Chapter - 1. Ratio & proportion, Indices, logarithms.

Ratio

1.) 3 : 2 3 → Anticedent | 1st term
2 → Consequent | 2nd term

2.) The new Quantity $= \frac{b \text{ of orignal quantity}}{a}$

Eg :- Rishab's weight is 56.7 kg If reduce is weight in the ratio 7 : 6, find his new weight

$a : b = 7 : 6$

$\text{New Quantity} = \frac{6 \times 56.7}{7}$

$= 48.6 \text{ Kg.}$

3.) Inverse Ratio ____ a : b is b : a

see 4.) Compound Ratio (multiply) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

5. Duplicate Ratio $a : b$ is $a^2 : b^2$

6. Sub-Duplicate Ratio $a : b$ is $\sqrt{a} : \sqrt{b}$

7. Triduplicate Ratio $a : b$ is $a^3 : b^3$

8. Sub Triduplicate Ratio $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$

9.) Continuous Ratio $\frac{a}{b} = \frac{b}{c} \rightarrow b^2 = ac$

Proportion

1.) First and fourth terms are called Extremes
Second and third terms are called Means

2.) If $a:b = c:d$ are in proportion then $a/b = c/d$ i.e $ad = bc$ $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

i.e product of Extremes = Product of Means

This is called Cross product rule

3.) $a:b = b:c$ i.e $\frac{a}{b} = \frac{b}{c} = \boxed{b^2 = ac}$

4.) If a, b, c in continuous proportion
$b^2 = ac$ i.e $b = \sqrt{ac}$

5.) Cross Multiplication

If $a:b = c:d$ than $\frac{a}{b} = \frac{c}{d}$

$\therefore ad = bc$

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5. Dividendo

If $a:b = c:d$ then $\frac{a}{b} = \frac{c}{d}$

$$= \frac{a-b}{b} = \frac{c-d}{d}$$

6. Componendo and Dividendo

If $a:b = c:d$ then $\frac{a}{b} = \frac{c}{d}$

$$\therefore \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

7. Addendo

If $a:b = c:d = e:f$

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

$$\frac{a+c+e}{b+d+f} = k$$

8. Subtrahendo

If ~~a~~ $a:b = c:d = e:f$

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \longrightarrow \frac{a-c-e}{b-d-f} = k$$

Law of Indices

1. $a^m \times a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$

3. $(a\,b)^n = a^n \cdot b^n$

4. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

5. $(a^m)^n = a^{m \times n}$

6. $a^0 = 1$

7. $\dfrac{1}{a^{-m}} = a^m$

8. $a^{-m} = \dfrac{1}{a^m}$

9. $a^m = b^m$
   $\therefore a = b$

10. $a^m = a^n$
    $\therefore m = n$

$\boxed{\text{Logarithm}}$ $\longrightarrow$ while solving always try to cut (remove) all log by below formulas

1. ~~$a \log_a n = x$~~

1. $a^x = n$

$\therefore \log_a n = x$

$\log_2 \frac{1}{2^4}$

$\log_2 2^{-4}$

$\boxed{-4}$

2. $\log a^m = m \log a$

3. $\frac{\log a}{\log b} = \log_b a$

4. $\log m \times n = \log m + \log n$

5. $\log \frac{m}{n} = \log m - \log n$

6. $\log_a a = 1$

7. $\log_a 1 = 0$

8. $\log_b a \times \log_a b = 1$

9. $\log_b a = \frac{1}{\log_a b}$

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10. $a^{\log \frac{x}{a}} = x \longrightarrow$ Inverse logarithm property

11. $\log 1 = 0$
$\log 10 = 1$
$\log 100 = 2$
$\log 1000 = 3$
$\log 10000 = 4$

* Formula

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 + b^2 = (a-b)^2 + 2ab$

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$

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Chapter - 2. Equations

1. Linear equation $\longrightarrow ax + bx = 0$
2. Simultaneous equation $\longrightarrow ax + by = c$
3. Quadratic equation $\longrightarrow ax^2 + bx + c = 0$
4. Cubic equation $\longrightarrow ax^3 + bx^2 + cx + d = 0$
   ↳ advice ⇒ 1, −1, 2, −2

1. Area of rectangle $= l \times b$ | Perimeter of rectangle $= 2(l+b)$

2. Saving = Income − Expense
   Expense = Income − Saving.

3.

Quadratic Equation

1. The quadratic equation $ax^2 + bx + c = 0$, $\underline{a \neq 0}$ has roots
$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

1st roots ← $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ OR $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ → 2nd roots

2. Sum of roots $= (\alpha + \beta) = \dfrac{-b}{a}$

3. Product of roots $(\alpha\beta) = \dfrac{c}{a}$

4. $x^2 -$ (sum of roots) $x +$ Product of roots $= 0$

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

$$x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\frac{x^2 + bx + c}{a} = 0 \rightarrow \frac{ax^2}{a} + \frac{bx + c}{a} = 0$$

$$\boxed{ax^2 + bx + c = 0}$$

* Some important expression in terms of sums & products $\longrightarrow$ (convert in $(\alpha+\beta)$ and $(\alpha.\beta)$)

1. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \boxed{\frac{\alpha+\beta}{\alpha\beta}}$

2. $\alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$\boxed{\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta}$

3. $\alpha^3 + \beta^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha+\beta) - 3\alpha\beta(\alpha+\beta$

$\boxed{\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}$

4. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \boxed{\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}}$

5. $\alpha^2 - \beta^2 = (\alpha+\beta)(\alpha-\beta)$

$= (\alpha+\beta)\sqrt{(\alpha-\beta)^2}$

$\boxed{\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}$

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6. $\alpha^3 - \beta^3 = \alpha^3 - \beta^3 - 3\alpha\beta(\alpha-\beta) + 3\alpha\beta(\alpha-\beta)$

$= (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

$$\boxed{\alpha^3 - \beta^3 = \left(\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\right)^3 + 3\alpha\beta\left(\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\right)}$$

* Nature of Roots

1. If $b^2 - 4ac = 0$ the roots are real and equal
2. If $b^2 - 4ac > 0$ the roots are real and unequal (or distinct)
3. If $b^2 - 4ac < 0$ the roots are imaginary
4. If $b^2 - 4ac$ is a perfect square $(\neq 0)$ the roots are real, rational and unequal (Distinct)
5. If $b^2 - 4ac > 0$ but not a perfect square the roots are real, irrational and unequal

① aap x^x, 3^x hai to use substitution.

Chapter:- 4. Time Value of Money.

A Simple Interest

1 Interest Amount (I) = Pit / Pin

2 Amount (A) = $P + I$

$A = P(1 + it)$

3 Interest Amount (I) = $A - P$

Annally = $n \times 1$, $\frac{i}{1}$

Sem-Annlly = $n \times 2$, $\frac{i}{2}$

Quatery = $n \times 4$, $\frac{i}{4}$

Monthly = $n \times 12$, $\frac{i}{12}$

B Compound Interest

1. Annualy = $P(1 + i)^n$

2. C.I = $A - P$

C.I = $P(1+i)^n - P$ → $P\boxed{(1+i)^n - 1}$

C Effective rate of interest

1. $E = (1+i)^n - 1$

D Depreciation

SLM → Same → $A = P(1 - it)$

WDV → Change → $A = P(1 - i)^n$ → कुछ नहीं बोला तो WDV.

