Do what they aren't willing to do and you'll get what they will never have.

★ Business Mathematics

Ratio

- First term - antecedant
  Second term - consequent
- Inverse Ratio $a:b \Rightarrow b:a$
- $a > b$, greater inequality
  $a < b$, less inequality
- Compound Ratio of $a:b$, $c:d \Rightarrow \frac{ac}{bd}$
- Duplicate Ratio $a:b \Rightarrow a^2:b^2$
  Sub-duplicate Ratio $a:b \Rightarrow \sqrt{a}:\sqrt{b}$
- Triplicate Ratio $a:b \Rightarrow a^3:b^3$
  Sub-triplicate Ratio $a:b \Rightarrow \sqrt[3]{a}:\sqrt[3]{b}$
- Ratio terms integers, called commensurable
  Ratio terms non-integers, called incommensurable
- Continued Ratio $\Rightarrow a:b:c:$ ____

Proportions

$a : b :: c : d$

→ a, d - extremes , $b; c$ - means

Product of extremes = Product of means

(cross product rule)

→ $a, b, c \Rightarrow a : b :: b : c$ (continuous proportion)

$b^2 = ac \Rightarrow b = \sqrt{ac}$

→ $a : b = c : d \Rightarrow ad = bc$

→ $a : b = c : d \Rightarrow b : a = d : c$ (Invertendo)

→ $a : b = c : d \Rightarrow a : c = b : d$ (Alternendo)

→ $a : b = c : d \Rightarrow a+b : b = c+d : d$ (Componendo)

→ $a : b = c : d \Rightarrow a-b : b = c-d : d$ (Dividendo)

→ $a : b = c : d \Rightarrow a+b : a-b = c+d : c-d$ (Comp. & Divid)

→ $a : b = c : d = e : f \Rightarrow \frac{a+c+e}{b+d+f} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

(Addendo)

→ $a : b = c : d = e : f \Rightarrow \frac{a-c-e}{b-d-f} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

(Subtrahendo)

→ Ratio of product included = $\frac{\text{High price} - C.P}{\text{Low Price} - CP}$

(Alligation)

Indices

→ $a^m \times a^n = a^{m+n}$

→ $\frac{a^m}{a^n} = a^{m-n}$

→ $(a^m)^n = a^{mn}$

→ $(ab)^n = a^n \times b^n$

→ $a^0 = 1$, $a^1 = a$

→ $a^{-m} = \frac{1}{a^m}$, $\frac{1}{a^{-m}} = a^m$

→ $a^x = a^y \Rightarrow x = y$

→ $x^a = y^a \Rightarrow x = y$

→ $x^a = y \Rightarrow x = y^{\frac{1}{a}}$

→ $\sqrt[a]{x} = x^{\frac{1}{a}}$, $\sqrt[2]{x} = x^{\frac{1}{2}}$, $\sqrt[3]{x} = x^{\frac{1}{3}}$

→ $(a+b)^2 = a^2+b^2+2ab$, $(a-b)^2 = a^2+b^2-2ab$

→ $a^2-b^2 = (a+b)(a-b)$

→ $(a+b)^3 = a^3+b^3+3ab(a+b)$

→ $(a-b)^3 = a^3-b^3-3ab(a-b)$

→ $a^3+b^3 = (a+b)(a^2+b^2-ab)$

→ $a^3-b^3 = (a-b)(a^2+b^2+ab)$

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Logarithm

→ If $a^n = x$ | $2^3 = 8$

then $\log_a x = n$ | $\log_2 8 = 3$

it is pronounced as log of x to the base a.

→ $a^{\log_a x} = x$

→ $\log_a(mn) = \log_a m + \log_a n$

→ $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

→ $\log_a m^n = n.\log_a m$

→ $\log_{a^b} m = \frac{1}{b}.\log_a m$

→ $\log_a m^n = \log_{a^{\frac{1}{n}}} m$

→ $\log_{a^b} m = \log_a m^{\frac{1}{b}}$

→ $\log_a b * \log_b a = 1 \Rightarrow \log_a b = \frac{1}{\log_b a}$

→ $\log_a 1 = 0$

→ $\log_a a = 1$

→ $\log_b a = \frac{\log_m a}{\log_m b}$ or $\frac{\log a}{\log b}$

Base 10 is considered as common logarithm.

→ $\log 1 = 0$, $\log 10 = 1$, $\log 100 = 2$, $\log 0{\cdot}1 = -1$.

Equations

Linear equation → $ax + b = 0$

Quadratic equation ⇒ $ax^2 + bx + c = 0$

Cubic equation ⇒ $ax^3 + bx^2 + cx + d = 0$

linear equation (two variables) ⇒ $ax + by + c = 0$

Simultaneous equations ⇒ $\begin{bmatrix} 2x + 3y = 5 \\ 3x + 5y = 8 \end{bmatrix}$

When $b = 0$, then it is pure quadratic equation

When $b \neq 0$, then it is affected quadratic equation.

Roots ⇒ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $= \dfrac{-b}{a}$, Product of Root $= \dfrac{c}{a}$

Quadratic equation ⇒ $a[x^2 - (\text{Sum of Roots})x + \text{Product of roots} = 0]$

$b^2 - 4ac$ is called discriminant

If $b^2 - 4ac = 0$ then roots are real & equal.

If $b^2 - 4ac > 0$, then roots are real & unequal

If $b^2 - 4ac < 0$, then roots are imaginary.

If $b^2 - 4ac$ = Perfect squar (rational) If not then it is irrational.

Irrational roots = $(m + \sqrt{n}), (m - \sqrt{n})$.

Time Value of Money -

$S.I = Pit = P \times \frac{R}{100} \times T$, $A = S.I + P$

$A = P(1+it)$

$C.I. = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$, $A = P\left(1+\frac{R}{100}\right)^n$

$CI = A - P$

SHORTCUTS -

S.I (i) A sum doubles in $\frac{100}{r}$ years.

(ii) A sum trebles in $\frac{200}{r}$ years.

C.I (i) A sum doubles in $\frac{72}{r}$ years.

(ii) A sum trebles in $\frac{114}{r}$ years

Compounded half-yearly $\Rightarrow A = P\left(1+\frac{R}{200}\right)^{2n}$

Compounded quarterly $\Rightarrow A = P\left(1+\frac{R}{400}\right)^{4n}$

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SHORTCUTS –

① When difference b/w S.I & C.I. is given for 2 years, then P is given by $P = \frac{d \times (100)^2}{r^2}$

② When difference b/w S.I & C.I. is given for 3 years, then P is given by $P = \frac{d \times (100)^3}{r^2 (r + 300)}$

③ If a sum becomes A_1 in t_1 years and A_2 in t_2 years, then P is
$P = \frac{A_2 t_1 - t_2 A_1}{t_1 - t_2}$

④ If a sum becomes n times in t years, then find ~~it becomes~~ in how many years it becomes m times.
Time $= \left(\frac{m-1}{n-1}\right) \times t$ years

(5) In case of WDV Depreciation,
Scrap value = (Present value) $\boxed{P\left(1-\frac{R}{100}\right)^n}$

Effective Rate of Interest

$$E = \left[(1+i)^n - 1\right] \times 100$$

Compound interest half-yearly, quarterly, monthly converted into annually.

Annuity

	Future Value (Investment)	Present Value (Loan Amount, other)
Annuity Regular	$A \times \left[\frac{(1+i)^n - 1}{i}\right]$	$A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$
Annuity Due/ Immediate	$A \times \left[\frac{(1+i)^n - 1}{i}\right](1+i)$	$A\left[\frac{(1+i)^{n-1} - 1}{i(1+i)^{n-1}}\right] + A$

Consider Annuity Regular unless stated otherwise.

Date ________

Perpetuity

Perpetuity Present Value = $\frac{\text{Annuity (A)}}{i}$

Growing Perpetuity

Present Value = $\frac{\text{Annuity } A}{i - g}$

i = interest rate $i = \frac{R}{100}$

g = growth rate of interest $\Rightarrow g = \frac{G \cdot R}{100}$

Compound Annual Growth Rate (CAGR)

$$CAGR(t_0, t_n) = \left[\frac{V(t_n)}{V(t_0)}\right]^{\frac{1}{t_n - t_0}} - 1$$

$V(t_0)$ = Beginning period

$V(t_n)$ = End period

t_n = last year

t_0 = first year

Permutations And Combinations

'n' should always be positive.

