

CHAPTER 1

Ratio, Proportion and Mixtures

Ratio

- A ratio is a fraction (either proper or improper) which compares two or more quantities of similar kind, which enables us to understand as to how many times one quantity is involved in the other.
- If $A : B \left(\frac{A}{B} \right)$ is a ratio, then the numerator A is called “**Antecedent**” and the denominator B is called the “**Consequent**”.
- Ratios must be expressed in the simplest possible form and we can calculate ratios only when the quantities are commensurable (fully quantifiable).
- Ratios act as a basis of apportionment of a given quantity in a specified order.
- Two or more ratios can be bridged in order to have a continuous comparison between more than two variables.
- Rule for bridging more than two ratios :

If ,a,b,c,d,e are five Quantities, and

$$\frac{a}{b} = \frac{N_1}{D_1}; \frac{b}{c} = \frac{N_2}{D_2}; \frac{c}{d} = \frac{N_3}{D_3}; \frac{d}{e} = \frac{N_4}{D_4}$$

Then, $a:b:c:d:e = N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

- If a quantity ‘Q’ increases in the ratio a:b (where b>a) then, the increased quantity will be given by $Q \times \frac{b}{a}$
- If a quantity ‘Q’ decreases in the ratio a:b (where b<a) then, the reduced quantity will be given by $Q \times \frac{b}{a}$

Let a : b is a ratio, then:

- $\frac{a}{b} > 1$ (Ratio of Greater Inequality)
- $\frac{a}{b} < 1$ (Ratio of Lesser Inequality)
- $\frac{a}{b} = 1$ (Ratio of Equality)
- $a^2 : b^2$ (Duplicate Ratio)
- $a^3 : b^3$ (Triplicate Ratio)
- $\sqrt{a} : \sqrt{b}$ (Sub-Duplicate Ratio)
- $\sqrt[3]{a} : \sqrt[3]{b}$ (Sub-Triplicate Ratio)
- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then the value of each ratio can be obtained by mean of any one of the following two operations;

a. Each ratio = $\frac{a+c+e+\dots}{b+d+f+\dots}$ (ADDENDO)

Or

b. Each ratio = $\frac{a-c-e-\dots}{b-d-f-\dots}$ (SUBTRANDENDO)

INVERSE RATIO:

- IR of a:b is b : a
- IR of a:b:c is bc : ac : ab
- IR of a:b:c:d is bcd : acd : abd : abc

COMPOUND RATIO:

The multiplying effect of all ratios given is known as compound ratio. If a:b and c:d are two ratios, then **ac : bd** is called the compounded ratio of the two.

Proportion

- Proportion is defined as the equality of two or more ratios. If $\frac{a}{b} = \frac{c}{d}$, in such a case the quantities a,b,c,d are said to be proportional, here 'd' is called the fourth proportional.
- If $\frac{a}{b} = \frac{b}{c}$, then a,b,c are said to be in continued proportion, where 'b' is called the mean proportional and 'c' is called third proportional.
- If $\frac{a}{b} = \frac{b}{c}$ or $b^2 = ac \therefore b = \sqrt{ac}$

IF	THEN	Property
$\frac{a}{b} = \frac{c}{d}$	$ad = bc$	PRODUCT OF EXTREMES = PRODUCT OF MEANS
	$\frac{b}{a} = \frac{d}{c}$	INVERTENDO
	$\frac{a}{c} = \frac{b}{d}$	ALTERNENDO
	$\frac{a+b}{b} = \frac{c+d}{d}$	COMPONENDO
	$\frac{a-b}{b} = \frac{c-d}{d}$	DIVIDENDO
	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	COMPONENDO & DIVIDENDO

Ratio: Basic Concepts

1. Two numbers are in the ratio 5 : 6. If 5 is subtracted from each number, the ratio becomes 4 : 5. The numbers are:
 - a) 15, 20
 - b) 5, 10
 - c) 10, 15
 - d) 25, 30

2. Two numbers are in the ratio 3 : 4. If 6 be added to each terms of the ratio, then the new ratio will be 4 : 5. The two numbers are:
 - a) 24, 32
 - b) 18, 24
 - c) 15, 20
 - d) 9, 12

3. The angles of a triangle are in the ratio 2 : 7 : 11. The angles are: (in degrees)
 - a) 20, 70, 110
 - b) 20, 70, 90
 - c) 30, 70, 80
 - d) 18, 63, 99

