Ratio $= \frac{A}{B} = \frac{\text{Antecedent}}{\text{Consequent}}$ → Comparison b/w quantities of Same Kind

Properties of Ratio

- **Order** of the terms in a ratio is **important.** [Ex: 3:4 is not same as 4:3].
- Ratio is written in **Simplest form** (lowest form). [Ex: $12:16 = \frac{12}{16} = \frac{3\times4}{4\times4} = \frac{3}{4} = 3:4$]
- **No impact** when terms of ratio are multiplied or divided by same (non-zero) number.
- **Inverse Ratio** of a: b = b: a [PC Note: Product of a ratio with its inverse Ratio = 1]
- **Compound Ratio** of two ratios a: b & c: d $= \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ [Multiplication of Ratios]

Q. Ratio compounded of (a+b): (a-b) and $a^2 - b^2 : (a + b)^2$ is ____.
(a) (a+b):1 (b) (a-b):1 (c) 1:1 (d) None

- **Duplicate Ratio** of a: b = $a^2 : b^2$ & **Triplicate Ratio** of a: b = $a^3 : b^3$
- **Sub-duplicate Ratio** of a: b = $\sqrt{a} : \sqrt{b}$ & **Sub-Triplicate Ratio** of a: b = $\sqrt[3]{a} : \sqrt[3]{b}$

Q. If (4x+3): (9x+10) is the Triplicate Ratio of 3: 4, then the value of x is ________.
(a) 9 (b) 7 (c) 6 (d) 5

Q. Ratio compounded of Duplicate Ratio of 4: 5, Triplicate of 1: 3, Sub Duplicate Ratio of 81: 256 and Sub Triplicate Ratio of 125: 512 is ____.
(a) 4:512 (b) 3:32 (c) 1:12 (d) 1:120

Q. p:q is a sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 =
(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{pq}$ (d) $\frac{pq}{p+q}$

- If original quantity increases or decreases in the ratio a:b, then

New Quantity = Original Quantity × $\frac{b}{a}$

Increase	Decrease
a < b	a > b
2:3	3:2

Q. Mr. PC weighs 56.7 kg. If he reduces his weight in the ratio 7: 6, find his new weight.

Class Note:

Number = 72
Part 1 = 45 Part 2 = 27

Ratio $= \frac{45}{27} = \frac{5}{3}$
$\frac{5 \times 9}{3 \times 9}$ [Common]

- To Calculate original numbers, Most Imp Step is to find out "Common".
- Common $= \frac{\text{Total of Numbers in Ratio}}{\text{Sum of Ratios}} = \frac{72}{8} = 9$

Q. Two numbers whose sum is 72 are in the ratio 5:3. Find the numbers.
- First Term = 5 x 9 = 45
- Second Term = 3 x 9 = 27

Q. Ratio of the number of boys to number of girls in a school of 1,200 Students is 7:5. If 20 boys are newly admitted in the school, find how many new girls may be admitted so that above ratio changes to 4: 3.
(a) 40 (b) 140 (c) 60 (d) 58

CONTINUED RATIO ⇒ A:B:C:D

	Question Asked	Method to Use
1	A:B:C:D	Use Option Method
2	A:D	$\frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$
3	A:C	$\frac{A}{C} = \frac{A}{B} \times \frac{B}{C}$
4	B:D	$\frac{B}{D} = \frac{B}{C} \times \frac{C}{D}$

Q. If 2A = 3B & 4B = 5C, then A:C is ________.
(a) 4:3 (b) 15:8 (c) 8:15 (d) 3:4

Q. If a: b = 3:4, value of (2a + 3b): (3a + 4b) = __.
(a) 18:25 (b) 8:25 (c) 17:24 (d) None

Q. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ = ________.
(a) 7 (b) 2 (c) 1/3 (d) 1/5

Q. A: B = 2: 3; B: C = 4: 5; & C: D = 6: 7, then A: B: C: D =
(a) 16: 22: 30: 35 (b) 16: 24: 15: 35
(c) 16: 24: 30: 35 (d) 18: 24: 30: 35

Q. If A: B = 2: 3, B: C = 4: 5, C: D = 6: 7; A: D =
(a) 35:16 (b) 16:35 (c) 2:7 (d) None

Proportion = Equality of two ratios

a, b, c, d are in proportion

a : b :: c : d

a : b = c : d

$\frac{a}{b} = \frac{c}{d}$

Product of means = Product of extremes

b x c = a x d

Continuous Proportion [3 Terms]

If $\frac{a}{b} = \frac{b}{c}$, then $b^2 = ac$; $b = \sqrt{ac}$

a → 1st proportional;
b → Mean proportional between a & c;
c → 3rd proportional

(Mean Proportional)^2 = 1st Proportional x 3rd Proportional

Q. If b is mean proportion between a & c, then mean proportion betn (a^2+b^2) & (b^2+c^2) is __.
(a) b (a + c) (b) a (b + c) (c) c (a + b) (d) abc

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ .... $= \frac{a+c+e.....}{b+d+f.....} = \frac{a-c-e.....}{b-d-f.....}$ = Original Ratio

Addendo (↑ first), Subtrahendo (↑ second)

Q. If a: b = c: d = 2.5:1.5, what are the values of (i) ad: bc & (ii) a+c: b+d?

DIRECTLY PROPORTION		INVERSELY PROPORTION	
a & b are directly proportional ⇒ a ↑ , b ↑ & a ↓ , b ↓		a & b are inversely proportional ⇒ a ↑ , b ↓ & a ↓ , b ↑	
Expressed as $a \propto b$	Mathematically a = k.b	Expressed as $a \propto \frac{1}{b}$	Mathematically $a = \frac{k}{b}$
PC Note: Calculate 'K' using one set of given value of x & y. Put value of 'K' & given variable to calculate unknown.			

Q. X varies inversely as y^2. Given that y = 2 for x = 1. Value of x for y = 6 will be _______.
(a) 3 (b) 9 (c) 1/9 (d) – 1/9

LAWS OF INDICES	
1. $a^0 = 1$	Ex: $5^0 = 1$
2. $a^m \times a^n = a^{m+n}$	Ex: $3^2 \times 3^1 = 3^{2+1} = 3^3$
3. $a^m \div a^n = a^{m-n}$	Ex: $3^2/3^1 = 3^{2-1} = 3^1$
4. $(a^m)^n = a^{mn}$	Ex: $(3^2)^2 = 3^{2\times 2} = 3^4$
5. $(a.b)^m = a^m.b^m$	Ex: $(3.2)^2 = 3^2.2^2$
6. $(a/b)^m = a^m/b^m$	Ex: $(4/2)^2 = 4^2/2^2$
7. $a^{-m} = \frac{1}{a^m}$ & $\frac{1}{a^{-m}} = a^m$	Ex: $x^{-1/4} = 1/x^{1/4}$
8. $x^a = x^b$ then $a = b$	Ex: $3^x = 9$; $3^x = 3^2$; $x = 2$
9. $x^a = y^a$ then $x = y$	Ex: $a^3 = 27$; $a^3 = 3^3$; $a = 3$

Q. $[x^{(-3)}.y^{(-4)}] \times (x^4.y^3) =$

(a) 1 (b) $\frac{x}{y}$ (c) $\frac{y}{x}$ (d) 0

Q. $\frac{2^{m+1}.3^{2m-n}.5^{m+n}.6^n}{6^m.10^{n+2}.15^m} =$ _____.

(a) 1 (b) $\frac{1}{50}$ (c) $\frac{1}{9}$ (d) 0

Q. Value of $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}} =$

(a) 102 (b) 105 (c) 107 (d) 109

Q. $\sqrt{a^{3/4}.b^{2/3}.c^4} \div \sqrt[3]{a^6.b^{-3}.c^6}$

(a) $a^{-13/8}b^{4/3}$ (b) $a^{-1/8}b^{1/3}$ (c) $a^{-8}b^3$ (d) 1

Q. $(x^{2^{n-1}} + y^{2^{n-1}}).(x^{2^{n-1}} - y^{2^{n-1}}) =$

(a) $x^{2^n} - y^{2^n}$ (b) $x^2 - y^2$

(c) $x^a - y^b$ (d) None

Some Useful Results

- $a^{1/n} = \sqrt[n]{a}$ & $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$
- $\sqrt{a\sqrt{a\sqrt{a\sqrt{a \ldots \infty}}}} = a$
- If $x = a^{1/3} - a^{1/3}$, then $(x^3 + 3x) = (a - 1/a)$
- If $x = a^{1/3} + a^{1/3}$, then $(x^3 + 3x) = (a + 1/a)$
- If $a^x b^y = a^m b^n$, then $x = m$ & $y = n$
- If $a^x = k$, then $a = k^{1/x}$ → K Method ⇒ Used when more than 2 variables are equal.

Q. If $2^x = \sqrt[3]{32}$ then $x =$ ________.

(a) 5 (b) 3 (c) 3/5 (d) 5/3

Q. If $x = 7^{1/3} + 7^{-1/3}$, then $7x^3 - 21x =$ ___.

(a) 49 (b) 50 (c) 48 (d) 51

Q. Find 'b' if $12^{2b+4} = 3^{3b} \times 4^{b+8}$

Type 1 - Condition is expressly given in the question	Type 2 - Condition is not given in question (but implied)
Q. If $a^p = b^q = c^r = d^s$ & $ab = cd$; then $\frac{1}{p} + \frac{1}{q} - \frac{1}{r} - \frac{1}{s} =$ (a) 1/a (b) 1/b (c) 0 (d) 1	Q. If $2^a = 3^b = (12)^c$ then $\frac{1}{c} - \frac{1}{b} - \frac{2}{a} =$ ________. (a) 1 (b) 0 (c) 2 (d) None

BASIC FORMULAE		
$(a + b)^2 = a^2 + b^2 + 2ab$	$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$	$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
$(a - b)^2 = a^2 + b^2 - 2ab$	$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$	$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
$a^2 - b^2 = (a + b)(a - b)$	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$	
If $(a + b + c) = 0$, then $a^3 + b^3 + c^3 = 3abc$	If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then $(a + b + c)^3 = 27abc$	

LOGARITHMS

Transformation Rule ⇒ If $a^x = b$ then $\log_a b = x$

Exponential Form	Logarithmic Form	Read as
$2^4 = 16$	$\log_2 16 = 4$	Log of 16 to the base 2 = 4
$10^3 = 1000$	$\log_{10} 1000 = 3$	Log of 1000 to the base 10 = 3
$3^{-4} = \frac{1}{81}$	$\log_3 \frac{1}{81} = -4$	Log of $\frac{1}{81}$ to the base 3 = - 4
$100^{1/2} = 10$	$\log_{100} 10 = 1/2$	Log of 10 to the base 100 is ½

Mentos Zindagi:

- Log apne side me positive logo ko hi rakhte hai **[a & b should be positive; a & b > 0; a ≠ 1]**
- Log 'x' ko apne se dur rakhte hai **[Therefore 'x' should be on other side of Log]**
- If **NO BASE** is given in the question, it is always **taken as 10 [In this chapter]**
- Base of Log > 1 [If Base = 1, then Value of b will always be 1 (1^x).]
- Number (b) > 0 [Log 0 → Does not Exist.]

