Unit - 2

Relations And function

⇒ Cartesian Product of two sets

ordered pair

$(2,4) \neq (4,2)$

↑ ordered pair ↑ unorder pair

(x,y) & (y,x) are two different things

$A = \{1,2\}$
$B = \{Alif, Laila\}$

$A \times B = \{(1, Alif), (1, Laila), (2, Alif), (2, Laila)\}$

$B \times A = \{(Alif, 1), (Alif, 2), (Laila, 1) (laila, 2)\}$

$$A \times B \neq B \times A$$

$A \times B \neq B \times A$ (In General)

$n(A \times B) = n(B \times A) = n(A) \times n(B)$

If $A \times B = \phi$ then $A = \phi$ or $B = \phi$

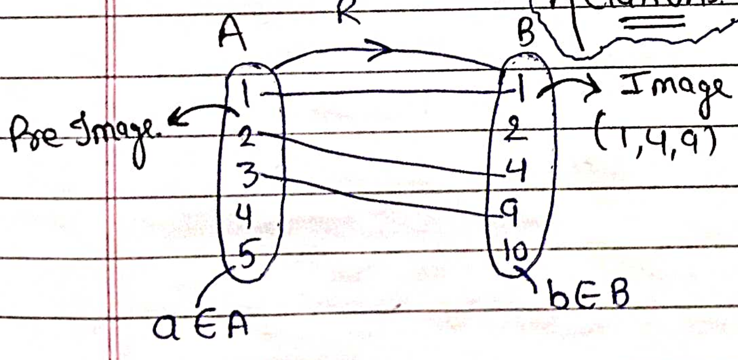
$A = B = \phi$

eg → $A = \{1,2,3\}$ $n(A) = 3$
$B = \{x, y\}$ $n(B) = 2$

$A \times B = \{(1,x) (1,y), (2,x), (2,y), (3,x), (3,y)\}$

$n(A \times B) = n(A) \times n(B) = 3 \times 2 = 6$

Relations



Generally denoted by R.

Relation will be defined between two sets.

$R : A \to B$,

$R = \{(a,b) : a \in A, b \in B, a^2 = b\}$

$R = \{(1,1), (2,4), (3,9)\}$

Domain of Relation

$D_R = \{1,2,3,4,5\}$

Co - Domain of Relation

$Co\text{-}D_R = \{1,2,4,9,10\}$

Range of Relation

$R_{R_1} = \{1,4,9\}$

Domain
↓
Set of all Pre Images.

Co-Domain
↓
Complete Set B

Range
↓
Set of all Images

$\boxed{\text{Range} \subseteq \text{Co-Domain}}$

Range is a subset of Co-Domain

Any Relation $R: A \to B$ is a subset of $A \times B$
↑
Cartesian Product.

If $n(A) = p$, $n(B) = q$
$n(A \times B) = p \cdot q$
Total No. of subset of $A \times B = 2^{p \cdot q}$

Total No. of Relations that can be defined in Set A and $B = 2^{m \times n}$

functions

Idea of functions.

\# functions are basically special type of Relations.

\# No extra button.

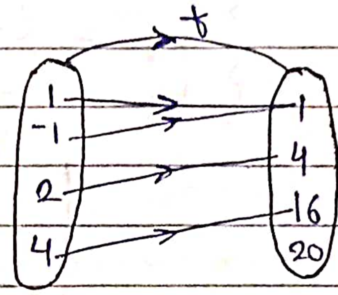
Define -

\# function can be defined between two sets.

\# $f: A \to B$

elements of set A → Inputs (x)

elements of set B → Output (y)



\# functions are the mappings which maps each and every elements of set 'A' to give a unique element of set B.

$f(x) = x^2$ or $y = x^2$

"All Relations are not functions but, All functions are Relation."

Types of function

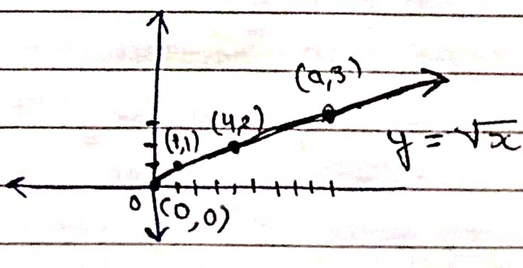
1. Square Root function. represented by $f(x) = \sqrt{x}$ / $y = \sqrt{x}$

$\sqrt{x}$ ← No Negative input
← No Negative output.

