CHAPTER 2

Indices, Surds and Logarithms

$a^x = N$

a = base

x = Power/Exponent/Index

N = Product

$\left[But, a \neq 0,1,\pm\infty\right]$

Theory of Indices deals with the various changes in power, during various mathematical operations.

Basic Rules

1. $a^m \times a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $\left(a^m\right)^n = a^{mn}$; m is added n times

4. $\left(ab\right)^m = a^m x b^m$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

6. $a^0 = 1$

7. $a^{-n} = \frac{1}{a^n}$

8. If $a^m = a^n \Rightarrow m = n;$ where, $a \neq 0,1,-1,\pm\infty$

9. $a^m = b^m$

   (i) $m \neq 0$

   (ii) $a = b$ (when m is odd)

   (iii) $a = \pm b$ (when m is even)

10. $a^x = N$

$\Rightarrow a = N^{\frac{1}{x}} = \sqrt[x]{N}$

11.$(i)0^a = 0$

$(ii)1^a = 1$

$(iii)a^1 = a$

$(iv)a^0 = 1$

$(v)0^0 has$ no meaning

Basic Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a+b)(a-b)$
4. $(a+b)^2 + (a-b)^2 = 2\left(a^2 + b^2\right)$
5. $(a+b)^2 - (a-b)^2 = 4ab$
6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
7. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$
8. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$
9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
11. If $a+b+c=0,$ then $a^3 + b^3 + c^3 = 3abc$
12. If $a^3 + b^3 + c^3 = 3abc, then \ \ either \ a+b+c=0$ or a = b = c
    but both the results cannot hold true simultaneously

Rational Numbers, Irrational Numbers & Surds

- A Rational Number is a number which can be expressed in the form p/q, where q ≠ 0; p & q are integers and p and q are prime to each other, i.e., there is no common factor between p & q, other than 1.
- Any terminating and recurring decimals are rational numbers.
- Thus any non-recurring and non-terminating decimals are irrational numbers, and when the irrational numbers are expressed in radical form (root form), it is known as "Surds".
- Thus all the surds are irrational, but all irrational numbers are not surds.
- The numbers whose perfect root can be evaluated are rational quantities and numbers for which perfect roots cannot be evaluated are irrational quantities.

Order of Surds

If $\sqrt[k]{m} = (m)^{\frac{1}{k}}$ is a surd, then, it is said to be a surd of order "k".

Pure Surds and Mixed Surds

In case of pure surds, entire expression is kept within the radical sign. In mixed surds, it is expressed as a product of one rational and one irrational quantity.

Example:

$\sqrt{7}$ is a pure surd; $\sqrt{12} = \sqrt{4x3} = 2\sqrt{3}$ is a mixed surd.

Conjugate of a Surd

If $\left(a + \sqrt{b}\right)$ or $\left(\sqrt{a} + \sqrt{b}\right)$ are surds, their respective conjugates would be given by,

$\left(a - \sqrt{b}\right)$ or $\left(\sqrt{a} - \sqrt{b}\right)$ and vice-versa.

Rationalization of Surds

Rationalization is a process, where we convert the irrational part of the surd into a rational quantity, with help of its conjugate.

Note: 1

- Rational + Rational = Rational
- Rational – Rational = Rational
- Rational x Rational = Rational
- Rational ÷ Rational = Rational

Note: 2

- Irrational + Irrational = Irrational
- Irrational – Irrational = Rational (only when the quantities are equal); otherwise –
- Irrational – Irrational = Irrational
- Irrational x Irrational = May be Rational or Irrational
- Irrational ÷ Irrational = May be Rational or Irrational

Note: 3

- Rational + Irrational = Irrational
- Rational – Irrational = Irrational
- Rational x Irrational = Irrational
- Rational ÷ Irrational = Irrational

Square Root of Surds

- The square root of a surd is always a surd.
- Every answer for square root must contain +ve or –ve sign and in the absence of +/- sign, "none of these" will be marked as answer.
- If the given surd, whose square root is to be evaluated is in the form $\left(a \pm \sqrt{b}\right)$, then the answer will also be in the form $\pm\left(x \pm \sqrt{y}\right)$.
- Square the options, in order to get the question back.