	LAWS OF LOGARITHMS	
1	$Log_M M = 1$ [Log of any number to same base = 1 (Since $a^1 = a$, $\log_a a = 1$)]	
2	Log 10 = 1 [Because since base is not given, it is taken as 10]	
3	Log 1 = 0 [Log of 1 to any Base = 0; (Since $a^0 = 1$, $\log_a 1 = 0$)]	
4	Log M + Log N = Log (M × N) [PC Note: Log M + Log N ≠ Log (M + N)]	
	Q. Log X + Log X^2= Log X.X^2 = Log X^3	Q. Value of Log $\frac{a^n}{b^n}$ + Log $\frac{b^n}{c^n}$ + Log $\frac{c^n}{a^n}$ =
4	Log M - Log N = Log (M/N) [PC Note: Log (M - N) ≠ Log M - Log N]	
	Q. Log 32/4 = Log 32 - Log 4	Q. Log (Logx2) - Log (Logx) =
	Master Question: If $Log_{10} y = 1 + 2 Log_{10} x - Log_{10} z$; then value of $\frac{yz}{x^2}$ is _______.	
5	Log (M^N) = N. Log M [PC Note: $(LogM)^N$ ≠ N. Log M]	
	Q. Log 25 = Log 5^2 = 2. Log 5	Q. If 2Log x = 4Log3, then x =
6	$Log_{N^b} M^a = (a \times \frac{1}{b}) \times Log_N M$	
	☞ Jo Number ka Log nikalna hai uska power "jaisa ka waise" bahar aayega. ☞ Base ka power "reciprocal" me bahar aayega.	
	Q. $Log_4 8$ =	
8	$Log_N M = \frac{Log M}{Log N}$ [Base Changing Rule.]	
	Q. $Log_4 8 = \frac{Log_2 8}{Log_2 4} = \frac{3 Log_2 2}{2 Log_2 2} = \frac{3}{2}$	Q. $\frac{\log_8 17}{\log_9 23} - \frac{\log_{2\sqrt{2}} 17}{\log_3 23}$ = ________.
9	$Log_C A = Log_B A \times Log_C B$	
	Q. Value of $\log_3 2$. $\log_4 3$. $\log_5 4$ .... $\log_{15} 14$. $\log_{16} 15$ =	
	Q. If Loge 2. Logb 625 = Log1016. Loge10, then b = _______.	

Values of Log (Base 10)

- Log 1 = 0
- **Log 2 = 0.3010**
- **Log 3 = 0.4771**
- Log 4 =
- Log 5 =
- Log 6 =
- **Log 7 = 0.8450**
- Log 8 =
- Log 9 = 0
- Log 10 = 1

Use in Questions of Log

1 = Log 10
2 = Log 100
3 = Log 1000
4 = Log 10,000

10	$Log_N M = \frac{1}{Log_M N}$	
	Q. $Log_5 10 = \frac{1}{Log_{10}5} = \frac{1}{Log_{10}(\frac{10}{2})} = \frac{1}{Log_{10}10 - Log_{10}2} = \frac{1}{1-0.3010} = \frac{1}{0.6990} = 1.43$ Q. If $\frac{1}{\log_x 10} + 2 = \frac{2}{\log_5 10}$, then the value of x is ______.	
11	$a^{\log_a x} = x$	Q. $a^{\log_a x} = x^{\log_a a} = x^1 = x$ [Inverse logarithm Property]
	Q. $a^{(\log_a b.\log_b c.\log_c d.\log_d t)} =$	

Q. Given that Log 2 = 0.3010, Log 3 = 0.4771, The value of $Log_8 81$ is ______.
(a) $\frac{9542}{4515}$ (b) $\frac{9442}{4515}$ (c) $\frac{4515}{9442}$ (d) None

Q. Log 0.0001 to the base 0.1 = _______.
(a) -4 (b) 4 (c) ¼ (d) None

Q. If $\frac{\log_8 17}{\log_9 23} - \frac{\log_{2\sqrt{2}} 17}{\log_3 23} =$ ________.
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 0

Q. Find the number of Digits in 2^{69}.

Q. If $Log_e M + Log_e N = Log_e (M + N)$, then find M as a function of N.
(a) 1/N (b) N^2 (c) N^2 x (N - 1) (d) N/ (N -1)

Q. Given that $Log_{10}{}^2 = x$ and $Log_{10}{}^3 = y$, the value of $Log_{10}{}^{60}$ is expressed as ______.
(a) x - y + 1 (b) x + y + 1 (c) x - y - 1 (d) None

EQUATIONS & ITS TYPES

	Power	One Variable	2 Variables	3 Variables [Option Method]
Linear	HD = 1	$3X - 4 = 11$ (OM)	$2x + 3y + 6 = 0$ (OM)	$2x - y + z = 3$
Quadratic	HD = 2	$x^2 - 2x - 15 = 0$ (TM)	$2x^2 - 3y^2 - 5x = 0$ (OM)	$2x^2 - 3y^2 - 5z^2 + 3y = 0$
Cubic	HD = 3	$x^3 + 1 = 28$ (OM)	$2x^3 + 3y^3 - y^2 = 0$ (OM)	$2x^3 - 6y^3 - 4z^3 + 2xy - 3x^2 = 0$

PC Note: OM ⇒ Option Method & TM ⇒ Tamiz Se – Pura Padhke Method 😍😍😍

- **Variable:** It is a quantity whose value varies (changes). Generally denoted by x, y, z.
- **Constant:** It is a quantity whose value does not change. Generally denoted by a, b, c.
- **Root/Solution:** Value of variable which satisfies the given equation. [LHS=RHS when substituted].

  PC Note: Number of Roots = Highest Degree of an Equation.

HOW TO SOLVE LINEAR EQUATION IN 2 VARIABLES:

1. **Substitution Method:** Any one variable is written in terms of another variable in any one equation & then obtained value is substituted in other equation.

   Q. Solve: $6x + 5y - 16 = 0$ and $3x - y - 1 = 0$ we get values of x, y as ____.

   Solution: $6x + 5y - 16 = 0$ --------(i) and $3x - y - 1 = 0$ --------------(ii)

   Now from (2), we get $y = 3x - 1$ ----------(iii);

   Substitute the value of y in (i), $6x + 5(3x - 1) - 16 = 0$.

   $6x + 15x - 5 - 16 = 0$; $21x - 21 = 0$; $21x = 21$; $x = 1$

   Now Put $x = 1$ in (iii); we get $y = 3(1) - 1 = 3 - 1 = 2$. Thus $(x, y) = (1, 2)$

2. **Solving Both Equations simultaneously**
   - Sign of variable with same co-efficient is opposite → Add the equations.
   - Sign of variable with same co-efficient is same → Subtract the equations.

   Q. Solve for (x,y): $7x - 2y = 45$ & $5x + y = 37$.

   Solution:

Q. If $a-b = p$ & $a+b = k$, then a^2-b^2 = ___	(a) pk	(b) $p^2 - k^2$	(c) p + k	(d) $\frac{p^2}{k^2}$
Q. If $b(x+2y) = 60$ and $by = 15$, Value of bx = __	(a) 20	(b) 25	(c) 30	(d) 45
Q. If $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x = 14$, then x is ___.	(a) 4	(b) 8	(c) 12	(d) 16

TEST OF CONSISTENCY FOR A SYSTEM OF EQUATIONS

- **Consistent System** → System having at least one Solution.
- **Inconsistent System** → System having NO Solution.

No. of Solutions	System of Equations	Lines intersect at
No solution	Inconsistent	Parallel
Unique Solution	Consistent	One Point
Infinite solutions	Consistent	Coincident

QUADRATIC EQUATION ⇒ Highest degree = 2 & thus No. of Roots = 2

- General format ⇒ $ax^2 + bx + c = 0$ [where $a \neq 0$ & a, b, c → Constant]
- x^2 – (sum of roots)x + Product of roots = 0
- PC Note: Sum of roots $(\alpha + \beta) = -\frac{b}{a}$ & Product of roots $(\alpha\beta) = \frac{c}{a}$
- Roots of QE ⇒ (1) $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ (2) $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ [$b^2 - 4ac$ ⇒ Discriminant]

NATURE OF THE ROOTS

Value of $b^2 - 4ac$	Nature of Roots	Example	$b^2 - 4ac$ =	Roots
Zero	Real, Equal & rational	$x^2 - 6x + 9 = 0$	36 – 4.1.9 = 0	3, 3
Perfect Square	Real, unequal & rational	$x^2 - 6x - 16 = 0$	36 – 4.1.(-16) = 100	8, -2
Not Perfect Square	Real, unequal & irrational	$x^2 - 6x + 7 = 0$	36 – 4.1.7 = 8	$(3 + \sqrt{2}), (3 - \sqrt{2})$
Negative	Imaginary (Complex No.)	$x^2 - 6x + 10 = 0$	36 – 4.1.10 = -4	No Solution

Important Properties

- Irrational roots occur in conjugate pairs. One root is $(a + \sqrt{b})$, other root will be $(a - \sqrt{b})$.
- Roots are equal in magnitude (value) but opposite in sign, Sum of roots = 0 & so $\frac{b}{a} = 0$ & b = 0.
- If one root is reciprocal of the other root, then their product is 1 & thus $\frac{c}{a} = 1$ i.e. a = c.
- If α & β are the roots of $ax^2 + bx + c = 0$, then 1/α, 1/β will be roots of $cx^2 + bx + a = 0$

Q. If α & β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ = __ (a) $2\sqrt{5}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) $-2\sqrt{5}$

SOME USEFUL RESULTS REQUIRED TO SOLVE QUESTIONS OF ROOTS OF QUADRATIC EQUATION

$a^2 + b^2 = (a+b)^2 - 2ab$	$a^2 - b^2 = (a+b)(a-b)$	$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$	$\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2 + b^2}{(ab)^2}$
$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$	$a^3 - b^3 = (a-b)^3 + 3ab(a-b)$	$a - b = \sqrt{(a+b)^2 - 4ab}$	$\frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$

Q. If one roots of $5x^2 + 13x + p = 0$ be reciprocal of the other, p =
(a) -5 (b) 5 (c) 1/5 (d) -1/5

Q. If one root of the equation is $2 - \sqrt{3}$, form the equation.
(a) $x^2 - 2x + 2 = 0$ (b) $x^2 - 3x + 1 = 0$
(c) $x^2 - 5x + 5 = 0$ (d) $x^2 - 4x + 1 = 0$

Q. If roots of equation $2x^2 + 8x - m^3 = 0$ are equal then m = __.
(a) -3 (b) -1 (c) 1 (d) -2

Q. The roots of the equation $x^2 + (2p - 1)x + p^2 = 0$ are real if __.
(a) $P \geq 1$ (b) $P \leq 4$ (c) $P \geq 1/4$ (d) $P \leq 1/4$

Q. The condition that one of the roots of $ax^2 + bx + c = 0$ is thrice the other is ____.
(a) $3b^2 = 16ca$ (b) $b^2 = 9ca$
(c) $3b^2 = -16ca$ (d) $b^2 = -9ca$

Q. Solve for z: $z^{10} - 33z^5 + 32 = 0$
(a) 1, 2 (b) 2, 3 (c) 2, 4 (d) 1, 4

Q. Solve for x: $4^x - 3.2^{x+2} + 2^5 = 0$
(a) 4, 8 (b) -2, -3 (c) 2, 6 (d) 2,3

Q. Solve for (x,y): $2^x.4^y = 32$ & $3^x \div 9^y = 3$.
(a) x = 3, y = 1 (b) x = y = 2
(c) x = y = 1 (d) x = y = 3

Q. Solve for z: $z + \sqrt{z} = \frac{6}{25}$.
(a) 1/5 (b) 2/5 (c) 1/25 (d) 2/25

Q. Solve for x: $\left(x - \frac{1}{x}\right)^2 - 10\left(x - \frac{1}{x}\right) + 24 = 0$
(a) 0 (b) 1 (c) -1 (d) None

Linear Inequality: Any linear function that involves an **inequality sign** (i.e <, >, ≤, ≥)

Solving an equation	Solving an inequality
Solve $$2x+1=9$$ $$2x=8$$ $$x=4$$ 4 is the only solution to this equation.	Solve $$2x+1<9$$ $$2x<8$$ $$x<4$$ x can be any value that is less than 4

Multiplying and dividing by a negative number

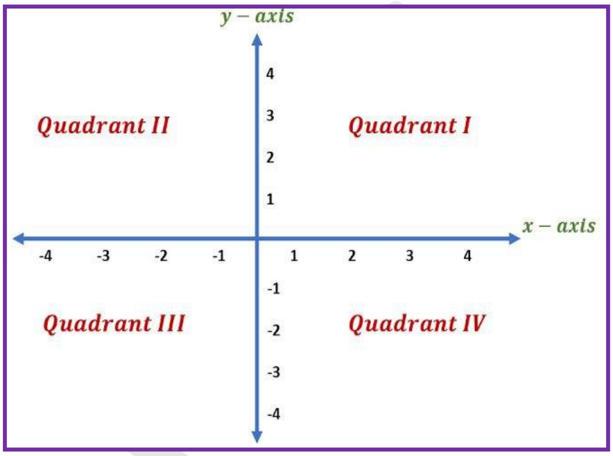
This **changes the direction** of the **inequality sign**.