① Domain$_f$
② Range$_f$
③ Graph$_f$

$$\left\{ \begin{array}{l} D_f = [0, \infty) \\ R_f = [0, \infty) \end{array} \right\}$$

Graph of functions :
⇒ Pictorial representation



x	y
0	0
1	1
4	2
~~6~~ 9	3

Variable

exponent | Power

2. Exponential function :- $\boxed{f(x) = a^{x}}$

(constant) base
positive hona chahiye
(−) nhi

$0^2 = 0$

0^{-2} = not define

0^0 = not define

$1^5 = 1$

$2^0 = 1$

$f(x) = a^x \quad (a > 0, a \neq 1)$

$f(x) = e^x$

$D_f = (-\infty, \infty)$

R_f : ~~$(-\infty, \infty)$~~ $(0, \infty)$

$2^{-\infty} = 0^+$

$\vdots \quad \vdots$

$2^{-2} = \frac{1}{4}$

$2^{-1} = \frac{1}{2}$

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

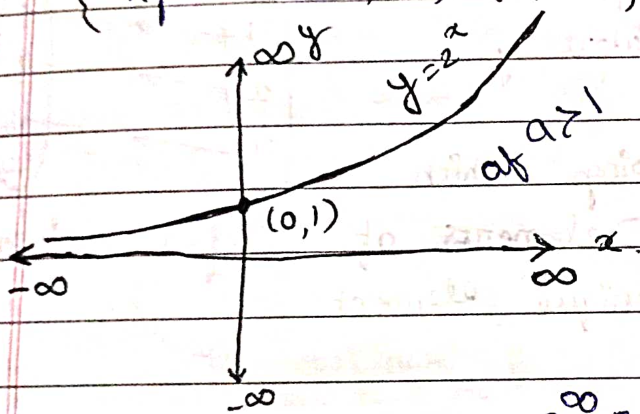
$\vdots \quad \vdots$

$2^{\infty} = \infty$

exponential function zero nhi hota

There is no negative output

output can not be zero.



$2^{\infty} = \infty$

$2^{-\infty} = 0$

$2^{-\infty} \neq 0$

$e^{-\infty} \approx 0$

$2^{\infty} = \infty$

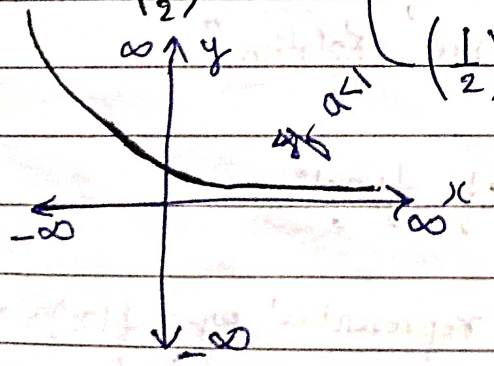
$2^{-\infty} = 0$

$\left(\frac{1}{2}\right)^{\infty} \to 0 \qquad \left(\frac{1}{2}\right)^{-\infty} = \infty$

$g(x) = \left(\frac{1}{2}\right)^x$

$D_f = (-\infty, \infty)$

$R_f = (0, \infty)$



If $\log_a x = y$

the $a^y = x$

2^0 = Not defined
Input of log can't be zero.

③ Logarithmic functions $\longrightarrow$ Inverse functions of exponentiation are called logarithmic function

$f(x) = \log_a x$ → Input/no. (x) → base of logarithmic (a)

$x > 0$; $a \neq 1$
$a > 0$

read as log 'x' with base 'a'.

Eg $g(x) = \log_2 x$

$\log_2 2 = 1$ (खोपड़ी) answer hai!!

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^0 = 1$

$\log_2 4 = 2$

$\log_2 8 = 3$ $\quad \log_2 \frac{1}{2} = -1$

$\log_2 16 = 4$ $\quad \log_2 \frac{1}{4} = -2$

$\log_2 1 = 0$

$\log_2 32 = 5$ $\quad \log_2 \frac{1}{16} = -4$ $\quad \log_{\frac{1}{2}} 4 = -2$

$\log_{\frac{1}{4}} \frac{1}{2} = \frac{1}{2}$ $\quad \log_2 \sqrt{2} = \frac{1}{2}$ $\quad \log_{(\frac{1}{2})} 2^{-2} = 4$

Input of log can'nt be negative

$\log_2 (-4)$ = Not defined

Input of logarithmic must always be (+ve).
Input can never be negative or zero

but output can be Positive, negative or zero.

