

## CHAPTER 7

### Mathematics of Finance

**Simple Interest**

Simple interest is charged on the principal amount and hence it is same for any every year.

A = Amount, P = principal, n = number of years, R = interest rate

$$SI = \frac{PTR}{100}$$

$$A = P + SI = P + \frac{PTR}{100} = P \left( 1 + \frac{TR}{100} \right)$$

**Notes:**

- If rate of interest is known, then sum of money will double itself in  $100/r$  years.
- If number of years is known, then sum of money will double itself @  $100/n$  %.
- A sum of money will become “n” times in  $\frac{(n-1) \times 100}{R}$  years.

Example:

In how many years a sum of money @10% p.a. SI will become (a) double, (b) triple, (c) N times.

(a) Double	(b) Triple	(c) N times
$\frac{(2-1) \times 100}{10} = 10$ years	$\frac{(3-1) \times 100}{10} = 20$ years	$\frac{(N-1) \times 100}{10} = 10(N-1)$ years

- If the sum of money becomes “n<sub>1</sub>” times in T<sub>1</sub> years and “n<sub>2</sub>” times in T<sub>2</sub> years, then

$$\frac{T_1}{T_2} = \frac{n_1 - 1}{n_2 - 1}$$

the ratio of their times is:

**Compound Interest**

- In case of compound interest, the interest is calculated on the amount of the succeeding years, i.e., principal keeps changing every year.
- Here interest on interest is also earned, thus money grow faster when Compounding is done
- If P is the principal, n = number of years for which interest is calculated and “i” (R/100) is the rate of interest, then, the amount A after n years will be given by:

$$A = P(1+i)^n$$

- In case of depreciation by diminishing balance method (WDV), if C = Cost of the machinery, I = rate of depreciation per annum and n = effective life of the machinery, then the depreciated value D after n years is :

$$D = C(1-i)^n$$

D is also known as the scrap value of the machinery.

- Compound Interest thus would be calculated as follows:

$$CI = A - P = P \left[ (1+i)^n - 1 \right]$$

- Depending upon the compounding style of interest rate, the effective formula for calculating Amount would be as follows:

Half Yearly or Semi Annually	Quarterly	Monthly
$A = P\left(1 + \frac{i}{2}\right)^{2n}$	$A = P\left(1 + \frac{i}{4}\right)^{4n}$	$A = P\left(1 + \frac{i}{12}\right)^{12n}$

- When differential interest rates are charged ( $i_1, i_2, i_3, \dots, i_n$ ), then:  
 $A = P(1 + i_1)(1 + i_2)(1 + i_3) \dots (1 + i_n)$
- Relationship between CI and SI
  - a) For the first year, CI = SI, i.e. for the first year difference is zero.
  - b) For two years, CI – SI =  $Pi^2$
  - c) For three years, CI – SI =  $Pi^2(i + 3)$

**Notes:**

1. A sum of money will double itself in approximately  $72/r$  years (known as Rule 72), where  $r$  is the rate of interest per annum.
2. A sum of money will triple itself in approximately  $114/r$  (known as Rule 114), where  $r$  is the rate of interest per annum.
3. If a sum of money becomes “ $n$ ” times in “ $t$ ” years, then, it will become  $n^m$  times in “ $mt$ ” years.  
 Example: If sum of money doubles itself in 3 years, then it will be 8 times ( $2^3$ ) in  $3 \times 3 = 9$  years at CI.

**Concept of Effective Rate of Interest**

1. When the compounding is done more than once a year, then, the net annual rate of interest is found to be slightly higher than the given annual rate of interest.
2. This new rate of interest is known as the effective rate of interest and the given annual rate is called the nominal rate of interest.
3. Effective rate of interest is denoted by  $E$  and is given by the formula:  

$$E = \left\{ (1 + i)^n - 1 \right\} \times 100$$
 Where “ $i$ ” is rate of interest, converted monthly, quarterly, half yearly and  $n$  is the number of conversion period per annum.
4. Effective rate of interest are particularly useful in making investment decisions when various options are given with differential interest rates.
5. Amongst various investment options, we shall choose that investment option, where effective rate of interest is maximum.

**Concept of Present Value**

Present Value is defined as the present worth of the money that would yield an amount  $A$  after  $n$  years at a specified rate of interest  $i$ .

