CA Foundation - Maths & LR Chart by Prof Jatin Dembla

RATIO AND PROPORTION

- comparison or simplified form of two quantities of the same kind is referred to as ratio
- Proportion is an equation which defines that the two given ratios are equivalent to each other

Duplicate Ratios

- If a:b is a ratio, then:
- a2:b2 is a duplicate ratio
- √a:√b is the sub-duplicate ratio
- a3:b3 is a triplicate ratio

Important Properties of Proportion

The following are the important properties of proportion
- Addendo – If a : b = c : d, then a + c : b + d
- Subtrahendo – If a : b = c : d, then a – c : b – d
- Dividendo – If a : b = c : d, then a – b : b = c – d : d
- Componendo – If a : b = c : d, then a + b : b = c+d : d
- Alternendo – If a : b = c : d, then a : c = b : d
- Invertendo – If a : b = c : d, then b : a = d : c
- Componendo and dividendo – If a : b = c : d, then a + b : a – b = c + d : c – d

LINEAR INEQUALITIES

In mathematics a linear inequality is an inequality which involves a linear function. A linear inequality contains one of the symbols of inequality:[1]. It shows the data which is not equal in graph form

- < less than
- > greater than
- ≤ less than or equal to
- ≥ greater than or equal to
- ≠ not equal to
- = equal to

A linear inequality looks exactly like a linear equation, with the inequality sign replacing the equality sign.

Properties Of Inequalities

- So far whatever properties that you have learned to solve linear equations will be applied in solving inequalities too. The only difference will be when you perform multiplication or division by a negative, you have to reverse the inequality sign as well.

Types Of Inequalities

There are Four Types of Inequalities, They are:
- **Strict:** The inequalities that have < or > symbol between the L.H.S and R.H.S.
- **Slack:** The inequalities that have ≤ or ≥ symbol between the L.H.S and R.H.S.
- **Linear:** The inequalities that have a degree 1. Example, 5x + 2y>10
- **Quadratic:** The inequalities that have a degree 2. Example, 5x² + 2y>10

Sets

A set is the representation of a collection of objects; distinct objects with one or more common properties.

Domain and Range

- The domain is the set of all first elements of R.
- The range is the set of all second elements of R.
Total no. of relations for n(A×B) are 2n(A)×n(B)

Relations and Functions

Relations and functions are the set operations that help to trace the relationship between the elements of two or more distinct sets or between the elements of the same set

Venn-Diagrams

Euler - Venn Diagrams make it easy to operate on sets.

In the above example we consider:
- The quadrilateral is a finite universal set which represents a set of ideas.
- Among those ideas we have two subsets which are finite as well:
- Set of ideas that are the truth, let's denote them as T and.
- Set of ideas that are either T or B or both.
- Their intersection set, knowledge is denoted as K.
- Here, T U B is the union of these two sets which is the set of ideas that are either T or B or both.
- T ∩ B is the intersection of the two sets which is the set of ideas that are both T and B i.e., K as already mentioned.
- Now, to find no. of elements in T U B we have, n(T U B) = n(T) + n(B) – n(T ∩ B).

Cartesian Product

If p ∈ P and q ∈ Q then the set of all ordered pair i.e., (p,q) is called the Cartesian product of P×Q. This means every first element of the ordered pair belongs to the set P and every second element belongs to the set Q.
- P and Q must be non-empty sets.
- P × Q is null if either P or Q is a null set.

TIME VALUE OF MONEY

Principal: The money borrowed or lent out for a certain period is called the principal or the sum.

Interest: Extra money paid for using other's money is called interest.

Simple Interest (S.I.):

If the interest on a sum borrowed for certain period is reckoned uniformly, then it is called simple interest. Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then :

Simple Interest = $\frac{P \times R \times T}{100}$

$P = \left(\frac{100 \times S.I.}{R \times T}\right)$ $R = \left(\frac{100 \times S.I.}{P \times T}\right)$ and $T = \left(\frac{100 \times S.I.}{P \times R}\right)$

Let Principal = P, Rate = R% per annum, Time = n years.
Amount = $P\left(1 + \frac{R}{100}\right)^n$
When interest is compound yearly:
Amount = $P\left(1 + \frac{R/2}{100}\right)^{2n}$
When interest is compounded Half yearly:
When interest is compounded Quarterly:
Amount = $P\left(1 + \frac{R/4}{100}\right)^{4n}$
When interest is compounded Annually but time is in fraction, say 3⅖ years:
Amount = $P\left(1 + \frac{R}{100}\right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100}\right)$
When Rates are different for different years, say R₁%, R₂%, R₃% for 1st, 2nd and 3rd year respectively.
Then, Amount = $P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$

ANNUITY : Annuities, in this sense of the word, break down into two basic types: ordinary annuities and annuities due.

