1.1. POLYNOMIALS:

Remainder Theorem: If a polynomial over the set of real numbers R is divided by $x - a$, where $a \in R$ then the remainder is $P(a)$.

Factor Theorem: If a polynomial $P(x)$ over the set of real numbers R is such that $P(a) = 0$ for $a \in R$, then $x - a$ is a factor of $P(x)$.

Formula:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= (a+b)^3 - 3ab(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= (a-b)^3 + 3ab(a-b)$$

1.2. SOLUTION BY QUADRATIC FORMULA:

1. Solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Formation of a quadratic equation:

$x^2 -$ (sum of the roots) x + Product of the roots $= 0$.

3. Nature of the roots:

	Discriminant $\Delta = b^2 - 4ac$	Nature of the roots.
(i)	$\Delta > 0$ but not a perfect squares	Real, unequal and irrational
(ii)	$\Delta > 0$ and a perfect square	Real, unequal and rational
(iii)	$\Delta = 0$	Real and equal.
(iv)	$\Delta < 0$	Unreal

1.3. SEQUENCE AND SERIES:

Sequence:

Any set of numbers obeying a particular law is called a sequence.

Eg.: 2, 6, 18, 54, ...

11, 6, 1, – 4, ...

Series:

When the terms of a sequence are connected by plus or minus signs they are said to form a series.

Arithmetic Progression:

A sequence of numbers in which the successive terms increase or decrease by a constant is called an Arithmetic progression and that constant number is called the common difference.

1. General form: $a, a+d, a+2d, \ldots, a+(n-1)d$
2. $t_n = a + (n-1)d$ (General term)

3. $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$
4. $n = \frac{l - a}{d} + 1$.

Arithmetic Series:

The series whose terms are in A.P is called in arithmetic series.

Eg.: $1 + 3 + 5 + 7 + \ldots$

Arithmetic Mean:

A is called the arithmetic mean of the numbers a and b if and only if a, A, b are in A.P. If A is the A.M. between a and b then a, A, b are in A.P.

$$A = \frac{a+b}{2}$$

Geometric Progression:

A sequence is said to be a geometrical progression if the ratio of any term to its preceding term is the same throughout. This ratio is called the common ratio of the geometric progression. It is denoted by 'r'.

1. General form: $a, ar, ar^2, \ldots, ar^{n-1}$
2. $t_n = ar^{n-1}$ (General term)
3. (i) $S_n = \frac{a(r^n - 1)}{r - 1}$ if $r > 1$

   (ii) $S_n = a\left(\frac{1 - r^n}{1 - r}\right)$ if $r < 1$

   (iii) $S_\infty = \frac{a}{1 - r}$, if $r < 1$ [sum of the infinite series]
4. Compound interest and G.P. $A = P(1 + i)^n$

Geometric Series:

The series whose terms are in G.P is called geometric series.

Geometric Mean:

G is called the geometric mean of the numbers a and b if and only if a, G, b are in G.P.

$$G = \pm \sqrt{ab}.$$

Harmonic Progression:

A sequence of non-zero numbers is said to be in harmonic progression if their reciprocals are in A.P.

Eg.: $1, \frac{1}{5}, \frac{1}{9}, \frac{1}{13}, \ldots$ is a H.P.

1. General form: $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots, \frac{1}{a+(n-1)d}$, where $a \neq 0$.
2. n^{th} term $= t_n = \frac{1}{a+(n-1)d}$

Harmonic Mean:

H is called the harmonic mean between a and b if a, H, b are in H.P.

$$H = \frac{2ab}{a+b}$$

Summation of Series:

1. $1+2+3+\ldots+n = \Sigma n = \frac{n(n+1)}{2}$
2. $1^2+2^2+3^2+\ldots+n^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1^3+2^3+3^3+\ldots+n^3 = \Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2$

1.4. SOME SPECIAL TYPE OF SERIES:

1. Binomial Series:

Binomial theorem for a Rational Index:

For any natural number n,

$$(x+a)^n = {}_nC_0\, x^n a^0 + {}_nC_1\, x^{n-1} a + {}_nC_2\, x^{n-2} a^2 + \ldots + {}_nC_r\, x^{n-r} a^r + \ldots + {}_nC_n\, x^0 a^n$$

1. General term: $T_{r+1} = {}_nC_r\, x^{n-r} a^r$
2. If n is even, then middle terms $= T_{\frac{n}{2}+1}$
3. If n is odd, then middle terms are

$$T_{\frac{n+1}{2}} \text{ and } T_{\frac{n+3}{2}}$$

4. If $n \in N$, in the expansion of $(1+x)^n$
   (i) sum of the binomial co-efficients $= 2^n$
   (ii) sum of the co-efficients of odd terms $=$ sum of the coefficients of even terms $= 2^{n-1}$

Some Particular Expansions:

1. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$
2. $(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$
3. $(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \ldots$
4. $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \ldots$

2. Exponential Series:

For all real values of x, $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

Results:

1. $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$

2. $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

3. $\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots.$

4. $\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \ldots$

5. $\frac{e - e^{-1}}{2} = 1 + \frac{1}{3!} + \frac{1}{5!} + \ldots.$

3. Logarithmic Series:

If $-1 < x < 1$, then $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots.$

Results:

1. $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots.$

2. $-\log(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$

3. $\log(1+x) - \log(1-x) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$

4. $\frac{1}{2}\log\left(\frac{1-x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots.$

4. Trigonometric Series:

1. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \infty$

2. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^5}{4!} + \ldots + \infty$

3. $\tan x = x + \frac{x^3}{3} + \frac{2}{15} x^5 + \ldots + \infty$

4. $\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \ldots + \infty$

Note:

1. $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + \infty$

2. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + \infty$

3. $\log \sec x = \frac{x^2}{2!} + 2\frac{x^4}{4!} + \ldots + \infty$

4. $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \infty$

1.5. LAWS OF LOGARITHMS:

1. Product Role: $\log(MN) = \log_x M + \log_x N$
2. Quotient Rule: $\log_x\left(\frac{M}{N}\right) = \log_x M - \log_x N$
3. Power Rule: $\log_x(M^n) = n \log_x M$
4. Change of Base Rule: $\log_b a = \log_c a \cdot \log_b c$

1.6. PERMUTATION:

The number of arrangements that can be made out of n things taking r at a time is called the number of permutations of n things taken r at a time.