* CI > S.I → $P[(1+i)^n - 1 - it]$ = C.I − S.I

E Annuity.

1. Future value of Annuity.

$$A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

A = Periodic payment

2 Present value of Annuity

$$A = \frac{V}{P(n,i)}$$, V = loan amount. $V = A \times P(n,i)$

$$P(n,i) = \frac{1}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right] \Big/ \underbrace{\frac{A}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]}$$

$$V = \frac{a}{i}$$ V =

Golden Rule.

1.) Amount बार - बार deposit kar na hai ya nahi?

Ans:- Hai → Annuity
Nahi → CI / SI

2.) Annuity → बड़ा Amount kab milne wala hai

Ans:- F.V.A → Future बड़ा Amount milega
P.V.A → अभी बड़ा Amount milega.

Chapter - 6. Sequence & Series. (AP & GP)

* Arithematic progression (A.P).

1.] A. The Sequence in the form $a, a+d, a+2d, \ldots$
Where a = first term (t_1) and d = Common difference is Called an Arthematic progression (AP).

2. n^{th} term $= t_n = a + (n-1)d$

3. Common difference $(d) = 2^{nd}$ Term $- 1^{st}$ term
$= T_n - T_{n-1}$

4. A.M $\rightarrow b = \dfrac{a+c}{2}$, $b - a = c - b$

5. 3 term are in A.P $= a-d, a, a+d$
4 term are in A.P $= a-3d, a-d, a+d, a+3d$
5 term are in A.P $= a-2d, a-d, a, a+d, a+2d$

6. Sum upto n^{th} term

i.) When Common difference given

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

ii.) When Last term is given

$$S_n = \frac{n}{2}\left[a + l\right]$$

iii.) Sum of first "n" odd number $= n^2$

iv.) Sum of first "n" even number $= n(n+1)$

v.) Sum of first "n" natural number $= \frac{n(n+1)}{2}$

$$\sum_{r=1}^{n} r$$

vi.) Sum of cubes of first "n" natural number $= \left[\frac{n(n+1)}{2}\right]^2$

$$\sum_{r=1}^{n} r^3$$

vii.) Sum of the square of the first "n" natural number $= \frac{n(n+1)(2n+1)}{6}$

$$\sum_{r=1}^{n} r^2$$

7. n^{th} term $= t_n = S_n - S_{n-1}$

8. $\sum_{p=1}^{n} 1 = n$

$$\sum_{r=1}^{n} r$$

n → ending point

r = 1 → starting point

r/i

Geometic progession (G.P)

1. The squence of the type $a, ar, ar^2, ar^3, \ldots\ldots$ where a = First term / t_1 and r = Common ratio is called geometic progession (G.P).

2. n^{th} term $= t_n = a(r)^{n-1}$

3. Common Ratio $(r) = \dfrac{2^{nd} \text{ term}}{1^{st} \text{ term}} = \dfrac{T_n}{T_{n-1}}$

4. 3^{rd} term in G.P $\Rightarrow \frac{a}{r}, a, ar$
   4^{th} term in G.P $\Rightarrow \frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
   5^{th} term in G.P $\Rightarrow \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

5. G.P $\neq$ G.M $= b^2 = ac \ldots\ldots \dfrac{b}{a} = \dfrac{c}{b}$

6. Sum upto n^{th} term of G.P

$$S_n = \frac{a(1-r^n)}{(1-r)}, \text{ where } \underline{r<1}$$

$$S_n = \frac{a(r^n-1)}{(r-1)}, \text{ where } \underline{r>1}$$

$$S_n = a \times n, \text{ where } \underline{r=1}$$

7. Sum upto infinit terms

$$S_\infty = \frac{a}{1-r}$$

Chapter :- 5. Basic Concepts of Permutations and Combination

(13) Permutations → Arrangement

A.] Factorial Notation

$$n! = \underline{|n} = 1 \times 2 \times 3 \times \ldots\ldots \times n$$
$$= (n)(n-1)(n-2)(n-3)\ldots\ldots 3 \times 2 \times 1$$
$$= n(n-1)!$$

B.] Note that

$$1! = 1 \ , \ 0! = 1$$

C.] $(n+1)! - n! = n \times n!$ → See

D.] AND = Multiplication
OR = Addition

E.] $^nP_r = \frac{n!}{(n-r)!}$

Always $n \geq r$
Rearrange = Arrange-1

F.] $^nP_n = n!$ | $^nP_{n-1} = n!$

G.] $^nP_1 = n$

H.] Circular permutations [for Same neigbours]

i) $(n-1)!$

ii.) $\frac{1}{2}(n-1)!$ ⟶ Different neigbour.

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I.] $(n-1)!\,(n-2) \longrightarrow$ two perticulars articular never come together.

J. $^{n-1}P_r \longrightarrow$ Perticulars not taken in any arrangement.

K $r.\,^{n-1}P_{r-1} \longrightarrow$ Perticulars is always taken in any arrangement.

L. $\boxed{\sum_{r=1}^{n} r.\,^{r}P_r} = \underline{\underline{^{n+1}P_{r+1}-1}}$

where Σ = Total

$^nP_r = \underbrace{^{n-1}P_r}_{\hookrightarrow J} + \underbrace{r.\,^{n-1}P_{r-1}}_{+ \quad \hookrightarrow K}$

⑥ Combination [Selection]

A.] $^nC_r = \frac{n!}{r!(n-r)!}$

$^nC_r \times r! = {^nP_r}$

B. Complimantary Combination

$\boxed{^nC_r = {^nC_{n-r}}}$ $\quad n = r + (n-r) \qquad r = n-r$

same

$\boxed{^nC_x = {^nC_y}}$ $\quad n = x + y \qquad x = y$

C. $^nC_r + {^nC_{r-1}} = {^{n+1}C_r} \longrightarrow$ Pascal's law

D. $^nC_{n-1} = n \longrightarrow$ eg:- $^{25}C_{24} = 25$

$^nC_1 = n$

$^nC_n = 1$

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E] Combination in Geometry.

i.) No. of Straight line $= {}^nC_2$ $n =$ No. of side / given Points

ii.) No. of triangle $= {}^nC_3$

iii.) No. of diagonals $= {}^nC_2 - n$

iv.) No. of Straight line $= {}^nC_2 - {}^mC_2 + 1$

v.) No. of triangles $= {}^nC_3 - {}^mC_3$

n = points, m = points are Co-linear

G Division into groups.

i) $\dfrac{\lfloor m+n+p}{\lfloor m \cdot \lfloor n \cdot \lfloor p}$ $\longrightarrow$ divided into 3 groups G

ii.) $\dfrac{1}{3!} \times \dfrac{\lfloor 3m}{(m!)^3}$ $\longrightarrow$ 3 group are equal G+E

$m = n = p$

$m = \dfrac{\text{Total}}{3}$

iii.) $\dfrac{\lfloor 3m}{(m!)^3}$ $\longrightarrow$ Divided into 3 persons equally.