Ordinary annuities: An ordinary annuity makes (or requires) payments at the end of each period. For example, bonds generally pay interest at the end of every six months.

Annuities due: With an annuity due, by contrast, payments come at the beginning of each period. Rent, which landlords typically require at the beginning of each month, is a common example.

FV Ordinary Annuity=C×[(1+i)^n−1/i]where: C=cash flow per period i=interest rate and n=number of payments

PV Ordinary Annuity=C×[1−(1+i)^−n/i]

FV Annuity Due=C×[(1+i)^n−1/i]×(1+i)

PV Annuity Due=C×[1−(1+i)^−n]×(1+i)
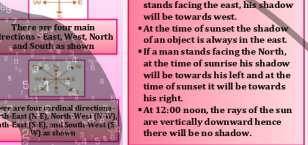
SEQUENCE AND SERIES

Sequence: If a1, a2, a3, ...an is an arrangement of 'n' numbers (more often non- zero numbers) as per a well defined order is called a sequence, and it is denoted by {an}. Here the numbers a 1, a 2, a 3, ...an are called the elements or terms of the sequence.

Series: If a1, a3, ...an is a sequence of 'n' terms then their sum a1+ a2+ a3+ ...... + an is called a finite series and it is denoted by Σn.

Progressions:

Special types of sequences are called progressions. Progressions are mainly classified in to three.
- Arithmetic Progressions (AP)
- Geometric Progressions (GP)
- Harmonic Progressions (HP)

Geometric Progression (GP)

A sequence of non-zero numbers a1, a2, a3,....an is called a GP, if the ratio of term and its preceding term is always a constant. ie. a2 / a1 = a3 / a2 = ... = an / an-1
This constant ratio is called the 'Common ratio' of the GP and denoted as 'r'. ie. r = an / an-1

Important results of Geometric Progression

- nth term of a GP
- If the first term of a GP is 'a' and its common ratio is 'r', then the GP is, a, ar, ar2, ar3,.....n-1
ie. The nth term = arn-1

Important results of Arithmetic Progression:

Let a1, a2, a3, ...an is an AP of 'n' terms
- Common difference (d) = an - an-1
- nth term (an) = a + (n - 1)d
- Number of terms {n} = [(an - a1)/ d] + 1
- Sum of 'n' terms = n/2 [2a + (n - 1)d] OR n/2 [a1 + an].
- Arithmetic Mean (AM) or Average of n terms of an AP = (a1 + an) / 2 i.e. AM = (a1 + an) / 2 = (a2 + an-1) / 2 = (a3 + an-2) / 2
- In an Arithmetic Progression, Mean = Median (where median is the middlemost term in the arrangement)
- In general, the mth AM among the 'n' AM's between 'a' and 'b' can be found by Am = a + m * {[(b-a) / (n+1)]}
- Insertion of single AM between 'a' and 'b'
- Let 'x' be the single AM between two numbers 'a' and 'b', then x = (a + b)/2

- nth term = arn-1, where 'a' is the first term and 'r' is the common ratio

DIRECTION TEST

There are four main directions- East, West, North and South as shown

- At the time of sunrise if a man stands facing the east, his shadow will be towards west.
- At the time of sunset the shadow of an object is always in the east.
- If a man stands facing the North, at the time of sunrise his shadow will be towards his left and at the time of sunset it will be towards his right.
- At 12.00 noon, the rays of the sun are vertically downward hence there will be no shadow.

There are four cardinal directions North-East (N-E), North-West (N-W), South-East (S-E), and South-West (S-W) as shown.

SYLLOGISM

All A are B
This phrase simply means that A is contained in B but not necessarily vice versa. This means A is a subset of B but B may not be a subset of A. The Venn diagram for this is:
In this diagram, it is clearly visible that circle A is inside the circle B which simply means that B contains the entire A i.e. All A are B.

