - x_i}{h}$$

$$\boxed{\bar{x} = A + \left(\frac{\Sigma d_i f_i}{\Sigma f_i} \times h\right)}$$ Step Deviation method

A → middle value of x_i

PROPERTIES OF AM

i) If the observations are say 'K' then AM is also 'K'.

ii) If the observations +, −, × by value 'K', then AM will also be +, −, × by value 'K'.

iii) The sum of deviation (diff.) from the mean is zero.

$$\Sigma(x_i - \bar{x}) = 0$$

iv)
$$\left.\begin{array}{l} y_i = x_i + a \\ y_i = x_i - a \end{array}\right\} \text{Origin shift}$$

$$\left.\begin{array}{l} y_i = bx_i \\ y_i = \dfrac{x_i}{b} \end{array}\right\} \text{Scaling}$$

$$\boxed{\bar{y} = a + b\bar{x}}$$

v) If there are 2 observations n_1 & n_2 with respective AM as $\bar{x}_1$ & $\bar{x}_2$, then combined AM is :

$$\bar{x} = \frac{n_1\bar{x}_1 + n_1\bar{x}_2 + \ldots}{n_1 + n_2 + \ldots}$$

CORRECTING INCORRECT MEAN

$$\boxed{\bar{x}_r = \bar{x}_w + \frac{x_r - x_w}{n}}$$

* Weighted Arithmetic mean

$$\bar{x} = \frac{\sum w_i x_i}{\sum w_i}$$

Arithmetic mean

Merits
- Rigidly defined
- Easy to calculate & understand
- Based on all observations
- Suitable for maths treatment further.
- Least affected by fluctuations of sampling

Demerits
- Affected by extreme values
- Open - end classes
- Not detected graphically
- Qualitative data X
- wrong conclusions if details not available

Median

→ middle value
Positional average

* Ungrouped data (Data in ascending value)

n is odd, $\left(\frac{n+1}{2}\right)^{th}$ term

n is even, $\dfrac{\frac{n}{2}^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$ term

$$\left(\frac{n+1}{2}\right)^{th} = 3.5^{th} = 3^{rd} + 0.5(4^{th} - 3^{rd})$$

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★ Discrete series

i) Find Cumulative Frequency

ii) Total of $\frac{N}{2}$ → last CF

iii) $Cf > \frac{N}{2}$

iv) Ans is x corresponding to $CF > \frac{N}{2}$

★ Continuous series

i) Find cumulative Frequency

ii) Total of $\frac{N}{2}$

iii) $CF > \frac{N}{2}$ ⇒ Median class

CF of class before

iv) $$\boxed{x_{med} = L.C.B + \frac{\frac{N}{2} - Cf_{-1}}{f} \times h}$$

PROPERTIES OF MEDIAN

i) x and y are two variables, $y = a + bx$, then the median of y is $\boxed{y_{median} = a + bx_{median}}$

ignore signs

ii) When the sum of absolute deviations is minimum, when deviation is taken from median.

$$\Sigma\,[x - x_{median}] \rightarrow \min.$$

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MERITS

- rigidly defined
- Simple to calculate
- Can deal with Open-end class
- Unaffected by extreme values
- Qualitative data
- Determined graphically

DEMERITS

- Not based on each and every item
- Not suitable for maths treatment
- Ungrouped data → even not suitable
- Sampling fluctuation
- need to arrange data

Partition Values

i) Quartiles
3 Quartiles → divide in 4 equal parts (25% each)

ii) Deciles
9 Deciles → divide in 10 equal parts (10% each)

iii) Percentiles
99 Percentiles → 100 equal parts (1% each)

Calculation of Quartiles [Individual Observations] :-

$$\boxed{K^{th} \text{ quartile} = \left[K\left(\frac{n+1}{4}\right)\right]^{th} \text{ value}}$$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th}$$

$$Q_2 = 2\left(\frac{n+1}{4}\right)^{th} = \left(\frac{n+1}{2}\right)^{th} \Rightarrow \boxed{\text{MEDIAN}}$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$$

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Calculation of Deciles :-

$$\boxed{D_K = \left[K\left(\frac{n+1}{10}\right)\right]^{th}} \qquad K = 1 \text{ to } 9$$

Calculation of Percentiles :-

$$\boxed{P_K = \left[K\left(\frac{n+1}{100}\right)\right]^{th}} \qquad K = 1 \text{ to } 99$$

☆ In case of Discrete series :-

Quartiles

$Q_K \rightarrow$ Value corresponding to $CF \geq \frac{KN}{4}$

Deciles

$D_K \rightarrow$ Value corresponding to $CF \geq \frac{KN}{10}$

Percentiles

$P_K \rightarrow$ Value corresponding to $CF \geq \frac{KN}{100}$

☆ In case of Continuous Series :-

Quartiles.

$$x_{median} = l + \frac{\frac{KN}{4} - CF_{-1}}{f} \times h \qquad CF \geq \frac{KN}{4}$$

Deciles

$$x_{median} = l + \frac{\frac{KN}{10} - CF_{-1}}{f} \times h \qquad CF \geq \frac{KN}{10}$$

Percentiles

$$x_{median} = l + \frac{\frac{KN}{100} - CF_{-1}}{f} \times h \qquad CF \geq \frac{KN}{100}$$

Mode → most occuring value

* Bi-model distribution – 2 modes
Multi-model distribution – more than 2 modes
No mode – 1 observation of each