$m = \dfrac{\text{Total}}{3}$ P+E

⑦ Standard Result in Permutations

1. ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

   ↳ I + ↳ K → permutation.

2. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ → Pascal's law

3. $P = \dfrac{n!}{n_1!\ n_2!\ n_3! \ldots}$ $\longrightarrow$ Things are exactly alike of one / two third kind.

   ↳ Ex 5 (B) ⇒ 1

No. Total letter!

Same letter! Same letter! ...

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Remeber r is...

4. $\underline{\underline{n^r}}$ ..... $r > n$ $\longrightarrow$ thing repated r times

5. ${}^nC_r = 2^n - 1$ $\longrightarrow$ $\{ {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 \mid 2^5 - 1 \}$

at least 1 — If 0 than ~~than~~ not -1 only

6. $\{(n_1+1)(n_2+1)(n_3+1)\ldots\ldots\} - 1$

$\longrightarrow S(C) \Rightarrow 12, 18$

~~7.~~

7. ${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$ $\longrightarrow$ r_2 & r_1 things are indepen

i) ${}^nC_n = 1$	i) ${}^nP_n = n!$
ii) ${}^nC_{n-1} = n$	ii) ${}^nP_{n-1} = n!$
iii) ${}^nC_1 = n$	iii) ${}^nP_1 = n$

16 Probability Distribution.

1. $P(A) = \frac{n(A)}{n(S)}$

2. $0 \leq P(A) \leq 1$

3. $P(A) = 0 \longrightarrow$ A is impossible event
$P(A) = 1 \longrightarrow$ A is sure event.

4. $P(A) = 1 - P(A')$

5. Some experiments and Sample Space are

i.) A Coin is tossed than $= S = \{H, T\}$ / $2^1 = 2$
ii.) 2 Coin is tossed than $S = \{HH, HT, TH, TT\}$
iii.) 3 Coin are tossed than $S = \{HHH, HHT, HTT, HTH, THH, THT, TTH, TTT\}$
$= 2^3 = 8$

No. of Possibility upon No. of Time $\leftarrow 2^{3}$ — 3 — No. of Time; 2 — No. of possibility

iv.) 6 faced die is thrown $= S = \{1, 2, 3, 4, 5, 6\}$ | 6^1

v.) 2 dice is thrown $= \{(1,1)\ (1,2) \ldots\ldots (6,5)\ (6,6)\}$
$= 6^2 = \boxed{36}$

vi.) 52 Cards → 26 Red, 26 Black
26 Red → 13 ♡ Heart, 13 ◇ Diamond
26 Black → 13 ♤ Spade, 13 ♧ Club

i) 13 Cards = 1 suit = {A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K}
ii.) Ace Card = 1
iii.) Picture Card = Court Card = 12 cards
4K, 4Q, 4J

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6. Type of Event

i) Certain Event | Sure Event | Sample Space

$$A = S$$
$$P(A) = P(S) = 1$$

ii. Impossible Event

$$\{\ \} = 0 \mid P(\phi) = 0$$

iii. Complementary Event

$$A' = \bar{A} = A^c = \text{Complementary event of } A$$

$$P(A) + P(A') = 1$$
$$P(A') = 1 - P(A)$$
$$P(\text{At least one}) = 1 - P(\text{none})$$

iv. Mutually exclusive event [Disjoint event]

$$A \cap B = \{\ \} = \phi$$
$$P(A \cap B) = 0$$
$$n(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

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v.) Non - Mutually exclusive event [Joint event]

$$P(A \cap B) \neq 0$$

$\boxed{or = \cup = +}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

vi) $(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$ } De-Morgans law

vii) $P(A) = P(A \cap B') + P(A \cap B)$
$P(B) = P(A' \cap B) + P(A \cap B)$

A - B = All elements of A but not of B
B - A = All elements of B but not of A.

viii) Exhaustive Event

(1) $S = \{1,2,3,4,5\}$, $A = \{1,2,3\}$, $B = \{4,5,6\}$
$A \cup B = \{1,2,3,4,5,6\}$
$\boxed{A \cup B = S}$
$\therefore$ A & B are exchaustive Event

(2) $S = \{1,2,3,4,5\}$, $A = \{1,2,3,4\}$ $B = \{4,5\}$
$A \cup B = \{1,2,3,4,5\}$
$\boxed{A \cup B = S}$
$\therefore$ A & B is exchaustive Event

(3) $A \cap A' = \phi$ = Mutually exclusive event
$\boxed{A \cap A' = S}$ = Exhaustive event.

$\therefore$ A & A' are Mutualy exclusive as well as Exhaustive events

and = $\cap$ = $\times$
~~or~~ = $\cup$ = $+$

$$* \quad P(A \cap B) = P(A \times B) = P(A \text{ and } B)$$
$$P(A \cup B) = P(A + B) = P(A \text{ or } B)$$

7. Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

A is dependent on B

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0$$

B is dependent on A

For dependent events

8. (i) Multiplication theorem for dependent events

Same

$$P(A \cap B) = P(A|B) \times P(B)$$
$$P(A \cap B) = P(B|A) \times P(A)$$

(ii) Multiplication theorem for Independent events

and = $\cap$ = $\times$ → $P(A \cap B) = P(A) \times P(B)$

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

(iii) If A & B are independent event than A & B', A' & B, A' & B' are also Independent event.

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9. Expected probability.

A Random variable 'x' assume the set values $x_1, x_2, x_3, \ldots\ldots x_n$ with respective probability.

$P_1, P_2, P_3, \ldots\ldots P_n$ then the escpected value of x.

1) $E(x) = \Sigma Px \; (P_1x_1 + P_2x_2 + P_3x_3 + \ldots\ldots P_nx_n)$

∴ Expected value of x is also Called Mean

→ Positive | Negative | zero.

2) $E(x^2) = \Sigma Px^2 \; \{ P_1x_1^2 + P_2x_2^2 + P_3x_3^3 + \ldots P_nx_n^2 \}$

3) $\bar{x} = \mu = E(x) = \text{Mean} \quad (\mu = M_e)$

4) $V(x) = \sigma x^2 = E(x-\mu)^2 \; / \; \boxed{E(x^2)} - \boxed{M^2}$

$\Sigma px^2 - (\Sigma Px)^2$

5) $SD = \sigma x = \pm\sqrt{V(x)}$

$\rightarrow \sqrt{\Sigma Px^2 - (\Sigma Px)^2}$

If $y = a + bx$ for 2 variable x & y and Constant A & B, then

$Ey = a + b\,Ex \longrightarrow \mu_y = a + b\mu_x$

- $\sigma y = |b| \cdot \sigma x$
- $V_y = b^2 \cdot Vx$

$\Sigma P = 1 \leftarrow$ Always.

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10. Property of expected probability.