If  $A = P(1 + i)^n$

$\therefore P = PV = \text{Principal} = \frac{A}{(1 + i)^n}$

or,  $PV = A(1 + i)^{-n}$

**Annuities**

- Annuity is defined as a series of payments (usually equal) which are made at regular intervals of time (usually a year).
- The period for which the payment continues is called the status or the term of the annuity.
- Unless otherwise stated, the first payment will fall due at the end of every year. This is known as “Ordinary Annuity”.
- When the payment falls due at the beginning of every year, i.e., immediately, it is called “Immediate Annuity”.
- When the status or term of the annuity is not fixed, i.e., the payment is to be continued for an indefinite period, these are known as “Perpetual Annuity or Perpetuity”.
- Hence forth, we shall maintain the following notation throughout. The regular annual payment i.e., annuity = P “i” = rate of interest and the period for which payment is made = n (status or term of the annuity).

- The amount of the ordinary annuity is given by:

$$A = \frac{P}{i} \left\{ (1+i)^n - 1 \right\}$$

- The amount of immediate annuity is obtained by multiplying amount obtained for ordinary annuity by (1 + i); hence the formula becomes:

$$A = \frac{P}{i} \left\{ (1+i)^n - 1 \right\} (1+i)$$

- Note:
  - When half yearly or quarterly or monthly payment is “P”, in such a case change “i” to i/2 or i/4 or i/12 and change “n” to 2n or 4n or 12n respectively.
  - When half yearly, quarterly or monthly rate of interest is “i”, in such a case, change P to P/2, P/4 or P/12 and change n to 2n or 4n or 12n respectively.
- The present value of an annuity payable over a period of n years is defined as the sum of the present value of all the future payments.

- The present value of an ordinary annuity is represented by V and given as follows:

$$V = \frac{P}{i} \left\{ 1 - (1+i)^{-n} \right\}$$

- If the term of the annuity is n years, then for evaluating the present value of the immediate annuity, first calculate the present value of the annuity for (n – 1) years and then add to it the initial or first payment.

$$A = \frac{P}{i} \left\{ (1+i)^{n-1} - 1 \right\} + P$$

- Present value of the perpetual annuity is given by,  $V = P/i$

**Simple Interest**

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1. Find the simple interest on ₹ 850 for 7 years at 9% p.a.
  - a) ₹ 500
  - b) ₹ 535.50
  - c) ₹ 453.05
  - d) ₹ 553.35
  
2. Find how much would be the simple interest on ₹ 1000 for 20 years @ 5% p.a.?
  - a) ₹ 1000
  - b) ₹ 1500
  - c) ₹ 1450
  - d) ₹ 1550
  
3. At what rate per cent per annum will ₹ 450 amount to ₹ 810 in 10 years?
  - a) 2%
  - b) 2.50%
  - c) 8%
  - d) 4%
  
4. At what rate per cent will simple interest on ₹ 8650 have simple interest of ₹ 5190 in 8 years?
  - a) 7%
  - b) 7.5%
  - c) 8%
  - d) None of the above
  
5. Find what sum of money will amount to ₹ 1401.80 in 7 years at 9% p.a. simple interest.
  - a) ₹ 860
  - b) ₹ 853
  - c) ₹ 845
  - d) ₹ 850
  
6. A sum of ₹ 720 amounts to ₹ 1044 in five years. What is the rate of interest per cent per annum? What sum will amount to ₹ 2363.50 in 7 years at the same rate?
  - a) 8%, ₹1450
  - b) 9%, ₹ 1450
  - c) 10%, ₹ 1450
  - d) 9%, ₹ 1500
  
7. The simple interest on ₹ 300 for 4 years together with that on ₹ 500 for 3 years is ₹ 162, the rate being same in both cases. Find the rate per cent of interest.
  - a) 6.5%
  - b) 5%
  - c) 6%
  - d) 5.5%
  
8. A person finds that a fall of interest from 4% to 3.5% p.a. diminishes his yearly income by ₹ 60. What is his capital?
  - a) ₹ 12000
  - b) ₹ 23000