A = B
In this case, the conclusion is similar to the first type i.e. "All A are B". Here not only "All A are B", but also "All B are A". This means A is a subset of B and is also a subset of A. The Venn diagram is:
Here A is contained in B and so is B contained in A. So, here A contains all B and again B also contains all A.

Some A are B
This is the case when some of A in B that is A and B are intersecting and thus some A are A will also be true. The Venn diagram depiction is as:
Here, the shaded portion simply indicates that some portion of A is contained in B while the unshaded portion is uncertain portion and does not indicate anything whether A's contained in B or not.

No A are B
It is simply understandable that B does not contain any of A and so A is not contained in B. This means that A and B are disjoint sets. The Venn diagram for this case is:
Here no part of A is present inside of B and is present in A. So neither A nor B contain any part of B or A respectively.

Some A are not B
This means that some portion of A is not included in B for sure while the other part of A is uncertain whether it is included in B or not. The Venn diagram is;
In this, some portion of A is surely not included in B while there is no surety whether the shaded region is included in B or not.

These are certain universal rules that should be followed while solving the syllogism questions. They are:
- Any "All" and "All" sentence will always imply an "All" conclusion.
- Any "All" and "No" sentence will always imply a "No" conclusion.
- Any "All" and "Some" sentence will always imply a "No" conclusion.
- Any "Some" and "All" sentence will always imply a "Some" conclusion.
- Any "Some" and "No" sentence will always imply a "Some not" conclusion.
- Any "Some" and "Some" sentence will always imply a "No" conclusion.

BLOOD RELATION

Simply blood relations means blood relationship among members of the family.

In-Laws Relations:
- Spouse's Father (Husband or Wife) : Father -in-law
- Spouse's Mother (Husband or Wife) : Mother -in-law
- Spouse's Brother (Husband or Wife) : Brother -in-law
- Spouse's Sister (Husband or Wife) : Sister-in-law
- Sister's Husband : Brother-in-law
- Brother's Wife: Sister-in-law
- Daughter's Husband : Son-in-law
- Son's Wife: Daughter-in-law

Different Types of Blood Relations

Father's Side	Paternal Relatives
Mother's Side	Maternal Relatives
Children of same parent	Siblings
Parent's Father	GRANDFATHER
Parent's Mother	GRANDMOTHER
Parent's Brother	UNCLE
Parent's Sister	AUNT
Parent's Sibling Child	COUSIN
Uncle's Son/ Daughter and Aunt's Son/ Daughter	COUSINS
My Siblings Son	NEPHEW (Male)
My Sibling's Daughter	NIECE (Female)

GENERATION

Three Generation Above

Two Generation Above

One Generation Above

Current Generation (Self)

One Generation Below

Two Generation Below

Three Generation Below

MALE
- Great Grandfather
- Maternal Grandfather
- Great Grandfather-in-law
- Grandfather
- Maternal Grandfather
- Grandfather-in-law
- Father
- Uncle
- Maternal Uncle
- Father-in-law
- Husband
- Brother
- Brother-in-law
- Cousin
- Son
- Nephew
- Son-in-law
- Grandson
- Grandson-in-law
- Great Grandson
- Great Grandson-in-law

FEMALE
- Great Grandmother
- Maternal Grandmother
- Great Grandmother-in-law
- Grandmother
- Maternal Grandmother
- Grandmother-in-law
- Mother
- Aunt
- Maternal Aunt
- Mother-in-law
- Wife
- Sister
- Sister-in-law
- Cousin
- Daughter
- Niece
- Daughter-in-law
- Granddaughter
- Granddaughter-in-law
- Great Granddaughter
- Great Granddaughter-in-law

EQUATION AND MATRICES

1. Linear equations - homogeneous variable (i.e. only one variable)
2. The standard form of a linear equation in one variable is represented as ax + b = 0 where, a ≠ 0 and x is the variable. A quadratic polynomial, when equated to zero, becomes a quadratic equation. The values of x satisfying the equation are called the roots of the quadratic equation.

Matrices
Any rectangular arrangement of numbers in m rows and n columns is called a matrix of order m×n.