1. $E(\text{Constant}) = \text{Constant}$, $E(K) = K$
2. $E(Kx) = K\,[E(x)]$
3. $E(x \pm y) = E(x) \pm E(y)$
4. $E(x - \bar{x}) = 0$

For x and y independent
$$E(xy) = E(x) \times E(y)$$

Theory + Sums

11.) $P(\text{odds in favour}) = \dfrac{x}{x+y}$

12. $P(\text{odds in against}) = \dfrac{y}{x+y}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A) = 1 - P(A')$$
$$P(A) = P(A \cap B') + P(A \cap B)$$

$$P(S) = P(A \cup B') + P(A' \cap B)$$

Chapter:- 17. Theoretical Distributions

* Binomial Distribution → Discrete distribution

1. $f(x) = P(X = x) = {}^nC_x \cdot p^x \cdot q^{n-x}$ for $x = 0, 1, 2$

$p + q = 1$ $\quad$ p = Success, q = Failer

Here mean, mode, median are not equal.

2. Mean of binominal distribution $= \mu = np$

3. $S.D = \sqrt{npq}$ or $V = npq$

4. $npq < np$
   Variance < Mean

5. mode = Larger interger contain in $\boxed{(n+1)p}$ → Never in points 10.9 than 10, 10.2 than also 10

Here 2 parameter are n & p $\therefore \mu = n - p$
Mean ≠ Median ≠ Mode

* Poission Distribution

1. $f(x) = P(X = x) = \dfrac{e^{-m} \cdot m^x}{x!}$ for $x = 0, 1, 2 \ldots n$

$e = 2.7183$

Poission distribution is knowns as <u>uniparametric distribution</u> as it is characterised by only one parameter m

Here 1 parameter is m
$\therefore \mu = m$

Sib x or mean diya hoga
tu poission distribution

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$E(x) = \mu = $ Mean

ii) Mean of poission distribution is m
i.e $\mu = m$

iii) Variance of passion distribution is given by
$\sigma^2 = m \mid \sigma = \sqrt{m}$
$\therefore V = m$

→ never in points 10.9 than 10, 10.2 than also 10

iv) Mode = Largest integer contained in m

Mode = Mean = Median

* Normal Distribution → Normal Distribution
→ Here $\mu = 0.5$, $\sigma = 0.5$

i.) $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(\bar{x}-\mu)^2/2\sigma^2}$$

ii) Mean of Normal Distribution is μ
i.e $\mu = m$

iii) Mean deviation of normal distribution is $\sigma\frac{\sqrt{2}}{\pi}$
$\therefore \sigma\frac{\sqrt{2}}{\pi} \cong 0.8\sigma$

→ $\frac{\sqrt{2}}{2.7} = 0.8$

iv) $Q_1 = \mu - 0.675\sigma$

v.) $Q_3 = \mu + 0.675\sigma$

So that, quartile deviation $= 0.675\sigma$

Mean = Median = Mode

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vi.) The mean, middian and mode of a normal distribution are $\boxed{\text{equal}}$

..o mai se Q attemp mai aaya hai

vii.) $S.D = 1$

$M.D = 0.8\sigma$

$Q.D = 0.675\sigma$

vi) Point of inflexcation of normal curve $= \mu - \sigma$ & $\mu + \sigma$

$S.D = \sqrt{\sigma_1^2 + \sigma_2^2}$ → eg 17.33
eg 17.32
17(B) = 24

Mean $= \mu_1 + \mu_2$

P (At least one) = 1 − P (none)

Maximum of Variance $= \frac{n}{4}$

Atmost 3 then $\boxed{P(x=0)} + P(x=1) + P(x=2) + P(x=3$

don't forget

Chapter - 15. Measures of Central Tendency and Dispersion

I Arithematic Mean

1 $A.M = \bar{x} = \frac{\Sigma x}{n}$ ⟶ Without frequency

2 $A.M = \bar{x} = \frac{\Sigma fx}{N} = \frac{\Sigma fx}{\Sigma f}$ ⟶ With frequency.

3. $A.M = \bar{x} = A + \left(\frac{\Sigma fd}{N} \times C\right)$ ⟶ With frequency (Group data)

A = Assumed Mean $C = l_2 - l_1$ $d = \frac{x-A}{c}$

↳ Midvalue

If all the observation are say k (i.e constant) then arthematic mean is also k.

If $y = a + bx$ then A.M of y is given $\bar{y} = a + b\bar{x}$

Combined $AM = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

The Algebaric sum of deviation of a set of observation for their Arthematic mean is Zero.

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2 II Median → ~~Most~~ I.M.P

Step :1 Arrange the observation in assending order

1) Odd no. of observation = Middle Most observation → Without frequency

OR

even no. of observation = Average of two middle Most observation → Without frequency

2. Median = $\left(\frac{N+1}{2}\right)^{th}$ observation → With frequency

3. Median = $l_1 + \left(\frac{\frac{N}{2} - N_L}{N_U - N_L} \times c\right)$ → With frequency (Group data)

4. If $y = a + bx$ then Median of $y_{me} = a + bx_{me}$

Note:- $\underbrace{\text{Median, Quartile, Deciles, Precentile,}}$

Arrange in assending order observation.

-- + (0.-) (Next term - old term)

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3. Quartile

Step :- 1 Arrange the observation in assending order

Without frequency

$$Q_1 = 1\left(\frac{n+1}{4}\right)^{th} \text{observation}$$

With frequency

$$\text{Quartile} = l_1 + \left(\frac{NP - N_L}{N_U - N_L} \times C\right)$$

$Q_1 = \frac{N}{4}$

$Q_2 = 2\frac{N}{4}$

⋮

$Q_4 = 4\frac{N}{4}$

4 Deciles

Step - 1 Arrange the observation in assending order.

Without frequency

$$D_1 = 1\left(\frac{n+1}{10}\right)^{th} \text{observation}$$

With frequency

cf

$$D = l_1 + \left(\frac{NP - N_L}{N_U - N_L} \times C\right)$$

$D_1 = \frac{N}{10}$

$D_2 = 2\frac{N}{10}$

⋮

$D_{10} = 10\frac{N}{10}$

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5 Precentile

Setep :- 1 Arrange the observation in Assending

Without frequency

$$P_1 = 1\left(\frac{n+1}{100}\right)^{th} \text{ observation}$$

With frequency

$$P = l_1 + \left(\frac{NP - N_L}{N_U - N_L} \times C\right) \qquad P_i = \frac{N}{100}$$

$P_{100} = 100$

VI Mode

1 Observation which is repeated Maximum number of time is called Mode — Without frequency

2. Maximum frequency is associated is called Mode

3. $$\text{Mode} = l_1 + \left[\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1}\right] \times C$$

f_0 = Highest frequency. → ~~C~~

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4. If $y = a + bx$ then mode of y i.e
i.e $y_{Mo} = a + b x_{Mo}$

Mean − Mode = 3 (Mean − Median)

VII] Geometric Mean

1 $GM = (x_1 \times x_2 \times x_3 \times, \ldots\ldots x_n)^{\frac{1}{n}}$
→ Without frequency

2. $G.M = (x_1^{F_1} \times x_2^{F_2} \times x_3^{f_3} \times, \ldots\ldots x_n^{f_n})^{\frac{1}{N}}$
→ With frequency

3. G.M of xy = G.M of x × G.M of y

4. G.M of $\frac{x}{y}$ = $\frac{\text{G.M of } x}{\text{G.M of } y}$

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8 VIII] Harmonic Mean

Reciprocal of A.M

1.) $H.M = \dfrac{n}{\Sigma\left(\frac{1}{x}\right)}$ → Without frequency

2.) $H.M = \dfrac{N}{\Sigma\left(\frac{f_i}{x_i}\right)}$ → With frequency

3 $H.M = \dfrac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$ → Combained H.M

*) $A.M \geq G.M \geq H.M$

*) $(G.M)^2 = A.M \times H.M$

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Dispersion

Range.