- c) ₹ 23500  
d) ₹ 24000
9. A pressure cooker is available for ₹ 250 cash or ₹ 100 cash down payment followed by ₹ 165 after 6 months. Find the rate of interest charged under the installment plan.  
a) 16%  
b) 15%  
c) 20%  
d) 14%
10. What sum will amount to ₹ 5200 in 6 years at the same rate of simple interest at which ₹ 1706 amount to ₹ 3412 in 20 years?  
a) ₹ 2000  
b) ₹ 3000  
c) ₹ 4000  
d) None of these
11. Moi lent ₹ 10000, partly at 10% and remaining at 12% interest. After one year he got ₹ 1050 as interest. How much did he lend at the lower rate?  
a) 7500  
b) 2500  
c) 3000  
d) 8000
12. At what rate will a sum of money double itself with simple interest in 16 years?  
a) 6.67%  
b) 6.25%  
c) 12.5%  
d) None of the above
13. A sum of ₹ 5700 is lent out in two parts in such a way that interest on one part @ 8% for 5 years is equal to that on the other part at  $\frac{1}{2}\%$  for 15 years. Find the sum lent at 8%. (simple interest being reckoned)  
a) 600  
b) 900  
c) 1200  
d) 1700
14. A sum of money amounts to ₹ 3576 in 4.67 years at 10.5 % simple interest. When will it double itself at the same rate?  
a)  $9\frac{11}{21}$   
b)  $9\frac{2}{3}$   
c)  $9\frac{12}{13}$   
d) None of the above
15. A sum of money doubles itself in 8 years. In how many years will it treble itself?  
a) 12 yrs  
b) 16 yrs

- c) 24 yrs  
d) None of the above
16. A sum of money trebles itself in 8 yrs. In how many years will it be five times? Simple interest being reckoned.  
a) 12 yrs.  
b) 16 yrs.  
c) 24 yrs.  
d) 13.33 yrs.
17. In what time will ₹ 8,000 amount to 40,000 at 4% per annum? (simple interest being reckoned)  
a) 100 years  
b) 50 years  
c) 110 years  
d) 160 years
18. Mr. Zen takes a loan of ₹ 525 at 4% p.a. S.I. from Mr. Moi. He pays Mr. Moi ₹ 250 at the end of 1st year. How much should he pay at the end of the 2<sup>nd</sup> year in order to clear his dues?  
a) 307  
b) 286.44  
c) 300.84  
d) 310.54
19. A person bought a motorbike under the following scheme: Down payment of ₹ 15,000 and the rest amount at 8% per annum for 2 years. In this way, he paid ₹ 28,920 in total. Find the actual price of the motorbike. (Assume simple interest).  
a) ₹ 26,000  
b) ₹ 27,000  
c) ₹ 27,200  
d) ₹ 26,500
20. Moizen borrows ₹ 7,000 at simple interest from the village moneylender. At the end of 3 years, he again borrows ₹ 3,000 and closes his account after paying ₹ 4615 as interest after 8 years from the time he made the first borrowing. Find the rate of interest.  
a) 3.5%  
b) 4.5%  
c) 5.5%  
d) 6.5%
21. At what rate per cent p.a. S.I. on as certain of money for 20 years be equal to  $\frac{4}{9}$ th of its amount for that period?  
a) 5%  
b) 4%  
c) 6%  
d) 7%

**Compound Interest**

1. Find the compound amount and the compound interest of ₹ 4000 invested for 7 years at 6% compounded annually.
  - a) 6014.52, 2014.52
  - b) 5014, 1014
  - c) 7296, 3296
  - d) None of the above.
2. What amount of money lent out at compound interest will amount to ₹ 1936 in 2 years at 10% p.a., interest being charged annually.
  - a. 1600
  - b. 1660
  - c. 1500
  - d. None
3. The value of a residential flat constructed at a cost of ₹ 100000 is depreciating at the rate of 10% per annum. Its value after 3 years of construction would be:
  - a. ₹ 92700
  - b. ₹ 72900
  - c. ₹ 80000
  - d. ₹ 85600
4. The population of a town is 20000. If the annual birth rate is 4% and annual death rate is 2%, the population after 2 years is:
  - a. 20000
  - b. 21000
  - c. 20808
  - d. 21808
5. A machine depreciates @ 10% p.a. for the first two years and then @ 7% p.a. for the next three years, depreciation being calculated on the diminishing value. If the value of the machine be ₹ 10000 initially, find the residual value at the end of the fifth year.
  - a.  $81 \times \frac{93^3}{10^4}$
  - b.  $81 \times \frac{93^2}{10^5}$
  - c.  $\left(\frac{9}{10}\right)^2 \left(\frac{93}{100}\right)^3$
  - d. None of the above
6. A person deposited ₹ 1000 in a bank at 5% compounded annually. After 5 years, the rate of interest was increased to 6% and after 4 more years, the rate was further increased to 7%. The money was withdrawn at the end of 12 years. Find the amount.
  - a) 1973.86
  - b) 1937.86
  - c) 1930.59
  - d) 1903.95
7. An amount of money grows up to ₹ 8000 in 2 yrs. and up to ₹ 8500 in 3 yrs. Find the compounded rate of interest.
  - a. 6.00%
  - b. 6.25%
  - c. 6.67%
  - d. 5.25%