For example,

$$1-2x<9$$
$$-2x<8$$
$$x>-4$$

x can be any value that is greater than -4

Sign Changes



Q1. - 6X < -18. If we divide both sides by -6, X > 3. Inequality sign will change.

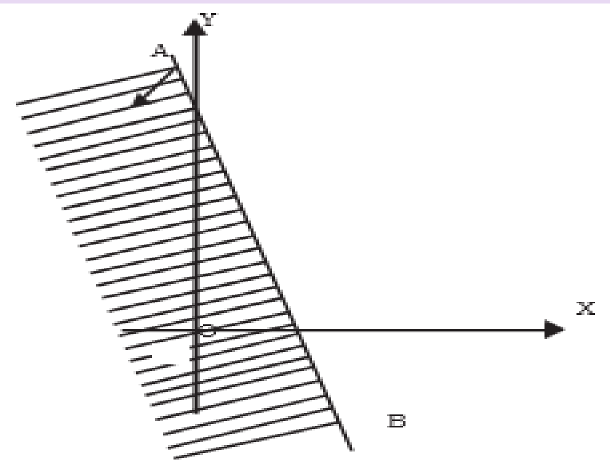
Q2. If a > b & c < 0, then ac < bc & a/c < b/c.

PC Note: 'NO CHANGE' in Inequality SIGN

- *If both sides are multiplied/divided by positive number [**Ex:** If a > b & c > 0, then ac > bc & a/c > b/c]*
- *If any number is added/subtracted to both sides [**Ex:** If a > b, then a + c > b + c & a - c > b – c]*

GRAPHICAL REPRESENTATIONS OF INEQUALITY - Steps to Plot linear inequalities in two variables

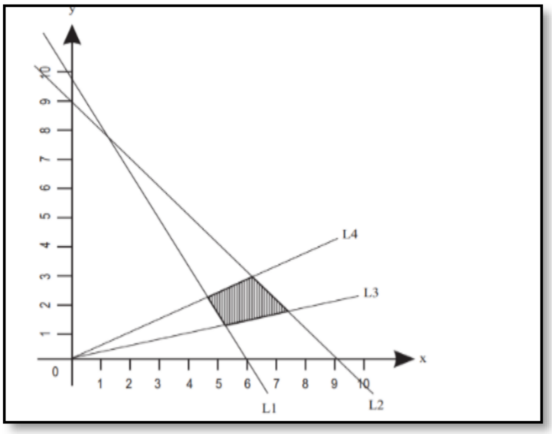
1. *Consider a linear inequality given by 3x + y < 6.*
2. *Replace the inequality by an equality & then you will get 3x + y = 6.*
3. *Now substitute two convenient values for x & y so that we get two points.*
   Let x = 0 so that y = 6. Let y = 0, so that x = 2. You will get two points (0,6) & (2, 0).
4. *Plot these points on co-ordinate plane & join them to get a line of the linear equation.*



PC NOTE

- ***If Plotted line is intersecting (touching) x & y axis, then** for*
  - *'Less than' inequality → Solution = **Part Below** the line.*
  - *'Greater than' inequality → Solution = **Part Above** the line.*
- ***If Plotted line is NOT intersecting (touching) both x & y** axis, then we take any point on either side of the line.*
  - *If that point satisfies the inequality, the part in which the point lies will be our solution.*
  - *If that point does not satisfies the inequality, the part on the other side of the point will be our solution.*

Q3. L1: 5x + 3y = 30; L2: x + y = 9; L3: y = x/3; L4: y=x/2. Common region refers to ___. (ICAI SM)



(a) $5x+3y \leq 30$; $x+y \leq 9$; $y \leq 1/5x$; $y \leq x/2$

(b) $5x+3y \geq 30$; $x+y \leq 9$; $y \geq x/3$; $y \leq x/2$; $x \geq 0$, $y \geq 0$

(c) $5x+3y \geq 30$; $x+y \geq 9$; $y \geq x/3$; $y \geq x/2$; $x \geq 0$, $y \geq 0$

(d) $5x+3y > 30$; $x+y < 9$; $y \geq 9$; $y \leq x/2$; $x \geq 0$, $y \geq 0$

HOW TO FORM INEQUATION FROM WORD PROBLEMS

Q4. A fertilizer company produces two types of fertilizers called Grade I & Grade II. Each of these types is processed through two critical chemical plant units. Plant A has maximum 120 hrs & Plant B has maximum of 180 hrs available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in Plant A and 4 hours in plant B. Manufacturing one bag of Grade II fertilizer requires 3 hrs in Plant A and 10 hours in Plant B.

Answer: Firstly, we need to identify the **key factor (factor having restrictions or conditions)**. Here we have limited Machine Hours & thus Machine hours becomes our Key Factor. Always arrange 'Key Factor' in columns.

Particulars	Machine A	Machine B
Chemical Grade I	6 hrs	4 hrs
Chemical Grade II	3 hrs	10 hrs
Maximum Available Time	**120 Hours**	**180 Hours**

Assume we will produce x units of Grade I & y units of Grade II. Thus, **$6x + 3y \leq 120$ & $4x + 10y \leq 180$.**

Q5. Solve for real 'x' if $5x - 2 \geq 2x + 1$ & $2x + 3 < 18 - 3x$.

(a) $1 < x < 3$ (b) $-1 > x > -3$ (c) $1 \leq x < 3$ (d) $x = 3$

Q6. If $p - q = -3$ then ______.

(a) $p < q$ (b) $p > q$ (c) $p = q$ (d) $p = 0$

Q7. If $x \leq 0$, then $2/x + 8/x$ is ______.

(a) $2 \leq x \leq 3$ (b) ≥ 0 (c) ≥ 4 (d) ≤ -1

Q8. What is the smallest integer value of x in $4 - 3x < 11$ = ____.

(a) -3 (b) -2 (c) -1 (d) 0

Q9. What is the largest integer value of p that satisfies the inequality $4 + 3p < p + 1$?

(a) -2 (b) -1 (c) 0 (d) 1

Q10. When $x > 0$, value of $|x|$ is ______.

(a) 0 (b) -x (c) x (d) 1

Q11. If $\left|x+\frac{1}{4}\right| > \frac{7}{4}$, then ____. (Nov 2006)

(a) $x < \frac{-3}{2}$ or $x > 2$ (b) $x < -2$ or $x > \frac{3}{2}$ (c) $-2 < x < \frac{3}{2}$ (d) None of these.

Q12. If $A = x - 2^{-1}$, $B = x + 2^{-1}$ and $A^2 - B^2 > 0$, then______.

(a) $x > 0$ (b) $x < 0$ (c) $x = 0$ (d) $x = A + B$

Q13. Rules demand that employer should employ not more than 5 experienced hands to 1 fresh one.

(a) $y \geq x/5$ (b) $5y \geq x$ (c) Both (a) and (b) (d) $5y \leq x$

SIMPLE INTEREST

PC Note: Interest हमेशा Principal ka % होता hai

- Simple Interest (SI) = Principal (P) × Rate of Interest (R in %) × Time in years (T).
- Accumulated Amount (A) = P + SI = P + PRT = P (1 + RT). [Not Recommended by PC]
- *Interest calculated on the original principal for the entire period of borrowing.* **No Interest on Interest.**

Variations in SI Formula	$T = \frac{SI}{P \times R \text{ (in \%)}}$	$R = \frac{SI}{P \times T}$	$P = \frac{SI}{T \times R \text{ (in \%)}}$

PC Note for Solving Directly on Calculator

Method 1: Calculate Interest/year (P x R%) & then multiply by Time OR

Method 2: Calculate Total Interest (in %) & then Multiply by Principle.

Q1. Calculate the simple interest on Rs. 50,000 at 12% simple interest for 5 years?

Solve using Method 1:

Solve using Method 2:

Q2. Sania Mirza deposited Rs. 50,000 in a bank for 20 years with interest rate of 5.5% p.a. How much interest would she earn? Find the final value of her investment.

Q3. Find rate of interest if amount owed after 6 months is Rs. 1050 & borrowed amount is Rs. 1000.

Q4. Katrina gave Rs. 70,000 as loan to Salman Khan @ 6.5% p.a. SI. She received Rs. 85,925 after the end of term. Find out the period for which loan was given by Katrina to Salman Khan.

Q5. Sharmaji deposited a particular amount in a bank for 7.5 years @ 6% p.a. SI. He received Rs. 1,01,500 at the end of the term. Compute initial deposit of Sharmaji.

Q6. In what time will Rs. 85,000 become Rs. 1,57,675 at 4.5 % p.a.?

PC Note: *Sometimes, we are given 2 different amounts for 2 time period & we have to find out interest, principal & Rate of Interest. Let two amounts be A_1 & A_2 & time period be T_1 & T_2* ☛ Interest per year = $\frac{A_2 - A_1}{T_2 - T_1}$

Q7. A sum of money amount to Rs.6,200 in 2 years and Rs. 7,400 in 3 years. Principal and rate of interest are __.
(a) Rs. 3,800, 31.57% (b) Rs. 3,000, 20% (c) Rs. 3,500, 15% (d) None

How to find Time or Rate when sum becomes Double, Triple etc [Concept se bhi ho jayega]

Particular	Sum is 1.5 times	Sum is Doubled	Sum is Trebled	Sum is 4 times
Time Required	$T = \frac{0.5}{R}$ yrs	$T = \frac{1}{R}$ yrs	$T = \frac{2}{R}$ yrs	$T = \frac{3}{R}$ yrs
Rate Required	$R = \frac{0.5}{T}$	$R = \frac{1}{T}$	$R = \frac{2}{T}$	$R = \frac{3}{T}$

Q8. A sum doubles itself in 10 years. Find interest rate. (a) 10 % (b) 12 % (c) 15 % (d) 20 %

Space for Class Note:

COMPOUND INTEREST

Amount (A) = P (1 + R)^T Interest (I) = A – P

- Interest of every year is added to principal & interest for next year is calculated on [Original Principal + Interest].
- In CI, Principal goes on changing every year & Interest is charged on Interest Earned.

		Under Simple Interest	Under Compound Interest
First year	Principal	₹ 100.00	₹ 100.00
	Interest 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹ (110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹ (120 + 10) = ₹ 130	₹ 133.10

Q9. PC deposited Rs. 1 crore in a nationalized bank for 3 years. If the rate of interest is 7% p.a. Calculate the interest after 3 years if interest is compounded annually. Also calculate the amount at the end of third year.