AND → Multiply
OR → Add.

1- $^nP_r = \frac{n!}{(n-r)!}$, $0! = 1$

2- $^nC_r = \frac{n!}{r!(n-r)!}$ | $^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2}$

3- $^nP_r = n(n-1)(n-2)(n-3)\ldots(n-r+1)$

4- $^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$

5- $^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n$

6- $^nC_r = {}^nC_{n-r}$ | $^{10}C_7 = {}^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2}$

7. $^nP_n = n!$, $^nP_0 = 1!$

8- $^nC_n = 1$, $^nC_0 = 1$

9- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

10- $\dfrac{^nC_{r-1}}{^nC_r} = \dfrac{r}{n-r+1}$, $\dfrac{^nC_r}{^nC_{r+1}} = \dfrac{r+1}{n-r}$

11- If $^nC_x = {^nC_y}$, then $x \neq y$, $x+y=n$

12- If $^nP_x = {^nP_y}$, then $x \neq y$, $x+y = 2n-1$

13- When two things are considered as one there is internal arrangement as well.

14- When one object is not ~~ta~~ taken in any arrangement.

$^{n-1}P_r$

15 - When one object is always included in the arrangement

$${}^{n-1}P_{r-1}$$

16 - When two thing out of 'n' should never come together

$$(n-1)!\,(n-2)$$

17 - Circular permutations = $(n-1)!$

In 6 men 6 women question (circular restriction) first use circula perm for first and then normal permutation for second for arrangement

18 - Necklaces with 'n' different beads

$$\frac{1}{2}(n-1)!$$

19 - If some or more of 'n' items are taken

$(2^n - 1)$ (for two choices)

$(3^n - 1)$ (for three choices)

$6! \times {}^{7}P_4$

<u>Note</u> - (6 Boys 4 Girls, No girls together) G B G B G B G B G B G B G

6

20 - To find the sum of numbers

Step 1: Total possibilities / Total digits

Step 2: Step 1 × Sum of digits

Step 3: Step 2 × 1111.

21 - No. of intersections in 'n' circles

$n(n-1)$

22 - No. of diagonals in a polygon of 'n' sides.

$\frac{1}{2} \times n \times (n-3)$

23 - Parallelograms formed by 'm' parallel lines intersecting 'n' parallel lines

${}^{m}C_2 \times {}^{n}C_2$

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Ask: - ZENITH → Formula ans' + 1
Rank

24 - No. of factors of a numbers.

Step 1: Express the number in $p^a . q^b . r^c$ form. where p, q, r are its prime factors

Step 2: $(a+1)(b+1)(c+1)\ldots$

Different factors other than own number = $(a+1)(b+1)(c+1)\ldots -1$

25 - Independent combinations

$^{n_1}C_{r_1} \times {}^{n_2}C_{r_2} \ldots$

26 - No. of straight lines formed from 'n' points of which 'm' are collinear.

$^nC_2 - {}^mC_2 + 1$

27 - No. of triangles formed from 'n' points of which 'm' are collinear.

$^nC_3 - {}^mC_3$.

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28 - Arrangement in Dictionary -

ZENITH $\qquad \overrightarrow{\underline{ZENITH}}^{\,5!}$

$\qquad\qquad \overleftarrow{\underline{EHINTZ}}_{\,5}$

$= 5!\,5 + 4!(0) + 3!(2) + 2!(1) + 1!(1)$

Rank of ZENITH $= 615 + \underline{1} = 616$

29 - When 'n' items are to be arranged in groups. (It is applied when items in a group are identical)

Equal $\qquad\qquad$ Unequal

~~Identical items~~ $\qquad$ ~~Different items~~

$\frac{n!}{(h!)^k}$ Identical	Different	
	No. of items in a group are equal	No. of items in a group are unequal
where n = total no.	$\frac{n!}{k!(h!)^k}$	$\frac{n!}{a!\,b!\,c!}$
h = items in a group		
k = no of groups		

30 - $\sum_{r=1}^{10} r \cdot {}^{r}P_{r} = (10+1)! - (1)!$

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SUN	MON	TUE	WED	THURS	FRI	SAT
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

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Sequence and Series

Imp

S_n, $S_{n/2}$ method
Calc. trick, Reverse Trick.

A.P 2, 4, 6, 8 ....

a = first term, d = common difference

n^{th} term of an AP = $t_n = a + (n-1)d$

$t_n = S_n - S_{n-1}$

Sum of first n terms = $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}(a + l)$

↳ last term

Assumptions of AP terms -

3 terms :- $(a-d), a, (a+d)$

4 terms :- $(a-3d), (a-d), (a+d), (a+3d)$

5 terms :- $(a-2d), (a-d), a, (a+d), (a+2d)$

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Note -

If a^2, b^2, c^2 are in AP then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are also in AP and vice-versa.

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Sum of first n natural numbers $= \frac{n(n+1)}{2}$

Imp. Results for an A.P.

if $t_n = An + B$, then $a = A + B$, $d = A$

if $S_n = An^2 + Bn$, then $a = A + B$, $d = 2A$

G.P. 2, 4, 8, 16, 32.......

a = first term, r = common ratio.

n^{th} term $= t_n = a \cdot r^{n-1}$

Sum of first n terms $= S_n = \frac{a[r^n - 1]}{[r-1]}$, $r > 1$

$S_n = \frac{a[1 - r^n]}{[1 - r]}$, $r < 1$

Sum of infinite terms of a G.P. $= S_\infty = \frac{a}{1-r}$

$(-1 < r < 1)$

when a^2, b^2, c^2 are in AP, then answer
$a = 1, b = 5, c = 7$.

(Best AP e.g. 1, 2, 3, 4.)

Sum of square of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

Assumption of G.P terms

3 terms :- $\frac{a}{r}, a, ar$

4 terms :- $\frac{a}{r^3}, \frac{a}{r}, a, ar^3$

5 terms :- $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Arithmetic Mean and Geometric Mean

AP → 1, 2, 3, 4
HP → $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

A.P

x, y, z

$y - x = z - y$

$2y = x + z$

$y = \frac{x+z}{2}$

G.P

x, y, z

$\frac{y}{x} = \frac{z}{y}$

$y^2 = xz$

$y = \sqrt{xz}$

AM > GM when terms are distinct

AM = GM when terms are equal.

Date

Sets, functions and Relations

Set- A collection of well-defined distinct objects.

$A = \{2,4,6,8,10\}$ - Roaster or Braces form.

$A = \{x : x$ is an even number less than $11\}$

- Set-builder form or Algebric form or rule method.

Property Method.

If,

$a \in A$, a is an element of A.

$b \notin B$, b is not an element of B.

$P \subset Q$, P is a subset of Q.

- When the elements of two sets are same and equal, they are called equal sets.
- When $n(A) = n(B)$, but elements are diff., called equivalent sets.

{ } or ϕ = Null / Void / Empty set.

Singeton Set - contains only one element.

Universal set - S, contains all elements.

Subsets formula = 2^n

Proper Subsets = All subgsets - Identical subsets

Proper subset formula = $2^n - 1$

Power set = collection of all subsets of a set.

