Business Mathematics, Logical Reasoning & Statistics

CA VINOD REDDY

calculator Tricks on page.No.19

Chapter 1

RATIO I PROPORTION LOGS & INDICES

CA VINOD REDDY

1 What is Ratio?

- Ratio is a fraction used for comparison of 2 or more quantities which are :
  - of same type
  - expressed in same unit of measurement
- Ratio can be expressed without unit of measurement

2 Find simplest form of 3.50 : 8.75.

$\Rightarrow$ • Generally Ratio is written in its simplest form

• $\frac{3.50}{8.75} = \frac{350}{875} = \frac{14}{35} = \frac{2}{5} = 2:5$

$\therefore$ simplest form is 2:5

3 5:7 can also be written as :

$5k : 7k = 50:70 = 5/k : 7/k$ where $k \neq 0$

All the terms of the ratio can be multiplied OR divided by same Non-zero number.

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Ratio	It's	Answer
5:7	Duplicate Ratio	25:49
8:3	Triplicate Ratio	$8^3 : 3^3 = 512:27$
11: 19	Inverse Ratio	19:11
64:625	Sub-Duplicate Ratio	$\sqrt{64} : \sqrt{625} = 8:25$
125:27	Sub-Triplicate Ratio	$\sqrt[3]{125} : \sqrt[3]{27} = 5:3$

5 Find compounded ratio of 5:7, a:b, x:y, 9:8

Compounded ratio = (product of antecedents : product of consequents)

Answer : $\boxed{45ax : 56by}$

6 3 : 8 : 9 : 11 is a continued ratio.

Ratio of 3 or more terms is known as continued ratio.

My Notes

- In a:b , a = First term = Antecedent
  b = second term = consequent
- simplest form of 81.81 : 36.36 is $\frac{8181}{3636} = \frac{909}{404}$
  $= 9:4$
- a:b can also be written as $am : bm = a/m : b/m$ where $m \neq 0$

7 Ratio of 3 or more terms is known as continued Ratio.

8:7:11:19 is a continued ratio.

8 Ratio is unit free. Ratio can be expressed without unit of measurement

9 First term of the ratio = Antecedent

Second term of the ratio = consequent

10 Find the ratio of 3kg : 35,000 grams

$= 3000 \text{ gms} : 35{,}000 \text{ gms} = 3:35 = \left(\frac{3}{35}\right)$

11 a:b can also be written as (ak : bk) or $\left(\frac{a}{k} : \frac{b}{k}\right)$ provided $k \neq 0$

5:8 can also written as $10:16 = 15:24 = 50:80 = 2.50:4$

$= \left(5/9 : 8/9\right) = \left(5/13 : 8/13\right)$

12 The order of the terms in a ratio is important.

13 Find simplest form of $2\frac{1}{3} : 3\frac{2}{3}$

$= \dfrac{2\frac{1}{3}}{3\frac{2}{3}} = \dfrac{\frac{7}{3}}{\frac{11}{3}} = \dfrac{7}{11} = 7:11$

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If the Ratio	then a:b is called as
a:b If	
a > b	Ratio of Greater inequality
a < b	Ratio of Lesser inequality
a = b	Ratio of Equality

15 Ratio exists only when 2 or more quantities are of same kind/type expressed in same unit of measurement.

16 Find simplest form of $\frac{1}{3}:\frac{1}{8}:\frac{1}{10}$

$\Longrightarrow \frac{1}{3} : \frac{1}{8} : \frac{1}{10} = 80:30:24 = 40:15:12$

17 Find simplest form of $\frac{3}{5}:\frac{2}{3}:\frac{8}{5}$

$\Longrightarrow \frac{3}{5} : \frac{2}{3} : \frac{8}{5}$

$= 9:10:24$

My Notes

Find simplest form of $8\frac{1}{7} : 9\frac{3}{11}$

$\Longrightarrow \dfrac{8\frac{1}{7}}{9\frac{3}{11}} = \dfrac{\frac{57}{7}}{\frac{102}{11}} = \frac{57}{7} \times \frac{11}{102} = \frac{627}{714} = \frac{209}{238}$

$= 209:238$

18. Ratios are unit - free

Ratio can be expressed without any unit of measurement. example: Ratio of Incomes of A, B, C is 2:3:8

19. If a:b = 2:3, b:c = 4:7, c:d = 8:1. Find a:b:c:d, a:d, b:d

a:b = 2:3 = 8:12

b:c = 4:7 = 12:21

∴ a:b:c = 8:12:21

c:d = 8:1

a:b:c = 64:96:168

c:d = 168:21

$\boxed{\therefore a:b:c:d = 64:96:168:21}$

∴ a:d = 64:21

b:d = 96:21 = 32:7

20. If Quantity increase or decreases in the ratio a:b then new quantity = b of original quantity = a

∴ New quantity = (original quantity X mutiplying ratio)

where multiplying ratio = (Reciprocal of given ratio)

original quantity = (new quantity X Given ratio)

A's Income ₹ 7 lakhs. his income changes in the ratio of 21:29. Find his new income?

⟹ New income = old income × multiply. ratio

$= 7 \times \frac{29}{21} = 9.666666$ lakhs

21. Population of a city is x then it changes in the ratio of p:q then find new population

⟹ New population = old population × Multiplying Ratio

$= x \times q/p = \left(\frac{qx}{p}\right)$

22.

Inverse ratio of Inverse ratio of a:b is = a:b

Duplicate ratio of sub duplicate ratio of p:q is = p:q

Triplicate ratio of sub triplicate ratio of m:n is = m:n

Sub triplicate ratio Triplicate ratio of x:y is = x:y

Sub duplicate ratio of duplicate ratio of u:v = u:v

23. Find Duplicate ratio of Inverse ratio of 5:7

⟹ Duplicate ratio of 7:5

= 49:25

24. Find Triplicate ratio of sub duplicate ratio of 25:49

⟹ Triplicate ratio of 5:7

= 125:343

My Notes :

25. **Find compounded ratio of Duplicate ratio of 2:3, Triplicate ratio of 9:4, Sub duplicate ratio of 81:64, sub duplicate ratio of 512:27**

$\Longrightarrow$ $4:9$, $729:64$, $9:8$, $\sqrt{512}:\sqrt{27}$

$$\text{compounded Ratio} = \frac{\cancel{4} \times \cancel{729}^{81} \times 9 \times \sqrt{512}}{\cancel{9} \times 64 \times \cancel{8}_{2} \times \sqrt{27}} = \frac{81 \times 9 \times \sqrt{256 \times 2}}{2 \times \sqrt{9 \times 3} \times 64}$$

$$= \frac{81 \times \cancel{9}^{\cancel{3}\sqrt{3}} \times \cancel{16} \times \sqrt{2}}{2 \times \cancel{3} \times \sqrt{\cancel{3}} \times \cancel{64}\,4} = \frac{81\sqrt{6}}{8}$$

My Notes :

$$\text{compounded Ratio} = \left(\text{product of antecedents} : \text{product of consequents}\right)$$

26 **When 4 quantities a,b,c,d are said to be in proportion?**

$\Longrightarrow$ • when $a:b = c:d$ then a,b,c,d are said to be in proportion.

when $a:b = c:d$ $\therefore \frac{a}{b} = \frac{c}{d}$ $\therefore ad = bc$

(product of extremes = product of means)

• If $ps = qr$ then p,q,r,s are said to be in proportion.

27 **When 4 quantities a,b,c,d are said to be in continued proportion?**

$\Longrightarrow$ when $a:b = b:c = c:d$ then a,b,c,d are said to be continued proportion.

• If $p:q = q:r = r:s$ then p,q,r,s are said to be in continued proportion

• 2,6,10,30 are in proportion but not in continued proportion.

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4 Quantities	Whether 4 Quantities are in	
	Continued Proportion?	Proportion?
2,6,18,54	Yes	Yes
3,8,12,32	NO	Yes
8,24,96,288	NO	Yes
8,5,80,45	NO	NO
4,6,9,13.50	Yes	Yes

29 **When 3 quantities a,b,c are said to be in proportion?**

• If $a:b = b:c$ then a,b,c are said to be in proportion as well as in continued proportion.

• 10, 30, 90 are in proportion as well as continued proportion.

30 **If a,b,c,d are in proportion i.e. $\frac{a}{b} = \frac{c}{d}$ then**

Invertendo : $\left(\frac{b}{a} = \frac{d}{c}\right)$

Alternendo : $\left(\frac{a}{c} = \frac{b}{d}\right)$

Componendo : $\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$

Addendo : $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

Dividendo : $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$

Subtrahendo : $\frac{a}{b} = \frac{c}{d} = \left(\frac{a-c}{b-d}\right) = \left(\frac{c-a}{d-b}\right)$

Componendo and Dividendo : $\left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$

If $\frac{50}{60} = \frac{5}{6}$ then

$\frac{50}{60} = \frac{5}{6} = \left(\frac{50+5}{60+6}\right) = \left(\frac{50-5}{60-6}\right)$

31 If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=\frac{i}{j}=k$, then

As per addendo k = $\left(\frac{a+c+e+g+i}{b+d+f+h+j}\right) = \frac{a}{b} = \left(\frac{a+3c+5e+8g+12i}{b+3d+5f+8h+12j}\right)$

As per subtrahendo k = $\left(\frac{a+c-e-g+i}{b+d-f-h+j}\right) = \left(\frac{a+10c-10e-25g+100i}{b+10d-10f-25h+100j}\right)$

32 If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$ then, Find value of $\left(\frac{4a+2b-3c}{5b}\right)$

$\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$

$\left(\frac{4a}{12}=\frac{2b}{8}=\frac{3c}{21}\right)=\frac{4a+2b-3c}{12+8-21}$

$\left(\frac{5b}{20}=\frac{4a+2b-3c}{-1}\right)$

$-1/20=\left(\frac{4a+2b-3c}{5b}\right)$

$\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=k$

$a=3k,\ b=4k,\ c=7k$

$\left(\frac{4a+2b-3c}{5b}\right)=\frac{12k+8k-21k}{20k}$

$=\frac{-k}{20k}=-1/20$

33 Find Fourth Proportional to 8, 12, 20

$\Longrightarrow$ 8, 12, 20, m are in proportion

$8m = 12 \times 20$

$m = 30$ $\therefore$ 4^{th} proportional to 8, 12, 20 is 30.

34 Find mean proportional to 9, 25

$\Longrightarrow$ 9, k, 25 are in proportion $\therefore$ $\frac{9}{k}=\frac{k}{25}$

$\therefore k^2 = 9\times25$ $\therefore k = 15$

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4 Quantities in Proportion	Value of k = ?
8, 9, k, 63	$504 = 9k \therefore k = 56$
58, -3k, 28, 85	$4930 = -84k \therefore k = -58.6904$
36, 60, 2k, 98	$3528 = 120k \therefore k = 29.40$
-3k, 86, 25, 63	$-189k = 2150 \therefore k = -11.37566$

36 Rules of Indices

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^{-m} = \left(\frac{1}{a^m}\right)$
5. $(a.b)^m = a^m \times b^m$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} = (a^m \times b^{-m})$
7. $a^{1/m} = \sqrt[m]{a}$
8. $[(a^m)^n]^p = a^{mnp}$
9. $(a^{m/n}) = (a^m)^{1/n} = \sqrt[n]{a^m}$
10. If $a^x = a^y$; then $x = y$
11. If $a^m = b^m$; then $a = b$

37 $2x^{1/2} \times 3x^{-1} = ?$ If $x = 4$

$= 2 \times (4)^{1/2} \times 3 \times (4)^{-1}$

$= \cancel{2} \times \cancel{2} \times 3 \times \frac{1}{\cancel{4}} = 3$

38 $\frac{6ab^2c^3}{2a^2bc^8} = \frac{3\,\cancel{6}\,\cancel{a}\cdot b\cdot\cancel{b}\cdot\cancel{c}\cdot\cancel{c}\cdot\cancel{c}}{\cancel{2}\,\cancel{a}\cdot a\cdot\cancel{b}\cdot\cancel{c}\cdot\cancel{c}\cdot\cancel{c}\cdot c^5} = \left(\frac{3b}{a\cdot c^5}\right)$

$= 3a^{-1}\cdot b\cdot c^{-5}$

39 $\frac{64 \times \sqrt[3]{128}}{\sqrt[5]{512}} = \frac{2^6 \times (2^7)^{1/3}}{(2^9)^{1/5}} = \frac{2^6 \times 2^{7/3}}{2^{9/5}} = (2)^{6+\frac{7}{3}-\frac{9}{5}}$

$= (2)^{\frac{90+35-27}{15}} = (2)^{98/15} = \sqrt[15]{2^{98}}$

40 $\frac{4x^{-1}}{x^{-1/3}} = \frac{4\cdot x^{1/3}}{x} = 4\cdot(x)^{\frac{1}{3}-1} = 4\cdot(x)^{-2/3}$

$= \left(\frac{4}{x^{2/3}}\right) = \frac{4}{\sqrt[3]{x^2}}$

41 $\frac{2a^{1/2} \times a^{2/3} \times a^{-7/3}}{9a^{-5/3} \times a^{3/2}} = ?$ If $a = 4$

$= \frac{2(4)^{1/2} \times (4)^{2/3} \times (4)^{-7/3}}{9 \times (4)^{-5/3} \times (4)^{3/2}}$

$= \frac{2}{9}(4)^{\frac{1}{2}+\frac{2}{3}-\frac{7}{3}+\frac{5}{3}-\frac{3}{2}}$

$= \frac{2}{9}(4)^{\frac{1}{2}-\frac{3}{2}}$

$= \frac{2}{9}(2^2)^{-1}$

$= \frac{2}{9} \times \frac{1}{4} = \frac{2}{36} = \frac{1}{18}$

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(1) $\frac{(a^m \times a^n \times a^p)}{a^x} = (a)^{(m+n+p-x)}$

(2) $\frac{64^{1/3}}{1024^{-8/3}} = \frac{(2^6)^{1/3}}{(2^{10})^{-8/3}} = \frac{2^{6/3}}{2^{-80/3}} = (2)^{\frac{6}{3}+\frac{80}{3}} = 2^{86/3}$

$= \sqrt[3]{2^{86}}$

43 $\sqrt[6]{a^{4b} \cdot x^{6} \ (a^{2/3} \cdot x^{-1})^{-b}} = ?$

$$= \left[a^{4b} \cdot x^{6} \cdot a^{-2b/3} \cdot x^{b}\right]^{1/6}$$

$$= \left(a^{4b-\frac{2b}{3}} \cdot x^{6+b}\right)^{1/6} = \left(a^{\frac{10b}{3}} \cdot x^{6+b}\right)^{1/6} = a^{\frac{5b}{9}} \cdot (x)^{1+\frac{b}{6}}$$

44 $(\sqrt{9})^{7} \times (\sqrt{3})^{-5} = 3^{k}$ then $k = ?$

$$3^{7} \times (3^{1/2})^{-5} = 3^{k}$$

$$3^{7} \times 3^{-5/2} = 3^{k}$$

$$(3)^{7-\frac{5}{2}} = (3)^{k}$$

$$(3)^{9/2} = (3)^{k}$$

$$\therefore k = 9/2$$

$$k = 4.50$$

45 $\frac{2^{5}}{2^{5}} = (2)^{5-5} = 2^{0} = 1 \quad \therefore (\text{Any Number})^{0} = 1$

46 $\left(\frac{81x^{4}}{y^{-8}}\right)^{1/4} = \frac{(81x^{4})^{1/4}}{(y^{-8})^{1/4}} = \frac{(3x)^{4\times\frac{1}{4}}}{y^{-8\times\frac{1}{4}}} = \frac{(3x)^{1}}{(y^{-2})} = 3xy^{2}$

47 $\left\{\frac{(3^{3})^{2} \times (4^{2})^{3} \times (5^{3})^{2}}{(3^{2})^{3} \times (4^{3})^{2} \times (5^{2})^{3}}\right\} = \frac{3^{6} \times 4^{6} \times 5^{6}}{3^{6} \times 4^{6} \times 5^{6}} = 1$

48 $y^{a-b} \cdot y^{b-c} \cdot y^{c-a} = ?$

$$= (y)^{a-b+b-c+c-a} = (y)^{0} = 1$$

49 $\left[1 - \left\{1 - (1-x^{2})^{-1}\right\}^{-1}\right]^{-1/2} =$

$$= \left[1 - \left\{1 - \frac{1}{1-x^{2}}\right\}^{-1}\right]^{-1/2} = \left[1 - \left\{\frac{-x^{2}}{1-x^{2}}\right\}^{-1}\right]^{-1/2} = \left[1 + \frac{1-x^{2}}{x^{2}}\right]^{-1/2} = \left[\frac{1}{x^{2}}\right]^{-1/2}$$

$$= (x^{-2})^{-1/2}$$

$$= x^{1} = x$$

50 $\left[(x^{n})^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}} = \left[(x^{n})^{\frac{n^{2}-1}{n}}\right]^{\frac{1}{(n+1)}}$

$$= \left[x^{\cancel{n} \times \frac{(n-1)(n+1)}{\cancel{n}}}\right]^{\frac{1}{n+1}}$$

$$= x^{(n-1)\cancel{(n+1)} \times \frac{1}{\cancel{(n+1)}}}$$

$$= (x)^{n-1}$$

$$= \left(\frac{x^{n}}{x}\right)$$

51 If $a^x = b$, $b^y = c$, $c^z = a$ then xyz = ?

$a^x = b$

$\log a^x = \log b$

$x.\log a = \log b$

$x = \dfrac{\log b}{\log a}$

$y = \dfrac{\log c}{\log b}$

$z = \dfrac{\log a}{\log c}$

$xyz = \dfrac{\cancel{\log b}}{\cancel{\log a}} \times \dfrac{\cancel{\log c}}{\cancel{\log b}} \times \dfrac{\cancel{\log a}}{\cancel{\log c}}$

$= 1$

$a^x = b$

$b^y = c$

$c^z = a$

$\therefore a = b^{1/x}$

$\therefore b = c^{1/y}$

$c = a^{1/z}$

$c = (b^{1/x})^{1/z}$

$c = (b)^{1/xz}$

$c = (c^{1/y})^{1/xz}$

$c^1 = (c)^{\frac{1}{xyz}}$

$\therefore 1 = \dfrac{1}{xyz}$

$\therefore xyz = 1$

52 $\left(\dfrac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \cdot \left(\dfrac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c^2+ac+a^2)}$ = ?

Pls. Remember: $\boxed{(a^3-b^3) = (a-b)(a^2+ab+b^2)}$

$= \left(x^{a-b}\right)^{a^2+ab+b^2} \cdot \left(x^{b-c}\right)^{b^2+bc+c^2} \cdot \left(x^{c-a}\right)^{c^2+ac+a^2}$

$= x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3} = (x)^{\cancel{a^3}-\cancel{b^3}+\cancel{b^3}-\cancel{c^3}+\cancel{c^3}-\cancel{a^3}} = x^0 = 1$

53 Log of number consist of 2 parts

Integer Part = characteristic

Fractional Part = Mantissa

54 Log x = characteristic of x + Mantissa of x

$\log b^a = a.\log b$

$\log m^{(ab)} = ab(\log m)$

$\log_m (a/b) = \log_m a - \log_m b$

If $\log_b a = k$; then $b^k = a$

If $x^y = z$; then $\log_x z = y$

$\log (a)^{-b} = -b.\log a$

$\log (ab/c) = \log a + \log b - \log c$

A.log (log x) = x

Log (A.log x) = x

$\log_a a = 1$ provided $a \neq 1$

$\log_b a \times \log_c b = \log_c a$

$\log_b a = \left(\dfrac{\log a}{\log b}\right)$

$\log_m (ab) = \log_m a + \log_m b$

$\log_{10} 10 = 1$

$\log_{10} 1 = 0$

$\log_{10} 100 = 2$

$\log_{10} 10{,}000 = 4$

$\log_{10} 1000 = 3$

common base of Logs : 10

Natural base of Logs : e

$\log_m abc = \log_m a + \log_m b + \log_m c$

$\boxed{\text{If } \log_q m = j \text{ then } q^j = m}$

$\boxed{\log x^z = z.\log x}$

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$$\frac{\log_3 8}{\log_9 16 \times \log_4 10} = \left[\frac{\log 8/\log 3}{\frac{\log 16}{\log 9} \times \frac{\log 10}{\log 4}}\right]$$

$$= \frac{\log 2^3}{\log 3} \times \frac{\log 3^2}{\log 2^4} \times \frac{\log 2^2}{\log 10}$$

$$= \frac{3.\cancel{\log 2}}{\cancel{\log 3}} \times \frac{\cancel{2}.\cancel{\log 3}}{\cancel{4}.\cancel{\log 2}} \times \frac{\cancel{2}.\log 2}{1}$$

$$= 3.\log 2 = \log 2^3 = \log 8$$

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Log x (where x > 0)

Characteristic of x (Integer part) + Mantissa of x (Fractional part)

- If x ≥ 1: (No. of digits before decimal point - 1)
- 1 > x > 0: (No. of zeros immediately after decimal point + 1) BAR

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x	Characteristic of x	x	characteristic of x
56.81	2 − 1 = 1	0.0081	$\bar{3}$
583.2	3 − 1 = 2	3.639	0
81.93	2 − 1 = 1	0.01181	$\bar{2}$
5.81	1 − 1 = 0	13.21	1
13.00	2 − 1 = 1	88888	4
0.008126	$\bar{3}$	33.63	1
0.5826	$\bar{1}$	7.819	0
8.5926	1 − 1 = 0	0.0021	$\bar{3}$

58 **How to find Log x on calculator?**

⟹ Enter x
then press √ 15 times
then deduct 1
then Multiply by 14230.9635
You will get Answer of Log x on calculator

$$\boxed{\begin{aligned}&\log 23.81\\ &= 1.3768260 7989\end{aligned}}$$

59 **How to find A.log y on calculator?**

⟹ Enter 'y'
then divide by 14230.9635
then Add 1
then (× =) 15 times

$$\boxed{\begin{aligned}&A.\log 1.3768260 7989\\ &= 23.81\end{aligned}}$$

60 How to find a^b on calculator? (Particularly when b is in fractions)

$\Longrightarrow$ How to find a^b

1) Enter 'a'
2) $\sqrt{\ }$ 12 times
3) Deduct 1
4) Multiply by 'b'
5) Add 1
6) $x =$ 12 times

$\sqrt[5]{600}$

$= 600^{1/5} = 600^{0.20}$

$= 3.59731$

61 Common base of Logs is : 10

Natural base of Logs is : e where e = exponential factor = 2.7183 (approx)

62 $\text{Log}_{\sqrt{2}} 64 = \frac{\text{Log} 64}{\text{Log} \sqrt{2}} = \frac{\text{Log} 2^6}{\text{Log} 2^{1/2}} = \frac{6 \cdot \cancel{\text{Log} 2}}{\frac{1}{2} \cancel{\text{Log} 2}} = 6 \times 2 = 12$

$\text{Log}_{\sqrt{2}} 64 = m \quad \therefore (\sqrt{2})^m = 64 \quad (2^{1/2})^m = 2^6 \quad \therefore 2^{m/2} = 2^6 \quad \therefore \frac{m}{2} = 6$

$\therefore m = 12$

63 $\text{Log}_2 \text{Log}_2 \text{Log}_2 16 = \text{Log}_2 \text{Log}_2 \left(\frac{\text{Log} 16}{\text{Log} 2}\right) = \text{Log}_2 \text{Log}_2 (4)$

$= \text{Log}_2 \left[\frac{2 \cancel{\text{Log} 2}}{\cancel{\text{Log} 2}}\right] = \text{Log}_2 2 = 1$

64 $\text{Log}_9 (1/3) = \frac{\text{Log}(1/3)}{\text{Log } 9} = \frac{\text{Log } 3^{-1}}{\text{Log } 3^2} = \frac{-1 \cdot \cancel{\text{Log} 3}}{2 \cdot \cancel{\text{Log} 3}} = -1/2 = -0.50$

65 $\text{Log}_{16} 32^{-8} = \frac{\text{Log}(32)^{-8}}{\text{Log}(16)} = \frac{\text{Log}(2^5)^{-8}}{\text{Log}(2)^4} = \frac{\text{Log}(2)^{-40}}{\text{Log}(2)^4}$

$= \frac{-40 \cancel{\text{Log} 2}}{4 \cancel{\text{Log} 2}} = -10$

66 Log x = (m + n) ; Log y = (m - n); then

$\text{Log}\left(\frac{10x}{y^2}\right) = \text{Log } 10x - \text{Log } y^2$

$= \text{Log} 10 + \text{Log } x - 2 \text{Log } y$

$= 1 + m + n - 2(m - n)$

$= 1 + m + n - 2m + 2n$

$= (-m + 3n + 1) = (3n - m + 1)$

67 2 Log 5 + Log 8 - (1/2) Log 4 =

$= \log 5^2 + \log 8 - \log (4)^{1/2}$

$= \log 25 + \log 8 - \log 2$

$= \log \left(\frac{25 \times 8}{2}\right) = \log 100 = 2.00$

68 $\sqrt[4]{729 \times \sqrt[3]{9^{-1} \times 27^{-4/3}}} = ?$

$= \left[3^6 \times \left(\frac{1}{9} \times \frac{1}{(3^3)^{4/3}}\right)^{1/3}\right]^{1/4} = \left[3^6 \times \left(\frac{1}{3^2 \cdot 3^4}\right)^{1/3}\right]^{1/4} = \left[3^6 \times (3^{-6})^{1/3}\right]^{1/4}$

$= \left[3^6 \times 3^{-2}\right]^{1/4} = \left[3^4\right]^{1/4} = 3^1 = 3.00$

69 $\log_{2\sqrt{2}} 64 = ?$

$= \frac{\log 64}{\log 2\sqrt{2}} = \frac{\log 2^6}{\log (2^1 \times 2^{1/2})} = \frac{\log 2^6}{\log 2^{3/2}} = \frac{6 \cdot \cancel{\log 2}}{\frac{3}{2} \cancel{\log 2}} = 6 \times \frac{2}{3} = 4$

70 Find 4th proportional to 2/3, 3/7, 4.

$\Longrightarrow \frac{2}{3}, \frac{3}{7}, 4, m$

$\therefore \frac{2m}{3} = \frac{3}{7} \times 4$

$\therefore m = \frac{3}{7} \times 4 \times \frac{3}{2}$

$m = \frac{36}{14} = \frac{18}{7}$

$\therefore$ 4th prop. to $\frac{2}{3}, \frac{3}{7}, 4$ is $\frac{18}{7}$

71 If $2^x = 3^y = 6^{-z}$; then $(1/x) + (1/y) + (1/z) = ?$

$2^x = 3^y = 6^{-z} = k$

$x \cdot \log 2 = y \cdot \log 3 = -z \cdot \log 6 = \log k$

$x = \frac{\log k}{\log 2}, \quad y = \frac{\log k}{\log 3}, \quad z = \frac{-\log k}{\log 6}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

$= \frac{\log 2}{\log k} + \frac{\log 3}{\log k} - \frac{\log 6}{\log k}$

$= \frac{\log 2 + \log 3 - \log 6}{\log k} = \frac{\log (2 \times 3) - \log 6}{\log k}$

$= (\log 6 - \log 6)/\log k = \text{zero}.$

72 Find in what ratio will the total wages of the workers of a factory be increased or decreased if there is reduction in no. of workers in the ratio of 17:12 and increment in wage rate per worker in the ratio of 24:29

$\Longrightarrow$

	old	New
No. of workers	x	$x \times \frac{12}{17} = \frac{12x}{17}$
wage rate per worker	y	$y \times \frac{29}{24} = \frac{29y}{24}$
Total wages	xy	$\frac{\cancel{12}x}{17} \times \frac{29y}{\cancel{24}_{2}} = \frac{29xy}{34}$

$\left(\frac{\text{old Total wages}}{\text{New Total wages}}\right) = \frac{xy}{\frac{29xy}{34}}$

$= 34:29$

73 What least number must be added to each one of 6, 14, 18, 38 to make them in proportion

a. 5 b. 3 ~~c.~~ 2 d. 4

8, 16, 20, 40 are in proportion

74 The incomes of X and Y are in the ratio of 3:2 and their expenditures are in the ratio of 5:3. If each saves ₹ 1500 then incomes of X and Y resp. are :

$\Longrightarrow$ Incomes of X, Y are $3x, 2x$

$$\frac{3x-1500}{2x-1500} = \frac{5}{3}$$

$9x - 4500 = 10x - 7500$

$\boxed{3000 = x}$

Incomes of X, Y are ₹9000, ₹6,000

75 In a sugar solution of 300 gms, the proportion of sugar is 40%. How much sugar should be added to make it 50%

$\Longrightarrow$ sugar $= 300 \times 40\% = 120$ gms

Let x gms of sugar be added

$50\%(300+x) = (120+x)$

$150 + 0.50x = 120 + x$

$30 = 0.50x$

$\boxed{x = 60 \text{ gms}}$

76 A mixture contains milk and water in the ratio of 5:1. On adding 5 litres of water, the ratio of milk to water becomes 5:2. The quantity of milk in the original mixture is :

$\Longrightarrow$ original mixture $\Rightarrow$ milk : water $= 5x : x$

$$\left(\frac{5x}{x+5}\right) = \frac{5}{2}$$

$\therefore 10x = 5x + 25$

$5x = 25$

$x = 5$ litres.

Qty of milk in original mixture $= 25$ litres

77 If the denominator of a fraction exceed the numerator by 8. If numerator and denominator are both increased by 5, then fraction becomes 3/5. Find the original fraction.

$\Longrightarrow$ $\left(\frac{x}{x+8}\right)$ be original fraction.

$$\left(\frac{x+5}{x+13}\right) = \frac{3}{5}$$

$\therefore 5x + 25 = 3x + 39$

$2x = 14$

$x = 7$

$\therefore$ original Fraction is : $7/15$

78 If $\text{Log}_{3/2} x = 3$, Find value of x

$\therefore \left(\frac{3}{2}\right)^3 = x$

$$\therefore x = \left(\frac{3^3}{2^3}\right) = \frac{27}{8}$$

$\boxed{\text{If } \text{Log}_b a = k \text{ then } b^k = a}$

79 $\text{Log}_{(1/9)} 243 = x$. Find x

$\Longrightarrow \left(\frac{1}{9}\right)^x = 243$

$\left(\frac{1}{3^2}\right)^x = 3^5$

$(3^{-2})^x = (3)^5$

$\therefore (3)^{-2x} = (3)^5$

$\therefore -2x = 5$

$\therefore x = -5/2$

OR

$$x = \frac{\text{Log } 243}{\text{Log } (1/9)}$$

$$x = \frac{\text{Log } 3^5}{\text{Log } 3^{-2}} = \frac{5.\cancel{\text{Log} 3}}{-2.\cancel{\text{Log} 3}}$$

$x = (-5/2)$

80 Log x^3 - 2 Log x - 2 = 0. Find x

$\Longrightarrow$ $3 \cdot \log x - 2\log x = 2$

$\log_{10} x = 2$

$\therefore 10^2 = x$

$\therefore x = 100$

81 $\log_a 3 = 2$, $\log_b 8 = 3$ then $\log_b a = ?$

$\Longrightarrow$ $\log_a 3 = 2$

$\therefore a^2 = 3$

$a = 3^{1/2}$

$\log_b 8 = 3$

$\therefore b^3 = 8$

$b = 2$

$\log_b a$

$= \frac{\log a}{\log b}$

$= \frac{\log 3^{1/2}}{\log 2}$

$= \frac{\frac{1}{2}\log 3}{\log 2}$

$= \frac{\log 3}{2\log 2} = \frac{\log 3}{\log 4}$

$= \log_4 3$

82 If 2 Log a + 3 Log b - 2 = 0 then a^2b^3 = ?

$\Longrightarrow$ $2\log a + 3\log b = 2$

$\log a^2 + \log b^3 = 2$

$\log_{10}(a^2b^3) = 2$

$\therefore 10^2 = a^2b^3$

$\therefore (a^2 \cdot b^3) = 100$

83 $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$

$= \log_2\left[\log_2\{\log_3(9)\}\right] = \log_2\left[\log_2\{2\}\right] = \log_2[1] = 0$

84 2 numbers are in the ratio of 3:4. If 6 is added to each term then the new ratio will be 4:5 then the numbers are

$\Longrightarrow$ Let 2 numbers be $3x, 4x$

$\frac{3x+6}{4x+6} = \frac{4}{5}$ $\therefore 15x + 30 = 16x + 24$

$\therefore 6 = x$

$\therefore x = 6$

$\therefore$ Numbers are $= 18, 24$

85 The sub-duplicate ratio of 1250:50 is :

simplest form of 1250 : 50 = 25 : 1   Answer : (5 : 1)

86 Dhrish earns ₹ 2,780 in 7 hrs and Vinod earns ₹ 990 in 12 hrs. Ratio of their earning per hour is :

$= \left(\frac{2780/7}{990/12}\right) = \frac{278\cancel{0}}{7} \times \frac{\cancel{12}^{\,4}}{\cancel{990}_{\,33}} = \frac{1112}{231} = (1112 : 231)$

87 P, Q, R are 3 cities. The ratio of avg. temp. of P, Q is 11:12 and that of P, R is 9:8. Find the ratio of avg temp. of Q:R.

$\Longrightarrow$ P : Q = 11 : 12 = 99 : 108

P : R = 9 : 8 = 99 : 88

$\therefore$ P : Q : R = 99 : 108 : 88

Q : R = 108 : 88 = 27 : 22

88 If 2s : 3t is the duplicate ratio of (2s-p) : (3t-p) then

~~a.~~ $p^2 = 6st$ b. p = 6st   c. 2p = 3st   d. None of these

$\Longrightarrow$ $\frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$

$\frac{2s}{3t} = \left(\frac{4s^2 - 4ps + p^2}{9t^2 - 6pt + p^2}\right)$

$18t^2s - \cancel{12pts} + 2p^2s = 12s^2t - \cancel{12pts} + 3p^2t$

$18t^2s - 12s^2t = 3p^2t - 2p^2s$

$6st(3t - \cancel{2s}) = p^2(3t - \cancel{2s})$

$\therefore p^2 = 6st$

89 If A = B/2 = C/5; then A:B:C is :

$$\Rightarrow \left(\frac{A}{1} = \frac{B}{2}\right) = \frac{C}{5} \qquad \therefore A:B:C = 1:2:5$$

$$A:B = 1:2, \quad B:C = 2:5$$

90 Log5 = 0.6990, Log 3 = 0.4771 then Log (50/300) = ?

$$\Rightarrow \text{Log}\left(\frac{50}{300}\right) = \text{Log}50 - \text{Log}300$$
$$= 1.6990 - 2.4771$$
$$= -0.7781$$

$$\text{Log}(50/300)$$
$$= \text{Log}(0.1666666)$$
$$= -0.7781$$

91 Log 2 = x; Log 3 = y; then Log 60 = ?

$$\Rightarrow \text{Log}(60) = \text{Log}(3 \times 2 \times 10)$$
$$= \text{Log}3 + \text{Log}2 + \text{Log}10$$
$$= (x + y + 1)$$

92 Log (1/81) to the base 9 is equal to :

$$\Rightarrow = \frac{\text{Log}(1/81)}{\text{Log } 9} = \frac{\text{Log}(9)^{-2}}{\text{Log } 9} = \frac{-2 \cdot \cancel{\text{Log } 9}}{\cancel{\text{Log } 9}} = -2$$

93 $\bar{4}.5671 + 7.8253 = ?$

$$= (-4 + 0.5671) + 7.8253$$
$$= -3.4329 + 7.8253 = 4.3924$$

$$\bar{8}.2386 + \bar{3}.2629$$
$$= -8 + 0.2386 - 3 + 0.2629$$
$$= -10.4985$$

94 $\sqrt[(a+b)]{\frac{x^{a^2}}{x^{b^2}}} \cdot \sqrt[(b+c)]{\frac{x^{b^2}}{x^{c^2}}} \cdot \sqrt[(c+a)]{\frac{x^{c^2}}{x^{a^2}}}$

$$= \left(x^{a^2-b^2}\right)^{\frac{1}{a+b}} \cdot \left(x^{b^2-c^2}\right)^{\frac{1}{b+c}} \cdot \left(x^{c^2-a^2}\right)^{\frac{1}{c+a}}$$

$$= x^{\cancel{(a+b)}(a-b) \times \frac{1}{\cancel{(a+b)}}} \cdot x^{(b-c)\cancel{(b+c)} \times \frac{1}{\cancel{(b+c)}}} \cdot x^{(c-a)\cancel{(c+a)} \times \frac{1}{\cancel{(c+a)}}}$$

$$= x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$$

$$= x^{a-b+b-c+c-a}$$

$$= x^0 = 1$$

95 What is a commensurable ratio and incommensurable ratio?

⟹

① If a ratio can be written in the form of ratio of integers, then it is said to be a commensurable ratio

i) $5:7$, $\sqrt{64}:\sqrt{121}$, $\sqrt{50}:\sqrt{32}$

ii) $1.50:3.75 = 150:375 = 6:15 = 2:5$

iii) $1.871:3.56 = 1871:3560$

iv) $1.8319:6.33691 = 183190:633691$

} commensurable ratios

② If a ratio can't be written in the form of ratio of integers, then it is said to be incommensurable ratio.

$\sqrt{2}:\sqrt{5}$, $\sqrt{11}:\sqrt{19}$ (incommensurable OR Non-commensurable)

96 A Dealer mixes tea costing ₹ 6.92 per kg with tea costing ₹ 7.77 per kg and sells the mixture at ₹ 8.80 per kg and earns profit of 17 1/2 % on sales price. In what proportion does he mix them ?

a. 2 : 3　　: 2　　5 : 2　　d. None of these

⟹

Total cost of $(x+y)$ kgs of Tea $=$ (6.92 per kg × x kgs) + (7.77 per kg × y kgs)

$7.26(x+y) = 6.92x + 7.77y$

$7.26x - 6.92x = 7.77y - 7.26y$

$0.34x = 0.51y$

$x/y = 51/34 = 3/2$

In the proportion of 3:2 tea has been mixed

Selling price per kg = 8.80

profit = 8.80 × 17.50% = 1.54 per kg

Total cost per kg = 8.80 − 1.54 = 7.26

97 If x : y = z : w = 8 : 7; then $\left(\frac{x+z}{y+w}\right) = ?$

$$\Longrightarrow \frac{x}{y} = \frac{z}{w} = \frac{x+z}{y+w} = \frac{8}{7} \text{ .... by using Addendo}$$

98 If $\left(\frac{5x - 3y}{5y - 3x}\right) = \frac{3}{4}$ then $x : y = ?$

$$\Longrightarrow \frac{5x-3y}{5y-3x} = \frac{3}{4}$$

$$20x - 12y = 15y - 9x$$

$$29x = 27y$$

$$\left(\frac{x}{y}\right) = \left(\frac{27}{29}\right)$$

$\therefore x : y = 27 : 29$

99 Find value of x if $x^2\sqrt{x} = (x\sqrt{x})^x$

$$\Longrightarrow x^2 . \sqrt{x} = (x\sqrt{x})^x$$

$$x^2 . x^{1/2} = (x^1 . x^{1/2})^x$$

$$x^{2+\frac{1}{2}} = (x^{1+\frac{1}{2}})^x$$

$$x^{5/2} = (x^{3/2})^x$$

$$x^{5/2} = x^{3x/2}$$

$$\therefore \frac{5}{2} = \frac{3x}{2}$$

$$3x = 5$$

$$x = (5/3)$$

100 $\frac{(3^3)^2 \text{ x } (4^2)^3 \text{ x } (5^3)^2}{(3^2)^3 \text{ x } (4^3)^2 \text{ x } (5^2)^3} = \frac{\cancel{3^6} \times \cancel{4^6} \times \cancel{5^6}}{\cancel{3^6} \times \cancel{4^6} \times \cancel{5^6}} = 1$

- If A's income is 200 crores. His income changes in the ratio of 2 : 3. Find New income ?

$\Longrightarrow$ New Income = old Income x multiplying ratio

where multiplying ratio = Inverse ratio of given ratio

New Income = 200 crores x $\frac{3}{2}$ = 300 crores

New quantity = old quantity x Multiplying ratio

old quantity = New quantity x Given ratio

Calculator Tricks

1 How to find a^b on calculator. (Mainly when b is a fractions)

Enter 'a'
$\sqrt{\ }$ 12 times
Deduct 1
Multiply by 'b'
Add 1
'x=' 12 times

Find -

1. $12^{0.35} =$ 2.38664531197
2. $286^{1.3528} =$ 2099.72850172
3. $1.0296^{0.3} =$ 1.00878947544
4. $878^{1.2896} =$ 6237.66084318
5. $\sqrt[5]{100} = 100^{1/5} = 100^{0.20} =$ 2.51292715552

2 How to find Log x on calculator

Enter 'x'
$\sqrt{\ }$ 15 times
Deduct 1
Multiply by 14230.9635

Find -

1. Log 35 = 1.5441518987
2. Log 896.8 = 2.95300220038
3. Log 0.008671 = −2.06178184957

3 How to find A.Log y on calculator

Enter 'y'
Divide by 14230.9635
Add 1
'x=' 15 times

Find -

1. A.Log 2.8935 = 782
2. A.Log 0.08613 = 1.21935329035
3. A.Log 5.8863 = 767506.389108
4. A. Log 1.2287 = 16.929600612

4

$1.0686^{90} =$ 392.071510773 — 1.0686 x = till step count comes 91

$1.0296^{56} =$ 5.12198023366 — 1.0296 x = till step count comes 57

$1.0811^{61} =$ 116.363803473 — 1.0811 x = till step count comes 62

Find Discounting factor @ 8.50% for Year 28
$\Longrightarrow$ 0.10185147683

5 How to find discounting factor on calculator?

1 ÷ (1+r) then '=' n times

$$\text{Discounting Factor} = \frac{1}{(1+r)^n}$$

Find Discounting Factor for 20% for Year 13
$\Longrightarrow$ 0.09346387896

Find annuity factor @ 13.50% for 30 years
$\Longrightarrow$ 7.241528 79703

Find annuity factor @ 18.12% for 48 years
$\Longrightarrow$ 5.5169003497

6 How to find annuity factor on calculator?

1 ÷ (1+r) then '=' n times and GT

7 8, 15, 22, 29............... Find t_{28}, t_{38}

$\Longrightarrow$ 8, 15, 22, 29

$t_{28} = 197$ $\quad t_{91} = 638$

$t_{38} = 267$

8

$5^2 = 25$ $\quad$ 5 × =

$15^2 = 225$ $\quad$ 15 × =

$28^2 = 784$ $\quad$ 28 × =

9

$\frac{1}{2 \times 2} = 0.25$

$\frac{1}{20} = 0.05$

$\frac{1}{5 \times 28} = 0.00714285714$

$\frac{1}{25 \times 4} = 0.01$

If you ÷ = on calculator then you will get reciprocal of the number on calculator screen

10

3 + 5 = 8

8 + 5 = 13

9 + 5 = 14

10 + 5 = 15

100 + 5 = 105

2086 + 5 = 2091

2246

3 + 5 =
then 8 =
9 =
10 =
100 =
2086 =

11

100 - 3 = 97

208 - 3 = 205

98 - 3 = 95

63 - 3 = 60

238 - 3 = 235

692

100 - 3 =
then 208 =
98 =
63 =
238 =

12

13 x 3 = 39
13 x 5 = 65
13 x 8 = 104
13 x 11 = 143
13 x 20 = 260

611

13 x 3 =
then 5 =
8 =
11 =
20 =

13

a. (1.20 x 5.36) + (28.96 + 15.92) + (28.11 x 18.63)
= 575.0013

b. (15.92 x 21.83) + (28.66 x 11.193) - (5.06 x 18.193)
= 576.2684

c. (17.33 x 18.21) + (11.35 x 19.81) − (3.86 x 12.86) − (13.22 x 11)
= 345.3632

14

$5^2 + 8^2 + 9^2 + 13^2 + 16^2 =$

① = (5 x M+) (8 x M+) (9 x M+) (13 x M+) (16 x M+)
then MRC
= 595

② $20^3 + 1.83^4 + 9.63^5 + 11.21^3 + 19.86^2$
= 92633.6064383

15

$(5 \times 9) + (33 \times 18) + (28 \times 93) - (16 \times 6) + 13^2$
= 3316

② (18.19 x 32.636) − (22.86 x −1.81) + (−33.86 x 11.81)
− (−11.83 x −1.81) + (3.86 x 111.92)

= 645.73774

③ (13.86 x −11.96) + (−33.96 x 55.81) + (55.331 x 123.861)

= 4792.279791

④ (20.536 x −11.9391) + (−6.36 x 28.53)

= −426.6321576

EXERCISE

1 Log 28.96 = 1.4618737332

2 A.Log 2.8592 = 722.62383164 5

3 1.20868592^{28} = 201.733008327

4 $883.9281^{1.5625}$ = 39960.8104092

5 68, 74, 80, 86............ Find t_{28}, t_{32}

t_{28} = 230

t_{32} = 254

6

	x	y	x^2	y^2	xy	x^2y	xy^2
	1.20	8.53					
	9.63	2.58					
	10.61	11.93					
Total	21.44	23.04	206.749	221.7422	161.6587	1594.529555	1661.481401

7 $\sqrt{\frac{63581}{8} - 56^2}$ = 69.3658777786

8 $\sqrt{(86 \times 93) + (59 \times 81) + (29 \times 63)}$ = ?

= 120.84701072

9 $\sqrt[10]{58263}$ = $(58263)^{1/10}$ = 2.9999

10 $\sqrt[15]{56298193}$ = $(56298193)^{1/15}$

= 3.29421172539

11

① $\dfrac{8}{\frac{1}{2}+\frac{1}{3}+\frac{1}{9}+\frac{1}{13}+\frac{1}{16}+\frac{1}{18}} = 7.02109704649$

② $\dfrac{13}{\left(\frac{3}{7}+\frac{8}{11}+\frac{9}{19}+\frac{2}{18}\right)} = 7.46851956896$

12

$\sqrt[8]{93} = 1.76222098474$

$\sqrt[4]{124} = 3.33699396547$

$\sqrt[16]{28963} = 1.90048839286$

$\sqrt[32]{58231} = 1.40900025029$

$\sqrt[64]{28,63,588} = 1.26150572879$

$\sqrt[11]{52,93,211} = (52,93,211)^{1/11} = 4.0954198426$

$\sqrt[20]{5,85,93,288} = (5,85,93,288)^{1/20} = 2.45018582765$

13

100 x 18% = 18

100 + 18% = 118

200 + 16% = 232

300 + 12% = 336

1050 + 16% = 1218

283 + 3.53% = 292.9899

18 + 2% = 18.36

200 - 3% = 194

300 - 2% = 294

14

① $\left|\frac{3}{5}+\frac{8}{7}+\frac{11}{9}+\frac{25}{8}\right| = 6.09007936507$

② $\dfrac{102}{\left(\frac{18}{17}+\frac{36}{53}+\frac{29}{81}-\frac{36}{27}-\frac{1}{8}\right)} = 159.934728958$

③ $\dfrac{28}{\left(\frac{1}{7}+\frac{8}{11}+\frac{3}{13}-\frac{9}{20}\right)} = 43.0174199997$

15

$15^2 + 8^3 + 3^4 + 18^2 + 2.82^3 + 9.53^4 = 9412.8616\,4481$

16

$$\frac{16}{(2/5)} + \frac{18}{(3/8)} + \frac{19}{(5/7)} + \frac{28}{(3/11)} = 217.2666666$$

$$\left(\frac{20}{8/7} + \frac{35}{1/8} + \frac{80}{2/7} + \frac{90}{8/11}\right) = 701.25$$

$$1.0193^{88} = 5.377471\,66356$$

Chapter 2

TIME VALUE OF MONEY

CA VINOD REDDY

1

Amount = Principle + Interest
Principle = Amount - Interest
Interest = Amount - Principle

2

Why is interest paid?

1. Time Value of Money
2. Opportunity Cost
3. Inflation
4. Liquidity Preference
5. Risk Factor

In other words interest is the rent to be paid by borrower to the lender for using lender's money.

3

Simple Interest = $p.n.r$

Amount = P + Simple Interest
$= p + pnr = p(1+nr)$

4

Compound Interest = $p\left[(1+r)^n - 1\right]$

Amount = $p(1+r)^n$

where
r = Rate of interest for the conversion period
n = No. of conversion periods

5 With Simple Interest

Amount Invested	Amount at the end of years						
	5	10	15	20	25	30	35
P	2P	3P	4P	5P	6P	7P	8P
P	3P	5P	7P	9P	11P	13P	15P

6 With Compound Interest

Amount Invested	Amount at the end of years					
	7	14	21	28	35	42
P	2P	4P	8P	16P	32P	64P
P	3P	9P	27P	81P	243P	729P
P	4P	16P	64P	256P	1024P	4096P

7

A = 50,00,000; r = 12% p.a.S.I; P = ?; n = 10 years

$\Longrightarrow A = p(1+nr)$

$50,00,000 = p\left[1 + (10 \times 0.12)\right]$

$\therefore p = ₹\ 22,72,727$

8 A = 50,50,000; r = 13.50% p.a.S.I; P = 20,00,000 ; n = ____ years

$\Longrightarrow$ $A = p(1 + nr)$

$50,50,000 = 20,00,000(1 + n \times 0.1350)$

$\boxed{n = 11.2963 \text{ Years (approx)}}$

9 A = ?; r = 18% p.a.S.I; P = 25,000 ; n = 8 years 3 months

$A = p(1+nr)$

$= 25,000\left[1 + (8.25 \times 0.18)\right]$

$= ₹\ 62,125/-$

10 A sum of money doubles itself with compound interest in 10 years. How many times it will become after 40 years?

Sum invested	After			
	10 Years	20 Years	30 Years	40 Years
P	2P	4P	8P	16P

Sum will become 16 times

11 Find the future value of ₹ 50,000 after 25 years @ 22% p.a.C.I

$\Longrightarrow$ Future value = present value $\times (1+r)^n$

$= 50,000 \times (1.22)^{25}$

$= ₹\ 72,10,506/-$ (approx)

My Notes

① Amount = principle amt $\times (1+r)^n$

② Future value = present value $\times (1+r)^n$

③ present value = Fututure value $\times \boxed{\dfrac{1}{(1+r)^n}}$

$\therefore$ present value = Future value $\times$ Discounting factor

12 Find present value of ₹ 20,00,000 receivable after 25 years if money is 18.50% effective.

⟹ Present Value = Future Value × Discounting Factor

= ₹20,00,000 × 0.01435625753

= ₹ 28,712.51506

Cross-check

Future value = $28712.51506 \times (1.1850)^{25}$

= 20,00,000/-

13 A = ?; r = 14% p.a.C.Q; P = 20,00,000 ; n = 3 years 9 months

$$\Longrightarrow \quad A = P(1+r)^n$$

$$A = 20{,}00{,}000\left(1+\frac{0.14}{4}\right)^{15}$$

$$A = 20{,}00{,}000 \times (1.035)^{15} = ₹\ 33{,}50{,}698/-$$

14 A = 80,00,000; r = 18.50% p.a.C.semiannually; P = ? ; n = 8 years 6 months

$$\Longrightarrow \quad A = P(1+r)^n$$

$$80{,}00{,}000 = P\left[1+\frac{0.1850}{2}\right]^{17} \qquad \therefore P = ₹\ 17{,}77{,}974/-$$

$$80{,}00{,}000 = P \times (1.0925)^{17}$$

15

Compounded	No. of conversion periods in a year
Annually	1
Semi-annually =	Half Yearly = Bi-annually = 2
Monthly	12
Quarterly	4
Weekly	52
Daily	365
Fortnightly	24

My Notes

① How to find discounting @ r% p.a.c.a. for n^{th} Year?

⟹ [1/(1+r)] then press '=' button

till step count comes (n+2)

② Find Discounting factor for Year 30 @ 12.57125% p.a.

⟹ 0.0286533459

16 P = 20,000; r = 20% p.q.c.w; n = 3 months; A = ?

$\Longrightarrow$ $A = P(1+r)^n$

$= 20,000\left(1+\frac{0.20}{52}\right)^{13}$

$= 20,000\,(1.003846153 84)^{13}$

$= ₹21,023/-$

17 A = 2,00,000; r = 18% p.a.C.Q; P = 80,000 ; n = x years

$\Longrightarrow$ $A = P(1+r)^n$

$2,00,000 = 80,000\left(1+\frac{0.18}{4}\right)^{4x}$

$2.50 = 1.045^{4x}$

$\text{Log } 2.50 = \text{Log } 1.045^{4x}$

$\text{Log } 2.50 = 4x \times \text{Log } 1.045$

$4x = \frac{\text{Log } 2.50}{\text{Log } 1.045}$

$4x = \left(\frac{0.39794541318}{0.01911616865}\right)$ $x = 5.20$ Years

18 A = 20,00,000; r = ______% p.a.C.Q ; P = 5,00,000 ; n = 8 years

$\Longrightarrow$ $A = P(1+r)^n$

$20,00,000 = 5,00,000\left(1+\frac{r}{4}\right)^{32}$

$\left(1+\frac{r}{4}\right)^{32} = 4$

$1+\frac{r}{4} = 4^{1/32}$

$r = 17.709512968\%$ p.a.C.Q.

19

1.01^{35} = 1.41660275588

1.1025^{38} = 40.774320 2164

1.10285^{45} = 81.8917474745

1.1826^{90} = 3592598.79256

My Notes

Question : A = 10,000 , P = 4000 , r = ____ % p.a.c.a.
n = 5 years

$\Longrightarrow$ $A = P(1+r)^n$

$10,000 = 4,000\,(1+r)^5$

$(1+r)^5 = 2.50$

$1+r = 2.50^{1/5}$

$\boxed{r = 20.114409001\% \text{ p.a.c.a.}}$

20

$A = P(1+r)^n$

A = Amount = principle + compound interest

P = principal amount = sum invested

r = Rate of interest for the conversion period

n = No. of conversion periods

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Discounting Factor = $1 \div (1+r)^n$

Present Value = (Future Value x Discounting Factor)

How to find discounting factor on calculator? (For n^{th} year)

$1 \div (1+r)$ then press '=' button

till step count comes 'n+2'

22

Simple Annuity is a series of payment / receipts where

- Time Gap between 2 consecutive payment/receipts must be same
- Amount paid/received in every period must be same

23

Effective rate of interest = $\left(1+\frac{r}{n}\right)^n - 1$

where r = Nominal rate of interest

n = No. of conversion periods in a year

P = ₹100, r = 12% p.a.c.q.

A = ?, n = 1 year

$\Rightarrow A = 100 \times (1.03)^4$

= ₹ 112.550881

My Notes

Find Effective rate for 20% p.a.c.m.

$\Rightarrow$ Effective rate

$= \left(1+\frac{0.20}{12}\right)^{12} - 1$

$= (1.01666666)^{12} - 1$

= 21.939108475% p.a.c.a.

P = 80,000, A = ?, n = 2 years

r = 20% p.a.c.m.

$A = 80{,}000 \times \left(1+\frac{0.20}{12}\right)^{24}$

$= 80{,}000 \times (1.0166666)^{24}$

= ₹ 1,18,953/-

r = 21.939108475% p.a.c.a.

$A = 80{,}000 \times (1.21939108475)^2$

= ₹ 1,18,953/-

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Nominal Rate of Interest	Effective Rate of Interest
12% p.a.c.q	$\left(1+\frac{0.12}{4}\right)^4-1 = 1.03^4-1 = 12.550881\%$
14.50% p.a.c.m	$\left(1+\frac{0.1450}{12}\right)^{12}-1 = 1.0120833333^{12}-1 = 15.5035\%$
18% p.a.c.semiannually	$\left(1+\frac{0.18}{2}\right)^2-1 = 1.09^2-1 = 18.81\%$
26.26% p.a.c.weekly	$\left(1+\frac{0.2626}{52}\right)^{52}-1 = 1.00505^{52}-1 = 29.9447\%$
22% p.a.c.monthly	$\left(1+\frac{0.22}{12}\right)^{12}-1 = 1.01833333^{12}-1 = 24.3597\%$

25

Effective Rate of Interest	Nominal Rate of Interest	
18% p.a.c.a.	16.90 % p.a.c.q	$0.18 = \left(1+\frac{r}{4}\right)^4-1 \therefore \left(1+\frac{r}{4}\right)=1.18^{\frac{1}{4}}$
20% p.a.c.a.	18.37 % p.a.c.monthly	$0.20 = \left(1+\frac{r}{12}\right)^{12}-1 \therefore \left(1+\frac{r}{12}\right)=1.20^{\frac{1}{12}}$
28.56% p.a.c.a.	26.77 % p.a.c. half yearly	$0.2856 = \left(1+\frac{r}{2}\right)^2-1$ $\therefore \left(1+\frac{r}{2}\right) = 1.2856^{\frac{1}{2}}$

26

18.50% p.a.c.monthly is equivalent to r % p.a.c.q

$\Longrightarrow$ 18.50 % p.a.c.m. = r % p.a.c.q

$$\left(1+\frac{0.1850}{12}\right)^{12}-1 = \left(1+\frac{r}{4}\right)^4-1$$

$$(1.0154166666)^{12} = \left(1+\frac{r}{4}\right)^4$$

$$1.2015212 3207 = \left(1+\frac{r}{4}\right)^4$$

$$1+\frac{r}{4} = (1.2015212 3207)^{\frac{1}{4}}$$

$r = 18.7867\%$ p.a.c.q.

27

20.86% p.a.c.q is equivalent to ________ % p.a.c. half yearly.

$\Longrightarrow$ 20.86 % p.a.c.q = r % p.a.c. half yearly

$$\left(1+\frac{0.2086}{4}\right)^4-1 = \left(1+\frac{r}{2}\right)^2-1$$

$$1.05215^4 = \left(1+\frac{r}{2}\right)^2$$

$$\left(1+\frac{r}{2}\right) = 1.05215^2$$

$r = 21.4039245\%$ p.a.c. half yearly

28

a. Future Value of annuity regular = Future value of ordinary annuity =

$$= \begin{pmatrix}\text{Periodical}\\ \text{amount}\end{pmatrix} \times \left[\frac{(1+r)^n - 1}{r}\right]$$

a. Future Value of annuity due = Future value of annuity immediate =

$$= \begin{pmatrix}\text{Periodical}\\ \text{amount}\end{pmatrix} \times \left[\frac{(1+r)^n - 1}{r}\right] \times (1+r)$$

29

Annuity Regular	Annuity Due / Immediate
↓ ordinary annuity	↓ Payment/receipt is at the BEGINNING of every period
Payment/receipt is at the END of every period	

If Type of annuity is not mentioned in question then By default it will be treated as annuity regular

30 **Present Value of Annuity Regular = (Periodical Amount x Annuity Factor)**

31 **Present Value of Annuity Due = (Periodical Amount x Annuity Factor) x (1+r)**

My Notes

x % p.a.c.m. is equivalent to 18.5672 % p.a.c.q. Find x.

$$\Rightarrow \left(1+\frac{x}{12}\right)^{12} - 1 = \left(1+\frac{0.185672}{4}\right)^{4} - 1$$

$$\left(1+\frac{x}{12}\right)^{12} = (1.046418)^{4}$$

$$1+\frac{x}{12} = (1.046418)^{4/12}$$

$$x = 18.2871\%\ \text{p.a.c.m.}$$

How to find annuity factor for n years @ r % p.a.

$$\Rightarrow = \frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots + \frac{1}{(1+r)^n}$$

1 ÷ (1+r) then press '=' button till step count comes (n+2) then press GT button.

32 Mr. A invested ₹ 500 at the end of each year for 30 years. Find amount to be received at the end of 30 years, if money is 16% effective.

$\Rightarrow$

$$\text{Future value of annuity regular} = \text{Periodical amt} \times \left[\frac{(1+r)^n - 1}{r}\right]$$

$$= ₹\,500 \times \left[\frac{(1.16)^{30} - 1}{0.16}\right]$$

$$= ₹\,2,65,156/-$$

33 A loan of ₹ 8,00,000 is to be repaid in 10 annual installments. Find amount of installment if interest rate is 12% p.a.

$\Rightarrow$ Present value of annuity regular = P.A. × A.factor

Loan amount = (installment amt × A.factor)

8,00,000 = installment amt × 5.65022302825

∴ Installment amount = ₹ 1,41,587/-

34 A person desires to create a sinking fund to be invested @12% p.a.c.I. by saving some amount at the end of each year for 30 years to buy house worth ₹ 30,00,000. Find amount to be saved at the end of each year.

$\Rightarrow$

$$\text{Future value of annuity Regular} = P.A. \times \left[\frac{(1+r)^n - 1}{r}\right]$$

$$30,00,000 = P.A. \times \left[\frac{1.12^{30} - 1}{0.12}\right]$$

$$P.A. = ₹\,12,431/-$$

He has to save ₹ 12,431/- at the end of each year.

My Notes

Find Effective rate of int. for 16.2624 % p.a.c.q.

$\Rightarrow$

$$\text{Eff. rate} = \left(1 + \frac{0.162624}{4}\right)^4 - 1$$

$$= (1.040656)^4 - 1$$

$$= 17.2812997\,\%\ p.a.c.a.$$

Effective rate of 38% is Equivalent to ____ % p.a.c.m.

$\Rightarrow$

$$0.38 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$1 + \frac{r}{12} = 1.38^{1/12}$$

$$r = 32.645642112\,\%\ p.a.c.m.$$

35 Rahul invested ₹ 70,000 in a bank at the rate of 6.50% p.a.S.I. he received ₹ 85,925 at the end of term. Find out the period for which the sum was invested by Rahul.

$\Longrightarrow$

$A = P(1+nr)$

$85925 = 70,000\,[1 + n \times 0.0650]$

$1.2275 = 1 + 0.0650n$

$n = 3.50$ Years

OR

$SI = P.n.r$

$85925 - 70,000 = Pnr$

$15925 = 70,000 \times n \times 6.50\%$

$15925 = 4550n$

$n = 3.50$ Years

36 Kapil deposited some amount in a bank for $7\frac{1}{2}$ years @ 6%p.a.S.I. Kapil received ₹ 1,01,500 at the end of term. Compute initial deposit of Kapil.

$\Longrightarrow$

$A = P(1+nr)$

$1,01,500 = P\,(1 + 7.50 \times 0.06)$

$1,01,500 = P \times 1.45$

$P = 70,000$

$\therefore$ Initial deposit by Kapil = ₹ 70,000/-

37 A sum of ₹ 46,875 was lent out at simple interest and at the end of 1 year and 8 months the total amount was ₹ 50,000. Find rate of interest p.a.

$\Longrightarrow$

$A = P(1+nr)$

$50,000 = 46,875\,(1 + 1.6666666666\,r)$

$r = 4.00\%$ p.a.S.I.

My Notes

A loan of ₹ 5,00,00,000 is to be repaid to 90 EMI's

Find EMI if r = 12% p.a.c.m.

$\Longrightarrow$ Present value of annuity regular = 5,00,00,000

EMI × A.factor = 5,00,00,000

EMI × 59.16088 14663 = 5,00,00,000

$\boxed{\text{EMI} = 8,45,153}$

38 What sum of money will produce ₹ 28,600 as an interest in 3 years and 3 months @2.50% p.a.S.I.

$\Longrightarrow$ Simple interest = p. n. r

₹ 28,600 = p × 3.25 years × 0.0250

p = ₹3,52,000/-

A sum of ₹ 3,52,000 will produce simple interest of ₹28,600 @ 2.50% p.a. in 3.25 years.

39 The sum required to earn monthly interest of ₹ 1,200 at 18% p.a.S.I is :

$\Longrightarrow$ SI = p. n. r

$₹\ 1200 = p \times \frac{1}{12}$ years × 18%

$p = \left(\frac{₹1200 \times 12}{18\%}\right) = ₹80,000/-$

40 Compute the compound interest on ₹ 40,000 for 1.5 years @10% p.a. compounded half yearly.

$\Longrightarrow$ $A = p(1+r)^n$

$= 40,000 \left(1 + \frac{0.10}{2}\right)^3$

$= 40,000 \times (1.05)^3$

= ₹ 46,305

compound interest

= ₹46,305 − ₹ 40,000

= ₹ 6,305/-

My Notes

A loan of ₹ 80,00,000 is to be repaid in 60 monthly installments. Find EMI if rate of interest is 13.50% p.a.c.m.

$\Longrightarrow$ present value of annuity regular = EMI × A.factor

80,00,000 = EMI × 43.459656316

EMI = ₹ 1,84,079/-

41 What rate of interest p.a. doubles the investment in 7 years at compounded interest?

$\Longrightarrow$ $A = P(1+r)^n$

$2P = P(1+r)^7$

$(1+r)^7 = 2$

$1+r = 2^{1/7}$ $\therefore r = 10.4097\%$ p.a.c.a.

money will be doubled in 7 years @ 10.4097% p.a.c.a.

42 In what time will ₹ 8,000 amount to ₹ 8,820 at 10% p.a. compounded half yearly?

$\Longrightarrow$ $A = P(1+r)^n$

$8{,}820 = 8{,}000\left(1+\frac{0.10}{2}\right)^{2n}$

$1.05^{2n} = 1.1025$

$1.05^{2n} = 1.05^2$ $\therefore 2n = 2$

$n = 1$ Year

43 A certain sum invested at 4% p.a. compounded semi-annually amounts to ₹ 78,030 at the end of one year. Find the sum.

$\Longrightarrow$ $A = P(1+r)^n$

$78{,}030 = P\left(1+\frac{0.04}{2}\right)^2$

$78{,}030 = P(1.02)^2$

$\boxed{P = 75{,}000}$

44 The population of a town increases every year by 2%. The number of years by which the total increase in population be 40% is

a. 7 years b. 10 years ~~c.~~ 17 years (approx.) d. None

$\Longrightarrow$ Suppose Today's Population = P

After some No. of Years = 1.40P = A

$A = P(1+r)^n$

$1.40P = P(1.02)^n$

$(1.02)^n = 1.40 = 1.02^{17}$

$\therefore n = 17$ Years

45 The difference between simple interest & compound interest on a certain sum of money invested for 3 years at 6% p.a. is ₹ 110.16. The principle is -

a. 3,000 b. 3,700 c. 12,000 ~~d.~~ 10,000 e. None

$\Longrightarrow$ comp. Interest − simple Interest = 110.16

$P\left[(1.06)^3 - 1\right] - P \times 3 \times 6\% = 110.16$

$0.191016P - 0.18P = 110.16$

$0.011016P = 110.16 \quad \therefore P = ₹\ 10{,}000/-$

46 The compound interest on ₹ 40,000 at 10% p.a. for 3 years when interest is payable quarterly is -

$\Longrightarrow A = P(1+r)^n = 40{,}000 \times \left(1 + \frac{0.10}{4}\right)^{12}$

$= 40{,}000 \times (1.025)^{12} = 53{,}796/-$

$CI = 53{,}796 - 40{,}000$

$= ₹\ 13{,}796$

47 Use calculator and find answers for the following questions :

$(1.0135)^{28}$ = 1.4556772 1669

$(1.20635)^{48}$ = 8141.78763281

$(1.10935)^{72}$ = 1757.67394446

$(1.089123)^{66}$ = 279.94 798697 5

present value of annuity regu. = Future value of annuity regular × Discounting factor

48 Present Value of Annuity Regular $= P.A \times \left[\frac{(1+r)^n - 1}{r}\right] \times \frac{1}{(1+r)^n}$

$= \frac{P.A.}{r}\left[\frac{(1+r)^n}{(1+r)^n} - \frac{1}{(1+r)^n}\right] = \frac{P.A}{r} \times \left[1 - \frac{1}{(1+r)^n}\right] = \frac{P.A}{r} \times \left[1 - (1+r)^{-n}\right]$

49 What is perpetuity?

Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually. (For unlimited time)

present value of perpetuity $= \left(\frac{\text{periodical amount}}{r}\right)$

50 The present value of annuity of ₹ 3,000 for 15 years @4.50% p.a.c.i is

$\Longrightarrow$ present value of annuity = P.A. × A.factor
= 3,000 × 10.739545 7256
= 32,219/-

OR present value of annuity $= \frac{P.A.}{r}\left[1-(1+r)^{-n}\right] = \frac{3000}{0.0450}\left(1-\frac{1}{(1.0450)^{15}}\right)$
= 32,219/-

51 A loan of ₹ 10,000 is to be paid back in 30 installments. The amount of each installment to cover principle and 4% p.a.c.i. is

a. 587.87 b. 587 c. 587.30 ~~d.~~ None of these

$\Longrightarrow$ present value of annuity regular = P.A. × A.factor
10,000 = installment amt × 17.2920332979
installment amt = ₹ 578.30

52 A person invests ₹ 500 at the end of each year @10% p.a. The amount standing to his credit one year after he has made his yearly investment for 12th time is:

~~a.~~ 11,761.36 b. 10,000 c. 12,000 d. None of these

$\Longrightarrow$ Future value of annuity at the end of 12 years $= 500 \times \left(\frac{1.10^{12}-1}{0.10}\right) = 10{,}692.14$

Amt to his credit after one year = 10692.14 + 10% = ₹ 11,761

53 A person bought a house paying ₹ 20,000 cash down & ₹ 4,000 at the end of each year for 25 years, at 5% p.a.c.i. The cash down price of house is :

a. ₹ 75,000 b. ₹ 76,000 ~~c.~~ ₹ 76,376 d. None of these

payment	present value
Today	20,000
4000 at the end of every year for 25 years	4000 × 14.0939445646 = 56376
cash down price	₹ 76,376

54 The difference between simple interest and compound interest at 5% p.a. for 4 years on ₹ 20,000 is ____________

$\Longrightarrow$ compound interest − simple interest
$= P\left[(1+r)^n-1\right] - P.n.r$
$= 20{,}000\left[1.05^4-1\right] - 20{,}000 \times 4 \times 5\%$
$= 4310.125 - 4000 = ₹\ 310.125$

55 The compound interest on half yearly rests on ₹ 10,000, if rate for 1st and 2nd year being 6% and for third year being 9% p.a. is ₹ ________

a. 2,200 b. 2,287 c. 2,285 ~~d.~~ None of these

$\Longrightarrow$ $A = \left[10{,}000 \times (1.03)^4\right] \times 1.045^2 = ₹\ 12{,}291/-$

compound Interest = ₹12,291 − ₹10,000 = ₹2291/-

56 Vinod borrows ₹ 6 lakhs housing loan at 6% p.a. repayable in 20 annual equal installments commencing at the end of first year. How much annual payment is necessary.

a. ₹ 52,420 b. ₹ 52,400 ~~c.~~ ₹ 52,310 d. None of these

$\Longrightarrow$ Present value of annuity regular = installment amt × Annuity factor

6,00,000 = installment amt × 11.4699212174

installment amt = 52,310/-

57 Raja aged 40 years wishes his wife Rani to have ₹ 40 lakhs at his death. If expectation of life is another 30 years & he starts making equal annual investments commencing now at 3% c.i.p.a. How much should he invest annually?

a. 88,448 b. 84,450 c. 84,449 ~~d.~~ 84,080

$\Longrightarrow$ Future value of annuity regular $= P.A. \times \left[\frac{(1+r)^n - 1}{r}\right]$

$40{,}00{,}000 = P.A. \times \left[\frac{1.03^{30} - 1}{0.03}\right]$ P.A. = ₹ 84077/-

58 A TV can be purchased by paying ₹ 10,000 now and ₹ 20,000, ₹ 50,000, ₹ 90,000, ₹ 80,000 at the end of years 1,2,3,4 respectively. Find cash down price of TV if money is 12% effective.

a. ₹ 1,83,816 ~~b.~~ ₹ 1,82,618 c. ₹ 1,86,218 d. ₹ 1,62,861

$\Longrightarrow$

At the end of Year	payment	present value
Now	10,000	10,000 x 1.00 = 10,000
1	20,000	20,000 x 0.89285 = 17857
2	50,000	50,000 x 0.79719 = 39860
3	90,000	90,000 x 0.71178 = 64060
4	80,000	80,000 x 0.63518 = 50841

= 1,82,618/-

59 Effective rate of 21.94% is equivalent to ________ % p.a.c.monthly

a. 21.94% ~~b.~~ 20% c. 20.66% d. 22.77%

$\Longrightarrow$ Eff. rate $= \left(1+\frac{r}{n}\right)^n - 1$

$0.2194 = \left(1+\frac{r}{12}\right)^{12} - 1$

$\left(1+\frac{r}{12}\right) = 1.2194^{1/12}$

r = 20% p.a.c.m.

$Rest = P - \frac{P}{3} - \frac{P}{6} = \frac{6P-2P-P}{6} = 3P/6 = P/2$

60 Out of certain money $(1/3)^{rd}$ is invested at 3% , $(1/6)^{th}$ is invested at 6% and rest at 8% for 2 years. Simple Interest from all these investments is ₹ 600. The original sum is :

a. ₹ 3,500 b. ₹ 4,000 ~~c.~~ ₹ 5,000 d. ₹ 4,500

$\Longrightarrow$

$P/3 \times 3\% \times 2 = 0.02P$

$P/6 \times 6\% \times 2 = 0.02P$

$P/2 \times 8\% \times 2 = 0.08P$

$Total = 0.12P$

$0.12P = 600$

$P = ₹ 5{,}000/-$

61 Population of a village is 10,000. If it increases at 10% p.a. What will be its population after 3 years?

~~a.~~ 13,310 b. 14,220 c. 17,908 d. 13,000

$\Longrightarrow$ $A = 10{,}000 \times (1.10)^3$

$= 13{,}310$

62 On a certain sum simple interest at the end of 6.25 years become $(3/8)^{th}$ of sum. The rate of interest is ________________

a. 7% b. 9% c. 5% ~~d.~~ 6%

$\Longrightarrow$ $SI = Pnr$

$\frac{3}{8}\cancel{P} = \cancel{P} \times 6.25 \times r$

$0.3750 = 6.25r$

$r = 0.06$

$r = 6\%$ p.a.S.I.

63 The amount of certain sum of money with simple interest at certain rate of interest is ₹ 2,660 in 3 years and ₹ 3,100 in 5 years. The rate of interest is :

a. 12% ~~b.~~ 11% c. ₹ 13% d. 10%

$P(1+3r) = 2660$

$P(1+5r) = 3100$

$\frac{\cancel{P}(1+5r)}{\cancel{P}(1+3r)} = \frac{3100}{2660}$

$2660 + 13300r = 3100 + 9300r$

$4000r = 440$

$r = 0.11$

$r = 11\%$ P.a.S.I.

My Notes

present value of annuity Regular

$= \left(\text{periodical amount} \times \text{Annuity Factor} \right) = \frac{\text{periodical amount}}{r} \times \left[1 - (1+r)^{-n}\right]$

64 At what rate of compound interest money will amount to 8 times in 20 years?

a. 12.75% b. 11.22% ~~c.~~ 10.96% d. None of these

$\Longrightarrow$ $A = P(1+r)^n$

$8P = P(1+r)^{20}$

$\therefore (1+r)^{20} = 8$

$1+r = 8^{1/20}$

$r = 10.9597\%$ p.a.c.I.

65 At what rate of simple interest money will become 8 times in 20 years?

~~a.~~ 35% b. 40% c. 30% d. None of these

$\Longrightarrow$ $A = P(1+nr)$

$8\cancel{P} = \cancel{P}(1+20r)$

$7 = 20r$

$r = 35\%$ p.a.S.I.

66 In what time ₹ 1,00,000 will become ₹ 8,00,000, If rate of interest is 10% p.a.s.i

a. 77 years b. 7 years ~~c.~~ 70 years d. 17 years

$\Longrightarrow$ $A = P(1+nr)$

$8{,}00{,}000 = 1{,}00{,}000\,(1 + n \times 0.10)$

$8 = 1 + 0.10n$

$7 = 0.10n$ $\therefore n = 70$ years

67 A sum of money triples itself with compound interest in 9 years. How many times it will become after 81 years?

a. 27 times b. 6,561 times c. 81 times ~~d.~~ 19,683 times

After _ Years									
	9	18	27	36	45	54	63	72	81
P	3P	9P	27P	81P	243P	729P	2187P	6561P	19683P

OR

$A = 3P = P(1+r)^9$

$\therefore (1+r)^9 = 3$

$A = P(1+r)^{81}$

$= P\left[(1+r)^9\right]^9 = P \times 3^9$

$= 19683P$

My Notes

A sum of money invested at compound interest becomes 5 times in 16 Years. Find Rate of interest.

$\Longrightarrow$ $A = P(1+r)^n$

$5P = P(1+r)^{16}$

$(1+r)^{16} = 5$

$1+r = 5^{1/16}$

$\boxed{r = 10.5823\%\ \text{p.a.c.I.}}$

68 A machine costs ₹ 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value of realisation of ₹ 25,000. What amount should be set aside every year if money is 3.50% effective?

a. ₹ 16,000 b. ₹ 16,564 ~~c. ₹ 16,046~~ d. ₹ 16,005

$\Longrightarrow$ (5,20,000 + 25% − 25,000) = 6,25,000

Future value of annuity regular = P.A. x $\left[\frac{(1+r)^n - 1}{r}\right]$

6,25,000 = P.A. x $\left[\frac{1.0350^{25} - 1}{0.0350}\right]$

P.A. = 16,046

69 A sum of ₹ 80,000 invested in a bank @10% p.a.s.i. for 5 years. Find amount, simple interest.

Year	Opening Balance (₹)	Interest (₹)	Closing Balance (₹)
1	80,000	80,000 x 10% = 8,000	88,000
2	88,000	80,000 x 10% = 8,000	96,000
3	96,000	80,000 x 10% = 8,000	1,04,000
4	1,04,000	80,000 x 10% = 8,000	1,12,000
5	1,12,000	80,000 x 10% = 8,000	1,20,000

Amount receivable at the end of 5 years = ₹ 1,20,000

Simple interest for 5 years = ₹1,20,000 − ₹80,000 = ₹40,000

₹40,000 = ₹80,000 x 10% x 5

SI = P.n.r

Amount = P + Pnr = P(1+nr)

My Notes

Simple interest = P.n.r

Amount when interest is simple = P(1+nr)

compound interest = P$\left[(1+r)^n - 1\right]$

Amount when interest is compound = P$(1+r)^n$

70 Mr. A deposited ₹ 80,000 in a bank @10% p.a.c.i. for 5 years. Find amount receivable after 5 years and compound interest.

Year	Opening Balance (₹)	Interest (₹)	Closing Balance (₹)
1	80,000	80,000 x 10% = 8,000	88,000
2	88,000	88,000 x 10% = 8,800	96,800
3	96,800	96,800 x 10% = 9,680	1,06,480
4	1,06,480	1,06,480 x 10% = 10,648	1,17,128
5	1,17,128	1,17,128 x 10% = 11712.80	1,28,840.80

Amount receivable at the end of 5 years = ₹ 1,28,840.80

Compound Interest = ₹ 1,28,840.80 – ₹ 80,000 = ₹ 48,840.80

1,28,840.80 = ₹ 80,000 x 1.10 x 1.10 x 1.10 x 1.10 x 1.10

$1,28,840.80 = 80,000 \times (1+0.10)^5$

$$\boxed{A = P \times (1+r)^n}$$

71 P = ₹ 1,00,000; r = 12% p.a.c.q; n = 2 years, A = ?

	Opening Balance (₹)	Interest (₹)	Closing Balance (₹)
Year 1 Q1	1,00,000	1,00,000 x 12% x $\frac{1}{4}$ = 3,000	1,03,000
Q2	1,03,000	1,03,000 x 3% = 3,090	1,06,090
Q3	1,06,090	1,06,090 x 3% = 3,183	1,09,273
Q4	1,09,273	3278	1,12,551
Year 2 Q1	1,12,551	3377	1,15,928
Q2	1,15,928	3478	1,19,406
Q3	1,19,406	3582	1,22,988
Q4	1,22,988	3689	1,26,677/-

Amount to be received after 2 years = ₹ 1,26,678

$$A = P(1+r)^n = 1,00,000\left(1+\frac{0.12}{4}\right)^{2\times 4} = 1,00,000 \times (1.03)^8 = 1,26,677/-$$

My Notes

Question :

$P = 8,50,000$, $r = 13\%$ p.a.c.q, $n = 9$ years 9 months

$$\longrightarrow A = P(1+r)^n$$

$$= 8,50,000 \times \left(1+\frac{0.13}{4}\right)^{39} = 8,50,000 \times (1.0325)^{39}$$

$$= ₹\ 29,58,907/-$$

72 You require ₹ 32,00,000 at the end of 9 years from now. Find the amount you should keep aside at the end of every year, if money is 14% effective

a. ₹ 2,20,819 b. ₹ 3,00,000 c. ₹ 3,55,556 ~~d.~~ None of these

$\Longrightarrow$ Future value of annuity regular = 32,00,000

$$P.A. \times \left[\frac{1.14^9 - 1}{0.14}\right] = 32,00,000$$

$\therefore$ P.A. = 1,98,939

73 Simple interest on ₹ 25,00,000 for 8 years and 4 months @ 19.25% p.a.s.i is

$\Longrightarrow$ Simple Interest = P.n.r

= 25,00,000 × 8.3333333 years × 19.25%

= ₹ 40,10,416.66666

74 A sum of ₹ 12,000 deposited at compound interest becomes double after 5 years. After 20 years it will become :

a. ₹ 1,44,000 b. ₹ 2,40,000 ~~c.~~ ₹ 1,92,000 d. None of these

original investment	After 5 years	After 10 years	After 15 years	After 20 years
12,000	24,000	48,000	96,000	1,92,000

75 A man deposits ₹ 2,000 @ 4% p.a. and ₹ 3,000 @ 14% p.a. Find average rate of return he is earning on whole sum?

~~a.~~ 10% b. 5% c. 14% d. None of these

$\Longrightarrow$ ₹ 2000 × 4% = ₹ 80

₹ 3000 × 14% = ₹ 420

₹ 5000 × r % = ₹ 500

$\therefore$ r = 10

SEQUENCE & SERIES

CA VINOD REDDY

1

Terms a, b, c, d, e, f, g are said to be in

AP, If	GP, If	HP, If
$b-a=c-b=d-c=e-d=f-e=g-f$ (succ. term − preceding term) is constant through-out the series. common Diff $= d = S.T. - P.T.$	$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=\frac{e}{d}=\frac{f}{e}=\frac{g}{f}$ (succ. term / prec. term) is constant throughout the series. common Ratio $= r = (S.T. / P.T.$	If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{1}{e}, \frac{1}{f}, \frac{1}{g}$ are in A.P. then a, b, c, d, e, f, g are said to be in H.P.

2

Progression	AP/GP/HP/None of these
8, 16, 32, 64, 128	Geometric progression with $r = 2$
80, 70, 60, 50, 40	Arithmetic Progression with $d = -10$
2, 8, 32, 128	Geometric Progression with $r = 4$
0.50, 0.25, 0.1666666, 0.125	$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ are in Harmonic progression as 2, 4, 6, 8 are in A.P.
$\frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{18}$	None of these
100, 97, 94, 91	Arithmetic progression with $d = -3$
4, 6, 9, 13.50	Geometric progression with $r = 1.50 = 3/2$
10,80,150,220	Arithmetic progression with $d = 70$
10,0,-10,-20,-30	Arithmetic progression with $d = -10$

My Notes

① 28, 28, 28, 28, 28 are in A.P. as well as G.P.

② a, b, c can be in AP as well as G.P. if and only if $a = b = c$

③ $\frac{1}{4}, \frac{1}{10}, \frac{1}{16}, \frac{1}{22}, \frac{1}{28}$ are in H.P. as 4, 10, 16, 22, 28 are in A.P

④ If $\frac{p}{q}, \frac{m}{n}, \frac{x}{y}, \frac{e}{j}, \frac{p}{v}$ are in H.P. then

$\frac{q}{p}, \frac{n}{m}, \frac{y}{x}, \frac{j}{e}, \frac{v}{p}$ must be in A.P.

⑤ If Few terms are in A.P. then their reciprocals must be in H.P. and vice versa.

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3

For	t_n	S_n
AP	$t_n = a + (n-1)d$	$S_n = \frac{n}{2}[2a + (n-1)d]$ OR $S_n = \frac{n}{2}(t_1 + t_n)$
GP	$t_n = a\cdot(r)^{n-1}$	$S_n = \frac{a(r^n - 1)}{(r-1)}$ when $r > 1$; $S_n = \frac{a(1-r^n)}{(1-r)}$ when $r < 1$

a = First term, d = common diff, r = common Ratio

4

80, 87, 94, 101, ... Find t_{30}, t_{80}, t_{125}, S_{45}, S_{100}, S_{125}

$\Longrightarrow$ In this A.P. a = First term = 80 & d = common diff = 7

① $t_{30} = a + 29d = 80 + (29 \times 7) = 283$

② $t_{80} = a + 79d = 80 + (79 \times 7) = 633$

③ $t_{125} = a + 124d = 80 + (124 \times 7) = 948$

④ $S_{45} = \frac{45}{2}[160 + (44 \times 7)] = 10,530$

⑤ $S_{100} = \frac{100}{2}[160 + (99 \times 7)] = 42,650$

⑥ $S_{125} = \frac{125}{2}[160 + (124 \times 7)] = 64,250$

5

5, 10, 20, 40, ... Find t_{12}, t_{10}, S_{16}, S_{22}

In this G.P. a = First term = 5 and
r = common ratio = 2

① $t_{12} = a \cdot r^{11} = 5 \times (2)^{11} = 10,240$

② $t_{10} = a \cdot r^{9} = 5 \times (2)^{9} = 2,560$

③ $S_{16} = \frac{a(r^{16} - 1)}{(r-1)} = \frac{5(2^{16} - 1)}{(2-1)} = 327675$

④ $S_{22} = \frac{a(r^{22} - 1)}{(r-1)} = \frac{5(2^{22} - 1)}{(2-1)} = 20971515$

6

1. Sum of infinite terms of G.P. where r > 1 = ∞
2. Sum of infinite terms of G.P. where 0< r < 1 = $\boxed{a/1-r}$

① $8+16+32+64+ \dots\dots \infty \text{ terms} = \infty$

② $1024+512+256+128+ \dots\dots \infty \text{ terms} = ?$

$$S_\infty = \frac{a}{1-r} = \frac{1024}{1-0.50} = \frac{1024}{0.50} = 2048$$

7

10 + 20 + 40 + 80 + ... ∞ terms = ?