1. Range = L − S L = Largest observation
 S = Smallest observation

2. Co-efficiate of Range $= \dfrac{L-S}{L+S} \times 100$

3. Range = upper class boundary − lower class boundary → continuous class

group data

= Difference between two extreme class boundary.

4. If $y = a + bx$ than $R_y = |b| \times R_x$

Expected value to find VI chapter-15

5. If all observation are constant say K then range is 0/zero. Range is also 0/zero for single observation.

Dispersion depends on scale (b) not on origin(a)

It is base on two observations, Hence it is not idel

It is quick to find, Hence use in quality control.

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Standard deviation

1. $S.D = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} \Big/ \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}} \rightarrow \bar{x} = \frac{\Sigma x}{n}$

2. $S.D = \sqrt{\frac{\Sigma f x^2}{N} - (\bar{x})^2} \Big/ \sqrt{\frac{\Sigma f (x-\bar{x})^2}{N}} \longrightarrow \bar{x} = \frac{\Sigma f x}{N}$

3. $S.D = \left(\sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}\right) \times C$

4. Variance $= (S.D)^2$
 $\sqrt{\text{variance}} = S.D$

5. Co-efficient of variation $= \frac{S.D}{A.M} \times 100$

6. S.D of first 'n' natural numbers

$$S.D = \sqrt{\frac{n^2 - 1}{12}}$$

7. If $y = a + bx$ then $S_y = |b| \times S_x$

8. For two numbers $S.D = \frac{1}{2}|a-b|$

S.D of $\frac{x-a}{b}$
$\therefore y = \frac{x-a}{b}$

variance $5-2x$
$\Downarrow$
$y = 5-2x$

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9. Combined S.D $= \sqrt{\dfrac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$

Where $d_1 = \bar{x}_1 - \bar{x}_c$, $d_2 = \bar{x}_2 - \bar{x}_c$

$$\bar{x}_c = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$$

Properties of S.D

1 If all observations are constant say K then S.D is $\boxed{\text{Zero}}$.

2. For single observation say K , then S.D is $\boxed{\text{Zero}}$

Add or less in all obsn S.D will remain unchange.

Mean deviation of Σ (Summation)

No

1. $M.D = \dfrac{\Sigma |x - A|}{n}$ ⟶ Without frequency

2. $M.D = \dfrac{\Sigma f |x - A|}{N}$ ⟶ With frequency

$A = \bar{x}$
A = Median
A = Mode

Simple no X100 // Co-efficient mai (— X 100) Always hoga.

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3. Co-efficient of M.D $= \frac{\text{M.D about A}}{A} \times 100$

4. If $y = a + bx$ then $M.D_y = |b| \times MD_x$

Properties of M.D

i) If all observation are constant say K than M.D is $\boxed{\text{zero}}$

ii) For ~~sig~~ single observation say K than M.D is $\boxed{\text{zer}}$

Quatile deviation

1.) Semi-inter quartile / or quartile deviation $= Qd = \frac{Q_3 - Q_1}{2}$

2. Co-efficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

$\frac{Qd}{\text{Median}} \times 100$

3. Inter quartile Range $= Q_3 - Q_1$

4. If $y = a + bx$ then $Qd_y = |b| \times Qd_x$

Chapter - 18. Correlation & Regression

I Karl Pearson's product moment Correlation co-efficient

1. $r = r_{xy} = \dfrac{Cov(x,y)}{S_x \times S_y}$

$$Cov(x,y) = \frac{\Sigma xy}{n} - \bar{x}\bar{y} \Bigg/ \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$$

$$S_x = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \Bigg/ \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$S_y = \sqrt{\frac{\Sigma y^2}{n} - \bar{y}^2} \Bigg/ \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$$r = \frac{n\Sigma xy - \Sigma x \times \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

Properties of Karl Pearson's Product moment Correlation co-efficient

$\dfrac{r \times b}{|b|}$

1. $-1 \leq r \leq 1$

2. $u = \dfrac{x-a}{b}$, $v = \dfrac{y-c}{d}$

'b' or 'd' ka sign alag hai tu rxy and ruv ka sign bhi alag hoga

$$r_{xy} = \frac{bd}{|b||d|} \times r_{uv}$$

rxy and ruv being the co-efficient of co-relation x and y and u and v respectively, the two co-relation coefficient remain equal and they would have oppsite sign only when 'b' and 'd' the two scale differ in sign

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Co-efficient of Correlation = r
Co-relation of Co-fficient = r

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III] Spearman's Rank Correlation Coefficient

1. Without tie

$$r_R = 1 - \frac{6[\Sigma d_i^2]}{n(n^2-1)} \quad \text{where, } d = x - y$$

2. With tie

$$r_R = 1 - \frac{6\left[\Sigma d^2 + \frac{\Sigma (t_j^3 - t_j)}{12}\right]}{n(n^2-1)}$$

III] Co-efficient of ConCurrent Deviation

1. $r_c = \pm \sqrt{\pm \frac{(2C-M)}{M}}$

$(2C-M) > 0 \Rightarrow$ positive
$(2C-M) < 0 \Rightarrow$ Negative

Where, C = No. of ConCurrent deviation
No. of positive sign

M = No. of pair of deviation

Note :- If $(2C-M) > 0$, then we take the positive sign both inside and outside the radical sign and If $(2C-M) < 0$ we take negative sign in both inside and outside the radical sign

~~2C~~ : 2C−M is positive than Answer is also positive and vice versa.

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2. $n = m + 1$ = Total observation
$m = n - 1$

IV) Regression line

$x \to$ given
$y = ?$

$\boxed{y \text{ on } x}$

1.) $y = a + bx$

2.) $b_{yx} = \frac{Cov(x,y)}{\sigma x^2}$

$Cov(x.y) = r \cdot \sigma x \cdot \sigma y$

$\therefore b_{yx} = \frac{r \sigma y}{\sigma x}$

3) $\bar{y} = a + b\bar{x}$

$\boxed{a = \cdot}$

y = given
$x = ?$

$\boxed{x \text{ on } y}$

1.) $x = a + by$

2.) $b_{xy} = \frac{Cov(x,y)}{\sigma y^2}$

$Cov(x.y) = r \cdot \sigma x \cdot \sigma y$

$\therefore b_{xy} = \frac{r \sigma x}{\sigma y}$

3.) $\bar{x} = a + b\bar{y}$

$\boxed{a = }$

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b

Regression Co efficient of x on y → bxy
Regression Coefficient of y on x → byx
Regression lines of y on x → $y = a + bx$
Regression lines of x on y → $x = a + by$

Properties of Regression line | Coefficient

1.) $-1 \leq r \leq 1$

2. $U = \frac{x - a}{p}$, $V = \frac{y - c}{q}$

$byx = \frac{q}{p} \times bvu$, $bxy = \frac{p}{q} \times buv$

3. $r = \pm\sqrt{byx \times bxy}$ | $r^2 = byx \times bxy$

Note:- If both regression coefficient (byx | bxy) are positive than r is positive and vice vesa

4. $\frac{byx + bxy}{2} > r$

5. $r = 0$ then Regression Co-efficient is also zero. bxy and byx = 0

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v) Probable Error & Standard Error

1 $SE ~~PE~~ = \frac{1-r^2}{\sqrt{n}}$

2 $PE = \frac{2}{3} \times \left(\frac{1-r^2}{\sqrt{n}}\right)$

0.66/0.67

Accounted → Co-efficient of determination $= r^2$

unAccounted → Co-efficient of non-determination $= 1-r^2$

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Chapter : 19. Index Numbers.