$\log_1 4$ = Not defined
→ base of log can never be '1'

$\log_{(-2)} 4$
→ base of log can never negative

Base of log must always be positive expt '1'

$\pi = 3.14 \ldots\ldots$
$e = 2.718 \ldots\ldots$

natural log.

$$\left\{ \begin{array}{l} D_f = (0, \infty) \\ R_f = (-\infty, \infty) \end{array} \right\}$$

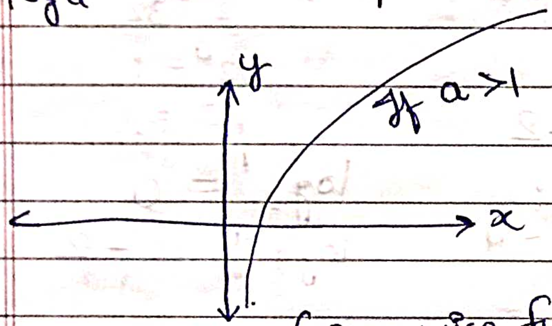
$\left.\begin{array}{l}\log \text{ natural } x \\ \ln x\end{array}\right\}$

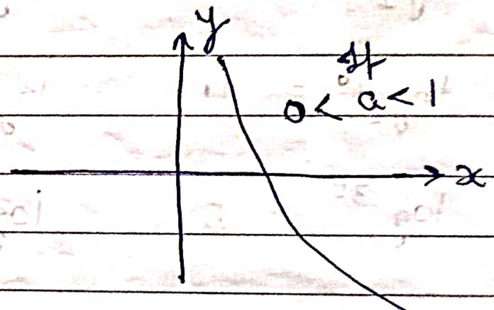
$\log_{10}{x} \} \to$ common log

Imp
$$\left\{ \begin{array}{l} \log_{10}{2} = 0.3010 \\ \log_{10}{3} = 0.4771 \\ \cancel{\log} \ \ln 10 = 2.303 \end{array} \right\}$$

Graph of Logarithmic function.

$y = \log_a{x}$





(Step wise function)

4 Greatest Integer function :- $f(x) = [x]$ ← Square bracket

[ ] ← Square brakets

This brackets [.] denotes G.I.f

$[2] = 2$ read as {greatest Integer 'x'} or {bracketed 'x'}
$[3] = 3$
$[4] = 4$
$[-5] = -5$
$[0] = 0$
$[2.1] = 2$
$[3.4] = 3$
$[-5.7] = -6$
$[-0.8] = -1$

$D_f = (-\infty, \infty)$
$R_f = (Z)$
↑ all Integer

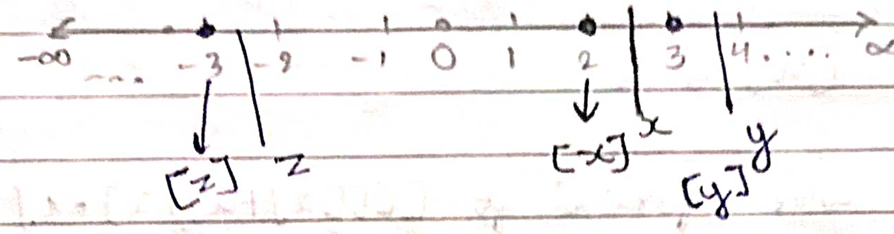
$[\sqrt{2}] = 1.414 = 2$ $\qquad [-2020.5] = -2021$

$[\sqrt{3}+1] = 1.732+1 = 2.$

3.1415 ← $[\pi] = 3$ $\qquad$ Visual Meaning of G.I.F.

$[e] = 2.$

Kisi bhi no. ka chotaa...