1. ${}_np_r = n(n-1)(n-2)\ldots.(n-(r-1))$
2. ${}_np_r = \frac{n!}{(n-r)!}$
3. ${}_np_n = n!$
4. ${}_np_0 = 1$

5. The number of mutually distinguishable permutations of n things, taken all at a time, of which p are alike of one kind, q alike of second kind such that $p+q=n$ is $\frac{n!}{p!\,q!}$

6. The number of circular permutations of n distinct objects is $(n-1)!$

7. If there are n things and if the direction is not taken into consideration, the number of circular permutations is $\frac{(n-1)!}{2}$

1.7. COMBINATION:

A selection of any r things out of n things is called a combination of n things r at a time.

1. ${}_nC_r = \frac{n!}{r!\,(n-r)!}$

2. ${}_nC_r = \frac{{}_nP_r}{r!}$

3. ${}_nC_n = 1$

4. ${}_nC_0 = 1$

5. ${}_nC_r = {}_nC_{n-r}$

6. If x and y are non-negative integers such that $x+y=n$ then ${}_nC_x = {}_nC_y$.

7. If n and r are positive integers such that $r \le n$, then ${}_nC_r + {}_nC_{n-r} = {}_{n+1}C_r$

8. If n and r are positive integers such that $1 \le r \le n$ then ${}_nC_r = \frac{n}{r}\,{}_{(n-1)}C_{(r-1)}$

9. For any positive integers x and y ${}_nC_x = {}_nC_y \Rightarrow x=y$ or $x+y=n$

Difference between Permutation and Combination:

1. In a combination only selection is made whereas in a permutation not only a selection is made but also an arrangement in a definite order is considered.
2. Usually the number of permutation exceeds the number of combinations.
3. Each combination corresponds to many permutations.

1.8. QUADRATIC INEQUIVALITIES:

Consider $ax^2+bx+c>0$.

Let $ax^2+bx+c = a(x-\alpha)(x-\beta)$

1. If $(x-\alpha)(x-\beta)>0$ then the values of x lies outside α and β.
2. If $(x-\alpha)(x-\beta)<0$ then the values of x lies between α and β.

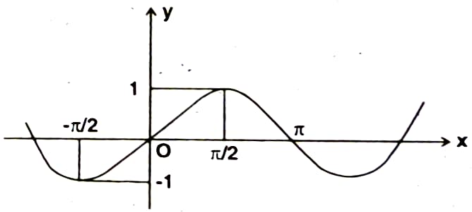
1.9. GRAPHS:

1. $y=\sin x$.

Domain $(-\infty, \infty)$

Range $[-1, 1]$

Principal domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

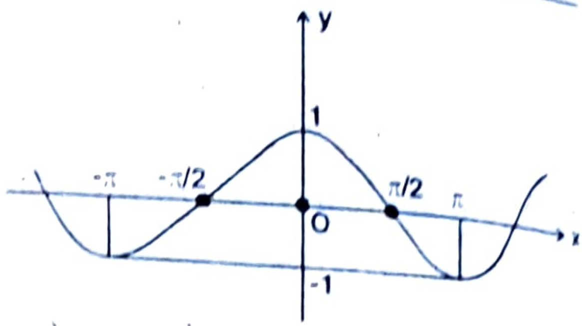


2. $y = \cos x$

   Domain $(-\infty, \infty)$

   Range $[-1, 1]$

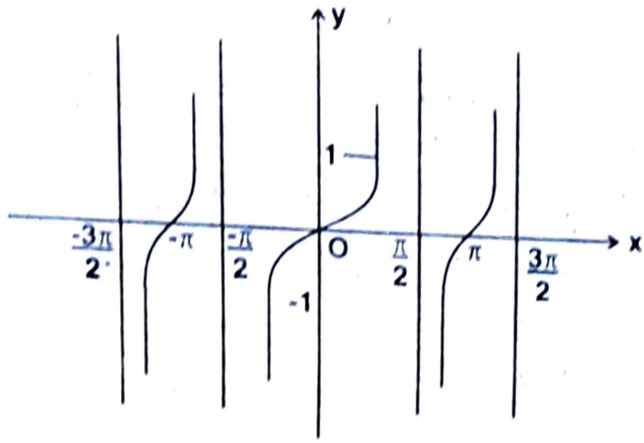
   Principal domain $[0, \pi]$.



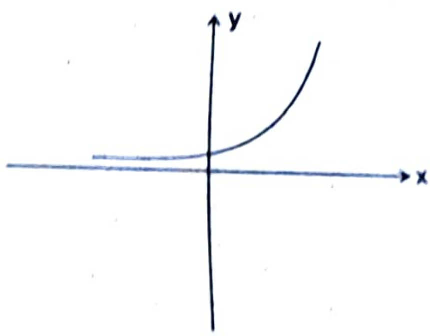
3. $y = \tan x$

   Domain $= R - \left\{ (2k+1)\dfrac{\pi}{2} \right\}, K \in Z$

   Range $= (-\infty, \infty)$



4. $y = e^x$ where 'e' is an irrational number whose value lies between 2 and 3 is called the exponential function.



5. $y = \log x$