Q10. On what sum will CI at 5% p.a. for 2 yrs compounded annually be Rs. 1,640?
(a) Rs. 16,000 (b) Rs. 17,000 (c) Rs. 18,000 (d) Rs. 19,000

Q11. A sum put at CI amount to Rs. 2,205 in 2 years & Rs. 2,315.25 in 3 years. Find R.
(a) 10% (b) 5 % (c) 8 % (d) 6 %

Q12. CI on a certain sum for 2 years is Rs. 41 & SI is Rs. 40. Find R.
(a) 4% (b) 5% (c) 6% (d) 8%

Q13. At what rate CI does a sum becomes four fold in 2 years?
(a) 150 % (b) 100 % (c) 200 % (d) 400 %

PC Note for Tricky Questions

Doubled	Tripled	Four-Fold
2 = (1 + R)^T	3 = (1 + R)^T	4 = (1 + R)^T

Q14. Time by which a sum would treble itself at 8 % p.a CI =
(a) 14.28 years (b) 14 years (c) 12 years (d) 15 years

PC Note:

- CI for 1^{st} year = SI for 1^{st} year. But then 2^{nd} year onwards, CI & SI will be different.
- CI for 2 years – SI for 2 years = PR^2
- CI for 3 years – SI for 3 years = $PR^2(R+3)$
- $R = \frac{2(CI_2 - SI_2)}{SI_2}$
- Years required for a sum to double $T = 0.35 + \frac{0.69}{R}$
- Different Interest Rate for different year (R_1, R_2, R_3) → Direct on Calculator: $A_n = P + R_1\% + R_2\% + R_3\%$

Q15. Difference between SI & CI on a certain sum invested for 2 years 5% p.a. is Rs. 30. Then the sum is ___.
(a) 10,000 (b) 12,000 (c) 13,000 (d) None

Q16. Compound interest at half-yearly rates on Rs. 10,000, the rate for 1^{st} & 2^{nd} years being 6% & for 3^{rd} year 9% p.a.
(a) Rs. 2,290 (b) Rs. 2,287 (c) Rs. 2,285 (d) Rs. 2,283

MORE THAN 1 COMPUNDING IN A YEAR

Conversion Period	Number of Conversion Period in a Year (K)	Formula to be used	Apply this TRICK & Use the Same CI Formula which we know [PC Special]
12 Months (Annually)	1	$A = P(1+R)^T$	Same as CI Formula
6 Month (Semi-annually)	2	$A = P(1+\frac{R}{2})^{2T}$	New R = 1/2 x Given R & New Time = 2 x Given Time
3 Months (Quarterly)	4	$A = P(1+\frac{R}{4})^{4T}$	New R = 1/4 x Given R & New Time = 4 x Given Time
1 Month (Monthly)	12	$A = P(1+\frac{R}{12})^{12T}$	New R = 1/12 x Given R & New Time = 12 x Given Time
1 Day (Daily)	365	$A = P(1+\frac{R}{365})^{365T}$	New R = 1/365 x Given R & New Time = 365 x Given Time

Formula (Don't Use – Apply the given Trick): Amount (A) = $P(1+\frac{R}{K})^{KT}$ ['K' = No. of conversion per year]

Q17. Rs. 10,000 is invested at annual rate of interest of 12%. What is the amount after 2 years if compounded?
(i) Annually = (ii) Semi-annually = (c) Quarterly = (d) Monthly =

EFFECTIVE RATE OF INTEREST

[Relevant when compounded more than once a year]

$E = (1+\frac{R}{K})^K - 1$ [E = Effective interest rate; R = Interest rate per annum; K = No. of conversion period]

Q18. Rs. 5,000 is invested in Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? What is effective rate of interest? [Interest = Rs. 306.82; E = 6.13%].

Q19. Which is better investment? (i) 3% p.a compounded monthly or (ii) 3.2% p.a SI. [$(1+0.0025)^{12}$ =1.0304]

Solution: K = 12 times; $E = (1+\frac{R}{K})^n - 1$; $E = (1+\frac{3}{12})^{12} - 1$; = 1.0304 -1 = 0.0304. Thus, **E = 3.04%**

Answer: Effective rate of interest < 3.2% & thus SI @ 3.2% per year is the better investment.

PC Note: CI formula can be used in case of uniform periodical increase at fixed rate like **population growth**. In case of uniform decrease like **depreciation** (W.D.V basis), **R is replaced by -R.** [MIND IT]

Present Value (आज ka Value)	Compounding: Multiply by $(1+R)^T$ → ← Discounting: Divide by $(1+R)^T$	Future Value (Future ka Value)
$PV = FV/(1+R)^T$		$FV = PV \times (1+R)^T$

Q20. Present value of Rs. 1 to be received after 2 years compounded annually at 10% is __.
(a) Rs. 0.9090 (b) Rs. 0.8264 (c) Rs. 0.7513 (d) Rs. 0.6830

Q21. Find PV of Rs. 10,000 to be required after 5 years if interest rate = 9%. [$(1.09)^5 = 1.5386$] [Ans: 6499.42]

Q22. Find PV of Rs. 500 due after 10 years (R= 10%) is compounded half yearly ___.
(a) Rs. 188.40 (b) Rs. 193.94 (c) Rs. 138.94 (d) Rs. 50.00

Q23. PC Sir invest Rs. 3,000 in a 2-year investment that pays you 12% pa. Calculate FV.
(a) Rs. 3,763.20 (b) Rs. 3,360.00 (c) Rs. 3,565.60 (d) Rs. 3,663.55

ANNUITY = INSTALMENT = PERIODIC PAYMENTS/RECEIPTS (SAME AMOUNT)

EXPLANATORY TABLE OF Rs. 1 invested for 4 years @ 6%

1	Rs. 1	$1\ (1 + 0.06)^3 = 1.191$
2	Rs. 1	$1\ (1 + 0.06)^2 = 1.124$
3	Rs. 1	$1\ (1 + 0.06)^1 = 1.060$
4	Rs. 1	$1\ (1 + 0.06)^0 = 1$
Future Value		4.375

Annuity [Regular – Payment @ End of the Year & Due – Payment @ Beginning of the Year]

FV of Annuity Regular	FV of Annuity Due	PV of Annuity Regular	PV of Annuity Due
$FV_{(R)} = P\left[\frac{(1+R)^n - 1}{R}\right]$ P = Amount deposited, R = Rate of Interest, N = No. of years	$FV_{(D)} = FY_{(R)} \times (1+R\%)$ PC Note: Calculate FY(R) + R(%)	$PV_{(R)} = A\left[\frac{(1+R)^n - 1}{R \times (1+R)^n}\right]$ A = Instalment Amount, R = Rate of Interest, n = No. of years	Compute PV(R) of (n-1) yrs + Add Initial payment/receipt to PV(R) of (n-1) yrs Refer Q29 on Next Page

PC Note: If Nothing is said in question, it is assumed as Annuity regular.

$$\text{PV of Annuity} = \frac{A}{(1+R)^1} + \frac{A}{(1+R)^2} + \frac{A}{(1+R)^3} + \frac{A}{(1+R)^4} + \ldots\ldots \frac{A}{(1+R)^N}$$

Space for PC Class Note:

Q24. Find FV of annuity of Rs. 500 made annually for 7 years @ 14% compounded annually. [(1.14)7 = 2.5023]

Q25. Find FV of annuity due of Rs. 500 made for 7 years at 14% compounded annually. [(1.14)7 = 2.5023]

Q26. Z invests Rs. 10,000 every year starting from today for next 10 yrs. Interest rate is 8% p.a compounded annually. Find FV of annuity. [(1 + 0.08)10 = 2.15892500] [Ans: Rs. 1,56,454.875]

Q27. S borrows Rs. 5,00,000 to buy a house. If he pays equal installments for 20 years and 10% interest on outstanding balance what will be the equal annual installment? [Ans: 58,730]

Q28. Rs. 5,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually? [Ans: 26,080]

Q29. Your mom decides to gift you Rs. 10,000 every year starting from today for the next 5 years. You deposit this amount in a bank as and when you receive and get 10% p.a compounded annually. Find PV of this annuity?

Solution: It is an annuity immediate. For calculating value of the annuity immediate following steps will be followed:

Step 1: Present value of the annuity as if it were a regular annuity for one year less i.e. for four years.

= Rs. 10,000 × P(4, 0.10); = Rs. 10,000 × 3.16987; = Rs. 31,698.70.

Step 2: Add initial cash deposit to the step 1 value: Rs. (31,698.70+10,000) = **Rs. 41,698.70.**

Q30. A person invests Rs. 500 at the end of each year with a bank which pays interest at 10% p.a. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is ______.

(a) Rs. 11,761.35 (b) Rs. 10,000 (c) Rs. 12,000 (d) None

PERPETUITY = Annuity where the **receipt (or payment)** takes place **forever.**

- FY of a Perpetuity – Cannot be computed.
- PV of Multi-period Perpetuity $PVA_\infty = \frac{P}{R}$ [P = Payment/Receipt each period; R = Rate of Interest]
- PV of Growing Perpetuity: Perpetuity which grows at constant rate. $PVA_\infty = \frac{P}{R-g}$ [g = Growth rate]

Q31. If I want to retire & receive Rs. 30,000 every month & I want my family to receive the same monthly payment after my death. I can earn an interest of 8% p.a. How much will I need to set aside to achieve my perpetuity goal? How much should I invest to get the amount from today itself? [Ans: Rs. 45,00,000]

Q32. I want to receive Rs. 10,000 forever. Interest rate is 8% & the rate at which perpetuity grows is 3%. Advise me the amount to be invested. [Ans: Rs. 2,00,000]

Answer: $PVA_\infty = \frac{P}{R-g} = \frac{10{,}000}{(8-3)\%} = \frac{10{,}000}{5\%}$ = **Rs. 2,00,000.**

COMPOUND ANNUAL GROWTH RATE (CAGR)

$$CAGR\ (t_0, t_n) = \left[\frac{V_n}{V_0}^{\left(\frac{1}{t_n - t_0}\right)}\right] - 1$$

Vn = Value in n^{th} year, Vo = Value in 0^{th} Year; t_0 = Starting period & t_n = Ending period

CQ33. Revenues of a company for 4 years, Calculate Compound annual Growth Rate.

Year	2013	2014	2015	2016
Revenues	100	120	160	210

Answer: $t_n - t_0$ = 2016 – 2013 = 3. $CAGR_{(0,3)}$ of Revenues = $\left[\frac{210}{100}\right]^{\frac{1}{3}}$ – 1 = 1.2806 – 1 = 0.2806 = **28.06%**

REAL LIFE APPLICATIONS OF ANNUITY

1 **Sinking Fund** - Fund **credited** for a **specified purpose** by way of **sequence of periodic payments**

Size of Sinking Fund Deposit (S) = $P \times \left[\frac{(1+R)^N - 1}{R}\right]$ S = Amount to be saved (FV) & P = Periodic Payment.

Q34. How much amount is required to be invested every year so as to accumulate Rs. 3,00,000 at the end of 10 years if interest is compounded annually at 10%? [Ans: Rs. 18,823.6]

2 **NET PRESENT VALUE (NPV)** = PV of Cash Inflow – PV of Cash Outflow

PC Note: If NPV > 0 → Accept Project; If NPV < 0 → Reject Project.

Q35. Compute NPV for a project with a net investment of Rs. 1,00,000 & net cash inflows for year 1, 2, 3 is Rs. 55,000, Rs. 80,000 & Rs. 15,000 resp. Cost of capital is 10%? [PVIF @ 10% for 3 years: 0.909, 0.826 & 0.751]

Solution: Since NPV of the project is positive, the company should accept the project.

Year	Net Cash Flows	PVIF @ 10%	Discounted Cash Flows
0	(1,00,000)	1.000	(1,00,000)
1	55,000	0.909	49,995
2	80,000	0.826	66,080
3	15,000	0.751	11,265
Net Present Value			**27,340**

3 **LEASING OR BUYING DECISION**

- If Cost of Asset > PV of lease rental → Lease
- If Cost of Asset < PV of lease rental → Buy

Q35. ABC Ltd. wants to lease out an asset costing Rs. 10 lacs for 5 years. It has fixed a rental of Rs. 3.1 lacs p.a payable annually starting from the end of first year. Suppose rate of interest is 12% p.a compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

Answer: Here we have to compute PV of the annuity of Rs. 3,10,000 for 5 years @ 12%p.a.

PV Factor for 5 years @ 12% = 3.604776. Thus, PV of Lease annuity = 3,10,000 × 3.604776 = Rs. 11,17,480.

Since PV of Lease annuity > initial cost of the asset, Leasing is favourable to the lessor.

Q36. A company is considering proposal of purchasing a machine either by making full payment of Rs. 4,000 or by leasing it for 4 years at lease rent of Rs. 1,250. Which option is preferable if R = 14% p.a.? [Ans: Lease]

4 **INVESTMENT DECISION**

- If PV of cash inflow > PV of cash outflow → Invest
- If PV of cash inflow < PV of cash outflow → Do NOT invest.

Q37. A machine with useful life of 7 years costs Rs. 10,000 while another machine with useful life of 5 years costs Rs. 8,000. The first machine saves labour expenses of Rs. 1,900 annually & second one saves labour expenses of Rs. 2,200 annually. Determine preferred course of action. Assume cost of borrowing as 10% p.a.

Answer: (i) PV of annual cost savings for 1st machine = Rs. 1,900 × 4.86842 = Rs. 9,250.

Cost of 1st machine = Rs. 10,000 & it saves Rs. 9,250. Thus, it costs Rs. 750 more than labour cost it saves.