2. Basic Definitions
- **Row matrix:** A matrix having one row is called a row matrix.
- **Column matrix:** A matrix having one column is called a column matrix.
- **Square matrix:** A matrix of order m×n is called square matrix if m = n.
- **Zero matrix:** A = [aij]m×n is called a zero matrix, if aij = 0 for all i and j.
- **Upper triangular matrix:** A = [aij]m×n is said to be upper triangular, if aij = 0 for i > j.
- **Lower triangular matrix:** A = [aij]m×n is said to be lower triangular, if aij = 0 for i < j.
- **Diagonal matrix:** A square matrix [aij]m×n is said to be diagonal, if aij = 0 for i ≠ j.
- **Scalar matrix:** A diagonal matrix A = [aij]m×n is said to be scalar, if aij = k for i = j.
- **Unit matrix (Identity matrix):** A diagonal matrix A = [aij] is a unit matrix, if aij = 1 for i = j.
- **Comparable matrices:** Two matrices A and B are comparable, if they have the same order.

General from: ax² + bx + c = 0
1. The roots of the quadratic equation: x = (-b ± √D)/2a. where D = b² - 4ac
2. Nature of roots:
- D > 0, roots are real and distinct (unequal)
- D = 0, roots are real and equal (coincident)
- D < 0, roots are imaginary and unequal
3. All cubic equations have either one real root, or three real roots. If the polynomials have the degree three, they are known as cubic polynomials. The cubic equation is of the form. ax^3+bx^2+cx+d=0

DETERMINANTS

Determinant is the numerical value of the square matrix. So, to every square matrix A = [aij] of order n, we can associate a number (real or complex) called determinant of the square matrix A. It is denoted by det A or |A|.

NOTE :
- Read |A| as determinant A not absolute value of A.
- Determinant gives numerical value but matrix do not give numerical value.
- A determinant always has an equal number of rows and columns, i.e. only square matrix have determinants.

BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS

Calculus is Divided into Two Categories:
Differential Calculus (Derivatives) | Integral Calculus (Anti Derivatives)
Method of Change | Method of Accumulation
Fundamental Theorem of Calculus (Connects: Differential and Integral Calculus)

Limits
The degree of closeness to any value or the approaching term. A limit is normally expressed using the limit formula as- limx→cf(x)=A
It is read as "the limit off of x as x approaches c equals A".

Differential Calculus Basics
Differential Calculus is concerned with the problem of finding the rate of change of a function with respect to the other variables. To get the optimal solution, derivatives are used to find the maxima and minima values of a function.

Instantaneous rate of change of a quantity with respect to the other. The derivative of a function is represented as:
limx→hf(x+h)-f(x)h=A
Derivatives are a fundamental tool of calculus. The derivative of a function of a real variable measures the sensitivity to change of a quantity, which is determined by another quantity. Derivative Formula is given as,
f1(x)=limΔx→0f(x+Δx)-f(x)/Δx

Integration
Integration is the reciprocal of differentiation. As differentiation can be understood as dividing a part into many small parts, integration can be said as a collection of small parts in order to form a whole. It is generally used for calculating area.

Definite Integral
A definite integral has a specific boundary within which function needs to be calculated. The lower limit and upper limit of the independent variable of a function is specified, its integration is described using definite integrals. A definite integral is denoted as: ∫ba f(x). dx = F(x)

Indefinite Integral
An indefinite integral does not have a specific boundary, i.e. no upper and lower limit is defined. Thus the integration value is always accompanied by a constant value (C). It is denoted as: ∫ f(x). dx = F(x) + C

Summary of Differential Formulas and Corresponding Integrals

$\frac{d}{dx}(c) = 0$ $\int dx = C$

$\frac{d}{dx}(x) = 1$ $\int 1 dx = x + C$

$\frac{d}{dx}(ax) = a$ $\int a dx = a x + C$

$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$ $\int u(x) + v(x) dx = \int u dx + \int v dx$

$\frac{d}{dx}(x^n) = n x^{n-1}$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ $\int \sqrt{x} dx = \frac{2}{3} x^{3/2} + C$

$\frac{d}{dx}(\sin x) = \cos x$ $\int \sin x dx = -\cos x + C$

$\frac{d}{dx}(\cos x) = -\sin x$ $\int \cos x dx = \sin x + C$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\int \sec^2 x dx = \tan x + C$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\int \sec x \tan x dx = \sec x + C$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\int \csc x \cot x dx = -\csc x + C$

$\frac{d}{dx}(\cot x) = -\csc^2 x$ $\int \csc^2 x dx = -\cot x + C$

$\frac{d}{dx}(e^x) = e^x$ $\int e^x dx = e^x + C$

SEATING ARRANGEMENT

Seating arrangement tricks to solve the problems
Questions on seating arrangement are generally asked in blocks of 4-5 questions. You are given some information and then there will 4-5 questions based on the information. These questions have two types of information.