Numerical Problems:

1. If "a" and "b" are whole numbers (a, b $\neq$ 1), such that $a^b = 49$, then the value of $\left(\frac{a-b}{a+b}\right)$ is:
   a) 5 / 9
   b) 5
   c) 9
   d) 1
   e)
2. If $3^a + 5^b = 34$ and $3^{a+1} + 5^{b+2} = 652$, then the values of "a" and "b" respectively are:
   a) 3, 3
   b) 5, 5
   c) 2, 2
   d) 1, 3

3. If $2^n - 2^{n-1} = 4$, then find the value of n^n.
   a) 8
   b) 27
   c) 125
   d) 1

4. Evaluate:. $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}} \cdot \left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}} \cdot \left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$
   a) 1
   b) – 1
   c) 3
   d) None of the above

5. Evaluate: $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\frac{x^a}{x^b}\right)^{(a+b-c)}$.

   a. x^0
   b. x^{abc}
   c. x^{a+b+c}
   d. (a) or (b) both

6. If $f^{abc} = f^a.f^b.f^c$, where a, b, c and f are all positive integers, then $a^2 + b^2 + c^2$ = ?

   a. 16
   b. 14
   c. 18
   d. 3

7. If $A = 2^{3^2}$, $B = \left(2^3\right)^2$, $C = (2)^{3^2}$, then which of the following is true?

   a) A = B
   b) A = C
   c) B = C
   d) A = B = C

8. Find the value of $\frac{1}{1+x+y^{-1}} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+x^{-1}}$; if xyz = 1.

   a. 0
   b. 1
   c. – 1
   d. 2

9. If E = $(10)^{150} \div (10)^{146}$, then E + 101 =?

   a. 11001
   b. 10101
   c. 100101
   d. 1000101

10. Evaluate: $(0.000064)^{5/6}$

    a) 0.0032
    b) 0.00032
    c) 0.0000032
    d) None of the above

11. The value of $\sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$ is:
    a. 3
    b. 2
    c. – 2
    d. (a) and (c) both

12. If $4^x = 8^y$, what is the value of $\frac{x}{y}$ - 1 ?
    a. 3
    b. 2
    c. 0.5
    d. – 0.5

13. If $2^x = 4^y = 8^z$, and 1/2x + 1/4y + 1/8z = 22/7, find (x, y, z).
    a) 1, 2, 3
    b) 16/7, 32/7, 48/7
    c) 7/16, 7/32, 7/48
    d) None of the above
    e)

14. If x + y + z = 0 and xyz ≠ 0, find the value of $\frac{x^3+y^3+z^3+(x+y)(y+z)(z+x)}{xyz}$
    a. – 2
    b. 0
    c. 2
    d. 3

15. If a + b + c = 0 find the value of $\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$
    a) 0
    b) 1
    c) 2
    d) 4

16. If a, b, c are real and distinct, then find the value of $\frac{27(a-b)(b-c)(c-a)}{a^2(b-c)+b^2(c-a)+c^2(a-b)}$
    a) – 27
    b) 0
    c) 27
    d) 81

17. Find the value of $(1.729)^3 + 3(1.729)(0.542) + (0.271)^3$
    a) 2
    b) 8
    c) 2.271
    d) 1.458

18. Find the value of a, if $2^{a+1} \cdot 5^{a} = 200$
    a. 1
    b. 2
    c. 3
    d. None of the above

19. If $27^{p} = \frac{9}{3^{p}}$, then the value of $\frac{1}{p^{2}}$ is?
    a. 9
    b. 4
    c. 16
    d. 1

20. Find the value of x, if $3^{5x}.9^{4x-2} = \frac{27^{3x-8}}{81^{-3x}}$
    a. 5
    b. 2
    c. 1
    d. 5/2

21. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is?
    a. 3
    b. 2
    c. 1
    d. 0