<u>De Morgan's Law</u> -

$(A \cup B)' = A' \cap B'$ | $(A \cap B)' = A' \cup B'$

→ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

→ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Product sets - $A = \{1, 2\}$, $B = \{3, 4, 5\}$

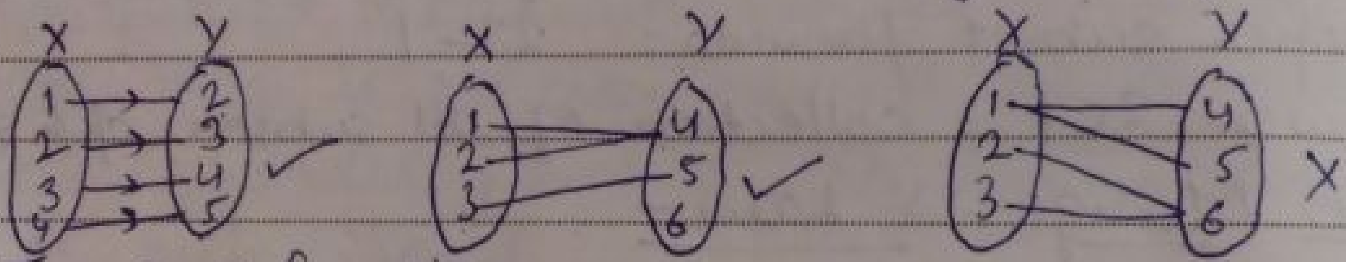
$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$

$n(A \times B) = n(A) \times n(B)$

$A \times B \neq B \times A$, but $n(A \times B) = n(B \times A)$

Relations and Functions

A relation is called a function when
- each element of A has a image in B.
- each element has one image only.



Types of functions

(One-one function) Injective function & Many-one function.

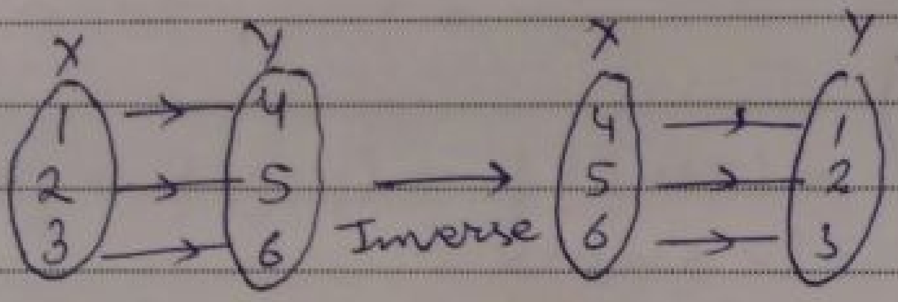
If each element of x has diff. image in y, then it is one-one function otherwise it is a many-one function.

Onto-function (Surjective) & Into-function.

If every element of Y has pre-image in X then it is onto function, otherwise it is into function.

Bijection function - One-one + onto-function.

Inverse function



Inverse can be found only if it is one-one function and onto function.

$$f(x) = 3 + 2x$$

$$y = 3 + 2x$$

$$y - 3 = 2x$$

$$\frac{y-3}{2} = x$$

Replacing y by x and x by $f^{-1}(x)$

$$\frac{x-3}{2} = f^{-1}(x)$$

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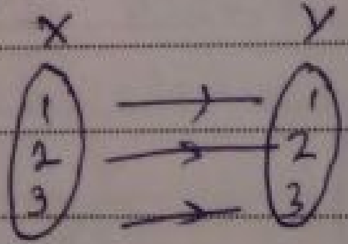
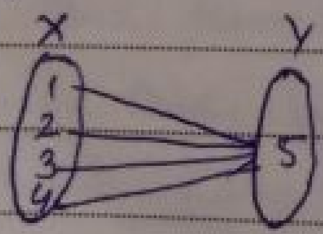
Identity function - One-one + Onto

$f(x) = x$



Constant function (Many one + Onto)

$f(x) = 5$



Equal function

$\left.\begin{array}{l} f(x) \\ g(x) \end{array}\right\}$ (If same domains)

$f(x) = g(x)$ [Then we get same value]

Composite Function

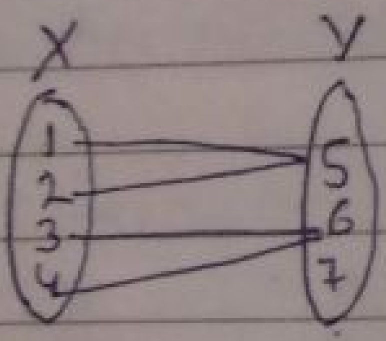
$f(x) = 2x + 5$
$g(x) = 3x + 5$

$$\begin{aligned} f\circ g = f(g(x)) &= f(3x+5) \\ &= 2(3x+5)+5 \\ &= 6x+10+5 \\ &= 6x+15 \end{aligned}$$

$$\begin{aligned} g\circ f = g(f(x)) &= g(2x+5) \\ &= 3(2x+5)+5 \\ &= 6x+15+5 \\ &= 6x+20 \end{aligned}$$

$$\begin{aligned} f\circ f = f(f(x)) &= f(2x+5) \\ &= 2(2x+5)+5 \\ &= 4x+10+5 \\ &= 4x+15 \end{aligned}$$

Date



Domain = [1,2,3,4]
Range = [5,6]
Co-domain = [5,6,7]

In Into-function, Range ≠ Co-Domain
In onto-function, Range = Co-domain.

<u>Types of Relations</u>

(1) Reflexive Relation

"is equal to". put <u>a</u> & <u>a</u>.

<u>a</u> is equal to <u>a</u>
True - Reflexive false - Not Reflexive.

"is brother of"
<u>a</u> is brother of <u>a</u>. false.

② Symmetric Relation

"is equal to" put a & b, b & a.
a is equal to b (It is true)
$\boxed{\text{b is equal to a}}$ True - Symmetric
False - x

③ ~~Sy~~ Transitive Relation

"is parallel to" put a & b, b & c, a & c.

a is ∥ to b [It is true)
b is ∥ to c (It is true)
$\boxed{\text{a is ∥ to c}}$ True - Transitive
False - x

④ Equivalence Relation

Reflexive + Symmetric + Transitive.

Differential Calculus

Imp - Try to solve $f(x)$ first to its simplest form.

Standard Results -

1 - $y = x^n \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^n) = n \cdot x^{n-1}$.

$\frac{d}{dx}(x) = 1$ | $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

2 - $y = k \Rightarrow \frac{dy}{dx} = \frac{d}{dy}(k) = 0$

3 - $y = a^x \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(a^x) = a^x \log a$.

4 - $y = e^x \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^x) = e^x$

5 - $y = \log x = \frac{dy}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$

Properties

$$\boxed{\begin{aligned} f(x) &= y \\ dy/dx &= f'(x) \end{aligned}}$$

$$\boxed{\frac{d^2y}{dx^2} = f''(x)}$$

constant

1- $y = k . u$

$$\frac{dy}{dx} = \frac{d}{dx}(k . u)$$

$$\boxed{\frac{dy}{dx} = k . \frac{du}{dx}}$$

2- $y = u + v$

$$\frac{dy}{dx} = \frac{d}{dx}(u+v)$$

$$\boxed{\frac{dy}{dx} = \frac{d}{dx}(u) + \frac{d}{dx}(v)}$$

3- $y = u - v$

$$\frac{dy}{dx} = \frac{d}{dx}(u-v)$$

$$\boxed{\frac{dy}{dx} = \frac{d}{dx}(u) - \frac{d}{dx}(v)}$$

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4 - $y = u \cdot v$.

$$\frac{dy}{dx} = \frac{d}{dx}(u \cdot v)$$

$$\boxed{\frac{dy}{dx} = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)}$$

5 - $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right)$$

$\cancel{\frac{dy}{dx}}$

$$\boxed{\frac{dy}{dx} = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}}$$

6 - $y = u \cdot v \cdot w$,

$$\frac{dy}{dx} = uv \cdot \frac{d}{dx}(w) + uw \cdot \frac{d}{dx}(v) + vw \cdot \frac{d}{dx}(u)$$

Chain Rule

$y = \log(1+x^2)$

Let $1+x^2 = t$

$y = \log t$

$\cancel{\frac{dy}{dx} = \log \frac{d}{dx}(\log t) = \frac{1}{x}\left(\frac{d}{dt}\right)}$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{1}{t} \times (0+2x)$$

$$= \frac{1}{1+x^2}(0+2x)$$

$$= \frac{2x}{1+x^2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\log(1+x^2) = \frac{1}{1+x^2} \times 2x = \frac{2x}{1+x^2}$$

Integral Calculus

Imp Try to solve $f'(x)$ to its simplest form.