$[x]$ is the greatest Integer less than or equal to 'x'

$[3.7] = 3, 2, 1, 0, -1 \ldots$

$[-4.5] = -5, -6, -7, -8 \ldots$

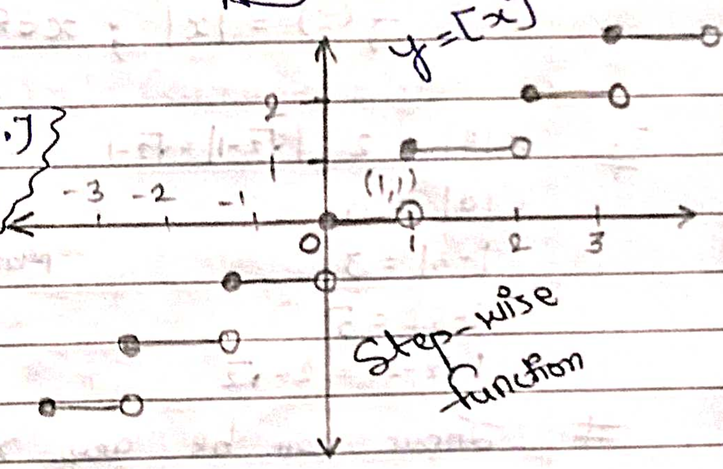
$f(x) = [x]$

\# $[x] \le x.$

$D_f = R$ [All real no.]

$R_f = Z$ [All Integer]

Graph →



Properties of G.I.F.

① $[[[x]]] = [x]$

eg. $[2.1] = 2$, $[[2.1]] = 2$

② $[x+I] = [x]+I$, if I is an Integer

eg. $[3.8+5] = [8.8] = 8$

$[3.8]+5 = 3+5 = 8$

③ $[x]+[-x] = \begin{cases} -1 & ; x \notin Z \text{ (Integer)} \\ 0 & ; x \in Z \end{cases}$

Eg. $[5]+[-5]=0$ $\quad$ $[3.1]+[-3.1]=-1$

$[x]+[-x]=0$ $\quad$ $[4.8]+[-4.8]=-1$

↓ $\quad$ ↓

Integer. $\quad$ Not an integer

(4) $[x] \leq x$

Que Solve for 'x' st $[[[[x]+x]+x]+x]>17$ where $[\cdot]$ denotes G.I.F.

5. Modulus function (Absolute value function)

↓

$f(x)=|x|$; $x \in R$

$|-2| \rightarrow$ Modulus.

$|2+5i| \rightarrow$ Magnitude of Complex No.

$|\vec{A}| \rightarrow$ Magnitude of vectors

$|A| \rightarrow$ Determinant. (A ← Matrix)

$|A| \rightarrow$ Cardinal no. of a set (A ← set)

Eg. $|2|=2$ $\quad$ $|\sqrt{2}-1|=\sqrt{2}-1$

$|0|=0$

$|-3|=3$

$|-5|=5$

$|\sqrt{2}-2|=2-\sqrt{2}$

Input can be any real No

Output must be non-negative

$2 \rightarrow$ positive value.

$-2 \rightarrow$ negative value.

$x^2 \rightarrow$ Non negative value

$x \rightarrow$ Positive, negative, zero.

$0 \rightarrow$ Neither positive or Negative.

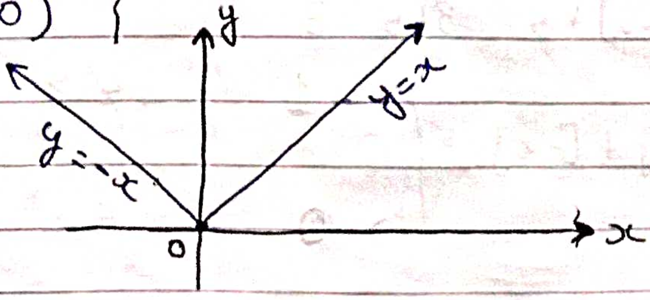
$-x \rightarrow$ Positive, negative, zero

$$f(x)=|x|=\begin{cases} x & ; \text{ if } x \geq 0 \\ -x & ; \text{ if } x<0 \end{cases}$$

Both are positive

$$\left\{\begin{array}{l} D_f=(-\infty, \infty) \\ R_f=[0, \infty) \end{array}\right\}$$