(ii) PV of annual cost savings of 2nd machine = Rs. 2,200 × 3.79079 = Rs. 8,339.74.

Cost of 2nd machine = Rs. 8,000 & it saves Rs. 8339.74. Thus, effective savings in labour cost = Rs. 339.74. Hence, the second machine is preferable.

5 **VALUATION OF BOND** = PV of Interest Paid + PV of Maturity Amount.

Q38. An investor intends purchasing a 3-year Rs. 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a return of 14%?

Answer: Interest on bond for every year = Rs. 100. Maturity Amount = Rs. 1,000.

PV of Bond = $\frac{100}{(1.14)^1} + \frac{100}{(1.14)^2} + \frac{100}{(1.14)^3} + \frac{1000}{(1.14)^3}$ = 87.719 + 76.947 + 67.497 + 674.972 = **Rs. 907.125**

Thus, the bond should be purchased @ Rs. 907.125 or less than it.

- *Permutation* = *Arrangement* + **Order** is **important** i.e (a, b) & (b, a) are different arrangements.
- *Combination* = *Selection* + **Order** is **not important** i.e (a, b) & (b, a) are same selection.

FUNDAMENTAL PRINCIPLES OF COUNTING

- *Multiplication Rule* ⇒ No. of ways of doing **BOTH things** one after another = (m×n) ways [Connector = AND]

  Q1. There are 4 routes for going from Dumdum to Sealdah & 5 routes for going from Sealdah to Chandni. In how many different ways can you go from Dumdum to Chandni Via Sealdah? (a) 9 (b) 1 (c) 20 (d) None

- *Addition Rule* ⇒ No. of ways of doing **2 Alternative things** = (m+n) ways [Connector = OR]

  Q2. If one wants to go school by bus where there are 5 buses or by auto where there are 4 autos, then total number of ways of going school is ___ [Ans: 5 + 4 = 9]

FACTORIAL ⇒ Denoted as !

- **Continuous Product** of all integers **from 1 to 'n'**.
- n! = 1.2.3.4.5.6……(n-2).(n-1).n
- **PC Note:** **While solving the question, all factorials in the question shall be reduced upto lowest factorial.**

0!	1!	2!	3!	4!	5!	6!	7!	8!	9!	10!
		2 × 1	3 × 2!	4 × 3!	5 × 4!	6 × 5!	7 × 6!	8 × 7!	9 × 8!	10 × 9!
1	1	2	6	24	120	720	5040	40320	362880	3828800

Q3. $\frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

Q4. Find n if $(n+1)! = 30(n-1)!$ [Ans = 5]

Q5. Find x if $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$ [Ans = 121]

PERMUTATIONS

- No. of Permutations of 'r' different object out of 'n' different object = $^nP_r = \frac{n!}{(n-r)!}$ $[0 \le r \le n]$

PC Note: *Draw 'r' blank lines & put n objects in them.* Ex: If n = 8 & r = 5, we will draw 5 lines.

Space for Class Note:

n ways = 8 Ways | (n-1) ways = 7 Ways | (n-2) ways = 6 Ways | (n-3) ways = 5 Ways | (n-4) ways = 4 Ways

Q6. How many 3 letter words can be formed using the letters of the words (a) SQUARE & (b) HEXAGON?

Ans: Since 'SQUARE' consists of 6 different letters, number of permutations of choosing 3 letters out of 6 = $^6P_3 = 6 \times 5 \times 4 = 120$. Since 'HEXAGON' contains 7 different letters, number of permutations is $^7P_3 = 7 \times 6 \times 5 = 210$.

Q7. There are 5 guests in a party & only 3 chairs are there. In how many ways can the guests be seated?

Ans: There are 3 chairs & 5 guests. It is obvious that 2 guest will not occupy same chair.

1st Chair → can be occupied by any 1 of the 5 guests = 5 ways &

2nd Chair → can be occupied by any 1 of remaining 4 guests = 4 ways &

3rd chair → can be occupied by any 1 of remaining 3 guests = 3 ways. Total number of ways = 5 × 4 × 3 = **60 ways**.

Q8. How many 4-digit numbers can be formed from 1, 2, 3, 4, 5. [Repetition not allowed]

Ans: 5 × 4 × 3 × 2 = 120 ways.

Q9. How many 4 digits numbers can be formed by using 1, 2, 3,4,5,6,7,8,9, no digit being repeated in any number?

Ans: We have 9 digits & we have to find number of permutations of these taken 4 at a time, which is $^9P_4$ =3024 ways.

Q10. If $^nP_3 : {}^nP_2 = 3:1$. Find n. (a) 7 (b) 4 (c) 5 (d) None

Q11. If $^{x+y}P_2 = 90$ and $^{x-y}P_2 = 30$ then ___. (a) x = 4y (b) x = 2 (c) x = y (d) 4x = y

Q12. How many 4-digits numbers can be formed out of the digits 1,2,3,5,7,8,9, if no digit is repeated in any number? (b) How many of these will be greater than 3000?

Ans: (a) 7 × 6 × 5 × 4 × = 840 numbers (b) $5 \times {}^6P_3 = 5 \times 6 \times 5 \times 4 = 5 \times 120 = 600$ numbers

Q13. Find total number of numbers greater than 2000 that can be formed with the digits 1, 2, 3, 4, 5 no repetition.

Ans: 5 Digit No. (All) → 120 + 4 Digit No. (>2000) → 4 x 4 x 3 x 2 = 96 = 120 + 96 = 216.

Permutations of n different things taken all 'n' things at a time = n! $[r = n \text{ Thus } {}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!]$

PERMUTATIONS WITH RESTRICTION		
SN	Scenario	Formula
1	Particular object is not included	${}^{(n-1)}P_r$
2	Particular object is always included [Person/object to be included is fix]	${}^{(n-1)}P_{(r-1)}$
3	Particular object is always included [Person/object to be included is not fix]	$r.{}^{(n-1)}P_{(r-1)}$
4	2 things are always together	(n-1)! × 2!
5	2 things are never together = Total ways – 'Always together' ways = n! – (n-1)! × 2!	(n-2) x (n-1)!

Q14. How many 4-digits numbers can be formed by using 1,2,3,4,5,6,7,8,9 such a that the numbers will begin with a specified digit & end with a specified digit?

Ans: Fixed (1 number) Any 7 numbers Remaining 6 numbers Fixed (1 way) = 7x6 = 42

Q15. In how many ways 10 examination papers can be arranged so that best & worst paper never come together?

Ans: Best & worst paper never together = Total ways – 'always together' = 10! - (9! × 2!) = 9! (10-2) = 8.9!

Q16. There are 6 books on Economics, 3 on Mathematics and 2 on Accountancy. In how many ways can these be placed on a shelf if the books on same subject are to be together? [Ans: 3! × 6! × 3! × 2! = 51,840]

Q17. In Monday, how many of this arrangement begin with A & end with D?

Ans: Suppose all words begin with A & end with D. Remaining 4 Places can be filled in ${}^4P_4$= 4! Ways = **24 Ways.**

Q18. In Q24, how many arrangements are there in which vowels A & O occur together?

Ans: Vowels are A & O. Assume them as one unit. remaining 5 letters can be arranged in ${}^5P_5$ = 120 ways. These two vowels can be arranged amongst themselves internally in 2! = 2 ways. **Total numbers of ways = 2 x 120 = 240 ways.**

Permutations when Repetition is Allowed (n,r)= n^r [Every place can be filled in 'n' ways since repetition is allowed]

Q19. How many telephones connections may be allotted with 8 digits from 0 to 9?

(a) 10^8 (b) 10! (c) ${}^{10}C_8$ (d) ${}^{10}P_8$

Permutation of SIMILAR THINGS taken all at a time

- No. of ways in which 'n' things can be arranged taking all at a time, when 'p' things are similar of one type, 'q' things are similar of 2nd type, 'r' things are similar of 3rd type & remaining things are different = $\frac{n!}{p! \times q! \times r!}$

Q20. How many permutations can be made out of the letters of the word?

(i) MATHEMATICS [Ans: 11!/2!2!2!] (ii) COMMERCE [Ans: 8!/2!2!2!] (iii) EXAMINATION = [11!/2!x2!x2!]

Q21. (i) How many different words can be formed with the letters of the word BHARAT?
(ii) How many of these begin with B and End T? (iii) In how many of these B and H are never together?

Ans: (i) 6! /2! =360 (ii) 4! /2! =12 (iii) 360 – 120 = 240

CIRCULAR PERMUTATIONS

- Clockwise & anti-clockwise are different arrangements: $(n-1)!$ [Used in 'Sitting arrangement of Person']
- Clockwise & anti-clockwise are same arrangements: $\frac{(n-1)!}{2}$. [Used in 'Necklace & garlands' examples]

Q22. Number of ways 5 boys & 5 girls can be seated at a round table, so no two boys are adjacent is: [July 2021]
(a) 2,550 (b) 2,880 (c) 625 (d) 2,476 [Ans: Girls = (5-1)! & then Boys = 5! = **2880**]

SUM OF ALL NUMBERS FORMED OUT OF 'n' DIGITS ⇒ (n-1)! × Sum of digits × (1111....n times)

Q23. Compute the sum of 4 digits numbers which can be formed with the four digits 1, 3, 5, 7, if each digit is used only once in each arrangement.
Ans: (n-1)! × Sum of digits × (1111....n times) = (4-1)! × (1 + 3 + 5 + 7) × 1111 = 6.16.1111 = 106656.

PC Note: If digits include 'ZERO', Answer = (i) Solve as per above given formula including '0' - Solve by ignoring '0'

Q24. A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways can they be seated if (i) all the sisters sit together, (ii) no two sisters sit together?
Ans: (i) **5! × 3! ways = 720 ways** (ii) **${}^5P_3$ × 4! = 60 × 24 = 1440 ways.**

Q25. 6 boys & 5 girls are to be seated for a photograph in a row such that no two girls sit together and no two boys sit together. Find the number of ways in which this can be done.
Ans: 6! × 5!.

Q26. How many words can be formed with letters of 'ORIENTAL so that A & E always occupy odd places **[Aug 2007]**
(a) 540 (b) 8640 (c) 8460 (d) 8450

COMBINATIONS = SELECTION

- No. of combinations of 'r' different object out of 'n' different object = ${}^nC_r = \frac{n!}{(n-r)!\ x\ r!}$ [0 ≤ r ≤ n]

PROPERTIES OF ${}^nC_r$

1	${}^nC_r = \frac{nPr}{r}$ **Q27.** If ${}^{10}P_r$ = 6,04,800 & ${}^{10}C_r$ = 120. Find r. **[Ans:** r = 7**]**
2	${}^nC_r = {}^nC_{n-r}$
3	${}^nC_n = 1$ & ${}^nC_0 = 1$. Here r = n, $[{}^nC_n = \frac{n!}{(n-n)! \times n!} = \frac{n!}{0! \times n!} = 1]$
4	${}^nC_x = {}^nC_y$ ⇒ Either x = y or x + y = n **Q28.** Find 'r' if ${}^{18}C_r = {}^{18}C_{r+2}$ **Ans:** r cannot be equal to r + 2. Therefore r + (r +2) = 18 ⇒ 2r + 2 = 18 ⇒ 2r = 16 ⇒ **r = 8.**
5	${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ **Q29.** Find x if ${}^{12}C_5 + 2.{}^{12}C_4 + {}^{12}C_3 = {}^{14}C_x$ **Ans:** ${}^{12}C_5 + 2.{}^{12}C_4 + {}^{12}C_3 = {}^{12}C_5 + {}^{12}C_4 + {}^{12}C_4 + {}^{12}C_3 = {}^{13}C_5 + {}^{13}C_4 = {}^{14}C_5$ Thus ${}^{14}C_5 = {}^{14}C_x$ ⇒ Either x = 5 or x = 9.
6	${}^nC_r = \frac{n}{r}.{}^{(n-1)}C_{(r-1)} \Rightarrow {}^{10}C_3 = \frac{10}{3}.{}^9C_2$
7	${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_{(n-1)} + {}^nC_n = 2^n$ **Q30.** ${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$ = ___ [Ans: 31]

Q31. A person has 12 friends of whom 8 are relatives. In how many ways can he invite 7 guests such that 5 of them are relatives? **[Ans:** ${}^8C_5 \times {}^4C_2 = 336$ ways]

Q32. A committee of 7 members is to be chosen from 6 CAs, 4 Economists & 5 Cost Accountants. In how many ways can this be done if in committee, there must be at least one member from each group and at least 3 CAs?