4. Daily earnings of two persons are in the ratio 4 : 5 and their daily expenses are in the ratio 7 : 9. If each saves ₹ 50 per day, their daily incomes in ₹ are
 - a) (40, 50)
 - b) (50, 40)
 - c) (400, 500)
 - d) None of these

5. The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are:
 - a. 40, 60, 50
 - b. 50, 45, 55
 - c. 55, 35, 60
 - d. 45, 50, 55

6. Moi earns ₹ 80 in 7 hours and Zen earns ₹ 90 in 12 hours. The ratio of their earnings is:
 - a) 32 : 21
 - b) 23 : 12
 - c) 8 : 9
 - d) None of the above

7. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms in 5 hrs, what is the speed of the first train?
- 10 km per hour
 - 70 km per hour
 - 50 km per hour
 - None of the above
8. The ratio of the speeds of two trains is 2 : 5. If the distances they travel are in the ratio 5 : 9, find the ratio of the times taken by them.
- 18 : 25
 - 5 : 4
 - 25 : 18
 - 1:1
9. If $a/3 = b/4 = c/7$, then find the value of $(a + b + c) / c$.
- 1
 - 2
 - 3
 - None of the above
10. If $x/2 = y/3 = z/7$, then find the value of $(2x - 5y + 4z) / 2y$.
- 6/23
 - 23/6
 - 3/2
 - None of the above
11. The ratio of the number of 50 paise, Re. 1 and ₹ 5 coins with Mr. Zen is 5 : 2 : 1. If the amount with him is ₹ 38, then the number of Re. 1 coins with him is:
- 4
 - 8
 - 12
 - 16
12. A boy has five rupee coins, two rupee coins and one rupee coins in the ratio 3 : 4 : 5. If he has an amount of ₹ 224, then find the numbers of one rupee coins with the boy.
- 40
 - 35
 - 20
 - 30

13. Moi, Zen and Moizen were in a cold country. They wanted to make a bonfire. Moi contributed 5 wood logs, Zen contributed 4 wood logs. Moizen had no wood logs, so he contributed ₹ 9 to Moi and Zen. What money Moi should get?
- ₹ 5
 - ₹ 4
 - ₹ 6
 - ₹ 3
14. If $(a + b) : (ab)^{1/2} = 4:1$, find the value of $(a/b)^{1/2} + (b/a)^{1/2}$.
- 1 : 4
 - 4 : 1
 - 2 : 1
 - 1 : 2
15. If $\frac{a}{b + c} = \frac{b}{c + a} = \frac{c}{a + b}$. Then find the value of each ratio.
- 1
 - $\frac{1}{2}$
 - $\frac{1}{20}$
 - None of the above
16. A precious stone worth ₹ 15,600 is accidentally dropped and broken into three pieces, the weights of which are respectively proportional to 2 : 3 : 5. The value of the stone of this variety varies as the cube of its weight. Calculate the percentage loss thus incurred by this breakage.
- 80%
 - 90%
 - 84%
 - 78%
17. An employer reduces the number of employees in the ratio of 19 : 16 and increases their wages in the ratio of 4 : 5. What is the ratio of the wage bill of the employer initially and now?
- 20 : 19
 - 17 : 16
 - 16 : 17
 - 19 : 20

Types of Ratios:

1. The duplicate ratio of 3 : 4 is
 - a) $\sqrt{3} : 2$
 - b) 4 : 3
 - c) 9 : 16
 - d) None of these

2. The sub duplicate ratio of 25 : 36 is
 - a) 6 : 5
 - b) 36 : 25
 - c) 50 : 72
 - d) 5 : 6

3. The triplicate ratio of 2 : 3 is
 - a) 8 : 27
 - b) 6 : 9
 - c) 3 : 2
 - d) None of these

4. The sub triplicate ratio of 8 : 27 is
 - a) 27 : 8
 - b) 24 : 81
 - c) 2 : 3
 - d) None of these

5. If $p : q$ is the sub duplicate ratio of $(p - x^2) : (q - x^2)$, then find the value of x^2 .
 - a) $p / (p + q)$
 - b) $q / (p + q)$
 - c) $pq / (p - q)$
 - d) None of the above

6. If $2s : 3t$ is the duplicate ratio of $(2s - p) : (3t - p)$, then:
 - a) $p = 6st$
 - b) $p^2 = 6st$
 - c) $2p = 3st$
 - d) None of the above