Direct information : This is the information which is clearly mentioned in the statement of the question. This is the information which you will use when you start solving the questions.

Indirect information: After filing the direct information you will look for the connection between different parts of the information. These connections form the indirect information.

While arranging the persons, the direction to which the persons are facing is very important. Let us take the case of linear arrangements. Here if it is stated that there are five persons sitting facing North then the arrangements will be like

If the persons are looking away from the centre then the right hand side will be in the clockwise direction and left hand side will be in the anti clockwise direction as shown below :

Similarly if the arrangement is a double row arrangement, then one group of people will be facing north and the second will face south and the directions will be facing north and the second will face south and the directions will be taken as similar to the above figures.
In case of circular arrangements, or rectangle arrangement, the persons may be facing the centre of the circle or they may be facing away from the center, if they are looking towards the center, then the right hand side will be in the anticlockwise direction and left hand side will be in the clockwise direction as shown below:

On the other hand if these persons are sitting South then the arrangements will be like

INDICES AND LOGARITHM

- am x an = am + n
- $\frac{a^m}{a^n} = a^{m-n}$
- (am)n = amn

Let a be a positive real number other than 1 and a = x, then we write m = logₐx and we say that the value of log x to the base a is m. (e.g) 10³ = 1000 ⇒ log₁₀1000 = 3

1. Logₐmn = logₐm + logₐn
(i.e) logarithm of the product of two quantities is equal to the sum of their logarithms taken separately.
2. logₐ m/n = logₐm - logₐn
(i.e) logarithm of the quotient of two numbers is equal to the difference of their logarithms.
3. logₐ(m)ⁿ = n logₐm
(i.e) logarithm of a power of a number is the product of the power and the logarithm of the number.
4. logₐm = 1
(i.e) logarithm of any number to itself as base is unity.
5. logₐ1 = 0
(i.e) logarithm of unity with respect to any finite quantity (other than zero) as base is zero.
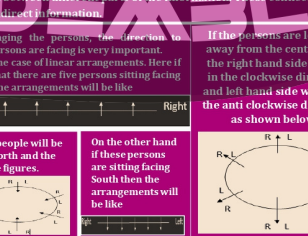6. logₐx = $\frac{1}{\log_x a}$

PERMUATATION AND COMBINATION

Permutation
In mathematics, permutation relates to the act of arranging all the members of a set into some sequence or order, or if the set is already ordered, rearranging its elements, a process called permutation.

Combination
The combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter.
- A permutation is the choice of r things from a set of n things without replacement and where the order matters.
nPr = (n!) / (n-r)!
- A combination is the choice of r things from a set of n things without replacement and where order does not matter.
nCr=n!/(n-r)!=npr/r!

circular permutation
There are two cases of circular-permutations:-
a. If clockwise and anti clock-wise orders are different, then total number of circular-permutations is given by (n-1)!
b. If clock-wise and anti-clock-wise orders are taken as not different, then total number of circular-permutations is given by (n-1)!/2!

Note : Number of circular-permutations of 'n' different things taken 'r' at a time:-
- If clock-wise and anti-clockwise orders are taken as different, then total number of circular-permutations = nPr /r.
- If clock-wise and anti-clockwise orders are taken as not different, then total number of circular – permutation = nPr/2r

DETERMINANTS

Properties of determinants:
- D = D'
- If a determinant has all the elements zero in any row or column, then D = 0
- If any two rows or columns of a determinant are interchanged, then D' = - D.
- If a determinant has any two rows or columns identical, then D = 0.
- If all the elements of any row or column are multiplied by the same number k, then D' = kD.

Cramer's rule: Two variables

(a) Consistent equations : Define an unique solution [Intersecting lines]
(b) Inconsistent equation: No solution. [Parallel line]
(c) Dependent equation: Infinite solutions. [Identical lines].
Let a₁ x + b₁ y + c₁ = 0 and a₂ x b₂ y + c₂ = 0 then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ = given equation are inconsistent.
If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ = given equations are dependent

Three variables :
Let a₁x + b₁y + c₁z = d₁ and a₂x + b₂y + c₂z = d₂ and a₃x + b₃y + c₃z = d₃ then x = D₁/D, y = D₂/D and z = D₃/D, where D =