Ans: The various methods of selecting the persons from the various groups are shown below:

	Committee of 7 members			
	C.A.s [Total 6]	Economists [Total 4]	Cost Accountants [Total 5]	Ways
Method 1	$3 \Rightarrow {}^6C_3$ ways = 20	$1 \Rightarrow {}^4C_1$ ways = 4	$3 \Rightarrow {}^5C_3$ ways = 10	800
Method 2	$3 \Rightarrow {}^6C_3$ ways = 20	$2 \Rightarrow {}^4C_2$ ways = 6	$2 \Rightarrow {}^5C_2$ ways = 10	1200
Method 3	$3 \Rightarrow {}^6C_3$ ways = 20	$3 \Rightarrow {}^4C_3$ ways = 4	$1 \Rightarrow {}^5C_1$ ways = 5	400
Method 4	$4 \Rightarrow {}^6C_4$ ways = 15	$1 \Rightarrow {}^4C_1$ ways = 4	$2 \Rightarrow {}^5C_2$ ways = 10	600
Method 5	$4 \Rightarrow {}^6C_4$ ways = 15	$2 \Rightarrow {}^4C_2$ ways = 6	$1 \Rightarrow {}^5C_1$ ways = 5	450
Method 6	$5 \Rightarrow {}^6C_5$ ways = 6	$1 \Rightarrow {}^4C_1$ ways = 4	$1 \Rightarrow {}^5C_1$ ways = 5	120

Therefore, total number of ways = 800 + 1200 + 400 + 600 + 460 + 120 = 3,570

Q33. A box contains 7 red, 6 white & 4 blue balls. How many selections of 3 balls can be made so that (a) all three are red (b) none is red (c) one is of each colour?

Ans: (a) ${}^7C_3 = 35$ **ways.** (b) ${}^{10}C_3 = 120$ **ways** (c) ${}^7C_1 \times {}^6C_1 \times {}^4C_1 = 7 \times 6 \times 4 = 168$ **ways.**

Q34. Find no. of ways of selecting 4 letters from word 'EXAMINATION'. **[Ans:** 136 ways]

SOME STANDARD RESULTS

- Total number of ways of forming a group by taking all of 'n' different things = $2^n - 1$
- Number of Diagonals of a polygon with 'n' sides = $\frac{n(n-3)}{2}$
- No. of Triangles from 'n' points = ${}^nC_3$
- No. of Triangles from 'n' points if 'm' points are collinear = ${}^nC_3 - {}^mC_3$
- No. of lines from 'n' points if 'm' points are collinear = ${}^nC_2 - {}^mC_2 + 1$.
- No. of parallelogram formed from 'm' parallel lines intersecting another 'n' parallel lines = ${}^mC_2 \times {}^nC_2$

FINDING RANK (POSITION) OF A WORD IN DICTIONARY [Trick will be given in Class]

Q35. Find the rank of 'KNIFE' in the dictionary.

Q36. If all permutations of word "CHALK" are written in a dictionary rank of this word will __.
(a) 30 (b) 31 (c) 32 (d) None

ARITHMETIC PROGRESSION (AP)

- A sequence in which **'difference between two consecutive terms'** is **same**. It is denoted by 'd'.
- **First term** is denoted by **'a'**. **Ex:** 2, 5, 8, 11, 14, 17 is an AP in which d = 3 is the common diference.
- **Common Difference** → $(T_2 - T_1)$ or $(T_3 - T_2)$ .... ⇒ $D = T_n - T_{n-1}$
- **Arithmetic Mean (AM)** → If a, b, c are in AP, then b - a = c – b. Thus b (AM) = $\frac{a+c}{2}$
- n^{th} **Term of AP** → $T_n = a + (n-1)d$ OR $T_n = S_n - S_{n-1}$ [Jo Number ki term nikalni hai, 'd' usse 1 kam rahega]

Q1. If the terms 2x, (x+10) and (3x+2) be in AP, the value of x is ____.

Q2. Arithmetic mean bet^n 33 & 77 = $\frac{33+77}{2}$ = 55.

Q3. Find the n^{th} term of the given AP 4,7,10...... [Ans: 3n+1]

Q4. If 10^{th} term of AP is twice the 4^{th} term & 23^{rd} term is 'k' times the 8^{th} term, then k = ______.

General Form of T_n = An + B [A & B are constants] → d = A & a = (A + B)

Q5. If T_n = 5n + 1, find AP.

PC Note: If 2 non-consecutive terms in AP (say T_m & T_n) & their values are given in question & you are asked to find out AP ⇒ $D = \frac{(T_m - T_n)}{m - n}$

Q6. If 5^{th} & 12^{th} terms of an AP are 14 & 35 respectively, find AP.

Q7. If 1^{st} term of AP is 5 & its 100^{th} term is -292, then T_{51} = (a) -142 (b) -149 (c) 155 (d) -145

INSERTION OF 'n' ARITHMETIC MEANS BETWEEN TWO NUMBERS

→ Total number of terms in the required AP will be (n+2).
→ Take 1^{st} given number as T_1 & 2^{nd} **given number as T_{n+2}** & use the above given note.

Q8. Two AMs between -7 & 14 is ___.

Ans: If we insert 2 AMs between -7 & 14, total number of terms will be 4. → **-7 AM_1 AM_2 14**

Take T_1 = -7; & T_{2+2} = 14; Thus T_4 = 14. Using the above note, (4-1)d = 14 – (-7) → 3d = 21 → **d = 7.**

AM_1 w= T_2 = a + d = -7 + 7 = 0 & $AM_2 = T_3$ = a + 2d = -7 + 2(7) = 7. So, 2 AM_s b/w -7 & 14 are **0 & 7.**

SUM OF FIRST 'N' TERM OF AP

$S_n = \frac{n}{2} \times (T_1 + T_n)$ OR $S_n = \frac{n}{2} \times [2a + (n-1)d]$

Q9. Sum of the series 9, 5, 1.... upto 100 terms = ___ [Ans: - 18900]

Q10. A sum of Rs. 6240 is paid off in 30 instalments such that each instalment is Rs. 10 more than the preceding instalment. The value of the 1^{st} instalment is______. (a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) None

Q11. Sum of AP whose first term is - 4 & last term is 146 is 7171. Find n. (a) 99 (b) 101 (c) 100 (d) 102

Q12. Sum of all natural numbers from 100 to 300 divisible by 4 or 5 = (a) 10200 (b) 15200 (c) 16200 (d) None

General Form of $S_n = An^2 + Bn$ [A & B are constants] → d = 2A & (a) = (A+B)

Q13. If S_n is $3n^2 + 5n$. Find AP.

Q14. P^{th} term of AP is $\frac{3p-1}{6}$. Sum of first n terms of AP is __. (a) n(3n+1) (b) $\frac{n}{12}$(3n+1) (c) $\frac{n}{12}$(3n-1) (d) None

Q15. If a, b, c are the sums of p, q, r terms respectively of AP, value of $\left(\frac{a}{p}\right)$(q-r) + $\left(\frac{b}{q}\right)$(r-p) + $\left(\frac{c}{r}\right)$(p-q) is ____.

(a) 0 (b) 1 (c) -1 (d) None

IMPORTANT SERIES ⇒ SUM OF	FORMULA	
1. 1st 'n' NATURAL No.	$\sum n = \frac{n(n+1)}{2}$	Q16. $1 + 2 + 3 + ...100 = \frac{n(n+1)}{2} = \frac{100(100+1)}{2}$
2. 1st 'n' ODD natural No.	$\sum(2n-1) = n^2$	Q17. $1 + 3 + 5 + 7 + 9 = 5^2 = 25$
3. 1st 'n' EVEN Natural No.	$\sum 2n = n(n+1)$	Q18. $2 + 4 + 6 + 8 + 10 = n(n+1) = 5(6) = 30$
4. SQUARE of 1st 'n' Natural No.	$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$	Q19. $1^2+2^2+...100^2 = \frac{n(n+1)(2n+1)}{6} = \frac{100(100+1)(200+1)}{6}$
5. CUBES of 1st 'n' Natural No.	$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$	Q20. $1^3+2^3+3^3...100^3 = \left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{100(100+1)}{2}\right]^2$

Q21. Value of $n^2 + 2n[1+2+3+ .....+(n-1)]$ is ______. (a) n3 (b) n2 (c) n (d) None

Q22. Value of $11^2 + 12^2 + 13^2 ...... 19^2 + 20^2 =$ (a) 3845 (b) 2485 (c) 2870 (d) 3255

- If 3 numbers are given in AP, Put 1st no = 1; 2nd no = 2; & 3rd no. = 3; (If necessary).
- If a, b, c are in AP → Put their value as 1, 2, 3 in options & get the answer.
- If a^2, b^2, c^2 are in AP → Put value as 1, 5, 7 in options & get answer [1,25,49 → AP]
- If we form a series from the reciprocal of all the terms of AP, it becomes HP.

Q23. If a, b, c are in AP, then value of $\frac{(a^3+4b^3+c^3)}{b(a^2+c^2)}$ (a) 1 (b) 2 (c) 3 (d) None

Properties of AP	Examples
1. If $S_n = S_m \rightarrow S_{(m+n)} = 0$	If $S_7 = S_{11} \rightarrow S_{18} = 0$
2. $T_p = \frac{1}{q}$ & $T_q = \frac{1}{p}$; → $T_{pq} = 1$ & $S_{pq} = \frac{pq+1}{2}$	$T_3 = \frac{1}{2}$ & $T_2 = \frac{1}{3}$; → $T_6 = 1$ & $S_6 = \frac{6+1}{2} = \frac{7}{2}$
3. If $S_p = q$ & $S_q = p \rightarrow S_{(p+q)} = -(p+q)$	If $S_7 = 11$ & $S_{11} = 7$, → $S_{18} = -(11+7) = -18$
4. If $T_p = q$ & $T_q = p$; then $T_r = (p + q - r)$	5. If $T_p = q$ & $T_q = p$; then $T_{(p+q)} = 0$.

GEOMETRIC PROGRESSION (GP)

- A sequence in which 'any term divided by its preceding term' is same. It is denoted by 'r'.
- Common Ratio → $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$ ----- = $\frac{T_n}{T_{n-1}}$
- $T_n = a \times r^{(n-1)}$
- Geometric Mean (GM) → If a, b, c are in GP, then b/a = c/b ⇒ $b^2 = a \times c$

Q24. Find 8th term of series 4, 8, 16 ...... is [**Ans:** 512]

Q25. 10th term of the G.P. $\frac{1}{2}$, 1, 2, 22, ... is [**Ans:** 256]

Q26. The last term of the series x^2, x, 1, ... to 31 terms is [**Ans:** $1/x^{28}$]

Q27. Which term of the G.P. series ¼, -1/2, 1....is -128?

Q28. The number of terms in 6,18,54, ... upto 1458 is _____.

Q29. If (k+9), (k-6) & 4 forms three consecutive terms of a G.P, then the value of 'k' is_____.

PC Note: If two non-consecutive terms in GP (say T_m & T_n) & their values are given in question & you are asked to find out GP $\Rightarrow r^{(m-n)} = \frac{T_m}{T_n}$

Q30. Find GP where T_3 is 36 & T_5 is 324. [GP = 4, ±12, ±36, ± 108.......]

Insertion of 'n' Geometric Means b/w 2 Numbers

→ Total number of terms in the required GP will be (n+2).

→ Take 1st given number as T_1 & **2nd given number as T_{n+2}** & use the above given note.

Q31. Insert 3 geometric means between 1/9 & 9.

Ans: Insert 3 GMs between 1/9 & 9, total number of terms will be 5 → 1/9, GM_1, GM_2, GM_3, 9.