Compound Ratio

1. The ratio compounded of $2 : 3$, $9 : 4$, $5 : 6$ and $8 : 10$ is
 - a) $1 : 1$
 - b) $1 : 5$
 - c) $3 : 8$
 - d) None of these

2. The ratio compounded of $4 : 9$ and the duplicate ratio of $3 : 4$ is
 - a) $1 : 4$
 - b) $1 : 3$
 - c) $3 : 1$
 - d) None of these

3. The ratio compounded of $4 : 9$, the duplicate ratio of $3 : 4$, the triplicate ratio of $2 : 3$ and $9 : 7$ is
 - a) $2 : 7$
 - b) $7 : 2$
 - c) $2 : 21$
 - d) None of these

4. The ratio compounded of the duplicate ratio of $4 : 5$, triplicate ratio of $1 : 2$, sub duplicate ratio of $81 : 256$ and sub triplicate ratio of $125 : 512$ is
 - a) $4 : 512$
 - b) $3 : 32$
 - c) $1 : 120$
 - d) None of these

5. Find the compounded ratio of $275 : 31$, inverse of $729 : 1331$, duplicate ratio of $2 : 5$, triplicate ratio of $9 : 11$, sub-duplicate ratio of $961 : 1296$, sub-triplicate ratio of $729 : 1331$.
 - a. $1 : 1$
 - b. $1 : 2$
 - c. $275 : 11$
 - d. $31 : 25$

Inverse Ratio

1. The Inverse ratio of 11 : 15 is
 - a) 15:11
 - b) $\sqrt{11} : \sqrt{15}$
 - c) 121 : 255
 - d) None of these

2. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is
 - a) 5
 - b) $\sqrt{5}$
 - c) 7
 - d) None of these

3. Divide ₹ 5130 among P, Q, R and S such that 3 times P's share = 4 times Q's share = 5 times R's share = 6 times S's share. What is the share of Q?
 - a. 1350
 - b. 1250
 - c. 1200
 - d. 1450

Joint Ratio

4. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$, the ratio a : b : c = ?
 - a) 4 : 6 : 15
 - b) 4 : 8 : 15
 - c) 8 : 12 : 15
 - d) 8 : 16 : 25

5. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 3 : 7, find A : B : C : D
 - a) 4 : 6 : 15 : 35
 - b) 4 : 12 : 15 : 35
 - c) 8 : 12 : 15 : 35
 - d) 8 : 16 : 25 : 35

6. If a : b = 3 : 5, b : c = 5 : 4, c : d = 2 : 3 and d is 50% more than e, find the ratio between a and e.
 - a) 2 : 3
 - b) 3 : 4
 - c) 3 : 5
 - d) 4 : 5

7. Aoi, Boi and Coi work in a company. The ratio of Aoi's age to Boi's age is 11 : 13 and Boi's age to Coi's age is 13:14. If the sum of their ages is 76, what are their respective ages?
- a) 33, 39, 42
 - b) 23, 27, 32
 - c) 22, 26, 28
 - d) 24, 28, 30
8. A and B have books in the ratio of 3 : 4, while B and C have books in the ratio 3 : 2. If the total number of books with the three of them is 116, how many books does A have?
- a) 36
 - b) 37
 - c) 32
 - d) 24
9. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$, then find the value of $\frac{a+b}{b+c}$
- a) 8 : 15
 - b) 20 : 27
 - c) 3 : 4
 - d) 27 : 20
10. Ahmedabad, Bombay and Calcutta are three cities. The ratio of average temperature between Ahmedabad and Bombay is 11 : 12 and the average between Ahmedabad and Calcutta is 9 : 8. Then the ratio between the average temperature of Bombay and Calcutta is:
- a) 22 : 27
 - b) 27 : 22
 - c) 32 : 33
 - d) None of the above
 - e)
11. A man distributes his property of ₹ 6,00,000 among his three sons. The share of his first son is thrice that of the second son's share and the share of the second son is twice that of the third son. Find the ratio in which sons share the property.
- a) 1 : 2 : 6
 - b) 3 : 4 : 5
 - c) 6 : 2 : 1
 - d) 2 : 4 : 6