$\Longrightarrow$ In this G.P. $a=10$, $r=2.00$

AS $r>1$ $S_\infty = \infty$ = Infinite.

8

200 + 100 + 50 + 25 + ... ∞ terms = ?

$\Longrightarrow$ In this G.P. $a=200$, $r=0.50$

$$S_\infty = \frac{a}{1-r} = \left(\frac{200}{1-0.50}\right) = 400$$

9

For AP $t_5 = 80$, $t_{15} = 580$

Find a, d, t_{80}, t_{100}, S_{80}

$\Longrightarrow$ ①
$$\begin{aligned} \cancel{a} + 4d &= 80 \\ \cancel{a} + 14d &= 580 \\ - \quad\quad - &\quad\quad - \\ \hline -10d &= -500 \\ d &= 50 \end{aligned}$$

② $a+4d = 80$

$a+(4\times50) = 80$

$\boxed{a=-120}$

③ $t_{80} = a+79d$

$= -120+(79\times50)$

$= 3,830$

④ $t_{100} = a+99d$

$= -120+(99\times50)$

$= 4,830$

⑤ S_{80}

$= \frac{80}{2}\left(t_1 + t_{80}\right)$

$= 40(-120+3830)$

$= 1,48,400$

OR

$S_{80} = \frac{80}{2}\left[-240+(79\times50)\right]$

$= 1,48,400$

10 For AP $t_3 = 15, S_3 = 30$

Find first term, common difference, S_{40}, S_{100}, t_{30}, S_{287}

$\Longrightarrow$

① $t_3 = a + 2d = 15$ ----- ①

$S_3 = \frac{3}{2}(2a + 2d) = 30$

$3a + 3d = 30$

$a + d = 10$ ---- ②

$a + 2d = 15$

$10 - d + 2d = 15$

$d = 5$

$\therefore a = 5$

② S_{40}

$= \frac{40}{2}[10 + (39 \times 5)] = 4{,}100$

③ S_{100}

$= \frac{100}{2}[10 + (99 \times 5)] = 25{,}250$

④ t_{30}

$= a + 29d$

$= 5 + (29 \times 5) = 150$

⑤ S_{287}

$= \frac{287}{2}[10 + (286 \times 5)]$

$= 2{,}06{,}640$/-

11 For AP $t_n = (3n+5)$. Find S_n

$\Longrightarrow$ $t_n = 3n + 5$

$\therefore t_1 = 3(1) + 5 = 8$

$S_n = \frac{n}{2}(t_1 + t_n)$

$= \frac{n}{2}(8 + 3n + 5)$

$= \frac{n}{2}(3n + 13) = \left(\frac{3n^2 + 13n}{2}\right)$

12 For AP $t_n = ?$, if $S_n = (8n^2 - 3n)$ $t_n = ?$

$\Longrightarrow$ $S_n = 8n^2 - 3n$

$S_1 = 8(1)^2 - 3(1) = 5$

$S_2 = 8(2)^2 - 3(2) = 26$

$a = 5$, $t_2 = 26 - 5 = 21$, $d = 16$

$t_n = a + (n-1)d$

$= 5 + (n-1)16$

$= 5 + 16n - 16$

$= 16n - 11$

13 For AP - Please Remember

1. If $S_m = S_n$, then S_{m+n} = zero
2. If $t_m = n$, and $t_n = m$, then t_{m+n} = zero
3. If m x t_m = n x t_n, then t_{m+n} = zero

For A.P.
① If $S_{30} = S_{50}$ then $S_{80} = 0$
② If $t_{81} = 19$, $t_{19} = 81$ then $t_{100} = 0$
③ If $13 \times t_{13} = 48 \times t_{48}$ then $t_{61} = 0$

14

For 2 observations x,y

AM = $\left(\frac{x+y}{2}\right)$

GM = $\pm\sqrt{x.y}$

HM = $\left(\frac{2}{\frac{1}{x}+\frac{1}{y}}\right)$

$= \left(\frac{2xy}{x+y}\right)$

15

① For 2 observations relation between AM, GM, HM is (positive)

$\Longrightarrow$ $GM^2 = AM \times HM$

② For any no. of observations relation between AM, GM, HM is

$AM \geq GM \geq HM$

③ For 'n' distinct observations : $AM > GM > HM$

16

① For 2 observations if GM = 10 and AM = 12, HM = ?

$\Longrightarrow$ $GM^2 = AM \times HM$

$10^2 = 12 \times HM$

$HM = (100/12) = 8.3333333$

② For 2 observation If AM = 12, GM = 15, HM = ?

$\Longrightarrow$ wrong data as Here GM > AM

17 Insert 2 A.means between -200 and 1600

$\Longrightarrow$ −200, 400, 1000, 1600

If a,b,c,d are in A.P. then b,c are 2 A.means between a & d

$a = -200$ $t_4 = 1600$

$a+3d = 1600$

$-200+3d = 1600$

$3d = 1800$

$d = 600$

$\therefore$ 400 & 1000 are 2 A.means between −200 & 1600

18 **Insert 3 A.means between 5000 and 8520.**

$\Longrightarrow$ $a = 5000$, $t_5 = 8520$

$a + 4d = 8520$

$5000 + 4d = 8520$

$d = 880$

$\therefore$ 3 A.Means are : 5880, 6760, 7640

19 **Insert one A.means between 100 and 250.**

$\Longrightarrow$ $A.M. = \left(\frac{100+250}{2}\right) = 175$

as 100, 175, 250 are in A.P.

175 is AM between 100 & 250.

20 **Insert 5 G.means between 500 and 8,000.**

$\Longrightarrow$ 500, 793.80, 1260.25, 2000, 3176.46, 5043, 8000

$t_1 = 500$

$t_7 = 8000 = a \cdot r^6$

$8000 = 500 \times r^6$

$r^6 = 16 \quad \therefore r = 16^{1/6} = 1.58760791558$

My Notes

① Insert 5 G.means between 20 & 227.8125

$\Longrightarrow$ $a = 20$, $t_7 = 227.8125$

$a \cdot r^6 = 227.8125$

$20 \times r^6 = 227.8125$

$r^6 = 11.390625$

$r = 11.390625^{1/6}$

$\boxed{r = 1.50}$

$\therefore$ 5 G.means are :

30, 45, 67.50, 101.25, 151.875

② Insert 7 A.means betn −30 & 2930

$\Longrightarrow$ $a = -30$

$t_9 = a + 8d = 2930$

$-30 + 8d = 2930$

$\boxed{d = 370}$

$\therefore$ 7 A.means are :

340, 710, 1080, 1450, 1820, 2190, 2560

21 a. Sum of first 'n' natural numbers = $1+2+3+4+ \cdots\cdots + n = \boxed{\frac{n(n+1)}{2}}$

b. Sum of first 'n' odd numbers = $1+3+5+7+ \cdots\cdots + n$ terms $= n^2$

c. Sum of squares of first 'n' natural numbers = $1^2+2^2+3^2+ \cdots\cdots + n^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}$

d. Sum of cubes of first 'n' natural numbers = $1^3+2^3+3^3+ \cdots\cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

e. Sum of first 'n' even numbers = $2+4+6+8+ \cdots\cdots + n$ terms

$= n(n+1) = (n^2+n)$

22 $19^2 + 20^2 + 21^2 + 22^2 + \ldots\ldots\ldots\ldots + 105^2$

$= (1^2+2^2+3^2+ \cdots\cdots + 105^2) - (1^2+2^2+ \cdots\cdots + 18^2)$

$= \left(\frac{105 \times 106 \times 211}{6}\right) - \left(\frac{18 \times 19 \times 37}{6}\right)$

$= 3,89,296/-$

23 $28^3 + 29^3 + 30^3 + \ldots\ldots\ldots\ldots + 62^3$

$\Longrightarrow = (1^3+2^3+3^3+ \cdots\cdots + 62^3) - (1^3+2^3+3^3+ \cdots\cdots + 27^3)$

$= \left(\frac{62 \times 63}{2}\right)^2 - \left(\frac{27 \times 28}{2}\right)^2 = 36,71,325/-$

My Notes

① $81^3+82^3+83^3+ \cdots\cdots + 120^3 = ?$

$\Longrightarrow (1^3+2^3+3^3+ \cdots\cdots + 120^3) - (1^3+2^3+3^3+ \cdots\cdots + 80^3)$

$= \left(\frac{120 \times 121}{2}\right)^2 - \left(\frac{80 \times 81}{2}\right)^2 = 4,22,10,000/-$

② $2 + 6 + 18 + 54 + \cdots\cdots$ Find S_{20}

$\Longrightarrow$ In this GP $a=2, r=3$

$S_{20} = \frac{2(3^{20}-1)}{(3-1)} = 3486784400$

24 1 + 3 + 5 + 7 + ... + 989 = ?

$\Longrightarrow$ In this A.P.

$a = 1$, $d = 2$, $t_n = 989$

$t_n = a + (n-1)d$

$989 = 1 + (n-1)2$

$n = 495$

S_{495}

$= \frac{495}{2}(1 + 989)$

$= 2,45,025$

(OR)

sum of first 495 odd numbers

$= 495^2 = 2,45,025$

25 4484 + 4488 + 4492 + ... + 16880 = ?

$\Longrightarrow$ In this A.P. $a = 4484$, $d = 4$, $t_n = 16880$

$t_n = a + (n-1)d$

$16880 = 4484 + (n-1)4$

$n = 3100$

$S_{3100} = \frac{3100}{2}(4484 + 16880)$

$= 33114200$

26 n^{th} term of sequence 1, 3, 5, 7, ... is

$\Longrightarrow$ In this A.P. $a = 1$, $d = 2$

$t_n = a + (n-1)d$

$= 1 + (n-1)2$

$= 1 + 2n - 2 = 2n - 1$

Q. 81, 86, 91, 96, -------- Find t_n.

$\Longrightarrow$ $t_n = 81 + (n-1)5 = 81 + 5n - 5 = (5n + 76)$

27 $\sum_{i=4}^{i=7} \sqrt{2i-1} =$

$\Longrightarrow$ $= \sqrt{2(4)-1} + \sqrt{2(5)-1} + \sqrt{2(6)-1} + \sqrt{2(7)-1}$

$= \sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$

$= 12.5679$

My Notes

For AP If $S_m = (8m^2 + 2m)$. Find t_k.

$\Longrightarrow$ $S_m = 8m^2 + 2m$

$S_1 = 8(1)^2 + 2(1) = 10$

$S_2 = 8(2)^2 + 2(2) = 36$

$a = 10$, $t_2 = 26$, $d = 16$

$t_k = 10 + (k-1)16$

$= 10 + 16k - 16 = (16k - 6)$

28 If $S_n = 2n^2 + 8n$, first 3 terms of AP are :

$\Longrightarrow S_n = 2n^2 + 8n$

$S_1 = 10 \quad \therefore a = 10$

$S_2 = 24 \quad t_2 = 14, \; d = 4$

$\therefore$ First 3 terms are :

10, 14, 18

For AP $S_n = (2n^2 - 33n)$ Find t_p

$\Longrightarrow S_1 = 2(1)^2 - 33(1) = -31$

$S_2 = 2(2)^2 - 33(2) = -58$

$a = -31, \; t_2 = -27, \; d = 4$

$t_p = -31 + (p-1)4$

$= -31 + 4p - 4 = 4p - 35$

29 For AP $t_1 = -4$, $t_n = 146$, $S_n = 7171$. The number of terms is :

Also find common diff.

$\Longrightarrow S_n = \frac{n}{2}(t_1 + t_n)$

$7171 = \frac{n}{2}(-4 + 146)$

$7171 = n \times 71$

$\therefore n = 101$

$t_{101} = 146$

$a + 100d = 146$

$-4 + 100d = 146$

$100d = 150$

$d = 1.50 = 3/2$

30 $3\frac{1}{2} + 7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots$ Find S_{17}, t_{998}

$\Longrightarrow 3.50 + 7 + 10.50 + 14 + \ldots\ldots\ldots$ Find S_{17}

In this A.P. $a = 3.50, \; d = 3.50$

$S_{17} = \frac{17}{2}\left[7 + (16 \times 3.50)\right] = 535.50$

$t_{998} = a + 997d = 3.50 + (997 \times 3.50) = 3493$

31 4 A.means between -2 & 23 are

$\Longrightarrow -2, \; 3, \; 8, \; 13, \; 18, \; 23$

$t_1 = a = -2$

$t_6 = a + 5d = 23$

$-2 + 5d = 23$

$5d = 25 \quad \therefore d = 5$

$\therefore$ 4 A.Means are

3, 8, 13, 18

My Notes

For AP $t_m = \left(\frac{8m+3}{11}\right)$. Find S_k.

$\Longrightarrow t_1 = 1$

$t_k = \left(\frac{8k+3}{11}\right)$

$S_k = \frac{k}{2}\left(\frac{11}{11} + \frac{8k+3}{11}\right)$

$= \frac{k}{2}\left(\frac{8k+14}{11}\right) = (8k^2 + 14k)/22$

$= (4k^2 + 7k)/11$

32 Find x such that 8x + 4, 6x - 2, 2x + 7, are in A.P

$\Longrightarrow \quad t_2 - t_1 = t_3 - t_2$

$6x - 2 - (8x+4) = 2x + 7 - (6x - 2)$

$6x - 2 - 8x - 4 = 2x + 7 - 6x + 2$

$-2x - 6 = -4x + 9$

$2x = 15$

$\therefore x = 15/2$

$x = 7.50 = 7\frac{1}{2}$

33 Find k such that (10k+8), (18k-19), (22k-81) are in A.P.

$\Longrightarrow \quad t_2 - t_1 = t_3 - t_2$

$18x - 19 - 10k - 8 = 22k - 81 - 18k + 19$

$8k - 27 = 4k - 62$

$4k = -35$

$\therefore k = -35/4 = -8.75$

34 4 A.means between -20 and 880 are

$\Longrightarrow \quad -20, 160, 340, 520, 700, 880$

$a = -20$

$t_6 = 880 = a + 5d$

$880 = -20 + 5d$

$900 = 5d$

$\therefore d = 180$

$\therefore$ 4 A.Means are :

160, 340, 520, 700

35 Insert 3 G.means between $\frac{1}{9}$ and 9.

$\Longrightarrow \quad \frac{1}{9}, 1/3, 1, 3, 9$

$a = \frac{1}{9}, \quad t_5 = 9$

$a \cdot r^4 = 9$

$\frac{1}{9} r^4 = 9$

$\therefore r^4 = 81$

$r = 3$

$\therefore$ 3 G.means are

$1/3, 1, 3$

My Notes

Insert 5 G.means between 20 & 227.8125

$\Longrightarrow$

$a = 20, \quad t_7 = a \cdot r^6 = 227.8125$

$20 \times r^6 = 227.8125$

$r^6 = 11.390625$

$\therefore$ 5 G.means are :

30, 45, 67.50, 101.25, 151.875

36 3 + 33 + 333 + ... n terms = ?

$\Longrightarrow = 3 + 33 + 333 + 3333 + \cdots\cdots$ n terms

$= 3(1 + 11 + 111 + 1111 + \cdots\cdots \text{ n terms})$

$= \frac{3}{9}[9 + 99 + 999 + 9999 + \cdots\cdots \text{ n terms}]$

$= \frac{3}{9}[(10-1) + (100-1) + (1000-1) + \cdots \text{ n terms}]$

$= \frac{3}{9}[(10 + 100 + 1000 + \cdots \text{ n terms}) - (1+1+1+1+\cdots + \text{ n terms})]$

Answer:

$= \frac{3}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

37 6, 12, 24, 48, ... Find t_{10}, S_{12}

$\Longrightarrow$ In this G.P. $a = 6$, $r = 2$

$t_{10} = a \times r^9 = 6 \times 2^9 = 3072$

$S_{12} = \left[\frac{a(r^{12}-1)}{r-1}\right] = \left[\frac{6(2^{12}-1)}{(2-1)}\right] = 24{,}570$

38 For GP $t_2 = 24$, $t_5 = 81$ then find common ratio.

$\Longrightarrow t_2 = a \cdot r = 24$

$t_5 = a \cdot r^4 = 81$

$a \cdot r \cdot r^3 = 81$

$24 \cdot r^3 = 81$

$r^3 = \frac{81}{24} = \frac{27}{8}$

$r = 3/2 = 1.50$

For G.P. $t_3 = 62.50$ & $t_6 = 976.5625$

Find a, r.

$\Longrightarrow a \cdot r^2 = 62.50$

$a \cdot r^5 = 976.5625$

$a \cdot r^2 \cdot r^3 = 976.5625$

$62.50\, r^3 = 976.5625$

$r^3 = 15.625$ $\boxed{r = 2.50}$

$\therefore \boxed{a = 10}$

39 Sum of first 20 terms of G.P. is equal to 244 times of sum of first 10 terms of G.P. then common ratio = ?

$\Longrightarrow S_{20} = 244 \times S_{10}$

$\frac{\cancel{a}(r^{20}-1)}{\cancel{(r-1)}} = \frac{244 \times \cancel{a} \times (r^{10}-1)}{\cancel{(r-1)}}$

$\cancel{(r^{10}-1)}(r^{10}+1) = 244\,\cancel{(r^{10}-1)}$

$r^{10} = 244 - 1 = 243$

$r^{10} = (3)^5 = [(\sqrt{3})^2]^5$

$r^{10} = (\sqrt{3})^{10}$

$r = \sqrt{3}$

My Notes

For AP $S_{30} = S_{80}$ Find S_{110}.

$\Longrightarrow S_{110} = 0$

If $S_m = S_n$ then $S_{m+n} = 0$

If $t_m = n$, $t_n = m$ then $t_{m+n} = 0$

If $m \times t_m = n \times t_n$ then $t_{m+n} = 0$

For A.P.

40 $1 + 2 + 4 + 8 + \dots\dots\dots = 8191.$
How many terms are there in the above G.P.?

$\Longrightarrow$ $S_n = \dfrac{a(r^n-1)}{(r-1)}$

$8191 = \dfrac{1(2^n-1)}{(2-1)}$

$\therefore 2^n - 1 = 8191$

$2^n = 8192$

$2^n = 2^{13}$

$\therefore n = 13$

41 4 G.Means between 4 and 972 are :

$\Longrightarrow$ 4, 12, 36, 108, 324, 972

$a = 4$

$t_6 = a \cdot r^5 = 972$

$4 \times r^5 = 972$

$r^5 = 243$

$\therefore r^5 = 3^5$

$r = 3$

$\boxed{\text{4 G.means are : } 12, 36, 108, 324}$

42 For G.P., Find $t_4 = x$, $t_{10} = y$, $t_{16} = z$ then $y^2 = xz$. True / False

$\Longrightarrow$ $t_4 = a \cdot r^3 = x$

$t_{10} = a \cdot r^9 = y$

$t_{16} = a \cdot r^{15} = z$

L.H.S. $= y^2 = (a \cdot r^9)^2 = a^2 \cdot r^{18}$

R.H.S. $= x \cdot z = a \cdot r^3 \cdot a \cdot r^{15} = a^2 \cdot r^{18}$

L.H.S. = R.H.S.

$y^2 = xz$ is True

43 Find sum of all odd numbers divisible by 9 between 5,000 and 15,000.

$\Longrightarrow$ $5013 + 5031 + 5049 + \dots\dots\dots + 14985$

In this AP $a = 5013$, $d = 18$, $t_n = 14985$

$14985 = 5013 + (n-1)18$

$n = 555$

$S_{555} = \dfrac{555}{2}(5013 + 14985)$

$= 55,49,445$

My Notes

Find sum of all natural numbers divisible by 7 between 30,000 & 50,000

$\Longrightarrow$ $30,002 + 30,009 + \dots\dots\dots + 49994 = ?$

$49994 = 30,002 + (n-1)7$

$n = 2857$

S_{2857}

$= \dfrac{2857}{2}(30002 + 49994)$

$= 114274286$

44 Find sum of all numbers divisible by 7 between 800 and 8000.

$\Longrightarrow$ $805 + 812 + 819 + \text{-------} + 7994 = ?$

$7994 = 805 + (n-1)7$

$n = 1028$

$S_{1028} = \frac{1028}{2}(805 + 7994) = 4522686$

45 $1.03 + 1.03^2 + 1.03^3 + \ldots\ldots\ldots\ldots$ Find S_{11}

$\Longrightarrow$ In this G.P. $a = 1.03$, $r = 1.03$

$S_{11} = \frac{a(r^{11}-1)}{(r-1)} = \frac{1.03(1.03^{11}-1)}{1.03-1} = 13.1920295603$

46 The n^{th} element of the sequence -1, 2, -4, 8 .... is

~~a.~~ $(-1)^n \times 2^{n-1}$ b. 2^{n-1} c. 2^n d. None of these

$\Longrightarrow$ In this G.P. $a = -1$, $r = -2$

$t_n = a \times (r)^{n-1}$

$= (-1) \times (-2)^{n-1}$

$t_n = (-1) \times \frac{(-2)^n}{(-2)^1}$

$= \frac{-1}{-2} \times (2 \times -1)^n$

$= \frac{1}{2^1} \times 2^n \times (-1)^n = (2)^{n-1} \times (-1)^n$

47 $\sum_{i=4}^{7} \sqrt{2i-1}$ can be written as : Refer Q.No. 27 on page No. 55

~~a.~~ $\sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$ b. $2\sqrt{7} + 2\sqrt{9} + 2\sqrt{11} + 2\sqrt{13}$

c. $\sqrt{7 + 9 + 11 + 13}$ d. None of these

My Notes

48 Which term of AP -1, -3, -5, ...is -39

a. 21st ~~b.~~ 20th c. 19th d. None of these

$\Longrightarrow$ $a = -1,\ d = -3-(-1) = -2$

$t_n = -39 = a + (n-1)d$ $\therefore n-1 = 19$

$-39 = -1 + (n-1)(-2)$ $n = 20$

$-38 = (n-1)(-2)$ $\therefore$ 20th term is -39

49 For AP $t_m = n$, $t_n = m$ then t_r = ?

a. m+n+r b. m+n-2r c. (m+n+r)/2 ~~d.~~ m+n-r

$\Longrightarrow$ $t_m = a + (m-1)d = a + md - d = n$

$t_n = a + (n-1)d = a + nd - d = m$

$\cancel{a} + md - \cancel{d} = n$	$d(m-n) = -(m-n)$	t_r
$\cancel{a} + nd - \cancel{d} = m$	$\boxed{d = -1}$	$= a + (r-1)d$
$md - nd = -(m-n)$	$a + m(-1) - (-1) = n$	$= m+n-1 + (r-1)(-1)$
	$a = m+n-1$	$= m+n-\cancel{1}-r+\cancel{1}$
		$= m+n-r$

50 $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + 8\frac{2}{3} + \ldots\ldots$Find S_{30}, S_{31}

~~a.~~ 155 b. 551 c. 1010 d. 305

$\Longrightarrow$ $\frac{30}{3}, \frac{29}{3}, \frac{28}{3}, \frac{27}{3}, \frac{26}{3}, \ldots\ldots$ $a = 10,\ d = -1/3$

$S_{30} = \frac{30}{2}\left[20 + (29) \times -\frac{1}{3}\right] = \frac{30}{2}\left(20 - \frac{29}{3}\right) = 15 \times \frac{31}{3}$

$= 155$

$S_{31} = \frac{31}{2}\left[20 + \left(30 \times -\frac{1}{3}\right)\right] = \frac{31}{2}(20-10) = 155$

For example

8, 6, 4, 2, 0

In this AP

$S_4 = 20$

$S_5 = 20$

51 2 A.means between terms -6 and 14 are

a. 2/3, 1/3 ~~b.~~ 2/3, 22/3 c. -2/3, -22/3 d. None of these

$\Longrightarrow$ $-6,\ 0.66666,\ 7.33333,\ 14$

$a = -6$

$t_4 = a + 3d = 14$ $\therefore$ 2 A.means are : 0.666666, 7.33333

$-6 + 3d = 14$ i.e. $2/3,\ 22/3$

$3d = 20$

$d = 6.666666$

My Notes

Insert 3 G.means between 4 and 20.25

$\Longrightarrow$ $t_1 = 4$ $4 \times r^4 = 20.25$ $\therefore$ 3 G.means are

$t_5 = 20.25$ $r^4 = 5.0625$ 6, 9, 13.50

$a \cdot r^4 = 20.25$ $r = 1.50$

52 The number of numbers between 74 and 25,556 divisible by 5 are:

a. 5090 ~~b.~~ 5097 c. 5095 d. None of these

$\Longrightarrow$ 75, 80, 85, ------, 25555

$t_n = a + (n-1)d$

$25555 = 75 + (n-1)5$

$n = 5097$ $\therefore$ There are 5097 terms

53 The 4 arithmetic means between -2 and 23 are :

a. 3, 13, 8, 18 b. 18, 3, 8, 13 ~~c.~~ 3, 8, 13, 18 d. None of these

$-2, 3, 8, 13, 18, 23$

as $-2, 3, 8, 13, 18, 23$ are in A.P.

54 $t_1 = -4$ and $t_n = 146$, $S_n = 7171$. Find n

~~a.~~ 101 b. 100 c. 99 d. None of these

$\Longrightarrow$ $S_n = \frac{n}{2}(t_1 + t_n)$ $\therefore 7171 = n \times 71$

$7171 = \frac{n}{2}(-4 + 146)$ $\therefore n = 101$

$7171 = \frac{n}{2} \times 142$

55 $x^2, x, 1$ .... $t_{31} = ?$

a. x^{28} b. $1/x$ ~~c.~~ $1/x^{28}$ d. $1/x^{35}$

$\Longrightarrow$ In this G.P. $a = x^2$, $r = 1/x$

$t_{31} = a \cdot r^{30}$

$= x^2 \cdot \left(\frac{1}{x}\right)^{30} = x^2 \times \frac{1^{30}}{x^{30}} = \frac{1}{x^{28}} = x^{-28}$

My Notes

$8\frac{1}{3}, 8\frac{2}{3}, 9, 9\frac{1}{3}, 9\frac{2}{3}$ Find S_{500}

$\Longrightarrow$ $\frac{25}{3}, \frac{26}{3}, \frac{27}{3}, \frac{28}{3}, \frac{29}{3}, \ldots\ldots$ $a = 25/3$ $d = 1/3$

$S_{500} = \frac{500}{2}\left[\frac{50}{3} + 499 \times \frac{1}{3}\right] = 250 \times \left[\frac{549}{3}\right] = 45,750/-$

56 For G.P. $t_2 = 24$, $t_5 = 81$. The series is,

a. 16, 36, 24, 54...... b. 24, 36, 53....... ~~c.~~ 16, 24, 36, 54....... d. None of these

$\Rightarrow t_2 = 24 = a \cdot r$

$t_5 = 81 = a \cdot r^4$

$81 = a \cdot r \cdot r^3$

$81 = 24 \cdot r^3$

$\therefore r^3 = \frac{81}{24} = \frac{27}{8}$

$r = \frac{3}{2}$

$a \times \frac{3}{2} = 24$

$\therefore a = 16$

$\therefore$ The series is, 16, 24, 36, 54, 81 ......

57 The sum of 3 numbers in G.P. is 39 and their product is 729. The numbers are :

knowledge

a. 3, 9, 27 b. 27, 9, 3 c. Both (a) & (b) d. None of these

3 terms $\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = 729$

$a^3 = 729$

$a^3 = 9^3$ $\boxed{\therefore a = 9}$

$\frac{9}{r} + 9 + 9r = 39$

$\frac{9}{r} + 9r = 30$

$9 + 9r^2 = 30r$

$9r^2 - 30r + 9 = 0$

$3r^2 - 10r + 3 = 0$

$3r^2 - 9r - r + 3 = 0$

$3r(r-3) - 1(r-3) = 0$

$(r-3)(3r-1) = 0$

$r = 3$ OR $r = 1/3$

$\therefore 3, 9, 27$

58 In a G.P, product of first 3 terms is 27/8. The middle term is

a. 2/3 ~~b.~~ 3/2 c. 9/8 d. None of these

$\Rightarrow$ Let those 3 terms be $\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = \frac{27}{8}$

$a^3 = \left(\frac{3}{2}\right)^3$ $\therefore a = \frac{3}{2}$ Middle term is 3/2

59 If you have 1 paise today, 2 paise next day, 4 paise succeeding day and so on. Total saving in 2 weeks will be :

a. ₹ 163 b. ₹ 183 ~~c.~~ ₹ 163.83 d. None of these

$\Rightarrow 1 + 2 + 4 + 8 + \ldots\ldots\ldots\ldots$ 14 terms

$S_{14} = \left[\frac{a(r^{14}-1)}{r-1}\right] = \left[\frac{1(2^{14}-1)}{2-1}\right] = 16{,}383$ paise

$=$ ₹ 163.83

My Notes

① In AP, any term can be obtained by adding common diff to its preceding term OR by deducting common diff from its succeeding term. $t_{25} = t_{24} + d = t_{26} - d$

② In GP, any term can be obtained by multiplying to its preceding term by common ratio OR dividing to its succeding term by common ratio. $t_{30} = t_{29} \times r = t_{31} \div r$

60 Sum of first 20 terms of G.P. is 244 times of sum of its first ten terms. The common ratio is :

~~a.~~ $\sqrt{3}$ b. 3 c. $1/\sqrt{3}$ d. None of these

$\Longrightarrow$ $S_{20} = 244 \times S_{10}$

$$\frac{\cancel{a}(r^{20}-1)}{\cancel{(r-1)}} = \frac{244 \times \cancel{a} \times (r^{10}-1)}{\cancel{(r-1)}}$$

$$\cancel{(r^{10}-1)}(r^{10}+1) = 244\,\cancel{(r^{10}-1)}$$

$r^{10} = 243 = 3^5$

$(r^2)^5 = 3^5$

$r^2 = 3 \quad \therefore r = \sqrt{3}$

61 The sum of the series 1 + 2 + 4 + 8 + ... + n terms is

~~a.~~ 2^n-1 b. 2n - 1 c. $(1/2^n) - 1$ d. None of these

$$\Longrightarrow \quad S_n = \frac{a(r^n-1)}{r-1} = \frac{1(2^n-1)}{2-1} = 2^n-1$$

62 The number of terms to be taken so that 1 + 2 + 4 + 8 + ... will be 8191 is :

a. 10 ~~b.~~ 13 c. 12 d. None of these

$\Longrightarrow$ $S_n = \dfrac{a(r^n-1)}{r-1}$

$8191 = \dfrac{1(2^n-1)}{2-1}$

$\therefore 8191 = 2^n - 1$

$8192 = 2^n$

$2^{13} = 2^n$

$\therefore n = 13$

63 Four Geometric means between 4 and 972 are

~~a.~~ 12,36,108,324 b. 12,24,108,320 c. 10,36,108,320 d. None of these

$\Longrightarrow$ 4, 12, 36, 108, 324, 972

must be in G.P.

My Notes

If 10th term of AP is 20 & 20th term is 10

Find 50th term of A.P.

$\Longrightarrow$ $t_{50} = 10 + 20 - 50 = -20$

Short cut to remember :

$$\boxed{\text{For AP If } t_m = n \;\&\; t_n = m \text{ then } t_r = (m+n-r)}$$

64 $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots \infty$ terms = ?

a. 0.75 ~~b.~~ 1.50 c. ∞ d. None of these

$\Longrightarrow$ $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ In this G.P. $a = 1$, $r = \frac{1}{3}$

As $0 < r < 1$ $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}}$

$= \frac{1}{2/3} = 3/2 = 1.50$

65 If p, q, r are in AP and x, y, z are in GP then $x^{q-r} \times y^{r-p} \times z^{p-q}$ =?

a. zero ~~b.~~ 1 c. -1 d. None of these

$\Longrightarrow$ $p = 2$ $q = 4$ $r = 6$

$x = 2$ $y = 4$ $z = 8$

$x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$

$= 2^{-2} \times 4^{4} \times 8^{-2} = 2^{-2} \times (2^2)^4 \times (2^3)^{-2} = 2^{-2} \times 2^{8} \times 2^{-6} = 2^0 = 1$

66 For G.P, $t_4 = x$, $t_{10} = y$, $t_{16} = z$. Then

a. $x^2 = y.z$ b. $z^2 = x.y$ ~~c.~~ $y^2 = x.z$ d. None of these

$\Longrightarrow$ $t_4 = a \cdot r^3 = x$

$t_{10} = a \cdot r^9 = y$

$t_{16} = a \cdot r^{15} = z$

$(a \cdot r^9)^2 = a \cdot r^3 \times a r^{15}$

$y^2 = x \cdot z$

$a^2 r^{18} = a^2 \cdot r^{18}$

67 A person saved ₹ 16,500 in 10 years. In each year after first year he saved ₹ 100 more than he did in preceeding year. The amount of money he saved in first year was

a. ₹ 1,000 b. ₹ 1,500 ~~c.~~ ₹ 1,200 d. None of these

$\Longrightarrow$ First Year saving $= a$

second Year saving $= a + 100$

$S_{10} = 16500 = \frac{10}{2}[2a + 9 \times 100]$

$3300 = 2a + 900$

$2a = 2400$

$a = ₹ 1200/-$

My Notes

Find sum of all natural odd numbers divisible by 7 betn 1000 & 10,000

$\Longrightarrow$ $1001 + 1015 + 1029 + \text{----------} + 9989 = ?$

$9989 = 1001 + (n-1) 14$

$n = 643$

$S_{643} = \frac{643}{2}(1001 + 9989) = 3533285$

68 Sum of first 30 even natural numbers is :

~~a.~~ 930 b. 465 c. 900 d. None of these

$\Longrightarrow$ $30 \times 31 = 930$

Sum of first 'n' Even natural numbers $= n(n+1)$

$= n^2 + n$

69 t_n for AP is $(8n + 3)$. Find S_n

a. $7n^2+7n$ b. $7n^2+4n$ ~~c.~~ $4n^2+7n$ d. $2n^2+7n$

$\Longrightarrow$ $t_n = 8n+3$

$t_1 = 8(1)+3$

$= 11$

$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(11+8n+3)$

$= \frac{n}{2}(8n+14) = n(4n+7)$

$= 4n^2 + 7n$

70 $101^3 + 102^3 + 103^3 + \ldots\ldots\ldots\ldots + 123^3 = ?$

a. 3,23,11,450 ~~b.~~ 3,26,53,376 c.3,15,45,295 d. None

$\Longrightarrow$ $(1^3+2^3+3^3+ \text{-----} + 123^3) - (1^3+2^3+ \text{-----} + 100^3)$

$= \left(\frac{123 \times 124}{2}\right)^2 - \left(\frac{100 \times 101}{2}\right)^2 = 32653376$

71 For A.P $t_9 = 40$ and $t_{40} = 9$ then $t_{49} = ?$

a. 49 b. ₹-98 ~~c.~~ zero d. None of these

$\Longrightarrow$ please Remember : For AP

$t_p = q$ & $t_q = p$ then $t_{(p+q)} = \text{zero}$

My Notes

72 If Loga, Logb, Logc are in AP, then

a. a, b, c are in G.P b. a^2, b^2, c^2 are in G.P ~~c.~~ Both of these d. None of these

$\Longrightarrow$ Loga, Logb, Logc are in A.P.

$\therefore$ Log b − Log a = Log c − Log b

$\log(b/a) = \log(c/b)$

$\frac{b}{a} = \frac{c}{b}$ $\therefore$ a, b, c are in G.P.

$\boxed{\text{If } a, b, c \text{ are in GP then } a^2, b^2, c^2 \text{ are also in GP}}$

73 For 2 positive observations G.M. is G.M of AM & HM

~~a.~~ True b. False

2 positive Numbers are : x, y

$AM = \frac{x+y}{2}$ $GM = \sqrt{xy}$ $HM = \left(\frac{2xy}{x+y}\right)$

$GM^2 = xy$

$AM \times HM = \frac{\cancel{(x+y)}}{\cancel{2}} \times \frac{\cancel{2}xy}{\cancel{(x+y)}}$

$= xy$

$\boxed{\therefore GM^2 = AM \times HM}$

74 For AP First term = common difference then ratio of m^{th} term to n^{th} term is -

~~a.~~ m:n b. n:m c. $m^2:n^2$ d. None

$\Longrightarrow$ $a = d$

$\frac{t_m}{t_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{\cancel{d}+md-\cancel{d}}{\cancel{d}+nd-\cancel{d}} = \frac{md}{nd} = m:n$

75 $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P, then x, y , z are in

a. A.P b. G.P c. Both d. H.P

$\Longrightarrow$ $a^{1/x} = b^{1/y} = c^{1/z} = k$

$a^{1/x} = k$

$a = k^x, b = k^y, c = k^z$

a, b, c are in G.P.

$b^2 = ac$

$(k^y)^2 = k^x \cdot k^z$

$k^{2y} = k^{x+z}$

$\therefore 2y = x+z$

$y = \frac{x+z}{2}$

$\therefore x, y, z$ are in A.P.

My Notes

76 $x = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots \infty$ terms, $y = 1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\ldots \infty$ terms. Find x, y.

~~a.~~ 2 | b. 1 | c. 8/9 | d. 1/2

$x = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots\cdots \infty$ terms, In this GP $a = 1, r = \frac{1}{3}$

$x = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = 3/2$, $y = \frac{1}{1-\frac{1}{4}} = \frac{1}{3/4} = 4/3$

$xy = \frac{3}{2} \times \frac{4}{3} = 2$

77 For AP if $t_7 : t_{10} = 5:7$, then $t_8 : t_{11} = ?$

a. 13:16 | ~~b.~~ 17:23 | c. 14:17 | d. 15:19

$\Longrightarrow \frac{t_7}{t_{10}} = \frac{a+6d}{a+9d} = \frac{5}{7}$ $\quad 7a + 42d = 5a + 45d$ $\quad \boxed{2a = 3d}$

$\frac{t_8}{t_{11}} = \frac{a+7d}{a+10d} = \frac{2a+14d}{2a+20d} = \frac{3d+14d}{3d+20d} = \frac{17d}{23d} = 17:23$

78 If G is GM of a, b then, $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2} = ?$

a. G^2 | b. $3G^2$ | ~~c.~~ $1/G^2$ | d. $2/G^2$

$\Longrightarrow G = \sqrt{ab}$

$G^2 = ab$

$\frac{1}{G^2-a^2} + \frac{1}{G^2-b^2}$

$= \frac{1}{ab-a^2} + \frac{1}{ab-b^2}$

$= \frac{1}{a(b-a)} + \frac{1}{b(a-b)}$

$= \frac{1 \times b}{ab(b-a)} - \frac{1 \times a}{ab(b-a)}$

$= \frac{\cancel{(b-a)}}{ab\cancel{(b-a)}} = \frac{1}{G^2}$

79 Find the product of $243 \times 243^{1/6} \times 243^{1/36} \times \ldots\ldots\ldots\ldots$

a. 1024 | b. 27 | ~~c.~~ 729 | d. 246

$\Longrightarrow 243^1 \times 243^{1/6} \times 243^{1/36} \times 243^{1/216} \times \ldots\ldots\ldots\ldots$

$= (243)^{1+\frac{1}{6}+\frac{1}{36}+\frac{1}{216}+\cdots\cdots} = (243)^{\frac{1}{1-\frac{1}{6}}} = (3^5)^{1/5/6} = (3^5)^{6/5}$

$= 3^{5 \times 6/5} = 3^6 = 729$

My Notes

n^{th} root of product of 'n' observations is the GM of 'n' observations.

Find GM of $x_1, x_2, x_3, \ldots\ldots x_n$

$\Longrightarrow GM = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot \ldots\ldots x_n} = (x_1 \cdot x_2 \cdot x_3 \cdots\cdots x_n)^{1/n}$

80 GM of P, P^2, P^3, P^4, ... P^n will be

a. P^{n+1} b. $P^{(n+1)/2}$ c. $P^{n(n+1)/2}$ d. None of these

$\Longrightarrow$ $GM = \left(p^1 \times p^2 \times p^3 \times \text{-----} \times p^n\right)^{1/n}$

$= \left(p^{1+2+3+4+\text{----}+n}\right)^{\frac{1}{n}}$

$= \left[p^{\frac{n(n+1)}{2}}\right]^{1/n}$

$= (p)^{\frac{n+1}{2}} = \sqrt{p^{n+1}}$

81 Find the numbers whose AM is 12.50 and GM is 10 :

~~a.~~ 20,5 b. 10,5 c. 5,4 d. None of these

$\Longrightarrow$ $AM = \frac{20+5}{2} = 12.50$, $\sqrt{20 \times 5} = 10 = GM$

For '2' observations x, y

$AM = \left(\frac{x+y}{2}\right)$ $GM = \sqrt{xy}$ $HM = \left(\frac{2xy}{x+y}\right)$

82 t_5 of GP = $3^{1/3}$ then product of the first 9 terms of GP is :

a. 8 ~~b.~~ 27 c. 243 d. 9

$t_5 = a \cdot r^4 = 3^{1/3}$

product of first 9 terms of GP

$= a \times ar \times ar^2 \times ar^3 \times \text{------} \times ar^8$

$= a^9 \cdot r^{\frac{8 \times 9}{2}} = a^9 \cdot r^{36}$

$= (a \cdot r^4)^9 = (3^{1/3})^9 = 3^3 = 27$

83 For AP $t_3 + t_9 = 8$. Find S_{11} for AP

~~a.~~ 44 b. 22 c. 19 d. 11

$a + 2d + a + 8d = 8$

$2a + 10d = 8$

$S_{11} = \frac{11}{2}[2a + 10d]$

$= \frac{11}{2} \times 8 = 44$

My Notes

For AP $t_8 = 40$. Find S_{15} for AP

$\Longrightarrow$ $t_8 = a + 7d = 40$

$S_{15} = \frac{15}{2}[2a + 14d]$

$= \frac{15}{2} \times 2 \times (a + 7d)$

$= 15 \times 40$

$= 600$

84 t_8 for AP is 15 then S_{15} = ?

a. 15 b. 0 ~~c.~~ 225 d. 225/2

$\Longrightarrow$ $a + 7d = 15$

$S_{15} = \frac{15}{2} \times \boxed{2a + 14d}$

$= \frac{15}{2} \times 2 \times (a + 7d)$

$= 15 \times 15 = 225$

85 Find first term of GP if second term is 2 and sum of infinite terms is 8.

a. 6 b. 3 ~~c.~~ 4 d. 1

$\Longrightarrow$ $S_\infty = \frac{a}{1-r}$

$8 = \frac{a}{1 - \frac{2}{a}}$

$8 = \frac{a}{\frac{a-2}{a}}$

$8 = \frac{a^2}{a-2}$

$8a - 16 = a^2$

$a^2 - 8a + 16 = 0$

$(a-4)^2 = 0$

$a = 4$

$a, 2$

$\therefore r = \frac{2}{a}$

86 If sum of 4^{th} term and 12^{th} term of AP is 8, what is the sum of first 15 terms?

~~a.~~ 60 b. 120 c. 110 d. 150

$\Longrightarrow$ $a + 3d + a + 11d = 8$

$2a + 14d = 8$

$S_{15} = \frac{15}{2}(2a + 14d)$

$= \frac{15}{2} \times 8 = 60$

87 In GP, t_6 = 729; r = 3, First term = ?

a. 2 b. 3 c. 4 d. 7

$\Longrightarrow$ $t_6 = a \cdot r^5 = 729$

$a(3)^5 = 729$

$a \times 243 = 729$

$a = 3$

My Notes

Find sum of all 4 digit numbers divisible by 7?

$\Longrightarrow$ $1001 + 1008 + 1015 + \text{----------} + 9996$

$t_n = 9996 = 1001 + (n-1)7$

$n = 1286$

$S_{1286} = \frac{1286}{2}(1001 + 9996)$

$= 70,71,071$

88 For AP $S_{13} = 143$, $t_3 = 5$, find first term.

a. 4 b. 7 c. 9 ~~d.~~ 2

$\Rightarrow S_{13} = \frac{13}{2}[2a+12d] = 143$

$a+2d = 5$

$2d = 5-a$ ------- ①

$2a+12d = 22$

$2a + 6 \times (5-a) = 22$

$2a+30-6a=22$

$-4a = -8$

$\boxed{a=2}$

89 If GM of a, b, c, d is 3 then GM of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ is "

~~a.~~ 1/3 b. 3 c. 81 d. 1/81

$GM = \sqrt[4]{abcd} = 3$

$abcd = 3^4$

GM of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$

$= \sqrt[4]{\frac{1}{a}\times\frac{1}{b}\times\frac{1}{c}\times\frac{1}{d}} = \sqrt[4]{\frac{1}{3^4}}$

$= (3^{-4})^{1/4} = 3^{-1} = 1/3$

90 Find common difference of AP, if a = 200 and sum of first 6 terms exceeds twice the sum of first 4 terms by 50

a. -10 b. -15 ~~c.~~ 150 d. None of these

$S_6 = 2 \times S_4 + 50$

$\frac{6}{2}(400+5d) = 2 \times \frac{4}{2} \times (400+3d) + 50$

$1200 + 15d = 1600 + 12d + 50$

$3d = 450$

$d = 150$

91 59 + 63 + 67 + 71 + ... + 107 = ?

a. 972 ~~b.~~ 1079 c. 1083 d. None of these

$\Rightarrow$ In this AP $a = 59$, $d = 4$

$107 = 59 + (n-1)4$

$n = 13$

$S_{13} = \frac{13}{2}(59+107)$

$= 1079$

My Notes

Find sum of all odd 5 digit numbers divisible by 11

$\Rightarrow 10{,}021 + 10{,}043 + 10{,}065 + \text{---------} + 99979 = ?$

$99979 = 10021 + (n-1)22$, $n = 4090$

$S_{4090} = \frac{4090}{2}[10021 + 99979] = 22{,}49{,}50{,}000$

92 If one AM 'A' and 2 G. means G_1 & G_2 are inserted between any 2 numbers then $(G_1^3+G_2^3)$ = ?

~~a.~~ $2AG_1G_2$ b. $2G_1G_2$ c. $2AG_1$ d. 2A

$\Longrightarrow$

x, A, y are in AP.

x, G_1, G_2, y are in GP.

$A = \frac{x+y}{2}$

$\therefore x+y = 2A$

$\frac{G_1}{x} = \frac{G_2}{G_1} = \frac{y}{G_2}$

$G_1^2 = xG_2$ & $G_2^2 = y \cdot G_1$

$G_1^3 + G_2^3 = G_1 \cdot G_1^2 + G_2 \cdot G_2^2$

$= G_1 \cdot xG_2 + G_2 \cdot yG_1$

$= G_1G_2(x+y) = 2A \cdot G_1 \cdot G_2$

93 If a, b, c are in G.P. a, x, b and b, y, c both are in A.P, then (a/x) + (c/y) = ?

a. 1 b. 0 ~~c.~~ 2 d. None of these

$a = 2$ $a = 2$ $b = 4$

$b = 4$ $x = 3$ $y = 6$

$c = 8$ $b = 4$ $c = 8$

$\frac{a}{x} + \frac{c}{y}$

$= \left(\frac{2}{3} + \frac{8}{6}\right) = \left(\frac{2}{3} + \frac{4}{3}\right) = \frac{6}{3} = 2.00$

94 For AP $(t_7 / t_3) = (12/5)$. Find (t_{13}/t_4) =?

a. 8:5 b. 9:4 c. 7:3 ~~d.~~ 10:3

$\frac{t_7}{t_3} = \frac{12}{5} = \frac{a+6d}{a+2d}$

$12a + 24d = 5a + 30d$

$7a = 6d$

$\frac{t_{13}}{t_4} = \frac{a+12d}{a+3d} = \frac{2a+24d}{2a+6d}$

$= \frac{2a + 4 \times 7a}{2a + 7a} = \frac{30a}{9a} = \frac{10}{3} = 10:3$

95 4^{th} term of AP is equal to 3 times of first term and 7^{th} term exceeds twice of third term by 1. Find first term.

~~a.~~ 3 b. 5 c. 7 d. 9

$t_4 = 3a$ and $t_7 = 2 \times t_3 + 1$

$a + 3d = 3a$

$3d = 2a$

$a = 1.50d$

$a + 6d = 2(a+2d) + 1$

$1.5d + 6d = 2(1.50d + 2d) + 1$

$7.50d = 7d + 1$

$0.50d = 1$

$d = 2$

$a = 1.50 \times 2$

$= 3$

My Notes

10, 8, 6.40, 5.12, ... Find sum of infinite terms.

$\Longrightarrow$ In this G.P. $a = 10$, $r = 0.80$

$S_\infty = \left(\frac{a}{1-r}\right) = \frac{10}{1-0.80} = 50$

96 $t_n = 1/243$. For 3, $\sqrt{3}$, 1 .... then n = ?

a. 12 ~~b.~~ 13 c. 14 d. 15

$\Longrightarrow$ $3, \sqrt{3}, 1$

Here $a = 3$

$r = \frac{1}{\sqrt{3}}$

$t_n = a \times r^{n-1} = \frac{1}{243}$

$3 \times \left(\frac{1}{\sqrt{3}}\right)^{n-1} = \frac{1}{3^5}$

$\left(\frac{1}{\sqrt{3}}\right)^{n-1} = \left(\frac{1}{3}\right)^6$

$(\sqrt{3})^{n-1} = 3^6 = [(\sqrt{3})^2]^6$

$(\sqrt{3})^{n-1} = (\sqrt{3})^{12}$

$n-1 = 12 \quad \therefore n = 13$

97 For GP $S_n = 4095$, $r = 2$, $t_n = 2048$. Find n

a. 10 b. 11 ~~c.~~ 12 d. 15

$\Longrightarrow$

$t_n = a \times (2)^{n-1}$

$2048 = a \times \frac{2^n}{2^1}$

$4096 = a \times 2^n$

$4095 = \frac{a(2^n - 1)}{2-1}$

$4095 \times 1 = a \times 2^n - a$

$4095 = 4096 - a$

$a = 1$

$4096 = a \times 2^n$

$4096 = 1 \times 2^n$

$4096 = 2^n$

$2^{12} = 2^n$

$\therefore n = 12$

98 Which term of AP 64, 60, 56, 52, ... is zero

a. 18th ~~b.~~ 17th c. 14th d. 15th

$\Longrightarrow$ In this A.P. $a = 64$, $d = -4$

$t_n = 64 + (n-1)(-4) = 0$

$(n-1)(-4) = -64$

$n-1 = 16$

$\therefore n = 16 + 1$

$n = 17$

99 Sum of all 2 digit natural numbers is

a. 4955 b. 4890 c. 3776 ~~d.~~ None of these

$\Longrightarrow$ $10 + 11 + 12 + \cdots\cdots + 99$

$= \left(\frac{99 \times 100}{2}\right) - \left(\frac{9 \times 10}{2}\right) = 4905$

My Notes

Sum of all 4 digit natural numbers is :

$\Longrightarrow$ $1000 + 1001 + 1002 + \cdots\cdots + 9999$

$= \left(\frac{9999 \times 10{,}000}{2}\right) - \left(\frac{999 \times 1000}{2}\right)$

$= 4{,}94{,}95{,}500/-$

100 1, y, 9 are in A.P, then value of y is

a. 3 b. -3 c. Either (a) or (b) ~~d.~~ None of these

$y = \left(\frac{1+9}{2}\right) = 5$

101 a, b, c are in AP as well as GP, then

~~a.~~ a = b = c b. a ≠ b = c c. a ≠ b ≠ c d. Wrong qs.

102 a,b,c,d,e,f are in AP then (e-c) = ?

a. 2 (c-a) b. 2 (f-d) ~~c.~~ 2(d-c) d. (d-c)

2 x common diff

$= 2 \times (d-c)$

103 The sum of first '2n' terms of AP 2, 5, 8 ...... is equal to sum of first 'n' terms of AP 57, 59, 61, ... then n = ?

a. 10 b. 12 ~~c.~~ 11 d. 13

$\Longrightarrow$ 2, 5, 8, --------- S_{2n} 57, 59, 61, --------- S_n

$\frac{\cancel{2}n}{\cancel{2}}\left[4+(2n-1)3\right] = \frac{n}{2}\left[114+(n-1)2\right]$

$4+6n-3 = 57+n-1$

$6n+1 = n+56$

$5n = 55$

$n = 11$

My Notes

104 If $a^x = b^y = c^z$ and x, y, z are in GP then loga, logb, logc are in

a. A.P ~~b.~~ G.P c. Both d. None of these

$\Rightarrow a^x = b^y = c^z$

$x.\log a = y.\log b = z.\log c$

$\frac{x}{y} = \frac{\log b}{\log a}, \quad \frac{y}{z} = \frac{\log c}{\log b}$

x, y, z are in GP

$\frac{y}{x} = \frac{z}{y}, \quad \frac{\log a}{\log b} = \frac{\log b}{\log c}$

$\left(\frac{\log b}{\log a}\right) = \left(\frac{\log c}{\log b}\right)$

$\therefore$ loga, logb, logc are in G.P.

105 (4x+5), (5x+7), (8x-1) are in A.P. then x = ?

~~a.~~ 5 b. 6 c. 7 d. 4

$5x+7-4x-5 = 8x-1-5x-7$

$x+2 = 3x-8$

$10 = 2x$

$x = 5$

106 3 numbers are in G.P. If we double the middle term, we get an A.P. then common ratio of G.P. is equal to

~~a.~~ $2 \pm \sqrt{3}$ b. $3 \pm \sqrt{2}$ c. $3 \pm \sqrt{5}$ d. $5 \pm \sqrt{3}$

$\frac{a}{r}, a, ar$ are in G.P.

$\frac{a}{r}, 2a, ar$ are in A.P.

$2a - \frac{a}{r} = ar - 2a$

$\cancel{a}\left(2-\frac{1}{r}\right) = \cancel{a}(r-2)$

$2r-1 = r^2-2r$

$r^2-4r+1=0$

$r = \frac{4 \pm \sqrt{16-4(1)(1)}}{2\times 1} = \frac{4\pm\sqrt{12}}{2}$

$r = \frac{4\pm 2\sqrt{3}}{2} = 2\pm\sqrt{3}$

107 $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots = ?$

a. 17/8 b. 9/2 c. 7/2 ~~d.~~ 4

$\Rightarrow$ In this G.P. $a=2, r=\frac{1}{2}$

$S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$

My Notes

108 In AP a,b,c,d,e,f,g,h common difference = k; then in A.P. a,c,e,g common diff. = ?

~~a.~~ 2k   b. k^2 c. k   d. None of these

6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, 76 are in A.P. with d = 5

6, 16, 26, 36, 46, 56, 66, 76 are also in A P. with d = 5×2 = 10

109 In G.P. a,b,c,d,e,f,g,h common ratio = m; then in G.P. a,c,e,g common ratio = ?

a. m   b. 2m   ~~c.~~ m^2 d. None of these

1, 5, 25, 125, 625, 3125 are G.P. with r = 5

1, 25, 625 are also in G.P. with $r = 5^2 = 25$

110 Shall we stop here for the day?

a. Yes   b. No

111 8,8,8,8,8 are in

a. A.P   b. G.P   c. H.P   ~~d.~~ All of these

My Notes

112 $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ are in

a. A.P b. G.P ~~c.~~ H.P d. All of these

113 $\frac{1}{8}, \frac{1}{m}, \frac{1}{18}$ are in H.P. then m = ?

a. 1/13 ~~b.~~ 13 c. 1/12 d. 144

8, m, 18 must be in A.P.

m = 13

114 3 , $\sqrt{m}$, 10 are in G.P.; then m = ?

a. $\sqrt{30}$ ~~b.~~ 30 c. 13 d. 13/3

$\Longrightarrow \quad (\sqrt{m})^2 = 3 \times 10$

$m = 30$

115 If a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q are in G.P with r as common ratio; then a,d,g,j,m,p are in GP. with common ratio = ?

a. r b. r^2 ~~c.~~ r^3 d. None of these

INEQUALITIES & EQUATIONS

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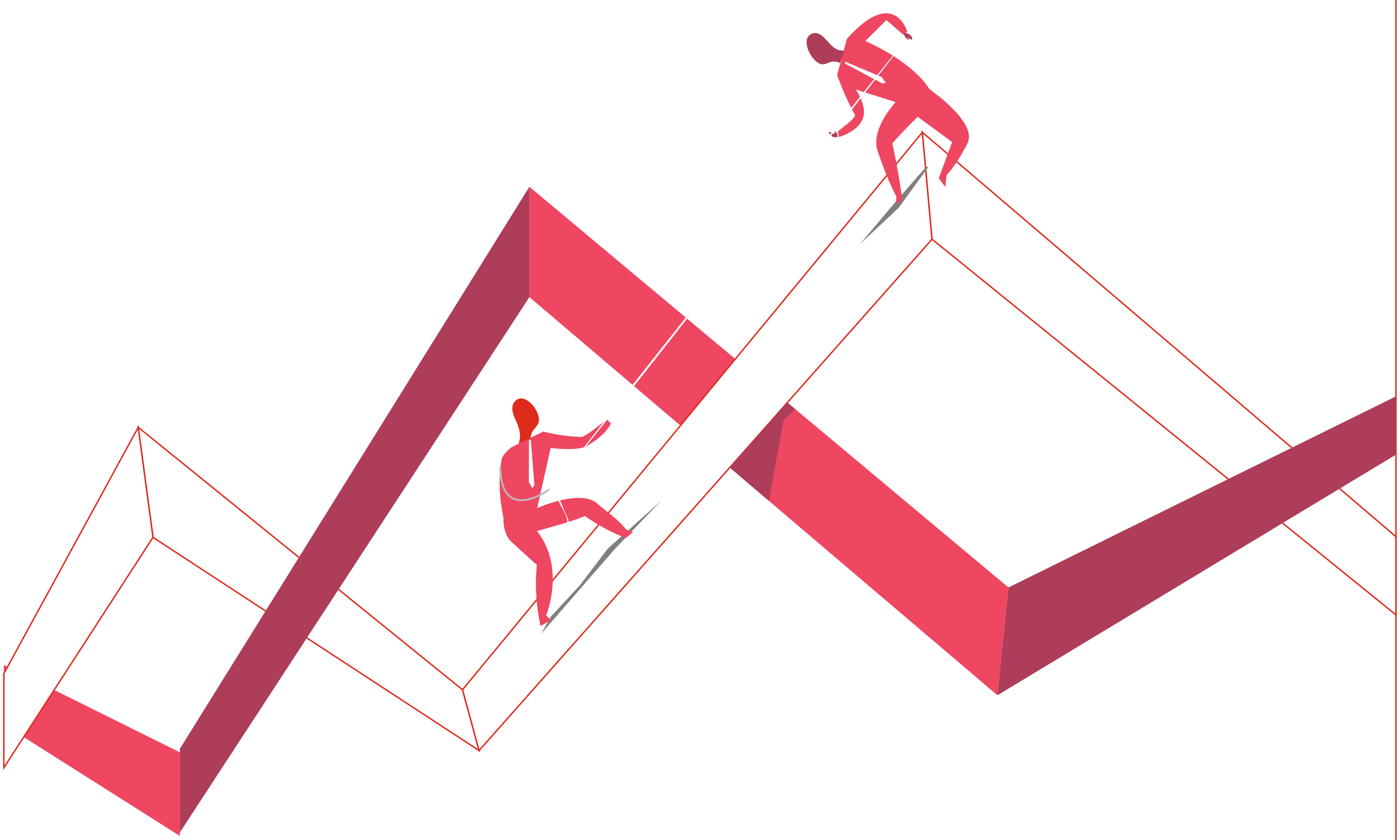


1

Locations	Points	Inequalities / Equations
1st Quadrant	$(+,+)$	$x>0,\ y>0$
2nd Quadrant	$(-,+)$	$x<0,\ y>0$
3rd Quadrant	$(-,-)$	$x<0,\ y<0$
4th Quadrant	$(+,-)$	$x>0,\ y<0$
X - Axis	$(\pm,0)$	$y=0$
Y - Axis	$(0,\pm)$	$x=0$ | Eqn of Y-axis : $x=0$
Origin	$(0,0)$	$x,y=0$ | Eqn of X-axis : $y=0$

2

The standard format of a linear equation is :

$ax+by+c=0$ where $a,b\neq 0$ at a time

OR $y=mx+c$ where $m=$ slope of the line

3

Graphical Presentation of a straight line is known as Linear Equation.

4

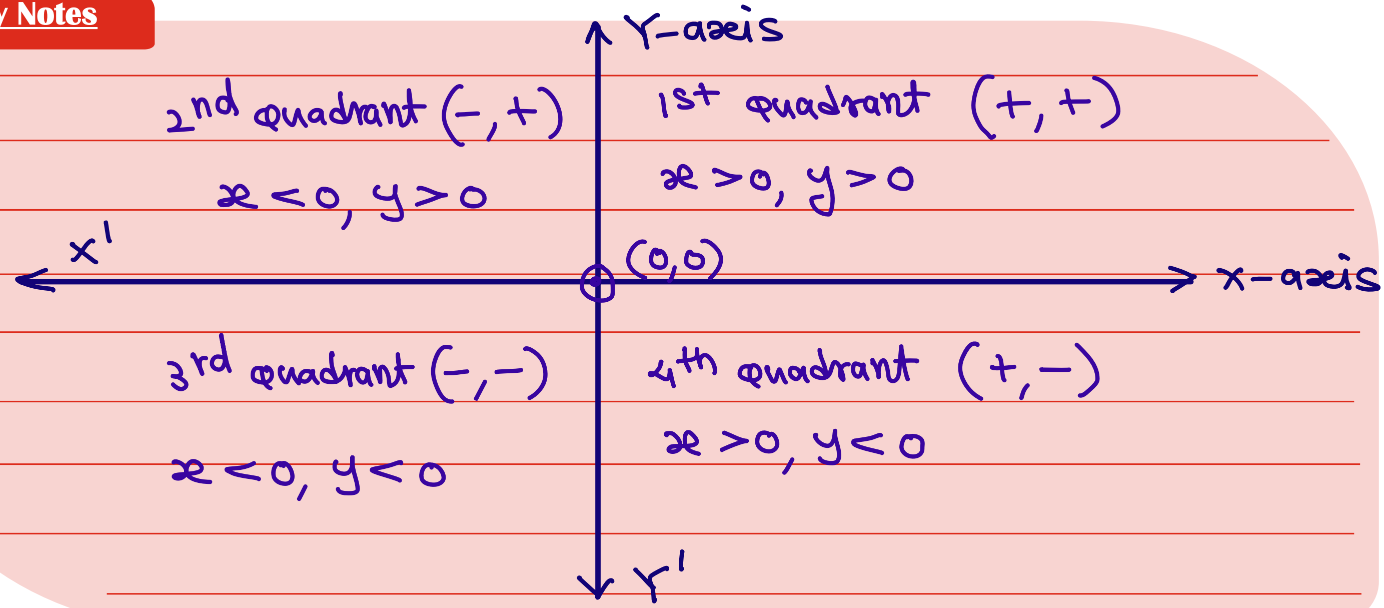
Line is a set / collection of infinite points satisfying the given linear equation.

5

Slope of the line ax + by + c = 0 is $-a/b$.

and slope of $y=mx+c$ is 'm'

My Notes



6

Equation of line	Slope of Line
$8x + 3y = 93$	$-8/3$
$3x - 11y = 51$	$-3/-11 = 3/11$
$-33x - 16y = -25$	$-33/16$
$3x = 83$, $3x + 0y = 83$	$-3/0$ = Not defined
$0x + 8y = 65$	$-0/8 = 0$ = zero
$px - qy = 80$	p/q
$2x + 6063y = 81$	$-2/6063$
$y = 8x + 13$	8
$y = -15x + 65$	-15

7

Equation of X-axis is : $y = 0$

Equation of Y-axis is : $x = 0$

Equation of || line to X-axis is : y = constant

Equation of || line to Y-axis is : x = constant

Slope of X-axis and all the lines || to X-axis is : zero

Slope of Y-axis and all the lines || to Y-axis is : Not defined

8

Equation of the line passing through points (x_1,y_1) and (x_2,y_2) is :

$$\Longrightarrow \quad \left(\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1} \right)$$

9

Slope of the line passing through points (x_1,y_1) and (x_2,y_2) is :

$$\Longrightarrow \quad \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$$

10

On solving 2 linear equations simultaneously if we get x=p and y=q, then

$\Longrightarrow$ (p,q) is the point of intersection of those 2 lines.

The point of intersection of $x+y=50$ $x-y=10$ is $(30,20)$. As if we put $x=30, y=20$ both eqⁿs are satisfied.

11

Equation	Number of roots
Linear	1
Quadratic	2
Cubic	3

value of x which satisfies the eqn is known as root of the Equation

12 $\frac{x+4}{4} + \frac{x-5}{3} = 11$; $x = ?$

$\Rightarrow \frac{3x+12+4x-20}{12} = 11$

$7x - 8 = 132$

$\therefore 7x = 140$

$x = 20$

13 $\frac{y+11}{6} - \frac{y+1}{9} = \frac{y+7}{4}$ then $y = ?$

$\frac{9y+99-6y-6}{54} = \frac{y+7}{4}$

$\frac{3y+93}{54} = \frac{y+7}{4}$

$12y + 372 = 54y + 378$

$-6 = 42y$

$\therefore y = -6/42 = -1/7$

14 $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$; then $x = ?$

$\frac{12x+1}{4} = \frac{45x^2-3x-15x+1+10x-25}{15x-5}$

$\frac{12x+1}{4} = \frac{45x^2-8x-24}{15x-5}$

$\cancel{180x^2}+15x-60x-5$

$= \cancel{180x^2}-32x-96$

$-45x-5 = -32x-96$

$91 = 13x$

$\therefore x = 7$

15 $\frac{x+24}{5} = 4 + \frac{x}{4}$; then $x = ?$

$\frac{x+24}{5} = \frac{16+x}{4}$

$4x+96 = 80+5x$

$16 = x$

$\therefore x = 16$

16 Find solution for $3x + 4y = 7$, $4x - y = 3$

$\Rightarrow$

$3x + \cancel{4y} = 7$

$16x - \cancel{4y} = 12$

$19x = 19$

$\boxed{x = 1}$

$3(1) + 4y = 7$

$4y = 4$

$\boxed{y = 1}$

$(1,1)$ is the point of intersection of 2 lines.

17 $x + 5y = 36$, $\frac{x+y}{x-y} = \frac{5}{3}$; then (x,y) = ?

$x + 5y = 36$ ----- ①

$3x + 3y = 5x - 5y$

$-2x + 8y = 0$

$\therefore 2x = 8y$

$x = 4y$

$4y + 5y = 36$

$\therefore 9y = 36$

$y = 4$

$\therefore x = 16$

(16,4) is the solution

18 $\frac{3}{x+y} + \frac{2}{x-y} = 3$ & $\frac{2}{x+y} + \frac{3}{x-y} = 3\frac{2}{3}$; then (x,y) = ?

$\Rightarrow \frac{3x - 3y + 2x + 2y}{x^2 - y^2} = 3$

$\therefore \frac{5x - y}{3} = x^2 - y^2$ ------- ①

$\frac{2x - 2y + 3x + 3y}{x^2 - y^2} = \frac{11}{3}$

$\frac{5x + y}{11/3} = x^2 - y^2$

$\frac{5x - y}{3} = \frac{(5x + y)3}{11}$

$55x - 11y = 45x + 9y$

$10x = 20y$

$\boxed{x = 2y}$

$\frac{9y}{3} = 3y^2$

$3y = 3y^2$

$y = 1$

$x = 2$

19 Monthly income of 2 persons is in the ratio of 4:5 and their monthly exp. are in the ratio of 7:9. If each saves ₹ 50 p.m. Find their monthly incomes.

$\Rightarrow$ Monthly incomes be : $4x, 5x$

$\frac{4x - 50}{5x - 50} = \frac{7}{9}$

$36x - 450 = 35x - 350$

$\therefore x = 100$

$\therefore$ Monthly incomes are : 400, 500

20 Standard format of a quadratic equation is :

$ax^2 + bx + c = 0$ where $a \neq 0$

OR $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

21 Find the roots of $x^2 - 9x + 20 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \left[\frac{9 \pm \sqrt{81 - 4(1)(20)}}{2 \times 1}\right]$

$= \frac{9+1}{2}$ OR $\frac{9-1}{2}$

$= 5, 4$ are roots

short-cut

$x^2 - 9x + 20 = 0$

$x^2 - 5x - 4x + 20 = 0$

$x(x-5) - 4(x-5) = 0$

$(x-5)(x-4) = 0$

$x = 5, x = 4$

super short-cut

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x = 5, x = 4$

22 First root of quadratic equation = $\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)$.

2^{nd} root of quadratic equation = $\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$.

Sum of roots = $\frac{-b + \sqrt{\cancel{b^2 - 4ac}} - b - \sqrt{\cancel{b^2 - 4ac}}}{2a} = \frac{-2b}{2a} = -b/a$

Product of roots = c/a.

23 Find roots of quadratic equation $3x^2 - 7x - 20 = 0$. Also find sum and product of roots.

$\Rightarrow$

$3x^2 - 7x - 20 = 0$

$3x^2 - 12x + 5x - 20 = 0$

$3x(x-4) + 5(x-4) = 0$

$(x-4)(3x+5) = 0$

$x = 4$ OR $x = -5/3$

Roots are : $4, -5/3$

cross-check

sum of roots

$= 4 + \frac{-5}{3}$

$= \frac{7}{3}$

product of roots

$= 4 \times -5/3$

$= -20/3$

sum of roots	$= -b/a = 7/3$
product of roots	$= c/a = -20/3$

24

Quadratic Equations	Sum of roots	Product of roots
$3x^2 + 2x + 11 = 0$	$-2/3$	$11/3$
$5x^2 - 19x - 13 = 0$	$19/5$	$-13/5$
$2kx^2 - 13px + 8p - 19 = 0$	$13p/2k$	$(8p-19)/2k$
$8x^2 - x - 63k + 25 = 0$	$1/8$	$(-63k+25)/8$
$2x^2 = 25$	0	$-25/2$
$8x^2 - 13x = 0$	$13/8$	0

25

$(a+b)^2 = a^2 + b^2 + 2ab$.

$(a-b)^2 = a^2 + b^2 - 2ab$.

$\boxed{(a^2+b^2) = (a+b)^2 - 2ab}$.

$\boxed{a^3 - b^3 = (a-b)(a^2+ab+b^2)}$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$.

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$.

$\boxed{(a^3+b^3) = (a+b)^3 - 3ab(a+b)}$.

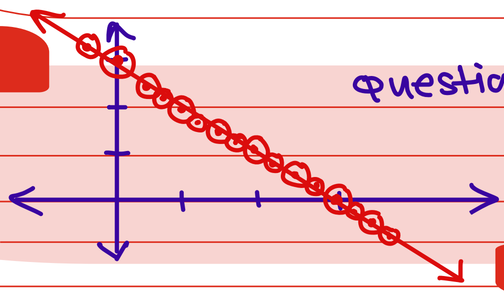
$(a^2-b^2) = (a-b)(a+b)$.

$(a+b+c)^2 = a^2+b^2+c^2 + 2ab + 2bc + 2ac$.

$a^3+b^3 = (a+b)(a^2-ab+b^2)$.

$\boxed{(a-b)^2 = (a+b)^2 - 4ab}$.

My Notes



question: Draw the Line $x+y=30$

$\Rightarrow$ points satisfying eqn $x+y=30$

(0,30), (30,0), (10,20), (20,10), (15,15), (5,25)

26 If α & β are roots of the quadratic equation $3x^2 + 7x + 12 = 0$, then

$\alpha\beta$ = product of roots $= c/a = 12/3 = 4$.

$\alpha+\beta$ = sum of roots $= -b/a = -7/3$.

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(-\frac{7}{3}\right)^2 - 2\times4 = \frac{49}{9} - \frac{72}{9} = -23/9$

$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \left(-\frac{7}{3}\right)^3 - 3\times4\times-\frac{7}{3} = \frac{-343}{27} + \frac{756}{27} = (413/27)$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = \frac{49}{9} - 16 = \frac{49}{9} - \frac{144}{9} = -95/9$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{-23/9}{4/1} = -\frac{23}{9}\times\frac{1}{4} = -23/36$

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{413/27}{4/1} = \left(\frac{413}{108}\right)$.

$\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = 4 \times -7/3 = -28/3$.

27

If $b^2 - 4ac$ =	Nature of roots
zero	Real, Rational, Equal
Negative	Unreal/complex/imaginary
Positive (perfect square)	Real, Rational, unequal
Positive (not a perfect square)	Real, Irrational, unequal

28

Value of $b^2 - 4ac$ =	Nature of roots
28	Real, Irrational, unequal
25	Real, Rational, unequal
-100	Unreal/complex/imaginary
0	Real, Rational, Equal
35	Real, Irrational, unequal
64	Real, Rational, unequal
729	Real, Rational, unequal
-35	Unreal/complex/imaginary
-0	Real, Rational, Equal

29

If roots of quadratic equation are	then
Equal	$b^2 - 4ac = 0$
Equal but opposite in sign	$b = 0$
Reciprocals of each other	$a = c$

30

If α & β are roots of the quadratic equation $x^2 + 7x + 12 = 0$, then quadratic equation whose roots are $(\alpha^2 + \beta^2)$ and $(\alpha - \beta)^2$ is

$\Longrightarrow$

$$\alpha + \beta = -7/1 = -7$$

$$\alpha\beta = 12/1 = 12$$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-7)^2 - 2(12) = 25$$

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-7)^2 - 4(12) = 1$$

∴ Question : Quadratic Eqn whose roots are 25, 1 is

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$

$$x^2 - (25+1)x + (25\times1) = 0$$

$$x^2 - 26x + 25 = 0$$

My Notes

If α, β are roots of quadratic Eqn $3x^2 - 2x + 7 = 0$. Find value of $\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)$

$\Longrightarrow$ $\alpha + \beta = 2/3$, $\alpha\beta = 7/3$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \left[\frac{(2/3)^3 - 3(7/3)(2/3)}{7/3}\right] = \frac{\frac{8}{27} - \frac{126}{27}}{\frac{63}{27}} = \frac{-118}{63}$$

31 If α,β are roots of the Qequation $2x^2 - 4x - 1 = 0$ then find values of

$\alpha+\beta =$ SUM of ROOTS $= -(-4)/2 = 2$

$\alpha\beta =$ PRODUCT of ROOTS $= -1/2$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 4 - 2\left(-\frac{1}{2}\right) = 4+1 = 5$

$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = 8-3(-1/2)(2) = 8+3 = 11$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = 2^2 - 4(-1/2) = 4+2 = 6$

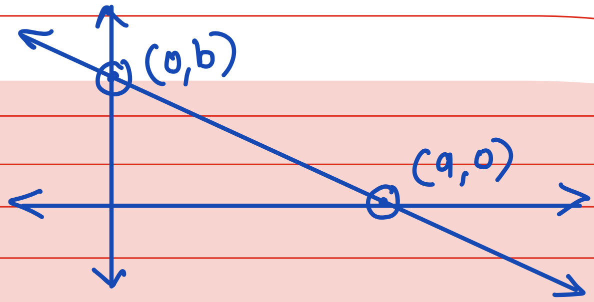
$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right) = \frac{11}{-1/2} = -22$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right) = \frac{5}{-1/2} = -10$

32 Intercept form of Equation of Line is -

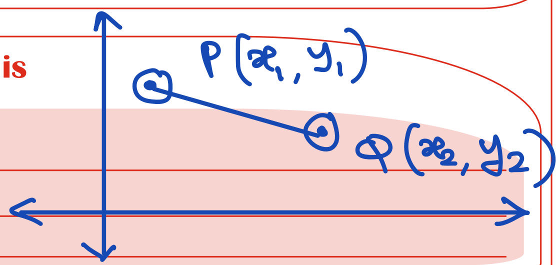
$$\left(\frac{x}{a}+\frac{y}{b}\right) = 1$$

where $a = x$ intercept
$b = y$ intercept



33 Length of segment drawn between points (x_1,y_1) and (x_2, y_2) is

$\Longrightarrow$ $l(PQ) = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$



34 If m_1 is slope of one line and m_2 is slope of other lines then lines are said to be

|| to each other if: $m_1 = m_2$

$\perp$ to each other, if: $m_1 \times m_2 = -1$

Oblique, if: $m_1 \neq m_2$

My Notes

If $3x+5y=83$ is $\perp$ to $7x-ky=93$
Find k.

$\Longrightarrow$ slope of 1st line $\times$ slope of 2nd line $= -1$

$-3/5 \times 7/k = -1$

$\therefore -21/5k = -1 \quad \therefore -21 = -5k \quad \therefore k = \left(\frac{21}{5}\right)$

35 The standard format of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$
dividing by 'a' on both sides

$$\Longrightarrow \quad ax^2 + bx + c = 0$$

$$\frac{ax^2 + bx + c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 - \left(-\frac{b}{a}\right)x + \left(\frac{c}{a}\right) = 0$$

$$x^2 - (\text{SUM OF ROOTS})x + (\text{PRODUCT OF ROOTS}) = 0$$

36 ① Find quadratic equation whose roots are 5, 8.

$$\Longrightarrow \quad x^2 - (5+8)x + (5 \times 8) = 0$$

$$x^2 - 13x + 40 = 0$$

② Find quadratic Eq^n whose roots are $(p+q), (p-q)$

$$\Longrightarrow \quad x^2 - 2px + (p^2 - q^2) = 0$$

37

Roots of quadratic equation	Quadratic Equation
8, 11	$x^2 - 19x + 88 = 0$
-19, 16	$x^2 + 3x - 304 = 0$
2, 20	$x^2 - 22x + 40 = 0$
3/8, 5/8	$x^2 - 1x + 15/64 = 0 \;\therefore\; 64x^2 - 64x + 15 = 0$
2/7, 5/2	$x^2 - 39/14\, x + 10/14 = 0 \;\therefore\; 14x^2 - 39x + 10 = 0$
$(5 + \sqrt{3}), (5 - \sqrt{3})$	$x^2 - 10x + (25-3) = 0 \;\therefore\; x^2 - 10x + 22 = 0$
$(8 + \sqrt{10}), (8 - \sqrt{10})$	$x^2 - 16x + 54 = 0$

38 Quad. Eq^n: $ax^2 + bx + c = 0$ where $a \neq 0$
Cubic Eq^n : $ax^3 + bx^2 + cx + d = 0$ where $a \neq 0$

For	Sum of roots	Product of roots
Quadratic Equation	$-b/a$	c/a
Cubic Equation	$-b/a$	$-d/a$

My Notes

Find quad. eq^n whose roots are $7/5$ & $11/19$

$$\Longrightarrow$$

$$x^2 - \left(\frac{7}{5} + \frac{11}{19}\right)x + \left(\frac{7}{5} \times \frac{11}{19}\right) = 0$$

$$x^2 - \frac{188}{95}x + \frac{77}{95} = 0$$

$$95x^2 - 188x + 77 = 0$$

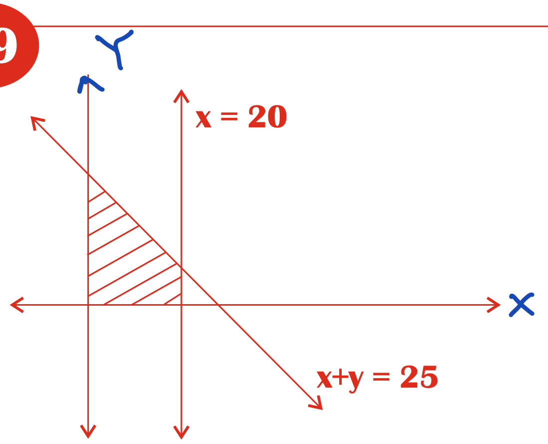
Roots → $7/5$ & $11/19$
Factors → $(5x-7)$ & $(19x-11)$

$$(5x-7)(19x-11) = 0$$

$$95x^2 - 55x - 133x + 77 = 0$$

$$95x^2 - 188x + 77 = 0$$

39

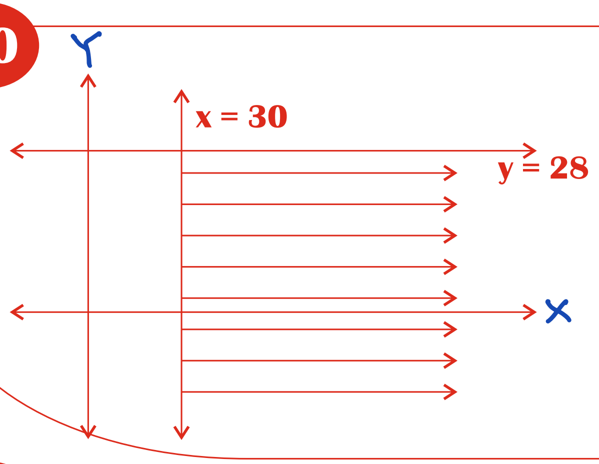


Inequalities representing shaded area are :

$x \le 20$, $x+y \le 25$, $x, y \ge 0$

$x, y \ge 0$ is known as Non-Negativity constraint which restricts feasible area in 1st quadrant

40



Inequalities representing shaded area are :

$x \ge 30$ & $y \le 28$

41 **Sum of 2 numbers is 52 and their difference is 2. The numbers are :**

a. 17, 15 **b. 12, 10** **~~c.~~ 27, 25** **d. None of these**

$\Longrightarrow$

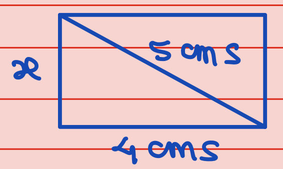
$$\begin{aligned} x + \cancel{y} &= 52 \\ x - \cancel{y} &= 2 \\ \hline 2x &= 54 \\ x &= 27 \end{aligned}$$

& $y = 25$

42 **Diagonal of a rectangle is 5 cms and one of the sides is 4 cms. Its area is ____ sq.cms**

a. 20 **b. 10** **~~c.~~ 12** **d. None of these**

$\Longrightarrow$



$5^2 = 4^2 + x^2$

$x^2 = 25 - 16$

$x^2 = 9$

$x = 3$ cms

Area of rectangle

= length × Breadth

= 4 cms × 3 cms

= 12 cm²

43 **4th part of a number exceeds sixth part by 4. The number is :**

a. 84 **b. 44** **~~c.~~ 48** **d. None of these**

Number = x

$\frac{x}{4} - \frac{x}{6} = 4$

$3x - 2x = 48$

$x = 48$

44 Ten years ago, age of father was 4 times of his son's age. Ten years hence age of the father will be twice that of his son. The present ages of father, son are :

~~a.~~ 50,20 b. 60,20 c. 55,25 d. None of these

$\Longrightarrow$

	Father	son
present age	x	y

$(x-10) = 4(y-10)$

$x-10 = 4y-40$

$x = 4y-30$

$x+10 = 2(y+10)$

$4y-30+10 = 2y+20$

$2y = 40$

$y = 20 \quad \therefore x = 50$

45 The number of which the half is greater than $(1/5)^{th}$ of the number by 15. The number is.

~~a.~~ 50 b. 40 c. 80 d. None of these

Number $= x$

$\frac{x}{2} - \frac{x}{5} = 15$

$\frac{5x-2x}{10} = 15$

$3x = 150$

$\therefore x = 50$

46 1.5x + 2.4y = 1.8 and 2.5(x + 1) = 7y have solution as :

a. 0.50,0.40 ~~b.~~ 0.40, 0.50 c. 1/2,1/5 d. None of these

$15x + 24y = 18$ ----- ①

$25x - 70y = -25$ --- ②

$\cancel{150x} + 240y = 180$

$\cancel{150x} - 420y = -150$

$- \quad + \quad +$

$660y = 330$

$y = \frac{1}{2} = 0.50$

$15x + 12 = 18$

$15x = 6$

$x = 6/15 = 2/5 = 0.40$

47 A 2 digit number is 5 times its sum of digits. If 9 is added digits are reversed. Find the number :

a. 54 b. 53 ~~c.~~ 45 d. 55

x	y
Ten's place	units place

$10x + y + 9 = 10y + x$

$9x - 9y = -9$

$x - y = -1$

$10x + y = 5(x+y)$

$5x - 4y = 0$

$5(-1+y) - 4y = 0$

$-5 + 5y - 4y = 0$

$\boxed{y = 5}$

$x = 4$

48 Wages of 8 men and 6 boys amount to ₹ 33. If 4 men earn ₹ 4.50 more than 5 boys. Determine wages of each man and boy.

a. ₹ 1.50, ₹ 3 ~~b.~~ ₹ 3, ₹ 1.50 c. ₹ 2.50, ₹ 2 d. ₹ 2, ₹ 2.50

man $= x$

Boy $= y$

$8x + 6y = 33$ ---- ①

$4x = 5y + 4.50$ --- ②

$8x = 10y + 9$ --- ③

$33 - 6y = 10y + 9$

$24 = 16y$

$\therefore y = 1.50$

$x = 3.00$

49 Of 2 numbers (1/5)th of the greater number is equal to (1/3)rd of the smaller & their sum is 16. The numbers are :

~~a.~~ 6,10 b. 9,7 c. 12,4 d. 11,5

$\frac{x}{5} = \frac{y}{3}$

$3x = 5y$

$x+y=16$

$3x+3y=48$

$5y+3y=48$

$\boxed{y=6}$

$\therefore x=10$

50 y is older than x by 7 years. 15 years back x's age was (3/4)th of y's age. The present ages are :

~~a.~~ 36,43 b. 50,43 c. 43,50 d. 40,47

$y = x+7$ ---- (1)

$\frac{3}{4}(y-15) = (x-15)$ ---- (2)

51 2 numbers are such that twice the greater number exceeds twice the smaller number by 18, & (1/3)rd of smaller number & (1/5)th of greater number are together 21. The numbers are

~~a.~~ 36,45 ~~b.~~ 45,36 c. 50,41 d. 55,46

Smaller No. = x

Greater No. = y

$2y = 2x+18$

$y = x+9$ ---- (1)

$\frac{x}{3} + \frac{y}{5} = 21$

$5x+3y = 315$

$5x+3(x+9) = 315$

$8x+27 = 315$

$\boxed{x=36 \quad y=45}$

52

Quadratic Equations	Value of (b^2-4ac)	Nature of roots
$x^2 - 8x + 16 = 0$	$(-8)^2-4(1)(16) = 0$	Real, Rational, Equal
$3x^2 - 8x + 4 = 0$	$(-8)^2-4(3)(4) = 16$	Real, Rational, unequal
$5x^2 - 4x + 2 = 0$	$(-4)^2-4(5)(2) = -24$	complex/imaginary
$2x^2 - 6x - 3 = 0$	$(-6)^2-4(2)(-3) = 60$	Real, Irrational, unequal

My Notes

If roots of $3kx^2 - 2x^2 + 18px - 22k + 15 = 0$ are reciprocals of each other. Find k.