Standard Results

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + k$

2. $\int dx = x + k$

3. $\int a^x \, dx = \frac{a^x}{\log a} + k$

4. $\int e^x \, dx = e^x + k$

5. $\int \frac{1}{x} \, dx = \log|x| + k$

6. $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + k$

7. $\int a^{px+q} \, dx = \frac{a^{px+q}}{p \log a} + k$

(8) $\int e^{px+q}\,dx = \frac{e^{px+q}}{p} + k$

(9) $\int \frac{1}{ax+b}\,dx = \frac{\log|ax+b|}{a} + k$

(10) $\int e^{x}\{f(x) + f'(x)\}\,dx = e^{x} f(x) + k$

(11) $\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + c$

(12), (13), (14) - Last page of chapter

Method of Substitution

$\int \frac{x^3}{(x^2+1)^3}\,dx$, Let $x^2+1 = t$

$2x = \frac{dt}{dx}$

$dx = \frac{dt}{2x}$

$\int \frac{x^3}{t^3} \frac{dt}{2x}$

$\frac{1}{2}\int \frac{x^2}{t^3}\,dt$

$\frac{1}{2}\int \frac{t-1}{t^3}\,dt$. Now Integrete this and then after put value in 'x' form.

Date

Integration By Parts

$$\int uv\,dx = u\int v\,dx - \int\left[\left\{\frac{du}{dx}\right\} \times \int v\,dx\right]dx$$

① ② ③
L A E

- Logarithmic $\log(x+3)$
- Algebric $x^2 a+b$
- e^{3x+4} exponential

Method of Partial Fraction

$$\int \frac{3x+2\,dx}{(x-2)(x-3)}$$

Let $\dfrac{3x+2}{(x-2)(x-3)} = \dfrac{A}{x-2} + \dfrac{B}{x-3}$

$\Rightarrow 3x+2 = A(x-3) + B(x-2)$

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Putting $x = 3$

$3 \times 3 + 2 = B(3-2)$

$9 + 2 = B$

$11 = B$

Putting $x = 2$

$3 \times 2 + 2 = A(2-3)$

$8 = -A$

$A = -8$

So,

$$\int \frac{3x+2}{(x-2)(x-3)}\,dx = \int\left(\frac{-8}{x-2} + \frac{11}{x-3}\right)dx$$

Find the equation of the curve where slope at (x, y) is $9x$ and which passes through the origin.

$\frac{dy}{dx} = 9x \quad \therefore \int 9x = \frac{9}{2} x^2 + c \Rightarrow 9x^2 = 2y$

$\because$ Konstant $= 0$

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Definite Integration

$$\int_0^2 x^5 \, dx$$

$$\int_0^2 x^5 \, dx = \left(\frac{x^6}{6}\right)_0^2$$

Putting 2 & 0 in place of x

$$\frac{2^6}{6} - \frac{0^6}{6}$$

$$= \frac{64}{6} - 0$$

$$= \frac{32}{3}$$

Note: In definite Integration the constant (C) should not be added.

Note: When we use substitution method In definite integration, we have to change the limits for 't' by putting value of x (limits).

(12) $\int k.u \, dx = k \int u \, dx$

(13) $\int (u+v) \, dx = \int u \, dx + \int v \, dx$

(14) $\int (u-v) \, dx = \int u \, dx - \int v \, dx$.

★ Statistics

Statistical Description of Data

Each & every line is important in icai module.
Plus MCQs.

Measures of Central Tendency

Arithmetic Mean

Direct Method

Individual Series, $\bar{x} = \frac{\sum x}{n}$

Discrete series, $\bar{x} = \frac{\sum fx}{\sum f}$

Continuous Series, $\bar{x} = \frac{\sum fx}{\sum f}$, x = mid-values

Assumed Mean Method

Individual series, $\bar{x} = A + \frac{\sum d}{n}$, $d = x - A$

Discrete series, $\bar{x} = A + \frac{\sum fd}{\sum f}$

Continuous series, $\bar{x} = A + \frac{\sum fd}{\sum f}$, x = mid-value

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Step-Deviation Method

~~Ind~~

Continuous series $= A + \frac{\Sigma f d'}{\Sigma f}$, where $d' = \frac{x - A}{C}$

and x = mid-values.

Properties of AM

1- If all the observations are k, then AM is also k.

2- If deviations are taken from ~~it~~ mean of a series, then sum of deviations is 0.

3- Combined Mean

$$\bar{x}_{12\ldots} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \ldots}{n_1 + n_2 + \ldots}$$

4- Change of origin & scale.

If $y = ax + b$, then $\bar{y} = a\bar{x} + b$.

Median

Individual Series, Median = $\left(\frac{n+1}{2}\right)$th term

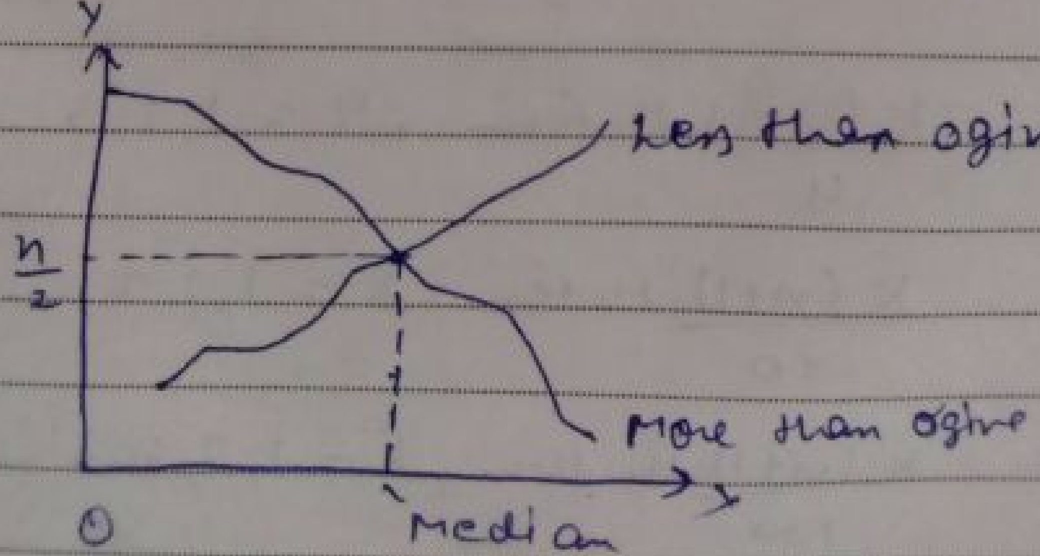
For 6.5th term, 6th term + 0.5 (7^{th} - 6^{th}) term

Descrete Series Median = $\left(\frac{n+1}{2}\right)$th term, + Use of cf

Continuous series, Median = $l + \frac{\frac{n}{2} - cf}{f} \times c$

Less than series - Upper limit + cf

More than series - Lower limit + cf (reverse)



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Properties of Median

1- If all the observations are k, then median is also k.

2- If $y = ax + b$, then $y_{me} = a x_{me} + b$.

3- When deviations are taken from median than the sum of absolute deviations is minimum.

Quartiles, Deciles & Percentiles

Individual series & (Discrete series + c.f.)