Take T_1 = 1/9; T_5 = 9. Thus r^{5-1} = 9/1/9; **r^4 = 81; & thus r = 3.**

GM_1 = 1/9 × 3 = **1/3**, GM_2 = 1/3 × 3 = **1**, GM_3 = 1 × 3 = **3.** **GP will be 1/9, 1/3, 1, 3, 9.**

SUM OF FIRST 'N' TERM OF GP		SUM OF INFINITE GP
If r < 1 $\Rightarrow S_n = a \times \frac{1-r^n}{(1-r)}$	r > 1 $\Rightarrow S_n = a \times \frac{r^n-1}{(r-1)}$	$S_\infty = \frac{a}{1-r}$

PC Note: If r = 1, it will be an Equal series [GP & AP also]. Sum (if r = 1) $\Rightarrow$ n.a [a is term of series]

Q32. Sum of first two terms of a GP is $\frac{5}{3}$ & sum to infinity of the series is 3. Common ratio =

Q33. Sum of first 20 terms of a GP is 244 terms the sum of its first 10 terms. Common ratio = [Ans: $\pm\sqrt{3}$]

Q34. Sum upto ∞ of the series $8 + 4\sqrt{2} + 4...$ = (a) $8(2 + \sqrt{2})$ (b) $8(2 - \sqrt{2})$ (c) $4(2 + \sqrt{2})$ (d) $4(2 - \sqrt{2})$

Q35. If $x = a + \frac{a}{r} - \frac{a}{r^2} + ... \infty$, $y = b - \frac{b}{r} + \frac{b}{r^2} ....\infty$, $z = c + \frac{c}{r} + \frac{c}{r^3} + ... \infty$; Value of $\frac{xy}{z} - \frac{ab}{c}$ is __

ASSUMPTIONS OF THE TERMS IN GP

If No. of terms given in question are	Middle Term	r	Examples of Terms
ODD No. of terms	a	r	**3 terms:** (a/r), a, (ar) **5 terms:** (a/r^2), (a/r), a, (ar), (ar^2)
EVEN No. of terms	(a/r) & (a.r)	r^2	**2 terms:** (a/r) & (ar) **4 terms:** (a/r^3), (a/r), (ar), (ar^3)

PC Note: But we will go by **OPTION METHOD** in such type of questions **TO SAVE TIME.**

Q36. Product of first three terms of GP is 27/8. Middle term = __. (a) 3/2 (b) 2/3 (c) 2/5 (d) None

- If a, b, c OR a^2, b^2, c^2 are in GP → Put a, b, c value as 1, 2, 4 in options & get the answer.
- Log of all terms of a GP, it will become AP.
- If there are 'n' terms in a GP, m^{th} term from the end will be $(m-n+1)^{th}$ term from the start.
  Ex: If there are 7 terms in a GP, 2^{nd} term from the end will be $(7-2+1)^{th}$ term from the start.

Q37. If a, b, c are in AP; a, x, b are in GP & b, y, c are in GP. Then x^2, b^2, y^2 are in __ (a) AP (b) GP (c) HP (d) None

Q38. Sum of n terms of the series 4 + 44 + 444 +.... = (a) $\frac{4}{9}\left\{\frac{10}{9}(10^n - 1) - n\right\}$ (b) $\frac{10}{9}(10^n - 1) - n$ (c) 1 (d) 0

Q39. Sum upto infinity of the series $(1+2^{-2}) + (2^{-1}+2^{-4}) + (2^{-2}+2^{-6}) + ...$ = (a) 7/3 (b) 3/7 (c) 4/7 (d) None

SET THEORY & VENN DIAGRAM

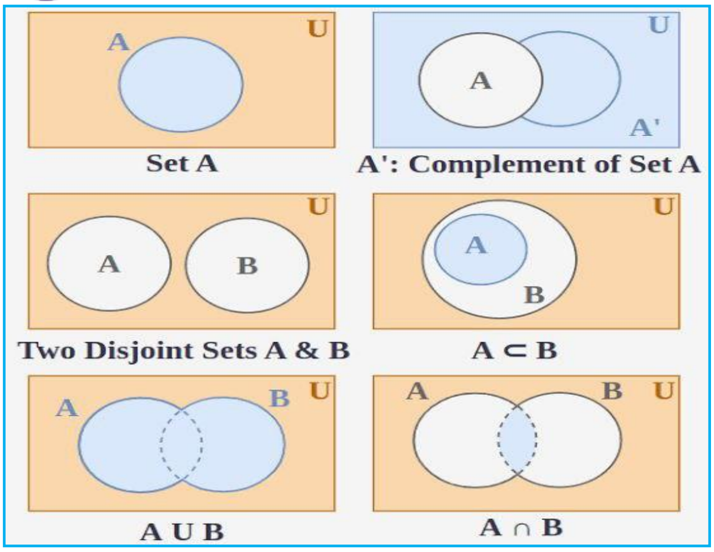
- *Sets:* A set is a *well-defined collection of objects.*
- *Element:* *Each object in a set* is called an element of the set.
- A *set* is denoted by *'capital letters'* & *their elements* are denoted by *'small letters'.*

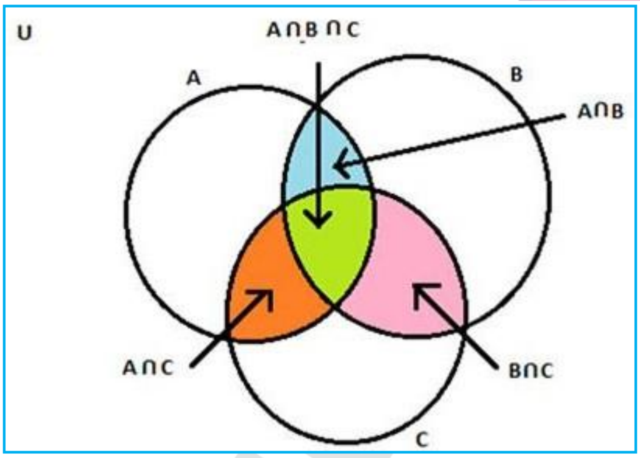
Ex: Set A = {a, e, i, o, u} 'a' is an element of Set A & we write a∈A & read as 'a' belongs to 'A'. But 3 is not an element of A & we write b ∉ B & read as '3' does not belong to 'B'.

> ☑ *Repetition* of elements in a set is *MEANINGLESS.*
> ☑ *Order* of the elements in a set is *NOT RELEVANT.*

TYPES OF SETS

1	Universal Set: A set containing *all possible elements.*
2	Null Set: Set having *NO Element* [Denoted by { } or ∅] (AKA - Empty set/void set) *Ex: A = {x: x is odd no. divisible by 2} = ∅*
3	Singleton Set: A set having *only one element* *Ex:* A = {5}
4	Equal Set: If *every element of A is in B* & *every element of B is in A*, A & B are equal sets. *Ex:* If A = {2, 4, 6} and B = {6, 2, 4} then Set A = Set B. *[Order of element is NOT relevant]*
5	Equivalent Set: If *Number of Elements* in Set A & Set B are *SAME.* [n(A) =n(B)] *Ex:* A = {a, b, c} & B = {1, 2, 3}; n(A) = 3 & n(B) = 3, A & B are equivalent sets.
6	Subset: *If all the elements of set A are present in Set B, A is a subset of B.* [A ⊆ B] *Ex:* A = {1, 2} & B = {1, 2 ,3} then A is subset of B. *[B is said to be a superset of A]* ⇨ PC Note: In subset, there exist an Equal set & Null set also. *Ex:* A = {1, 2, 3} *Subset of A include* {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3} & { } Number of Subsets of a set = 2^n [where 'n' = Number of elements]
7	Proper Subset: If *Set A* is a *subset of Set B but not equal set.* [A ⊂ B] *Ex:* A = {1, 2, 3}; *Proper Subset of A includes* {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3} & { }. ⇨ PC Note: Proper Subset does not include Equal set. *Thus, A Null set does not have a Proper subset.* Number of Subsets of a set = $2^n - 1$ [where 'n' = Number of elements]
8	Power Set: *Set of all subsets of a set is called Power set.* *Power set of A* = { {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, { } }.
9	Disjoint Sets: If Set A & Set B has NO Common element, they are disjoint Sets. [A∩B = ∅]
10	Union of Sets (A∪B): It contains all elements which are EITHER in Set A OR in Set B.
11	Intersection of Sets (A∩B): It contains all the elements which are in Set A AND Set B.
12	Complimentary Set (A'): Set of elements which are in Universal set but not in Set A.
13	Difference of Sets (A-B): Set of elements which are in Set A but not in Set B [Sirf A me hona] B – A: Set of elements which are in Set B but not in Set A. [Sirf B me hona] ⇨ PC Note: n(A-B) = n(A) - A∩B & n(B-A) = n(B) - A∩B *Ex:* If A = {1, 2, 3, 5, 7} & B = {1, 3, 6, 7, 15} *A − B* = {2, 5} & *B − A* = {6, 15}





A Δ B = (A-B) ∪ (B-A) [Symmetric Difference of A&B]

Q1. U = {1, 2, 3, 4, 5, 6, 7, 8, 9}; P = {2, 4, 6, 8}; Q = {1, 2, 3, 4, 5}

(i) P∪Q = {1, 2, 3, 4, 5, 6, 8}; (ii) (P∪Q)' = {7, 9} (iii) P∩Q = {2, 4} (iv) P' = {1, 3, 5, 7, 9}

(v) (P∩Q)' = {1, 3, 5, 6, 7, 8, 9}; (vi) Q' = {0, 6, 7, 8, 9}; (vii) P − Q = {6, 8} (viii) Q − P = {1, 3, 5}

A∪B = B∪A	(A∪B)∪C = A∪(B∪C)	A∩(B∪C) = (A∩B) ∪ (A∩C)	(A∪B)' = A' ∩ B'	A ∩ A' = ∅
A∩B = B∩A	(A∩B)∩C = A∩(B∩C)	A∪(B∩C) = (A ∪ B) ∩ (A ∪ C)	(A∩B)' = A' ∪ B'	A ∪ A' = U
A ∩ A = A	A ∪ A = A A ∪ ∅ = A	A ∩ U = A	PC DON'T RECOMMEND LEARNING THESE FORMULA	

Q2. If A = {a, b, c, d, e, f} & B = {a, e, i, o, u} & C = {m, n, o, p, q, r, s, t, u} then

(i) A∪B = _____ (ii) A∪C = _____ (iii) B∪C = _____ (iv) A - B = _____ (v) A∩B = _____

(vi) B∩C = _____ (vii) A∪(B - C) = _____ (viii) A∪B∪C = _____ (ix) A∩B∩C = _____.

VENN DIAGRAM

[Do Not Use these Formula, Use Venn Diagram]

- **n(A∪B) = n(A) + n(B) – n(A∩B)** OR n(A∪B) = n(A-B) + n(B-A) + n(A∩B)
- **n(A∪B∪C) = n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(C∩A) + n(A∩B∩C).**
- **n(A) = n(A-B) + n(A∩B)** & n(B) = n(B-A) + n(A∩B)
- **n(A Δ B)** = No. of elements which belongs to **exactly one** of A or B = n(A) + n(B) - 2n(A∩B).
- No. of elements in **exactly two** of the sets A, B, C = n(A∩B) + n(B∩C) + n(C∩A) - 3n(A∩B∩C).
- No. of elements in **exactly one** of three sets = n(A) + n(B) + n(C) – 2n(A∩B) – 2n(B∩C) – 2n(C∩A) + 3n(A∩B∩C)

Q3. 74% of Indians like grapes, 68% like bananas. What % of Indians like both grapes & bananas? [Ans: 42%]

Q4. In a class of 60 students, 40 students like Maths, 36 like Science & 24 like both the subjects. Find the number of students who like (i) Maths only (ii) Science only (iii) Maths or Science (iv) Not Maths & Science.

Ans: (i) Maths only = 16 (ii) Science only = 12 (iii) Maths or Science = 52 (iv) Not Maths & Science = 8

SCAN ME

VENN DIAGRAM – MOST LOGICAL EXPLANATION

JUST WATCH THIS LECTURE & YOU WILL NOT HAVE TO LEARN ANY OF THE FORMULA OF SET THEORY TO SOLVE THE QUESTION.