$\Longrightarrow (3k-2)x^2 + 18px - 22k+15 = 0$

$a = c$ ------- As Roots are reciprocals of each other

$3k-2 = -22k+15$

$25k = 17$

$\therefore k = (17/25)$

53 If α,β are roots of the quadratic equation $2x^2-4x-1=0$. Find the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

$\Longrightarrow \quad \alpha+\beta = -(-4)/2 = 2, \quad \alpha\beta = -1/2$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \left[\frac{2^3-3(-\frac{1}{2})(2)}{-1/2}\right] = \frac{8+3}{-1/2} = \frac{11}{-1/2} = -22$$

54 $4^x - 3 \times 2^{(x+2)} + 32 = 0$; then x = ?

a. 2 b. 3 ~~c.~~ 2 or 3 d. None of these

$4^x - 3 \times 2^{x+2} + 32 = 0$

By options

$4^2 - 3 \times 2^{2+2} + 32 = 16 - 48 + 32 = 0$

$4^3 - 3 \times 2^{3+2} + 32 = 64 - 96 + 32 = 0$

55 $2^{(x-2)} + 2^{(3-x)} = 3$; then x = ?

a. 2 b. 3 ~~c.~~ 2 or 3 d. None of these

By options

$2^{2-2} + 2^{3-2} = 2^0 + 2^1 = 1 + 2 = 3$

$2^{3-2} + 2^{3-3} = 2^1 + 2^0 = 2 + 1 = 3$

56 Find the quadratic equation whose one root is $(8+\sqrt{7})$

a. 2 b. 3 c. 2 or 3 d. None of these

$\Longrightarrow$ If one root is $(8+\sqrt{7})$, other must be $(8-\sqrt{7})$

$\therefore$ Quad. eq^n whose roots are $(8+\sqrt{7})$ & $(8-\sqrt{7})$ is,

$x^2 - 16x + (64-7) = 0$

$x^2 - 16x + 57 = 0$

57 If one root of $5x^2+13x+p=0$ be reciprocal of other; then value of p is

a. -5 ~~b.~~ 5 c. 1/5 d. -1/5

When Roots are reciprocals of each other

$a = c$

$5 = p$

$\therefore p = 5$

58 If p,q are roots of $x^2+2x+1=0$; then find (p^3+q^3)

a. 2 ~~b.~~ -2 c. 4 d. None of these

$\Longrightarrow$ $p+q = -2/1 = -2$

$pq = 1/1 = 1$

$p^3+q^3 = (p+q)^3 - 3pq(p+q)$

$= (-2)^3 - 3(1)(-2)$

$= -8 + 6 = -2$

59 If one root of the equation is $x^2 - 8x + m = 0$; exceeds the other by 4. m = ?

a. 10 b. 11 c. 9 ~~d.~~ 12

$x^2 - 8x + m = 0$

$x^2 - (\text{sum of roots})\,x + (\text{product of roots}) = 0$

sum of roots = 8

Roots are 6, 2

product of roots = m

$= 2 \times 6$

$= 12$

60 Five times of a positive whole number is 3 less than twice the square of the number. The number is :

~~a.~~ 3 b. 4 c. -3 d. 2

Number $= x$

$5x = 2x^2 - 3$

$0 = 2x^2 - 5x - 3$

$2x^2 - 6x + x - 3 = 0$

$2x(x-3) + 1(x-3) = 0$

$(x-3)(2x+1) = 0$

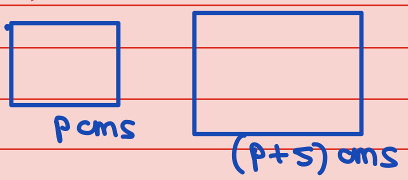
$x = 3$ OR $x = -1/2$

61 Two squares have sides p cms and (p+5) cms respectively. The sum of their squares is 625 sq. cms. The sides of the squares are :

a. 10 cms, 30 cms b. 12 cms, 25 cms

~~c.~~ 15 cms, 20 cms d. None of these

$\Longrightarrow$



$625 = p^2 + (p+5)^2$

$625 = p^2 + p^2 + 10p + 25$

$0 = 2p^2 + 10p - 600$

$\therefore\ p^2 + 5p - 300 = 0$

$(p+20)(p-15) = 0$

$\therefore\ p = -20,\ p = 15$

62 $x + y = 50$ and $(1/x) + (1/y) = (1/12)$; then x,y are

a. 24, 26 b. 28, 22 c. 27, 23 ~~d.~~ 20, 30

with options

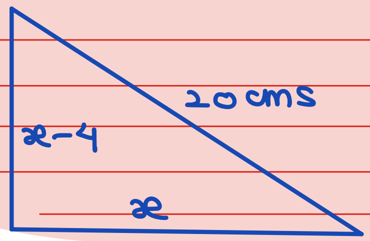
$20 + 30 = 50$

$\frac{1}{20} + \frac{1}{30} = \frac{20+30}{600} = \frac{50}{600} = \frac{1}{12}$

63 The hypotenuse of a right angled triangle is 20 cms. The diff. between other 2 sides is 4 cms. The sides are :

a. 11 cms, 15 cms ~~b.~~ 12 cms, 16 cms

c. 20 cms, 24 cms d. None of these



$20^2 = x^2 + (x-4)^2$

$400 = x^2 + x^2 - 8x + 16$

$0 = 2x^2 - 8x - 384$

$x^2 - 4x - 192 = 0$

$(x-16)(x+12) = 0 \quad \therefore x = 16, \; x = -12$

64 The sum of 2 numbers is 45 and mean proportional between them is 18. The numbers are

a. 15,30 b. 32,13 ~~c.~~ 36,9 d. 25,20

$x \quad y$

$x + y = 45$

$\sqrt{xy} = 18$

$xy = 324$

$\downarrow$ As $36 + 9 = 45$

$36 \times 9 = 324$

65 The sum of 2 irrational numbers multiplied by the larger one is 70 and their diff is multiplied by smaller one is 12; 2 numbers are :

a. $3\sqrt{2}, 2\sqrt{3}$ b. $5\sqrt{2}, 3\sqrt{5}$ ~~c.~~ $2\sqrt{2}, 5\sqrt{2}$ d. None of these

$5\sqrt{2}(2\sqrt{2} + 5\sqrt{2})$

$= 5\sqrt{2} \times 7\sqrt{2}$

$= 35 \times 2 = 70$

$2\sqrt{2}(5\sqrt{2} - 2\sqrt{2})$

$= 2\sqrt{2} \times 3\sqrt{2}$

$= 6 \times 2 = 12$

66 The solution of a cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ is given by

a. (-1,1,-2) ~~b.~~ (1, 2, 3) c. (-2,2,3) d. (0,4,-5)

$\Longrightarrow x^3 - 6x^2 + 11x - 6 = 0$

$\downarrow$

sum of roots = 6

product of roots = 6

For cubic Eq^n

$ax^3 + bx^2 + cx + d = 0$

$\downarrow$

sum of roots $= -b/a$

product of roots $= -d/a$

67 The cubic equation $x^3 + 2x^2 - x - 2 = 0$ has 3 roots namely.

a. 1, -1, 2 ~~b.~~ (-1, 1, -2) c. (-1, 2, -2) d. None of these

$\Longrightarrow$ Cubic Eq^n $x^3 + 2x^2 - x - 2 = 0$

sum of roots $= -2$

product of roots $= 2$

68 The roots of cubic equation $x^3 + 7x^2 - 21x - 27 = 0$ are :

a. (-3,-9,-1) ~~b.~~ (3,-9,-1) c. (3,9,1) d. (-3,9,1)

$\Longrightarrow$ $x^3 + 7x^2 - 21x - 27 = 0$

sum of roots $= -7$

product of roots $= 27$

$3 + -9 + -1 = -7$

& $3 \times (-9) \times (-1) = 27$

69 If $4x^3+8x^2-x-2 = 0$; then $(2x+3)$ = ?

~~a.~~ 4,-1,2 b. -4,2,1 c. 2,-4,-1 d. None of these

$\Longrightarrow$ $4x^3 + 8x^2 - x - 2 = 0$

$4x^2(x+2) - 1(x+2) = 0$

$(x+2)(4x^2-1) = 0$

$(x+2)(2x-1)(2x+1) = 0$

$x = -2,\ x = \frac{1}{2},\ x = -\frac{1}{2}$

$x = -2 \quad x = \frac{1}{2} \quad x = -\frac{1}{2}$

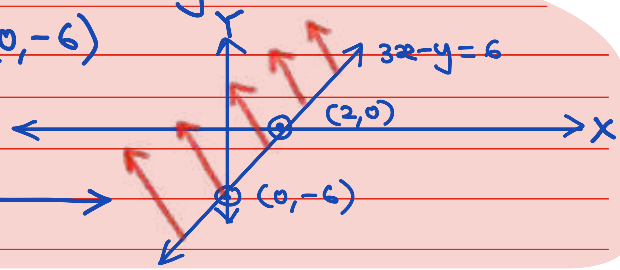
$2x+3 \Rightarrow -1,\ 4,\ 2$

70 $x, y \geq 0$ is known as Non-Negativity constraint will restrict the feasible region in First quadrant.

71 Find feasible area for $(3x-y) \leq 6$

$\Longrightarrow$ First we will draw the line $3x - y = 6$ by joining points $(2,0), (0,-6)$

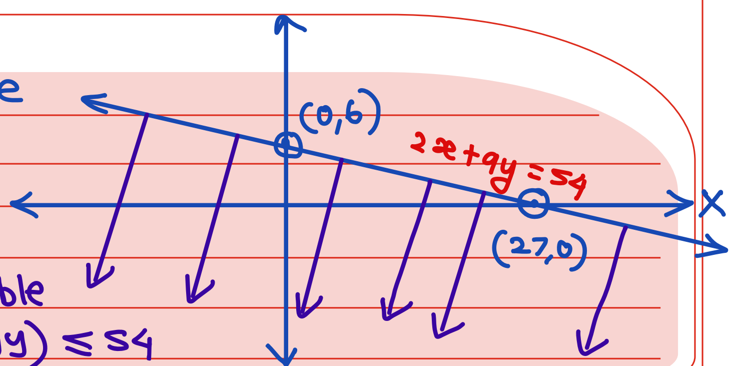
Unbounded Feasible area for $3x - y \leq 6$



72 Find feasible area for $2x + 9y \leq 54$

$\Longrightarrow$ First we will draw the line $2x + 9y = 54$ by joining the points $(27,0)$ & $(0,6)$

unbounded feasible area for $(2x+9y) \leq 54$



My Notes

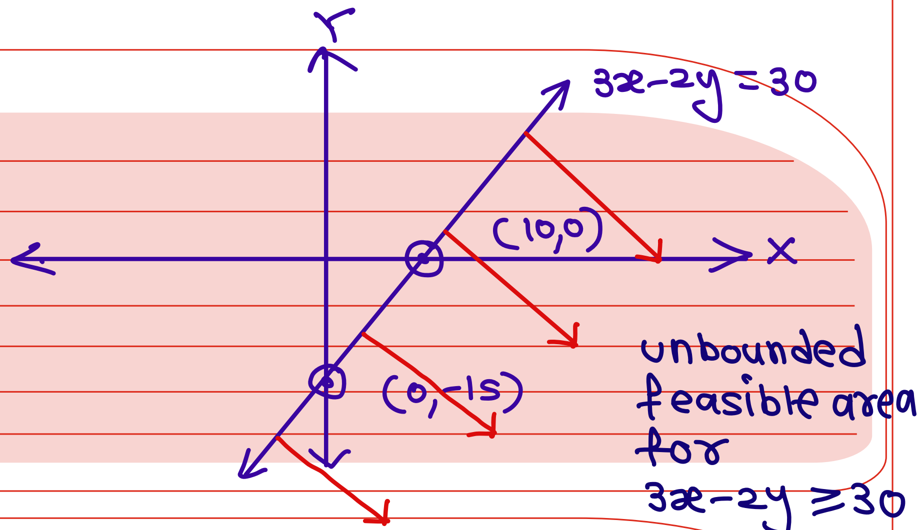
Find 15 points satisfying the Linear Equation $x + 2y = 120$

$\Longrightarrow$ (120,0), (0,60), (20,50), (30,45), (-10,65), (60,30), (90,15), (1.20,59.40), (-100,110), (2,59), (40,40), (100,10), (50,35), (-60,90), (-50,85), (-200,160), (-25,72.50)

73 Find feasible area for $3x - 2y \geq 30$

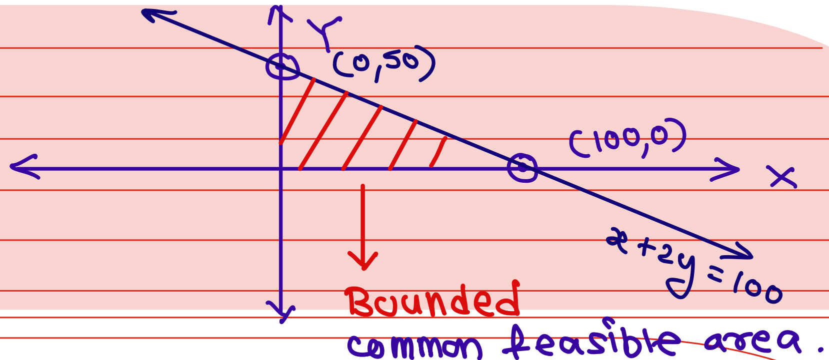
$\Longrightarrow$

First we will draw the line $3x-2y=30$ by joining $(10,0)$ & $(0,-15)$



74 Find common feasible area for $x+2y \leq 100$; $x,y \geq 0$

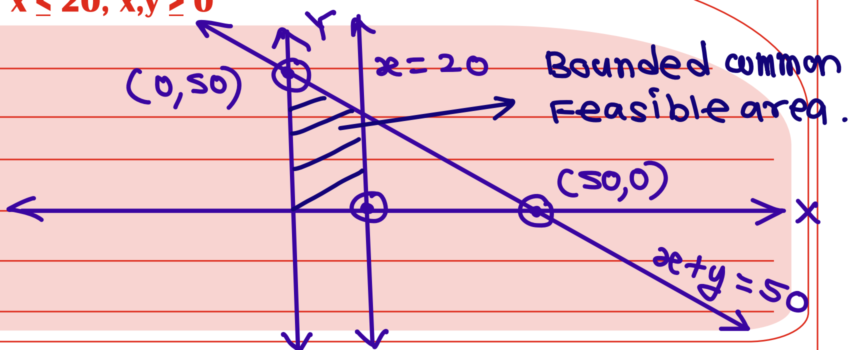
First we will draw the line $x+2y=100$ by joining points $(0,50)$ & $(100,0)$



75 Find common feasible area for $x+y \leq 50$, $x \leq 20$, $x,y \geq 0$

First we will draw the line $x+y=50 \Rightarrow (50,0),(0,50)$

$x=20 \Rightarrow (20,0)$, || line to Y-axis



76

$8x+3y = 24$
$x=81$
$3x-5y=63$
$8y = \left(\frac{-81}{5}\right)$
$3x-22y=635$

} Linear Equations Or Linear Equalities

$3x+2y \leq 50$
$8x-y \geq 60$
$x+y < 90$
$2x-y > 65$
$y > 90$
$x < 35$
$x \leq 35$

} Linear Inequations Or Linear Inequalities

My Notes

- Linear Equality is shown by a Line on Graph paper whereas Linear inequality is shown by an area on Graph paper
- Graphical presentation of a Linear equation is a Line whereas Graphical presentation of a Linear inequation is an area.

77 Point of intersection of lines 5x+3y = 150 and 3x+5y= 350 lie in ________ quadrant,

a. 1st ~~b.~~ 2nd c. 3rd d. 4th

$\Longrightarrow$

$$\cancel{15x} + 9y = 450$$
$$\cancel{15x} + 25y = 1750$$
$$- \quad - \quad -$$
$$-16y = -1300$$
$$\boxed{y = 81.25}$$

$$5x + 3(81.25) = 150$$
$$\boxed{x = -18.75}$$

(−18.75, 81.25) is the point of intersection.

78 One of the point on line 2x + 5y = 100 is

a. (20,30) ~~b.~~ (60,-4) c. (8,12) d. All of these

$$2(60) + 5(-4) = 100$$

79 Point of intersection of lines (3x+5y=120) and (3x+2y=10) is

a. (-30,90) b. (-90,30) c. (90,-30) ~~d.~~ None of these

$\Longrightarrow$

$$\cancel{3x} + 5y = 120$$
$$\cancel{3x} + 2y = 10$$
$$- \quad - \quad -$$
$$3y = 110$$

$$y = 36.6666$$
$$3x + 5(36.666666) = 120$$
$$x = -21.111111$$

(−21.1111, 36.666666) is the point of intersection.

80 The point (0,60), (0,90), (0,-80), (0,-66) lie on

a. X-Axis ~~b.~~ Y-axis c. Origin d. Can't say

81 The inequalities representing second quadrant are :

a. x>0, y>0 ~~b.~~ x<0, y>0 c. Both d. None

82 The line y = 80 is parallel to

~~a.~~ X-Axis b. Y-axis c. Origin d. Can't say

My Notes

1. Slope of x-axis is zero
2. Slope of y-axis is Not defined
3. Eqn of x-axis : $y = 0$
4. Eqn of y-axis : $x = 0$
5. The Line $x = 100$ is $\perp$ to x-axis
6. The Line $y = -35$ is $\parallel$ to x-axis

83 In number 78,534 place value of 8 is

a. 8   ~~b.~~ 8,000   c. 800   d. 80,000

70000
8000
500
30
4
78,534

84 The lines (2x+3y) = 60 and (10x+15y) = 238 have

~~a.~~ No solution   b. Unique solution   c. 2 solution   d. None of these

slope of 1st line = $-2/3$

slope of 2nd line = $-10/15 = -2/3$

These line are || to each other

∴ They have No solution

85 Factors of quadratic equation $(x^2-5x-6) = 0$ are

a. (x-3) (x-2)   b. (x+6) (x-1)   c. (x-3) (x+2)   ~~d.~~ (x-6) (x+1)

$\Longrightarrow$ $x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

Factors are : $(x-6)$ & $(x+1)$

Roots are : $6, -1$

86 Formulae to remember -

$(a+b)^2 = a^2+b^2+2ab$.

$(a+b)^3 = a^3+b^3+3ab(a+b) = a^3+b^3+3a^2b+3ab^2$.

$(a-b)^2 = a^2+b^2-2ab$.

$(a-b)^3 = a^3-b^3-3ab(a-b) = a^3-b^3-3a^2b+3ab^2 = a^3-b^3+3ab(b-a)$.

$(a^2-b^2) = (a-b)(a+b)$.

$(a^3-b^3) = (a-b)(a^2+ab+b^2)$.

$(a^3+b^3) = (a+b)(a^2-ab+b^2) = (a+b)^3-3ab(a+b)$.

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$.

$(a-b+c)^2 = a^2+b^2+c^2-2ab-2bc+2ac$.

$(a-b-c)^2 = a^2+b^2+c^2-2ab+2bc-2ac$.

$(a^2+b^2) = (a+b)^2-2ab$.

$(a-b)^2 = (a+b)^2-4ab$.

$(a+b)^2+(a-b)^2 = 2(a^2+b^2)$.

My Notes

If p, q are roots of $3x^2-7x+17=0$

Find value of $\left(\frac{p^2}{q}+\frac{q^2}{p}\right)$

$\Longrightarrow$ $p+q = 7/3$, $pq = 17/3$

$$\frac{p^2}{q}+\frac{q^2}{p} = \frac{p^3+q^3}{pq} = \frac{(p+q)^3-3pq(p+q)}{pq} = \frac{\frac{343}{27} - 3\times\frac{17}{3}\times\frac{7}{3}}{17/3}$$

$$= \left(\frac{343/27 - 1071/27}{153/27}\right) = -728/153$$

87 $(1/5)^{th}$ of one half of a number is 11. The number is

a. 11 b. 55 c. 22 ~~d.~~ 110

$\Longrightarrow \frac{1}{5} \times \frac{1}{2} \times x = 11 \quad \therefore x = 110$

88 Find the quadratic equation whose roots are (-2/3) and (5/17)

a. $15x^2+19x+10=0$ b. $15x^2+19x-10=0$ ~~c.~~ $51x^2+19x-10=0$ d. None of these

$x^2 - \left(\frac{-2}{3} + \frac{5}{17}\right)x + \left(-\frac{2}{3} \times \frac{5}{17}\right) = 0, \quad x^2 + \frac{19}{51}x - \frac{10}{51} = 0, \quad 51x^2 + 19x - 10 = 0$

89 If p,q are roots of quadratic equation $10x^2-x-7=0$. Find the quadratic equation whose roots are (p+q), pq

a. $10x^2+6x-7=0$ b. $100x^2+60x+7=0$ ~~c.~~ $100x^2+60x-7=0$ d. None of these

$\Longrightarrow (p+q) = 1/10$

$(pq) = -7/10$

Question is: Find the quad. eq^n whose roots are 1/10 & -7/10

Answer is: $x^2 - \left(\frac{-6}{10}\right)x + \left(\frac{-7}{100}\right) = 0$

$100x^2 + 60x - 7 = 0$

90 If p,q are roots of quadratic equation $3x^2-19x-1=0$. Find the quadratic equation whose roots are (p/q), (q/p)

a. $3x^2-19x-1=0$ ~~b.~~ $3x^2+367x+3=0$ c. $3x^2+367x-3=0$ d. None of these

$\Longrightarrow p+q = 19/3, \quad pq = -1/3$

$x^2 - \left(\frac{p}{q} + \frac{q}{p}\right)x + \left(\frac{p}{q} \times \frac{q}{p}\right) = 0$

$x^2 - \left(\frac{p^2+q^2}{pq}\right)x + 1 = 0$

$x^2 - \left(\frac{\frac{361}{9} + \frac{2}{3}}{-1/3}\right)x + 1 = 0$

$x^2 + \left(\frac{367/9}{3/9}\right)x + 1 = 0$

$x^2 + \frac{367}{3}x + 1 = 0$

$3x^2 + 367x + 3 = 0$

91 Equation of Y-Axis is

~~a.~~ x=0 b. y=0 c. x.y=0 d. x/y=0

92 Cubic Equation whose roots are p, q, r is

$(x-p)(x-q)(x-r) = 0$

$(x^2 - qx - px + pq)(x-r) = 0; \quad x^3 - qx^2 - px^2 + pqx - rx^2 + qrx + prx - pqr = 0$

$x^3 - (p+q+r)x^2 + (pq+qr+pr)x - pqr = 0$

My Notes

Find cubic eq^n whose roots are $\frac{1}{2}, \frac{7}{2}, -\frac{17}{2}$

$\Longrightarrow x^3 - \left(\frac{1}{2} + \frac{7}{2} - \frac{17}{2}\right)x^2 + \left(\frac{7}{4} + \frac{-119}{4} + \frac{-17}{4}\right)x - \left(\frac{-119}{8}\right) = 0$

$x^3 + \frac{9}{2}x^2 - \frac{129}{4}x + \frac{119}{8} = 0$

$8x^3 + 36x^2 - 258x + 119 = 0$

93

Roots of quadratic equation	Factors of Quadratic Equation	Quadratic Eqn
3, -2	$(x-3)(x+2)$	$x^2-1x-6=0$
-8, -9	$(x+8)(x+9)$	$x^2+17x+72=0$
$-3/2, 8/7$	(2x + 3) (7x - 8)	$14x^2+5x-24=0$
$\frac{3}{8}, \frac{-9}{17}$	$(8x-3), (17x+9)$	$136x^2+21x-27=0$
$-10/7$ & $11/8$	(7x + 10) (8x - 11)	$56x^2+3x-110=0$
$\frac{-2}{3}, \frac{8}{13}$	$(3x+2), (13x-8)$	$39x^2+2x-16=0$
1, -1	$(x-1)(x+1)$	$x^2-1=0$
$1, \frac{-8}{27}$	$(x-1)(27x+8)$	$27x^2-19x-8=0$
$33, -31/8$	(x - 33) (8x + 31)	$8x^2-233x-1023=0$
$\frac{1}{2}, \frac{-1}{2}$	$(2x-1)(2x+1)$	$4x^2-1=0$

94 Roots of quadratic eqn $3kx^2 - 2x^2 + 19x - 3k + 63 = 0$ are reciprocals of each other. Find k.

a. 7/13 ~~b.~~ 65/6 c. 21 d. None of these

$\Longrightarrow (3k-2)x^2 + 19x + (-3k+63) = 0$

As Roots are reciprocals of each other $a = c$

$3k-2 = -3k+63 \quad \therefore 6k = 65 \quad \therefore k = 65/6$

95 Roots of quadratic eqn $3x^2 - 2kx + 21x - 35 = 0$ are equal but opposite in sign. Find k.

~~a.~~ 21/2 b. 35/3 c. 2/21 d. None of these

$\Longrightarrow 3x^2 + (-2k+21)x - 35 = 0$

As Roots are Equal but opposite in sign, $b = 0$

$-2k+21 = 0 \quad \therefore 2k = 21 \quad \therefore k = \frac{21}{2} = 10\frac{1}{2} = 10.50$

My Notes

Roots of quadratic equation

$3kx^2 - 22x + 88 = 0$ are Equal. Find value of k

$\Longrightarrow (-22)^2 - 4(3k)(88) = 0$ ..... As Roots are equal, $b^2 - 4ac = 0$

$484 - 1056k = 0$

$484 = 1056k$

$\therefore k = 484/1056 = (121/264) = 0.45833333$

96 Y = Total cost, x = No. of units produced. Fixed Cost = ₹ 3,80,000 & Variable cost p.u. = ₹ 10; then

a. y = 10x - 3,80,000 ~~b.~~ y = 3,80,000 + 10x
c. y = 3,80,000x + 10 d. None of these

Total cost = Total Fixed cost + (Variable cost p.u. x No. of units produced)

$y = 3,80,000 + 10x$

97 If p,q are roots of quadratic equation $x^2+2x+1=0$ then quadratic equation whose roots are (1/p), (1/q) is :

a. $x^2-2x-1=0$ ~~b.~~ $x^2+2x+1=0$ c. $x^2-2x+1=0$ d. None of these

$p+q=-2$
$pq=1$

quadratic eqn whose roots are $\frac{1}{p}, \frac{1}{q}$ is

$x^2-\left(\frac{1}{p}+\frac{1}{q}\right)x+\left(\frac{1}{p}\times\frac{1}{q}\right)=0$

$x^2-\left(\frac{p+q}{pq}\right)x+\left(\frac{1}{pq}\right)=0$

$x^2-\frac{-2}{1}x+\frac{1}{1}=0$

$x^2+2x+1=0$

98 $a^2+b^2 = 45$ and $ab = 18$; then (1/a) + (1/b) = ?

a. 1/3 b. 2/3 ~~c.~~ 1/2 d. None of these

$(a+b)^2 = a^2+b^2+2ab$
$= 45+2(18)$
$(a+b)^2 = 81$
$\therefore (a+b) = 9$

$\frac{1}{a}+\frac{1}{b} = \left(\frac{a+b}{ab}\right) = \frac{9}{18} = \frac{1}{2}$

99 $\left[\frac{0.7214 \times 20.37}{69.80}\right]^{1/3} = ?$

a. 1.5948 ~~b.~~ 0.5949 c. 0.2348 d. None of these

$= (0.21052891117)^{1/3} = 0.5949$

100 Find average of first 30 multiples of 5

~~a.~~ 77.50 b. 87.50 c. 75 d. None of these

$= \frac{5+10+15+20+ \text{.......... } 30 \text{ terms}}{30} = \frac{\frac{\cancel{30}}{2}[10+(29\times5)]}{\cancel{30}} = 77.50$

101 A cricketer scored 180, 258 runs in first & second test respectively. How many runs he should score in third test so that his average score of 3 tests would be 230.

a. 219 b. 242 ~~c.~~ 252 d. 334

$\frac{180+258+x}{3} = 230$

$x = 252$

He should score 252 runs in third test so that his avg would be 230 runs.

102 A number is added to another number, the total becomes 150% of second number. What is the ratio of first and second number?

~~a. 1:2~~ b. 1:3 c. 2:3 d. None of these

First number $= x$

Second number $= y$

$x+y = \frac{150}{100}(y)$

$100x + 100y = 150y$

$100x = 50y$

$\therefore \frac{x}{y} = \frac{50}{100} = \frac{1}{2}$

$x:y = 1:2$

103 Calculate the number such that it is equal to 3 times its difference from 56.

a. 14 b. 28 ~~c. 42~~ d. 178

$\Longrightarrow$ $x = 3 \times (56-x)$

$x = 168 - 3x$

$4x = 168$ $\therefore x = 42$

104 kx + 2y = 5 and 3x + y = 21 have unique solution if : means they have point of intersection

a. k = 6 ~~b. k ≠ 6~~ c. k = ±6 d. None of these

$kx+2y=5$ & $3x+y=21$ are not $\parallel$ to each other

$-\frac{k}{2} \neq -\frac{3}{1}$ $k \neq 6$

105 For what value of k, the equation 9x + 4y = 9 and 7x + ky = 5 has no solution.

~~a. 28/9~~ b. 36/7 c. 23/9 d. 7

$\Longrightarrow$ They have No solution means Lines are $\parallel$ to each other

slope of 1st line = slope of 2nd line

$-\frac{9}{4} = -\frac{7}{k}$ $\therefore 9k = 28$

$k = 28/9$

106 If $b^2 > 4ac$ then roots of quadratic equation are

a. imaginary ~~b. Real, unequal~~ c. Real, Rational d. None

$b^2 > 4ac$ $b^2 - 4ac$ is positive

$b^2 - 4ac > 0$ $\therefore$ Roots are Real, unequal

107 For what value of k the equation $x^2+4kx+k+2 = 0$ has no one root as zero

a. 2 b. 4 ~~c. -2~~ d. -1/2

As one root is 'zero'. If we put $x=0$, eqn must be satisfied

$0^2 - 4k(0) + k + 2 = 0$

$k = -2$

108 If equation $x^2 - (p+4)x + 2p + 5 = 0$ has equal roots then p = ?

a. ±1 ~~b. ±2~~ c. 2 d. -2

$[-(p+4)]^2 - 4(1)(2p+5) = 0$

$p^2 + \cancel{8p} + 16 - \cancel{8p} - 20 = 0$

$\therefore p^2 - 4 = 0$

$p^2 = 4$

$\therefore p = \pm 2$

109 If total cost of 10 units, 20 units is ₹ 15,000 and ₹ 20,000 respectively. Find total cost of 30 units?

a. ₹ 30,000 b. ₹ 35,000 c. ₹ 25,000 d. None of these

$\Rightarrow$ variable cost p.u. = $\left(\frac{\text{change in cost}}{\text{change in quantity}}\right) = \frac{5,000}{10 \text{ units}} = 500$ p.u.

Total fixed cost = $15,000 - (10 \times 500) = 10,000$

Total cost of 30 units = $10,000 + (500 \times 30 \text{ units}) = 25,000$

110 Find the quadratic equation whose roots are 5, -5

a. $x^2 + 10x + 25 = 0$ b. $x^2 - 10x + 25 = 0$ c. $x^2 - 5 = 0$ d. $x^2 - 25 = 0$

$\Rightarrow x^2 - (5 + -5)x + (5 \times -5) = 0$

$x^2 - 0x - 25 = 0$

$x^2 - 25 = 0$

111 If p,q are roots of quadratic equation $3x^2 + 6x + 9 = 0$ then value of $(p^2 + q^2 + 2pq)$ is :

a. 4 b. -4 c. 3 d. 9

$\Rightarrow p + q = -6/3 = -2$

$p^2 + q^2 + 2pq = (p+q)^2 = (-2)^2 = 4$

112 If roots of quadratic equation $(x^2 - px + 8p - 15 = 0)$ are equal then p = ?

a. 3 or 5 b. 2 or 5 c. 2 or 30 d. None of these

$\Rightarrow (-p)^2 - 4(1)(8p - 15) = 0$

$p^2 - 32p + 60 = 0$

$(p - 30)(p - 2) = 0$

$\therefore p - 30 = 0$ OR $p - 2 = 0$

$p = 30$ OR $p = 2$

113 Out of 3 numbers, sum of first and second is 24. Sum of second & third is 30, sum of first and third is 26. The smallest number is :

a. 18 b. 14 c. 16 d. 10

$1^{st} = x$, $2^{nd} = y$, $3^{rd} = z$

$x + y = 24$

$y + z = 30$

$x + z = 26$

$x + 30 - y = 26$

$x + 30 - (24 - x) = 26$

$x - 24 + x = -4$

$2x = 20$

$\boxed{x = 10,\ y = 14,\ z = 16}$

114 Find slope of the line ⊥ to the line 2x + 78y = 1234

a. -2/78 b. 2/78 c. 39 d. -39

$\Rightarrow 2x + 78y = 1234 \Rightarrow$ slope of this line $= -2/78$

slope of its ⊥ line $= 78/2 = 39$

115 The point (-2, -1/3) lie in ________ quadrant.

a. 1^{st} b. 2^{nd} c. 3^{rd} d. 4^{th}

Permutations & Combinations

CA VINOD REDDY

1

Permutation = is an arrangement where order of objects is important.

Combination = is a selection where order of objects is NOT important.

pqr, qrp, prq, qpr, rpq, rqp are 6 permutations but only one selection.

2

$\boxed{8! = \underline{|8}}$

0! = 1

1! = 1

2! = 2

3! = 6

4! = 24

5! = 120 = 5 × 4!

6! = 720 = 6 × 5 × 4 × 3!

7! = 5040

8! = 40,320

9! = 3,62,880

10! = 36,28,800

11! = 11 × 10! = 39916800

12! = 12 × 11 × 10! = 479001600

n! = n(n−1)(n−2)(n−3) .... n terms

a. $\frac{19!}{18!} = \frac{19 \times \cancel{18!}}{\cancel{18!}} = 19$

b. $\frac{16!}{14!3!} = \frac{16 \times 15 \times \cancel{14!}}{\cancel{14!} \times 6} = 40$

c. $\frac{x!}{(x-1)!} = \frac{x\cancel{(x-1)!}}{\cancel{(x-1)!}} = x$

d. $\frac{(x+3)!}{(x+2)!} = \frac{(x+3)\cancel{(x+2)!}}{\cancel{(x+2)!}} = (x+3)$

e. $\frac{(x-3)!}{(x-1)!} = \frac{\cancel{(x-3)!}}{(x-1)(x-2)\cancel{(x-3)!}} = \frac{1}{(x-1)(x-2)}$

3 **In how many ways 3 students can stand in a line for a photograph?** (A, B, C)

ABC BAC CAB

ACB BCA CBA

6 diff photographs

$\underbrace{3P_1}_{} \times \underbrace{2P_1}_{} \times \underbrace{1P_1}_{}$ = 6 ways

3 students can stand in a line in 3! diff ways = 6

4 **In how many ways 4 students can stand in a line for a photograph?**

⇒ 4 students can stand in a line in $4P_4$ diff ways. ($4P_4 = 4! = 24$ ways)

My Notes

In How many ways 2 students out of 4 students A, B, C, D can be arranged for a photograph?

$\underbrace{4P_1}_{} \times \underbrace{3P_1}_{}$ = 12 diff ways

AB, AC, AD, BC, BD, CD

BA, CA, DA, CB, DB, DC

= 12 diff photos

'r' objects out of 'n' can be arranged in nP_r diff ways

$4P_2 = 4 \times 3 = 12$

5 There are 5 students A, B, C, D, E in how many ways 2 of them can be

Selected $= {}^5C_2$ ways
$= \frac{5!}{2!\,3!} = \frac{120}{2\times 6} = 10$

Arranged $= {}^5P_2$ ways
$= \frac{5!}{3!} = \frac{5\times4\times3!}{3!} = 20$

AB, AC, AD, AE, BC,
BD, BE, CD, CE, DE

10 diff. selections

${}^5C_2 \times 2! = {}^5P_2$

${}^nC_r \times r! = {}^nP_r$

${}^nC_r = n! / r!(n-r)!$

AB, AC, AD, AE, BC,
BD, BE, CD, CE, DE.
BA, CA, DA, EA, CB
DB, EB, DC, EC, ED

6 ${}^nP_r = \frac{n!}{(n-r)!}$ where n = positive integer & $n \geq r \geq 0$

${}^nP_r = n(n-1)(n-2)$ .... r tems

${}^nP_0 = 1$

${}^nP_1 = n$

${}^nP_2 = n(n-1)$

${}^nP_3 = n(n-1)(n-2)$

${}^nP_4 = n(n-1)(n-2)(n-3)$

${}^nP_5 = n(n-1)(n-2)(n-3)(n-4)$

${}^nP_n = n!$

${}^{18}P_3 = 18\times17\times16 = 4896$

${}^{100}P_2 = 100\times 99 = 9900$

${}^{50}P_4 = 50\times49\times48\times47 = 5527200$

${}^{25}P_1 = 25$

${}^{20}P_5 = 20\times19\times18\times17\times16 = 1860480$

${}^{24}P_8 = 24\times23\times22\times21\times20\times19\times18\times17$
$= 29654190720$

7 $\frac{{}^{18}P_3 \times {}^{16}P_3}{{}^{19}P_4 \times {}^{17}P_2} =$

$$= \frac{\cancel{18}\times\cancel{17}\times\cancel{16}\times\cancel{16}\times15\times14}{19\times\cancel{18}\times\cancel{17}\times\cancel{16}\times17\times\cancel{16}} = \left(\frac{210}{323}\right)$$

8 $\frac{9!}{6!2!} \times {}^5P_2 =$

$$= \frac{9\times8\times7\times\cancel{6!}\times5\times4}{\cancel{6!}\times2} = 5040 = 7!$$

9 AND ⟹ Multiply
OR ⟹ Add

10 n! can also be written as $\underline{|n}$

11 How many different words can be formed by using letters of word :

SQUARE :

$= 6! = 720$ diff words

HEXAGON :

$= 7! = 5040$ diff words

MISSISSIPPI :

$= \frac{11!}{4!\,4!\,2!} = 34,650$ diff words

Total no. of letters = 11
I – 4 times P – 2 times
S – 4 times

BOSTON :

$= \frac{6!}{2!} = \frac{720}{2} = 360$ diff words

MANAGEMENT

$= \frac{10!}{2!\,2!\,2!\,2!} = 2,26,800$ diff words

PERMUTATION

$= \frac{11!}{2!} = 1,99,58,400$ diff words

BANANA :

$= \frac{6!}{3!\,2!} = \frac{720}{6 \times 2} = 60$ diff words

My Notes

In How many diff ways Letters of word 'ACCOUNTING' can be arranged if vowels should be together ?

$\Longrightarrow$ A, O, U, I C C N T N G

$= \frac{7!}{2!\,2!} \times$ Internal arrangements

$= \frac{7!}{2!\,2!} \times 4!$

$= 30,240$ diff words

12 How many different words can be formed by using letters of word ________ if all vowels should be kept together.

BANANA : AAA B, N, N

No. of diff words

$= \frac{4!}{2!} \times \frac{3!}{3!} = 12$

PERCEPTION : E, E, I, O P R C P T N $= \frac{7!}{2!} \times \frac{4!}{2!} = 30,240$ diff words

JAYARAMAN : AAAA J, Y, R, M, N

$= 6! \times \frac{4!}{4!} = 720$ diff words

STATISTICS : A, I, I S, T, T, S, T, C, S $= \frac{8!}{3!\,3!} \times \frac{3!}{2!} = 3360$ diff words.

COMPUTER : OUE C, M, P, T, R

$= 6! \times 3! = 4,320$ diff words

CALCULATOR :

AUAO C, L, C, L, T, R $= \frac{7!}{2!\,2!} \times \frac{4!}{2!} = 15,120$ diff words

TATED : A, E T, T, D $= \frac{4!}{2!} \times 2! = 24$ diff words

13 In how many ways 'n' students can stand in a line for a photograph if r of them

Want to be always together?

$= (n-r+1)! \times r!$

Want to be never together?

= All possible arrangements − Arrangements where they are always together

$= n! - (n-r+1)!\,r!$

14 In how many ways 3 letter words can be formed by using letters of the word

SQUARE	HEXAGON	COMPUTER
$= {}^6P_3$	$= {}^7P_3$	$= {}^8P_3$
$= 120$	$= 210$	$= 336$
diff words	diff words	diff words

15 In how many ways 12 students can stand in a line for a photograph if

2 of the want to be always together?

$= 11! \times 2!$

$= 79833600$ ways

2 of them want to be never together?

= (All possible arrangements − Arrangements where those 2 students are always together)

$= 12! - 11!2! = 479001600 - 79833600$

$= 399168000$ ways

OR $10! \times {}^{11}P_2 = 399168000$ ways

16 If $6 \times {}^nP_3 = 7 \times {}^{(n-1)}P_3$. Find n.

$\Longrightarrow 6 \times {}^nP_3 = 7 \times {}^{(n-1)}P_3$

$6 \times n\cancel{(n-1)}\cancel{(n-2)} = 7 \times \cancel{(n-1)}\cancel{(n-2)}(n-3)$

$6n = 7n - 21$ $\quad \therefore n = 21$

$21 = n$

17 If ${}^nP_4 = 12 \times {}^nP_2$. then n = ?

$\Longrightarrow {}^nP_4 = 12 \times {}^nP_2$

$\cancel{n}\cancel{(n-1)}(n-2)(n-3) = 12 \times \cancel{n} \times \cancel{(n-1)}$

$(n-2)(n-3) = 12$

$\boxed{n = 6}$

My Notes

There are 15 students. How many diff photographs can be taken if 2 of them want to be ______

ⓐ always together

$= 14! \times 2!$

$= 174356582400$

diff. photographs

ⓑ Never together

$= 15! - 14!2! = 15 \times 14! - 14!2! = 14! \times 13$

= 14 × 13 × 12 × 11 × 36288 × 100 × 13

= 11333177856000

OR $13! \times {}^{14}P_2 = 14 \times 13 \times 13 \times 12 \times 11 \times 36288 \times 100$

$= 11333177856000$

18 ${}^nP_3 : {}^nP_2 = 3:1$; then n = ?

$\Rightarrow \frac{\cancel{n}(\cancel{n-1})(n-2)}{\cancel{n}(\cancel{n-1})} = \frac{3}{1}$

$\therefore n-2 = 3 \quad \therefore n = 5$

19 ${}^5P_r = 60$; then r = ?

$\Rightarrow {}^5P_r = 60 = 5 \times 4 \times 3 = {}^5P_3$

${}^5P_r = {}^5P_3 \qquad \therefore r = 3$

20 The no. of ways in which letters of word 'TRIANGLE' can be arranged if word 'ANGLE' is always present.

$\Rightarrow$ ANGLE T, R, I

= 4! = 24 diff words can be formed

21 In how many ways 5 students can form a

Line

$= {}^5P_5$

$= 5! = 120$ ways

Circle

$= \frac{{}^5P_5}{5} = 4!$

= 24 ways

The no. of ways in which letters of word TRIANGLE can be arranged so that letters of word ANGLE are always together?

$\Rightarrow 4! \times 5! = 2880$ diff. words

22 In how many different ways 12 students can form a

Line

$= {}^{12}P_{12} = 12!$

= 479001600

Circle

$= \left(\frac{{}^{12}P_{12}}{12}\right)$

$= 11! = 39916800$ ways

My Notes

① In How many diff ways 'n' students can form a ① Line = $n!$

② circle = $(n-1)!$

② In How many ways 12 diff diamonds can form a necklace?

$= \frac{1}{2} \times (11!) = 19958400$ ways

23 In how many ways ________ of 7 students can be formed out of 12 students

Line

$= {}^{12}P_7 = 3991680$ ways

Circle

$= \left(\frac{{}^{12}P_7}{7}\right) = 570240$ ways

In how many ways ________ of r students can be formed out of n students

Line

$= {}^{n}P_r$

Circle

$= \left(\frac{{}^{n}P_r}{r}\right)$

24 The no. of ways in which 'n' diamonds can form a necklace.

$\Longrightarrow \frac{1}{2} \times (n-1)!$

$= \left[\frac{(n-1)!}{2}\right]$ ways

25 The number of ways of arranging 'n' persons along a round table so that no person has the same 2 neighbours

$= \frac{1}{2} \times (n-1)!$

- In How many 'n' people can sit along round dining table?

$\Longrightarrow (n-1)!$

26 No. of different necklaces can be formed with 'n' beads of different colours = ?

$\Longrightarrow \left[\frac{1}{2}(n-1)!\right]$

My Notes

There are 10 students. In How many ways photograph of 4 students can be taken if a particular favourite student should

always be there?

$= {}^{4}P_1 \times {}^{9}P_3$

$= 2016$ ways

Never be there?

$= {}^{9}P_4$

$= 3024$ ways

27 Permutation of 'n' distinct things taken 'r' at a time when a particular object is

Always there?

$= {}^{r}P_1 \times {}^{(n-1)}P_{(r-1)}$

$= r \times {}^{(n-1)}P_{(r-1)}$

Never there?

$= {}^{(n-1)}P_r$

28 How many 4 digit numbers can be formed by using 0,1,2,3,4,5 if repetition of digits is

Allowed

$\underline{{}^5P_1} \times \underline{{}^6P_1} \times \underline{{}^6P_1} \times \underline{{}^6P_1}$

= 1080 Numbers

Not allowed

$= \underline{{}^5P_1} \times \overbrace{\underline{\quad}\ \underline{\quad}\ \underline{\quad}}^{{}^5P_3}$

= 300 Numbers

29 How many even numbers of 5 digits can be formed by using 2,3,4,5,6,7,8 if repetition of digits is

Not allowed

$\overbrace{\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}}^{{}^6P_4}\ \underline{{}^4P_1}$

= 1,440 Numbers

Allowed

$\underline{{}^7P_1} \times \underline{{}^7P_1} \times \underline{{}^7P_1} \times \underline{{}^7P_1} \times \underline{{}^4P_1}$

= 9,604 Numbers

30 How many 5 digit numbers greater than 23,000 can be formed by using 1,2,3,5,8,9

If question is silent about repetition ~~of~~ digits then Repetition of digits is not allowed

$\Longrightarrow$ starting with 2

$\underline{{}^1P_1} \times \underline{{}^4P_1} \times \overbrace{\underline{\quad}\ \underline{\quad}\ \underline{\quad}}^{{}^4P_3} = 96$

OR

starting with 3,5,8,9

$\underline{{}^4P_1} \times \overbrace{\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}}^{{}^5P_4} = 480$

576

31 How many 4 digit numbers greater than 4700 can be formed by using 2,3,4,5,8 if repetition of digits is

Allowed

① Starting with 4

$1P_1 \times 1P_1 \times 5P_1 \times 5P_1 = 25$

② starting with 5,8

$2P_1 \times 5P_1 \times 5P_1 \times 5P_1 = 250$

275

Not allowed

① Starting with 4

$1P_1 \times 1P_1 \times 3P_2 = 6$

② starting with 5,8

$2P_1 \times 4P_3 = 48$

54

32 ^nC_r = (No. of selections of 'r' objects out of 'n') × r! = (No. of arrangements of 'r' objects out of 'n')

$$^nC_r \times r! = {}^nP_r = \frac{n!}{(n-r)!}$$

$$\boxed{\therefore \; ^nC_r = \frac{n!}{r!\,(n-r)!}}$$

where $n \geq r \geq 0$

n = positive Integer

33 Formulae on combinations

$^nC_r = \left[\frac{n!}{r!\,(n-r)!}\right]$ where $n \geq r \geq 0$

$^nC_r = \left[{}^nP_r / r!\right] = \left[\frac{n(n-1)(n-2)\ldots\ldots \text{ r terms}}{r!}\right]$

$^nC_0 = 1$

$^nC_1 = n$

$^nC_2 = \frac{n(n-1)}{2!}$

$^nC_3 = \left[\frac{n(n-1)(n-2)}{3!}\right]$

$^nC_4 = \left[\frac{n(n-1)(n-2)(n-3)}{4!}\right]$

$^nC_n = 1$

$^nC_{n-r} = {}^nC_r$

$^nC_2 + {}^nC_3 + {}^nC_4 + \ldots\ldots + {}^nC_n$

$= 2^n - {}^nC_0 - {}^nC_1 = \left[(2^n) - 1 - n\right]$

$^nC_r + {}^nC_{r-1} = \left[{}^{n+1}C_r\right]$

$^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots\ldots + {}^nC_n = (2)^n$

$^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots\ldots + {}^nC_n = 2^n - {}^nC_0$

$= 2^n - 1$

$^{45}C_{20} = {}^{45}C_{45-20}$

$\frac{45!}{20!\,25!} = \frac{45!}{25!\,20!}$

34 $^{18}C_r = {}^{18}C_{r+2}$ then r = ?

$^{18}C_r = {}^{18}C_{r+2}$

$r = r + 2$ OR $r + r + 2 = 18$

This is impossible

$2r = 16$

$r = 8$

35 $^{45}C_x = {}^{45}C_y$ then

$^{45}C_x = {}^{45}C_y$ then

$x = y$ OR $x + y = 45$

36 $^{15}C_{11} = \dfrac{15!}{11!\,(15-11)!} = \dfrac{15!}{11!\,4!} = 1365$ ----(1)

$^{15}C_4 = \dfrac{15!}{4!\,(15-4)!} = \dfrac{15!}{4!\,11!} = 1365$ ----(2) Pls. Remember

$\boxed{{}^nC_x = {}^nC_{n-x}}$

$^{15}C_{11} = {}^{15}C_{15-11} = {}^{15}C_4$

37 **In how many ways 52 cards can be equally divided in 4 groups?**

$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} = \left(\dfrac{52!}{13!\,13!\,13!\,13!}\right) = \dfrac{52!}{(13!)^4}$

38 **In how many different ways 10 mangoes can be divided among 3 people such that they will get 2,3,5 mangoes**

$= {}^{10}C_2 \times {}^8C_3 \times {}^5C_5$

$= 45 \times 56 \times 1 = 2520$ ways

$= \dfrac{10!}{2!\,3!\,5!} = \dfrac{3628800}{2 \times 6 \times 120}$

$= 2,520$ ways

39 $\boxed{\dfrac{^nP_r}{^nC_r} = r!}$ $\boxed{\dfrac{^nC_r}{^nP_r} = \dfrac{1}{r!}}$

$^5P_r = \dfrac{5!}{(5-r)!}$

$^5C_r = \dfrac{5!}{(5-r)!\,r!}$

$\therefore \dfrac{^5P_r}{^5C_r} = \dfrac{5!/(5-r)!}{5!/(5-r)!\,r!} = \dfrac{\cancel{5!}}{\cancel{(5-r)!}} \times \dfrac{\cancel{(5-r)!}\,r!}{\cancel{5!}}$

$= r!$

40 P (8, 3) = $^8P_3 = 8 \times 7 \times 6 = 336$

C (12, 4) = $^{12}C_4 = \left(\dfrac{12 \times 11 \times 10 \times 9}{4!}\right) = 495$

$\underline{|12} = 479001600$

41 (1) $\dfrac{^{18}P_3 \times {}^{17}C_2}{^{19}P_2 \times {}^{18}C_2} = \dfrac{18 \times 17 \times 16 \times 17 \times 16 \times 2!}{2! \times 19 \times 18 \times 18 \times 17} = \left(\dfrac{2176}{171}\right)$

(2) $\dfrac{^{20}P_4 \times {}^{25}P_5 \times {}^{20}C_2}{^{22}C_4 \times {}^{21}C_3 \times {}^{26}C_5} = \dfrac{20 \times 19 \times 18 \times 17 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 \times 24 \times 6 \times 120}{2 \times 22 \times 21 \times 20 \times 19 \times 21 \times 20 \times 19 \times 26 \times 25 \times 24 \times 23 \times 22}$

$= \left(\dfrac{3 \times 17 \times 12 \times 3 \times 120}{7 \times 13 \times 11}\right) = \left(\dfrac{220320}{1001}\right)$

42 $\dfrac{^{20}P_3 \times {}^{21}P_4 \times {}^{22}C_4}{^{23}C_3 \times {}^{22}P_3 \times {}^{21}P_2} =$

$= \dfrac{20 \times 19 \times 18 \times 21 \times 20 \times 19 \times 18 \times 22 \times 21 \times 20 \times 19 \times 6}{24 \times 23 \times 22 \times 21 \times 22 \times 21 \times 20 \times 21 \times 20}$

$= \left(18{,}51{,}930/1771\right)$

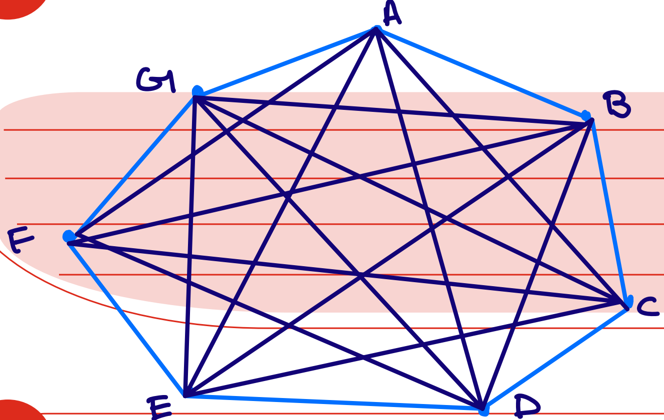
43 In a party of x people if everyone hand shakes with other. How many handshakes will take place

$\Longrightarrow \quad {}^{x}C_2$

44 How many diagonal can be drawn in a polygon having : 7 sides $= {}^7C_2 - 7 = 14$

8 sides $= {}^8C_2 - 8 = 20$

10 sides $= {}^{10}C_2 - 10 = 35$



$= \left(\text{No. of sides \& diagonals}\right) - \left(\text{No. of sides}\right)$

$= {}^7C_2 - 7 = 14$ diagonals can be drawn in a Heptagon.

45 In a group of 100 people, if everyone sends a greeting to other, How many cards will be used in total?

$\Longrightarrow \quad {}^{100}P_2 = 9900$ cards

OR

$99 + 99 + 99 + \cdots\cdots$ 100 people $= 9900$ cards will be used

46 In a plane of 20 non-collinear points

If a straight line can pass through all the points then points are said to be collinear

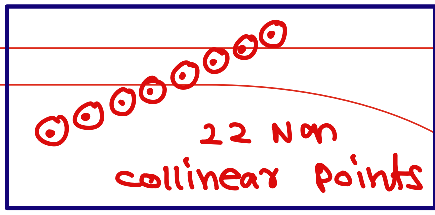
How many different straight lines can be drawn?

$= {}^{20}C_2$

How many different triangles can be obtained?

$= {}^{20}C_3$

47 In a plane there are 30 points out of which 8 are collinear



How many different straight lines can be drawn?

$= ({}^{22}C_2) + ({}^{8}C_1 \times {}^{22}C_1) + 1$

$= 231 + 176 + 1 = 408$

OR $\;{}^{30}C_2 - {}^{8}C_2 + 1 = 408$

How many different triangles can be obtained?

$= {}^{22}C_3 + ({}^{22}C_1 \times {}^{8}C_2) + ({}^{22}C_2 \times {}^{8}C_1)$

$= 1540 + 616 + 1848 = 4004$ triangles

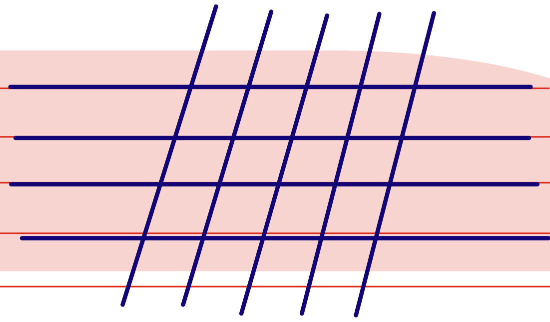
OR $\;{}^{30}C_3 - {}^{8}C_3 = 4004$ diff. triangles

48 There are 4 parallel lines intersecting with another set of 5 parallel lines. How many parallelograms can be drawn?

$= {}^{4}C_2 \times {}^{5}C_2$

$= 6 \times 10$

$= 60$ diff. parallelograms



49 8 Red; 3 Pink; 6 White Balls -

How many different selections of 3 balls are possible with

All Red balls

$= {}^{8}C_3 \times {}^{9}C_0$

$= 56$ diff ways

2 Red balls

$= {}^{8}C_2 \times {}^{9}C_1$

$= 252$ diff ways

Atleast 2 white balls

$= ({}^{6}C_2 \times {}^{11}C_1) + ({}^{6}C_3 \times {}^{11}C_0)$

$= 165 + 20$

$= 185$ diff ways

No pink balls

$= ({}^{3}C_0 \times {}^{14}C_3)$

$= 364$ diff ways

50 4 CA's; 3 Engineers; 8 Doctors -

How many ways a commitee of 4 members cane be formed with

Atleast 1 doctor

$= \left({}^8C_1 \times {}^7C_3\right) + \left({}^8C_2 \times {}^7C_2\right) + \left({}^8C_3 \times {}^7C_1\right) + \left({}^8C_4 \times {}^7C_0\right)$

$= 280 + 588 + 392 + 70$

$= 1330$ ways

OR

$= {}^{15}C_4 - \left({}^8C_0 \times {}^7C_4\right)$

$= 1365 - 35 = 1{,}330$ ways

Atleast 1 person of each profession

$= \left({}^4C_1 \times {}^3C_1 \times {}^8C_2\right) + \left({}^4C_1 \times {}^3C_2 \times {}^8C_1\right) + \left({}^4C_2 \times {}^3C_1 \times {}^8C_1\right)$

$= 336 + 96 + 144$

$= 576$ ways

51 There are 8 males & 11 females. In how many ways a committee of 5 members can be formed with

No restriction

$= {}^{19}C_5$

$= 11{,}628$ ways

Atleast 4 Females

$= \left({}^{11}C_4 \times {}^8C_1\right) + \left({}^{11}C_5 \times {}^8C_0\right)$

$= 2640 + 462$

$= 3102$ ways

Atmost 1 Female

$= \left({}^{11}C_1 \times {}^8C_4\right) + \left({}^{11}C_0 \times {}^8C_5\right)$

$= 770 + 56$

$= 826$ ways

3 Females

$= \left({}^{11}C_3 \times {}^8C_2\right)$

$= 4620$ ways

52 ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

a. True　　b. False

Let's check with numbers : $n = 5$, $r = 2$

L.H.S. $= {}^5P_2 = 20$

R.H.S. $= {}^4P_2 + 2 \times {}^4P_1 = 12 + (2 \times 4) = 12 + 8 = 20$

53 A supreme court bench consist of 7 judges. In how many ways majority decision can be taken?

$= {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7$

$= 35 + 21 + 7 + 1 = 64$ ways majority can be formed.

54 A question paper has 8 questions. In how many ways atleast one question can be solved?

$= (2^8) - 1 = 255$ ways

OR $\quad {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + \cdots + {}^8C_8 = 2^8 - 1 = 255$

55 A question paper has 8 questions (each one has alternatives). In how many ways one or more questions can be solved?

$= (3)^8 - 1$

$= 6560$ ways

56 No. of ways in which 9 things can be divided in 3 groups containing 2,3,4 things respectively.

$\Longrightarrow$

$= {}^9C_2 \times {}^7C_3 \times {}^4C_4 \qquad = \dfrac{9!}{2!\,3!\,4!} = 1260$ ways

$= 1260$ ways

57 Number of odd numbers greater than 500 can be formed by using 3, 1, 2, 8

3 digit : $\underline{{}^1P_1} \times \underline{{}^2P_1} \times \underline{{}^2P_1} = 4$

4 digit : $\underbrace{\underline{\quad}\ \underline{\quad}\ \underline{\quad}}_{}$ ${}^3P_3 \times \underline{{}^2P_1} = 12$

$\overline{\underline{16 \text{ Numbers}}}$

58 $\dfrac{{}^nP_r}{{}^{n-1}P_{r-1}} =$

$$= \frac{n!/(n-r)!}{(n-1)!/(n-\cancel{1}-r+\cancel{1})!} = \frac{n!}{\cancel{(n-r)!}} \times \frac{\cancel{(n-r)!}}{(n-1)!} = \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$= n$

59 A man has 12 friends in how many ways he can invite ________ for dinner

2 of them

$= {}^{12}C_2$

$= 66$ ways

Atleast 10 of them

$= {}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12}$

$= 66 + 12 + 1$

$= 79$ ways

5 of them

$= {}^{12}C_5$

$= 792$ ways

Atleast one of them

$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_4 + \dots\dots + {}^{12}C_{12}$

$= 2^{12} - 1$

$= 4096 - 1$

$= 4095$ ways

Atmost 10 of them

$= {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + \dots\dots + {}^{12}C_{10}$

$= 2^{12} - {}^{12}C_{11} - {}^{12}C_{12}$

$= 4096 - 12 - 1$

$= 4083$ ways

5 OR 9 of them : ${}^{12}C_5 + {}^{12}C_9 = 792 + 220$

$= 1012$ ways

60 In a paper there are 2 sections A, B containing 5, 8 questions respectively. In how many ways total 5 questions can be solved with atmost 3 questions of any one of the section.

→ Section A – 5 questions   section B – 8 questions

${}^5C_2 \times {}^8C_3 = 560$

OR ${}^5C_3 \times {}^8C_2 = 280$

$\underline{840 \text{ ways}}$

61 $\dfrac{{}^xP_2 \cdot {}^xP_3}{{}^xP_4 \cdot {}^xP_1} = \dfrac{\cancel{x}(x-1)\,\cancel{x}(x-\cancel{1})(x-\cancel{2})}{\cancel{x}(x-\cancel{1})(x-\cancel{2})(x-3)\,\cancel{x}}$

$= \left(\dfrac{x-1}{x-3}\right)$

62 $\dfrac{{}^{10}P_3 \times {}^2P_1}{{}^{11}P_4} = \dfrac{\cancel{10} \times \cancel{9} \times \cancel{8} \times 2}{11 \times \cancel{10} \times \cancel{9} \times \cancel{8}} = \left(2/11\right)$

$\dfrac{{}^{18}P_5}{{}^{19}C_5} = \dfrac{\cancel{18} \times \cancel{17} \times \cancel{16} \times \cancel{15} \times 14 \times 120}{19 \times \cancel{18} \times \cancel{17} \times \cancel{16} \times \cancel{15}}$

$= (1680/19)$

$\dfrac{{}^xP_3 \times {}^{(x-1)}P_4}{{}^{(x+1)}P_6 \times {}^{(x-3)}P_1} = \dfrac{\cancel{x}(x-\cancel{1})(x-\cancel{2})(x-1)(x-2)(x-\cancel{3})(x-\cancel{4})}{(x+1)\,\cancel{x}\,(x-\cancel{1})(x-\cancel{2})(x-\cancel{3})(x-\cancel{4})(x-3)}$

$= \boxed{\dfrac{(x-1)(x-2)}{(x+1)(x-3)}}$

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63 $\frac{{}^{18}P_2 \times {}^{20}P_3}{{}^{21}P_3 \times {}^{19}P_3} = \frac{\cancel{18}^{\,6} \times \cancel{17} \times \cancel{20} \times \cancel{19} \times \cancel{18}}{\cancel{21}_{\,7} \times \cancel{20} \times \cancel{19} \times 19 \times \cancel{18} \times \cancel{17}} = \left(6/133\right)$

$\frac{16P_2 \times 18C_3}{18C_5} = \frac{16 \times \cancel{15} \times \cancel{18} \times \cancel{17} \times \cancel{16} \times \cancel{120}^{\,\cancel{20}\,10}}{\cancel{6} \times \cancel{18} \times \cancel{17} \times \cancel{16} \times \cancel{15} \times \cancel{14}_{\,7}} = \left(\frac{160}{7}\right)$

64 **In how many ways 10 students can be arranged in a line if 4 of them want to be always together?**

$\Longrightarrow$

$= (10-4+1)! \times$ Internal arrangements

$= 7! \times 4! = 5040 \times 24 = 1,20,960$ ways

65 **There are 9 students, in how many ways they can stand in a line if 2 of them want to be never together?**

= (All possible arrangements of 9 students − arrangements where 2 of them are always together)

$= 9! - (8!\,2!)$

$= 362880 - 80640$

$= 2,82,240$ ways

OR

$= 7! \times 8P_2$

$= 2,82,240$ ways

66 **In how many ways letters of word 'DAUGHTER' can be arranged if all vowels should always be together?**

$\Longrightarrow$ A, U, E D, G, H, T, R

$= 6! \times$ Internal arrangements $= 6! \times 3! = 4320$ ways

67 **In how many ways letters of word 'CALCULATOR' can be arranged if all consonants should always be together?**

$\Longrightarrow$

C, L, C, L, T, R A, U, A, O

$= \frac{5!}{2!} \times$ Internal arrangements

$= \frac{5!}{2!} \times \frac{6!}{2!\,2!}$

$= 10,800$ diff words

68 How many 3 digit numbers can be formed by using 1,2,3,4,5 if repetition of digits is

Allowed

$\underline{5P_1} \times \underline{5P_1} \times \underline{5P_1}$

$= 125$ Numbers

Not allowed

$\underline{5P_1} \times \underline{4P_1} \times \underline{3P_1}$

$= 60$ Numbers

Please Note

If question is silent about whether repetition of digits is allowed or not then REPETITION OF DIGITS IS NOT ALLOWED

69 How many 4 digit numbers greater than 5000 can be formed by using 3,5,8,2,1 if repetition of digits is

Not allowed

$\underline{2P_1} \times \underbrace{\_\_\ \_\_\ \_\_}_{4P_3}$

$= 48$ Numbers

Allowed

$\underline{2P_1} \times \underline{5P_1} \times \underline{5P_1} \times \underline{5P_1}$

$= 250$ Numbers

70 How many numbers greater than 8000 can be formed by using 1,2,7,8,9 if repetition of digits is

Not allowed

4 digit : $\underline{2P_1} \times \underbrace{\_\_\ \_\_\ \_\_}_{4P_3} = 48$

5 digit : $\underbrace{\_\_\ \_\_\ \_\_\ \_\_\ \_\_}_{5P_5} = 120$

$\overline{168}$

Allowed

4 digit : $\underline{2P_1} \times \underline{5P_1} \times \underline{5P_1} \times \underline{5P_1} = 250$

5 digit : $\underline{5P_1} \times \underline{5P_1} \times \underline{5P_1} \times \underline{5P_1} \times \underline{5P_1} = 3125$

$\overline{3{,}375}$

Note : we have restricted the answer to 5 digit Number.

71 How many 5 digit numbers divisible by 5 can be formed by using 0,2,3,4,5,8,9, if repetition of digits is

Not allowed

Ending with 5

$\underline{5P_1} \times \underbrace{\_\_\ \_\_\ \_\_}_{5P_3} \ \underline{1P_1} = 300$

Ending with 0

$\underbrace{\_\_\ \_\_\ \_\_\ \_\_}_{6P_4} \times \underline{1P_1} = 360$

$\overline{660 \text{ Numbers}}$

Allowed

$\underline{6P_1} \times \underline{7P_1} \times \underline{7P_1} \times \underline{7P_1} \times \underline{2P_1}$

$= 4116$ Numbers

My Notes

How many 4 digit numbers greater than 4000 and divisible by 5 can be formed with 0,2,3,4,5,8 ?

Ending with 0 $\quad \underline{3P_1} \times \underline{4P_2} \ \_\_ \times \underline{1P_1} = 36$

Ending with 5 $\quad \underline{2P_1} \times \underline{4P} \ \_\_ \times \underline{1P_1} = 24$

$\overline{60 \text{ Numbers}}$

OR

start with 4, 8 $\quad \underline{2P_1} \times \underline{4P_2} \times \underline{2P_1} = 48$

start with 5 $\quad \underline{1P_1} \times \underline{4P_2} \times \underline{1P_1} = 12$

$\overline{60}$

72 How many 5 digit numbers greater than 34,000 can be formed by using 3,1,2,7,8,0

$\Longrightarrow$ Starting with 3 : $^1P_1 \times {}^2P_1 \times {}^4P_3 = 48$

Starting with 7,8 : $^2P_1 \times {}^5P_4 = 240$

288 Numbers

73 In how many ways 5 sisters & 6 brothers can stand in a line for a photograph if no 2 sisters or no 2 brothers should stand together?

B S B S B S B S B S B

$\Longrightarrow$ $^6P_6 \times {}^5P_5 = 86,400$ diff photographs

74 How many 2 digit numbers can be formed with atleast one digit as 7?

Starting with 7

$^1P_1 \times {}^{10}P_1 = 10$

Ending with 7

$^8P_1 \times {}^1P_1 = 8$

18 Numbers

OR

17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 87, 97

75 In how many ways 11 players out of 16 players can be selected if -

There is No restriction

$= {}^{16}C_{11}$

$= 4368$ diff selections

2 Particular players must be included?

$= {}^{14}C_9 \times {}^2C_2$

$= 2002$ diff selections

3 Particular players must be excluded?

$= {}^{13}C_{11} \times {}^3C_0$

$= 78$ diff selections

2 Particular players must be excluded & 4 particular players must be included?

$= {}^2C_0 \times {}^4C_4 \times {}^{10}C_7$

$= 120$ diff selections

In How many 5 Brother & 5 sisters can form a line for a photograph If

(a) All Brother should be together

$= 6! \times 5!$

$= 86,400$

(b) All sisters should be together

$= 6! \times 5!$

$= 86,400$

(c) All Brothers & All sisters should be together

$= 2! \times 5! \times 5!$

$= 28,800$

My Notes

(d) No 2 brother OR 2 sisters should be together

$= (5! \times 5!) + (5! \times 5!)$

$= 28,800$

76 $\dfrac{{}^{20}P_2 \times {}^{21}C_3}{{}^{18}C_5 \times {}^{21}P_2} = ?$

~~a.~~ $\dfrac{1805}{12852}$ b. $\dfrac{1826}{18562}$ c. $\dfrac{1528}{17882}$ d. None of these

$$= \frac{\cancel{20} \times 19 \times \cancel{21} \times \cancel{20} \times 19 \times \cancel{120}}{\cancel{6} \times 18 \times 17 \times \cancel{16} \times \cancel{15} \times 14 \times \cancel{21} \times \cancel{20}} = \frac{5 \times 19 \times 19}{18 \times 17 \times 3 \times 14}$$

$$= (1805/12852)$$

77 There are 8 men and 7 women, in how many ways a committee of 4 members can be formed :

Without any restriction

$= {}^{15}C_4$

$= 1365$ ways

With 2 Men

$= {}^8C_2 \times {}^7C_2$

$= 28 \times 21$

$= 588$ ways

With Atleast 3 Men

$= ({}^8C_3 \times {}^7C_1) + ({}^8C_4 \times {}^7C_0)$

$= 392 + 70$

$= 462$ ways

With Atmost 1 Woman

$= ({}^7C_1 \times {}^8C_3) + ({}^7C_0 \times {}^8C_4)$

$= 392 + 70$

$= 462$

78 8 Red, 3 White, 4 Pink Balls - How many different selections of 4 balls are possible with atleast one ball of each colour?

$\Longrightarrow ({}^8C_1 \times {}^3C_1 \times {}^4C_2) + ({}^8C_1 \times {}^3C_2 \times {}^4C_1) + ({}^8C_2 \times {}^3C_1 \times {}^4C_1)$

$= 144 + 96 + 336$

$= 576$ diff selections

79 There are 'm' points in a plane out of which 'k' are collinear

How many different straight lines can be drawn by joining them?

$= ({}^mC_2 - {}^kC_2 + 1)$

How many different triangles can be obtained by joining them?

$= ({}^mC_3 - {}^kC_3)$

My Notes

There are 50 points in a plane out of which 18 are collinear. How many _____ can be formed by joining them?

Lines

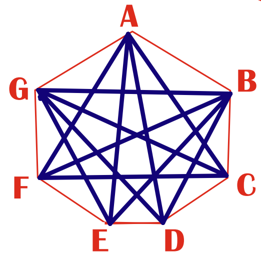
$= ({}^{50}C_2 - {}^{18}C_2 + 1)$

$= 1073$

Triangles

$= ({}^{50}C_3 - {}^{18}C_3) = 18{,}784$

80 How many diagonals can be drawn in a Heptagon?
(Heptagon = A polygon having 7 sides)



No. of diagonals that can be drawn in a heptagon

= (No. of sides & diagonals) – No. of sides

$= {}^7C_2 - 7$

$= 21 - 7 = 14$ diagonals

81 A man has 13 friends. In how many ways he can invite ____________ for dinner

Atleast one of them

$= {}^{13}C_1 + {}^{13}C_2 + \text{--------} + {}^{13}C_{13}$

$= 2^{13} - 1$

$= 8192 - 1$

$= 8191$ ways

4 of them

$= {}^{13}C_4$

$= 715$ ways

4 or 7 of them

$= {}^{13}C_4 + {}^{13}C_7$

$= 715 + 1716$

$= 2431$ ways

Atmost 11 of them

$= {}^{13}C_0 + {}^{13}C_1 + {}^{13}C_2 + \cdots\cdots + {}^{13}C_{11}$

$= 2^{13} - \left({}^{13}C_{12} + {}^{13}C_{13}\right)$

$= 8192 - (13+1)$

$= 8178$ ways

82 There are 4 papers in an exam. in how many ways student can pass the exam if passing in all papers is compulsory to pass the exam?

~~a.~~ 1 b. 15 c. 16 d. None of these

$\Longrightarrow$ PPPP $= {}^4C_4$

83 There are 5 papers in an exam. in how many ways a student can pass the exam if student passes the exam if he passes in atleast one paper?

a. 32 ~~b.~~ 31 c. 1 d. None of these

$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= 2^5 - 1 = 32 - 1 = 31$

My Notes

How many 4 digit even numbers greater than 4000 can be formed with 0, 2, 5, 3, 6, 8, 4

$\Longrightarrow$

starting with 4, 6, 8 : $\underline{{}^3P_1} \times \underline{\quad} \; \underline{{}^5P_2} \; \underline{\quad} \times \underline{{}^4P_1} = 240$

starting with 5 : $\underline{{}^1P_1} \times \underline{\quad} \; \underline{{}^5P_2} \; \underline{\quad} \times \underline{{}^5P_1} = 100$

$\overline{340 \text{ Numbers}}$

84 There are 5 multiple choice questions with 4 options each. How many different sequences of answer are possible?

$\Longrightarrow \quad {}^4P_1 \times {}^4P_1 \times {}^4P_1 \times {}^4P_1 \times {}^4P_1$

$= 1024$ diff sequences

85 There are 6 multiple choice questions. First 4 questions have 4 options each and last two questions have 5 options each. How many different sequences of answer are possible?

$\Longrightarrow \quad {}^4P_1 \times {}^4P_1 \times {}^4P_1 \times {}^4P_1 \times {}^5P_1 \times {}^5P_1$

$= 6400$ diff sequences

86 There are 8 males and 5 females. In how many ways a committee of 5 members can be formed so that males are in majority?

$\Longrightarrow \quad ({}^8C_3 \times {}^5C_2) + ({}^8C_4 \times {}^5C_1) + ({}^8C_5 \times {}^5C_0)$

$= 560 + 350 + 56 = 966$ ways

87 ① No. of arrangements of 'n' different things taken 'r' at a time in which a particular thing

Alway there

$= {}^rP_1 \times {}^{n-1}P_{(r-1)}$

Never there

$= {}^{(n-1)}P_r$

② No. of selections of 'n' different things taken 'r' at a time in which a particular thing is

always there

$= {}^{n-1}C_{r-1} \times {}^1C_1$

Never there

$= {}^{(n-1)}C_r \times {}^1C_0$

My Notes

How many 5 digit odd numbers greater than 50,000 can be formed with 0, 1, 2, 3, 4, 6, 8, 9 if repetition of digits is allowed?

$\Longrightarrow \quad {}^3P_1 \times {}^8P_1 \times {}^8P_1 \times {}^8P_1 \times {}^3P_1$

$= 4608$ Numbers

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88 No. of selections of 'n' different things taken 'r' at a time in which a particular thing

Is alway there

$= {}^{(n-1)}C_{(r-1)} \times {}^{1}C_{1}$

$= {}^{(n-1)}C_{(r-1)}$

Is never there

$= {}^{n-1}C_{r} \times {}^{1}C_{0}$

$= {}^{(n-1)}C_{r}$

89 Find sum of all 4 digit numbers formed by using 1,3,5,7

~~a.~~ 1,06,656 b. 1,78,252 c. 1,78,282 d. None

$= \frac{24}{4}(1111 + 3333 + 5555 + 7777)$

$= 1,06,656$

1357	3157	5137	7135
1375	3175	5173	7153
1537	3517	5317	7315
1573	3571	5371	7351
1735	3715	5713	7513
1753	3751	5731	7531

90 ${}^{n}P_{r}$ can also be written as :

a. $\frac{n!}{r!}$ ~~b.~~ $\frac{\underline{|n}}{\underline{|(n-r)}}$ c. $\frac{\underline{|n}}{\underline{|r}\,\underline{|(n-r)}}$ d. None

${}^{n}P_{r} = \frac{n!}{(n-r)!}$, ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ where $n \geq r \geq 0$

n = positive integer

91 There are 6 Books on Eco, 3 on Maths, 2 on stats. In how many ways all books can be placed on a shelf if books on same subject are to be always together?

a. 1,06,656 b. 1,78,252 c. 1,78,282 ~~d.~~ None

6 Eco. 3 maths 2 stat

$= 3! \times \left(\text{Internal arrangements}\right) = 3! \times (6! \times 3! \times 2!) = 51,840$ ways

My Notes

In How many ways letters of word 'NECESSITY' can be arranged if all the consonants & all the vowels should be together?

$\Longrightarrow$ NCSSTY, EEI

$= 2! \times \frac{6!}{2!} \times \frac{3!}{2!}$

$= 2160$ diff words

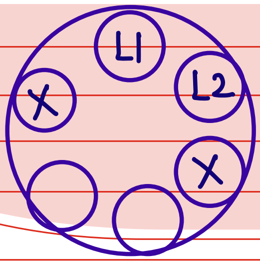
92 The number of ways in which 7 girls can form a ring is :

a. 700 b. 710 ~~c.~~ 720 d. 360

$\Longrightarrow \dfrac{7!}{7} = 6! = 720$ ways

93 3 Ladies and 3 gents are to be seated on a round table so that 2 and only 2 ladies should sit together. The number of arrangements are :

a. 70 b. 27 ~~c.~~ 72 d. None of these



$= \left(2! \times {}^2P_1 \times 3!\right) \times 3$ $\quad 3 = (L_1, L_2)$, (L_2, L_3), (L_1, L_3)

$= 72$ diff arrangements

94 In a group of boys the no. of arrangements of 4 boys is 12 times the number of arrangements of 2 boys. The no. of boys in the group is

a. 10 b. 8 ~~c.~~ 6 d. None of these

no. of Boys $= n$

${}^nP_4 = 12 \times {}^nP_2$

$\cancel{n}(\cancel{n-1})(n-2)(n-3) = 12 \times \cancel{n} \times (\cancel{n-1})$

$(n-2)(n-3) = 12$

$\boxed{n = 6}$

95 $\sum_{r=1}^{r=10} r \cdot {}^rP_r = ?$ $\quad \sum_{r=1}^{r=10} r \cdot {}^rP_r$

a. ${}^{11}P_{11}$ ~~b.~~ ${}^{11}P_{11} - 1$ c. ${}^{11}P_{11} + 1$ d. None of these

$\Longrightarrow$

$= (1 \times {}^1P_1) + (2 \times 2!) + (3 \times 3!) + \text{--------} + (10 \times 10!)$

$= 39916799$

${}^{11}P_{11} - 1 = 39916800 - 1$

$= 39916799$

My Notes

In How many ways 20 objects can be Equally divided among 4 people ?

$\Longrightarrow {}^{20}C_5 \times {}^{15}C_5 \times {}^{10}C_5 \times {}^5C_5$ $\quad$ OR $\quad \dfrac{20!}{5!\,5!\,5!\,5!}$

$= 15504 \times 3003 \times 252 \times 1$ $\quad = 11732745024$

$= 11732745024$

96 There are 10 trains plying between Latur and Pune, The no. of ways in which a man can go from Latur to Pune and return by different train is

a. 99 ~~b.~~ 90 c. 80 d. 100

$\Longrightarrow$ $^{10}C_1 \times {^9C_1}$

$= 90$

97 The number of ways in which six '+' and four '-' signs can be arranged in a line such that no '-' signs occur together is

a. 7!/3! b. (6! x 7!) / 3! ~~c.~~ 35 d. None of these

0 + 0 + 0 + 0 + 0 + 0 + 0

$= \frac{6! \times {^7P_4}}{6!\,4!} = 35$

98 The number of ways in which letters of word 'MOBILE' be arranged so that consonants always occupy the odd places is :

~~a.~~ 36 b. 63 c. 30 d. None of these

$\Longrightarrow$ $\underline{1^{st}}$ ___ $\underline{3^{rd}}$ ___ $\underline{5^{th}}$ ___

M, B, L $= 3! \times 3! = 36$ diff words

99 5 persons are sitting along a round table in such a way that tallest person is always to the immediate right of shortest person. The no. of such arrangements are :

~~a.~~ 6 b. 8 c. 24 d. None of these

$\Longrightarrow$ $= 3! = 6$ ways

100 In how many ways 17 balls can be arranged in a line if 7 of them are Black, 6 are Red and 4 are White?

$\Longrightarrow$ $\frac{17!}{7!\,6!\,4!} = 4{,}084{,}080$

My Notes

How many words can be formed by using B, B, B, B, B, B, K, K, K, K if No 2 K's should occur together?

$\Longrightarrow$ $\frac{6! \times {^7P_4}}{6!\,4!} = 35$ diff words

101 The number of different words that can be formed with 12 consonants and 5 vowels by taking 4 consonants and 3 vowels in each word are

a. $^{12}C_4 \times {}^5C_3$ b. $^{17}C_7$ ~~c.~~ 4950 x 7! d. None of these

$= \left({}^{12}C_4 \times {}^5C_3\right) \times 7!$

102 How many different words can be formed by using all letters of word 'ASSASSINATION' if all vowels should be together?

$\Longrightarrow$ A A I A I O SSSSNTN

$= \left(\frac{8!}{4!\,2!} \times \frac{6!}{3!\,2!}\right) = 50{,}400$ diff words.

103 How many numbers greater than a million can be formed with the digits 0,4,4,5,5,5,3?

a. 420 ~~b.~~ 360 c. 7! d. None of these

$\Longrightarrow \frac{{}^6P_1 \times {}^6P_6}{2!\,3!} = 360$

104 $4 \times {}^nP_3 = 5 \times {}^{(n-1)}P_3$; then value of 'n' is

a. 12 b. 13 c. 14 ~~d.~~ 15

$4 \times n \times \cancel{(n-1)}\cancel{(n-2)} = 5 \times \cancel{(n-1)}\cancel{(n-2)}(n-3)$

$4n = 5n - 15$

$15 = n$

105 The number of ways in which 8 examination papers can be arranged so that best and worst paper never come together are :

~~a.~~ 8! - 2!7! b. 8! - 7! c. 8! d. None of these

$\Longrightarrow$ [All possible arrangements − arrangements where Best & worst are always together]

$= 8! - (7!\,2!)$

$= 8! - 2!\,7!$

My Notes

In How many ways 7 Brothers & 11 sisters can form a line so that All brothers are together?