Reuben → $\frac{P_1 = P_0}{P_0 = P_0}$

Price Ind Number

- Weighted
  - Simple Aggergative
  - Average price Relative → find C.L.I
- Unweights
  - Simple Aggerative
  - Average price Relative

Unweighted:

Simple Aggerative: $P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$

Average price Relative: $P_{01} = \frac{1}{N}\left[\left(\Sigma \frac{P_1}{P_0}\right) \times 100\right]$ — Arithmetic Mean of P.R.

Weighted — Simple Aggergative:

1] Laspeyres

$L = P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$ — $\times 100$ mat bulna

— Base are q_0

2] Paasche's

$P = P_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100$

— Base are q_1

3] Fisher's ideal Index

$I = \sqrt{L \times P}$ → do not forget $\sqrt{\ }$ at the time of solving

$I = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}$

I = G.M of L & P

Weighted — Average price Relative:

1)

$P_{01} = \frac{\Sigma P w}{\Sigma w} \times 100$

where, $P = \frac{P_1}{P_0}$

$$\boxed{P_{01} = \frac{\Sigma\left(\frac{P_1}{P_0}\right) w}{\Sigma w} \times 100}$$

2) Group Index

Cost of living (C.I.L) $= \frac{\Sigma I W}{\Sigma W}$

$IW = W \times$ Index number

Where, $I = \frac{P_1}{P_0} \times 100$

$$\boxed{\text{Cost of living (C.I.L)} = \frac{\Sigma\left(\frac{P_1}{P_0} \times 100\right) \times W}{\Sigma W}}$$

→ See 20 Sum in B.K

5 Marshal edgeworth Index $= \left(\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} + \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right) \times 100$ — (L + P) $\times 100$

6 Dorbist bowley Index $= \frac{1}{2}(L + P)$

$=$ A.M of L & P

7 Price Index number $= \frac{\Sigma P_1}{\Sigma P_0} \times 100$ — $\times 100$ mat bulna

8 Quantity Index number $= \frac{\Sigma q_1}{\Sigma q_0} \times 100$

9 Value Index number $= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} \times 100$

10 $C.L.I = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$ → laspeyres

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Quantity Index Number

q_{01} are obtain form those of laspeyre's, paasChi's and Fisher's by interChanging P and q here suffix are not Charge.

1] Laspeyre's quantity Index number

$$= \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100$$

2] Paasche's ~~index~~ quantity Index number

$$= \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1} \times 100$$

3] Fisher's quantity Index number

$$= \sqrt{L \times P}$$

4] Real wages $= \frac{\text{Actual wages}}{\text{CLI}} \times 100$

general Index number is also known as CLI

$$CLI = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100 \quad \text{--- (1) ---}$$

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* There are $\boxed{\text{four}}$ test in Index number

1. Unit Test = Formula should be independent.

This test is satisfied by all formula except simple unweighted
SUW

2. Time Reversal Test

$$\boxed{P_{01} \times P_{10} = 1}$$

Where, P_{01} = Index for time 1 on Zero
P_{10} = Index fore time Zero on 1

(1) Fisher's ideal Index, (2) simple Aggergative, (3) simple gometric price mean of relative, (4) Marshal edge worth (L+P)/100 index satisfy time Reversal test.
F, SA, SG, ME.

Except :- Laspery's & Passarys don't pass time Reversal test.
L & P

3. Factoral Resersal Test

$$\boxed{P_{01} \times Q_{01} = V_{01}} \quad \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} = \text{Value Index Number}$$

This test is passed by (1) fisher's Index number and also by (2) simple aggergated method
F & SA Expect L & P

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4 Circular Test

$$I_{01} \times I_{12} \times I_{23} \times, \ldots \times I_{(n-1)n} \times I_{n \times 0}$$

This test is passed by - (1) Simple geometry mean of a price relative.
and
(2) Simple Aggrerative index and (3) weight aggerated formula with fixed weight.

SG, SA, WA

5 Simple geometric Mean of price relative

$$I_{01} = \sqrt[n]{\left(\frac{P_1}{P_0} \times 100\right)\left(\frac{P_1}{P_0} \times 100\right) \times \ldots \ldots \text{(n times)}}$$

6 Shipted price Index = $\dfrac{\text{Orignal Price index}}{\text{Price index of year on which its has to be Shipted}}$

Shipted year Ka Price Index

7 Chain Index = $\dfrac{\text{Link relative of Current Year} \times \text{Chain Index of Previous Year}}{100}$

See Crash Book

classmate
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7. Sets, Functions & Relations.

A] i.) Natural number set

$N = \{1, 2, 3, \ldots\ldots\}$

ii.) Whole number set

$W = \{0, 1, 2, 3, \ldots\ldots\}$

iii.) Integer set

$I = \{\ldots\ldots, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$

iv.) Rational number set

$= \left\{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\right\}$

$Z = \left\{\frac{3}{5}, -5, \frac{-5}{-8}, \frac{0}{9}, \cancel{\frac{9}{0}}\right\}$

v.) Prime number set

$P = \{2, 3, 5, 7, \ldots\}$

vi.) even number set

$E = \{2, 4, 6, 8, \ldots\ldots\}$

vii.) odd number set

$O = \{1, 3, 5, 7, \ldots\ldots\}$

viii.) Null set / Empty set $= \{\ \} / \phi$ - Phi

ix.) Real set = {All rational $\cup$ All irreational}

Real number — Union

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \log_2 3, 3+\sqrt{2}, 4\pi, 6\sqrt{3}$ etc.

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1 Marks
CA Exam

B.] i) If every element of set 'P' is ~~alwa~~ also an element of set 'Q'. we say that P is Subset of Q

Eg:- P = {1, 2}, Q = {1, 2, 3}

Here, P ⊂ Q or P ⊆ Q → P is subset of P
Here Q is Superset of P.

ii) If P is a subset of Q but P is not equal to Q then P is called proper subset of Q.

Eg:- P = {1, 2}, Q = {1, 2, 3}

Here, P is proper subset of Q

iii) ∅ has no proper subset

iv) A set containing n elements has 2^n subsets.