This QR Scanner Contains the Link of a Lecture of our Full Course in which Venn Diagram Along with **15 Past Exam Questions** has been Discussed.

CARTESIAN PRODUCT SET

- **Ordered Pair:** Two elements 'a' & 'b', listed in a specific order, form an ordered pair. It is denoted by (a, b).
- **Set of all ordered pairs** (a, b) such that $a \in A$ & $b \in B$, is called Cartesian product of A & B. It is denoted by $A \times B$. Thus, $A \times B = \{(a, b) \text{ such that } a \in A \text{ \& } b \in B\}$.
- **PC Note:** $(a, b) \neq \{a, b\}$. If $(a, b) = (c, d)$, it means that $a = c$ & $b = d$.
- In set, repetition of elements is meaningless. But for ordered pairs, (5, 5) means 5 belongs in both the sets.
- Cardinal Number = Number of elements in a set → Denoted by $n(A)$ → $n(A \times B) = n(A) \times n(B)$

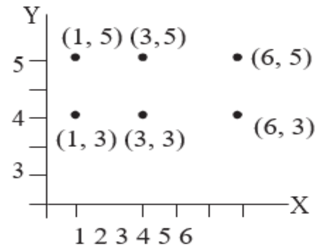
Q1. If P = {1, 3, 6} & Q {3, 5}. Find P × Q & Q × P.

Ans: $P \times Q = \{(1, 3), (1, 5), (3, 3), (3, 5), (6, 3), (6, 5)\}$;

$Q \times P = \{(3, 1), (3, 3), (3, 6), (5, 1), (5, 3), (5, 6)\}$

It is noted that ordered pairs (3, 5) & (5, 3) are not equal.

So, $P \times Q \neq Q \times P$; but $n(P \times Q) = n(Q \times P)$.



RELATIONS

- **Any subset of the product set A × B** is called a relation from A to B. It is denoted by R. $R \subseteq A \times B$
- **Domain of a Relation** = Set of all first elements of ordered pair.
- **Range of a Relation** = Set of all second elements of ordered pair.

⇒ **Dom (R) = {a: (a, b) ∈ R}**
⇒ **Range (R) = {b: (a, b) ∈ R}**

Q2. Set A = {1, 2, 3} & Set B = {2, 4, 6}

☞ $A \times B = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)\}$
☞ Every subset of the product set A × B is called a relation from A to B.
☞ Now, we consider the relation which is the subset of A × B. Let R = {(1, 2), (1, 4), (3, 2), (3, 4)}.
☞ **Domain of R = 1st Elements = {1, 3} & Range of R = 2nd Element = {2, 4}**

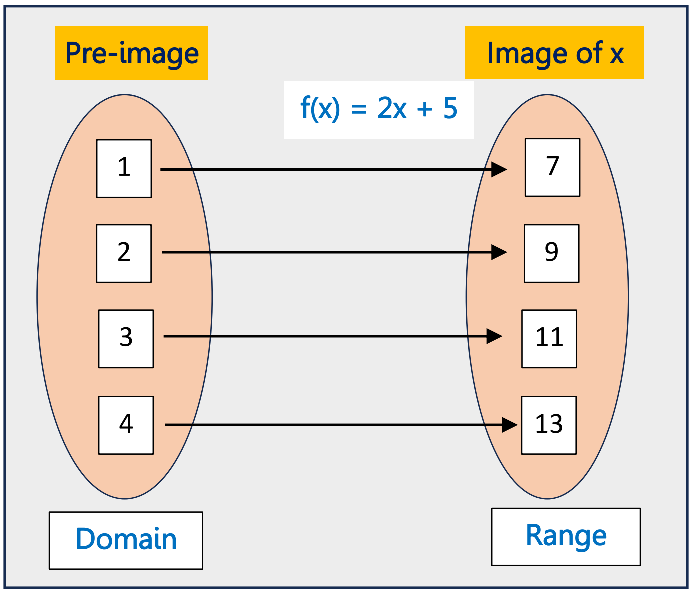
TYPES OF RELATIONS

1	**Identity Relation:** If both elements of ordered pairs are same, it is an identity relation. [I = {(a, a): a ∈ A}] Ex: Let A = {1, 2, 3} then I = {(1, 1), (2, 2), (3, 3)}
2	**Reflexive Relation:** R is reflexive relation if (a, a) ∈ R & a = a. **PC Note:** R is reflexive if it contains **ALL POSSIBLE ORDERED PAIRS** of the type (x, x). Ex: Let A = {1, 2, 3}; If R = {(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)}; It is a reflexive relation because all possible ordered pair of the form (x, x) are present in the given relation. If R = {(1,1), (1,3), (2,3), (3,1), (3,3)} is NOT a reflexive relation because (2,2) is missing in R.
3	**Symmetric Relation:** If (a, b) ∈ R; then (b, a) should also ∈ R. [PC NOTE: Reverse pair bhi hona Relation me] Ex: R = {(1,1), (1, 3), (1,2), (2,1), (3,1)}.
4	**Transitive Relation:** R is transitive relation if (a, b) ∈ R & (b, c) ∈ R, then (a, c) should ∈ R.
5	**Equivalence relation:** A relation which is reflexive, symmetric & transitive. [Ex: Parallel & Is Equal to]
6	**Inverse Relation:** R is a relation from A to B, then relation R^{-1} from B to A = {(b, a): (a, b) ∈ R}. Dom of (R^{-1}) = Range of (R) & Range of (R^{-1}) = Dom of (R). Ex: Let A = {1, 2, 3} & If R = {(1, 2), (2, 2), (3, 1), (3, 2)}; then R^{-1} = {(2, 1), (2, 2), (1, 3), (2, 3)} Dom of (R) = {1, 2, 3} & Range of (R) = {2, 1} & Dom (R^{-1}) = {2, 1} & Range (R^{-1}) = {1, 2, 3}
7	**Void Relation:** A relation R is a void relation if R = ∅ Ex: Let A = {7, 11} and B = {3, 5}. Let R = [(a, b): a ∈ A, b ∈ B, a − b is odd}, then R = ∅
	PC Note: A partial order relation is any relation that is reflexive, antisymmetric, and transitive.

FUNCTIONS

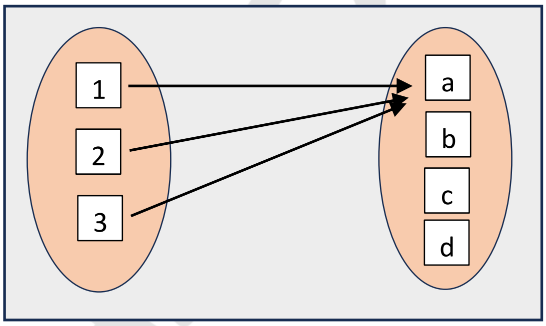
सबकी Image होना & Sirf 1 होना

- *Function = Any relation from X to Y in which* ***two different ordered pairs should not have same first element.***
- *If any ordered pair of a relation have same first element, then such relation is not a function.* ***[f: A→B]***



Function/Mapping [f: x → y]

1. **All Elements in 'x' ⇒ सबकी Image होना 'y' me.**
2. **All Elements in 'x' ⇒ Sirf 1 Image होना 'y' me.**



Ex: Let A = {1, 2, 3, 4} & B = {1, 2, 3}.
R = Subset {(1, 2), (1, 3), (2, 3)} ***is a relation on A×B.*** *So, it is "less than" relation since A < B in all ordered pairs*
This relation is ***not a function*** *because it includes two different ordered pairs (1,2), (1,3) have same 1st element.*

Q3. Which of these is a function from A → B A = {x, y, z} B = {a, b, c, d} **[Ans: C]**

(a) {(x,a) (x,b) (y,c)} (b) {(x,a) (x,b) (y,c) (z,d)} (c) {(x,a) (y,b) (z,d)} (d) {(a,x) (b,z) (c,y)}

Q4. If $f(x) = x^2 - 5$, evaluate f (3), f (-4), f (5) and f(1). **[Ans: C]**

(a) 0, 11, 20, 4 (b) -4, 11, -2, 4 (c) 4, 11, 20, - 4 (d) 4,10, 20, 5

Q5. If $f(x) = 2^x$, then f (x+y) = __ [Nov 2007]

(a) f(x) + f(y) (b) f(x).f(y) (c) f(x) ÷ f (y) (d) None

Q6. f(x) = (2x + 3), then the value of f(2x) - 2f(x) + 3 =

(a) 3 (b) 2 (c) 1 (d) 0

Q7. If $f(x-1) = x^2-4x + 8$, then f(x+1) = __

(a) x^2+8 (b) x^2+7 (c) x^2+4 (d) $x^2- 4x$

Q8. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ = __

(a) f(x) (b) 2f(x) (c) 3f(x) (d) -f(x)

Q9. If f(x) = then $\frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)}$ = _____

(a) x/y (b) y/x (c) -x/y (d) -y/x

TYPES OF FUNCTIONS

1	One - One function (Injective function) ⇒ Every element in Set A have different images in Set B. Ex: Let A = {1, 2, 3} & B = {2, 4, 6}. ⇒ Thus, function is f: A→B: **f(x) = 2x.**
2	Many-one function ⇒ If two or more elements in A have same image in B. Ex: $f(x) = x^2$; x ∈ R ⇒ $f(1) = (1)^2 = 1$ & $f(-1) = f(-1)^2 = 1$
3	Onto function (Surjective function) ⇒ If all the element in B has at least one pre-image in A. Ex: A = {1, 2, 3} & B = {a, b}. Let f = {(1, a), (2, a), (3, a)}. Since 'b' does not have any pre image, it is **not onto** function. It is an **Into** Function.
4	Into function ⇒ If at least one element in B has no pre-image in A.
5	Bijective Function ⇒ One-One onto function.
6	Constant Function ⇒ All the elements in 'A' have same image in 'B'. ▪ Range of a constant function = Singleton set. Ex: Let f(x) = {(1, 3), (2, 3), (3, 3), (4, 3)}.
7	Identity function ⇒ If every element in A is mapped to itself (has same image), it is an identity function. ▪ It is a one-to-one onto function with domain A and range A. Ex: Let x = {1, 2, 3, 4} then f(1) = 1; f(2) = 2; f(3) = 3; f(4) = 4 is an identity function.
8	Equal Function ⇒ Two functions f(x) & g(x) are said to be equal if (i) they have same domain; (ii) f(x) = g(x). Ex: Let $f(x) = x^2$, ∀ x ∈ R & $g(y) = y^2$, ∀ y ∈ R. Then two function f & g are equal.
9	Inverse Function ⇒ If f(x) = y; then $f^{-1}(y) = x$. [Don't even dare to see the definition] PC Tips to find Inverse Function 1. Substitute f(x) = y. 2. Find the value of x in terms of y. 3. Replace 'x' with $f^{-1}(x)$ & 'y' with x. 4. The resultant will be the answer. Q9. f(x) = 2x. Find $f^{-1}(x)$. Ans: Step 1: Let f(x) = y. Thus y = 2x; Step 2: x = y/2; Step 3: $f^{-1}(x) = x/2$. Q10. If $f(x) = \frac{2+x}{2-x}$, then $f^{-1}(x)$: June [2008] (a) $\frac{2(x-1)}{x+1}$ (b) $\frac{2(x+1)}{x-1}$ (c) $\frac{x+1}{x-1}$ (d) $\frac{x-1}{x+1}$
10	Composite Function ⇒ Function of a Function PC Tips to find Composite Function ➔ **f[g(x)]:** Replace 'x' with g(x) in f(x). ➔ **g[f(x)]:** Replace 'x' with f(x) in g(x). Q11. Let f(x) = 2x & $g(x) = 3x^2$. Find f[g(x)] & g[f(x)]. Ans: (i) f[g(x)] = Replace 'x' with g(x) in f(x); $f[g(x)] = 2(3x^2) = 6x^2$. (ii) g[f(x)] = Replace 'x' with f(x) in g(x); $g[f(x)] = 3(2x)^2 = 12x^2$. Q12. If $f(x) = x^2 - 1$ & $g(x) = \|2x + 3\|$, then fofg(3) – gof(-3) = ? [July 2021] (a) 71 (b) 61 (c) 41 (d) 51