$\Longrightarrow 12! \times 7!$

$= 12 \times 11 \times 36288 \times 100 \times 504 \times 10$

$= 2414168064000$

106 How many 6 digit numbers can be formed out of 4,5,6,7,8,9 if no digit being repeated?

a. 6!-5! ~~b~~. 6! c. 6!+5! d. None of these

107 There are 50 stations on a railway line, How many different kinds of tickets to be printed to enable a passenger to travel from one station to another station?

a. 2500 ~~b~~. 2450 c. 2400 d. None of these

o o o o o o o ..... 50 stations

$\Longrightarrow$ $^{50}P_2 = 2450$ diff. tickets

108 In ^nP_r, ^nC_r; n is always

~~a~~. positive integer b. an integer c. zero d. None of these

109 If all permutations of word 'CHALK' are written in a dictionary sequence. the rank of word 'CHALK' is

a. 30 b. 31 ~~c~~. 32 d. None of these

ACHKL
ACHLK
ACKHL
⋮

$\boxed{\text{CA}}$HKL : 25th
$\boxed{\text{CA}}$: 26th
⋮ 6 words

31st : CHAKL
32nd : CHALK

110 How many words can be formed by using letter A thrice, letter B twice. letter C once.

~~a~~. 60 b. 120 c. 90 d. 6

A A A B B C $= \dfrac{6!}{3!\,2!\,1!} = 60$ diff words

My Notes

If all permutations of word 'CUT' are written in dictionary sequence. what is rank of word 'TCU'

$\Longrightarrow$ CTU, CUT, $\boxed{\text{TCU}}$ ← 3rd

111 If different permutations of the word 'EXAMINATION' are listed in a dictionary, How many items are there in the list before the first word starting with E?

a. 9,06,200 ~~b.~~ 9,07,200 c. 9,08,200 d. 9,05,200

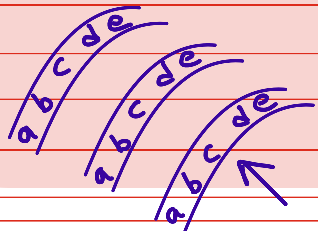
No. of words starting with 'A' $= \dfrac{10!}{2!\,2!} = 9,07,200$ words

EXMINATION

112 A letter lock consist of 3 rings marked with 5 different letters. Number of maximum unsuccessful attempts to open the lock is :

~~a.~~ 124 b. 125 c. 120 d. 75

$\Longrightarrow$



Total No. of passwords possible $= {}^5P_1 \times {}^5P_1 \times {}^5P_1 = 125$

Max. No. of unsuccessful attempts $= 125 - 1 = 124$

113 The number of 5 letter words that can be formed using letters of word 'DELHI' which begin and end with vowel, when repetitions are allowed is

a. 125 b. 625 ~~c.~~ 500 d. 1350

$\underline{{}^2P_1} \times \underline{{}^5P_1} \times \underline{{}^5P_1} \times \underline{{}^5P_1} \times \underline{{}^2P_1} = 500$ diff words

114 The number of ways in which 20 persons be seated along a round table if there are 7 seats is :

a. ${}^{20}P_7$ b. ${}^{20}P_7 / 7!$ c. ${}^{20}C_7$ ~~d.~~ ${}^{20}P_7 / 7$

$\Longrightarrow \left[\dfrac{{}^{20}P_7}{7}\right]$

115 ${}^nP_r = 120 \times {}^nC_r$, then n = ?

~~a.~~ 5 b. 120 c. 24 d. 4

${}^nP_r = 120 \times {}^nC_r$

$\dfrac{{}^nP_r}{{}^nC_r} = 120$

$r! = 5!$

$r = 5$

My Notes

If ${}^{45}C_n = {}^{45}C_{20}$ then ${}^nC_2 = ?$

(a) 190 (b) 300 ~~(c)~~ a or b (d) None of these

$\Longrightarrow {}^{45}C_n = {}^{45}C_{20} \therefore n = 20, n = 25$ ${}^{20}C_2$ or ${}^{25}C_2$

$= 190$ or 300

116 In how many ways letters of the word 'BALLOON' be arranged so that 2 L's do not come together is :

(LL) B A O O N

~~a.~~ 900 b. 1200 c. 800 d. 600

= All possible arrangements − Arrangements where 2 L's are always together

$= \left(\frac{7!}{2!\,2!}\right) - \left(\frac{6!}{2!} \times \frac{2!}{2!}\right) = 1260 - 360 = 900$ words

117 $^{15}C_{11} / {}^{15}C_{10} = ?$

a. 15/11 b. 15/10 c. 5/10 ~~d.~~ None of these

$\Longrightarrow \frac{^{15}C_{11}}{^{15}C_{10}} = \left[\frac{\frac{15!}{11!\,4!}}{\frac{15!}{10!\,5!}}\right] = \frac{\cancel{15!}}{11 \times \cancel{10!} \times \cancel{4!}} \times \frac{\cancel{10!} \times 5 \times \cancel{4!}}{\cancel{15!}} = \left(\frac{5}{11}\right)$

118 How many even numbers greater than 300 can be formed with digits 1,2,3,4,5. No repetition being allowed

a. 112 ~~b.~~ 111 c. 113 d. 121

$\Longrightarrow$

3 digit : Ending with 2 : $\underline{^3P_1} \times \underline{^3P_1} \times \underline{^1P_1} = 9$

Ending with 4 : $\underline{^2P_1} \times \underline{^3P_1} \times \underline{^1P_1} = 6$

4 digit : $\underline{^4P_3} \times \underline{^2P_1} = 48$

5 digit : $\underline{^4P_4} \times \underline{^2P_1} = 48$

$\overline{\quad 111 \quad}$

119 $^{43}C_{(r-6)} = {}^{43}C_{(3r+1)}$, then r = ?

~~a.~~ 12 b. 8 c. 6 d. 10

$r - 6 + 3r + 1 = 43$

$4r - 5 = 43$

$4r = 48$

$r = 12$

OR

$r - 6 = 3r + 1$

$-5 = 2r$

This is impossible

My Notes

If $^nC_x = {}^nC_y$ then

Either $x = y$ OR $x + y = n$

120 A committee of 3 ladies and 4 gents to be formed out of 8 ladies and 7 gents. Mrs. X refuses to serve in a committee if Mr. Y is there. Number of such committees are :

a. 1530 b. 1500 c. 1520 d. 1540

$\Longrightarrow \left({}^8C_3 \times {}^7C_4\right) - \left({}^1C_1 \times {}^1C_1 \times {}^7C_2 \times {}^6C_3\right)$

$= 1960 - 420 = 1540$ ways

121 What is rank of word 'TALK' if all words by using letters of word are arranged in a dictionary sequence ?

~~a. 20~~ b. 18 c. 19 d. None of these

$\Longrightarrow$ AKLT – 6 words starting with A
– 6 words starting with K
– 6 words starting with L

19th TAKL 20th : TALK

122 How many 3 digit odd numbers can be formed by using 1,3,5, if repetition of digits is allowed?

~~a. 3^3~~ b. 3! c. (3x3x4) d. None of these

$\Longrightarrow \underline{{}^3P_1} \times \underline{{}^3P} \times \underline{{}^3P_1} = 3^3$

123 ${}^{56}P_{(r+6)} : {}^{54}P_{(r+3)} = 30{,}800 : 1$; then r = ?

a. 42 ~~b. 41~~ c. 45 d. None of these

$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = 30{,}800$ | $\frac{\frac{56!}{(56-r-6)!}}{\frac{54!}{(54-r-3)!}} = 30{,}800$ | $\frac{56 \times 55 \times \cancel{54!}}{\cancel{(50-r)!}} \times \frac{(51-r)(50-r)\cancel{!}}{\cancel{54!}} = 30{,}800$

$(51-r) = 10$ $\boxed{r = 41}$

124 There are 6 questions in section A and 7 in section B. In how many ways 8 questions can be attempted with atmost 6 questions from any section are :

a. 360 ~~b. 1281~~ c. 6 d. 42

A – 6 questions		B – 7 questions		
${}^6C_2$	×	${}^7C_6$	=	105
${}^6C_3$	×	${}^7C_5$	=	420
${}^6C_4$	×	${}^7C_4$	=	525
${}^6C_5$	×	${}^7C_3$	=	210
${}^6C_6$	×	${}^7C_2$	=	21
				1281

My Notes

125 How many words can be formed by using all letters of word 'REDDY'

a. 120 ~~b.~~ 60 c. 240 d. None of these

$\Longrightarrow$ $= \frac{5!}{2!} = 60$ diff words

My Notes

Sets, Functions, Relations

CA VINOD REDDY

1 Set is a collection of welldefined and Distinct objects

Roster Form / Braces Form / List form

A = {1,2,3,4,5,6,7,8,9,10}

Algebric Form / Rule Form / Property Form / Set Builder Form / Description form

A is a set of first 10 natural numbers

OR

A = {x : where x ∈ N and x ≤ 10}

2 In mathematics everything in this world whether living or non-living, is called as an object oR Elements

3 A = {5,8,9,10,13} Explain : ∈

$5 \in A$: 5 belongs to set A

$10 \in A$: 10 belongs to set A

$200 \notin A$: 200 does not belong to set A.

$13 \in A$: 13 belongs to set A

$m \notin A$: 'm' does not belong to set A

B = {Accounts, maths, Law, Law, Law, maths, Accounts, Eco, Eco, Eco, Eco, Eco}

4 No. of distinct elements of a set is known as cardinal value $n(B) = 4$

5 Types of sets on the basis of elements

- Null set oR Empty set oR void set $= \phi$ oR { } → cardinal value is 'zero'
- singleton set → cardinal value is '1'
- Finite set → cardinal value is finite / Limited / countable
- Infinite set → cardinal value is infinite / unlimited / uncountable

6 Generally name of the set is denoted by capital letters.

Order of object is Not Relevant.

Repetition is of no use

A = {1,2,3,4,5} B = {5,4,4,5,1,2,2,3,4,5,4}

sets A and B are name sets

7 Equivalent sets : $A = \{5, 8, 9, 13\}$ $B = \{VR, P, m, 38\}$

Here $n(A) = 4$ & $n(B) = 4$

$\therefore$ A, B are Equivalent sets

If cardinal value of 2 OR more sets is same then sets are said to be Equivalent sets.

8 Subset :

$A = \{2, 3, 5, 8\}$ $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Here Every obs^n of set A belongs to set B also

$\therefore$ A is a subset of B OR B is a superset of A.

9 Superset : $A = \{a, b, c\}$, $B = \{a, b, c, d, e\}$, $D = \{a, b, c\}$

A is a subset of B : A is a proper subset of B.

D is a subset of B : D is a proper subset of B

A is a subset of D : A is an improper subset of D

10 Proper Subset : $A = \{5, 10\}$ $B = \{5, 10, 18\}$

A is a proper subset of B $\Rightarrow A \subset B$

11 Improper Subset :

$X = \{10, 18, 20\}$, $Y = \{18, 20, 10\}$

X is a subset but not a proper subset of Y

i.e. X is improper subset of Y.

My Notes

Find All possible subsets of $A = \{10, 20, 30\}$

$\Longrightarrow$

$\{10\}, \{20\}, \{30\}, \{10, 20\}, \{10, 30\}, \{20, 30\}, \phi$ — proper subsets

$\{10, 20, 30\}$ — Improper subset

12 Find all possible subsets of A = {5,7,8}

proper subsets : $\{5\}, \{7\}, \{8\}, \{7,5\}, \{5,8\}, \{7,8\}, \phi$

Improper subset : $\{5,7,8\}$

13 For set B = {a,b,c}

All possible subsets : $2^3 = 8$

All possible proper subsets : $2^3 - 1 = 7$

All possible improper subsets : 1

All possible empty subsets : 1

All possible non-empty subsets : $2^3 - 1 = 7$

All possible Non-empty proper subsets $= 2^3 - 2 = 6$

Find all subsets of $\{p\}$

$\Rightarrow \{p\}, \phi$

14 If cardinal value of a set = n; then

No. of subsets : 2^n

No. of proper subsets : $(2^n) - 1$

No. of improper subsets : 1

No. of empty subsets : 1

No. of non-empty subsets : $(2^n) - 1$

No. of non-empty proper subsets : $(2^n) - 2$

Find all subsets of $\{p, q\}$

$\Rightarrow \{p\}, \{q\}, \phi, \{p,q\}$

Null set is a subset of any other set

15 When 2 sets are said to be equivalent sets? Equal sets ?

① $A = \{2,3,4,8,9\}$ $B = \{2,3,4,8,9\}$

Here A is a subset of B & B is a subset of A

$\therefore$ A, B are said to be Equal sets

My Notes

As A, B are Equal sets, they are Equivalent

also as $n(A) = n(B) = 5$

② $M = \{2, 15, 100, p, m\}$, $N = \{a, b, p, m, q\}$

As $n(M) = n(N) = 5$

$\therefore$ M, N are Equivalent sets.

All Equal sets are Equivalent also but All Equivalent sets are not necessarily Equal sets

16 When 2 or more sets are said to be equal sets?

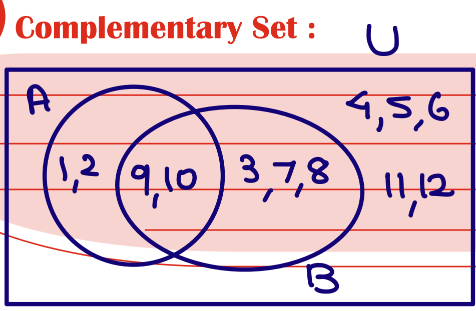
$\Longrightarrow$ If $A \subseteq B$ & $B \subseteq A$ then A, B are said to be Equal sets.

∴ All equal sets are equivalent but all equivalent sets are not necessarily equal sets.

17 Universal Set :

- A set of all observations under the scope of study is known as universal set.
- universal set is the superset of any other set
- universal set is denoted by 'U' or 'S'
- It is denoted as 'Rectangle' in venn diagram.

18 Complementary Set :



$A = \{1,2,9,10\}$

$A' = A^c$ = complementary set of A
$= \{3,7,8,4,5,6,11,12\}$

$\therefore n(A) + n(A') = n(U)$

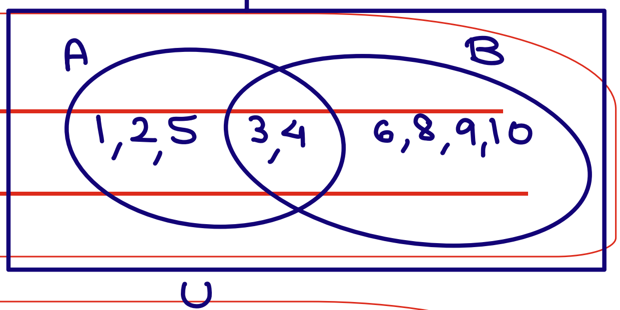
$B = \{9,10,3,7,8\}$

$B' = \{1,2,4,5,6,11,12\}$

19 If A = {1,2,3,4,5} B = {3,4,6,8,9,10}

Find (A union B) = $\{1,2,5,3,4,6,8,9,10\} = (A \cup B)$

Find (A intersection B) = $\{3,4\} = (A \cap B)$



union = OR   Intersection = and

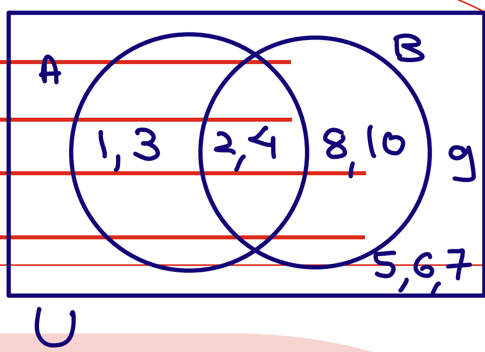
20 If A = {1,2,3,4} B = {2,4,8,10} U = {1,2,3,4,5,6,7,8,9,10}

Find A' = $\{5,6,7,8,9,10\}$

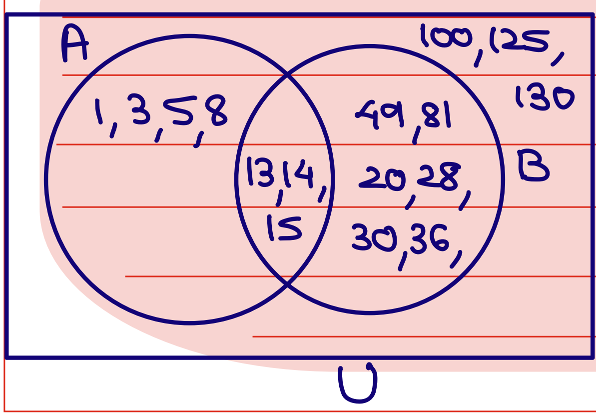
B' = $\{1,3,5,6,7,9\}$

(A ∪ B) = $\{1,2,3,4,8,10\}$

(A ∩B) = $\{2,4\}$



My Notes

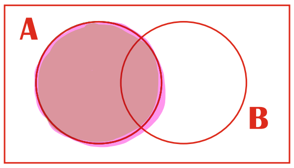


$n(A) = 7,\ n(B) = 9,\ n(A \cap B) = 3$

$n(A \cup B) = 13$

$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

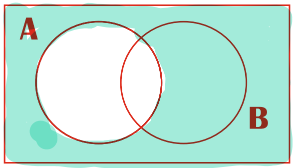
21 Find Set A



$n(A) = n(U) - n(A')$

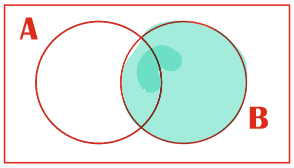
$n(A) + n(A') = n(U)$

22 Find Set A'



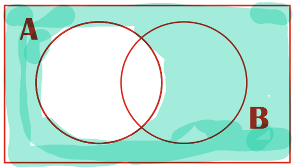
$n(A') = n(U) - n(A)$

23 Find Set B



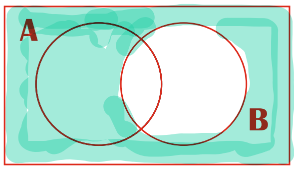
$n(B) = n(U) - n(B')$

24 Find Set A'



$n(A') = n(U) - n(A)$

25 Find Set B'

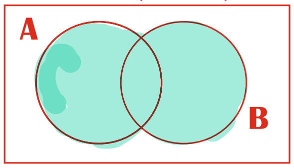


$n(B') = n(U) - n(B)$

My Notes

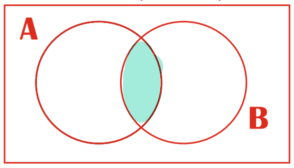
1. $n(A) + n(A') = n(U)$
2. $n(B') = n(U) - n(B)$
3. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
4. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
5. $n(A - B) = n(A \cap B') = n(A) - n(A \cap B)$
6. $n(B - A) = n(B \cap A') = n(B) - n(A \cap B)$
7. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
8. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

26 Find Set (A ∪ B)



$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

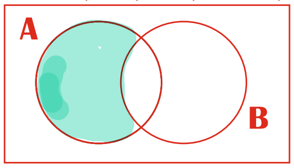
27 Find Set (A ∩ B)



$n(A\cap B) = n(A) + n(B) - n(A\cup B)$

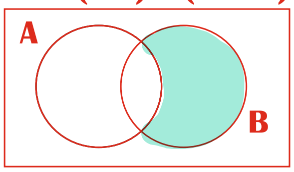
$n(A\cap B) = n(A) - n(A-B)$

28 Find (A-B) = (A ∩ B')



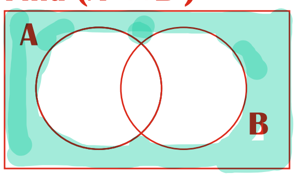
$n(A-B) = n(A\cap B') = n(A) - n(A\cap B)$

29 Find (B-A) = (B ∩ A')



$n(B-A) = n(B\cap A') = n(B) - n(A\cap B)$

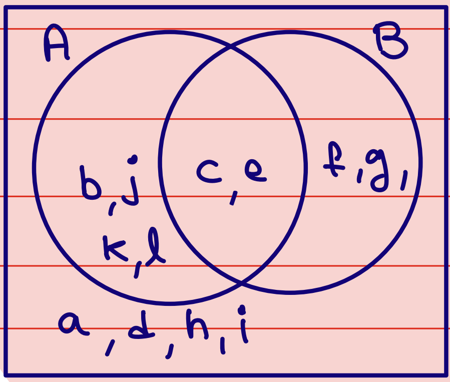
30 Find (A' ∩ B')



$n(A'\cap B') = n(A\cup B)'$
$= n(U) - n(A\cup B)$

De-morgan's rule on sets

My Notes



U = universal set

① $A = \{b, j, k, l, c, e\}$ ② $B = \{c, e, f, g\}$

③ $A' = \{f, g, a, d, h, i\}$ ④ $B' = \{b, j, k, l, a, d, h, i\}$

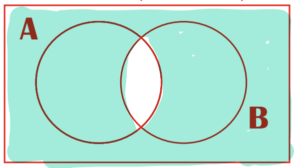
⑤ $A\cap B = \{c, e\}$ ⑥ $(A\cup B) = \{b, j, k, l, c, e, f, g\}$

⑦ $A - B = \{b, j, k, l\}$ ⑧ $B - A = \{f, g\}$

⑨ $(A'\cap B') = \{a, d, h, i\} = (A\cup B)'$

⑩ $A\Delta B = \{b, j, k, l, f, g\}$

31 Find Set (A' ∪ B')

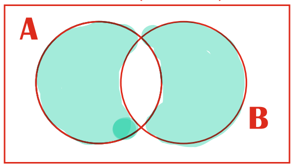


$n(A' \cup B') = n(A \cap B)'$

$= n(U) - n(A \cap B)$

$\boxed{(A' \cup B') = (A \cap B)'}$ De morgan's Rule on sets

32 Find Set (A △ B)

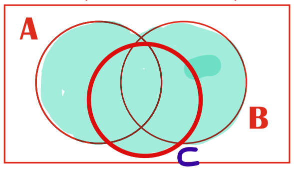


$(A \Delta B) = [(A-B) \cup (B-A)] = [(A \cap B') \cup (B \cap A')]$

$n(A \Delta B) = n(A-B) + n(B-A)$

$= n(A \cup B) - n(A \cap B)$

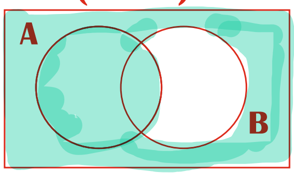
33 Find (A ∪ B ∪ C)



$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

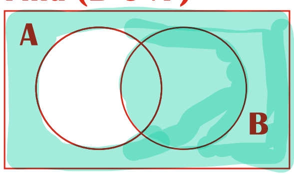
34 Find (A ∪ B')



$n(A \cup B') = n(U) - n(B - A)$

$= n(A) + n(A' \cap B')$

35 Find (B ∪ A')

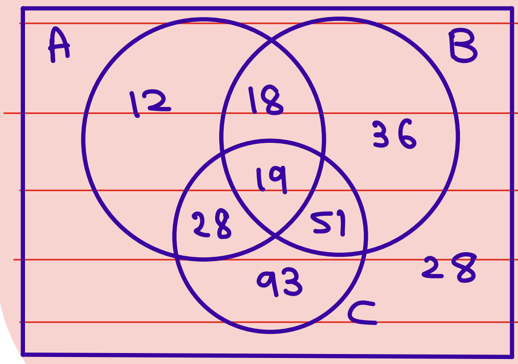


$n(B \cup A') = n(U) - n(A-B)$

$= n(B) + n(A' \cap B')$

$n(A \cup B \cup C) = n(U) - n(A' \cap B' \cap C')$

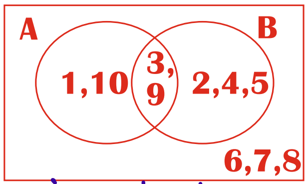
My Notes



U = universal set

$n(A) = 77$	$n(A \cap B \cap C) = 19$
$n(B) = 124$	$n(U) = 285$
$n(C) = 191$	$n(A \cup B \cup C) = 285 - 28 = 257$
$n(A \cap B) = 37$	$= n(A) + n(B) + n(C) - n(A \cap B)$
$n(B \cap C) = 70$	$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$n(A \cap C) = 47$	$= 77 + 124 + 191 - 37 - 70 - 47 + 19$
	$= 257$

36



U = universal set

Find A = { 1, 10, 3, 9 }

Find B = { 3, 9, 2, 4, 5 }

Find A' = { 2, 4, 5, 6, 7, 8 }

Find B' = { 1, 10, 6, 7, 8 }

Find A ∪ B = {1, 10, 3, 9, 2, 4, 5}

Find A ∩ B = {3, 9}

Find A - B = {1, 10}

Find B - A = {2, 4, 5}

Find A ∪ B' = {1, 10, 3, 9, 6, 7, 8}

Find A' ∩ B' = {6, 7, 8}

Find A' ∪ B' = {1, 10, 2, 4, 5, 6, 7, 8}

Find B ∪ A' = {3, 9, 2, 4, 5, 6, 7, 8}

37 Formulae of sets at one place

n(A') = n(U) − n(A)

n(B') = n(U) − n(B)

n(A∪B) = n(A) + n(B) − n(A∩B)

n(A∩B) = n(A) + n(B) − n(A∪B)

n(A - B) = n(A) − n(A∩B)

n(B - A) = n(B) − n(A∩B)

n(A'∩B') = n(A∪B)' = n(U) − n(A∪B)

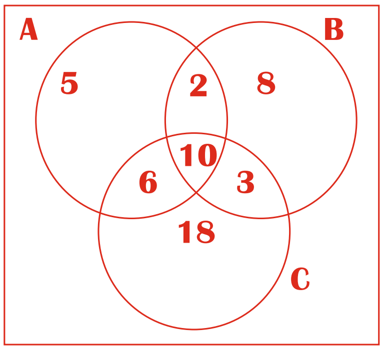
n(A △ B) = n(A−B) + n(B−A) = n(A∪B) − n(A∩B)

n(A'∪ B') = n(A∩B)' = n(U) − n(A∩B)

n(A ∪ B') = n(B−A)' = n(U) − n(B−A)

n(B ∪ A') = n(A−B)' = n(U) − n(A−B)

38



n(A ∪ B ∪ C) = 52

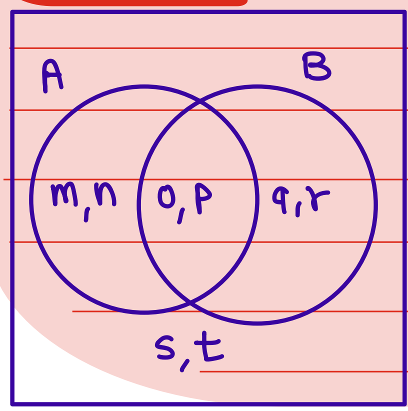
n(A∪B∪C) =

n(A) + n(B) + n(C) - n (A∩ B) - n(A∩C) - n(B∩C) + n(A∩B∩C) =

= 23 + 23 + 37 − 12 − 16 − 13 + 10

= 52

My Notes



A = {m, n, o, p}

B = {o, p, q, r}

A' = {q, r, s, t}

B' = {m, n, s, t}

A ∩ B = {o, p}

A ∪ B = {m, n, o, p, q, r}

A − B = {m, n}

B − A = {q, r}

A Δ B = {m, n, q, r}

(A' ∩ B') = {s, t}

(A' ∪ B') = {m, n, q, r, s, t}

cartesian product of sets

39 If A = {1,2,3} B = {8,9}

Find (A x B) = $\{(1,8), (1,9), (2,8), (2,9), (3,8), (3,9)\}$

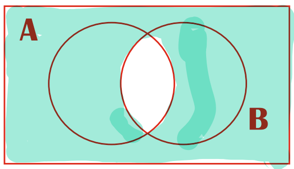
Find (B x A) = $\{(8,1), (8,2), (8,3), (9,1), (9,2), (9,3)\}$

$(A\times B) \neq (B\times A)$
but
$n(A\times B) = n(B\times A) = 6$
$(A\times B)$ & $(B\times A)$ are Equivalent sets but not equal sets.

40 A is a subset of B : Notation : $A \subseteq B$

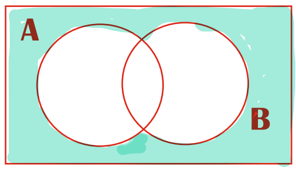
A is a proper subset of B : Notation : $A \subset B$

41 Demorgan's Rules of Sets



$(A' \cup B') = (A \cap B)'$

$n(A' \cup B') = n(U) - n(A \cap B)$



$(A \cup B)' = (A' \cap B')$

$n(A' \cap B') = n(U) - n(A \cup B)$

My Notes

If $n(A) = 537$, $n(B) = 1081$, $n(A \cap B) = 238$, $n(U) = 1980$

Find

$\Longrightarrow$ ① $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 1380$

② $n(A \cap B') = n(A) - n(A \cap B) = 537 - 238 = 299$

③ $n(B \cap A') = n(B) - n(A \cap B) = 1081 - 238 = 843$

④ $n(A' \cap B') = n(U) - n(A \cup B) = 1980 - 1380 = 600$

⑤ $n(A \Delta B) = n(A \cap B') + n(B \cap A') = 299 + 843 = 1142$

⑥ $n(A' \cup B') = n(U) - n(A \cap B) = 1980 - 238 = 1742$

42

$A \cup A = A$

$A \cap A = A$

$A \cup \phi = A$

$A \cap \phi = \phi$

$A \cup A' = U$

$A \cap A' = \phi$

$A \cup U = U$

$\phi' = U$

$A \cup (A \cup B) = (A \cup B)$

$(A \cup B) \cup (A \cap B) = (A \cup B)$

$(A \cup B) \cup (A' \cap B') = U$

$A \cup (A \triangle B) = (A \cup B)$

$A \cup (A \cap B') = A$

$(A \cap B') \cup (A \cap B) = A$

$(A \triangle B) \cup (A \cap B) = (A \cup B)$

$U' = \phi$

43

Any subject of the product set X.Y is said to define a relation from X to Y, and any relation from X to Y in which no 2 different ordered pairs have the same first element is called as function.

In f : A $\longrightarrow$ B

the element f(x) of B is called as image of x while x is called as pre-image of f(x).

44

There are 4 types of relations

1. one to one
2. one to many
3. many to one
4. many to many

out of these 4 relations only

i) one to one
ii) many to one

$\Big\}$ Relations are functions

- Every function is a Relation but every Relation is not necessarily a function.

45

If $f(x) = 3x^2+2x+1$

Find f(3), f(8), f(-9), f(10)

$\Longrightarrow f(x) = 3x^2 + 2x + 1$

$f(3) = 3(3)^2 + 2(3) + 1 = 34$

$f(-9) = 3(-9)^2 + 2(-9) + 1 = 226$

$f(10) = 3(10)^2 + 2(10) + 1 = 321$

& $f(8) = 3(8)^2 + 2(8) + 1$
$= 209$

When y is a function of x
then y = Dependent variable
x = Independent variable

46

If $f(x) = 8x+11$; $g(x) = 2x+9$

Find $f(3) = 8(3) + 11 = 35$

$g(8) = 2(8) + 9 = 25$

$g(p) = 2p + 9$

$g(y) = 2y + 9$

$f(-13) = 8(-13) + 11 = -93$

$f(20) = 8(20) + 11 = 171$

$g(2k) = 2(2k) + 9 = 4k + 9$

$f[g(10)] = f[2(10)+9]$
$= f(29)$

$f \cdot g(10) = 8(29) + 11 = 243$

$g \cdot f(3) = g[8(3)+11] = g(35)$
$= 2(35) + 9 = 79$

Function of Function is known as : composite Function

47 If f(x) = 10x+15; g(x) = 7x - 13 **Find f.g(x), g.f(x)**

$\Longrightarrow$ ① $f.g(x) = f(7x-13) = 10(7x-13)+15$

$= 70x - 130 + 15 = 70x - 115$

② $g.f(x) = g(10x+15) = 7(10x+15) - 13$

$= 70x + 105 - 13 = 70x + 92$

48 If f(x) = 2x+11 Find $f^{-1}(y)$, $f^{-1}(x)$, $f^{-1}(p)$

$\Longrightarrow$ $y = f(x) = 2x + 11$

$y - 11 = 2x$

$x = \left(\frac{y-11}{2}\right)$

$f^{-1}(y) = \frac{y-11}{2}$

$f^{-1}(x) = \left(\frac{x-11}{2}\right)$

$f^{-1}(p) = \left(\frac{p-11}{2}\right)$

If Demand (y) is a function of price (x) then price is the Inverse Function of demand

49 If $f(x) = \frac{2x+13}{8x-2}$; Find $f^{-1}(y)$, $f^{-1}(20)$, $f^{-1}(p)$, $f^{-1}(p+1)$

$\Longrightarrow$

$f^{-1}(20) = \frac{2(20)+13}{8(20)-2} = \frac{53}{158}$

$f^{-1}(p) = \left(\frac{2p+13}{8p-2}\right)$

$f^{-1}(p+1) = \frac{2(p+1)+13}{8(p+1)-2} = \left[\frac{2p+15}{8p+6}\right]$

$y = \frac{2x+13}{8x-2} = f(x)$

$8xy - 2y = 2x + 13$

$8xy - 2x = 2y + 13$

$x(8y-2) = 2y + 13$

$x = \left(\frac{2y+13}{8y-2}\right)$

$f^{-1}(y) = \left(\frac{2y+13}{8y-2}\right)$

If $y = f(x)$ then $x = f^{-1}(y)$

50 If $f(x) = \frac{1}{1-x}$; Find f(10), f(2), f(13), f(p), $f^{-1}(10)$, $f^{-1}(p)$

$\Longrightarrow$ $y = f(x) = \frac{1}{1-x}$

$y - xy = 1$

$y - 1 = xy$

$\therefore x = \frac{y-1}{y}$

$f^{-1}(y) = \frac{y-1}{y}$

$f(10) = 1/-9$
$f(2) = -1$
$f(13) = 1/-12$
$f(p) = 1/(1-p)$
$f^{-1}(10) = 9/10$
$f^{-1}(p) = (p-1)/p$

51 If $g(x) = \frac{x-1}{x}$; Find g(-1/2), $g^{-1}(y)$

$g\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2} - 1}{-\frac{1}{2}} = \frac{-1-2}{-1} = 3$

$y = g(x) = \frac{x-1}{x}$

$xy = x - 1$

$xy - x = -1$

$x(y-1) = -1$

$x = \frac{-1}{y-1} = \left(\frac{1}{1-y}\right)$

$\therefore g^{-1}(y) = \frac{1}{1-y}$

52 If f(2x+3) = 8x + 7. Find f(x), f(30)

$\Longrightarrow$ $f(2x+3) = 8x + 7$

$f(2x+3) = 4(2x+3) - 12 + 7$

$f(2x+3) = 4(2x+3) - 5$

$f(p) = 4p - 5$

$\therefore f(x) = 4x - 5$

$f(30) = 4(30) - 5 = 115$

If $f(x) = 4x - 5$ Find $f(2x+3)$

$\Longrightarrow f(2x+3) = 4(2x+3) - 5 = 8x + 7$

53 **Domain and Range of {(1,5),(2,8),(3,9),(4,18)}**

$\Longrightarrow$ Domain = set of all first elements = $\{1, 2, 3, 4\}$

Range = set of all second elements = $\{5, 8, 9, 18\}$

54 **f(x-1) = x². Find f(x), f(x+1)**

$f(x-1) = x^2$

$f(x+1-1) = (x+1)^2$

$f(x) = (x+1)^2$

$f(x-1) = x^2$

$f(x-1) = (x-1)^2 + 2x - 1$

$f(x-1) = (x-1)^2 + 2(x-1) + 2 - 1$

$f(p) = p^2 + 2p + 1 = (p+1)^2$

$f(x) = (x+1)^2, \ f(x+1) = (x+2)^2$

55 **When a relation is said to be**

Symmetric If (a,b) is present in Relation then (b,a) should also be present

Reflexive (a,a)

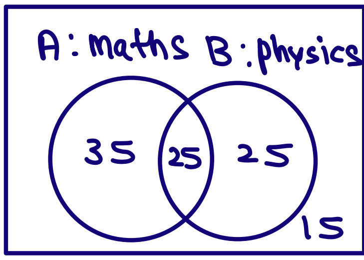
Transitive If (a,b), (b,c) ∈ R then (a,c) ∈ R

56 **Relation of Equivalence** If a Relation is Reflexive, symmetric, Transitive then it is said to be Relation of Equivalence.

57 **'Is perpendicular to' is a** symmetric Relation

58 **'Is the reciprocal of' is a** symmetric Relation

59 **In a class of 100 students 60 like maths 50 like physics 25 like both subjects. Find how many students :**



a. Like maths or physics $= n(A \cup B) = 85$

b. Like maths but not physics $= n(A - B) = n(A \cap B') = 35$

c. Like physics but not maths $= n(B - A) = n(B \cap A') = 25$

d. Neither like maths nor like physics $= n(A' \cap B') = 15$

e. Not like atleast one of 2 subjects $= n(A' \cup B') = n(A \cap B)' = 75$

f. Like one and only one subject $= n(A \Delta B) = 60$

A, B are said to be
Equal sets If $A \subseteq B$ & $B \subseteq A$
Equivalent sets If $n(A) = n(B)$

60 $A = \{5,8,9,10\}$; $B = \{8,5,9,10\}$; $C = \{a,b,c,d\}$

A, B are Equal Sets; Therefore Equivalent Also.

A, C are Equivalent Sets; but not Equal sets.

B, C are Equivalent sets but not Equal sets.

61 Set of cubes of a natural numbers is ________ set

a. Finite   b. Infinite   c. Singleton   d. Null

set of cubes of natural numbers $= \{1^3, 2^3, 3^3, 4^3, \ldots\ldots\}$

62 $\{x : [(1) - (-1)^x]\}$ for all integer values of x then x =

a. {0}   b. {2}   c. {0,2}   d. None of these

$1 - (-1)^x$: $x = 1 \Rightarrow 1 - (-1)^1 = 2$
$x = 2 \Rightarrow 1 - (-1)^2 = 0$
$x = 3 \Rightarrow 1 - (-1)^3 = 2$
$x = 4 \Rightarrow 1 - (-1)^4 = 0$

$\{2, 0\}$

63 E is a set of all even natural numbers and O is a set of all odd natural numbers then

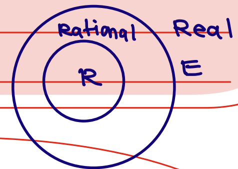
$(E \cup O) = \{1,2,3,4,5,6,7,8 \ldots\ldots\} = N$

$(E \cap O) = \phi = \{\ \}$

64 If R is a set of positive rational numbers and E is a set of all real numbers then

a. $R \subseteq E$ b. $R \subset E$ c. $E \subseteq R$ d. $E \subset R$

All Rational numbers are Real but all Real numbers are not necessarily rational.



$\therefore$ R is a proper subset of E.

65 If N is a set of all natural numbers and I is a set of positive integers then

a. N = I   b. $N \subset I$ c. $N \subseteq I$ d. $I \subseteq N$

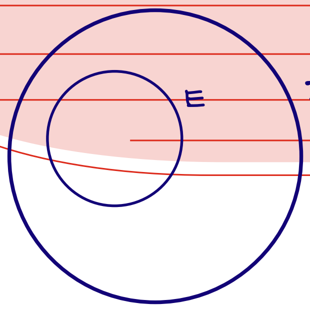
$N = \{1,2,3,4,5 \ldots\ldots\}$

$I = \{1,2,3,4,5, \ldots\ldots\}$

N, I are Equal sets

66 If I is a set of all isosceles triangles and E is a set of all equilateral triangles, then

a. $I \subset E$ b. $E \subset I$ c. E = I   d. None of these



All equilateral triangles are isosceles triangles but All isosceles triangles are not necessarily equilateral triangles.

67 {[n(n+1)/2] where n is a positive integer} is a ________________

a. Finite set ~~b.~~ Infinite set c. An empty set d. Singleton

68 If A = {1,2,3,4,5} B = {x^2 : x ∈ A} then -

a. n(A) > n(B) b. n(A) < n(B) ~~c.~~ n(A) = n(B) d. None

$A = \{1,2,3,4,5\}$

Here $n(A) = n(B) = 5$

$B = \{1,4,9,16,25\}$

69 Let f :A ⟶B then A is called as domain of f, while B is called as co-domain of f.
Then set f(A) = {f(x) : x ∈ A} is called as

Range of f

$A = \{1,5,9,12\}$ $B = \{1, 25, 81, 144, 169, 225\}$

$f : A \to B$ where $f(x) = x^2$ Domain = {1,5,9,12}

co-domain = {1,25,81,144, 169,225}

Range = {1,25,81,144}

70 Let A = {1,2,3,4,5} B = {1,4,9,16,25,36,49}, we consider the rule f(x) = x^2

then f (1) = 1
f (2) = 4
f (3) = 9
f (4) = 16
f (5) = 25

Clearly each element of A has unique image in B so

f : A→B : f(x) = x^2 is a function from A to B

where domain = {1,2,3,4,5}

Range = {1,4,9,16,25}

As in set A pre-image of 36,49 is not there it is 'INTO' function.

If each element of 'B' has atleast one pre-image in set A then function is said to be 'ONTO' function.

71 A one-one onto function is said to be bijective. A bijective function is also known as one to one correspondence.

Let f : A→B, defined in such a way that all elements in A have the same image in B, then f is said to be constant function

Two functions f and g are said to be equal written as f = g if they have the same domain and they satisfy the condition f(x) = g(x) for all values of x.

72 Inverse function is possible only when function is one to one onto

A = {Pune, chennai} B = {MH, Bihar, TN}

A × B = {(Pune, MH), (Pune, Bihar), (Pune, TN), (chennai, mH), (chennai, Bihar) (chennai, TN)}

Every subset of A × B is a relation

will form a relation

If $y = h(x)$
then $x = h^{-1}(y)$

73 Inverse $h^{-1}(x)$ when $h(x) = \log_{10}x$ is :

a. $\log_{10}x$ ~~b.~~ b. 10^x c. $\log_{10}(1/x)$ d. None of these

$\Longrightarrow$ $y = h(x) = \log_{10} x = y$ $x = 10^y$

$\therefore \; 10^y = x$ $h^{-1}(y) = 10^y$

$\therefore \; h^{-1}(x) = 10^x$

74 For the function $h(x) = 10^{(1+x)}$ the domain of real values of x where $0 \le x \le 9$, the range is -

~~a.~~ a. $10 \le h(x) \le 10^{10}$ b. $0 \le h(x) \le 10^{10}$ c. $0 \le h(x) \le 10$ d. None

$0 \leq x \leq 9$ $\therefore \; 10^1 \leq h(x) \leq 10^{10}$

$10^{1+0} \leq h(x) \leq 10^{1+9}$

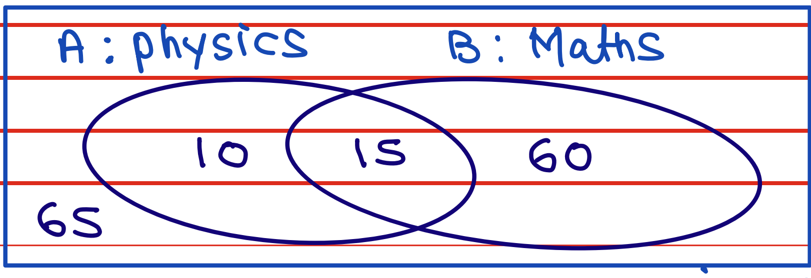
75 Let S = {a,b,c, ... } be any set then the relation R is a subset of the product set (S x S)

1. If R contains all ordered pairs (a,a) in (S x S) then R is said to be Reflexive
2. If (a,b) ∈ R, then (b,a) ∈ R. For every (a,b) ∈ S then R is said to be Symmetric
3. If (a,b) ∈ R, and (b,c) ∈ R; then (a,c) ∈ R. For every a,b,c ∈ S then R is said to be Transitive

A relation which is reflexive symmetric as well as transitive is called as Equivalence relation OR Relation of Equivalence

76 In a class of 150 students 25 like physics, 75 like maths. 135 students dislike atleast one subject then find no. of students

1. Who like physics but not maths : $n(A-B) = n(A \cap B') = 10$
2. Who like maths but not physics : $n(B-A) = n(B \cap A') = 60$
3. Who like both subjects : $n(A \cap B) = 15$
4. Who like neither maths nor physics : $n(A' \cap B') = 65$
5. Who like one and only one subject : $n(A \Delta B) = 10 + 60 = 70$



U = universal set

77 “Is smaller than” over the set of eggs in a box is :

~~a.~~ a. Transitive b. Symmetric c. Reflexive d. Equivalence

As, $\boxed{\text{If } a < b \;\&\; b < c} \Rightarrow \boxed{\text{then } a < c}$

78 A = {2,3,8,9,11} B = {3,10,13} C = {5,10,13,15,19} Find A x (B∩C)

$\Longrightarrow$ $A = \{2,3,8,9,11\}$ $B \cap C = \{10,13\}$

$A \times (B \cap C) = \{(2,10),(2,13),(3,10),(3,13),(8,10),(8,13),(9,10),(9,13),(11,10),(11,13)\}$

79 A = {2,8} B = {2,8} Find (AxB), (BxA), [(AxB) ∪ (BxA)], [(AxB)∩(BxA)]

$A \times B = \{(2,2),(2,8),(8,2),(8,8)\}$

$B \times A = \{(2,2),(2,8),(8,2),(8,8)\}$

$(A \times B) \cup (B \times A) = \{(2,2),(2,8),(8,2),(8,8)\}$

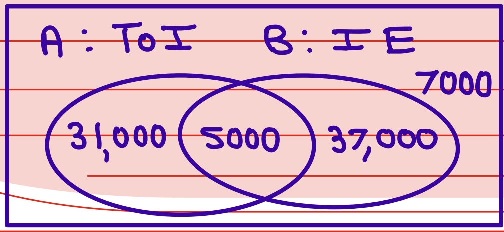
$(A \times B) \cap (B \times A) = \{(2,2),(2,8),(8,2),(8,8)\}$

If A, B are Equal sets then

$(A \times B) = (B \times A) = (A \times B) \cup (B \times A)$

$= (A \times B) \cap (B \times A)$

80 A town has total population of 80,000. Out of it 36,000 read TOI, 42,000 read IE, 5000 read both, then find no. of persons who read one and only one newspaper?



U = universal set

$n(A \Delta B)$

$= n(A-B) + n(B-A)$

$= 31{,}000 + 37{,}000 = 68{,}000$

81 If $f(x) = 1/(1-x)$ the $f^{-1}(x) = ?$

a. (1-x) ~~b.~~ b. (x-1)/x c. x/(x-1) d. None of these

$y = f(x) = \dfrac{1}{1-x}$

$y(1-x) = 1$

$y - xy = 1$

$y - 1 = xy$

$\therefore x = \left(\dfrac{y-1}{y}\right)$

$\therefore f^{-1}(y) = \left(\dfrac{y-1}{y}\right)$

$\therefore f^{-1}(x) = \left(\dfrac{x-1}{x}\right)$

82 Null set is represented by

a. {ϕ} or 0 ~~b.~~ b. { } or ϕ c. ϕ or {0} d. None of these

Null set is also known as Empty set OR void set which is denoted by : { } OR ϕ

My Notes

y = vinod Reddy's Income

x = No. of students in his Batches

m = No. of students pursuing CA course in India

k = Demand of CA's in India

l = Industrial & service sector Growth rate of India

$y = f(x)$

$y = f[g(m)]$

$y = f.g.h(k)$

$y = f.g.h.d(l)$

83 If $f(x) = x^2$ then it is

a. Odd function ~~b.~~ Even function c. Both of these d. None of these

$\Longrightarrow$ $f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

Here $f(x) = f(-x)$ then $f(x)$ is said to be an Even function

84 f(x) is said to be an

Odd Function if
$f(x) = -f(-x)$ OR $-f(x) = f(-x)$

Even Function if
$f(x) = f(-x)$

If $f(x) = x^3 + x^5$ then

$f(-x) = (-x)^3 + (-x)^5 = -x^3 - x^5 = -(x^3 + x^5) = -f(x)$

$f(-x) = -f(x)$ Here $f(x)$ is an odd function.

85 If $f(x) = \text{Log}\left(\frac{1+x}{1-x}\right)$ then function f is said to be

~~a.~~ Odd function b. Even function c. Both of these d. None of these

$f(x) = \text{Log}\left(\frac{1+x}{1-x}\right) = \text{Log}(1+x) - \text{Log}(1-x)$ ----1

$f(-x) = \text{Log}\left(\frac{1-x}{1+x}\right) = \text{Log}(1-x) - \text{Log}(1+x) = -\left[\text{Log}(1+x) - \text{Log}(1-x)\right]$

$f(-x) = -f(x)$

86 If $f(x) = 8x + 4$ the $f^{-1}(x) = ?$

a. $1/(8x+4)$ ~~b.~~ $(x-4)/8$ c. $(8x+4)/(4-8x)$ d. None of these

$y = f(x) = 8x + 4$

$\therefore x = \frac{y-4}{8} = f^{-1}(y)$

$f^{-1}(x) = \left(\frac{x-4}{8}\right)$

87 If $h(x) = \left(\frac{px-q}{qx-p}\right)$ then $x = ?$

a. $h(1/y)$ b. $h(-y)$ ~~c.~~ $h(y)$ d. None of these

$\Longrightarrow$ $y = h(x) = \frac{px-q}{qx-p}$

$qxy - py = px - q$

$qxy - px = py - q$

$x(qy - p) = py - q$

$x = \left(\frac{py-q}{qy-p}\right)$ 1

$h(x) = \left(\frac{px-q}{qx-p}\right)$

$h(y) = \left(\frac{py-q}{qy-p}\right)$ ----2

As per eqns 1 & 2

$x = h(y)$

My Notes

88 A set of intelligent students in a class is ____________

a. Null set

b. Singleton set

c. An infinite set

~~d. Not a well defined collection~~ (As 'intelligence' is a relative concept)

89 If $f(x+1) = f(x-1)$ where $f(x) = x^2 - 2x + 3$ then $x = ?$

~~a. 1~~ b. 2 c. 3 d. None of these

$f(x+1) = f(x-1)$

$(x+1)^2 - 2(x+1) + \cancel{3} = (x-1)^2 - 2(x-1) + \cancel{3}$

$\cancel{x^2} + 2x + \cancel{1} - \cancel{2x} - 2 = \cancel{x^2} - 2x + \cancel{1} - \cancel{2x} + 2$

$\therefore 4x = 4$

$x = 1$

90 If $f(x+1) = f(x+2)$ where $f(x) = 1 + x - x^2$ then $x = ?$

a. 2 b. 0 c. 1 ~~d. -1~~

$f(x+1) = f(x+2)$

$\cancel{1} + \cancel{x} + 1 - (x+1)^2 = \cancel{1} + \cancel{x} + 2 - (x+2)^2$

$\cancel{1} - \cancel{x^2} - 2x - \cancel{1} = 2 - \cancel{x^2} - 4x - 4$

$-2x = -4x - 2$

$2x = -2$

$\boxed{x = -1}$

91 If $f(x) = 3x + 4$ then $f[(x-4)/3] = ?$

a. 1 ~~b. x~~ c. zero d. None of these

$\Longrightarrow \quad f\left(\frac{x-4}{3}\right) = \cancel{3} \times \frac{(x-4)}{\cancel{3}} + 4$

$= x - 4 + 4 = x$

92 If $f(x+1) = 4x + 5$; find $f(x)$

a. 3x+4 ~~b. 4x+1~~ c. 4x+3 d. None of these

$f(x+1) = 4x + 5$

$f(x-1+1) = 4(x-1) + 5 = 4x - 4 + 5 = 4x + 1$

$f(x) = 4x + 1$

93 If $f(x-1) = x^3$; find $f(x)$

~~a. $(x+1)^3$~~ b. $(x+1)^2$ c. x^3 d. $(x-1)^3$

$f(x-1) = x^3$

$f(x+1-1) = (x+1)^3 \quad \therefore f(x) = (x+1)^3$

94 $f(x) = 3x + 5$; $g(x) = 6x + 100$. Find $g[f(2x)] = ?$

a. 16x + 200 b. 9x - 300 c. f(x) ~~d. None of these~~

$\Longrightarrow \quad g \cdot f(2x) = g\left[3(2x) + 5\right] = g(6x+5)$

$= 6(6x+5) + 100$

$= 36x + 130$

95 If S = {0,1,5,4,7,9,10} then $\therefore n(S) = 7$

No. of subsets = $2^7 = 128$

No. of proper subsets = $2^7 - 1 = 127$

No. of non empty subsets = $2^7 - 1 = 127$

No. of non empty proper subsets = $2^7 - 2 = 126$

96 If $A \subseteq B$ then

a. $A' \subseteq B'$ b. $A' = B'$ ~~c.~~ $B' \subseteq A'$ d. None of these

$A = \{1,2,3\}$

$B = \{1,2,3,4,5\}$

$U = \{1,2,3,4,5,6,7,8\}$

$A' = \{4,5,6,7,8\}$

$B' = \{6,7,8\}$

$\boxed{\text{When } A \subseteq B \text{ then } B' \subseteq A'}$

97 If 'A' is any set then

a. $A \cup A' = \phi$ ~~b.~~ $A \cap A' = \phi$ c. $A \cup \phi = A'$ d. None

98 $f(x-1) = 2x-2$ then $f(16)$ is

a. 16 b. 15 ~~c.~~ 32 d. Insufficient information

$f(x-1) = 2x-2$

$f(x-1) = 2(x-1)$

$f(p) = 2p$

$\therefore f(16) = 2 \times 16$

$= 32$

99 If A = {1,2,3,5,7}, B = {1,3,6,10,15} and universal set = U = {1,2,3,4,5,.......,15} then cardinal value of

$(A \cap B) = 2$

$(A \cup B) = 8$

$(A - B) = 3$

$(B - A) = 3$

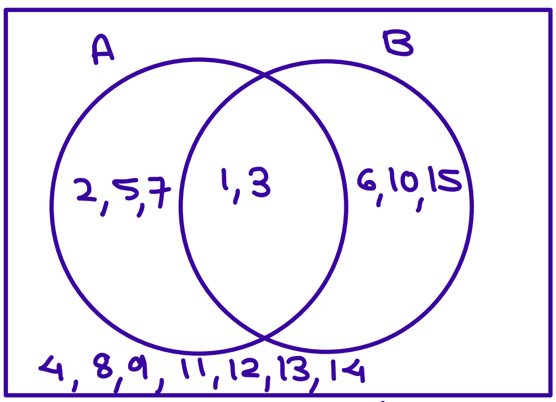
$(A' \cap B') = 7$

$(A \triangle B) = 6$

$(A \cup B') = 12$

$(B \cup A') = 12$

$(A' \cup B') = 13$



U = universal set

$n(U) = 15$

100 Null set don't have a proper subset

~~a.~~ True b. False

Null set has only one subset i.e. Null set only. which is an improper subset.

101 Find All subsets of A = {5,8,9,12}

$\{5\}, \{8\}, \{9\}, \{12\}, \{5,8\}, \{5,9\}, \{5,12\}, \{8,9\}, \{8,12\}, \{9,12\}, \{5,8,9\}, \{5,8,12\}, \{5,9,12\}, \{8,9,12\}, \{\ \}$ — proper subsets

$\{5,8,9,12\}$ — Improper subset

102 Find power set of A if A = {2,8,9}

power set of A $= \{\{2\}, \{8\}, \{9\}, \{2,8\}, \{2,9\}, \{8,9\}, \{\ \}, \{2,8,9\}\}$

Set of all possible subsets is known as power set

103 If universal set U = {1,2,3,4,5,..........,25}; A= {2,6,8,10,12,......,24} B = {4,8,10,14} then

~~a.~~ (A ∩ B)' = (A' ∪ B') b. (A ∩ B)' = A' ∩ B' c. A' ∩ B' = A' d. (A' ∪ B') = A'

104 P set has 3 elements, Q set has 4 elements then the set (PxQ) contains __________ elements

a. 34 b. 7 c. 1 ~~d.~~ 12

$n(P \times Q) = n(P) \times n(Q)$
$= 3 \times 4 = 12$

105 If f(x) = 2^x then function is

~~a.~~ one-one b. one-many c. many-one d. many-many

106 If f(x) = e^x then f(p-q) is

a. f(p) + f(q) b. f(p) - f(q) c. f(p) x f(q) ~~d.~~ f(p) / f(q)

$f(x) = e^x$
$f(p) = e^p$
$f(q) = e^q$

$f(p-q) = e^{p-q} = \left(\frac{e^p}{e^q}\right) = \boxed{\frac{f(p)}{f(q)}}$

My Notes

If $f(x) = x^2 + x^3$ then $f(x)$ is

(a) odd function (b) Even function ~~(c)~~ Neither odd nor even (d) a & b

$f(x) = x^2 + x^3$
$\Longrightarrow$ $f(-x) = (-x)^2 + (-x)^3 = x^2 - x^3$

$f(2) = 2^2 + 2^3 = 12$
$f(-2) = (-2)^2 + (-2)^3 = -4$

107 If A = {x : x < 1 and x > 1} then set A is

~~a.~~ Null set b. Singleton set c. Infinite set d. Power set

$A = \{ \quad \}$

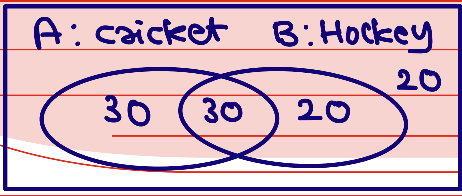
108 Set of Even Prime natural numbers is

a. Null set ~~b.~~ Singleton set c. Infinite set d. Power set

Set of Even prime natural numbers $= \{2\}$

109 In a class of 100 students 60 play Cricket, 50 play Hockey and 30 play both. Then no. of students who don't play atleast one of 2 games is :

~~a.~~ 70 b. 50 c. 10 d. None of these



$n(A' \cup B') = 70$

$n(A' \cup B') = n(A') + n(B') - n(A' \cap B')$

$= 40 + 50 - 20 = 70$

110 If f(x) = (x+1) / (x-1); then $f^{-1}(30)$ = ?

a. 23/12 b. 30/8 ~~c.~~ 31/29 d. None of these

$\Rightarrow y = f(x) = \dfrac{x+1}{x-1}$

$xy - y = x + 1$

$xy - x = y + 1$

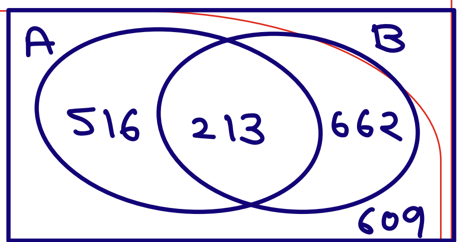
$\therefore x = \left(\dfrac{y+1}{y-1}\right)$

$f^{-1}(y) = \dfrac{y+1}{y-1}$

$f^{-1}(30) = \dfrac{30+1}{30-1}$

$= 31/29$

111 n(A) = 729, n(B) = 875, n(A∩B) = 213, n(U) = 2000. Find -



n(A') = 1271

n(B') = 1125

n(A - B) = 516

n(B - A) = 662

n(A'∩B') = 609

n(A △ B) = 1178

n(A'∪ B') = 1787

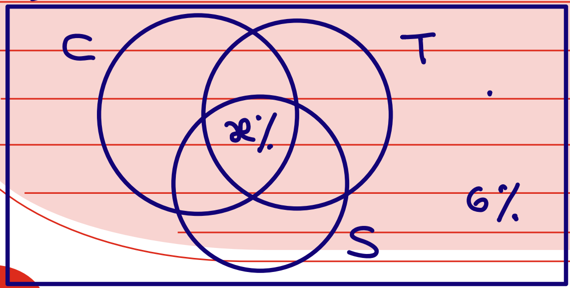
n(A ∪ B') = 1338

n(B ∪ A') = 1484

My Notes

112 Out of 2000 employees in an office 48% preferred coffee (C) and 54% liked Tea (T) and 64% used to smoke (S) 28% used C & T. 32% used T & S. 30% preferred C & S. Only 6% did none of these. The number having all three is :

~~a.~~ 360 b. 300 c. 380 d. None of these



$n(C \cup T \cup S) = 94\%$

$0.48 + 0.54 + 0.64 - 0.28 - 0.32 - 0.30 + x = 0.94$

$0.76 + x = 0.94$

$x = 18\%$

Number having all three $= 18\% (2000) = 360$

113 P set has 11 elements & Q set has 12 elements then (P x Q) has ___________ elements

a. 1 b. 23 ~~c.~~ 132 d. 11/12

$n(P \times Q) = n(P) \times n(Q) = 11 \times 12 = 132$

114 If A = {5,7,8} B = {7,5,8} show that (AxB) = (BxA) = (AxB) ∪ (BxA) = (AxB) ∩ (BxA)

$A \times B = \{(5,7), (5,5), (5,8), (7,7), (7,5), (7,8), (8,7)\ (8,5)\ (8,8)\}$

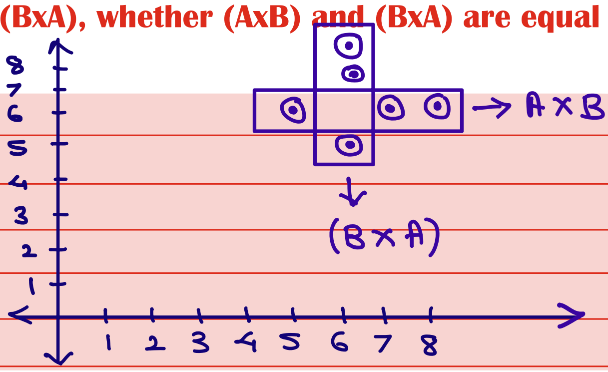
$= (B \times A) = (A \times B) \cup (B \times A) = (A \times B) \cap (B \times A)$

Therefore, (A x B) and (B x A) are equal as well as equivalent sets are.

115 If A = {5,7,8} B = {6}; Find (AxB), (BxA), whether (AxB) and (BxA) are equal sets? Equivalent sets?

$A \times B = \{(5,6), (7,6), (8,6)\}$

$B \times A = \{(6,5), (6,7), (6,8)\}$



(A×B) & (B×A) are Equivalent sets but NOT equal sets

116 In (AxB) = {(a,b) : where a ∈ A , b ∈ B }

If A = ϕ or B = ϕ then we defined (AxB) or (BxA) as ϕ

A × B is a set of all ordered pairs (i, j) where i ∈ A & j ∈ B

My Notes

(B × A) is a set of all ordered pairs (x, y) where x ∈ B and y ∈ A

117 If A = {12,10,16} B = {5,8,12,13} C = {8,11,10,25,16}

Find a. A x (B ∩ C)

b. B x (A ∩ C)

① $A \times (B \cap C) = \{12, 10, 16\} \times \{8\} = \{(12,8), (10,8), (16,8)\}$

② $B \times (A \cap C) = \{5,8,12,13\} \times \{10,16\}$

$= \{(5,10), (8,10), (12,10), (13,10), (5,16), (8,16), (12,16), (13,16)\}$

118 If f(x) = (x+1) / (x-1). Find f(-3/2), f(7/3)

$f(x) = \left(\frac{x+1}{x-1}\right)$, $f\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}+1}{-\frac{3}{2}-1} = \frac{-3+2}{-3-2} = \frac{-1}{-5} = (1/5)$

$f(7/3) = \left(\frac{7/3+1}{7/3-1}\right) = \frac{7+3}{7-3} = \frac{10}{4} = (5/2)$

119 If $g(x) = \left(\frac{2x+1}{3x+8}\right)$; f(x) = 8x + 5; Find f.g(10); g.f(-2); $g[f^{-1}(5)]$

$\Longrightarrow$ ① $f.g(10) = f\left[\frac{2(10)+1}{3(10)+8}\right] = f(21/38)$

$= 8\left(\frac{21}{38}\right) + 5 = \frac{168}{38} + 5 = \frac{358}{38}$

$= \left(\frac{179}{19}\right)$

② $g.f(-2) = g\left[8(-2)+5\right] = g(-11)$

$= \left[\frac{2(-11)+1}{3(-11)+8}\right] = \frac{-21}{-25} = 21/25$

③ $y = f(x) = 8x + 5 \quad \therefore \quad x = \frac{y-5}{8} = f^{-1}(y)$

$f^{-1}(5) = \frac{5-5}{8} = 0$

$g[f^{-1}(5)] = g(0) = \frac{2(0)+1}{3(0)+8} = 1/8$

My Notes

If $f(x) = 3x + 13$, $g(x) = 10x + 21$. find $g[f^{-1}(12)]$

$\Longrightarrow y = f(x) = 3x + 13$

$f^{-1}(y) = x = \frac{y-13}{3}$

$f^{-1}(12) = \frac{12-13}{3}$

$= -1/3$

$g[f^{-1}(12)] = g(-1/3)$

$= 10\left(-\frac{1}{3}\right) + 21$

$= 21 - \frac{10}{3} = (53/3)$

120 f(x) = 1/(1-x). Find f(-1)

a. 1 ~~b.~~ 1/2 c. Not defined d. 2

$$f(x) = \frac{1}{1-x}$$

$$f(-1) = \frac{1}{1-(-1)} = \frac{1}{2}$$

121 {(x,y) : x < y and x,y ∈ R} is

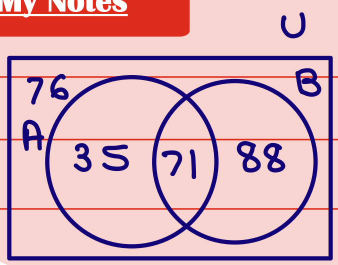
~~a.~~ not a function b. a function c. one-one mapping d. None of these

$$A = \{(5,7), (5,8), (5,10), (5,33), (2,50) \ldots\ldots\}$$

122

1. A∪A = A
2. A ∪ A' = U
3. A ∩ A'= ϕ
4. A ∪ U = U
5. A ∪ Φ = A
6. A ∩ Φ = ϕ
7. Φ ∪ A' = A'
8. Φ ∩ U = ϕ
9. (A-B)∩(B-A) = ϕ
10. (A∪B) ∪ (A∩B) = $(A\cup B)$
11. (A∪B) ∩ (A∩B) = $(A\cap B)$
12. (A∪B) ∪ A = $(A\cup B)$
13. (A∪B) ∩ A = A
14. (A∩B) ∪ A = A
15. (A∩B) ∩ A = $(A\cap B)$
16. (A ∪ B) ∪ A' = U
17. (A ∪ B) ∩ (A'∩B') = ϕ
18. (A△B) ∪ (A∩B) = $(A\cup B)$
19. (A'∪ B') ∪ (A∩B) = U
20. (A-B) ∪ (A∩B) = A
21. (B-A) ∪ B = B
22. (A△B) ∪ (A'∩B') = $(A\cap B)' = (A'\cup B')$
23. (A'∩B') ∪ (A△B) = $(A\cap B)' = (A'\cup B')$
24. (A△ B) ∪ A = $(A\cup B)$

My Notes

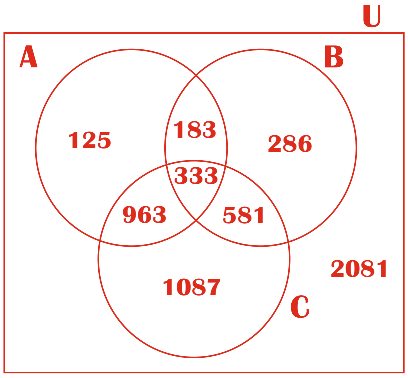


$$n[(A\triangle B)\cup A] = n(A\cup B) = 194$$

$$n[(A\cup B)\cup(A'\cap B')] = n(U) = 270$$

$$n(A\cap A') = 0$$

123



1. $n(A) =$ 1604
2. $n(B) =$ 1383
3. $n(C) =$ 2964
4. $n(A') =$ 4035
5. $n(B') =$ 4256
6. $n(C') =$ 2675
7. $n(U) =$ 5639
8. $n(A \cap B) =$ 516
9. $n(B \cap C) =$ 914
10. $n(A \cap C) =$ 1296
11. $n(A \cup B) =$ 2471
12. $n(B \cup C) =$ 3433
13. $n(A \cup C) =$ 3272
14. $n(A-B) =$ 1088
15. $n(B-A) =$ 867
16. $n(A-C) =$ 308
17. $n(C-A) =$ 1668
18. $n(B-C) =$ 469
19. $n(C-B) =$ 2050
20. $n(A' \cap B') =$ 3168

21. $n(B' \cap C') =$ 2206
22. $n(A' \cap C') =$ 2367
23. $n(A \triangle B) =$ 1955
24. $n(B \triangle C) =$ 2519
25. $n(A \triangle C) =$ 1976
26. $n(A \cup B') =$ 4772
27. $n(B \cup A') =$ 4551
28. $n(A \cup C') =$ 3971
29. $n(C \cup A') =$ 5331
30. $n(B \cup C') =$ 3589
31. $n(C \cup B') =$ 5170
32. $n(A' \cup B') =$ 5123
33. $n(B' \cup C') =$ 4725
34. $n(A' \cup C') =$ 4343
35. $n(A \cup B \cup C) =$ 3558
36. $n(A \cap B \cap C) =$ 333
37. $n(A' \cap B' \cap C') =$ 2081
38. $n(A \cap B' \cap C') =$ 125
39. $n(A' \cap B \cap C') =$ 286
40. $n(C \cap A' \cap B') =$ 1087

My Notes

If $n(A) = 5281$, $n(B) = 7863$, $n(A \cap B) = 4448$

Find $n(A \triangle B)$, $n(A \cup B')$

$\Longrightarrow n(A \triangle B) = n(A \cup B) - n(A \cap B)$

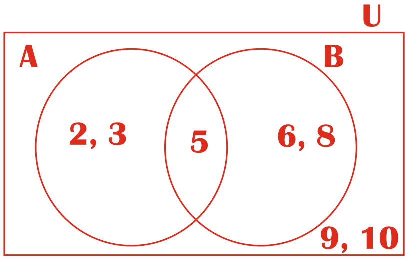
$= n(A) + n(B) - n(A \cap B) - n(A \cap B)$

$= 5281 + 7863 - 4448 - 4448 = 4248$

$n(A \cup B') = n(U) - n(B-A)$

= Insuffi. data

124



Find Sets :

1. A = {2,3,5}
2. B = {5,6,8}
3. (A ∩ B) = {5}
4. (A ∪ B) = {2,3,5,6,8}
5. (A ∩ B') = {2,3}
6. (B ∩ A') = {6,8}
7. (A' ∩ B') = {9,10}
8. (A △ B) = {2,3,6,8}
9. (A ∪ B') = {2,3,5,9,10}
10. (B ∪ A') = {5,6,8,9,10}
11. (A' ∪ B') = {2,3,6,8,9,10}

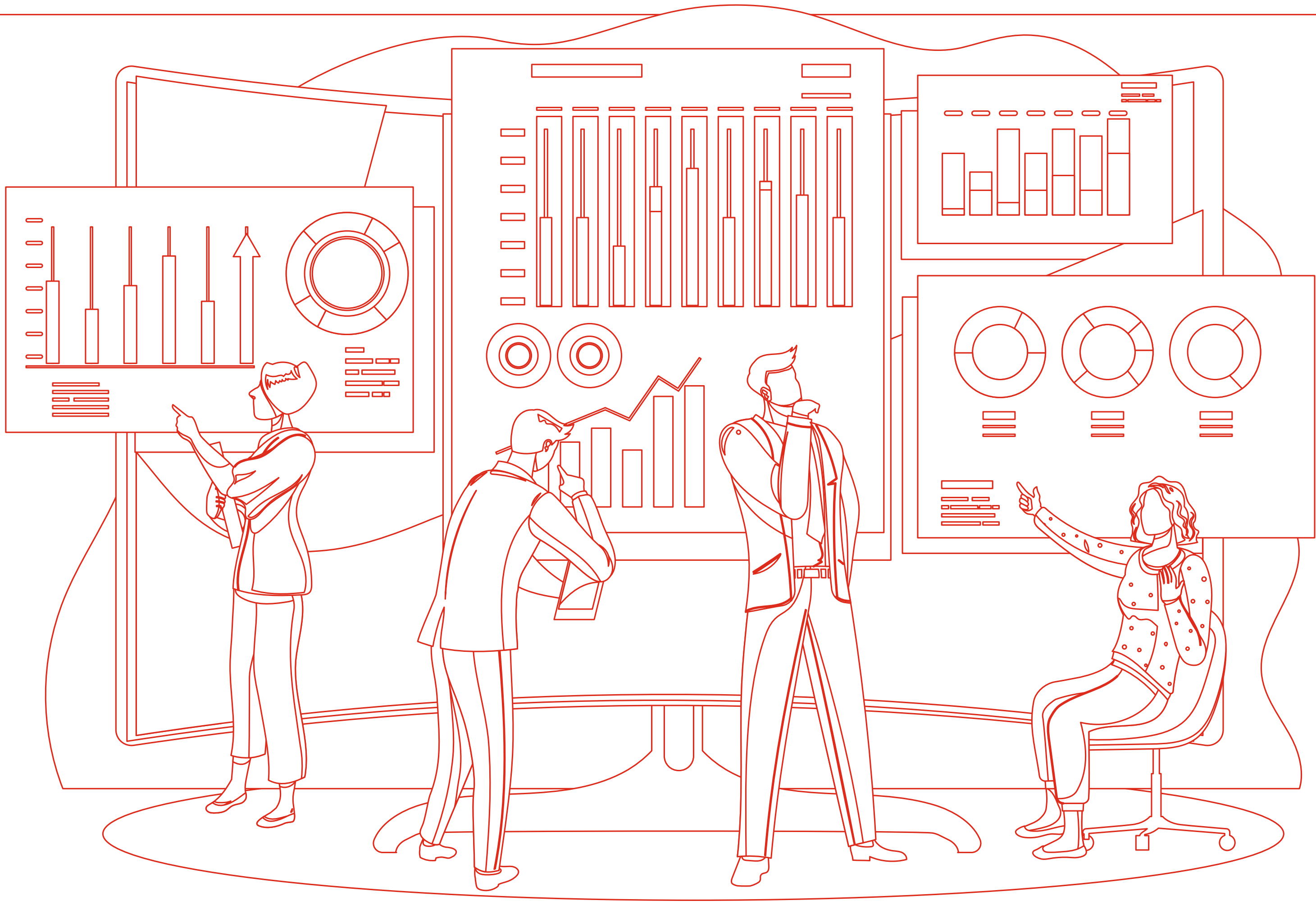
125

B = {8,9,3,6,8,9,6,6,8,9,11,13,8,9,9,15}

Cardinal Value of Set B is 7.

B = {8,9,3,6,11,13,15}

As there are 7 distinct observations in set B.



Statistical Description of Data

CA Vinod Reddy

Statistical Description of Data

1 The word statistics is derived from :

Latin word Status
Italian word Statista
German word Statistik
French word Statistique

2 We may define statistics in singular and plural sense

3 Statistics is useful in -

① Business Economics ② Business management
③ commerce & related fields ④ medical, Engineering fields
⑤ Investment managements ⑥ Insurance, portfolio mgmt
⑦ politics, sports etc.

4 5 Steps in Statistics -

① Collection of data
② Scrutiny of data
③ Classification of data
④ Presentation of data
⑤ Analysis of data

(CSCPA)

5 Collection of Data

Primary data	Secondary data
↓	↓
First hand data	Second hand data
↓	↓
Data originally collected	Data not originally collected
↓	↓
collected & used by same agency	collected as used by diff agencies

6 Following methods can be used for collection of primary data

1. Questionnaire Method
2. Mailed questionnaire Method
3. Interview Method
4. Observation Method

7 Sources of Secondary Method

1. International sources WHO, IMF, World Bank, etc.
2. Govt. Sources
3. Private Sources
4. Unpublished Data etc

8 Checking the data for it Accuracy and consistency is known as scrutiny of data

9 **Methods of Classification of Data**

1. Geographical classification.
2. Chronological classification. (Temporal classification)
3. qualitative classification.
4. quantitative classification.
5.

10 **Methods of Presentation of Data**

1. Textual presentation : Most lengthy method
2. Tabular presentation : Best method
3. Diagrammatic presentation : most attractive method

11 Table No. 678 — Table number

Course wise No. of students at PERCEPT → Title

Captions (Year 2022) → Head-Note

Box Head → Stubs →

Students / Course	Boys	Girls	Total
CA Foundation	250	200	450
CA Inter	1000	1200	2200
CA Final	900	200	1100
Total	2,150	1,600	3,750

→ Body

Foot Note → Source : www.perceptforca.com

12 The best method of data presentation is Tabular presentation

13 The most attractive method of data presentation is Diagrammatic presentation → Graphs, pie-charts, Histograms, Bar diagrams, ogives (cumu. freq. curves), pictorial presentation, Tree Diagrams

14 Stubs are : Left part of the table describing the rows.

15 **Captions are :** upper part of table describing columns, sub columns.

16 **Diagrammatic Presentation**

- Line Diagrams OR Histograms
- Bar Diagrams
  - Horizontal Bar diagrams
  - Vertical Bar diagrams
- Pie Chart

17 **Simple data on marks of 20 students :**
6, 3, 8, 11, 19, 23, 24, 18, 11, 13, 16, 15, 19, 11, 20, 16, 8, 9, 2, 3, 5, 4, 9, 2, 13

Grouped & classified data

C.I.	Frequency (No. of times an observation occurs)
0-5	~~IIII~~ = 5
5-10	~~IIII~~ I = 6
10-15	~~IIII~~ = 5 $\Sigma f = N = 25$
15-20	~~IIII~~ I = 6
20-25	III = 3

Tally marks are used represent Frequency

18 **8, 9, 8, 10, 8, 10, 10, 9, 10, 10, 9, 8, 8, 8, 10, 10, 10, 9, 8**

x	f
8	7
9	4
10	8
	19

simple data ⇓ Grouped data

$\therefore \Sigma f = N = 19$

19

LCB = LCL − (D/2)

UCB = UCL + (D/2)

Relative Frequency = (Freq. of the class OR observation / Total Frequency)

Percentage Frequency = (Freq. of the class OR observation / Total Frequency) × 100

Class Width = (UCB − LCB)

Class-mark = $\left(\frac{LCL + UCL}{2}\right) = \left(\frac{LCB + UCB}{2}\right)$

Frequency Density = (Freq. of the class / class width)

Less than type of cumulative frequency = [Freq. of the class + sum of freq. of all preceding classes]

Greater than type of cumulative frequency = [Freq. of the class + sum of freq. of all succeding classes]

20 exclusive classification : preferred for continuous variable

C.I.	Freq.	LCL	UCL	LCB	UCB	Relative Freq.	% Freq.	Freq. Density	Class Mark	Class Width	less than type c.f.	greater than type c.f.
10-20	5	10	20	10	20	0.10	10%	0.50	15	10	5	50
20-60	8	20	60	20	60	0.16	16%	0.20	40	40	13	45
60-80	7	60	80	60	80	0.14	14%	0.35	70	20	20	37
80-100	20	80	100	80	100	0.40	40%	1.00	90	20	40	30
100-120	3	100	120	100	120	0.06	6%	0.15	110	20	43	10
120-140	7	120	140	120	140	0.14	14%	0.35	130	20	50	7

Inclusive classification : preferred for Discrete variable

21

C.I.	Freq	LCL	UCL	LCB	UCB	Relative Freq.	% Freq.	Class Mark	Class Width	less than type c.f.	greater than type c.f.
10-18	2	10	18	9	19	0.04	4%	14	10	2	50
20-38	8	20	38	19	39	0.16	16%	29	20	10	48
40-98	10	40	98	39	99	0.20	20%	69	60	20	40
100-168	13	100	168	99	169	0.26	26%	134	70	33	30
170-218	8	170	218	169	219	0.16	16%	194	50	41	17
220-318	5	220	318	219	319	0.10	10%	269	100	46	9
320-398	4	320	398	319	399	0.08	8%	359	80	50	4

22 **Graphical Presentation**

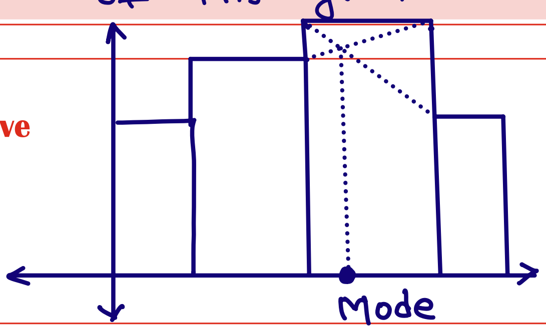
- Area Diagrams OR Histograms
- Frequency Polygon
- Cumulative frequency curves OR Ogives

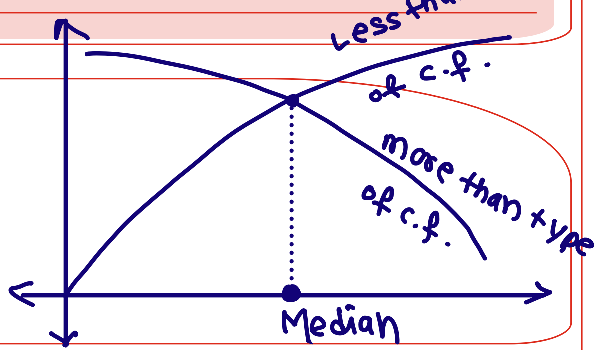
23 Median can be graphically obtained with the help of cumulative frequency curves OR ogives.

Mode can be graphically obtained with the help of Histogram.

24 **Frequency Curves**

1. Bell shaped curve
2. U-shaped curve
3. J-shaped curve
4. Mixed curve





25 **Data**

- A qualitative characteristic is known attribute
- A quantitative characteristic is known as variable
  1. Discrete variable
  2. continuous variable

26 **Discrete Variable :** Within a given interval, if a variable can assume Finite/limited values, It is said to be a Discrete variable. examples: No. of students, No. of errors in a Book

Continuous Variable : Within a given interval if a variable can assume Infinite/unlimited values, It is said to be a continuous variable examples: Age, Height, Income, marks, weight etc

27 **Definition of Statistics**

Plural sense → **Quantitative and qualitative data collected usually with a view of having statistical analysis**

Singular sense → **Scientific method that is employed for collecting, analysing and presenting data leading finally to drawing statistical interferences.**

28 **Limitation of Statistics**

~~a.~~ Deals with aggregate, an individual has no statistical significance.
~~b.~~ Mostly concerned with quantitative data
~~c.~~ Based on assumptions, so projections are likely to be inaccurate
~~d.~~ Based on random sampling.

29 **Methods of Collection of primary data**

Interview Method	Mailed Questionnaire Method	Observation Method	Questionnaire filled and sent by enumeraters
a. Personal interview b. Indirect interview c. Telephonic interview			

30 a. In personal interview investigator meets to the respondent directly and collects the information.

b. If there are some practical problems in reaching the respondents directly then we may go for indirect interview when investigator collects the information from the persons associated with the problem.

c. Telephonic interview is quick and non expensive method to collect primary data.

First 2 methods are inapplicable when there is large data. The amount of non-response is maximum for third method of data collection.

My Notes

$LCB = LCL - (D/2)$

$UCB = UCL + (D/2)$

Freq. density $= \left(\dfrac{\text{Freq. of the class}}{\text{class width}}\right)$

	f	Relative Freq.	% Freq
Boys	70	0.35	35%
Girls	130	0.65	65%
		1.00	100%

31 **Scrutiny of Data :**

Since statistical analysis are made only on the basis of data, it is necessary to check whether the data under consideration are accurate and consistent.

No hard and fast rules can be applied for scrutiny of data. One must apply his intelligence, patience and experience while scrutinising the given information.

32 **Textual Presentation :**

This method comprises presenting data with the help of paragraphs.

Advantage of this method lies in its simplicity, a layman can also present data under this method.

Textual presentation, however not preferred as it is Dull, Monotonous, Lengthy.

33 **Tabular Presentation :**

It may be defined as systematic presentation of data with the help of a statistical table having no. of rows, columns and complete ref. no., title, description of rows and columns, foot notes, if any.

a. Caption is the upper part of the table describing column and sub-columns.

b. Stubs are left part of table providing description of rows.

c. Body is the main part of the table that contains numerical figures.

d. Addi. info, Disclaimer, source of data in mentioned in : Foot Note

e. units of measurement can be written in : Box-Head.

34 **Diagrammatic Presentation of Data**

a. Another alternative and attractive method is with the help of charts, graphs, pictures, etc.

b. Any hidden trend can be understood with the help of this method.

c. However, as compared to tabulation, this method is less accurate. So if priority is accuracy of data, we have to recommend tabulation.

• The Best method of data presentation is : Tabular presentation

35 We are going to consider the following types of diagrams

a. Line diagram / histogram

b. Bar diagram

c. Pie chart / pie diagram / circle diagram.

My Notes

Stubs are : Left part of table describing rows

captions are : upper part of table describing columns & sub-columns.

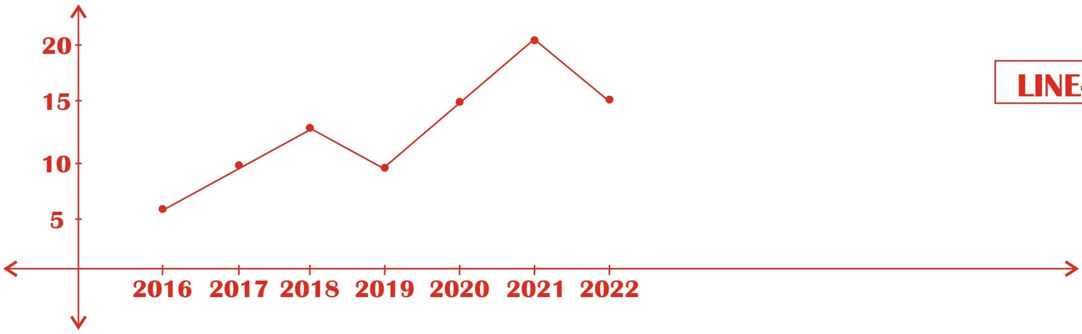
36

- Line diagram that uses logs is known as Ratio-chart.
- Multiple Line chart is used for representing 2 or more related time series data expressed in same unit.
- Multiple Axis chart in somewhat similar situations if variables are expressed in different units.