Eg:- P = {1, 2, 3) $2^n = 2^3 = 8$ | {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3}, { }

v) A set containing n elements has $2^n - 1$ proper subset, How the set {a, b, c} has

$$\{ \overbrace{\underbrace{\phi, a, b, c, ab, bc, ac}_{\text{proper subset}}, abc}^{\text{Subset}} \}$$

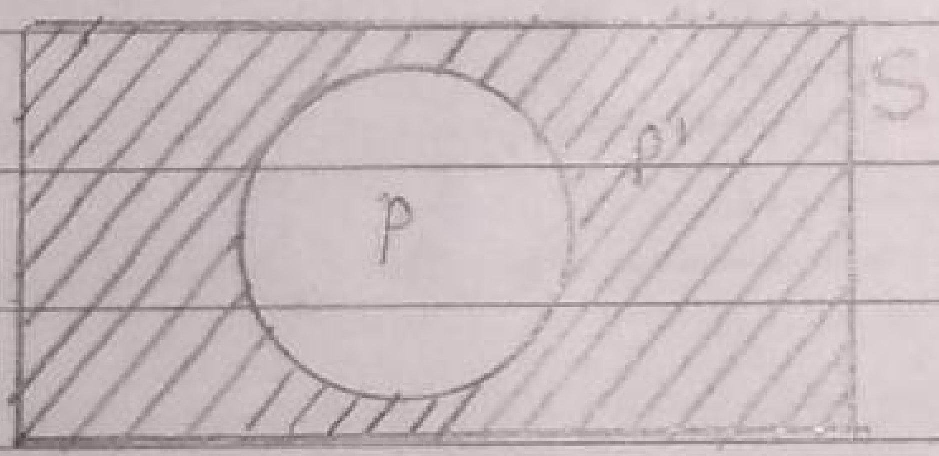
vi.) $A \cap B = \phi \Rightarrow$ A and B are disjoint set

$A \cap B \neq \phi \Rightarrow$ A and B are intersecting or overlapping set.

vii.) $\{\ \} \longrightarrow$ Empty set $= \phi =$ phi

viii.) $\{0\} \longrightarrow$ Non-empty set

ix.)



S = universal set

$P \subset S =$

$P' = P^c = S - P =$ = Complement set of P.

$P \cup P' = S$

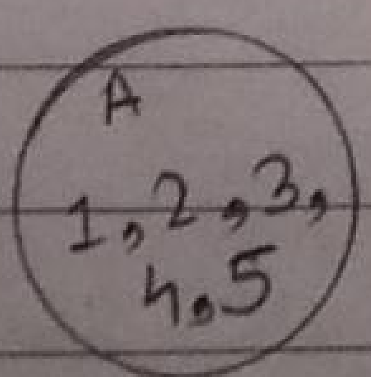
$P \cap P' = \phi = \{\ \}$

x.) De-morgan's law

1.) $(P \cap Q)' = P' \cup Q'$

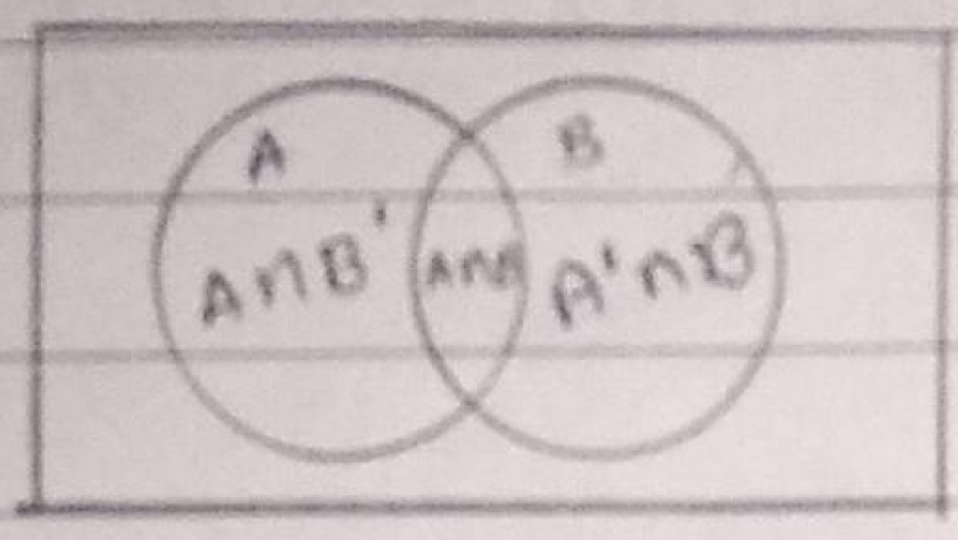
2.) $(P \cup Q)' = P' \cap Q'$

xi.)