* Discrete series
↓
max. frequency

* Continuous series

$$x_{mod} = LCB + \frac{f_0 - f_{-1}}{2f_0 - f_1 - f_{-1}} \times h$$

LIMITATIONS :-

i) Exclusive type series
ii) Same length of class interval

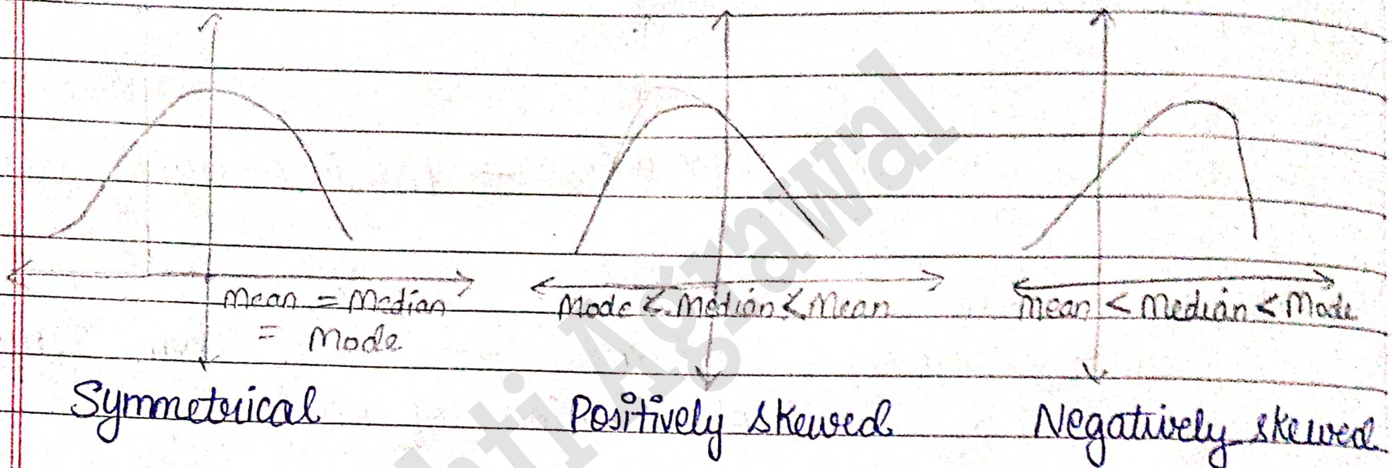
PROPERTIES

i) Mean − Mode = 3 (Mean − Median)
3 Median = 2 Mean + Mode

Empirical formula

↓

Moderately Skewed Distribution
~~Symmetrical Distribution~~ X



ii) $y = a + bx$, then $\boxed{y_{mod} = a + bx_{mod}}$

MERITS

- Simple and easy to calculate.
- Located by inspection
- Graphically by histogram.
- Not affected by extreme values.
- Open end class not affected

DEMERITS

- Not rigidly defined (1-2 values)
- Not based on all observations
- Not further for mathematical treatment
- Affected by sampling
- May have bi-modal distribution.

Most popular — Arithmetic mean
Most reasonable — Geometric mean

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Geometric Mean

▷ $G.M = (X_1 * X_2 * \ldots * X_n)^{\frac{1}{n}}$
where n = no. of observations

▷ $\log GM = \frac{1}{n} \Sigma \log x_i$

$GM = \text{antilog}\left(\frac{\Sigma \log x_i}{n}\right)$

Individual series

▷ $GM = \left(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_2} \ldots \times x_n^{f_n}\right)^{\frac{1}{n}}$

~~log~~ $GM = \text{antilog}\left(\frac{\Sigma f_i \log x_i}{N}\right)$

Discrete series

▷ Continous series — same (except x_i = mid-point)

Uses :-

i) When ratio of consecutive terms remains constant.
ii) Most appropriate average in index number.
iii) Give weightage to smaller items.

PROPERTIES

i) If observations are 'K' then GM is also 'K'.
ii) If $Z = xy$, then, $GM_Z = GM_x \times GM_y$
iii) If $Z = \frac{x}{y}$, then $GM_Z = \frac{GM_x}{GM_y}$

- If all observations are k, then GM = HM = AM = k
But otherwise AM > GM > HM

Harmonic Mean

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}} = \frac{n}{\Sigma \frac{1}{x_i}}$$

✱ Discrete Series

$$HM = \frac{\Sigma f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots \frac{f_n}{x_n}} = \frac{\Sigma f_i}{\Sigma \frac{f_i}{x_i}}$$

✱ Continuous Series

$$HM = \frac{\Sigma f_i}{\Sigma \frac{f_i}{x_i}}$$ where x_i = class mid-point

Uses :-

i) If observations are 'k' then HM is also 'k'

ii) n_1 n_2
HM_1 HM_2

$$HM_{combined} = \frac{n_1 + n_2}{\frac{n_1}{HM_1} + \frac{n_2}{HM_2}}$$

Weighted Mean

$$AW_W = \frac{\Sigma W_i x_i}{\Sigma W_i}$$

$GM_w = \text{antilog} \frac{\sum w_i \log x_i}{\sum w_i}$

$HM_w = \frac{\sum w_i}{\sum \frac{w_i}{x_i}}$

* Sum of 1^{st} n natural numbers $= \frac{n(n+1)}{2}$

* Relationship b/w AM, GM & HM

If all observations are equal, $GM = HM = AM = K$
Else $AM > GM > HM$

$\boxed{GM^2 = AM \times HM}$ for two observations

* Sum of squares of 1^{st} n natural numbers
$= \frac{n(n+1)(2n+1)}{6}$

* Sum of cubes of 1^{st} n natural numbers
$= \frac{n^2(n+1)^2}{2^2}$

* If n terms are there, $\sum(x_i - a) = b$
$\sum x_i - an = b$