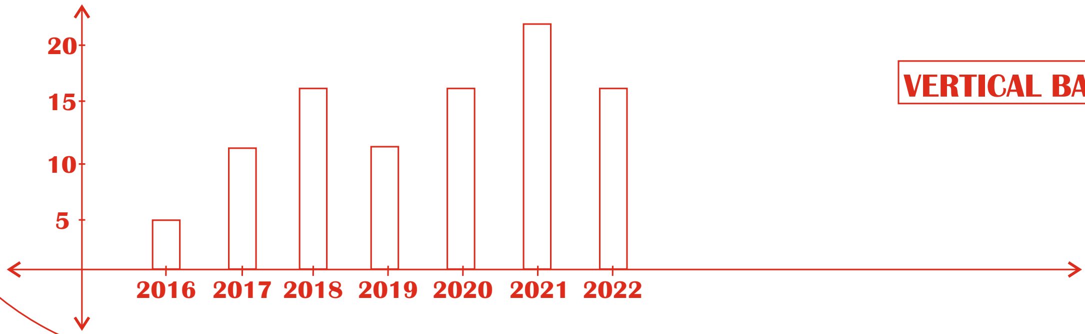
37

- Horizontal bar diagram issued for qualitative data.
- Vertical bar diagram is associated with quatitative data OR time series data

38

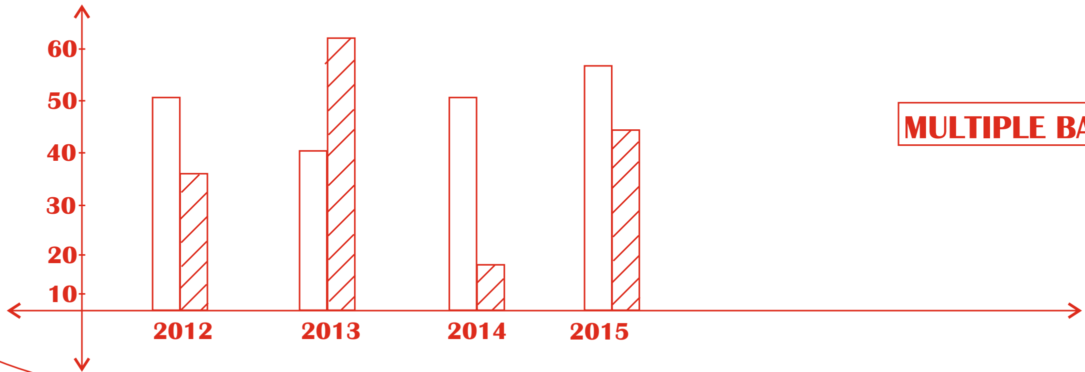


LINE-CHART



VERTICAL BAR-CHART

39



MULTIPLE BAR-CHART

My Notes

40

Draw the appropriate diagram for presentation the of following data :

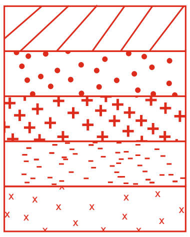
Source	Revenue in Millions (₹)
Customs	80
Excise	190
Income-Tax	160
Corporate Tax	75
Misc	35
Total	540

Angle in a pie-chart

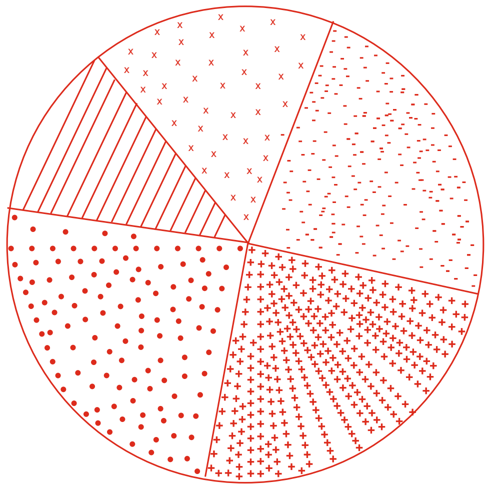
$$= \left(\frac{\text{value of a component}}{\text{Total value}}\right) \times 360^\circ$$

⟹

Source	Angle in Pie chart
Customs	(80/540) x 360 = 53° (approx.)
Excise	(190/540) x 360 = 127°
Income-Tax	(160/540) x 360 = 107°
Corporate Tax	(75/540) x 360 = 50°
Misc	(35/540) x 360 = 123°



Customs
Excise
Income-Tax
Corporate Tax
Misc



Pie-chart OR
Pie- Diagram

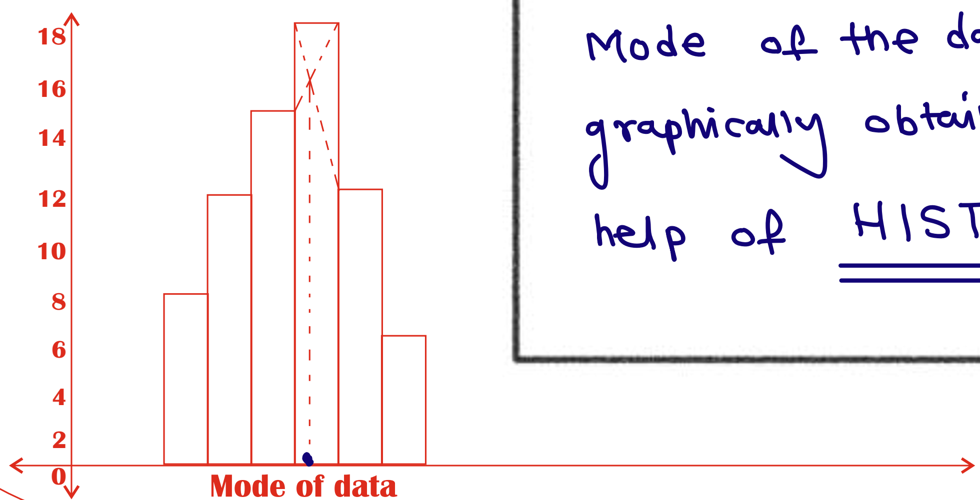
41

Graphical Presentation of Frequency Distribution

- Area Diagrams OR Histograms
- Frequency Polygon
- Cumulative frequency curves OR Ogives

42

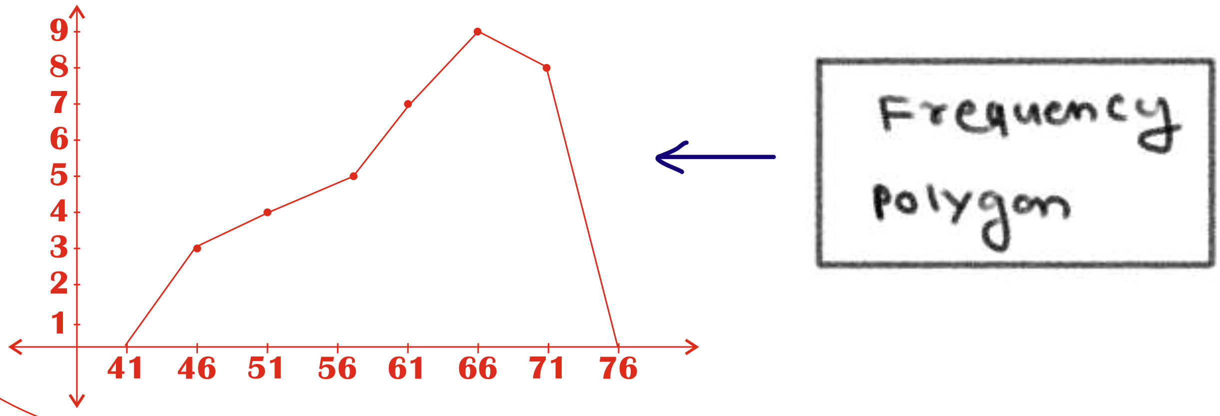
Histogram



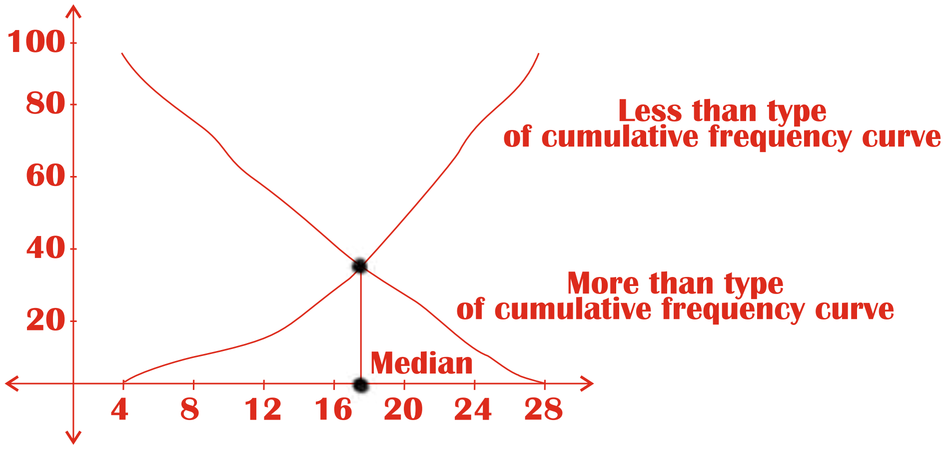
Mode of the data can be graphically obtained with the help of HISTOGRAM

43 Frequency Polygon

Mid-points	46	51	56	61	66	71
No. of student	3	4	5	7	9	8



44 Ogives OR Cumulative frequency curves



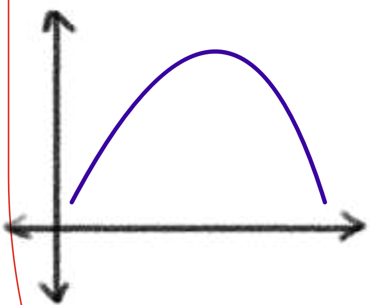
1. Median can be graphically obtained with the help of cumulative frequency curves / ogives
2. Mode can be graphically obtained with the help of histogram.

My Notes

45 There are 4 types of frequency curves

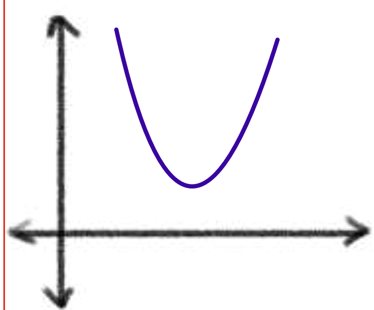
Bell shaped curve

Distribution of height, weight income generally belong this category. Freq. distribution starts with low becomes maximum at centre then gradually reaches to lowest value to other extremity.



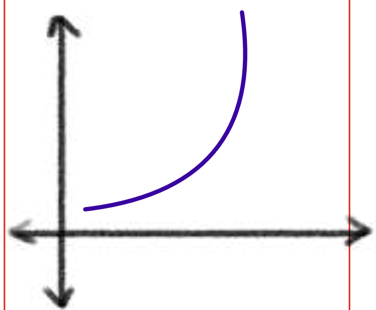
U shaped curve

Frequency distribution is minimum near the central part and freq. slowly and steadily reaches maximum at after two extremities.



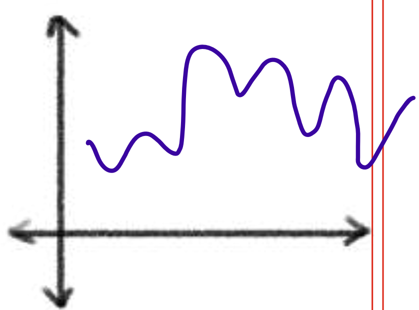
J shaped curve

It starts with the minimum frequency and then gradually reaches to maximum frequency at other extremity.



Mixed curve

We may have combination of these frequency curves. No specific shape for mixed curve.



46 The primary data is collected by

a. Interview Method
b. Observation Method
c. Questionnaire Method
~~d.~~ All of these

47 The quickest method to collect primary data is :

a. Personal Interview
b. Indirect Interview
~~c.~~ Telephonic interview
d. Observation Method

My Notes

scruteny of data

Checking the data for its accuracy & consistency & Authenticity is known as Scruteny of data

48 In case of Rail accident, the appropriate method of data collection is by :

a. Personal Interview
b. Direct Interview
c. Indirect Interview
d. All of these

49 Which method of data collection covers widest area

a. Telephonic interview
b. Mailed Questionnaire Method
c. Direct Interview Method
d. All of these

50 The amount of non-responses are maximum in case of

a. Mailed Questionnaire Method
b. Interview Method
c. Observation Method
d. All of these

51 The accuracy and consistency of data can be verified by -

a. Internal checking
b. External checking
c. Scrutiny
d. None of these

52 The unit of measurement in tabulation is shown in the

a. Box Head
b. Body
c. Caption
d. Stub

53 In tabulation, source of the data if any is shown in the

a. Foot-Note
b. Body
c. Caption
d. Stub

54 Hidden trend, if any, in a data can be noticed by

a. Textual presentation
b. Tabulation
c. Diagrammatic Presentation
d. None of these

55 The most accurate (Best) method of data presentation is :

a. Diagrammatic Presentation
b. Tabulation
c. Textual presentation
d. None of these

My Notes

56 The chart used logarithms of a variable is known as :

a. Line chart ~~b.~~ Ratio chart

c. Multiple line chart d. Pie chart

57 Pie diagram is used for?

~~a.~~ Comparing diff. components and their relation to total

b. Representing qualitative data in a circle

c. Representing quantitative data in a circle

d. b or c

58 A frequency distribution

a. Arranges observations in increasing order

~~b.~~ Arranges observations in number of groups

c. is for time pass

d. All of these

59 Frequency distribution of a continuous variable is known as

~~a.~~ Grouped frequency distribution b. Simple frequency distribution

c. a or b d. a and b

60 The distribution of shares is an example of frequency distribution of :

~~a.~~ A discrete variable b. A continuous variable

c. An attribute d. None of these

61 The distribution of profits of a blue chip company relates to :

a. A discrete variable ~~b.~~ A continuous variable

c. An attribute d. None of these

62 Mutually exclusive classification

a. Excludes both the class limits ~~b.~~ Excludes UCL but includes LCL

c. Includes UCL and excludes LCL d. None of these

10 – 20

$\boxed{20 - 30}$ includes LCL (20) & excludes UCL (30)

30 – 40

My Notes

- For exclusive classification,

LCB = LCL

UCB = UCL

63 Out of 1000 workers, 25% were industrial workers and rest were agricultural workers. 300 persons enjoyed world cup matches on T.V, 30% of people who had not watched world cup matches were industrial workers. What is agri. no. of workers who had enjoyed world cup matches on T.V.?

~~a. 260~~ b. 240 c. 230 d. 250

workers	Industrial	Agri.	Total
watched world cup on TV	40	260	300
Not watched	210	490	700
Total	250	750	1,000

Textual to Tabular presentation

64 The number of accident for 7 days in a locality are given below :

No. of accidents	0	1	2	3	4	5	6
Frequency	15	19	22	31	9	3	2

What is no. of cases when 3 or less accidents occur?

a. 56 b. 6 c. 68 ~~d. 87~~

15 + 19 + 22 + 31
= 87 cases

65 The follow data relates to income :

Income	500 - 999	1000 - 1499	1500 - 1999	2000 - 2499
No. of persons	15	28	36	7

What is % of persons earning more than ₹ 1500?

a. 43% ~~b. 50%~~ c. 40% d. None of these

$(36+7) = 43$ $\left(\frac{43}{86} \times 100\right) = 50\%$

66 The following data relate to the marks of group of students :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Students	15	38	65	84	100

How many students have marks more than 30?

a. 65 b. 184 ~~c. 35~~ d. None of these

My Notes

C.I. (Marks)	No. of students (f)
0–10	15
10–20	23
20–30	27
30–40	19
40–50	16

19+16 = 35

67 Find number of observations between 250 and 300 from the following data :

Value	More than 200	More than 250	More than 300	More than 350
No. of Observations	56	38	15	0

a. 56 ~~b.~~ 23 c. 15 d. 8

C.I.	frequency
200 – 250	56 − 38 = 18
250 – 300	38 − 15 = 23
300 – 350	15 − 0 = 15

68 Cost of sugar in a month under the heads material, labour, expenses, overheads are ₹ 12,20,35,23 respectively. What is diff between central angles for the largest and smallest components of cost of sugar?

a. 72° b. 48° c. 56° ~~d.~~ 92°

Largest Angle $= \frac{35}{90} \times 360 = 140°$

smallest Angle $= \frac{12}{90} \times 360 = 48°$

Diff $= 140 - 48$ $= 92°$

69 The distribution of profits of a company generally follows :

a. J shaped freq. curve
b. U-shaped frequency curve
~~c.~~ Bell shaped freq. curve
d. None of these

70 The distribution most commonly used is :

a. Mixed b. U-shaped ~~c.~~ Bell shaped d. None of these

71 Graph is a ________________

~~a.~~ Line diagram b. Bar-diagram c. Pie-diagram d. Pictogram

72 (Class frequency / class width) is defined as ________________

~~a.~~ Frequency density b. Frequency distribution c. Both d. None

My Notes

73 Tally Marks determines

a. Class width
b. Class boundary
c. Class limit
~~d.~~ Class Frequency

74 An area diagram is

~~a.~~ Histogram
b. Frequency Polygon
c. Ogives
d. None

75 Ogive is a ________________

~~a.~~ Line diagram
b. Bar diagram
c. Both
d. None

76 Unequal width of classes in a frequency distribution do not cause any difficulty in construction of ____________

a. Ogive
b. Frequency Polygon
~~c.~~ Histogram
d. None of these

77 Graphical presentation of cumulative frequency distribution is called as ___________

a. Histogram
~~b.~~ Ogive
c. Both
d. None of these

78 The most common form of diagrammatic presentation of a grouped frequency distribution is

a. Ogive
~~b.~~ Histogram
c. Frequency Polygon
d. None of these

79 Vertical Bar diagram may appear somewhat alike -

~~a.~~ Histogram
b. Frequency Polygon
c. Ogive
d. None of these

80 Number of types of cumulative frequency is :

a. One
~~b.~~ Two
c. Three
d. Four

My Notes

81 A representative value of a class interval for the calculation of Mean, SD, MD, etc. is

a. Class interval b. Class limit
c. Class mark d. None

82 In all statistical calculations & diagrams involving end points of classes ______ are used.

a. Class Boundaries b. Class Values
c. both d. None

83 Upper boundary of a class coincide with Lower boundary of next class.

a. True b. False
c. Both d. None

84 The lower extreme point of a class is called as ________________

a. Lower Class Limit b. Lower Class Boundary
c. Both d. None

Below 10
10-30
30-80
80-above

Data with open class intervals

85 When one end of the class is not specified, the class is called as ________

a. Open end class b. Close end class
c. Both d. None of these

86 When all classes have equal width, the heights of rectangles in histogram will be numerically equal to the _____________ .

a. Class Frequencies b. Class Boundaries
c. Both d. None of these

87 To find 'Mode of data' graphically we use __________ .

a. Ogives b. Frequency Polygon
c. Histogram d. None of these

88 In representing simple frequency distributions of a discrete variable __________ is useful.

a. Ogives b. Histogram
c. Frequency Polygon d. None of these

My Notes

89 Diagrammatic presentation of cumulative frequency distribution is ____________ .

a. Frequency Polygon
~~b.~~ Ogives
c. Histogram
d. None of these

90

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	6	4

For the class 20-30 cumulative frequency is :

a. 20 b. 13 c. 15 ~~d.~~ 28

91 Breadth of rectangle is equal to length of class interval in ___________ .

a. Ogives ~~b.~~ Histogram c. Line diagram d. None

92 In Histogram classes are taken _________________

~~a.~~ Overlapping b. None Overlapping c. Both d. None

93 There are ____________ methods of classification of data.

~~a.~~ 4 b. 3 c. 2 d. 1

94 There are ____________ methods of presentation of data.

a. 4 ~~b.~~ 3 c. 2 d. 1

95 For the overlapping classes 0-10, 10-20, 20-30, etc. the class mark of 0 - 10 is

~~a.~~ 5 b. 4.50 c. 4 d. 10

96 For the classes 0-9, 10-19, 20-29, 30-39, the class mark of 10-19 is _________

~~a.~~ 14.50 b. 15 c. 20 d. 16

97 Mutually inclusive classification is meant for _______________

~~a.~~ Discrete variable b. Continuous variable c. Both d. None

My Notes

98 Mutually exclusive classification is meant for ____________

a. Discrete variable ~~b.~~ Continuous variable c. Both d. None

99 LCB is ________________

a. Latur Crime Branch

b. Lower Class Branch

~~c.~~ Lower Class Boundary

d. a or c

100 Relative Frequency of a particular class

~~a.~~ Lies between 0 and 1

b. Lies between -1 and 1

c. Lies between -1 and zero

d. None of these

101

Characteristic	Discrete / Continuous Variable / Attribute
a. Income	a continuous variable
b. Profit	a continuous variable
c Blue-colour	an attribute
d. Honesty	an attribute
e. Nationality	an attribute
f. No. of shares	a Discrete variable
g. Age	a continuous variable
h. No. of members	a Discrete variable
i.Drinking habit	an attribute
j. Beauty	an attribute
k. Children in a family	a Discrete variable
l. Love	an attribute
m. Batch size	a Discrete variable

102

Class - Interval	Frequency
0 - 10	5
10 - 20	8
20 - 40	9
40 - 60	10

Mutually Exclusive Classification

preferred for: ↓ continuous variable

Class - Interval	Frequency
0 - 9	25
10 - 29	28
30 - 89	35
90 - 189	40

Mutually Inclusive Classification

↓ Discrete variable

My Notes

Measures of Central Tendency And Dispersion

CA VINOD REDDY

1 5 Measures of Central Tendency are :

1. Arithmetic Mean (A.M.)
2. Harmonic Mean (H.M.)
3. Geometric Mean (G.M.)
4. Median
5. Mode

2 AM of simple data = $\left[\Sigma x / n\right] = \left[\dfrac{\text{sum of all observations}}{\text{Number of observations}}\right]$

AM of grouped data = $\left[\Sigma f.x / \Sigma f\right] = \left(\dfrac{\Sigma f x}{N}\right)$ $N = \Sigma f$ = Total Frequency

AM of grouped & classified data = $\left[\Sigma f.x / \Sigma f\right] = \left(\dfrac{\Sigma f x}{N}\right)$ where x = class-mark

3 Find AM of : 80,63,90,101,65,73,88,100.

$\Rightarrow$ AM of simple data $= \left(\dfrac{\text{sum of 8 observations}}{8}\right) = \left(\dfrac{660}{8}\right)$

$= 82.50$

4 Find AM of

x	20	30	40	50	60
f	28	52	68	72	80

$\Rightarrow$ AM of Grouped data $= \dfrac{\Sigma f.x}{\Sigma f} = \dfrac{13240}{300}$

$= 44.1333333$

My Notes

5 **Find AM of**

C.I	10-20	20-40	40-80	80-120
f	15	18	23	84

AM of Grouped & classified data $= \dfrac{\Sigma f.x}{\Sigma f} = \dfrac{10545}{140} = 75.3214285714$

where x = class-mark

6 **Find AM of**

C.I	10-19	20-39	40-69
f	33	32	85

AM of Grouped & classified data $= \dfrac{\Sigma f.x}{\Sigma f} = \left(\dfrac{6055}{150}\right) = 40.366666666$

where x = class-mark

7 **AM is magnitude-wise central number**

Median is position wise central Number. positional avg of data.

Mode is Most likely observation. (i.e. observation having highest Frequency)

8 **Find Median for 81,36,25,35,38,43,50**

$\Longrightarrow$ Median for simple data $= \left[\frac{1}{2}(n+1)\right]^{th}$ term

Ascending order: 25, 35, 36, 38, 43, 50, 81

Median $= \left[\frac{1}{2}(7+1)\right]^{th}$ term $= 4^{th}$ term $= 38$

9 **Find Median for 80,60,28,90,81,100,103,115**

$\Longrightarrow$ Ascending order: 28, 60, 80, $\boxed{81, 90}$, 100, 103, 115

Median $= \left[\frac{1}{2}(8+1)\right]^{th}$ term $= 4.50^{th}$ term

$= 4^{th}$ term $+ 0.50\,(5^{th} - 4^{th})$ term

$= 81 + 0.50\,(90-81) = 85.50$

My Notes

Find median of 40, 43, 43, 45, 47, 51, 93, 80, 20, 35

$\Longrightarrow$ Ascending order 20, 35, 40, 43, $\boxed{43, 45}$, 47, 51, 80, 93

Median = AM of 2 middlemost obs^n s $= \dfrac{43+45}{2}$

$= 44$

10 Median - If No. of observations are

Odd

middlemost observation is the median

Even

AM of 2 middlemost obs^ns is the median.

• Median = positional avg of data = positional measure of central tendency

11 Find Median, Mode for

C.I	10-20	20-30	30-40	40-60	60-100
f	15	18	33	22	16

① Find CB Less than type cf

median class

CB	10	20	30	40	60	100
cf	0	15	33	66	88	104=N

N_l N_u

$$\text{Median} = LCB + \left(\frac{\frac{1}{2}N - N_l}{N_u - N_l} \times c\right)$$

$$= 30 + \left(\frac{52-33}{66-33} \times 10\right) = 35.757575$$

C.I.	10-20	20-30	30-40	40-60	60-100
Freq.	15	18	33	22	16

30-40 ⇒ Modal class c = 10

$f_0 = 33,\ f_{-1} = 18,\ f_1 = 22$

$$\text{Mode} = LCB + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times c\right)$$

$$= 30 + \left(\frac{33-18}{66-18-22} \times 10\right) = 35.76923$$

12 Find AM, Median, Mode for

80, 60, 90, 90, 80, 90, 50, 90, 10, 5, 18, 16, 12, 16, 55

⟹ ① $AM = \frac{\Sigma x}{n} = \frac{762}{15} = 50.80$

② $\text{Median} = \left[\frac{1}{2}(n+1)\right]^{th}\text{term} = \left[\frac{1}{2}(15+1)\right]^{th}\text{term}$

$= 8^{th}\text{ term} = 55$

5, 10, 12, 16, 16, 18, 50, 55, ________

③ Mode = 90 as it has highest frequency.

My Notes

• Mode is also known as most likely obs^n

•

x	10	30	50	80	90
f	115	118	123	138	122

mode of data = 80

This is uni-modal data as there is only one mode

• Data can have 2 OR more modes

• Data having 2 OR more modes : Multi-modal data

• AM, Median is uniquely defined whereas mode is not uniquely defined.

13 Empirical relation between Mean, Median, Mode

For a moderately skewed data

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{median})$$

$$3\,\text{Median} - \text{Mode} = 2.\text{Mean}$$

14

Fractiles	Divides the data in ___ equal parts	No. of fractiles	Notations
Median	2	1	Me
Quartiles	4	3	Q_1, Q_2, Q_3
Deciles	10	9	$D_1, D_2, \ldots\ldots, D_9$
Percentiles	100	99	$P_1, P_2, \ldots\ldots, P_{99}$

15 For Simple data - Formulae

(DATA must be ASCENDING order)

Median = $\left[\frac{1}{2}(n+1)\right]^{th}$ term

Q_1 = $\left[\frac{1}{4}(n+1)\right]^{th}$ term

Q_3 = $\left[\frac{3}{4}(n+1)\right]^{th}$ term

D_6 = $\left[\frac{6}{10}(n+1)\right]^{th}$ term

P_{71} = $\left[\frac{71}{100}(n+1)\right]^{th}$ term

$D_5 = \left[\frac{5}{10}(n+1)\right]^{th}$ term

$P_{50} = \left[\frac{50}{100}(n+1)\right]^{th}$ term

$$\boxed{\text{Median} = Q_2 = D_5 = P_{50}}$$

$$\boxed{Q_1 = P_{25},\ Q_2 = P_{50},\ Q_3 = P_{75}}$$

$$\boxed{D_1 = P_{10},\ D_2 = P_{20},\ \ldots\ldots\ldots,\ D_9 = P_{90}}$$

16 For Grouped and Classified data

Median = $LCB + \left[\dfrac{\frac{1}{2}N - N_l}{N_u - N_l} \times c\right]$

Q_3 = $LCB + \left[\dfrac{\frac{3}{4}N - N_l}{N_u - N_l} \times c\right]$

D_2 = $LCB + \left[\dfrac{\frac{2}{10}N - N_l}{N_u - N_l} \times c\right]$

P_{80} = $LCB + \left[\dfrac{\frac{80}{100}N - N_l}{N_u - N_l} \times c\right]$

$$Q_i = LCB + \left(\dfrac{\frac{i}{4}N - N_l}{N_u - N_l} \times c\right)$$

where $i = 1$ to 3

$$D_m = LCB + \left(\dfrac{\frac{m}{10}N - N_l}{N_u - N_l} \times c\right)$$

where $m = 1$ to 9

$$P_j = LCB + \left(\dfrac{\frac{j}{100}N - N_l}{N_u - N_l} \times c\right)$$

where $j = 1$ to 99

17 Find P_{85} for

C.I	10-18	20-38	40-98	100-168
f	28	36	56	28

Also find D_3

CB	cf
9	0
19	28
39	64
99	120
169	148=N

$$\left.\begin{matrix}28\\64\end{matrix}\right\} D_3 \qquad \left.\begin{matrix}120\\148=N\end{matrix}\right\} P_{85}$$

$$D_3 = LCB + \left(\frac{\frac{3}{10}N - N_l}{N_u - N_l} \times C\right)$$

$$D_3 = 19 + \frac{44.40 - 28}{64 - 28} \times 20 = 28.11111$$

$$P_{85} = LCB + \left(\frac{\frac{85}{100}N - N_l}{N_u - N_l} \times C\right)$$

$$P_{85} = 99 + \left(\frac{125.80 - 120}{148 - 120} \times 70\right) = 113.50$$

18

Measure	Simple Data	Grouped Data
AM	$\frac{\Sigma x}{n}$	$\frac{\Sigma f.x}{\Sigma f} = \frac{\Sigma fx}{N}$
GM	$(x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdots\cdots x_n)^{1/n}$	$(x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \cdots\cdots x_n^{f_n})^{1/N}$
HM	$\left[n / \Sigma(1/x)\right]$	$\left[\Sigma f / \Sigma(f/x)\right]$
Median	$\left[\frac{1}{2}(n+1)\right]^{th}$ obs^n.	$LCB + \left(\frac{\frac{1}{2}N - N_l}{N_u - N_l} \times C\right)$
Mode	The obs^n having highest frequency	$LCB + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times C\right)$
Q_1	$\left[\frac{1}{4}(n+1)\right]^{th}$ term	$LCB + \left(\frac{\frac{1}{4}N - N_l}{N_u - N_l} \times C\right)$
D_7	$\left[\frac{7}{10}(n+1)\right]^{th}$ term	$LCB + \left(\frac{\frac{7}{10}N - N_l}{N_u - N_l} \times C\right)$
P_{61}	$\left[\frac{61}{100}(n+1)\right]^{th}$ term	$LCB + \left(\frac{\frac{61}{100}N - N_l}{N_u - N_l} \times C\right)$

My Notes

Find P_{63} for

81, 93, 100, 107, 112, 193, 201, 223, 281, 333, 545, 781

$$\Longrightarrow P_{63} = \left[\frac{63}{100}(12+1)\right]^{th} \text{term} = 8.19^{th} \text{ term}$$

$$= 8^{th} \text{ term} + 0.19\left(9^{th} - 8^{th}\right) \text{term}$$

$$= 223 + 0.19(281 - 223) = 234.02$$

19 Find AM, GM, HM for - 2,6,8,9,3,13,20,18

$\Longrightarrow$

① $AM = \frac{\Sigma x}{n} = \frac{79}{8} = 9.875$

② $GM = (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{1/n} = (2 \times 6 \times 8 \times 9 \times 3 \times 13 \times 20 \times 18)^{1/8}$

$= (12130560)^{1/8} = 7.6821911$

③ $HM = \frac{n}{\Sigma \frac{1}{x}} = \left[\frac{8}{\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{3}+\frac{1}{13}+\frac{1}{20}+\frac{1}{18}\right)}\right] = 5.6394$

20 Properties of AM

① AM of 30, 30, 30, 30, 30 -------- is 30

If all obsns are same, their AM is that obsn only

② sum of Algebric deviations of all observations from their AM is always zero. i.e. $\Sigma(x - AM) = zero = 0$

(Effect of positive & Negative deviations will get nullified)

③ AM is affected by shift of origin as well as by change in scale

If $y = a + bx$ then $(AM\ of\ y) = a + b(AM\ of\ x)$

If $px + qy + r = 0$ then $p(AM\ of\ x) + q(AM\ of\ y) + r = 0$

④ combined AM = pooled mean of 2 groups = $\left(\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}\right)$

My Notes

Find AM, GM, HM of

p, q, x, y, r, m

$AM = \left(\frac{p+q+x+y+r+m}{6}\right)$	$GM = \sqrt[6]{pqxyrm}$	$HM = \frac{6}{\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{x}+\frac{1}{y}+\frac{1}{r}+\frac{1}{m}\right)}$

21 If $\bar{x}_1 = 80$, $\bar{x}_2 = 120$ and Combined AM = 103. Find $n_1 : n_2$

$$\text{combined AM} = \left(\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}\right)$$

$$103 = \frac{80n_1 + 120n_2}{n_1+n_2}$$

$$103n_1 + 103n_2 = 80n_1 + 120n_2$$

$$23n_1 = 17n_2$$

$$\frac{n_1}{n_2} = \frac{17}{23}$$

Short-cut

$\bar{x}_1 = 80$ $\bar{x}_2 = 120$

$\bar{x} = 103$

Diff $= 23 : 17 = n_2 : n_1$

$n_1 : n_2 = 17 : 23$

22 Best Measure of Central Tendency = Arithmetic Mean

Best Measure of Dispersion = S.D. = standard Deviation

For Open Class interval
Best Measure of Central Tendency = Median

Best measure of disp. for comparison purpose = coeffi. of variation

23 For n observations = AM $\geq$ GM $\geq$ HM

For n distinct observations = AM > GM > HM

For 2 Observations = $GM^2 = AM \times HM$

→ GM is GM of AM & HM

24

Observations	AM	GM	HM
p,q	$\left(\frac{p+q}{2}\right)$	$\sqrt{pq}$	$\frac{2}{\left(\frac{1}{p}+\frac{1}{q}\right)} = \left(\frac{2pq}{p+q}\right)$
a,b,c,d	$\left(\frac{a+b+c+d}{4}\right)$	$\sqrt[4]{abcd}$	$\frac{4}{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)}$
60,20,80	$\left(\frac{60+20+80}{3}\right) = 53\frac{1}{3}$	$\sqrt[3]{96000} = 45.9524$	$\frac{3}{\frac{1}{60}+\frac{1}{20}+\frac{1}{80}} = 37.8947$
5,10,20,0	$\left(\frac{5+10+20+0}{4}\right) = 8.75$	zero	Not defined

If one of the observation is zero then :
GM = zero
HM = Not defined

25 Find GM, HM, AM for

x	5	6	7	8
f	1	2	2	3

① $AM = \frac{\Sigma fx}{\Sigma f} = \frac{55}{8} = 6.875$

② $GM = \left(5^1 \times 6^2 \times 7^2 \times 8^3\right)^{1/8} = (4515840)^{1/8} = 6.789571$

③ $HM = \left(\frac{\Sigma f}{\Sigma f/x}\right) = \frac{8}{\left(\frac{1}{5}+\frac{2}{6}+\frac{2}{7}+\frac{3}{8}\right)} = 6.6999$

26

	For 2 Groups
Combined AM =	$\left(\frac{n_1A_1 + n_2A_2}{n_1+n_2}\right)$
Combined GM =	$\left[G_1^{n_1} \times G_2^{n_2}\right]^{\left(\frac{1}{n_1+n_2}\right)}$
Combined HM =	$\left[\frac{n_1+n_2}{(n_1/H_1) + (n_2/H_2)}\right]$

27

	For 3 Groups
Combined AM =	$\left[\frac{n_1A_1 + n_2A_2 + n_3A_3}{n_1+n_2+n_3}\right]$
Combined GM =	$\left[G_1^{n_1} \times G_2^{n_2} \times G_3^{n_3}\right]^{\left(\frac{1}{n_1+n_2+n_3}\right)}$
Combined HM =	$\left[\frac{n_1+n_2+n_3}{n_1/H_1 + n_2/H_2 + n_3/H_3}\right]$

28 $n_1 = 30$; $n_2 = 20$; $S_1 = 3$; $S_2 = 4$; $\bar{x}_1 = 40$, $\bar{x}_2 = 50$. Find combined SD.

$\Longrightarrow$ n_1 = No. of obs^ns in Group 1 = 30
n_2 = No. of obs^ns in Group 2 = 20
S_1 = SD of Group 1 = 3 $\quad$ $\bar{x}_1$ = AM of Group 1 = 40
S_2 = SD of Group 2 = 4 $\quad$ $\bar{x}_2$ = AM of Group 2 = 50
d_1 = Diff betn AM of Group 1 & combined AM = 4
d_2 = Diff betn AM of Group 2 & combined AM = 6

combined AM
$= \frac{30(40) + 20(50)}{30+20}$
$= 44$

$$\text{combined SD} = \sqrt{\frac{n_1S_1^2 + n_2S_2^2 + n_1d_1^2 + n_2d_2^2}{n_1+n_2}}$$

$$= \sqrt{\frac{30(9) + 20(16) + 30(16) + 20(36)}{30+20}} = 5.9833$$

& combined variance = 35.80

29 **Dispersion means :** Scatterness of data or Deviation of data which is opposite of concentration of data

Measures of dispersion are used to measure : scatterness of data or concentration of data.

30 **Measures of Dispersion**

For comparison purpose we use Relative measures of dispersion.

Absolute measures of dispersion	**Relative** measures of dispersion
① Range	① coeffi. of Range
② Mean Deviation (MD) ⓐ MD about AM ⓑ MD about Median ⓒ MD about Mode	② coeffi. of Mean deviation ⓐ coeffi. of MD about AM ⓑ coeffi. of MD about Median ⓒ coeffi. of MD about Mode
③ Standard Deviation (SD) Best	③ coeffi. of variation
④ Quartile Deviation (QD)	④ coeffi. of quartile deviation
Best measure of disp. for data with open class intervals	Best measure of Dispersion for comparison purpose is coeffi. of variation

31 **For simple data :**

Range = $(L - S)$ where L = Largest observation & S = smallest observation.

M.D = $\left[\frac{\Sigma|x - AM|}{n}\right]$

S.D = $\sqrt{\frac{\Sigma(x-AM)^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

Q.D = $\left(\frac{Q_3 - Q_1}{2}\right)$ = Semi-interquartile Range

My Notes

Find Range, SD, MD, QD for 10, 30, 25, 35

$\Longrightarrow$ ① Range = $L - S = 35 - 10 = 25$

②

x	$\lvert x - AM \rvert$	$(x-AM)^2$
10	15	225
30	5	25
25	0	0
35	10	100
	30	350

$MD = \frac{\Sigma|x - AM|}{n} = \frac{30}{4} = 7.50$

$SD = \sqrt{\frac{\Sigma(x-AM)^2}{n}} = \sqrt{\frac{350}{4}} = 9.3541$

③ 10, 25, 30, 35 $Q_3 = \left(\frac{3}{4} \times 5\right)^{th}$ term $= 3.75^{th}$ term

$Q_1 = \left(\frac{1}{4} \times 5\right)^{th}$ term $= 10 + 0.25(25 - 10)$

$= 30 + 0.75(5) = 33.75$

$= 13.75$ $\quad = 30 + 0.75(5) = 33.75$

$$Q.D. = \left(\frac{33.75 - 13.75}{2}\right) = 10$$

32 Find Range, M.D, S.D, Q.D for - 20,28,35,40,48,60,65,68

① Range = L − S = 68 − 20 = 48

②

x	$\lvert x - AM \rvert$
20	25.50
28	17.50
35	10.50
40	5.50
48	2.50
60	14.50
65	19.50
68	22.50
	118

AM = 45.50

$$\text{Mean Deviation} = \frac{\sum \lvert x - AM \rvert}{n} = \frac{118}{8}$$

$$= 14.75$$

③ $SD = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} = \sqrt{\frac{18762}{8} - 45.50^2} = \sqrt{275}$

$= 16.5831$

④ $Q_3 = \left[\frac{3}{4}(8+1)\right]^{th}$ term $= 6.75^{th}$ term $= 60 + 0.75(65-60)$

$= 63.75$

$Q_1 = \left[\frac{1}{4}(8+1)\right]^{th}$ term $= 2.25^{th}$ term $= 28 + 0.25(35-28)$

$= 29.75$

$$Q.D. = \left(\frac{Q_3 - Q_1}{2}\right) = \left(\frac{63.75 - 29.75}{2}\right) = 17.00$$

33 For Grouped data :

Range = $(L - S)$ where L = UCB of Last CI, S = LCB of First CI

M.D = $\left[\frac{\sum f \lvert x - AM \rvert}{N}\right]$

S.D = $\sqrt{\frac{\sum f (x - AM)^2}{N}} = \sqrt{\frac{\sum f \cdot x^2}{N} - (\bar{x})^2} = \sqrt{\frac{\sum f x^2}{N} - \left(\frac{\sum f x}{N}\right)^2}$

Q.D = $\left(\frac{Q_3 - Q_1}{2}\right)$ where $Q_3 = LCB + \left(\frac{\frac{3N}{4} - N_l}{N_u - N_l} \times c\right)$ & $Q_1 = LCB + \left[\frac{\frac{1}{4}N - N_l}{N_u - N_l} \times c\right]$

My Notes

x	10	20	40	80
f	2	3	11	19

Find SD

x	x^2	f	$f \cdot x$	$f \cdot x^2$
10	100	2		
20	400	3		
40	1600	11		
80	6400	19		
		35	2040	1,40,600

SD

$$= \sqrt{\frac{1,40,600}{35} - \left(\frac{2040}{35}\right)^2}$$

$$= 24.8982$$

34 Find Range, M.D, S.D, Q.D

CI	10-20	20-30	30-40	40-50
f	5	7	2	6

$\Rightarrow$

① Range $= L - S = 50 - 10 = 40$

②

x	f	fx	$\lvert x-AM\rvert$	$f\lvert x-AM\rvert$
15	5		14.50	
25	7		4.50	
35	2		5.50	
45	6		15.50	
		590		208

$AM = \frac{590}{20} = 29.50$

Mean deviation about AM

$= \frac{\sum f\lvert x-AM\rvert}{N} = \left(\frac{208}{20}\right) = 10.40$

③

x	x^2	f	$f.x^2$
15	225	5	
25	625	7	
35	1225	2	
45	2025	6	
			20,100

$SD = \sqrt{\frac{20,100}{20} - (29.50)^2}$

$= 11.60819$

④

CB	cf
10	0
20	5
30	12
40	14
50	20

$Q_3 = 40 + \frac{15 - 14}{20-14} \times 10$

$= 41.66666$

$Q_1 = 20$

$Q.D. = \left(\frac{41.66666 - 20}{2}\right)$

$= 10.83333$

35 Find Missing Frequency if median = 32

CI	0-10	10-20	20-30	30-40	40-50	50-60
f	10	m	25	30	25 - m	10

Total Frequency = 100

$\Rightarrow$ sum of missing frequencies $= 100 - 75 = 25$

CB	cf
0	0
10	10
20	10+m
30	35+m ← N_l
40	65+m ← N_u
50	90
60	100 = N

Median class : 30-40

$\text{Median} = LCB + \left(\frac{\frac{1}{2}N - N_l}{N_u - N_l} \times C\right)$

$32 = 30 + \left(\frac{50 - 35 - m}{65 + \cancel{m} - 35 - \cancel{m}} \times 10\right)$

$2 = \left(\frac{15 - m}{30} \times 10\right)$

$6 = 15 - m$

$\boxed{m = 9}$

$\therefore$ Missing frequencies are 9, 16

36 If Mode = 66. Find missing frequency

Modal class

CI	30-40	40-50	50-60	60-70	70-80	80-90
f	8	16	22	28	-	12

$f_{-1} = 22 \quad f_0 = 28 \quad f_1 = m$

$$\text{Mode} = LCB + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times c\right)$$

$$66 = 60 + \left(\frac{28-22}{56-22-m} \times 10\right)$$

$$6 = \frac{6}{34-m} \times 10$$

$$34 - m = \left(\frac{6}{6} \times 10\right) = 10$$

$\boxed{m = 24}$ $\quad \therefore$ Missing Frequency $= 24$

37 S.D of 2 Observations = $\left(\frac{\text{Range of 2 observation}}{2}\right)$

Range of 30, 40 = 10 $\quad \therefore$ SD of 30, 40 = 5

S.D of 1st 'n' natural numbers =

SD of 1, 2, 3, 4, 5, 6, ... n $= \sqrt{\left(\frac{n^2-1}{12}\right)}$

$\boxed{\text{SD for 2 observations is always half of the Range}}$

38

M.D about	Simple Data	Grouped Data
AM	$\left[\frac{\sum \lvert x - AM \rvert}{n}\right]$	$\left[\frac{\sum f \lvert x - AM \rvert}{N}\right]$
Median	$\left[\frac{\sum \lvert x - Median \rvert}{n}\right]$	$\left[\frac{\sum f \lvert x - Median \rvert}{N}\right]$
Mode	$\left[\frac{\sum \lvert x - Mode \rvert}{n}\right]$	$\left[\frac{\sum f \lvert x - Mode \rvert}{N}\right]$

39

Q.D = Semi inter Quartile Range = $\left(\frac{Q_3 - Q_1}{2}\right)$

Coefficient of Quartile Deviation = $\left(\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100\right)$

Inter quartile Range = $(Q_3 - Q_1)$

40

If y = a + bx then

AM of y = a + b(AM of x)

Median of y = a + b (Median of x)

Mode of y = a + b (Mode of x)

AM, Median, Mode are affected by change of origin as well as by change in scale

$R_y = |b| \times R_x$

$SD_y = |b| \times SD_x$

$MD_y = |b| \times MD_x$

$QD_y = |b| \times QD_x$

Range, MD, SD, QD are affected only by change in scale & Not by shift of origin

41

	Old Data	If 15 is subtracted from each obs^n	If every observation is increased by 5	If every observation is mutliplied by 10	If every observation is divided by 20
AM	30	15	35	300	1.50
Median	50	35	55	500	2.50
Mode	60	45	65	600	3.00
Range	70	70	70	700	3.50
MD	28	28	28	280	1.40
SD	36	36	36	360	1.80
QD	55	55	55	550	2.75

My Notes

Data	Every obs^n is increase by 5	Every obs^n is decreased by 30	Multiplied oR divided by k
coeffi. of variation $= \frac{SD}{AM} \times 100$	Decreases	Increases	Remains same

42 Impact on coefficient of variation :

If 20 is added to each observation	coeffi. of variation decreases
If 30 is subtracted from each observation	coeffi. of variation increases
If every observation is multiplied by 80	coeffi. of variation remains same
If every observation is divided by 100	coeffi. of variation remains same

43

Runs of last 8 innings

Batsman A 80, 60, 65, 85, 75, 40, 35, 20

Batsman B 35, 25, 50, 25, 55, 60, 25, 15

Who is more consistent?

	Batsman A	Batsman B
AM	57.50	36.25
SD	$\sqrt{\frac{30,300}{8} - 57.50^2}$ $= 21.9374$	$\sqrt{\frac{12450}{8} - 36.25^2}$ $= 15.5624$
coeffi. of variation $= \frac{SD}{AM} \times 100$	38.152	42.9308

Lesser coeffi. of variation represents Greater consistency

Batsman A is more consistent as compared to Batsman B as he has lesser coeffi. of variation

My Notes

Batsman	A	B	C	D	E
SD of Runs	20	5	60	31	1
Avg of Runs (AM)	103	65	125	40	60
coeffi. of variation	19.417	7.6923	48	77.50	1.6666
Rank of consistency	3rd	2nd	4th	5th	1st

44

Best measure of dispersion	Standard Deviation
For comparison purpose	coeffi. of variation
For Open Class Intervals	Quartile Deviation

positional measure of central tendency is : MEDIAN.
positional measure of dispersion is : quartile deviation.

45 Find S.D, Variance, Coefficient of Variation for 18,19,20,28,35.

$\Longrightarrow$ variance $= \dfrac{\Sigma(x-\bar{x})^2}{n} = \dfrac{\Sigma x^2}{n} - \bar{x}^2$

$= \dfrac{3094}{5} - 24^2 = 42.80$

$SD = \sqrt{42.80} = 6.542171$

coeffi. of variation $= \left(\dfrac{SD}{AM} \times 100\right) = \left(\dfrac{6.542171}{24} \times 100\right)$

$= 27.259045$

variance $= (S.D.)^2$

$S.D. = \sqrt{\text{variance}}$

coeffi. of variation : SD of data expressed as % of AM of data is known as coeffi. of variation

46

Observations x,y,z

AM = $\left(\dfrac{x+y+z}{3}\right)$

GM = $\sqrt[3]{xyz}$
$= (xyz)^{1/3}$

HM = $\left[\dfrac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right]$

47 Find Range & Coefficient of range for : ₹90, ₹80, ₹60, ₹30, ₹10, ₹5, ₹65, ₹78

$\Longrightarrow$ ① Range = L − S = ₹90 − ₹5 = ₹85

~~(a)~~ ₹85 (b) 85 (c) 89.4737 (d) ₹89.4737

② coeffi. of Range $= \left(\dfrac{L-S}{L+S}\right) \times 100 = \dfrac{₹85}{₹95} \times 100$

My Notes

$= 89.47$

(a) ₹85 (b) 85 ~~(c)~~ 89.4737 (d) ₹89.4737

- Relative measures of dispersion are unit-free. whereas absolute measures of dispersion are Not unit-free

85 cms, 50cms, 150 cms → Range = 150cms − 50 cms = 100 cms

coeffi. of Range $= \dfrac{100\text{cms}}{200\text{ cms}} \times 100 = 50.00$

48 If $3x + 5y = 85$; AM of x = 3; SD of x = 0.75. Find AM of y, S.D of y

$\Longrightarrow \quad 3x + 5y = 85$

$3(\text{AM of } x) + 5(\text{AM of } y) = 85$

$3(3) + 5(\text{AM of } y) = 85$

$\therefore 5 \times \text{AM of } y = 76$

$\text{AM of } y = 15.20$

First we will bring eqn in the form of $y = a + bx$

$5y = 85 - 3x$

$y = \frac{85}{5} - \frac{3}{5}x$

$SD_y = |b| \times SD_x$

$= \frac{3}{5} \times 0.75 = 0.45$

49 **Properties of Median**

1. If $y = a + bx$, then (Median of y) = a + b(Median of x)

If $ax + by + c = 0$; then a(median of x) + b(median of y) + c = 0

2. For a set of observations, the sum of absolute deviations is minimum when deviations are taken from median.

$$\Sigma | x\text{-median} | = \text{minimum}$$

50 For 2 Observations GM = 9; AM = 10. Find HM.

$\Longrightarrow \quad GM^2 = AM \times HM$ ------- For 2 observations

$9^2 = 10 \times HM$

$HM = 81/10 = 8.10$

$\boxed{\text{For 2 observations GM is GM of AM \& HM}}$

$\boxed{AM = 10,\ GM = 9.00,\ HM = 8.10}$ $\Longrightarrow$ They will form a G.P.

51 AM : ~~a.~~ is Best measure of central tendency.
~~b.~~ is rigidly defined.
~~c.~~ based on all observations
~~d.~~ easy to comprehend, easy to calculate
~~e.~~ amenable to mathematical properties.

However drawback of AM is - it is very much affected by sampling fluctuation and AM can't be calculated for data with open-end classification.

My Notes

Find AM, Median, Mode for

80, 60, 90, 90, 90, 30, 20, 15

$\Longrightarrow$ ① $AM = \frac{475}{8} = 59.375$

② $Median = \left(\frac{60+80}{2}\right) = 70$ $\quad$ 15, 20, 30, $\boxed{60, 80}$, 90, 90, 90

③ Mode = 90

AM, Median $\Rightarrow$ uniquely defined

Mode $\Rightarrow$ NOT uniquely defined as data can have 2 OR more modes.

52 Median

~~a~~. Median is also rigidly defined.

~~b~~. Easy to comprehend and calculate.

~~c~~. It is positional average of data.

~~d~~. It is the central number when data is arranged is ascending or descending order of their magnitude. (Median is positional avg of data)

~~e~~. Median is not based on all observations. (Median is not affected by extreme observations)

~~f~~. Most appropriate measure of central tendency for open-end classification.

54 Measures of central tendency for a given set of observations measures

a. Scatterness of Observations
~~b~~. Central location of observations
c. Both of these
d. None of these

55 While computing AM from a grouped frequency distribution, we assume that

a. The classes are of equal length
b. The classes have equal frequency
~~c~~. All the values of a class are equal to mid value of class.
d. None of these

	class mark
10-20	15
20-50	35
50-100	75

we assume that all obs^n s are equal to 15

we assume that all obs^n's are equal to 35.

56 Which of the following is true

a. Usually AM is Best measure of dispersion
~~b~~. Usually SD is Best measure of dispersion
c. Both of these
d. None of these

My Notes

① AM : Best measure of central tendency

② SD : Best measure of Dispersion

③ QD : Best / Most appropriate/suitable Measure of dispersion for data with open class intervals

④ Median : Best / Most appropriate/suitable Measure of central tendency for data with open class intervals

⑤ coeffi. of variation : Best measure of dispersion for comparison purpose.

57 **Which of the following is not uniquely defined**

a. Mean b. Median ~~c.~~ Mode d. All of these

Find mode of 80, 90, 60, 90, 80, 100, 120, 120, 180, 185

$\Longrightarrow$ Modes are : 80, 90, 120

this is Tri-modal data.

58 **Weighted average are considered when**

Weighted avg = $\left(\frac{\Sigma wx}{\Sigma w}\right)$

a. The data are not classified

b. The data are put in the form of grouped freq. distribution.

c. All observations are not of equal importance

d. All of these

where 'w' represents weight = Relative importance

59 **Which of the following is correct for a set of 'n' district positive observations.**

a. AM $\geq$ GM $\geq$ HM ~~b.~~ AM > GM > HM

c. GM < AM < HM d. None of these

60 **When the firm registers both profits and losses then, which of the following measure of central tendency can not be considered?**

a. AM ~~b.~~ GM c. Median d. Mode

61 **Quartiles are the values dividing given set of observations into**

a. 2 equal parts b. 4 parts ~~c.~~ 4 equal parts d. 3 parts

- 3 quartiles divides the data in 4 equal parts

My Notes

① Stubs are : Left part of table describing rows

② captions are : upper part of table describing columns & sub-columns

③ Numerical figures are written in : Body of table

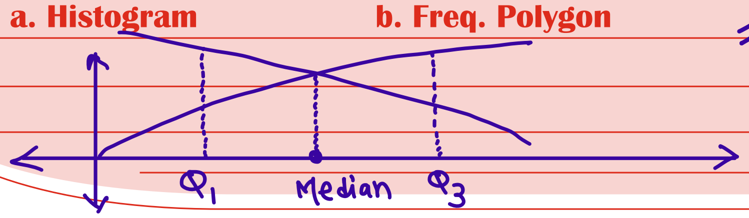
④ unit of measurement is mentioned in : Box-Head

⑤ source of data is mentioned in : Foot-Note

⑥ Head Note : Addi. info about title

62 **Quartiles can be determined graphically using**

a. Histogram b. Freq. Polygon ~~c.~~ Ogives d. Pie-charts



Median is also known as
2nd quartile = Q_2
Lower quartile = Q_1
upper quartile = Q_3

63 **Which of the following measure satisfy linear relationship between 2 variables.**

a. AM b. Median c. Mode ~~d.~~ All of these

64 **What is GM of 8, 24, 40**

a. 24 b. 12 ~~c.~~ $8 \times \sqrt[3]{15}$ d. $\sqrt[5]{7680}$

$$GM = \sqrt[3]{8 \times 24 \times 40} = \sqrt[3]{8 \times 8 \times 3 \times 8 \times 5} = \sqrt[3]{8^3} \times \sqrt[3]{15}$$
$$= 8 \times \sqrt[3]{15}$$

65 **H.M of 2, 3, 5 is**

a. 2.00 b. 3.33 ~~c.~~ 2.90 d. $\sqrt[3]{30}$

$$HM = \left(\frac{3}{\frac{1}{2} + \frac{1}{3} + \frac{1}{5}}\right) = 2.90$$

66 **AM and HM of 2 numbers are 5 and 3.20 resp. then GM will be**

a. 16.00 b. 4.10 c. 4.05 ~~d.~~ 4.00

$$GM^2 = AM \times HM$$
$$= 5 \times 3.20 = 16$$
$$\therefore GM = \sqrt{16} = 4.00$$

67 **Find value of first/lower quartile for 15, 18, 10, 20, 23, 28, 12, 16**

a. 17 b. 16 ~~c.~~ 12.75 d. None of these

$\Longrightarrow$ 10, 12, 15, 16, 18, 20, 23, 28

$$Q_1 = \left[\frac{1}{4}(8+1)\right]^{th} \text{term} = 2.25^{th} \text{ term}$$
$$= 12 + 0.25(15-12)$$
$$= 12.75$$

My Notes

Find D_6 for 10, 13, 16, 19, 21, 23, 28, 35, 38, 40, 48, 50, 53, 59, 63, 68, 73, 78

$$\Longrightarrow D_6 = \left[\frac{6}{10}(18+1)\right]^{th} \text{term} = 11.40^{th} \text{ term}$$
$$= 48 + 0.40(50-48)$$
$$= 48.80$$

68 Third decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12 is

a. 13 b. 10.70 c. 11 d. 11.50

$\Longrightarrow$ 9, 10, 11, 12, 15, 18, 20, 25

$$D_3 = \left[\frac{3}{10}(8+1)\right]^{th} \text{term} = 2.70^{th} \text{ term} = 10 + 0.70(11-10) = 10.70$$

69 If average salary of unskilled workers is ₹ 10,000 and that of group of skilled workers is ₹ 12,000, what is the % of skilled workers?

a. 40% b. 50% c. 60% ~~d.~~ None of these

(Insufficient data)

skilled $= n_1$

unskilled $= n_2$

70 If there are 2 groups with 75, 65 as Harmonic Mean and containing 15,13 observations then combined HM is given by

a. 65 b. 70.36 c. 70.81 ~~d.~~ None of these

$$\text{combined HM} = \left(\frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}\right) = \left[\frac{15+13}{\left(\frac{15}{75}+\frac{13}{65}\right)}\right] = 70$$

71 What is HM of 1, 1/2, 1/3, 1/4,........1/n

a. n b. 2n ~~c.~~ 2/(n+1) d. n(n+1)/2

$$HM = \frac{n}{\sum \frac{1}{x}} = \frac{n}{1+2+3+4+\dots+n} = \frac{\cancel{n}}{\frac{\cancel{n}(n+1)}{2}} = \frac{2}{(n+1)}$$

72 An aeroplane flies from A to B at a speed of 500 kms/hr and comes back from B to A at 700 kms/hr. The avg, speed of entire journey is :

a. 600 kms/hr ~~b.~~ 583.33 kms/hr c. 100 $\sqrt{35}$ kms/hr d. None

HM of 500 & 700

$$= \frac{2}{\frac{1}{500}+\frac{1}{700}} = 583.33333333 \text{ kms/hr}$$

My Notes

Avg speed of Journey can be calculated with the help of HM of data.

73 If the variable assumes the values 1,2,3,4,5 with frequencies 1,2,3,4,5 then what is AM?

~~a.~~ 11/3 b. 5 c. 4 d. 4.50

$\Longrightarrow$

x	1	2	3	4	5
f	1	2	3	4	5
$f.x$	1	4	9	16	25

$AM = \frac{\Sigma fx}{\Sigma f} = \frac{55}{15}$

$= 11/3$

74 GM of x is 10 and GM of y is 10 then GM of x.y is

a. 150 b. log10 x log15 c. log150 ~~d.~~ None of these

75 If AM and GM for 10 observations are both 15, then value of HM is :

a. Less than 15 b. More than 15 ~~c.~~ 15 d. None of these

(All observations are same)

76 Find Range of 65 cms, 20 cms, 100 cms, 90 cms, 81 cms

~~a.~~ 80 cms b. 80 c. 66.66666cms d. 66.66666

Range = 100 cms − 20 cms

= 80 cms

77 Find Coefficient of Range for 65 cms, 20 cms, 100 cms, 90 cms, 81 cms

a. 80 cms b. 80 c. 66.66666cms ~~d.~~ 66.66666

coeffi. of Range = $\left(80\text{cms}/120\text{cms}\right) \times 100 = 66.666666$

78 Find S.D and Range for 80 cms, 20 cms.

$\Longrightarrow$ Range = 80 cms − 20 cms = 60 cms

S.D. = $\left(\frac{60 \text{ cms}}{2}\right)$ = 30 cms

$\boxed{\text{Pls Remember : For 2 obs}^n\text{s SD is always half of Range}}$

79 Find S.D of first 25 natural numbers is :

$\Longrightarrow$ SD of first 'n' natural numbers $= \sqrt{\frac{n^2-1}{12}}$

SD of first 'n' natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{25^2-1}{12}} = 7.2111$

My Notes

SD of first n natural numbers is 5.76628129733.

Find 'n'.

$\Longrightarrow \sqrt{\frac{n^2-1}{12}} = 5.76628129733$

$\frac{n^2-1}{12} = 33.25$

$n^2 - 1 = 399$

$n^2 = 400$

$n = 20$

If $y = a + bx$ then
$SD_y = |b| \times SD_x$
variance of $y = b^2 \times$ vari. of x

80 Properties of SD

~~1~~. If all observations are same then SD is zero.

~~2~~. SD is unaffected by change of origin but affected by change in scale.

~~3~~. Combined SD = $\sqrt{\dfrac{n_1S_1^2 + n_2S_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

$d_1 = |\bar{x}_1 - \bar{x}|$

$d_2 = |\bar{x}_2 - \bar{x}|$

81 If AM and coeffi. of variation of x are 10,40 resp. what is the variance of (15-2x)?

$\Longrightarrow$ coeffi. of variation $= \dfrac{SD}{AM} \times 100$

$40 = \dfrac{SD \text{ of } x}{10} \times 100$

SD of $x = 4$

$y = 15 - 2x$

$SD_y = |b| \times SD_x = 2 \times 4 = 8$

$\therefore$ variance of $y = 64$

variance of $(15-2x) = 64$

82 Range is quickest to compute. However range is based on only 2 observations and affected too much by presence of extreme observations.

83 If profit of the company remains the same for last 10 months then SD of profit would be

~~a~~. zero   b. positive   c. negative   d. a or c

84 Which measure of dispersion is considered for finding a pooled measure of dispersion after combining several groups :

a. MD   ~~b~~. SD   c. QD   d. Range

85 If all observations are increased by 25 then

AM	increases by 25.
Median	increases by 25.
Mode	increases by 25.
Range	Remains same.
MD	Remains same.
SD	Remains same.
QD	Remains same.
Coeff. of Variation	Decreases.

My Notes

Find MD, SD, QD, Range, AM for

80, 80, 80, 80, 80, 80, 80, ----------

$\Longrightarrow$ AM = 80

Range = MD = SD = QD = 0 = Zero.

86 If all observations are multiplied by 10 then

AM	becomes 10 times
Median	becomes 10 times
Mode	becomes 10 times
Range	becomes 10 times
MD	becomes 10 times
SD	becomes 10 times
QD	becomes 10 times
Coeff. of Variation	Remains same

87 If y = -8x+500 and Range of x = 45, Range of y = ?

$y = 500 - 8x$

$R_y = |b| \times R_x$

$R_y = 8 \times 45 = 360$

88 If all observations are multiplied by -8 then Range becomes

a. -8 times ~~b.~~ 8 times c. $(1/8)^{th}$ d. None of these

Range, SD, MD, QD : Are always Non-Negative.

89 Find coefficient of MD about AM for first 9 natural numbers.

a. 200/9 b. 80 ~~c.~~ 400/9 d. None of these

x	1	2	3	4	5	6	7	8	9
$\lvert x-AM\rvert$	4	3	2	1	0	1	2	3	4

$\sum |x - AM| = 20$

MD about AM $= \frac{20}{9}$

$= 2.22222$

coeffi. of MD about AM

$= \left(\frac{\text{MD about AM}}{\text{AM}}\right) \times 100$

$= \left(\frac{2.22222}{5} \times 100\right) = 44.44444$

My Notes

coeffi. of MD about AM $= \left(\frac{\text{MD about AM}}{\text{AM}} \times 100\right)$

coeffi. of MD about Median $= \left(\frac{\text{MD about Median}}{\text{Median}} \times 100\right)$

coeffi. of MD about Mode $= \left(\frac{\text{MD about Mode}}{\text{Mode}} \times 100\right)$

simple and Grouped data

90 If 2x - 3y = -7; AM of x = 1; MD of x = 0.30. Find coeff. of MD about AM for y.

a. 12 b. 50 c. 4 ~~d.~~ None of these

$\Rightarrow 2x - 3y = -7$

$2(\text{AM of } x) - 3(\text{AM of } y) = -7$

$2(1) + 7 = 3 \times \text{AM of } y$

AM of $y = 3$

$2x - 3y = -7$

$-3y = -7 - 2x$

$y = \frac{7}{3} + \frac{2}{3}x$

$MD_y = \frac{2}{3} \times 0.30$

$= 0.20$

coeffi. of MD abt AM for y

$= \frac{0.20}{3} \times 100$

$= 6.666666$

91 Find MD about Mode for : 4/11, 6/11, 8/11, 9/11, 12/11, 8/11

~~a.~~ 1/6 b. 1/11 c. 6/11 d. 5/11

Mode $= 8/11$

x	$\lvert x - \text{mode}\rvert$
4/11	4/11
6/11	2/11
8/11	0
9/11	1/11
12/11	4/11
8/11	0

$11/11 = 1.00$

MD about mode

$= \frac{\sum \lvert x - \text{mode}\rvert}{n} = \frac{1.00}{6}$

$= 1/6$

92 What is standard deviation of 5, 5, 9, 9, 9, 10, 5, 10, 10

a. $\sqrt{14}$ ~~b.~~ $\sqrt{(42)}/3$ c. 4.50 d. None of these

$SD = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{618}{9} - 8^2} = 2.160246$

93 AM and SD of x are a, b resp. then SD of [(x - a) / b] is

a. -1 ~~b.~~ 1 c. ab d. a/b

AM of $x = a$

SD of $x = b$

$y = \frac{x - a}{b}$

$\boxed{y = -\frac{a}{b} + \frac{1}{b}x}$

$SD_y = |b| \times SD_x$

$= \frac{1}{b} \times b$

$= 1.00$

My Notes

If $8x + 13y = 28x - 19y + 20$ &

$QD_y = 5.75$, $QD_x = ?$

$\Rightarrow 32y = 20 + 20x$

$y = \frac{20}{32} + \frac{20}{32}x$

$QD_y = \frac{20}{32} \times QD_x$

$5.75 = \frac{20}{32} \times QD_x$

$\therefore QD_x = 9.20$

94 If quartiles of a variables are 45, 52, 65 resp. Find quartile deviation.

~~a. 10~~ b. 20 c. 25 d. 8.30

$\Longrightarrow$ $Q_1 = 45$ Q.D.

$Q_2 = 52$ $= \left(\frac{Q_3 - Q_1}{2}\right) = \left(\frac{65-45}{2}\right) = 10.00$

$Q_3 = 65$

95 Standard Deviation of first 'n' natural number is 2 then find 'n'

a. 2 ~~b. 7~~ c. 6 d. 5

$\Longrightarrow$ $\sqrt{\frac{n^2-1}{12}} = 2$

$\frac{n^2-1}{12} = 4$

$n^2 - 1 = 48$

$\therefore n^2 = 49$

$n = 7$

$\boxed{\text{SD of first 'n' natural numbers} = \sqrt{\left(\frac{n^2-1}{12}\right)}}$

96 If $n_1 = 30$, $n_2 = 20$, $\bar{x}_1 = 55$, $\bar{x}_2 = 60$, $S_1 = 4$, $S_2 = 5$; Find combined SD.

a. 5.00 ~~b. 5.06~~ c. 5.23 d. 5.35

$\Longrightarrow$ combined AM $= \frac{30(55) + 20(60)}{30+20} = 57$

$d_1 = |55-57| = 2$

$d_2 = |60-57| = 3$

$\therefore$ combined SD $= \sqrt{\frac{30(16) + 20(25) + 30(4) + 20(9)}{30+20}}$

$= \sqrt{25.60} = 5.0596$

97 The mean and SD of sample of 100 observations were calculated as 40 and 5.10 respectively. one observation was taken as 50 instead of 40 by mistake. The correct SD is -

a. 4.90 ~~b. 5.00~~ c. 4.88 d. 4.85

$\Longrightarrow$

wrong AM $= 40 = \frac{\Sigma x}{100}$

wrong $\Sigma x = 4000$

correct $\Sigma x = 4000 - 50 + 40$

$= 3990$

correct AM $= 3990/100$

$= 39.90$

$5.10 = \sqrt{\frac{\Sigma x^2}{100} - 40^2}$

wrong $\Sigma x^2 = 162601$

correct $\Sigma x^2 = 162601 - 50^2 + 40^2$

$= 1,61,701$

correct SD $= \sqrt{\frac{161701}{100} - 39.90^2} = 5.00$

98 The words "mean" or "average" only refers to

~~a.~~ AM | b. GM | c. HM | d. None of these

99 Mean is of ____________ types.

a. 5 | b. 4 | ~~c.~~ 3 | d. None of these

100 AM is never less than GM.

(AM ≥ GM ≥ HM)

~~a.~~ True | b. False

101 AM is always more than HM.

(Because AM & HM can be equal sometimes)

a. True | ~~b.~~ False

102 GM of set of 'n' observations is the _______ root of their product.

a. $(n/2)^{th}$ | b. $(n/4)^{th}$ | ~~c.~~ n^{th} | d. $(n-1)^{th}$

103 GM of 8, 4, 2 is

$GM = \sqrt[3]{8 \times 4 \times 2} = \sqrt[3]{64} = 4.00$

~~a.~~ 4 | b. 2 | c. 8 | d. None of these

104 Median is unaffected by extreme values.

~~a.~~ True | b. False

105 When all observations occur with equal frequency ______ does not exist.

a. AM | b. Median | ~~c.~~ Mode | d. HM

106 Find Mode of 8, 8, 3, 3, 8, 3, 8, 8, 8, 3, 3, 3

a. 8 | b. 3 | c. 8 & 3 | ~~d.~~ No mode for this data

107 Find Mode of 8,8,3,3,8,3,8,8,8,3,3,3,10 is

a. 8 | b. 3 | ~~c.~~ 8 & 3 | d. No mode for this data

108 Simple average is sometimes called as

a. Weighted Avg. | ~~b.~~ Unweighted Avg. | c. Both | d. None

109 Multiplying the values of the variables by their corresponding weights and then dividing the sum by sum of weights is _________

weighted avg $= (\Sigma wx / \Sigma w)$

a. Simple Avg. | ~~b.~~ Weighted Avg. | c. Both | d. None

110 Simple and Weighted Average are equal when all weights are equal.

~~a.~~ True | b. False

when Importance of all observations is not same then concept of weighted avg. is useful

111 Frequencies are generally used as

a. Range | ~~b.~~ Weights | c. Mean | d. None

112 The values of all items are taken into consideration in calculation of

~~a.~~ AM | b. Median | c. Mode | d. None of these

113 GM can be calculated only when all observations have same sign & none is zero.

~~a~~. True b. False

If one of the obs^n is zero then HM = Not defined

114 HM is defined when No observation is

a. 3 b. 2 c. 1 ~~d~~. zero

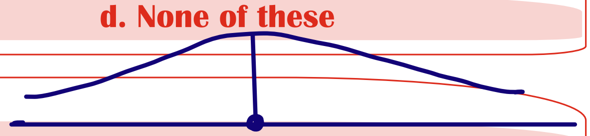
115 The class in which 'mode' belongs is known as :

a. Median Class b. Mean Class ~~c~~. Modal Class d. Backward Class

116 For calculation of ____________ we need to find cumulative frequency.

a. AM ~~b~~. Median c. Mode d. None of these

117 When distribution is symmetrical mean, median, mode



~~a~~. Coincide b. Do not coincide c. Both d. None

AM = median = mode

118 The no. of observations smaller than ________ is equal to no. of observations larger than it.

~~a~~. Median b. Mode c. Mean d. None of these

119 ____________ quartile is known as upper quartile.

Q_3 = upper quartile

a. First b. Second ~~c~~. Third d. Fourth

120 Second quartile is also known as

Q_2 = Median = D_5 = P_{50}

a. Lower quartile b. Upper quartile ~~c~~. Median d. Mode

121 Median = 2nd quartile = 5th Decile = 50th Percentile

~~a~~. True b. False

122 10th Percentile = ?

$P_{10} = D_1$

~~a~~. 1st Decile b. 1st Quartile c. Median d. None

123 25th Percentile = ?

$P_{25} = Q_1$

~~a~~. Q_1 b. D_{25} c. Q_3 d. Median

124 In ogive, abscissa corresponding to ordinate (N/2) is

~~a~~. Median b. 1st Quartile c. 3rd Quartile d. None

125 In ogive, abscissa corresponding to ordinate (3N/4) is

a. Median b. 1st Quartile ~~c~~. 3rd Quartile d. None

126 For 600, 300, 500, 300, 800, 200, 300, 550, 450, 350 rank of median is

a. 5 ~~b~~. 5.50 c. 5.05 d. 600

Rank of median $= \frac{1}{2}(n+1) = \frac{1}{2}(10+1) = 5.50$

127 For 81,23,51,93,103,28,36 rank of 1st Quartile is

a. 3 b. 1 ~~c~~. 2 d. 7

$Q_1 = \frac{1}{4}(7+1) = 2^{nd}$ term

128 Standard deviation is denoted by

a. $\bar{x}$ ~~b.~~ σ c. σ^2 d. None of these

129 The square of SD is known as __________ .

~~a.~~ Variance b. MD c. QD d. Square Man

130 $\frac{\sigma}{\bar{x}}$ x 100 = ?

$(\sigma/\bar{x}) \times 100$ = coeffi. of variation

a. AM b. MD c. QD ~~d.~~ Co-efficient of Variation

131 Find AM, GM, HM, for the data : a,b,c,d,e,f,g

$$AM = \left(\frac{a+b+c+d+e+f+g}{7}\right) \qquad GM = (abcdefg)^{1/7}$$

$$HM = \left[\frac{7}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}+\frac{1}{f}+\frac{1}{g}}\right]$$

132 For Observations : 18,18,18,18,18,18

AM = 18

HM = 18

GM = 18

Median = 18

Mode = No mode

Range = 0

MD = 0

SD = 0

QD = 0

coeffi. of variation = 0

My Notes

① Find mode of 8,8,3,3,3,8,6,6,6,10,10,10

$\Longrightarrow$ No mode as all observations have Equal Frequency.

② Find mode of 8,8,3,3,3,8,6,6,6,10,10,10,18

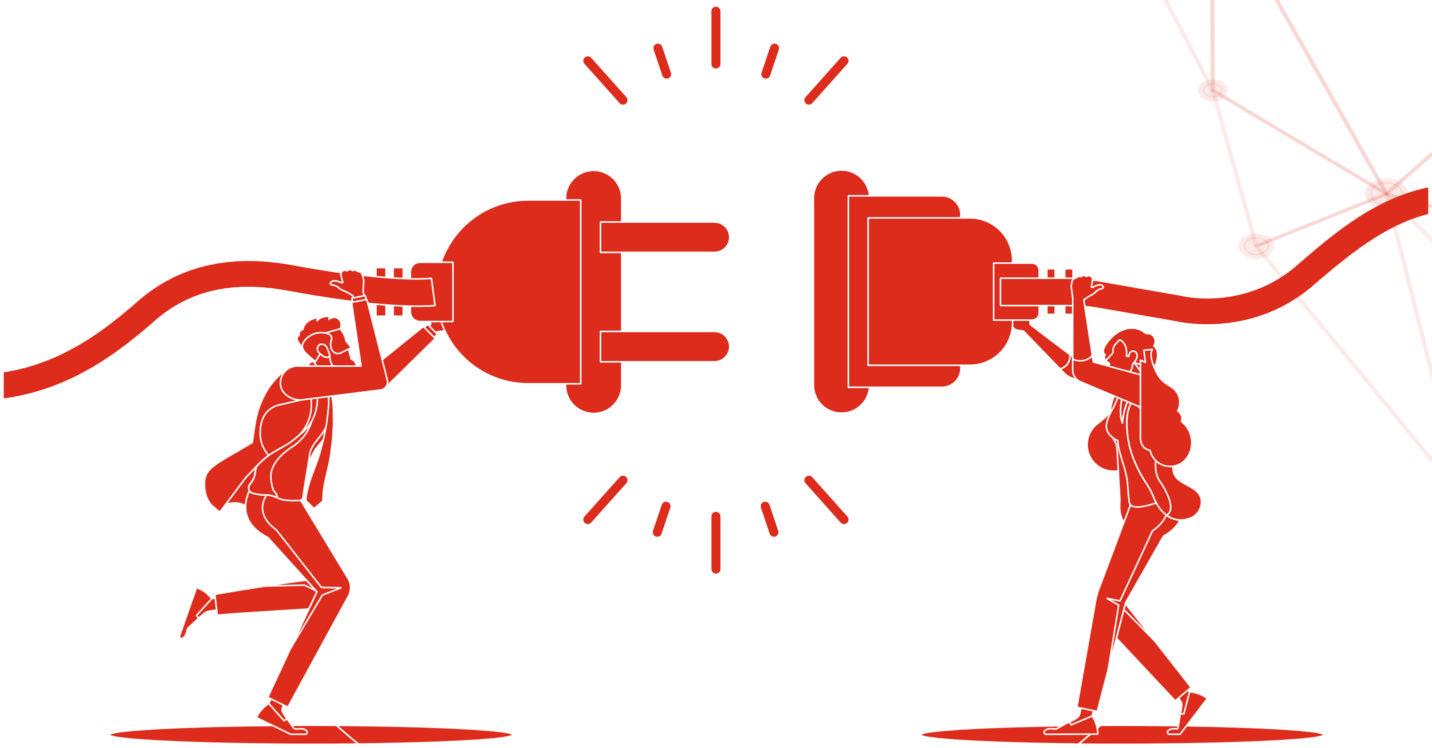
$\Longrightarrow$ Modes are : 8, 3, 6, 10

③

x	10	20	30	40	50
f	20	20	20	20	20

$\Rightarrow$ There is No mode for this data as all obs^{n}s have equal frequency.

CORRELATION AND REGRESSION ANALYSIS



CA VINOD REDDY

1 What is correlation and what is regression?

⟹ • correlation is the process of establishing relationship/association betn 2 OR more variables although they are not in proportion.

• Regression is the process of Determining the value of one variable on the basis of other.

• correlation is the pre-requisite to study Regression.

2 Whether correlation between 2 variables exists or not?

- Yes
  - ① What is the type / Nature of correlation?
    - positive

x	y
↑	↑
↓	↓

    - Negative

x	y
↑	↓
↓	↑

- No
  - ② What is the degree of correlation?
    - Perfect
    - small OR weak OR poor
    - Moderate
    - High OR strong

3 Methods to measure correlation between 2 variables :

a. Scatter Diagram
b. Spearman's rank correlation coefficient (r)
c. coeffi. of concurrent deviation (r)
d. karl pearson's product moment correlation coefficient (r) (Best)

My Notes

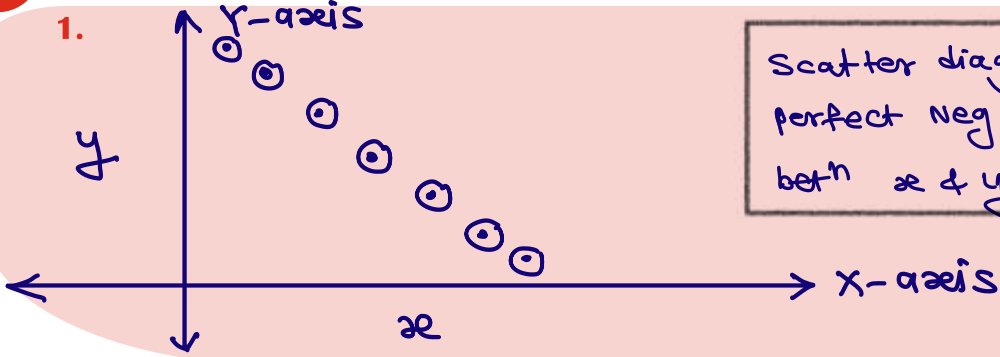
• studying association betn 2 variables :
Bi- variate correlation

• studying association betn 3 OR more variables :
Multi-variate correlation

• examples of positive correlation	examples of negative correlation
① price & supply	① Demand & price
② Temp, sale of cold drinks	② Temp, sale of Tea/coffee
③ industry Growth rate, Demand for CA's etc	③ No. of claims, profit of Insu. company

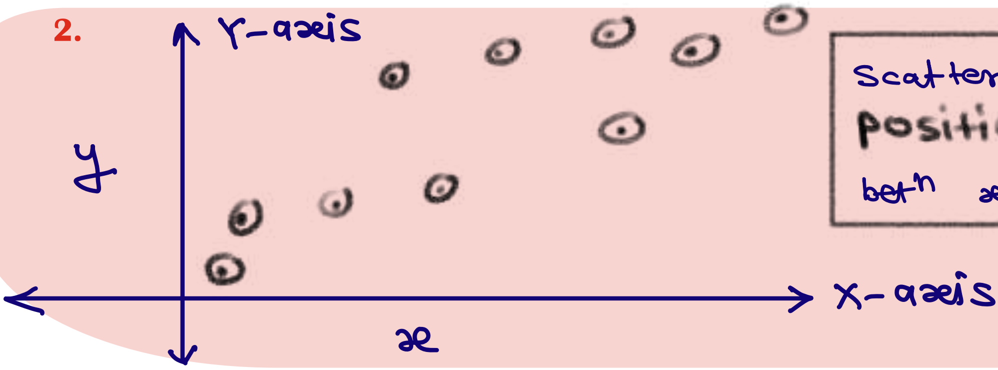
4 Scatter diagram showing

1.



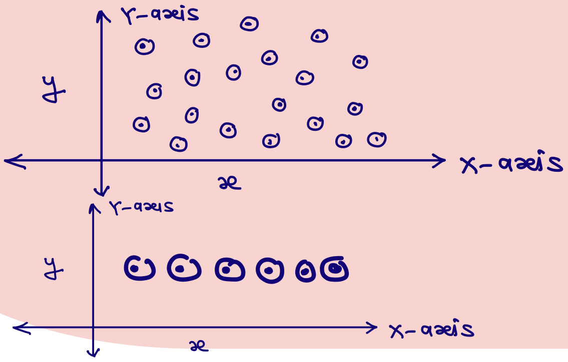
Scatter diagram is showing perfect Negative correlation betn x & y

2.



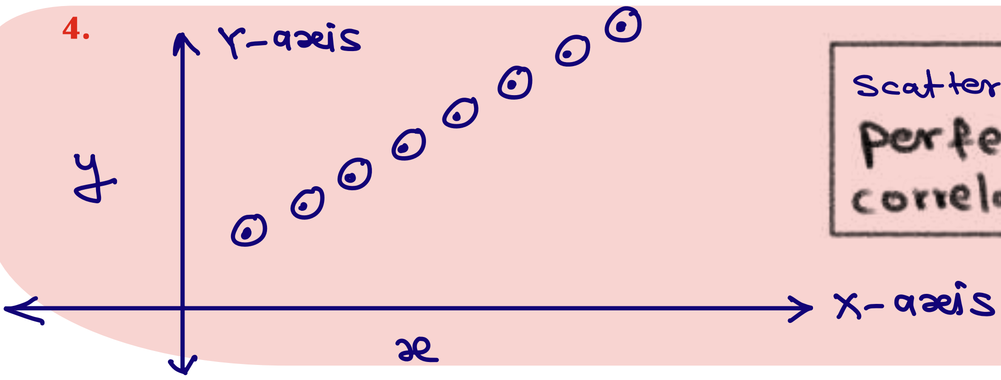
Scatter diagram is showing positive correlation betn x & y

3.



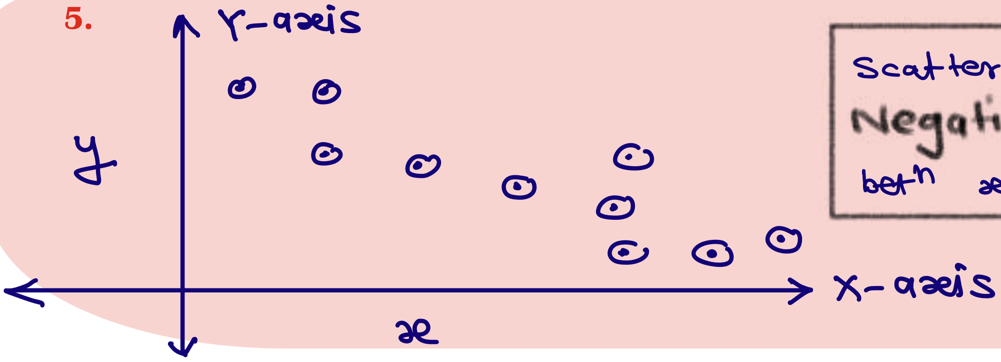
Scatter diagram is showing NO correlation betn x & y

4.



Scatter diagram is showing perfect position correlation betn x & y

5.



Scatter diagram is showing Negative correlation betn x & y

Scatter diagrams can give an idea about type of correlation but it can't give exact degree of correlation.