Quartile $\frac{k(n+1)}{4}$ th term, $k = 1, 2, 3$

Decile $\frac{k(n+1)}{10}$ th term, $k = 1, 2, 3, \ldots, 9$

Percentile $\frac{k(n+1)}{100}$ th term, $k = 1, 2, 3 \ldots 99$

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Continuous Series

Quartile $l + \frac{\frac{kn}{4} - cf}{f} \times c$ | $\frac{kn}{4}$

Decile $l + \frac{\frac{kn}{10} - cf}{f} \times c$ | $\frac{kn}{10}$

Percentile $l + \frac{\frac{kn}{100} - cf}{f} \times c$ | $\frac{kn}{100}$

(like $\frac{n}{2}$)

<u>Mode/Modal</u>

Individual & Discrete series

Mode = variable with highest frequency

Continuous Series

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

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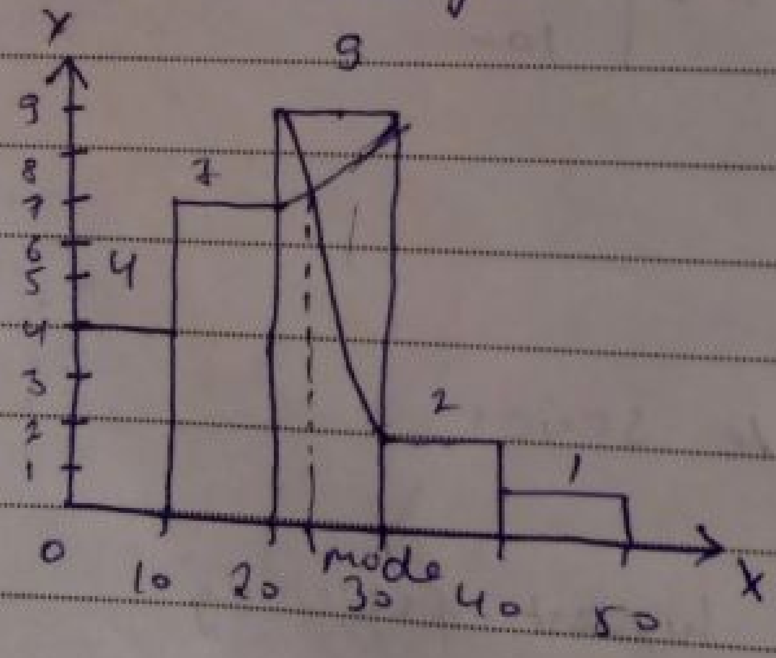
Properties of Mode -

1- If all the observations are k, then mode is also k.

2- If $y = ax + b$, then $y_{mo} = ax_{mo} + b$.

Mode with Diagram



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Geometric Mean

Individual series

$$G = (x_1 \times x_2 \times x_3 \times \ldots - x_n)^{\frac{1}{n}}$$

Discrete & Continuous series

$$G = (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \_\_\_ \times x_n^{f_n})^{\frac{1}{n}}$$

Properities of GM

1- ~~Logof~~ $\log G = \dfrac{\sum \log x}{n}$

2- If all the observations are k, then GM is also k.

3- If $z = xy$, then
GM of z = (GM of x) × (GM of y)

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4- If $z = \frac{x}{y}$, then

$$GM \text{ of } z = \frac{GM \text{ of } x}{GM \text{ of } y}$$

Harmonic Mean

Individual Series

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}} = \frac{n}{\Sigma\left(\frac{1}{x_i}\right)}$$

Discrete & Continuous Series

$$HM = \frac{n}{\Sigma\left(\frac{f}{x}\right)}$$

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Properties of HM

1- If all the observations are K, then HM is also K.

2- Combined HM = $\dfrac{n_1+n_2}{\dfrac{n_1}{H_1}+\dfrac{n_2}{H_2}}$

Points	Mean	Median	Mode	GM, HM
Unique	✓	✓	X	✓
Mathematical properties	✓	X	X	✓
Affected by sampling fluctuations	✓	X	✓	✓
Open-end classifications	X	✓	X	X

Date

Some Important Notes -

- AM is best measure
- Median deals with only 50% of central values.
- Mode is a popular measure.
- GM & HM are difficult to compute and are used in average rates & ratios.
- Open-end classification - when lower and upper limit of extreme classes are not given.
- Uniquely defined means one mean & median for a data.
- In Mean, inclusive as well as exclusive series can be used, but in median and mode only exclusive series can be used.
- 3 Median = 2 Mean + Mode
- When one thing is constant then we use harmonic mean (aeroplane speed example)
- For two positive numbers in a series $AM \times HM = GM^2$

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SUN	MON	TUE	WED	THURS	FRI	SAT
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

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~~dohera~~ Weighted AM, GM, HM, use weights instead of frequencies.

AM ≥ GM ≥ HM

ut, when all obeservations are same

AM = GM = HM

ut, when ~~all~~ observations are distinct (positive)

AM > GM > HM

Dispersion

To measure the scatterness.

- Two sets of observations may have same or diff. central tendencies and dispersion measures.

Absolute Measures of Dispersion

1- Range 2- Quartile Deviation
3- Mean Deviation 4- Standard Deviation

Relative Measures of Dispersion

1- Coefficient of Range
2- Coefficient of Quartile Deviation
3- Coefficient of Mean Deviation
4- Coefficient of Variation.

Diff. b/w absolute measures & relative measures

Absolute	Relative
1- Depend on unit of variable	Unit free
2- Cannot be used for comparison	Can be used for comparison.
3- Easy to compute	Difficult to compute.

RANGE

Range = L - S

Coefficient of Range = $\frac{L-S}{L+S} \times 100$

II In Continuous series

Only exclusive series

Range = Largest UCB - Smallest LCB

Coefficient of Range = $\frac{\text{Largest UCB} - \text{Smallest LCB}}{\text{Largest UCB} + \text{Smallest LCB}} \times 100$

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MD about Mean = $\frac{n^2-1}{4n}$

MEAN DEVIATION

$MD_{\bar{x}} = \frac{\Sigma |x-\bar{x}|}{n}$ | $MD_{\bar{x}} = \frac{\Sigma f|x-\bar{x}|}{\Sigma f}$

$MD_{Me} = \frac{\Sigma |x-Me|}{n}$ | $MD_{Me} = \frac{\Sigma f|x-Me|}{\Sigma f}$

$MD_{Mo} = \frac{\Sigma |x-Mo|}{n}$ | $MD_{Mo} = \frac{\Sigma f|x-Mo|}{\Sigma f}$

Coefficient of MD = $\frac{\text{MD about } (\bar{x}/Me/Mo)}{(\bar{x}/Me/Mo)} \times 100$

Coefficient of MD from Mean = $\frac{MD_{\bar{x}}}{\bar{x}} \times 100$

Coefficient of MD from Median = $\frac{MD_{Me}}{Me} \times 100$

Coefficient of MD from Mode = $\frac{MD_{Mo}}{Mo} \times 100$

QUARTILE DEVIATION

Or semi-inter-quartile range.

$$QD = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

STANDARD DEVIATION

$$s \text{ or } \sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

$$s \text{ or } \sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}$$

$$\text{Variance} = (s)^2$$

Date

Coefficient of variation = $\frac{SD}{\bar{X}} \times 100$

- Combined Standard Deviation

$$s_{12} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$d_1 = \bar{X}_{12} - \bar{X}_1$

$d_2 = \bar{X}_{12} - \bar{X}_2$

- For two numbers a and b, standard deviation is given by $\left|\frac{a-b}{2}\right|$.

- Standard deviation of first n natural numbers is given by $\sqrt{\frac{n^2-1}{12}}$.

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Common Properties of Measures of Dispersion

1- If all the observations are same, then range, QD, MD, SD, all are 0.

2- Measures of dispersion are unaffected by change in origin but change in same proportion by change in scale with modulus function.

If $y = ax + b$

then,

$R_y = |a| \times R_x$

$MD_y = |a| \times MD_x$

$QD_y = |a| \times QD_x$

$SD_y = |a| \times SD_x$

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Important Question

The mean and SD of a sample of 100 observations were calculated as 40 and 5.1 respectively by a CA student who took one of the observation as 50 instead of 40 by mistake. The current value of SD would be.

Sol -

$\bar{X} = 40$, $n = 100$

$\Sigma x = \bar{x} \times n = 40 \times 100 = 4000$

$$\text{Corrected } \bar{X} = \frac{\Sigma x - \text{Incorrect value} + \text{Correct value}}{n - \text{excluded} + \text{included}}$$

$$= \frac{4000 - 50 + 40}{100 - 1 + 1}$$

$$= \frac{3990}{100}$$

$$= 39.9$$

$s = 5.1$, $\bar{x} = 40$, $n = 100$

$$s = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

$$5.1 = \sqrt{\frac{\Sigma x^2}{100} - (40)^2}$$

$$26.01 = \frac{\Sigma x^2}{100} - 1600$$

$$1626.01 = \frac{\Sigma x^2}{100}$$

$$\Sigma x^2 = 1{,}62{,}601$$

Corrected $\Sigma x^2 = \Sigma x^2 - (\text{Incorrect})^2 + (\text{Correct})^2$

$$= 162601 - 50^2 + 40^2$$

$$= 162601 - 2500 + 1600$$

$$= 161701$$

Correct SD $= \sqrt{\frac{161701}{100} - (39.9)^2}$

$$= \sqrt{1617.01 - 1592.01}$$

$$= \sqrt{25} = 5$$

Date

<u>Range</u>

- Quickest to compute
- Based on only two observations
- Too much affected by extreame observation
-

<u>Mean Deviation</u>

- Rigidly defined
- Based on all observation
- Not much affected by sampling fluctuations
- Difficult to compute
- Do not have mathematical properties.