22. Find the value of x, when $\left[x.x^{1/2}\right]^{x} = (x)^{\left[x.x^{1/2}\right]}$
    a. 3/2
    b. 9/4
    c. 27/8
    d. 81/16

23. If $A = \frac{1}{1+a^{n-m}} + \frac{1}{1+a^{m-n}}$, then the value of (A + 1) is?
    a) 0
    b) 2
    c) 5
    d) 11

24. If x = $\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$, then find the value of x^{x}.
    a. 0
    b. 1
    c. 4
    d. 9

25. If $M = m^{\frac{1}{3}} + \frac{1}{m^{\frac{1}{3}}}$, then the value of $M^3 - 3M$.
    a. 2m
    b. $m + m^{-1}$
    c. 0
    d. None of the above

26. If $x = 5^{\frac{1}{3}} + 5^{-\frac{1}{3}}$, then find the value of $x^3 - 3x$.
    a. 5
    b. 5.10
    c. 5.20
    d. 5.50

27. $2^x = 3^y = 6^z$, then the value of z is ?
    a. xy
    b. $x + y$
    c. $x - y$
    d. $\frac{xy}{x + y}$

28. If $(7.77)^x = (.777)^y = 1000$, then the value of $\frac{1}{x} - \frac{1}{y}$ is?
    a) 1
    b) 2
    c) 3
    d) None of the above

29. If $p^a = q^b = r^c$ and pqr = 1, then which of the following is true?
    a) a + b + c = 0
    b) a = b + c
    c) b = c + a
    d) ab +bc + ca = 0

30. Simplify: $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$.
    a) 1
    b) 2
    c) 3
    d) None of the above

31. Simplify: $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}$.
    a) 1
    b) 2
    c) 3
    d) 0

32. If $a=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then find the value of $a^2 + b^2$.
    a) 98
    b) 99
    c) 100
    d) None of the above

33. If $a=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then find the value of $a^{-2} + b^{-2}$.
    a) 10
    b) 100
    c) 98
    d) 99

34. If $x=\frac{\sqrt{2}+1}{\sqrt{2}-1}, \ y=\frac{\sqrt{2}-1}{\sqrt{2}+1}$ and $E = x^2 + xy + y^2$, Then E – 5 =?
    a) 33
    b) 30
    c) 12
    d) 35

35. If $x = 3 + \sqrt{8}$, then the value of $x^4+\frac{1}{x^4}$ is?
    a) 1100
    b) 1
    c) 1154
    d) 1145

36. If $x=\frac{5-\sqrt{21}}{2}$, then the value of $\left(x^3+\frac{1}{x^3}\right)-5\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)$ is?
    a) 1
    b) 0
    c) 3
    d) 2

37. If $a=\sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$ then find the value of $[a(a - 14)]^2$.
    a) 1
    b) 2
    c) 7
    d) 14

38. If cube root of 2, 6^{th} root of 3, 9^{th} root of 4 are to be arranged in descending order of magnitude, the new order will be:
    a. $\sqrt[3]{2}, \sqrt[9]{4}, \sqrt[6]{3}$
    b. $\sqrt[9]{4}, \sqrt[6]{3}, \sqrt[3]{2}$
    c. $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$
    d. None

39. $\sqrt{28-5\sqrt{12}}$ = ?

a. $5+\sqrt{3}$

b. $5-\sqrt{3}$

c. $5+\sqrt{6}$

d. None

40. $\sqrt{10+2\sqrt{6}+2\sqrt{15}+2\sqrt{10}}$ = ?