$\longrightarrow n(A) = 5$

(.) xi.) Important diagram



$A = (A \cap B') \cup (A \cap B)$

$B = (A' \cap B) \cup (A \cap B)$

$A \cup B = (A \cap B') \cup (A \cap B) \cup (A' \cap$

B. Domain and Range of a function

$F : A \longrightarrow B$

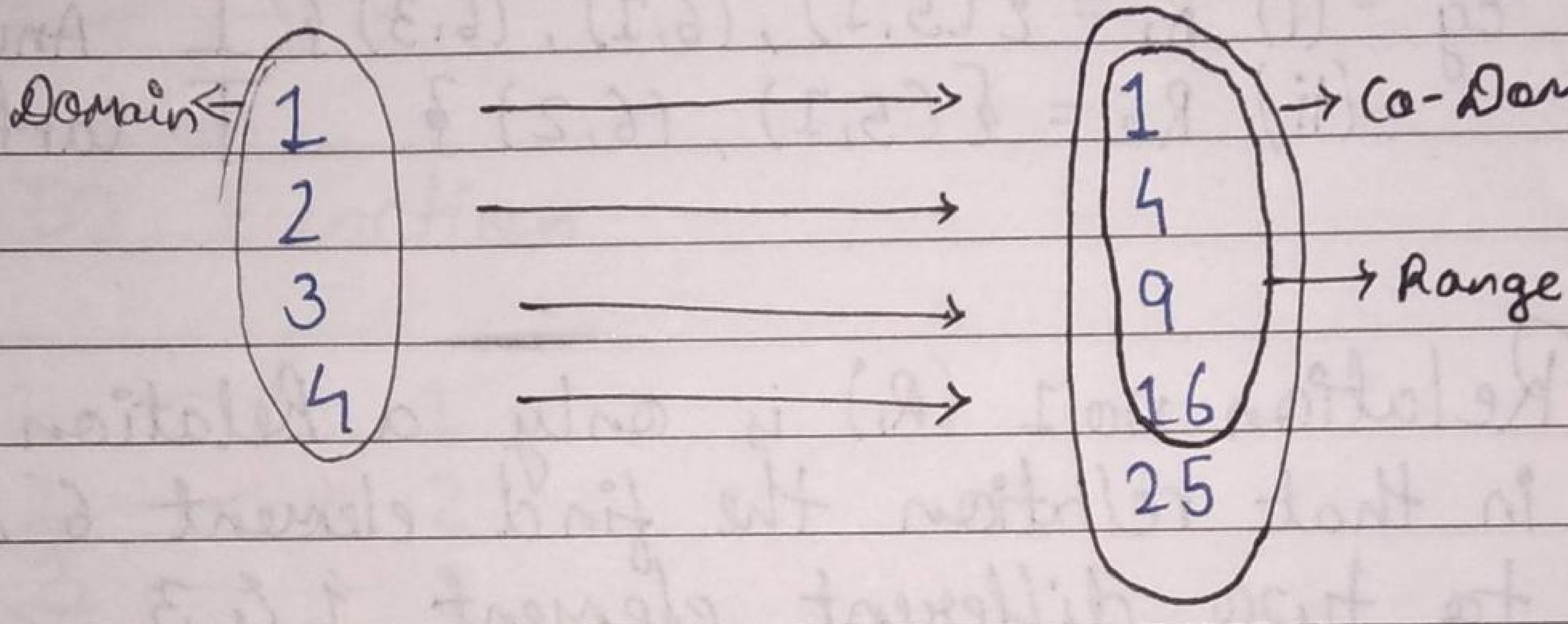
Here,

A = Domain of A

$A = f(x)$

B = Co - Domain of B

$B = f(x^2)$



$y = f(x) = x^2$

$f(1) = (1)^2 = 1$

$f(2) = (2)^2 = 4$

$f(3) = 3^2 = 9$

$f(4) = 4^2 = 16$

Domain of $f = \{1, 2, 3, 4\}$

Co-Domain of $f = \{1, 4, 9, 16, 25\}$

Range of $f = \{1, 4, 9, 16\}$

The range of the functions is subset of its Co-domain. [Range $\subset$ Co-Domain]

C. One to One functions

$f(x) = 3x$

1	3
2	6
3	9

$f(1) = 3(1) = 3$

$f(2) = 3(2) = 6$

$f(3) = 3(3) = 9$

Range of f = Co - domain of f → ONTO FUNCTION

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A.) Relations & functions

Eg - $P = \{5, 6\}$, $Q = \{1, 2, 3\}$

Cantension Product = $P \times Q = \{(5,1), (5,2), (5,3)$
$(6,1), (6,2), (6,3)\}$

Eg - (i) $R_1 = \{(5,1), (6,1), (6,3)\}$
(ii) $R_2 = \{(5,1), (6,2)\}$
Any subset of Cartesian Product

Relation no 1 (R_1) is only a Relation because in that relation the find element 6 is related to two different element 1 & 3. → (6,2), (6,3)

Relation no. 2 (R_2) becomes a function because in the relation find element is different in each pair (5,1) (5,2) (5,3) or (6,1), (6,2), (6,3)

Hence we write

$$f : P \longrightarrow Q$$
$$: \{(5,1), (6,2)\}$$

Here, 1, 2 are called image of 5 & 6 respectively and 5 is pre-image of 1 and 6 is pre-image of 2

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One - one function and onto function is said to be byjective function

d.) Identily function

$1 \longrightarrow 1$
$2 \longrightarrow 2$
$3 \longrightarrow 3$

$\boxed{LHS = RHS}$

e.) INTO Function

$1 \longrightarrow 1$
$2 \longrightarrow 2$
$3 \longrightarrow 3$
4

f.) Constant function

$1 \longrightarrow 5$
$2 \longrightarrow 5$
$3 \longrightarrow 5$

The range of a Constant function is also sigle tanset

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g.) Composite function

$$f \circ g = f[g(x)]$$
$$g \circ f = g[f(x)]$$
$$f \circ f = f[f(x)]$$
$$g \circ g = g[g(x)]$$

h.) Equal functions

$f(x) = g(x) \longrightarrow$ Same domain

I.) Inverse functions

If $y = f(x)$ then $x = f^{-1}(y)$

Step to solve

1) formula
2.) $y = f(x)$
3) find x
4.) $x = f^{-1}(y)$
5.) Replace x by y

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Chapter :- 8. Basic Concepts of Differential and Integral Calculus

A] Differential Calculus

i) $\frac{d}{dx} x = 1$

ii) $\frac{d}{dx}$ (constant) $= 0$

iii) $\frac{d}{dx} x^n = n(x)^{n-1}$

iv) $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

v) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

vi) $\frac{d}{dx} e^x = e^x \cdot \log e = e^x$, $\log e = 1$

vii) $\frac{d}{dx} e^{Mx} = M \cdot e^{Mx}$

upper x hai toh log

viii) $\frac{d}{dx} a^{x} = a^x \cdot \log a$

ix) $\frac{d}{dx} a^{Mx} = M \cdot a^{Mx} \log a$

x) $\frac{d}{dx} \log_e x = \frac{1}{x \cdot \log e} = \frac{1}{x}$

xi) $\frac{d}{dx} \log_a x = \frac{1}{x \log_a}$

xii) $\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$

xiii) $\frac{d}{dx}\frac{u}{v} = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$

xiv) $\frac{\cancel{d}}{\cancel{dt}}\ x = f(t)$, $y = f(t)$

$\frac{d}{d\cancel{x}t}x = \frac{d}{dt}f(t)$, $\frac{dy}{d\cancel{x}t} = \frac{df(t)}{dx}$

$$\frac{\frac{dy}{d\cancel{t}}}{\frac{dx}{dt}} = \frac{dy}{dx}$$

Golden Rule

1.) See question OC / IC

Simple / U.V / $\frac{U}{V}$ / t wala / $f(x) = f'(x)$

$= \frac{d}{dx}\log y$

$= \frac{1}{y}\left(\frac{dy}{dx}\right)$

The ordinate and the abscissa are equal =

The gradient of the curve $\rightarrow \boxed{\frac{dy}{dx}}$

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B.] Integral Calculus

i. $\int 1\,dx = x + c$

ii.) $\int \frac{1}{x}\,dx = \log x + c$

iii.) $\int e^{x}\,dx = e^{x} + c$

iv.) $\int x^{n}\,dx = \frac{x^{n+1}}{n+1} + c$

v.) $\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

vi.) $\int a^{x}\,dx = \frac{a^{x}}{\log a} + c$

vii.) $\int a^{Ma}\,dx = \frac{a^{Ma}}{\log a} + c$

viii.) $\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$

ix.) $\int UV\,dx = U\int V\,dx - \int \left[\int V\,dx \cdot \frac{d}{dx} u\right] dx$

L I A T E

x.) $\int \frac{1}{x^2+a^2}\, dx = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

xi) $\int \frac{1}{a^2+x^2}\, dx = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

xii) $\int \frac{1}{\sqrt{x^2+a^2}} = \log \left| x + \sqrt{x^2+a^2} \right| + c$

xiii) $\int \frac{1}{\sqrt{x^2-a^2}} = \log \left(x + \sqrt{x^2-a^2}\right) + c$

xiv) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \log \left(x+\sqrt{x^2+a^2}\right)$

xv) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2} \sqrt{x^2+a^2} - \frac{a^2}{2} \log \left(x+\sqrt{x^2-a^2}\right)+$

xvi) $\int e^x \{f(x) + f'(x)\}\, dx = e^x f(x) + c$

xvii) $\int \frac{f'(x)}{f(x)}\, dx = \log f(x) + c$

xviii) $\int_a^b f(x)\, dx = f(b) - f(a)$

$\therefore \int F(x)\, dx = f(x)$