<u>Standard Deviation</u>

- Most widely and commonly used
- have mathematical properties
- Based on all observations.

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Quartile Deviation

- Rigidly Defined
- Easy to compute
- Not much affected by sampling fluctuations.
- Based on central 50% of observation.
- No mathematical properties.
- Best measure for open-end classification.

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Correlation

Bivariate Data

When data are collected on two variables simultaneously at the same time, they are known as bivariate data and the corresponding frequency distribution derived from it, is known as bivariate frequency distribution.

It can be classified in two ways-

- Marginal Distribution:
Frequency distribution of one variable across other variables full range of values.
- Conditional Distribution:
Frequency distribution of one variable across a particular sub-population of other variable.

- In $(p+q)$ classification, total no. of cells is pq.
- Some of the cell frequencies may be 0,
- For $p \times q$ bivariate frequency table, maximum no. of marginal distributions is 2 and maximum no. of conditional distributions is $p+q$.

Correlation

When $r = 1$, perfect positive correlation
$r = -1$, perfect negative correlation.
$r > 0$, positive correlation
$r < 0$, negative correlation
$r = 0$, no correlation, non-sense correlation, 'spurious' correlation

If $y = a + bx$, $b > 0$, then $r = 1$, $a \cdot 24$

If $y = a + bx$, $b < 0$, then $r = -1$

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Types of Methods

1 - Scatter Diagram -
It can be applied for linear as well as non-linear i.e. curvilinear.

2 - Karl Pearson's or Product Moment Coefficient of Correlation -
It can be applied only when relationship is linear.
It is the best measure for correlation.

$$r = \frac{cov(x,y)}{\sigma x \times \sigma y} , \quad cov(x,y) = r \cdot \sigma x \cdot \sigma y$$

$$cov(x,y) = \frac{\Sigma xy}{n} - \bar{x} \cdot \bar{y}.$$

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

JUNE 2018

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Properties of Correlation Coefficient

(i) - Unit free measure.
(ii) ~~to~~ Change in origin - no affect
change in scale - changes sign under a particular situation.

If $u = ax + b$, $v = cy + d$. and
if a & c have same signs then there is no change in correlation.
but if a & c have diff. signs, then correlation changes its sign.

(iii) $-1 \leq r \leq 1$.

- Since there is no affect in correlation due to change in origin, we use it in questions to reduce large values to smaller values.

– No. of blind persons percentage = $\frac{\text{No. of blinds}}{\text{No. of persons}} \times 100$

3- <u>Spearman's</u> <u>Rank</u> <u>Coefficient</u> <u>of</u> <u>Correlation</u>

It is used when qualitative characteristic is concerned.

$$r_R = 1 - \frac{6 \Sigma d^2}{n(n^2-1)} \quad , d = R_x - R_y$$

Smallest rank is given to largest number and vice versa.

$$r_R = 1 - \frac{6\left[\Sigma d^2 + \frac{\Sigma(m^3 - m)}{12}\right]}{n(n^2-1)}$$

where m = no. of times a rank repeat

4 - Coefficient of (Con-current Deviations)

no. of deviations with same signs.

It is the quickest method of correlation and used when we don't consider magnitude of variables.

$$r_c = \pm \sqrt{\pm \frac{(2c-m)}{m}},$$

where c = no. of con-current deviations
$m = n-1$

If $2c-m$ is '+ve', then we take positive signs both places.
If $2c-m$ is '-ve', then we take negative sign both places.

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Probable Error

Probable error is a method of obtaining correlation coefficient of population.

$$P.E = \frac{2}{3} \times \frac{1-r^2}{\sqrt{N}}$$

$$P.E = \frac{2}{3} \times S.E \text{ (Standard Error)}$$

$$S.E = \frac{1-r^2}{\sqrt{N}}$$

Limit $= r \pm P.E$

(i) If $r < P.E$, then there is no correlation.
(ii) If $r > 6P.E$, then correlation is certain.
(iii) Since $-1 \leq r \leq 1$, P.E is never negative.

Coefficient of determination $= r^2$

Coefficient of non-determination $= 1 - r^2$.

Regression

In regression analysis, we estimate the value of one variable with the help of another variable.

Difference between observed value and estimated value is known as error or residue. Errors can be positive, negative or zero.

Least Squares -

$$\boxed{b_{yx} = \frac{cov(x,y)}{(\sigma_x)^2}} \qquad \boxed{b_{xy} = \frac{cov(x,y)}{(\sigma_y)^2}}$$

$$= \frac{r.\sigma_x\sigma_y}{(\sigma_x)^2} \qquad = \frac{r.\sigma_x\sigma_y}{(\sigma_y)^2}$$

$$\boxed{b_{yx} = r.\frac{\sigma_y}{\sigma_x}} \qquad \boxed{b_{xy} = r.\frac{\sigma_x}{\sigma_y}}$$

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$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad b_{xy} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

Regression line of y on x - minimisation of vertical distances.

$$(Y - \bar{Y}) = b_{yx}(X - \bar{X})$$

Regression line of x on y - minimisation of horizontal distances.

$$(X - \bar{X}) = b_{xy}(Y - \bar{Y})$$

Properties of Regression Lines

(i) The regression coefficients doesn't change due to change in origin but change due to change in scale.

If $u = px + a$ and $v = qy + b$

then, $b_{uv} = \frac{p}{q} \times b_{xy}$, $b_{vu} = \frac{q}{p} \times b_{yx}$.

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(ii) Two regression lines intersect at their means.
Solution of x & y are $\bar{X}$ and $\bar{Y}$.

(iii) Coefficient of correlation is the geometric mean of two regression coefficients.

$$r = \pm\sqrt{b_{xy} \times b_{yx}}$$

If b_{xy} and b_{yx} are '+ve', r is (+ve)
If b_{xy} and b_{yx} are '-ve', r is (-ve)

- Product of b_{xy} and b_{yx} should be less than 1.
- B_{xy} and B_{yx} should have same sign.

When $r = -1$ or 1, two regression lines coincide.
When $r = 0$, Regression lines are perpendicular to each other.

Date

Probability

Probability - Subjective or objective

Events -

(i) Simple or Elementary
- Cannot be further divided.

(ii) Composite or Compound
- Can be further divided.

Mutually Exclusive or Incompatible Events -

Nothing is common.

$P(A \cap B) = 0$, $P(A \cup B) = P(A) + P(B)$

Exhaustive Events

No other event ~~ego~~ can occur

$P(A \cup B) = 1$

Equally Likely or Mutually Symmetric or Equi-Probable Events

$P(A) = P(B) = P(C)$

Classical Definition limitations -

(i) Events shoulds be finite.
(ii) Events should be equally likely.
(iii) Limited field of application where events are known.

→ $0 \leq P(A) \leq 1$

→ $P(A') = 1 - P(A)$

→ odds in favour of A = success : failure

→ odds against A = failure : success.

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Addition Theorems

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

Independent Events

$$P(A\cap B) = P(A) \times P(B)$$

Independent events can never be mutually exclusive, and vice-versa.

Conditional Probability

$$P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$$

Provided $P(A) > 0$ i.e. A is not an impossible event.

If A and B are independent, then then pairs are also independent -

(i) A' and B

(ii) A and B'

(iii) A' and B'

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

<u>Compound Probability</u>

$$P(A \cap B) = P\left(\frac{B}{A}\right) \times P(A) = P\left(\frac{A}{B}\right) \times P(B)$$

Provided, $P(A) \geq 0$, $P(B) > 0$.

$$P(A \cap B \cap C) = P\left(\frac{C}{A \cap B}\right) \times P\left(\frac{B}{A}\right) \times P(A)$$

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Random Variable

→ Discrete random variables-
based on discrete series, have finite numbers.