a. $\sqrt{2}+\sqrt{3}+\sqrt{5}$

b. $\pm\left(\sqrt{2}+\sqrt{5}+\sqrt{7}\right)$

c. $-\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)$

d. None

41. Find the value of $\sqrt{\sqrt{18}-\sqrt{16}}$

a. $\pm\sqrt[6]{2}\left[\sqrt{2}-1\right]$

b. $\pm\sqrt[4]{2}\left[\sqrt{2}-1\right]$

c. $\pm\sqrt{2}\left[\sqrt{2}-1\right]$

d. None of the above

42. Simplify: $\dfrac{\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}}{\sqrt{5+2\sqrt{6}}+\sqrt{5-2\sqrt{6}}}$.

a) 3

b) 2

c) 1

d) None of the above

43. Simplify: $\frac{1}{\sqrt{12-\sqrt{140}}}-\frac{1}{\sqrt{8-\sqrt{60}}}-\frac{2}{\sqrt{10+\sqrt{84}}}$.

a) 0
b) 1
c) 2
d) None of the above

44. Find the cube root of $9\sqrt{3}+11\sqrt{2}$

a) $3\sqrt{3}\left(1+\sqrt{\frac{2}{3}}\right)$
b) $\sqrt{3}\left(1+\sqrt{\frac{2}{3}}\right)$
c) $3\sqrt{3}\left(1-\sqrt{\frac{2}{3}}\right)$
d) $\sqrt{3}\left(1-\sqrt{\frac{2}{3}}\right)$

45. If x = 0.6, then find the value of $\left[1-\left\{1-\left(1-x^5\right)^{-1}\right\}^{-1}\right]^{-\frac{2}{5}}$.

a. 0.36
b. 0.6
c. 0.625
d. 0.63

46. If $x = 2 + \sqrt{3}$, then the value of $x^3 - 2x^2 - 7x + 4$ is?

a) 3
b) 2
c) 1
d) 0

47. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ then find the value of $2a^4 - 21a^3 + 12a^2 - a + 1$.

a. 21
b. 1
c. 12
d. None of the above

Logarithms

If $a^x = N$, then $x = \log_a N$; * $a \neq 0,1,\pm\alpha$ and for the purpose of log, any negative quantity.

* x is called the logarithm of N (product) to the base "a".

Base "a"

- The base "a" of log can be any positive real number except 1.
- The base of log can be clearly divided into two parts:
- 0 < a < 1 (the proper fraction)
- a > 1 (positive integer / mixed fraction)
- Unless otherwise specified, the base of log is always taken to be 10 and this is known as Common Logarithm.
- For theoretical purpose, the base is always taken to be "e", where "e" is a constant and this is known as "Natural Logarithm".
- Common Logarithms are used for numerical calculations and Natural Logarithms are used in calculus.

Basic Rules

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a 0 =$ Undefined
7. $\log_a -ve =$ Undefined
8. $\log_a m = \log_a n \Rightarrow m = n$

Change of Base in Logarithms

1. $\log_a b = \frac{\log_m a}{\log_m b}$ (m can be any common base) (m $\neq 0,1, \pm\alpha$,-ve value)
2. $\log_a b = \frac{1}{\log_b a}$
3. $a^{\log_a x} = x$

Nature of Log Values

- All the values which are obtained from log tables are irrational numbers provided the numbers are not 10 or in the form of 10^n.
- $\log_b a$ is a rational quantity only when, $\frac{\log a}{\log b}$ is rational.

Number of Digits

Rule for finding out the number of digits in an expression of the form a^b, where a and b are integers, using logarithms.