8. The difference between C.I. and S.I. on a certain sum of money is ₹ 40 for first two years and ₹ 122 for first three years. Find the sum, if the rate is same in both the cases.
- 8000
  - 12000
  - 15000
  - 16000
9. The C.I. on a certain sum for 2 yrs. is ₹ 71.40 and the S.I. is ₹ 70. What is the rate of interest?
- 3%
  - 4%
  - 5%
  - 6%
10. If the difference between C.I. and S.I. on a certain sum of money for 3 yrs at 10% p.a. is ₹ 248, find the sum.
- 6000
  - 7000
  - 8000
  - 9000
11. Approximately how long will it take to triple an investment at 10% compounded annually?
- 16.5 years
  - 15 years
  - 11.5 years
  - 15.5 years
12. How long will it take for a principal to double if money is worth 6% compounded yearly?
- 16.5 years
  - 15 years
  - 11.9 years
  - 14.5
13. At what annual rate of interest, compounded yearly, will a money be double in 8 years?
- 6.5%
  - 7.5%
  - 8.5%
  - 9%
14. How long will it take for ₹ 4000 to amount to Rs.7000 if it is invested at 7% compounded yearly?
- 6 years
  - 5 years
  - 7 years
  - 8 years
15. A sum of money doubles itself in 5 years. In how many years will it become four fold (if interest is compounded)?
- 15
  - 10
  - 20
  - 12

16. A certain sum of money doubles itself at a certain rate of compound interest in 3yrs. in how many years will the ratio of the principal to the compound interest be 1:3?
- 5 yrs.
  - 6 yrs.
  - 9 yrs.
  - None
17. A sum of money placed at compound interest doubles itself in 3 years. In how many years will it amount to 8 times itself?
- 9 years
  - 8 years
  - 27 years
  - 7 years

**Effective Rate of Interest**

18. Find the effective rate equivalent to the nominal rate of 8% converted quarterly.
- 8.00%
  - 8.14%
  - 8.24%
  - 8.50%
19. What is the effective rate of interest corresponding to the rate of 10% per annum, if interest is compounded half-yearly?
- 10.25%
  - 10.50%
  - 10.00%
  - 10.15%
  - e)
20. Which is better investment?  
A: 9% per annum compounded half yearly.  
B: 9.1% per year simple interest.
- Option A
  - Option B
  - Either a) or b)
  - None of the above

**Present Value Concepts**

21. What is the present value of ₹ 10,000 due in 2 years @ 8% pa CI, according as the interest is paid yearly?
- ₹ 8574
  - ₹ 8500
  - ₹ 9574
  - ₹ 9,800
22. Find the present value of ₹ 40,000 due in 15 years hence if the interest rate is 6% pa CI compounded semi-annually.
- ₹ 16,520
  - ₹ 16.479
  - ₹ 17,520
  - ₹ 17,800

23. What is the present value of Re. 1 to be received after 2 years compounded annually at 10%?
- Re. 0.83
  - Re. 0.78
  - ₹ 0.90
  - ₹ 0.71
  - e)

**Future Value of Annuities**

24. The amount of an annuity if payment of ₹ 500 is made annually for 3 years at interest rate of 6 compounded annually would be: Given  $(1.06)^3 = 1.1910$ .
- ₹ 1951.66
  - ₹ 1591.80
  - ₹ 1659.65
  - ₹ 1966.51
25. A man decides to deposit ₹ 500 at the end of each year in a bank, which pays 8% p. a. a compound interest. If the installment are allowed to accumulate, what will be the total accumulation at the end of 7 years?
- ₹ 4461.40
  - ₹ 4546.25
  - ₹ 4600.25
  - ₹ 4325.25
26. A person invests ₹ 1000 every year with a company, which pays interest at 10% p.a. He allows his deposits to accumulate with the company at compound rate. Find the amount standing to his credit one year after he has made his yearly installments for the tenth time.
- ₹ 17531.17
  - ₹ 17429.60
  - ₹ 17000.00
  - ₹ 17829.20
27. The amount of an ordinary annuity of ₹ 800 payable at the end of each 3 months for 3 years, if the money is worth 8% compounded quarterly is: Given  $(1.02)^{12} = 1.2682$ .
- ₹ 8000
  - ₹ 10729.67
  - ₹ 8050
  - ₹ 8500
28. A machine costs a company ₹ 80000 and its effective life is estimated to be 20 years. A sinking fund is created for replacing the machine at the end of its effective life time when its scrap realizes a sum of ₹ 5000 only. Calculate to the nearest hundreds of rupees, the amount which should be provided, every year, for the sinking fund, if it accumulates at 9% p.a. compounded annually.
- ₹ 1500
  - ₹ 15000
  - ₹ 1700
  - 1465.99