→ Continuous random variable -
- based on continuous series, have uncountably infinite numbers.

(i) $p_i \geq 0$ $\qquad f(X) \geq 0$

(ii) $\sum p_i = 1$ $\qquad \sum_x f(X) = 1$

Expected Value

Discrete random variable-

$E(x) = \mu = \sum px$

$E(x^2) = \sum px^2$

$E[g(x)] = \sum p\,g(x)$

$$\text{Variance}(x) = \sigma^2 = E(x-\mu)^2$$

$$= E(x^2) - \cancel{E}\,\mu^2$$

$$= E(x^2) - [E(x)]^2$$

If $y = a + bx$, then

$$\mu_y = a + b\mu_x$$

$$\sigma_y = |b| \times \sigma_x$$

Continuous random variable

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\sigma^2 = E(x^2) - \mu^2$$

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Properties of Expected values

(1) - $E(k) = k$, k is any constant

2 - $E(x+y) = E(x) + E(y)$

3 - $E(k \cdot x) = k \cdot E(x)$

4 - $E(x, y) = E(x) \times E(y)$,
provided x and y are independent.

AND = Multiply, when two events occur simultaneously.

OR = Add, when either events occur.

Date

Theoretical Distributions

Discrete Probability Distributions
-1- Binomial Distributions
2) Poisson Distributions

Continuous Probability Distribution
3- Normal or Gaussian Distribution.

BINOMIAL DISTRIBUTION

Parameters - n and p.
Denoted by - $x \sim B(n,p)$
Probability mass function
$f(x) = P(X=x) = {}^nC_x\, p^x q^{(n-x)}$, $x = 0,1,2,3 \ldots n$

where p = success with value n = 1.
q = failure
n = total trials
x = success required.

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Imp. points

(i) $f(x) \geq 0$.

(ii) $p+q=1$, $p=1-q$, $q=1-p$

(iii) $\Sigma f(x) = f(0) + f(1) + f(2) + ... + f(n) = 1$

(iv) Mean $= \mu = np$.

(v) It may have unimodal or bi-modal.
Mode $= \mu_0 =$ largest integer in $(n+1)p$ (In case $(n+1)p$ is non-integer)
Mode $= \mu_0 = (n+1)p$ and $(n+1)p-1$ [In case $(n+1)p$ is an integer]

(vi) Variance $= \sigma^2 = npq$
Variance of a binomial variable is always less than its mean.

Variance of X is maximum when $p=q=0.5$.
Maximum value of Variance $= \frac{n}{4}$

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(viii) Additive property of binomial distributions

if $X \sim B(n_1, p)$

and $Y \sim B(n_2, p)$

Then $(X+Y) \sim B(n_1+n_2, p)$

POISSON DISTRIBUTION

(i) p is very low.

(ii) n is very large.

Parameters - m (λ)

Denoted by $X \sim P(m)$

Probability Mass function

$f(x) = P(X=x) = \dfrac{e^{-m} . m^x}{x!}$, $x = 0, 1, 2 \ldots n.$

where m = mean = np

x = success required.

$e = 2.71828$

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Imp. Points

(i) $f(x) \geq 0$

(ii) $\sum f(x) = f(0) + f(1) + f(2) + \dots + f(n) = 1.$

(iii) Uniparametric – m.

(iv) Mean $= \mu = m = np.$

(v) Variance $= \sigma^2 = m$

(vi) $\sigma = \sqrt{m}$

(vii) Modal, same as binomial distribution.

μ_0 = largest integer in m if m is non-integer

= m and m-1 if m is an integer.

(viii) $B(n, p) \cong P(m)$

(ix) Additive property of poisson distribution.

$X \sim P(m_1)$

$Y \sim P(m_2)$

then $(X+Y) \sim P(m_1+m_2)$

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NORMAL OR GAUSSIAN DISTRIBUTION

Parameters = μ and σ^2

Denoted by $X \sim N(\mu, \sigma^2)$

Probability Density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad , -\infty < x < \infty$$

Imp. Points

(i) $f(x) \geq 0 \quad , x \in (-\infty, \infty)$

(ii) $\int_{-\infty}^{+\infty} f(x) = 1$

(iii) Normal distribution is applicable when x is continuous.

iv) The area between $-\infty$ to μ = area between μ to ∞ = 0.5

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(v) Normal distribution is bell shaped.

(vi) It is unimodal.

(vii) Mean = Median = Mode

(viii) 4 SD = 5 MD = 6 QD

4 Start = 5 Middle = 6 Khatam

(ix) MD = 0.8σ

(x) QD = 0.675σ

(xi) Q_1 and Q_3 are equidistant from the median.

$Q_1 = \mu - 0.675\sigma$ QD

$Q_3 = \mu + 0.675\sigma$

(xii) μ = Median = Mode = $\frac{Q_3 + Q_1}{2}$

(xiii) The normal distribution is symmetric about its mean. Therefore its skewness is zero, i.e. neither positively skewed nor negatively skewed.

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(xiv) The normal curve $y = f(x)$ has two points of inflexion, $x = \mu - \sigma$ and $x = \mu + \sigma$, where normal curve changes its curvature from concave to convex and convex to concave.

(xv) Area under normal curve,

$\mu - \sigma$ to $\mu + \sigma$ = 0.6828 or 68.28%.

$\Rightarrow -1 < z < 1$ = 0.6828 or 68.28%.

$\mu - 2\sigma$ to $\mu + 2\sigma$ = 0.9545 or 95.45%.

$\Rightarrow -2 < z < 2$ = 0.9545 or 95.45%.

$\mu - 3\sigma$ to $\mu + 3\sigma$ = 0.9973 or 99.73%.

$\Rightarrow -3 < z < 3$ = 0.9973 or 99.73%.

x

Outside these limits = 0.0027.

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xvii) Additive Property of Normal Distribution

if $X \sim N(\mu_1, \sigma_1^2)$
$Y \sim N(\mu_2, \sigma_2^2)$
then $(X+Y) \sim N(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2})$

Standard Normal Distribution

It is denoted by $Z \sim N(0,1)$
Probability Density Function -

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Imp. Points

(i) $\mu = 0$, $\sigma = 1$

(ii) Z has mean, median, mode all $= 0$.

(iii) Since $\sigma = 1$, MD = 0.8 and SD = 0.675

(iv) Standard normal distribution is symmetrical about $z = 0$.

(v) Inflexion points = -1 and 1.

(vi) The two tails of standard normal curve never touch the horizontal axis.

Index Numbers

An index number is a ratio or an average of ratios expressed as a percentage, having two or more time periods, one of the time periods is base period.

Issues Involved in Index numbers

(a) Selection of data
(b) Base period
(c) Selection of weights
(d) Use of Averages
(e) Choice of Variables
(f) Selection of Formula

Price Relative = $\frac{P_n}{P_o}$ or $\frac{P_n}{P_o} \times 100$

Quantity Relative = $\frac{Q_n}{Q_o}$

Value Relative = $\frac{V_n}{V_o} = \frac{P_n Q_n}{P_o Q_o} = \frac{P_n}{P_o} \times \frac{Q_n}{Q_o}$

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Link relative = $\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \frac{P_n}{P_{n-1}}$

Chain Relatives = $\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \frac{P_n}{P_0}$

Simple Aggregative P.I = $\frac{\Sigma P_n}{\Sigma P_0} \times 100$

Simple Average Relative = $\frac{\Sigma \left(\frac{P_1}{P_0}\right)}{N} \times 100$

Weighted Aggregative Method :-

Laspeyres's Index = $\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$

Paasche's Index = $\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n} \times 100$

Marshall-Edgeworth Index = $\frac{\Sigma P_n (Q_0 + Q_n)}{\Sigma P_0 (Q_0 + Q_n)} \times 100$

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$$\text{Fisher's Index} = \sqrt{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}} \times 100$$

$$\text{Weighted Average of Relative} = \frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$$

$$\text{Bowley's Index no.} = \frac{\text{Laspeyre's} + \text{Paasche's}}{2}$$

$$\text{Chain Index} = \frac{\text{Link Relative of current year} \times \text{Chain Index of Previous Year}}{100}$$