To know type as well as degree of correlation
↓
we need to obtain correlation coefficient

5 Find Spearman's rank correlation coefficient.

x	30	80	45	63	91	28	222
y	101	111	93	123	86	65	79

x	y	Rank of x	Rank of y	d^2
30	101	6	3	9
80	111	3	2	1
45	93	5	4	1
63	123	4	1	9
91	86	2	5	9
28	65	7	7	0
222	79	1	6	25
			$\sum d^2 =$	54

Spearman's Rank correlation coefficient

$$= 1 - \left[\frac{6\sum d^2}{n(n^2-1)}\right]$$

$$= 1 - \left[\frac{6 \times 54}{7 \times (7^2-1)}\right]$$

$$= 1 - \left(\frac{324}{336}\right)$$

$$= 0.0357$$

There is weak/poor/Low degree of positive correlation.

6 Find Spearman's rank correlation coefficient.

x	58	92	63	63	65	65	63	58
y	20	25	28	25	28	25	30	38

x	y	Rank-x	Rank-y	d^2
58	20	7.50	8	0.25
92	25	1	6	25
63	28	5	3.50	2.25
63	25	5	6	1
65	28	2.50	3.50	1
65	25	2.50	6	12.25
63	30	5	2	9
58	38	7.50	1	42.25
			$\sum d^2$	93

t = No. of observations involved in a 'tie'

$= 2, 3, 2, 2, 3$

$$\sum\left(\frac{t^3-t}{12}\right) = \left(\frac{2^3-2}{12}\right) + \left(\frac{3^3-3}{12}\right) + \left(\frac{2^3-2}{12}\right) + \left(\frac{2^3-2}{12}\right) + \left(\frac{3^3-3}{12}\right)$$

$= 0.50 + 2 + 0.50 + 0.50 + 2$

$= 5.50$

$$r = 1 - \left[\frac{6\left(\sum d^2 + \sum \frac{t^3-t}{12}\right)}{n(n^2-1)}\right]$$

$$r = 1 - \left[\frac{6(93+5.50)}{8(8^2-1)}\right]$$

$$r = 1 - \left(\frac{591}{504}\right) = -0.172619$$

There is weak degree of Negative correlation betn x & y

6 Find spearman's correlation coefficient for

x	10	18	26	10
y	36	14	22	22

$\Longrightarrow$

Rank of x : 3.50 2 1 3.50

Rank of y : 1 4 2.50 2.50

d^2 : 6.25 4 2.25 1

$t = 2, 2$ $\Sigma d^2 = 13.50$

$$\Sigma\frac{t^3-t}{12} = \left(\frac{2^3-2}{12} + \frac{2^3-2}{12}\right)$$
$$= 0.50 + 0.50$$
$$= 1.00$$

$$r = 1 - \frac{6\left(\Sigma d^2 + \Sigma\frac{t^3-t}{12}\right)}{n(n^2-1)}$$

$$r = 1 - \frac{6(13.50 + 1.00)}{4(4^2-1)}$$

$$r = 1 - \left(\frac{87}{60}\right) = -0.45$$

There is moderate degree of Negative correlation

7 **Spearman's Rank Correlation Coefficient.**

without tie

$$r = 1 - \left[\frac{6\Sigma d^2}{n(n^2-1)}\right]$$

with tie

$$r = 1 - \left[\frac{6\left(\Sigma d^2 + \Sigma\frac{t^3-t}{12}\right)}{n(n^2-1)}\right]$$

where d = Diff of Rank

n = No. of pairs of observations

t = No. of observations involved in a tie.

8 **Find Coefficient of Concurrent Deviation for -**

x	60	90	28	36	51	58	90	95	101	63
y	28	111	93	28	63	78	53	28	99	100

x	y	Deviation of x	Deviation of y	product
60	28			
90	111	+	+	+
28	93	−	−	+
36	28	+	−	−
51	63	+	+	+
58	78	+	+	+
90	53	+	−	−
95	28	+	−	−
101	99	+	+	+
63	100	−	+	−

c = No. of concurrent deviations

= No. of '+' signs in product columns

= 5

$m = n - 1 = 10 - 1 = 9$

$$r = \pm\sqrt{\pm\left(\frac{2c-m}{m}\right)}$$

$$r = \sqrt{\frac{2(5)-9}{9}}$$

$$r = \sqrt{\frac{1}{9}}$$

$$r = 1/3 = 0.3333333$$

8

In the product column : No. of positive signs = x

No. of negative signs = y

x > y	r is positive	→ $0 < r \le 1.00$
x < y	r is negative	→ $-1 \le r < 0$
x = y	r = 0	→ $r = 0$

9

Find Karl Pearson's product moment correlation coefficient

x	8	3	11	9	6
y	5	8	13	20	28

x	y	x^2	y^2	xy
8	5			
3	8			
11	13			
9	20			
6	28			
37	74	311	1442	555

co-variance of x, y

$= cov(x,y)$

$= \dfrac{\Sigma xy}{n} - \bar{x}\cdot\bar{y}$

$= \dfrac{555}{5} - (7.40 \times 14.80)$

$= 1.48$

$\bar{x} = 7.40$

$\bar{y} = 14.80$

$SD_x = \sqrt{\dfrac{311}{5} - 7.40^2} = 2.72764$

$SD_y = \sqrt{\dfrac{1442}{5} - 14.80^2} = 8.3283$

Karl pearson's corr. coeffi.

$= \left[\dfrac{cov(x,y)}{\sigma_x \cdot \sigma_y}\right] = \dfrac{1.48}{2.72764 \times 8.3283}$

$= 0.065151$

10

Covariance of (x,y) = $\left(\dfrac{\Sigma xy}{n} - \bar{x}\cdot\bar{y}\right) = \left[\dfrac{\Sigma[(x-\bar{x})(y-\bar{y})]}{n}\right]$

$SD_x = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}}$

$SD_y = \sqrt{\dfrac{\Sigma y^2}{n} - \bar{y}^2} = \sqrt{\dfrac{\Sigma(y-\bar{y})^2}{n}}$

My Notes

Karl pearson's product moment correlation coefficient

$$r = \left[\frac{cov(x,y)}{\sigma_x \cdot \sigma_y}\right] = \left[\frac{\dfrac{\Sigma xy}{n} - \bar{x}\cdot\bar{y}}{\sqrt{\dfrac{\Sigma x^2}{n} - (\bar{x})^2} \times \sqrt{\dfrac{\Sigma y^2}{n} - (\bar{y})^2}}\right]$$

$$r = \left[\frac{\Sigma[(x-\bar{x})(y-\bar{y})]}{\sqrt{\Sigma(x-\bar{x})^2} \times \sqrt{\Sigma(y-\bar{y})^2}}\right]$$

$$-1.00 \leq r \leq 1.00$$

11

r	Type of Correlation
r = 1.00	perfect positive correlation.
0.30 < r < 0.80	Moderate degree of positive correlation.
0.80 < r <1.00	strong/High degree of positive correlation.
r = 0	No correlation.
r = -1.00	perfect Negative correlation
-1.00 < r < -0.80	strong/High degree of Negative correlation.
-0.80 < r < -0.30	Moderate degree of Negative correlation.
0 < r < 0.30	weak/Low degree of positive correlation.
-0.30 < r < 0	weak/Low degree of Negative correlation.

12

If $v = 3x+8$; $u = 8y-19$; $r_{xy} = 0.80$

$r_{uv} = r_{xy} = 0.80$

Correlation coefficient is unaffected by change / shift of origin as well as by change in scale.

13

If $u = -3x+53$; $v = -18y+99$; $r_{xy} = 0.70$

$r_{uv} = r_{xy} = 0.70$

14

If $u = -18x+55$; $v = 16y+100$; $r_{xy} = 0.85$

$r_{uv} = -r_{xy} = -0.85$

15

If $u = -8x+19$; $v = -16y-33$; $r_{xy} = -0.56$

$r_{uv} = r_{xy} = -0.56$

16

Find Karl Pearson's Coefficient for -

x	30	60	90	50
y	20	30	40	80

$\Longrightarrow$

$\bar{x} = 57.50$, $\bar{y} = 42.50$

$$\sigma_x = \sqrt{\frac{15100}{4} - 57.50^2} = 21.651$$

$$\sigma_y = \sqrt{\frac{9300}{4} - 42.50^2} = 22.7761$$

$$\text{cov}(x,y) = \frac{10{,}000}{4} - (57.50 \times 42.50)$$

$$= 56.25$$

$$r = \frac{\text{cov}(x,y)}{\sigma_x \cdot \sigma_y} = \frac{56.25}{21.651 \times 22.7761}$$

$$r = 0.1141$$

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$
30	20	−27.50	−22.50	618.75
60	30	2.50	−12.50	−31.25
90	40	32.50	−2.50	−81.25
50	80	−7.50	37.50	−281.25
				225

$\Sigma(x-\bar{x})^2 = 1875$

$\Sigma(y-\bar{y})^2 = 2075$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2} \times \sqrt{\Sigma(y-\bar{y})^2}}$$

$$r = \frac{225}{\sqrt{1875} \times \sqrt{2075}} = 0.1141$$

17

x	100	80
y	30	60

Find r by Karl Pearson's Method :

x	y	xy
100	30	3000
80	60	4800

$cov(x,y) = \frac{\Sigma xy}{n} - \bar{x}\cdot\bar{y}$

$= \frac{7800}{2} - (90\times45)$

$= -150$

$\sigma_x = 10$

$\sigma_y = 15$

$r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$r = \frac{-150}{10\times15} = -1.00$

x	y
2	30
7	15

$\Rightarrow r = -1.00$

x	y
70	800
20	100

$\Rightarrow r = 1.00$

when $n = 2$ then

$r = 1.00$ OR $r = -1.00$

x	y
200	300
250	480

$\Rightarrow r = 1.00$

x	y
80	595
70	625

$\Rightarrow r = -1.00$

18

Regression Analysis

After studying correlation between 2 variables, the process of estimating the value of one variable on the basis of other is known as regression analysis

x = Given
y = ?

Regression of y on x

Eqn of Regression line of y on x

$(y-\bar{y}) = b_{yx}(x-\bar{x})$

y = Given
x = ?

Regression of x on y

Eqn of Reg line of x on y

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

r, b_{yx}, b_{xy} all are unit-free

Reg line of y on x is : $(y-\bar{y}) = b_{yx}(x-\bar{x})$

Reg line of x on y is : $(x-\bar{x}) = b_{xy}(y-\bar{y})$

Reg coefficient of y on x is = b_{yx} = $r\cdot\frac{SD_y}{SD_x} = r\cdot\frac{\sigma_y}{\sigma_x}$

Reg coefficient of x on y is = b_{xy} = $r\cdot\frac{SD_x}{SD_y} = r\cdot\frac{\sigma_x}{\sigma_y}$

My Notes

(1) $u = 18x - 258$ $v = -16y + 520$

$r_{xy} = -0.8578$, $r_{uv} = ?$

$\Longrightarrow r_{uv} = 0.8578$

19 If $\bar{x} = 30$, $\bar{y} = 90$, $\sigma_x = 5$, $\sigma_y = 8$, $r = 0.80$

Find a. Reg line of x on y

b. Reg line of y on x

c. If x =25, y =?

d. If y =85, x =?

$\Longrightarrow$

① Eq^n of Regression line of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - \bar{x} = r \cdot \frac{\sigma x}{\sigma y}(y - \bar{y})$$

$$x - 30 = 0.80 \times \frac{5}{8} \times (y - 90)$$

$$b_{xy} = r \cdot \frac{\sigma x}{\sigma y} = 0.80 \times \frac{5}{8} = 0.50$$

$$x - 30 = 0.50(y - 90)$$

$$x - 30 = 0.50y - 45$$

$$\boxed{x = -15 + 0.50y}$$

② Eq^n of Regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - \bar{y} = r \cdot \frac{\sigma y}{\sigma x}(x - \bar{x})$$

$$\boxed{b_{yx} = r \cdot \frac{\sigma y}{\sigma x} = 0.80 \times \frac{8}{5} = 1.28}$$

$$y - 90 = 0.80 \times \frac{8}{5} \times (x - 30)$$

$$y - 90 = 1.28x - 38.40$$

$$\boxed{y = 51.60 + 1.28x}$$

③ $x = 25$, $y = ?$

$$y = 51.60 + 1.28x$$
$$= 51.60 + (1.28 \times 25) = 83.60$$

④ $y = 85$, $x = ?$

$$x = -15 + 0.50y$$
$$= -15 + (0.50 \times 85)$$
$$= 27.50$$

$$\boxed{b_{yx} = r.\frac{\sigma_y}{\sigma_x}}$$

$$\boxed{b_{xy} = r.\frac{\sigma_x}{\sigma_y}}$$

Therefore, $b_{yx} . b_{xy}$

$$= r.\frac{\sigma_y}{\sigma_x} \times r.\frac{\sigma_x}{\sigma_y}$$

$$= r^2$$

$$r = \sqrt{b_{yx} . b_{xy}}$$

$$\boxed{r^2 = b_{yx} . b_{xy}}$$

Square of correlation coefficient is equal to product of 2 regression coefficients.

Correlation coefficient 'r' is G.M. of 2 regression coefficients $b_{yx} . b_{xy}$

20

$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \times \frac{\sigma_y}{\sigma_x} = \frac{cov(x,y)}{\text{variance of } x}$

$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \times \frac{\sigma_x}{\sigma_y} = \frac{cov(x,y)}{\text{variance of } y}$

$b_{yx} \cdot b_{xy} = r^2$

$y = ?$, $x = \text{Given}$ } we should use Reg. of y on x

Therefore 'r' is G.M. of b_{yx} & b_{xy}

$r^2 = (b_{yx} \cdot b_{xy})$

$\boxed{1.00 \geq r \geq -1.00}$

$\boxed{1.00 \geq r^2 \geq 0.00}$

r	b_{yx}	b_{xy}
0	0	0
+	+	+
-	-	-

$x = ?$, $y = \text{Given}$ } we should use Reg. of x on y

21

If Reg. line of y on x is written in the form of $y = a + bx$ then 'b' represents 'b_{yx}'

If Reg. line of y on x is 3x+5y=83. Find b_{yx}

$3x + 5y = 83$

$5y = 83 - 3x$

$y = \frac{83}{5} - \frac{3}{5}x$

$\boxed{\therefore y = 16.60 - 0.60x}$

$\therefore b_{yx} = -0.60$

22

If Reg. line of x on y is written in the form of $x = a + by$ then 'b' represents 'b_{xy}'

If Reg. line of x on y is 2x-3y=95. Find b_{xy}

$2x - 3y = 95$

$2x = 95 + 3y$

$x = \frac{95}{2} + \frac{3}{2}y$

$x = 47.50 + 1.50y$

$\boxed{\therefore b_{xy} = 1.50}$

$0 \leq b_{yx} \cdot b_{xy} \leq 1.00$

as

$0 \leq r^2 \leq 1.00$

23

On solving 2 regression lines simultaneously. If we get x = 50 and y = 90, then

$\Longrightarrow$ (50,90) is the point of intersection of 2 regression lines $= (\bar{x}, \bar{y})$

$\bar{x} = 50, \ \bar{y} = 90$

24

Probable Error = 0.674 x $\frac{(1 - r^2)}{\sqrt{N}}$ = [0.674 × standard error]

Standard Error = $\frac{(1 - r^2)}{\sqrt{N}}$

Coefficient of determination = (r^2) = (Explained variance / Total variance)

Coefficient of Non-determination = $(1 - r^2)$ = (unexplained variance / Total variance)

n = sample size

where N = population size

25 2 regression lines become identical i.e. they coincide when r = -1 or r = 1.

i.e. 2 Regression lines coincide when there is perfect positive OR perfect negative correlation.

26 If r = 0; then regression lines are ⊥ to each other.

When there is no correlation between 2 variables then regression lines will be ⊥ to each other.

27

Particulars	Maths (x)	Stats (y)
AM	88	92
SD	10	12
r	0.75	

Find 1. Reg. line of y on x
2. Reg. line of x on y
3. If x = 95, y = ?
4. If y = 90, x = ?

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = 0.75 \times \frac{12}{10} = 0.90$$

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = 0.75 \times \frac{10}{12} = 0.625$$

① Regression line of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 92 = 0.90(x - 88)$$
$$y - 92 = 0.90x - 79.20$$
$$\boxed{y = 12.80 + 0.90x}$$

② Reg. line of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$x - 88 = 0.625(y - 92)$$
$$x - 88 = 0.625y - 57.50$$
$$\boxed{x = 30.50 + 0.625y}$$

③ $y = 12.80 + 0.90(95)$
$y = 98.30$
when $x = 95$ then exp. value of $y = 98.30$

④ when $y = 90$, $x = ?$
$x = 30.50 + 0.625(90) = 86.75$
esti. value of x when $y = 90$ $= 86.75$

$y = 12.80 + 0.90x$
$x = 30.50 + 0.625y$
Let's solve these eqns simultaneously

$$y = 12.80 + 0.90(30.50 + 0.625y)$$
$$y = 12.80 + 27.45 + 0.5625y$$
$$0.4375y = 40.25$$
$$\boxed{y = 92}$$

$\therefore x = 30.50 + 0.625(92)$
$\boxed{x = 88}$

point of intersection of 2 regression lines $\equiv (88, 92) \equiv (\bar{x}, \bar{y})$

28

$1.00 \geq r \geq -1.00$

$1.00 \geq r^2 \geq 0.00$

$1.00 \geq (b_{yx} . b_{xy}) \geq 0.00$

Minimum value of $r = -1.00$
Maximum value of $r = 1.00$
Minimum value of $r^2 = 0.00$
Maximum value of $r^2 = 1.00$

29

If $b_{yx} > 0$; then $b_{xy} < 0$ ...... This statement is incorrect

sign of byx, bxy and r will always be same

30

If $b_{yx} = 2.50$, $r = 0.80$, $b_{xy} = ?$

$\Longrightarrow$ $r^2 = byx \times bxy$

$0.80^2 = 2.50 \times bxy$

$bxy = 0.256$

31

If $b_{xy} = -1.56$, $b_{yx} = -0.20$, $r = ?$

$\Longrightarrow$ $r^2 = byx . bxy = -0.20 \times -1.56 = 0.312$

$r = \sqrt{0.312} = -0.55856960175$

32

If $b_{xy} = -1.5281$, $b_{yx} = 0.2381$, $r = ?$

$\Longrightarrow$ $r^2 = byx . bxy = 0.2381 \times -1.5281 = -0.36384061$

As $0 < r^2 < 1.00$ $\therefore$ Given data is wrong.

33

If $b_{yx} = 1.82$, $b_{xy} = 0.90$, $r = ?$

$\Longrightarrow$ $r^2 = byx . bxy$

$= 1.82 \times 0.90$

$r^2 = 1.638$

As r^2 can not be more than 1.00, This is wrong data.

My Notes

If $r = 0.60$ then

(coefficient of Determination)
$= r^2 = 0.60^2 = 0.36$
$= 36\%$
$=$ Explained variance

(coefficient of Non-determination)
$= 1 - r^2 = 1 - 0.60^2 = 1 - 0.36 = 0.64$
$= 64\% =$ unexplained variance

34 If $\bar{x} = 90$, $\bar{y} = 80$, $r = -0.85$, $\sigma_x = 10$, $\sigma_y = 18$

1. If x = 35, y = ?

$y - 80 = -0.85 \times \frac{18}{10} \times (35-90)$

$\boxed{y = 164.15}$

2. If y = 98.70, x = ?

$x - 90 = -0.85 \times \frac{10}{18} \times (98.70 - 80)$

$\boxed{x = 81.1694}$

35 If r = 0.75. Find coefficient of determination and coefficient of non-determination.

$\Rightarrow$ ① coefficient of Determination $= r^2 = 0.75^2 = 0.5625$

$\therefore$ Explained variance = 56.25%

② coeffi. of Non-determination $= 1 - r^2 = 1 - 0.5625 = 0.4375$

$\therefore$ unexplained variance = 43.75%

36

x	y
35	480
28	410

Find 'r'

$\Rightarrow$ pls remember that for 2 pairs of obs^ns either $r = 1.00$ OR $r = -1.00$

Here both x & y are decreasing

$\therefore r = 1.00$

37

x	y
200	500
180	600

$\Rightarrow r = -1.00$

x	y
200	800
250	703

$\Rightarrow r = -1.00$

x	y
10	30
28	36

$\Rightarrow r = 1.00$

38 If C = 5, m = 11. Find coefficient of concurrent deviation.

coeffi. of concurrent deviation $= \pm\sqrt{\pm\left(\frac{2c-m}{m}\right)} = \pm\sqrt{\pm\frac{2(5)-11}{11}} = -\sqrt{-\left(-\frac{1}{11}\right)}$

$= -0.3015$

My Notes

If $\left(\frac{2c-m}{m}\right) = 0$ then $r = 0$

$\left(\frac{2c-m}{m}\right) > 0$ then $0 < r \leq 1.00$

$\left(\frac{2c-m}{m}\right) < 0$ then $-1 \leq r < 0$

39 If cov (x,y) = 0, then r = 0

If cov (x,y) = positive, then $\Longrightarrow$ $1.00 \geq r > 0$

If cov (x,y) = negative, then $\Longrightarrow$ $-1.00 \leq r < 0$

As $r = \dfrac{\text{covariance of }(x,y)}{SD_x \times SD_y} = \left[\dfrac{cov(x,y)}{\sigma_x \cdot \sigma_y}\right]$

40 **Karl Pearson's product moment correlation coefficient is the ratio of cov (x,y) to product of standard deviations of x & y**

41 Prepare a bi-variate frequency table for the following data relating to marks in stats (x) and maths (y).

(12,18) (2,16) (12,3) (19,12) (5,8) (8,2) (13,14)
(2,6) (13,19) (6,10) (2,12) (14,2) (18,5) (20,1)

x \ y		Marks in Maths (y) 0-10	10-20	Total
Marks in Stats (x)	0-10	||| = 3	||| = 3	6
	10-20	|||| = 4	|||| = 4	8
Total		7	7	14

Bi-variate Frequency Table

Find Marginal Distribution of x : Distri. of x over all values of y

x	0-10	10-20
f	6	8

Find Marginal Distribution of y : Distri. of y over all values of x

y	0-10	10-20
f	7	7

Find conditional Distribution of x when y is 10-20:

x	0-10	10-20
f	3	4

Find conditional Distribution of y when x is 0-10:

y	0-10	10-20
f	3	3

42

'Marginal Distribution' is the frequency distribution of one variable (x or y) across the other variable's full range of values.

'Conditional Distribution' is the frequency distribution of one variable (x or y) across the particular sub-population of other variable.

43

x \ y	0-10	10-20	20-30	30-40	40-50	Total
0-10	5	20	22	23	25	95
10-20	8	30	26	28	42	134
20-30	9	20	29	38	48	144
30-40	13	50	36	39	56	194
40-50	26	60	28	19	26	159
Total	61	180	141	147	197	726

Bi-variate Frequency Table

Find Marginal Distribution of x :

x	0-10	10-20	20-30	30-40	40-50
f	95	134	144	194	159

Find Marginal Distribution of y :

y	0-10	10-20	20-30	30-40	40-50
f	61	180	141	147	197

Find conditional Distribution of x when y is 30-40:

x	0-10	10-20	20-30	30-40	40-50
f	23	28	38	39	19

Find conditional Distribution of y when x is 20-30:

y	0-10	10-20	20-30	30-40	40-50
f	9	20	29	38	48

My Notes

① $b_{yx} = b_{xy}$	ⓐ correct	~~ⓑ~~ incorrect
② $r_{xy} = r_{yx}$	~~ⓐ~~ correct	ⓑ incorrect
③ $b_{yx} \cdot b_{xy} = r^2$	~~ⓐ~~ correct	ⓑ incorrect

44 If 2 variables move in same direction i.e. an increase on the part variable introduces an increase on the part of other variable and
Decrease on the part of one variable introduces decrease on the part of other variable also, then 2 variables are known to be positively correlated.

45 If 2 variables move in opposite direction i.e. an increase on the part variable introduces an decrease on the part of other variable and
Decrease on the part of one variable results in increase on the part of other variable, then 2 variables are known to be Negatively correlated.

46 2 variables are known to be uncorrelated
if movement on the part of one variable does not produce any measureable movement on the part of other variable.

47 1. Correlation coefficient (r) is unit free.
2. Correlation coefficient remains same in value, not necessarily in sign after shift of origin and change in scale.
3. Correlation coefficient lies between -1 and 1, including both limiting values.
4. b_{yx} & b_{xy} are also unit-free
5. b_{yx}, b_{xy} and r will always have same sign.

48 For a group of 8 students, the sum of squares of diff. in ranks for maths & stats marks was found to be 50. What is the value of rank correlation coefficient?

$\Longrightarrow$ $\sum d^2 = 50$, $n = 8$

$$r = 1 - \left[\frac{6\sum d^2}{n(n^2-1)}\right] = 1 - \left[\frac{6 \times 50}{8 \times (8^2-1)}\right] = 1 - \frac{300}{504}$$

$$r = 0.40476$$

49 For a number of towns, correlation coefficient between people living below poverty line and increase of population is 0.50. If sum of squares of diff. in rank awarded to these factors are 82.50. Find number of towns.

$\Longrightarrow$ $\sum d^2 = 82.50$, $r = 0.50$, $n = ?$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)} \qquad 0.50 = 1 - \left[\frac{6 \times 82.50}{n(n^2-1)}\right]$$

$$\left[\frac{495}{n(n^2-1)}\right] = 0.50$$

$$\therefore \quad n(n^2-1) = 990$$

$$\boxed{n = 10}$$

My Notes

No. of towns = 10

50 While computing rank correlation coefficient between profit and investments for 10 years of a firm, the diff of rank of one observation was taken as 7 instead of 5 and rank correlation coefficient was 0.80. What is correct value of rank correlation coefficient?

~~a.~~ 0.95 b. 0.78 c. -0.80 d. None of these

$0.80 = 1 - \frac{6\Sigma d^2}{10 \times (10^2-1)}$

$\frac{6\Sigma d^2}{990} = 0.20$

wrong $\Sigma d^2 = 33$

correct $\Sigma d^2 = 33 - 7^2 + 5^2$
$= 9$

$r = 1 - \frac{6 \times 9}{990}$

$= 1 - \frac{54}{990} = 0.94545$

51 Regression equations are derived from method of least squares.

52 Regression coefficient remain unchanged by shift of origin but affected due to change in scale.

a. If $u = 3 + x$, $v = y - 18$ then $b_{vu} = b_{yx}$ and $b_{uv} = b_{xy}$ → $r_{uv} = r_{xy}$, $\sigma_u = \sigma_x$, $\sigma_v = \sigma_y$ →

$b_{vu} = r_{uv}\frac{\sigma_v}{\sigma_u}$
$= r_{xy} \cdot \frac{\sigma_y}{\sigma_x}$
$b_{vu} = b_{yx}$
$b_{uv} = b_{xy}$

b. If $u = x + 17$, $v = y + 30$ then $b_{vu} = b_{yx}$, $b_{uv} = b_{xy}$

c. If $u = 3x + 18$, $v = 8y - 19$ then $b_{vu} =$, $b_{uv} =$ ⇒

$b_{vu} = r_{uv} \times \frac{\sigma_v}{\sigma_u} = r_{xy} \times \frac{8 \times \sigma_y}{3 \times \sigma_x}$

$b_{vu} = \frac{8}{3} \times r_{xy} \cdot \frac{\sigma_y}{\sigma_x} = \frac{8}{3} \times b_{yx}$

$b_{uv} = \frac{3}{8} \times b_{xy}$

d. If $u = 18x + 17$, $v = 2y - 20$ then $b_{vu} =$, $b_{uv} =$ ⇒

$b_{vu} = \frac{2}{18} \times b_{yx}$

$b_{uv} = \frac{18}{2} \times b_{xy}$

53 Two regression lines i.e. $(y - \bar{y}) = b_{yx}(x - \bar{x})$ and $(x - \bar{x}) = b_{xy}(y - \bar{y})$ intersect at point $(\bar{x}, \bar{y})$

2 Reg. line will always intersect at point (AM of x, AM of y)

My Notes

① If $u = 3x - 19$, $v = -8y - 63$ then
$r_{uv} = -r_{xy}$

② If $u = 8x + 50$, $v = 19y + 83$ then
$b_{uv} = \frac{8}{19} \times b_{xy}$
$b_{vu} = \frac{19}{8} \times b_{yx}$

54

r	b_{yx}	b_{xy}
0.80	5.80	0.1103448275$8$
0.75	0.20	2.8125
-0.60	−0.26471	-1.36
−0.93808	-0.80	-1.10
0.2819	1.23619	0.06428430095

$r^2 = b_{yx} \cdot b_{xy}$

55 There are some cases when we may find a correlation between 2 variables although 2 variables are not casually related. This is due to existence of third variable which is related to both the variables under consideration, such a correlation is known as spurious correlation.

OR Non-sense correlation

56 Bi-variate data are data collected for :
a. 2 variables.
b. More than 2 variables.
~~c.~~ 2 variables at same point of time.
d. 2 variables at diff. point of time.

57 If plotted paints in a scatter diagram lie from

Upper left to lower right then → NEGATIVE correlation.

Upper right to lower left then → POSITIVE correlation.

58 If plotted points in a scatter diagram are evenly distributed without depicting any pattern then there is NO correlation betn 2 variables.

59 If plotted points in a scatter diagram lie on a single line then correlation is
a. Perfect Positive b. Perfect Negative ~~c.~~ a or b d. None of these

60 The correlation between shoe-size and intelligence is
a. Positive b. Negative ~~c.~~ Zero d. None of these

My Notes

$u = \frac{3x+18}{23}$ & $v = \frac{8y-93}{21}$ then

$\Longrightarrow$ $= \frac{18}{23} + \frac{3}{23}x$ $\quad v = -\frac{93}{21} + \frac{8}{21}y$

$b_{uv} = \frac{3/23}{8/21} \times b_{xy} = \frac{63}{184} \times b_{xy}$

$b_{vu} = \frac{8/21}{3/23} \times b_{yx} = \frac{184}{63} \times b_{yx}$

Value of 'r' helps us to find Type/Nature of correlation as well as degree of correlation

61 Product moment correlation coefficient is considered for ________________.

a. Finding nature of correlation
b. Finding degree of correlation
~~c.~~ Both of these
d. None of these

62 If r is positive then points in a scatter diagram tend to cluster :

~~a.~~ From lower left corner to upper right corner
b. From lower left corner to lower right corner
c. From lower right corner to upper left corner
d. None of these

63 The co-variance between 2 variables is :

a. Strictly positive
b. Strictly negative
c. Always zero
~~d.~~ Either positive, negative or zero

Similarly SD = zero or positive

Variance = zero or positive

64 To find degree of agreement about beauty between 2 judges in a beauty contest, we use :

a. Scatter Diagram
b. Product moment correlation coefficient
~~c.~~ Spearman's rank correlation coefficient
d. Coefficient of concurrent deviation

65 The diff. between observed value and estimated value in a regression analysis is known as Error or Residue.

66 What are the limits of 2 regression coefficient ?

a. No limit
b. Both must be positive
c. One positive & other negative
~~d.~~ Product of 2 regression coefficients must be numerically less than unity.

67 Regression coefficients remain unchanged due to :

~~a.~~ Shift of origin
b. Change of scale
c. Both a and b
d. Either a or b

My Notes

68 Correlation coefficient between 2 variables is -0.90, then coefficient of determination is :

a. 0.90 b. -0.81 c. 0.19 d. 0.81

69 Correlation coefficient between 2 variables is 0.70, then % of variation unaccounted for is :

a. 70% b. 49% c. 51% d. 100%

unaccounted variation $= 1 - 0.70^2 = 1 - 0.49 = 0.51$

70 If cov (x,y) = 15, then $\sigma_x \cdot \sigma_y$

$\Rightarrow \quad \sigma_x \cdot \sigma_y \geq 15$

71 If u + 5x = 6 and 3y - 7v = 20. $(r)_{xy} = 0.58$ then $(r)_{uv}$ = ?

a. 0.58 b. -0.58 c. 0.84 d. -0.84

$u = 6 - 5x$

$u = -5x + 6$

$3y - 7v = 20$

$3y - 20 = 7v$

$v = -\frac{20}{7} + \frac{3}{7}y$

$r_{uv} = -r_{xy}$

$= -0.58$

72 If sum of squares of diff. in ranks, given by 2 judges A and B of 8 students is 21, what is the value of rank correlation coefficient?

a. 0.70 b. 0.65 c. 0.75 d. 0.80

$\Longrightarrow \quad r = 1 - \left[\frac{6 \times 21}{8 \times (8^2 - 1)}\right] = 1 - \frac{126}{504} = \frac{378}{504} = 0.75$

73 For 10 pairs of observations, No. of concurrent deviations found to be 4. What is coefficient of concurrent deviation?

a. $\sqrt{0.20}$ b. $-\sqrt{0.20}$ c. 1/3 d. -1/3

$n = 10, \quad m = 9, \quad c = 4$

$r = \pm\sqrt{\pm\frac{2c - m}{m}} = \sqrt{\frac{2(4) - 9}{9}} = -\frac{1}{3}$

My Notes

$\frac{1}{\sqrt{3}} = \sqrt{\frac{2c - m}{m}}$

$\sqrt{\frac{1}{3}} = \sqrt{\frac{2(6) - m}{m}}$

$\frac{1}{3} = \frac{12 - m}{m}$

$m = 36 - 3m$

$4m = 36$

$m = 9$

$\therefore n = 10 = p$

74 The coefficient of concurrent deviation for 'p' pairs of observations was found to be $1/\sqrt{3}$ If no. of concurrent deviations was found to be 6. Value of 'p' is :

~~a.~~ 10 b. 9 c. 8 d. None of these

75 If y = 4 + 3x is regression line of y on x. AM of x = -1; AM of y = ?

~~a.~~ 1 b. -1 c. 7 d. None

AM of y $= 4 + 3\,(\text{AM of } x)$

$= 4 + 3(-1)$

$= 1$

76 2 regression lines are y = -2x+3 and 8x = -y+3. Find value of r.

a. 0.50 ~~b.~~ - 0.50 c. $-1/\sqrt{2}$ d. None of these

$y = -2x + 3$ | $8x = 3 - y$

$y = 3 - 2x$ | $x = \frac{3}{8} - \frac{1}{8}y$

$b_{yx} = -2$ | $b_{xy} = -\frac{1}{8}$

$r^2 = b_{yx} \times b_{xy} = -2 \times -1/8 = 1/4$

$r = -\frac{1}{2} = -0.50$

77 Given the following equations 2x - 3y = 10 and 3x + 4y = 15, which one is the regression equation of x on y.

a. 3x+4y=15 b. 2x-3y=10 c. Both ~~d.~~ None

x on y

$2x - 3y = 10$

$2x = 10 + 3y$

$x = 5 + \frac{3}{2}y \quad \therefore b_{xy} = \frac{3}{2} = 1.50$

y on x

$3x + 4y = 15$

$4y = 15 - 3x$

$y = \frac{15}{4} - \frac{3}{4}x$

$b_{yx} = -0.75$

78 2 regression lines are given by : 8x + 10y = 25 and 16x + 5y = 12. & Variance of x = 25, SD of y = ?

a. 16 ~~b.~~ 8 c. 64 d. 4 e. None of these

Reg. line of y on x

$8x + 10y = 25$

$10y = 25 - 8x$

$y = \frac{25}{10} - \frac{8}{10}x$

$b_{yx} = -0.80$

Reg. line of x on y

$16x + 5y = 12$

$16x = 12 - 5y$

$x = \frac{12}{16} - \frac{5}{16}y$

$b_{xy} = -0.3125$

$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

$-0.80 = -0.50 \times \frac{\sigma_y}{5}$

$\sigma_y = 8$

$r^2 = -0.80 \times -0.3125$

$r = -0.50$

79

Variables	Nature of Correlation
1. Profit of insurance company and no. of claims	Negative
2. Demand for goods and their prices under normal circumstances	Negative
3. Years of education and Income	Positive
4. Amount of rainfall and Yield of crop	Positive
5. Sale of woollen garments and temperature	Negative

80 For the bivariate data [(20,5), (21,4), (22,3)] the correlation coefficient between x and y is

a. zero b. 1 ~~c.~~ c. -1 d. 0.50

x	20	21	22
y	5	4	3

81 r = 0.48, cov (x,y) = 36, SD of x = 16, SD of y = ?

a. 18.75 b. -18.75 c. 16.75 ~~d.~~ d. None of these

$$r = \frac{\text{cov}(x,y)}{\sigma_x \cdot \sigma_y} \qquad 0.48 = \frac{36}{16 \times \sigma_y} \qquad \therefore \sigma_y = 4.6875$$

82 r = 0.52, cov (x,y) = 7.80, Variance of x = 16, SD of y = ?

a. 2.85 b. 3.25 c. 1.25 ~~d.~~ d. 3.75

$$r = \frac{\text{cov}(x,y)}{\sigma_x \cdot \sigma_y} \qquad 0.52 = \frac{7.80}{4 \times \sigma_y} \qquad \therefore \sigma_y = 3.75$$

83 If r = 0.40 then coefficient of determination and coefficient of non-determination are resp.

~~a.~~ a. 0.16, 0.84 b. 0.36,0.64 c. 0.60,0.40 d. None

coeffi. of Determination $= r^2 = 0.40^2 = 0.16$

coeffi. of Non-determination $= 1 - r^2 = 0.84$

84 Simple correlation is known as :

~~a.~~ a. Linear correlation b. Non-linear correlation

c. Non-sense correlation d. None of these

85 Slope of regression equation of x on y is :

$\boxed{\text{slope of the line } px + qy + r = 0 \text{ is } -p/q}$

a. b_{xy} b. b_{yx} ~~c.~~ c. $1/b_{xy}$ d. $1/b_{yx}$

Reg. line of x on y is

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$x - \bar{x} = b_{xy} \cdot y - b_{xy} \cdot \bar{y}$$

$$1 \cdot x - b_{xy} \cdot y = \bar{x} - b_{xy} \cdot \bar{y}$$

$$\text{slope} = \frac{-1}{-b_{xy}} = \frac{1}{b_{xy}}$$

86 Slope of regression equation of y on x is :

a. b_{xy} ~~b.~~ b_{yx} c. $1/b_{xy}$ d. $1/b_{yx}$

Reg. line of y on x

$y-\bar{y} = b_{yx}(x-\bar{x})$

$y-\bar{y} = b_{yx}.x - b_{yx}.\bar{x}$

$b_{yx}.\bar{x}-\bar{y} = b_{yx}.x - y$

$b_{yx}.x - y = b_{yx}.\bar{x}-\bar{y}$

Slope of the line $= -b_{yx}/-1 = b_{yx}$

87 $(r)_{xy} = (r)_{yx}$

~~a.~~ correct b. wrong c. can't say d. None of these

88 b_{yx} is always same as b_{xy}

a. correct ~~b.~~ wrong

89 Covariance measures Joint variation between 2 variables.

~~a.~~ Joint b. Common c. Relative d. None of these

variance of $(x,x) = \frac{\Sigma x^2}{n} - \bar{x}^2 = \frac{\Sigma x.x}{n} - \bar{x}.\bar{x}$

co-variance of $(x,y) = \frac{\Sigma x.y}{n} - \bar{x}.\bar{y}$

90 Karl Pearson's Product Moment Correlation Coefficient $= \left[\frac{\text{covariance of }(x,y)}{\sigma_x.\sigma_y}\right]$

$$= \left[\frac{\frac{\Sigma xy}{n} - \bar{x}.\bar{y}}{\sqrt{\frac{\Sigma x^2}{n}-\bar{x}^2} \times \sqrt{\frac{\Sigma y^2}{n}-\bar{y}^2}}\right] = \left[\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2} \times \sqrt{\Sigma(y-\bar{y})^2}}\right]$$

Spearman's Rank Correlation Coefficient = without tie : $1 - \left[\frac{6\Sigma d^2}{n(n^2-1)}\right]$

with tie : $1 - \left[\frac{6\left(\Sigma d^2 + \Sigma\frac{t^3-t}{12}\right)}{n(n^2-1)}\right]$

Coefficient of Concurrent Deviation $= \pm\sqrt{\pm\left(\frac{2c-m}{m}\right)}$

91 $b_{yx} = 1.20\ b_{xy} = 0.90$; then $r = ?$

a. 1.039 b. -1.039 c. 1.08 ~~d.~~ Wrong data

$r^2 = b_{yx} \times b_{xy} = 1.20 \times 0.90 = 1.08$

As max. value of $r^2 = 1.00$, This is wrong data.

92 If $\bar{x} = 30$, $\bar{y} = 90$, $\sigma_x = 8$, $\sigma_y = 5$, $r = -0.75$. Find Reg. equation of y on x.

a. Joint b. Common c. Relative d. None of these

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 90 = -0.75 \times \frac{5}{8} \times (x - 30)$

$y - 90 = -0.46875\ (x - 30)$

$y - 90 = -0.46875x + 14.0625$

$\boxed{y = 104.0625 - 0.46875x}$

93 If $\sum(x-\bar{x})(y-\bar{y}) = 30$, $n = 3$. Find cov (x,y)

$$\text{cov}(x,y) = \frac{\sum(x-\bar{x})(y-\bar{y})}{n} = \frac{30}{3} = 10$$

94 If cov (x,y) = 36, $\sigma_x = 9$, $\sigma_y = 4$. Find r

~~a.~~ 1.00 b. -1.00 c. 0 d. None

$$r = \frac{\text{cov}(x,y)}{\sigma_x \cdot \sigma_y} = \frac{36}{9 \times 4} = 1.00$$

95 correlation coefficient is also known as measure of association between 2 variables.

96 Karl pearson's product moment correlation coefficient is the best method to obtain correlation between 2 variables.

My Notes

97 If Reg line of y on x is 3x + 8y = 13y - 63x + 103. Find b_{yx}

$\Longrightarrow$ $3x + 8y = 13y - 63x + 103$

$-5y = 103 - 66x$

$y = -\frac{103}{5} + \frac{66}{5}x$ $\quad \therefore b_{yx} = \frac{66}{5} = 13.20$

98 If Reg line of x on y is 16x - y = 93x - 21y + 83. Find b_{xy}

$\Longrightarrow$ $16x - y = 93x - 21y + 83$

$-77x = 83 - 20y$

$x = -\frac{83}{77} + \frac{20}{77}y$ $\quad \therefore b_{xy} = 20/77$

$= 0.25974$

99 If r = -0.63812, b_{yx} = -1.36822, b_{xy} = ?

$r^2 = b_{yx} \cdot b_{xy}$

$(-0.63812)^2 = -1.36822 \times b_{xy}$ $\quad \therefore b_{xy} = -0.297611$

100 Correlation between temperature of city and sale of cold drinks is :

~~a.~~ Positive b. Negative c. Zero d. Can't say

Probability

CA VINOD REDDY

1 **Probability is the** 'chance' OR 'possibility' of happening OR Non-happening of any Future uncertain event.

2 **Classical Definition of Probability**

$$= \left(\frac{\text{No. of outcomes in favour}}{\text{No. of all possible equally likely outcomes}} \right)$$

3 **Coin** : There are 2 equally likely outcomes
H, T

Dice/Die : There are 6 equally likely outcomes
1 point, 2 points, 3 points, 4 points, 5 points, 6 points

Card 52 cards → 26 Black cards → 13 clubs & 13 spades
→ 26 Red cards → 13 Diamonds & 13 Hearts
13 cards : 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace

4 **A coin is tossed 2 times what is probability of getting**

All possible equally likely outcomes : HH, HT, TH, TT

2 heads	**1 head**	**Atleast 1 head**	**Atmost 1 head**
HH	HT, TH	HT, TH, HH	HT, TH, TT
$= \frac{1}{4}$ $= 0.25$ $= 25\%$	$= \frac{2}{4} = \frac{1}{2} = 0.50$ $= 50\%$	$= \frac{3}{4} = 0.75$ $= 75\%$	$= \frac{3}{4} = 0.75$ $= 75\%$

My Notes

5 coins are tossed. what is probability of getting

All heads	All Tails	Atleast 1 Head	exactly 1 head
$= \frac{1}{32}$ $= 3.125\%$	$= \frac{1}{32}$ $= 3.125\%$	$= \left(\frac{31}{32}\right)$ $= 96.875\%$	$= \left(\frac{5}{32}\right) = 15.625\%$

unbiased = Fair
Biased = unfair

5

An unbaised coin is tossed 3 times. Find the probability of getting

2 tails	Atleast 2 heads	No tails	Atmost 2 tails
HTT, TTH, THT	HHT, HTH, THH, HHH	HHH	All except TTT
$= 3/8 = 37.50\%$	$= 4/8 = 1/2$	$= 1/8$	$= 7/8 = 0.8750$
$= 0.3750$	$= 0.50 = 50\%$	$= 0.1250$	$= 87.50\%$
		$= 12.50\%$	

All possible equally likely outcomes : HHH HHT HTH HTT
: TTT TTH THT THH

6

An unbaised coin is tossed 4 times. What is the probability of getting

2 heads	3 tails	Atleast 3 tails	Atmost 3 tails
$= 6/16 = 37.50\%$	$= 4/16 = 25\%$	$= 5/16 = 31.25\%$	$= 15/16 = 93.75\%$

THHH	THTH	TTHH	TTTH
THHT	THTT	TTHT	TTTT
HHHH	HHTH	HTHH	HTTH
HHHT	HHTT	HTHT	HTTT

7

A dice is rolled once. What is the probability of getting

3 Points	4 Points	1 Point	Atmost 3 Points	Atleast 5 Points	Odd Number as point	Prime Number as point
$= \frac{1}{6}$	$= \frac{1}{6}$	$= \frac{1}{6}$	$= \frac{3}{6}$	$= \frac{2}{6}$	$= \frac{3}{6}$	$= \frac{3}{6}$
$= 16\frac{2}{3}\%$	$= 16\frac{2}{3}\%$	$= 16\frac{2}{3}\%$	$= 50\%$	$= 33\frac{1}{3}\%$	$= 50\%$	$= 50\%$

My Notes

What are all possible outcomes when 2 dice are rolled ?

$\Rightarrow$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

} 36 possible equally likely outcomes

8 A dice is rolled twice what is the probability of getting

→ 7 points as sum $= 6/36 = 1/6 = 16.666666\% = 16\frac{2}{3}\% = 0.166666$

→ 8 points as sum $= 5/36 = 13.88888\%$

→ 9 or more points $= \frac{4+3+2+1}{36} = \frac{10}{36} = \frac{5}{18} = 27.77777\%$

→ Atleast 3 points $= \frac{35}{36} = 97.222222\%$

→ Odd points on both dice $= \frac{9}{36} = \frac{1}{4} = 25\%$

→ Odd points on atleast one dice $= \frac{27}{36} = \frac{3}{4} = 75\%$

→ Even points on both dice $= \frac{9}{36} = \frac{1}{4} = 25\%$

→ 5 or 7 points $= \frac{4+6}{36} = \frac{10}{36} = \frac{5}{18} = 27.77777\%$

→ Sum as prime number $= \frac{1+2+4+6+2}{36} = \frac{15}{36} = \frac{5}{12} = 41\frac{2}{3}\%$

→ Odd points on atleast one dice $= \frac{3}{4} = 75\%$

→ Sum as odd number $= \frac{18}{36} = \frac{1}{2} = 50\%$

→ Sum as even number $= \frac{18}{36} = \frac{1}{2} = 50\%$

9 A card is drawn from a well shuffled pack of 52 cards. What is probability of getting :

a. A diamond = $\left({}^{13}C_1 / {}^{52}C_1\right) = \left(13/52\right) = 1/4 = 25\%$

b. A King = $\left({}^{4}C_1 / {}^{52}C_1\right) = 4/52 = 1/13 = 7.6923\%$

c. A Black Card = $\left({}^{26}C_1 / {}^{52}C_1\right) = 26/52 = 1/2 = 50\%$

d. A Black Queen = $\left({}^{2}C_1 / {}^{52}C_1\right) = 2/52 = 1/26 = 3.84615\%$

e. A Jack = $\left({}^{4}C_1 / {}^{52}C_1\right) = 4/52 = 1/13 = 7.6923\%$

10

P(A∪B) = $P(A) + P(B) - P(A \cap B)$

P(A') = $1 - P(A)$

P(B') = $1 - P(B)$

P (A∩B) = $P(A) + P(B) - P(A \cup B)$

P(A-B) = $P(A \cap B') = P(A) - P(A \cap B)$

P(B-A) = $P(B \cap A') = P(B) - P(A \cap B)$

P(A'∩B') = $P(A \cup B)' = 1 - P(A \cup B)$

P(A∪B') = $P(B-A)' = 1 - P(B-A)$

P(B∪A') = $P(A-B)' = 1 - P(A-B)$

P(A △ B) = $P(A-B) + P(B-A) = P(A \cup B) - P(A \cap B)$

P(A∪B∪C) = $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

My Notes

$P(A) = 0.35 \quad P(B) = 0.47 \quad P(A \cap B) = 0.13$

Find

① $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.69$

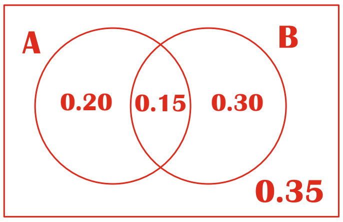
② $P(A \triangle B) = P(A \cup B) - P(A \cap B) = 0.69 - 0.13 = 0.56$

③ $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.69 = 0.31$

④ $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.13 = 0.87$

⇒ De-morgan's Rules

11



P(A) = 0.35

P(B) = 0.45

P(A')= 0.65

P(B')= 0.55

P(A∪B) = 0.65

P(A∩B) = 0.15

P(A-B) = 0.20

P(B-A) = 0.30

P(A'∩B') = 0.35

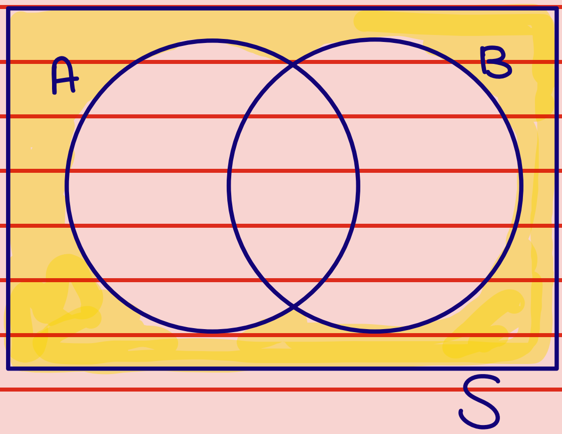
P(A △ B) = 0.50

$P(A \cup B') = 0.70$

$P(B \cup A') = 0.80$

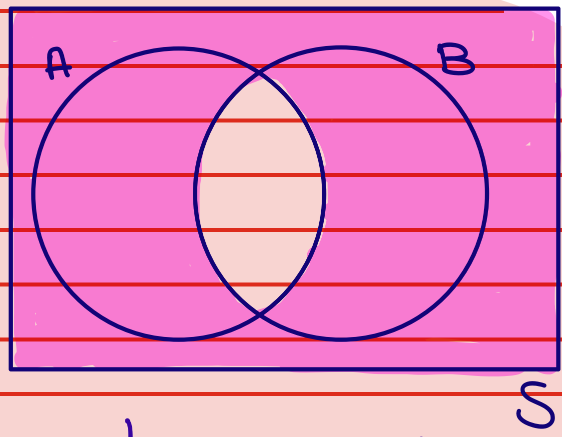
$P(A' \cup B') = 0.85$

12 De-morgan's rule of probability (with diagram)



$P(A \cup B)' = P(A' \cap B')$

$P(A' \cap B') = 1 - P(A \cup B)$



$P(A \cap B)' = P(A' \cup B')$

$P(A' \cup B') = 1 - P(A \cap B)$

My Notes

$P(A-B) = 0.20$, $P(A \Delta B) = 0.35$, $P(A' \cup B') = 0.88$

Find

$P(A) = 0.32$

$P(B) = 0.27$

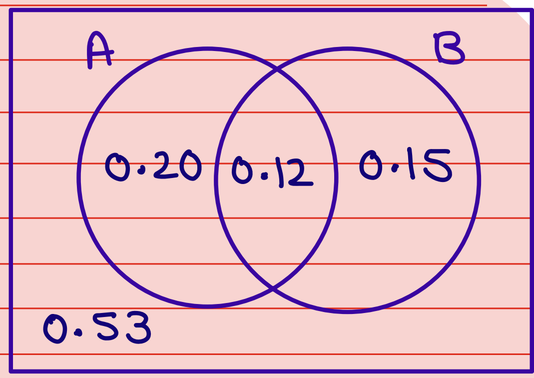
$P(B-A) = 0.15$

$P(A' \cap B') = 0.53$

$P(A' \cup B) = 0.80$

$P(A') = 0.68$

$P(A \cup B') = 0.85$

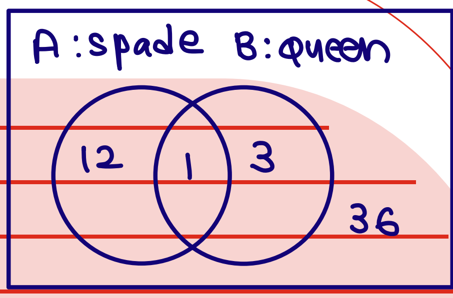


CA Vinod Reddy - vinod.reddy.ca@gmail.com

13 If 2 dice are rolled then

Sum of points on 2 dice	Probability
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

14 A card is drawn from a well shuffled pack of 52 cards then what is probability that it is a -



a. Spade = $P(A) = \frac{13}{52}$

b. Queen = $P(B) = \frac{4}{52}$

c. Spade and Queen = $P(A \cap B) = \frac{1}{52}$

d. Spade or Queen = $P(A \cup B) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$

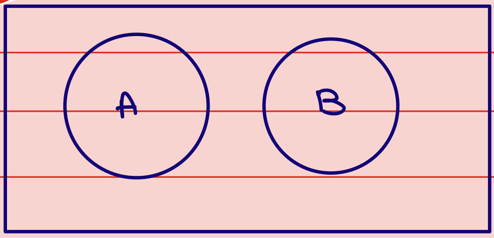
e. Spade but not Queen = $P(A - B) = P(A \cap B') = P(A) - P(A \cap B) = \frac{13}{52} - \frac{1}{52} = \frac{12}{52}$

f. Queen but not Spade = $P(B - A) = P(B \cap A') = P(B) - P(A \cap B) = \frac{4}{52} - \frac{1}{52} = \frac{3}{52}$

g. Neither Spade nor Queen = $P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{16}{52} = \frac{36}{52}$

My Notes

15 A, B are said to be mutually exclusive events then :



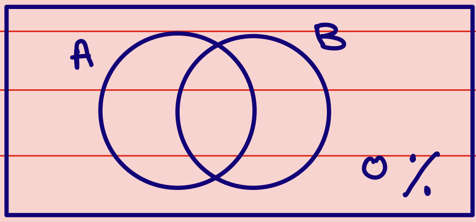
Here,

$P(A \cap B) = 0$

$\therefore$ A, B are said to be Mutually exclusive events

- when happening of event A prevents happening of event B & vice versa. (i.e. Both events can't occur at a time) then events are said to be Mutually exclusive

16 A, B are said to be mutually exhaustive events then :



Events are said to be mutually exhaustive If Atleast one of them must necessarily occur.

when $P(A \cup B) = 1.00$
i.e. $P(A' \cap B') = 0$ } A, B are said to be Mutually exhaustive events.

17 A, B are said to be independent events when :

$P(A \cap B) = P(A) \times P(B)$

when Happening OR Non happening of event A doesn't affect probability of happening OR Non-happening of event B, then A, B are said to be Independent events

18

Events A & B are said to be	If
Mutually Exclusive events	P (A∩B) = 0
Mutually Exhaustive events	P (A∪B) = 1.00
Independent events	P (A∩B) = P(A) x P(B)
Equally likely events	P(A) = P(B)

My Notes

$P(A) = 0.30$, $P(B) = 0.80$. A, B are independent events. Find

$\Longrightarrow$ $P(A \cap B) = P(A) \times P(B) = 0.30 \times 0.80 = 0.24$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.30 + 0.80 - 0.24 = 0.86$

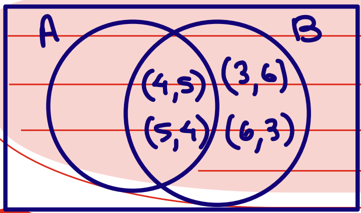
$P(A \cap B') = P(A) - P(A \cap B) = 0.30 - 0.24 = 0.06$

$P(B \cap A') = P(B) - P(A \cap B) = 0.80 - 0.24 = 0.56$

$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.86 = 0.14$

19 2 dice are rolled. It is observed that sum of points is 9. What is probability that 4 has appeared on one of the dice?

$\Longrightarrow$



A : event that 4 has appeared on one of the dice

B : sum of points on 2 dice is 9.

$P(A/B)$ = probability of A provided 'B' has already occured $= \dfrac{P(A\cap B)}{P(B)}$

$= \left(\dfrac{2/36}{4/36}\right) = \dfrac{2}{4} = \dfrac{1}{2} = 50\%$

20

P (A/B) = $P(A\cap B)/P(B)$

P (A'/B) = $P(A'\cap B)/P(B)$

P (B/A) = $P(B\cap A)/P(A)$

P (A'/B') = $P(A'\cap B')/P(B')$

P (A/B') = $P(A\cap B')/P(B')$

P (B'/A) = $P(B'\cap A)/P(A)$

P (B/A') = $P(B\cap A')/P(A')$

P (B'/A') = $P(B'\cap A')/P(A')$

21 If A, B are independent events then :

1. $P(A\cap B) = P(A) \times P(B)$
2. $P(A\cap B') = P(A) \times P(B')$
3. $P(A'\cap B) = P(A') \times P(B)$
4. $P(A'\cap B') = P(A') \times P(B')$
5. $P(A/B) = P(A)$
6. $P(B/A) = P(B)$
7. $P(A/B') = P(A)$
8. $P(A'/B) = P(A')$
9. $P(B'/A') = P(B')$
10. $P(B'/A) = P(B')$

If A, B are independent events then

(a) A, B'
(b) B, A'
(c) A', B'

$\Bigg\}$ are also independent event

22

8 Red
6 White
5 Black

3 balls are drawn. What is probability of getting

2 Red balls

$= \left(\dfrac{{}^8C_2 \times {}^{11}C_1}{{}^{19}C_3}\right)$

$= \left(\dfrac{308}{969}\right) = 31.7853\%$

Atleast 2 white Balls

$= \dfrac{({}^6C_2 \times {}^{13}C_1) + ({}^6C_3 \times {}^{13}C_0)}{{}^{19}C_3}$

$= \left(\dfrac{195 + 20}{969}\right) = \left(\dfrac{215}{969}\right)$

$= 22.1878\%$

Atmost 1 Black Ball

$= \dfrac{({}^5C_1 \times {}^{14}C_2) + ({}^5C_0 \times {}^{14}C_3)}{{}^{19}C_3}$

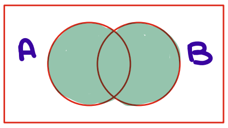
$= \dfrac{455 + 364}{969}$

$= \left(\dfrac{819}{969}\right)$

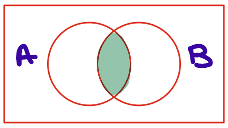
$= 84.5201\%$

23

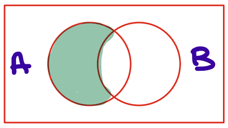
1. $P(A \cup B)$



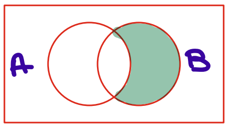
2. $P(A \cap B)$



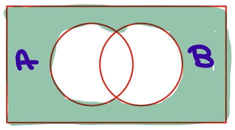
3. $P(A \cap B')$



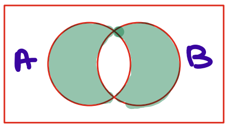
4. $P(B \cap A')$



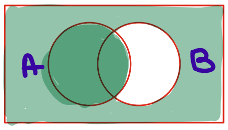
5. $P(A' \cap B')$



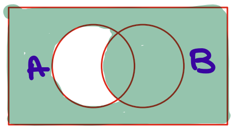
6. $P(A \triangle B)$



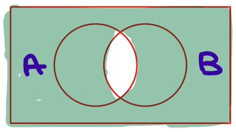
7. $P(A \cup B')$



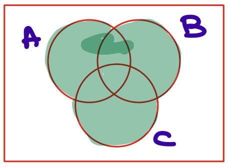
8. $P(B \cup A')$



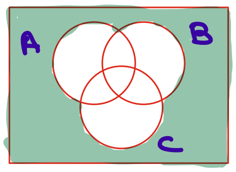
9. $P(A' \cup B')$



10. $P(A \cup B \cup C)$



11. $P(A' \cap B' \cap C')$



24 If $P(A) = 0.30$, $P(B) = 0.40$, $P(A \cap B) = 0.15$. Find

$P(A') = 1 - P(A) = 1 - 0.30 = 0.70$

$P(B') = 1 - P(B) = 1 - 0.40 = 0.60$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.55$

$P(A-B) = P(A) - P(A \cap B) = 0.15$

$P(B-A) = P(B) - P(A \cap B) = 0.25$

$P(A' \cap B') = 1 - P(A \cup B) = 0.45$

$P(A \cup B') = 1 - P(B-A) = 0.75$

$P(B \cup A') = 1 - P(A-B) = 0.85$

$P(A \triangle B) = P(A-B) + P(B-A) = 0.40$

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.15}{0.40} = 0.3750$

$P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.15}{0.30} = 0.50$

$P(A/B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{0.15}{0.60} = 0.25$

$P(A'/B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{0.45}{0.60} = 0.75$

25 $P(A) = 0.30$, $P(B) = 0.40$, A, B are independent events, then find

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = 0.30$

$P(B/A) = P(B) = 0.40$

$P(A/B') = P(A) = 0.30$

$P(B/A') = P(B) = 0.40$

$P(A'/B') = P(A') = 0.70$

$P(B'/A') = P(B') = 0.60$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.58$

$P(A-B) = P(A \cap B') = P(A) \times P(B') = 0.30 \times 0.60$
$= 0.18$

$P(B-A) = P(B \cap A') = P(B) \times P(A') = 0.40 \times 0.70$
$= 0.28$

$P(A' \cap B') = 1 - P(A \cup B) = 0.42 = P(A') \times P(B')$
$= 0.70 \times 0.60$

$P(A' \cup B') = 1 - P(A \cap B)$
$= 1 - 0.12$
$= 0.88$

26 In a leap year selected at random what is probability of getting

- 53 Mondays $= 2/7$
- 52 Mondays $= 5/7$
- Atleast 52 Mondays $= 100\%$
- 54 Mondays $= 0\%$

27 In a non-leap year selected at random what is probability of getting

- 53 Sundays $= 1/7$
- 52 Sundays $= 6/7$
- Atleast 52 Sundays $= 100\%$
- 54 Sundays $= 0\%$

28 In a year selected at random what is the probability of getting

52 Tuesdays

$$= \left(\frac{1}{4} \times \frac{5}{7}\right) + \left(\frac{3}{4} \times \frac{6}{7}\right) = \frac{5}{28} + \frac{18}{28} = \left(23/28\right)$$

53 Tuesdays

$$= \left(\frac{1}{4} \times \frac{2}{7}\right) + \left(\frac{3}{4} \times \frac{1}{7}\right) = \frac{2}{28} + \frac{3}{28} = \left(5/28\right)$$

29 What is probability that 15th day of a randomly selected month is Sunday?

$= (1/7)$

what is probability that 18^{th} day of a randomly selected month is a Non-Friday ?

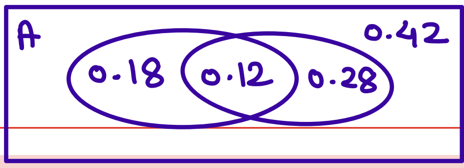
$\Rightarrow 6/7$

30 Probability of A passing exam is 0.30. and B passing exam is 0.40.
What is the probability that

(A, B are independent events)

- Both will pass $= P(A \cap B) = 0.12$
- Only A will pass $= P(A \cap B') = P(A - B) = 0.18$
- Only B will pass $= P(B \cap A') = P(B - A) = 0.28$
- Atleast one will pass $= P(A \cup B) = 0.58$
- One & Only One will pass $= P(A \Delta B) = 0.46$
- Atleast one will fail $P(A' \cup B') = 0.88$

Both will fail $= P(A' \cap B') = 0.42$



My Notes

$P(A) = 0.4528$, $P(B) = 0.9568$ A, B are Indep. events.

Find

① $P(A' \cap B') = P(A') \times P(B') = 0.5472 \times 0.0432 = 0.02363904$

② $P(A \cap B') = P(A) \times P(B') = 0.4528 \times 0.0432 = 0.01956096$

③ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.97636096$

31

x	30	60	90	120	150
Prob. x	0.20	0.30	0.10	0.15	0.25

Find E(x), SD_x, Variance of x

$E(x) = \sum x \cdot prob(x)$

variance of $x = \sum x^2 \cdot p(x) - \left[\sum x \cdot p(x)\right]^2 = E(x^2) - [E(x)]^2$

$\Rightarrow$

x	$p(x)$	x^2	$x \cdot p(x)$	$x^2 \cdot p(x)$
30	0.20	900		
60	0.30	3600		
90	0.10	8100		
120	0.15	14,400		
150	0.25	22,500		
	1.00		88.50	9855

Mean of $x = E(x) = \dfrac{\sum x \cdot p(x)}{\sum p(x)} = \dfrac{\sum x \cdot p(x)}{1.00} = \sum x \cdot p(x) = 88.50$

variance of $x = \dfrac{\sum x^2 \cdot p(x)}{\sum p(x)} - [E(x)]^2 = \sum x^2 \cdot p(x) - [E(x)]^2$

$= E(x^2) - [E(x)]^2 = 9855 - (88.50)^2 = 2022.75$

SD of $x = \sqrt{\sum x^2 \cdot p(x) - \left[\sum x \cdot p(x)\right]^2} = \sqrt{2022.75} = 44.9750$

32

x	10	20	30	40	50
Prob. x	0.20	3k	5k	7k	k

Find E(x), SD_x, Variance of x

$\Rightarrow \sum prob(x) = 0.20 + 16k = 1.00$

$16k = 0.80$

$k = 0.05$

x	$p(x)$	x^2	$x \cdot p(x)$	$x^2 \cdot p(x)$
10	0.20	100		
20	0.15	400		
30	0.25	900		
40	0.35	1600		
50	0.05	2500		
			29.00	990

① $E(x) = \sum x \cdot p(x) = 29$

$E(x^2) = \sum x^2 \cdot p(x) = 990$

② variance of $x = E(x^2) - [E(x)]^2 = 990 - 29^2 = 149$

③ SD of $x = \sqrt{149} = 12.2065$

33 **If odds in favour of event A are 3 : 8. Find P(A), P(A')**

$\Longrightarrow$ $P(A) = \frac{3}{3+8} = \frac{3}{11}$

$P(A') = \frac{8}{3+8} = \frac{8}{11}$

$P(A) + P(A') = \frac{3}{11} + \frac{8}{11} = \frac{11}{11} = 1.00 = 100\%$

34 **If odds against event B are 8 : 13. Find P(B), P(B')**

$P(B') = \frac{8}{21}$

$P(B) = \frac{13}{21}$

35 **If odds in favour of event A are 3 : 11; Odds against event B are 2 : 15; A,B are independent events, then find :**

P(A) = $3/14$

P(B) = $15/17$

P(A∩B) = $P(A) \times P(B) = (45/238)$

P(A∪B) = $\frac{3}{14} + \frac{15}{17} - \frac{45}{238} = \frac{261}{238} - \frac{45}{238} = \frac{216}{238} = \frac{108}{119}$

P(A'∩ B') = $1 - P(A \cup B) = 1 - \frac{108}{119} = (11/119)$

P(A-B) = $P(A) - P(A \cap B) = \frac{3}{14} - \frac{45}{238} = \frac{51}{238} - \frac{45}{238} = \frac{6}{238} = \frac{3}{119}$

P(B-A) = $P(B) - P(A \cap B) = \frac{15}{17} - \frac{45}{238} = \frac{210}{238} - \frac{45}{238} = \frac{165}{238}$

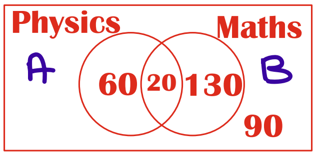
My Notes

If $P(A) = 8/93$ then

odds against event A are $\Rightarrow$ 85 : 8

odds in favour of event A are $\Rightarrow$ 8 : 85

36



Find probability that a student likes

a. Maths if it is known that he likes physics = $P(B/A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{20/300}{80/300}$

$= 20/80 = 1/4 = 0.25 = 25\%$

b. Physics if it is known that he doesn't likes maths =

$= P(A/B') = \dfrac{P(A\cap B')}{P(B')} = \dfrac{60/300}{150/300} = \dfrac{60}{150} = \dfrac{2}{5} = 0.40$
$= 40\%$

37

1 Ball

10 Red 8 White	2 Red 3 White

1 ball is drawn.
What is the probability that it is a red ball?

Transfer

Red : $\dfrac{10}{18} \times \dfrac{3}{6} = \dfrac{30}{108}$

white : $\dfrac{8}{18} \times \dfrac{2}{6} = \dfrac{16}{108}$

Total $= \dfrac{46}{108} = \left(\dfrac{23}{54}\right) = 42.5925\%$

38

2 Balls

5 Red 6 White	3 Red 11 White

1 ball is drawn from it,
What is the probability that it is a white ball?

2 Red : $\dfrac{5c_2}{11c_2} \times \dfrac{11c_1}{16c_1} = \dfrac{10}{55} \times \dfrac{11}{16} = \dfrac{110}{880}$

2 white : $\dfrac{6c_2}{11c_2} \times \dfrac{13c_1}{16c_1} = \dfrac{15}{55} \times \dfrac{13}{16} = \dfrac{195}{880}$

1 R, 1 W : $\dfrac{5c_1 \times 6c_1}{11c_2} \times \dfrac{12c_1}{16c_1} = \dfrac{30}{55} \times \dfrac{12}{16} = \dfrac{360}{880}$

Final answer : $665/880 = (133/176)$
$= 75.5681818181\%$

My Notes

1 ball

11 Red 5 white	3 Red 8 Black

what is the chance that it's a Red ball?

$\Longrightarrow \dfrac{11}{16} \times \dfrac{4}{12} = \dfrac{44}{192}$

$\dfrac{5}{16} \times \dfrac{3}{12} = \dfrac{15}{192}$

Answer : $\dfrac{59}{192} = 30.7291666\%$

39

Information	Whether A,B are	
	Mutually Exclusive Events?	Mutually Exhaustive Events?
$P(A) = 0.30$; $P(B) = 0.60$ $P(A \cap B) = 0.10$	No	No
$P(A) = 0.60$; $P(B) = 0.50$ $P(A \cap B) = 0.10$	No	Yes
$P(A) = 0.30$; $P(B) = 0.40$ $P(A \cap B) = 0$	Yes	No
$P(A) = 0.65$; $P(B) = 0.35$ $P(A \cap B) = 0$	Yes	Yes

40

Two Broad divisions of Probability are

Subjective Probability

Subjective Probability is basically dependent on personal judgement and experience.

It may be influenced by personal belief, attitude and bias.

Objective Probability

It is not based upon personal judgement.

41

An experiment may be described as a performance that produces certain results.
The result or outcome of a random experiment are known as events.

42

Events are of 2 types

Simple or Elementary Event

Getting Head when One Coin is tossed

Composite or Compound Event

Getting Head when Two Coins are tossed

43

Equally likely events are also known as Mutually Symmetric Events or Equi-probable events.
If $P(A) = 0.30$, $P(B) = 0.30$ then A,B are equally likely events OR Equi-probable events OR Mutually Symmetric events

My Notes

$P(A) = 0.36$, $P(B) = 0.68$, $P(A \cap B) = 0.04$

Here A, B are

(a) mutually exclusive events
(b) mutually exhaustive events
(c) Independent events
(d) None of these

44 If P(A) = 1.00 = 100% then event A is said to be a sure event O.R certain event.

45 If P(B) = 0.00 = 0% then event B is said to be a Impossible event.

46

Total No. of workers ↓

$N = \Sigma f = 261$

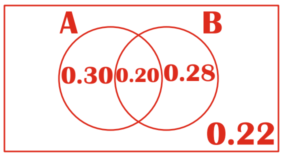
Wages in ₹	100-200	200-300	300-400	400-500
No. of workers	23	57	88	93

If a worker is selected at random, what is the probability that

1. He earns more than ₹ 300 = $\left(\frac{88+93}{261}\right) = \frac{181}{261} = 69.3487\%$
2. He earns more than ₹ 400 = $(93/261) = (31/87) = 35.6321\%$
3. He earns between ₹ 200 - ₹ 400 = $\left(\frac{57+88}{261}\right) = (145/261) = 55.5555\%$
4. He earns less than ₹ 300 =

$$= \left(\frac{23+57}{261}\right) = \left(\frac{80}{261}\right) = 30.6513\%$$

47



S = Sample Space
= Set of all possible outcomes

<u>For above diagram. Find</u>

P(A) $= 0.50$

P(B) $= 0.48$

P(A') $= 0.50$

P(B') $= 0.52$

P(A∪B) $= 0.78$

P(A∩B') $= P(A-B) = 0.30$

P(B∩A') $= P(B-A) = 0.28$

P(A∪B') $= 0.72$

$P(A \triangle B) = 0.58$

$P(A' \cap B') = 0.22$

P(B∪A') $= 0.70$

P(A'∪B') $= 0.80$

P(A/B) $= P(A \cap B)/P(B) = 0.41666666$

P(B/A) $= P(B \cap A)/P(A) = 0.40$

P(A'/B')

$= P(A' \cap B')/P(B') = 0.42307692307$

P(B'/A')

$= P(B' \cap A')/P(A') = 0.44$

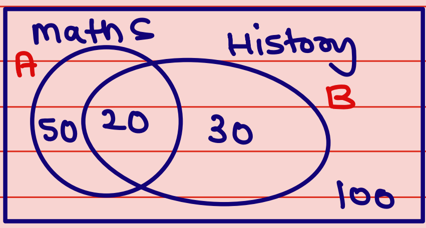
P(A'/B)

$= P(A' \cap B)/P(B) = 0.58333333$

P(B/A')

$= P(B \cap A')/P(A') = 0.56$

<u>My Notes</u>



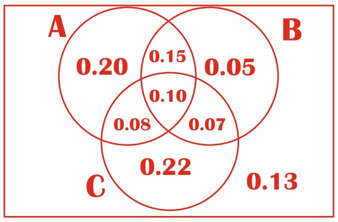
what is prob. that a student randomly selected doesn't like History if it is known that He doesn't like maths

$\Rightarrow \frac{100}{130} = 10/13 = 76.9231\%$

$P(B'/A') = \frac{P(B' \cap A')}{P(A')}$

$= \left[(100/200) / \quad\right] = \frac{100}{130} = \frac{10}{13}$

48



From this Venn Diagram : Find

P(A) = 0.53
P(B) = 0.37
P(C) = 0.47
P(A') = 0.47
P(B') = 0.63
P(C') = 0.53
P(A∩B) = 0.25
P(B∩C) = 0.17
P(A∩C) = 0.18
P(AUB) = 0.65

P(BUC) = 0.67
P(AUC) = 0.82
P(A-B) = 0.28
P(B-A) = 0.12
P(A-C) = 0.35
P(C-A) = 0.29
P(B-C) = 0.20
P(C-B) = 0.30
P(AUBUC) = 0.87
P(A∩B∩C) = 0.10

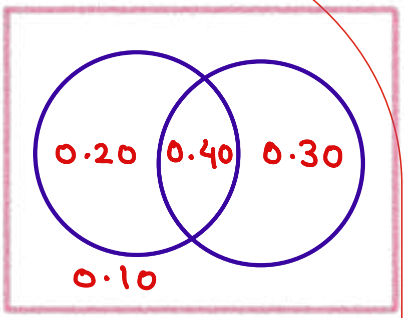
P(A∩B'∩C') = 0.20
P(B∩A'∩C') = 0.05
P(C∩A'∩B') = 0.22
P(A'UB') = 0.75
P(B'UC') = 0.83
P(A'UC') = 0.82
P(A'∩B'∩C') = 0.13
P(A△B) = 0.40
P(B△C) = 0.50
P(A△C) = 0.64

49

P(A-B) = 0.20, P(B-A) = 0.30, P(A'∩B') = 0.10. Find

P(A) = 0.60
P(B) = 0.70
P(AUB) = 0.90
P(A∩B) = 0.40
P(A△B) = 0.50
P(AUB') = 0.70
P(BUA') = 0.80
P(A'UB') = 0.60
P(A') = 0.40
P(B') = 0.30

1. P(A∪B) = 1 − P(A'∩B')
   = 0.90
2. P(A△B) = 0.50
3. P(A∪B) − P(A∩B) = 0.50
   0.90 − 0.50 = P(A∩B) = 0.40
4. P(A−B) = P(A) − P(A∩B)
   0.20 = P(A) − 0.40
   P(A) = 0.60
5. P(B−A) = P(B) − P(A∩B)
   0.30 = P(B) − 0.40
   P(B) = 0.70



My Notes

50 P(A) = 0.30, P(B) = 0.20, P(C) = 0.60, P(A∩B) = 0.10, P(B∩C) = 0.15, P(A∩C) = 0.18, P(A∩B∩ C) = 0.05, Find P(A∪B∪C) and P(A'∩B'∩C'), P(A∪B), P(B∪C) P(A∩C'), P(B∪C')

① $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$= 0.30 + 0.20 + 0.60 - 0.10 - 0.15 - 0.18 + 0.05$

$= 0.72$

② $P(A'\cap B'\cap C') = 1 - P(A\cup B\cup C) = 1 - 0.72 = 0.28$

③ $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.30 + 0.20 - 0.10 = 0.40$

④ $P(B\cup C) = P(B) + P(C) - P(B\cap C) = 0.20 + 0.60 - 0.15 = 0.65$

⑤ $P(C\cup A') = 1 - P(A - C) = 1 - [P(A) - P(A\cap C)] = 1 - (0.30 - 0.18)$

$= 0.88$

$P(A\cap C') = P(A) - P(A\cap C) = 0.30 - 0.18 = 0.12$

⑥ $P(B\cup C') = 1 - P(C - B) = 1 - [P(C) - P(C\cap B)]$

$= 1 - (0.60 - 0.15) = 0.55$

51 Odds in favour of an event are 2:3 and odds against another event are 3:7. Find the probability that only one of two events occurs. (Two events are independent)

$P(A) = \frac{2}{5} \quad P(B) = \frac{7}{10}$

$P(A\cap B) = \frac{2}{5} \times \frac{7}{10} = \frac{14}{50} = \frac{7}{25}$

$P(A\Delta B) = P(A\cup B) - P(A\cap B) = P(A) + P(B) - P(A\cap B) - P(A\cap B)$

$= \frac{2}{5} + \frac{7}{10} - \frac{14}{50} - \frac{14}{50} = \frac{55}{50} - \frac{28}{50} = \left(\frac{27}{50}\right) = 54\%$

52 There are 3 boxed with composition of balls :

5 Red 8 Blue	6 Red 3 Blue	8 Red 2 Blue

If one box is selected at random and one ball is drawn, what is the probability that it is a red ball?

$\Longrightarrow = \left(\frac{1}{3} \times \frac{5}{13}\right) + \left(\frac{1}{3} \times \frac{6}{9}\right) + \left(\frac{1}{3} \times \frac{8}{10}\right)$

$= \left(\frac{5}{39} + \frac{6}{27} + \frac{8}{30}\right) = 61.7094\%$

53 In a business venture, a man can make profit of ₹ 50,000 or incur a loss of ₹ 10,000. The probability of making profit or incurring loss from past experience are known to be 0.75 and 0.25 respectively. What is his expected profit?

x	$prob(x)$	$x \times prob(x)$
50,000	0.75	37,500
−10,000	0.25	−2,500
	$E(x) =$	35,000

Expected profit $= \sum x . prob(x) =$ ₹ 35,000

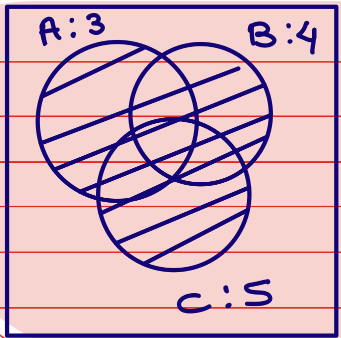
54 Ashwat draws 2 balls from a bag containing 3 white and 5 red balls. He gets ₹ 500 if he draws a white ball and ₹ 200 if he draws a red ball. What is his expectation?

3 white
5 Red

	x	$P(x)$
2 red	400	${}^5C_2 / {}^8C_2 = \frac{10}{28}$
2 white	1,000	${}^3C_2 / {}^8C_2 = \frac{3}{28}$
1 Red, 1 white	700	$\frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28}$

Expected earning for Ashwat $= \sum x . prob(x) =$ ₹625/-

55 A number is selected form first 1000 natural numbers, what is probability that number is divisible by 3 or 4 or 5.



$P(A) = \frac{333}{1000}$, $P(B) = \frac{250}{1000}$, $P(C) = \frac{200}{1000}$

$P(A \cap B) = \frac{83}{1000}$ $P(B \cap C) = \frac{50}{1000}$ $P(A \cap C) = \frac{66}{1000}$

$P(A \cap B \cap C) = 16/1000$

$$P(A \cup B \cup C) = \frac{(333 + 250 + 200 - 83 - 50 - 66 + 16)}{1000}$$

$= (600/1000) = 0.60 = 60\%$

56 The probability of an event lies between 0 and 1, both inclusive.

0 ≤ Probability (Any event) ≤ 1.00

57 A : Vinod is a minor

B : Vinod is a major

Here A, B are (a) Mutually exclusive events

(b) Mutually exhaustive events

~~(c)~~ Both

(d) None

58

A : Ashwat is an Indian
B : Ashwat is an American

Here A, B are Mutually exclusive but not mutually exhaustive.

59

All general Formulae at one place :

1. $P(A) = 1 - P(A')$
2. $P(B') = 1 - P(B)$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
5. $P(A-B) = P(A) - P(A \cap B)$
6. $P(B-A) = P(B) - P(A \cap B)$
7. $P(A \cup B') = 1 - P(B-A)$
8. $P(B \cup A') = 1 - P(A-B)$
9. $P(A \triangle B) = P(A-B) + P(B-A)$
10. $P(A' \cap B') = 1 - P(A \cup B)$
11. $P(A' \cup B') = 1 - P(A \cap B)$
12. $P(A \cup B \cup C) = [P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)]$
13. $P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C)$
14. $P(A/B) = P(A \cap B)/P(B)$
15. $P(B/A) = P(B \cap A)/P(A)$
16. $P(A/B') = P(A \cap B')/P(B')$
17. $P(B/A') = P(B \cap A')/P(A')$
18. $P(A'/B) = P(A' \cap B)/P(B)$
19. $P(A'/B') = P(A' \cap B')/P(B')$
20. $P(B'/A') = P(B' \cap A')/P(A')$
21. $P(B'/A) = P(B' \cap A)/P(A)$

60

When A,B are mutually exclusive events

$\boxed{P(A \cap B) = 0}$

$P(A \cup B) = P(A) + P(B)$

$P(A-B) = P(A)$

$P(B-A) = P(B)$

$P(A' \cup B') = 1.00 = 100\%$

$P(A/B) = 0\%$

$P(B/A) = 0\%$

$P(A \triangle B) = P(A \cup B)$

$P(A \cup B') = P(B')$

$P(B \cup A') = P(A')$

61

When A,B are mutually exhaustive events then :

$\boxed{P(A \cup B) = 1.00}$

$P(A' \cap B') = 0\%$

$P(A \cap B) = P(A) + P(B) - 1$

$P(A/B') = 1.00 = 100\%$

$P(B/A') = 1.00 = 100\%$

$P(A \triangle B) = 1 - P(A \cap B)$

$= P(A \cap B)' = P(A' \cup B')$

62 When A,B are independent events then, $\boxed{P(A \cap B) = P(A) \times P(B)}$

$P(A \cap B') = P(A) \times P(B')$

$P(B \cap A') = P(B) \times P(A')$

$P(A' \cap B') = P(A') \times P(B')$

$P(A \cup B) = P(A) + P(B) - P(A).P(B)$

$P(A/B) = P(A)$

$P(B/A) = P(B)$

$P(A/B') = P(A)$

$P(B/A') = P(B)$

$P(A'/B) = P(A')$

$P(A'/B') = P(A')$

$P(B'/A) = P(B')$

$P(B'/A') = P(B')$

63 2 dice are rolled, what is probability that points on first dice are more than points on second dice?

outcomes in favour

(2,1) (3,1) (3,2) (4,1) (4,2) (4,3) (5,1) (5,2) (5,3) (5,4)

(6,1) (6,2) (6,3) (6,4) (6,5)

$$\text{probability} = \frac{15}{36} = \frac{5}{12} = 41\tfrac{2}{3}\,\% = 41.666666\,\%$$

64 A committee of 5 members is formed from 8 ladies and 9 gents. What is probability that ladies form the majority?

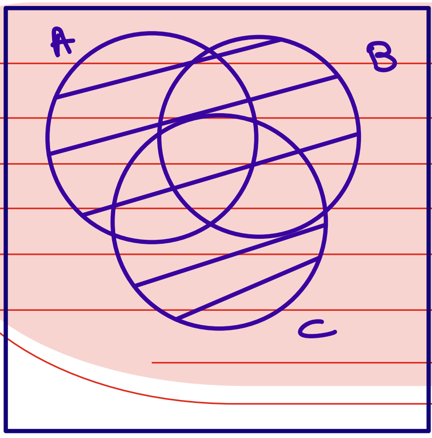
$$= \left[\frac{\left({}^8C_3 \times {}^9C_2\right) + \left({}^8C_4 \times {}^9C_1\right) + \left({}^8C_5 \times {}^9C_0\right)}{{}^{17}C_5}\right]$$

$$= \left(\frac{2016 + 630 + 56}{6188}\right)$$

$$= \left(\frac{2702}{6188}\right)$$

$$= 43.6652\,\%$$

65 A problem of maths was given to 3 students, chances of solving it are 1/3, 1/5, 1/2 respectively. What is the probability that problem gets solved?



$$P(A \cup B \cup C) = \frac{1}{3} + \frac{1}{5} + \frac{1}{2} - \frac{1}{15} - \frac{1}{10} - \frac{1}{6} + \frac{1}{30}$$

$$= \left(\frac{10 + 6 + 15 - 2 - 3 - 5 + 1}{30}\right) = \frac{22}{30}$$

$$= 11/15 = 73\tfrac{1}{3}\,\%$$

$$P(A \cup B \cup C) = 1 - P(A' \cap B' \cap C')$$

$$= 1 - \left(\frac{2}{3} \times \frac{4}{5} \times \frac{1}{2}\right) = 1 - \frac{8}{30} = \frac{22}{30}$$

$$= 11/15$$

66 8 identical balls are placed at random in 3 bags. What is the probability that first bag contains 3 balls?