Graph $=f(x)=|x|$



Q. find the value of 'x' satisfying ① $|x-2|=4$ ② $||x|-2|=5$

Q. Draw the graph of $f(x)=|x-4|$

Q. find the value of 'x' satisfying $|x+4|-|x|=4$
$[0,\infty)$

Q. find 'x' : $|x+6|+|x|=8$
$x=-7,1$

Q. If $|x|+|x-4|+|x-6|=10$ then find x.

Q. If $f(x)=|x|+2|x-1|$ the Draw the graph $y=f(x)$

Q. Draw the graph of $f(x)=|x+2|+2|x|+|x-5|$

6. Signum function :-

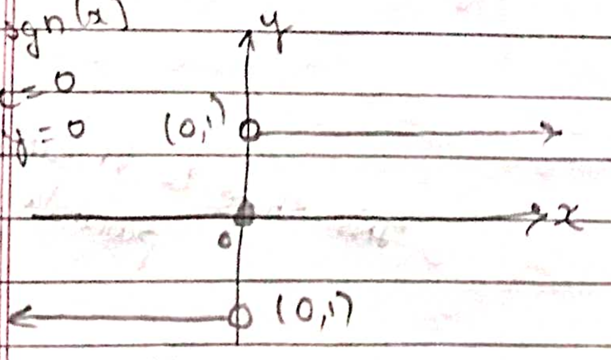
$f(x)=$ Signum (x)
or $Sgn(x)$

$D_f=(-\infty,\infty)$
$R_f=(-1,0,1)$

$Sgn(5)=1$
$Sgn(7)=1$
$Sgn(8)=1$
$Sgn(-1)=-1$
$Sgn(-\sqrt{2})=-1$
$Sgn(0)=0$

$Sgn(\pi)=1$
$Sgn(\sqrt{2}+\sqrt{3}-1)=1$
$Sgn(\underbrace{1-\sqrt{2}}_{\text{negative}})=-1$
$Sgn(1+x^2)=1$

$y=Sgn(x)$
$x=0$
$y=0$



$$Sgn(x)\begin{cases}\to 1 & \text{If } x>0\\ \to 0 & \text{If } x=0\\ \to -1 & \text{If } x<0\end{cases}$$

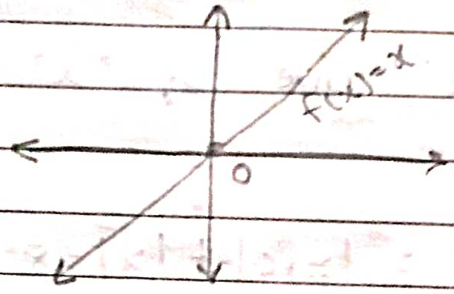
\# $Sgn(x)=$ sgn sgn x $=$ Sgn Sgn Sgn x $=\ldots$
\# $Sgn(-x)=-Sgn(x)$

Q. If sgn $(x+1)=1$ then find 'x' $f(x)=x$
$y=x$

7. Identity functions.

$f(x)=x$

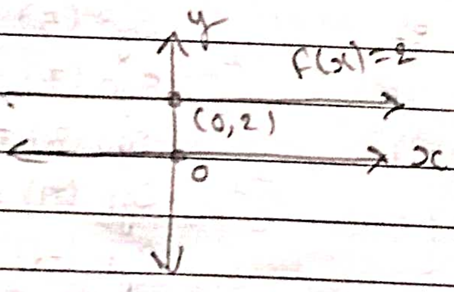
$D_f=(-\infty,\infty)$
$R_f=(-\infty,\infty)$

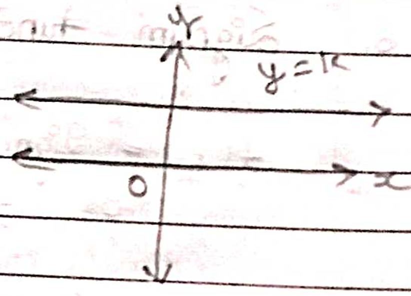


8. Constant function.

$f(x)=2$ $f(x)=k$ (where k is constant)

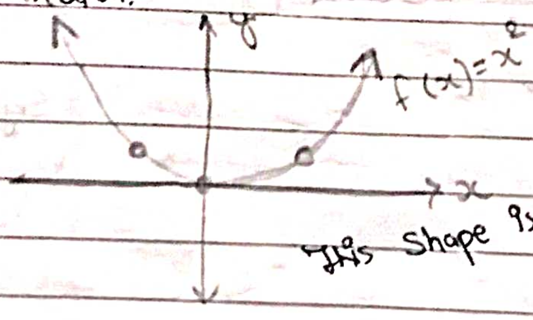
$D_f=(-\infty,\infty)$ $D_f=(-\infty,\infty)$
$R_f=\{2\}$ $R_f=\{k\}$