Quantity Index Numbers

1- Simple Aggregate of Quantities $= \frac{\Sigma Q_n}{\Sigma Q_0} \times 100$

2- Simple Average of Quantity Relative $= \frac{\Sigma\left(\frac{Q_n}{Q_0}\right)}{N} \times 100$

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3- Weighted Aggregative Quantity Index

i, Laspeyre's Index = $\frac{\Sigma Q_n P_o}{\Sigma Q_o P_o}$

ii, Paasche's Index = $\frac{\Sigma Q_n P_n}{\Sigma Q_o P_n}$

(iii) Fisher's Index = $\sqrt{\frac{\Sigma Q_n P_o}{\Sigma Q_o P_o} \times \frac{\Sigma Q_n P_n}{\Sigma Q_o P_n}} \times 100$

4- Weighted Average of Quantity Relatives = $\frac{\Sigma Q_n P_o}{\Sigma Q_o P_o} \times 100$

Value Index = $\frac{V_n}{V_o} = \frac{\Sigma P_n Q_n}{\Sigma P_o Q_o}$

Deflated Value = $\frac{\text{Current Value}}{\text{Price Index of Current Year}}$

Deflated value = Current Value $\times \frac{\text{Base Price}}{\text{Current Price}}$

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$$\text{Shifted Price Index} = \frac{\text{Original Price Index}}{\text{Price Index of the year on which it has to be shifted}} \times 100$$

Test of Adequacy

(i) Unit Test:

The formula should be unit free.
All test satisfy this test except simple aggregative index.

(ii) Time Reversal Test:

It means $P_{01} \times P_{10} = 1$

Laspeyre's and Paasche's method do not satisfy this test.
Only Fisher's Method satisfy this test.

iii) Factor Reversal Test: $P_{01} = 1$ on 0

It means $P_{01} \times Q_{01} = V_{01}$

Fisher's method satisfies this test.

Fisher's Index number satisfies both (ii) and (iii) test, thus it is called an Ideal Index Number.

iv) Circular Test:

$$\frac{P_2}{P_0} = \frac{P_2}{P_1} \times \frac{P_1}{P_0}$$

This test is not satisfied by Laspeyre's or Paasche's or Fisher's ideal index.

Simple geometric mean of price relatives and weighted aggregative with fixed weights meet this test.

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Important Questions of Mathematics

Ratio, Proportion, Indices and Logarithms	
Examples	**Example-2** (Page - 1.3), (Page – 1.5) **Example-3** (Page – 1.5)
Exercise- 1A	2, 11, 16, 19, 20, 21, 23, 24 Example -3 (Page – 1.11)
Exercise- 1B	6, 7, 10, 14, 24, 28, 30 Example -9 (Page – 1.18)
Exercise- 1C	6, 8, 11, 13, 17, 18, 21, 22, 27, 30
Exercise- 1D	8, 10, 15, 16, 19, 21, 22
	Equation
Exercise- (A)	2, 4, 5, 8
Exercise- (B)	4, 8, 11
Exercise- (C)	2, 5, 6, 9
Exercise- (D)	6, 7, 9
Exercise- (E)	2, 5, 10, 11
Exercise- (F)	2, 3, 11
Exercise- (G)	4, 7, 10
Exercise- (H)	2, 4, 8
Exercise- (I)	5, 6, 8, 10

Time Value of Money	
Example	6, 8, 9, 16, 18, 20, 24, 33, 37
Exercise- 4A	7, 8, 9, 10
Exercise- 4B	4, 7, 8, 12, 13
Exercise- 4C	3, 9, 10, 11, 13
Exercise- 4D	5, 7, 8
Additional Question Bank	2, 5, 7, 9, 11, 12, 13, 14, 15
Permutation and Combination	
Exercise- 5A	5, 8, 14, 16, 20, 23
Examples	Example-8 (Page-9.12), Example-10 (Page-9.13), Example-6 (Page-5.25)
Exercise- 5B	3, 5, 7, 10, 17, 18, 19
Exercise- 5C	9, 10, 12, 17, 19, 21, 22
Exercise- 5D	1, 5, 12, 14, 19
Sequence & Series	
Exercise- 6A	5, 8, 9, 14, 18, 19, 22, 24
Examples	**Example-5** (Page-6.13)
Exercise- 6B	6, 7, 9, 10, 14, 16, 17, 22, 23, 24
Exercise- 6C	2, 4, 8, 14, 15, 19, 25, 27
Additional	12, 14, 24, 32, 41, 59, 63, 73, 85, 93, 99
Sets Relations & Functions	
Exercise- 7A	2, 5, 8, 13, 17, 18, 20, 24, 32
Exercise- 7B	8, 9, 11, 12, 16, 19, 20
Exercise- 7C	1, 3, 5, 8, 12, 16, 21
Differentiation and Integration Calculus	
Examples	Example-1 (Page-8.15), Example-2 (Page-8.16), Example-4 (Page-8.17)
Exercise- 8A	2, 5, 7, 11, 13, 15, 18, 19, 21, 23, 27, 28, 33, 37, 46, 49, 50
Exercise- 8B	3, 6, 8, 11, 13, 16, 21, 23, 28, 33, 35, 39, 42, 45, 47

Important questions of Logical Reasoning and Statistics

Important Questions of Logical Reasoning

Number Series	
Exercise	1, 3, 6, 7, 8, 9, 10, 14, 15, 17, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 32, 33, 34, 38, 40, 42, 47, 49, 51, 54, 56, 59, 61
Examples	4, 7, 8, 9, 13, 16
Direction Test	
Exercise	3, 4, 7, 10, 11, 12, 13, 16, 17, 19, 20, 22, 24, 25, 27, 31, 33, 37, 39, 42, 46, 48, 50, 53
Example	3
Seating Arrangement	
Exercise	1, 4, 5, 7, 8, 9, 12, 13, 15, 16, 19, 21, 22, 24, 26, 27
Examples	Example – 6, Example – 8, Example – 9, Example – 10, Example – 11, Example - 12
Blood Relations	
Exercise	1, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 23, 25, 30, 31, 32, 36, 37, 39, 42, 46, 47, 50, 54, 55
Examples	5, 8, 10, 13
Syllogism	
Exercise	2, 3, 5, 7, 9, 10, 12, 13, 14, 17, 18, 20, 21, 23, 24

Important questions of Statistics

Measure of Central Tendency	
Exercise Set-B	1, 4, 5, 6, 7, 10, 11, 12, 13, 14, 19, 20

Important questions of Statistics

Measure of Central Tendency	
Exercise Set-B	1, 4, 5, 6, 7, 10, 11, 12, 13, 14, 19, 20
Examples	15.1.2, 15.1.4, 15.1.5, 15.1.7, 15.1.8, 15.1.11, 15.1.12, 15.1.14, 15.1.16, 15.1.17, 15.1.20
Measure of Dispersion	
Exercise Set-B	2, 4, 6, 7, 9, 10, 12, 16, 20
Examples	15.2.2, 15.2.3, 15.2.6, 15.2.7, 15.2.8, 15.2.9, 15.2.10, 15.2.13, 15.2.14, 15.2.15, 15.2.16, 15.2.17, 15.2.20
Probability	
Exercise Set-B	1, 2, 6, 8, 12, 13, 15, 17, 19, 21
Examples	16.1, 16.4, 16.6, 16.7, 16.8, 16.10, 16.11, 16.12, 16.13, 16.14, 16.15, 16.16, 16.18, 16.19, 16.21, 16.23, 16.27, 16.29, 16.31
Theoretical Distribution	
Exercise Set-B	1, 4, 7, 8, 9, 11, 12, 15, 17, 20, 21, 22
Examples	17.1, 17.2, 17.5, 17.8, 17.9, 17.10, 17.11, 17.12, 17.13, 17.15, 17.18, 17.21, 17.22, 17.23, 17.28, 17.30, 17.32
Correlation and Regression	
Exercise Set-B	1, 2, 5, 6, 7, 9, 12, 14, 16, 17, 18, 21, 23
Examples	18.2, 18.3, 18.6, 18.8, 18.11, 18.13, 18.15, 18.17, 18.18, 18.19, 18.20, 18.21, 18.22
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