$\Longrightarrow$ $n = 8$, $p = \frac{1}{3}$, $q = \frac{2}{3}$, $x = 3$

$$\text{prob}(x=3) = {}^8C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^5$$

$$= 56 \times \frac{1}{27} \times \frac{32}{243} = \left(\frac{1792}{6561}\right) = 27.31291\%$$

67 $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, Find $P(A'/B')$

$\Longrightarrow$ $$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right)}{1 - \frac{1}{3}} = \frac{1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}}{\frac{2}{3}} = \frac{12 - 6 - 4 + 3}{8}$$

$$= \left(5/8\right) = 62.50\%$$

68 The probability that there is atleast one error in an account statement prepared by 3 persons A,B,C are 0.20, 0.30, 0.10 respectively. If A, B, C prepare 60, 70, 90 such statements. Find expected number of correct statements.

a. 170 b. 176 c. 178 d. 180

	x	$p(x)$	$x \cdot p(x)$
A	60	0.80	
B	70	0.70	
C	90	0.90	
		$\Sigma x \cdot p(x)$	178

Expected No. of correct statements

$= \Sigma x \cdot p(x)$

$= 178$

My Notes

x	20	50	30
prob(x)	0.20	0.70	0.10

Find $E(x)$, variance of x, SD of x

$\Longrightarrow$

x	prob(x)	x^2	$x \cdot p(x)$	$x^2 \cdot p(x)$
20	0.20	400		
50	0.70	2500		
30	0.10	900		
	1.00		42.00	1920

$E(x) = \Sigma x \cdot \text{prob}(x) = 42$

$E(x^2) = \Sigma x^2 \cdot \text{prob}(x) = 1920$

$\text{vari. of } x = E(x^2) - [E(x)]^2$

$= 1920 - 42^2$

$= 156$

$\text{SD of } x = \sqrt{156}$

$= 12.49$

69

x	1	2	4	6	8
Prob. x	k	2k	3k	3k	k

Find Expected Value of x, SD of x, Variance of x.

$\Longrightarrow \quad \sum prob(x) = 1.00 = 10k$

$k = 0.10$

x	$prob(x)$	$x.p(x)$	$x^2.p(x)$
1	0.10		
2	0.20		
4	0.30		
6	0.30		
8	0.10		
		4.30	22.90

$E(x) = \sum x.prob(x) = 4.30$

$E(x^2) = \sum x^2.prob(x) = 22.90$

variance of x $= E(x^2) - [E(x)]^2$

$= 22.90 - 4.30^2$

$= 4.41$

SD of $x = \sqrt{4.41} = 2.10$

70

5 Red
6 White
4 Black

→ **4 Balls are drawn. What is the proabibility that there is atleast one ball of each colour?**

$$\Longrightarrow \left[\frac{(5c_1 \times 6c_1 \times 4c_2) + (5c_1 \times 6c_2 \times 4c_1) + (5c_2 \times 6c_1 \times 4c_1)}{15c_4}\right]$$

$= (180 + 300 + 240)/1365$

$= 720/1365 = \frac{144}{273} = \left(\frac{48}{91}\right) = 52.74725\%$

71

5 Red
12 Blue
3 Pink

→ **5 Balls are drawn. What is the proabibility that there is atleast one ball of each colour?**

$$\Longrightarrow \left[\frac{(5c_1 \times 12c_1 \times 3c_3) + (5c_1 \times 12c_3 \times 3c_1) + (5c_3 \times 12c_1 \times 3c_1) + (5c_2 \times 12c_2 \times 3c_1) + (5c_2 \times 12c_1 \times 3c_2) + (5c_1 \times 12c_2 \times 3c_2)}{20c_5}\right]$$

$$= \left(\frac{60 + 3300 + 360 + 1980 + 360 + 990}{15504}\right) = \left(\frac{7050}{15504}\right)$$

$= 45.4721\%$

72 The expected number of heads in 100 tosses of an unbaised coin is :

$\Longrightarrow \quad 100 \times 0.50 = 50$

73 A man can kill a bird once in 3 shots. The probability that bird is not killed is

a. 1/3 ~~b.~~ 2/3 c. 1.00 d. 0

74 If on an average 9 ships out of 10 return safely to the port, the probability that one ship returns to the port safely is

a. 1/10 ~~b.~~ 9/10 c. 8/10 d. None of these

75 A family has 2 children. The probability that both of them are boys if it is known that one of them is a boy is :

a. 1.00 ~~b.~~ 1/2 c. 3/4 d. None of these

76 Probability of throwing an odd number with an ordinary six faced die is?

~~a.~~ 1/2 b. 1.00 c. -1/2 d. 1/6

77 When none of the outcomes is favourable to the event then event is said to be

a. Certain b. Sample ~~c.~~ Impossible d. None

78 What is probability that 4 children selected at random would have different birthdays?

~~a.~~ 98.36% b. 100% c. 99.82% d. 0%

$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365}$

79 For 2 independent events A, B, P(AUB) = 2/3, P(A) = 2/5, P(B) = ?

a. 4/15 ~~b.~~ 4/9 c. 5/9 d. 7/18 e. None

$\frac{2}{3} = \frac{2}{5} + P(B) - \left(\frac{2}{5} \times P(B)\right) \quad \frac{4}{15} = P(B) \times \frac{3}{5} \quad \therefore P(B) = \frac{4}{15} \times \frac{5}{3} = \frac{4}{9}$

80 What is chance of throwing atleast 7 in a single cast with 2 dice?

a. 5/12 ~~b.~~ 7/12 c. 1/4 d. 17/36 e. None

$= (6+5+4+3+2+1)/36 = 21/36 = (7/12)$

81 Expected value of a random variable

a. Is always positive b. May be positive or negative
~~c.~~ May be positive, negative or zero d. Can never be zero

82 P(A) = 8/17, then odds against event A is .

a. 8:17 b. 17:8 c. 8:9 ~~d.~~ 9:8

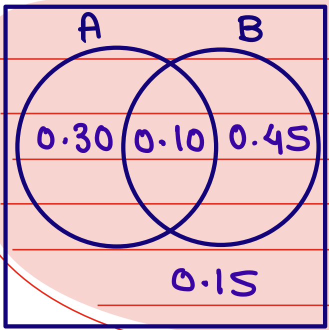
83 Initially probability was branch of

a. Physics b. Chemistry c. Statistics ~~d.~~ Mathematics

84 Subjective probability may be used in

a. Mathematics b. Statistics ~~c.~~ Management d. Biology

85 $P(A-B) = 0.30$, $P(A \cap B) = 0.10$, $P(A' \cap B') = 0.15$.
Find $P(A)$, $P(A \cup B)$, $P(A' \cup B')$, $P(B)$, $P(A \triangle B)$, $P(B-A)$, $P(A/B)$, $P(B'/A')$



$P(A) = 0.40$

$P(A \cup B) = 0.85$

$P(A' \cup B') = 0.90$

$P(B) = 0.55$

$P(A \triangle B) = 0.75$

$P(B-A) = 0.45$

$P(A/B) = \left(\frac{0.10}{0.55}\right) = 18.181818\%$

$P(B'/A') = \frac{0.15}{0.60} = 25\%$

86 P(A/B) is defined only when

$P(A/B) = \left[P(A \cap B) / P(B)\right]$ ..... This is defined only when $P(B) \neq 0$

a. B is a sure event b. B is an impossible event
c. B is not a sure event ~~d.~~ B is not an impossible event

87 P(A/B') is defined only when

$P(A/B') = \left[P(A \cap B') / P(B')\right]$ ... This is defined only when $P(B') \neq 0$ i.e. $P(B) \neq 1$

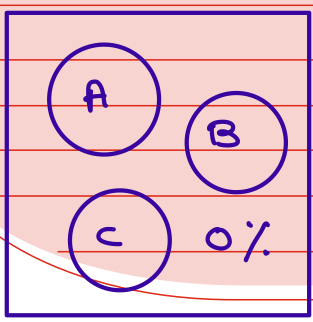
a. B is a sure event b. B is an impossible event
~~c.~~ B is not a sure event d. B is not an impossible event

88 P(X/Y) is defined only when : Y is not an impossible event.

P(X/Y') is defined only when : Y is not a sure event.

89 If A,B,C are 3 mutually exclusive and exhaustive events such that
$P(A) = 2.P(B) = 3.P(C)$ then $P(B) = ?$

a. 6/11 ~~b.~~ 3/11 c. 1/6 d. 1/3



$P(A \cup B \cup C) = P(A) + P(B) + P(C) - 0 - 0 - 0 + 0$

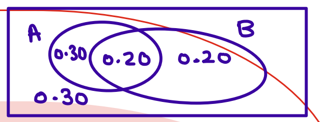
$1.00 = 2.P(B) + P(B) + \frac{2}{3}P(B)$

$1 = P(B) \times \left[2 + 1 + \frac{2}{3}\right]$

$1 = P(B) \times \left(\frac{11}{3}\right) \quad \therefore P(B) = 3/11$

90 $P(A-B) = 0.30$, $P(A \triangle B) = 0.50$, $P(A' \cup B') = 0.80$
Find $P(A' \cap B')$

OR



$\Rightarrow$

$P(A \triangle B) = P(A-B) + P(B-A)$

$0.50 = 0.30 + P(B-A)$

$P(B-A) = 0.20$

$P(A' \cup B') = 1 - P(A \cap B)$

$0.80 = 1 - P(A \cap B)$

$P(A \cap B) = 0.20$

$P(A-B) = P(A) - P(A \cap B)$

$0.30 = P(A) - 0.20$

$P(A) = 0.50$

$P(B-A) = P(B) - P(A \cap B)$

$0.20 = P(B) - 0.20$

$P(B) = 0.40$

$P(A \cup B)$

$= 0.50 + 0.40 - 0.20$

$= 0.70$

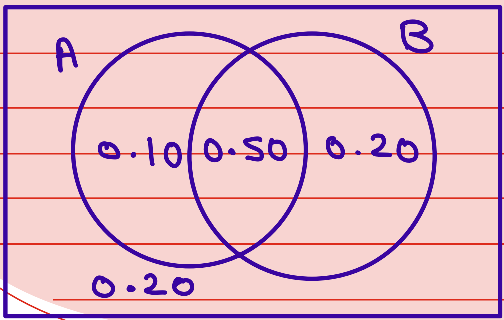
$\therefore P(A' \cap B')$

$= 1 - P(A \cup B)$

$= 1 - 0.70 = 0.30$

91 P(A) = 0.60, P(B) = 0.70, P(A'∩B') = 0.20
Find P(A-B), P(B-A), P(A∩B)

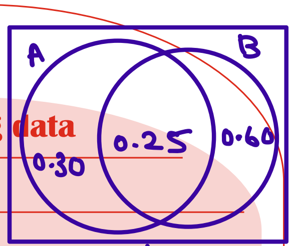
S



$P(A-B) = 0.10$
$P(B-A) = 0.20$
$P(A\cap B) = 0.50$
$P(A'\cup B') = 0.50$
$P(A\Delta B) = 0.30$

92 P(A-B) = 0.30, P(B-A) = 0.60, P(A) = 0.55
Find P(A∪B)

a. 1.15   b. 0.15   c. 0.85   ~~d.~~ Wrong data



this is wrong data

$P(A-B) = P(A) - P(A\cap B)$
$0.30 = 0.55 - P(A\cap B)$
$P(A\cap B) = 0.25$

$P(B-A) = P(B) - P(A\cap B)$
$0.60 = P(B) - 0.25$
$P(B) = 0.85$
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$= 0.55 + 0.85 - 0.25$
$= 1.15$

93 2 dice are rolled, what is probability that sum of points is a prime number?

$\Longrightarrow$ outcomes in favour: (1,1), (1,2), (2,1), (1,4), (2,3), (3,2), (4,1), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), (6,5)

$= \left(15/36\right) = 5/12 = 41\frac{2}{3}\% = 41.666666\%$

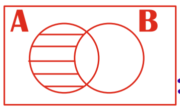
94 One card is drawn from each of 2 packs of 52 cards. What is probability that atleast one of them is an ace?

a. 8/104   b. ${}^{8}C_2/{}^{104}C_2$ ~~c.~~ 25/169   d. 1/169   e. None

$n = 2,\ p = \frac{1}{13},\ q = \frac{12}{13},\ x = 1, 2$

$P(x\geq 1) = 1 - P(x=0) = 1 - {}^{2}C_{0}\left(\frac{1}{13}\right)^{0}\left(\frac{12}{13}\right)^{2}$
$= 1 - \frac{144}{169} = \left(25/169\right)$

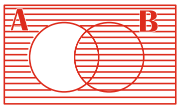
95



Shaded area represents

~~a.~~ (A-B)
b. (B-A)
c. (A∪B')
d. (A'∪B')

96



Shaded area represents

a. (A-B)
b. (A+B)
c. (AUB')
d. (BUA')

97 **A number is selected from first 100 natural numbers, what is the probability that**

It is a perfect square?

$= \left(\frac{{}^{10}C_1}{{}^{100}C_1}\right) = \frac{1}{10} = 0.10$

$= 10\%$

It is a perfect cube?

$= \left(\frac{{}^{4}C_1}{{}^{100}C_1}\right)$

$= 1/25 = 0.04 = 4\%$

It is an odd number?

$= \left(\frac{{}^{50}C_1}{{}^{100}C_1}\right)$

$= \frac{1}{2} = 0.50 = 50\%$

98 **2 cards are drawn one after other from a pack of 52 cards, what is the probability that both cards are kings if cards are drawn**

Without Replacement

$= \frac{{}^{4}C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{6}{1326} = \frac{1}{221}$

OR $\frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \left(\frac{1}{221}\right)$

With Replacement

$= \left(\frac{4}{52} \times \frac{4}{52}\right)$

$= \frac{1}{13} \times \frac{1}{13} = \left(1/169\right)$

99 **2 numbers are selected from first 50 natural numbers, find the probability that both are divisible by 3?**

$= \frac{{}^{16}C_2 \times {}^{34}C_0}{{}^{50}C_2} = \frac{120}{1225} = \frac{24}{245} = 9.7959\%$

100 **Mr. A says to Mr. B "If it rains today I will give you ₹ 50,000 but if it doesn't rain today you have to pay me ₹ 80,000". Find expected gain / (loss) for Mr. B if probability of raining is 0.20**

	x	$P(x)$	$x \cdot P(x)$
Rain	50,000	0.20	10,000
No Rain	−80,000	0.80	−64,000

Expected Gain for Mr. B = −54,000

= Loss of ₹ 54,000

101 **A and B tossed 3 coins each. What is probability that both of them will get same number of heads?**

0 heads	$1/8 \times 1/8 = 1/64$
1 head	$3/8 \times 3/8 = 9/64$
2 heads	$3/8 \times 3/8 = 9/64$
3 heads	$1/8 \times 1/8 = 1/64$

$20/64 = 5/16 = 31.25\%$

Theoretical Distribution

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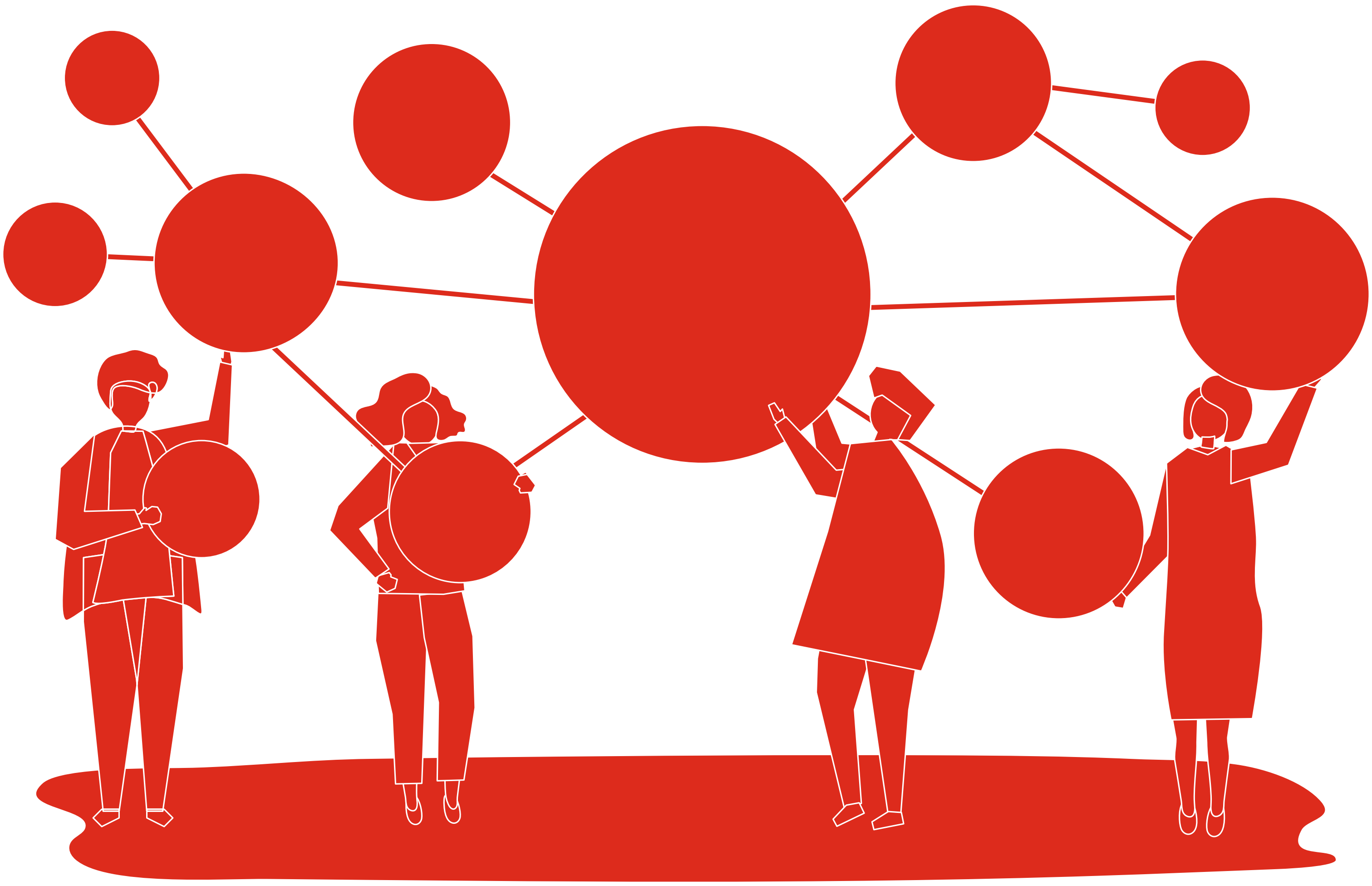


1 Theoretical Distributions OR probability distribution

Discrete probability distribution
- Binomial's Distribution
- Poisson's Distribution
- Rectangular
- Geometric
- Multinomial's

Continuous probability distribution
- Normal Distribution
- chi-square Distribution
- t – distribution (students)
- F – distribution.

2 Binomail's Distribution (Derived from Burnoulli's trials)

prob (x) = $^nC_x \cdot (p)^x \cdot (q)^{n-x}$

where, n = No. of Trials = Trials are finite, Independent.

p = probability of success in a single trial

x = Number of successes in 'n' trials

q = probability of failure in a single trial

Here $(p+q) = 1.00 = 100\%$

3 4 coins are tossed. What is probability of getting 3 heads

Classical Approach

No. of all possible equally likely outcomes $= 2^4 = 16$

outcomes in favour = HHHT, HHTH, HTHH, THHH

probability $= \left(\frac{4}{16}\right) = \frac{1}{4}$

$= 0.25 = 25\%$

OR

Binomial's Approach

$n = 4, \; p = \frac{1}{2}, \; q = \frac{1}{2}, \; x = 3$

$P(x=3)$

$= {}^4C_3 \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^1$

$= 4 \times \frac{1}{8} \times \frac{1}{2} = \frac{4}{16} = \frac{1}{4}$

$= 0.25 = 25\%$

My Notes

8 dates are selected at random, what is probability of getting 4 sundays?

$\Longrightarrow \quad n = 8, \; p = \frac{1}{7}, \; q = \frac{6}{7}, \; x = 4$

$prob(x=4) = {}^8C_4 \left(\frac{1}{7}\right)^4 \left(\frac{6}{7}\right)^4 = 70 \times \frac{1296}{5764801} = \frac{90720}{5764801}$

$= 1.5737\%$

4 5 coins are tossed. What is probability of getting 3 heads

Classical Approach OR **Binomial's Approach**

Classical Approach

No. of All possible equally likely outcomes = 32

HHHTT, HHTHT, HHTTH, HTHHT, HTHTH, HTTHH, THHHT, THHTH, THTHH, TTHHH

probability $= \frac{10}{32} = \frac{5}{16} = 31.25\%$

Binomial's Approach

$n = 5,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 3$

prob $(x = 3)$

$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= 10 \times \frac{1}{8} \times \frac{1}{4} = \frac{10}{32} = \frac{5}{16}$

$= 31.25\%$

5 Mode of Binomial's distribution =

Largest integer contained in (n+1)P if (n+1)P is non integer. Data is uni-modal.

If (n+1)P is an integer, then data is bi-modal.

Modes are (n+1)P and (n+1)P-1

6 Freq (x) = N x $^nC_x\, p^x.q^{n-x}$

where 'n' trial are repeated 'N' times

6 coins are tossed 1,60,000 times. Find expected freq. of getting (a) 4 heads (b) 3 Tails (c) atleast 5 heads (d) atmost 3 tails

$\Longrightarrow$ (a) $n = 6,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 4$

prob$(x=4)$ $= {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 = \left(\frac{15}{64}\right) = 0.234375$

$\frac{0.234375}{1} = \frac{?}{1,60,000}$

Freq $(x=4)$

$= 1,60,000 \times 0.234375$

$= N \times$ prob$(x=4)$

$= 37,500$

(b) Freq$(x=3) = N \times$ prob$(x=3) = 1,60,000 \times {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3$

$= 50,000$

My Notes

(c) Freq $(x \geq 5) = N \times [P(x=5) + P(x=6)]$

$= 1,60,000 \left[{}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0\right]$

$= 1,60,000 \times \left(\frac{1}{2}\right)^6 (6+1) = 17,500$

(d) Freq $(x \leq 3) = 1,60,000 \times \frac{1}{2^6} \left({}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3\right)$

$= 1,60,000 \times \left(\frac{42}{64}\right) = 1,05,000$

7 8 Coins are tossed 40,000 times. Find expected frequency of at most 7 heads?

$\Longrightarrow$ $n=8,\ p=\frac{1}{2},\ q=\frac{1}{2},\ x=0,1,2,3,4,5,6,7,\ N=40{,}000$

$$Freq\ (x\le 7) = N \times prob\ (x \le 7)$$
$$= N \times \left[1 - P(x=8)\right]$$
$$= 40{,}000 \times \left[1 - {}^{8}C_{8}\left(\tfrac{1}{2}\right)^{8}\left(\tfrac{1}{2}\right)^{0}\right]$$
$$= 40{,}000 \times \left(1 - \frac{1}{256}\right) = 40{,}000 \times \frac{255}{256}$$
$$= 39843.75$$

8 10 coins are tossed. Find probability of getting

a. 2 heads $= {}^{10}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{8} = (45/1024)$

b. 3 heads $= {}^{10}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{7} = (120/1024)$

c. 3 tails $= {}^{10}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{7} = (120/1024)$

d. 4 tails $= {}^{10}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{6} = (210/1024)$

e. 5 or 7 heads $= {}^{10}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{5} + {}^{10}C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{10}\left({}^{10}C_{5} + {}^{10}C_{7}\right)$
$= (372/1024)$

f. 4 or 5 or 6 heads $= \left(\frac{1}{2}\right)^{10}\left({}^{10}C_{4} + {}^{10}C_{5} + {}^{10}C_{6}\right) = (672/1024)$

g. Atmost 9 heads $= 1 - P(x=10) = 1 - {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{0} = 1 - \frac{1}{1024}$
$= (1023/1024)$

h. Atleast 2 heads $= 1 - \left(\frac{1}{2}\right)^{10}\left({}^{10}C_{0} + {}^{10}C_{1}\right) = 1 - \frac{11}{1024} = (1013/1024)$

I. Atleast 1 tails $= 1 - {}^{10}C_{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{10} = 1 - \frac{1}{1024} = (1023/1024)$

j. Atmost 2 tails $= \left(\frac{1}{2}\right)^{10}\left({}^{10}C_{0} + {}^{10}C_{1} + {}^{10}C_{2}\right) = \frac{56}{1024}$

My Notes

3 dice are rolled. what is prob. of getting 4 points on 2 dice?

$\longrightarrow$ $n=3,\ p=\frac{1}{6},\ q=\frac{5}{6},\ x=2$

$$prob\ (x=2) = {}^{3}C_{2}\left(\tfrac{1}{6}\right)^{2}\left(\tfrac{5}{6}\right)^{1} = 3 \times \frac{1}{36} \times \frac{5}{6} = \left(\frac{15}{216}\right) = \frac{5}{72}$$

$= 6.944444\%$

9 2 dice are rolled what is probability of getting odd points on atleast one dice

Classical Approach

outcomes in favour

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,3), (2,5) (3,1) (3,2) (3,3)
(3,4) (3,5) (3,6) (4,1) (4,3), (4,5)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,3) (6,5)

probability $= \left(\frac{27}{36}\right) = \frac{3}{4} = 75\%$

OR

Binomial's Approach

$n = 2,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 1, 2$

$P(x \geq 1) = 1 - P(x = 0)$

$= 1 - {}^2C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^2$

$= 1 - \frac{1}{4} = \frac{3}{4} = 0.75$

$= 75\%$

10 2 dice are rolled. What is the probability of getting 5 points on atmost 1 dice?

Classical Approach

outcomes in favour

$\Longrightarrow$ All except (5,5)

probability $= \left(\frac{35}{36}\right)$

Binomial's Approach

$n = 2,\ p = \frac{1}{6},\ q = \frac{5}{6},\ x = 0, 1$

prob $(x \leq 1)$

$= 1 - P(x = 2)$

$= 1 - {}^2C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^0$

$= 1 - \frac{1}{36} = \frac{35}{36}$

11 5 dice are rolled. What is the probability of getting 3 points on 4 dice?

$n = 5,\ p = \frac{1}{6},\ q = \frac{5}{6},\ x = 4$

prob $(x = 4) = {}^5C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^1$

$= 5 \times \frac{1}{1296} \times \frac{5}{6} = \left(\frac{25}{7776}\right)$

$= 0.321502057\%$

My Notes

8 dice are rolled. what is probability of getting 3 points on 3 dice?

$\Longrightarrow = {}^8C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^5 = 56 \times \frac{3125}{16,79,616} = \frac{1,75,000}{16,79,616}$

$= 10.419\%$

12 5 dice are rolled. What is the probability of getting at least 5 points on atmost 1 dice?

$\Longrightarrow \quad n=5, \; p=\frac{1}{3}, \; q=\frac{2}{3}, \; x=0,1$

$p(x \le 1) = p(x=0) + p(x=1)$

$= 5c_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5 + 5c_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4$

$= \left(\frac{32}{243} + \frac{80}{243}\right) = \left(\frac{112}{243}\right) = 46.090535\%$

13 4 dice are rolled. What is the probability of getting atleast 3 points on atleast 3 dice?

$\Longrightarrow \quad n=4, \; p=\frac{4}{6}=\frac{2}{3}, \; q=\frac{1}{3}, \; x=3,4$

$p(x \ge 3) = 4c_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^1 + 4c_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^0$

$= \left(4 \times \frac{8}{27} \times \frac{1}{3}\right) + \left(1 \times \frac{16}{81} \times 1\right) = \left(\frac{48}{81}\right) = \frac{16}{27}$

$= 59.25926\%$

14 15 dates are selected at random. What is the probability of getting 4 Sundays?

$\Longrightarrow \quad n=15, \; p=\frac{1}{7}, \; q=\frac{6}{7}, \; x=4$

$prob(x=4) = 15c_4 \left(\frac{1}{7}\right)^4 \left(\frac{6}{7}\right)^{11}$

$= 1365 \times \frac{1}{2401} \times \frac{362797056}{1977326743}$

$= 10.4\%$

15 4 x prob(x=4) = prob(x=2) for Binomial's distribution and n=6. Find values of p,q?

$\Longrightarrow \quad 4 \times prob(x=4) = prob(x=2)$

$4 \times 6c_4 \, p^4 \cdot q^2 = 6c_2 \, p^2 q^4$

$4 \times 1\cancel{5} \times \cancel{p^2} \times p^2 \times \cancel{q^2} = 1\cancel{5} \times \cancel{p^2} \times \cancel{q^2} \times q^2$

$4p^2 = q^2$

$(2p)^2 = (q)^2$

$2p = q$

$2p = 1-p$

$3p = 1$

$\therefore p = \frac{1}{3}, \; q = \frac{2}{3}$

$p+q=1.00$, Mean $= n.p$, $SD=\sqrt{npq}$, variance $= npq$

16

n	p	q	Mean	SD	Variance
20	0.20	0.80	4.00	1.788854	3.20
80	0.40	0.60	32.00	4.38178	19.20
120	0.05	0.95	6.00	2.38747	5.70
200	0.225	0.775	45	5.90551	34.875
250	0.20	0.80	50	6.324555	40.00
100	0.80	0.20	80	4.00	16.00
500	0.20	0.80	100	8.944272	80
60	0.35	0.65	21.00	3.6946	13.65
2,000	0.95	0.05	1,900	9.74679	95.00
8,000	0.02	0.98	160	12.5220	156.80
10,000	0.63	0.37	6,300	48.280431	2331.00

17 Summary of Binomial's Distribution.

① It is a discrete probability distribution derived from Burnoulli's trials

② $prob(x) = {}^{n}C_{x} \cdot (p)^{x} \cdot (q)^{n-x}$

③ $p+q=1.00$

④ Trials are finite & independent

⑤ Mean $= n.p.$

⑥ variance $= n.p.q$

⑦ $S.D. = \sqrt{npq}$

⑧ Mode = Find value of $(n+1)p$

Integer → Bi-modal → modes $= (n+1)p$ and $(n+1)p-1$

Fraction → uni-modal → [Largest integer contained in value of $(n+1)p$]

⑨ success & failure of mutually exclusive & mutually exhaustive outcomes

⑩ $p,q \neq 0$ There must be some probability of success, failure in each trial

⑪ If $p=q=0.50$ the Binomial's distri. is said to be symmetrical

⑫ If $p \neq q$ then Binomial's distri. is said to be asymmetrical OR Non-symmetrical

⑬ n, p are parameters of Binomial's distri ∴ It is Bi-parametric distri.

⑭ $x = 0,1,2,3,4,-----,n$

⑮ As $np > npq$ we can say that mean is always greater than variance

⑯ Binomial's distribution attains maximum variance of $0.25n = n/4$ when $p=q=0.50$

⑰ $Freq(x) = N \times prob(x)$
$= N \times {}^{n}C_{x} (p)^{x} (q)^{n-x}$

My Notes

18 Prob (x) as per poisson's model =

$$prob(x) = \left(\frac{e^{-m} \times m^{x}}{x!}\right)$$

where
e = exponential factor = 2.7183 (approx)
m = mean or variance of poisson's variate.
$x = 0, 1, 2, 3, \ldots\ldots \infty$

19 If m = 4. Find prob (x=5) for poisson's variate.

$$prob(x=5) = \frac{e^{-4} \times 4^{5}}{5!}$$

$$= \frac{8.5333333}{2.7183^{4}} = 15.6289\%$$

'm' is the only parameter of poisson's distribution
∴ poisson's distribution is uni-parametric.

20 If SD of poisson's variate is 2. Find probability (-2.30 < x ≤ 1)

$\Longrightarrow \sqrt{m} = 2, \; m = 4$

$$prob(-2.30 < x \le 1)$$

$$= p(x=0) + p(x=1)$$

$$= \frac{e^{-m} \times m^{0}}{0!} + \frac{e^{-m} \times m^{1}}{1!}$$

$$= e^{-m}\left(\frac{4^0}{0!} + \frac{4^1}{1!}\right) = \left(\frac{5}{2.7183^4}\right)$$

$$= 9.15757457\%$$

For poisson's variate

Mean = m
variance = m
S.D. = $\sqrt{m}$

21 If m = 3, for poisson's variate. Find prob (x ≥ 1), prob (x > 1), prob (3 ≤ x ≤ 5)

① $prob(x \ge 1) = p(x=1) + p(x=2) + p(x=3) + \ldots\ldots$

$$= 1 - p(x=0)$$

$$= 1 - \left(\frac{e^{-3} \times 3^0}{0!}\right) = 1 - \frac{1}{2.7183^3} = 95.021\%$$

② $prob(x > 1) = p(x=2) + p(x=3) + p(x=4) + \ldots\ldots$

$$= 1 - [p(x=0) + p(x=1)] = 1 - \left[e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!}\right)\right]$$

$$= 1 - \frac{4}{2.7183^3} = 80.08557\%$$

③ $prob(3 \le x \le 5) = p(x=3) + p(x=4) + p(x=5)$

$$= e^{-3}\left(\frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}\right) = \frac{9.90}{2.7183^3} = 49.28821\%$$

22 n = 200, p = 0.01, find prob (x=2)

Binomial's Model

$$= {}^{n}C_{x} (p)^{x} (q)^{n-x}$$

$$= {}^{200}C_{2} (0.01)^{2} (0.99)^{198}$$

$$= 19900 \times 0.0001 \times 0.1366333938$$

$$= 27.20\%$$

Poisson's Model

$$m = 200 \times 0.01 = 2$$

$$p(x=2) = \frac{e^{-m} \times m^{x}}{x!}$$

$$= \frac{e^{-2} \times 2^{2}}{2!} = \frac{2}{2.7183^{2}} = 27.10\%$$

23 Difference between Binomial's & Poisson's Distribution.

Binomial's Distribution	Poisson's Distribution
① Formula $prob(x) = {}^{n}C_{x} (p)^{x} (q)^{n-x}$	$prob(x) = \left(\frac{e^{-m} \times m^{x}}{x!}\right)$
② value of x $x = 0, 1, 2, 3, \text{-------}, n$	$x = 0, 1, 2, 3, \text{-------}, \infty$
③ Mean = $n \cdot p$	③ Mean = m
④ variance = $n \cdot p \cdot q$	④ variance = m
⑤ $SD = \sqrt{npq}$	⑤ $SD = \sqrt{m}$
⑥ Relation betn mean & variance Mean > Variance as $np > npq$	⑥ Mean = variance = m
⑦ parameters are n, p	⑦ parameter = m
⑧ Biparametric	⑧ uni-parametric
⑨ Mode = $(n+1)p$	⑨ Mode = Find value of 'm' Integer → modes are = m, (m−1) → Bi-modal Fraction → Largest integer in value of 'm' → uni-modal

24 If m = 5. Find prob (-8 ≤ x ≤ 1.56) for poisson's variate

$\Longrightarrow$ prob $(-8 \le x \le 1.56)$

$= p(x=0) + p(x=1)$

$= \left(\frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!}\right) = e^{-5}(1+5)$

$= 6/2.7183^5 = 4.0426\%$

25 p(x=3) = p(x=4). Find mean of Poisson's Distribution.

$\Longrightarrow p(x=3) = p(x=4)$

$\frac{\cancel{e^{-m}} \times \cancel{m^3}}{3!} = \frac{\cancel{e^{-m}} \times \cancel{m^4}\, m}{4!}$

$\frac{1}{6} = \frac{m}{24}$ $\quad \therefore m = 4$ $\quad$ Mean = variance = 4 = m

SD = $\sqrt{4} = 2$

26 Summary of Poisson's Distribution

1. It is a Discrete probability distribution
2. prob$(x) = \left(\frac{e^{-m} \times m^x}{x!}\right)$

   where $x = 0, 1, 2, 3, \ldots \infty$

   $e = 2.7183$ (approx)

   m = mean of Distribution
3. Mean = variance = m
4. SD = $\sqrt{m}$
5. It is uniparametric distri. as only parameter is 'm'
6. Mode = Find value of m'
   - Integer → modes are = m, (m−1) → Bi-modal
   - Fraction → Largest integer in value of 'm' → uni-modal
7. poisson's distri. can be uni-modal or Bi-modal
8. It is used when
   (a) probabi. of success or failure is very small or

   There are extremely Large No. of trials.

My Notes

If n=15 and p=0.25 Find mode of Binomial's distribution

$\Longrightarrow (n+1)\times p = (15+1) \times 0.25$

$= 4$

$\therefore$ Modes are = 4, 3

If n=23 and p=0.23 Find mode of Binomial's distribution

$\Rightarrow (n+1)p = (23+1) \times 0.23$

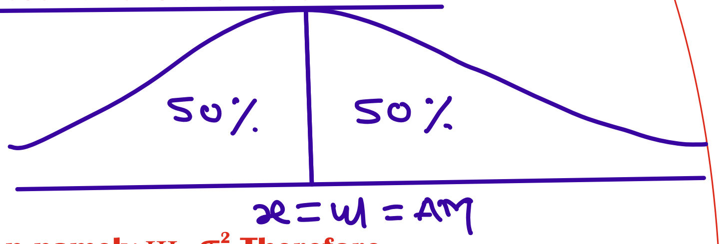
$= 5.52$

Mode = 5

(19) points of inflexion are $(\mu-\sigma)$ & $(\mu+\sigma)$

27 Normal Distribution

1. It is applicable only for distribution of a 'continous variable' like age, Height, income, salary, weight etc
2. Derived by Karl Gauss :- known as Gaussian's theorem.
3. It is based on assumption of Normality.
4. As per assumption of Normality a variable is said to be normally distributed if 50% observations are less than AM and 50% of the observations are more than AM.
5. $\boxed{\text{prob}(x < \mu) = 50\% \quad \text{Prob}(x > \mu) = 50\%}$



6. Z = Normal curve wefficient = $\frac{(x-\mu)}{\sigma}$
7. There are 2 parameters of normal distribution namely μ, σ^2 Therefore It is a Bi-parametric distribution

μ = AM of Normal Distribution

σ = SD of Normal Distribution

8. Normal curve is a Bell-shaped curve, symmetrical about AM.
9. In probability distribution of this type:

   Prob($x \leq 50$) = prob($x < 50$)

   Prob($x \geq 85$) = prob($x > 85$)

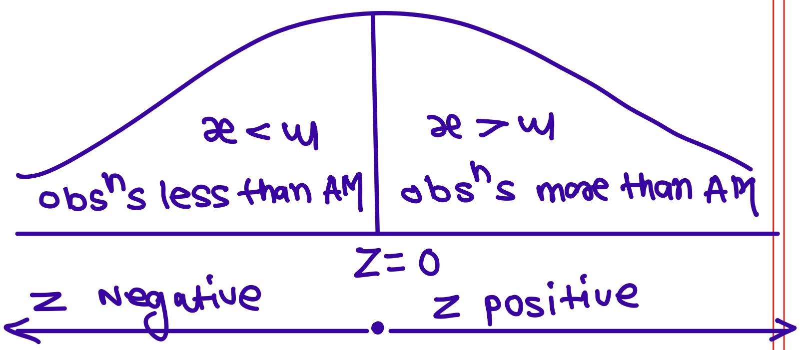
Therefore, we can say that : probability that a particular variable will assume a specific value is always 0.

10. AM= Median= Mode

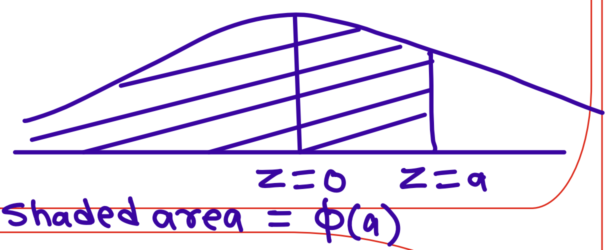
(For Normal Distri. AM, Median, Mode coincide)

$Z = \left(\frac{x-\mu}{\sigma}\right)$

11. Median = $\frac{(Q_3+Q_1)}{2}$ = μ = mode
12. $\boxed{\text{Q.D.} = \frac{(Q_3-Q_1)}{2} = 0.6750 \times \text{SD}}$



13. $\boxed{\text{MD} = 0.80 \times \text{SD}}$
14. $\boxed{Q_3 = \mu + 0.675\sigma \quad Q_1 = \mu - 0.675\sigma}$
15. Φ(a) represents area from $-\infty$ to a.
16. Total area covered by normal curve = 1.00= 100%
17. Expected frequency = N x probability
18. For normal distribution : SD > MD > QD



28 For Normal Distribution

1. Relation between MD & SD

   $\boxed{\text{MD} = 0.80 \times \text{SD}}$

2. Relation between QD & SD

   $\boxed{\text{QD} = 0.6750 \times \text{SD}}$

3. Relation between MD & QD

   $\boxed{\text{MD} = 1.1852 \times \text{QD}}$

   $\boxed{\text{Q.D.} = 0.84375 \times \text{MD}}$

Binomial's	Biparametric	n, p
poisson's	uniparametric	m
Normal	Bi-parametric	μ, σ^2

QD = 0.6750 x SD
MD = 0.80 x SD

29

Q_3	Q_1	Q.D.	M.D.	S.D.
50	20	15.00	17.7777	22.22222
69.60	29.40	20.10	23.8222	29.77777
86	40	23.00	27.259256	34.07407
91.80	40.63	25.585	30.322296	37.9037
81.88	43.63	19.125	22.66666	28.333
28.93	12.13	8.40	9.95555	12.44444
60.86	12.98	23.94	28.373333	35.46666

30 QD < MD < SD OR SD > MD > QD

31 We have thrown 6 bombs at a building. 2 bombs are sufficient to destroy the building. Find the probability of destruction of building if chance that bomb hitting the target is 0.20.

$\Longrightarrow$ $n = 6,\ p = 0.20,\ q = 0.80,\ x = 2,3,4,5,6$

$p(x \geq 2) = 1 - [p(x=0) + p(x=1)]$

$= 1 - \left[{}^6C_0 (0.20)^0 (0.80)^6 + {}^6C_1 (0.20)^1 (0.80)^5\right]$

$= 1 - (0.262144 + 0.393216) = 0.34464 = 34.464\%$

32 An overall 70% students passed in the exam. Find the probability that out of 10 students randomly selected atleast 8 have passed the exam?

$\Longrightarrow$ $n = 10,\ p = 0.70,\ q = 0.30,\ x = 8,9,10$

$prob(x \geq 8) = p(x=8) + p(x=9) + p(x=10)$

$= {}^{10}C_8 (0.70)^8 (0.30)^2 + {}^{10}C_9 (0.70)^9 (0.30)^1 +$

${}^{10}C_{10} (0.70)^{10} (0.30)^0$

$= 38.2782786 4\%$

33 8 coins are tossed 409600 times. Find the expected frequency of atleast 6 tails?

$\Longrightarrow$ $n = 8,\ N = 409600,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 6,7,8$

$Freq(x \geq 6) = N \times prob(x \geq 6)$

$= 409600 \times \left(\frac{1}{2}\right)^8 \left({}^8C_6 + {}^8C_7 + {}^8C_8\right)$

$= 409600 \times \frac{1}{256} \times 37 = 59{,}200$

34 There are 12,800 families with 5 children each. How many of them are expected to have atleast 4 boys?

$\Longrightarrow$ $n=5$, $N=12800$, $p=\frac{1}{2}$, $q=\frac{1}{2}$, $x=4,5$

$\text{Freq}(x\geq 4) = N \times \text{prob}(x\geq 4)$

$= 12800 \times \left(\frac{1}{2}\right)^5 \left({}^5C_4 + {}^5C_5\right)$

$= 12,800 \times \frac{1}{32} \times 6 = 2400$ families are expected to have atleast 4 boys.

35 5% of total bulbs are known to be defective. 6 bulbs are selected at random, what is the probability of getting 3 defective bulbs?

$p=0.05$, $q=0.95$, $n=6$, $x=3$

$\text{prob}(x=3) = {}^6C_3 (0.05)^3 (0.95)^3$

$= 0.21434375\%$

36 60% of total students passed in exams. Find the probability that in the group of 7 students atleast 5 have passed the exam.

$\Longrightarrow$ $p=0.60$, $q=0.40$, $n=7$, $x=5,6,7$

$\text{prob}(x\geq 5) = p(x=5) + p(x=6) + p(x=7)$

$= {}^7C_5 (0.60)^5 (0.40)^2 + {}^7C_6 (0.60)^6 (0.40)^1 + {}^7C_7 (0.60)^7 (0.40)^0$

$= 41.9904\%$

37 5 coins are tossed 512 times. Find out expected frequency of getting 0,1,2,3,4,5 heads. Also, Find mean, SD, variance of the distribution.

x = No. of heads	prob(x)	Expected frequency = $N \times P(x)$
0	${}^5C_0 (\frac{1}{2})^0 (\frac{1}{2})^5 = 1/32$	16
1	${}^5C_1 (\frac{1}{2})^1 (\frac{1}{2})^4 = 5/32$	80
2	${}^5C_2 (\frac{1}{2})^2 (\frac{1}{2})^3 = 10/32$	160
3	${}^5C_3 (\frac{1}{2})^3 (\frac{1}{2})^2 = 10/32$	160
4	${}^5C_4 (\frac{1}{2})^4 (\frac{1}{2})^1 = 5/32$	80
5	${}^5C_5 (\frac{1}{2})^5 (\frac{1}{2})^0 = 1/32$	16

Mean $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{1280}{512} = 2.50$

Mean $= n.p$

$= 5 \times \frac{1}{2} = 2.50$

variance $= n.p.q$

$= 1.25$

SD $= \sqrt{1.25} = 1.118033$

38

p	q	Variance = n.p.q
0.90	0.10	0.09n
0.80	0.20	0.16n
0.37	0.63	0.2331n
0.50	0.50	$\boxed{0.25n}$
0.85	0.15	0.1275n
0.89	0.11	0.0979n
0.09	0.91	0.0819n
0.02	0.98	0.0196n

Therefore, Variance attains its max value of 0.25n = n/4, when p = q = 0.50

39

Variance in case of Binomial's distribution attains its max value in case of symmetrical Binomial's distribution. $(p = q = 0.50)$

40

In Binomail's distribution, no. of successes, no. of trials must be a whole number. It cannot be in fractions. $x = 0, 1, 2, 3, 4, \text{------}, n$

41 Find p, q. If n = 6 and 4 x prob(x = 4) = prob(x = 2), Find mode.

$\Longrightarrow$ $4\ prob(x=4) = prob(x=2)$

$4 \times {}^6C_4\ p^4 . q^2 = {}^6C_2\ p^2 q^4$

$4 \times 15 \times \cancel{p^2} . p^2 . \cancel{q^2} = 15 \times \cancel{p^2} . \cancel{q^2} . q^2$

$(2p)^2 = (q)^2$

$2p = 1 - p$

$3p = 1$

$\therefore p = \frac{1}{3}, \quad q = \frac{2}{3}$

$(n+1)p$

$= (6+1) \times \frac{1}{3}$

$= 2.333333$

Mode = 2

& Distribution is uni-modal

42 2% of the bulbs manufactured are known to be defective. 200 bulbs are selected at random, what is the probability of getting 3 defective bulbs?

$\Longrightarrow$ $m = 200 \times 2\% = 4$

$prob(x=3) = \left(\frac{e^{-4} \times 4^3}{3!}\right) = \frac{10.6666666}{2.7183^4} = 19.5362\%$

OR

$= {}^{200}C_3 (0.02)^3 (0.98)^{197} = 19.59\%$

My Notes

If $n = 17$, $p = \frac{1}{3}$ Find mode of Binomial's Distri.

$\Longrightarrow$ $(n+1)p = (17+1) \times \frac{1}{3} = 6$

$\therefore$ Modes are $= 6, 5$ $\quad \therefore$ Distribution is Bi-modal.

43 An experiment succeeds twice as it fails. If the experiment is repeated 5 times, what is the probability having no success at all?

$\Rightarrow$ $p = \frac{2}{3}$, $q = \frac{1}{3}$, $n = 5$, $x = 0$

$$\text{prob}(x=0) = {}^5C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5 = \left(\frac{1}{243}\right)$$

$= 0.411522633\%$

44 We generally think of using Poisson's model instead of Binomial's model when :

i) Probability of finding success is very small OR

ii) Probability of having more than one success in this time interval is very low OR

iii) when Number are extremely large No. of trials

45 Like Binomial's distribution, Poisson's distribution could also be uni-modal or bi-modal depending upon the value of m

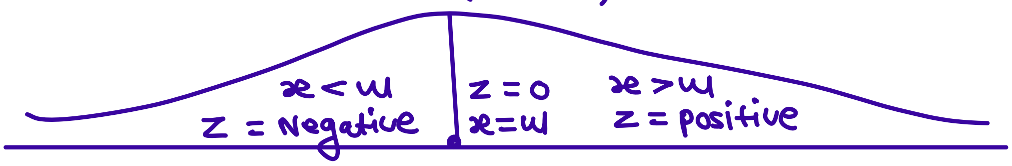
If 'm' is an integer then it is bi-modal
modes are = (m) and (m-1)
↓
Bi-modal

If 'm' is non-integer then it is uni-modal
mode is = Largest integer contained in 'm'
↓
uni-modal

46

When	z
x = μ	zero
x > μ	positive
x < μ	Negative

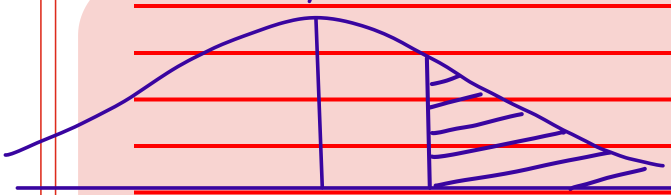
Normal curve coefficient $(z) = \left(\frac{x - \mu}{\sigma}\right)$



47 Wages of workers in a factory are normally distributed with AM and SD of ₹ 8000, ₹ 750 respectively. If 1 worker is selected at random. Find the probability that he earns a) more than ₹ 8500; b) Less than ₹ 8250; c) between ₹ 7500 and ₹ 8750

$\phi(-0.67) = 0.25143$
$\phi(0.67) = 0.74857$
$\phi(0.33) = 0.62930$
$\phi(1.00) = 0.84134$

$\mu = ₹8000$, $\sigma = ₹750$



$x = 8000$ $x = 8500$

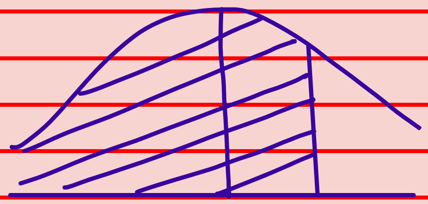
$z = 0$ $z = \frac{8500 - 8000}{750}$

$z = 0.67$

$\text{prob}(x > 8500) = 1 - 0.74857$

$= 0.25143$

i.e. 25.143%



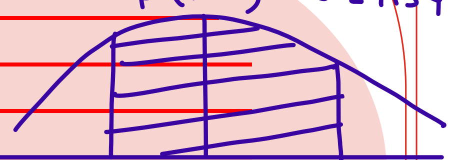
$x = 8000$ $x = 8250$

$z = 0$ $z = 0.33$

$\text{prob}(x < 8250)$

$= 0.62930$

$= 62.93\%$



$x = 7500$ $x = 8000$ $x = 8750$

$z = -0.67$ $z = 0$ $z = 1.00$

$\text{prob}(7500 < x < 8750)$

$= 0.84134 - 0.25143$

$= 0.58991$

i.e. 58.991%

$\phi(a)$ represents area from $-\infty$ to a

48 Characteristics of population are known as Parameters

Characteristics of sample are known as Statistic.

49 For normal distribution, Probability density function =

50 The normal distribution is symmetrical when $x = \mu$. When $x = \mu$, then skewness of normal curve is zero; i.e. neither inclined to move towards the right (Negatively skewed) nor towards the left (Positively skewed)

51 The normal curve has 2 points of inflexion to be given by $x = \mu - \sigma$ and $x = \mu + \sigma$ i.e. at these points, the normal curve changes its curvature from concave to convex and from convex to concave.

52 The theoretical probability distribution:-

a) Does not exists

~~b~~) Exists only in theory

c) Exists in real life

d) None of these

53 The probability distribution may be ________

a. Discrete

b. Continuous

~~c~~. a or b

d. None of these

54 An example of parameter is ____________

a. Sample SD

b. Sample mean

c. Sample mode

~~d~~. Population mean

55 A trial is an attempt to _________________________

a. Make something possible

b. Make something impossible

c. Prosecute in court of law

~~d~~. Produce an outcome that is neither certain nor impossible

56 The important characteristics of Bernoulli's trials are :

a. Each trial is associated with just 2 possible outcomes.

b. Trials are independent

c. Trials are infinite

~~d~~. Both a & b

My Notes

57 If x is a Binomial Variable with parameters n,p then x can assume

a. Any value between 0 and n
b. Any value between 0 and n, both inclusive
~~c.~~ Any whole no. between 0 and n, both inclusive
d. Any number between 0 and infinity

58 For a symmetrical binomial's distribution If n = 60.
Find mean, SD, variance of the distribution

$\Longrightarrow$ $n = 60, \; p = 0.50, \; q = 0.50$

Mean $= n.p = 60 \times 0.50 = 30$

Variance $= n.p.q = 60 \times 0.50 \times 0.50 = 15$

SD $= \sqrt{n.p.q} = \sqrt{15} = 3.87298$

59 Which of the following is Bi-parametric distribution :

a. Binomial's Distribution
b. Normal Distribution
~~c.~~ Both of these
d. None of these

60 Which of the following is Uni-parametric distribution :

a. Binomial's Distribution
~~b.~~ Possion's Distribution
c. Normal Distribution
d. None of these

61 The most important continuous probability distribution is known as ______________ .

a. Binomial's Distribution
~~b.~~ Normal Distribution
c. Chi-square Distribution
d. Sampling Distribution

62 The total area of a normal curve is

~~a.~~ 1.00 (i.e. 100%)
b. 50%
c. 0.25
d. 0.00

63 The normal curve is

~~a.~~ Bell-shaped
b. U-shaped
c. J-shaped
d. V-shaped

64 The normal curve is

a. Positively skewed
b. Negatively skewed
~~c.~~ Symmetrical about AM
d. None of these

Median $= \left(\dfrac{Q_3 + Q_1}{2}\right)$

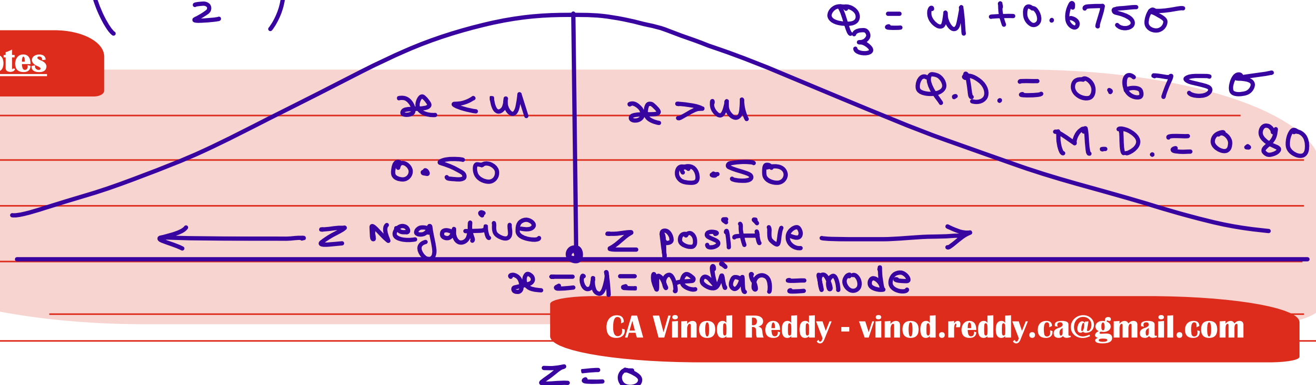
$Q_1 = \mu - 0.675\sigma$

$Q_3 = \mu + 0.675\sigma$

$Q.D. = 0.675\sigma$

$M.D. = 0.80\sigma$

My Notes



65 What is SD of number of recoveries among 48 patients when probability of recovering is 0.75

$SD = \sqrt{npq} = \sqrt{48 \times 0.75 \times 0.25} = \sqrt{9} = 3$

a. 36 b. 81 c. 9 ~~d.~~ 3

66 If x ~ B (n,p), what would be the greatest value of variance of x, when n = 16

a. 2 ~~b.~~ 4 c. 8 d. $\sqrt{5}$

Greatest variance $= 0.25n = 0.25 \times 16 = 4$

67 If x is a binomial variate with n = 15 and p = 1/3. What is the mode of the distribution

a. 5 and 6 ~~b.~~ 5 c. 5.50 d. 6

$\text{Mode} = (n+1)p = (15+1) \times \frac{1}{3} = 5.3333$

68 For Binomial's distribution n = ?, mean = 3, SD = 1.50

a. 2 b. 4 c. 8 ~~d.~~ 12

$np = 3, \; npq = 1.50^2 = 2.25, \; 3 \times q = 2.25 \; \therefore q = 0.75$

$p = 0.25$

$np = 3$

$n \times 0.25 = 3$

$n = 12$

69 What is probability of 5 correct guesses in 12 true-false questions?

$\Longrightarrow \; {}^{12}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^7$

$= \left(\frac{792}{4096}\right) = 19.3359375\%$

70 If Q_3 = 25.40, Q_1 = 14.60. Find SD of normal distribution.

$QD = (Q_3 - Q_1)/2 = 5.40$

$SD = \left(\frac{5.40}{0.675}\right) = 8$

a. 9 b. 6 c. 10 ~~d.~~ 8

71 Points of inflexion of a normal curve are 40,60 respectively. Find mean of normal distribution.

a. 8 b. 45 ~~c.~~ 50 d. 60

$\mu - \cancel{\sigma} = 40$

$\mu + \cancel{\sigma} = 60$

$2\mu = 100$

$\boxed{\mu = 50}$

$\therefore$ AM of Normal Distribution $= 50$

72 Q1 = 13.25, MD = 8 for a Normal distribution then, find mode of distribution

~~a.~~ 20 b. 10 c. 15 d. 12

$MD = 8$

$SD = \frac{8}{0.80} = 10$

$QD = 10 \times 0.675 = 6.75 = \left(\frac{Q_3 - 13.25}{2}\right)$

$Q_3 = 26.75$

My Notes

$26.75 = \mu + 0.675\sigma$

$26.75 = \mu + 6.75$

$\therefore \boxed{\mu = 20}$ $\therefore$ AM = Median = Mode = 20

73 **If it is known that the probability of missile hitting the target is 1/8, what is the probability that out of 10 missiles fired, atleast 2 will hit the target?**

a. 0.4258 b. 0.3968 c. 0.5238 ~~d.~~ 0.3611

$\Longrightarrow \quad n = 10, \quad p = \frac{1}{8}, \quad q = \frac{7}{8}, \quad x = 2, 3, 4, ----, 10$

$$\text{prob}(x \geq 2) = 1 - \left[p(x=0) + p(x=1) \right]$$

$$= 1 - \left[{}^{10}C_0 \left(\tfrac{1}{8}\right)^0 \left(\tfrac{7}{8}\right)^{10} + {}^{10}C_1 \left(\tfrac{1}{8}\right)^1 \left(\tfrac{7}{8}\right)^9 \right]$$

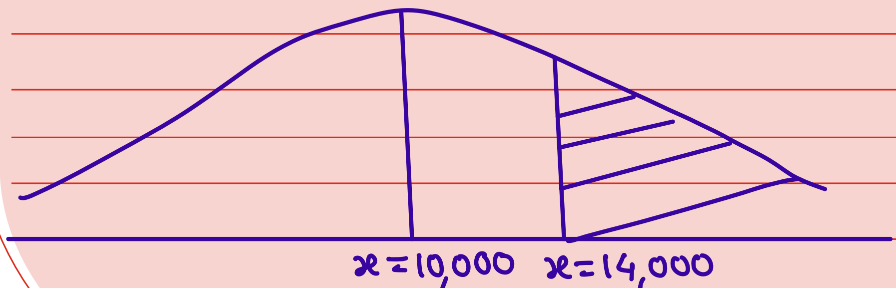
$$= 1 - \left[0.875^{10} + 1.25 \times 0.875^9 \right] = 0.3611$$

74 **Salary of workers in a factory is normally distribution with AM & SD of ₹ 10,000 & ₹ 2000 respectively. If 50 workers receive salary more than ₹ 14,000. Find Total no. of workers.** $\phi(2) = 0.97725$

a. 2193 b. 2000 c. 2581 d. None of these ~~e.~~ 2198

$AM = \mu = 10{,}000$

$SD = \sigma = 2{,}000$



$z = 0 \qquad z = \frac{14000 - 10{,}000}{2000}$

$z = 2.00$

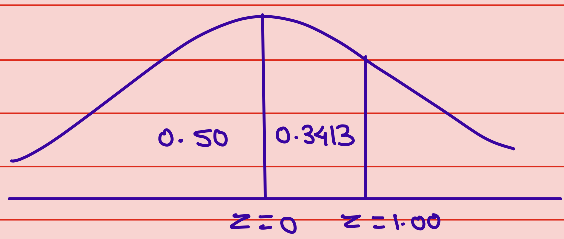
$\text{prob}(x > 14000)$
$= 1 - 0.97725 = 0.02275$
$= 2.275\%$

Total No. of workers $\times 2.275\% = 50$

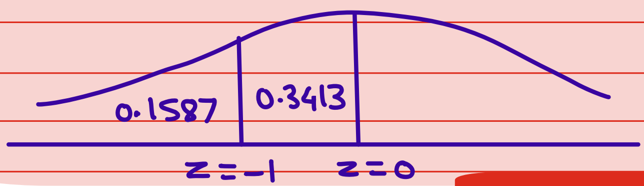
Total No. of workers $= 2198$ workers

75 **Area of a normal curve between z = 0 & z = 1 is 0.3413, then value of Φ(1) is ________**

a. 0.50 b. 0.1587 c. 0.8413 d. -0.3413



$\therefore \phi(1) = 0.50 + 0.3413$
$= 0.8413$



$\phi(-1.00) = 0.1587$

76 The Mean Deviation of a normal distribution is 16. What is quartile deviation of the distribution?

a. 10 ~~b.~~ 13.50 c. 15.00 d. 12.05

$MD = 16$

$SD \times 0.80 = MD$

$SD = \frac{16}{0.80} = 20$

$QD = 0.6750 \times SD = 0.6750 \times 20 = 13.50$

77 For a poisson's distribution, if prob (x=2) = 3 x prob(x=4), What is the variance of x.

~~a.~~ 2 b. 4 c. 3 d. $\sqrt{2}$

$\Longrightarrow$ $prob(x=2) = 3 \cdot prob(x=4)$

$\frac{\cancel{e^{-m}} \times \cancel{m^2}}{2!} = \frac{3 \times \cancel{e^{-m}} \times \cancel{m^4}\, m^2}{4!}$

$\frac{1}{2} = \frac{3m^2}{24}$

$\frac{1}{2} = \frac{m^2}{8}$

$\therefore m^2 = 4$

$m = 2$

Mean = variance = 2

78 If SD of poisson's variate is 2 then Find prob (1.50 < x < 2.90).

a. 0.231 b. 0.158 c. 0.15 ~~d.~~ 0.144

$\Longrightarrow$ $\sqrt{m} = 2$

$m = 4$

$prob(1.50 < x < 2.90)$

$= prob(x=2)$

$= \left(\frac{e^{-4} \times 4^2}{2!}\right) = \frac{8}{2.7183^4} = 14.6521\%$

My Notes

79 **If mean of poisson's variable x is 1, What is p(x = takes the value atleast 1) =**

a. 0.456 **b. 0.821** **~~c.~~ 0.632** **d. 0.254**

$\Longrightarrow$ $m = 1$

$prob\,(x \geq 1) = 1 - p(x=0)$

$= 1 - \left(\frac{e^{-1} \times 1^0}{0!}\right) = 1 - \frac{1}{e^1}$

$= 1 - \left(\frac{1}{2.7183}\right) = 0.6321$

80 **Out of 12800 families with 4 children each, How many families are expected to have all boys?**

$\Longrightarrow$ $n = 4,\ p = \frac{1}{2},\ q = \frac{1}{2},\ x = 4,\ N = 12800$

$Freq\,(x=4) = N \times prob\,(x=4)$

$= 12800 \times {}^4C_4 \times \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$

$= 12800 \times 1 \times \frac{1}{16} \times 1$

$= 800$ families

81 **For Binomial's distribution if mean = 9, variance = 2.25, then probability of a failure in a single trial = ?**

a. 0.75 **~~b.~~ 0.25** **c. 0.50** **d. None of these**

$\Longrightarrow$ $n.p = 9$ variance $= npq = 2.25$

$9 \times q = 2.25$

$\boxed{q = 0.25}$

My Notes

82 For Binomial's distribution, Match the following :

a. Mean	a. n,p
b. SD	b. n.p
c. Variance	c. npq
d. Parameters	d. $\sqrt{npq}$

83 A listing of possible outcomes of an experiment and their corresponding probability is called as

a. Random Variable
b. Frequency distribution
c. Probability Distribution
d. Contingency table

84

Distribution	Discrete / Continuous	Parameters	Types
Binomial's	Discrete	n, p	Bi parametric
Poisson's	Discrete	m	Uni-parametric
Normal	continuous	μ, σ^2	Bi-parametric

85 5 dice are rolled what is probability of getting 3 points on 4 dice?

$\Longrightarrow$ $n = 5, \; p = \frac{1}{6}, \; q = \frac{5}{6}, \; x = 4$

$$P(x=4) = {}^5C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^1$$

$$= 5 \times \frac{1}{6^4} \times \frac{5}{6} = \left(\frac{25}{7776}\right)$$

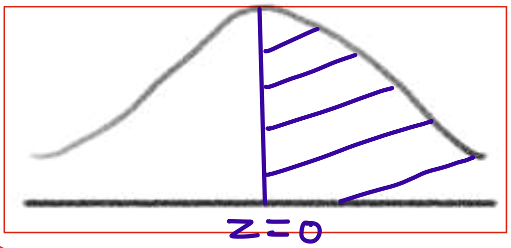
$= 0.3215\%$

My Notes

86 Match the following

a. Mean of Binomial's distri.	a. 0.6750 x SD
b. Mean of Poisson's distri.	b. Symmetrical Binomial's distri.
c. QD of Normal distri.	c. m
d. MD of Normal distri.	d. μ, σ^2
e. Variance of Poisson's distri.	e. is always greater than variance
f. SD of Binomial's distri.	f. Bell shaped
g. Parameters of Normal distri.	g. is always equal to variance
h. When p = q = 0.50	h. $\sqrt{npq}$
I. Shape of Normal Curve	I. 0.80 x SD
j. Binomial's & Poisson's distri.	j. can be uni-modal or bi-modal.

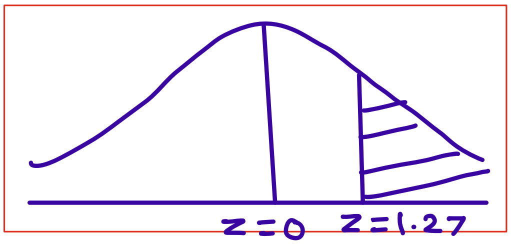
87



Shaded Area = 0.50 = 50%

= prob (x > μ)

88



$\phi(1.27) = 0.89796$

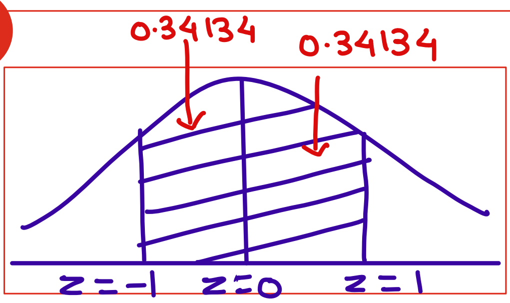
Shaded Area =

= 1 − 0.89796

= 0.10204 i.e. 10.204%

My Notes

89



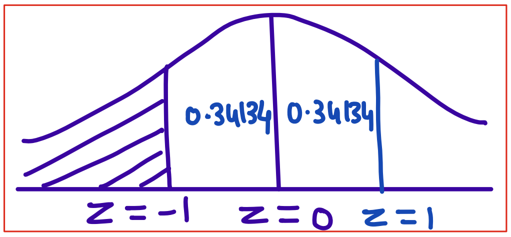
$\phi(1) = 0.84134$

Shaded Area =

$= 0.34134 + 0.34134$

$= 0.68268$ (i.e. 68.268%)

90



$\phi(1) = 0.84134$

Shaded Area =

$= 0.50 - 0.34134$

$= 0.15866$ (i.e. 15.866%)

Derivatives & Integration

CA VINOD REDDY

If $y = f(x)$, $\frac{dy}{dx} = f'(x)$

1 What is Derivative or Differential function?

$\Rightarrow y = f(x)$ — y = Dependent variable, x = Independent variable

Demand = f (price)
with the small change in price, what is the impact on demand
$\Rightarrow$ [Derivative of qty demanded with respect to price.] $\Rightarrow \frac{d}{dp}(Q)$

2 Derivative of f(x) is f'(x)

f'(x) by first principle = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x} = \left[\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right]$

3

y = f(x)	f'(x) = dy/dx
x	1
x^2	$2x$
x^3	$3x^2$
x^n	$n.(x)^{n-1}$
Log x	$1/x$
a^x	$a^x . \text{Log}\, a$
e^x	$e^x . \text{Log}_e e = e^x$
$\sqrt{x}$	$1/2\sqrt{x}$
constant = k	zero

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^9	$9.x^8$
x^{98}	$98.x^{97}$
x^m	$m.x^{m-1}$
5^x	$5^x . \text{Log}\, 5$
$x^{1/2}$	$\frac{1}{2} x^{-1/2} = 1/2\sqrt{x}$

4

u, v are diff functions of x

$\frac{d}{dx}(u + v) = \left(\frac{du}{dx} + \frac{dv}{dx}\right)$

$\frac{d}{dx}(u - v) = \left(\frac{du}{dx} - \frac{dv}{dx}\right)$

$\frac{d}{dx}(u \times v) = \left[u.\frac{dv}{dx} + v.\frac{du}{dx}\right]$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \left[\frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}\right]$

$\frac{d}{dx}(a^x + e^x) = \frac{d}{dx}a^x + \frac{d}{dx}e^x$

$= a^x.\text{Log}\, a + e^x$

$\frac{d}{dx}(x^5 \times \text{Log}\, x)$

$= x^5 \times \frac{d}{dx}\text{Log}\, x + \text{Log}\, x . \frac{d}{dx} x^5$

My Notes

$\frac{d}{dx}(x^5) = \frac{d}{dx}(x^3.x^2)$

$= x^3 \times \frac{d}{dx}x^2 + x^2.\frac{d}{dx}x^3$

$= x^3 \times 2x + x^2 \times 3x^2$

$= 2x^4 + 3x^4$

$= 5x^4$

$\frac{d}{dx}(x^8) = \frac{d}{dx}\left(\frac{x^{11}}{x^3}\right)$

$= \left[\frac{x^3.\frac{d}{dx}x^{11} - x^{11}.\frac{d}{dx}x^3}{(x^3)^2}\right]$

$= \left(\frac{x^3 \times 11x^{10} - x^{11}.3.x^2}{x^6}\right) = \left(\frac{11x^{13} - 3x^{13}}{x^6}\right)$

$= \frac{8x^{13}}{x^6} = (8x^7)$

5 Find $\frac{dy}{dx}$ if

$\frac{d}{dx}[k \cdot f(x)]$

$= k \cdot f'(x) + f(x) \cdot 0$

$= k \times f'(x)$

$\frac{d}{dx}(3x^2) = 6x$
$\frac{d}{dx} 8 \log x = \frac{8}{x}$
$\frac{d}{dx} 19 \times x^8 = 19 \times 8x^7$
$\frac{d}{dx} 5.e^x = 5.e^x$
$\frac{d}{dx} 8\sqrt{x} = 8 \times \frac{1}{2\sqrt{x}}$

a) $y = 3x^2 + 5x - 2$

$\frac{dy}{dx} = 6x + 5$

b) $y = a^x + x^a + a^a$

$\frac{dy}{dx} = a^x . \log a + a . x^{a-1}$

c) $y = \frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3$

$\frac{dy}{dx} = \frac{1}{3} \cdot 3x^2 - 10x + 6 - \frac{2}{x} + 0 = \left(x^2 - 10x + 6 - \frac{2}{x}\right)$

d) $y = \frac{e^x}{\log x}$

$\frac{dy}{dx} = \frac{\log x \times e^x - e^x . \frac{1}{x}}{(\log x)^2} = \frac{e^x\left(\log x - \frac{1}{x}\right)}{(\log x)^2}$

e) $y = \frac{2x}{3x^3+7}$

$\frac{dy}{dx} = \frac{(3x^3+7)2 - (2x \times 9x^2)}{(3x^3+7)^2} = \frac{6x^3 + 14 - 18x^3}{(3x^3+7)^2} = \left[\frac{14 - 12x^3}{(3x^3+7)^2}\right]$

f) $y = 2^x . \log x$

$\frac{dy}{dx} = 2^x \times \frac{1}{x} + \log x \times 2^x . \log 2 = 2^x\left(\frac{1}{x} + \log x . \log 2\right)$

g) $y = 5^x . x^{10}$

$\frac{dy}{dx} = 5^x \times \frac{d}{dx} x^{10} + x^{10} . \frac{d}{dx} 5^x = 5^x . 10x^9 + x^{10} . 5^x . \log 5$

$= 5^x . x^9 (10 + x . \log 5)$

h) $y = \frac{3x + 5}{5x + 8}$

$\frac{dy}{dx} = \frac{(5x+8)3 - (3x+5)5}{(5x+8)^2} = \frac{\cancel{15x} + 24 - \cancel{15x} - 25}{(5x+8)^2} = \frac{-1}{(5x+8)^2}$

My Notes

① $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

② $\frac{d}{dx}[f(x) \times g(x)] = f(x) . g'(x) + g(x) . f'(x)$

③ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \left[\frac{g(x) . f'(x) - f(x) . g'(x)}{[g(x)]^2}\right]$

④ $\frac{d}{dx}(k) = 0$ where k = constant

6 **Chain Rule** Find $\frac{dy}{dx}$ if $y = a^{(2x+3)}$

$y = a^{2x+3}$

$t = 2x+3, \quad y = a^t$

$\frac{dt}{dx} = 2, \quad \frac{dy}{dt} = a^t . \text{Log}a$

As per chain Rule

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= a^t . \text{Log}a \times 2$

$= 2 \times a^{2x+3} . \text{Log}a$

Short-cut

$\frac{d}{dx} a^x = a^x . \text{Log}a . \frac{d}{dx}(x)$

$\frac{d}{dx} a^{f(x)} = a^{f(x)} . \text{Log}a . f'(x)$

$\frac{d}{dx}\left(a^{2x+3}\right)$

$= a^{2x+3} . \text{Log}a \times 2$

7 Find $\frac{dy}{dx}$ if

a. $y = 5^{(2x+3)}$ $\quad \frac{dy}{dx} = 5^{2x+3} \times \text{Log}5 \times 2$

b. $y = (8x+3)^2$ $\quad \frac{dy}{dx} = 2 \times (8x+3)^1 \times 8 = 16(8x+3) = 128x + 48$

c. $y = e^{\log x}$ $\quad \frac{dy}{dx} = e^{\text{Log}x} \times \frac{1}{x}$

d. $y = \sqrt{(5x+13)}$ $\quad \frac{dy}{dx} = \frac{1}{2\sqrt{5x+13}} \times 5$

e. $y = \sqrt{2x^2 + 5x + 3}$ $\quad \frac{dy}{dx} = \frac{1}{2\sqrt{2x^2+5x+3}} \times (4x+5)$

f. $y = \sqrt{\text{Log}x}$ $\quad \frac{dy}{dx} = \frac{1}{2\sqrt{\text{Log}x}} \times \frac{1}{x}$

8

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$f(x)^n$	$n \times [f(x)]^{n-1} \times f'(x)$
$a^{f(x)}$	$a^{f(x)} . \text{Log}a . f'(x)$
$e^{f(x)}$	$e^{f(x)} . f'(x)$
$\text{Log}\ [f(x)]$	$\frac{1}{f(x)} \times f'(x)$
$\sqrt{f(x)}$	$\frac{1}{2\sqrt{f(x)}} \times f'(x)$

My Notes

① $\frac{d}{dx}(7x+8)^{33} = 33 \times (7x+8)^{32} \times 7 = 231 \times (7x+8)^{32}$

② $\frac{d}{dx}\text{Log}(9x+18) = \frac{1}{9x+18} \times 9 = \left(\frac{9}{9x+18}\right)$

③ $\frac{d}{dx} e^{5x^2+13x+2} = e^{5x^2+13x+2} \times (10x+13)$

9 $y = at^3$, $x = 2bt$. Find $\frac{dy}{dx}$

$y = at^3$

$\frac{dy}{dt} = a \times 3t^2$

$= 3at^2$

$x = 2bt$

$\frac{dx}{dt} = 2b$

As per chain Rule

$\frac{dy}{dx} = \left(\frac{dy/dt}{dx/dt}\right) = \left(\frac{3at^2}{2b}\right)$

10 $y = x^x$. Find $\frac{dy}{dx}$

$\Longrightarrow \quad y = x^x$

Taking log on both sides,

$\log y = \log x^x$

$\log y = x.\log x$

Taking deri. on both sides w.r.to x

$\frac{1}{y} \cdot \frac{dy}{dx} = x \times \frac{1}{x} + \log x \times 1$

$\frac{dy}{dx} = y\left(1 + \log x\right) = x^x\left(1 + \log x\right)$

$\frac{d}{dx} x^n = n.x^{n-1}$ $\left(\text{variable}^{\text{constant}}\right)$
$\frac{d}{dx} a^x = a^x.\log a$ $\left(\text{constant}^{\text{variable}}\right)$
$\frac{d}{dx} a^a = 0$ $\text{constant}^{\text{constant}}$
$\frac{d}{dx} x^x = x^x(1 + \log x)$ $\text{variable}^{\text{variable}}$

11 $y = \sqrt{\frac{1-x}{1+x}}$ Find $\frac{dy}{dx}$

$\Longrightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2} \times \sqrt{\frac{1+x}{1-x}} \times \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2} = \frac{1}{2}\sqrt{\frac{1+x}{1-x}} \times \frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}$

$= \frac{1}{\cancel{2}} \times \frac{(1+x)^{1/2}}{(1-x)^{1/2}} \times \frac{-\cancel{2}}{(1+x)^2} = \frac{-1 \times (1+x)^{-3/2}}{(1-x)^{1/2}}$

My Notes

You think about the rule

$\text{variable}^{\text{constant}}$	$\Longrightarrow$	$\frac{d}{dx} x^n = n.x^{n-1}$, $\frac{d}{dx}(f(x))^n = n\left[f(x)\right]^{n-1} . f'(x)$
$\text{constant}^{\text{variable}}$	$\Longrightarrow$	$\frac{d}{dx} a^x = a^x.\log a$, $\frac{d}{dx} a^{f(x)} = a^{f(x)}.\log a.f'(x)$
$\text{variable}^{\text{variable}}$	$\Longrightarrow$	Logarithmic differentiation

Higher order Derivatives

12 Find $\frac{d^2y}{dx^2}$ If $y = 16x^3 - 22x^2 + 18x + 54$

$y = 16x^3 - 22x^2 + 18x + 54$

$\Rightarrow$ Diff. both sides w.r.to x

$\frac{dy}{dx} = 48x^2 - 44x + 18$

Diff. both sides w.r.to x

$\frac{d^2y}{dx^2} = 96x - 44$

$y = f(x) = 5x^4 - 18x^3 + 20x + 15$

$\frac{dy}{dx} = f'(x) = 20x^3 - 54x^2 + 20$

$\frac{d^2y}{dx^2} = f''(x) = 60x^2 - 108x$

$\frac{d^3y}{dx^3} = f'''(x) = 120x - 108$

13 Find the gradient of curve $y = 3x^2 - 5x + 4$ at point (1,2)

(slope)

$\Rightarrow \frac{dy}{dx} = 6x - 5$

$\left(\frac{dy}{dx}\right)_{x=1,\, y=2} = 6(1) - 5 = 1$

Find slope of the line

$3x + 5y = 83$

$\Rightarrow -3/5$

If $3x + 5y = 83$ Find $\frac{dy}{dx}$

$\Rightarrow 3 + 5 \cdot \frac{dy}{dx} = 0$

$5 \cdot \frac{dy}{dx} = -3$

$\frac{dy}{dx} = -3/5$

14 $x = 2t + 5$, $y = t^2 - 2$; Find $\frac{dy}{dx}$

$x = 2t + 5 \qquad y = t^2 - 2$

Diff. both sides w.r.t. 't'

$\frac{dx}{dt} = 2, \quad \frac{dy}{dt} = 2t$

As per chain Rule

$\frac{dy}{dx} = \left[\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right] = \frac{2t}{2} = t$

15 $x = 3t^2 - 1$, $y = t^3 - t$; Find $\frac{dy}{dx}$

$x = 3t^2 - 1, \quad y = t^3 - t$

Diff. both sides w.r.t. 't'

$\frac{dx}{dt} = 6t, \quad \frac{dy}{dt} = 3t^2 - 1$

As per chain Rule

$\frac{dy}{dx} = \left[\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right] = \left(\frac{3t^2 - 1}{6t}\right)$

16 If $f(x) = x^k$ and $f'(1) = 10$; then value of k is

$$\Longrightarrow \quad f(x) = x^k$$

$$f'(x) = k \cdot (x)^{k-1}$$

$$f'(1) = k \cdot (1)^{k-1} = k \times 1 = k = 10 \qquad \boxed{\therefore k = 10}$$

17 $y = e^{\sqrt{2x}}$ Find $\frac{dy}{dx}$

$$\Longrightarrow \quad y = e^{\sqrt{2x}}$$

$$\frac{dy}{dx} = e^{\sqrt{2x}} \times \frac{d}{dx}\sqrt{2x}$$

$$= e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}} \times \frac{d}{dx} 2x$$

$$= e^{\sqrt{2x}} \times \frac{1}{\sqrt{2x}} = \left[\frac{e^{\sqrt{2x}}}{\sqrt{2x}}\right]$$

18 $f(x) = \frac{3x^2 - 2x + 5}{2x + 1}$ Find $f'(x)$

$$f'(x) = \left[\frac{(2x+1)(6x-2) - (3x^2 - 2x + 5)(2)}{(2x+1)^2}\right]$$

$$= \left[\frac{12x^2 - 4x + 6x - 2 - 6x^2 + 4x - 10}{(2x+1)^2}\right] = \left[\frac{6x^2 + 6x - 12}{(2x+1)^2}\right]$$

$$= \left[\frac{6(x^2 + x - 2)}{(2x+1)^2}\right]$$

19 If $y = x^{x^{x^{x^{\cdots \infty \text{ terms}}}}}$ Find $\frac{dy}{dx}$

$$\Longrightarrow \quad y = (x)^y$$

$$\log y = y \cdot \log x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = y \times \frac{1}{x} + \log x \cdot \frac{dy}{dx}$$

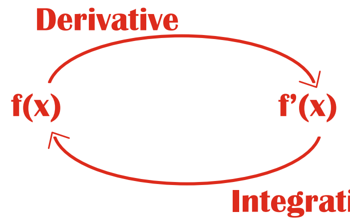
$$\frac{1}{y}\frac{dy}{dx} - \log x \cdot \frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1 - y \cdot \log x}{y}\right) = \frac{y}{x}$$

$$\frac{dy}{dx} = \left[\frac{y^2}{x(1 - y \cdot \log x)}\right]$$

20



Therefore, Integration is anti-derivative

$$\int 1 \cdot dx = x + c$$

$$\int 5 \cdot dx = 5x + c$$

$$\int x^2 \cdot dx = \frac{x^3}{3} + c$$

$$\int x^3 \cdot dx = \frac{x^4}{4} + c$$

21

$\int x^n.dx = \left(\frac{x^{n+1}}{n+1}\right) + c$

$\int k.dx = kx + c$

$\int a^x.dx = \left(\frac{a^x}{\log a}\right) + c$

$\int 1 .dx = x + c$

$\int e^x.dx = e^x + c$

$\int \frac{1}{x} .dx = \log x + c$

$$\int \sqrt{x}.dx = \frac{x^{3/2}}{3/2} + c$$

22

$\int\sqrt{x} .dx = \frac{x^{3/2}}{3/2} + c$

$\int\frac{1}{\sqrt{x}} .dx = \int x^{-1/2}.dx = \frac{x^{1/2}}{1/2} + c = 2\sqrt{x} + c$