9. Simple Parabolic function.

$f(x)=x^2$
$D_f=(-\infty,\infty)$
$R_f=[0,\infty)$



This shape is known as parabola

10. Polynomial functions. $f(x)=5$ Constant Polynomial

$f(x)=x+1$ → Linear Polynomial
$f_2(x)=x^2+x+1$ → Quadratic Poly..
$f_3(x)=x^3-3x^2+4x-5$ → Cubic Poly...
$f_4(x)=6x^4-9x^3+13$ → Bi quadratic poly..

Difference b/w Polynomial & Multinomial

General Polynomial.

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$

$n \in W$

$\underbrace{a_0, a_1, a_2 \dots a_n}_{\text{Coefficient}} \in R$ $\quad a_0 \rightarrow$ leading coefficient of the polynomial

$D_f = R$ $\quad$ Degree of polynomial $= n$ $(a_0 \neq 0)$

$R_f =$ depends

$F(x) = 0 \rightarrow$ zero polynomial $\quad g(x) = 2 \rightarrow$ constant polynomial

Degree $= 0$

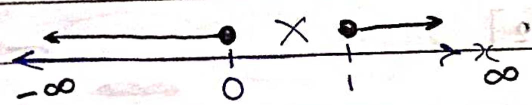
[Degree is not defined for ploynomial] $\quad$ # Degree of constant polynomial is zero (except $F(x) = 0$)

Wavy curve Method

Que. Find the domain of $f(x) = \sqrt{x^2 - x}$.

$x^2 - x \geq 0$

$x(x-1) \geq 0$



$x \in (-\infty, 0] \cup [1, \infty)$

$D_f = (-\infty, 0] \cup [1, \infty)$

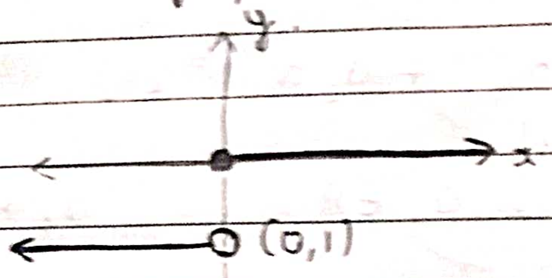
To solve Inequalities

Step-I
Make linear factor in LHS & zero in RHS

Step-II
find the critical point of each linear factor & plot them on Real no. line

Step-III Now observe each part of the number line & Hence write Solutions of the inequality.

Q. Draw the graph of $f(x) = \text{Sgn}(x - |x|) = \begin{cases} 0 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$



Q. find the range of $f(x) = \dfrac{2-x^2}{1+x^2}$

Sol^n $f(x) = \dfrac{2-x^2}{1+x^2} = \dfrac{3}{1+x^2} - \dfrac{1+x^2}{1+x^2} = \dfrac{3}{1+x^2} - \dfrac{(1+x^2)}{(1+x^2)}$

Method-1

$f(x) = \dfrac{3}{1+x^2} - 1$

Range by using step by step Method

$x^2 \in [0, \infty)$

$1+x^2 \in [1, \infty)$

$\dfrac{3}{1+x^2} \in (0, 3]$

$\dfrac{3}{1+x^2} - 1 \in (-1, 2]$

$f(x) \in (-1, 2]$

Range

Method-2

$f(x) = \dfrac{2-x^2}{1+x^2}$

Let $f(x) = y$

$y = \dfrac{2-x^2}{1+x^2}$

$y + yx^2 = 2 - x^2$

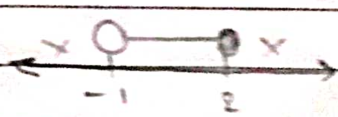
$(y+1)x^2 = 2 - y$

$x^2 = \dfrac{2-y}{y+1}$

Express 'x' in term of y then proceed

$\dfrac{(2-y)}{(y+1)} \geq 0$

Value of $y \rightarrow$ Range



$y \in (-1, 2]$

$R_f \in (-1, 2]$