- If K is a number, then its log value, logK can be divided into two parts: a) Integral Part, b) Fractional Part.
- The integral part is called "Characteristics" and the fractional part is called "Mantissa".
- The integral characteristics part can be positive or negative or zero but not a fraction.
- The values of mantissa are always positive fractions.
- The values for mantissa are obtained from log tables.
- Characteristics are to be calculated before we evaluate mantissa from the log table.
- Value of characteristics = number of significant digits before decimal – 1
- Thus Number of digits = value of characteristics + 1

1. $\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000$ is

   a) 15

   b) $\log_{10} 11111$

   c) $\log_{10} 1111$

   d) $14\log_{10} 100$

2. If $\log\left(\frac{a}{b}\right)+\log\left(\frac{b}{a}\right)=\log(a+b)$, then which of the following is true?

   a) a + b = 1

   b) a + b = 0

   c) a = b

   d) a – b = 1

3. Find the value of $\log_{10}\left(\frac{4}{25}\right)+\log_{10}\left(\frac{125}{7}\right)-\log_{10}\left(\frac{2}{7}\right)$.

   a) 1

   b) 4

   c) 41

   d) None of the above

4. Evaluate : $\left[\log\left(\frac{a^2}{bc}\right)+\log\left(\frac{b^2}{ac}\right)+\log\left(\frac{c^2}{ab}\right)\right]$

a) 1

b) 2

c) – 1

d) 0

5. $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$ equals

a) 18

b) 1

c) 3

d) None of the above

6. $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}$ =

a. log 2

b. log 3

c. log 5

d. None of the above

7. If $\log_{10}$ [98 + $\sqrt{x^2-12x+36}$] = 2, then x =

a. 4

b. 8

c. 12

d. 4, 8

8. $\frac{\log 49\sqrt{7}+\log 25\sqrt{5}-\log 4\sqrt{2}}{\log 17.5}$ =

a. 5

b. 2

c. 5/2

d. 3/2

9. If $\log_5\left(x^2+x\right)-\log_5(x+1)=2$; then find the value of x.

a. 5

b. 1/5

c. 5^2

d. None of the above

10. $\dfrac{3+\log_{10} 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)}$ =

a. 3

b. $\frac{3}{2}$

c. 2

d. 1

11. If $\left(\frac{21}{10}\right)^x = 2$, then x = ?

a. $\dfrac{\log 2}{\log 3+\log 7+1}$

b. $\dfrac{\log 2}{\log 3+\log 7-1}$

c. $\dfrac{\log 2}{\log 7+\log 3+2}$

d. None of the above

12. Evaluate: $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y}$.

a. 0

b. 1

c.2

d. – 1

13. Evaluate: $(bc)^{\log b/c} \cdot (ca)^{\log c/a} \cdot (ab)^{\log a/b}$.

a. 0

b. 1

c. – 1

d. None of the above

14. The value of is $\mathbf{a^{\log b/c} \cdot b^{\log c/a} \cdot c^{\log a/b}}$

a) 0

b) 1

c) -1

d) None

15. Given log2 = 0.3010 and log3 = 0.4771, find the value of log6.

a) 0.9030

b) 0.9542

c) 0.7781

d) None of the above

16. Given that $\log_{10}2 = x$ and $\log_{10}3 = y$, the value of $\log_{10} 60$ is expressed as:
    a. x + y + 1
    b. x – y + 1
    c. x – y – 1
    d. None of the above

17. Given logx = m + n and logy = m – n, the value of log $(10x/y^2)$ is expressed in terms of m and n as:
    a. 1- m + 3n
    b. m – 1 + 3n
    c. m + 3n + 1
    d. None of the above

18. If $\log(\frac{x+y}{5}) = \frac{1}{2}(\log x + \log y)$, then $\frac{x}{y} + \frac{y}{x}$ =
    a. 20
    b. 23
    c. 22
    d. 21

19. If $\log(x+y) = \log\left(\frac{3x-3y}{2}\right)$, then log x – log y =
    a. log 2
    b. log 3
    c. log 5
    d. log 6

20. If $\log a = \frac{1}{2}\log b = \frac{1}{5}\log c$, then find the value of $a^4b^3c^{-2}$.
    a. 0
    b. – 1
    c. 1
    d. None of the above

21. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then find the value of $x^xy^yz^z$?
    a. 0
    b. 1
    c. 2
    d. None of the above

22. Find the value of logarithm of 64 to the base $2(2)^{0.5}$.
    a. 1
    b. 2
    c. ½
    d. None of the above

23. Find the value of $\log_{2\sqrt{3}} 1728$.
    a. 2
    b. 6
    c. 1
    d. None of the above

24. log (a-9) + loga = 1, the value of a is given by
    a) 0
    b) 10
    c) -1
    d) None

25. On solving the equation logt + log (t - 3) = 1 we get the value of t as
    a) 5
    b) 2
    c) 3
    d) 0

26 For any three consecutive integers x, y, z. the equation log (1 + xz) – 2logy = 0 is:
    a. True
    b. False
    c. Sometimes true
    d. Cannot be determined in case of cyclic order

27. If $\log_2 (\log_3 (\log_2 x)) = 1$, then x =
    a. 512
    b. 128
    c. 12
    d. 0

28. If $\log_{0.5} (\log_x (\log_4 32)) = 2$, then x =
    a. 5/2
    b. 625/16
    c. 25/4
    d. None of the above

29. If $x = \log_a bc$; $y = \log_b ca$; $z = \log_c ab$, then the value of xyz – x – y – z is:
    a. 1
    b. 2
    c. – 1
    d. 0

30. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ba$, then the value of $\frac{1}{x+1}+\frac{1}{z+1}+\frac{1}{1+y}$ is….
    a. 3
    b. 1
    c. 2
    d. 0
31. Find the value of $\log_5 5.\log_4 9.\log_3 2$.
    a. 1
    b. 2
    c. 5
    d. None of the above
32. Find the value of $(\log_b a \times \log_c b \times \log_a c)^3$
    a. 1
    b. 2
    c. 3
    d. None of the above
33. If $\log_4 x + \log_2 x = 6$, then the value of x is
    a. 2
    b. 4
    c. 8
    d. 16
34. If $\log_{10\sqrt{x}} = 2 \log_x 10$, then a possible value of x is given by:
    a. 10
    b. $\frac{1}{100}$
    c. $\frac{1}{1000}$
    d. None of the above
35. Evaluate : $a^{\frac{1}{\log_b a}}$
    a. a
    b. b
    c. a + b
    d. None of the above
36. Find the value of the following expression: $a^{\log_a b.\log_b c.\log_c d.\log_d t}$
    a. t
    b. abcdt
    c. a + b + c + d + t
    d. None of the above

37. The value of $\left[\frac{1}{\log_{\frac{p}{q}} x}+\frac{1}{\log_{\frac{q}{r}} x}+\frac{1}{\log_{\frac{r}{p}} x}\right]$ is …..?

a. 3

b. 2

c. 1

d. None of the above

38. $\log_2 \log_{\sqrt{2}} \log_3 81$ = ?

a. 3

b. 2

c. 1

d. 0

39. If MOI = $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$, then MOI equals:

a. 3

b. 5

c. 7

d. 25

40. Given log (1 + 2 + 3) = log1 + log2 + log3. Using the given rule calculate log (3 + 4 + 5).

a. log 3 + log 4

b. log 3 + log 4 + log 5

c. a) or b)

d. Can't be determined

41. If $x=\frac{e^y-e^{-y}}{e^y+e^{-y}}$, then find the value of y.

a) $\log\frac{1+x}{1-x}$

b) $\log_e\frac{1+x}{1-x}$

c) $\frac{1}{2}\log_e\frac{1-x}{1+x}$

d) $\frac{1}{2}\log_e\frac{1+x}{1-x}$

42. Given log2 = 0.30103, the number of digits in 2^{50} is

a. 14

b. 16

c. 18

d. 25

43. $\log_2 5$

a) An integer

b) A rational number

c) An irrational number

d) A prime number

44. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$

a) log 2

b) 1

c) 0

d) None of the above