$\int e^{-3x} .dx = \frac{e^{-3x}}{-3} + c$

$\int 3^x .dx = \left(\frac{3^x}{\log 3}\right) + c$

$\int x\sqrt{x} .dx = \int x^{3/2}.dx = \left(\frac{x^{5/2}}{5/2} + c\right)$

$$\boxed{\begin{aligned} &\int (u+v).dx = \int u.dx + \int v.dx \\ &\int (u-v).dx = \int u.dx - \int v.dx \end{aligned}}$$

23

$\int\left(x + \frac{1}{x^2}\right) .dx = \int x.dx + \int \frac{1}{x^2}.dx = \int x.dx + \int x^{-2}.dx$

$= \left(\frac{x^2}{2} + \frac{x^{-1}}{-1} + c\right) = \left(\frac{x^2}{2} - \frac{1}{x} + c\right)$

24

$\int\sqrt{x}\,(x^3+2x-3).dx =$

$= \int x^{1/2}\left(x^3 + 2x - 3\right).dx = \int\left(x^{7/2} + 2x^{3/2} - 3x^{1/2}\right)dx$

$= \int x^{7/2}.dx + 2\int x^{3/2}.dx - 3\int x^{1/2}.dx$

$= \left(\frac{x^{9/2}}{9/2} + \frac{2x^{5/2}}{5/2} - \frac{3 \cdot x^{3/2}}{3/2}\right) + c = \left(\frac{2x^{9/2}}{9} + \frac{4x^{5/2}}{5} - \frac{2x^{3/2}}{1}\right) + c$

25

$\int (e^{3x} + e^{-4x}).dx = \int e^{3x}.dx + \int e^{-4x}.dx$

$= \left(\frac{e^{3x}}{3} + \frac{e^{-4x}}{-4}\right) + c$

26

$\int\left(\frac{x^2}{x+1}\right).dx = \int \frac{x^2-1+1}{x+1}.dx = \int \frac{x^2-1}{(x+1)} + \frac{1}{(x+1)}\,dx$

$= \int \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}.dx + \int \frac{1}{(x+1)}.dx = \frac{x^2}{2} - x + \log(x+1) + c$

27 $\int \frac{x^3+5x^2-3}{x+2}.dx =$

$$= \int \left(\text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}} \right) .dx$$

$$= \int x^2+3x-6 + \frac{9}{(x+2)} .dx$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} - 6x + 9.\text{Log}(x+2) + c$$

$$\frac{44}{7} = ? \qquad \frac{44}{7} = 6 + \frac{2}{7}$$

6 → Quotient

→ 7) 44 → Dividend

Divisor

42

2 – Remainder

$$\begin{array}{r} x^2+3x-6 \\ x+2 \overline{) \cancel{x^3} + 5x^2 - 3} \\ \underline{\cancel{x^3} + 2x^2} \\ - \quad - \\ 3\cancel{x^2} - 3 \\ \underline{3\cancel{x^2} + 6x} \\ - \quad - \\ -3 - \cancel{6x} \\ \underline{-12 - \cancel{6x}} \\ + \quad + \\ 9 \end{array}$$

28 $\int \frac{x^3}{(x^2+1)^3}.dx =$

(Solve by Method of Substitution)

$$\Longrightarrow \int \frac{x^3}{(x^2+1)^3} .dx$$

Suppose $t = x^2+1 \quad \therefore x^2 = t-1$

$$\frac{dt}{dx} = 2x$$

$$\therefore dx = \frac{dt}{2x}$$

$$I = \int \frac{x^2.\cancel{x}}{t^3} . \frac{dt}{2\cancel{x}}$$

$$I = \frac{1}{2} \int \frac{t-1}{t^3} .dt$$

$$I = \frac{1}{2} \left[\int \frac{t}{t^3} - \frac{1}{t^3} .dt \right]$$

$$= \frac{1}{2} \left[\int t^{-2}.dt - \int t^{-3}.dt \right]$$

$$= \frac{1}{2} \left[\frac{t^{-1}}{-1} + \frac{t^{-2}}{2} \right]$$

$$= \frac{1}{2} \left(\frac{1}{-t} + \frac{1}{2t^2} \right)$$

$$= \frac{1}{2t} \left(-1 + \frac{1}{2t} \right)$$

$$= \frac{1}{2(x^2+1)} \left(-1 + \frac{1}{2(x^2+1)} \right)$$

29 $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \text{Log} \left| \frac{x-a}{x+a} \right| + c$

30 $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \text{Log} \left| \frac{a+x}{a-x} \right| + c$

31 $\int \frac{1}{\sqrt{x^2+a^2}} dx = \text{Log} \left| x + \sqrt{x^2+a^2} \right| + c$

32 $\int \frac{1}{\sqrt{x^2-a^2}} dx = \text{Log} \left| x + \sqrt{x^2-a^2} \right| + c$

33 $\int e^x [f(x) + f'(x)].\, dx = e^x .\, f(x) + c$

34 $\int \sqrt{x^2+a^2}.\, dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \text{Log} \left| x + \sqrt{x^2+a^2} \right| + c$

35 $\int\sqrt{x^2-a^2}\,.\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\,\text{Log}\left|x+\sqrt{x^2-a^2}\right| + c$

36 $\int \frac{f'(x)}{f(x)}\,dx = \text{Log } f(x) + c$

37 **Integration by parts**

$\int (u.v)\,.\,dx = u\int v.dx - \int\left[\frac{du}{dx} \times \int v.dx\right].dx$

38 If $\int f(x).dx = g(x) + c$; then

$\int_a^b f(x) = g(b) - g(a)$

$\int \text{Log}x\,.\,dx$

$\int(\text{Log}x \times 1)\,.\,dx$

$= \text{Log}x \times \int 1.dx - \int\left[\frac{d}{dx}\text{Log}x \times \int 1.dx\right]dx$

$= \text{Log}x \times x - \int\left(\frac{1}{x}\times x\right).dx$

$= x.\text{Log}x - \int 1.dx$

$= x\,\text{Log}x - x + c$

$= x(\text{Log}x - 1) + c$

39 Find $\frac{dy}{dx}$ If $x^2y^2 + 3xy + y = 0$

Derivatives of implicit functions

$\implies x^2y^2 + 3xy + y = 0$

Taking derivative on both sides with respect to x

$\frac{d}{dx}x^2y^2 + 3.\frac{d}{dx}xy + \frac{d}{dx}y = 0$

$x^2 \times 2y.\frac{dy}{dx} + y^2 \times 2x + 3\left(x.\frac{dy}{dx} + y\right) + \frac{dy}{dx} = 0$

$2x^2y.\frac{dy}{dx} + 2xy^2 + 3x.\frac{dy}{dx} + 3y + \frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x^2y + 3x + 1) = -3y - 2xy^2 \quad \therefore \frac{dy}{dx} = \boxed{\frac{-(2xy^2+3y)}{2x^2y+3x+1}}$

40 Find $\frac{dy}{dx}$ If $y = \text{Log}(x + \sqrt{x^2+a^2})$

$\int\frac{1}{\sqrt{x^2+a^2}}.dx = \text{Log}\; x+\sqrt{x^2+a^2} + c$

$\implies y = \text{Log}\left(x + \sqrt{x^2+a^2}\right)$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \times \frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$

$= \frac{1}{x+\sqrt{x^2+a^2}} \times \left(1 + \frac{1}{2\sqrt{x^2+a^2}} \times 2x\right)$

$= \frac{1}{x+\sqrt{x^2+a^2}}\left(1 + \frac{x}{\sqrt{x^2+a^2}}\right) = \frac{1}{\cancel{(x+\sqrt{x^2+a^2})}} \times \frac{\cancel{(x+\sqrt{x^2+a^2})}}{\sqrt{x^2+a^2}}$

$= \boxed{\frac{1}{\sqrt{x^2+a^2}}}$

41 If $y = (a.e^{mx} + b.e^{-mx})$. Find $\frac{d^2y}{dx^2}$

$\Longrightarrow$ $y = a.e^{mx} + b.e^{-mx}$

Taking derivative on both sides with respect to x

$$\frac{dy}{dx} = a.e^{mx} \times m + b.e^{-mx} \times -m$$

Taking derivative on both sides with respect to x

$$\frac{d^2y}{dx^2} = am.e^{mx}.m - bm.e^{-mx} \times (-m)$$
$$= am^2.e^{mx} + bm^2.e^{-mx} = m^2\left(a.e^{mx} + b.e^{-mx}\right) = m^2y$$

42 If $y = \sqrt{x+1}$. Find $\frac{dy}{dx}$

a. $1/\sqrt{x+1}$ b. $-1/\sqrt{x+1}$ ~~c.~~ $1/2\sqrt{x+1}$ d. None of these

43 If $f(x) = e^{(ax^2+bx+c)}$ Find $f'(x)$

a. $e^{(ax^2+bx+c)}.(ax+b)$ b. $e^{(ax^2+bx+c)}$ ~~c.~~ $e^{(ax^2+bx+c)}.(2ax+b)$ d. $(ax^2+bx+c) \times e^{ax+b}$

$$f'(x) = e^{ax^2+bx+c} \cdot \frac{d}{dx}(ax^2+bx+c)$$
$$= e^{ax^2+bx+c} \cdot (2ax+b)$$

44 If $f(x) = \frac{x^2+1}{x^2-1}$ then $f'(x) = ?$

~~a.~~ $-4x/(x^2-1)^2$ b. $4x/(x^2-1)^2$ c. $x/(x^2-1)^2$ d. None of these

$\Longrightarrow$ $f(x) = \frac{x^2+1}{x^2-1}$

$$f'(x) = \frac{(x^2-1)2x - (x^2+1)2x}{(x^2-1)^2} = \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2}$$

$$= \frac{-4x}{(x^2-1)^2}$$

45 $y = x(x-1)(x-2)$; Find $\frac{dy}{dx}$

~~a.~~ $3x^2 - 6x + 2$ b. $-6x + 2$ c. $3x^2 + 2$ d. None of these

$$\Rightarrow y = x(x-1)(x-2)$$
$$y = (x^2 - x)(x-2)$$
$$y = (x^3 - 3x^2 + 2x)$$
$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

$$\frac{d}{dx}[uvw] = \frac{d}{dx}[(uv) \times w]$$
$$= uv \cdot \frac{dw}{dx} + w \cdot \frac{d}{dx}(uv)$$
$$= uv \cdot \frac{dw}{dx} + w\left(u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}\right)$$
$$= \left[uv \cdot \frac{dw}{dx} + uw \cdot \frac{dv}{dx} + v \cdot w \frac{du}{dx}\right]$$

46 If $xy = 1$; then $y^2 + \frac{dy}{dx} =$

a. 1 ~~b.~~ 0 c. -1 d. None of these

$$\Rightarrow xy = 1$$
$$x \cdot \frac{dy}{dx} + y = 0$$
$$x \cdot \frac{dy}{dx} = -y$$
$$\therefore \frac{dy}{dx} = -\frac{y}{x}$$
$$= -\frac{y}{1/y} = -y^2$$

As $xy = 1$
$x = \frac{1}{y}$

$$y^2 + \frac{dy}{dx} = y^2 + (-y^2) = 0$$

47 $y = \sqrt{x + \sqrt{x}}$ then $\frac{dy}{dx} = ?$

a. $\frac{1}{2\sqrt{x+\sqrt{x}}}$ b. $\frac{1}{2\sqrt{x+\sqrt{x}}} \times (1 + \sqrt{x})$

c. $\frac{2}{\sqrt{x+\sqrt{x}}}$ ~~d.~~ $\frac{1}{2\sqrt{x+\sqrt{x}}} \times \left(1 + \frac{1}{2\sqrt{x}}\right)$

$$y = \sqrt{x + \sqrt{x}}$$
$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x}}} \times \frac{d}{dx}(x + \sqrt{x}) = \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$$

48 $e^{-xy} - 4xy = 0$; then $\frac{dy}{dx} = ?$

~~a.~~ $-y/x$ b. y/x c. x/y d. None of these

$$\Rightarrow e^{-xy} - 4xy = 0$$
$$e^{-xy} = 4xy$$
$$-xy \log e = \log 4 + \log x + \log y$$

$$-\left[x \cdot \frac{dy}{dx} + y\right] = 0 + \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$
$$-x \cdot \frac{dy}{dx} - y - \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx}\left(-x-\frac{1}{y}\right)=\frac{1}{x}+y$$

$$\frac{dy}{dx}-1\times\frac{xy+1}{y}=\frac{1+xy}{x}$$

$$\frac{dy}{dx}=\frac{(1+\cancel{xy})}{x}\times\frac{-y}{\cancel{(xy+1)}}$$

$$=-y/x$$

49 $x^3 + y^3 - 3axy = 0$; then $\frac{dy}{dx}$

a. $\frac{ay - x^2}{y^2 + ax}$ ~~b.~~ $\frac{ay - x^2}{y^2 - ax}$ c. $\frac{ay + x^2}{y^2 + ax}$ d. None of these

$$x^3+y^3-3axy=0$$

$$3x^2+3y^2\cdot\frac{dy}{dx}-3a\left(x\cdot\frac{dy}{dx}+y\right)=0$$

$$3x^2+3y^2\cdot\frac{dy}{dx}-3ax\cdot\frac{dy}{dx}-3ay=0$$

$$\frac{dy}{dx}\left(3y^2-3ax\right)=3ay-3x^2$$

$$\therefore\ \frac{dy}{dx}=\frac{3(ay-x^2)}{3(y^2-ax)}$$

$$\frac{dy}{dx}=\frac{ay-x^2}{y^2-ax}$$

50 $x = 2t + 5$ & $y = t^2 - 2$; then $\frac{dy}{dx}$ = ?

~~a.~~ t b. -1/t c. 1/t d. None of these

$$\Longrightarrow\quad x=2t+5 \qquad y=t^2-2$$

$$\frac{dx}{dt}=2 \qquad \frac{dy}{dt}=2t$$

$$\frac{dy}{dx}=\left(\frac{dy/dt}{dx/dt}\right)=\left(\frac{2t}{2}\right)=t$$

51 If $y = 1/\sqrt{x}$; then $\frac{dy}{dx}$ = ?

a. $1/2x\sqrt{x}$ b. $-1/x\sqrt{x}$ ~~c.~~ $-1/2x\sqrt{x}$ d. None of these

$$\Longrightarrow\quad y=\frac{1}{\sqrt{x}}=x^{-1/2}$$

$$\frac{dy}{dx}=-\frac{1}{2}(x)^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-3/2}=\frac{-1}{2x^{3/2}}=\frac{-1}{2x^{1/2}\cdot x}$$

$$=\frac{-1}{2x\sqrt{x}}$$

52 If $x = 3t^2 - 1$ and $y = t^3 - t$; then $\frac{dy}{dx} = ?$

~~a.~~ $\frac{3t^2 - 1}{6t}$ b. $3t^2 - 1$ c. $\frac{3t - 1}{6t}$ d. None of these

$\Longrightarrow$ $x = 3t^2 - 1$, $y = t^3 - t$

$\frac{dx}{dt} = 6t$, $\frac{dy}{dt} = 3t^2 - 1$

$\frac{dy}{dx} = \left(\frac{dy/dt}{dx/dt}\right)$

$= \left(\frac{3t^2 - 1}{6t}\right)$

53 For the curve $x^2 + y^2 + 2gx + 2hy = 0$; the value of $\frac{dy}{dx}$ at (0,0) is

~~a.~~ -g/h b. g/h c. h/g d. None of these

$\Longrightarrow$ $x^2 + y^2 + 2gx + 2hy = 0$

Taking deri. on both sides w.r.to x

$2x + 2y \cdot \frac{dy}{dx} + 2g + 2h \cdot \frac{dy}{dx} = 0$

Let's put $x = 0, y = 0$

$0 + 0 + 2g + 2h \cdot \frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{-2g}{2h} = -g/h$

54 Given $x = t + t^{-1}$ & $y = t - t^{-1}$ then $\frac{dy}{dx}$ for t = 2 is

a. 3/5 b. -3/5 ~~c.~~ 5/3 d. None of these

$\Longrightarrow$ $x = t + t^{-1}$

$\frac{dx}{dt} = 1 + -1 \times t^{-2}$

$= 1 - t^{-2}$

$y = t - t^{-1}$

$\frac{dy}{dt} = 1 - (-1)t^{-2}$

$= 1 + t^{-2}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{1 + t^{-2}}{1 - t^{-2}} = \frac{1 + 2^{-2}}{1 - 2^{-2}}$

$= \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}} = \frac{4+1}{4-1} = 5/3$

55 $x^3 - 2x^2y^2 + 5x + y - 5 = 0$ then $\frac{dy}{dx}$ at x = 1 and y = 1 is :

~~a.~~ 4/3 b. -4/3 c. 3/4 d. None of these

$\Longrightarrow$ $3x^2 - 2\left(x^2 \times 2y \cdot \frac{dy}{dx} + y^2 \times 2x\right) + 5 + \frac{dy}{dx} - 0 = 0$

$3(1)^2 - 2\left(2(1)^2(1) \cdot \frac{dy}{dx} + (1)^2 \times 2(1)\right) + 5 + \frac{dy}{dx} = 0$

$3 - 2\left(2 \cdot \frac{dy}{dx} + 2\right) + 5 + \frac{dy}{dx} = 0$

$8 - 4 \cdot \frac{dy}{dx} - 4 + \frac{dy}{dx} = 0$

$-3 \cdot \frac{dy}{dx} = -4$

$\therefore \frac{dy}{dx} = 4/3$

56 $y = x^2 . \text{Log } x$. Find $\frac{dy}{dx}$

a. 1 + 2 Log x ~~b.~~ x (1 + 2 Logx) c . 2 logx d. None of these

$\Longrightarrow$ $y = x^2 . \text{Log} x$

$\frac{dy}{dx} = x^2 \times \frac{1}{x} + \text{Log} x \times 2x = x + 2x . \text{Log} x$

$= x(1 + 2 . \text{Log} x)$

57 If $x = at^2$ and $y = 2at$; then $\left.\frac{dy}{dx}\right|_{t=2}$ is

~~a. 1/2~~ b. -2 c. -1/2 d. None of these

$\Longrightarrow$ $x = at^2$

$\frac{dx}{dt} = 2at$

$y = 2at$

$\frac{dy}{dt} = 2a$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$

$\left(\frac{dy}{dx}\right)_{t=2} = \frac{1}{2}$

58 If $f(x) = x^2 - 6x + 8$. Find $f'(5) - f'(8)$

a. $f'(2)$ ~~b. $3f'(2)$~~ c. $2f'(2)$ d. None of these

$\Longrightarrow$ $f(x) = x^2 - 6x + 8$

$f'(x) = 2x - 6$

$f'(5) - f'(8) = [2(5) - 6] - [2(8) - 6]$

$= 4 - 10 = -6$

$3 \times f'(2) = 3[2(2) - 6] = -6$

59 If $f(x) = x^k$ and $f'(1) = 0$then k = ?

a. 10 b. -10 c. 1/10 ~~d. None of these~~

$f(x) = x^k$

$f'(x) = k.(x)^{k-1}$

$f'(1) = k(1)^{k-1} = k = 0$

Definite Integrals

60 $\int_3^5 x^2.dx = ?$

$= \left[\frac{x^3}{3}\right]_3^5$

$= \frac{5^3}{3} - \frac{3^3}{3}$

$= \frac{125-27}{3} = \left(\frac{98}{3}\right)$

If $\int f(x) = g(x) + c$

then $\int_a^b f(x) = g(b) - g(a)$

61 If $\int f(x)\,dx = g(x) + c$; then

$$\int_a^b f(x) = g(b) - g(a)$$

62 $\int_2^3 (2x^2 + 5x + 3).dx = ?$

$$= \left[\frac{2x^3}{3} + \frac{5x^2}{2} + 3x\right]_2^3$$

$$= \left[\frac{2}{3}(3)^3 + \frac{5}{2}(3)^2 + 3(3)\right] - \left[\frac{2}{3}(2)^3 + \frac{5}{2}(2)^2 + 3(2)\right]$$

$$= \left(18 + \frac{45}{2} + 9\right) - \left(\frac{16}{3} + 10 + 6\right)$$

$$= 49.50 - 21.33333 = 28.1666666 = 28\frac{1}{6} = \left(\frac{169}{6}\right)$$

63 $\int_7^{10} a^{2x}.dx = ?$

$$\Rightarrow \left[\frac{a^{2x}}{2.\log a}\right]_7^{10}$$

$$= \frac{a^{20}}{2.\log a} - \frac{a^{14}}{2\log a} = \left(\frac{a^{20} - a^{14}}{2\log a}\right) = \left[\frac{a^{14}(a^6 - 1)}{2.\log a}\right]$$

64 $\int_0^4 \sqrt{3x + 4}\,.dx = ?$

a. 9/112 ~~b.~~ 112/9 c. 11/9 d. None of these

$$\int_0^4 (3x+4)^{1/2}.dx$$

$$= \left[\frac{(3x+4)^{3/2}}{3/2 \times 3}\right]_0^4 = \frac{16^{3/2}}{9/2} - \frac{4^{3/2}}{9/2} = \frac{(2^4)^{3/2} \times 2}{9} - \frac{(2^2)^{3/2} \times 2}{9}$$

$$= \frac{128}{9} - \frac{16}{9} = \frac{112}{9}$$

65 $\int_0^2 \left(\frac{x+2}{x+1}\right).dx = ?$

a. $2 + Log_e 2$ ~~b.~~ $2 + Log_e 3$ c. $Log_e 3$ d. None of these

$$= \int \frac{x+1+1}{x+1}.dx$$

$$= \int \left(\frac{x+1}{x+1} + \frac{1}{x+1}\right).dx$$

$$= \int \left(1 + \frac{1}{x+1}\right).dx$$

$$= \left[x + Log(x+1)\right]_0^2$$

$$= 2 + Log3 - (0 + Log1)$$

$$= 2 + Log3 - 0 - 0 = 2 + Log3$$

66 $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}.dx = ?$

a. $51\frac{1}{5}$ b. 48/5 c. 48 ~~d.~~ $55\frac{7}{15}$

$$= \int_0^4 \left(\frac{x^2+5x+4}{x^{1/2}}\right).dx$$

$$= \int_0^4 \left(\frac{x^2}{x^{1/2}} + \frac{5x}{x^{1/2}} + \frac{4}{x^{1/2}}\right).dx$$

$$= \int_0^4 \left(x^{3/2} + 5x^{1/2} + 4x^{-1/2}\right)dx$$

$$= \left[\frac{x^{5/2}}{5/2} + \frac{5x^{3/2}}{3/2} + \frac{4.x^{1/2}}{1/2}\right]_0^4$$

$$= \frac{4^{5/2}}{5/2} + \frac{5(4)^{3/2}}{3/2} + \frac{4(4)^{1/2}}{1/2} - (0+0+0)$$

$$= \frac{64}{5} + \frac{80}{3} + 16 = 55.4666$$

67 $\int Log\, x^2 .dx = ?$

a. $x(\log x - 1) + k$ ~~b.~~ $2x(\log x - 1) + k$
c. $2(\log x - 1) + k$ d. None of these

$$= \int 2.Log x.dx$$

$$= 2\int Log x.dx$$

$$= 2\int Log x \times 1\, dx$$

$$= 2\left[Log x \times x - \int\left(\frac{1}{x} \times x\right).dx\right]$$

$$= 2(x.Log x - x) + c$$

$$= 2x(Log x - 1) + c$$

Integration by parts

$$\int u.v.dx$$

$$= u.\int v.dx - \int\left[\frac{du}{dx}\int v.dx\right]dx$$

68

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x	1
x^2	$2x$
x^3	$3x^2$
$4x^3$	$12x^2$
$5x^4 + 2x^2$	$20x^3 + 4x$
$8x^3 - 9x^{10}$	$24x^2 - 90x^9$
$10x^3 + 16x^2 + 18x$	$30x^2 + 32x + 18$
35	0
$8x^2 - 35x + 18$	$16x - 35$
a^x	$a^x . \text{Log } a$
a^{2x}	$a^{2x} . \text{Log } a . 2$
a^{2x+5}	$a^{2x+5} . \text{Log } a . 2$
5^{8x+9}	$5^{8x+9} . \text{Log } 5 . 8$
e^x	e^x
e^{2x+5}	$e^{2x+5} . 2$
e^{5x^2+2x+5}	$e^{5x^2+2x+5} . (10x+2)$
$\log_e x$	$\frac{1}{x}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sqrt{2x+5}$	$\frac{1}{2\sqrt{2x+5}} \times 2 = 1/\sqrt{2x+5}$
$\sqrt{2x^2+3x+9}$	$\frac{1}{2\sqrt{2x^2+3x+9}} \times (4x+3)$
$\text{Log } x$	$1/x$
$\text{Log } (2x+3)$	$\left(\frac{1}{2x+3} \times 2\right)$

68

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\text{Log}\sqrt{x}$	$\frac{1}{\sqrt{x}} \times \frac{1}{2\sqrt{x}} = \frac{1}{2x}$
$\text{Log}(5x^2 + 2x + 3)$	$\frac{1}{5x^2+2x+3} \times (10x+2)$
$a^x . e^x$	$a^x.e^x + e^x.a^x.\text{Log}\,a = a^x.e^x(1+\text{Log}\,a)$
$\text{Log of } x.\sqrt{x}$	$\frac{1}{x\cancel{\sqrt{x}}} \times \left(\frac{3}{2}\cancel{x^{1/2}}\right) = \frac{3}{2x}$
$(3x + 2) . 5^x$	$(3x+2).5^x.\text{Log}\,5 + 5^x \times 3$

$$= 5^x\left[(3x+2)\cdot\text{Log}\,5 + 3\right]$$

LOGICAL REASONING

CA VINOD REDDY

1 Series is classified into

A. Number Series.

B. Alphabet Series.

C. Letter Series.

2 2, 7, 16, ?, 46, 67, 92

~~a.~~ 29 b. 31 c. 41 d. None

⟹ $2+5=7$ $29+17=46$

$7+9=16$ $46+21=67$

$\boxed{16+13=29}$ $67+25=92$

3 2, 5, 10, 17, ?, 37

a. 30 b. 21 c. 25 ~~d.~~ 26

⟹ $2+3=5$ $10+7=17$ $26+11=37$

$5+5=10$ $\boxed{17+9=26}$

4 1, 1, 4, 8, 9, 27, 16, ?

a. 32 b. 48 ~~c.~~ 64 d. 50

⟹ $1^2, 1^3, 2^2, 2^3, 3^2, 3^3, 4^2, \boxed{4^3=64}$

5 120, 99, 80, 63, ?

~~a.~~ 48 b. 40 c. 30 d. None

⟹ $11^2-1=120$ $9^2-1=80$ $7^2-1=48$

$10^2-1=99$ $8^2-1=63$

6 10, 11, 22, 23, 46, 47, 94, 95

a. 96 b. 110 ~~c.~~ 190 d. 180

$10+1=11$ $22+1=23$ $46+1=47$ $94+1=95$

$11\times2=22$ $23\times2=46$ $47\times2=94$ $\boxed{95\times2=190}$

7 1000, 500, 250, 125, ?

a. 69 b. 25 c. 60 ~~d.~~ 62.5

$1000\div2=500$ $250\div2=125$

$500\div2=250$ $125\div2=62.50$

8 6, 11, 17, 24, 32, ?

~~a.~~ 41 | b. 40 | c. 64 | d. None of these

6 + 5 = 11   17 + 7 = 24 $\boxed{32 + 9 = 41}$

11 + 6 = 17   24 + 8 = 32

9 1, 9, 25, 49, 81, ?

a. 169 | ~~b.~~ 121 | c. 225 | d. 289

$1^2, 3^2, 5^2, 7^2, 9^2, \boxed{11^2 = 121}$

10 10, 12, 36, 38, 114, 116, 348, ?

~~a.~~ 350 | b. 1050 | c. 1044 | d. None of these

10 + 2 = 12   36 + 2 = 38   114 + 2 = 116 $\boxed{348 + 2 = 350}$

12 × 3 = 36   38 × 3 = 114   116 × 3 = 348

11 5760, 960, 192, ?, 16, 8

~~a.~~ 48 | b. 64 | c. 384 | d. None of these

5760 ÷ 6 = 960 $\boxed{192 \div 4 = 48}$ 16 ÷ 2 = 8

960 ÷ 5 = 192   48 ÷ 3 = 16

12 5, 25, 36, 6, 8, 64, 625, ?

~~a.~~ 25 | b. 390625 | c. 125 | d. 5

$5, 5^2, 36, \sqrt{36}, 8, 8^2, 625, \boxed{\sqrt{625} = 25}$

13 2, 3, 5, 7, 11, 13, ?

a. 19 | ~~b.~~ 17 | c. 15 | d. 21

series of prime Numbers : 2, 3, 5, 7, 11, 13, 17

My Notes

8, 10, 30, 33, 132, 136, ?

$\Longrightarrow$ 8 + 2 = 10   30 + 3 = 33   132 + 4 = 136

10 × 3 = 30   33 × 4 = 132 $136 \times 5 = \boxed{680}$

14

Coding

Letter Coding ← → Number Coding

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

15 If MENTION is coded as NFOUJPO then EXPERT will be coded as -

M E N T I O N
plus:1
N F O U J P O

E X P E R T
+1
$\boxed{\text{F Y Q F S U}}$

16 If VINOD is coded as WHONE then SUSHEEL will be coded as -

⟹ V I N O D
+1 −1 +1 −1 +1
W H O N E

S U S H E E L
+1 −1 +1 −1 +1 −1 +1
$\boxed{\text{TTTGFDM}}$

17 If TAP is coded as QBU then GREEN will be coded as -

T A P → Reverse PAT plus1 ⇒ QBU

GREEN → Reverse NEERG plus1 → $\boxed{\text{OFFSH}}$

18 If MOBILE is coded as NQEMQK then ASHWAT will be coded as -

M O B I L E
+1 +2 +3 +4 +5 +6
N Q E M Q K

A S H W A T
+1 +2 +3 +4 +5 +6
$\boxed{\text{B U K A F Z}}$

19 If MAT is coded as 34 then PILLAR will be coded as -

MAT ⟹ 13 + 1 + 20 = 34

PILLAR ⟹ 16 + 9 + 12 + 12 + 1 + 18 = 68

My Notes

If Book is coded as YLLP then VINOD is coded as :

From end

⟹ B O O K : 2nd, 15th, 15th, 11th : YLLP

VINOD : 22nd, 9th, 14th, 15th, 4th : ERMLW

20 Find the odd man out -

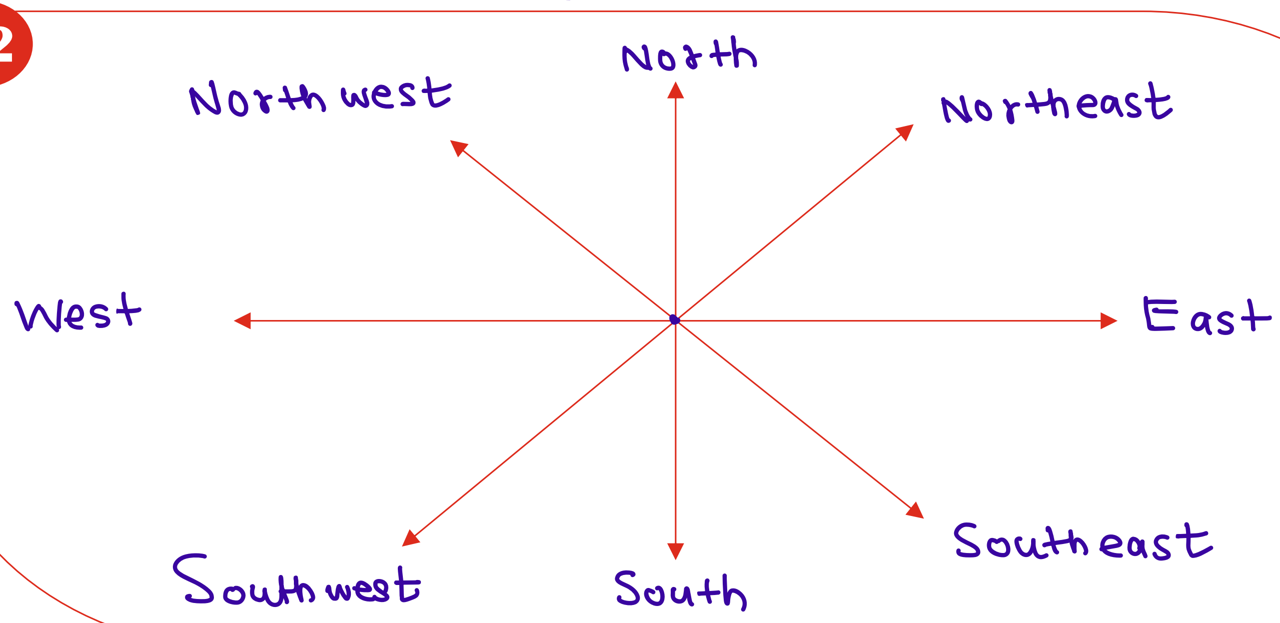
Find the odd man out -	odd man out
i. January, May, December, April	April : only month of 30 days
ii. 10, 14, 16, 28, 17, 30, 38, 42	17 : only odd number
iii. 25, 49, 35, 81, 121, 64, 4	35 : only Non perfect square
iv. 78, 91, 26, 52, 130, 117, 82, 143, 39	82 : only Number Non divi. by 13
v. 1, 64, 27, 16, 125, 343	16 : only Non perfect cube
vi. Physics, Biology, Chemistry, Accounts	Accounts : only subject from comm.
vii. Book, Pen, Pencil, Bike	Bike : only Non-educational stationery.

21 Find the odd man out - 49, 39, 36, 225

39 : only Non perfect square.
225 : only 3 digit Number.
36 : only even Number.
49 : only Non divisible by 3.

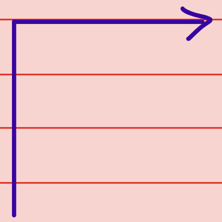
$\Big\}$ ambiguity

22



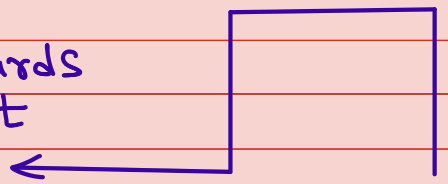
My Notes

23 North then right



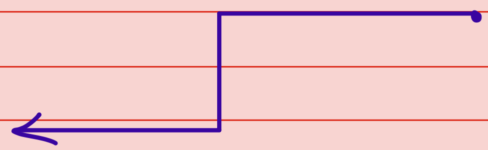
Towards east

24 North, left then left then right



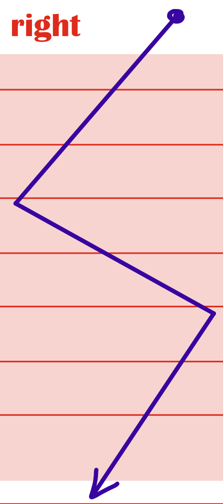
Towards west

25 West then left then right



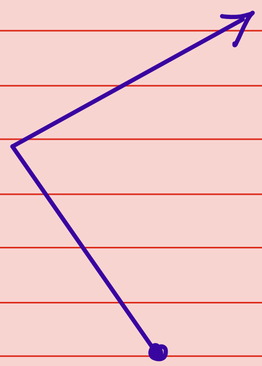
Towards west

26 Southwest then left then right



Towards southwest

27 Northwest then right



Towards Northeast

28 Seating arrangements are classified into

i) Linear arrangements (in a Line)

ii) circular arrangements

iii) polygonic arrangements

29

↑ ↑ ↑ ↑ ↑

P Q R S T facing north

Who are to the left of R : P, Q

Who is to the immediate left of S : R

Who is to the immediate right of P : Q

30 A B C D E F facing south

↓ ↓ ↓ ↓ ↓ ↓

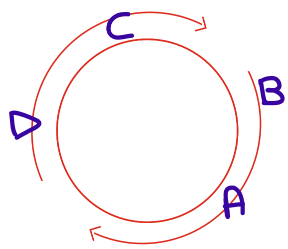
Who is to the right of A : No one.

Who is to the left of B : C, D, E, F

Who is to the immediate right of C : B

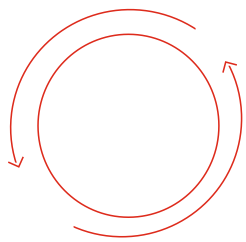
Who is to the immediate left of E : F

31



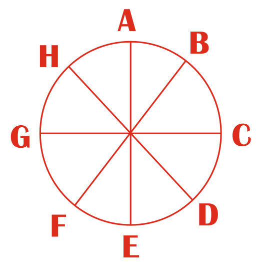
Clockwise

If they are facing centre : Right : Anticlockwise
Left : clockwise



Anti-clockwise

32



If A, B, C, D, E, F, G, H are facing centre, then

A facing	:	South
B	:	south west
C	:	west
D	:	North west
E	:	North
F	:	North east
G	:	East
H	:	southeast

My Notes

33

1. Father's Father : paternal Grandfather
2. Father's Mother : paternal Grand mother
3. Father's Brother : uncle
4. Father's Sister : Aunt
5. Children of Uncle : cousins
6. Wife of Uncle : Aunt
7. Children of Aunt : cousins
8. Husband of Aunt : uncle
9. Mother's Father : Maternal Grandfather
10. Mother's Mother : Maternal Grand mother
11. Mother's Brother : Maternal uncle
12.Mother's Sister : Maternal Aunt
13. Children of Maternal Uncle : cousins
14. Wife of Maternal Uncle : Aunt
15. Grandfather's Son : Father OR uncle
16. Grandfather's Only son : Father
17. Mother or Father's Mother : Grand mother
18. Grandmother's Mother : Great Grand mother
19. Grandmother's Father : Great Grand father
20. Grandson's Daughter : Great Grand daughter
21. Grandson's Son : Great Grandson
22. Grand-daughter's Son : Great Grandson
23. Grand-daughter's Daughter : Great Grand daughter
24. Daughter's Husband : son in Law
25. Son's Wife : Daughter in Law
26. Husband's Father : Father in Law
27. Husband's Mother : Mother in Law
28. Wife's Brother : Brother in Law
29. Wife's Sister : Sister in Law
30. Wife's Father : Father in Law
31. Wife's Mother : Mother in Law
32. Brother's Son : Nephew
33. Brother's Daughter : Niece
34. Sister's Son : Nephew
35. Sister's Daughter : Niece
36. Brother's Wife : sister in Law
37. Sister's Husband : Brother in Law
38. My father's son is my : Brother
39. My father's daughter is my : sister

33

40. My father's father is my : Grand father
41. My mother's brother is my : uncle (maternal)
42. My daughter's husband is my : son in law
43. My son's wife is my : Daughter in law
44. My Brother's wife is my : sister in law
45. My brother's daughter is my : Niece
46. My brother's son is my : Nephew
47. My wife's father is my : Father in law
48. My wife's mother is my : Mother in law
49. My wife's sister is my : sister in law
50. My wife's brother is my : Brother in law
51. My father's wife is my : mother
52. My mother's husband is my : Father
53. My son's daughter is my : Grand daughter
54. My daughter's son is my : Grand son.

34

6, 11, 21, 36, 56, ?

a. 42 b. 51 ~~c.~~ 81 d. 91

$6+5=11$ $21+15=36$ $\boxed{56+25=81}$

$11+10=21$ $36+20=56$

35

10, 100, 200, 310, ?

a. 400 b. 410 c. 420 ~~d.~~ 430

$10+90=100$ $200+110=310$

$100+100=200$ $\boxed{310+120=430}$

36

11, 13, 17, 19, 23, 25, ?

a. 27 ~~b.~~ 29 c. 31 d. None of these

$11+2=13$ $17+2=19$ $23+2=25$

$13+4=17$ $19+4=23$ $\boxed{25+4=29}$

37 6, 12, 21, 33, ?

a. 36 b. 39 c. 45 ~~d.~~ 48

$6+6=12$ $21+12=33$

$12+9=21$ $33+15=48$

38 2, 5, 9, 14, ?, 27

~~a.~~ 20 b. 16 c. 18 d. None of these

$2+3=5$ $9+5=14$ $20+7=27$

$5+4=9$ $\boxed{14+6=20}$

39 11, 21, ?, 56, 81

a. 42 ~~b.~~ 36 c. 91 d. 51

$11+10=21$ $36+20=56$

$\boxed{21+15=36}$ $56+25=81$

40 10, 18, 28, 40, 54, ?, 88

~~a.~~ 70 b. 86 c. 72 d. 80

$10+8=18$ $28+12=40$ $\boxed{54+16=70}$

$18+10=28$ $40+14=54$ $70+18=88$

41 195, 168, 143, 120, ?, 80

a. 100 ~~b.~~ 99 c. 105 d. None of these

$14^2-1=195$ $12^2-1=143$ $\boxed{10^2-1=99}$

$13^2-1=168$ $11^2-1=120$ $9^2-1=80$

42 8, 10, 40, 42, 168, 170, 680, 682, ?

a. 684 b. 1528 ~~c.~~ 2728 d. None of these

$8+2=10$ $40+2=42$ $168+2=170$ $680+2=682$

$10\times4=40$ $42\times4=168$ $170\times4=680$ $\boxed{682\times4=2728}$

43 28, 33, 31, 36, 34, ?

a. 38 ~~b.~~ 39 c. 42 d. None of these

28+5 = 33 31+5 = 36 $\boxed{34+5 = 39}$
33−2 = 31 36−2 = 34

44 5760, 960, 192, ?, 16, 8

a. 96 ~~b.~~ 48 c. 32 d. None of these

5760 ÷ 6 = 960 $\boxed{192 \div 4 = 48}$ 16 ÷ 2 = 8
960 ÷ 5 = 192 48 ÷ 3 = 16

45 2, 3, 3, 5, 10, 13, 39, 43, ?, 177

a. 46 ~~b.~~ 172 c. 48 d. None of these

2+1 = 3 3+2 = 5 10+3 = 13 39+4 = 43 172+5
3×1 = 3 5×2 = 10 13×3 = 39 $\boxed{43 \times 4 = 172}$ = 177

46 If RAMAN is written as 12325 and DINESH is written as 675489,
How HAMAM is written?

~~a.~~ 92323 b. 92233 c. 93292 d. None of these

R	A	M	A	N	D	I	N	E	S	H	HAMAM
1	2	3	2	5	6	7	5	4	8	9	92323

47 If RED is coded as 6720 then GREEN would be coded as

a. 9207716 b. 167129 ~~c.~~ 1677209 d. 1672091

R E D ⇒ Reverse D E R ⇒ 6720
4th 5th 18th +2

G R E E N ⇒ Reverse N E E R G ⇒ 1677209
14th 5th 5th 18th 7th

My Notes

48 If BROTHER is coded as 2456784, SISTER is coded as 919684, what is coded as BORBERS?

~~a.~~ 2542849 b. 2542898 c. 2454889 d. 2524889

BORBERS ⇒ 2542849

49 If DELHI is coded as 73541 & CALCUTTA as 82589662, How is CALICUT coded?

a. 5279431 b. 5978213 ~~c.~~ 8251896 d. 8543962

CALICUT ⇒ 8251896

50 If CLOCK is coded as 34235 and TIME as 8679, what will be the code for MOTEL?

~~a.~~ 72894 b. 77684 c. 72964 d. 27894

MOTEL ⇒ 72894

51 In a certain code NAME is written as 4258 then what would be the code for MEAN?

a. 2458 b. 5842 c. 8524 ~~d.~~ 5824

N	A	M	E
4	2	5	8

MEAN
5824

52 If GOLD is written as IQNF then WIND would be coded as

a. VHMC ~~b.~~ YKPF c. XJOE d. DNIW

G	O	L	D
+2	+2	+2	+2
I	Q	N	F

W	I	N	D
+2	+2	+2	+2

$\boxed{\text{Y K P F}}$

My Notes

53 **If ROSE is written as TQUG, How BISCUIT can be written in that code**

~~a.~~ DKUEWKV b. CJTDVJU c. DKVEWKV d. DKUEWKY

R O S E
+2 +2 +2 +2
T Q U G

BISCUIT
plus 2

$\boxed{\text{DKUEWKV}}$

54 **If DELHI is coded as CCIDD then How would you code BOMBAY?**

a. AJMTVT ~~b.~~ AMJXVS c. MJXVSU d. None of these

D E L H I
−1 −2 −3 −4 −5
C C I D D

B O M B A Y
−1 −2 −3 −4 −5 −6

$\boxed{\text{AMJXVS}}$

55 **If PALAM is given with a code number of 43 then, what will be the code number for SANTACRUZ?**

~~a.~~ 123 b. 85 c. 120 d. 125

PALAM ⇒ 16+1+12+1+13 = 43

SANTACRUZ ⇒ 19+1+14+20+1+3+18+21+26
= 123

56 **If 256 means you are good**
637 means we are bad
358 means good and bad
which of the following represents 'and' in that code

a. 2 b. 5 ~~c.~~ 8 d. 3

My Notes

2 : you
5 : Good
6 : are
3 : Bad
7 : we
8 : and

57 Find odd man out from : Avni, Ishani, Esha, Usha, Veena

~~a. Veena~~ b. Esha c. Usha d. Avni

only Name starting with consonant.

58 Find the odd man out from : 64, 32, 512, 243, 1024, 8, 2048

a. 2048 ~~b. 243~~ c. 64 d. 8

243 is the only odd Number

59 Find the odd man out from AB, MN, YZ, VU

a. AB b. MN c. YZ d. VU

VU ⇒ only Reverse in order

60 If A = 1, PAT = 37, then TAP =

a. 73 ~~b. 37~~ c. 36 d. 38

PAT ⇒ 16 + 1 + 20 = 37

61 If D = 4, BAD = 7, then what is the value of ANT?

a. 8 b. 17 c. 35 d. 37

BAD ⇒ 2 + 1 + 4 = 7

ANT ⇒ 1 + 14 + 20 = 35

62 If MATHEMATICS = 12345123678, then MAHATMA = ?

~~a. 1242312~~ b. 12345123 c. 12345678 d. 12425341

MAHATMA ⇒ $\boxed{1242312}$

63 If D = 4, COVER = 63, then BASIS = ?

a. 55 ~~b.~~ 50 c. 49 d. 54

COVER = 3 + 15 + 22 + 5 + 18 = 63

BASIS = 2 + 1 + 19 + 9 + 19 = 50

64 If HKUJ means FISH, what does UVCD means?

a. STAR ~~b.~~ STAB c. STAL d. None of the these

H K U J — U V C D

−2 −2 −2 −2 — −2 −2 −2 −2

F I S H — $\boxed{\text{S T A B}}$

65 If NOIDA is written as STNIF, How MEERUT can be written in that code?

a. QIIVYX b. RJJWZV ~~c.~~ RJJWZY d. RIIVYX

NOIDA — MEERUT

plus 5: STNIF — plus 5 : $\boxed{\text{RJJWZY}}$

66 If 'BEQUICK' is coded as ZCOSGAI then INDIAN is coded as

B E Q U I C K — I N D I A N

minus - 2 — minus : 2

Z C O S G A I — GLBGYL

67 If RAMAYANA is coded as PYKYWYLY then MAHABHARATA can be written in that code as

R A M A Y A N A — M A H A B H A R A T A

Minus : 2 — Minus : 2

P Y K Y W Y L Y — KYFYZFYPYRY

68 In a certain code HYDROGEN is coded as JCJZYSSD then how can ANTIMONY be coded?

H Y D R O G E N — A N T I M O N Y

+2 +4 +6 +8 +10 +12 +14 +16 — +2 +4 +6 +8 +10 +12 +14 +16

J C J Z Y S S D — $\boxed{\text{CRZQWABO}}$

69 In certain language PLAYER is coded as QNDCJX then how SINGER will be coded in that language

P L A Y E R
+1 +2 +3 +4 +5 +6
Q N D C J X

S I N G E R
+1 +2 +3 +4 +5 +6
$\boxed{\text{TKQKJX}}$

70 In certain code MONKEY is written as XDJMNL. How TIGER is written in that code?

M O N K E Y
Reverse: Y E K N O M then minus 1
X D J M N L

TIGER
Reverse REGIT
minus 1
$\boxed{\text{QDFHS}}$

71 If BAT can be written as DCV, then MAN can be written as

B A T
plus 2: D C V

MAN
plus 2 : $\boxed{\text{OCP}}$

72 If CAT can be written as CNANT, then GOD can be written as

C A T
CNANT

GOD
$\boxed{\text{GNOND}}$

73 If SIR can be written as PSPIPR, then MAN can be written as

S I R
PSPIPR

MAN
$\boxed{\text{PMPAPN}}$

My Notes

74 If TIMBER is written as BERMIT then how would BANTER be written in that code

TIMBER → Reverse BER MIT

BAN TER → Reverse $\boxed{\text{TER NAB}}$

75 In a certtain code COURSE is written as ESRUOC. How BREATH can be written in the same code

COURSE → Reverse the order of letters ESRUOC

BREATH will be coded as $\boxed{\text{HTA ERB}}$

76 In a certain code 493 means 'Friendship difficult challenge'; 961 means 'Struggle difficult exam'; 178 means 'Exam believable subject', then which digit is used for believable?

(a) 7 (b) 8 ~~(c)~~ a or b (d) None of these

4
9 : Difficult
3
6
1 : Exam
7 } Believable and subject
8 }

77 Vehicle is coded as Book, Book as Flower, Flower as House, House as Calculator. then where is the treasure of huge amount of knowledge hidden?

a. Book b. Calculator ~~c.~~ Flower d. Vehicle

My Notes

78 If TALENT is written as LATENT then how would EXOTIC be written in that code?

a. EXOTIC ~~b.~~ OXETIC c. TICOXE d. None of these

T A L E N T EXOTIC

Reverse

LATENT OXETIC

79 In a certtain language NOITCELES represents SELECTION then AIDNI represents-

a. AIDSI b. HINDI ~~c.~~ INDIA d. None

NOITCELES ⇒ Reverse in order

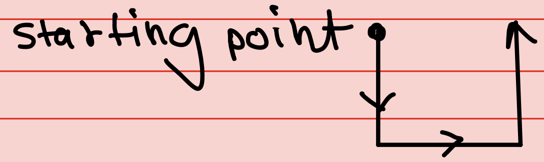
SELECTION

AIDNI represents ⇒ INDIA

My Notes

80 Mohan starts walking from point A & 1 km towards south, turns left and walks 1 km. Then he turns left and again walks 1 km. Now he is facing ...

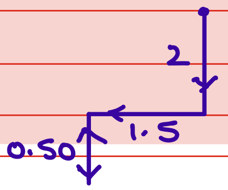
a. East b. West c. South d. North



Now Mohan is facing North

81 Suresh starts from a point walks 2 miles towards south, turns right and walks $1\frac{1}{2}$ miles, turns left and walks $\frac{1}{2}$ miles and then he turns back. What direction is he facing now?

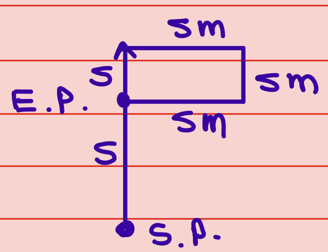
a. East b. West c. South d. North



Finally he is facing North

82 A man facing East, then he turns left and goes 10 m, then he turns right and goes 5 m then goes 5 m to the south and from there 5 m to the west. In which direction he is from original place?

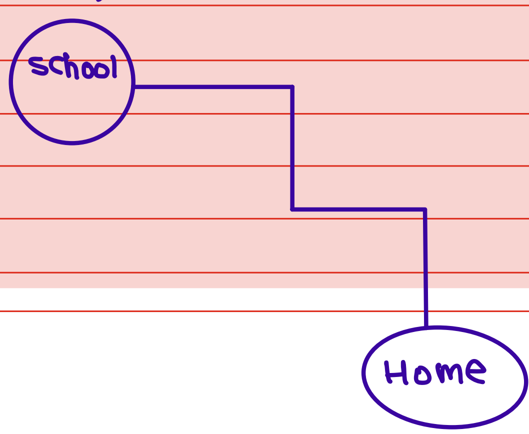
a. East b. West c. South d. North



S.P. → starting point
E.P. → End point

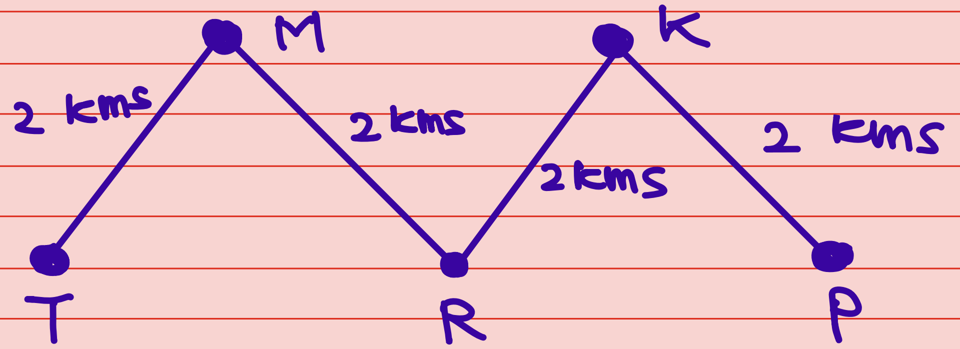
83 From her home Avni Reddy wishes to go to school. From home she goes towards north & then turns left & then turns right and finally she turns left and reaches school. In what direction her school is situated with respect to her home?

a. North-East b. North-West c. South-East d. South-West



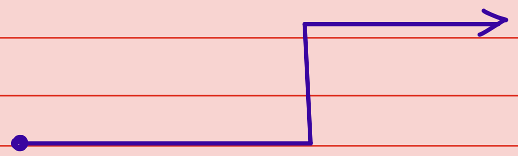
84 K is a place which is located 2 kms away in north-west direction from the capital P. R is another place that is located 2 kms away in south-west direction from K. M is another place that is located 2 kms away in north-west direction from R. T is another place that is located 2 kms away in south-west direction from M. In which direction T is located in relation to P.

a. South-West b. North-West c. West d. North



85 I started walking down a road in the morning facing sun. After walking for sometime I turned to my left then I turned to my right. In which direction was I going then?

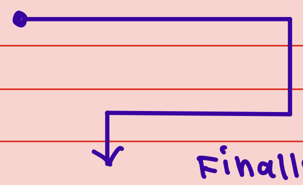
a. East b. West c. North d. South



Finally going in east direction

86 You are going straight, first eastwards then turn to the right, then right again, then left. In which direction would you be going now?

a. East b. West c. South d. North

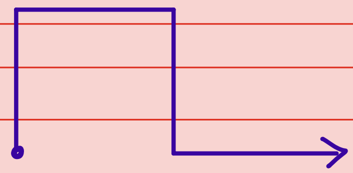


Finally going in south direction

My Notes

87 You go North, turn right, then right again, & then go to the left. Which direction are you facing now?

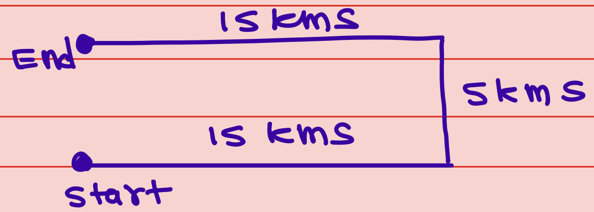
~~a~~. East b. West c. South d. North



Finally facing East direction.

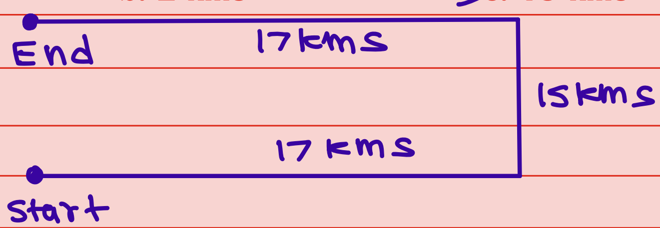
88 Ashwat Reddy traveled 15 kms eastwards, then turned left and travelled 5 kms then turned left and travelled 15 kms. How far is he from starting point?

a. 30 kms b. 35 kms c. 15 kms ~~d~~. 5 kms



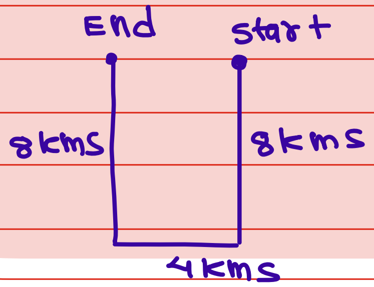
89 Hari travelled 17 kms to the east, he turned left and went 15 kms, he again turned left and went 17 kms. How far is he from starting point?

a. 17 kms b. 2 kms ~~c~~. 15 kms d. 32 kms



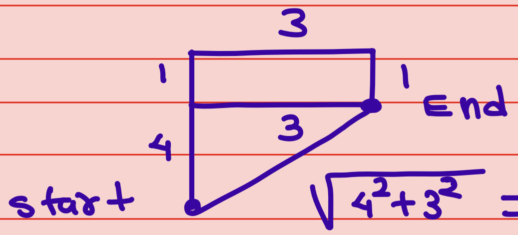
90 Sudha travels 8 kms to the south, then she turns to the right and walks 4 kms. Then again she turns to her right and moves 8 kms forward. How many kms away is she from starting point?

a. 7 kms b. 6 kms ~~c~~. 4 kms d. 8 kms



91 From my house I walked 5 kms towards North. I turned right and walked 3 kms. Again I went 1 km to the south. How far am I from my house?

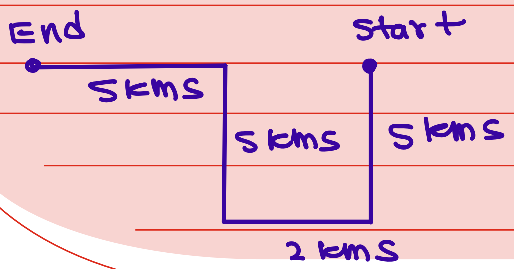
a. 7 kms   b. 6 kms   c. 4 kms   ~~d.~~ 5 kms



$\sqrt{4^2+3^2} = \sqrt{25} = 5$ kms

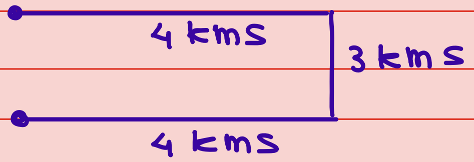
92 Rama left home and walked 5 kms southwards. turned right and walked 2 kms and turned right and walked 5 kms and turned left and walked 5 kms. How many kilometers will she have to walk to reach her home straight?

a. 5 kms   ~~b.~~ 7 kms   c. 17 kms   d. 15 kms



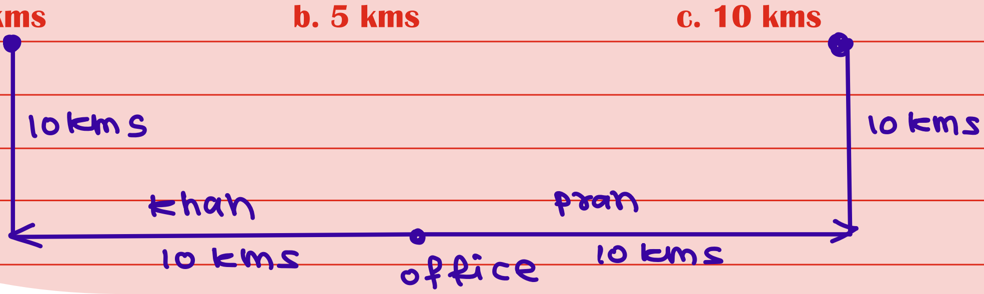
93 Facing the East Gopi walks straight 4 kms, turns left and walks 3 kms and again turns left and walked 4 kms. How far is he now from the starting point?

a. 2 kms   ~~b.~~ 3 kms   c. 10 kms   d. 11 kms



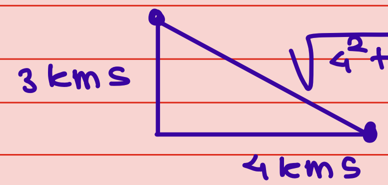
94 Pran and Khan start from their office and walk in opposite direction each travelling 10 kms. Pran then turns left and walks 10 kms while Khan turns right and walks 10 kms. How far are they from each other?

a. 0 kms   b. 5 kms   c. 10 kms   ~~d.~~ 20 kms



95 A starts walking from a point 'P'. A goes westward and covers a distance of 4 kms and then turns to his right and walks 3 kms. How far 'A' is from starting point.

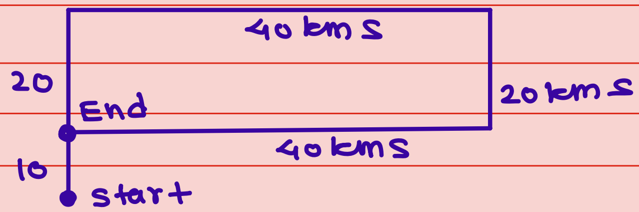
a. 7 kms b. 9 kms c. 2 kms ~~d.~~ 5 kms



$\sqrt{4^2+3^2} = \sqrt{25} = 5$ kms

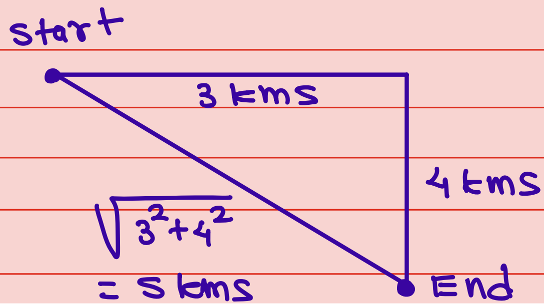
96 A cyclist goes 30 kms to North and turning to the East he goes 40 kms. Again he turns to his right and goes 20 kms. After this he turns to his right and goes 40 kms. How far is he from his starting point?

a. 20 kms ~~b.~~ 10 kms c. 25 kms d. 40 kms



97 A man travels 3 kms in the East and turns to south and moves 4 kms. How far is he from starting point?

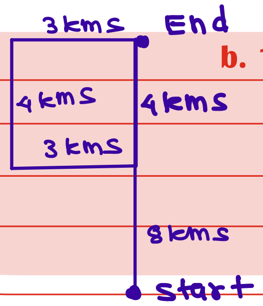
~~a.~~ 5 kms b. 6 kms c. 2 kms d. 10 kms



My Notes

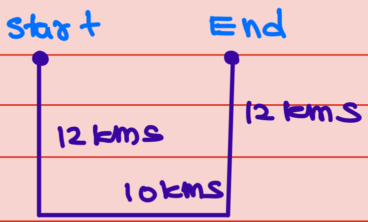
98 Dhrish travels 8 kms towards the North, turns left and travels 3 kms and then again turns right and covers another 4 kms then turns right and travels another 3 kms. How far is he from starting point?

a. 18 kms b. 11 kms ~~c.~~ 12 kms d. 15 kms



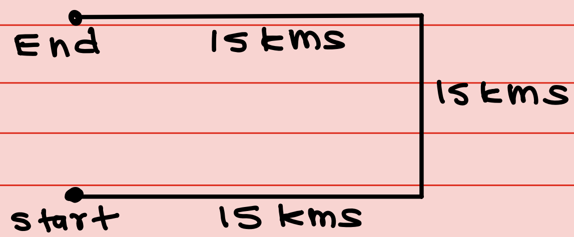
99 Mitan travelled 12 kms southwards and turned left and travelled 10 kms, then turned left and travelled 12 kms. How far was Mitan from starting point?

a. 8 kms ~~b.~~ 10 kms c. 12 kms d. 14 kms



100 Ashwat travelled 15 kms towards East then turned towards North and travelled 15 kms and turned west & travelled 15 kms. How far is he from starting point?

~~a.~~ 15 kms b. 30 kms c. 45 kms d. zero kms

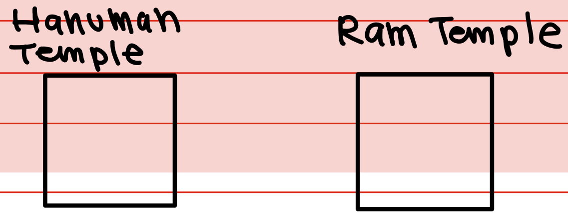


My Notes

101 Daily in the morning the shadow of Ram temple falls on Hanuman temple, and in the evening shadow of Hanuman temple falls on Ram temple. In which direction Hanuman temple is from Ram temple?

a. East ~~b.~~ West c. South d. North

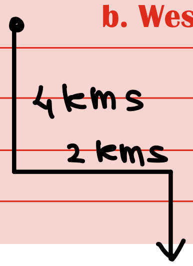
west



Sun - East

102 A man on a moped starts from a point and rides 4 kms south then turns left and rides 2 kms and turns again to the right to ride to go more. In which direction is he moving now?

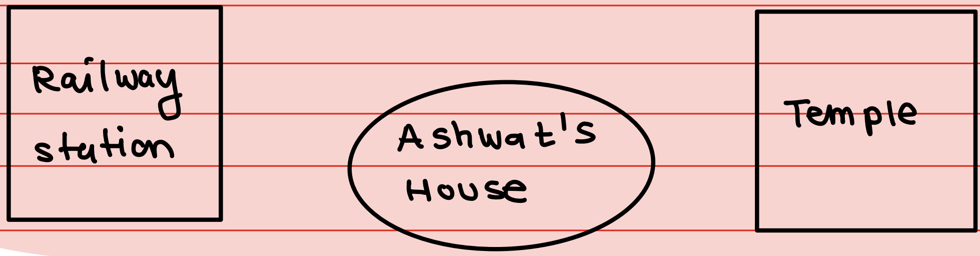
a. East b. West ~~c.~~ South d. North



Finally he is moving in south direction

103 If Ashwat sees rising sun behind the temple and setting sun behind railway station from his house. What direction of temple from railway station?

~~a.~~ East b. West c. South d. North



My Notes

104 A is B's daughter. B is C's mother. D is C's brother. How is D related to A?

a. Father b. Grandfather c. Brother d. Son

B mother

C, A-daughter D-Brother

105 P is Q's brother. R is Q's mother. S is R's father. T is S's mother. How is P related to T?

a. Grand-daughter b. Great grandson c. Grandson d. Grandmother

T-Mother

S Father

R Mother

P Brother, Q

106 A is B's brother. C is D's father. E is B's mother A & D are brothers. How is E related to C?

a. Sister b. Sister in law c. Niece d. Wife

E Mother, C Father

A Brother, B, D Brother

107 Given that A is mother of B, C is son of A, D is brother of E, E is daughter of B, who is the grandmother of D?

a. A b. B c. C d. D

A Mother

B C - son

Brother D, E Daughter

My Notes

108 M is the son of P. Q is grand daughter of O who is husband of P.
How is M related to O?

a. Son | b. Daughter | c. Mother | d. Father

wife P, O husband
↓
M - son
↓
Q - Daughter

109 A is B's sister . C is B's mother. D is C's father. E is D's mother.
How is A related to D?

a. Grandmother | b. Grandfather | c. Daughter | d. Grand-daughter

E mother
↓
D - Father
↓
C - mother
A sister, B

110 A is father B & C, B is son of A. But C is not son of A. How is C related to A?

a. Niece | b. Son-in-law | c. Daughter | d. Grandson

A Father
son B   C Daughter

111 A is father B , C is daughter of B, D is brother of B. E is son of A.
What is relationship between C and E?

a. Brother & sister | b. Cousins | c. Niece & uncle | d. Uncle & Aunt

A Father
↓
son - E, B, D Brother
↓
C Daughter

My Notes

112 If P is husband of Q and R is mother of S & Q. What is R to P?

a. Mother b. Sister c. Aunt ~~d.~~ Mother in law

R mother

husband P, Q wife

S

113 X is husband of Y. W is daughter of X. Z is husband of W. N is daughter of Z. What is the relation of N to Y?

a. Cousin b. Niece c. Daughter ~~d.~~ Grand-daughter

X husband, Y-wife

W-daughter, Z-husband

N-Daughter

114 A's mother is sister of B and she has daughter C who is 21 years old. How is B related to C?

a. Niece ~~b.~~ Maternal Uncle c. Daughter d. Uncle

A's mother, B

A, C

115 A is D's brother. D is B's father. B & C are sisters. How is C related to A?

a. Cousin ~~b.~~ Niece c. Aunt d. Nephew

A Brother, D-Father

B, C sisters

My Notes

116 A is B's brother, C is A's mother. D is C's father, E is B's son. How is E related to A?

a. Cousin ~~b~~. Nephew c. Uncle d. Grandson

D Father
C Mother
↓
A Brother, B
↓
E son

117 A and B are brothers. E is daughter of F. F is wife of B.
What is relationship of E to A?

a. Sister b. Daughter ~~c~~. Niece d. Sister-in-law

Brother A, B, F married couple
↓
E Daughter

118 M and F are a married couple. A and B are sisters. A is sister of F. Who is B to M?

~~a~~. Sister-in-law b. Sister c. Mother d. Niece

M, F, A, B married couple
↓
sisters

119 Q is son of P. X is daughter of Q. R is aunty (Bua) of X and L is son of R, then what is L to P?

~~a~~. Grandson b. Granddaughter c. Daughter d. Nephew

P
↓
Q son, R-sister
↓ ↓
X Daughter L-son

My Notes

120 Rajiv is brother of Atul. Sonia is sister of Sunil. Atul is son of Sonia. How is Rajiv related to Sonia?

a. Nephew ~~b.~~ Son c. Brother d. Father

Sunil, Sonia

Rajiv, Atul Brothers

121 There are 2 film stars, one is father of other's son. what is relation of two with each other?

a. Grandfather-Grandson b. Grandfather-son

~~c.~~ Husband-Wife d. Father & Son

122 Ramu's mother said to Ramu : "My mother has a son whose son is Ashwat". How is Ashwat related to Ramu?

a. Uncle ~~b.~~ Cousin c. Brother d. Nephew

mother

Ramu's mummy, son

Ramu Ashwat

My Notes

123 There are 5 houses P,Q,R,S,T. P is right of Q and T is left of R and right of P. Q is right of S. Which house is in the middle?

~~a.~~ P b. Q c. R d. T

↑ ↑ ↑ ↑ ↑
S Q P T R

we have assumed that they are facing North.

124 Five friends are sitting on bench. A is to the left of B but on the right of C, D is to the right of B but on the left of E. Who are at the extremes?

a. A,B b. A,D ~~c.~~ C,E d. B,D

C A B D E

Facing North

125 In a college party, 5 girls are sitting in a row. P is to the left of M and to the right of O. R is sitting to the right of N but to the left of O, who is sitting in the middle?

~~a.~~ O b. R c. P d. M

N R O P M

126 5 boys A,B,C,D,E are standing in a row. D is to the right of E, B is on the left of E but on the right of A. D is to the left of C, who is standing on extreme right. Who is standing in the middle?

a. B b. C c. D ~~d.~~ E

A B E D C

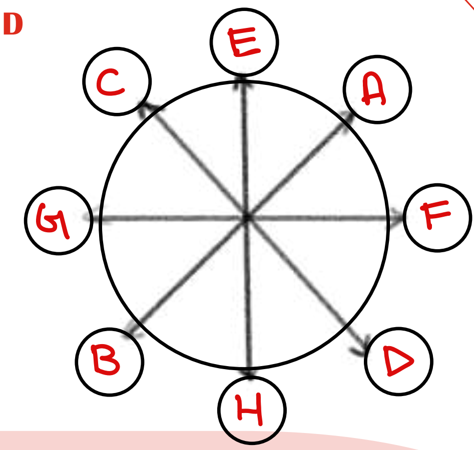
My Notes

127 4 Ladies A,B,C,D and 4 gentlemen E,F,G,H are sitting in a circle around a table facing each other - I. No 2 ladies or gentlemen are sitting side by side

II. C who is sitting between G, E facing D

III. F is between D and A and facing G

IV. H is to the right of B



Qs. 1 Who is sitting to the left of A?

a. E b. F c. G d. H

Qs. 2 E is facing whom?

a. F b. B c. G d. H

Qs. 3 Who are immediate neighbours of A?

a. G,H b. E,F c. E,H d. F,H

128 P to W are sitting in front of one another in two rows. Each row has 4 persons. P is between U and V and facing North. Q, who is immediate left of S is facing W. R is between T and S and W is to the immediate right of V.

Qs. 1 Who is sitting in front of R?

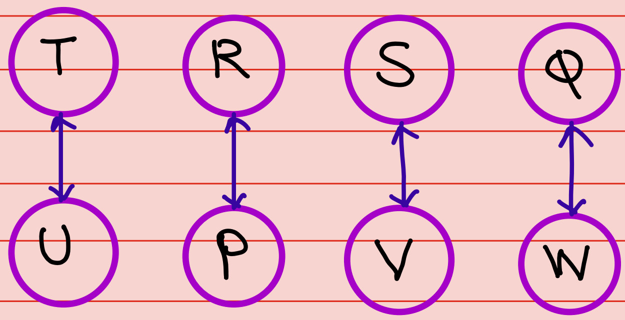
a. U b. Q c. V d. P

Qs. 2 Who is to the immediate right of R?

a. S b. U c. T d. None of these

Qs. 3 In which of the following pairs, persons are sitting in front of each other?

a. S,V b. R,V c. T,V d. U,R



My Notes

129 A to H are seated in a straight line facing North. C sits 4th left of G. D sits second right of G. Only 2 people sit between D and A. B and F are immediate neighbour of each other. B is not immediate neighbour of A. H is not neighbour of D.

a. Who among the following sits third to the right of C?

a. B b. F ~~c.~~ A d. E

b. Which of the following represents persons seated at 2 extreme ends of line?

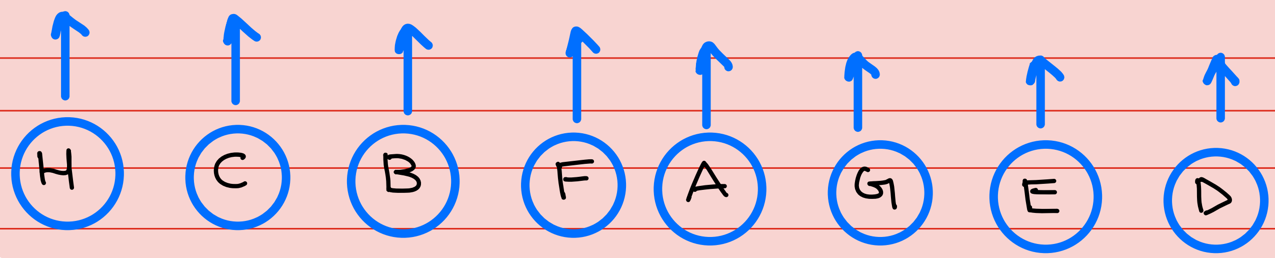
a. C,D b. A,B c. B,G ~~d.~~ D,H

c. What is position of H with respect to F?

~~a.~~ 3rd to left b. immediate right c. 2nd to right d. 4th to left

d. How many persons are seated between A & E?

~~a.~~ 1 b. 2 c. 3 d. 4



My Notes

130 **Given that A is mother of B. C is son of A. D is brother of E. E is daughter of B. Who is grandmother of D?**

a. A b. B c. C d. D

A Mother

B C - son

E Daughter, D Brother

My Notes

Index Numbers

CA VINOD REDDY

1 Often we encounter news of price rise, GDP growth, production growth, etc. It is important for student of chartered accountancy to learn techniques of measuring growth / rise or decline of various economic & business data and reporting it with the help of index numbers.

2 **Definition of Index Numbers :**

1. Index number is ratio or avg of ratios of prices, quantities, values where 2 or more time periods are involved, one of which is the base period.
2. The value at base time period serves as standard point for comparison.

Examples : Sensex, CII, HDI, CPI, etc.

3 There are 2 broad types of index numbers

a.

b.

Simple index number is computed for one variable where as composite index number is calculated from 2 or more variables. Most index numbers are composite in nature.

4 All index Numbers are UNIT FREE.

5 Issues involved in the construction of index numbers

a. Selection of data.
b. Base period.
c. Selection of weights.
d. Use of averages
e. Choice of variable
f. Selection of formula

6 Price Relative = ______________________

____

Quantity Relative = ______________________

____

Value Relative = ______________________

____

My Notes

7

Year (B.Year) 2016	Price of commodity A	Quantity of commodity A	Value of commodity A	Relatives		
				Price	Qty.	Value
2016	50	8				
2017	103	13				
2018	68	16				
2019	98	21				
2020	111	28				
2021	125	35				

8

Simple Aggregative Price Index Number =

Simple Aggregative Quantity Index Number =

Simple Aggregative Value Index Number =

9

Commodities	Year		
	2021	2022	2023
Cheese (per 10 gms)	12	15	16.80
Egg (per piece)	3	3.60	3.30
Potato (per kg)	5	6.00	5.70
Aggregate	20		
Simple Aggregative Price Index Number			

Commodities	Year		
	2021	2022	2023
Cheese (per 100 gms)			
Egg (per dozen)			
Potato (per 20 kg)			
Aggregate			
Simple Aggregative Price Index Number			

9 Observations from above two tables :

10 Simple Aggregative Index Numbers do not satisfy <u>unit test</u>

⇓

To overcome this limitation of simple aggregative index number we have introduced

11 Weighted Aggregative Index Numbers :

While finding weighted aggregative price index numbers we use weight as :

While finding weighted aggregative quantity index numbers we use weight as :

12

Weighted Aggregative Index Numbers	Price	Quantity
1. Laspeyre's		
2. Paasche's		
3. Marshall Edgeworth's		
4. Fisher's Ideal Indices		

13

$$\text{Chain Index} = \frac{\text{(Link relative of Current Year x Chain Index of Previous Year)}}{100}$$

My Notes

14

Year	Price	Link Relatives	Chain Indices
2011	50		
2012	60		
2013	62		
2014	65		
2015	70		
2016	78		
2017	82		
2018	84		
2019	88		
2020	90		
2021	103		
2022	108		

15 **Limitations of Index Numbers :**

1. Indices are collected mostly from samples.
2. They depict only broad trend and not real picture
3. There are many methods employed from constructing index numbers, the result gives diff values and this at times creates confusion.

16

Year	Wholesale Price Index	GNP at current Prices	Real GNP
2021	113.10	7499	
2022	116.30	7935	
2023	121.20	8657	
2024	127.70	9323	

$$\text{Deflated Value} = \frac{\text{Current Value}}{\text{Price Index of current year}}$$

My Notes

17

Year	Original Price Index	Shifted Price Index to the base 2020
2010	100	
2011	104	
2012	106	
2013	107	
2014	110	
2015	112	
2016	115	
2017	117	
2018	125	
2019	131	
2020	140	
2021	147	

$$\text{Shifted Price Index} = \frac{\text{Original Price Index}}{\text{Price Index of the year on which base has to be shifted}} \times 100$$

18

Tests of Adequacy : a. ____________________

b. ____________________

c. ____________________

d. ____________________

19

When unit test of index numbers is said to be satisfied?

My Notes

20 **When time reversal test is said to be satisfied?**

- You will notice that Laspeyre's & Paasche's method do not satisfy Time-reversal test but Fisher's formula satisfy Time-reversal test.
- While selecting an appropriate index formula, the time reversal test and factor reversal test are considered necessary in testing the consistency.

21 **When factor reversal test is said to be satisfied?**

Fisher's formula satisfy time reversal test as well as factor reversal test.
Therefore, it is called as ideal index number.

22 **When circular test of index numbers is said to be satisfied?**

23 **A series of numerical figures which show the relative position is called as ________________ .**

24 **Index number for the base period is always taken as**

a. 200 b. 50 c. 150 d. 100

My Notes

25 ________ play very important part in construction of index numbers.

a. Weights b. Classes c. Estimations d. Students

26 ________ is particulars suitable for construction of index numbers.

a. AM b. GM c. HM d. None of these

27 Index number show ________ changes rather than absolute amounts of change.

a. Relative b. Percentage c. Major d. Minor

28 The ________ makes index numbers time reversible

a. AM b. GM c. HM d. Mode

29 The ________ of group indices gives General Index.

a. AM b. GM c. HM d. None of these

30 ________ Test is extension to time reversal test.

a. Factor Reversal Test b. Circular Test c. Both d. None

31 Factor Reversal Test is satisfied by :

a. Fisher's Index b. Laspeyre's Index c. Paasche's Index d. None of these

32 Laspeyre's formula does not satisfy :

a. Factor Reversal Test b. Time Reversal Test c. Circular Test d. All of these

33 The value at ________ time period serves as standard point for comparison

a. Base b. High c. My d. Past

34 Index numbers are often constructed from ________

a. Frequency b. Class c. Sample d. Temple

35 The ratio of price of a single commodity in a given period to its price in the base year is called as

a. Price Relative b. Close Relative

c. Cousin d. Price

36 $$\frac{\text{Sum of prices of all commodities}}{\text{Sum of prices of all commodities in base year}} \times 100 = ?$$

a. Price Relative b. Quantity Relative

c. Simple aggregative price index number d. Weighted aggregative price index number

37 P_{01} is the index for time.

a. 1 on 0 b. 0 on 1 c. 1 on 1 d. 0 on 0

38 P_{10} is the index for time.

a. 1 on 0 b. 0 on 1 c. 1 on 1 d. 0 on 0

39 P_{ab} is the index for time.

a. a on b b. b on a c. a on a d. b on b

40 When product of price index number and quantity index number is equal to corresponding value index number then the test that holds good is ______________ .

a. Unit Test
b. Time Reversal Test
c. Circular Test
d. Factor Reversal Test

41 The formula should be independent of the unit in which or for which prices and quantities are quoted in :

a. Unit Test
b. Time Reversal Test
c. Circular Test
d. Factor Reversal Test

42 Fisher's ideal formula does not satisfy _______ test.

a. Unit Test b. Circular Test c. Time Reversal Test d. None of these

43 Laspeyre's and Paasche's method __________ time reversal test.

a. Satisfy b. Do not satisfy c. Sometimes satisfy d. Can satisfy

44 There is no such thing as unweighted index numbers.

a. True b. False

45 Theoretically GM is best avg in construction of index numbers but in practice, mostly AM is used -

a. True b. False

46 The number of tests of adequacy are ________

a. 3 b. 4 c. 8 d. 2

47 We use price index numbers ___________________

a. To measure and compare prices
b. To measure prices
c. To compare prices
d. None of these

48 If price of all commodities in a place have increased 1.25 times in comparison to their base period, the index number of prices of that place now is :

a.125 b. 25 c. 150 d. 225

49 If index number of prices at a place in 2022 is 250 with 2005 as base year then prices have increased on avg by

a. 250% b. 150% c. 350% d. 50%

50 If prices of all commodities in a place have decreased 35% over the base period prices, then index number of prices of that place now is ,

a. 35 b. 135 c. 65 d. None of these

51 Link relative index number is expressed for the period of 'n' is :

a. $\frac{P_n}{P_{(n+1)}}$ b. $\frac{P_n}{P_{(n-1)}}$ c. $\frac{P_{(n+1)}}{P_{(n-1)}}$ d. None of these

52 Fisher's ideal Price Index

$= \sqrt{\quad}$

53 Fisher's ideal Quantity Index

$= \sqrt{\quad}$

54 Consumer price index for the year 1957 was 313 with 1940 as the base year. The avg. monthly wages in 1957 of the workers in the factory be ₹ 160, their real wages is :

a. ₹ 48.40 b. ₹ 51.12 c. ₹ 40.30 d. None of these

55

Bowley's Index = $\frac{\text{Lasp. Index + Paasche's Index}}{2}$

My Notes

56

Commodity	Base Year		Current Year		
	Price	Quantity	Price	Quantity	
	20	125	22	150	
	28	163	32	170	
	30	128	32	150	
	38	193	42	200	
	42	186	42	193	
	45	176	48	192	
	60	185	56	198	
	70	198	75	210	

Find Lasp. Price Index =

Paasche's Price Index =

Marshall Edgeworth's. Price Index =

Fisher's Ideal Price Index =

Dorbish-Bowley's Price Index =

My Notes

Lasp. Quantity Index =

Paasche's Quantity Index =

Fisher's Quantity Index =

Marshall Edgeworth's. Quantity Index =

Dorbish-Bowley's Quantity Index =

57

Circular test is not met by Laspeyre's and Paasche's index OR Fisher's index.
The simple geometric mean of price relatives and weighted aggregative with fixed weights meets this test.

58

In 1980, the net monthly income of an employee was ₹ 800 p.m. The consumer price index was 160 in 1980. It rises to 200 in 1984. If he has to be rightly compensated the additional amount to be paid to employee is

a. ₹ 175 **b. ₹ 185** **c. ₹ 200** **d. ₹ 125**

My Notes

59

Uses of Index Numbers

a. Framing suitable policies in economics & business.

b. They reveal trends and tendencies.

c. They are used for forecasting the future.

d. They are useful in deflating.

e. Useful to measure changes in cost of living.

60

The purpose determines the type of index numbers to use.

a. True

b. False

My Notes

CLASSY is when you have
a lot to say
but you CHOOSE to remain
SILENT in front of fools

CA VINOD REDDY

People don't care for you,
when you are alone
They just care for you
when they are alone!