29. The accumulation in a provident fund is invested at the end of every year to earn 10% pa. A person contributes 12.5% of his salary to which his employer adds 10% every month. Find how much the accumulation will amount to at the end of 30 years of his service, for every 100 rupees of his monthly salary. Give your answer to the nearest rupees.
- ₹ 44,000
  - ₹ 44,442
  - ₹ 44,222
  - ₹ 44,022
30. Zen invests ₹ 10000 every year starting from today for next 10 years, Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity.
- ₹ 156454.875
  - ₹ 156555.875
  - ₹ 145698.750
  - ₹ 160544.875
31. M plans to send his son for higher studies abroad after 10 years. He expects the cost of these studies to be ₹ 100000. How much should he save at the beginning of each year to have a sum of ₹ 100000 at the end of 10 years, if the interest rate is 12% compounded annually?
- ₹ 5087.87
  - ₹ 4800
  - ₹ 4268
  - ₹ 4842

**Present Value of Annuities**

32. Find the present value of an annuity of ₹ 10,000 per annum for 12 years, the interest being 4% pa CI.
- ₹ 93,700
  - ₹ 95,000
  - ₹ 100,000
  - ₹ 98,000
33. A wagon is purchased on installment basis such that ₹ 5000 is to be paid on signing the contract and four-yearly installments of ₹ 3000 each payable at the end of first, second, third and fourth year, if interest is charged at 5% p.a. what would be the cash down price?
- ₹ 15500
  - ₹ 15638
  - ₹ 15800
  - ₹ 15744

34. A dealer advertises that a tape-recorder is sold at ₹ 450 cash down followed by two yearly installments of ₹ 680 and ₹ 590 at the end of first year and second year respectively. If the interest charged is 18% p.a. compounded annually, find the cash price of the tape-recorder?
- ₹ 1500
  - ₹ 1550
  - ₹ 1450
  - ₹ 1400
35. A person takes a loan on compound interest and returns it in 2 equal annual installments. If the rate of interest is 16% p.a. and the yearly installment is ₹ 1682, find the principal?
- ₹ 3000
  - ₹ 2900
  - ₹ 2800
  - ₹ 2700
36. A person purchases a house worth ₹ 70,000 on a hire purchase scheme. At the time of gaining possession he has to pay 40% of the cost of the house and the rest amount is to be paid in 20 equal annual installments. Of CI is reckoned at 7.5% pa, what should be the value of each installment?
- ₹ 4120
  - ₹ 5520
  - ₹ 4520
  - None of the above
37. The price of a tape-recorder is ₹ 1561. A person purchased it by paying cash of ₹ 300 and the balance, with due interest in 3 half-yearly installment. If the dealer charges interest at the rate of 12% p.a. compounded half-yearly, find the value of each installment.
- ₹ 430.05
  - ₹ 471.75
  - ₹ 400
  - ₹ 430
38. A man buys an old piano for ₹ 500, agreeing to pay ₹ 100 down and the balance in equal monthly installment of ₹ 20 with interest at 6% p.a. compounded monthly. How long will it take him to complete payments.
- 20 months
  - 21 months
  - 22 months
  - 24 months
39. A firm likes to spend ₹ 5000 in repairing a machine, which will generate incomes of ₹ 1000, ₹ 1500, ₹ 2000, ₹ 2500 over the next consecutive 4 years. If interest is 9% p.a., find whether the expenditure in repairing the machine is profitable or not
- Profitable
  - Loss
  - Can't say
  - Data insufficient

40. A person retires at the age of 58 and earns a pension of ₹ 6000 a year. He wants to commute one-fourth of his pension to ready money. If the expectation of life at this age be 12 years, find the amount he will receive when money is worth 4% pa CI / it is assumed that pension for a year is due at the end of the year.
- a) ₹ 14,000      b) ₹ 15,000  
b) c) ₹ 14,047      d) ₹ 14,078

**Perpetual Annuity**

41. The annual rent of a free-hold estate is ₹ 1000. What is its current value, if the compound interest rate is 4% p.a.?
- a. ₹ 30000    b) ₹ 20000    c) ₹ 25000    d) None of the above
42. A person desires to create an endowment fund to provide for a prize of ₹ 300 every year. If the fund can be invested at 10% pa CI, find the amount of the endowment.
- a) ₹ 3,000    b) ₹ 6,000    c) ₹ 2,500    d) None of the above