Proportion

1. The fourth proportional to 4, 6, 8 is
 - a) 12
 - b) 32
 - c) 48
 - d) None of these

2. The third proportional to 12, 18 is
 - a) 24
 - b) 27
 - c) 36
 - d) None of these

3. The mean proportional between 25, 81 is
 - a) 40
 - b) 50
 - c) 45
 - d) None of these

4. The number which has the same ratio to 26 that 6 has to 13 is
 - a) 11
 - b) 10
 - c) 21
 - d) None of these

5. The fourth proportional to $2a$, a^3 & c is
 - a) $ac/2$
 - b) ac
 - c) $2/ac$
 - d) None of these

6. If four numbers $1/2$, $1/3$, $1/5$, $1/x$ are proportional then x is
 - a) $6/5$
 - b) $5/6$
 - c) $15/2$
 - d) None of these

7. The mean proportional between $12x^2$ and $27y^2$ is
 - a) $18xy$
 - b) $81xy$
 - c) $8xy$
 - d) None of these

8. If $x / y = z / w$, implies $y / x = w / z$, then the process is called
- Dividendo
 - Componendo
 - Alternendo
 - None of these.
9. If $p / q = r / s = p - r / q - s$, the process is called
- Subtrahendo
 - Addendo
 - Invertendo
 - None of these.
10. If $a/b = c/d$, then the process $(a+b)/(a-b) = (c+d)/(c-d)$, is called
- Componendo
 - Dividendo
 - Componendo and Dividendo
 - None of these.
11. If $u / v = w / p$. then the process $(u-v) / (u+v) = (w-p) / (w+p)$, is called
- Invertendo
 - Alternendo
 - Addendo
 - None of these.
12. If $\frac{a}{4} = \frac{b}{5}$ then
- $\frac{a+4}{a-4} = \frac{b-5}{b+5}$
 - $\frac{a+4}{a-4} = \frac{b+5}{b-5}$
 - $\frac{a-4}{a+4} = \frac{b+5}{b-5}$
 - None of these
13. What should be added to 3, 15, 38 and 134 so that the number become proportionate to each other.
- 3
 - 5
 - 7
 - 2

14. Four integers p, q, r, s are in continued proportion. If $p : r :: 9 : 4$, find p/s .
- a) $\frac{9}{4}$
 - b) $\frac{27}{4}$
 - c) $\left(\frac{9}{4}\right)^{\frac{3}{2}}$
 - d) 81

Mixtures and Alligation

15. In what proportion must rice @ ₹ 3.10/kg be mixed with rice @ ₹ 3.60/kg to make the mixture worth ₹ 3.25/kg?
- a. 3 : 5
 - b. 5 : 3
 - c. 3 : 7
 - d. 7 : 3
16. On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.
- a. 2 : 3
 - b. 3 : 5
 - c. 5 : 3
 - d. 3 : 2
17. A merchant has 100 kg of sugar, part of which he sells at 7% profit and the rest at 17% profit. He gains 10% on the whole. Find how much is sold at 7% profit.
- a. 30 kg
 - b. 70 kg
 - c. 55 kg
 - d. 45 kg
18. Two solutions 90% and 97% purity are mixed resulting in 21 litres of mixture of 95% purity. What is the quantity of 97% purity in this mixture?
- a) 13 litres
 - b) 14 litres
 - c) 14.5 litres
 - d) 15 litres

19. A sum of ₹ 6.25 is made up of 80 coins which are either 10p or 5p. How many are there of each kind?
- 60, 20
 - 50, 30
 - 45, 35
 - 40, 40
20. 300gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 50%?
- 50gm
 - 60gm
 - 70gm
 - 100gm
21. An alloy is to contain copper and Zinc in the ratio 9:4. Zinc required to melt with 24kg of copper is:
- $10\frac{2}{3}$ kg
 - $10\frac{1}{3}$
 - $9\frac{2}{3}$
 - 9kg
22. Three vessels having volumes in the ratio of 1:2:3 are full of a mixture of coke and soda. In the first vessel, ratio of coke and soda is 2 :3, in second, 3:7 and in third, 1:4. If the liquid in all the three vessels were mixed in a bigger container, what is the resulting ratio of coke and soda?
- 4:11
 - 5:7
 - 7:11
 - 11:7
23. Three equal glasses are filled with a mixture of spirit and water. The proportion of spirit to water in each glass is respectively: 2 : 3, 3 : 4 and 4 : 5. The contents of the three glasses are emptied into a single vessel. What is the proportion of spirit and water in it?
- 4 : 5
 - 429 : 533
 - 401 : 544
 - 1 : 1

